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AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D.C 20523

DATE: 8/3/88

MEMORANDUM

TO: AID/PPC/CDIE/DI, room 209 SA-18

FROM: AID/SCI, Victoria Ose VO

SUBJECT: Transmittal of AID/SCI Progress Report(s)

Attached for permanent retention/proper disposition is the following:

AID/SCI Progress Report No. C5-269
4th PR 1/1-6/30/88

Attachment

CS-467

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AID PROJECT DPE-G-SS-6006-00
"IDENTIFICATION OF AQUIFER HYDRAULIC PROPERTIES UNDER UNCERTAINTY"

FOURTH PROGRESS REPORT
FOR THE PERIOD JANUARY 1 - JUNE 30, 1988

1. Activity of the Israeli team.

During this period the Israeli team has carried out the folowing tasks, along the project plan:

(i) The theoretical approach to identify the aquifer properties and the natural recharge under unsteady conditions developed during the preceding period has been applied to a portion of the Israeli Coastal Aquifer. The analysis and the result are presented in the first Appendix of this report. The material has been accepted and is due to be published as an article in the journal Water Resources Research.

(ii) Collaborative work has been carried out with the Portugaese team during its visit in Israel toward application of the methodology to the Rio Maior aquifer in Portugal. A full account of this activity and of results is given below in the Portugaes team report.

(iii) Prof. G. Dagan has visited LNEC in Lisbon in March, 1988. Besides intensive talks about the project, a series of lectures on the inverse problem and its solution by the methodology developed in the frame of the project has been given. These lectures have been attended by the research team as well as by other Portugaese hydrologists interested in the subject.

2. Activity of the Portugaese team (reported by Drs. Lobo Ferreira and J. Rodriquez).

During the 6-month period starting january 1, 1988, the Portuguese team worked mainly in the following areas:

(i) Collection and analysis of piezometric data of the Rio Maior Aquifer (January and February).

(ii) Completion of the geology and hydrogeology description of the Rio Maior Aquifer. A synthesis of this description is presented in Appendix 2.

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(iii) Application of the theory developed by the Israeli team to the Rio Maior Aquifer. The data information selected for this case-study includes a 19 month sequential set of 18 piezometric head data and a set of 3 (out of 4 available) transmissivity values. This data corresponds to the Northern region of the Rio Maior aquifer, which in this area is a confined aquifer. The Southern part of the aquifer has different characteristics, being an unconfined aquifer. The January 1988 piezometric data was also analyzed, being considered as an extra reference value.

The results of the case-study will be part of a paper submitted to The Journal of Hydrology. They are presented in Appendix 3.

(iv) Two further applications of the theory were carried out during this period. The first one analyzes the Southern part of the Rio Maior Aquifer. The second one deals with the deep quaternary aquifer of AVEIRO. The results otained were not considered satisfactory, due to scarcity of hydrogeologic data. The two case-studies will be further developed during the next research period.

(v) Dr. Lobo Ferreira and Dr. Delgado Rodrigues visited for a month and for three weeks, respectively, the University of Tel-Aviv, working with Prof. G. Dagan and with Dr. Y. Rubin, mainly on the case-study described in item 3.

Reported by Drs. J.P. Lobo Ferreira and J. Delgado Rodrigues.

3. Joint activity.

Reported under items 1 (ii), (iii) and 2 (v) above.

Gedeon Dagan
Professor of Fluid Mechanics

/tpp. /

STOCHASTIC IDENTIFICATION OF RECHARGE, TRANSMISSIVITY AND STORATIVITY IN AQUIFER UNSTEADY FLOW: 2. CASE STUDY.

C5-No 9

by

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ABSTRACT

The stochastic-analytic solution to the inverse problem in aquifer flow under transient conditions, which was developed in Part 1 (Dagan and Rubin, this issue) is applied to a section of the Israeli coastal aquifer. This is a heavily pumped aquifer and the input data consists of measured monthly heads, a few transmissivities and wells rates of pumpage and recharge. In the first stage, identification of the model parameters by a maximum-likelihood procedure (MLP), based on head and transmissivity measurements is carried out. Relatively small parameters' variances prove the suitability of the approach to the case at hand. The model parameters then serve to estimate the monthly natural recharges, which are compared to recorded precipitation. Furthermore, the MLP estimates include that of storativity. Results are found to be in fair agreement with previously obtained estimates. In the second stage the spatial distributions of head and logtransmissivity through their conditional first two moments are determined, which completes the solution of the inverse problem. The selected aquifer is characterized by few measurements of transmissivity (five) and a larger number of heads (thirty five). It is found that under these

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conditions the solution of the inverse problem is extremely beneficial in obtaining sound estimates for the aquifer parameters.

INTRODUCTION

A solution to the inverse problem in unsteady groundwater flow is presented in an accompanying paper (Dagan and Rubin, this issue). The solution includes the identification of recharge, transmissivity and storativity under transient conditions.

The main distinctive feature of the model is its reliance on an analytic solution of the stochastic flow equation, as compared with the numerical study of Carrera and Neuman (1986) and other works reviewed by Yeh (1986). Closed form, simple analytic expressions were obtained for Γ_H, the time dependent head residuals variogram, and for C_{YH}, the time dependent logtransmissivity-head crosscovariance, for a given exponential logtransmissivity covariance function C_Y. An attempt has been made to include as small as possible number of parameters in these expressions, and consequently, a parsimonious model parametrization was achieved.

The main aim of this part of the study is to apply the newly proposed method to a real-life case. This task is done stagewise, as follows: In Section 2, the hydrogeology of a section of Israel's coastal aquifer, which serves as our case study, is briefly described. In Section 3, the details of the methodology are elucidated. Section 4 contains the main results, followed by a summarizing discussion in Section 5.

2. HYDROGEOLOGICAL BACKGROUND

As a case study we chose to concentrate on a section of Israel's coastal aquifer. The coastal aquifer, of length 120 Km and width 12 to 20 Km, stretches along the Mediteranean coast. The aquifer is wedge-shaped in a

vertical cross-section perpendicular to the coast, reaching there its maximal depth of 180 m'. Our case study is located in the Be'er Tuvia region, which is on the south-eastern periphery of the aquifer - about 9 Km east of the coastline, and covers a rectangular - shaped area of dimension 7x7 Km^2.

Figure 1 depicts a typical vertical cross-section of the aquifer, in a direction perpendicular to the coastline. The aquifer consists mainly of medium to coarse-grained sand and sandstone of the Pliopleistocene age, with sub-layers and lenses of clay and loam. The aquifer is underlain by an impervious clay bottom (Mercado, 1970), and is covered by a semipervious loam layer.

The aquifer is replenished by rain and by artificial recharge through wells. The direction of the flow is generally seaward, with some exceptions in areas where head depressions were caused by heavy pumping. The Be'er Tuvia region is such an area, in which intensive pumpage in summertime, is followed by artificial recharge through wells in wintertime. The aquifer is constantly monitored by Tahal - Israel Water Planning Ltd., and the Israeli Hydrological Service. We have chosen to analyze here a period of 7 months, starting October 1970 until april 1971. For this period Tahal has provided us with records of heads and pumping rates (see Sect. 4).

3. DETAILS OF THE METHODOLOGY APPLIED TO THE COASTAL AQUIFER

The observations vector is defined by:

$$\underline{Z}=(Y_i,\ h_{l,k}-h_{N,k}) \qquad i=1,..M;\ l=M+1,..,N-1;\ k=1,..,P \tag{1}$$

$$h_{l,k}=H_{l,k}-\langle H_{l,k}\rangle\ ;\ H=H_t-H_w.$$

where H_t (the total head), H_w (the well induced head) and H (the difference

between the first two) are defined in eqs.(3-5) in Part 1. It is reminded that M is the number of points at which the transmissivity $T_i = \ln Y_i$ has been measured, whereas N-M is the number of points at which heads were monitored. Angle brackets denotes the expectation operator, whereas single index indicates location in space. When double indexing is used, the second one (k) denotes in running order the month in which the head measurement was taken. The size of Z is L=M+(N-1-M)P, and head residuals spatial increments are employed rather than h alone for reasons given in Dagan (1985), and Rubin an Dagan (1987). Thus Z is of a dimension lesser by unity than the number of head measurements.

The observation vector Z is employed to identify with the aid of a maximum likelihood procedure (MLP) the optimal estimator $\hat{\underline{\theta}}$ of the vector $\underline{\theta}$, which is defined as follows.

$$\theta_1 = m_Y;\ \theta_2 = \bar{\sigma}^2_Y;\ \theta_3 = W;\ \theta_4 = I_Y;\ \theta_5 = S;$$
$$\theta_{6,k} = J_{0x,k};\ \theta_{7,k} = J_{0y,k};\ \theta_{8,k} = c_{11,k};\ \theta_{9,k} = 2c_{12,k};$$
$$\theta_{10,k} = c_{22,k};\ \theta_{11,k} = H_{0,k};\quad (k=1,..,P) \tag{2}$$

The first four parameters pertain to the statistical structure of Y, which is defined in eq.2 of Part 1. The others are the parameters of the time-dependent quadratic polynomial which describes the mean recharge-induced flow given by eq.9 in Part 1. P is the number of periods over which heads were measured in the field.

Posing the inverse problem as a maximum-likelihood procedure (MLP) was first suggested by Kitanidis and Vomvoris (1983), and later adopted in the studies of Dagan (1985), Carrera and Neuman (1986) and Rubin and Dagan (1987). MLP estimators are unbiased, consistent, asymptotically unique, of minimum variance, and normally distributed (Mood and Graybill, 1963).

MLP, in our case, requires the maximization of the Gaussian likelihood function

$$F=(2\pi)^{\frac{-1}{2}L}\,|\underline{\underline{Q}}|^{\frac{-1}{2}}\exp\left[-\frac{1}{2}(Z-\langle Z\rangle)^T\underline{\underline{Q}}^{-1}(Z-\langle Z\rangle)\right] \tag{3}$$

where $\underline{\underline{Q}}$ is the covariance matrix defined as $\underline{\underline{Q}}=\langle(Z-\langle Z\rangle)(Z-\langle Z\rangle)^T\rangle$, and is of order L^2. Superscript T denotes the transpose operator. Also in eq.3 we define:

$$\langle Z_i\rangle=\begin{cases}\theta_i & i=1,\ldots,M\\ 0 & i=M+1,\ldots,N-1 \text{ and all } k\end{cases} \tag{4}$$

The matrix $\underline{\underline{Q}}$ is comprised of submatrices as follows.

$$\underline{\underline{Q}}=\begin{cases}Q_0 & W_1\ldots\ldots W_k\ldots\ldots\ldots W_P\\ W_1^T & \vdots\\ & \vdots\\ W_k^T & \ldots\ E_{kk}\\ \vdots & \\ W_P^T & \ldots\ldots E_{PP}\end{cases} \tag{5}$$

The submatrix Q_0 is the covariance matrix of the Y-measurements, which is time independent, and its terms are

$$(Q_0)_{il}=\langle Y_iY_l\rangle=C_Y(\mathbf{x}_i,\mathbf{x}_l) \qquad (i,l=1,\ldots,M) \tag{6}$$

W_k is the crosscovariance matrix of logtransmissivities and the heads residuals increments at time $t=t_k$, and its terms are

$$(W_k)_{il}=\langle Y_i(h_{lk}-h_{Nk})\rangle \qquad (i=1,\ldots M;\ l=M+1,\ldots,N-1) \tag{7}$$

$E_{kk'}$ is the covariance matrix of the head residuals increments at t_k and $t_{k'}$, and its terms are:

$$(E_{kk'})_{pl}=\langle(h_{pk}-h_{Nk})(h_{lk'}-h_{Nk'})\rangle \qquad (p,l=M+1,..,N-1) \tag{8}$$

The covariance structure given in (5) is required for a simultaneous identification of all the parameters in $\underline{\theta}$. The MLP in this case may, however, be quite cumbersome. One reason is the heavy computational burden associated with the manipulation and inversion of matrices of the order L^2.

