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PRODUCTION RISK AND OPTIMAL FERTILIZER RATES: AN APPLICATION OF THE RANDOM COEFFICIENT MODEL

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PRODUCTION RISK AND OPTIMAL FERTILIZER RATES: AN APPLICATION OF THE RANDOM COEFFICIENT MODEL[1]

ABSTRACT

This paper investigates the effect of risk aversion on fertilizer use. A Random Coefficient Model estimated the probability distribution of yields at different nitrogen rates. This technique allows coefficients of explanatory variables to vary temporally or spatially. It is thus useful for modeling N response, which varies from year to year due to interactions with stochastic disturbances such as weather, and insect and disease infestation.

The procedure used data from 5 yr of N response trials by the Agronomy Department at IRRI under rainfed conditions. The results showed only a slight effect of risk aversion on fertilizer use. Application of risk-neutral and risk-averse decision-making models predicted that moderately risk-averse farmers would apply only 7 kg less than the risk-neutral, profit-maximizing N rate of 42 kg/ha. Thus, the results do not support the idea that risk is a major reason why farmers apply low levels of nitrogen to rice in favorable production environments.

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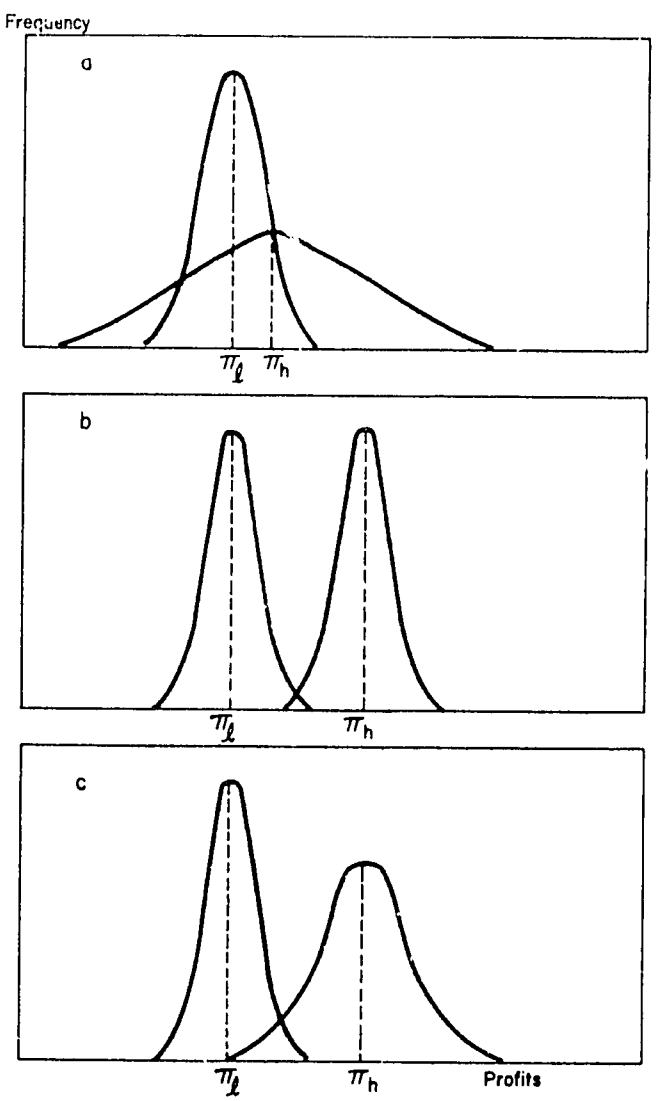
PRODUCTION RISK AND OPTIMAL FERTILIZER RATES: AN APPLICATION OF THE RANDOM COEFFICIENT MODEL

Risk as an impediment to progress has been a recurrent theme in development work for the last 15 yr (8, 24). Some writers apply risk theory to rice production and argue that modern varieties have not realized their full potential because year-to-year yield variability makes it risky for farmers to apply economically optimal levels of inputs (2). The argument is illustrated in Figure 1, which depicts the probability distribution of profit at high (π_h) and low (π_l) input levels.

In Figure 1a, both the mean and variance of profit are less at the lower input level. If farmers were risk neutral, ie, indifferent to risk or profit variability, they would use the higher input level because mean profit is higher for π_h than for π_l. Farmers, however, are postulated to be risk averse, ie, they are more anxious to avoid low incomes than to attain high profit levels (19). Therefore, they might prefer π_l because the chance of making very low profits is less when they apply the lower level of inputs. The probability of making really large gains is also lower for π_l but this is relatively less important to a risk-averse farmer. When risk-averse farmers apply suboptimal input levels, lower yields represent a social cost.



1. Probability distribution of profits at different N rates. (Adapted from [7]).

The critical point in this argument is how profit variability changes as inputs increase. If the profit distribution at high input levels is π_h (Fig. 1b), ie, if the variance does not change, a risk-averse farmer will not gain by choosing the lower application rate. This case presents no conflict between risk and profit maximization.

An intermediate case is shown in Figure 1c. Profit variability is greater at the higher input level. However, the increased variance is small relative to the increase in mean profits. Risk is, therefore, unlikely to significantly change input use.

Research has measured how risk aversion alters input use, particularly of fertilizer. The evidence is conflicting. While Moscardi and de Janvry (16) and Ryan and Perrin (20) conclude that risk aversion substantially lowers input levels, Roumasset (18) and Rosegrant and Herdt (17) show that profit variability is not responsible for the low observed levels of input use.

This conflicting evidence raises the question, "To what extent are methodological differences responsible for the disparity in conclusions?" Risk studies have two key methodological components: the yield distributions of alternative strategies and the farm decision-making framework. Incorporating the parameters of the yield distribution into the hypothesized risk-averse, decision-making model provides estimates of input levels applied by rational risk-averse farmers. Comparing these risk-averse input levels with risk-neutral levels permits inferences about the effect of risk on input use.

