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FOREIGN TRADE AND FACTOR INCOMES IN PORTUGAL: AN APPLICATION OF A GENERAL EQUILIBRIUM MODEL

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Abt Associates, Inc.
4250 Connecticut Ave., N.W.
Suite 500
Washington, D.C. 20008

Subcontractor:

Department of Agricultural Economics
Oklahoma State University
Stillwater, OK 74078

Prepared by:
Roger D. Norton
Pasquale L. Scandizzo
and
Linda W. Zimmerman

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ABSTRACT

This paper addresses the question of how to construct a policy-oriented model on the basis of a social accounting matrix (SAM). Starting from a Portuguese SAM, a general equilibrium model is developed step by step, and where it is appropriate options are indicated for the model's specification. The model is then applied with Portuguese data to the computation of general income multipliers, in order to provide a preliminary assessment of the aggregative and distributional effects on Portugal of entry into the European Economic Community. Increases in the availability of foreign exchange are found to affect urban incomes more than rural incomes, and to affect the lower income groups more than the upper income groups in both rural and urban areas. The general equilibrium model developed here contains production functions of the process analysis type, labor supply functions, possibilities for substitution among types of labor and between labor and capital, export and import functions, and a simple set of government accounts.

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Foreign Trade and Factor Incomes in Portugal: An Application of a General Equilibrium Model

Roger D. Norton, Pasquale L. Scandizzo, and Linda W. Zimmerman*

1. Introduction

The origin of this paper lies in two methodological concerns and one empirical concern. The first methodological issue has an empirical basis: how to efficiently incorporate into general equilibrium models the data set which is provided by social accounting matrices. These matrices extend interindustry accounts in the direction of institutional accounts and estimates of household incomes. They are static concepts, however, and to be useful for policy analysis they must be applied in a comparative statics or dynamic framework, in which change is contemplated.

The second topic in methodology is partly computational in origin. It has been demonstrated recently (Norton and Scandizzo, 1981) that a kind of general equilibrium in product and factor markets may be computed via a single-pass optimization solution, using linear programming algorithms. However, that demonstration was made in the context of a very simple illustrative economy, with no investment or savings, no government, and no foreign trade. Thus, one purpose of this paper is to extend that class of computable general equilibrium model to a more meaningful macroeconomic specification.

The paper's empirical concern is to estimate income multipliers associated with foreign trade for Portugal. Given that country's recent

*The authors are, respectively, Professor of Economics at the University of New Mexico; Senior Advisor to the Budget Commission of Parliament, Italy; and Adjunct Professor of Economics at Temple University. Neither the Portuguese government nor any other official agency is responsible for the views expressed.

decision to enter the European Economic Community, it is pertinent to inquire as to the aggregate income effects and distributional effects of changes in Portuguese trade variables. The estimates presented here include direct and indirect effects, and as such they may be called general equilibrium income multipliers.

As might be expected in a multi-purpose undertaking, not all of the paper's objectives have been fulfilled completely. Probably the most adequately treated issue is the second methodological one, of extending the computable general equilibrium models. Second in order of completeness is the issue of incorporating a SAM into a price-endogenous economy-wide model, but here we find that a SAM alone does not provide all the information required, and further there are several alternative paths of departure from the SAM. Finally, the Portuguese multipliers should be regarded as very preliminary estimates; they are found to be sensitive in particular to specifications of import functions and to factor pricing rules.

2. Social Accounting Matrices and Economic Models

There are two motivations for attempting to develop more systematic procedures for utilizing social accounting matrices, or SAMs, in economic models: the increasing availability of such accounts in many countries, and the conviction that accounting identities play a crucial role in any model exercise (see Taylor et al., 1980).

In the simplest sense, a SAM provides the link that closes a set of input-output accounts. Closure in this regard refers to recognizing the relations which must exist between factor incomes and consumption levels. Since factor incomes are generated by production activities,

introduction of this link into an input-output model provides a second relation between production and final demand, the first being provided by the material balance identities. This double linkage suggests the possibility, although not the necessity, of over-determination of the model. This topic is explored extensively later in the paper.

Final demand vectors in input-output tables normally are disaggregated into separate vectors for private consumption, government consumption, investment, exports, and imports. Therefore, to complete the link between factor incomes and final demand, a SAM must specify the following: (a) the formation of household incomes from factor incomes, i.e., the income distribution process; (b) government revenue collections and the government budget identity; (c) saving by institution and the intermediation of saving into capital formation; and (d) the role of the foreign sector in generating household incomes (such as flows of net factor income from abroad) and foreign saving. In the process of doing this, a SAM records four additional classes of identities beyond the input-output material balances: household budget identities, a government budget identity, the balance of payments identity, and the aggregate saving-investment identity. The latter may be a redundant equation, for it is implicit in the national accounts identity, which in turn is implied by the combination of the material balance identities and rules for generation of gross factor incomes for production.[1]

These identities are central elements in the Portuguese SAM and hence they are basic to the model presented in this paper. The material balance identity usually is stated in constant-price terms (in quantity indexes), although, following an early suggestion of Klein,

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[1]This statement assumes that net factor incomes from abroad are defined elsewhere in the system.

it may be stated in current-value terms by imposing the assumption of unitary derived demand elasticities for intermediate inputs. The other identities which were mentioned hold ex post in current prices, and they are not necessarily true in constant prices. Thus, in order to link the SAM relationships with basic input-output accounting in the context of a model, the presence of price variables is essential. The required prices refer to products (sectors) and factors.

For purposes of price determination, input-output accounts offer another identity: the Leontief pricing equation, or the input-output cost decomposition. This identity, too, is implicit in the Portuguese SAM, and it is incorporated in the model. The remaining relationships required for the model, such as a demand-side relationship between product prices and consumption, must be supplied from outside the SAM. In the course of the paper, the minimum requirements for data beyond the SAM are spelled out. In a general sense, the task of building a SAM-based model depends on decisions as to which of the SAM elements are to be regarded as stable (as system parameters) and which are regarded as variables. Explanatory equations for the latter then are introduced.

Useful references on SAMs are found in Pyatt and Roe (1977) and Bell, Hazell, and Slade (1982).

3. The Portuguese Social Accounts

Social accounts for Portugal have been estimated by GEBEI, which is a special study group of the Ministry of Planning, for 1974. It was aggregated to a "mini-SAM" by the World Bank and then updated to 1974. For this study, we use the 1974 mini-SAM (Table 1). It has two producing

sectors (agriculture, non-agriculture), six domestic factors of production, and four households. The households are defined as rural poor, rural non-poor, urban poor, and urban non-poor. There also is a corporate sector which generates saving but not final consumption demands. The rural households obtain part of their incomes from urban activities, and vice-versa. Factor incomes from abroad and government transfer payments contribute to the formation of disposable income in all household groups.

The Portuguese social accounts may be presented compactly as a series of sub-matrices (Table 1) whose elements are transactions, or assignments of economic flows, in current prices. These sub-matrices describe the economic relationships between the following sets: products (sectors), subscripted i,j; final goods, k; factors, f; and households, h. The sub-matrices describe input-output production relations, tranformation of output prices into final goods prices via indirect taxation, income distribution, and the expenditures of each institutional group, or sector, in the economy.

4. From the Social Accounts to an Economic Model

The process of formulating a model from the SAM information consists of three steps: (a) deciding which elements (cells) of the SAM are to be regarded as variable (endogenous); (b) specifying equations or constraints for those elements; and (c) deciding how to close the model, by omitting some equations or adding others. The list of equations begins with the identities previously noted. None of these may be omitted for the sake of closure.

