9. ABSTRACT

The need for measuring the social opportunity cost of resources arises whenever such resources are to be diverted from current uses (or non-use) to a government project or activity. The economic acceptability of any proposed project depends upon the cost and benefit streams anticipated over the life of the project, and a crucial aspect of any cost-benefit analysis is how to assign values to the resources required as inputs. Since this is a public endeavor, social profitability is the relevant consideration and social costs and benefits are the relevant measures.

The question to be discussed here is how to approach the estimation of the social opportunity cost of labor (SOCL) when faced with such a variance in observed market wages, the possible inequality between wage and marginal product, and open unemployment (involuntary). This task is further complicated by the fact that labor is not homogeneous and that the market conditions vary according to skill level. This discussion tends to focus on the unskilled labor category but would be applicable to any labor category which is abundant relative to demand. Of course, many types of skilled labor are scarce in labor surplus economies and would not fit the market pattern described above.
The Measurement of the Social Opportunity Cost of Labor in a Labor Surplus Economy

William H. Meyers
PREFACE

Several term papers prepared by graduate students enrolled in Agricultural and Applied Economics 8-264 in the Fall of 1973 were of excellent quality. Because of their value to students of resource economics problems, several of these are being issued in the Staff Paper Series of the Department of Agricultural and Applied Economics.

This paper by William H. Meyers tackles a difficult but relevant and important problem for developing countries—estimating the shadow price for labor in a labor-surplus economy. The first two papers in this series were:


K. William Easter
Lee R. Martin
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THE MEASUREMENT OF THE SOCIAL OPPORTUNITY COST
OF LABOR IN A LABOR SURPLUS ECONOMY

William H. Meyers*

1. Introduction

The need for measuring the social opportunity cost of resources arises whenever such resources are to be diverted from current uses (or non-use) to a government project or activity. The economic acceptability of any proposed project depends upon the cost and benefit streams anticipated over the life of the project, and a crucial aspect of any cost-benefit analysis is how to assign values to the resources required as inputs. Since this is a public endeavor, social profitability is the relevant consideration and social costs and benefits are the relevant measures.

If perfect market conditions are assumed and there are no externalities and no tax-induced distortions, the market price (or private cost) of an input accurately measures its social opportunity cost as well. But when actual market conditions diverge from perfect market assumptions, it becomes important to look for alternative measures of social cost or for appropriate adjustments to the market price.

One very clear-cut example of divergence from perfect market conditions is the labor market in a labor-surplus economy. Such an economy--typical of many low-income countries--is characterized by an abundance of

* I would like to thank Phillip Aust, Dr. K. William Easter, Dr. Edward M. Foster and Dr. Lee R. Martin for many valuable suggestions; I claim any remaining errors as my own.
labor, especially unskilled labor, relative to other factors of production. Frequently one or more of the following conditions can be observed in a labor-surplus economy:

1. Minimum-wage laws and capital subsidies which raise wages and reduce employment in the modern urban sector while wages and productivity in the "traditional" urban sector and the rural sector remain relatively close to subsistence levels.

2. Substantial factor transfer costs.

3. Large portions of the labor force--especially in agriculture--employed in family enterprises and thus not directly entering the market.

4. Sticky, institutionalized wages in the agriculture sector with large seasonal variations.

As a result of these conditions it is typical for these economies to manifest significant differences between nominal wages in the modern and traditional sectors of the economy, seasonal and regional differences in rural sector wages, open unemployment \(^{1/} \) in the urban sector and disguised unemployment in the traditional sectors, especially in rural family enterprises, where the marginal product may be below the wage.

The question to be discussed here is how to approach the estimation of the social opportunity cost of labor (SOCL) when faced with such a variance in observed market wages, the possible inequality between wage and marginal product, and open unemployment (involuntary). This task is further complicated by the fact that labor is not homogeneous and that the market conditions vary according to skill level. This discussion

\(^{1/} \) It has been pointed out in the UNIDO, Guidelines for Project Evaluation (1972), that underemployment in the traditional sector is a sufficient condition for surplus labor even in the absence of open unemployment.
will tend to focus on the unskilled labor category but would be applicable to any labor category which is abundant relative to demand. Of course, many types of skilled labor are scarce in labor surplus economies and would not fit the market pattern described above.²/

A frequently used framework for calculating the social opportunity cost of labor or accounting wage rate (AWR) is of the form:

\[ \text{AWR} = \text{DOC} + \text{IC} \]

where DOC is the opportunity cost of the worker and IC is the indirect cost to the economy brought about by a combination of (1) a sub-optimal savings rate and (2) an increase in consumption and drop in savings when additional workers are hired. The latter rests upon the assumption that the proportion of the wage bill saved is less than the proportion of government revenues diverted from savings.

Different measures of IC have been proposed by Marglin (1967), Sen (1968), Little and Mirrlees (1969) and UNIDO (1972) and a succinct catalogue of the implicit assumptions of each has been made by Warr (1973). Mishan (1971) raises a fundamental question about the inclusion of IC and suggests that it is an unjustified addition to the Pareto criterion and would lead to the exclusion of some projects offering potential Pareto improvement.

While further investigation of questions bearing upon the use and measurement of IC would be interesting, the scope of this paper will be

²/ For such skill categories, the market wage may well be a good measure of the SOCL. Bussery (1973) has suggested that limits to management salaries may even create a situation where the SOCL is greater than the market wage for very scarce executive talent.
limited to the direct opportunity cost (DOC). The measurement of DOC is often treated sparingly. Perhaps this is because, as noted in UNIDO (1972), it is "the most tractable from the conceptual point of view and the least tractable from the point of view of empirical measurement."

It will be demonstrated, however, that the decision about which measure to use for DOC—which hereafter is referred to as the social opportunity cost of labor (SOCL)—has important implications and that tractable measurements are available which are consistent with a realistic model of labor market behavior in labor surplus economies.

2. "Direct" Social Opportunity Cost

The social opportunity cost is here defined as the social value of the direct sacrifice required to add a worker to the project payroll. In the broadest sense this includes the foregone marginal utility of the worker as well as the marginal product foregone, though the former is often overlooked. The problem of measuring the social opportunity cost is a subject of considerable debate in the economic literature, which can be summarized under three general headings: "shadow price," "foregone product" and "supply price." These are not mutually exclusive but rather indicate the emphasis of different authors.

**Shadow Price.** If a giant programming model of the economy could be developed which included all alternative uses for labor and had maximization of national income as the objective function, then the solution to the dual minimization problem would generate a shadow price for labor. Even if labor were a homogeneous factor the data requirements for such a model would be too demanding, and trying to accommodate various labor skill categories would only compound the difficulty. Even early discussions of
this technique by Tinbergen (1958) and Qayum (1960) freely acknowledged its practical limitations. Chakravarty (1964) proposed using a more aggregative model to approximate shadow prices of foreign exchange and the rate of interest, but even performing such an approximation for the relevant labor skill categories may very well entail an inefficient allocation of scarce economists, given the data limitations in low-income countries.

