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## GENERALIZED DISCHARGE RELATIONS FOR CUTTHROAT FLUMES

By Gaylord V. Skogerboe,<sup>1</sup> M. ASCE, Ray S. Bennett,<sup>2</sup>  
and Wynn R. Walker,<sup>3</sup> A. M. ASCE

## INTRODUCTION

Water measurement is essential to water resources management. Flowing water must be measured to determine the amount received from a source and to control the amounts distributed to various uses in conformance with legal requirements or water supply contracts. The flume is the most widely accepted and used device for measuring open channel flows. Of the flumes in present day use, the most common is the Parshall flume (2) developed by Ralph Parshall.

The Cutthroat flume (4) is a new flume designed to measure flows in flat gradient streams. The flume was originally limited to one length with width as a variable. The different sized flumes were therefore not geometrically similar. This study presents groups of flumes which are geometrically similar.

## DEVELOPMENT OF CUTTHROAT FLUME

The Cutthroat flow measuring flume has a horizontal floor, with an entrance section and an exit section of equal width, but no throat length (Fig. 1). Thus, this flume has been called a Cutthroat flume by its developers (4).

The Cutthroat flume has certain beneficial operating characteristics in the following areas: (1) It operates well under submerged flow conditions and (2) it has a low head loss because of the level floor. Also the head loss is further

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<sup>1</sup>Assoc. Prof., Dept. of Agr. Engrg., Coll. of Engrg., Colorado State Univ., Fort Collins, Colo.

<sup>2</sup>Grad. Res. Asst., Dept. of Agr. Engrg., Coll. of Engrg., Colorado State Univ., Fort Collins, Colo.

<sup>3</sup>Res. Assoc., Dept. of Agr. Engrg., Coll. of Engrg., Colorado State Univ., Fort Collins, Colo.

reduced under submerged flow conditions. Because the angles of convergence and divergence remain the same for all flumes, the flume size is changed by merely moving the walls in or out, sideways, thereby allowing ratings for intermediate sized flumes to be developed from interpolation of existing ratings because of this simple geometric change between flume sizes. The most obvious advantage of a Cutthroat flume is its simplicity. Fabrication is facilitated by the flat-bottom (horizontal floor) and the absence of a throat section.

The initial investigations (4) were confined to a flume length,  $L$ , of 9 ft (2.7 m) with throat widths,  $W$ , varying from 1 ft (0.3 m) to 6 ft (1.8 m). The

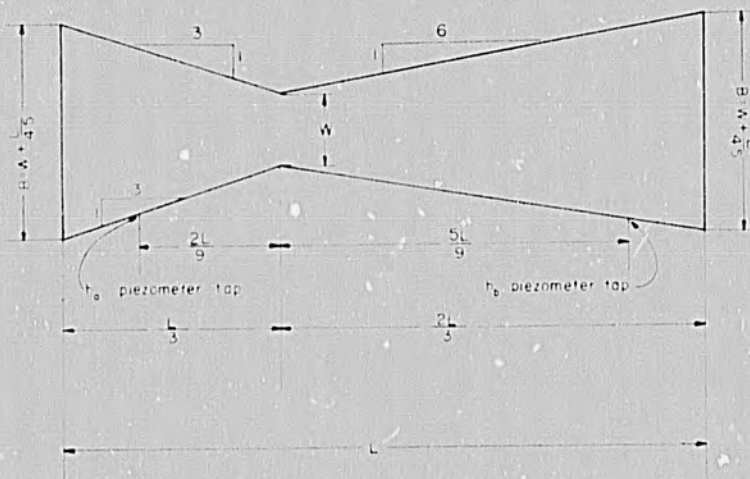


FIG. 1.—DEFINITION SKETCH OF CUTTHROAT FLUME

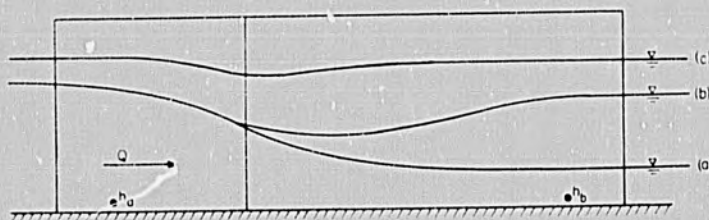


FIG. 2.—FLOW CONDITIONS IN CUTTHROAT FLUME

studies reported herein (1) extend the range of flume lengths and throat widths which can be used under both free flow and submerged flow conditions.

#### METHOD OF FLOW ANALYSIS

The Cutthroat flume can be used to measure flow rates under two different flow conditions; namely, free flow and submerged flow. The flow equation and the method of flow analysis are different for each type of flow.

*Free Flow.*—Under free flow conditions, critical depth occurs in the vicinity of the flume throat. Thus, the flow rate can be determined from the upstream depth,  $h_a$ , alone, because whenever critical depth occurs in the flume, the upstream depth,  $h_a$ , is not affected by changes in the downstream depth,  $h_b$ , as shown in Fig. 2 [water surface profiles (a) and (b)]. Thus, for free flow there is but one value of the discharge,  $Q$ , for each upstream flow depth,  $h_a$ .

