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A THEORY OF THE COMBINED MOLE-TILE DRAIN SYSTEM

by

Tariq Naji Kadir

A thesis submitted in partial fulfillment
of the requirements for the degree

of

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in

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"Irrigation and Drainage Engineering"

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Tariq Naji Kadir

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LIST OF SYMBOLS

1. A — cross-sectional area (L^2),

   or $= \int_0^{S_t} f(x) \text{ Sin } \frac{\pi x}{S_t} dx \quad (L^2)$

2. A_n — Fourier series coefficients (dimensionless)
3. A_{mn} — Fourier series coefficients (dimensionless)
4. a — diameter of mole drains (L),

   or variable

5. B — $= \frac{S_m S_t}{4} K_1 \quad (L^3)$
6. b — variable
7. b_0 — statistical model coefficient (L)
8. b_1 — statistical model coefficient (LT^{-1})
9. C_n — polynomial constants
10. Cos — trigonometric function
11. Cosh — hyperbolic function
12. c — $= S_t - x_o \quad (L)$
13. D_n — Fourier series coefficients (dimensionless)
14. d — differential operator,

    or $= d_2 + d_3 \quad (L)$

15. d_1 — height of initial water surface above mole drains (L)
16. d_2 — height of mole drains above tile drains (L)
17. d_3 — depth of impermeable layer below tile drains (L)
18. d_e — equivalent depth of impermeable layer below mole drains =

    $$\frac{d}{1 + \frac{d}{L}\left(\frac{8}{\pi}\frac{d}{a} - \alpha'\right)} \quad (L)$$

19. E $= \frac{2A}{S_t}$ (L)

20. E_n Fourier series coefficients (dimensionless)

21. e base of natural logarithms

22. $F(x, y)$ shape of the initial water surface at the beginning of the drainout period (L)

23. f average specific yield of saturated soil by volume (dimensionless)

24. $f(x)$ profile of the water surface along the mole drain boundary (L)

25. $g(x, y)$ $= (d_1 + d_2) - v(x, y)$ (L)

26. H $= u - d_2$ (L)

27. h cartesian coordinate (L)

28. $h_1(x)$ shape of the water surface profile along the mole drain boundary, $0 < x < x_o$ (L)

29. $h_2(x)$ shape of the water surface profile along the mole drain boundary, $(S_t - x_o) < x < S_t$ (L)

30. K_1 $= \frac{16(d_1 + d_2)}{\pi^2} - \frac{4d_2x}{\pi}$ (L)

31. K_2 $= 2d_2\psi x$ (L)

32. K_x hydraulic conductivity of saturated soil in the x-direction (LT^{-1})

33. K_y hydraulic conductivity of saturated soil in the y-direction (LT^{-1})

34. k average hydraulic conductivity of saturated soil (LT^{-1})

35. L spacing of mole drains while correcting for convergence of flow ($= S_{mc}$) (L)

36. l_n natural logarithm

37. $\dot{M}_x$ ground water flux in the x-direction $(FL^{-2}T)$

38. m summation index

39. n summation index, or degree of polynomial

40. P $= \frac{8}{\pi} \sum_{n=1, 3,...}^{\infty} \frac{1}{n} \cdot \text{Sin}\, \frac{n\pi}{2} \cdot \frac{\text{Sinh}\, \frac{n\xi}{2}}{\text{Sinh}\, n\xi}$ (dimensionless)

41. r regression coefficient (dimensionless)

42. S_m spacing of mole drains (L)

43. S_{mc} spacing of mole drains corrected for convergence of flow (L)

44. S_t spacing of tile drains (L)

45. Sin trigonometric function

46. Sinh hyperbolic function

47. t time (T)

48. t_1 time necessary for the highest point on the water surface to drop from its initial position (t = 0) to the mole drain elevation during Stage I (T)

49. t_2 time necessary for the highest point on the water surface to drop from the mole drain elevation to the tile drain elevation during Stage II (T)

50. U_x water velocity in the x-direction (LT^{-1})

51. U_y water velocity in the y-direction (LT^{-1})

52. u height of water surface above the tile drains at the mid-point of the system (L)

53. u(x, y, t) solution to second order Heat equation with non-homogeneous boundary conditions (L)

54. u(x) shape of water surface profile along the mole drain boundary for Case 6 (L)

55. $u_n(x)$ shape of water surface profile along the mole drain boundary for Cases 1-5 (L)

56. v constant rainfall rate (LT^{-1})

57. $v(x, y)$ $= v_1(x, y) + v_2(x, y)$ (L)

58. $v_1(x, y)$ solution of Laplace's equation in rectangular coordinates (L)

59. $v_2(x, y)$ solution of Laplace's equation in rectangular coordinates (L)

60. $w(x, y, t)$ solution to the second order Heat equation with homogeneous boundary conditions (L)

61. X $= t$ (T)

62. x cartesian coordinate (L)

63. x_o distance between tile drain and the point along the mole drain where water surface drops from the mole drain elevation (L)

64. Y $= u - K_2$ (L)

65. y_1 cartesian coordinate (L)

66. α $= \frac{f}{kd_3}$ $(L^{-2}T)$

67. α' $= 3.55 - \frac{1.6d}{L} + 2(\frac{d}{L})^2$ (dimensionless)

68. β $= \frac{x_o}{S_t}$ (dimensionless)

69. Δ increment in

70. δ $= \frac{d_2}{\sin \beta}$ (L)

71. ζ $= \frac{\pi^2}{\alpha S_m^2}$ (T^{-1})

72. ξ $= \frac{\pi S_m}{S_t}$ (dimensionless)

73. π $= 3.14159.......$

74. Σ summation operator

75. σ specific weight (FL^{-3})

76. ϕ $= \frac{\pi^2}{\alpha S_t^{\ 2}}$ (T^{-1})

77. χ $= \frac{2A}{S_t d_2}$ (dimensionless)

78. ψ $= \frac{\text{Sinh } \frac{\xi}{2}}{\text{Sinh } \xi}$ (dimensionless)

79. $\int$ integral of

80. ∂ partial differential operator

ABSTRACT

A Theory of the Combined Mole-Tile Drain System

by

Tariq Naji Kadir

Utah State University, 1973

Major Professor: Dr. Komain Unhanand
Department: Agricultural and Irrigation Engineering

A theory is presented to describe the stages of flow of water in the soil in a combined mole-tile drain system.

Based on the theory and along with some assumptions to simplify the complexity of the mathematical calculations involved, two general equations are derived for the spacing of the tile drains and the mole drains, respectively. Six different boundary conditions are considered, and the solutions for each presented. Some of the theoretical equations are compared with field data. A method is presented whereby the equations can be corrected for convergence of flow at the drains.

Finally, a procedure is presented whereby the theoretical equations could be used in designing a combined mole-tile drain system.

(96 pages)

INTRODUCTION

Drainage is one of many factors that influence crop growth and soil conservation. It helps create in the soil the best conditions for crop root growth and to keep the land surface free from excess water so that farm operations can be conducted effectively.

Drainage can be carried out using one or a combination of the following systems, (Donnan and Houston, 1967): (1) open drains, (2) covered drains, (3) wells and pumps and (4) sumps. An advantage of the covered drains system over the other systems is that it does not interfere with the farm operations.

Covered drains consist of a series of channels below the ground surface which may be connected to each other or discharge separately, either into an open channel or other point of disposal. Two common types of covered drains are tile drains and mole drains. Mole drains are underground channels, lined or unlined, formed by pulling a bullet shaped cylinder through the soil (Soil Conservation Service, 1973 and Donnan and Fouss, 1962). Tile drains are either perforated continuous pipes (e.g., plastic drains) or short sections of porous pipes butted together (e.g., clay or tile drains), placed underground usually in a trench and surrounded by a filter material, (Donnan and Houston, 1967). The trench is then backfilled to the ground surface.

The advantages of tile drains over the mole drairs are that they can be used in any soil, have a long working life and low maintenance costs. The main disadvantage cf this system of drains is the high initial cost, especially in heavy textured soils (soils with a high percentage of clay) in which close spacing is necessary due to the low hydraulic conductivity of the soils.

The mole drains, on the other hand, have the advantages of being much less expensive and simpler to install. The main disadvantage of mole drains is the short working life, and its use is restricted to somewhat heavy textured soils in which the unlined mole channels retain their shape after moling.

Consequently, drainage of excess water from heavy soils, particularly in humid areas, using tile drains alone requires close drain spacing and therefore may not be economical if the agricultural return from the land does not offset the initial and maintenance costs of installing these drains. However, by using a combination of the mole drainage and the tile drainage systems, it may be possible to obtain the advantages of both systems with few of the disadvantages. The moling operation causes numerous cracks in the soil which increase the hydraulic conductivity, and the mole channels work more or less as collecting drains, enabling the tile drains to be laid farther apart, thus reducing the initial cost of the drainage system (Theobald, 1963).

Need for the Study

Extensive investigations have been carried out to determine the spacing of tile drains, based on both theoretical and practical analysis.

From the theoretical point of view, the problem of tile drain spacing has been approached from both the steady and the transient (time dependent) flow concepts, with the latter receiving a wider acceptance in practice (Luthin, 1957, 1973). There has been no theoretical analysis made concerning mole drain spacing, and therefore the spacing is based mainly on practical experience (Soil Conservation Service, 1973). As for the combined mole-tile drain system, there have been no theoretical studies made up to this date to determine the spacing of the mole drains and the spacing of the tile drains, thus preventing a sound economic feasibility study of the system as compared to the ordinary tile drainage system.

Objectives

The main objectives of this study are:

1. To set up a mathematical model for the combined mole-tile drain system under transient conditions.

2. To obtain solutions for the above model for particular initial and boundary conditions.

3. To compare part of the solutions obtained in Step 2 with data obtained from a field experiment.

REVIEW OF LITERATURE

The combined mole-tile drain system is being used in various parts of the world, particularly Europe and the Far East. Successful use of the combined system has been reported in Austria, Federal Republic of Germany, Hungary, Ireland, United Kingdom and Yugoslavia (FAO, 1971). Use of and studies made on the system have also been reported in Japan (Tomita, 1971 and Tomita et al., 1968).

Generally speaking, the combined system is composed of two networks of underground drains: tile drains and mole drains. The two networks are laid at different elevations and are more or less orthogonal to each other, with the mole drains lying above the tile drains (Figure 1).

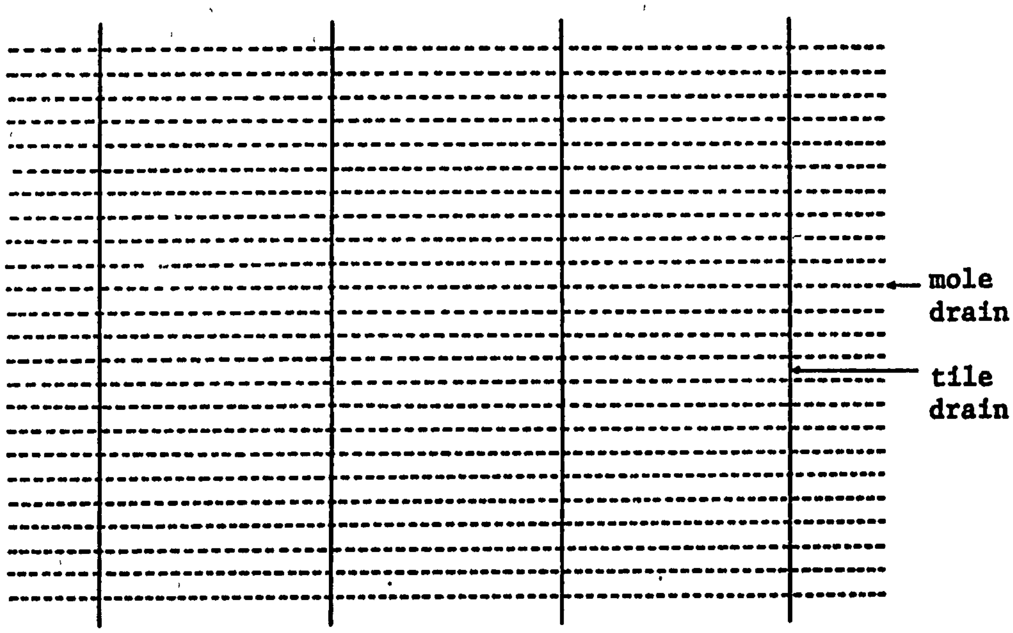

Figure 1. Plan view of the combined mole-tile drain system.

The tile drains, usually spaced at 30 m to 150 m and laid at a depth of about 100 cm below the ground surface, are placed in trenches and surrounded by a graded filter (Figure 2) and then backfilled to the ground surface with top soil. The mole plow, a bullet shaped cylinder connected to a sharp blade, is then pulled through the soil across the tile drains but at a higher elevation, forming the mole channels. These channels are usually spaced at 2 m to 5 m at a depth of about 60 cm below the ground surface. It should be emphasized again that the soil texture must be such that the mole drains will retain their shape long after the moling process.

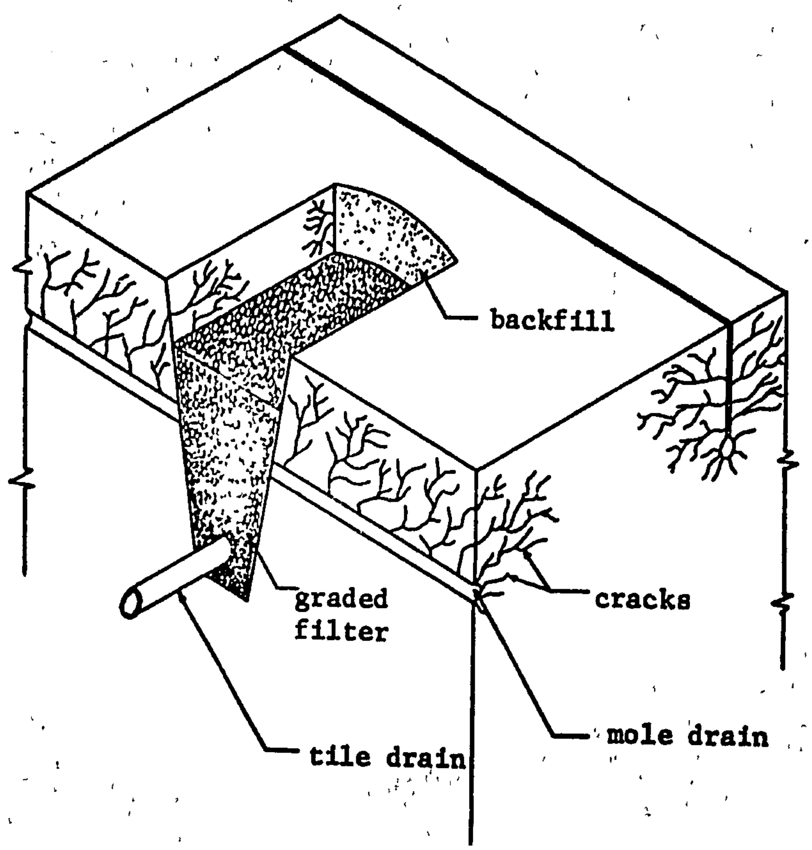

Figure 2. Illustrative diagram of the combined mole-tile drain system.

During the moling operation, cracks or fissures are developed on the sides of both the mole channel and the slit formed by the mole plow blade. These cracks form secondary drainage channels, making it easier for the water to flow in the soil. The filter material surrounding the tile drains allow the water in the mole channels to escape into the tile drains which in turn remove the water from the system.

The spacing and depth, however, of the drains in the combined system are based mainly on practical experience (FAO, 1971). In fact, the only indication of published theoretical work on the combined system is a paper published in Japan (Tomita, 1971), which dealt with a computer solution to the Three-Dimensional Steady State equation (i.e., Laplace equation) of flow in the combined system for ponded water using a digital computer. His results for homogeneous soils were presented in the form of graphical solutions with the ratio of the discharge per unit drain length to the hydraulic conductivity as the ordinate and the drain spacing as the absissa.

To the best of the author's knowledge, the analysis to be presented herein is the first of its kind to this date.