This approach will not be discussed further here except to note that if it could be assumed that any particular skill category of labor would not be fully employed in the optimal solution of a programming model, the Kuhn-Tucker conditions guarantee that the shadow price of labor would be zero for that category. However, even the presence of visible unemployment in a country is not sufficient evidence to conclude that unemployment would occur in a programming model where technology is variable and perfect markets are assumed.

Foregone Product. The basic consideration in the foregone product approach is to take the current technology and market structure as given and try to determine the value of the marginal product foregone when a worker is added to the public payroll. By definition, foregone product excludes any costs associated with the relocation and resettlement of the worker or any loss in utility experienced by the worker as a consequence of his new environment and work. Usually the latter elements of cost are acknowledged as in the case of Beyer (1972), who accepts a downward bias in his measurement because these elements are too difficult to estimate.

In the absence of perfect markets the marginal product of labor is not equal in different sectors of the economy, so the foregone product
depends on which activity labor is diverted from. The ECLA Manual (1958) argues that if the workers hired for a project were previously unemployed or if their former positions were filled by unemployed persons, society loses nothing and the SOCL is zero. Alternatively, if the new workers were withdrawn from agricultural production or if their former positions were filled with persons withdrawn from agriculture, society loses the value of agricultural production foregone, which the ECLA Manual equates to the agricultural wage.

In the simplest case of a project located in the rural sector which draws its workers only from the region in which it is located, the marginal product of agricultural labor for the region would appear to be a good measure of the foregone product. Whether or not the agricultural wage is an accurate reflection of the marginal product would still be a crucial question.

If a project is large enough to draw labor from several regions with the possibility that labor market conditions and hence the marginal product of labor may vary among regions, then the measure suggested above is hardly adequate. This is particularly true for an urban project, which may hire migrants from any or all regions of the country in unknown proportions. The attempt to add the costs of relocation and the disutility to the worker further complicates the measurement problem. The latter costs are seldom dealt with explicitly in the literature and indeed are practically impossible to quantify except in the context of the "supply price" approach discussed below.

The two project evaluation manuals developed by the United Nations Industrial Development Organization (UNIDO, 1972) and by Little and
Mirrlees (1969) suggest methods of approximating the marginal productivity of labor in agriculture. Both are set in the context of an urban industrial project drawing upon unskilled labor from the traditional sectors. UNIDO suggests that in the absence of detailed production function analysis, an approximation could be made by using the incomes of individuals in traditional sectors "who possess only their own labor power." An example would be the wages of landless agricultural laborers. Admittedly this wage might be above the marginal productivity of a family worker in a household enterprise, but with an appropriate amount of hand-waving it is considered a suitable approximation. It is also suggested that in many cases unemployment or underemployment may be so widespread that foregone product is zero.

Little and Mirrlees suggest a first approximation using total agricultural production and agricultural labor force to calculate average productivity and then say "one may not go far wrong" by taking half the average product as a measure of marginal productivity. This seems very arbitrary indeed and would only be a good approximation when production relationships satisfy the condition $0.5AP = MP$ for labor. For a Cobb-Douglas production function, for example, a labor coefficient of 0.5 would be required to satisfy this condition. Alternatively, they propose that a better approximation could be obtained by taking the season-specific wages ($w_t$) times the number of "full employment days" ($D_{wt}$) when each wage prevails. The $w_t$ might be zero at some times of the year and highest during peak demand seasons, e.g., harvest-time. The product sum $\sum w_tD_{wt}$ is an estimate of the amount per year farmers are willing to pay for an extra worker and thus approximates the marginal product of a man-year. By definition this value is less than or equal to the annual income of a
landless laborer and could be considered a lower bound on their "foregone product." Since full employment is assumed over each period, this value also appears to be a lower bound on the marginal product of family workers. However, to determine with any degree of accuracy what numbers to choose for the $D_{w_t}$'s may require more detailed employment data than are available in most low-income countries.

The foregone product approximations are a useful tool in the context of a rural sector project drawing labor only from a limited region, but there are deficiencies in this approach when considering an urban project:

1. If there are significant regional differences in the marginal product of labor and the project draws directly or indirectly on labor from several regions, what can be said about the product foregone? It depends upon where the labor originates.

2. As distance between project site and labor origin increases, relocation costs--if borne by the worker--become more important as do the effects of rural-urban price differentials. If these factors are significant, it would not be desirable to use the foregone product alone as the SOCL for an urban project.

3. The foregone marginal utility of the worker is likely to be more important as the distance to the project increases, and there is no practical way of adding this to the foregone product measure.

Supply Price. If a planner were blessed with omniscience, he would know which individuals were going to be hired for a given project and who--if anyone--was going to replace those attracted away from current jobs and could perceive the supply price inside the head of each. With this information the omniscient planner could find the ideal measure of the social opportunity cost of the total project labor input.

\[3/\] In addition to the peak seasons, a landless laborer usually must find other work in the non-agricultural sector in order to survive.
The "voluntary supply price" is simply defined by Harberger (1971) as the amount for which a worker is willing to present himself for work at a specific project site. It is an ideal measure in the sense that it is location-specific and implicitly includes the value of foregone product as well as the value of all other monetary and non-monetary sacrifices that the worker makes when he presents himself at the project site. Thus it will be shown that none of the deficiencies listed above for the foregone product measure are encountered in the voluntary supply price.  

The catch, of course, is that an omniscient planner has not yet appeared on the scene, so how are these supply prices to be determined? For simplicity let it be assumed that regions can be defined in such a way that the supply price of unskilled labor is constant within each region, though it may vary from region to region. It should be clear that a project which draws only upon unskilled labor within the region of its location, has the regional supply price as its relevant SOCL. In order to determine this supply price Harberger first makes the reasonable assumption that "employers do not wittingly pay workers more than they (the employers) believe the incremental contribution of each worker to the value of output to be." Then if a substantial market for hired labor exists and there is no "abnormal unemployment," the supply price (and the

---

4/ Even the problem of divergence between average product and marginal product of a family worker vanishes if it is assumed that the household rather than the individual member decides whether or not he should "present himself" for work. For, while the individual decision would be based on his foregone income--essentially the average product of household labor--the household decision would most likely be based on his foregone marginal product. This is not an unreasonable assumption in the context of the close-knit rural households found in most low-income countries.
SOCL) of unskilled labor is best measured by the going wage. He argues that even in rural India, where 25 percent of the labor force are landless laborers, these assumptions are not unrealistic.5/

Thus in the simplest case of a rural project which draws workers only from within the specified region, the foregone product measure and the supply price measure would, in general, be identical. This is not surprising since relocational costs, interregional price differentials and foregone utility would probably be insignificant in this case. The two exceptions to this general equality in measures are:

1. Non-comparable working conditions between the project and the alternative employment may exist, in which case the supply price would rise or fall to account for the loss or gain in the "pleasantness" of work.

2. The higher wages offered by the project may attract persons into the labor force who have a supply price greater than the market wage.6/

Where such factors are significant, adjustments could be made in the SOCL, though it would again be difficult to determine the magnitude of the adjustment.