For free flow operation, a plot on logarithmic paper of flow rate,  $Q$ , against upstream depth,  $h_a$ , with  $Q$  as the ordinate and  $h_a$  as the abscissa, will fall on a straight line, as shown in Fig. 3. The equation for this line is

$$Q = C h_a^{n_1} \dots \dots \dots (1)$$

in which  $Q$  = flow rate, in cubic feet per second;  $C$  = free flow coefficient;  $h_a$  = upstream flow depth, in feet; and  $n_1$  = free flow exponent. The coefficient

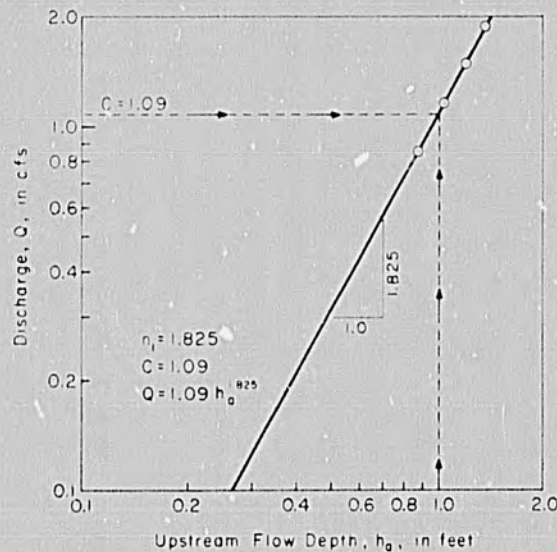


FIG. 3.—TYPICAL FREE FLOW RATING CURVE SHOWING ACTUAL DATA POINTS AND DEVELOPMENT OF FREE FLOW EQUATION

$C$  = the value of  $Q$  for  $h_a = 1.0$ , and the exponent  $n_1$  = the slope of the rating curve.

The values of the free flow coefficient,  $C$ , and the free flow exponent,  $n_1$ , were determined with the help of a digital computer program. The values of  $Q$  and  $h_a$  were read into the computer and the best fit rating curve determined using a mathematical regression. The values of  $n_1$  and  $C$  were then calculated and printed out by the computer. These were then plotted by hand and compared with the values obtained for the other flumes used herein.

*Submerged Flow.*—When the flow conditions are such that the downstream flow depth,  $h_b$ , is raised to the extent that the flow depths at every point through the structure become greater than the critical depth, resulting in a change in the upstream depth, the flume is operating under submerged flow conditions.

as shown in Fig. 2 [water surface profile (c)]. A flume operating under submerged flow conditions requires that two flow depths be measured, one upstream,  $h_a$ , and one downstream,  $h_b$ , from the flume throat. Submergence  $S$  is the ratio, often expressed as a percentage, of the downstream depth to the upstream depth:

$$S = \frac{h_b}{h_a} \dots \dots \dots (2)$$

Submerged flow calibration curves are determined for the Cutthroat flume by preparing three-dimensional plots of the parameters describing submerged flow. The data are plotted on logarithmic paper with the discharge,  $Q$ , as the ordinate; difference in upstream and downstream depths of flow,  $h_a - h_b$ , as the abscissa; and the submergence,  $h_b/h_a$ , as the varying parameter. Lines are then drawn connecting points of equal submergence. These are straight lines having a slope identical to the slope of the free flow rating curve, which is  $n_1$ , for the same geometry.

An equation has been developed (3) which describes the flow rate through the Cutthroat flume under submerged flow conditions (see Fig. 4):

$$Q = \frac{C_1 (h_a - h_b)^{n_1}}{(-\log S)^2} \dots \dots \dots (3)$$

in which  $h_b$  = downstream flow depth, in feet;  $C_1$  = submerged flow coefficient; and  $n_2$  = submerged flow exponent.

The value of  $C_1$  and  $n_2$  must be determined from a plot of the submerged flow data. For  $h_a - h_b = 1.0$  ( $\Delta h = 1.0$ ), Eq. 3 can be reduced to

$$Q_{\Delta h} = C_1 (-\log S)^{-n_2} \dots \dots \dots (4)$$

By plotting  $Q_{\Delta h}$  against  $-\log S$  on logarithmic paper as shown in Fig. 5, a linear relationship should result, in which  $C_1$  = the value of  $Q_{\Delta h}$  at  $-\log S = 1$  and  $n_2$  = the slope of the straight line.

The preceding procedure can be carried out by hand, but for this study it was accomplished using a digital computer. Having determined the values of the constants in the submerged flow equation, it is now possible to evaluate the flow rate for any combination of upstream and downstream flow depth that might be encountered.

The transition submergence,  $S_t$ , is the value of submergence at which the discharge passes from free flow to submerged flow, or vice versa [Fig. 2, water surface profile (b)]. Under this unique condition, both the free flow equation and the submerged flow equation will predict the same value of discharge.

