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INTERACTIONS BETWEEN THE GOVERNMENT AND THE PRIVATE SECTOR: AN ANALYSIS OF GOVERNMENT EXPENDITURE POLICY AND THE REFLECTION RATIO

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I. INTRODUCTION

Theoretical models of underdeveloped countries often draw policy conclusions concerning various development strategies without explicitly taking into account the role of the government. The focus is usually on the relationship between agriculture and industry rather than between the private and public sectors. Yet to ignore the specific contribution of the government as a provider of crucial development inputs or to fail to consider the government as a decision maker having its own set of preferences is to omit an important part of the development model. The purpose of this paper is to introduce the government as a sector having its own set of objectives, instruments, and constraints and to explore the resulting interactions between the government and the private sector.

There are a number of important characteristics of the government sector in underdeveloped countries that deserve special attention. First, a significant share of government activity in developing countries has a directly productive effect on other sectors of the economy. Government financed infrastructure and education, for example, often form a major part of the physical and human capital stock of the country. Government services in transportation, communications, research, peace and order, etc. are intermediate goods which affect the level of productivity in the private sector. Expenditure policy is thus a crucial instrument of development strategy.

Second, the capacity of the government to earn revenue is limited severely by the costs of collecting taxes and by political and ideological constraints on the tax structure. In many underdeveloped countries, the

largest share of revenue is derived from indirect taxes on a limited number of exported or imported commodities. The revenue of the government depends therefore upon the growth of taxable sectors.

Finally, the government sector can appropriately be viewed as an institution within society having its own goals and preferences some of which may be in harmony with the objectives of the private sector and some of which may be in conflict. These goals are determined by the specific political process of the country and reflect the interests and power of various pressure groups as well as the desires of the state bureaucracy and ambitions of the ruling elite. In technical terms, we cannot assume the government is in all cases attempting to achieve Pareto efficiency for the country as a whole but instead we must view the government as maximizing specific goals of its own subject to specific constraints.*

These principles of productive expenditure, limited tax capacity, and specific government preference functions, taken together, imply a quasi-market mechanism to determine the growth of the government sector and its impact on the private sector. If government expenditure policies fail to stimulate the growth of the economy, and in particular those sectors from which it derives its taxes, government revenue ceases to grow, and its expansion must come to a halt. For survival and growth, the government must allocate some of its resources in directions that will generate income. This, however, sets limits on government behavior within which it chooses according to its preference function.

*See C.P. Kindleberger, "Group Behavior and International Trade".

The Reflection Ratios

Formally, we may derive the relevant relationship between the private and public sectors as follows. The size of the government sector is constrained by its budget equation

(1.1) $G = R + B$

where G equals total expenditures, R total revenue and B net borrowing. Ignoring B for the moment, the size of G and its rate of growth through time depends upon the level and rate of growth of R. The point of departure for this article is that there is a functional dependence of R upon G which may be called the reflection ratio.

Our first principle noted above says that the level of activity of various sectors of the economy is functionally related to the expenditure policy of the government. This relationship can be written as

(1.2) $X = F(g)$

where X is a vector of indices of economic private economic activity, and g a government expenditure vector whose elements $(g_1, g_2, \ldots g_n)$ denote the level of activity of a particular government function.*

The second principle states that government revenue will depend upon the vector of private economic activities

(1.3) $R = tX$

where R equals total revenue and t is a tax vector whose elements are the given tax rates associated with each private economic activity. We assume for this paper that the tax structure represented by this vector

*We assume the following conditions:

$$X = \bar{X} \text{ if } g = 0, \frac{\partial X}{\partial g} > 0, \frac{\partial^2 X}{\partial g^2} < 0.$$

tends to be stable over time. Our primary concern is to analyze the effect of changing g, given t as a constraint. In underdeveloped countries, it can reasonably be argued that governments have only limited scope for changing t within a given economic structure. In the short run it can thus be viewed as exogeneous. An analysis of changes in t, especially the discontinuous jumps that occur with economic revolution, is beyond the scope of the present paper.*

Combining these equations we obtain the reflection ratio

(1.4) $G = t\,F(g) + B$

which indicates that the level of government expenditures is functionally determined by its composition.

Another type of reflection ratio can be devised as follows. The government sector requires certain inputs from the rest of the economy, e.g., imported goods, labor, raw materials, etc. But government expenditure influences the supply curve of these inputs. Government help to export industries, for example, increases the supply of foreign exchange, while government help to agriculture lowers the price of food and hence the supply price of labor and intermediate goods, and government expenditure on education increases the supply of skilled personnel. These rela-

*Although we are assuming this feature as a stylized fact of underdeveloped countries, considerable empirical estimation remains to be done. This hypothesis implies that a regression of revenue on the level of activity in key sectors would yield stable parameters and a high correlation coefficient over long periods of time. It is to be expected that the structure might shift at given points of time such as when a country moves from colonial to independent status but that it would remain stable within a given period. Data exist for testing this hypothesis, though the relevant investigations have not yet been made.

tionships generate a second type of feedback of government expenditure on government expenditure.

This general relationship between governmental inputs and its own expenditure can be illustrated in the following simple model. Assume the government uses only one factor of production, labor (L), and the amount it can employ is equal to total revenue (R) divided by the wage rate (w). If we define the productivity of each worker as a, the total output of the government sector is then given by

(1.5) $G = \frac{a}{w} R.$*

A certain portion of total government expenditure, say, g_2 is assumed to have a direct effect on either the productivity of government labor (a) or its cost (w). The second type of reflection ratio can then be derived as

(1.6) $\frac{a}{w} = \rho(g_2).$

A Model of the Two Types of Reflection Ratios

We can now summarize our basic relationship between the private and public sectors in the following simplified set of equations:†

(2.1) $G = \frac{a}{w} R$

(2.2) $g_0 = G - g_1 - g_2$

(2.3) $R = \rho_1(g_1)$

(2.4) $\frac{a}{w} = \rho_2(g_2).$

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*Formally, we may consider the government having a cost constraint $R = wL$ and a production relationship $G = aL$. Solving we derive (1.5).

†We have ignored net borrowing of the government (B) in this model.

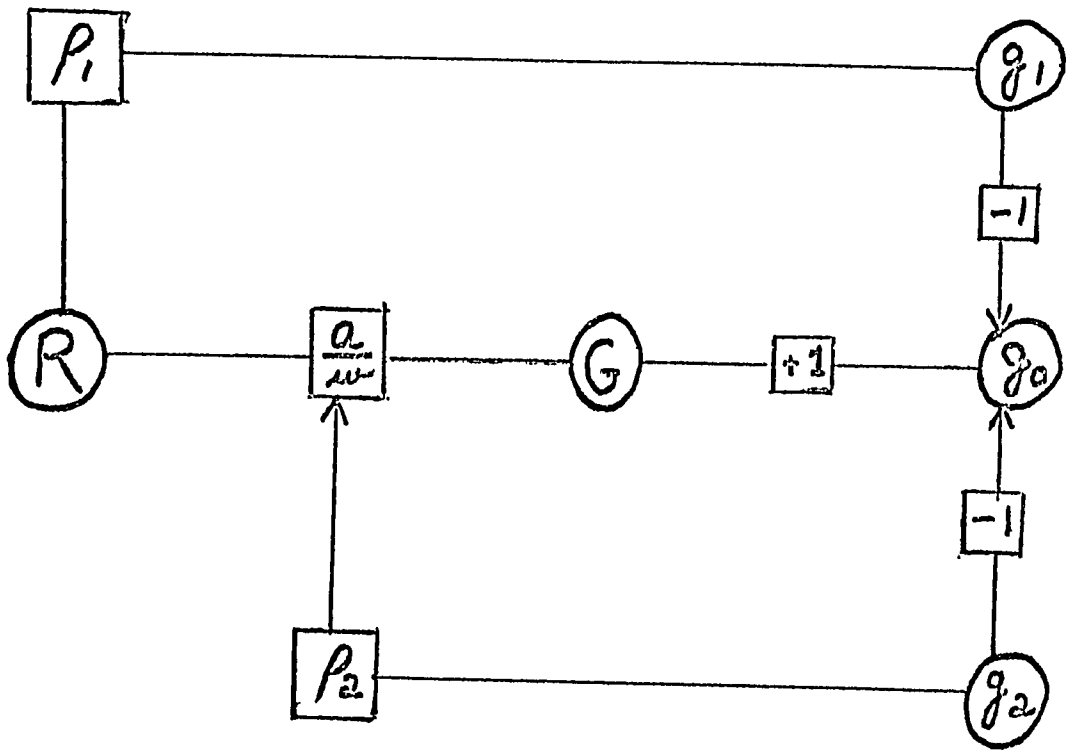
Equation (2.2) states that government activity can be divided into three kinds: g_0 which has no directly productive effect on the economy in the period under consideration but is either a government consumption item or a long range development activity, g_1 which has a direct effect on output in the private sector and hence on the government's revenue as described by equation (2.3); and g_2 which has a direct effect on either the productivity of labor in the government sector or its cost [equation (2.4)]. The total output of the government as given by (2.1) can then be rewritten as

$$G = \rho_2(g_2)\ \rho_1(g_1).$$

This model can be seen schematically in Figure 1 which demonstrates the two feedback loops from government expenditure to government expenditure. This illustrates, for example, that even if the government is interested in maximizing development expenditure such as g_0, it must spend certain sums on g_1 and g_2 because of their indirect effects in producing g_0.