The important divergence between the supply price and foregone product measures occurs when a project draws laborers directly or indirectly from a region (or regions) with a market wage lower than that in which the project is located. The obvious example is an urban project drawing upon rural labor. As seen earlier, the foregone product measure takes the

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5/ This percentage may not be as high in every region, so the conclusion may not hold for specific regions.

foregone agricultural product as the SOCL or at best adds on the monetary costs incurred by the migrant. The voluntary supply price, however, implicitly includes non-monetary as well as monetary costs perceived by the migrant.

3. A "Supply Price" Model

The problem of estimating the supply price in this situation is best approached through a realistic model of labor migration. Such a model is described by Harberger, and a slightly different but more rigorous model developed by Todaro (1969 and 1971) is also useful in the present context though its original purpose was to analyze employment policy. Both authors assume that a utility-maximizing individual will make the migration decision on the basis of the difference in expected real income between sectors. In the Todaro formulation total migration of unskilled labor in the present time period, \( M(0) \), can be represented as a function of the difference in the present value of the two expected real income streams minus the fixed cost of migration.\(^1\)

\[
M(0) = F[V(0)] \\
F' > 0
\]

where

\[
V(0) = \int_0^T \left[ p(t)Y_u(t) - Y_R(t) \right] e^{-rt} dt - C(0)
\]

\( p(t) \) = the probability of having income \( Y_u(t) \) in period \( t \)

\( Y_u(t) \) = the net urban real income in period \( t \) including the value of urban amenities

\(^1\) Research in areas as diverse as Colombia, Ghana and the United States have given empirical support to this type of model by demonstrating that migration rates are highly responsive to economic and environmental factors. See Beals (1967), Greenwood (1969) and Schultz (1971).
$Y_R(t)$ = the net expected rural real income in period $t$

$T$ = the time horizon of potential migrants

$r$ = the discount rate reflecting the degree of consumption time preference of potential migrants

$C(0)$ = the cost of migration and resettlement

A micro model of migrant behavior consistent with this aggregate model is:

$$V_i(0) = \int_0^T [p_i(t)Y_{ui}(t) - Y_{Ri}(t)]e^{-rt}dt - C_i(0)$$

(2)

where the $i$th individual in the rural sector will migrate if $V_i(0) > 0$, but would not consider migration an attractive investment if $V_i(0) < 0$ and hence would not migrate. If the equality holds the individual would be indifferent between the two sectors.

For the purpose of this discussion, this expression can be simplified by considering only a one period time horizon. In addition two other adjustments are made in this individual decision function. First, it seems preferable to separate the value of amenities from urban income term because it is a non-monetary parameter and also it can be argued that most amenities (e.g., schools and public services) are location specific and therefore independent of the probability term. Second, it will be assumed that all rural workers in the same skill category face the same expected urban real income, denoted $p*I_u$, but not necessarily the same rural income, due to regional differences. Likewise, the migration costs and perception of net amenities are likely to differ among individuals.

The decision function of the $i$th individual in a specified labor skill category is now defined as:
\( Z_i = p^* I_u + A_i - I_{ri} - C_i \)

where

\( p^* \) = the probability of obtaining an urban job in the current time period

\( I_u \) = the urban real income associated with the current nominal wage \( w_u \)

\( A_i \) = the real value of net amenities (positive or negative) expected by the \( i \)th individual

\( I_{ri} \) = the expected rural real income of the \( i \)th individual

As before, the \( i \)th individual will not migrate unless:

\( Z_i \geq 0 \)

that is

\( p^* I_u \geq I_{ri} + C_i - A_i \)  
(4)

If the equality in (4) holds for the marginal migrant, then \( p^* I_u \) is the value which exactly compensates him for foregone earnings (\( I_{ri} \)) as well as monetary and non-monetary sacrifices (\( C_i - A_i \)). Rural-urban price differentials are accounted for by the use of real income levels.

Within the simple framework of equation (4), Harberger's conclusions can be discussed with more clarity. He discusses three cases, the third of which will be shown to generate his conclusions only under restrictive assumptions. The first case assumes that the entire urban sector is "unprotected" (no minimum wage or other wage constraints) and that there is no abnormal unemployment (only "functional" and "seasonal" unemployment). The second assumes that the entire urban sector is "protected" by a minimum wage above the full employment equilibrium and thus experiences chronic unemployment. The third and most general case assumes the combined existence of a protected sector, an unprotected sector and chronic unemployment.
It will be assumed initially in every case that the project or projects under consideration have no non-marginal effects on the parameters of the above decision function (4). It should also be noted that, although the urban sector is the example used here, the same results would obtain for any project location which draws labor from outside its own region.

Case I.

If the entire urban sector is unprotected, it is assumed that full employment exists, so \( p^* = 1 \). Let the real income in the unprotected urban sector be \( I_{uu} \) so the decision function becomes:

\[
I_{uu} = I_{ri} + C_i - A_i
\]  

(4a)

Thus the marginal migrant has a real income expectation of \( I_{uu} \), which is equal to the real income of the "native" workers in the same skill category. The relevant supply price is a nominal concept and is equal to the nominal urban wage \( w_{uu} \) associated with \( I_{uu} \). Therefore, it is not necessary to know whether the workers hired by the project are "natives" or migrants or to know where the migrants originated, since \( SOCL = w_{uu} \) in any event. For any marginal migrant, \( w_{uu} \) provides just enough real income (\( I_{uu} \)) to compensate him for the monetary and non-monetary sacrifices he made.

Figure 1 represents the corresponding aggregate relationship between the unprotected urban and rural sector labor markets in the simplified

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8/ Throughout this paper income per period, \( I \), is defined as \( I = kw/P \), where \( P \) is the price level, \( w \) the nominal wage per day (or hour) and \( k \) is the constant number of days (or hours) which define a worker-period. (e.g., 200 days = 1 man-year equivalent).
Figure 1. Representing the labor market relationships in Case I.*

Figure 2. Representing the labor market relationships in Case II.**

* See Appendix I for an algebraic specification of this model.

** See Appendix II for an algebraic specification of this model.
world where these are the only two regions in the economy. The labor supply and demand relations for the urban and rural sectors are shown in the extreme left and right diagrams of Fig. 1, where \( N \) is labor units and \( I \) is real income per labor unit for a given period. The excess supply of rural labor \( (ES = S_r - D_r) \) and the excess demand for urban labor \( (ED = D_{uu} - S_{uu}) \) are in the center diagram, where \( M \) is migrant labor units.