To determine the transition submergence,  $S_t$ , the free flow and submerged flow equations are solved simultaneously to yield:

$$Ch_a^{n_1} = \frac{C_1 (h_a - h_b)^{n_1}}{-\log \left( \frac{h_b}{h_a} \right)^2} \dots \dots \dots (5)$$

Dividing both sides of Eq. 5 by  $h_a^{n_1}$  in order to obtain an expression containing only the submergence and known values of coefficients and exponents, an

equation for the transition submergence is obtained:

$$-\log (S_t)^{n_2} = \frac{C_1}{C} (1 - S_t)^{n_1} \dots \dots \dots (6)$$

Eq. 6 can be solved by trial and error to obtain a value of the transition submergence.

In order to determine whether free flow or submerged flow conditions exist in a Cutthroat flume, or any flow measuring flumes, it is necessary to calculate the submergence, which is then compared with the transition submergence to

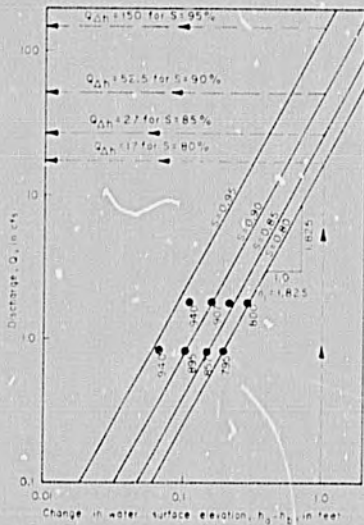


FIG. 4.—TYPICAL SUBMERGED FLOW RATING CURVE

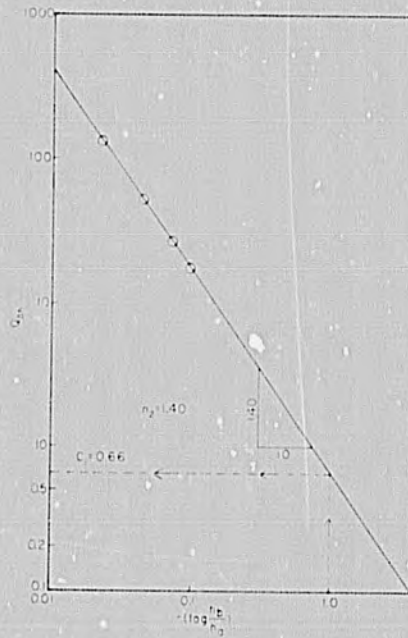


FIG. 5.—TYPICAL PLOT FOR DEVELOPING SUBMERGED FLOW COEFFICIENT  $C_1$  AND SUBMERGED FLOW EXPONENT  $n_2$

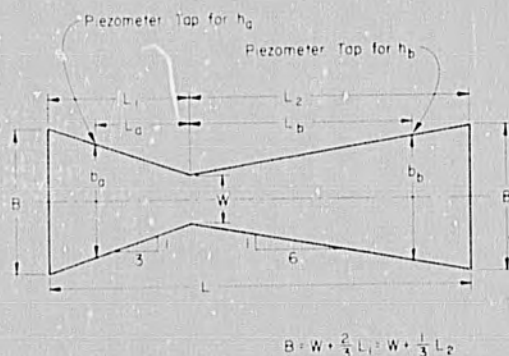
determine which flow equation should be used. If the submergence is less than the transition submergence, then free flow conditions exist; but the flume is operating under submerged flow conditions if the submergence is greater than the transition submergence.

EXPERIMENTAL DESIGN

The geometry of the Cutthroat flume is simple. The only independent dimensions are flume length,  $L$ , and flume width,  $W$  (Fig. 6). For any given

flume length, the size of flume is changed by simply moving the walls of the flume, which changes the flume width. All dimensions except those dealing with the width of the flume remain constant for any given flume length (Fig. 6).

Because of the simplicity in geometric design for Cutthroat flumes, it is possible to develop laboratory discharge ratings for a few sizes and then prepare the ratings for intermediate sizes by interpolation. Thus, in order to develop generalized discharge relationships for Cutthroat flumes, it is only necessary to rate some flumes which cover the desired range of flume length and throat width.



$$B = W + \frac{2}{3}L_1 + W + \frac{1}{3}L_2$$

Flume	W	L <sub>1</sub>	L <sub>2</sub>	L <sub>d</sub>	L <sub>b</sub>	L	B
3" x 4.5'	3"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	1'-3"
6" x 4.5'	6"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	1'-6"
12" x 4.5'	12"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	2'-0"
24" x 4.5'	24"	1'-6"	3'-0"	1'-0"	2'-5½"	4.5'	3'-0"
2" x 3.0'	2"	1'-0"	2'-0"	0'-8"	1'-7½"	3.0'	0'-10"
4" x 3.0'	4"	1'-0"	2'-0"	0'-8"	1'-7½"	3.0'	1'-0"
8" x 3.0'	8"	1'-0"	2'-0"	0'-8"	1'-7½"	3.0'	1'-4"
16" x 3.0'	16"	1'-0"	2'-0"	0'-8"	1'-7½"	3.0'	2'-0"
1" x 1.5'	1"	0'-6"	1'-0"	0'-4"	0'-9½"	1.5'	0'-5"
2" x 1.5'	2"	0'-6"	1'-0"	0'-4"	0'-9½"	1.5'	0'-6"
4" x 1.5'	4"	0'-6"	1'-0"	0'-4"	0'-9½"	1.5'	0'-8"
8" x 1.5'	8"	0'-6"	1'-0"	0'-4"	0'-9½"	1.5'	1'-0"