For decision (a), most of the cells in the mini-SAM in Table 1 are made endogenous, such as quantities of goods purchased by each household

Table 1. 1974 Mini-SAM for Portugal (In Million Escudos)

			WANTS		FACTORS							INSTITUTIONS - CURRENT						REST OF WORLD		INVEST-MENT	PRODUCTION		IND. TAXES	Σ
			1.1	1.2	2.1	2.2	2.3	2.4	2.5	2.6	2.7	3.1	3.2	3.3	3.4	3.5	3.6	4.1	4.2	5	6.1	6.2	7	
1.1	WANTS	Food										18,932	41,608	2,394	21,449	-	-							84,383
1.2		Non-Food										17,556	77,978	3,168	56,291	-	-							155,033
2.1	FACTORS	Unskilled Ag. Labor																-	-		15,674	-		15,674
2.2		Skilled Ag. Labor																-	-		6,956	-		6,956
2.3		Unskilled Non-Ag. Labor																-	-		-	94,993		94,993
2.4		Skilled Non-Ag. Labor																-	-		-	59,410		59,410
2.5		Expatriates																22,087	11,399		-	-		33,486
2.6		Own Acct. & Small Enterprises																-	-		14,459	85,714		100,173
2.7		Capital																856	4,306		12,160	19,596		36,918
3.1	INSTITUTIONS - CURRENT	Household group 1: Rural Poor			34,385	-	788	289	5,308	14,382	-	-	-	-	-	-	1,376	-	-				-	36,528
3.2		Household group 2: Rural Non-Poor			-	5,956	54,768	17,043	19,436	68,914	-	-	-	-	-	2,370	6,313	-	-				-	174,800
3.3		Household group 3: Urban Poor			1,289	-	412	528	1,128	1,180	-	-	-	-	-	-	1,025	-	-				-	5,562
3.4		Household group 4: Urban Non-Poor			-	1,000	39,025	39,808	7,614	15,697	-	-	-	-	-	969	9,500	-	-				-	113,633
3.5		Enterprises			-	-	-	-	-	-	32,901	-	-	-	-	-	863	-	-				-	33,764
3.6		Government			-	-	-	-	-	-	2,134	-	20,244	-	13,160	6,543	-	100	88				27,873	70,142
4.1	REST OF WORLD	E.E.C.			-	-	-	105	-	-	824	-	-	-	-	-	264				1,609	53,697		56,499
4.2		Non-E.E.C.			-	-	-	1,637	-	-	1,059	-	-	-	-	-	370				18,587	64,778		86,431
5		Investment										-	36,970	-	22,733	23,362	1,295	2,000	18,921					103,781
6.1	PRODUCTION	Agriculture	33,640	1,659								-	-	-	-	-	90	1,292	2,704	3,130	5,464	46,679		88,658
6.2		Non-Agriculture	45,799	142,222								-	-	-	-	-	49,046	30,164	59,013	99,620	14,333	160,800		590,997
7		Indirect Taxes	4,944	11,152																1,031	-584	11,330		27,873
Σ		TOTAL	84,383	155,033	15,674	6,956	94,993	59,410	33,486	100,173	36,918	36,528	174,800	5,562	113,633	33,764	70,142	56,499	86,431	103,781	88,658	590,997	27,873	

group; and in general the row and column totals of the SAM also are treated as endogenous variables, such as total income for each household group.

The only items in the mini-SAM which are treated as exogenous are the entries in the "rest-of-world" columns which are found in rows 2.5 (expatriate earnings), 2.7 (private international capital flows), and 3.6 (public international capital flows). In addition, an upper bound has been placed on foreign borrowing (intersections of rest-of-world columns with row 5). The pairs of rest-of-world columns and rows, EEC and non-EEC, are aggregated into a single account for transactions with other countries. Therefore, if the 103 entries in the modified mini-SAM (excluding row totals since they duplicate column totals), 99 have become endogenous variables in the model and one is upper-bounded at the SAM value. Expatriate earnings and capital flows could have been made endogenous as well had there been a reasonable basis for supposing that their levels are determined by other variables in the SAM system.

To summarize, the variables chosen for the model are outputs, factor incomes, household and corporate and government incomes, government expenditures, saving by institution and quantities consumed, investment, foreign trade activities, and prices of outputs, of final goods, and of factors.

The equations of the model, called GAMA, are given in condensed form in the following section. Appendices give the demand parameters and base-year values of all variables. The remainder of this section discusses their content and the decisions which had to be made to bring additional information into GAMA, apart from that provided by the mini-SAM. The discussion is organized in terms of modules, or model components.

Module I. Aggregate Identities

There are two commodity balance restrictions here, one for agriculture and one for non-agriculture. Each provides for market-clearing behavior in its product market, abstracting from short-run inventory fluctuations. Total domestic production plus imports must be equal to the total sources of demand: intermediate input-output demands, household consumption, uses in government consumption, uses as capital goods, and exports. Each equation is expressed in real (constant-price) terms, although it holds in current prices as well, and each is written as an inequality which in principle permits free disposal of excess supply, but to be acceptable a model solution should contain no excess supply. These equations are direct borrowings of rows 6.1 and 6.2 in the mini-SAM.

Module II. Production, Factor Use, and Factor Incomes

The basic decision for this module concerned the nature of factor substitution in production. If the production elements of the mini-SAM are regarded as fixed in constant prices, then there is only one combination of the various labor skills and capital which will produce a sector's output. This may be a reasonably accurate statement in the very short run, but clearly it is unrealistic for a medium- to long-run adjustment period. In the spirit of requiring the minimal amount of information from outside the SAM, rather than launching a study in the estimation of production functions it was decided to adopt the SAM production elements as fixed in current prices. This assumption implies constant factor shares, i.e., a Cobb-Douglas expression in a linear model, a grid linearization was employed. Discrete technological alternatives along an isoquant are computed with the aid of the production function parameters, and the

model chooses among those alternatives, or some linear combination thereof, and it also chooses the scale at which they are to be operated.

Choice of isoquant points also implies the factor intensities of production and hence the factor use levels are immediately derived via other linear equations.

As regards factor incomes, two cases may be distinguished. In the first case, there are no more factors than goods; correspondingly, factor prices and hence factor incomes are determined by the production cost functions. In the more general case, which is illustrated by the Portuguese mini-SAM, there may be more factors than goods. In that case, in order to close the model, additional information must be introduced regarding the determination of factor prices. For N goods and F factors, F-N factor prices remain undetermined without the additional information. Since the model implicitly contains derived demand functions for factors, which determine factor demand prices, it was decided to provide the additional information in the form of factor supply functions. For simplicity of exposition, they have been posited to have unitary elasticities of supply, and for each factor a maximum utilization level of 5% more than the SAM level was assumed to be achievable. For the various categories of labor, this increase in labor supply could be attained, e.g., via increased labor force participation rates. Another grid linearization is applied to determine factor prices and factor incomes simultaneously with factor use levels.

For both commodities and factors, the base year physical use levels were established by the simple device of setting all prices equal to unity. The only exception occurs in the case of food and non-food prices, which must differ from the unitary prices for agricultural and non-agricultural

output owing to the presence of sales taxes. This procedure implies that the units of measure differ for each commodity and each factor, but that is of no particular importance, especially since the model is applied primarily to determine percentage changes from the base-year behavior.

Module III. Income Distribution

The task of this module is to determine household incomes on the basis of domestic factor incomes, factor incomes from abroad, and government transfer payments. The pertinent equations are taken directly from the mini-SAM, which is assumed to reflect patterns of asset ownership. For example, row 3.2 of the mini-SAM states that household group 2 received 174,800 million escudos in 1974, and of that total, 5,956 million escudos derived from the earnings of skilled agricultural labor, 54,768 from the earnings of unskilled non-agricultural labor, etc. To utilize this information, it has been assumed that the relevant column proportions are fixed. For example, of the skilled agricultural labor income, 5,956/6,956, or 85.62%, is destined for household group 2 and the remainder to household group 4. These proportions reflect the socio-economic status of laborers of each skill, and they are assumed to be relatively stable over the medium term. The proportions implicit in column 3.6 (rows 3.1 to 3.5) are the share of government transfer payments, as a proportion of the total government budget accruing to each household group. These are regarded as parameters fixed by policy action, but they may be varied to simulate the consequences of changes in transfers policy (see results section).