Following the discussion in Section 4 of Part 1, this problem is reduced considerably if we replace the process of simultaneous identification of $\underline{\theta}$ at all times by a sequence of P sub-processes, where in each one we identify only a part of $\underline{\theta}$. Hence, instead of identifying $\underline{\theta}$ as given in (2), we identify a sequence of vectors $\underline{\theta}_k$ where $k=1,..,P$ and each vector pertains to one of the P periods during which head measurements were taken. $\underline{\theta}_k$ is given by:

$$\underline{\theta}(t_k)=(\ m_Y,\ w,\ \bar{\sigma}^2_Y,\ I_Y,\ S,\ J_{o,k},\ c_{i,j,k},\ H_{o,k})\ (i,j=1,2) \tag{9}$$

and contains only 11 terms instead of 5+6P in $\underline{\theta}$ (2).

The objective function is obtained now by substituting L=N-1 in (3). The vector of expected values $\langle Z\rangle$ remains as given in (4). However, it contains now only N-1 terms, instead of the [M+(N-1-M)P] terms it contained before. The covariance matrix reduces to:

$$\underline{\underline{Q}}=\begin{cases} Q_o & W \\ W^T & E \end{cases} \tag{10}$$

The structure of Q_o is given by (6), with C_Y given by eq.2 in Part 1. W is given by:

$$W_{il}=C_{YH}(\mathbf{x}_i,\mathbf{x}_l,t_k)-C_{YH}(\mathbf{x}_i,\mathbf{x}_N,t_k) \quad (i=1,..,M \quad l=M+1,..,N-1) \tag{11}$$

where $C_{YH}(\mathbf{x},\mathbf{x}',t_k)$ is expressed, with the aid of $\underline{\theta}_k$, by eq.16 in Part 1. The computation of the submatrix E in (10) reduces to that of any one of the submatrices on the diagonal of $\underline{\underline{Q}}$ in (5), and its terms are given by:

$$\begin{aligned}(E)_{ij}=&C_H(\mathbf{x}_i,\mathbf{x}_j,t_k)-C_H(\mathbf{x}_i,\mathbf{x}_N,t_k)\\ &-C_H(\mathbf{x}_N,\mathbf{x}_j,t_k)+C_H(\mathbf{x}_N,\mathbf{x}_N,t_k) \quad (i,j=M+1,N-1)\end{aligned} \tag{12}$$

C_H in (12) is formally replaced by the variogram given in eq.19 of Part 1, leading to:

$$(E)_{ij}=\Gamma_H(\mathbf{x}_i,\mathbf{x}_N,t_k) + \Gamma_H(\mathbf{x}_j,\mathbf{x}_N,t_k) - \Gamma_H(\mathbf{x}_i,\mathbf{x}_j,t_k) \tag{13}$$

$\underline{\underline{C}}$ in (10) is of the order $(N-1)^2$ rather then the higher order $[M+(N-1-M)P]^2$ of $\underline{\underline{Q}}$ in (5). It has to be computed repeatedly for each of the P identification subprocesses.

The basis for the quasi-steady analysis is discussed in Section 4 of Part 1, and it is realized that some loss of information (in the sense of Shannon) concerning the parameters might be incurred due to the neglect of the inter-time correlation structure, i.e. by considering $E_{kk'}$ of (5) to be a null matrix for $k \neq k'$. However, the assumption that this loss is minimal is supported by two observations. First, the major information concerning the values of

$\theta_1,..,\theta_{11}$ at time t_k can be extracted from the head measurements of that period. Second, the head measurements of consecutive periods may hardly be considered as independent realizations of a stochastic process. On the contrary, they are very strongly correlated over time, and thus add little or no information at all with regard to $\theta_1,..,\theta_5$ when considered jointly. The same conjecture was reported by Carrera and Neuman (1986b, pp. 232), based on their numerical simulations. Thus it is assumed here that the possible loss of information is a worthwhile trade-off versus the considerable reduction in computational cost and burden, and the possibility to obtain a quick estimate of the hydraulic parameters of the aquifer.

4. RESULTS

THE DATA BASE.

The data which serves for the purpose of identification is extracted from 39 wells and only in 5 of them pumping tests were conducted. 10 of the 39 wells are observation wells, and do not serve for pumpage or recharge purposes. The rest 29 wells are pumping wells, 6 of which are also recharging wells at times. Heads were measured monthly in 35 of the 39 wells. Employing the terminology of eq.A.2 of Part 1, we have: N_w=29, of which M_1=5, and M_2=24. Refering to eq. (1), we have M=5, and N=40. The head records are available for many years. However, in our study we concentrated over the data collected during the 7 winter months starting October 1970 up to April 1971, i.e. P=7. This data is reported in detail in the dissertation of Rubin (1988).

A recovery period of 24-48 hours is usually allowed before heads are measured, and the monthly pumpages or recharges were recorded when measuring

the heads.

MLP ESTIMATES OF THE PARAMETERS.

Table 1 comprises the MLP estimates of $\theta_1=m_Y$, $\theta_2=W$, $\theta_3=\bar{\sigma}^2_Y$, $\theta_4=I_Y$ and $\theta_5=S$, which were obtained by employing the two different approaches discussed in Part 1. The first is the simplified approach, described in Appendix B of Part 1, in which the effect of well pumpage or recharge was smeared over the aquifer and incorporated in R_{ef}, the effective recharge. The second is the general approach, in which the wells effects are incorporated explicitely, by using the stepwise time records of Q_{ik}. The results for the other parameters for both methods are given in tables 2 and 3.

The important finding is the fact that the incorporation of the head measurements in the identification process, in addition to the measured transmissivities, enabled to obtain meaningfull estimates for the parameters of the random Y field. This task would have been otherwise impossible due to the fact that only five transmissivity measurements are at our disposal in the present case, unlike the abundant data available in the Avra Valley aquifer case-study (Clifton and Neuman, 1982, and Rubin and Dagan, 1987).

Inspection of the Tables 1,2 and 3 leads to the following conclusions:

1. The relatively small variance of the MLP estimates indicates an appropriate model parametrization, and the suitability of the approach to the case at hand.
2. The MLP estimates of $\theta_1,\ldots,\theta_4$ obtained by both methods are quite close. The 'smearing' of the effect of the point source/sink terms of the flow equation by the simplified approach did not cause any pronounced bias in the MLP estimates. This is mostly due to the fact that a recovery period was allowed before taking head measurements. Furthermore, the wells are scattered quite

evenly over the aquifer, and their pumping regimes are quite similar.

3. Close parameters estimates were obtained for $\theta_1,\ldots,\theta_4$ when applying the method for different months. This indicates that the quasi-steady analysis is justified in the present case.

4. The MLP estimates of consecutive months display a certain variation, i.e. dispersion around their arithmetic mean. This is, however, of the order of the spread indicated by their variance of estimation.

5. The aquifer is of relatively low variability with variance of correlated residuals $\bar{\sigma}^2_Y$ about 0.19 and nugget effect w around 0.11 , which is well within the range for which the first-order approximation of the flow equation is a valid aproximation, as indicated in the study of Dagan (1985b). The integral scale is estimated $\hat{\theta}$=0.44 km, which is relatively small compared to the values provided by Hoeksema and Kitanidis (1985). However, it is larger than the minimal distance between measurement points (r_{min}=0.1 km). Its coefficient of variation is about 0.2.

6. The results for θ_5=S indicate that the storativity is a parameter of low variability. This conclusion is in agreement with the study of Freeze (1975). Butbul (1983) found the values of S of that region to vary between 0.15-0.22 as compared to S=0.2 found here.

7. The results given in Tables 2 and 3 lend evidence to two previously-stated assumptions. First, the proximity of the parameters' estimates of consecutive months supports our assumption that the data they are based on do not represent independent realizations, i.e. different flow regimes. Consequently, little information can be gained by a simultaneous identification of $\underline{\theta}$ (2) at various t_k. Second, the small change of these estimates from month to month is indicative of the gradual change of the head, and is in agreement with the assumptions underlying the quasi-steady analysis of the time-dependent

quadratic mean flow field given by eq.9 of Part 1. Eq.12 in Part 1 states the criterion for the head variogram in two-dimensional flow to adapt to its steady state expression, which was obtained by Dagan (1982). This criterion can now be checked based on the data in Table 1 and 5, dH_0/dt being given in the last. Typical values for τ are about 10 months. For a typical distance of 2500 m', we get values of 9 and 2 for the left hand and right hand sides of the inequality (12) of Part 1, respectively, and the condition for quasi-steady analysis is met.

8. The analysis based on the general model, which accounts for the local effects of the point source/sink recharge is in general favorable, in the sense of reducing model uncertainty, and this is shown below.

The parameters of Table 2 and 3 differ in their physical meaning, since $\theta_6,\ldots,\theta_{10}$ in the simplified approach include the effect of the wells. Consequently, it is best to measure the reduction of the parameters' variances globally, by employing Information Theory, which regards variance reduction as gain in information, rather then by comparing separately and directly the variance of each parameter. Table 4 contains Shannon's (1948) index of information pertaining to the seven months of the analysis. This index is defined by $I=\log \det(\Sigma)$, where Σ is the estimation error covariance matrix of the parameters ($\theta_5=S$ excluded, since it is not common to both methods). Information is gained upon minimizing I.

It is evident from Table 4 that ignoring the local effect of the point source/sink terms leads to some lose of information, expressed by larger I. Furthermore, the additional computational complexity of the general model is evidently worthwhile in light of the gain in information made available without resorting to additional measurements in the field. There is an additional gain in information, namely the estimates of the natural recharge and of the

storativity and their variances, which are available only in the general model.

ESTIMATION OF THE NATURAL RECHARGE

The estimation of natural recharge is carried out based on eq.10 of part 1 and on the procedure outlined there in Section 6. The details and results are given in Table 5, covering the winter period between October 1970 and April 1971.

The results show that the water table at that area rose steadily by a total of 1.39 m' during that period. The total natural recharge for that period is estimated in Table 5 to be 0.055 m'± 0.0053 m'. The precipitation in that area can be estimated according to the data collected by the Be'er Tuvia meteorologic station. The total precipitation as measured by the station during the winter of 1970/71 was 0.50 m'.

It is instructive to compare our recharge estimates with those obtained by Tahal-Israel Water Planning Ltd. Tahal employed two methods for that purpose (Mercado, 1970): The first is based on multi-seasonal and regional water balances. These data served to calibrate the model $R_i=\alpha(P_i-\beta)$, where α and β are two constants. β is sometimes refered to in the literature as treshold rain, R_i and P_i are the yearly natural recharge and precipitation, respectively. For that area it was found that $\alpha=0.267$ and $\beta=260$ (mm/year). According to these values, the anticipated natural recharge for the winter of 1970/71 is 0.064 m. Different values for α and β were found when calibrating the coastal aquifer by an analogue computer in which each cell represented a smaller zone. There it was found $\alpha=0.1$ and $\beta=15$ mm/year, which leads to an estimated R of 0.0488 m'. In a later report by Tahal (Butbul, 1983), in which the Coastal aquifer was calibrated by an indirect numerical method, the calibration is based on α

only, β being neglected. There it was found $\alpha=0.1$, which gives $R=0.0503$ m'. Thence, the three different values are quite close to the one we found.