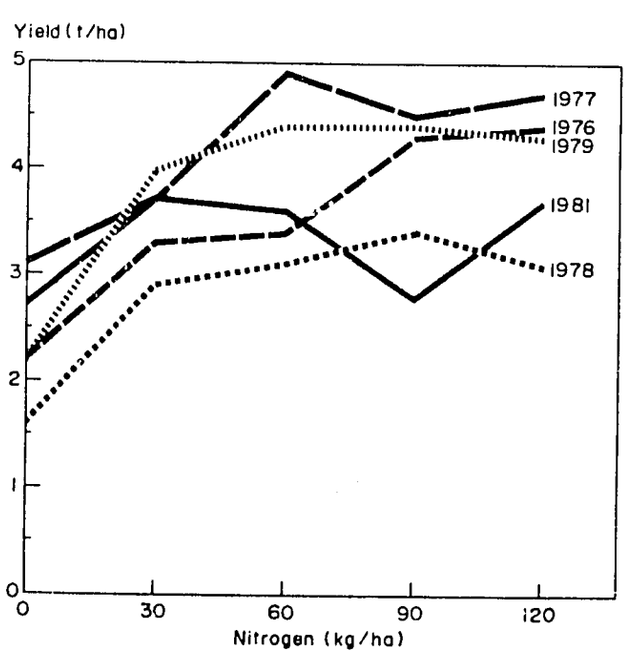
This paper estimates the yield distribution by applying the Random Coefficient Model (RCM), an econometric procedure explicitly designed for situations where the parameters of the estimated relationship vary either temporally or spatially. Therefore, the model is particularly suitable for estimating the uncertain yield response to nitrogen (N) in rice production. The RCM was first used by Young and Mount (25) to investigate the variability of crop response (25). Huysman later applied the RCM to survey data from Iloilo Province, Philippines (13).

A brief discussion of methodological issues related to yield distributions will highlight the model's appropriateness. This is followed by the mathematical specification and the estimation procedure. The final section applies a number of decision-making models to quantify the effect of risk on input use.

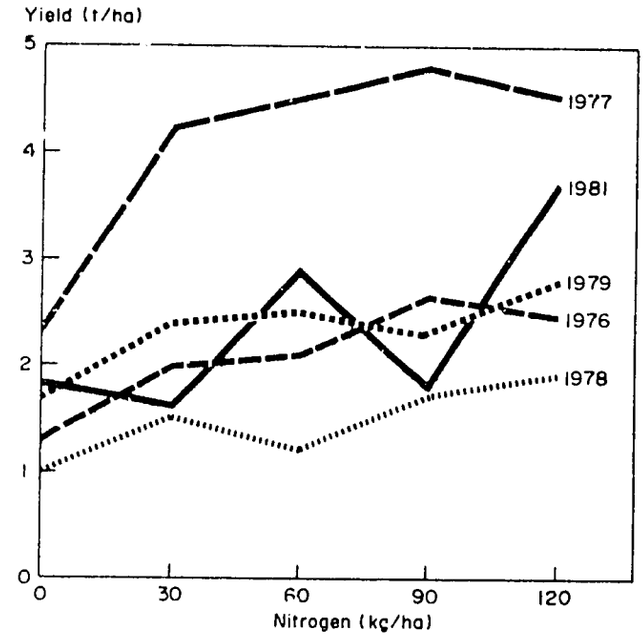
METHODOLOGICAL ISSUES RELATED TO YIELD DISTRIBUTIONS

In estimating yield distributions an important question is whether to use subjective or objective probability distributions. Directly questioning farmers produces subjective probabilities, ie, decision makers' beliefs about the likelihood of particular outcomes (1). In practice, such elicitation is difficult. Farmers' perceptions of yield variability are not usually based on deliberate information processing. The problems of capturing such preattentive processes has been documented by anthropologists (5, 10). Herath et al (11) report that only 50% of the subjective yield distributions they elicited were usable. Considering such difficulties, most researchers have used objective yield distributions derived from production functions. The implicit assumption is that experienced farmers are knowledgeable about actual year-to-year variations in yields.

Another major issue is how to specify the production function. Risk analysis rests on the existence of variability. Year-to-year variability of nitrogen response is accepted as an agronomic fact (Fig. 2, 3). Several approaches are available for specifying production functions under yield variability. The analytical approach accounts for variability by incorporating interaction terms between N and the hypothesized stochastic influences. The joint probability distribution of the stochastic variables generated by simulation (17) or by direct elicitation (18) provides an estimate of yield variability. Because of the difficulty of including all stochastic effects, Anderson et al (1) recommend *gross* approaches which automatically pick up all variability without explicitly accounting for its sources.



2. Nitrogen response of IR42, 1976-81. (Source: Agronomy Department, IRRI).



3. Nitrogen response of IR36, 1976-81. (Source: Agronomy Department, IRRI).

The RCM is a gross approach appropriate for investigating N variability over time. This model explicitly specifies that the coefficients of the explanatory variables vary randomly over time or space. In this research, N responses comprising the data are deemed to be a random sample from the population of responses. The RCM provides estimates of the mean and variance of the response population. A unit increase in the explanatory variable produces a random increase, with a specified mean and variance, in the dependent variable (23). The methodology is, therefore, useful where the researcher primarily wishes to measure variability.

The randomness assumption is reasonable, since variability in N response is caused by factors such as weather and insect and disease infestation, which, assuming management uniformity, are generally regarded as random.

THE RANDOM COEFFICIENT MODEL

Given P observations in each of T-time periods the model for the ith time period can be written as:

$$Y_i = X_i \beta_i + e_i \ (i = 1, \ldots T) \qquad (1)$$

where Y_i is a Px1 vector of observations on the dependent variable in time period i,

X_i is a PxK matrix of observations on K independent variables in time period i, and

β_i is a Kx1 vector of random coefficients in time period i.

For a N response function this indicates that the response to N varies randomly over time periods.

e_i is a Px1 random disturbance vector in time period i.

$E(e_i) = 0$ $\qquad E(e_i e_i') = \sigma_{ii} I$

$E(e_i e_j) = 0 \ (i \neq j)$

Each e_i captures disturbances to N response within a particular time period. The heteroscedastic, uncorrelated nature of the e_i's imply that:

1. the variability of N response within time period i need not equal variability in time period j. This is plausible because some sources of variability, such as weed infestation, interact with N levels whereas others, such as typhoons, affect all plots uniformly.