FIG. 6.—DIMENSIONS OF CUTTHROAT FLUMES USED IN EXPERIMENTAL DESIGN

Three flume lengths were chosen for this study; namely, 1.5 ft (0.45 m), 3.0 ft (0.90 m), and 4.5 ft (1.35 m). The 9.0 ft-long (2.7-m) flume of the initial study (4) is a member of this series, and thus it was possible to use the results of this earlier study in the analysis. Four throat width-to-length ratios ( $W/L$ ) were used, which were  $1/18$ ,  $1/9$ ,  $2/9$ , and  $4/9$ . These four ratios applied to the three flume lengths yielded the 12 combinations listed in Fig. 6.

The throat widths of the flumes for any one length vary from each other by a factor of two. Also, the flumes of a given width-to-length ratio are scale models of each other with a length ratio of two. For example, the 2 in. by 1.5 ft flume is a  $1/2$  model of the 4 in. by 3 ft flume; a  $1/3$  model of the 6 in. by 4.5 ft flume; and a  $1/6$  model of the 12 in. by 9 ft flume. Varying the width of

the flume while holding the length constant permits determining the effect of flume proportions on the coefficient. Varying the size of the flume while holding the proportion constant permits detection of any scale effect.

#### EXPERIMENTAL FACILITIES

The data for this study were collected using a test channel located in the Fluid Mechanics Laboratory of the Engineering and Physical Science Building at Utah State University, Logan, Utah. The test channel is 5 ft wide, 5 ft deep, and 100 ft long. The water is supplied from a sump located under the building and is circulated by four deep well turbine pumps and one propeller pump. Each pump can be operated individually or in parallel, which allows for a fairly large range of flow rates. The discharge from the pumps varies only slightly with the head, which minimizes fluctuations in the flow rate due to the water level in the sump.

The water is transported from the pumps to the head of the test channel by a 12-in. diam pipeline which is located along the ceiling of the laboratory. The flow is then dropped vertically in the pipeline into the test channel. Excessive turbulence in the water emerging from the pipe, which could have caused large fluctuations in flow depth throughout the length of the channel, was removed by installing a wire basket filled with gravel across the test channel, just below the pipeline outlet.

The flumes were each equipped with piezometer taps located at the bottom of the flume wall, as shown in Fig. 1. These piezometer taps were connected by means of rubber hose to stilling wells, which were used to measure the flow depths,  $h_a$  and  $h_b$ , in each flume.

Each stilling well was 1 ft in diameter. The piezometer taps on the flume were 1/2 in. in diameter and provided satisfactory damping of the water level fluctuations in the stilling well. The water level in the stilling wells was measured using a hook gage equipped with a vernier which could be read to an accuracy of 0.001 ft.

The test channel was fitted with an adjustable overflow device near the downstream end. This consisted of a gate fastened to the channel floor with a hinge. The gate was raised using a winch and could be set at any level desired. By varying the height of this gate, the submergence on the Cutthroat flume could be varied over the desired range of interest for this study.

The water, after passing through the test channel, was directed into one of two weighing tanks. Each of these tanks has a capacity of 26,000 lb and the scale is accurate to the nearest 5 lb. The time required to fill the tank was determined using a stop watch. Five readings were taken and the times averaged. The flow rate was then calculated to the nearest 0.01 cfs. With this facility, and using reasonable care, it was possible to obtain very accurate discharge data.

#### DEVELOPMENT OF RATINGS

*Free Flow Discharge Relations.*—The free flow rating curve was developed for a measuring flume by using Eq. 1. Two methods of grouping the data were

TABLE 1.—FREE FLOW COEFFICIENTS AND EXPONENTS FOR EXPERIMENTAL CUTTHROAT FLUMES

Flume	12 in. × 9.0 ft	3 in. × 4.5 ft	2 in. × 3.0 ft	1 in. × 1.5 ft
$C$	3.50	0.960	0.719	0.494
$n_1$	1.560	1.720	1.840	2.150
$K$	3.500	3.980	4.500	6.100
Flume	24 in. × 9.0 ft	6 in. × 4.5 ft	4 in. × 3.0 ft	2 in. × 1.5 ft
$C$	7.11	1.960	1.459	0.974
$n_1$	1.560	1.720	1.840	2.150
$K$	3.500	3.980	4.500	6.100
Flume	48 in. × 9.0 ft	12 in. × 4.5 ft	8 in. × 3.0 ft	4 in. × 1.5 ft
$C$	14.49	3.980	2.970	1.975
$n_1$	1.560	1.720	1.840	2.150
$K$	3.500	3.980	4.500	6.100
Flume	72 in. × 9.0 ft	24 in. × 4.5 ft	16 in. × 3.0 ft	8 in. × 1.5 ft
$C$	22.0	8.010	6.040	4.030
$n_1$	1.560	1.720	1.840	2.150
$K$	3.500	3.980	4.500	6.100

