ENERGY MANAGEMENT TRAINING PROGRAM

ENERGY PLANNING FOR DEVELOPING COUNTRIES: AN INTRODUCTION TO QUANTITATIVE METHODS

Peter M. Meier

August 1982

INSTITUTE FOR ENERGY RESEARCH
STATE UNIVERSITY OF NEW YORK AT STONY BROOK

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1. INTRODUCTION

1.1 WHAT IS ENERGY SYSTEMS ANALYSIS?

Energy systems analysis is a young field, with no long history of academic scholarship. Indeed, it is not even clear that energy planning and policy analysis itself has much recognition as an academic discipline, and the body of such knowledge as might be termed energy planning remains ill defined. Moreover, the body of knowledge that is germane to the unique problems of the third world is extremely fragmented, much of it not generally available in the places where lessons learned in one country could be usefully applied in another.

Debates about the legitimacy of new fields of study, especially in cases that cross the traditional academic delineations, are of course not new. The debate over what is geography has occupied the pages of learned journals for decades; the evolution of comprehensive land use planning in the 1950's and 1960's, and environmental engineering in the 1960's and 1970's, faced similar definitional problems. These debates were not entirely semantic, since their resolution had a great deal to do with where and how such fields were taught in universities, and the directions of research that followed. Moreover, particularly in reference to the third world, there are those who view energy planning as a simple extension of economic development planning. Others would stress the scientific and engineering basis. However, there appears to be evolving a broad consensus among both educators and practioners that energy planning for developing countries is indeed a discipline in its own right, that cannot simply be viewed as an extension of traditional development planning.

The essence of energy planning, as opposed to the more traditional sectoral planning activities--electric sector planning, refinery and petroleum sector planning, or industrial development planning--lies in the comprehensiveness of analysis; in the understanding of inter-fuel substitution (rather than just identifying the most effective delivery system for a single fuel); in the understanding of the interaction of energy with economic development (rather than energy needs merely emerging from a sectoral development plan); and in the understanding of the competition of energy sector investment needs with non-energy sector investment needs (rather than the traditional isolation in which the capital intensive electric sector defined its expansion program). Energy systems analysis, then, is simply the quantitative treatment of such problems;[1] its relationship to energy planning is analogous to the position of environmental systems analysis to environmental planning. It rests on the integration of a number of traditional disciplines - economics, engineering, mathematics - into a coherent analytical framework that stresses the interaction of the different components of the system.[2]