FIGURE 1

Two Feedback Loops from G onto G



Model

$$G = \frac{a}{w} R$$

$$g_0 = G - g_1 - g_2$$

$$R = \rho_1(g_1)$$

$$\frac{a}{w} = \rho_2(g_2)$$

II. THE GOVERNMENT'S CHOICE

The problem confronting the government in choosing the optimal level and allocation of expenditure is illustrated in Figure 2. For the present we are considering only the first type of reflection ratio, i.e., ρ_1 or the feedback from increased tax revenue. As before, B is set equal to zero. It is further assumed in the background that there are three sectors: X_1, a taxed export or manufacturing sector; X_2, a non-taxed large agrarian and service sector which supplies an unlimited amount of labor at a constant wage; and G, the government sector whose activity affects X_1.

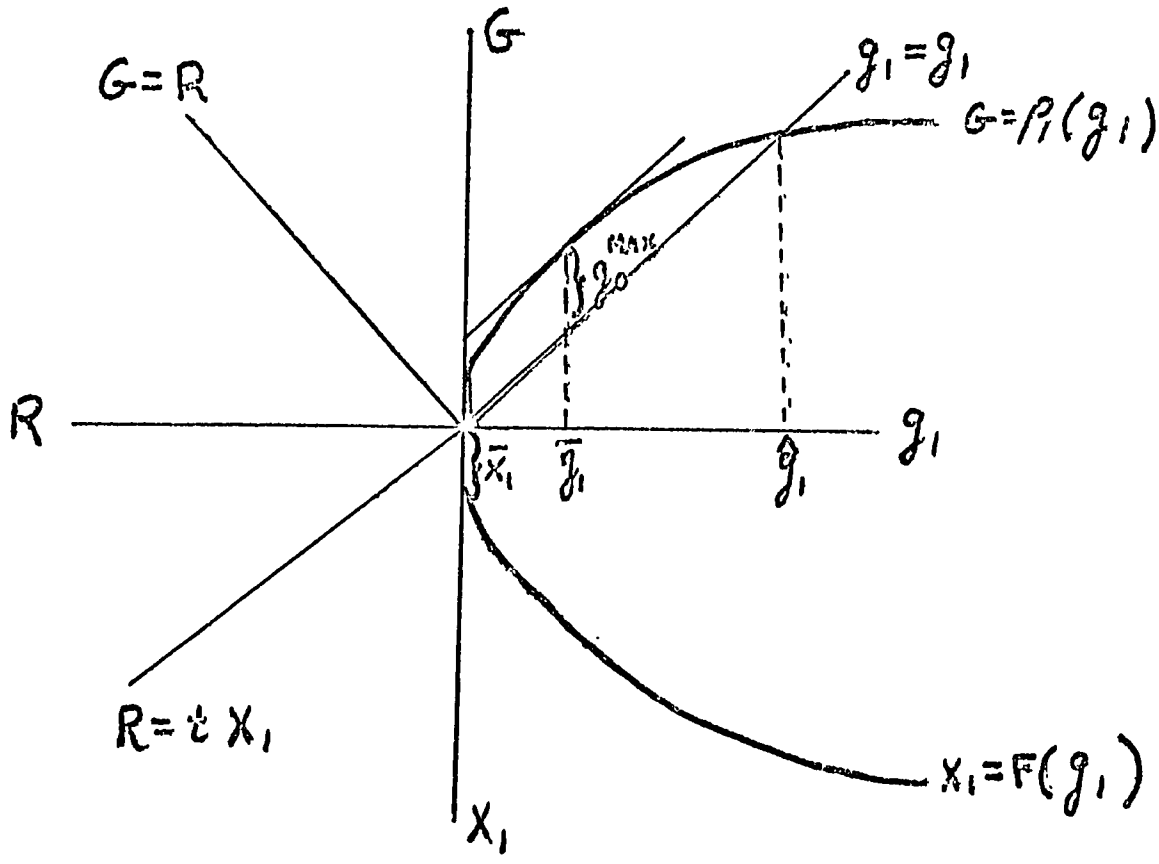
The reflection curve is pictured in quadrant I which shows the total level of government expenditure as a function of the amount allocated to g_1. It is derived as follows:

Quadrant IV shows the productivity of the government on the private sector according to $X_1 = F(g_1)$ where the curve is concave downward due to diminishing returns, $F' > 0$, $F'' < 0$. If the government set $g_1 = 0$, it is assumed that the level of private output would be $X_1 = \bar{X}_1$.

Quadrant III indicates the relationship between activity in the private sector and the tax revenue of the government. We have assumed taxes are a constant proportion of activity in X_1 but could easily explore the case where taxes are an increasing or decreasing proportion. It should be noted that we have assumed that taxes have no disincentive effect on production. This is not realistic but could be relaxed by making the revenue function concave to the X_1 axis thereby changing the shape of the reflection curve in the first quadrant.

FIGURE 2

The Government's Choice



Model

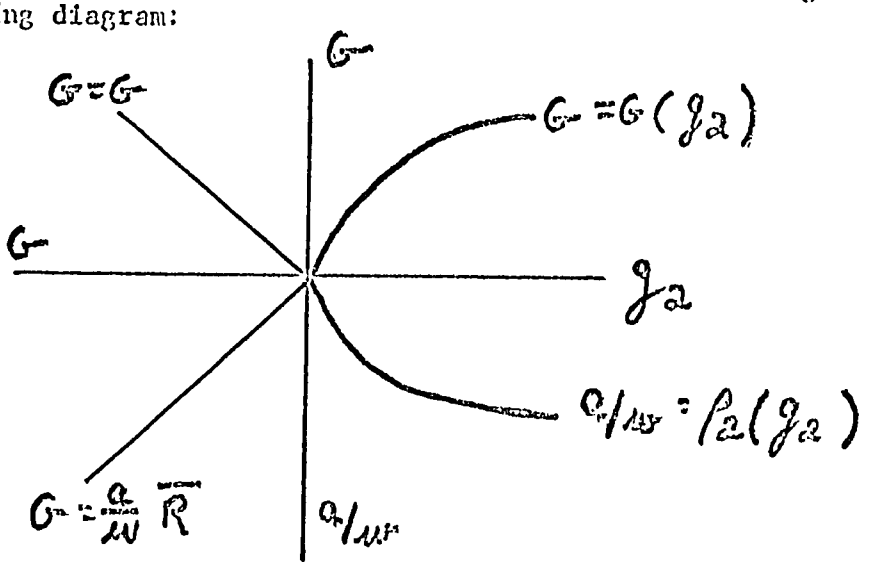
$X_1 = F(g_1)$	Productivity of Government ($F' > 0$, $F'' < 0$)
$R = t\,X_1$	Revenue Function
$R = G$	Balanced Budget ($B = 0$)
$G = \rho_1(g_1)$	Reflection Curve ($\rho_2 = 0$)

The second quadrant shows the relationship between revenue and government expenditure. Assuming a balanced budget, $R = G$, the relationship is a straight line with a 45° slope.

The reflection curve in quadrant I tells us the total amount of government expenditure associated with any level of expenditure on g_1. It is derived by choosing various initial levels of g_1 which determine X_1, then R, and finally back onto G.* The horizontal difference between the reflection curve and a 45° line indicates the surplus available to the government for expenditure on g_0 ($g_0 = \rho_1(g_1) - g_1$).

What is the optimum point for the government? It is immediately evident that there is no obvious single best point in the absence of a social welfare function to evaluate the desirabilities of various combinations of government and private activity. Thus we must introduce

*Given our assumptions, the reflection curve is the mirror image of the productivity function in quadrant IV, or $\rho_1' > 0$, $\rho_1'' < 0$, and $G = \bar{G}$ when $g_1 = 0$. We may also note that our second type of reflection relationship $\frac{a}{w} = \rho_2(g_2)$, could be derived in a somewhat similar manner given $\bar{R}$ as in the following diagram:



our third principle of government behavior. It is unrealistic to assume that the government in underdeveloped countries always maximizes some vague notion of "general welfare" representing somehow the combined interests and views of the population as a whole. It is also unrealistic to assume that the government always strives to achieve Pareto optimality and then redistributes using lump sum taxes and transfers. A particular government is pushed and pulled by its own views of the world and by political pressures of various groups both internal and external. We assume instead that the government (i.e. the state) in an underdeveloped country has its own welfare function possibly different from a large section of the private sector. It is appropriate therefore, to analyze problems in terms of the implications and contradictions of various possible social welfare functions.