2. variability in one period is uncorrelated with variability in other periods. Since the causes of variability are expected to occur randomly, this assumption appears reasonable.

The β_i are distributed with mean $\bar{\beta}$ and variance Δ,

ie, $\beta_i = \bar{\beta} + u_i$ (2)

where u_i is a Kxl vector of random elements:

$E(u_i) = 0.\ E(u_i u_i') = \Delta.\ E(u_i u_j) = 0\ (i \neq j)$

The distribution of the u_i's captures the variability of N response over time periods. N response over time has a fixed variance (Δ). N response in different time periods is independent.

Pooling observations over time periods gives

$$Y = X\beta + \epsilon \tag{3}$$

$$\text{where } Y = \underset{(PTx1)}{\begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ \cdot \\ Y_T \end{bmatrix}} \qquad X = \underset{(PTxK)}{\begin{bmatrix} X_1 \\ X_2 \\ \cdot \\ \cdot \\ X_T \end{bmatrix}}$$

$$\epsilon = \underset{(PTxKT)}{\begin{bmatrix} X_1 & 0 & \cdots & 0 \\ 0 & X_2 & \cdots & 0 \\ & & & \cdot \\ & & & \cdot \\ 0 & \cdots & & X_T \end{bmatrix}} \underset{(KTx1)}{\begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ \cdot \\ u_T \end{bmatrix}} + \underset{(PTx1)}{\begin{bmatrix} e_1 \\ e_2 \\ \cdot \\ \cdot \\ e_T \end{bmatrix}}$$

The covariance matrix of the disturbance is a block diagonal matrix of dimension PTxPT with the ith diagonal given by $X_i \Delta X_i' + \sigma_{ii} I$.

Since the diagonal elements of the variance covariance matrix of ϵ are unequal, OLS (Ordinary Least Squares) will give unbiased but inefficient estimates. GLS (Generalized Least Squares) provides efficient estimates of $\bar{\beta}$.

Swamy (22) shows that a GLS estimator of $\bar{\beta}$ is given by

$$\bar{\beta} = \sum_{i=1}^{T} W_i \hat{\beta}_i \tag{4}$$

where $\hat{\beta}_i = (X_i'X_i)^{-1} X_i'Y_i$ is the OLS estimator of β_i from equation 1.

$$W_i = \left[\sum_{j=1}^{T} \left\{\Delta + \sigma_{jj} (X_j'X_j)^{-1}\right\}^{-1}\right]^{-1} \left\{\Delta + \sigma_{ii} (X_i'X_i)^{-1}\right\}^{-1} \tag{5}$$

As in the GLS estimator without random coefficients, equation 4 weights the estimate in each year by the inverse of its variance.

Swamy also shows that the variance covariance matrix of the GLS estimator is given by:

$$\left[\sum_{j=1}^{T} \left\{\Delta + \sigma_{jj} (X_j'X_j)^{-1}\right\}^{-1}\right]^{-1} \tag{6}$$

which is the reciprocal of the denominator of the weight W_i.

Swamy proposes unbiased estimators for the unknown parameters, Δ and σ_{ii}, as follows:

$$\hat{\sigma}_{ii} = (Y_i - X_i\hat{\beta}_i)'(Y_i - X_i\hat{\beta}_i)/P-K \tag{7}$$

which is the error mean square of the OLS regression in year i. Finally, the estimate of $\hat{\Delta}$, the variance covariance matrix of the β_i, is

$$\hat{\Delta} = \frac{1}{T-1}\left[\sum_{i=1}^{T} (\hat{\beta}_i - \bar{\hat{\beta}})(\hat{\beta}_i - \bar{\hat{\beta}})'\right] - \frac{1}{T}\sum_{i=1}^{T} \hat{\sigma}_{ii} (X_i'X_i)^{-1} \tag{8}$$

The first term of equation 8 is a variance covariance matrix measuring variability of OLS coefficients across years. The ij^{th} element is given by

$$\frac{1}{T-1}\left[\sum_{i=1}^{T} (\hat{\beta}_{ij} - \bar{\hat{\beta}}_j)(\hat{\beta}_{ik} - \bar{\hat{\beta}}_k)\right]$$

where $\hat{\beta}_{ij}$ and $\hat{\beta}_{ik}$ are the OLS estimates of β_j and β_k, respectively, in year i.

$\bar{\hat{\beta}}_j$ and $\bar{\hat{\beta}}_k$ are the means of the OLS estimates for β_j and β_k over the T-time periods.

The second term, $\hat{\sigma}_{ii}(X_i'X_i)^{-1}$, is the variance covariance matrix of the OLS coefficients in year i.

$\hat{\Delta}$ and $\hat{\sigma}_{ii}$ are substituted into equation 5 and then into equation 4 to compute the GLS estimates of the mean coefficient vector.

EMPIRICAL ESTIMATION[2]

A random coefficient model of N rice response was estimated using data from rainfed N trials conducted at the International Rice Research Institute by the Agronomy Department. N rates were varied from 0 to 120 kg N/ha, at 30-kg intervals. A split plot experimental design, replicated 3 times, was used with fertilizer as the main plot and varieties as subplots. The model was estimated with two cultivars, IR36 and IR42, the most widely grown varieties in the Philippines. Thirty observations on each variety were available for 5 yr (1976-79, 1981).

Mean N response at each N level, by variety and year, are graphed in Figures 2 and 3. These confirm that N response varied widely across years. The intercept of IR36 was consistently lower than that of IR42, with the difference varying over time. This was captured in the estimated model by incorporating a dummy variable to distinguish the two varieties. In the figures, the IR36 slopes appear

[2]The computer program for estimating the random coefficient model is given in the Appendix.

flatter than the IR42 slopes. However, interaction terms between N and variety, included in the initial model version, turned out statistically insignificant and were dropped from the analysis. Of the different functional forms estimated with the annual data, the quadratic provided the best fit.

$$Y_{ij} = a + b\,N_{ij} + c\,N^2_{ij} + dV_{ij} + e_{ij} \qquad (9)$$

where Y_{ij} = yield (kg/ha) of observation j in time period i;
N_{ij} = nitrogen application (kg/ha);
V_{ij} = variety dummy for observation j in time period i; 1 for IR42; 0 for IR36; and
e_{ij} = error term for observation j in time period i.