THEORY

Description of the Theory

Consider a field with a combined mole-tile drain system laid out infinitely in all directions to guarantee that the field boundary conditions will have no effect on the model under study. Because of symmetry, it is possible to choose as a model of study two parallel tile drains overlain orthogonally by two parallel mole drains (Figure 3). These drains now form the rectangular boundaries of the model.

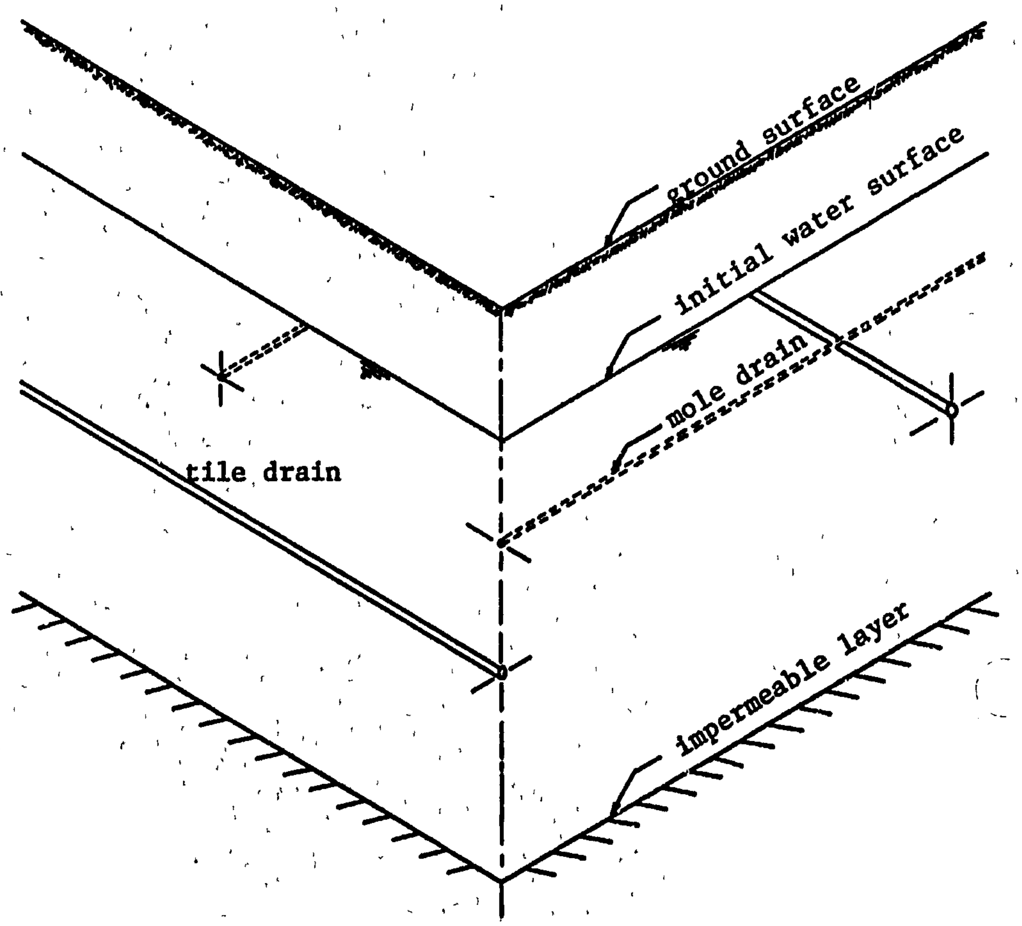

Figure 3. Flat water surface at time t = 0, Stage I.

The exact nature of the water movement in the combined system is not yet fully known. However, based on what is known and on the observations of a field experiment, a hypothetical description of the water movement in the system is presented below. It will be assumed, henceforth, that the Dupuit-Forchheimer assumptions are valid.

To begin with, assume that all the drain outlets were closed and the water table was built up to a uniform height above the mole drains, forming a flat water surface (Figure 3). If now all the drain outlets were opened simultaneously, the water surface, because of the hydraulic gradient components formed by the presence of the drains, quickly drops along the boundaries (i.e., the drains) to the drain levels and forms a curved surface within the model (Figure 4). The curves b-a-c and d-a-e (Figure 3) represent the water surface profiles at the sections midway between the mole drains and the tile drains, respectively. Point "a," lying at the intersection of the two curves, represents the highest point on the water surface in the system since it is least affected by the drains.

The curves b-a-c and d-a-e divide the water surface into four symmetrical regions, I, II, III, and IV. Since the flow is symmetrical in all four regions, it is possible to concentrate on one region, say region I (Figure 5) to describe the water movement. The direction of flow at any point on the curve a-b (e.g., point "1" in Figure 5) is along the curve itself (i.e., towards the tile drain only) since the velocity vector component toward the mole drain is zero. In fact, the largest velocity vector component towards the tile drain in region I exists along a-b. Similarly, the direction of flow at any point on the curve a-e (e.g., point "2" in Figure 5) will be towards

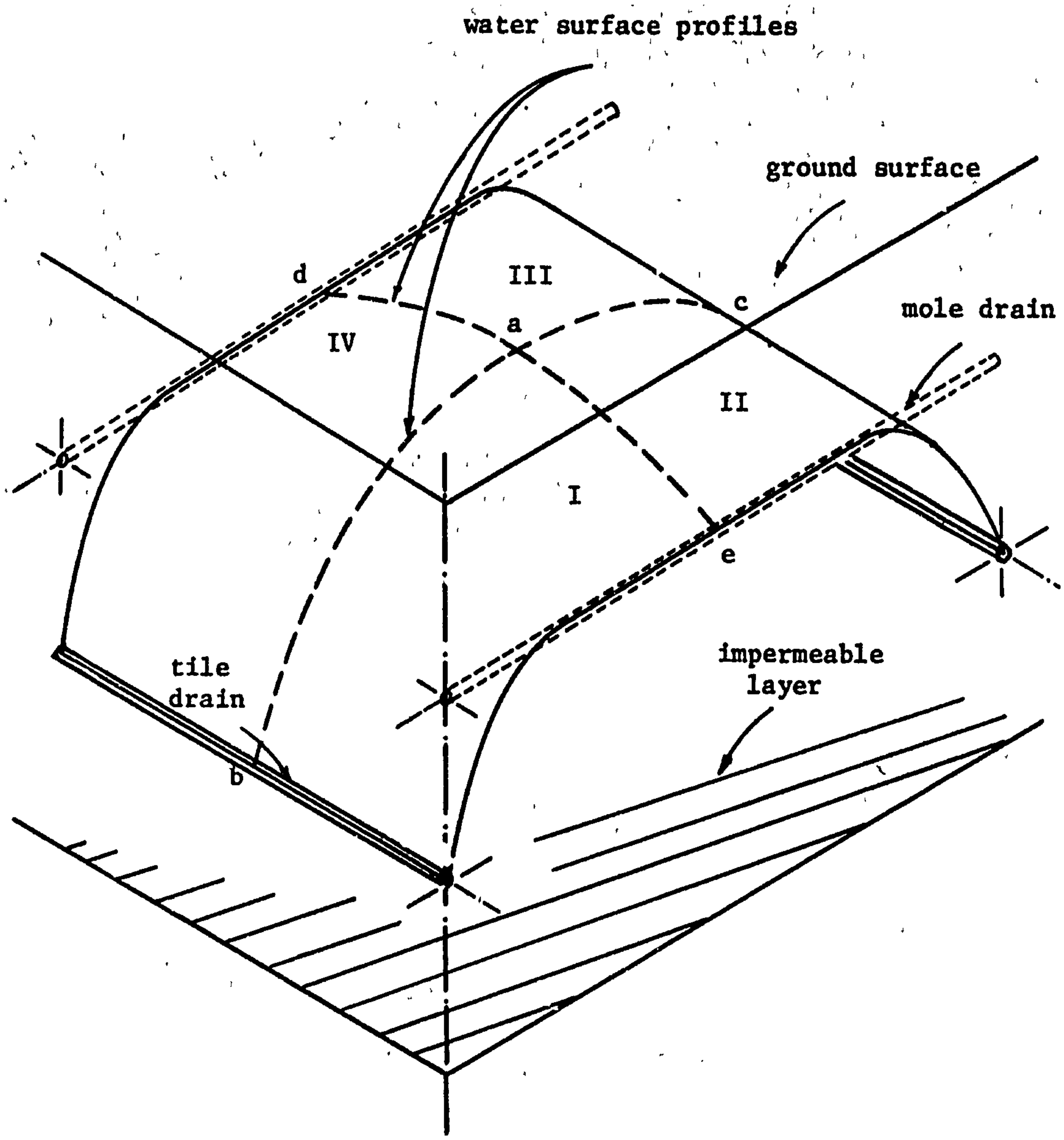

Figure 4. Symmetric regions of the water surface

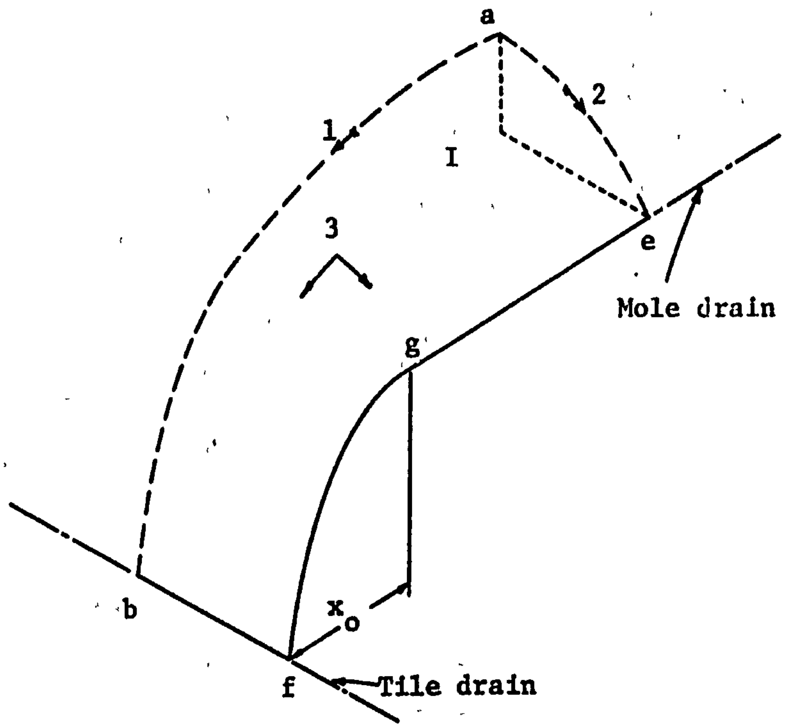

Figure 5. Flow of groundwater - Stage I.

the mole drain only because the largest velocity vector component towards the mole drain in region I exists along the curve itself. Any point lying on the surface within the region and not on the curves a-b or a-e or the boundaries will be affected by the velocity vector components in two directions (e.g., point "3" in Figure 5).

Along the tile drain, assuming that the drain is always half full with water, the water level is constant at the drain center line elevation. Along the mole drain, the problem is a little more complicated. From the field experiment, the discharge of the mole drain in the region of the tile drain was found to be small and quickly diminished with time even though the water surface was still above the mole drain elevation. This seems to indicate that while water enters and flows in the mole drain, the most part of it seeps out from the channel at some point "g" a distance x_o from the tile drain due to the velocity vector component towards the tile drain and begins flowing in a curved path toward the tile drain (i.e., the curve g-f in Figure 5). A further complication of the problem is that the distance x_o is not constant but increases with time (i.e., time dependent). After a certain amount of time has elapsed, a situation will be reached where the velocity vector component towards the mole drain will be very small compared to the velocity vector component towards the tile drain and flow towards the tile drain will dominate. At that point in time, the mole drain almost ceases to function, and the water level along the mole drain boundary begins to drop below the mole drain level. The water surface then gradually flattens out until the velocity vector is completely towards the tile drain, causing the flow to be one dimensional, the condition upon which the ordinary tile drain theories are based (Figure 6).

Stages of Water Movement in the System

The study model, shown in three dimensions in Figure 7, consists of two tile drains spaced at S_t, overlain orthogonally by two mole drains spaced at S_m. The vertical distance between the mole drains and the tile drains is d_2. The impermeable layer lies at a distance d_3 below the tile drains. The three cartesian axes u, x, and y are taken as shown in Figure 7.

The two stages of water movement in the combined system are:

Stage I - Water surface above the mole drains

During this stage, both the mole drains and the tile drains act together in a combined fashion resulting in a two-dimensional flow

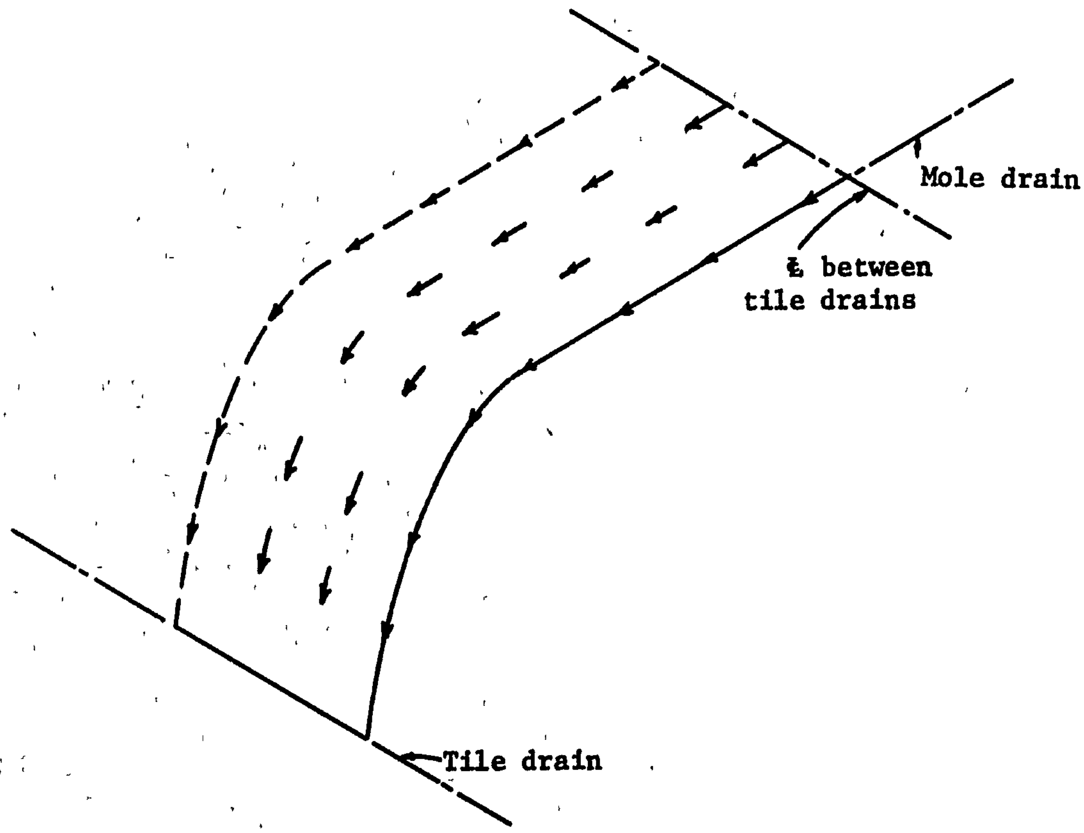

Figure 6. Flow of groundwater - Stage II.