Suppose we make the crude assumption that the government's only interest is g_0. The X_1 sector, for example, may be a foreign firm operating in the export sector of no interest to the government except for the revenue it provides through taxes which can then be spent on armies, monuments, or development. The government would then choose the point $\bar{g}_1$, where g_0 is a maximum.*

Another crude assumption, with quite different effects, is that the government's only interest is in its total size. It may, for example, try to maximize G regardless of composition because of the employment generating aspects. The government would then chose the point $\bar{g}_1$ where g_0 is equal to zero. This is the point which maximizes the total size of X_1 as well

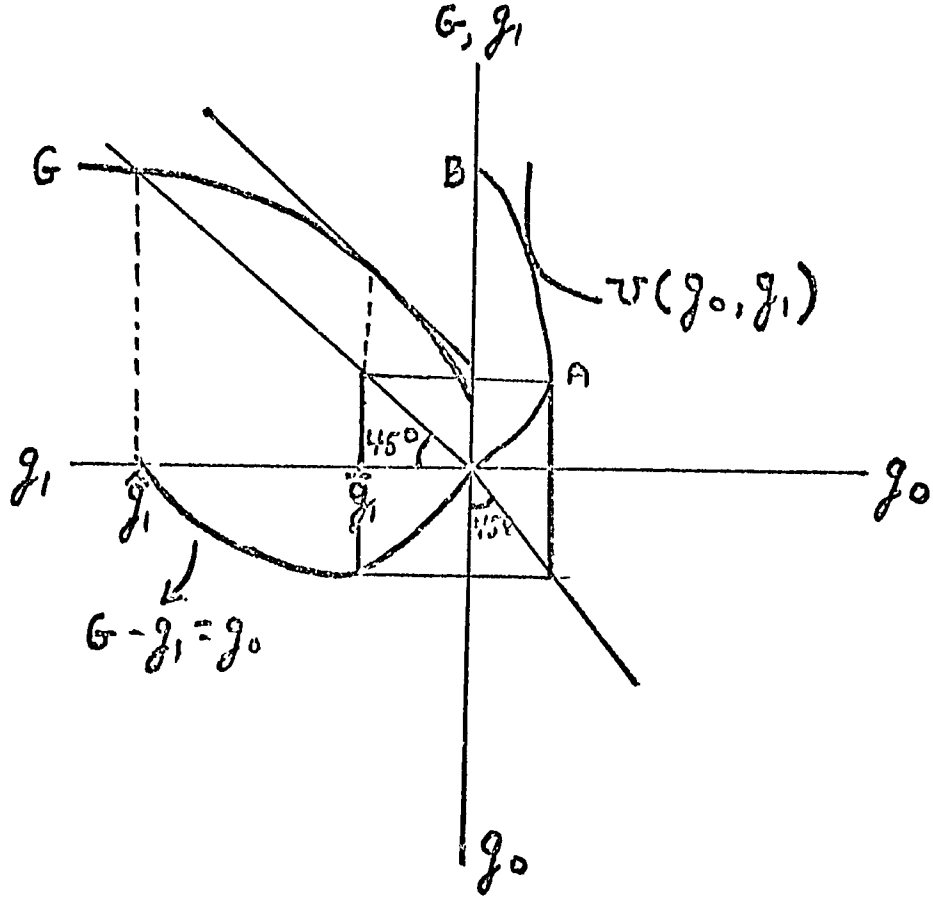
*$g_0 = \rho_1 (g_1) - g_1$
g_0 is at a maximum when $\frac{dg_0}{dg_1} = 1$ or when $g_1 = \bar{g}_1$.

because of the particular assumptions of this model. A government choosing this policy would therefore obtain the largest possible combined employment in the export plus government sector, at the expense of the rest of the economy if g_0 were considered to be partly development expenditures with a long gestation period.

In Figure 3, we can summarize the various distributions between g_0 and g_1 (quadrant I) from the government's point of view. A social welfare function, $U(g_0, g_1)$ is drawn to indicate one possible solution equating the marginal rates of substitution and transformation. Our two limiting points, A and B, are indicated to show the range of the government's choice.

Neither of these extremes, however, is sufficient to describe government behavior in a complex world. In actual fact, the government will assign utility weights to a number of objectives: employment, output, size of the private sector, degree of openness of the economy, etc. The proposition remains empirically empty as long as we do not know the content of the government's preference function. Nonetheless, the above analysis contains an important lesson for research on the structure and performance of economies and the evaluation of national income. The economic record of a country does not merely reflect technological production functions and factor supplies but also the tastes of the government. Models which omit this latter feature, and this is the case in most theoretical and empirical models of underdeveloped countries, are therefore ~~un~~inis specified to the extent that the government sector is an important force in the economy.

Figure 3



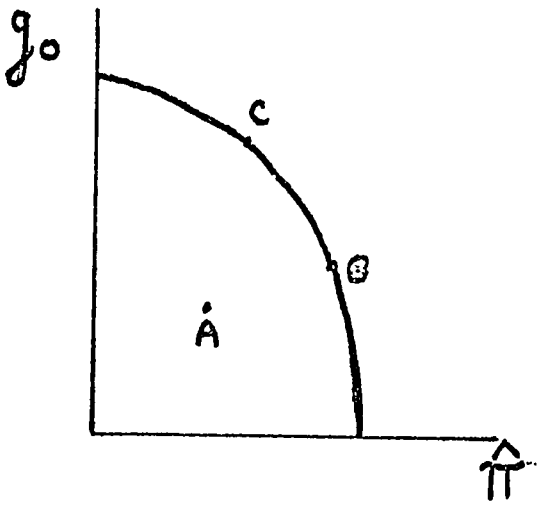
III. A BARGAINING MODEL

The reflection ratio as derived in the ~~proceeding~~ preceding sections focuses on the allocation of government expenditure solely from the point of view of the government itself. For a given tax rate, the government surplus g_0, rose to a maximum and then fell as increasing amounts were spent on "productive" activities, g_1 or g_2. Given the government's preference function, we were able to indicate the choice of the policy instrument, g_0, which maximized the government's objective function.

The government, however, does not act in a vacuum since its choice of expenditure policy has a direct effect on output and profits in the private sector. A simple bargaining model, taking into account the preferences of the private sector, can illustrate the regions of conflict and complementarity between the government and the private sector in the choice of policy instruments.

In Figure 4, we have drawn an opportunity locus or bargaining curve between various combinations of the private surplus (net of taxes), $\hat{\pi}$, and public surplus, g_0. It is obvious from our preceding analysis that variations in t and g_1 will affect the surplus of both the government and private sector. If the economy is within the frontier, say at point A, then a change in t or g_1 will make both sectors better off by moving to, say, point B on the frontier. There is then a complementary relationship between the two surpluses for given changes in t or g_1. Once at point B, however, a trade-off between private and public surplus exists and a potential movement to point C must involve us with a political bargaining process or the specification of a social welfare function, $U(g_0, \hat{\pi})$, for the entire

Figure 4



economy. In the following discussion, we will derive this opportunity locus and provide some possible reasons why certain underdeveloped countries might end up within the frontier.

The bargaining model is characterized by two equations relating the government surplus (g_0) and the private surplus ($\hat{\pi}$) to the two policy instruments, the rate of tax on profits (t) and the level of productive expenditure (g_1). The government surplus is defined as the excess of revenue over expenditure on g_1 and the private surplus as after tax profits:

(1) Government surplus equation $\quad g_0 = t\pi - g_1$

(2) Private surplus equation $\quad \hat{\pi} = (1 - t)\pi$

where the range of the variables is restricted so that t lies between 0 and 1, and g_0 is always positive.

The family of government <u>iso surplus curves</u> will be U-shaped as pictured in Figure 5 (the diagram has been drawn to scale using specific analytical functions described in the appendix). The slope of this curve is defined as follows:

$$\frac{dt}{dg_1} = -\frac{\frac{\partial g_0}{\partial g_1}}{\frac{\partial g_0}{\partial t}}$$

The denominator of this expression, $\frac{\partial g_0}{\partial t}$, is always positive since for a given expenditure on g_1, an increase in the tax rate will increase revenue and hence the government surplus. The numerator is positive for low values of g_1 and then becomes negative. As we saw in Figure 2, the government surplus at first increases for a given tax rate as more is spent on g_1, but then decreases after the point where the marginal produc-

tivity of g_1 $(\frac{\partial \pi}{\partial g_1})$ falls below $\frac{1}{t}$. This can be shown algebraically from equation 1:

$$\frac{\partial g_0}{\partial g_1} = t\frac{\partial \pi}{\partial g_1} - 1$$

$$\therefore \frac{\partial g_0}{\partial g_1} \gtreqless 0 \text{ as } \frac{\partial \pi}{\partial g_1} \gtreqless \frac{1}{t}.$$

It should be noted in Figure 5 that the turning point occurs at large values of g_0 the higher is t. The shape of the <u>iso government surplus curve</u> is thus negative and then positive as the numerator changes sign with increasing g_1. The turning point shifts upward and to the right for higher <u>iso government surplus curves</u> (the reader is again referred to the appendix for a formal derivation using specific analytical functions).