Another method employed by Tahal (Mercado, 1970) is based on the fact that the relationship between R_i and P_i depends mainly on the texture of the earth cover layer. For the coastal aquifer it was found that this relationship is given by $R_i/P_i=6.378*10^{-3}(\eta_1)+4.558*10^{-3}(\eta_2)+3.456*10^{-4}(\eta_3)+0.0207$ where η_1, η_2 and η_3 are the share (in %) of sand, red loam and clays in the layer which constitutes the upper cover, respectively. For that area $\eta_2=10$ and $\eta_3=90$, which leads to $R_i/P_i=0.097$ or $R=0.048$ m' for the winter under consideration. All these estimates are in close agreement with our results.

The data of Table 1 and 3 offer the possibility to estimate the monthly recharge to the aquifer in the area under consideration, which is given in Table 5. The results are illustrated in fig.2, in which we also have represented the monthly measured rainfall, and they permit one to learn about the effects of the upper soil cover on the recharge. First, it can be seen that although the monthly rain vary from month to month, the rate of infiltration is quite steady on the average, which is generally expected due to the buffer effect of the unsaturated zone. It is interesting to note that the relatively heavy rain of April 1970 did not contribute to the recharge of that month and the following one, and seems to have turned into floods. This again can be explained by the texture of the earth cover mentioned above.

ESTIMATION OF THE CONDITIONAL HEAD MEAN FIELD.

The conditional head mean field and its generalized conditional variance are computed by following the procedure given in Section 3 of the study of Rubin and Dagan (1987a,b), with the conditioning equations given in Dagan (1984). The

computation of the conditional head mean field for each month is carried out based on the head measurements of that month only, following the discussion in Section 7 of Part 1 and verification of the criterion for quasi- steady analysis given there in section 4. The conditional mean head is calculated repeatedly for 112 points of a regular grid, which then serve to plot contour maps (conditioning is an exact interpolator since the value of the conditional mean head at a measurement point is equal to the measured value and the conditional variance is zero).

To illustrate the results, the head contour maps for the months of October 1970 and april 1971 are given in figure 3. These maps demonstrate the rise of the water table during the winter months. The water table gradient is steeper in the east, corresponding to the wedge-shaped vertical cross-section.

The effects of the recharging wells on the mean flow field can be studied through these maps. Artificial recharge during that hydrologic year took place mainly between November 1970 and February 1971. The locations of the recharging wells are in the center and western sections of the aquifer. Consequently, the rise in the water table is generaly more pronounced in these sections of the aquifer, rather than in its eastern section.

Figure 4 is the contour map of the heads for April 1971 as obtained by the simplified method described in Appendix B of Part 1. The quality of estimation of fig.4 and that of 3b (both pertain to the same month) can be compared by analysing the maps of the conditional head variance as obtained by both methods, given in fig.5 and 6, respectively. The variance of estimation in these figures emanates from two sources. First, the uncertainy which stems from the variability of the heads and that of transmissivities, as reflected through their variogram, and in addition, the variance that is caused by the uncertainty associated with the MLP estimation of the parameters of $\underline{\theta}$. As can

be seen from Tables 1-3, the ratio $\sigma^2(\theta_i)/\theta_i^2 << 1$ is valid for $i=1,..,10$, and thus justifying the first order analysis of the generalized conditional variance, which is described in detail in section 3 of the study of Rubin and Dagan (1987a).

The results of these maps are succintly summarized in fig.7. This figure is a hystogram of the frequencies of the head conditional variance all over the aquifer, without regard to specific location. It can be concluded that the general method provides more accurate results, with an average variance of 0.0071 m^2 vs. 0.05 m^2 in the simplified method. The reasons for that are two: first, the general model is more accurate because the local effects of pumpage and recharge are accounted for; second, the variances of the parameters' estimates (Table 1) in the general model are smaller.

ESTIMATION OF THE CONDITIONAL LOGTRANSMISSIVITY MEAN FIELD

The estimation of $\tilde{Y}^c$, the conditional expected logtransmissivity and its generalized conditional variance $\sigma^{2,c}_Y(\mathbf{x})$ is carried out according to the procedure outlined in Section 3 of the study of Rubin and Dagan (1987a). These values are computed repeatedly on a regular grid, and the results serve to plot contour maps.

Figures 8 and 9 illustrate the contour lines of $\tilde{Y}^c$ obtained by employing the simplified and general methods, respectively, based on the data of January, 1971. Both maps display close features, and both methods captured the tendency of $\tilde{Y}^c(x)$ to grow in the northwest direction. This is in agreement with the wedge-like shape of the coastal aquifer, as illustrated in fig.1.

Figures 10 and 11 depict the contour lines of the conditional logtransmissivity variance $\sigma^{2,c}_Y(\mathbf{x})$ based on the simplified and general methods,

respectively. The quality of estimation of both methods is compared in figure 12, which depicts the frequencies of $\sigma^{2,c}_{Y}(\mathbf{x})$ as computed on 112 grid points. It is evident that the general model performs better that the reduced model as far as reducing the conditional generalized variance.

5. SUMMARY

In the present sequence of two papers, a stochastic method to identify the natural recharge, storativity and transmissivity under transient conditions is proposed. In its development, four main assumptions were adopted: Y is a normal random space function, the aquifer is unbounded, a first-order approximation of the flow equation is adopted and the transients are slowly-varying.

An advantage of the proposed method is its relatively easy implementation, which requires only a non-linear search algorithm. An additional one stems from the fact that the flow equation is solved analytically, thus saving the need to carry out a numerical solution of the flow equation. Furthermore, the estimation variances of the parameters are accounted for when estimating the spatial distributions of H and Y, resulting in an unbiased estimation.

A unique feature of the method is the possibility to identify the storativity and monthly natural recharge by a simple analytical mean, especially in light of the difficulties in obtaining sound estimates of natural recharge, as discussed recently in the study of Johansson (1987).

The method's main limitations are:

(1) The requirement $\bar{\sigma}^2_Y \leq 1$ (see Dagan 1985), which prevents its application to formations of large variability.

(2) The assumption of unbounded aquifer, made in order to simplify the analytic

solution of the flow equation. In many real-life cases the aquifer's boundaries and boundary conditions are poorly defined, making this assumption as good as any. It can be circumvented though, by conditioning the head by a series of points for boundaries of given head, or by images of measurements points for a boundary of no flux.

(3) The assumption that the average head is slowly-varying in time. It is recalled that this head is obtained after filtering the drawdowns due to point pumpage/discharge, and it describes the flow due solely to natural recharge or changing boundary conditions, which has a time scale of the order of many months.

The role of the inverse solution in obtaining the above estimates was discussed in a previous study (Rubin and Dagan, 1987b). It was shown there that upon conditioning of Y on the measured heads in addition to the measured transmissivities, the generalized variance $\sigma^{2,c}_{Y}(\mathbf{x})$ is reduced compared to the case where head measurements are not incorporated. The present example illustrates this point in a dramatic way. Indeed, while in the former case of the Avra Valley aquifer (Rubin and Dagan, 1987b) we had at our disposal about 50 transmissivity measurements and approximately the same number of head measurements, in this case we have only 5 transmissivity measurements and 35 head measurements. These numbers by no means allows one to carry out any meaningfull statistical inference based solely on the transmissivity data. Therefore, the role of head data in the present case is not merely to improve estimates, but to permit the identification of the transmissivity field. It is believed that scarcity of transmissivity measurements, as encountered in the present case, is rather the rule for many aquifers.

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Figure Captions.

Figure 1. - Typical vertical cross section of the Coastal aquifer in Be'er Tuvia region in a direction perpendicular to the coastline.

Figure 2. - Monthly rain and natural recharge in Be'er Tuvia region.

Figure 3. - Conditional head mean field as estimated by the general method, (a) October 1970 (b) April 1971. (in m')

Figure 4. - Conditional head mean field as estimated by simplified method, April 1970. (in m')

Figure 5. - The generalized variance of estimation of the conditional head mean field as computed by the general method for April 1971 (in m^2).

Figure 6. - The generalized variance of estimation of the conditional head mean field as computed by the simplified method for April 1971 (in m^2).

Figure 7. - Histogram of the conditional head generalized variance.

Figure 8. - Conditional logtransmissivity mean field as computed by the simplified method.

Figure 9. - Conditional logtransmissivity mean field as computed by the general method.

Figure 10. - The generalized variance of estimation of the conditional logtransmissivity mean field as computed by the simplified method.

Figure 11. - The generalized variance of estimation of the conditional logtransmissivity mean field as computed by the general method.

Figure 12. - Histogram of the conditional logtransmissivity generalized variance.

Month	$\theta_1 = m_Y$		$\theta_2 = w$		$\theta_3 = \bar{\sigma}_Y^2$		$\theta_4 = I_Y(km)$		$\theta_5 = S$	
	$\hat{\theta}_1$	Σ_{11}	$\hat{\theta}_2$	$\Sigma_{2,2}$	$\hat{\theta}_3$	$\Sigma_{3,3}$	$\hat{\theta}_4$	$\Sigma_{4,4}$	$\hat{\theta}_5$	$\Sigma_{5,5}$
10/70	(1) 7.054	0.042	0.106	0.013	0.133	0.0022	0.448	0.017	-	-
	(2) 7.055	0.007	0.04	0.0094	0.24	0.0048	0.480	0.01	0.220	$0.68 \cdot 10^{-3}$
11/70	(1) 7.053	0.045	0.08	0.022	0.242	0.0107	0.421	0.020	-	-
	(2) 7.056	0.0080	0.121	0.017	0.199	0.0039	0.460	0.014	0.2020	$0.52 \cdot 10^{-3}$
12/70	(1) 7.056	0.056	0.169	0.026	0.180	0.0039	0.403	0.013	-	-
	(2) 7.055	0.0078	0.108	0.016	0.205	0.0041	0.462	0.016	0.2002	$0.56 \cdot 10^{-3}$
1/71	(1) 7.054	0.05	0.134	0.019	0.193	0.0045	0.415	0.013	-	-
	(2) 7.055	0.0080	0.122	0.018	0.199	0.0039	0.458	0.015	0.2007	$0.88 \cdot 10^{-3}$
2/71	(1) 7.056	0.052	0.137	0.022	0.193	0.0053	0.417	0.015	-	-
	(2) 7.055	0.0081	0.134	0.02	0.193	0.0038	0.454	0.015	0.2009	$0.93 \cdot 10^{-3}$
3/71	(1) 7.058	0.045	0.108	0.017	0.209	0.0049	0.425	0.014	-	-
	(2) 7.056	0.0082	0.173	0.051	0.155	0.0026	0.426	0.016	0.2202	$0.14 \cdot 10^{-2}$
4/71	(1) 7.054	0.048	0.119	0.019	0.20	0.0048	0.428	0.014	-	-
	(2) 7.050	0.0086	0.182	0.027	0.165	0.0029	0.443	0.016	0.2009	$0.1 \cdot 10^{-2}$

Table 1: (1) - simplified method - (2) General method
$\hat{\theta}$ stands for the MLP estimates, whereas Σ_{ii} are variances of estimation (the diagonal elements of the inverse Fisher matrix).