The OLS estimates of the linear and quadratic N parameters change considerably from year to year (Table 1). The coefficient sizes in all years except 1981 are plausible, ie, consistent with expectations based on previous research. The low and insignificant estimates for 1981 are not surprising given the widely oscillating pattern of response in that year (Fig. 2, 3).

Swamy (22) proposes the following test for the homogeneity of coefficients across years:
Given the null hypothesis:

$$H_0 = \beta_1 = \beta_2 = \ldots\ldots \beta_5 = \bar{\beta} \text{ (where } \bar{\beta} = \text{GLS estimate without random coefficients)}$$

the homogeneity statistic H_β is given by

$$\sum_{i=1}^{T} (\hat{\beta}_i - \hat{\bar{\beta}})' \frac{X_i'X_i}{\hat{\sigma}_{ii}} (\hat{\beta}_i - \hat{\bar{\beta}})$$

$$\text{where} \quad \hat{\bar{\beta}} = \left[\sum_{i=1}^{T} \frac{X_i'X_i}{\hat{\sigma}_{ii}}\right]^{-1} \sum_{i=1}^{T} \frac{X_i'X_i}{\hat{\sigma}_{ii}} \hat{\beta}_i \qquad (10)$$

The asymptotic distribution of $H_\beta/K\,(T-1)$ can be approximated by F with K (T-1), T (P-K) degrees of freedom.

The calculated value of F was 13.08 with 16 and 130 degrees of freedom. The data do not support the hypothesis that the coefficients are fixed over time. Therefore, a varying parameter model is appropriate for this data set.

Year-to-year variations in N response are assumed to be random. The following regression pooling data over years was estimated to test whether a significant trend existed.

$$Y_{ij} = a + b\,N_{ij} + c\,N^2_{ij} + d\,V_{ij} + f\,T_i + e_{ij} \qquad (11)$$

where T_i = time trend
= 1 . . . 5 as year = 1976 . . . . 1979, 1981.

The trend variable was not statistically significant (Table 2). Figures 2 and 3 also do not reveal any systematic effect of time.

GLS estimates with random parameters

Equation 8 was used to obtain an unbiased estimate of Δ which measures the variability of the OLS coefficients over time (Table 3). Weights for each element of the β_i's were computed as follows:

$$W_{ik} = \left[\sum_{j=1}^{T} \left\{\hat{\Delta}_{kk} + \hat{\sigma}_{jj}\,(X_j'X_j)^{-1}\right\}^{-1}\right]^{-1} \left\{\hat{\Delta}_{kk} + \hat{\sigma}_{ii}\,(X_i'X_i)^{-1}\right\}^{-1} \qquad (12)$$

Table 2. Quadratic nitrogen response function pooled over all years and including time trend.[a]

Independent variable	Coefficient estimate	Standard error
Intercept	1869.84**	252.71
Nitrogen (kg/ha)	27.50**	6.66
Nitrogen squared (kg/ha)	-0.13*	0.05
Dummy for variety (IR42=1)	1096.75**	160.40
Time trend (1976=1)	-109.37	56.71
R^2	0.39	

[a]As in equation 11. ** = statistically significant at the 1% level, * = statistically significant at the 5% level.

Table 3. Unbiased estimator of $\Delta = \hat{\Delta}$.[a]

	Intercept	Nitrogen	Nitrogen squared	Dummy for variety
Intercept	484638.00	6942.55	-47.07	-327210.00
Nitrogen	6942.55	137.99	-0.96	-3373.88
Nitrogen squared	-47.07	-0.96	0.01	24.02
Dummy for variety	-327210.00	-3373.88	24.02	204016.00

[a]Estimated as in equation 8.

Table 1. Quadratic nitrogen response functions, 1976-81.[a]

Independent variable	Coefficient estimate and standard error[b]				
	1976	1977	1978	1979	1981
Intercept	1124.13**	2677.82**	724.72**	1300.65**	1881.38**
	(352.37)	(261.32)	(240.46)	(291.03)	(352.74)
Nitrogen (kg/ha)	27.55*	48.51**	22.03**	34.33**	5.11
	(12.57)	(9.32)	(8.58)	(10.38)	(12.58)
Nitrogen squared (kg/ha)	-0.11	-0.29**	-0.11	-0.19*	0.03
	(0.10)	(0.07)	(0.07)	(0.08)	(0.10)
Dummy for variety (IR42=1)	1352.60**	243.33	1361.27**	1552.80**	973.73**
	(302.47)	(224.31)	(206.41)	(249.82)	(302.79)
R^2	0.59	0.63	0.70	0.69	0.39

[a]As in equation 9. [b]** = statistically significant at the 1% level, * = statistically significant at the 5% level.

where W_{ik} is the weight for the kth parameter in year i and the subscripts of $\hat{\Delta}$ indicate the relevant elements of the $\hat{\Delta}$ matrix. The weighted estimates are aggregated over years to give the mean coefficient vector of GLS estimates with random parameters (Table 4). The magnitudes of all coefficients are reasonable. On an average, maximum yield is at 102 kg N/ha, which is plausible considering that Figures 2 and 3 show a downturn between N rates of 90 and 120. The marginal product at the mean N level of 60 kg/ha is 12 kg grain/kg N, reasonable under rainfed conditions. The base yields of IR42 are about a ton more than those of IR36. This reflects agronomic characteristics of IR42, a high yielder even without fertilizer (14).

The asymptotic standard errors are listed in parenthesis below the coefficient estimates. These were obtained by taking the square root of the diagonal elements of $\hat{\Delta}/5$ (22). All estimates are significant at the 1% level. The OLS estimates for the pooled data set given in the second column of Table 4 are similar to the GLS estimates.