The <u>iso profits curve</u> is much simpler to derive because an increase in g_1 always has a positive effect on profits after tax while an increase in t always has a negative effect. The slope of the <u>iso profit curve</u> is therefore always positive (see Figure 6)*

$$\frac{dt}{dg_1} - \frac{\frac{\partial \hat{\pi}}{\partial g_1}}{\frac{\partial \hat{\pi}}{\partial t}}$$

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*From equation 2, we have $d\hat{\pi} = \frac{\partial \hat{\pi}}{\partial g_1} dg_1 + \frac{\partial \hat{\pi}}{\partial t} dt$

$$= (1 - t)\frac{\partial \pi}{\partial g_1} dg_1 - \pi\, dt$$

Setting $d\hat{\pi} = 0$ to derive our <u>iso profit curve</u>, we have $- \frac{(1 - t)\frac{\partial \pi}{\partial g_1}}{-\pi}$ which is clearly positive. Figure 6 is drawn to scale according to the derivation found in the appendix.

Figure 5

Iso-Surplus Curves

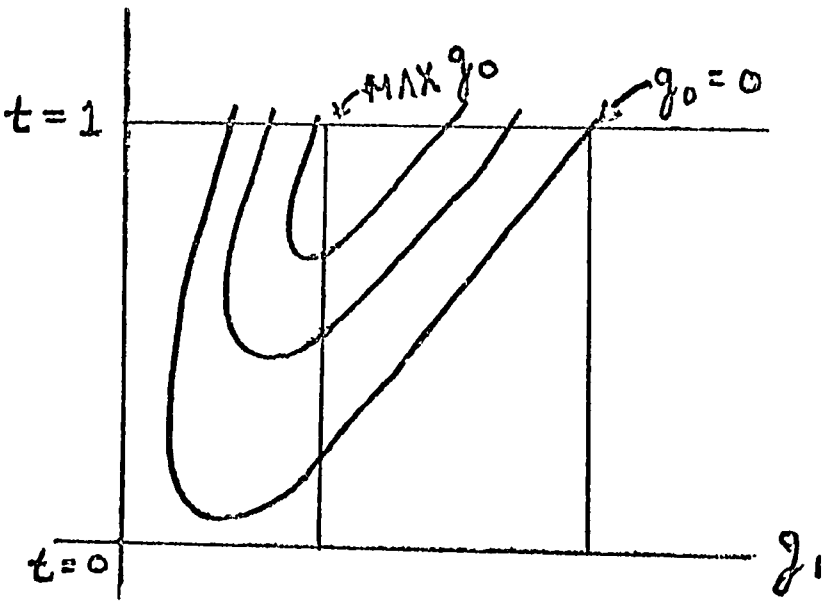
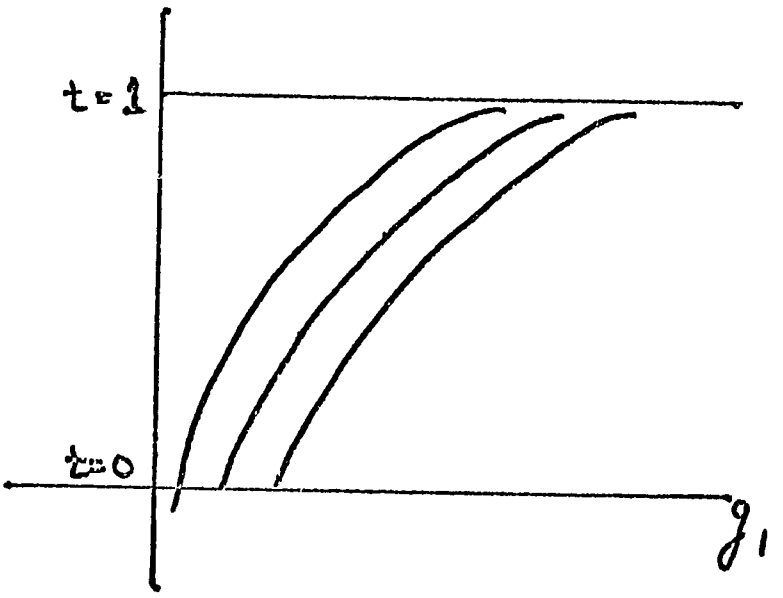


Figure 6

Iso-Profit Curves



The iso government surplus curve and the iso profit curve can be superimposed on an Edgeworth Bowley type diagram (Figure 7). The tangencies of iso profit and iso surplus curves yield a contract curve showing the trade-off between $\hat{\pi}$ and g_0 with optimal combinations of t and g_1. If we map the points on this contract curve onto a $\{\hat{\pi}, g_0\}$ space, we then derive the opportunity locus as in Figure 4.

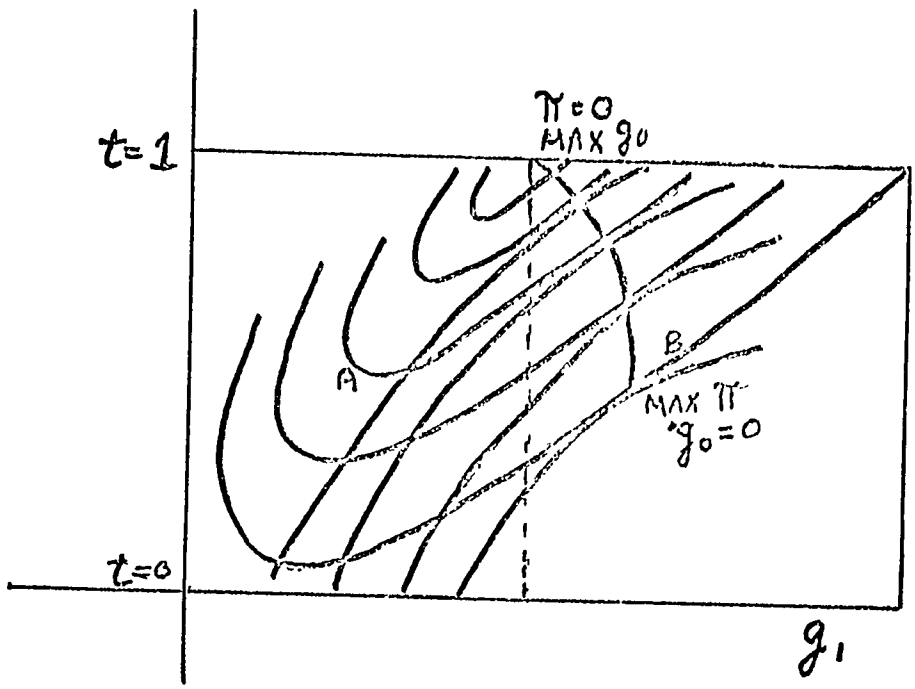
A theory of bargaining as well as a theory of politics would be necessary to predict the eventual resting point. We may for the moment confine ourselves to one case to illustrate that many countries may not be on the contract curve.

Suppose we begin with a given tax rate $\hat{t}$. The government's expenditure policy is then a straight line parallel to the g_1 axis and perpendicular to the t axis. As g_1 increases, g_0 increases up to point A and $\hat{\pi}$ increases up to point B which is beyond A. Suppose the government chooses to maximize g_0 by resting at A. It is obvious that both parties could be made better off by increasing t and g_1 in some combination that moves the economy to the contract curve. Will such a move necessarily occur? The private sector may very well resist it. It may prefer a lazy incompetent government to an efficient one. An efficient government would move to the contract curve, but once there, might decide to move along it by squeezing profits. It may be in the private sector's interest to keep the government as a satisficer by giving it enough g_0 to keep it stable and content, even though this sacrifices efficiency.

This simple analysis covers only two variables. In the real world, the government would no doubt be interested in other targets (employment, output, etc.). These also vary as g_1 varies. A specification of social

Figure 7

Contract Curve Between Private and Public Sectors



welfare functions would be necessary to analyze the more complex case. For the moment we may merely note that the derivatives $\frac{dX_1}{dg_1}$, $\frac{dL}{dg_1}$, etc. all have different values and there is no unique maximum for the society.