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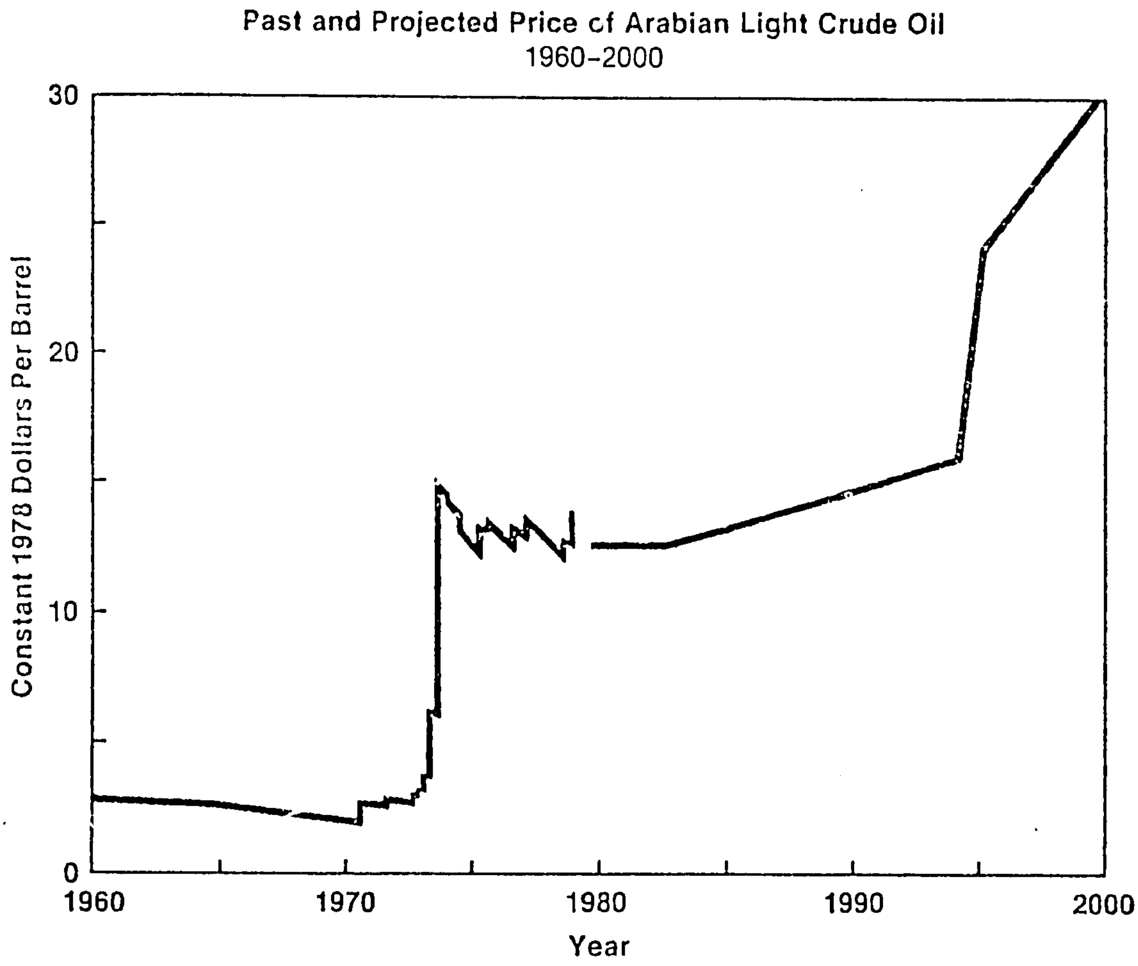
[1]Actually the original use of the term systems analysis did not necessarily imply quantitative, mathematical treatment of the system under study: some of the classic texts contain not a single equation. Systems analysis was meant simply as a way of approaching complex problems, emphasising comprehensive rather than partial analysis. In general, however, systems analysis has come to imply the use of formal mathematical techniques, and it is in this sense that we shall use the term here.

[2]The definitional difficulties here are typified by the National Research Council Report, "Systems Analysis and Operations Research: A Tool for Policy and Program Planning for Developing Countires," Commission on International Relations National Academy of Sciences, Washington, D.C., which defines applied systems analysis in the following terms: "Applied Systems Analysis is not merely a technique or group of techniques such as probability theory or mathematical problems; rather it can be thought of as a broad research strategy one that involves the use of techniques, concepts, and a scientific, systematic approach to the solution of complex problems. It is a framework of thought designed to help decision makers choose a desirable (or in some cases a "best") course of action," (p. 65). Systems analysis, then, is not easy to define in a few words.

Digression 1.1: The Necessity for Judgement.

Some of the more egregious failures of energy systems modelling are to be attributed less to the shortcomings of the models themselves as much as the mode in which they are used, and the failure to appreciate the necessity for judgement in the assumptions that are used to drive the scenarios. An excellent example is the assumption made for world oil prices in the 1979 Annual Report to Congress by the Energy Information Administration of the U.S. Department of Energy. The figure below shows the world oil price trajectory used as a basis for this study. To be sure, this was prepared in October 1978, prior to the events in Iran that sparked the price increases in 1979. But the report, appearing in mid 1979, was widely criticized; and the models used in its preparations castigated for failing to "predict" the price increases. Of course it was not a failure of model prediction at all, since the world oil price was a matter of exogenous assumption; rather it was a failure of judgement or, perhaps less charitably, an unwillingness to go against the conventional wisdom at the time, which held that since oil prices had stayed constant in real terms over the interval 75 - 78, it would be some years until prices again rose in real terms.