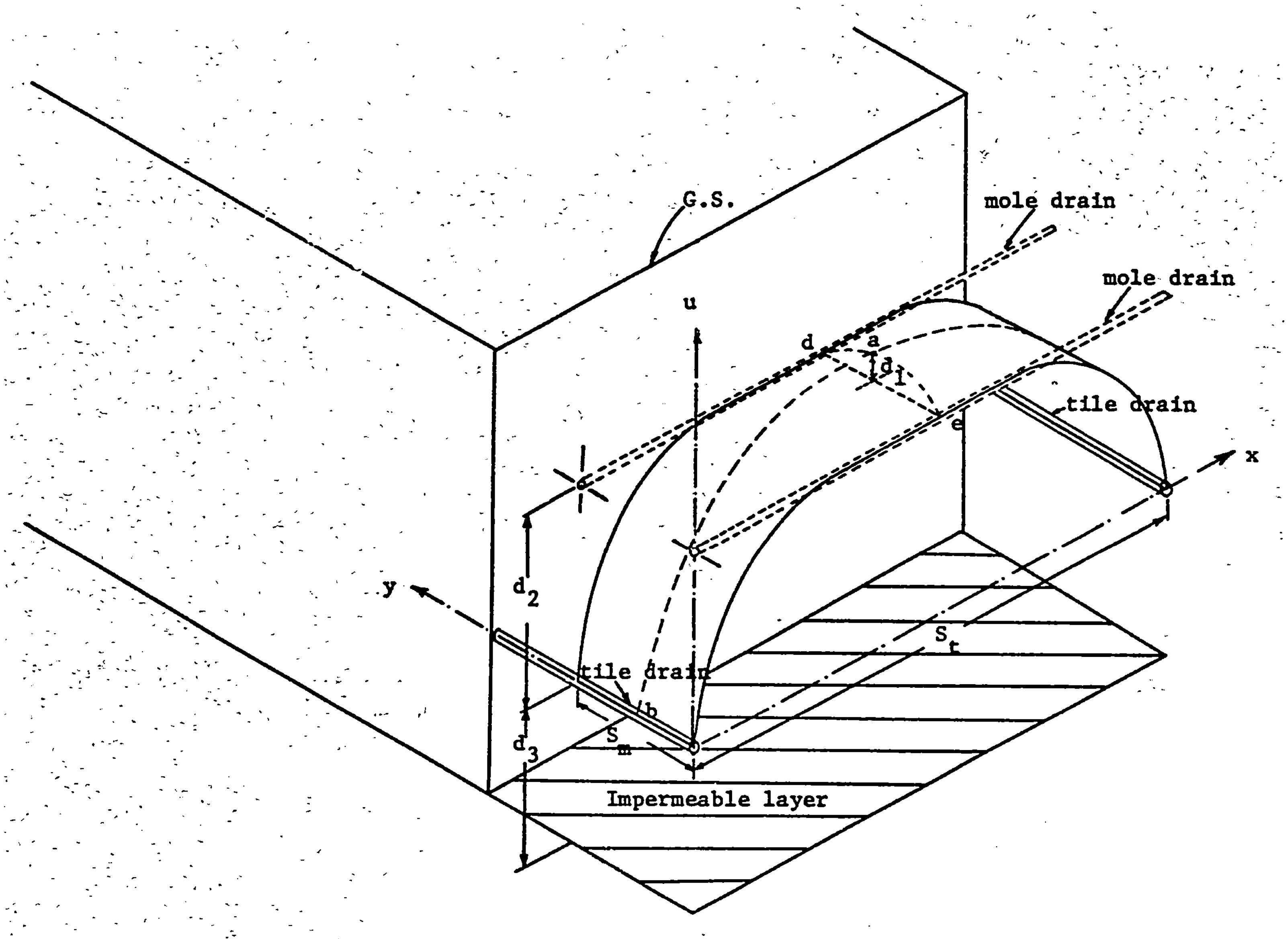

Figure 7. Study model of the combined mole-tile drain system.

pattern due to the velocity vector which can be broken up into two components, one towards the mole drains and the other towards the tile drains.

At the time $t = 0$ with all drain outlets closed, the water table is horizontal at a distance d_1 above the mole drains. Then it will be assumed that during an infinitesimal amount of time after all drain outlets are opened simultaneously, the water surface re-orients itself into a curved surface within the boundaries (i.e., the drains) and along the boundaries the water level drops to the drain level. Furthermore, the shape of the water surface profile along any boundary will be assumed to be independent of time. Along the tile drains the water level is at a constant elevation (i.e., $u = 0$). Along the mole drains, the water surface profile takes on a constant shape which will be assumed later.

Once all the points on the water surface are at an elevation equal to or below that of the mole drains (i.e., $u \leq d_2$), the mole drains will cease to function and Stage II begins.

Stage II - Water surface between the mole drains and the tile drains

During this stage only the tile drains are operating. The component of the velocity vector towards the mole drains is assumed to be zero. Therefore, the velocity vector will be only towards the tile drains, resulting in a one-dimensional flow pattern toward the tile drains (Figure 6).

The shape of the water surface at any section between the mole drains at the outset of this stage is assumed to be identical to the constant water surface profile along the mole drains assumed in Stage I. The shape, however, during this stage will be time dependent.

Assumptions

The following are the assumptions that will be used throughout the theoretical analysis.

1. Soil is homogeneous and isotropic.
2. Specific yield and hydraulic conductivity of the soil are constant.
3. Dupuit-Forchheimer assumptions are valid.
4. Darcy's law is applicable.
5. Flow is under a Transient State condition.
6. Flow is completely gravitational (no upward flow).
7. Land slope is small such that it has no effect on water movement.
8. Height of the water surface at any point at any time above the tile drains is very small as compared to the distance between the tile drains and the impermeable layer.
9. Tile drains are parallel. Mole drains are parallel. Tile drains are orthogonal to the mole drains.
10. Flat water surface as an initial condition.
11. Shape of the water surface profile along the boundaries is independent of time.
12. First term of the infinite Fourier series is sufficient for convergence.
13. Spacing of the mole drains S_m is small as compared to the spacing of the tile drains S_t such that $(1/S_t^2)$ can be neglected as compared to $(1/S_m^2)$.
14. The duration of Stage I is the time necessary for the highest point on the water surface to drop from its initial position to

an elevation just above the mole drains where it can be assumed that these drains no longer have a significant effect on the system flow (i.e., Stage II begins).

Setting Up the Mathematical Model

Stage I - Water surface above the mole drains

The basic two-dimensional continuity equation governing the flow of water through the soil is expressed as follows (see the derivation in Appendix A and Figure 7 for the study model):

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \alpha \frac{\partial u}{\partial t} \tag{1}$$

where

u is the height of the water surface above the tile drains at any time t

t is the time

$\alpha = \dfrac{f}{kd_3}$

f is the average specific yield of the soil (by volume)

k is the average hydraulic conductivity of the soil

d_3 is the vertical distance between the tile drains and the impermeable layer, $d_3 >> u$

The boundary conditions (B.C.) and the initial condition (I.C.) are as follows:

B.C.

$u(x, 0, t) = f(x)$ $\qquad$ $u(0, y, t) = 0$

$u(x, S_m, t) = f(x)$ $\qquad$ $u(S_t, y, t) = 0$

I.C.

$u(x, y, 0) = d_1 + d_2$ (Flat Water Table)

where

$f(x)$ represents the constant shape of the water surface profile along the mole drain boundary.

As shown in Appendix B, the solution $u(x, y, t)$ of Equation (1) is the sum of two solutions $v(x, y)$ and $w(x, y, t)$.

$$u(x, y, t) = v(x, y) + w(x, y, t) \tag{2}$$

where

$v(x, y)$ is the Steady State solution of Laplace's equation in rectangular regions,

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{3}$$

B.C.

$v(x, 0) = f(x)$ $\quad$ $v(0, y) = 0$

$v(x, S_m) = f(x)$ $\quad$ $v(S_t, y) = 0$

and $w(x, y, t)$ is the Transient State solution of

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha \frac{\partial w}{\partial t} \tag{4}$$

B.C.

$w(x, 0, t) = 0$ $\quad$ $w(0, y, t) = 0$

$w(x, S_m, t) = 0$ $\quad$ $w(S_t, y, t) = 0$

I.C.

$w(x, y, 0) = (d_1 + d_2) - v(x, y)$

where

$v(x, y)$ is the solution obtained from Equation (3).

Furthermore, $v(x, y)$ is the sum of two solutions (Powers, 1972):

$$v(x, y) = v_1(x, y) + v_2(x, y) \tag{5}$$

where

$v_1(x, y)$ is the solution of:

$$\frac{\partial^2 v_1}{\partial x^2} + \frac{\partial^2 v_1}{\partial y^2} = 0 \tag{6}$$

B.C.

$v_1(x, 0) = f(x)$ $\qquad$ $v_1(0, y) = 0$

$v_1(x, S_m) = 0$ $\qquad$ $v_1(S_t, y) = 0$

and $v_2(x, y)$ is the solution of:

$$\frac{\partial^2 v_2}{\partial x^2} + \frac{\partial^2 v_2}{\partial y^2} = 0 \tag{7}$$

B.C.

$v_2(x, 0) = 0$ $\qquad$ $v_2(0, y) = 0$

$v_2(x, S_m) = f(x)$ $\qquad$ $v_2(S_t, y) = 0$

Solving Equation (6) for $v_1(x, y)$ results in the following infinite Fourier series (Kreider et al., 1966):

$$v_1(x, y) = \sum_{n=1}^{\infty} A_n \operatorname{Sin} \frac{n\pi x}{S_t} \operatorname{Sinh} \frac{n\pi}{S_t} (S_m - y) \tag{8}$$

where

$$A_n = \frac{2}{S_t \operatorname{Sinh} \frac{n\pi S_m}{S_t}} \int_0^{S_t} f(x) \operatorname{Sin} \frac{n\pi x}{S_t} dx \tag{9}$$

Similarly, solving Equation (7) for $v_2(x, y)$:

$$v_2(x, y) = \sum_{n=1}^{\infty} D_n \operatorname{Sin} \frac{n\pi x}{S_t} \operatorname{Sinh} \frac{n\pi y}{S_t} \tag{10}$$

where

$$D_n = \frac{2}{S_t \operatorname{Sinh} \dfrac{n\pi S_m}{S_t}} \int_0^{S_t} f(x) \operatorname{Sin} \frac{n\pi x}{S_t} dx \tag{11}$$

Summing up $v_1(x, y)$ and $v_2(x, y)$ and noting that $A_n = D_n$, results in:

$$v(x, y) = \sum_{n=1}^{\infty} A_n \operatorname{Sin} \frac{n\pi x}{S_t} [\operatorname{Sinh} \frac{n\pi}{S_t} (S_m - y) + \operatorname{Sinh} \frac{n\pi y}{S_t}] \tag{12}$$

where

$$A_n = \frac{2}{S_t \operatorname{Sinh} \dfrac{n\pi S_m}{S_t}} \int_0^{S_t} f(x) \operatorname{Sin} \frac{n\pi x}{S_t} dx \tag{13}$$

Using the trigonometric identity:

$$\operatorname{Sinh} a + \operatorname{Sinh} b = 2 \operatorname{Sinh} \frac{a+b}{2} \cdot \operatorname{Cosh} \frac{a-b}{2}$$

Equation (12) can be re-written as:

$$v(x, y) = 2 \sum_{n=1}^{\infty} A_n \operatorname{Sin} \frac{n\pi x}{S_t} \operatorname{Sinh} \frac{n\pi S_m}{2S_t} \operatorname{Cosh} \frac{n\pi}{2S_t} (S_m - 2y) \tag{14}$$

Solving Equation (4) for $w(x, y, t)$, (Kreider et al., 1966):

$$w(x, y, t) = \sum_{\substack{m=1 \\ n=1}}^{\infty} A_{mn} \operatorname{Sin} \frac{n\pi x}{S_t} \operatorname{Sin} \frac{m\pi y}{S_m} e^{-\frac{\pi^2}{\alpha}\left(\frac{m^2}{S_m^2} + \frac{n^2}{S_t^2}\right)t} \tag{15}$$

where

$$A_{mn} = \frac{4}{S_m S_t} \int_0^{S_m} \int_0^{S_t} g(x, y) \, \text{Sin} \frac{n\pi x}{S_t} \, \text{Sin} \frac{m\pi y}{S_m} \, dx \, dy \qquad (16)$$

$$g(x, y) = [d_1 + d_2] - v(x, y) \qquad (17)$$

At this point, a closer study of the results obtained so far is necessary. Once v(x, y) is obtained from Equation (14) it is substituted into Equation (17) to obtain g(x, y) which in turn is substituted into Equation (16) to obtain the constant values A_{mn}. Equation (16) involves the integration of an infinite series of terms. Therefore, an assumption to simplify the problem is necessary. If it is assumed that the first term of the infinite Fourier series for v(x, y) of Equation (14) is sufficient for convergence (Appendix C), Equations (14) and (13) reduce to:

$$v(x, y) = \left(\frac{4}{S_t}\right) (\psi) (A) \, \text{Sin} \frac{\pi x}{S_t} \, \text{Cosh} \frac{\pi}{2S_t} (S_m - 2y) \qquad (18)$$

where

$$A = \int_0^{S_t} f(x) \, \text{Sin} \frac{\pi x}{S_t} \, dx \qquad (19)$$

$$\psi = \frac{\text{Sinh} \frac{\xi}{2}}{\text{Sinh} \, \xi}$$

$$\xi = \frac{\pi S_m}{S_t}$$

Once Equation (15) is solved for w(x, y, t), it is possible to find the general solution u(x, y, t) from Equation (2). However, since Equation (15) involves an infinite series of terms, so will Equation (2). In other words u(x, y, t) will contain the sum of an infinite series.

Generally speaking, the number of terms necessary for sufficient convergence beyond which the infinite series is truncated depends on the behavior of the series itself. In the problem under consideration, including more than one term of the infinite series involves extensive mathematical calculations, the results of which are not suitable for practical design purposes. Therefore, the earlier assumption applied to Equation (14) that the first term is sufficient for convergence will be used again. Therefore, taking the first term only, Equations (15) and (16) reduce to:

$$w(x, y, t) = \left(\frac{4}{S_m S_t}\right) (B) \operatorname{Sin} \frac{\pi x}{S_t} \operatorname{Sin} \frac{\pi y}{S_m} e^{-\frac{\pi^2}{\alpha}\left(\frac{1}{S_m^2} + \frac{1}{S_t^2}\right)t} \tag{20}$$

where

$$B = \int_0^{S_m} \int_0^{S_t} g(x, y) \operatorname{Sin} \frac{\pi x}{S_t} \operatorname{Sin} \frac{\pi y}{S_m} \, dx \, dy \tag{21}$$

Carrying out the integration on the right-hand side of Equation (21) reduces it to the following (Appendix D):

$$B = \frac{S_m S_t}{4} (K_1) \tag{22}$$

where

$$K_1 = \frac{16(d_1 + d_2)}{\pi^2} - \frac{4d_2}{\pi} (\chi) \tag{23}$$

$$\chi = \frac{2A}{S_t d_2} \tag{24}$$

In summary, the general equations controlling the water surface height at any point above the tile drains during Stage I are:

$$u(x, y, t) = v(x, y) + w(x, y, t) \qquad 0 < t < t_1 \tag{2}$$

where

t_1 is the time necessary for the highest point on the water surface to drop to the mole drains elevation during the drainout period.

$$v(x, y) = \left(\frac{4}{S_t}\right) (\psi) (A) \text{ Sin } \frac{\pi x}{S_t} \text{ Cosh } \frac{\pi}{2S_t} (S_m - 2y) \tag{18}$$

$$w(x, y, t) = K_1 \text{ Sin } \frac{\pi x}{S_t} \text{ Sin } \frac{\pi y}{S_m} e^{-\frac{\pi^2}{\alpha}\left(\frac{1}{S_m^2} + \frac{1}{S_t^2}\right)t} \tag{25}$$

A and K_1 are defined in Equations (19) and (23), respectively.

Stage II - Water surface between the mole drains and the tile drains

During this stage, the flow is basically one-dimensional. Figure 8 shows the shape of the water surface profile at the initialization time (t = 0) of this stage.

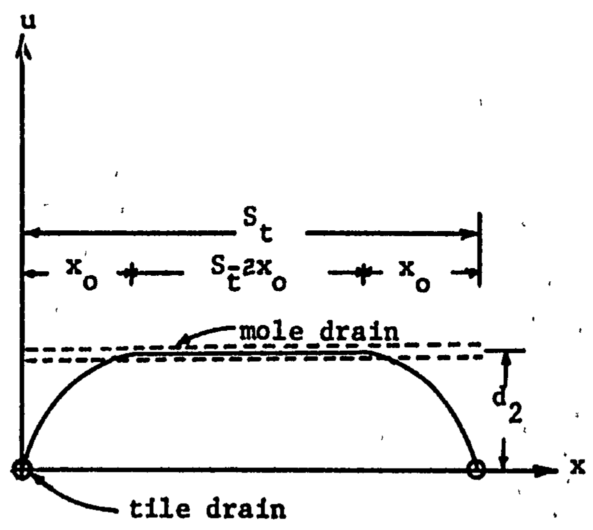

Figure 8. Water surface profile along the mole drains at time t = 0, Stage II.