If \( C - A = 0 \); that is, if the transfer of labor is costless, equilibrium could be achieved when real income in both sectors are equal \( (I_{uu} = I_r) \). However, if \( C - A > 0 \), intersectoral equilibrium would be achieved when \( I_{uu} - I_r = C - A \). This point is determined in Fig. 1 by the intersection of \( L = C - A \) and \( F = ED^{-1} - ES^{-1} \), where \( F \) is the vertical distance between the ED and ES curves and shows how the rural-urban income difference is influenced by the level of migration. With \( L = L_o \), the equilibrium level of migration is \( M_o \), the real income levels are \( I_{uu}^o \) and \( I_r^o \) and the units of labor remaining in the rural and urban sectors are \( N_r \) and \( N_u \), respectively. If this were an accurate representation of the state of the labor market before the project is undertaken and the project has no non-marginal effects, then it is clear that \( I_{uu}^o \) is the appropriate measure of the real income necessary to attract workers to the project whether they be migrants or urban sector workers.

It is not likely that equilibrium is actually realized in a dynamic world, but this static model demonstrates the point that from society's point of view \( I_{uu}^o \) is a better approximation of the real opportunity cost of marginal migrants than is \( I_r^o \), and likewise the urban wage \( w_{uu} \) is a better measure of the SOCL than is the foregone product in agriculture.
This is the same result derived above when the micro decision function (4a) was considered.

Case II.

Let the protected urban sector wage be denoted by \( w_m \) and the associated real income by \( I_m \). It is now assumed that \( w_m \) is above the full employment equilibrium and there is chronic unemployment, so it is clear that the probability of being hired is less than one and the decision function becomes:

\[
p^*I_m = I_{ri} + C_i - A_i
\]

It is useful to assume that the probability of being hired \( (p^*) \) is the same for migrants as for the currently unemployed "natives" in the same skill category; and in fact there are empirical data to support such an assumption. This provides some justification for accepting Todaro's simplifying assumption that hiring is a random selection process, where \( p^* \) equals the number of jobs available \( (n) \) divided by the number of unemployed natives and migrants \( [p^* = n/(U_o + M)] \).

The marginal migrant in this case has a real income expectation of \( p^*I_m \) but the social opportunity cost per hired worker is not \( p^*w_m \) but rather the protected sector wage \( w_m \). This can be demonstrated best by considering the social opportunity cost of \( M \) migrants. The opportunity cost to each migrant at the margin is equal to \( p^*I_{pu} \), so the total opportunity cost of \( M \) migrants is \( Mp^*I_{pu} \). The number of migrants actually

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hired for the project and other protected sector jobs is by our assumptions equal to \( p^*M \), and thus the social opportunity cost per hired worker is:

\[
\frac{M_p I_m}{p^*M} = I_m
\]

This means that in nominal terms \( \text{SOCL} = w_m \), which reflects the losses incurred by all migrants whether hired or not.

To demonstrate that \( w_m \) is also the correct supply price of the individual migrant, the difference \( (I_m - p^*I_m) \) can be thought of as a risk premium. Since the migrant is not certain of being employed, he will only present himself for work at the project site if he is offered a wage of \( w_m \) (real income of \( I_m \)) such that his expected real income is \( p^*I_m \). At the margin then:

\[
I_m = I_{ri} + C_i - A_i + (1 - p^*)I_m, \quad (4c)
\]

which is equivalent to (4b) but makes the risk premium explicit.

The relation \( \text{SOCL} = w_m \) naturally applies to any worker diverted from other protected sector employment, but it also applies to those hired from the pool of urban unemployed, whether they are natives or migrants of previous periods. The reasoning—similar to that used in the foregone product argument—is that those hired from the ranks of the currently unemployed will be quickly replaced by new migrants (whose supply price is \( w_m \)). In fact, Todaro has demonstrated that, in general, the number of migrants will exceed the number of new urban jobs created, since there is an equilibrium unemployment rate which is quite stable in the short run.\(^{10/}\)

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\(^{10/}\) Todaro (1969) concludes that only a rise in \( I_{ri} \) relative to \( I_m \) and \( A_i \) would be effective in reducing unemployment rates.
Figure 2 represents the corresponding aggregate relationship between the protected urban sector and rural sector labor markets. The two basic differences between Fig. 1 and Fig. 2 are:

1. The "excess demand" in Fig. 2 is perfectly elastic at \( I_m \), where \( I_m \) is predetermined by the minimum wage \( w_m \). The existence of \( p^* < 1 \) means that not all workers are in fact accepted at the minimum wage. The influence of \( p^* \) is captured in the cost function \( L \).

2. The cost function \( L \) in Fig. 2 includes the required risk premium \((1 - p^*) I_m\) in addition to \( C - A \). This cost function is positively sloped because a higher level of migration, \( I_m \), increases unemployment, reduced \( p^* \) and thus increases the risk premium (see Appendix 2).  

Under the assumption of Case II the equilibrium level of migration would be at \( M_1 \) where the "cost" of migration \( L \) equals the maximum sustainable rural-urban income difference \( r \). Since \( I_m \) is predetermined, the native urban labor force \( N_{us} \) and urban employment \( N_{ud} \) are independent of the level of migration; but the equilibrium migration \( M_1 \) determines \( I_r \), \( N_{rs} \), \( N_{rd} \), total urban labor force \( N_u = N_{us} + M_1 \) and the equilibrium level of unemployment \( N_u - N_{ud} = U_1 + M_1 \), where \( U_1 \) is the number of unemployed before migration. If this were an accurate representation of the state of the labor market before the project is undertaken and the project has no non-marginal effects, then it is clear that \( I_m \) is the level of real income necessary to attract workers to the project regardless of the origin of these workers.

Although the static equilibrium represented in Fig. 2 may never be fully realized, it is again argued that \( I_m = I_r + L_1 \) is a better approximation.

\[ \text{Note that } I_r + L \text{ corresponds to the right-hand-side of (4c) and when } p^* = 1, \text{ it corresponds to the right-hand-side of (4a) where no risk premium is required to induce migration.} \]
of the real opportunity cost to society of hiring a project worker than is $I^r_r$, and thus the corresponding minimum wage $w_m$ is a better approximation of the SOCL than is the foregone product in agriculture.

**Case III.**

This is the general case where the urban sector is assumed to have protected and unprotected sub-sectors as well as chronic unemployment. Complexities are introduced in this case as a result of the mutual existence of these three groups in one sector. It has already been shown that those employed in an urban protected sector have a supply price of $w_m$, while those employed in an urban unprotected sector have a supply price of $w_{uu}$. In case II migrants to a protected sector have a supply price of $w_m$ and those to an unprotected sector have a supply price of $w_{uu}$ (Case I.). The complexity arises because those in the unprotected urban sector and in the rural sector both appear to have a choice between certain employment ($p^* = 1$) with real income $I_{uu}$ and entering the pool of urban unemployed with an expected real income of $p^*I_m$.

Harberger assumes that those who choose to be unemployed while looking for protected sector employment do so because their supply price is above the unprotected sector wage $w_{uu}$, and he concludes that the SOCL for a protected sector project is greater than $w_{uu}$ and approaches $w_m$ as the level of unemployment increases relative to the number employed in the unprotected sector. Case II would be the limiting case where there is no urban unprotected sector employment and the SOCL is $w_m$.