	$\theta_6 = J_{0,x}\left(\frac{m}{km}\right)$		$\theta_7 = J_{0,y}\left(\frac{m}{km}\right)$		$\theta_8 = c_{11}\left(\frac{m}{km^2}\right)$		$\theta_9 = 2c_{12}\left(\frac{m}{km^2}\right)$		$\theta_{10} = c_{22}\left(\frac{m}{km^2}\right)$	
Month	$\hat{\theta}_6$	Σ_{66}	$\hat{\theta}_7$	$\Sigma_{7,7}$	$\hat{\theta}_8$	$\Sigma_{8,8}$	$\hat{\theta}_9$	$\Sigma_{9,9}$	$\hat{\theta}_{10}$	$\Sigma_{10,10}$
10/70	-0.420	0.0073	0.868	0.0084	-0.824	0.0020	0.196	0.0014	-0.790	0.0031
11/70	-0.425	0.011	1.081	0.015	-0.833	0.0032	0.027	0.0020	-0.704	0.0047
12/70	-0.304	0.0058	0.556	0.0058	-0.684	0.0015	0.159	0.0011	-0.723	0.0026
1/71	-0.365	0.0049	0.431	0.0051	-0.597	0.0014	0.194	0.0011	-0.622	0.0029
2/71	-0.049	0.0048	0.480	0.0037	-0.487	0.0013	0.163	0.0009	-0.619	0.0026
3/71	-0.210	0.0059	0.688	0.0072	-0.686	0.0019	0.128	0.0012	-0.615	0.0027
4/71	-0.263	0.0053	0.660	0.0061	-0.612	0.0015	0.135	0.0011	-0.624	0.0025

Table 2: MLP estimates and variances of quadratic polynomial (see eq. 9, Part 1), by simplified method. ($\hat{\theta}$ and Σ_{ii} defined in Table 1).

	$\theta_6 = J_{0,x}\left(\frac{m}{km}\right)$		$\theta_7 = J_{0,y}\left(\frac{m}{km}\right)$		$\theta_8 = c_{11}\left(\frac{m}{km^2}\right)$		$\theta_9 = 2c_{12}\left(\frac{m}{km^2}\right)$		$\theta_{10} = c_{22}\left(\frac{m}{km^2}\right)$	
Month	$\hat{\theta}_6$	Σ_{66}	$\hat{\theta}_7$	$\Sigma_{7,7}$	$\hat{\theta}_8$	$\Sigma_{8,8}$	$\hat{\theta}_9$	$\Sigma_{9,9}$	$\hat{\theta}_{10}$	$\Sigma_{10,10}$
10/70	-0.720	0.0064	0.540	0.0068	-0.480	0.0012	0.170	0.0010	-0.540	0.0021
11/70	-0.690	0.0045	0.457	0.0043	-0.442	0.0008	0.151	0.0007	-0.513	0.0016
12/70	-0.718	0.0053	0.530	0.0055	-0.466	0.0008	0.096	0.0007	-0.519	0.0017
1/71	-0.620	0.0049	0.467	0.0045	-0.446	0.0007	0.120	0.0007	-0.541	0.0017
2/71	-0.602	0.0042	0.394	0.0035	-0.406	0.0006	0.126	0.0006	-0.522	0.0015
3/71	-0.550	0.0022	0.582	0.0034	-0.473	0.0007	-0.033	0.0004	-0.360	0.0011
4/71	-0.601	0.0033	0.507	0.0030	-0.383	0.0005	0.057	0.0004	-0.471	0.0011

Table 3: MLP estimates and variances of quadratic polynomial (see eq. 9, Part 1), by general method ($\hat{\theta}$ and Σ_{ii} defined in Table 1).

Month	10/70	11/70	12/70	1/71	2/71	3/71	4/71
General Method	-28.26	-28.09	-28.13	-28.11	-27.21	-27.19	-27.88
Simplified Method	-20.67	-19.04	-20.63	-20.95	-20.19	-20.69	-20.88

Table 4: Shannon's I = log (det(Σ)) (θ_5=S excluded)

•	$\frac{dH_o}{dt}$ (m/month)	$S \frac{dH_o}{dt}$ (m/month)	$(c_{11}+c_{22})\ T_G$ (m/month)	R (m/month)	V(R) (m/month)
9/70	0.21	0.042	-0.035	0.007	$4.5 \cdot 10^{-6}$
10/70	0.20	0.04	-0.032	0.008	$4.4 \cdot 10^{-6}$
12/70	0.18	0.036	-0.033	0.003	$4.0 \cdot 10^{-6}$
1/71	0.20	0.040	-0.034	0.006	$4.7 \cdot 10^{-6}$
2/71	0.20	0.04	-0.032	0.008	$4.1 \cdot 10^{-6}$
3/71	0.21	0.042	-0.028	0.014	$3.4 \cdot 10^{-6}$
4/71	0.19	0.038	-0.029	0.009	$3.5 \cdot 10^{-6}$
				0.055	

Table 5: Estimation of monthly natural recharge 10/70 - 4/71

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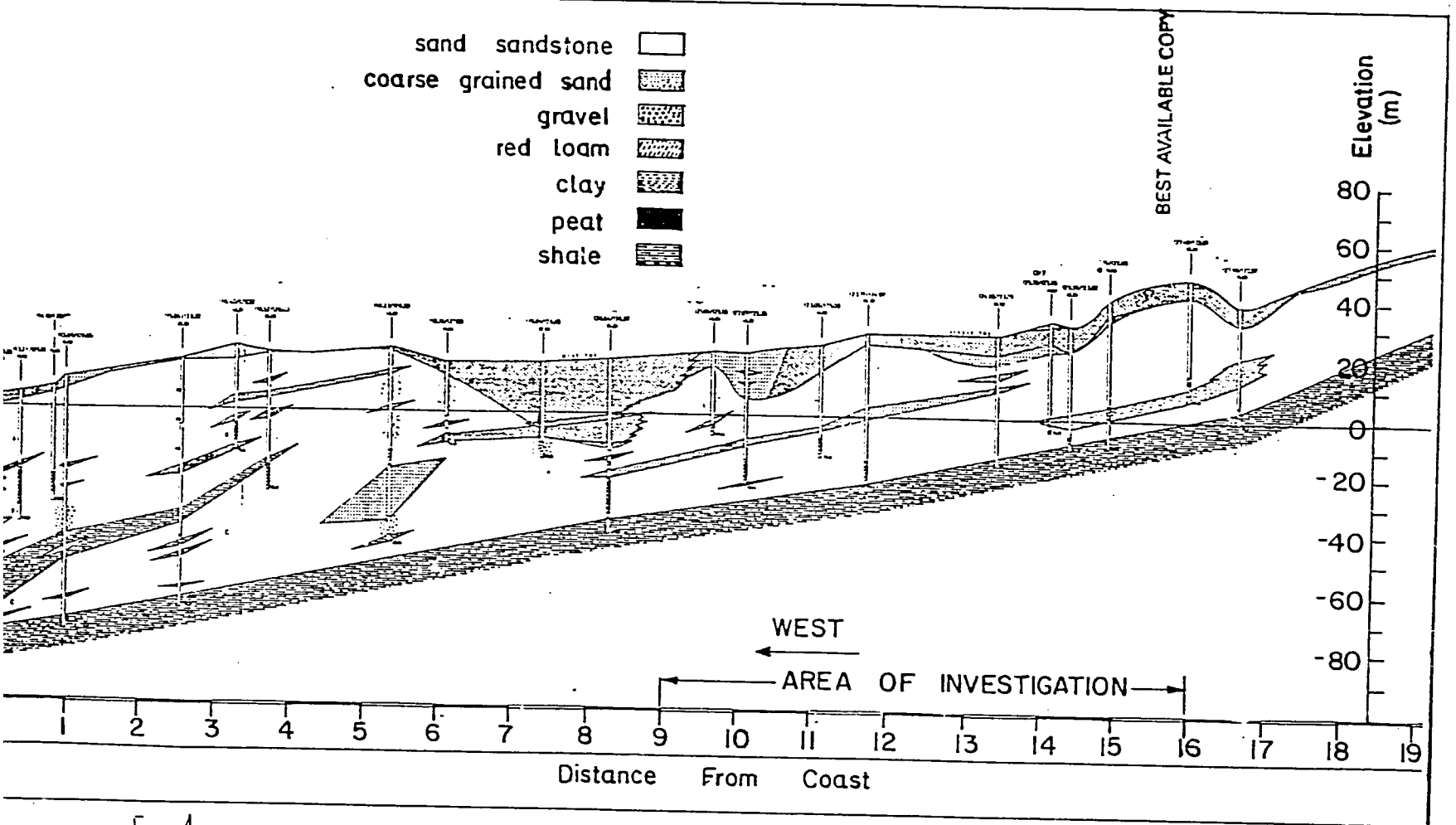
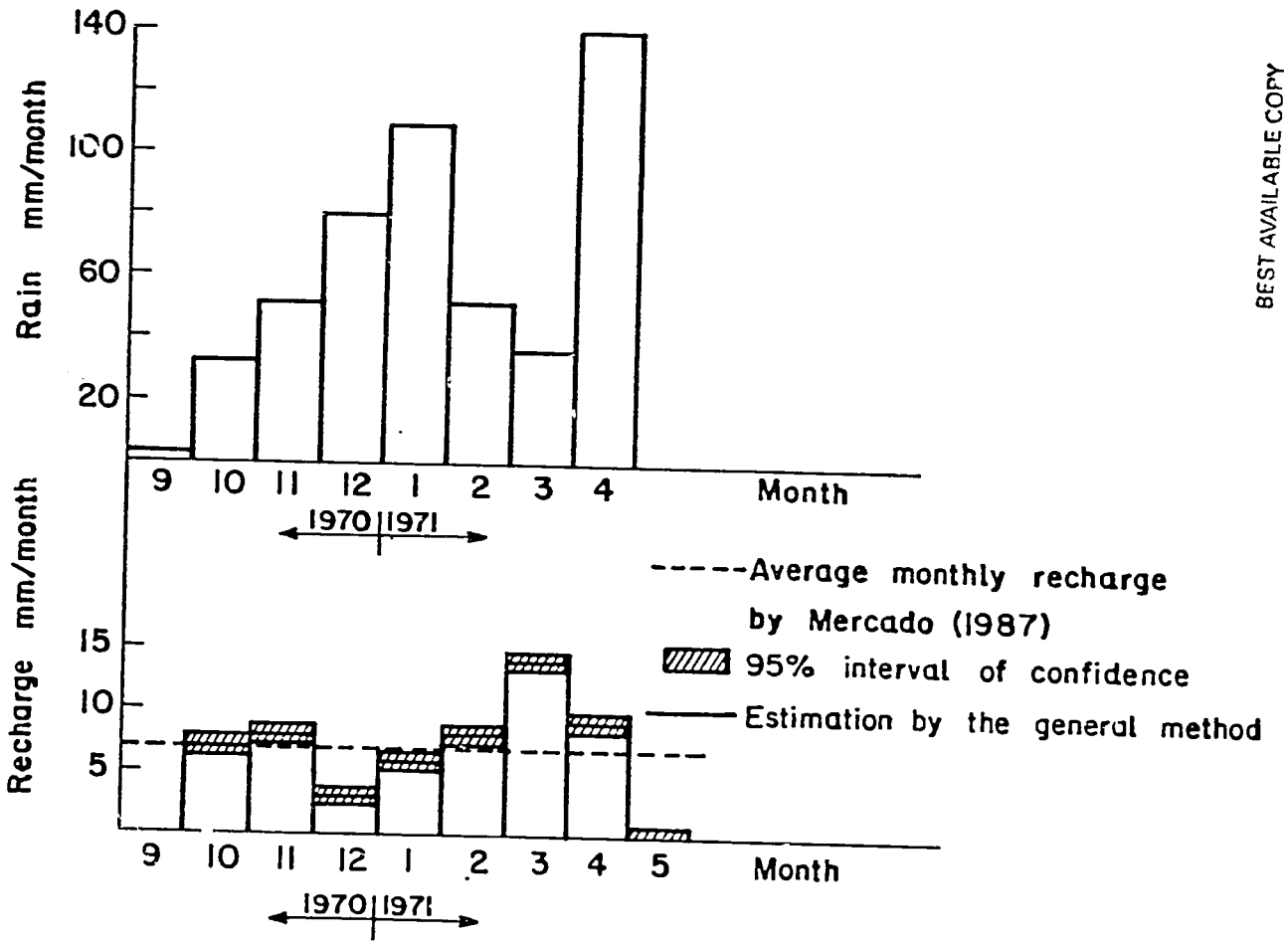