By taking $\frac{\partial^2 u}{\partial y^2} = 0$ in Equation (1), the basic continuity equation for the one-dimensional flow is similar to Equation (1) and may be written as:

$$\frac{\partial^2 u}{\partial x^2} = \alpha \frac{\partial u}{\partial t} \tag{26}$$

B.C.

$u(0, t) = 0$ $\qquad\qquad$ $u(S_t, t) = 0$

I.C.

$u(x, 0) = f(x)$

Two points should be noted here. First, the term t being initialized at the outset of Stage II is independent of t of Stage I. Second, the shape of the water surface profile f(x) is no longer constant but varies with time. Theoretically speaking, f(x) represents the shape of the water surface profile of the intersection of a vertical plane in the x - direction with the water surface at any point $o \leq y \leq S_m$, at $t = 0$.

The method of solving Equation (26) is similar to those solved by R. E. Glover (Dumm, 1954, 1964).

Solving Equation (26) (Kreider et al., 1966):

$$u(x, t) = \sum_{n=1}^{\infty} E_n \operatorname{Sin} \frac{n\pi x}{S_t} e^{-\frac{n^2\pi^2}{\alpha S_t^2} t} \tag{27}$$

$$E_n = \frac{2}{S_t} \int_0^{S_t} f(x) \operatorname{Sin} \frac{n\pi x}{S_t} dx \tag{28}$$

Assuming that the first term of the infinite Fourier series of Equation (27) is sufficient for convergence:

$$u(x, t) = E \operatorname{Sin} \frac{\pi x}{S_t} e^{-\phi t} \qquad 0 < t < t_2 \tag{29}$$

where

$$E = \frac{2}{S_t} A \tag{30}$$

$$\phi = \frac{\pi^2}{\alpha S_t^{\,2}}$$

t_2 is the time necessary for the highest point on the water surface to drop from the mole drains elevation to the tile drains elevation

Shape of the Water Surface Profile Along the Mole Drains, f(x)

So far, nothing has been mentioned about the term f(x) which represents the shape of the water surface profile along the mole drains during Stage I and the shape of the water surface at time t = 0 during Stage II. Since there are no published studies or field data from which the shape could be approximated, it will have to be assumed for theoretical purposes.

Considering Figure 7, it was pointed out earlier that the water may flow in the mole drain for a considerable distance to either side of the center line lying midway between the tile drains. Once the water reaches the point g (or g') it drops from the mole drain level and begins flowing towards the tile drain along a curved path g - o (or g' - o').

Since f(x) was also assumed to be independent of time during Stage I, it can be expressed mathematically as follows:

$$f(x) = \begin{cases} h_1(x) & 0 \leq x \leq x_o \\ d_2 & x_o \leq x \leq (S_t - x_o) \\ h_2(x) & (S_t - x_o) \leq x \leq S_t \end{cases}$$

where $h_1(x)$ and $h_2(x)$ are algebraic expressions for the curves o - g and o' - g', respectively.

Six cases will be studied. In the first five cases, a polynomial will be assigned to the curves o - g and o' - g'. In the sixth case, a sine wave equation will be assigned.

Polynomial equations (Case 1 - Case 5)

The general form of the n^{th} degree polynomial equation can be expressed as:

$$u = C_0 + C_1x + C_2x^2 + \ldots + C_nx^n \tag{31}$$

where C_0, C_1, C_2, ... C_n are constants evaluated when considering the boundary conditions. Five cases will be considered:

Case 1. Zero degree polynomial

Case 2. First degree polynomial

Case 3. Second degree polynomial

Case 4. Third degree polynomial

Case 5. Fourth degree polynomial

Each case listed above represents the polynomial that will be assigned to the curves o - g and, symmetrically, o' - g'. Figure 9 shows the shapes of the polynomials relative to one another.

The boundary conditions necessary to evaluate the constants C_0, C_1, C_2, ..., C_n of Equation (31) are:

B.C. (curve o - g)

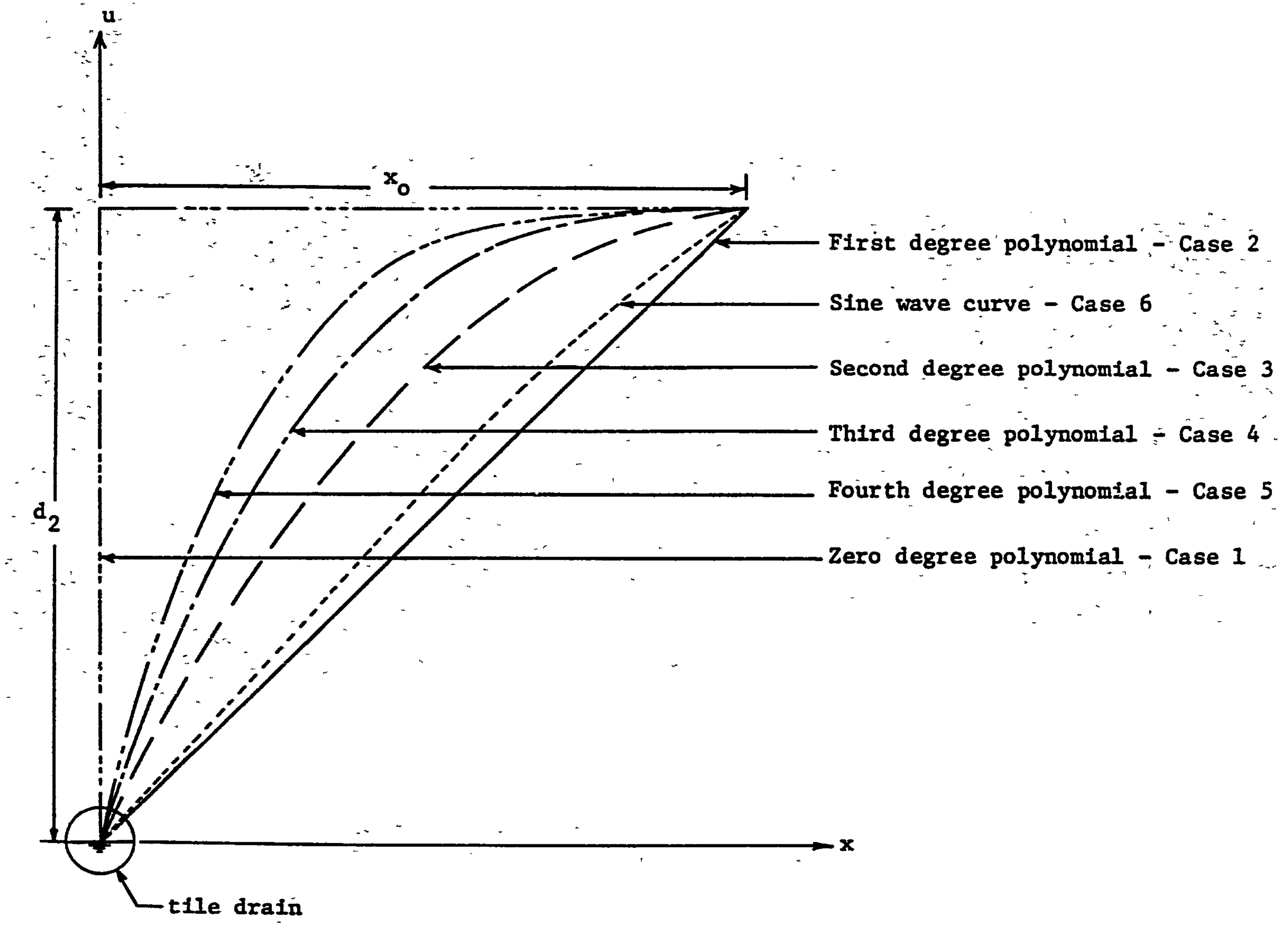

Figure 9. Relative shapes of the water surface profile along the mole drain for the six cases considered.

$u(0) = 0$

$u(x_o) = d_2$

$u'(x_o) = u''(x_o) = \ldots = u^{(n)}(x_o) = 0$

I.C. (curve o' - g')

$u(S_t) = 0$

$u(S_t - x_o) = d_2$

$u'(S_t - x_o) = u''(S_t - x_o) = \ldots = u^{(n)}(S_t - x_o) = 0$

Equating high order derivatives to zero at a point generates a smoother curve at that point.

Solving Equation (31) for the constants $C_0, C_1, C_2, \ldots, C_n$ using the above listed boundary conditions and grouping the terms together in a more compact form as follows:

$$u_n(x) = \begin{cases} d_2 - (-1)^n \dfrac{d_2}{x_o^{\,n}} (x - x_o)^n & 0 \le x \le x_o \quad (32) \\ d_2 & x_o \le x \le c \quad (33) \\ d_2 - \dfrac{d_2}{(S_t - c)^n} (x - c)^n & c \le x \le S_t \quad (34) \end{cases}$$

where

$c = S_t - x_o$

n = degree of polynomial considered

Equations (32), (33) and (34) represent f(x) for the five cases of polynomial forms of o - g or o' - g'.

Sine wave equation (Case 6)

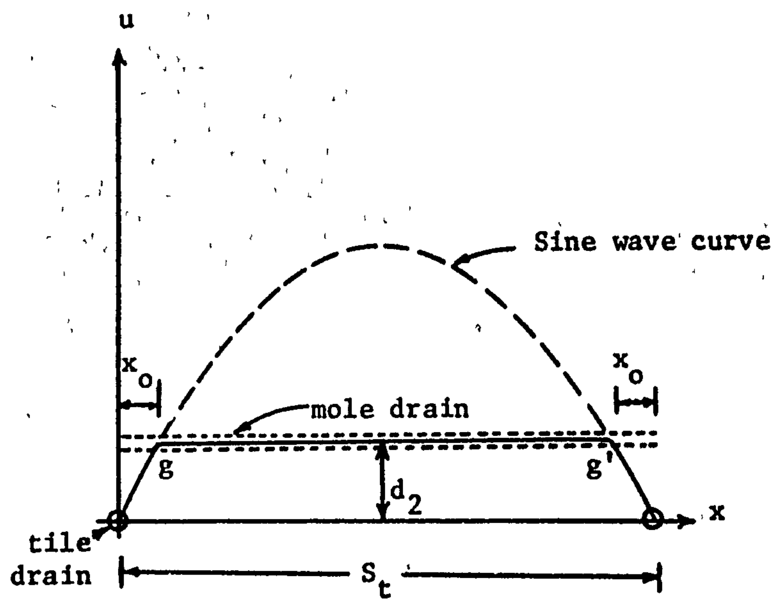

Figure 10. Water surface profile along the mole drain as a sine curve.

Figure 10 shows a sine curve intersecting the mole drain center line at "g" and "g'". The period of this curve is S_t. In order to find the equation of this curve and consequently that of the curves o - g and o' - g', consider the general sine wave equation:

$$u(x) = \delta \, \mathrm{Sin} \frac{\pi x}{S_t} \tag{35}$$

and the boundary conditions

$$u(0) = 0$$

$$u(S_t) = 0$$

$$u(x_o) = d_2$$

Solving equation (35) using the given boundary conditions:

$$u(x) = \frac{d_2}{\mathrm{Sin} \frac{\pi x_o}{S_t}} \mathrm{Sin} \frac{\pi x}{S_t} \tag{36}$$

Therefore

$$u(x) = \begin{cases} \delta \text{ Sin } \dfrac{\pi x}{S_t} & 0 \le x \le x_o \quad (37) \\ d_2 & x_o \le x \le c \quad (38) \\ \delta \text{ Sin } \dfrac{\pi x}{S_t} & c \le x \le S_t \quad (39) \end{cases}$$

where

$$\delta = \frac{d_2}{\text{Sin } \beta}$$

$$\beta = \frac{\pi x_o}{S_t}$$

$$c = S_t - x_o$$

It should be noted that in the cases of the First Degree Polynomial and the Sine Wave Equation there is a discontinuity at both points g and g'. This is impossible under practical conditions. However, it will not effect the analysis since the particular points g and g' will not be considered.

It should be pointed out here before proceeding with finding the solutions that assigning different algebraic and trigonometric expressions to the curves o - g and o' - g' without knowing the actual shape of these curves may not have that great an effect on the problem. Under actual field conditions, the distance d_2 between the mole drains and the tile drains is very small compared to the tile drain spacing S_t. Since it is also believed that the water travels in the mole drain a considerable distance before dropping from the mole drain level (i.e., x_o is small), the particular shape of the water surface profile along o - g and o' - g' will probably have little effect on the whole problem, especially if concentration is restricted to point "a" which represents the highest point on the water surface (Figure 7).

Solutions

The equations derived earlier describing the height of the water surface above the tile drains were general. They all had integrals containing f(x) which could be any one of the six cases assumed. By carrying out the integration for each case, a particular solution is obtained which is then evaluated at the particular point

$$x = \frac{S_t}{2},\ y = \frac{S_m}{2}$$

which represents the highest point on the water surface (see point "a", Figure 7) at any time t.

Stage I

General steps for obtaining the solution

The general steps for obtaining the solution for each case are as follows:

1. Obtain the algebraic or trigonometric expression of f(x) for the particular case considered from Equations (32) through (34) or Equations (37) through (39).

2. Substitute f(x) into Equation (19) and evaluate the integral to find A.

3. Substitute A into Equation (18) to find v(x, y).

4. Substitute A into Equation (24) to find χ.

5. Substitute χ into Equation (23) to find K_1.

6. Substitute K_1 into Equation (25) to find w(x, y, t).

7. Add w(x, y, t) to v(x, y) from Step 3 to obtain u(x, y, t), Equation (2).

8. Evaluate u(x, y, t) at the particular point $x = \frac{S_t}{2}$ and $y = \frac{S_m}{2}$.

If the term $\frac{1}{S_t^2}$ is neglected in the sum $(\frac{1}{S_m^2} + \frac{1}{S_t^2})$ while computing for B and w(x, y, t) the calculation work could be reduced and the form of the solution is simplified. The error introduced by doing so is not significant, considering the fact that in practice S_m = 2 m to 5 m and S_t = 30 m to 150 m.

Given below are the general solution u(x, y, t) and the solution evaluated at the mid-point $u(\frac{S_t}{2}, \frac{S_m}{2}, t)$ for each of the six cases considered.

Particular solutions

The following are the solutions of the six particular cases of Stage I. All the terms represented by Greek letters are listed in Table 1 (page 34).