Although this conclusion seems sensible, it is inconsistent with the reasoning applied in Case II; and further analysis indicates that Harberger's
conclusion would only hold in a special case that will be noted below. It can be inferred from the previous discussion of equation (4b) that for those who stay in the rural sector, $p^* I_m$ is not enough; that is, those who choose not to enter the urban pool of unemployed job hunters have a supply price higher than $w_m$. But this is of no consequence to the project planner, since they will not be among those hired. But Harberger is assuming that urban unemployed always have the option of taking a job in the unprotected sector if they run out of money, time or patience; and when they do so their supply price drops to $w_{uu}$. Is it not more consistent to conclude that those who remain in—or return to—the unprotected sector do so because $p^* w_m$ is not sufficiently high? Clearly a rise in $w_m$, given $p^*$, will draw more individuals out of the unprotected sector and the rural sector—the incremental migrants having a higher supply price than those who left during the previous period. This suggests a decision function similar to equation (4) to represent the choices open to an individual in the urban unprotected sector. Let $v_i$ represent the individual aversion to risk-taking,\(^{12/}\) where $0 \leq v_i \leq 1$, $v_i = 1$ for individuals who are neutral to risk and $v_i = 0$ for individuals with infinite risk aversion. The relevant function for the $i$th individual working in the urban unprotected sector can then be written,

$$v_i p^* I_m \geq I_{uu} \quad (5)$$

\(^{12/}\) This coefficient could be used to reflect other individual differences as well, but risk could well be an important one and is used illustratively. Such a coefficient is used to represent risk aversion, non-random labor turnover and financing difficulties by W. M. Corden and R. Findlay, "Urban Unemployment, Inter-Sectoral Capital Mobility and Development Policy," mms 1973.
In this model an individual with a higher risk aversion—either because of family circumstances or personal traits—will tend to have a higher supply price to the protected sector. The variable $v_i$ could also be introduced into equation (4) along with the choice between protected and unprotected sector expected incomes, which results in,

$$\text{Max}(v_i p_m, I_{uu}) \geq I_{ri} + C_i - A_i$$

The interpretation of (4d) is that a rural resident weighs his current income $I_{ri}$ plus his monetary and psychic costs against two alternatives and chooses the maximum welfare. One alternative is certain employment in the unprotected sector with income $I_{uu}$. As seen in Case I, this choice is represented by:

$$I_{uu} \geq I_{ri} + C_i - A_i$$

Risk aversion is not relevant here since both are certain outcomes. The second alternative is represented by Case II. If the risk aversion coefficient is introduced, (4b) becomes:

$$v_i p_m \geq I_{ri} + C_i - A_i$$

The decision to migrate will be based on whether either alternative offers an increase in welfare and the choice of destination will be based upon which alternative provides the greater welfare.

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13/ Empirical data for several labor-surplus economies indicate that the highest levels of urban unemployment are found among persons between the ages of 15 and 25 and with more than 5 but less than 12 years of education. This is consistent with the notion that young, educated and unmarried individuals have relatively low levels of risk aversion and thus by equation (5) would be more likely to leave the unprotected sector. (See D. Turnham, op. cit. pp. 58-64.)
In the simplified world represented by decision functions (4d) and (5), by the same reasoning applied in Case I and Case II, the marginal worker entering the protected sector has a supply price of $w_m$ as in Case II regardless of whether s/he originates in the unprotected urban sector or the rural sector. If the project is located in the urban unprotected sector the relevant supply price is $w_{uu}$ as in Case I.

There seems to be one special circumstance in which the protected sector supply price for individuals from the unprotected sector would equal $w_{uu}$ rather than $w_m$. That is when individuals could apply for work in the protected sector without leaving their current employment. Thus there is no loss to the individual or to society until he is lucky enough to be hired in the protected sector, at which time he bears no risk by leaving his old job; and his supply price and the SOCL equals his current wage of $w_{uu}$. This is a plausible situation and it would introduce the uncertain supply price between $w_m$ and $w_{uu}$, since it cannot be known how many project employees will come from the unprotected sector in this way and how many will come from the pool of unemployed or from new migrants. In this special case $w_{uu} \leq \text{SOCL} \leq w_m$, and this may be a common situation for some skill categories but it is dependent upon the assumption that the worker can work and job-hunt simultaneously. This possibility was specifically ruled out in Harberger's argument, so his Case III result does not appear to be consistent with this model.

A final consideration which should be mentioned regarding Case III is that some particular labor skill categories in the urban unprotected sector may have no parallels in the protected sector, and certainly the reverse is true. So the occurrence of Case III may be limited to certain skill categories.
4. Implications of the Supply Price Approach

Using a supply price approach, consistent reasoning has led to the conclusion that when the project has no non-marginal effects on the market, a good approximation of the SOCL is the going wage for the sector in which the project is located. Many public projects would fit this category, and in most countries these wage data exist or are obtainable at reasonable expense for specific regions, seasons and labor skills.

For projects located in the rural sector, there is very little difference between using a "supply price" or a "foregone product" approach, except in cases with non-comparable working conditions or when project wages are above the local market wage. In such cases, the supply price approach calls for an upward adjustment in the SOCL if project working conditions are less favorable than in existing jobs, or if higher project wages are attracting new entrants to the labor force. However, determining the size of such adjustments remains a problem.

The implications of the supply price approach are most important for projects which are either located in urban areas or are not location specific. In all cases discussed above, including the special exception to Case III, the supply price estimate of SOCL was greater than the foregone product measure and the greatest divergence occurred when chronic unemployment was assumed. The latter is a common occurrence in labor surplus economies, so this will be used to illustrate the point.

Consider the situation represented by Fig. 2. Using the minimum wage \( w_m \) as a measure of the SOCL could lead to the rejection of projects which show a handsome return when the foregone product in agriculture is used. The reason, of course, is that the foregone product measure fails
to account for the loss to the economy represented by those who were
induced by $w_m$ to migrate but were not hired.\textsuperscript{14/}

More important, perhaps, is the project which is not unique to a
specific location, such as a public works project. If gross benefits
and non-labor costs are insensitive to location, the foregone product
measure would indicate no location preference since it would not be
influenced by the project location. A decision may well be made to locate
in the urban sector to help absorb unemployment. However, the supply price
approach would show a higher payoff for the rural sector location, since
the benefits would be equal and the SOCL substantially lower.

The supply price approach would, in general, lead to the acceptance
of more projects for low-wage regions and fewer projects for high-wage
regions, simply on the basis of social profitability. The effect of this
overall shift in emphasis, as implied by the migration model, would be a
relative decline in migration and possibly a relative decline in urban
unemployment.

The implications of the migration model for the choice of location
are illustrated by considering the effects of locating a public works
project in the rural versus the urban sector. This example also provides
an opportunity to look at a case involving non-marginal effects. It will
be assumed that the project hires the same number of persons in either
sector and pays the going wage of the sector in which it is located. The
graphical analysis uses a replication of Fig. 2 so that the results can be

\textsuperscript{14/} Of course, it also fails to measure $C - A$, but that may be very
insignificant in comparison to the cost of increased unemployment.
interpreted as a change in the static equilibrium as a result of the project. The algebraic analysis which supplements and formalizes the graphics is based on the reduced form equations derived from the model (Appendix II).