Fig. 1



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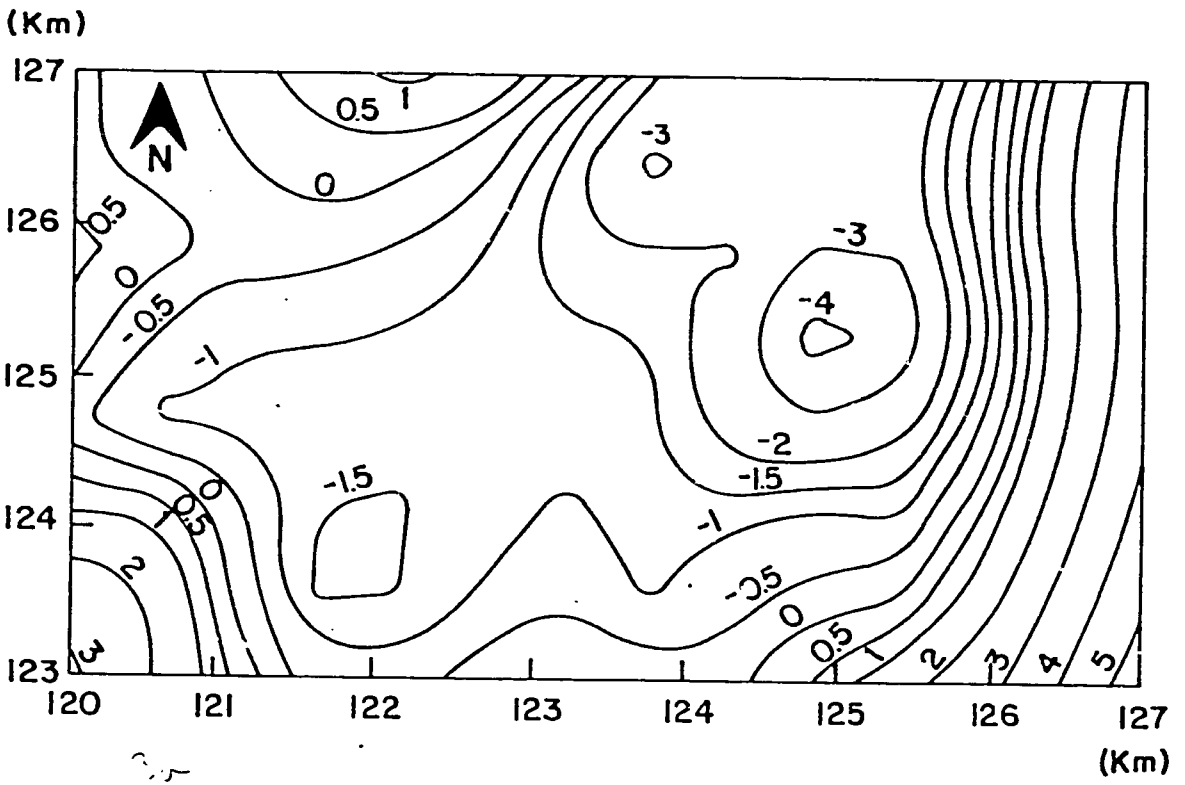

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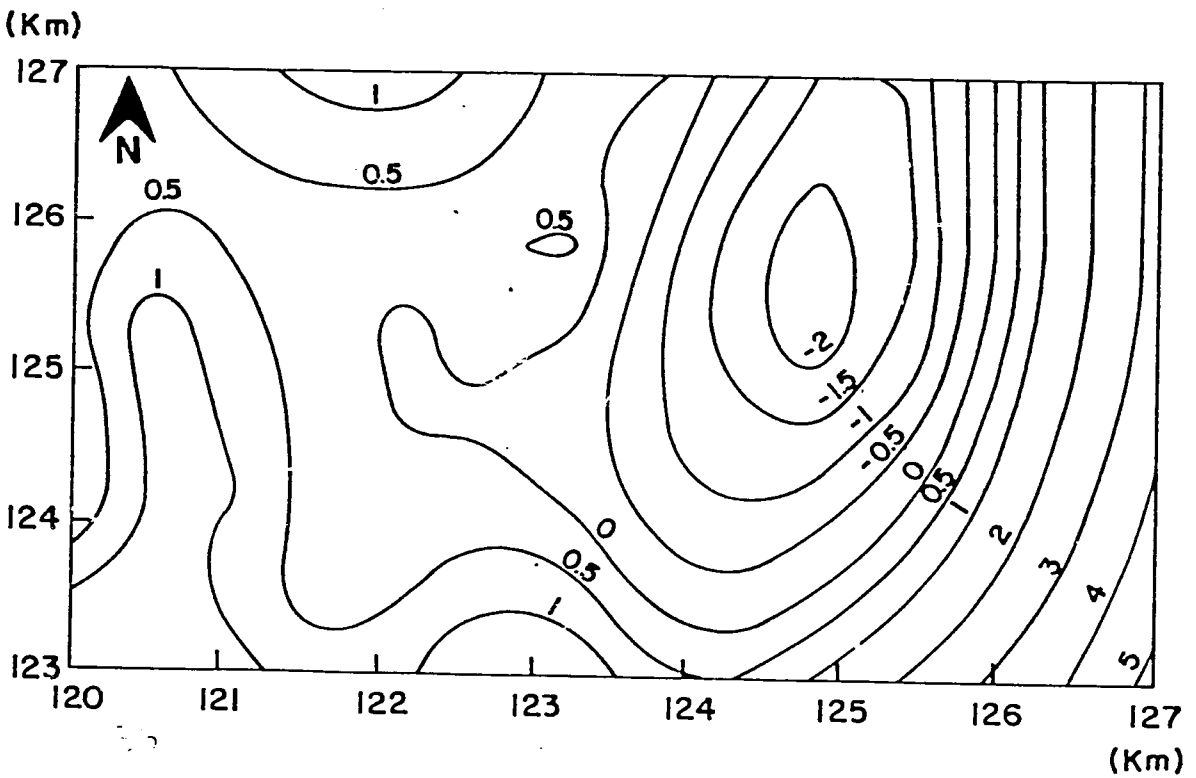

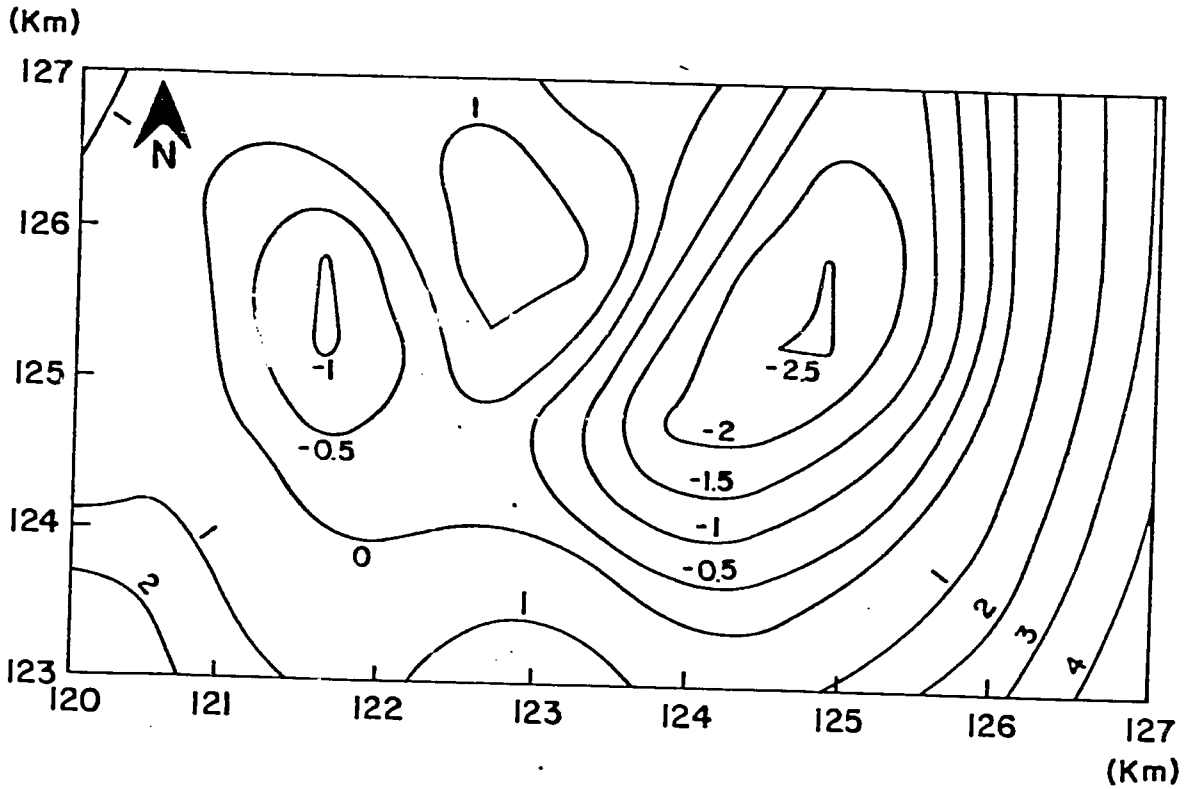

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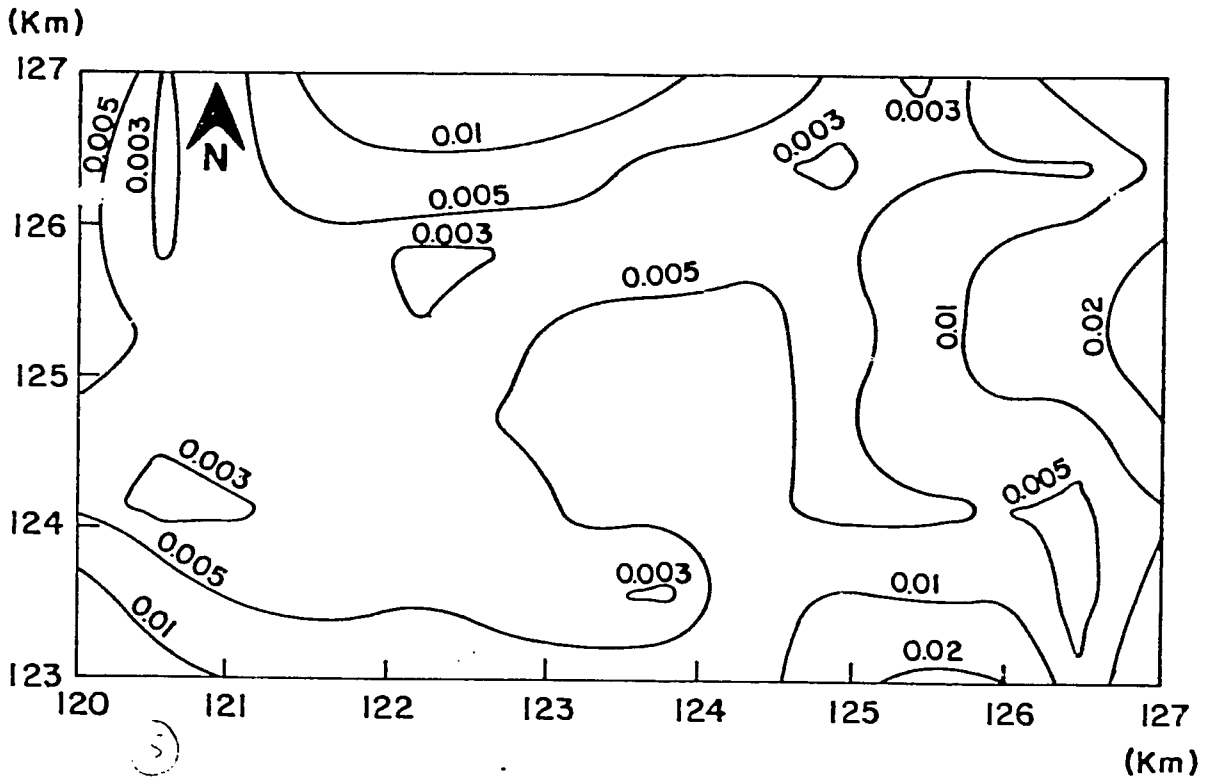