IV. A DYNAMIC MODEL

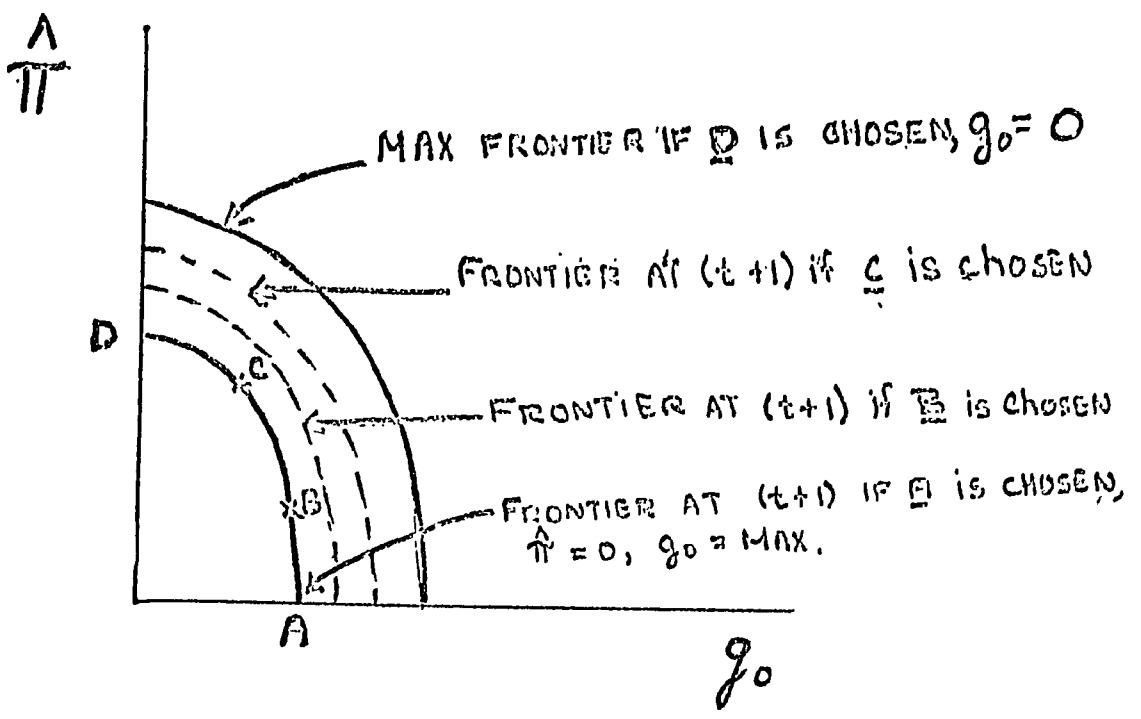
Movements along the efficiency frontier for g_0 and $\hat{\pi}$ have important dynamic implications which should be taken into account when choosing the appropriate government fiscal policy. Profits are one of the major sources of private savings in underdeveloped countries and the level of $\hat{\pi}$ becomes an important determinant of the rate of private capital formation. In a similar vein, the government uses some part of its surplus, g_0, for capital formation and development. A particular combination of $\hat{\pi}$ and $\hat{g}_0$ in one period determines the level and mix of private and public investment and hence the rate of growth of the economy.

Suppose, for example, government investment is zero and that the private sector reinvests some fraction s_1 of its net profits. The greater the level of $\hat{\pi}$ permitted the private sector, the greater the rate of capital formation and hence the greater the <u>outward shift in the efficiency frontier</u>. This is illustrated in Figure 8 which shows the efficiency frontier of period (t + 1) corresponding to a choice of point A, B, C, or D in period (t). If point A is chosen so that $\hat{\pi} = 0$ and g_0 is a maximum, no capital formation occurs and the efficiency frontier remains stationary. If point D is chosen so that g_0 is zero and $\hat{\pi}$ a maximum, the efficiency frontier shifts to the maximum possible extent. B and C are intermediate choices.

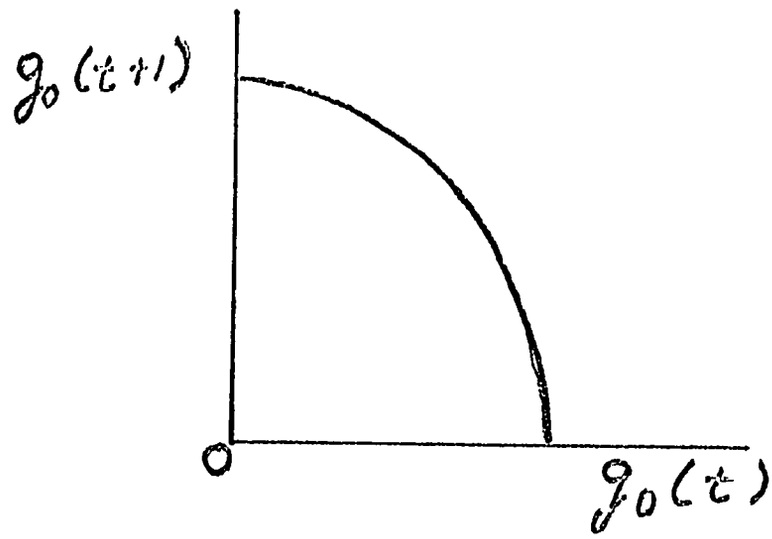
The government's choice of g_0 in one period thus affects its possibility of choice in the next period and so on <u>ad infinitum</u>. The optimum choice from the government's point of view depends upon its horizon and time preference. Suppose, for example, the government's time horizon extends only one period and it derives no utility from $\hat{\pi}$. We assume then that at (t + 1)

Figure 8

Efficiency Frontier for g_0 and $\hat{\pi}$



the government will choose the point where $\hat{\pi}(t + 1) = 0$ and $g_0(t + 1)$ is a maximum. A one period Fisher production possibilities curve can then be derived from Figure 8 showing for each g_0 at time (t), the amount of g_0 obtainable at (t + 1):*



*The well-known formula for deriving the present value of g_0 now and g_0 next period is

$$V = g_0(t) + \frac{g_0(t + 1)}{(1 + i)}$$

where i is the discount rate. This will be maximized when

$$\frac{dV}{dg_0} = \frac{(1 + i) + F'(g_0)}{(1 + i)} = 0$$

or, $i = - [F'(g_0) + 1]$.

A more interesting model allows both the public and private sectors to contribute to capital formation. There are two types of capital stock used by the private sector: K_1 which is the private capital stock consisting of plant, equipment, etc., and K_2 which is the public capital stock consisting of infrastructure, human capital, etc. Private investment is a function of profits and public investment is a function of revenue. The basic model is as follows:

(3.1)* $Y = F(K_1, K_2, L)$

(3.2) $I_1 = s\hat{\pi} = s\ (1 - t)\ \pi$

(3.3) $I_2 = g\ t\ \pi$

(3.4) $g_0 = G - I_2$

where:

Y = total private output

K_1 = private capital stock

K_2 = public capital stock

L = labor employed in Y

I_1 = private investment

I_2 = ~~private~~ public investment

s = private savings rate

g - government savings rate

t = tax rate on profits (π)

$\hat{\pi}$ = private profits net of taxes

g_0 = public surplus

G = total government expenditure

*(3.1) is assumed to be a constant returns to scale production function.

Differentiating (3.1) totally, we have

(3.5) $dY = f_1\, dK_1 + f_2\, dK_2 + f_3\, dL$

but $dK_1 = I_1$, $dK_2 = I_2$ and $f_3 = w$ where w is the wage rate assumed given (i.e., we assume a perfectly elastic supply of labor at the given w).*

(3.5) can then be rewritten as

(3.6) $dY - w\, dL = f_1\, I_1 + f_2\, I_2$, or

(3.7) $d\pi = f_1\, s(1 - t)\, \pi + f_2\, g\, t\pi$

where we have used equations (3.2) and (3.3).

(3.7) can be converted into a growth equation showing the rate of growth of private profits in terms of the two instrumental variables, t and g, as follows:

(3.8) $\frac{d\pi}{\pi} = \pi^* = f_1\, s(1 - t) + f_2\, gt.$

The government, however, is interested in its surplus (g_0). There is then a relationship between π^* and the relative public private surplus ratio $(\frac{g_0}{\pi})$ as follows:

By definition, $g_0 = (1 - g)\, t\, \pi$ where $t\pi = G$ [see equation (3.4)], and $gt = t - \frac{f_0}{\pi}$. Substituting this into the growth equation (3.8) we have

(3.9) $\pi^* = f_1\, s(1 - t) + f_2(t - \frac{g_0}{\pi}).$

For a given t, $\pi^* = F(\frac{g_0}{\pi})$ where $\frac{\partial \pi^*}{\partial \frac{g_0}{\pi}} < 0.$

---

*The partial derivatives, f_i, indicate the relevant marginal productivities of the private and public capital, and labor.

These growth equations can be used to illustrate the growth paths associated with different levels of the instrumental variables g and t. To anticipate our results, the model shows that the government must choose among growth paths such as the ones depicted in Figure 9. Path A has a higher initial level of g_0 than Path B but a lower rate of growth. Path B sacrifices present g_0 but generates a higher rate of growth given a higher initial g or lower t than Path A.

Let us now turn to the derivation of the government's decision rules for a given $(\frac{g_0}{\pi})$. Differentiating equation (3.9) partially with respect to t reveals that for a given $(\frac{g_0}{\pi})$ the growth rate of π and g_0 rises or falls as t increases depending on whether $f_1 s \lesseqqgtr f_2$, or

$$(4.0) \quad \frac{\partial \pi^*}{\partial t} = - f_1 s + f_2$$

where $\frac{\partial \pi^*}{\partial t} \lesseqqgtr 0$ as $f_1 s \gtreqqless f_2$.

This result can be given a straightforward interpretation. $\underline{f_2}$ is the productivity of a dollar's worth of investment in public capital formation. $\underline{f_1 s}$ is the productivity of a dollar's worth of tax reduction to the private sector taking into account both the productivity of private capital and the leakage into private consumption. For a given level of g_0, the government will wish to have all capital formation taking place either in K_1 or K_2 depending upon whether $f_1 s \gtrless f_2$.