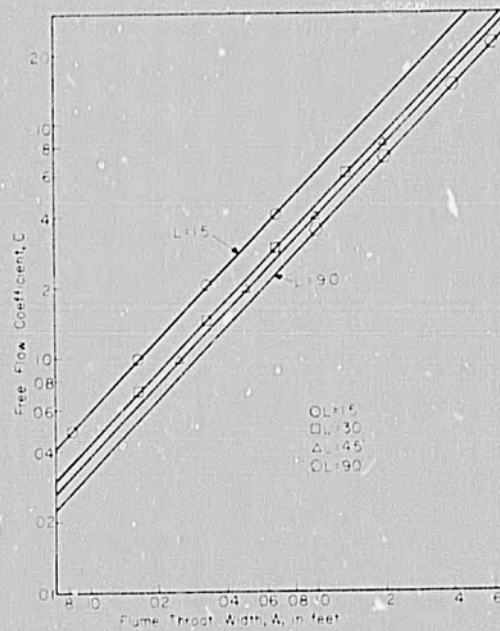


FIG. 7.—FREE FLOW RELATIONSHIPS BETWEEN FLUME LENGTH, FLUME THROAT WIDTH, AND FREE FLOW COEFFICIENT



tried; namely, grouping by flume length  $L$  and by throat width  $W$ . It was found that the slope,  $n_1$ , of the free flow curve was a constant for all flumes of equal length. Therefore,  $n_1$  is dependent only on the length of the flume. Consequently,  $L$  was used as the parameter for grouping the data for the analyses that follow.

The next step in the analysis was to determine if there was a consistent relationship among the values of the free flow coefficient,  $C$ , for the various flumes. A plot was made on logarithmic paper of  $C$  against  $W$ , with  $C$  as the ordinate and  $W$  as the abscissa. It was found that the point plotted as a straight line for flumes of equal length. Furthermore, with small changes, the lines for all four flume lengths could be made parallel. This adjustment was made and a new free flow rating curve prepared for each flume using the adjusted  $C$  and  $n_1$  values. The new graphs were then compared with the original data. The entire process was repeated until the difference between the original plots and the corrected plots was minimized.

As a further check, the free flow plots for the 9-ft Cutthroat flumes reported by Skogerboe and Hyatt (4) were also compared with those obtained in this study, with very satisfactory agreement.

For each flume length, there are three of the four flume widths for which the ratings fit the data points very closely, with one rating in each group having a larger error. However, this error is still small for all flumes and was attributed to scale effects resulting from very curvilinear flow and nonhydrostatic pressure distribution, because it was most apparent in the small flumes. The values of  $n_1$  and  $C$  for the various flumes tested are shown in Table 1.

The final adjusted curves showing the relationships between the free flow coefficient,  $C$ , and throat width  $W$  are shown in Fig. 7. From these curves, the equation for determining the value of  $C$  to be used for a given flume size can be written

$$C = KW^{1.025} \dots \dots \dots (7)$$

in which  $K$  = free flow flume length coefficient and  $W$  = flume throat width, in feet. The value of  $K$ , which is a constant for any particular flume length, is listed in Table 1 for the flume lengths studied.

The free flow rating for any size of Cutthroat flume can now be developed by interpolating to find the value of  $K$  for the desired flume length from Fig. 8 and then using Eq. 7 to calculate the free flow coefficient,  $C$ , for this flume. The value of the free flow exponent,  $n_1$ , can also be determined from Fig. 8 for any chosen flume length. These values of  $C$  and  $n_1$  are then used in Eq. 1 to calculate the flow rate through the flume for any given upstream flow depth,  $h_a$ .

*Submerged Flow Discharge Relations.*—The general method shown in Figs. 4 and 5, for developing a submerged flow rating was used to analyze the data collected in this study. However, certain refinements in the analysis were made possible by the use of a digital computer program to remove some of the error due to human judgment.

The procedure for the submerged flow analysis is to first of all determine values of  $C_1$  and  $n_2$  for each experimental Cutthroat flume; then, attempt to develop generalized relations for  $C_1$  and  $n_2$ .

Because the value of  $n_1$  was already known from the free flow tests for each flume, the method of analysis was to write a simple equation for each data point having the form:

$$Q = Q_{\Delta h} (\Delta h)^{n_1} \dots \dots \dots (8)$$

With  $Q$ ,  $\Delta h$ , and  $n_1$  known, a value of  $Q_{\Delta h}$  can be computed for the data point. Knowing  $Q_{\Delta h}$  and computing the value of  $-\log S$  from the known values of  $h_a$  and  $h_b$ , the data point can be represented on a plot containing the two variables (Fig. 5). This procedure can be repeated for each data point, thereby producing the straight-line relationship on logarithmic paper between the two variables ( $Q_{\Delta h}$  and  $-\log S$ ), which allows a determination to be made of the value of both  $n_2$  and  $C_1$  in Eq. 4.