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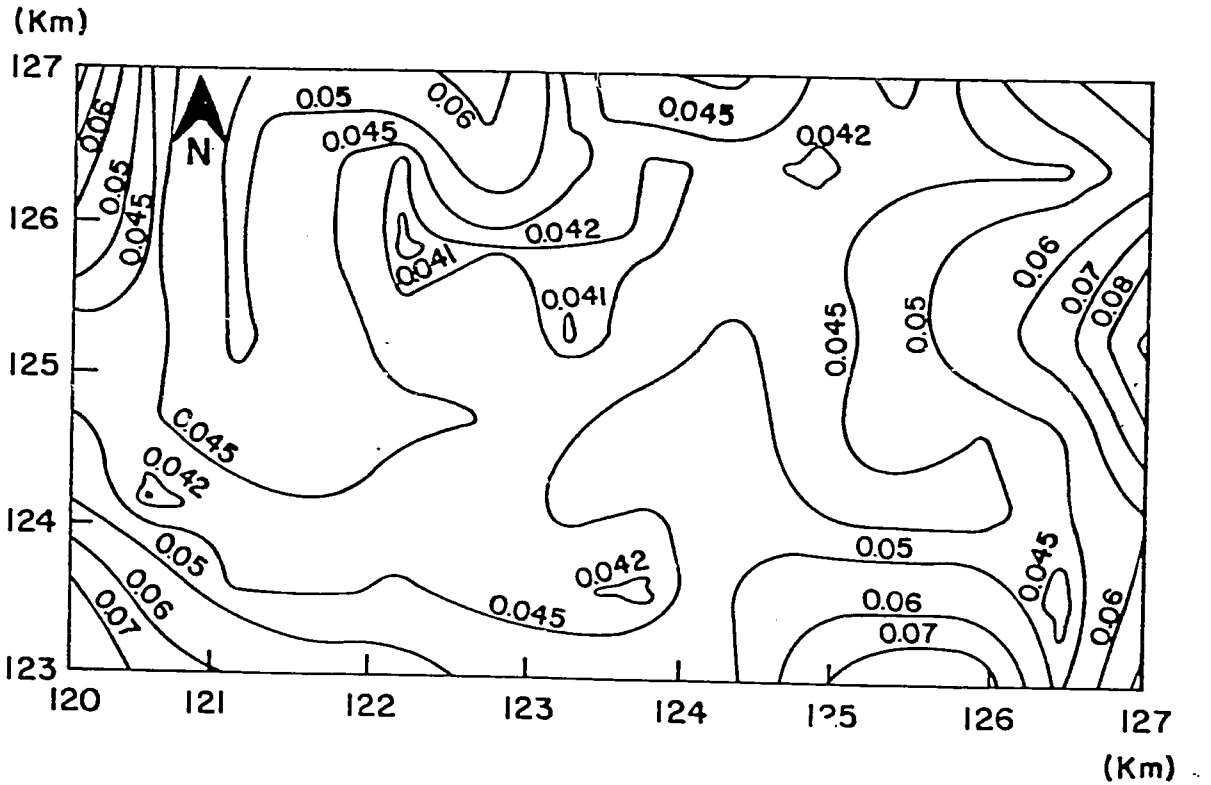
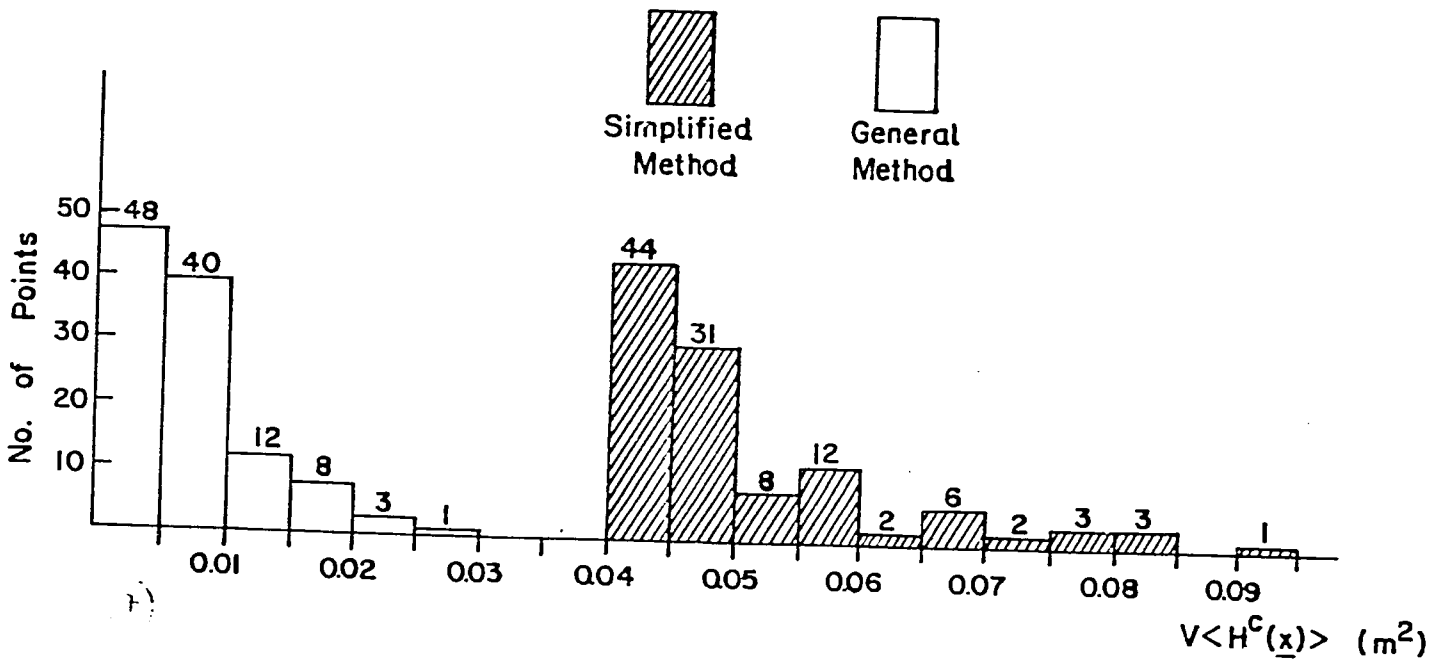
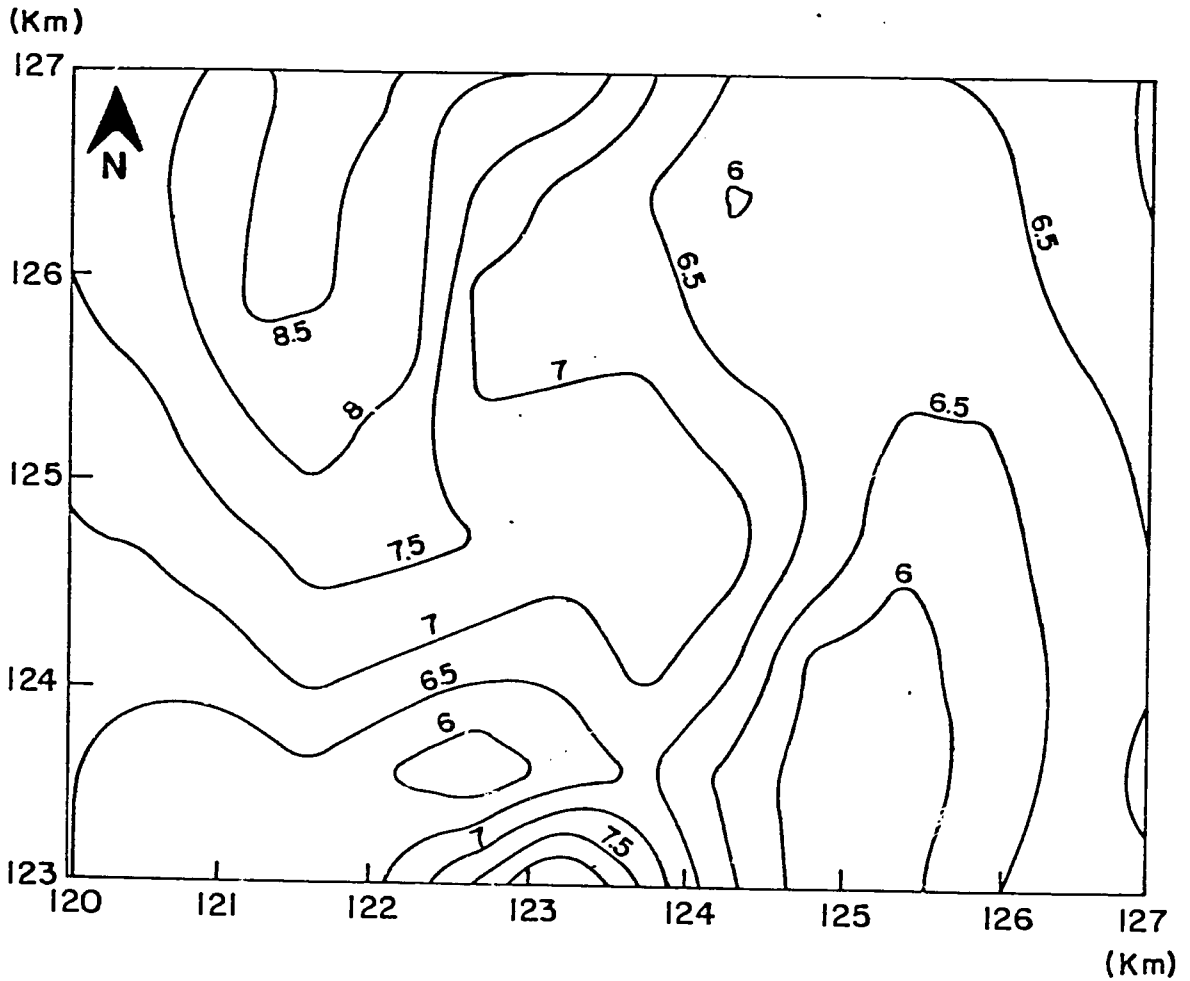

(Km)
127
126
125
124
123
120
121
122
123
124
125
126
127
(Km)
N
0.05
0.06
0.045
0.042
0.041
0.07
0.08


Simplified Method
General Method
No. of Points
50
40
30
20
10
48
40
12
8
3
1
44
31
8
12
2
6
2
3
3
1
0.01
0.02
0.03
0.04
0.05
0.06
0.07
0.08
0.09
V<H^c(x)> (m^2)