Case 1 - Zero degree polynomial.

$$u(x, y, t) = \frac{16d_1}{\pi^2} \text{Sin} \frac{\pi x}{S_t} \text{Sin} \frac{\pi y}{S_m} e^{-\zeta t} + [\frac{8d_2}{\pi}] [\psi] \text{Cosh} \frac{\pi}{2S_t} \cdot (S_m - 2y) \text{ Sin} \frac{\pi x}{S_t} \qquad (40)$$

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = \frac{16d_1}{\pi^2} e^{-\zeta t} + [\frac{8d_2}{\pi}] [\psi] \qquad (41)$$

Case 2 - First degree polynomial.

$$u(x, y, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{4}{\pi} \frac{\text{Sin } \beta}{\beta})] \text{ Sin} \frac{\pi x}{S_t} \cdot \text{Sin} \frac{\pi y}{S_m} e^{-\zeta t} +$$

$$[2d_2]\ [\psi]\ [\frac{4}{\pi} \cdot \frac{\text{Sin } \beta}{\beta}]\ \text{Cosh } \frac{\pi}{2S_t} (S_m - 2y)\ \text{Sin } \frac{\pi x}{S_t} \cdot \tag{42}$$

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{4}{\pi} \frac{\text{Sin } \beta}{\beta})]e^{-\zeta t} + [2d_2]\ [\psi] \cdot$$
$$[\frac{4}{\pi} \cdot \frac{\text{Sin } \beta}{\beta}] \tag{43}$$

<u>Case 3 - Second degree polynomial.</u>

$$u(x, y, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2})]\ \text{Sin } \frac{\pi x}{S_t} \cdot$$
$$\text{Sin } \frac{\pi y}{S_m}\ e^{-\zeta t} + [2d_2]\ [\psi]\ [\frac{8}{\pi} \cdot \frac{1\text{-Cos } \beta}{\beta^2}]\ \text{Cosh } \frac{\pi}{2S_t} (S_m - 2y) \cdot$$
$$\text{Sin } \frac{\pi x}{S_t} \tag{44}$$

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2})]e^{-\zeta t} + [2d_2] \cdot$$
$$[\psi]\ [\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2}] \tag{45}$$

<u>Case 4 - Third degree polynomial.</u>

$$u(x, y, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\text{Sin } \beta}{\beta}))]\ \text{Sin } \frac{\pi x}{S_t} \cdot$$
$$\text{Sin } \frac{\pi y}{S_m} \cdot e^{-\zeta t} + [2d_2]\ [\psi]\ [\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\text{Sin } \beta}{\beta})]\ \text{Cosh } \frac{\pi}{2S_t} \cdot$$
$$(S_m - 2y) \cdot \text{Sin } \frac{\pi x}{S_t} \tag{46}$$

$$u(\frac{S_p}{2}, \frac{S_m}{2}, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\sin\beta}{\beta}))] e^{-\zeta t} + [2d_2] [\psi] [\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\sin\beta}{\beta})] \quad (47)$$

Case 5 - Fourth degree polynomial.

$$u(x, y, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \cos\beta)}{\beta^2}))] \cdot \sin\frac{\pi x}{S_t} \cdot \sin\frac{\pi y}{S_m} \cdot e^{-\zeta t} + [2d_2] [\psi] [\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \cos\beta)}{\beta^2})] \cdot \cosh\frac{\pi}{2S_t} (S_m - 2y) \sin\frac{\pi x}{S_t} \quad (48)$$

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \cos\beta)}{\beta^2}))] e^{-\zeta t} + [2d_2] [\psi] [\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \cos\beta)}{\beta^2})] \quad (49)$$

Case 6 - Sine wave equation.

$$u(x, y, t) = [\frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\frac{2}{\pi} \cdot (\frac{\beta}{\sin\beta} + \cos\beta))] \sin\frac{\pi x}{S_t} \cdot \sin\frac{\pi y}{S_m} \cdot e^{-\zeta t} + [2d_2] [\psi] [\frac{2}{\pi} \cdot (\frac{\beta}{\sin\beta} + \cos\beta)] \cdot \cosh\frac{\pi}{2S_t} \cdot (S_m - 2y) \sin\frac{\pi x}{S_t} \quad (50)$$

$$u\left(\frac{S_t}{2}, \frac{S_m}{2}, t\right) = \left[\frac{16(d_1 + d_2)}{\pi^2} - \left(\frac{4d_2}{\pi}\right)\left(\frac{2}{\pi} \cdot \left(\frac{\beta}{\sin\beta} + \cos\beta\right)\right)\right] e^{-\zeta t} + [2d_2]\,[\psi]\,\left[\frac{2}{\pi} \cdot \left(\frac{\beta}{\sin\beta} + \cos\beta\right)\right] \quad (51)$$

Table 1. List of the terms represented by Greek letters in the solutions, for both Stages I and II.

Greek Letter	Term
α (alpha)	$\frac{f}{kd_3}$
β (beta)	$\frac{\pi x_o}{S_t}$
δ (delta)	$\frac{d_2}{\sin\beta}$
ζ (zeta)	$\frac{\pi^2}{\alpha S_m^{\ 2}}$
ξ (xi)	$\frac{\pi S_m}{S_t}$
ϕ (phi)	$\frac{\pi^2}{\alpha S_t^{\ 2}}$
ψ (psi)	$\frac{\sinh\frac{\xi}{2}}{\sinh\xi}$

Stage II

General steps for obtaining the solution

The general steps for obtaining the solution for each case are as follows:

1. Obtain the algebraic or trigonometric expression of f(x) for the particular case considered from Equations (32) through (34) or Equations (37) through (39).

2. Substitute f(x) into Equation (19) to obtain A.

3. Substitute A into Equation (30) to obtain E.

4. Substitute E into Equation (29) to obtain u(x, t).

5. Evaluate u(x, t) at the particular point $x = \frac{S_t}{2}$ which lies mid-way between the tile drains.

Similar to Stage I, u(x, t) at the particular point $x = \frac{S_t}{2}$ obtained from Step 5 is of most importance since it represents the highest point on the water surface during Stage II.

Given below are the values of u(x, t) and $u(\frac{S_t}{2}, t)$ for each of the six cases considered.

Particular solutions

The solutions for the six particular cases of Stage II are as follows. The terms represented by Greek letters are listed in Table 1.

Case 1 - Zero degree polynomial.

$$u(x, t) = [d_2] \, [\tfrac{4}{\pi}] \, \mathrm{Sin} \, \frac{\pi x}{S_t} \, e^{-\phi t} \tag{52}$$

$$u(\frac{S_t}{2}, t) = [d_2] \, [\tfrac{4}{\pi}] \, e^{-\phi t} \tag{53}$$

Case 2 - First degree polynomial.

$$u(x, t) = [d_2] \, [\tfrac{4}{\pi} \cdot \frac{\mathrm{Sin}\,\beta}{\beta}] \, \mathrm{Sin} \, \frac{\pi x}{S_t} \, e^{-\phi t} \tag{54}$$

$$u(\frac{S_t}{2}, t) = [d_2] \, [\tfrac{4}{\pi} \cdot \frac{\mathrm{Sin}\,\beta}{\beta}] \, e^{-\phi t} \tag{55}$$

Case 3 - Second degree polynomial.

$$u(x, t) = [d_2] \left[\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2}\right] \text{Sin } \frac{\pi x}{S_t} e^{-\phi t} \tag{56}$$

$$u(\frac{S_t}{2}, t) = [d_2] \left[\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2}\right] e^{-\phi t} \tag{57}$$

Case 4 - Third degree polynomial.

$$u(x, t) = [d_2] \left[\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\text{Sin } \beta}{\beta})\right] \text{Sin } \frac{\pi x}{S_t} e^{-\phi t} \tag{58}$$

$$u(\frac{S_t}{2}, t) = [d_2] \left[\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\text{Sin } \beta}{\beta})\right] e^{-\phi t} \tag{59}$$

Case 5 - Fourth degree polynomial.

$$u(x, t) = [d_2] \left[\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \text{Cos } \beta)}{\beta^2})\right] \text{Sin } \frac{\pi x}{S_t} e^{-\phi t} \tag{60}$$

$$u(\frac{S_t}{2}, t) = [d_2] \left[\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \text{Cos } \beta)}{\beta^2})\right] e^{-\phi t} \tag{61}$$

Case 6 - Sine wave equation.

$$u(x, t = [d_2] \left[\frac{2}{\pi} \cdot (\frac{\beta}{\text{Sin } \beta} + \text{Cos } \beta)\right] \text{Sin } \frac{\pi x}{S_t} e^{-\phi t} \tag{62}$$

$$u(\frac{S_t}{2}, t) = [d_2] \left[\frac{2}{\pi} \cdot (\frac{\beta}{\text{Sin } \beta} + \text{Cos } \beta)\right] e^{-\phi t} \tag{63}$$

Relationship Between the Solutions

It is interesting to note that the particular solutions derived in the previous section for each stage, either the general equations or the equations evaluated at the mid-point of the system are very much related to each other. In fact, these solutions differ from one another by a constant only, as will be shown below.

Stage I - General form of the solution evaluated at the mid-point of the system

The general form may be expressed as:

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = K_1 e^{-\zeta t} + K_2 \tag{64}$$

where

$$K_1 = \frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\chi) \tag{23}$$

$$K_2 = (2d_2) (\psi) (\chi) \tag{65}$$

The value of χ, a constant, in Equations (23) and (65) depends on the particular case under consideration. Table 2 lists the values of χ for the six particular cases considered.

Table 2. Values of χ for the six particular cases for both Stage I and Stage II.

Case	χ
Case 1 - Zero Degree Polynomial	$\frac{4}{\pi}$
Case 2 - First Degree Polynomial	$\frac{4}{\pi} \cdot \frac{\text{Sin } \beta}{\beta}$
Case 3 - Second Degree Polynomial	$\frac{8}{\pi} \cdot \frac{1 - \text{Cos } \beta}{\beta^2}$
Case 4 - Third Degree Polynomial	$\frac{24}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{\text{Sin } \beta}{\beta})$
Case 5 - Fourth Degree Polynomial	$\frac{48}{\pi} \cdot \frac{1}{\beta^2} \cdot (1 - \frac{2(1 - \text{Cos } \beta)}{\beta^2})$
Case 6 - Sine Wave Equation	$\frac{2}{\pi} \cdot (\frac{\beta}{\text{Sin } \beta} + \text{Cos } \beta)$

By re-arranging the terms, Equation (64) may be transformed into the following form:

$$S_m = \sqrt{\frac{\pi^2 t}{\alpha \ln\left(\frac{K_1}{u - K_2}\right)}} \tag{66}$$

The spacing of the mole drains, S_m, may be computed by substituting K_1 and K_2 into Equation (66). Values of K_1 and K_2 are computed from Equations (23) and (65), respectively, using the appropriate χ.

<u>Stage II - General form of the solution evaluated at the mid-point of the system</u>

Proceeding in a manner similar to Stage I, the general form of the solution is:

$$u\left(\frac{S_t}{2}, \frac{S_m}{2}, t\right) = \chi d_2 \, e^{-\phi t} \tag{67}$$

The terms in Equation (67) may be re-arranged in the following form:

$$S_t = \sqrt{\frac{\pi^2 t}{\alpha \ln\left(\frac{\chi d_2}{u}\right)}} \tag{68}$$

Where the value of χ for the particular case considered is given in Table 2. Therefore, by substituting the particular value of χ into Equation (68), it is possible to obtain the spacing of the tile drains, S_t.

FIELD EXPERIMENT

The experiment was carried out during the summer of 1972, on the Utah State University Drainage Farm located northwest of Logan, Utah.

General Field Layout

The general field layout is shown in Figure 11. Four 240 ft. long perforated plastic drains, 4 in. in diameter, were laid out parallel to each other in an east-west direction at a 120 ft. spacing. Each drain was laid in a trench about 3 ft. deep at about 6 in. above the bottom of the trench. The width of each trench was 1.5 ft. The trench was then filled with very permeable graded gravel to a level about 1 ft. below the ground surface. Finally, the trench was filled to the ground surface with top soil (Figure 12). All the drains discharge into an open ditch drain located on the east side of the field.

Next, ten single mole drains and ten double mole drains (Unhanand, 1972) were drawn across the field above and orthogonal to the tile drains in a north-south direction (Figure 11). The moling process was done using special mole plows mounted on a tractor. The mole drains are 3 in. in diameter, drawn at a 6 ft. spacing, and at a depth of about 22 in. below the ground surface (Figure 12).

Then, five waterproof manholes, each 4 ft. in diameter and 6 ft. deep, were constructed in the locations shown in Figure 11. The walls of the manhole were made with corrugated metal sheets and the bottom sealed with a concrete slab to prevent any seepage of water.

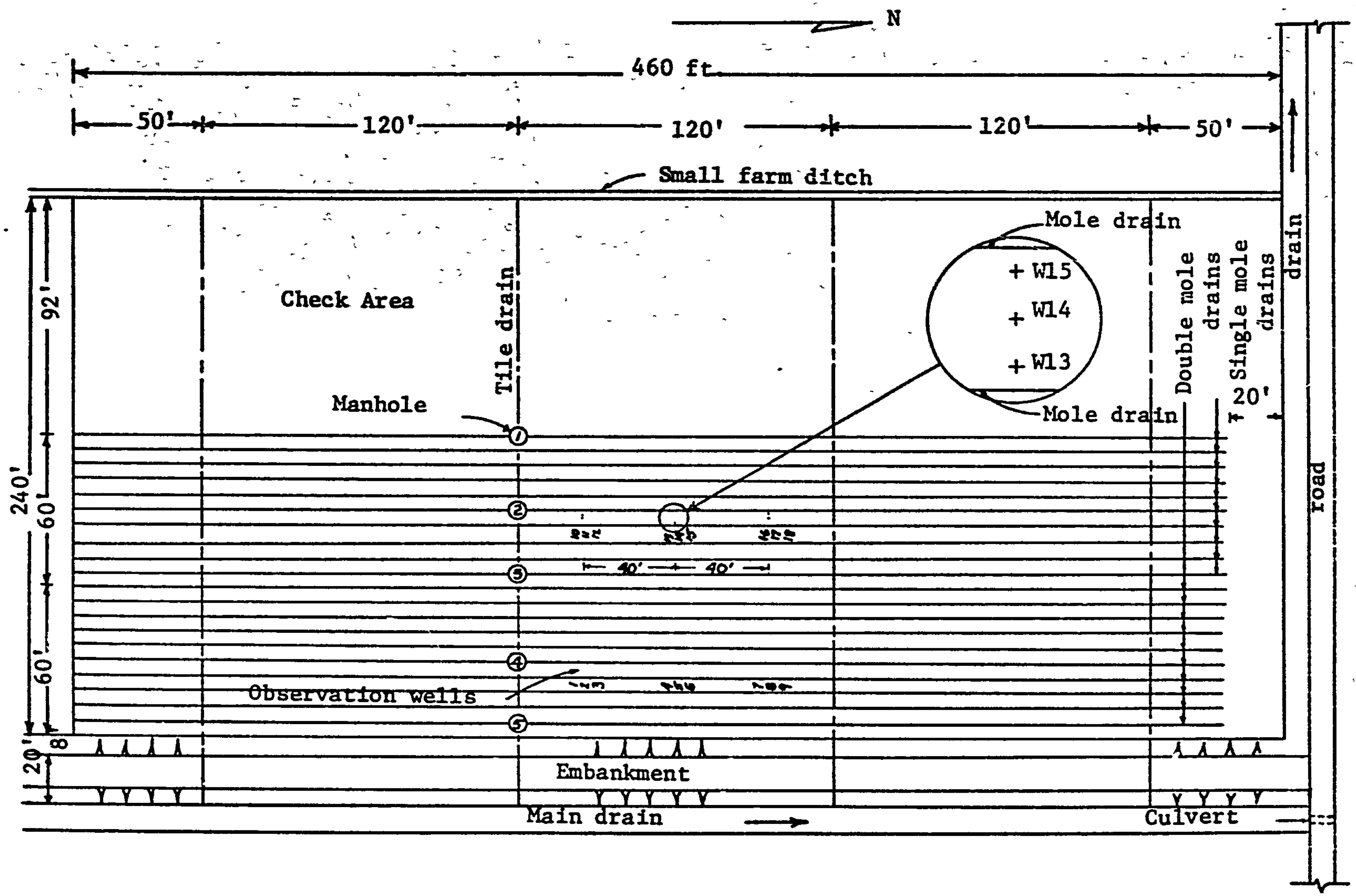

Figure 11. General field layout of the combined mole-tile drain system for the field experiment.

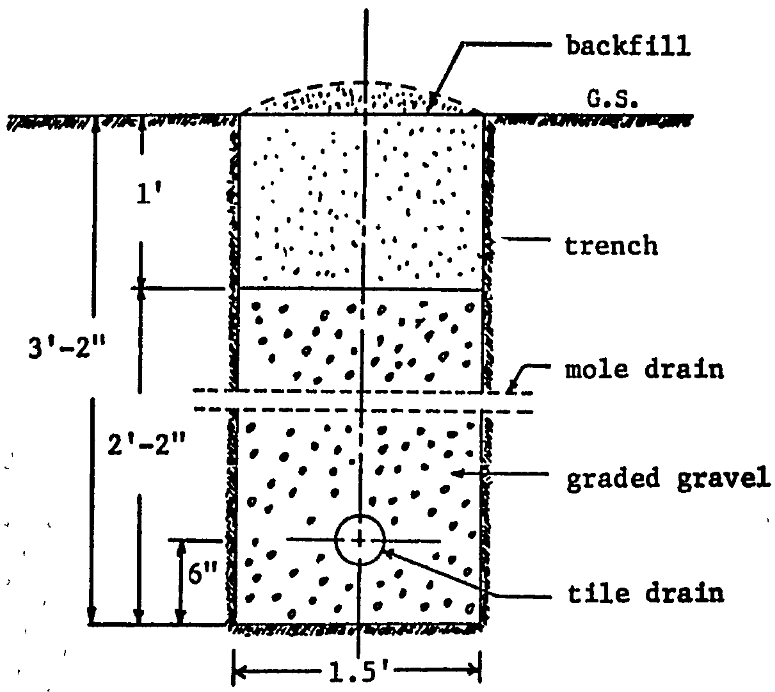

Figure 12. Tile drain trench design.