In Fig. 3 the effects of the urban public works project are illustrated. Asterisks indicate the new urban demand curve, the new loss function \((L)\), and the new equilibrium levels of migration \((M^*)\) and rural income \((I^*_r)\). As a result of the urban public works project the probability of urban employment increases, shifting \(L\) downward and increasing the equilibrium level of migration from \(M_1\) to \(M^*\) and the equilibrium rural wage from \(I^*_r\) to \(I^*_r\). The higher rural wage \((I^*_r)\) induces increased migration both by reducing employment in the rural sector and by attracting to the market new entrants whose rural supply price is between \(I^*_r\) and \(I^*_r\).

The social cost of total labor for the project in real terms is the sum of three components: the foregone product of workers withdrawn from rural sector employment (shaded area \(A\)), the foregone utility of new entrants from the rural sector (shaded area \(B\)),\(^{15/}\) and the change in monetary and psychic costs of migration (shaded area \(C - D = L^*M^* - L_1M_1\)).

It can be seen in Fig. 3 that

\[
C - D > 0 \text{ if } |\partial F/\partial M| < 1
\]

where \(\partial F/\partial M = -\beta_1\), the slope of eqn. (7), Appendix II. If the project employs \(n\) units of labor, the social real cost per unit is

\[
\frac{A + B + C - D}{n}
\]

\(^{15/}\) Each point on the rural labor supply curve between \(I^*_r\) and \(I^*_r\) can be considered as the reservation income for one unit of labor. The reservation income is a measure of the value of leisure foregone when a new unit of labor enters the market.
Figure 3. Project located in the urban sector (functions given in Appendix II).

Figure 4. Project located in the rural sector (functions given in Appendix II).

* or ** indicates the "after project" level of a curve or variable.
Disregarding area D for a moment, it can be shown that

\[ \frac{A + B + C}{n} < I_m \]

when the increase in migration \((M^* - M_1)\) is no greater than \(n\), that is, when

\[ \frac{\partial M}{\partial D^*} \leq \frac{\partial N_{ud}}{\partial D^*} \]

From Appendix II, eqn. (2), it can be seen that \(\frac{\partial N_{ud}}{\partial D^*} = D_2\) and from the impact multiplier \((m5)\)

\[ \frac{\partial M}{\partial D^*} = D_2 \left[ \frac{I_m (p_2 - p_1)}{I_m p_3 - \beta_1} \right] \]

so the increased migration (from the rural area) will be greater than increased (urban) employment only if

\[ \frac{I_m (p_2 - p_1)}{I_m p_3 - \beta_1} > 1 \]

This depends upon the nature of the probability function, the elasticities of the rural sector supply and demand equations and the level of \(I_m\).\(^{16/}\)

From the impact multiplier \((m8)\)

\[ \frac{\partial I_T}{\partial D^*} = \beta_1 D_2 \left[ \frac{I_m (p_2 - p_1)}{I_m p_3 - \beta_1} \right] < 0 \]

\(^{16/}\) Using Todaro's \(p^*_t = S - N\), where \(r\) is the rate of job creation and urban unemployed \((U)\) and new migrants \((M)\) make up the total unemployment \((S-N = U + M)\); \(\frac{\partial p^*_t}{\partial U} = \frac{\partial p^*_t}{\partial M} = p_3 = p_2\) and \(\frac{\partial p^*_t}{\partial N} = p_1 = r/\mu (U + M)\), where \(\mu = (U + M)/S\). Thus, \(I_m (p_3 - p_1)/(I_m p_3 - \beta_1) > 1\) only if \(I_m p_1 > \beta_1\) or \(p_1 > 1/I_m (s_1 - d_1)\). But this is quite improbable since \(p_1\) is very small.
It is already clear from Fig. 3 that \( \frac{\partial I_r}{\partial D^*} > 0 \), since rural income \( I_r \) must increase with increased migration. However, an interesting result in the reduced form equations (r1 and r2, Appendix II) is that the impact on \( I_r \) of the same project located in the rural sector tends to be smaller. Note from (m7) that

\[
\frac{\partial I_r}{\partial D^*} = \beta_1 d_2 \left[ \frac{I_m p_3}{I_m p_3 - \beta_1} \right] > 0
\]

If the project employment is assumed to be the same in both locations, \( d_2 = D_2 \). If, as is often assumed, the number of unemployed natives and the number of migrants have the same marginal effect on \( p^* \), then \( p_2 = p_3 \) and

\[
\frac{\partial I_r}{\partial D^*} > \frac{\partial I_r}{\partial D^*} \quad \text{since} \quad p_1 > 0.
\]

In the case of a public works project, however, the constraint may be constant expenditures rather than constant employment. Constant expenditures would lead to a greater number of jobs in the rural location due to lower wages, so that \( d_2 > D_2 \). In this case the rural location generates a greater increase in \( I_r \) when

\[
d_2 > D_2 \left( \frac{p_2 - p_1}{p_3} \right)
\]

The effects of placing the public works project in the rural sector are illustrated in Fig. 4, where asterisks indicate the new rural demand curve, the new excess supply function, the new F curve, and the new equilibrium levels of migration \( (M_e) \) and rural income \( (I_r) \). The increased level of rural demand causes fewer migrants to respond to any given rural-urban income differential. Thus the equilibrium level of migration is reduced
by $M_1 - M_*$. From the reduced form equations,

$$\frac{\partial M}{\partial d^*} = d_2 \frac{\beta_1}{I^m p_3 - \beta_1} < 0$$

In this instance $\beta_1 < \vert I^m p_3 - \beta_1 \vert$

so necessarily $\frac{\partial M}{\partial d^*} < d_2$,

i.e., the reduction in migration is less than the number of new jobs created, so $I_r$ must rise. This rise in $I_r$ again draws new entrants to the market and attracts some labor from current rural sector jobs. Since $N_us$ and $N_ud$ are constant and migration is reduced, total unemployment and the unemployment rate in the urban sector are reduced. Such a favorable impact on the urban sector is possible but not assured when the project is located in the urban sector.