Module IV. Consumption and Savings

The mini-SAM provides average propensities to save and consumption budget shares. For example, food accounts for $\frac{18,932}{36,528}$ = 51.83% of the total budget of the rural poor (compare row 1.1, column 3.1, with the total row ($\sum$), column 3.1). However, to introduce the price and income responsiveness of consumption, demand elasticities must be obtained from other studies. For this investigation, international norms have been used for the elasticity values (see Appendix B).

The composition of government consumption in terms of agricultural, non-agricultural and imported goods (column 3.6, rows 6.1, 6.2, and 4.1 and 4.2) is assumed unchanged with respect to changes in government spending levels. If the SAM contained separate columns for different types of government spending programs, then the effects via the goods market of changes in the composition of government outlays could be simulated with the model. The level of government saving is assumed to be a linear function of the total size of the government budget.

Module V. Investment and Capital Stock

The savings-investment identity in the model is row 5 of the mini-SAM. Total investment (the total of row 5 and column 5) has to be converted into incremental capital, that is, an increase in investment above the SAM level of 103,781 has to lead to an increase in the factor endowment of domestic capital above the 31,750 reported in row 2.7.[1a] To do this requires a distinction between capital stock and the productive services of capital. Investment is an increment to capital stock, and production utilizes annual flows of capital services.

We have chosen to regard the figure of 31,756 as the total value of

[1a] Total capital available is 36,918, but of that total, 5.162 is invested abroad.

the flow of domestic capital services in 1974, and we have assumed a 5% annual real rate of return to capital for the economy as a whole. This implies that the base-year capital stock was $\frac{31,756}{.05} = 635,120$, or, put another way, that the base-year level of investment adds $.05 \times 103,781 = 5,189$ to the potential flow of capital services. Thus the capital restriction on production has been expressed as

$$\text{(capital uses)} \leq 26,567 + .05I$$

where $26,567 = 31,756-5,189$ and I is the investment variable. Since the model is used in a comparative statics sense, without reference to time elapsed between the base year and the solution period, there does not appear to be an obvious way to account for depreciation of capital stock.

Module VI. The Labor Force

The mini-SAM specifies five types of labor: unskilled and skilled workers in agriculture and non-agriculture, and own-account workers. The first four are sector-specific, but the last one may move between agriculture and non-agriculture. GAMA allows for indirect movement of labor by permitting substitution in production between the sector-specific labor types and both own-account workers and capital (module II). As noted, labor is assumed to be supplied with unitary elasticity up to a 5% increment over the base level (except in the model experiments directed toward analysis of long-run growth patterns).

An alternative labor supply specification is to incorporate institutionally-mandated wage minima for certain categories of labor. Also, as of this writing, a variant of GAMA is being developed which includes intersectoral labor migration possibilities.

Module VII. The Foreign Sector

As regards foreign trade, the mini-SAM shows only the value of agricultural and non-agricultural exports and the value of imports by domestic destination. In this module, the SAM is particularly lacking in information on appropriate functional relationships in a model.

For exports, downward-sloping export demand curves have been introduced with elasticities of -2.0. Without the possibility of export price variation, domestic output prices cannot vary as long as exports are non-zero. International studies of export demand elasticities generally concur in values in the range of -2.0 to -4.0, for commodities in which the exporting does not dominate world markets, but clearly this is a topic which deserves further study.

A variety of import specifications are likewise possible, but only time series analysis can reveal appropriate formulations. The extreme formulations are infinite elasticity of substitution between imported and domestic goods, and a zero elasticity. In input-output terminology, these correspond to competitive and complementary (or non-competitive) imports. The former specification, when incorporated in an economy-wide model, tends to lead to unrealistically large reallocations of imports over sectors, and it also prevents domestic relative price changes. Therefore, for the present version of the model an adaptation of the latter formulation has been used.

Most of the imports are dependent on domestic production levels, but not with fixed average propensities to import. An intercept term has been introduced along with fixed marginal propensities to import, so that the ratio of imports to domestic production changes as the level of economic activity changes. (Alternative import formulations will

continue to be investigated.) Imports which are used for government consumption and as capital goods are determined by fixed average coefficients in the government consumption and investment vectors.

The foreign capital flows and expatriate earnings from abroad are assumed to be determined exogenously at the levels in the mini-SAM, for purposes of comparative statics experiments. However, in some of the experiments reported in this paper, the expatriate earnings levels are varied, in order to study their effect on the economy. Foreign borrowing is governed by an upper bound, as mentioned previously.

Module VIII. The Government Accounts

The mini-SAM entries for government revenues (row 3.6) imply certain coefficients of revenue collection with respect to the corresponding column variables in the table. These coefficients are adopted for use in the government budget identity in GAMA. Government expenditures (column 3.6) may be divided into transfer payments (rows 3.1 to 3.5), government saving (row 5), and government consumption (row 4.1, 6.1, 6.2).

Transfer payment coefficients are defined with respect to the total size of the government budget, e.g., transfers to the rural poor are ($\frac{1,376}{70,142}$ = 1.96%) of the budget. However, as noted, these coefficients may be varied in different solutions to simulate the consequences of, say, a redistributional fiscal policy. Government saving is regarded as an exogenous variable. Total government consumption is endogenous, and the shares of government consumption by type of good are given by coefficients from the mini-SAM.

Module IX. Prices

Relative factor and commodity prices are determined in part by the optimization aspect of the model; the relevant theory is developed in Norton and Scandizzo (1981) and is elaborated upon in the following section. It should be noted that the exchange rate is assumed to be fixed, and therefore this model would not be valid under circumstances which imply very large changes in the balance of payments. Some of the experiments reported later in the paper deal with the effects of taxation and factor pricing policy, and other experiments (e.g., that of technical progress) result in relative price changes.

5. Properties of the Model

In this section, a simplified version of GAMA is set out in order to evaluate some of its properties at optimality. To facilitate the analysis, attention is restricted to the case in which there are no more factors than domestic goals; this makes it possible to proceed without including explicit equations for factor price formation, in other words, without factor supply functions.

Other key simplifications include omitting the government and corporate sectors and investment. This model extends the Norton-Scandizzo model (1981) by incorporation of foreign trade variables; in that sense, it resembles the model of Hong (1980). However, none of the cited studies present first-order conditions, and so in this paper we provide a modest initial step in evaluating this class of models in terms of their first-order conditions.

The sets of variables in GAMA, and the corresponding subscripts, are as follows:

goods	$i,j = 1,\ldots,N$
households	$h = 1,\ldots,H$
resources (factors)	$f = 1,\ldots,F$
production technologies per good	$t = 1,\ldots,T$
demand function segments	$s = 1,\ldots,S$

In this specification of the model, extensive use is made of demand segment variables, sometimes called grid interpolation variables, along with associated convex combination constraints. Prior information on the relevant range of the demand functions is utilized in order to establish an efficient linearization of nonlinear terms. The procedure is described in Norton and Scandizzo (1981, especially appendix B)[2]. The quantity demanded of good i by household h ($x_{i,h}$) is written in terms of demand segment variables $w_{i,h,s}$ as

$$\sum_s \theta_{i,h,s} w_{i,h,s} \quad ,$$

and likewise the bilinear form $x_{i,h}p_i$ is written

$$\sum_s \rho_{i,h,s} w_{i,h,s} \quad .$$

Similarly, revenues from export sales are written in terms of export demand segment variables $z_{i,s}$ as

$$\sum_s \overline{v}_{i,s} z_{i,s} \quad .$$

The export segment variables must sum to unity for a given good i, and if changes from the base-year values in household incomes and prices

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[2]The segmentation procedure is used in a partial equilibrium context in Duloy and Norton (1975).

are zero, then the domestic demand segment variables also sum to unity over the index s. When income levels and prices change, then the convexity constraints impose the appropriate rotation on the demand functions. This is accomplished by allowing the $w_{i,h,s}$ to sum to an endogenous value which is different than unity.