In order to be able to measure the water surface elevation, 18 observation wells were installed; nine in the double mole drains area and nine in the single mole drains area (Figure 11). Each well was made by drilling a 4 in. auger hole 4 ft. deep and then placing a 1 in. perforated plastic pipe about 5 ft. long in the center of the augered hole and filling the gap with graded gravel.

Procedure

1. A sprinkler irrigation system was installed on the field to be used in building up the ground water table.

2. All drain outlets leading into the manholes and those located along the open ditch drain were closed.

3. The sprinkler system was turned on and allowed to run until the ground water table in the field reached the ground surface. Then

the sprinkler system was turned off. A short period was allowed for the water to redistribute itself into a flat shaped water surface before the experiment was started.

4. Depth of the water surface in each observation well was measured at the outset of the experiment.

5. All drain outlets were opened simultaneously. A short period was allowed for the water accumulated in the drains to flush out.

6. Successive measurements were taken of the water surface elevation in each observation well along with the corresponding time each measurement was taken. The period between two successive readings was increased with time because of the decrease of the rate of recession of the water surface.

7. Data were collected from September 21 to September 27, 1972, and then stopped because of rain.

Data Collected

The physical parameters measured in the field which will be used in the study of the comparison between the theoretical solutions and the field data are as follows (see Figure 7 for the notation):

S_m = 6.0 ft.

S_t = 120.0 ft.

d_1 = 1.75 ft. (= height of ground surface above mole drain at t = 0)

d_2 = 1.02 ft.

d_3 = 3.23 ft.

k = 0.74 ft./day (hydraulic conductivity)

f = 0.045 (specific yield)

a = 3 in. (diameter of mole drains)

The specific yield f shown above was not measured in the field but was obtained from a relationship between specific yield and hydraulic conductivity (Dumm, 1968).

Tables 7, 8, 9 and 10 (Appendix E) contain the values of height of the water surface above the tile drains in each observation well along with the time each measurement was taken.

COMPARISON OF THE THEORY TO THE FIELD EXPERIMENT

Probably the best way to compare the theory developed with the data collected from the field experiment is to investigate the validity of Equation (66) which expresses the spacing of the mole drains, S_m, in an implicit form, and Equation (68) which expresses the spacing of the tile drains, S_t, also in an implicit form. However, since the data given in Tables 8-11, Appendix E, do not include values when the water surface dropped below the mole drains, only Equation (66) can be investigated.

The original form of Equation (66) is Equation (64) which is given below:

$$u(\frac{S_t}{2}, \frac{S_m}{2}, t) = K_1 e^{-\zeta t} + K_2 \tag{64}$$

where

$$K_1 = \frac{16(d_1 + d_2)}{\pi^2} - (\frac{4d_2}{\pi}) (\chi) \tag{23}$$

$$K_2 = (2d_2) (\psi) (\chi) \tag{65}$$

Equation (64) represents the height of the water surface u evaluated at the mid-point of the system (point "a" in Figure 7) as a function of time. Transferring K_2 in Equation (64) to the left-hand side and taking the natural logarithms of both sides,

$$\ln Y = b_0 + b_1 X \tag{69}$$

where

$Y = u - K_2$

$X = t$

b_0 and b_1 are constants

Equations (69) represents the statistical model of Equation (64). By substituting values for X and Y into Equation (69) and then running a regression analysis, it is possible to obtain a regression coefficient r which represents to what degree the data fit the model.

The observation well "W14" (Figure 11) is located at the mid-point of the single mole drains area, and therefore identical to point "a" in Figure 7. For the simplicity of computations, the first case, Zero Degree Polynomial, will be investigated. The terms for χ and ψ, and the field values of S_m, S_t, and d_2 are substituted into Equation (65) to get K_2:

$$K_2 = (2d_2)\ (\psi)\ (\chi) \tag{65}$$

$$= (2 \cdot 1.02)\ \left(\frac{\text{Sinh}\ \frac{\pi \cdot 6}{2 \cdot 120}}{\text{Sinh}\ \frac{\pi \cdot 6}{120}}\right)\ \left(\frac{4}{\pi}\right)$$

$$= 1.30 \text{ ft.}$$

Table 3 gives the values of u, u − K_2 (i.e., Y), and t(i.e., X) for observation well W14.

Using the statistical model, Equation (69), and running a computerized regression analysis (Snedecor and Cochran, 1972) using the data of Table 3, the following results were obtained:

$b_0 = 1.108$

$b_1 = -0.813$

$r^2 = 0.890$

Table 3. Values of u, Y, X for observation well W14.

u (ft.)	Y (ft.)	X (days)
2.920	1.620	0.094
2.910	1.610	0.226
2.890	1.590	0.388
2.860	1.560	0.640
2.840	1.540	1.059
2.750	1.450	1.194
2.610	1.310	1.381
2.460	1.160	1.963
1.900	0.600	2.407
1.680	0.380	3.211
1.550	0.250	3.374
1.470	0.170	4.067
1.350	0.050	4.391
1.320	0.020	5.076
1.290	(-)*	5.366

*neglect

The above r^2 gives a regression coefficient $r = 0.940$, which indicates that the field data agree very well with the general form of the solution of Equation (64) for the particular case of the Zero Degree Polynomial.

Recalling that the derivation of the solutions involved terminating an infinite series, it is only logical to assume that K_1 and K_2 which appear on the right-hand side of Equation (64) will affect the solution.

If time was allowed to approach infinity in Equation (64), the first term on the right-hand side of the equation approaches zero. The second term, K_2, in Equation (64) should, theoretically speaking, equal the value of u at $t = \infty$ which is d_2 (i.e., $u\,(\frac{S_t}{2}, \frac{S_m}{2}, \infty) = d_2$). A regression analysis was made similar to the one above except that K_2 was now taken as d_2. Table 4 shows the values of u, Y (i.e., $u - d_2$), and X (i.e., t).

Table 4. Values of u, adjusted Y, X for observation well W14.

u (ft.)	Y (ft.)	X (days)
2.920	1.900	0.094
2.910	1.890	0.226
2.890	1.870	0.388
2.860	1.840	0.640
2.840	1.820	1.059
2.750	1.730	1.194
2.610	1.590	1.381
2.460	1.440	1.963
1.900	0.880	2.407
1.680	0.660	3.211
1.550	0.530	3.374
1.470	0.450	4.067
1.350	0.330	4.391
1.320	0.300	5.076
1.290	0.270	5.366

Running the regression analysis again using the data in Table 4, the following results were obtained:

$b_0 = 0.887$

$b_1 = -0.414$

$r^2 = 0.967$

The above r^2 gives a regression coefficient $r = 0.98$. This value is larger than the corresponding one for the previous case (i.e., larger than 0.89), thus indicating a stronger correlation between the field data and the statistical model.

Since a strong correlation was found between the field data and the statistical model as expressed in Equation (69), the next logical step is to investigate Equation (66) itself. The spacing of the mole drains, S_m, is expressed implicitly in Equation (66):

$$S_m = \sqrt{\frac{\pi^2 t}{\alpha \ln\left(\frac{K_1}{u - K_2}\right)}} \tag{66}$$

in which the right-hand side of Equation (66) also contains S_m as a variable in the term K_2

$$K_2 = (2d_2)\ (\psi)\ (\chi) \tag{65}$$

where, in this case

$$\chi = \frac{4}{\pi} \quad \text{and} \quad \psi = \frac{\mathrm{Sinh}\,\frac{\pi S_m}{2S_t}}{\mathrm{Sinh}\,\frac{\pi S_m}{S_t}}$$

However, Equation (66) can be solved numerically for S_m.

First, the field values of d_1, d_2, d_3, S_t, k and f were substituted into Equation (66). Then three values of u and t were selected to cover the whole range of the field data. Equation (66) was then solved numerically for S_m corresponding to the values of u and t using the Fixed Point Iteration Technique (Stark, 1970). The results are shown in Table 5.

The basic differential equation, Equation (1), does not account for the convergence of flow at the drains. In order to compensate for this, Hooghoudt's equivalent depth is usually used to replace the actual depth.

to the impermeable layer (Moody, 1966). In the combined system, however, there are two sets of orthogonal drains, and since the exact nature of the convergence of flow at the drains is not known yet, it is not possible to apply Hooghoudt's equivalent depth concept directly.

Table 5. Results of solving Equation (66) numerically for the case of the Zero Degree Polynomial.

u (ft.)	t (days)	S_m (ft.)
2.860	0.640	24.557
1.90	2.407	29.963
1.29	5.366	27.696

Given below is a procedure, although not exact, whereby Hooghoudt's equivalent depth concept is applied to correct the spacing of the mole drains in the combined system.

The first step is to neglect the presence of the tile drains and assume that the system is composed of mole drains only, with all the physical parameters of the field remaining the same. The next step is to transform this system of mole drains which is under transient state conditions, for any time t, to a system of mole drains under steady state conditions (subjected to a constant rainfall rate) using Hooghoudt's steady state equation (Luthin, 1973).

$$S_m^{\ 2} = \frac{4kH}{v} (H + 2d) \tag{70}$$

where

S_m = theoretical spacing of mole drains obtained from Equation (66)

k = hydraulic conductivity

v = constant rate of rainfall

$H = u - d_2$

$d = d_2 + d_3$

d_2 and d_3 are as defined previously (Figure 7)

Replacing S_m by the corrected spacing S_{mc} and d by the equivalent depth d_e, Equation (70) may be written as:

$$S_{mc}^{\ 2} = \frac{4kH}{v} (H + 2d_e) \tag{71}$$

where the equivalent depth d_e may be expressed as (Moody, 1966):

$$d_e = \frac{d}{1 + \frac{d}{S_{mc}} \left(\frac{8}{\pi} \ln \frac{d}{a} - \alpha'\right)} \qquad 0 \leq \frac{d}{S_{mc}} \leq 0.3 \tag{72}$$

$$\alpha' = 3.55 - 1.6 \frac{d}{S_{mc}} + 2\left(\frac{d}{S_{mc}}\right)^2 \tag{73}$$

a = diameter of mole drain

Equations (70), (71), (72) and (73) are then used as follows:

1. Obtain the theoretical spacing S_m from Equation (66).

2. Substitute in the values of S_m, d and H into Equation (70) to obtain the value of the constant ratio $\frac{k}{v}$.

3. Assume S_{mc} and find α' from Equation (73).

4. Substitute α' into Equation (72) to obtain d_e.

5. Substitute d_e into Equation (71) to obtain a corrected spacing S_{mc}.

6. Substitute S_{mc} (Step 5) into Equation (73) and repeat Steps 3 through 6 until both values of S_{mc} are equal.

It is clear that Steps 3 through 6 is a trial-and-error procedure. These steps can be combined in a form such that S_{mc} can be obtained numerically. Equations (71), (72) and (73) can be combined into

$$S_{mc} = \sqrt{(4H)\left(\frac{k}{v}\right)\left(H + \frac{2d}{1 + \frac{d}{S_{mc}}\left[\frac{8}{\pi}\ln\frac{d}{a} - 3.55 + \frac{1.6d}{S_{mc}} - 2\left(\frac{d}{S_{mc}}\right)^2\right]}\right)} \quad (74)$$

Equation (74) can be solved quite easily numerically using the Fixed Point Iteration Technique (Stark, 1970).

The above procedure waf used to correct the theoretical values of S_m (Table 5). The results are shown in Table 6.

Table 6. Spacing of the mole drains corrected for convergence of flow.

S_m (ft.)	S_{mc} (ft.)	S_{mc}/S_m (%)
24.557	17.818	72.56
29.963	21.671	72.33
27.696	18.633	67.28

The above procedure may also be used to correct the spacing of the tile drains in the combined system.

RESULTS AND DISCUSSION

The equations derived for the spacing of the mole drains S_m and the spacing of the tile drains S_t are:

$$S_m = \sqrt{\frac{\pi^2 t}{\alpha \ln\left(\frac{K_1}{u - K_2}\right)}} \tag{66}$$

$$S_t = \sqrt{\frac{\pi^2 t}{\alpha \ln\left(\frac{\chi d_2}{u}\right)}} \tag{68}$$

where

$$K_1 = \frac{16(d_1 + d_2)}{\pi^2} - \left(\frac{4d_2}{\pi}\right)(\chi) \tag{23}$$

$$K_2 = (2d_2)\ (\psi)\ (\chi) \tag{65}$$

The terms represented by the Greek letters in the above equations are listed in Table 1. The value of χ, depending on the particular case considered, is given in Table 2. It should be emphasized once again that t in Equation (68) is initialized (i.e., t = 0) at the outset of Stage II.

The equation for the spacing of the mole drains S_m, Equation (66), was compared with the field data collected. The case of the Zero Degree Polynomial or Case 1, being the simplest of all cases, was investigated. A very good correlation between the field data and the statistical model of the equation (69) was obtained. However, when the field data were substituted into Equation (66) directly, and S_m

was obtained numerically (Table 5), it over estimated the actual field spacing of the mole drains by about four to five times.

By applying the concept of Hooghoudt's equivalent depth in an approximate way, it was possible to reduce the spacings of the mole drains obtained from Equation (66) by about 30%.

The above mentioned overestimation can be attributed to one or more of the following:

1. Sensitivity of the terms of Equation (66) such that any inaccuracy on the part of the field data could result in an over-estimation or under-estimation of S_m.

2. The error introduced by terminating the infinite series during the derivation of the equation and restricting it to the first term only.

3. The effect of the convergence of flow at the drains.

It should be noted here that the value of the spacing of the tile drains, S_t, in Equation (66) was not taken from Equation (68) but was taken as being equal to the actual field value.

For the remaining cases of Equation (66), correlation with the field data was not possible because the new term contains x_o which depends on Equation (68). In order to obtain the spacing of the tile drains S_t from Equation (68) x_o would have to be assumed beforehand. The two (i.e., S_t and x_o) would then be substituted into Equation (66) to obtain the spacing of the mole drains S_m. Since the field data for Stage II is not available, and since Equation (68) depends on Stage II (i.e., water surface below the mole drains but above the tile drains), the correlation of Equation (68) to field data was not possible.

FIELD DESIGN PROCEDURE

The following is a suggested procedure whereby the theoretical equations may be used in finding the spacings of the mole drains and the tile drains in a combined system once the depths of these drains below the ground surface have been established.

1. Assume the profile of the water surface along the mole drains (i.e., Case 1 through Case 6).

2. Assume x_o ($x_o \leq \frac{S_t}{2}$). For the case of the Zero Degree Polynomial $x_o = 0$.

3. Find the value of χ in terms of S_t (see Table 2). For the case of the Zero Degree Polynomial χ is a constant.

4. Solve Equation (68) numerically for the spacing of the tile drains S_t. Note that the time t in the equation starts when the water surface begins to drop from the mole drain elevation and not the time when the intitial water surface starts to recede. If $x_o > \frac{S_t}{2}$, repeat Steps 2 through 4. This does not apply to the case of the Zero Degree Polynomial, because for this case $x_o = 0$.

5. Correct S_t for the convergence of flow neglecting the presence of the mole drains.

6. Find the value of χ (Step 3) after substituting in the value of S_t obtained in Step 5. For the case of the Zero Degree Polynomial χ remains the same.

7. Solve Equation (66) numerically for the spacing of the mole drains S_m. The term t in the equation represents the time of drop of the water surface from its initial position.