Variance of yield distributions

Estimates of the relationship between yield variability and N rates are given in this section. The variance of Y at each level of N is based on the covariance matrix of the disturbance term (25):

$$\mathrm{Var}\,(Y|N) = \bar{X}\,\hat{\Delta}\,\bar{X}' + \hat{\sigma} \qquad (13)$$

where N = the value of N at which the variance of Y is to be evaluated,

$\bar{X}$ = a lxK vector of independent variables with N = N and all others valued at the sample means

$\hat{\sigma}$ = the mean error sum of squares over years, given by

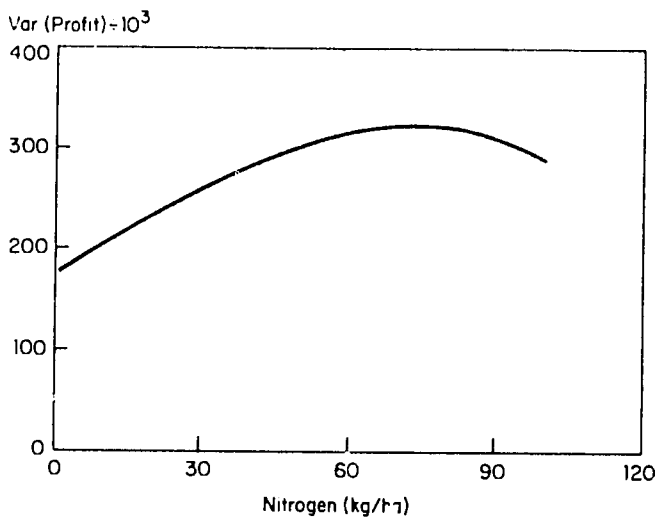
$$\frac{\sum_{i=1}^{T} SSE_i}{\sum_{i=1}^{T} (P_i - K_i)}$$

where SSE_i = sum of squared errors from the OLS regression in year i.

Table 4. Generalized Least Squares (GLS) and Ordinary Least Squares (OLS) estimates of quadratic nitrogen response function, 1976-81 pooled data.[a]

Independent variable	GLS estimate and asymptotic standard error[b]	Pooled OLS estimate and standard error[c]
Intercept	1541.37	1541.74
	(311.33)	(188.60)
Nitrogen (kg/ha)	28.50	27.50
	(5.25)	(6.73)
Nitrogen squared (kg/ha)	-0.14	-0.13
	(0.04)	(0.05)
Dummy for variety (IR42=1)	1090.19	1096.75
	(201.99)	(161.89)

[a]All coefficients are significant at the 1% level. [b]GLS estimates as in equation 4. [c]OLS estimates as in equation 9.



4. Profit variability, IR36.

Table 5. Mean production and price data for 12 farmers, Batangas Province, Philippines.

Farm size (ha)	1.5
Yield (t/ha)	2.4
Nitrogen application (kg/ha)	44
Palay price (₱/kg)	1.3
Nitrogen (urea) price: (₱/kg)	5.1
Interest rate on production credit (% per year)	137
Expenditure on inputs other than fertilizer (₱/ha)	600

OPTIMAL FERTILIZER RATES UNDER UNCERTAINTY

A comparison of optimal N rates under risk-neutral and risk-averse decision-making models quantifies the effect of risk aversion on fertilizer use.

Farm costs for the optimality analysis were obtained by interviewing a small sample of farmers in Barrio Taysan, Batangas Province (Table 5), a rainfed rice growing area climatically and topographically similar to the IRRI experimental site from which the production data were obtained.

Farm size in Taysan is small, averaging 1.5 ha. All farmers grew modern rices: C4 and a number of IR varieties. Average yields were 2.4 t/ha with a mean N rate of 44 kg/ha. At the same fertilizer application rate, the mean parameter estimates of the RCM predict a slightly higher yield of 2.5 t/ha for IR36.

Share tenancy was predominant. Landlords did not contribute to production costs. They received one-third of the output net of harvesters/threshers' share (one-fifth) as land rent.

Risk-neutral model

A rational risk-neutral farmer maximizes expected profits. The decision rule adjusted to reflect the institutional environment is:

$$\text{Max } E(\pi) = w_1 P_Y E(Y) - w_2 P_N N - C \quad (14)$$

where π = net returns/ha,

P_Y = reported price of rough rice/kg,

$E(Y)$ = f (N) is the mean production function estimated by the RCM,

P_N = reported price of nitrogen/kg from urea,

N = nitrogen rate (kg/ha), and

C = expenditure per ha on inputs other than nitrogen.

C was taken as fixed and independent of the fertilizer rate. Reported production and prices were adjusted to consider other costs farmers incurred. w_1, the adjustment parameter for production, was set at 0.39, to account for output shares paid to landlords and harvesters. It also incorporated transport and storage losses and cleaning and drying costs. w_2, the adjustment parameter for nitrogen, was set equal to 1.6 to represent fertilizer, fertilizer application, and transport costs (Table 6).

Risk-averse full optimality model

In this model, farmers are assumed to maximize the expected value of a concave utility function which consistently orders outcomes according to underlying preferences. Using a Taylor series expansion, Anderson et al (1) show that the utility of (π) can be written as:

$$U(\pi) = U[E(\pi)] + U_2 [E(\pi)] M_2(\pi)/2 + U_3 [E(\pi)] M_3(\pi)/6 + \ldots\ldots \quad (15)$$

where U_i is the ith derivative of the utility function and M_i is the ith moment of the probability distribution of profits.

The variance of yield (obtained from equation 13) was used to compute the variance of profit as follows (15):

$$\text{Var}(\pi) = (w_1 P_Y)^2 \text{ Var}(Y) \quad (16)$$

As N rates increased, profit variance increased at a declining rate, reaching a maximum at 70 kg N/ha (Fig. 4). This is consistent with the pattern of variability exhibited by the data set (Fig. 2, 3).