Making use of the above conventions, the primal equations of the model may be written as follows (variables and parameters are defined immediately after the equations):

<u>Maximand</u> [1][3]

$$\text{Max } Z = \sum_{i,h,s} \rho_{i,h,s} w_{i,h,s} - \sum_{f,h} \bar{b}_{f,h} p_f + \sum_{i,s} \bar{v}_{i,s} z_{i,s} \tag{1}$$

$$\text{Maximize} \left\{ \begin{bmatrix} \text{gross revenue} \\ \text{from sales to} \\ \text{household con-} \\ \text{sumption} \end{bmatrix} - \begin{bmatrix} \text{factor} \\ \text{costs} \end{bmatrix} + \begin{bmatrix} \text{gross revenue} \\ \text{from export} \\ \text{sales} \end{bmatrix} \right\}$$

subject to

<u>Commodity balances</u> [N]

$$[\pi_i] \qquad \sum_{h,s} \theta_{i,h,s} w_{i,h,s} + \sum_{s} \bar{u}_{i,s} z_{i,s} - \sum_{j,t} \bar{a}^*_{i,j,t} q_{j,t} - m_i \leq 0 \tag{2}$$

$$\begin{bmatrix} \text{quantities sold} \\ \text{to household} \\ \text{consumption} \end{bmatrix} + \begin{bmatrix} \text{quantities} \\ \text{sold on} \\ \text{export} \\ \text{markets} \end{bmatrix} - \begin{bmatrix} \text{production} \\ \text{net of inter-} \\ \text{mediate sales} \end{bmatrix} - [\text{imports}] \leq 0$$

[3]The symbol in brackets after the equation name gives the number of equations in each set; symbols in brackets before the equations are dual variables.

Resource constraints [F]

$$[\lambda_f] \qquad \sum_{i,t} \bar{r}_{f,i,t} q_{i,t} \leq \bar{s}_f \tag{3}$$

$$\begin{bmatrix}\text{requirements}\\ \text{for resource}\\ \text{f in production}\end{bmatrix} \leq \begin{bmatrix}\text{available}\\ \text{supply of}\\ \text{resource f}\end{bmatrix}$$

Domestic demand functions [N x H]

$$[\alpha_{i,h}] \qquad -\frac{1}{x_{i,h,o}} \sum_s \rho_{i,h,s} w_{i,h,s} + \sum_j \left(\frac{\eta_{i,j,h}}{p_{j,o}}\right) p_j$$

$$+ \left(\frac{\varepsilon_{i,h}}{y^d_{h,o}}\right)\left(1-\bar{a}_h\right) y_h = -1 \tag{4}$$

$$\begin{bmatrix}\text{quantity demanded}\\ \text{relative to base}\\ \text{year quantity}\\ \text{demanded}\end{bmatrix} + \begin{bmatrix}\text{elasticity}\\ \text{adjusted term}\\ \text{in all product}\\ \text{prices}\end{bmatrix}$$

$$\begin{bmatrix}\text{income relative to base-}\\ \text{year income, adjusted for}\\ \text{Engel elasticity and savings}\\ \text{propensity}\end{bmatrix} = -1$$

Income distribution [H]

$$[\gamma_h] \qquad -\sum_f \bar{b}_{f,h} p_f + y_h = 0 \tag{5}$$

$$\begin{bmatrix}\text{value of returns}\\ \text{to factor endowments}\\ \text{held by household h}\end{bmatrix} = \begin{bmatrix}\text{income of}\\ \text{household}\\ \text{h}\end{bmatrix}$$

Household budget constraints [H]

$$[\phi_h] \qquad -y_h(1 - \bar{a}_h) + \sum_{i,s} \rho_{i,h,s} w_{i,h,s} \leq 0 \qquad (6)$$

[disposable income of household h] $\geq$ [value of consumption expenditures by household h]

Marginal cost pricing [N x T]

$$[\mu_{i,t}] \qquad \bar{a}^*_{i,i,t} p_i + \sum_j \bar{a}^*_{j,i,t} p_j - \sum_f \bar{r}_{f,i,t} p_f \leq 0 \qquad (7)$$

[unit value of output] $\leq$ [intermediate goods costs, per unit of output] + [factor costs per unit of output]

Balance of payments constraint [1]

$$[\delta] \qquad \sum_i \bar{p}^m_i m_i - \sum_{i,s} \bar{v}_{i,s} z_{i,s} \leq B \qquad (8)$$

[value of imports] - [value of exports] $\leq$ [net available foreign capital]

Import functions [N]

$$[\beta_i] \qquad m_i - \bar{m}^*_i \sum_{j,t} q_{j,t} - \bar{m}_i = 0 \qquad (9)$$

[imports depend on total gross output plus an autonomous component]

Domestic demand convexity constraints [4] [N x H]

$$[\sigma_{i,h}] \quad \sum_s w_{i,h,s} - \left(\frac{\varepsilon_{i,h}}{y^d_{h,o}}\right)(1-\bar{a}_h)y_h - \sum_{j\neq i}\left(\frac{\eta_{i,j,h}}{p_{j,o}}\right)p_j$$

$$\leq 1 + \eta_{i,i,h} \tag{10}$$

$$\begin{bmatrix}\text{sum of demand}\\ \text{segment variables,}\\ \text{good i, household h}\end{bmatrix} \leq 1 + \begin{bmatrix}\text{term in the}\\ \text{growth rate of}\\ \text{disposable}\\ \text{income}\end{bmatrix} + \begin{bmatrix}\text{sum of}\\ \text{terms in}\\ \text{rates of}\\ \text{change of}\\ \text{prices of}\\ \text{other goods}\end{bmatrix}$$

---

[4] The derivation of this equation is as follows, where dotted variables indicate percentage changes:

$$\sum_s w_{i,h,s} \leq 1 + \varepsilon_{i,h}\dot{y}^d_h + \sum_{j\neq i}\eta_{i,j,h}\dot{p}_j$$

$$\sum_s w_{i,h,s} \leq 1 + \varepsilon_{i,h}\left(\frac{y^d_h}{y^d_{h,o}} - 1\right) + \sum_{j\neq i}\eta_{i,j,h}\left(\frac{p_j}{p_{i,o}} - 1\right)$$

$$\sum_s w_{i,h,s} \leq 1 + \left(\frac{\varepsilon_{i,h}}{y^d_{h,o}}\right)y^d_h - \varepsilon_{i,h} + \sum_{j+i}\left(\frac{\eta_{i,j,h}}{p_{j,o}}\right)p_j$$

$$- \sum_{j\neq i}\eta_{i,j,h}$$

and, by the homogeneity condition of demand theory,

$$-\varepsilon_{i,h} - \sum_{j\neq i}\eta_{i,j,h} = \eta_{i,i,h}$$

Export convexity constraints

$$[\psi_i] \qquad \sum_s z_{i,s} \leq 1 \tag{11}$$

Variables

$w_{i,h,s}$ = segment choice variables on domestic demand functions

p_f = factor price

z_i = segment choice variables on export demand functions

$q_{j,t}$ = production (of commodity j under technology t)

m_i = imports

p_i, p_j = commodity price

y_h = household income

Parameters

$\rho_{i,h,s}$ = gross revenue (price times quantity) from sales to consumers

$\bar{b}_{f,h}$ = initial endowments of resource (factor) f held by household h

$\bar{v}_{i,s}$ = gross revenue from export sales

$\theta_{i,h,s}$ = quantity demanded in domestic markets

$\bar{u}_{i,s}$ = quantity exported

$\bar{a}^*_{i,j,t}$ = element of a rectangular (I-A) Leontief matrix which has multiple technologies per good

$\bar{r}_{f,i,t}$ = resource requirements per unit of production

$\eta_{i,j,h}$ = cross-price elasticity of demand

$\varepsilon_{i,h}$ = Engel elasticity

$\bar{a}_h$ = average propensity to save

$\bar{p}_i^m$ = import price

$\bar{m}_i'$ = marginal propensity to import with respect to domestic production

$\bar{m}_i$ = intercept term in import function

Initial conditions on quantities demanded, prices, and disposable incomes are denoted by the symbols $s_{i,h,o}$, $p_{j,o}$, and $y^d_{h,o}$, respectively. Without loss of generality, the input-output coefficients are normalized so that $\bar{a}^*_{i,i,t} = 1.0$ and $\bar{a}^*_{i,j,t} < 0$, $i \neq j$.