We can summarize the results of this model in the following two decision rules:

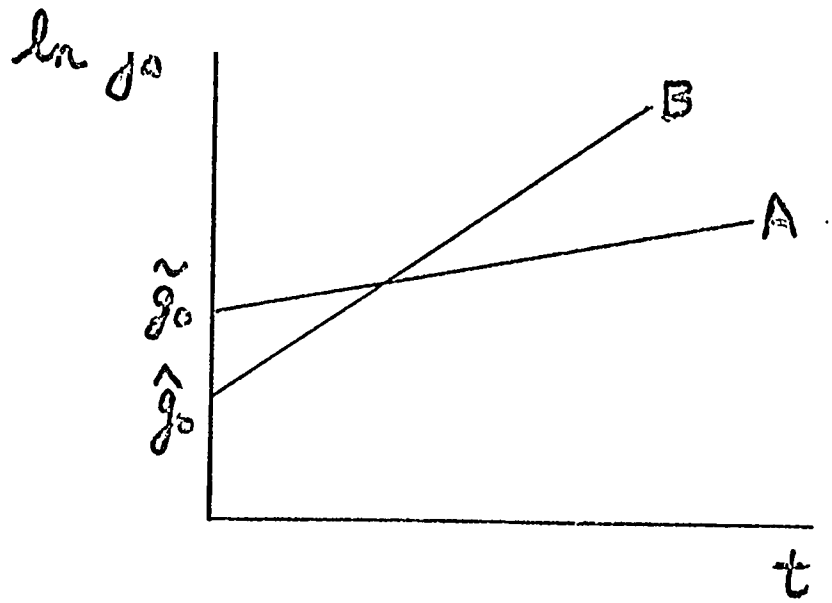
<u>Case 1</u>. If $f_2 > f_1 s$, the government sets t at a maximum, i.e. equal to 1, thus reducing private investment to zero. The growth equation then becomes

$$\pi^* = f_2(1 - \frac{g_0}{\pi}).$$

The higher the level of g_0 the lower the rate of growth of π^* and hence of g_0.

Figure 9

Alternative Paths of g_0



Path A: Higher initial $\hat{g}_0$ but lower rate of growth.

Path B: Sacrifice present $\hat{g}_0$ but higher rate of growth as higher initial g_1 or lower t.

If g_0 is spent only on consumption, then problem only of time preference.

$$\text{Max} \int_0^\infty U(g_0)dt \qquad \text{S.T. } g_0^* = f[g_0(t)]$$

Case 2. If $f_1s > f_2$, the government sets public capital formation at zero and raises taxes only for g_0, i.e., $t = \frac{g_0}{\pi}$. The growth equation then becomes

$$\pi^* = f_1s\ (1 - \frac{g_0}{\pi})$$

and again there is a trade-off between the share of profits devoted to g_0 and the rate of growth, the higher the t the lower the rate of growth.

These two cases, however, illustrate only partial solutions, since they assume f_1s and f_2 will remain constant over time. In fact, they wil vary as the ratio of $\frac{K_1}{K_2}$ changes. In Case 1, $K_1^* = 0$ and $K_2^* > 0$, hence $\frac{K_1}{K_2}$ will fall and $\frac{f_1s}{f_2}$ will rise until $f_1s = f_2$. In Case 2, $K_2^* = 0$ and $K_1^* > 0$, therefore $\frac{K_1}{K_2}$ rises and $\frac{f_1s}{f_2}$ will fall.

The equilibrium growth path will always, therefore, tend to what we call Case 3 where $f_1s = f_2$. Along the equilibrium growth path, $\frac{K_1}{K_2}$ will equal $\bar{K}^*$, the particular public private capital ratio which equates f_1s to f_2. The ratio of I_1 to I_2 will also have to be equal to $\bar{K}^*$ to maintain the growth path. We can then solve for t along this equilibrium path as follows:

Solving (3.9) for the equilibrium growth rate yields

$$\bar{K} = \frac{(1 - t)}{(t - \frac{g_0}{\pi})}.$$

Therefore, $t = \frac{1 + \bar{K} \cdot \frac{g_0}{\pi}}{(1 + \bar{K})}$.

Our major conclusion from this model that the government must choose between $(\frac{g_0}{\pi})$ and π^* still holds. This can easily be seen by once again turning to equation (3.9) and letting $f_1s = f_2$ for equilibrium. This

yields

$$\pi^* = f_2(1 - t + t - \frac{g_0}{\pi}) = f_1 s\ (1 - t + t - \frac{g_0}{\pi})$$
$$= f_2(1 - \frac{g_0}{\pi}) = f_1 s\ (1 - \frac{g_0}{\pi})$$

and the government's choice between π^* and $(\frac{g_0}{\pi})$ is again evident.

We may now briefly examine some of the factors which enter into the government's choice of growth paths. First, let us suppose g_0 is spent entirely on public consumption in the interest of either the nation as a whole or some particular group in control. The optimization problem is then simply one of time preference. Given a time rate of discount, the government can choose the income stream that maximizes the present discounted value of a stream of g_0 with initial value $\bar{g}_0$ and a rate of growth g_0^*.†

It is, however, more interesting and relevant to assume that g_0 is used, at least in part, for general developmental purposes or for some other productive activity. Suppose g_0 is used as an investment in another sector Y_2 which will also feed back revenue to the government when it becomes productive. Suppose that this alternate outlet for investment funds has a rate of return of r_2. The flow of funds to the government is now composed of two streams: the first is $g_0 e^{r_1 t}$, the surplus generated by the sector Y_1 analyzed above; the second stream is $g_0 e^{r_2 t}$, the stream generated by investing g_0 in a development program. The funds available to the govern-

†We would calculate the present discounted value of $\int_0^T g_0(0)e^{(g_0^*-r)t}dt$ where r is the discount rate, and T the end of the planning period. Integrating we have $\frac{g_0(0)}{g^*-r}\ [e^{(g_0^*-r)T}-1]$.
Given that $g_0^* = F(\bar{g}_0)$, the maximum could be calculated from the point of view of the government.

ment at some future point will therefore be:

$$g_0 e^{r_1 t} + g_0 e^{r_2 t} = g_0 (e^{r_1 t} + e^{r_2 t}).$$

The government will maximize the discounted value of this stream, keeping in mind that r_1 is a declining function of g_0. (It is also likely that r_2 will be a declining function of g_0 if there are diminishing returns. A more realistic variant, too complicated to analyze here, is to assume that the development program has a long gestation period so that for the first n years it yields zero return.

Finally, we explore a model in which the government invests in a capital stock which increases the productivity of labor in the government sector itself. We assume that there is a government production function relating output of the government sector to its own capital stock and to labor employed by the government

(4.1) $G = G\ (K,\ L)$.*

Labor is available in unlimited amounts at a fixed wage rate $\bar{w}$. Government investment is the surplus of revenue over wages

(4.2) $I = R - \bar{w}L$.

We further assume that R is determined autonomously and grows at a constant rate R*. A balanced growth path is then defined in which all variables are growing at the same rate:

$$G^* = K^* = L^* = I^* = R^*.$$

---

*The government production function is assumed to be a constant returns to scale function.

In this model, the government's instrumental variable is its savings rate, i.e., the fraction of total revenue in each period which it devotes to its own investment. The choice is illustrated in Figure 10 for arbitrary levels of R. We assume that the government chooses an expansion path implying a constant savings rate $\frac{I}{R}$. It is easy to show that given an exogeneously determined rate of growth of R, there is one optimum savings rate that provides the highest possible growth path for G. There exists then a golden rule for government investment along a balanced growth path equal to R* which is the analogue to the natural growth rate.

We know that along the balanced growth path, capital grows at the same rate as revenue or $I = KR^*$. Substituting this in equation (4.2) above, we obtain for any point of time

$$(4.3) \quad R = R^* K + \bar{w}L.$$

This equation provides the government with the opportunity cost of capital and labor. The government can vary its capital labor ratio by varying its savings rate as long as it satisfies equation (4.3).

The problem for the government is to choose the K and L which maximizes G (equation (4.1)) subject to the constraint that $R = R^* K + \bar{w}L$. The solution is illustrated graphically in Figure 11. The maximum occurs where the ratio of the marginal productivity of labor and capital, $\frac{f_1}{f_2}$, equals $-\frac{w}{R}$. This is the golden rule for the government.