RCM provides estimates of the first two moments of the yield distribution. This is no problem if yields are normally distributed. Theoretically the Central Limit Theorem (9) can be invoked to support the assumption of normality. On the other hand, Day (6) points out that since crop yields are necessarily nonnegative the normality assumption is not plausible. The real issue is whether the normal distribution provides an adequate approximation for the data set under consideration. The distribution of sample yields at each N rate was tested for normality by using the Kolmogorov-Smirnov D-statistic. The hypothesis of normality could not be rejected (Table 7). (If initial tests indicate that yields are not normally distributed, the Pearson curve fitting system can be used to identify the distribution and its higher moments.)

The normality assumption allowed equation 15 to be estimated with all, except the first two, moments of the profit distribution set equal to zero. The utility function used was the Constant Partial Risk Averse Function (CPRA) specified by Binswanger (3) as

$$U(\pi) = (1 - S) R^{(1-S)} \quad (17)$$

where S is the partial risk aversion coefficient[3] and R is the stochastic profit at each level of N application. This function has been estimated in different parts of the developing world; results indicate most farmers are moderately risk averse (4). In the Philippines, CPRA function estimates are available from Sillers' work (21) in Nueva Ecija. Seventy-eight per cent of farmers fell into two intermediate categories of risk aversion. The S value used in this paper (0.8) is the common endpoint of these two categories.

The CPRA function is a local approximation derived from a utility function in which risk aversion increases with the size of the prospect at stake (3). Ideally, therefore, the value of S should be increased as farm size increases. Because there were no indications of the magnitude of change, this paper attempted no such adjustment.

Table 6. Adjustment parameters applied to reported data: gross revenue and fertilizer cost.

	Adjustment parameter
Gross revenue	
Harvester's/thresher's share	.20
Landlord's share	.27
Transport/storage losses, cleaning/drying costs	.14
Total adjustment ($1 - w_1$)	.61
Fertilizer cost	
Interest (137%/yr for 5 mo)	.57
Fertilizer application and transport costs	.03
Total adjustment ($w_2 - 1$)	.60

Table 7. Test of normality: experimental data, IRRI, Los Baños, Philippines: 30 observations at each N level.

H_0: Data values are a random sample from a normal distribution.

N level (kg/ha)	Kolmogorov-Smirnov D-statistic	Significance level
0	0.13	>.20
30	0.15	<0.11
60	0.07	>.20
90	0.08	>.20
120	0.09	>.20

RESULTS

The effect of risk aversion on optimal N rates is much smaller than previously believed. The optimal choice in the risk-averse model is 35 kg N/ha, compared with the risk-neutral optimum of 42 kg/ha (Table 8). This 17% reduction in fertilizer use lowers yields by 122 kg/ha. These results

[3]Partial risk aversion traces the behavior of an individual when the amount at risk increases but wealth remains the same.

Table 8. Risk-neutral and risk-averse optimal N rates.

	kg/ha
Risk-neutral optimal N rate[a]	42
Risk-averse optimal N rate[b]	35
Effect of risk on fertilizer use	7
Effect of risk on yield	122

[a]Estimated as in equation 14. [b]Estimated as in equation 15.

are consistent with the findings of risk studies conducted in irrigated areas of the Philippines. Roumasset (18) and Rosegrant and Herdt (17) reported that risk did not significantly lower fertilizer levels in irrigated areas. This paper shows it may be possible to extend this conclusion to favorable shallow rainfed areas.

Roumasset (18) noted no conflict between risk aversion and profit maximization if yield variability remained constant or increased relatively slowly as N rates were increased. This might account for the conflicting evidence on the effect of risk aversion. In Roumasset's data set, risk (defined as the probability of a negative rate of return) increased slowly with changes in fertilizer application. By contrast, Ryan and Perrin (20) found that the variance of profits in potato cultivation increased sharply with N rates. Consistent with this, they reported that risk aversion substantially reduced input levels.

CONCLUSIONS

This paper investigates the effect of risk aversion on fertilizer use. A Random Coefficient Model estimated the probability distribution of yields at different nitrogen rates. This technique allows coefficients of explanatory variables to vary temporally or spatially. It is thus useful for modeling N response, which varies year to year because of interactions with stochastic disturbances such as weather, and insect and disease infestation.

The procedure used data from 5 yr of N response trials by the Agronomy Department at IRRI under rainfed conditions. The results showed only a slight effect of risk aversion on fertilizer use. Application of risk-neutral and risk-averse decision-making models predicted that moderately risk-averse farmers would apply only 7 kg less than the risk-neutral, profit-maximizing N rate of 42 kg/ha.

These results should be interpreted cautiously because experiment station data may not reflect conditions prevailing on farmer fields. Results, however, are consistent with other risk studies in irrigated environments in the Philippines (17, 18).

Fertilizer is largely responsible for the gap between potential and actual yields. Using data from 6 Asian countries, Herdt and Mandac (12) showed that 0.4 t of the average yield gap could be attributed to fertilizer. The risk studies cited above provide increasing evidence that risk aversion accounts for only a small proportion of this gap, at least in irrigated and favorable shallow rainfed sites. Production conditions may not be risky enough to significantly affect input levels.

These conclusions are valid only for favorable production environments. Clearly, further work is required in harsh environmental conditions where risk could have a greater impact on input use. However, the crucial factor is the *rate* at which yield variability increases as N rates go up. If, in unfavorable environments, absolute yield variability is higher but the rate at which variability increases with N rates is relatively slight, the reported results on the effect of risk aversion would still be valid. If, however, yield variability is found to increase faster, risk aversion could substantially affect inputs. If strong interactions exist between N and stochastic factors such as weather, risk aversion could impede progress. The larger these interactions, the more significant would be the conflict between risk and profit maximization.

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Appendix. Computer program for estimating the Random Coefficient Model

The computer package Statistical Analysis System (SAS 1979) was used to estimate the model. A complete program listing appears on the left hand side; explanatory comments appear on the right.