The parameters are required to be defined so that, for a base-year set of values of the variables, all equations hold as equalities except for those equations in (7) which represent unused technologies. The demand parameters must be derived in a manner which ensures their consistency with the three basic tenets of demand theory:

<u>Engel aggregation</u>

$$\sum_i \varepsilon_i \left(\frac{p_{i,o} x_{i,h,o}}{y_{h,o}} \right) = 1 \tag{12}$$

Homogeneity

$$\sum_j \eta_{i,j,h} = -\varepsilon_{i,h} \tag{13}$$

Cournot aggregation

$$\sum_i \left(\frac{p_{i,o} x_{i,h,o}}{y_{h,o}} \right) \eta_{i,j,h} = - \left(\frac{p_{j,o} x_{j,h,o}}{y_{h,o}} \right) \tag{14}$$

Before developing the implications of the system (1) - (11), it is useful to restate it to eliminate the exact equalities. Hence the system's constraint set becomes [6]

$$[\pi_i] \qquad \sum_j \eta_{i,j,h} x_{i,h,o} p_j + \sum_h \left(\frac{\varepsilon_{i,h} x_{i,h,o}}{y^d_{h,o}} \right) \sum_f \overline{b}_{f,h} p_f$$

$$+ \sum_s \overline{u}_{i,s} z_{i,s} - \sum_{j,t} \overline{a}^*_{i,j,t} q_{i,t} - m_i \leq \sum_h x_{i,h,o} \tag{15}$$

$$[\lambda_f] \qquad \sum_{i,t} \overline{r}_{f,i,t} q_{i,t} \leq \overline{s}_f \tag{16}$$

$$[\gamma_h] \qquad -\left(\sum_f \overline{b}_{f,h} p_f \right) + \sum_{i,s} \rho_{i,h,s} w_{i,h,s} \leq 0 \tag{17}$$

$$[\mu_{i,t}] \qquad p_i + \sum_{j \neq i} \overline{a}^*_{j,i,t} p_j - \sum_t \overline{r}_{f,i,t} p_f \leq 0 \tag{18}$$

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[6] This derivation utilizes the conventions that all $p_{j,o} = 1.0$ and that all $\overline{a}^*_{i,i,t} = 1.0$.

$$[\sigma_{i,h}] \quad \sum_s w_{i,h,s} - \left[\frac{\varepsilon_{i,h}}{y^d_{h,o}} \right] \sum_f \overline{b}_{f,h} p_f - \sum_{j \neq i} \eta_{i,j,h} p_j$$

$$\leq 1 + \eta_{i,i,h} \tag{19}$$

$$[\delta] \quad \sum_i \overline{p}_i \overline{m}_i \sum_{j,t} q_{j,t} + \sum_i \overline{p}_i \overline{m}_i - \sum_{i,s} \overline{v}_{i,s} z_{i,s} \leq B \tag{20}$$

$$[\phi_i] \quad \sum_s z_{i,s} \leq 1 \tag{21}$$

For the primal variables in the system (1), (15)-(21), the conditions of a competitive market equilibrium are written into the primal constraints. Therefore, if a feasible and optimal solution exists with all constraints binding, except for (20) and (21), and for (18) in the case of unused technologies, then the primal variables conform to the conditions of a competitive equilibrium.

The first-order conditions for (1) and (15)-(21) are as follows:

$$\left(\frac{\partial L}{\partial w_{i,h,s}} \right) w_{i,h,s} = \left[\rho_{i,h,s}(1-\gamma_h) - \sigma_{i,h} \right] w_{i,h,s} = 0 \tag{22}$$

$$\left(\frac{\partial L}{\partial p_f}\right)p_f = \Bigg[- \sum_h \overline{b}_{f,h} - \sum_{i,h}\left(\frac{\varepsilon_{i,h}x_{i,h,o}}{y^d_{h,o}}\right)\overline{b}_{f,h}\pi_i$$

$$+ \sum_h \overline{b}_{f,h}\gamma_h + \sum_{i,t}\overline{r}_{f,i,t}\mu_{i,t}$$

$$+ \sum_{i,h}\left(\frac{\varepsilon_{i,h}\overline{b}_{f,h}}{y^d_{h,o}}\right)\sigma_{i,h}\Bigg]p_f = 0 \tag{23}$$

$$\left(\frac{\partial L}{\partial z_{i,s}}\right)z_{i,s} = \left[\overline{v}_{i,s} - \overline{u}_{i,s}\pi_i + \overline{v}_{i,s}\delta - \phi_i\right]z_{i,s} = 0 \tag{24}$$

$$\left(\frac{\partial L}{\partial q_{j,t}}\right)q_{j,t} = \Bigg[\pi_j - \sum_{i\neq j}\overline{a}^*_{i,j,t}\pi_i - \sum_f \overline{r}_{f,j,t}\lambda_f$$

$$-\left(\sum_i \overline{p}_i\overline{m}_i'\right)\delta\Bigg]q_{j,t} = 0 \tag{25}$$

$$\left(\frac{\partial L}{\partial p_j}\right)p_j = \Bigg[- \sum_{i,h}\eta_{i,j,h}x_{i,h,o}\pi_i - \mu_{j,t} + \sum_{i\neq j}\overline{a}^*_{j,i,t}\mu_{i,t}$$

$$+ \sum_{\substack{i\neq j\\ h}}\eta_{i,j,h}\sigma_{i,h}\Bigg]p_j = 0 \tag{26}$$

In the following paragraphs, it is shown that the solution

$$S_1 = \left\{ \pi_i = p_i > 0 \; ; \; \lambda_f = p_f > 0 \; ; \right.$$

$$\left. \gamma_h = 0 \; ; \; \mu_{i,t} = q_{i,t} \; ; \; \sigma_{i,h} = p_i x_{i,h} \right\} ,$$

where $x_{i,h}$ connotes the quantity consumed, is a sufficient condition for all primal equations (15) - (19) to hold as equalities and for the solution to replicate the initial conditions in the variables.

First, substitute relations S_1 into (23) and cancel:

$$\sum_{i,t} \bar{r}_{f,i,t} q_{i,t} = \sum_h \bar{b}_{f,h} = \bar{s}_f \tag{27}$$

which shows that (16) must hold as an equality.

From (22), multiplying out and summing,

$$\sum_s w_{i,h,s} = \frac{p_i x_{i,h}}{\sigma_{i,h}} = 1 \tag{28}$$

and therefore all $y_h = y_{h,o}$ and all $p_j = p_{j,o}$ (see note above on the derivation of (10) and hence of (19)). By (4), then, $x_{i,h} = x_{i,h,o}$.

For departures from the initial conditions, primal-dual proportionality may hold but exact equality cannot hold.