When the relationship between  $Q_{\Delta h}$  and  $-\log S$  for all flumes of the same length were plotted, a series of nearly parallel lines resulted, with each line representing a particular value of  $W$ . Therefore, it was assumed that  $n_2$  was also a constant with flume length.

A plot was then made on logarithmic paper between the submerged flow coefficient,  $C_1$ , and the flume throat width,  $W$ , with  $C_1$  as the ordinate and  $W$  as the abscissa. The best fit straight line was drawn through the points and the value of  $C_1$  redetermined for each flume. The plot relating  $Q_{\Delta h}$  and  $-\log S$  was again prepared using the new value of  $C_1$ , which required computing  $n_2$ . This plot was then compared to the original one. The process was repeated until the discrepancy between the two types of plots was minimized.

A summary of the values of  $n_2$  and  $C_1$  determined for each flume is listed in Table 2. This same analysis was performed on the data reported by Skogerboe and Hyatt (4) for the 9-ft flume length.

The final plot of  $C_1$  against  $W$  is shown in Fig. 9. From this figure, the general equation for  $C_1$  can be written

TABLE 2.—SUBMERGED FLOW COEFFICIENTS AND EXPONENTS FOR EXPERIMENTAL CUTTHROAT FLUMES

Flume	12 in. × 9.0 ft	3 in. × 4.5 ft	2 in. × 3.0 ft	1 in. × 1.5 ft
$C_1$	1.688	0.548	0.413	0.261
$n_2$	1.390	1.410	1.480	1.741
$K_1$	1.700	2.250	2.580	3.250
Flume	24 in. × 9.0 ft	6 in. × 4.5 ft	4 in. × 3.0 ft	2 in. × 1.5 ft
$C_1$	3.430	1.120	0.837	0.516
$n_2$	1.390	1.410	1.480	1.741
$K_1$	1.700	2.250	2.580	3.250
Flume	48 in. × 9.0 ft	12 in. × 4.5 ft	8 in. × 3.0 ft	4 in. × 1.5 ft
$C_1$	6.970	2.275	1.705	1.048
$n_2$	1.390	1.410	1.480	1.741
$K_1$	1.700	2.250	2.580	3.250
Flume	72 in. × 9.0 ft	24 in. × 4.5 ft	16 in. × 3.0 ft	8 in. × 1.5 ft
$C_1$	10.600	4.575	3.465	2.140
$n_2$	1.390	1.410	1.480	1.741
$K_1$	1.700	2.250	2.580	3.250

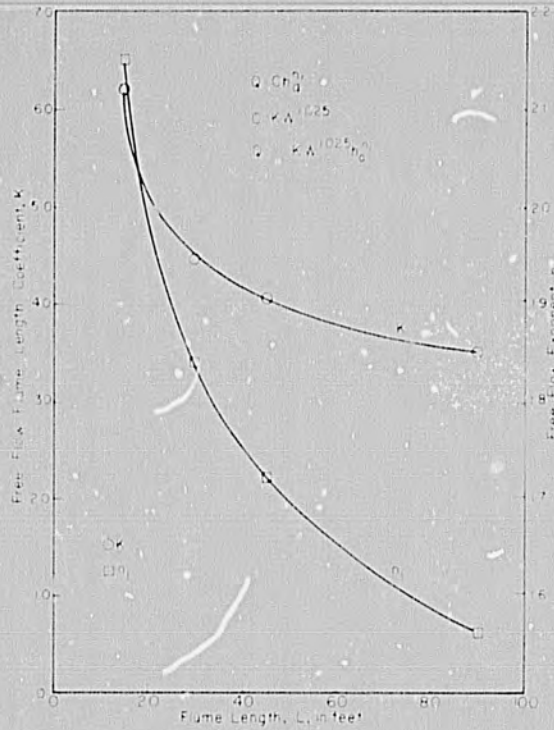


FIG. 8.—GENERALIZED FREE FLOW RATINGS FOR CUTTHROAT FLUMES

$$C_1 = K_1 W^{1.025} \dots \dots \dots (9)$$

in which  $K_1$  = submerged flow flume length coefficient.

The value of  $K_1$  for each experimental Cutthroat flume is shown in Table 2.

The submerged flow rating curves can now be determined for any size of Cutthroat flume ranging in length from 1.5 ft to 9 ft. The value of  $n_1$  is determined from Fig. 8. The values of  $n_2$  and  $K_1$  are obtained from Fig. 10. Thus, the value of  $C_1$  can be computed using Eq. 9. Knowing  $n_1$ ,  $n_2$ , and  $C_1$ , the discharge,  $Q$ , can be calculated by using Eq. 3 for any combination of  $h_u$  and  $h_b$ .