8. Correct S_m for convergence of flow neglecting the presence of the tile drains.

SUMMARY AND CONCLUSIONS

A theory was presented to predict the drop of the water surface in a combined mole-tile drain system.

Based on several assumptions, general equations for the spacing of the mole drains and the spacing of the tile drains in the combined system were derived.

Six water surface profiles along the mole drains were assumed, and the particular solutions for the spacing of the mole drains and the spacing of the tile drains in the combined system were derived.

The equation expressing the spacing of the mole drains for a particular water surface profile along the mole drains was compared with data from a field experiment.

A procedure whereby the equations derived could be used in finding the field spacings of the mole drains ard the tile drains was also presented.

In conclusion, the theory which has been presented is only a first step in the design of combined systems. Extensive investigations are necessary, both on the equations derived and on the nature of water flow in a combined system, before it may be possible to apply the theory to actual field design problems.

SUGGESTIONS FOR FURTHER STUDY

1. Extensive field studies are necessary to determine the applicability of the theoretical equations derived.

2. Field investigations are necessary to determine the shape of the water surface profile along the mole drains and its variability with time.

3. The effect of x_o in the theoretical equations should be studied more extensively.

4. An economic feasibility study is necessary to determine the cost of installing a combined system as compared to other covered drain systems in heavy soils.

BIBLIOGRAPHY

Carslaw, H. S., and J. C. Jaeger. 1959. Conduction of heat in solids. Second edition. Oxford University Press. Cambridge, England.

Donnan, William W., and James E. Fouss. 1962. Plastic-lined mole drains. Agricultural Engineering 43:512-515.

Donnan, William W., and Clyde E. Houston. 1967. Drainage related to irrigation management. In R. M. Hagan, H. R. Haise, and T. W. Edminster, eds., Irrigation of agricultural lands, Agronomy II. American Society of Agronomy. Madison, Wisconsin. p. 974-987.

Dumm, Lee D. 1954. Drain spacing formula. Agricultural Engineering 35:726-730.

Dumm, Lee D. 1964. Transient flow concept of subsurface drainage. Transactions of the American Society of Agricultural Engineers 7:142-146, 151.

Dupuit, Jules. 1863. Etudes theoriques et pratiques sur le mouvement des eaux. Second edition. Dunod, Paris.

Food and Agricultural Organization of the United Nations. 1971. Drainage of heavy soils. Rome, Italy.

Forchheimer, Philipp. 1930. Hydraulik. Third edition. Teubner, Leipzig and Berlin.

Kreider, Donald L., Robert G. Kuller, Donald R. Ostberg, and Fred W. Perkins. 1966. An introduction to linear analysis. Addison-Wesley Publication Company, Inc. Reading, Massachusetts. 773 p.

Longwell, Paul A. 1966. Mechanics of fluid flow. McGraw-Hill Book Company, Inc. New York, New York. 433 p.

Luthin, James N. (Ed.) 1957. Drainage of agricultural lands, Agronomy 7. American Society of Agronomy. Madison, Wisconsin. 620 p.

Luthin, James N. 1973. Drainage engineering. Robert E. Krieger Publishing Co., Inc. Huntington, New York. 250 p.

Moody, William T. 1966. Nonlinear differential equation of drain spacing. Journal of the Irrigation and Drainage Division, ASCE, Vol. 92, No. IR2, June, 1966.

Powers, David L. 1972. Boundary value problems. Academic Press. New York, New York. 238 p.

Snedecor, George W., and William G. Cochran. 1972. Statistical methods. Sixth Edition. The Iowa State University Press. Ames, Iowa. 593 p.

Soil Conservation Service. 1973. Drainage of agricultural land. Water Information Center, Inc. Port Washington, New York. 430 p.

Stark, Peter A. 1970. Introduction to numerical methods. The Mac-Millan Company. New York, New York. 334 p.

Theobald, G. H. 1963. Methods and machines for tile and other tube drainage. Food and Agricultural Organization of the United Nations. Rome, Italy. 104 p.

Tomita, Masahiko. 1971. On enlargement of the object of analysis with digital computer in ground-water movement. Scientific Reports of Shiga Prefectural Junion College, No. 12. Japan.

Tomita, Masahiko, Kengo Nagahama, Sanji Tejima, and Hiroshi Taniguchi. 1968. On the underdrainage of reclaimed land from the sea - The substantial study on the mechanism of underdrainage (III). Transactions of JJIDRE. Japan.

Unhanand, Komain. 1972. Irrigation and drainage by mole systems. Progress report. Agricultural and Irrigation Engineering Department, Utah State University, Logan, Utah. 52 p.

Van Schilfgaarde, J., D. Kirkham, and R. K. Frevert. 1956. Physical and mathematical theories of tile and ditch drainage and their usefulness in design. Research Bulletin 436. Agricultural Experiment Station, Iowa State College, Ames, Iowa. February.

APPENDIXES

Appendix A

Derivation of the Continuity Equation

Appendix A

Derivation of the Continuity Equation

(Transient State Condition)

The derivation of the continuity equation which governs the height of the water surface in the soil based on the Dupuit-Forchheimer theory (Dupuit, 1863 and Forchheimer, 1930) is well known and can be found in different literature (Luthin, 1957 and Van Schilfgaarde, Kirkham and Frevert, 1956). However, a more elaborate derivation based on (Longwell, 1966) is given below.

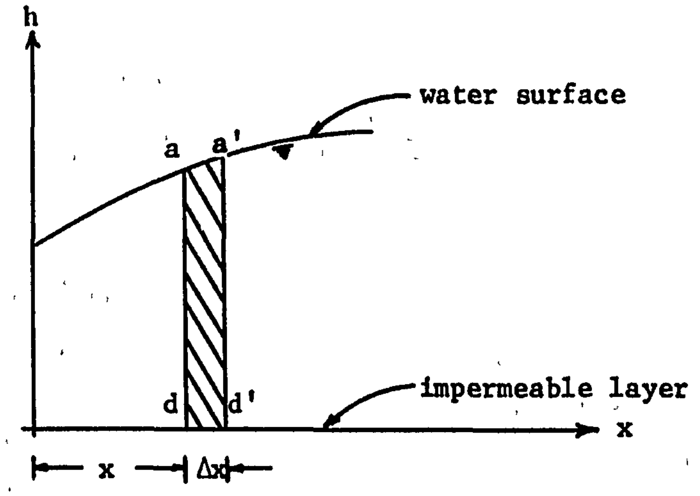

Figure 13. Groundwater flow system in dynamic equilibrium.

Figure 13 shows a two dimensional sketch of the water table overlying an impermeable soil layer. Consider a saturated soil column aa'dd' lying between the water table and the impermeable layer. Figure 14 shows the soil column in three dimensions with the base cc'dd' lying on the impermeable layer and a surface top aa'bb'.

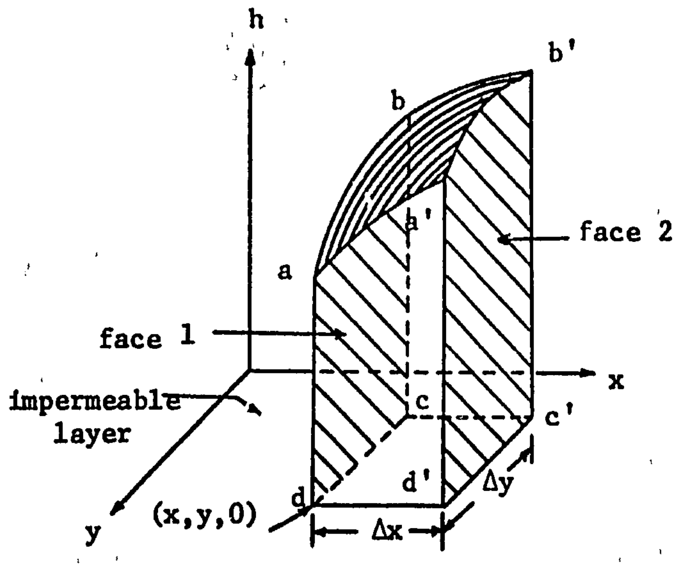

Figure 14. Flow of water through the soil column.

The basic assumptions of the Dupuit-Forchheimer theory state that:

1. All streamlines of gravity flow towards a shallow sink are horizontal.

2. The velocity along these streamlines is proportional to the slope of the free water surface, but independent of depth.

Therefore, based on the first assumption, the water will flow in the x-direction and the y-direction only (i.e., no flow in the h-direction).

The law of conservation of mass states that "material is neither created nor destroyed" and can be expressed simply as:

$$\text{In} - \text{Out} + \text{Source} - \text{Sink} = \text{accumulation} \tag{75}$$

Let

U_x = velocity in the x-direction

U_y = velocity in the y-direction

The water flux in the x-direction is:

$\dot{M}_x = \sigma U_x$

where

σ is the specific weight; unit weight per unit volume

The amount of water crossing face 1 (Figure 14) per unit time in the x-direction is:

$$(\dot{M}_x \; \Delta A)_1 = \dot{M}_x \; U_x \; \Delta y = \sigma U_x \; h \Delta y$$

Also, the amount of water crossing face 2 per unit time is:

$$(\dot{M}_x \; \Delta A)_2 = \sigma U_x \Delta h \; y + \frac{\partial}{\partial x} \; (\sigma U_x h) \; \Delta y \Delta x$$

$$(\text{In} - \text{Out})_x = (\dot{M}_x \; \Delta A)_1 - (\dot{M}_x \; \Delta A)_2 = -\frac{\partial}{\partial x} \; (\sigma U_x h) \; \Delta y \Delta x$$

Similarly in the y-direction:

$$(\text{In} - \text{Out})_y = -\frac{\partial}{\partial y} \; (\sigma U_y h) \; \Delta x \Delta y$$

Amount of water in the soil column = $f \sigma h \Delta y \Delta x$

where

f is the specific yield of the soil

$$\text{Rate of accumulation} = \frac{\partial}{\partial t} \; (f \sigma h \Delta y \Delta x)$$

$$= \frac{\partial}{\partial t} \; (\sigma f h) \; \Delta y \Delta x$$

Since there is no source or sink, and since the flow is incompressible (i.e., σ = constant), substituting into Equation (75):

$$-\frac{\partial}{\partial x} \; (U_x h) \; \sigma y \Delta x - \frac{\partial}{\partial y} \; (U_y h) \; \sigma \Delta x \Delta y = f \sigma \Delta x \Delta y \frac{\partial h}{\partial t}$$

$$-\frac{\partial}{\partial x} \; (hU_x) - \frac{\partial}{\partial y} \; (hU_y) = f \frac{\partial h}{\partial t} \tag{76}$$

From the second Dupuit-Forchheimer assumption:

$$U_x = -K_x \frac{\partial h}{\partial x} \qquad U_y = -K_y \frac{\partial h}{\partial y}$$

where

K_x = hydraulic conductivity in the x-direction

K_y = hydraulic conductivity in the y-direction

Substituting into Equation (76):

$$-\frac{\partial}{\partial x}\left(-K_x h \frac{\partial h}{\partial x}\right) - \frac{\partial}{\partial y}\left(-K_y h \frac{\partial h}{\partial y}\right) = f \frac{\partial h}{\partial t}$$

$$\frac{\partial}{\partial x}\left(K_x h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(K_y h \frac{\partial h}{\partial y}\right) = f \frac{\partial h}{\partial t} \tag{77}$$

For a homogeneous and isotropic soil, $K_x = K_y = k$, a constant. Therefore, Equation (77) reduces to:

$$\frac{\partial}{\partial h}\left(h \frac{\partial h}{\partial x}\right) + \frac{\partial}{\partial y}\left(h \frac{\partial h}{\partial y}\right) = \frac{f}{k}\frac{\partial h}{\partial t} \tag{78}$$

Equation (78) is a non-linear, second order partial differential equation which can be linearized as shown below.

Figure (15) shows two cartesian reference frames x-y-h and x-y-u. Translation from the first to the second is quite simple. The x and y directions remain the same. Now letting

$$h = d_3 + u$$

where d_3 is the distance between the x-y planes in both frames, and assuming that d_3 is much larger numerically than values of u measured from the x-y-u frame of reference, then

$$h \simeq d_3$$

and

$$\frac{\partial h}{\partial t} = \frac{\partial u}{\partial t}, \frac{\partial h}{\partial x} = \frac{\partial u}{\partial x}, \frac{\partial h}{\partial y} = \frac{\partial u}{\partial y}$$

Therefore, Equation (78) reduces to:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x^2} = \alpha \frac{\partial u}{\partial t} \tag{79}$$

where

$$\alpha = \frac{f}{kd_3} \quad \text{(constant)}$$

Equation (79) is identical in form to the two-dimensional heat flow equation (Carslaw and Jaeger, 1959), and therefore, the techniques used in solving that type of equation will be used.

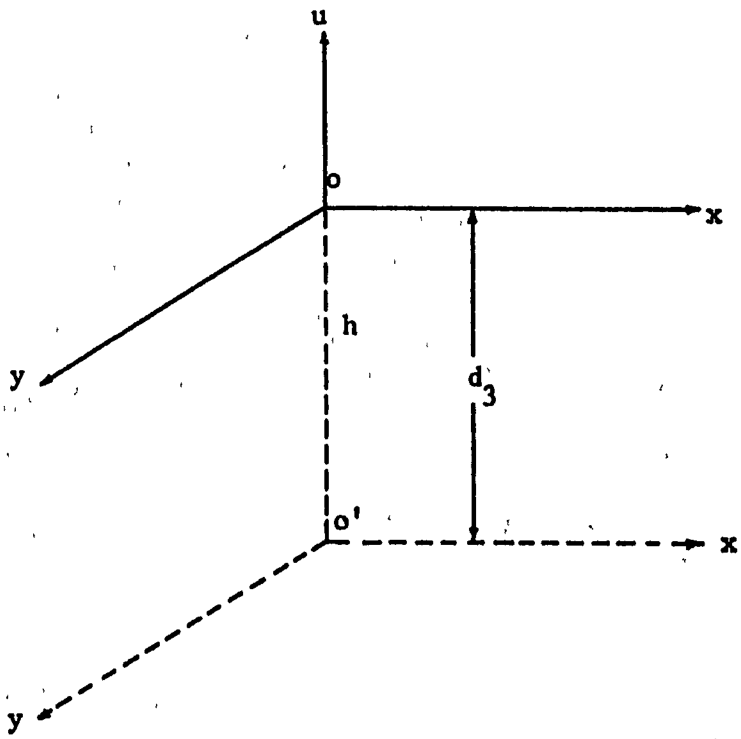

Figure 15. Translation of cartesian axes.

Appendix B

Solution of the Two-Dimensional Heat Flow Equation with Non-Homogeneous Boundary Conditions

Appendix B

Solution of the Two-Dimensional Heat Flow Equation with Non-Homogeneous Boundary Conditions

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \alpha \frac{\partial u}{\partial t} \tag{80}$$

B.C.

$u(x, o, t) = f(x)$ $\qquad$ $u(o, y, t) = 0$

$u(x, S_m, t) = f(x)$ $\qquad$ $u(S_t, y, t) = 0$

I.C.

$u(x, y, o) = F(x, y)$ $\qquad$ (general case)

The solution of Equation (80) can be expressed as the sum of two solutions:

$$u(x, y, t) = v(x, y) + w(x, y, t) \tag{81}$$

where $v(x, y)$ is the solution to Laplace's equation

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{82}$$

B.C.

$v(x, o) = f(x)$ $\qquad$ $v(o, y) = 0$

$v(x, S_m) = f(x)$ $\qquad$ $v(S_t, y) = 0$

and $w(x, y, t)$ is the solution of:

$$\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} = \alpha \frac{\partial w}{\partial t} \tag{83}$$

B.C.