It is interesting to relate this to other formulations of the golden rule. By Euler's theorem,

$$G = f_1 L + f_2 K.$$

and by equation (4.3) above,

$$R = \bar{w}L + R^*K.$$

Figure 10

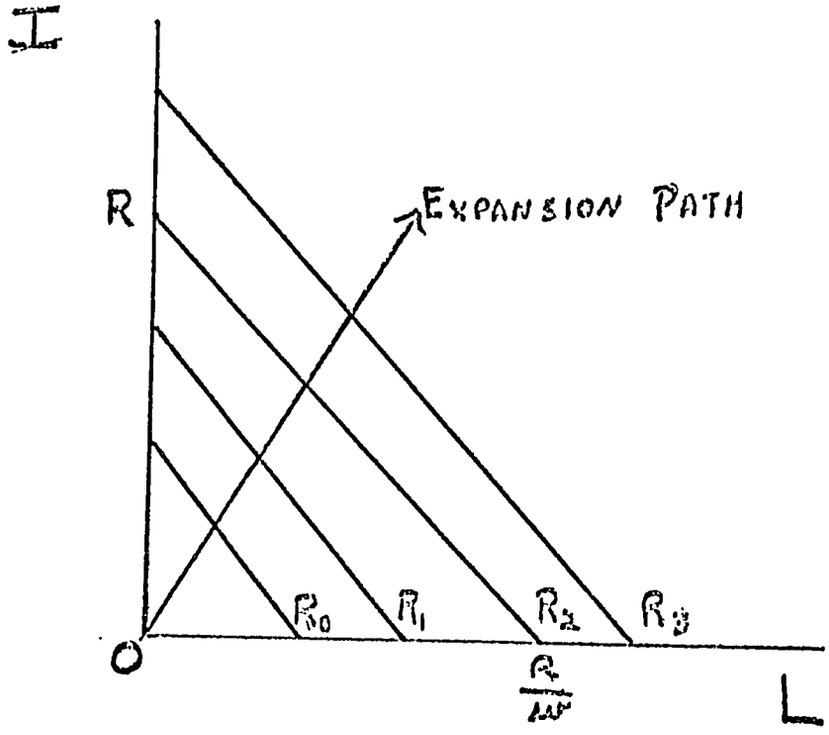
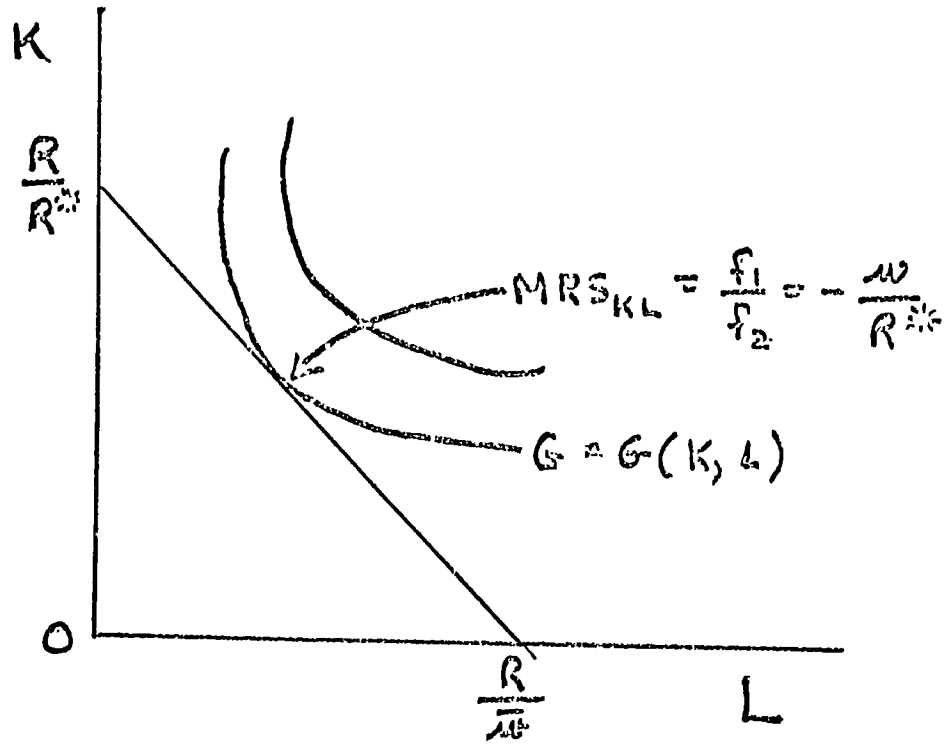


Figure 11



Suppose we assume that we can convert the government's equation to monetary terms by multiplying through by P_g such that $P_g G = R$. In other words, we assume (as is the usual practice) that the value of government output is equal to the value of total revenue and to expenditure by the government in investment and on labor. Our equations would then read:

$$P_g G = P_g f_1 L + P_g f_2 K$$

$$R = \bar{w}L + R^* K.$$

Since $\frac{f_1}{f_2} = \frac{w}{R^*}$, we conclude that

$$w = P_g f_1$$

$$R^* = P_g f_2.$$

Along the golden rule path, the marginal revenue product of capital equals the growth rate and the marginal revenue product of labor equals the wage rate. It is important to note that in order to obtain this result, we assumed that the value of government output in any year equalled the value of current expenditures plus capital expenditures. The true definition of total value should be current expenditure, wL, plus imputed capital costs. Our formula requires the assumption that capital costs should be imputed at the rate of growth R*.

INTERACTION BETWEEN THE GOVERNMENT AND THE PRIVATE SECTOR

Appendix

to

Section III

The bargaining model can be written as follows (definition of variables are found in the text):

1. $X_1 = g_1^{\alpha} L^{\beta} K^{\gamma}$
2. $K = \bar{K}$
3. $w = \frac{\beta X_1 P}{L}$
4. $R = t\pi = t(1 - \beta)X_1$
5. $\hat{\pi} = (1 - t)\pi$
6. $R - G = g_0 + g_1$

Equation 1 describes the production function for the private sector. It is assumed to be Cobb-Douglas. In this production function, the effect of g_1 is like neutral technological change in the sense that it does not affect the marginal rates of substitution between K and L. For many purposes, it would be more interesting and relevant to explore the possibility that government expenditure on, say, research or education is biased towards capital or labor. Note that g_1 is assumed to be a flow whereas many government activities, e.g., roads and dams are better viewed as a capital stock. The model might be viewed as describing periods of time longer than one year, or if viewed as a short-run model, as covering only the recurrent expenditure of government on maintaining roads, providing information, etc.

Equation 2 assumes that the private capital is fixed in the period of consideration.

Equation 3 indicates that labor is hired up to the point where the wage rate equals the marginal product. Because of the Cobb-Douglas as-

sumption and the assumption of constant wages and prices, this yields an expression for labor as a simple non-linear function of X_1:

$$L = \frac{P\beta}{w} X_1$$

Equation 4 shows total revenue for the government (equal to total expenditure) as a constant ratio of profits. Profits before tax is the residual after paying wages and because of the Cobb-Douglas assumption is a constant share of output.

Equations 5 and 6 derive respectively profits after tax ($\hat{\pi}$) and total R and G.

EQUATIONS 1.-6.

SOLVING IN TERMS OF g_1:

7. $X_1 = A\, g_1^{\lambda}$

8. $L = B\, g_1^{\lambda}$

9. $R = tC\, g_1^{\lambda}$

10. $\hat{\pi} = (1-t)\, C\, g_1^{\lambda}$

11. $g_0 = t\, C\, g_1^{\lambda} - g_1$

where:

$$A = \left(\frac{P\beta}{w}\right)^{\frac{\beta}{1-\beta}} K^{\frac{\alpha}{1-\beta}}$$

$\lambda = \frac{\alpha}{1-\beta}$ and $\frac{\alpha}{1-\beta} < 1$ since $\alpha + \beta < 1$

$$B = \frac{P\beta}{w} A$$

$$C = (1-\beta) A$$

DERIVATION OF ISO-SURPLUS CURVES

$$g_0 = t\,c\,g_1^{\lambda} - g_1$$

$$t = \frac{g_0}{c\,g_1^{\lambda}} + \frac{g_1}{c\,g_1^{\lambda}}$$

$$\frac{dt}{dg_1} = g_1^{-\lambda}\left[\frac{-\lambda g_0}{c\,g_1} + \frac{(1-\lambda)}{c}\right]$$

$$\frac{dt}{dg_1} \lesseqqgtr 0 \quad \text{or} \quad g_1 \lesseqqgtr \frac{\lambda g_0}{(1-\lambda)}$$

Since G_0,

$$\lim_{g_1 \to 0} t = +\infty \quad \text{and} \quad \lim_{g_1 \to \infty} t = +\infty$$