PROGRAM	REMARKS
DATA IN;	The DATA statement creates a SAS data set. The word following DATA (in this case IN) is the name given to the data set.
INPUT YEAR YIELD N VARIETY;	The INPUT statement reads the data into the data set. The words following INPUT are the variables in the data set.
D42 = Ø; IF VARIETY='IR42' THEN D42=1; NSQ=N*N; INT=_N_/_N_;	Mathematical operations create new variables in the data set.
CARDS;	The CARDS statement specifies that all cards read hereafter are data cards until a new SAS command is met.
DATA YEAR76; SET IN; IF YEAR=76; DATA YEAR77; SET IN; IF YEAR=77; DATA YEAR78; SET IN; IF YEAR=78; DATA YEAR79; SET IN; IF YEAR=79; DATA YEAR81; SET IN; IF YEAR=81;	Subsetting IF creates a new data set whose observations satisfy the condition(s) in the IF statement. The observations come from the data SET (IN) specified in the SET statement.
PROC SYSREG DATA=YEAR76 OUTSSCP=XXQ1 OUTEST=BETAQ1; MODEL YIELD=N NSQ D42/COVB; PROC SYSREG DATA=YEAR77 OUTSSCP=XXQ2 OUTEST=BETAQ2; MODEL YIELD=N NSQ D42/COVB; PROC SYSREG DATA=YEAR78 OUTSSCP=XXQ3 OUTEST=BETAQ3; MODEL YIELD=N NSQ D42/COVB; PROC SYSREG DATA=YEAR79 OUTSSCP=XXQ4 OUTEST=BETAQ4; MODEL YIELD=N NSQ D42/COVB; PROC SYSREG DATA=YEAR81 OUTSSCP=XXQ5 OUTEST=BETAQ5; MODEL YIELD=N NSQ D42/COVB;	PROC statement invokes the procedure specified after PROC. DATA option in the procedure specifies the data set to be used in the procedure. OUTSSCP and OUTEST options output the crossproducts matrix into the data set specified by OUTSSCP= and the vector of parameter estimates into the data set specified by OUTEST= MODEL statement in the SYSREG procedure specifies the regression model. COVB option in the MODEL statement of the SYSREG procedure prints the variance-covariance matrix of the OLS coefficient estimates. SYSREG procedure computes OLS parameter estimates.
DATA SPVARCOV; SET BETAQ1 BETAQ2 BETAQ3 BETAQ4 BETAQ5;	Specifying 2 or more data sets in a SET statement vertically concatenates the data sets.
PROC CORR COV;	When no DATA option is given in any PROC statement, the data set immediately preceding the procedure is used. In this case, PROC COPR will use SPVARCOV.

Appendix continued

PROGRAM	REMARKS
DATA FORMEANS; SET IN; PROC SORT; BY N;	By statement in any procedure performs the procedure at each value of the variable(s) specified in the BY statement, Thus, the BY statement in the SORT procedure groups the data at each value (level) of N.
PROC MEANS; BY N; VAR INT N NSQ D42; OUTPUT OUT=BARNQ MEAN=INTMEAN NMEAN NSQMEAN D42MEAN;	MEANS procedure computes the means of the variables specified in the VAR statement. OUTPUT statement outputs data into the data set specified by OUT= which contains the variables specified by MEAN=
DATA BARØ; SET BARNQ; IF N=Ø; DATA BAR3Ø; SET BARNQ; IF N=3Ø; DATA BAR6Ø; SET BARNQ; IF N=6Ø; DATA BAR9Ø; SET BARNQ; IF N=9Ø; DATA BAR12Ø; SET BARNQ; IF N=12Ø;	

GLS ESTIMATION OF THE RCM COEFFICIENTS

- Before attempting to estimate any model under the set of assumptions implied by equation 2, test whether β_i are all fixed and are all equal.
- The MATRIX procedure was used in the empirical estimation.

I. ASYMPTOTIC TESTS OF EQUALITY BETWEEN FIXED COEFFICIENT VECTORS IN t RELATIONS WITH HETEROSCEDASTIC DISTURBANCES – equation 10 (HB)

PROGRAM	REMARKS
PROC MATRIX;	
FETCH XPRIMEX1 DATA=XXQ1; FETCH XPRIMEX2 DATA=XXQ2; FETCH XPRIMEX3 DATA=XXQ3; FETCH XPRIMEX4 DATA=XXQ4; FETCH XPRIMEX5 DATA=XXQ5;	FETCH statement in the MATRIX procedure inputs the data set specified in the DATA= option into the matrix specified by the FETCH command. $XPRIMEX_t$ is a (k+1) x (k+1) matrix.
	Delete row 2 and column 2 of each $XPRIMEX_t$. They contain crossproducts for the dependent variable.
XX1=XPRIMEX1 (1 3 4 5, 1 3 4 5); XX2=XPRIMEX2 (1 3 4 5, 1 3 4 5); XX3=XPRIMEX3 (1 3 4 5, 1 3 4 5); XX4=XPRIMEX4 (1 3 4 5, 1 3 4 5); XX5=XPRIMEX5 (1 3 4 5, 1 3 4 5);	This statement creates the matrix XX_t with 4 rows equal to rows 1 3 4 and 5 of $XPRIMEX_t$ and 4 columns equal to columns 1 3 4 and 5 of $XPRIMEX_t$. The general term for this operation is Matrix Reshaping. XX_t is a kxk matrix.
C1=XX1 #/MSE1; C2=XX2 #/MSE2; C3=XX3 #/MSE3; C4=XX4 #/MSE4; C5=XX5 #/MSE5;	Divide each XX_t by its corresponding MSE_t obtained from the OLS estimation.
D=C1+C2+C3+C4+C5; E=INV(D)	E is the first term of $\hat{\tilde{\beta}}$ of equation 10.
FETCH A1 DATA=BETAQ1; FETCH A2 DATA=BETAQ2; FETCH A3 DATA=BETAQ3; FETCH A4 DATA=BETAQ4; FETCH A5 DATA=BETAQ5;	A_t is a 1 x(k+2) vector.
	Delete columns 1 and 2 of each A_t. Move the last column of the vector to column 1. The last column given by OUTEST contains the intercept.
AVECTOR1=A1 (1, 6 3 4 5); AVECTOR2=A2 (1, 6 3 4 5); AVECTOR3=A3 (1, 6 3 4 5); AVECTOR4=A4 (1, 6 3 4 5); AVECTOR5=A5 (1, 6 3 4 5);	This statement creates $AVECTOR_t$ with 1 row equal to row 1 of A_t and 4 columns equal to columns 6 3 4 and 5 of A_t.
A1=AVECTOR1'; A2=AVECTOR2'; A3=AVECTOR3'; A4=AVECTOR4'; A5=AVECTOR5';	Each A_t is a kx1 vector. A_t is $\hat{\beta}_i$ in equation 6. 'denotes transpose of the matrix.