(Km)
127
126
125
124
123
120
121
122
123
124
125
126
127
(Km)
N
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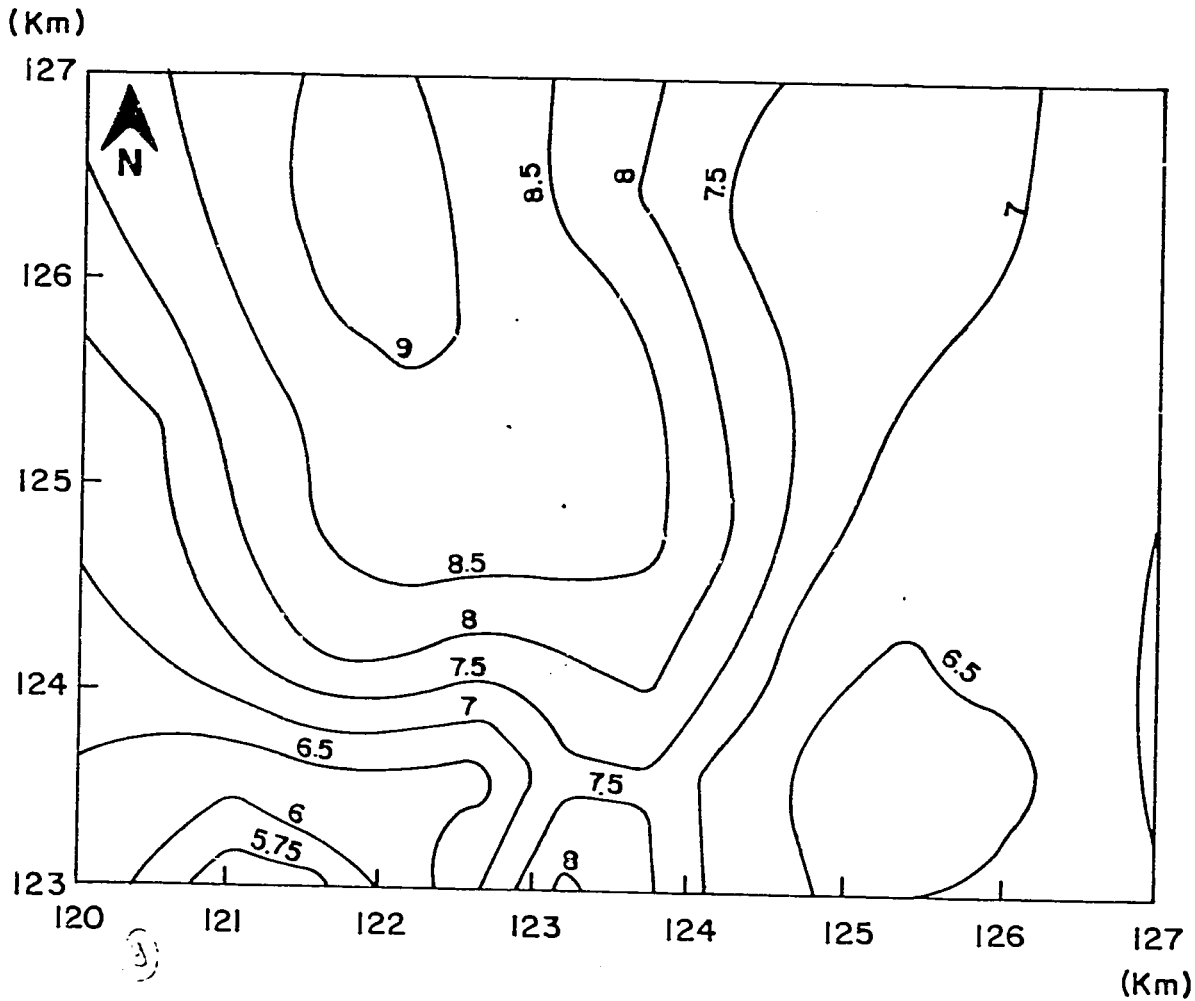
6.5
6.5
6.5
6


(Km)
127
126
125
124
123
120
121
122
123
124
125
126
127
(Km)
N
9
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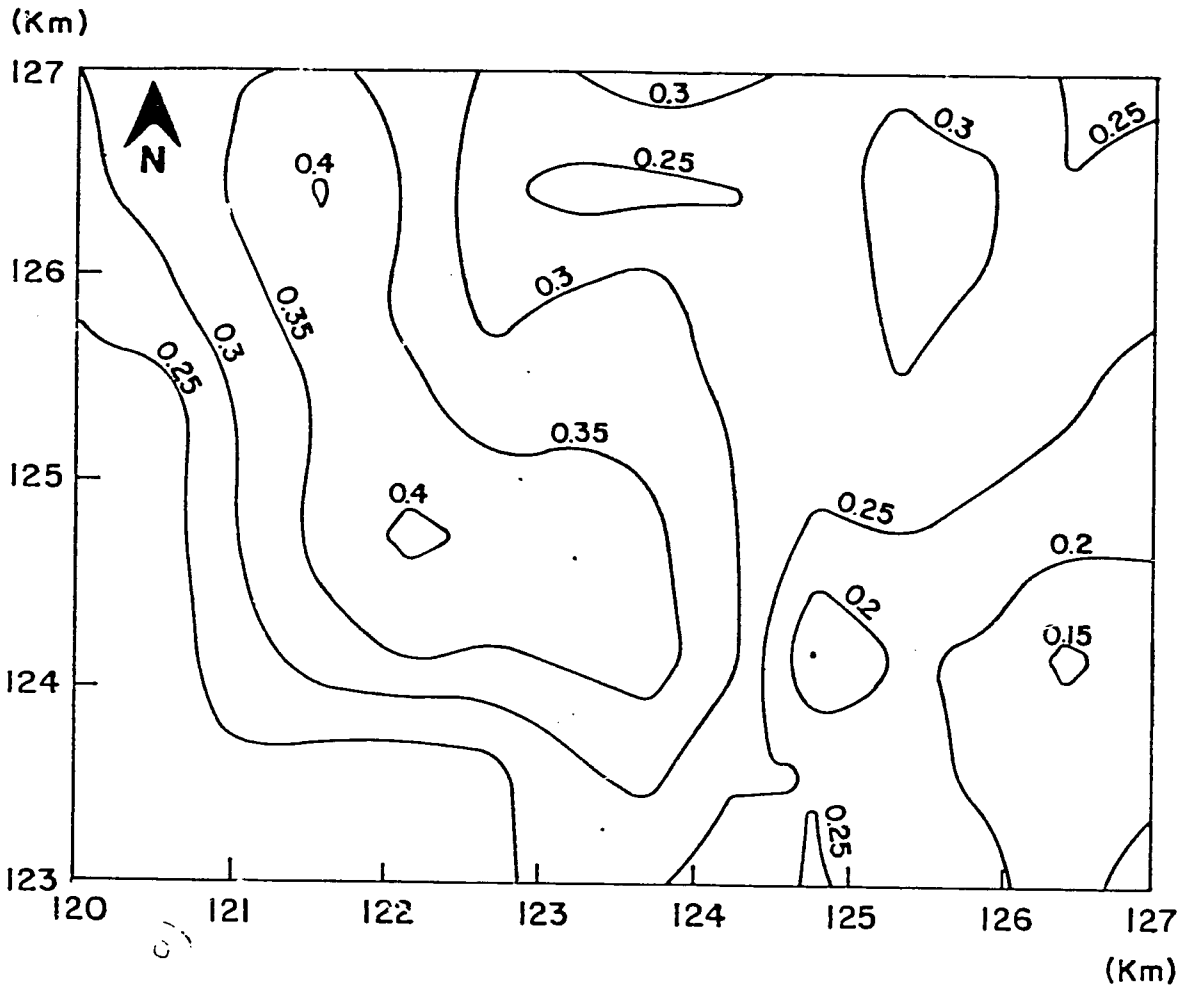
7.5
7
6.5
6
5.75


(Km)
127
126
125
124
123
120
121
122
123
124
125
126
127
(Km)
N
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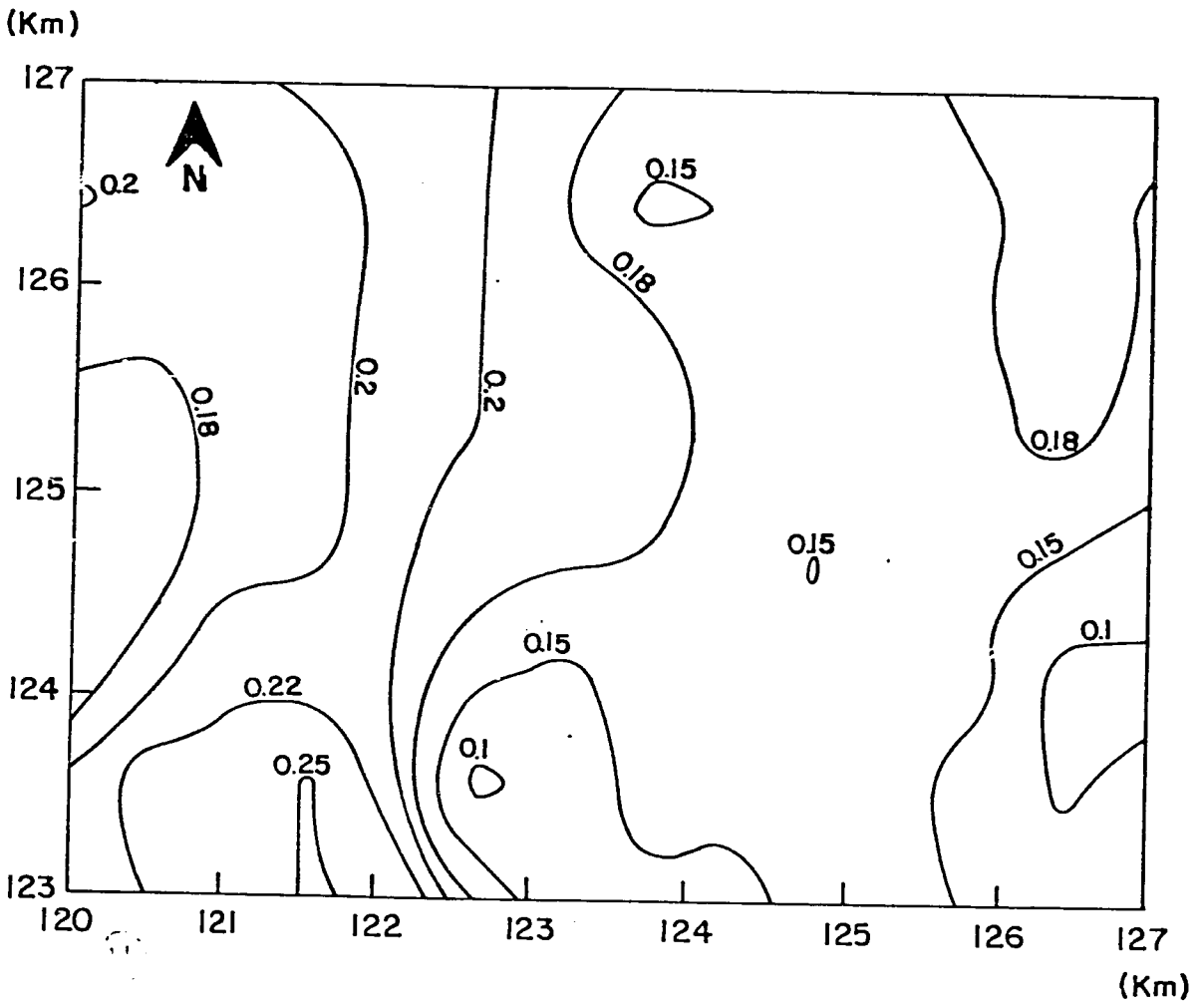
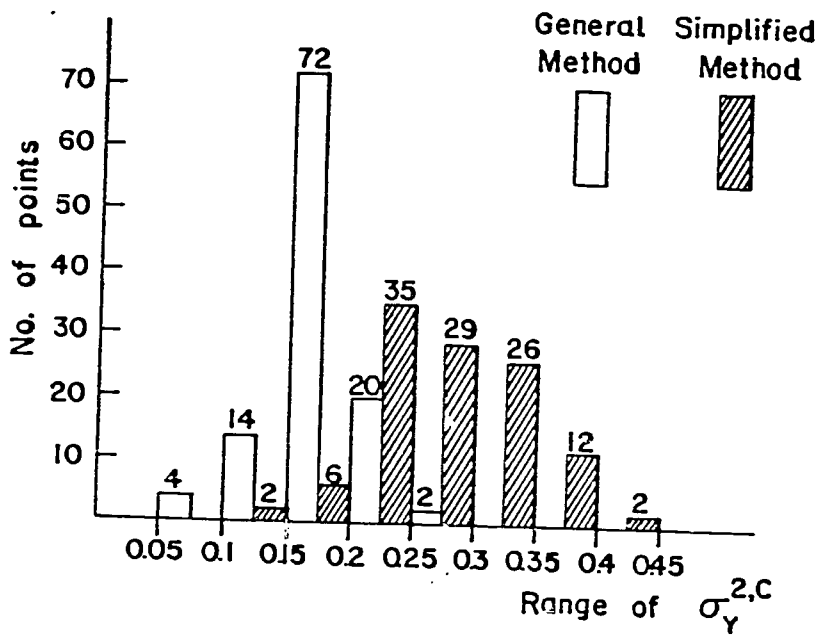
0.3
0.25
0.3
0.25
0.3
0.35
0.25
0.3
0.35
0.4
0.25
0.2
0.2
0.15
0.25