Pricing and resource valuation relationships emerge from the dual solution. From (25)

$$\pi_j = \sum \overline{a}^*_{i,j,t}\pi_i + \sum_f \overline{r}_{f,j,t}\lambda_f + \hat{m}\delta \tag{29}$$

where: a) the first right-hand term represents intermediate input costs;

b) the second right-hand term represents the marginal opportunity cost of fixed resources; and

c) the third right-hand term represents the cost of the incremental imports required by an increase in output;

$\hat{m} = \sum_i \overline{p}^m_i \overline{m}'_i$ (marginal import value at world prices)

δ = shadow exchange rate (shadow price on the foreign borrowing constraint).

Also, from the (unstated) first-order conditions associated with system (1) - (11),

$$\pi_i = \frac{v^e_i(1 + \delta) - \psi_i}{q^e_i} = p^e_i(1 + \delta) - \psi_i \tag{30}$$

where v_i^e is the value of Portuguese exports of sector i at world prices, p_i^e is the (endogenous) export prices, and ψ_i is the shadow price of the external export market limitation which fixes the position of the demand curve for Portugal's exports. (Shifting that demand curve rightward would reduce the value of ψ_i).

6. Model Results

The numerical results from the model are summarized in Tables 2-5. In Table 2, the base solution is compared with the base year values from the mini-SAM; it can be seen that the comparison is quite close, as would be expected from the fact that the base-year accounts have been made consistent in order to define market-clearing conditions. Table 3 defines the eight experiments which were specified for this paper, and Table 4 reports their principal results. In Table 5 some income multipliers and elasticities are calculated with respect to changes in foreign exchange availability.

Because the model contains upper limits on resource availability, the parametric changes in the experiments generally have been defined in a negative direction. In other words, increasing one of the resource endowments would not lead to a very significant increase in total economic activity, because other resource constraints would be binding. Factor substitution would offset this consideration to some extent, but nevertheless a more complete measurement of multipliers can be obtained by making the changes in a negative direction. The exception to this rule was experiment number 1, in which positive technological progress was simulated.

Table 2. Base Solution

	Base Values From SAM	Base Solution
XA	88.658	88.69931
XN	590.997	591.40077
EFUA	15.674	15.79562
EFSA	6.956	6.95924
EFUN	94.993	95.05790
EFSN	59.410	59.410
EFOW	100.173	100.20494
EK	31.756	31.7438
CAPITL	103.781	103.53696
YRPR	36.528	36.76342
YRNPR	174.800	174.87130
YUPR	5.562	5.58142
YUNPR	113.633	113.65772
EXPA	3.996	3.996
EXPN	79.177	79.29788
IMP	142.930	142.98967
FBOR	20.921	20.921
PRNF	1.0775	1.07606
PRF	1.0622	1.06041
PFUA	1.0	1.00776
PFSA	1.0	1.00047
PFUN	1.0	1.00068
PFSN	1.0	1.0
PFOW	1.0	1.00032
PK	1.0	0.97846

Note: See appendix 2 for definition of the names of variables.

Table 3. Description of Experiments

#	Description	Procedure
1	Negative Technological Progress in Non-Agriculture	(XA, TBXA) 1.0 → 0.95 (PRAGR, $MCOSTA_t$) 0.9156 → 0.96138
2	Reduce Agricultural Exports to Zero	(RHS, CCEA) 1.0 → 0.0
3	Reduce Non-Agricultural Exports by 3.996 (Shift Demand Function to the Left)	(RHS, CCEN) 1.0 → 0.94953
4	Reduce Expatriate Income to Rural Poor by 3.996	(RHS, LEXPAT1) 5.308 → 1.312
5	Reduce Expatriate Income to Rural Non-poor by 3.996	(RHS, LEXPAT2) 19.436 → 15.440
6	Reduce Foreign Borrowing Limit by 3.996	(RHS, LFBOR) 20.921 → 16.925
7	Redistribute Government Transfer Payments from Rural Non-poor to Rural Poor	(GOVT, T1) 0.0196 → 0.0500 (GOVT, T2) 0.0900 → 0.0596
8	Shift Supply of Unskilled Agricultural Labor to the Right by 15%	FSUA1 / FSUA2 / FSUA3 ESUA 14.42008 / 18.0251 / 18.92636 YSUA 11.53606 / 18.0251 / 19.87267

Table 3. Description of Experiments

#	Description	Procedure
1	Technological Progress in Agriculture	(XA,TBXA) 1.0 → 0.95 (PRAGR,MCOSTA$_i$) 0.91562 → 0.96138 (XA,CBALA) 0.91562 → 0.96138
2	Reduce Agricultural Exports to Zero	(RHS,CCEA) 1.0 → 0.0
3	Reduce Non-Agricultural Exports by 3.996 (Shift Demand Function to the Left)	(RHS,CCEN) 1.0 → 0.94953
4	Reduce Expatriate Income to Rural Poor by 3.996	(RHS,LEXPAT1) 5.308 → 1.312
5	Reduce Expatriate Income to Rural Non-poor by 3.996	(RHS,LEXPAT2) 19.436 → 15.440
6	Reduce Foreign Borrowing Limit by 3.996	(RHS,LFBOR) 20.921 → 16.925
7	Redistribute Government Transfer Payments from Rural Non-poor to Rural Poor	(GOVT,T1) 0.0196 → 0.0500 (GOVT,T2) 0.0900 → 0.0596
8	Shift Supply of Unskilled Agricultural Labor to the Right by 15%	FSUA1 / FSUA2 / FSUA3 ESUA 14.42008 / 18.0251 / 18.92636 YSUA 11.53606 / 18.0251 / 19.87267
9	With Shifted Supply Function of #8, Add an Institution Wage Floor of 1.0	See Above {LPFUA} PFUA $\geq$ 1.0

Table 4. Results of Experiments

	BASE	#1	#2	#3	#4	#5	#6	#7	#8
XA	88.69931	88.54291	82.32759	86.41406	85.66936	86.12904	86.28330	88.94601	88.62287
XN	591.40077	591.22224	574.67418	571.67752	571.94436	571.60192	571.72440	591.18813	591.01017
EFUA	15.79562	15.38317	15.15245*	15.27729*	15.14563	15.22690	15.25417	15.81334	16.60782
EFSA	6.95924	6.72172	6.45932*	6.77994*	6.72151	6.75758	6.76968	6.97860	6.95324
EFUN	95.05790	95.97578*	94.993	94.993	94.993	94.99300	94.993	95.02372	94.993
EFSN	59.410	59.41000	57.76915*	57.46791*	57.49473	57.46031	57.47262	59.41000	59.40984
EFOW	100.20494	100.17300	99.66095	100.02338	99.97437	99.95152	100.01478	100.26776	100.173
EK	31.7438	28.80125*	25.07176*	25.52468*	25.43935	25.48538*	25.50970	31.71931	30.87227
CAPITL	103.53696	103.87406	100.33431	100.65189	100.60329	99.78251	96.64788	103.04726	103.83861
YRPR	36.76342	36.06484	35.47340	35.77355	31.54137	35.65733	35.73221	38.93095	36.34262
YRNPR	174.87130	175.58174	172.09466	172.92496	172.75283	168.73244	172.89575	172.80387	174.79368
YUPR	5.58142	5.52859	5.41364	5.43714	5.41239	5.41736	5.43290	5.58216	5.54468
YUNPR	113.65772	114.39677	110.96030	110.81444	110.77239	110.67886	110.80812	113.61964	113.62397
EXPA	3.996	3.996	0	3.996	3.996	3.996	3.996	3.996	3.996
PEXPA	1.0	1.0	-	1.0	1.0	1.0	1.0	1.0	1.0
EXPN	79.29788	79.177	79.177	75.74881	79.49315	79.53834	79.70073	79.29082	79.177
PEXPN	0.99898	1.0	1.0	0.99496	0.99733	0.99695	0.99559	0.99904	1.0
EMP	142.98967	142.950	138.954	139.14028	139.05774	139.07257	139.12586	142.98735	142.950
FBOR	20.921	20.921	20.921	20.921	20.921	20.92100	16.925	20.921	20.921
PRNF	1.07606	1.07746	1.07584	1.07716	1.07666	1.07669	1.07707	1.07597	1.07724
PRF	1.06041	1.06442	1.06040	1.06416	1.06271	1.06334	1.06391	1.06041	1.05696
PFUA	1.00776	.98145	0.96673	0.97469	0.96629	0.97148	0.97322	1.00839	0.92137
PFSA	1.00047	.96632	0.92860	0.97469	0.96629	0.97148	0.97322	1.00325	0.9996
PFUN	1.00068	1.01035	1.0	1.0	1.0	1.0	1.0	1.00032	1.0
PFSN	1.0	1.00000	0.97238	0.96731	0.96776	0.96718	0.96739	1.0	1.0
PFOW	1.00032	1.00000	0.99489	0.99851	0.99802	0.99779	0.99842	1.00095	1.0
PK	0.97846	1.07592	1.08503	1.08564	1.08615	1.08550	1.08573	0.97566	1.02826

Note: an asterisk (*) signifies significant underemployment of a factor.