Appendix continued

PROGRAM	REMARKS
F1=C1 * A1; F2=C2 * A2; F3=C3 * A3; F4=C4 * A4; F5=C5 * A5; F=F1 + F2 + F3 + F4 + F5;	F is the second term in $\hat{\bar{\beta}}$ of equation 10.
B = E * F;	B is $\hat{\bar{\beta}}$ in equation 10.
X1=A1−B; X2=A2−B; X3=A3−B; X4=A4−B; X5=A5−B;	X_t is $(\hat{\beta}_i-\hat{\bar{\beta}})$ in equation 10.
X1PRIME=X1'; X2PRIME=X2'; X3PRIME=X3'; X4PRIME=X4'; X5PRIME=X5';	
Y1=X1PRIME * C1 * X1; Y2=X2PRIME * C2 * X2; Y3=X3PRIME * C3 * X3; Y4=X4PRIME * C4 * X4; Y5=X5PRIME * C5 * X5;	
HB = Y1 + Y2 + Y3 + Y4 + Y5;	HB is the homogeneity statistic given by equation 10.

II. ESTIMATION OF THE RANDOM COEFFICIENTS – equations 4, 5, 6, 7, and 8 (BETA)

PROGRAM	REMARKS
VARCOVQ1 = vc111 vc112...vc11k/ vc121 vc122...vc12k/ . . . vc1k1 vc1k2...vc1kk;	Input the variances-covariances matrix of the OLS coefficient estimates for each year.
VARCOVQ2 = vc211 vc212...vc21k/ vc221 vc222...vc22k/ . . . vc2k1 vc2k2...vc2kk;	$VARCOVQ_t$ is a kxk matrix. The vc_{tij} element is the covariance of the ith and the jth parameter in the model for year t. For i=j, vc_{tij} is the variance of the ith parameter. Input also the special variance-covariance matrix (VARCOVQ) printed by the PROC CORR COV statement in the preceding procedure.
. . . VARCOVQ5 = vc511 vc512...vc51k/ vc521 vc522...vc52k/ . . . vc5k1 vc5k2...vc5kk;	
VARQ1 = DIAG (VARCOVQ1); VARQ2 = DIAG (VARCOVQ2); VARQ3 = DIAG (VARCOVQ3); VARQ4 = DIAG (VARCOVQ4); VARQ5 = DIAG (VARCOVQ5);	Get the matrix of parameter variances and set covariances to zero. The DIAG (argument) statement creates a matrix whose diagonal elements are equal to the diagonal elements of the argument, and whose off-diagonal elements are zeroes.
VARQ = DIAG (VARCOVQ);	VARQ is the first term in $\hat{\Delta}$ of equation 8.
Z2 = (VARQ1 + VARQ2 + VARQ3 + VARQ4 + VARQ5) #/5;	Sum the variance matrices across T years and divide by T. Z2 is the second term in $\hat{\Delta}$ of equation 8.
Z3 = VARQ − Z2;	Z3 is $\hat{\Delta}$ in equation 8.
W111 = Z3 + VARQ1; W112 = Z3 + VARQ2; W113 = Z3 + VARQ3; W114 = Z3 + VARQ4; W115 = Z3 + VARQ5;	

Appendix continued

PROGRAM	REMARKS
W11 = INV (W111); W12 = INV (W112); W13 = INV (W113); W14 = INV (W114); W15 = INV (W115);	$W1_t$ is the second term in W_i of equation 5.
W = W11 + W12 + W13 + W14 + W15;	W is the first term in W_i of equation 5.
W1 = W11 * INV (W); W2 = W12 * INV (W); W3 = W13 * INV (W); W4 = W14 * INV (W); W5 = W15 * INV (W);	W_t is W_i in equation 5.
BETA = W1 * A1 + W2 * A2 + W3 * A3 + W4 * A4 + W5 * A5;	The A_t's in BETA are the same A_t's used in computing HB. BETA is $\bar{\beta}$ in equation 4.

III. ASYMPTOTIC STANDARD ERRORS – square roots of the relevant elements in the variance-covariance matrix given by equation 8 divided by T (ASE)

PROGRAM	REMARKS
ZASE = Z3 #/5;	Divide each element of $\hat{A}$ by T.
DIAGZASE = DIAG (ZASE);	Create a new matrix whose diagonal elements are those of ZASE and whose off-diagonal elements are zeroes.
ASE = DIAGZASE ## 0.5;	ASE is the matrix of the standard errors of the random coefficients.

IV. VARIANCE OF Y AT DIFFERENT N LEVELS – equation 13. ($VARN_\ell$)

PROGRAM	REMARKS
FETCH XØMEAN DATA = BARØ; FETCH X3ØMEAN DATA = BAR3Ø; FETCH X6ØMEAN DATA = BAR6Ø; FETCH X9ØMEAN DATA = BAR9Ø; FETCH X12ØMEAN DATA = BAR12Ø;	X_ℓ MEAN is $\bar{X}$ in equation 13 where $\ell = N$.
VARNØ = XØMEAN * Z3 * XØMEAN' + 488947.61; VARN30 = X3ØMEAN * Z3 * X3ØMEAN' + 488947.61; VARN60 = X6ØMEAN * Z3 * X6ØMEAN' + 488947.61; VARN90 = X9ØMEAN * Z3 * X9ØMEAN' + 488947.61; VARN120 = X12ØMEAN * Z3 * X12ØMEAN' + 488947.61;	$VARN_\ell$ is the variance of Y when N = ℓ.

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