If $t = 1$, $g_0 = 0$ when $g_1 = c^{\frac{1}{1-\lambda}}$

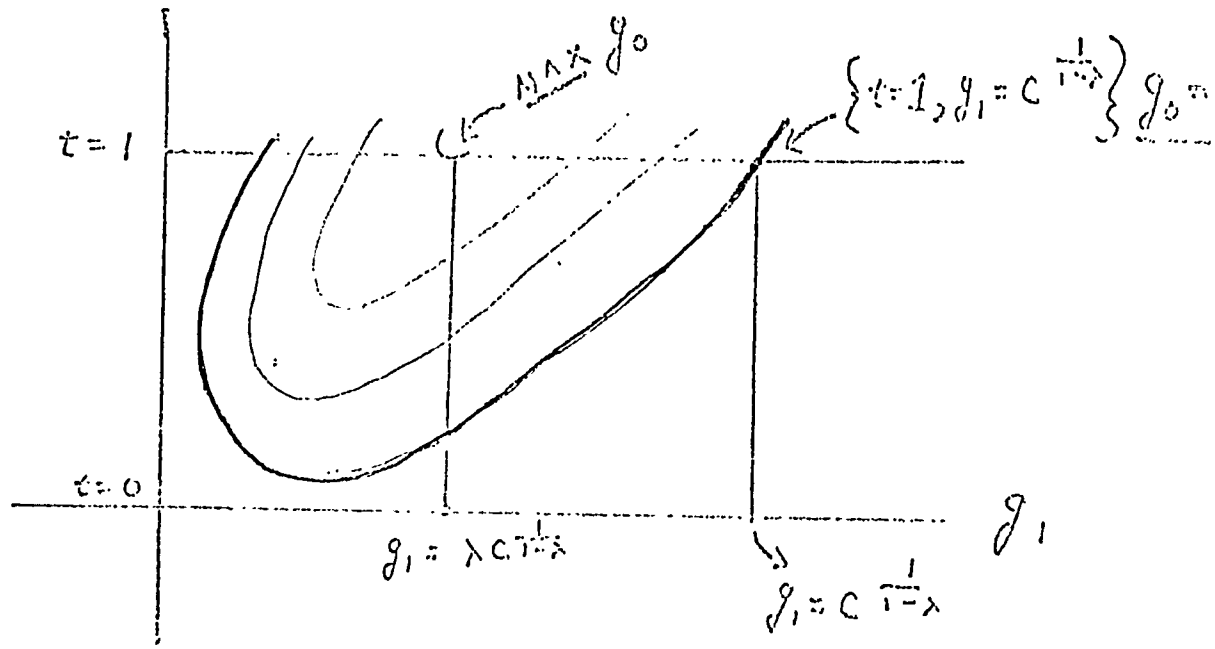
To max g_0, $\dfrac{dg_0}{dg_1} = \lambda t\, c\, g_1^{\lambda-1} - 1 = 0$

and when $t = 1$, $g_0 = \text{MAX}$

when $g_1 = (\lambda c)^{\frac{1}{1-\lambda}}$

FIGURE 5A

- ISO - SURPLUS CURVES -



FAMILY OF U-SHAPED CURVES WHOSE MAX POINT SHIFTS UPWARD TO RIGHT AS G_0 INCREASES, AND IS AT MAX WHEN $g_1 = \lambda c^{\frac{1}{1-\lambda}}$; LOWER BOUND IS SHOWN WHERE $g_0 = 0$ IN HEAVY LINE.

Derivation of Iso-Profit Curves

$$\hat{\pi} = (1-t)\, c\, g_1^{\lambda}$$

$$t = 1 - \frac{\pi}{c\, g_1^{\lambda}}$$

$$\frac{dt}{dg_1} = \lambda \pi c^{-1} g_1^{-\lambda-1} = \frac{\lambda \pi}{c\, g_1^{\lambda+1}} > 0$$

$$\frac{d^2 t}{dg_1^2} = (-\lambda - 1)\, \lambda \pi / c\, g^{\lambda+2} < 0$$

$\therefore$ concave downward

Given $\hat{\pi}$,

$$\lim_{g_1 \to 0} t = -\infty \quad \text{and} \quad \lim_{g_1 \to \infty} t = 1$$

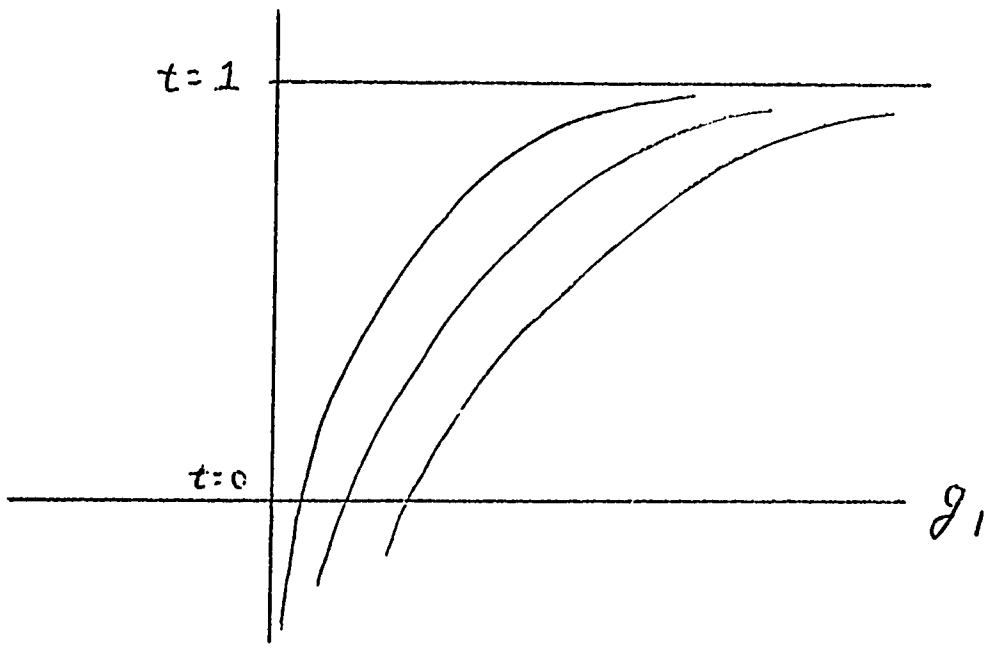
If $t = 0$, $g_1 = \left(\frac{\pi}{c}\right)^{\frac{1}{\lambda}}$

$t = 1$; $\pi = 0$

$\therefore$ zero profit curve, i.e., the limiting case occurs, when either $t = 1$ or $g_1 = 0$ and i.e. of vertical ships.

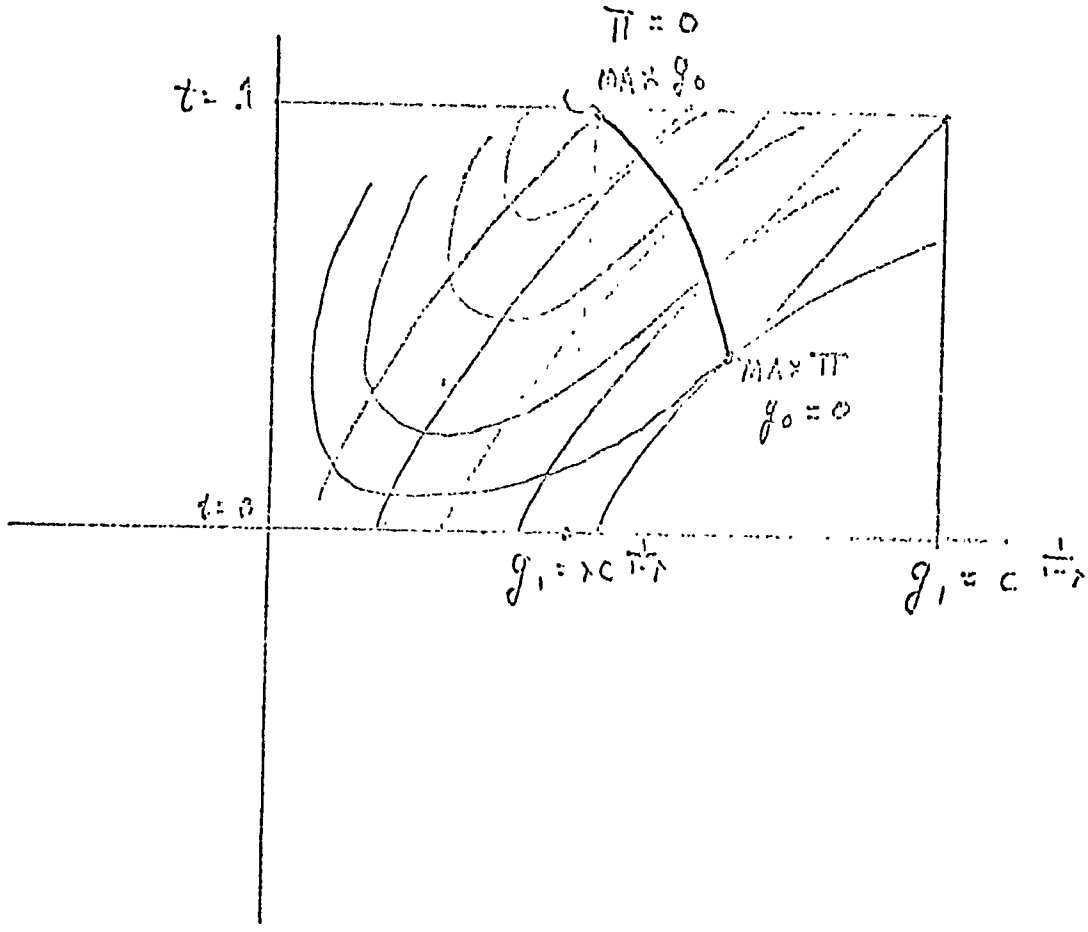
FIGURE 6A

- ISO - PROFIT CURVES -



FAMILY OF HYPERBOLAS OF DEGREE λ CONCAVE DOWN WHERE PROFITS INCREASE FROM LEFT TO RIGHT; $\pi = 0$ WHEN $\tau = 1$, OR $g_1 = 0$.

The two families can be combined on a single diagram as in Figure 7A. The tangencies of iso-profit and iso-surplus curves yield the contract curve for the specific model in this appendix. As noted, the general case is found in the text.

FIGURE 7A

- CONTRACT CURVE BETWEEN PRIVATE AND PUBLIC SECTORS --