*Transition Submergence.*—In order to determine the values of  $S_f$  for the various experimental Cutthroat flumes, a computer program was written which performed the trial and error solution of Eq. 6. The value of  $S_f$  was found to be a constant for each flume length. The values of  $S_f$  were 60% for a flume length,  $L = 1.5$  ft; 65% for  $L = 3.0$  ft;  $S_f = 70\%$  for  $L = 4.5$  ft; and  $S_f = 80\%$  for the 9-ft flume length.

METRIC RATINGS

Because the preceding analysis is general, the development of free flow and submerged flow ratings can be easily accomplished (5). The free flow

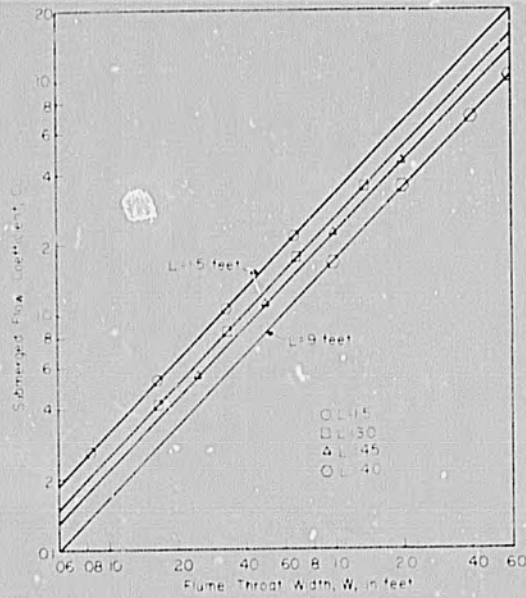


FIG. 9.—SUBMERGED FLOW RELATIONSHIPS BETWEEN FLUME LENGTH, FLUME THROAT WIDTH, AND SUBMERGED FLOW COEFFICIENT

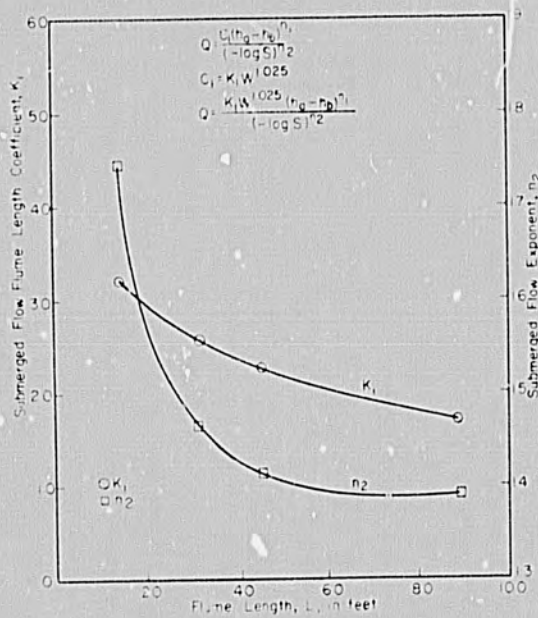


FIG. 10.—GENERALIZED SUBMERGED FLOW RATINGS FOR CUTTHROAT FLUMES

equation, Eq. 1, can be rewritten in metric units as

$$\dot{Q}^* = C^* h_a^{*n} \dots \dots \dots (10)$$

in which  $\dot{Q}^*$  = flow rate, in cubic meters per second;  $C^*$  = free flow coefficient for metric units; and  $h_a^*$  = upstream flow depth, in meters. The following expression can be written for the free flow coefficient

$$C^* = K^* W^{1.025} \dots \dots \dots (11)$$

in which  $K^*$  = free flow flume length coefficient for metric units and  $W^*$  = throat width, in meters. The value of  $K^*$  can be obtained from Fig. 11.

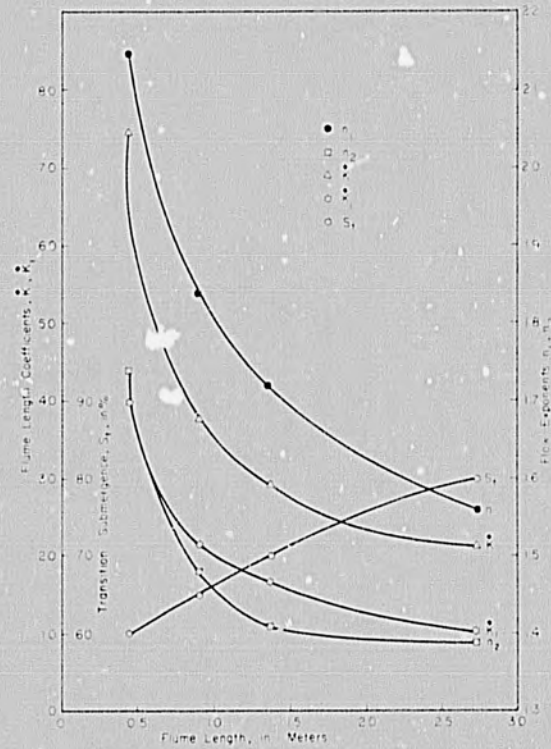


FIG. 11.—GENERALIZED FREE FLOW AND SUBMERGED FLOW RATINGS FOR CUTTHROAT FLUMES IN METRIC UNITS

The submerged flow equation (Eq. 3) can be rewritten in metric units as

$$\dot{Q}^* = \frac{C_s^* (h_a^* - h_f^*)^{\frac{n}{2}}}{(-\log S)^2} \dots \dots \dots (12)$$