Two components of the social cost of total labor are the same as before, though the levels are lower. The foregone product of workers withdrawn from rural sector employment (shaded area $A'$) is less than $A$ if $I_r^{**} < I_r^*$ as indicated for the case of equal employment. For the same reason the foregone utility of new entrants (shaded area $B'$) is less than $B$. Since there is a reduction rather than an increase in migration, the third component is a gain rather than a loss to society. Thus the shaded area $C'$, where $C' = L_1 M_1 - L_* M_*$, could be interpreted as the losses foregone as a result of reduced migration. If the project employs $n$ workers,

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The implication here that lower unemployment is desirable must be based on the assumption that not all externalities have been internalized in this model. If all externalities were accounted for in the model, it could not be said that an equilibrium solution with a lower level of unemployment is more beneficial to society than equilibrium at a higher level of unemployment. An example of an omitted externality might be urban congestion.
the social real cost per worker is
\[
\frac{A' + B' - C'}{n} \quad \text{(7)}
\]
For the equal employment case
\[
\frac{A' + B' - C'}{n} < \frac{A + B + C - D}{n} \quad \text{if } C' < C - D \quad \text{(8)}
\]
For the equal expenditure case, the direction of this inequality is not clear. Let expenditures be such that A, B, C, D, and n are the same as before. As already noted, constant expenditure in the rural sector leads to \(d_2 > D_2\), i.e., a greater number of jobs are created. This higher project employment would generate a higher \(I^{**}\) and lower \(N^*_r\), meaning that \(A', B',\) and \(C'\) would be greater than in the equal employment case. The increases in \(n\) and \(C'\) would tend to further reduce the left hand side of the inequality (8), but the effect of higher \(A'\) and \(B'\) would be in the opposite direction.

The nominal social costs derived from expressions (6) and (7) would be the appropriate values for SOCL based on the supply price concept. Without being able to take the integrals necessary to calculate these areas, it is not clear how the results would compare with the sector wage rates which were used in cases with no non-marginal effects.

For the case of equal employment, however, there seems to be several advantages to locating the project in the rural sector:

1. Lower social opportunity cost per worker.
2. Lower project wage bill \(\left(\frac{I^{**}}{r} < \frac{I}{m}\right)\).
3. Assurance of a lower rate of urban unemployment and a corresponding reduction in any undesirable externalities associated with such unemployment and not accounted for in the model.
4. Although the analysis indicates that the equilibrium rural income would have a smaller increase, this could be offset by a more rapid rate of adjustment to the equilibrium when the project is located in the rural sector. For an urban-based project, the rate of adjustment in rural income would be dependent upon the rate at which migration would occur.

Note that this analysis also is relevant for the comparison of wage subsidies which would have an equivalent effect on the sectoral demand functions.

In summary, although many individual projects can be assumed to have no non-marginal effects, a general change in the pattern of project selection and location could have significant non-marginal effects on migration, unemployment patterns, wage differentials and other economic variables. This analysis has given some indication of the nature of these effects. For projects with no non-marginal effects the previous conclusions still hold. Although the supply price approach at first may look like a reversion to the unthinking use of market prices to measure the SOCL, it is firmly grounded in a rational economic model of labor market behavior.
APPENDIX I

Model of the Rural and Urban Unprotected Sectors

(1) \( N_{us} = S(I_u, S^*) \quad S_1, S_2 > 0 \)
(2) \( N_{ud} = D(I_u, D^*) \quad D_1 < 0, D_2 > 0 \)
(3) \( M_d = D(I_u, D^*) - S(I_u, S^*) = M_d(I_u, S^*, D^*) \)
(4) \( N_{rs} = s(I_r, s^*) \quad s_1, s_2 > 0 \)
(5) \( N_{rd} = d(I_r, d^*) \quad d_1 < 0, d_2 > 0 \)
(6) \( M_s = s(I_r, s^*) - d(I_r, d^*) = M_s(I_r, s^*, d^*) \)
(7) \( M_s = M_d \)
(8) \( M = M_s \)
(9) \( L = C - A \)
(10) \( F = M_d^{-1} - M_s^{-1} = F(M, S^*, D^*, s^*, d^*) \)
(11) \( F = L \)

From (3),

(12) \( \frac{dI_u}{dM_d} - \frac{S_2}{D_1 - S_1} \frac{dM_d}{ds^*} - \frac{D_2}{D_1 - S_1} \frac{dD^*}{dd^*} \)

(12a) \( = a_1 dM_d + a_2 ds^* - a_3 dD^* \)

where \( a_1 = \frac{1}{D_1 - S_1} \), \( a_2 = S_2 a_1 \), \( a_3 = D_2 a_1 \); \( a_1, a_2, a_3 < 0 \)

From (6),

(13) \( \frac{dI_r}{ds_1} = \frac{1}{s_1 - d_1} \frac{dM_s}{ds} - \frac{s_2}{s_1 - d_1} \frac{ds}{ds^*} + \frac{d_2}{s_1 - d_1} \frac{dd}{dd^*} \)

(13a) \( = \beta_1 dM_s - \beta_2 ds^* + \beta_3 dd^* \)

\(^1/\) Except for \( a_i \) and \( \beta_i \), a numeral subscript denotes the first partial derivative of the function with respect to the \( i \)th independent variable.
where $\beta_1 = \frac{1}{s_1 - d_1}$, $\beta_2 = s_2 \beta_1$, $\beta_3 = d_2 \beta_1$; $\beta_1, \beta_2, \beta_3 > 0$

From (7) and (8),

(14) $dM = dM_s = dM_d$

From (10),

(15) $dF = dI_u - dI_r$

From (11),

(16) $dF = dL$

(17) $dL = (a_1 - \beta_1)dM + a_2 dS^* - a_3 dD^* + \beta_2 dS^* - \beta_3 dD^*$

(18) $dM = \frac{1}{a_1 - \beta_1} dL - \frac{a_2}{a_1 - \beta_1} dS^* + \frac{a_3}{a_1 - \beta_1} dD^* - \frac{\beta_2}{a_1 - \beta_1} dS^*$

$+ \frac{\beta_3}{a_1 - \beta_1} dD^*$
APPENDIX II

Model of the Rural and Urban Protected Sectors

(1) \( N_{us} = S(I_u, S^*) \quad S_1, S_2 > 0 \)
(2) \( N_{ud} = D(I_u, D^*) \quad D_1 < 0, D_2 > 0 \)
(3) \( I_u = I_m \)
(4) \( N_{rs} = s(I_r, s^*) \quad s_1, s_2 > 0 \)
(5) \( N_{rd} = d(I_r, d^*) \quad d_1 < 0, d_2 > 0 \)
(6) \( M = s(I_r, s^*) - d(I_r, d^*) = M(I_r, s^*, d^*) \)
(7) \( F = I_m - M^{-1} = F(M, s^*, d^*, I_m) \)
(8) \( L = C - A + (1 - p^*)I_m \)
(9) \( F = L \)
(10) \( p^* = p(N_{ud}, U, M) \quad p_1 > 0, p_2, p_3 < 0 \)
(11) \( U = S(I_m, S^*) - D(I_m, D^*) \)

Endogenous variables

\( N_{us}, N_{ud}, N_{rs}, N_{rd}, M, F, L, I_r, I_u, p^*, U \)

Exogenous variables

\( S^*, D^*, s^*, d^*, I_m, C, A \)

\(^1/\) Except for \( \beta_4 \), a numeral subscript denotes the first partial derivative of a function with respect to the \( i \)th independent variable.
From (10),

\[ dp^* = p_1 d_{Nud} + p_2 dU + p_3 dM \quad p_1 > 0; \quad p_2, \quad p_3 < 0 \]

From (2),

\[ dp = D_1 dI_m + D_2 dD^* \quad D_1 < 0, \quad D_2 > 0 \]