$w(x, o, t) = 0$ $\qquad$ $w(o, y, t) = 0$

$w(x, S_m, t) = 0$ $\qquad$ $w(S_t, y, t) = 0$

I.C.

$$w(x, y, o) = F(x, y) - v(x, y)$$

<u>Proof</u>

1. To check if the initial equation, Equation (80), is satisfied:

$$u(x, y, t) = v(x, y) + w(x, y, t) \tag{81}$$

$$\frac{\partial^2 u}{\partial x^2} = \frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 w}{\partial y^2} \tag{84}$$

$$\frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 w}{\partial y^2} \tag{85}$$

$$\frac{\partial u}{\partial t} = \frac{\partial w}{\partial t} \tag{86}$$

Summing up Equations (84) and (85) and noting that

$$\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2} = 0 \tag{82}$$

results in:

$$\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}$$

$$= \alpha \frac{\partial w}{\partial t} \quad \text{(From Equation 83)}$$

$$= \alpha \frac{\partial u}{\partial t} \quad \text{(From Equation 86)}$$

2. To check if the original boundary and initial conditions are satisfied:

$$u(x, y, t) = v(x, y) + w(x, y, t) \tag{81}$$

$$u(x, o, t) = v(x, o) + w(x, o, t) = f(x) \tag{87}$$

$$u(x, S_m, t) = v(x, S_m) + w(x, S_m, t) = f(x \tag{88}$$

$$u(o, y, t) = v(o, y) + w(o, y, t) = 0 \tag{89}$$

$$u(S_t, y, t) = v(S_t, y) + w(S_t, y, t) = 0 \tag{90}$$

Equations (87) through (90) guarantee the satisfaction of the boundary conditions.

$$\begin{aligned} u(x, y, o) &= v(x, y) + w(x, y, o) \\ &= v(x, y) + F(x, y) - v(x, y) \\ &= F(x, y) \end{aligned} \tag{91}$$

Equation (91) guarantees the satisfaction of the initial condition.

Appendix C

Analysis of Truncating the Infinite Fourier Series of $v(x, y)$, Case 1

Appendix C

Analysis of Truncating the Infinite Fourier Series of v(x, y), Case 1

$$v(x, y) = 2 \sum_{n=1}^{\infty} A_n \, \text{Sin} \frac{n\pi}{S_t} x \, \text{Sinh} \frac{n\pi S_m}{2S_t} \, \text{Cosh} \frac{n\pi}{2S_t} (S_m - 2y) \tag{14}$$

where

$$A_n = \frac{2}{S_t \, \text{Sinh} \dfrac{n\pi S_m}{S_t}} \int_0^{S_t} f(x) \, \text{Sin} \frac{n\pi}{S_t} x dx \tag{13}$$

For the case of the zero degree polynomial (Case 1):

$$f(x) = d_2$$

$$\int_0^{S_t} f(x) \, \text{Sin} \frac{n\pi}{S_t} x dx = - \frac{d_2 S_t}{n\pi} \text{Cos} \frac{n\pi}{S_t} x \Bigg|_0^{S_t}$$

$$= \frac{2d_2 S_t}{n\pi} [1 - \text{Cos } n\pi]$$

$$\text{But } (1 - \text{Cos } n\pi) = \begin{cases} 2 & n = 1, 3, 5, \ldots \\ 0 & n = 2, 4, 6, \ldots \end{cases}$$

$$A_n = \frac{4d_2}{n\pi \, \text{Sinh} \dfrac{n\pi S_m}{S_t}} \qquad n = 1, 3, 5, \ldots$$

$$v(x, y) = \frac{8d_2}{\pi} \sum_{n=1, 3, 5, \ldots}^{\infty} \frac{1}{n} \cdot \text{Sin} \frac{n\pi}{S_t} x \cdot \frac{\text{Sinh} \frac{n\xi}{2}}{\text{Sinh } n\xi} \cdot \text{Cosh} \frac{n\pi}{2S_t} (S_m - 2y) \tag{92}$$

where

$$\xi = \frac{\pi S_m}{S_t}$$

Evaluating v(x, y) in Equation (83) at $x = \frac{S_t}{2}$ and $y = \frac{S_m}{2}$:

$$v(\frac{S_t}{2}, 0) = d_2 \frac{8}{\pi} \sum_{n=1, 3, 5, \ldots}^{\infty} \frac{1}{n} \cdot \mathrm{Sin} \frac{n\pi}{2} \cdot \frac{\mathrm{Sinh} \frac{n\xi}{2}}{\mathrm{Sinh}\ n\xi} \tag{93}$$

Let

$$P = \frac{8}{\pi} \sum_{n=1, 3, 5, \ldots}^{\infty} \frac{1}{n} \cdot \mathrm{Sin} \frac{n\pi}{2} \cdot \frac{\mathrm{Sinh} \frac{n\xi}{2}}{\mathrm{Sinh}\ n\xi} \tag{94}$$

$$v(\frac{S_t}{2}, \frac{S_m}{2}) = P \cdot d_2 \tag{95}$$

The right-hand side of Equation (93) should converge to d_2 as $n \to \infty$. In other words, P of Equation (94) should converge to 100% as $n \to \infty$.

Equation (94) is a function of ξ which in turn is a function of the ratio $\frac{S_m}{S_t}$. In order to cover all practical field spacings of the mole drains and tile drains in the combined system, six values of $\frac{S_m}{S_t}$ were investigated. In other words, Equation (94) was investigated five times, each time using a different value of ξ.

Table 7 lists the results. Columns 2, 3, 4 and 5 indicate the number of terms summed up in the infinite series of Equation (94).

As indicated by the results of Table 7, Equation (93) has an oscillating behavior slowly converging as the number of terms of the infinite series is increased. Thus, if only the first term of the infinite series of Equation (94) were taken and the rest of the terms of the series were neglected, on the average v(x, y) would be over estimated by over 27%.

Table 7. Convergence of the infinite Fourier series of Equation (94).

$\frac{S_m}{S_p}$	P(%)			
	1 term	2 terms	3 terms	4 terms
0.0167	127	85	110	92
0.0250	127	85	109	91
0.0500	130	88	113	97
0.0750	126	86	108	95
0.1500	126	92	106	99
Average	127	87	109	95

However, the mathematical calculations involved in finding u(x, y, t) in Equation (1) would be so complex if more than one term of Equation (93) is considered, particularly if it is noted that the case considered is the simplest of all cases (i.e., zero degree polynomial), that a loss of accuracy of about 30% should be accepted for the sake of simplicity.

Appendix D

Reducing B to a Function of A

Appendix D

Reducing B to a Function of A

$$B = \int_0^{S_m} \int_0^{S_t} g(x, y) \text{ Sin } \frac{\pi x}{S_t} \text{ Sin } \frac{\pi y}{S_m} \, dx \, dy \tag{21}$$

$$g(x, y) = (d_1 + d_2) - v(x, y) \tag{17}$$

$$v(x, y) = (\frac{4}{S_t}) (\psi) (A) \text{ Sin } \frac{\pi x}{S_t} \text{ Cosh } \frac{\pi}{2S_t} (S_m - 2y) \tag{18}$$

Substituting v(x, y) into Equation (17):

$$g(x, y) = [d_1 + d_2] - (\frac{4}{S_t}) (\psi) (A) \text{ Sin } \frac{\pi x}{S_t} \text{ Cosh } \frac{\pi}{2S_t} (S_m - 2y) \tag{96}$$

Substituting g(x, y) into Equation (21):

$$B = \int_0^{S_m} \int_0^{S_t} ((d_1 + d_2) - v(x, y)) \text{ Sin } \frac{\pi x}{S_t} \text{ Sin } \frac{\pi y}{S_m} \, dx \, dy$$

$$= \int_0^{S_m} \int_0^{S_t} (d_1 + d_2) \text{ Sin } \frac{\pi x}{S_t} \text{ Sin } \frac{\pi y}{S_m} \, dx \, dy - \int_0^{S_m} \int_0^{S_t} (\frac{4}{S_t}) (\psi) (A)$$

$$\text{Sin}^2 \frac{\pi x}{S_t} \text{ Sin } \frac{\pi y}{S_m} \text{ Cosh } \frac{\pi}{2S_t} (S_m - 2y) \, dx \, dy$$

$$= \frac{4S_t S_m}{\pi^2} (d_1 + d_2) - 2\psi A \int_0^{S_m} \text{ Sin } \frac{\pi y}{S_m} \text{ Cosh } (\frac{\pi S_m}{2S_t} - \frac{\pi y}{S_t}) \, dy$$

$$= \frac{4S_t S_m}{\pi^2} (d_1 + d_2) - (2\psi A) (\frac{2S_m}{\pi}) (\text{Cosh } \frac{\pi S_m}{2S_t}) \tag{97}$$

$$\text{But} \quad \psi \text{ Cosh } \frac{\pi S_m}{2S_t} = \frac{\text{Sinh } \frac{\pi S_m}{2S_t} \cdot \text{Cosh } \frac{\pi S_m}{2S_t}}{\text{Sinh } \frac{\pi S_m}{S_t}} = 1/2$$

Therefore Equation (97) reduces to:

$$B = \frac{4S_t S_m}{\pi^2}(d_1 + d_2) - \frac{2A\ S_m}{\pi} \tag{98}$$

Equation (98) shows that B is a function of A where:

$$A = \int_0^{S_t} f(x)\ \mathrm{Sin}\ \frac{\pi x}{S_t}\ dx \tag{19}$$

Letting $\chi = \dfrac{2A}{S_t d_2}$

and $K_1 = \dfrac{16(d_1 + d_2)}{\pi^2} - \dfrac{4d_2}{\pi}\chi$

Equation (98) reduces to:

$$B = \frac{S_m S_t}{4} K_1 \tag{22}$$

Appendix E

Field Experiment Data

Appendix E

Field Experiment Data

Table 8. Height of the water surface above the tile drains in the observation wells - September 21, 1972.

Observation Well	September 21							
	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)
	A.M.		A.M.		P.M.		P.M.	
W1	9:27	2.93	11:34	2.92	2:46	2.90	6:40	2.89
W2	9:27	2.92	11:35	2.91	2:46	2.90	6:40	2.88
W3	9:27	2.93	11:35	2.91	2:47	2.90	6:41	2.88
W4	9:29	2.92	11:37	2.90	2:49	2.78	6:44	2.60
W5	9:29	2.93	11:37	2.95	2:50	2.89	6:44	2.59
W6	9:29	2.95	11:38	2.91	2:50	2.81	6:45	2.62
W7	9:31	2.89	11:40	2.87	2:52	2.56	6:46	2.33
W8	9:31	2.88	11:40	2.79	2:53	2.50	6:46	2.33
W9	9:31	2.88	11:41	2.72	2:54	2.44	6:47	2.25
W10	9:33	2.93	11:51	2.92	3:03	2.90	6:55	2.89
W11	9:33	2.91	11:51	2.90	3:03	2.89	6:56	2.87
W12	9:33	2.92	11:52	2.90	3:04	2.89	6:56	2.88
W13	9:35	2.93	11:49	2.91	3:00	2.90	6:52	2.89
W14	9:35	2.93	11:50	2.92	3:01	2.91	6:53	2.89
W15	9:35	2.93	11:50	2.92	3:01	2.90	6:54	2.88
W16	9:37	2.93	11:46	2.92	2:57	2.90	6:50	2.89
W17	9:37	2.92	11:47	2.91	2:58	2.90	6:50	2.89
W18	9:37	2.92	11:47	2.91	2:58	2.90	6:51	2.89

Table 9. Height of the water surface above the tile drains in the observation wells - September 22, 1972.

Observation Well	September 22							
	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)
	A.M.		A.M.		P.M.		P.M.	
W1	12:41	2.77	10:43	2.87	2:04	2.74	6:29	2.62
W2	12:41	2.79	10:45	2.83	2:04	2.82	6:30	2.80
W3	12:41	2.78	10:45	2.84	2:05	2.82	6:30	2.80
W4	12:45	2.48	10:49	2.31	2:05	2.20	6:32	2.05
W5	12:45	2.47	10:49	2.31	2:06	2.19	6:32	2.04
W6	12:45	2.49	10:50	2.32	2:07	2.19	6:33	2.04
W7	12:48	2.24	10:51	2.12	2:08	2.01	6:35	1.86
W8	12:48	2.21	10:52	2.08	2:09	1.98	6:35	1.84
W9	12:48	2.19	10:53	2.06	2:09	1.96	6:35	1.84
W10	1:00	2.87	11:02	2.85	2:17	2.83	6:45	2.82
W11	1:00	2.87	11:03	2.84	2:18	2.83	6:46	2.81
W12	1:00	2.85	11:03	2.84	2:19	2.79	6:46	2.34
W13	12:57	2.88	11:00	2.84	2:15	2.76	6:43	2.62
W14	12:57	2.86	11:00	2.84	2:15	2.75	6:43	2.61
W15	12:57	2.84	11:01	2.84	2:16	2.74	6:44	2.58
W16	12:59	2.88	10:57	2.85	2:13	2.83	6:40	2.70
W17	12:59	2.87	10:57	2.84	2:13	2.83	6:41	2.67
W18	12:59	2.87	10:58	2.84	2:14	2.81	6:41	2.51

Table 10. Height of the water surface above the tile drains in the observation wells - September 23-24, 1972.

Observation Well	September 23				September 24			
	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)
	A.M.		P.M.		P.M.		P.M.	
W1	8:28	2.22	7:09	1.87	2:35	1.61	6:20	1.55
W2	8:29	2.25	7:10	1.84	2:35	1.59	6:21	1.54
W3	8:29	2.28	7:10	1.86	2:36	1.63	6:22	1.58
W4	8:31	1.97	7:12	1.70	2:32	1.57	6:23	1.53
W5	8:31	1.97	7:12	1.70	2:32	1.57	6:24	1.54
W6	8:32	1.98	7:13	1.70	2:33	1.57	6:24	1.53
W7	8:34	1.81	7:14	1.61	2:30	1.50	6:25	1.46
W8	8:35	1.82	7:14	1.53	2:31	1.44	6:26	1.39
W9	8:35	1.82	7:15	1.54	2:31	1.40	6:28	1.31
W10	8:44	2.41	7:24	1.81	2:37	1.56	6:35	1.47
W11	8:45	2.29	7:24	1.71	2:38	1.51	6:36	1.41
W12	8:45	2.03	7:25	1.62	2:38	1.44	6:36	1.32
W13	8:40	2.48	7:21	1.92	2:39	1.69	6:33	1.56
W14	8:41	2.46	7:21	1.90	2:39	1.68	6:33	1.55
W15	8:41	2.44	7:21	1.88	2:40	1.66	6:34	1.53
W16	8:38	2.44	7:19	1.95	2:41	1.59	6:30	1.44
W17	8:38	2.35	7:19	1.86	2:42	1.56	6:31	1.42
W18	8:39	2.26	7:20	1.79	2:42	1.54	6:32	1.41

Table 11. Height of water surface above the tile drains in the observation wells - September 25-26, 1972.

Observation Well	September 25				September 26			
	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)	Time	u (ft.)
	A.M.		P.M.		A.M.		P.M.	
W1	10:59	1.37	6:45	1.33	11:10	1.24	6:07	1.22
W2	11:00	1.34	6:46	1.31	11:11	1.23	6:07	1.20
W3	11:00	1.39	6:46	1.33	11:11	1.26	6:08	1.22
W4	11:01	1.44	6:47	1.35	11:13	1.33	6:08	1.28
W5	11:02	1.45	6:48	1.36	11:14	1.33	6:09	1.29
W6	11:02	1.45	6:48	1.35	11:14	1.33	6:10	1.28
W7	11:04	1.35	6:50	1.32	11:16	1.28	6:13	1.26
W8	11:05	1.28	6:50	1.23	11:17	1.20	6:13	1.19
W9	11:05	1.23	6:51	1.18	11:18	1.15	6:14	1.13
W10	11:12	1.27	7:00	1.21	11:26	1.14	6:24	1.10
W11	11:12	1.24	7:00	1.18	11:27	1.11	6:24	1.08
W12	11:13	1.23	7:01	1.11	11:28	1.09	6:25	1.04
W13	11:10	1.49	6:57	1.36	11:24	1.32	6:21	1.29
W14	11:11	1.47	6:58	1.35	11:24	1.32	6:22	1.29
W15	11:11	1.46	6:58	1.33	11:25	1.31	6:23	1.27
W16	11:08	1.37	6:55	1.20	11:21	1.19	6:19	1.13
W17	11:08	1.36	6:55	1.20	11:21	1.19	6:20	1.13
W18	11:09	1.36	6:56	1.21	11:22	1.20	6:20	1.13

VITA

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