in which  $C_1^*$  = submerged flow coefficient for metric units and  $h_2^*$  = downstream flow depth, in meters. The expression for the submerged flow coefficient is

$$C_1^* = K_1^* W^{1.025} \dots \dots \dots (13)$$

in which  $K_1^*$  = submerged flow flume length coefficient for metric units. The value of  $K_1^*$  can be obtained from Fig. 11.

#### CONCLUSIONS

The Cutthroat flume is an accurate open channel flow measuring device, which can be used either in the laboratory or in the field. The accuracy is satisfactory under both free flow and submerged flow conditions.

Generalized discharge rating curves can be easily developed. This is attributed to the simplicity of the structure and the geometric similarity between flume sizes. Consequently, it is possible for both free flow and submerged flow ratings to be developed for all intermediate flume sizes by merely interpolating on the appropriate graphs.

The flume is both simple and economical to construct. Now, based upon the results of this study, the range of sizes has been extended from a length of 9 ft to 18 in., while throat widths varying from 1 in. or 2 in. to 6 ft can be used. However, scale effects resulting from curvilinear flow and nonhydrostatic pressure distribution become apparent in the small flume sizes. Therefore, based upon this study, flumes less than 3 ft in length are satisfactory for free flow operations, but are not recommended for submerged flow operation.

In order to obtain the best rating accuracy, based upon analysis of data scatter of laboratory ratings, it is recommended that flumes with throat width to length ratios between 0.1 and 0.4 be used. This range of throat width to flume length ratios corresponds to a range on constriction ratios (throat width divided by entrance, or exit, width,  $W/B$ ) of 1/4 to 2/3.

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 APPENDIX II.—NOTATION
 

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The following symbols are used in this paper:

- $B$  = entrance and exit width for Cutthroat flume;  
 $C$  = free flow coefficient for English units;  
 $C^*$  = free flow coefficient for metric units;  
 $C_1$  = submerged flow coefficient for English units;  
 $C_1^*$  = submerged flow coefficient for metric units;  
 $h_a$  = upstream flow depth, in feet;  
 $h_a^*$  = upstream flow depth, in meters;  
 $h_b$  = downstream flow depth, in feet;  
 $h_b^*$  = downstream flow depth, in meters;  
 $K$  = free flow flume length coefficient for English units;  
 $K^*$  = free flow flume length coefficient for metric units;  
 $K_1$  = submerged flow flume length coefficient for English units;  
 $K_1^*$  = submerged flow flume length coefficient for metric units;  
 $L$  = length of the Cutthroat flume;  
 $L_a$  = length from throat of flume to upstream measuring point;  
 $L_b$  = length from throat of flume to downstream measuring point;  
 $L_1 = L/3$  = length of converging inlet section;  
 $L_2 = 2L/3$  = length of diverging outlet section;  
 $M = W/B$  = constriction ratio for Cutthroat flume;  
 $n_1$  = free flow exponent;  
 $n_2$  = submerged flow exponent;  
 $Q$  = discharge through flume, in cubic feet per second;  
 $Q^*$  = discharge through flume, in cubic meters per second;  
 $S = h_b/h_a$  = submergence;  
 $S_t$  = transition submergence;  
 $W$  = flume throat width, in feet;  
 $W^*$  = flume throat width, in meters; and  
 $\Delta h = h_a - h_b$  = difference in flow depth.

9438 DISCHARGE RELATIONS FOR CUTTHROAT FLUMES

KEY WORDS: Drainage; Flow measurement; Flumes; Hydraulics; Hydraulic structures; Irrigation; Measuring instruments; Open channel flow; Subcritical flow

ABSTRACT: A group of Cutthroat flumes were rated under both free flow and submerged flow conditions. Generalized discharge rating curves can be developed due to the simplicity of the structure and the geometric similarity between flume sizes. Twelve flumes were used in this study, including three flume lengths of 1.5 ft (0.45 m), 3 ft (0.90 m), and 4.5 ft (1.35 m), with four different throat widths for each length. In addition, the flume sizes were selected so as to permit correlation with the initial Cutthroat flume studies, wherein a flume length of 9 ft (2.7 m), and throat widths varying from 1 ft (0.3 m) to 6 ft (1.8 m), were studied.

REFERENCE: Skogerboe, Gaylord V., Bennett, Ray S., and Walker, Wynn R., "Generalized Discharge Relations for Cutthroat Flumes," *Journal of the Irrigation and Drainage Division, ASCE*, Vol. 98, No. IR4, Proc. Paper 9438, December, 1972, pp. 569-583