From (11),

\[ dU = (S_1 - D_1) dI_m + S_2 dS^* - D_2 dD^* \quad S_1, \quad S_2 > 0 \]

\[ dp^* = [p_1 D_1 + p_2 (S_1 - D_1)] dI_m + p_2 S_2 dS^* + (p_1 - p_2) D_2 dD^* + p_3 dM \]

From (8),

\[ L = C - A \]

\[ dL = dL_o + (1 - p^*) dI_m - I_m dp^* \]

\[ dL = dL_o + [1 - p^* - I_m (p_1 D_1 + p_2 (S_1 - D_1))] dI_m - I_m [p_2 S_2 dS^* + (p_1 - p_2) D_2 dD^* + p_3 dM] \]

From (6),

\[ dI_r = \beta_1 dM - \beta_2 dS^* + \beta_3 dd^* \]

where: \[ \beta_1 = \frac{1}{s_1 - d_1}, \quad \beta_2 = s_2 \beta_1, \quad \beta_3 = d_2 \beta_1; \quad \beta_1, \quad \beta_2, \quad \beta_3 > 0 \]

From (7),

\[ dF = dI_m - dI_r \]

\[ dF = dI_m - \beta_1 dM + \beta_2 dS^* - \beta_3 dd^* \]

From (9),

\[ dF = dL \]

\[ (I_m p_3 - \beta_1) dM = dL_o - [p^* + I_m (p_1 D_1 + p_2 (S_1 - D_1))] dI_m - I_m p_2 S_2 dS^* - I_m (p_1 - p_2) D_2 dD^* - \beta_2 dS^* + \beta_3 dd^* \]
Reduced form equations for migration and rural income were obtained from (21) and (18).

(r1) \[ \frac{1}{I_m p_3 - \beta_1} \frac{dM}{dL_o} - \frac{p^* + I_m [p_1 D_1 + p_2 (S_1 - D_1)]}{I_m p_3 - \beta_1} \frac{dI_m}{dL_o} \]

\[ - \frac{I_m p_2 S_2}{I_m p_3 - \beta_1} \frac{dS^*}{dL_o} - \frac{I_m (p_1 - p_2) D_2}{I_m p_3 - \beta_1} \frac{dD^*}{dL_o} - \frac{\beta_2}{I_m p_3 - \beta_1} \frac{ds^*}{dL_o} \]

\[ + \frac{\beta_3}{I_m p_3 - \beta_1} \frac{dd^*}{dL_o} \]

(r1a) \[ dM = r_1 dL_o - r_2 dI_m - r_3 dS^* - r_4 dD^* - r_5 ds^* + r_6 dd^* \]

(r2) \[ dI_x = \beta_1 r_1 dL_o - \beta_1 r_2 dI_m - \beta_1 r_3 dS^* - \beta_1 r_4 dD^* - (\beta_1 r_5 + \beta_2) ds^* \]

\[ + (\beta_1 r_6 + \beta_3) dd^* \]

where \[ r_1 = \frac{1}{I_m p_3 - \beta_1} , \quad r_2 = \frac{p^* + I_m [p_1 D_1 + p_2 (S_1 - D_1)]}{I_m p_3 - \beta_1} , \]

\[ r_3 = \frac{I_m p_2 S_2}{I_m p_3 - \beta_1} , \quad r_4 = \frac{I_m (p_1 - p_2) D_2}{I_m p_3 - \beta_1} , \quad r_5 = \frac{\beta_2}{I_m p_3 - \beta_1} , \]

\[ r_6 = \frac{\beta_3}{I_m p_3 - \beta_1} \]

Impact multipliers were obtained from reduced form equations (r1) and (r2).

(ml) \[ \frac{\delta M}{\delta L_o} = \frac{1}{I_m p_3 - \beta_1} = r_1 < 0 \]

(m2) \[ \frac{\delta M}{\delta I_{pu}} = -\frac{[p^* + I_m [p_1 D_1 + p_2 (S_1 - D_1)]]}{I_m p_3 - \beta_1} = -r_2 > 0 \]

if \( p^* < I_m [p_1 D_1 + p_2 (S_1 - D_1)] \)
(m3) \( \frac{\delta M}{\delta s^*} = \frac{-I_m p_2 s_2}{I_m p_3 - \beta_1} = -r_3 < 0 \)

(m4) \( \frac{\delta M}{\delta s} = \frac{-\beta_2}{I_m p_3 - \beta_1} = s_2 \frac{-\beta_1}{I_m p_3 - \beta_1} > 0; \text{ but also } \frac{\delta M}{\delta s^*} < s_2 \)

(m5) \( \frac{\delta M}{\delta d^*} = \frac{\beta_3}{I_m p_3 - \beta_1} = d_2 \frac{-\beta_1}{I_m p_3 - \beta_1} < 0; \text{ but also } \left| \frac{\delta M}{\delta d^*} \right| < d_2 \)

(m6) \( \frac{\delta M}{\delta D^*} = \frac{-I_m (p_1 - p_2) D_2}{I_m p_3 - \beta_1} = -r_4 > 0 \)

\[
= D_2 \frac{I_m (p_2 - p_1)}{I_m p_3 - \beta_1} < D_2 \text{ if } |I_m (p_2 - p_1)| < |I_m p_3 - \beta_1|
\]

(m7) \( \frac{\delta I_r}{\delta d^*} = \beta_1 \frac{\beta_3}{I_m p_3 - \beta_1} + \beta_3 = \beta_3 \left( \frac{I_m p_3}{I_m p_3 - \beta_1} \right) = \beta_1 d_2 \left( \frac{I_m p_3}{I_m p_3 - \beta_1} \right) > 0 \)

(m8) \( \frac{\delta I_r}{\delta D^*} = \beta_1 r_4 = \beta_1 D_2 \left( \frac{I_m (p_2 - p_1)}{I_m p_3 - \beta_1} \right) > 0 \)
APPENDIX III

Definition of Variables

Endogenous variables:

\[ N_{us} \] - urban labor supply
\[ N_{ud} \] - urban labor demand
\[ N_{rs} \] - rural labor supply
\[ N_{rd} \] - rural labor demand
\[ M_d \] - demand for migrant labor
\[ M_{s,M} \] - supply of migrant labor (M in Appendix II)
\[ I_u \] - urban real income per worker
\[ I_r \] - rural real income per worker
\[ p^* \] - probability of obtaining an urban protected sector job
\[ U \] - number of unemployed urban natives
\[ F \] - rural-urban income difference
\[ L \] - monetary and non-monetary (private) costs of migration

Exogenous variables:

\[ S^* \] - vector of urban sector labor supply shift variables
\[ D^* \] - vector of urban sector labor demand shift variables
\[ s^* \] - vector of rural sector labor supply shift variables
\[ d^* \] - vector of rural sector labor demand shift variables
\[ I_m \] - minimum real income associated with protected sector minimum wage
\[ C \] - monetary real cost of migration and resettlement per migrant
\[ A \] - real value of net amenities gained (urban minus rural) per migrant
Bibliography