(Km)
127
126
125
124
123
120
121
122
123
124
125
126
127
(Km)
N
0.2
0.18
0.2
0.2
0.15
0.18
0.18
0.15
0.15
0.1
0.15
0.22
0.25
0.1


General Method
Simplified Method
No. of points
70
60
50
40
30
20
10
4
14
2
72
6
20
35
2
29
26
12
2
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
Range of $\sigma_Y^{2,C}$

App 2

C5-M9

2. GEOLOGY AND HYDROGEOLOGY OF THE RIO-MAIOR AQUIFER

The Rio Maior basin is located approximately 80 km north of Lisbon, on the border of the lower Tagus Cenozoic basin. It is a small graben-like depression of a rough rectangular shape running northwest to southeast. The depression is filled up with Pliocen and Quaternary sediments, with a thick layer of sands at the bottom which, in the deepest part of the basin is overlaid by a lens-shaped and very anisotropic sequence of lignites, clays, sands and diatomites, commonly referred to as the lignite-diatomite complex. These lignites have a certain economic value and they are the main justification for the studies that have been carried out at the site. The underlaying layer is a quite homogeneous body and is composed of medium to fine-grained siliceous, white sands containing around 10 percent of kaolinite. This layer outcrops in a dew places at the basin border where it is exploited for ceramic and glass industry. It also constitutes the main aquifer of the basin from where the data used in this study were obtained.

Some boreholes were drilled down to the bottom of this sand aquifer showed it to reach a thickness of about 100 m in its deepest part, and to terminate abruptly against the graben faults.

The boundary formations range from Jurassic to Miocene and they are composed mainly of siltstones, claystones, sandstones, marls and limestones. Fig. 1 shows a sketch geological map of the region and Fig. 2 presents a transversal cross-section of the basin.

Beneath lignite-diatomite complex, the aquifer is of the leaky type, as inferred from its behavior during the pumping tests. Outside this zone it is unconfined although no pumping test was carried in this area. Further information on the hydrogeology of this basin is provided in the paper by Delgado Rodrigues [1983].

The area is crossed by the small and intermitent Caniceira creek, which joins the main water course, the Maior river, in the southwestern border of the Graben basin. The relation between the Caniceira creek and the aquifer is not clearly established but no major contributions are expected to

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come from this source due to the small extension of the creek over the aquifer and to its intermitent character. On the other hand, it is clearly established that the Maior river is the main discharge boundary of the aquifer and permanent flow of not less than 140 liter/sec is estimated to discharge through it. The mean annual precipitation in Rio Maior region is about 800 mm and it occurs mainly between October and April.

Possible recharge sources of the aquifer are precipitation over the unconfined areas, flow from the lateral boundaries and leakage from the upper formations. The water flowing in the overlaying complex is likely to move preferentially in horizontal directions discharging into the underlaying aquifer mostly at the border of the lens-shaped complex where it merges into the underlaying sands.

7. RESULTS

7.1 General Comments

The previously described methodology was applied to the northern region of the Rio Maior white-sand aquifer, which is, in its majority, a confined aquifer (see Section 2). For an overview we have inserted the region under study in a square which has an area of 9 km².

The aims of the study are the following: (i) evaluation of the vector of parameters Θ which models the random Y and H joint field, based on the limited transmissivity data and the sequence of head measurements taken periodically; (ii) Evaluation of the effective recharge of the aquifer; (iii) Drawing of the regional maps of $\langle Y^c \rangle$, $\sigma^{2,c}_{Y}$, $\langle H^c \rangle$ and $\sigma^{2,c}_{H}$, based on the Y and H measurement values.

7.2 The Data

The available data for this study consists of a 19 months sequential set of 18 piezometric head data measurements and also a set of (only) 3 transmissivity values.

The above data, which consists of measurements, constitutes our 'hard-core' data. In addition, there is also the so-called 'soft data' (see Journel, 1986), based on the experience gathered by the authors about this aquifer. This refers to the flat piezometric surface of this aquifer (indicating small recharge coming from the top confining aquitard), and above all to the high level of homogeneity of the hydraulic conductivity, the main heterogeneity source being the thickness variation. The incoorporation of this data in the evaluation process is described in Section 6, and summarized below.

From the initial set of 18 head measurement points a first set of 14 head points was selected. This was done because some of the initial points are located very near to each other, forming a cluster of points which do not provide additional head information. Finally, from the set of 14 points a set of 9 points was selected, by eliminating 5 supposedly outliers, as indicated in Section 6.

The final selected information is formed by the sequence of 9 head measurements on a monthly basis covering the period from January 1985 to July 1986 (19 months), and 3 transmissivity measurements. The limited information on the experimental evaluated transmissivities of the aquifer does not allow for a reliable evaluation of the regional log-transmissivity field based on the usual kriging procedures, and suggests employing the inverse method which can interpret also head measurements.

7.3 *Unconditional Stage*

The unconditional stage of the study was divided in the following steps: (i) Identification of the parameters of the random Y field, namely $(Y)_i$, w_i, $\sigma^2_{Y_i}$, l_i, $(i=1,..,19)$ and the covariance matrix, for each of the 19 months; (ii) Averaging the parameters of Y of the 19 months, namely:

$$(Y)=\sum_{i=1}^{19}\frac{(Y)_i}{19}\,;\ \sigma^2_Y=\sum_{i=1}^{19}\frac{\sigma^2_{Y_i}}{19}\ ;$$
$$l=\sum_{i=1}^{19}\frac{l_i}{19}\,;\ w=\sum_{i=1}^{19}\frac{w_i}{19}\,; \tag{11}$$
$$\Sigma_{11}=\sum_{i=1}^{19}\frac{\Sigma_{11i}}{19^2}\,;$$

etc., all other $\Sigma_{k\ell}$ being $(\Sigma_{k\ell})=\sum_{i=1}^{19}\frac{(\Sigma_{k\ell})_i}{19}$. Here $\Sigma_{k\ell}$ stands for the covariance of estimation of parameters θ_k and θ_ℓ, determined by the ML procedure outlined in Section 4. The results of step (i) are showed in Figs.3, 4 and 5, which show the time series of the maximum likelihood estimators of (Y), σ^2_Y and l, respectively. It is evident that the differences between months are very small, as should be for these structural parameters. Such differences are due not only to variations of the piezometric surface of the aquifer, but also to possible head measurement errors. The small variations with time presented before strengthens the confidence in the model which renders the averaged variances of (Y), σ^2_Y, l and w.

The final results of the unconditional parameters estimates and the covariance matrix of the averaged estimators are given in Table 1. It is emphasized that the smallness of the variances of estimation reflects the accuracy of the ML identification procedure on one hand, and the effect of averaging over 19 months in Eq.11.

7.4 *Recharge Evaluation*

According to Table 1, the net distributed effective recharge R_{ef} from the top confining aquitard is almost zero. This is due to the fact that the parameters related to the recharge, i.e. c_{11} amd c_{22}, have small or zero values. The effective recharge R_{ef} of the aquifer is evaluated through the relationship (see Eq.6):

$$R_{ef} = R - S\,\frac{dH_0}{dt} = T_G\,(c_{11}+c_{22}) \tag{12}$$

where H_0 is the expected value of the piezometric head of the aquifer at the origin and S is the storativity. In our case, since c_{11} and c_{22} are very small, this expression may be reduced to:

$$R = S\,\frac{dH_0}{dt} \tag{13}$$

which indicates that the recharge through leakage is counterbalanced by the increase in storage. The value of S was obtained by averaging the 3 available storativity measurements (located at the same points where the transmissivities were measured). Those values of S are very close to each other, yielding the average value $S = 1.1\times10^{-3}$. The values H_0 were calculated for each one of the 19 months, based on the head measurements for each month.

The recharge was computed by using (13), in which dH_0/dt has been replaced by $\Delta H_0/\Delta t$, $\Delta t = t_k - t_{k-1} = 1$ month, and $H_{0,k}$ has been identified in each month by the ML procedure. Fig.6 shows the time series of the evaluated recharge for the Rio Maior aquifer, according to Eq.12. Fig.7 shows the time series of the precipitation in Rio Maior. Comparing both figures one observes a good agreement between the shapes of the recharge and of the precipitation, with a delay of about two months for the recharge.

7.5 Conditioning of the logtransmissivity Y

The conditioning of Y was done on the measurements of Y and H for each one of the 19 months (for details of the procedure see Dagan, 1985). For this purpose, use was made of the parameters of Y and of the covariance matrix averaged over 19 months, shown in Table 1. In contrast, for the head H use was made of the estimates of H_0, J_x, J_y, c_{11}, c_{22} and c_{12} from each of the 19 months, since these parameters naturally depend on head measurements. Finally, the evaluated monthly results for Y were averaged for the 19 months.

Fig.8 shows the final regional log-transmissivity map of the aquifer, based on the final averaged parameters. The similarity between this map and the map of Rio Maior's bed rock isolines, Fig.9, is emphasized. Notice that the top of the aquifer is almost horizontal and, as referred to previously, the hydraulic conductivity of the aquifer is considered to be a very homogeneous one. Thus, the transmissivity of the aquifer is strongly corelated with the thickness of the aquifer.

Fig.10 show the regional maps of $\sigma^{2,c}_{Y}$, i.e. the conditional generalized variance of Y (i.e., including the parameters estimation errors in the conditioning procedure) following the procedure of Rubin and Dagan (1987).

7.6 Conditioning of the head H

To complete the study, the conditioning of the piezometric heads of the aquifer was carried out for two different months, one in the winter time (January 1986) and the other in the summer time (July 1986).

This conditioning was done with the averaged values of Y parameters, shown in Table 1, and the values of H parameters evaluated for each one of the two referred months.

Figs.11 and 12 show, respectively, the maps of the expected values of the piezometric heads obtained for January 1986 and those obtained for July 1986. As expected the piezometric surface of the aquifer, in both cases, is rather flat, showing also little differences in the piezometric heads between

summer time and winter time.

Finally, Fig.13 presents the variances of the generalized conditional H for the month of July, 1986. It is seen that the values of $\sigma^{2,c}_{H}$ are negligible in the majority of the region under study, showing a good knowledge on the piezometric heads of this aquifer.