Table 5. Income Multipliers and Elasticities Associated with Changes in Foreign Exchange Availability (absolute values)

Experiment		ΔForeign Exchange	ΔY	ΔYR	ΔYU	ΔYPR	ΔYNPR
#2		3.9960	6.9319	4.0667	2.8652	1.4578	5.4741
	(M)	n.a.	1.7345	1.0177	0.7170	0.3648	1.3699
	(%)	2.80%	2.10%	1.92%	2.40%	3.44%	1.90%
	(ε)	n.a.	0.750	0.686	0.857	1.229	0.679
#3		3.4282	5.9238	2.9362	2.9876	1.1342	4.7896
	(M)	n.a.	1.7280	0.8565	0.8715	0.3308	1.3971
	(%)	2.40%	1.79%	1.39%	2.51%	2.68%	1.66%
	(ε)	n.a.	0.746	0.579	1.046	1.117	0.692
#4		3.9960	10.3949	7.3405	3.0544	5.3911	5.0038
	(M)	n.a.	2.6013	1.8370	0.7644	1.3491	1.2522
	(%)	2.80%	3.14%	3.47%	2.56%	12.73%	1.73%
	(ε)	n.a.	1.121	1.239	0.914	4.546	0.618
#5		3.996	10.3879	7.2450	3.1429	1.2702	9.1178
	(M)	n.a.	2.5996	1.8131	0.7865	0.3179	2.2817
	(%)	2.80%	3.14%	3.42%	2.64%	3.00%	3.16%
	(ε)	n.a.	1.121	1.221	0.943	1.071	1.129

Note: See Tables 3 and 4 for definitions of the experiments and more detail on them.

Key: Y, total household income
YR, rural household income
YU, urban household income
YPR, income of the poor
YNPR, income of the non-poor

M, multiplier
%, percentage change
ε, elasticity with respect to the change in foreign exchange availability

In experiment number 1 unskilled non-agricultural labor is made 5% more productive. This change has the expected result that the relative price of non-agricultural goods decreases (Table 4). Also, given the impossibility of raising aggregate output, the change leads to underemployment of that factor. In this case, underemployment means moving upward on the implicit derived demand curve for the factor, and so its marginal productivity and price actually rise. Much of the increased factor productivity also consitutes higher returns to capital, and so capital's price rises and its level of use also drops. The total set of factor price and quantity changes leads to an increase in the incomes of the non-poor and a deterioration of the income distribution, even though it was an unskilled labor group which became more efficient. Entrepreneurs appropriate most of the increased returns.

Experiments 2 through 6 are concerned with the consequences of reductions in foreign exchange availability. In those five experiments, the reduction is effected in different ways: via exports, via worker remittances from abroad, and via foreign borrowing. In all cases, there is an output reduction, and, as expected, reduction of agricultural exports leads to a proportionately greater reduction in agricultural output. Relative output prices change in the expected directions, according as agricultural or non-agricultural exports are reduced (Table 4, rows PRNF and PRF, columns for experiments #2 and #5). In all cases, with less foreign exchange available, capital is underutilized, and, since its price is purely a reflection of marginal value product, that price rises.

The corresponding income changes are reported in Table 5. The figures in the rows labelled (M) may be called "general equilibrium multipliers."

They take into account both direct and indirect effects in the computation of the full adjustment of income to reduced availability of foreign exchange. Interestingly, the multipliers vary according to the source of the change in foreign exchange. A decrease in worker remittances from abroad has a greater impact on Portuguese GDP than does a decrease in exports or foreign borrowing. This is understandable in light of the fact that the value added in exports is significantly less than the sales value of the exports, but it points out the importance of net factor earnings from abroad for economies like Portugal's. The multipliers range from about 1.7 (exports and foreign borrowing) to 2.6 (remittances).

The calculated elasticities reflect the sensitivity of different components of the economy to foreign exchange availability. A pattern emerges in which the urban sector is more sensitive to foreign exchange availability than the rural sector is, and also the poor are more sensitive than the non-poor. Even when agricultural exports are reduced (experiment #2), the elasticity of income change is greater for the urban income groups than it is for the rural groups. Likewise, the elasticity is always greater for the poor than for the non-poor, except when expatriate earnings for the non-poor are reduced (experiment #5). Even then, the elasticities of income response for the poor and non-poor are almost identical.

Although they are not shown, Table 5 also reveals elasticities of total income change with respect to rural income change. In experiments #4 and #5, the expatriate earnings for the rural poor and the rural non-poor, respectively, are altered. Urban incomes change as well, owing to reduced household demand for goods and services, and the total chain of effects gives a total-to-rural income multiplier of 1.42 - 1.43.

Experiment 7 (Table 4) is directed to the question of the effects of income redistribution via government transfer payments policies. The magnitudes of change are not very great, but redistributing income from the rural poor to the rural non-poor has a slight tendency to increase the relative price of food and to increase total household income in the economy.

These experiments represent one kind of application of this style of model. From the model's structure, it is clear that many other kinds of experiments are possible as well.

Appendix 1. GAMA Demand Parameters

$\tilde{w}$ = -4, POOR

$\tilde{w}$ = -2, NPR

Rural Poor

α_i	ε_i	
.5183	0.8	food
.4817	1.2152	nonfood

$$e_{ij} = \begin{bmatrix} -.5317 & -.2683 \\ -.5039 & -.7113 \end{bmatrix} \begin{matrix} f \\ nf \end{matrix}$$

Rural Non-poor

α_i	ε_i	
.3479	0.5	food
.6521	1.2668	nonfood

$$e_{ij} = \begin{bmatrix} -.3805 & -.1195 \\ -.3306 & -.9362 \end{bmatrix} \begin{matrix} f \\ nf \end{matrix}$$

Urban Poor

α_i	ε_i	
.4304	0.8	food
.5696	1.1511	nonfood

$$e_{ij} = \begin{bmatrix} -.4755 & -.3245 \\ -.3963 & -.7548 \end{bmatrix} \begin{matrix} f \\ nf \end{matrix}$$

Urban Non-poor

α_i	ε_i	
.2759	0.5	food
.7241	1.1905	nonfood

$$e_{ij} = \begin{bmatrix} -.3535 & -.1465 \\ -.2463 & -.9442 \end{bmatrix} \begin{matrix} f \\ nf \end{matrix}$$

$\tilde{w}$ = Frisch money flexibility parameters

GAMA:

Variables and Their Base Year Values

Variable	Value	Description	SAM Location
XA	88.658	gross agricultural output	(CTOTAL, PAGRIC)
XN	590.997	gross non-agricultural output	(CTOTAL, PNAGR)
VA	49.249	agricultural value added	(PAGRIC,FUAGR) + PAGRIC,FSAGR) + (PAGRIC,FOWNACC) + (PAGRIC,FOTHERS)
VN	259.713	non-agricultural value added	(PNAGR,FUNAG) + (PNAGR,FSNAG) + (PNAGR,FOWNACC) + (PNAGR,FOTHERS)
EXPA	3.996	agricultural exports	(PAGRIC,REEC) + (PAGRIC, RNEEC)
EXPN	79.177	non-agricultural exports	(PNAGR,REEC) + (PNAGR,RNEEC)
GSAV	1.295	government savings	(IGOVT,CCAPITL)
GOVT	70.142	total government expenditure	(CTOTAL,IGOVT)
GCON	49.770	government consumption (includes government imports)	(IGOVT,REEC) + (IGOVT,RNEEC) + (IGOVT,PAGRIC) + (IGOVT,PNAGR)
CPEXP	5.162	private capital exports	(REEC,FOTHERS) + (RNEEC,FOTHERS)
CGEXP	0.188	government capital exports	(REEC,IGOVT) + (RNEEC,IGOVT)
FBOR	20.921	foreign borrowing	(REEC,CCAPITL) + (RNEEC,CCAPITL)
EXPAT1	5.308	expatriate earnings abroad by rural poor	(FEXPAT,IRPOOR)
EXPAT2	19.436	expatriate earnings abroad by rural non-poor	(FEXPAT,IRNPR)
EXPAT3	1.128	expatriate earnings abroad by urban poor	(FEXPAT,IUPOOR)
EXPAT4	7.614	expatriate earnings abroad by urban non-poor	(FEXPAT,IUNPR)
CAPITL	103.781	total capital formation	(CTOTAL,CCAPITL)
YUA	15.674	factor income, unskilled agricultural labor	(PAGRIC,FUAGR)
YSA	6.956	factor income, skilled agricultural labor	(PAGRIC,FSAGR)
YUN	94.993	factor income, unskilled urban labor	(PAGRIC,FUNAG)
YSN	59.410	factor income, skilled urban labor	(PAGRIC,FSNAG)
YOW	100.173	factor income, own-account workers	(PAGRIC,FOWNACC) + (PNAGR,FOWNACC)
YK	31.756	factor income, capital	(PAGRIC,FOTHERS) + (PNAGR,FOTHERS)
FEXPAT	33.486	total expatriate earnings	(REEC,FEXPAT) + (RNEEC,FEXPAT)
YRPR	36.528	income of rural poor	(CTOTAL,IRPOOR)
YRNPR	174.800	income of rural non-poor	(CTOTAL,IRNPR)
YUPR	5.562	income of urban poor	(CTOTAL,IUPOOR)
YUNPR	113.633	income of urban non-poor	(CTOTAL,IUNPR)
G1T	1.376	government transfers to rural poor	(IGOVT,IRPOOR)
G2T	6.313	government transfers to rural non-poor	(IGOVT,IRNPR)
G3T	1.025	government transfers to urban poor	(IGOVT,IUPOOR)
G4T	9.500	government transfers to urban non-poor	(IGOVT,IUNPR)
G5T	0.863	government transfers to corporations	(IGOVT,ISOCIET)
CRPR	36.528	total value of consumption by rural poor	(IRPOOR,WFOOD) + (IRPOOR,WNFOOD)
CRNPR	119.586	total value of consumption by rural non-poor	(IRNPR,WFOOD) + (IRNPR,WNFOOD)
CUPR	5.562	total value of consumption by urban poor	(IUPOOR,WFOOD) + (IUPOOR,WNFOOD)
CUNPR	77.740	total value of consumption by urban non-poor	(IUNPR,WFOOD) + (IUNPR,WNFOOD)

GAMA:

Variables and Their Base Year Values

Variable	Value	Description	SAM Location
CVAL1F	18.932	value of food consumption by rural poor	(IRPOOR,WFOOD)
CVAL2F	41.608	value of food consumption by rural non-poor	(IRNPR,WFOOD)
CVAL3F	2.394	value of food consumption by urban poor	(IUPOOR,WFOOD)
CVAL4F	21.449	value of food consumption by rural non-poor	(IUNPR,WFOOD)
CVAL1NF	17.596	value of non-food consumption by rural poor	(IRPOOR,WNFOOD)
CVAL2NF	77.978	value of non-food consumption by rural non-poor	(IRNPR,WNFOOD)
CVAL3NF	3.168	value of non-food consumption by urban poor	(IUPOOR,WNFOOD)
CVAL4NF	56.291	value of non-food consumption by urban non-poor	(IUNPR,WNFOOD)
W1F	17.8234	quantity of food consumption by rural poor	(IRPOOR,WFOOD)/1.0622
W2F	39.1719	quantity of food consumption by rural non-poor	(IRNPR,WFOOD)/1.0622
W3F	2.2538	quantity of food consumption by urban poor	(IUPOOR,WFOOD)/1.0622
W4F	20.1930	quantity of food consumption by urban non-poor	(IUNPR,WFOOD)/1.0622
W1N	16.0344	quantity of non-food consumption by rural poor	(IRPOOR,WNFOOD)/1.0775
W2N	72.3694	quantity of non-food consumption by rural non-poor	(IRNPR,WNFOOD)/1.0775
W3N	2.9401	quantity of non-food consumption by urban poor	(IUPOOR,WNFOOD)/1.0775
W4N	52.2422	quantity of non-food consumption by urban non-poor	(IUNPR,WNFOOD)/1.0775
WhFs	n.a.	food demand segment variables, household h, segment s	
WhNs	n.a.	non-food demand segment variables, household h, segment s	
PRNF	1.0775	price of non-food	(CTOTAL,WNFOOD)/(WNFOOD,PAGRIC) +(WNFOOD,PNAGR)
PRF	1.0622	price of food	(CTOTAL,WFOOD)/(WFOOD,PAGRIC) + (WNFOOD,PNAGR)
EFUA	15.674	employment of unskilled agricultural workers	(PAGRIC,FUAGR)
EFSA	6.956	employment of skilled agricultural workers	(PAGRIC,FSAGR)
EFUN	94.993	employment of unskilled non-agricultural workers	(PNAGR,FUNAG)
EFSN	59.410	employment of skilled non-agricultural workers	(PNAGR,FSNAG)
EFOW	100.173	employment of own-account workers	(CTOTAL,FOWNACC)
EFOWA	14.459	employment of own-account workers in agriculture	(PAGRIC,FOWNACC)
EFOWN	85.714	employment of own-account workers in non-agriculture	(PNAGR,FOWNACC)
EK	36.918	total use of capital (including abroad)	(CTOTAL,FOTHERS)
EKA	12.160	use of capital in agriculture	(PAGRIC,FOTHERS)
EKN	19.596	use of capital in non-agriculture	(PNAGR,FOTHERS)

GAMA:

Variables and Their Base Year Values

Variable	Value	Description	SAM Location
PFUA	1.0	factor price: unskilled agricultural workers	(by definition)
PFSA	1.0	factor price: skilled agricultural workers	(by definition)
PFUN	1.0	factor price: unskilled non-cultural workers	(by definition)
PFSN	1.0	factor price: skilled non-agricultural workers	(by definition)
PFOW	1.0	factor price: own-account workers	(by definition)
PK	1.0	factor price: capital	(by definition)
IMP	142.930	total imports	(CTOTAL,REEC) + (CTOTAL,RNEEC)
MA	13.8691	agricultural imports	n.a.
MN	124.8221	non-agricultural imports	n.a.
PFA	1.0	weighted sum of factor prices in agriculture	n.a.
PFN	1.0	weighted sum of factor prices in non-agriculture	n.a.

Notational conventions

a) Prefix D denotes percentage change variable.

b) Suffix 1 denotes a non-negative percentage change variable; suffix 2 denotes a non-positive percentage change variable.

c) Variables beginning with A and B are adjustment variables, bounded to very small values, which allow flexibility in the demand equations and the balance of payments equation.

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