1.2 APPLICABILITY OF ENERGY SYSTEMS ANALYSIS TO DEVELOPING COUNTRIES.

Given that one may well question the effectiveness of current energy planning models as they have been applied in developed countries such as the United States (see Digression 1.1), to what extent are the techniques of Energy Systems Analysis applicable to developing countries? Clearly, given the heterogeneous nature of the third world, there are no simple answers. In some countries the capability for comprehensive energy planning, including the necessary data collection and computer support structures, is already in place, or can be expected to be in place within the next few years. In others, such capabilities may be a decade or more away. But as noted earlier, energy systems analysis is above all an approach to the analysis of energy problems that rests on a comprehensive view of the interaction of different parts of the system itself, and the interaction to the overall economic and social framework, rather than a discipline necessarily dependent on computers. Indeed, many of the techniques discussed in this book, such as Reference Energy Systems Analysis, can be implemented quite satisfactorily by manual means.

In any event, whatever the applicability of the type of models presented here to operational decision-making, there is considerable pedagogic value to the presentation of systems analysis as part of training programs for energy planning for developing countries. Regardless of ones ability to implement a model to the point of operational application and effectiveness, model building in general forces one to conceptualize relationships, to think of causalities, and to think through the requirements for data. Model building and systematic analysis in energy planning requires in addition the application of quantitative rigor in an arena not particularly distinguished for objective analysis; rhetoric abounds in an arena whose political ramifications have become so important. Especially in oil importing developing countries, the balance of payments as well as the success of the entire development strategy may be affected by, say, petroleum product pricing decisions, or the allocation of capital investment funds to a particular energy project. Yet. for example, the intuitive rules of thumb prevalent throughout the world in the electric utility industry for capital outlay justification will simply no longer suffice regardless of any claim of positive benefit/cost based solely on some partial analysis. Indeed, there can be no doubt that the use of quantitative, comprehensive energy analysis will increase in the future -- a circumstance reflected in the creation over the past few years of a high level energy planning agency in almost every developing country, cutting across the traditional institutional lines that have long frustrated coherent and rational energy planning.

1.3 A FRAMEWORK FOR NATIONAL ENERGY PLANNING

Figure 1 depcits the general process of national energy planning (which in overall concept applies to developed countries as much as it does to developing countries). The ultimate objective of the planning process is the development of a plan which, given the absence of simple panaceas, will likely contain a diversity of elements -- energy pricing, the substitution of technologies, the development of indigenous resources, direct Government investment in certain areas, tax incentives, and so forth. Clearly, there will be strong interactions between the individual measures in the portfolio -- to take but one example, beyond direct capital cost, the desirability of investment in solar industrial process heat is a function of the price of alternative fossil fuels, of process technology in a potential industry, of tax incentives, among others -- and thus the ability to structure a set of comprehensive and coordinated policy measures is a formidable task. It is the objective of Energy Systems Analysis to give the planning process an analytical coherence, and to replace an ad hoc, piecemeal procedure by a systematic process directed toward a development of a credible and supportable plan.[3]

Let us dwell on Figure 1.1, for all of the models and techniques elaborated in this book relate to one or more of the indicated steps. Macroanalysis is the process of assembling national scale energy supply-demand balances, and identifying their relationship to the overall economic development plan. The analytical framework is therefore focussed on macroeconomic impacts, and ought to provide the ability to quantify the links between general energy policy strategies and the important macroeconomic indicators. At its simplest level is the Reference Energy System (RES) technique of Chapter 3, which provides a systematic physical description of the energy flows in an economy, and provides for a way of identifying at least some important macroeconomic parameters -- such as oil imports -- as a function of fuel substituion, conservation, and major additions to the capital stock. What is the impact, say of increasing energy efficiency in the steel industry by percent on oil and coke imports, or the what is the

[3]For some other views of the National Energy Planning proces in the context of developing countries, see e.g., "An Analytical Framework for the Assessment of Energy Resource and Technology Options for Developing Countries," BNL 50800, Brookhaven National Laboratory, New York, NY, February 1978.

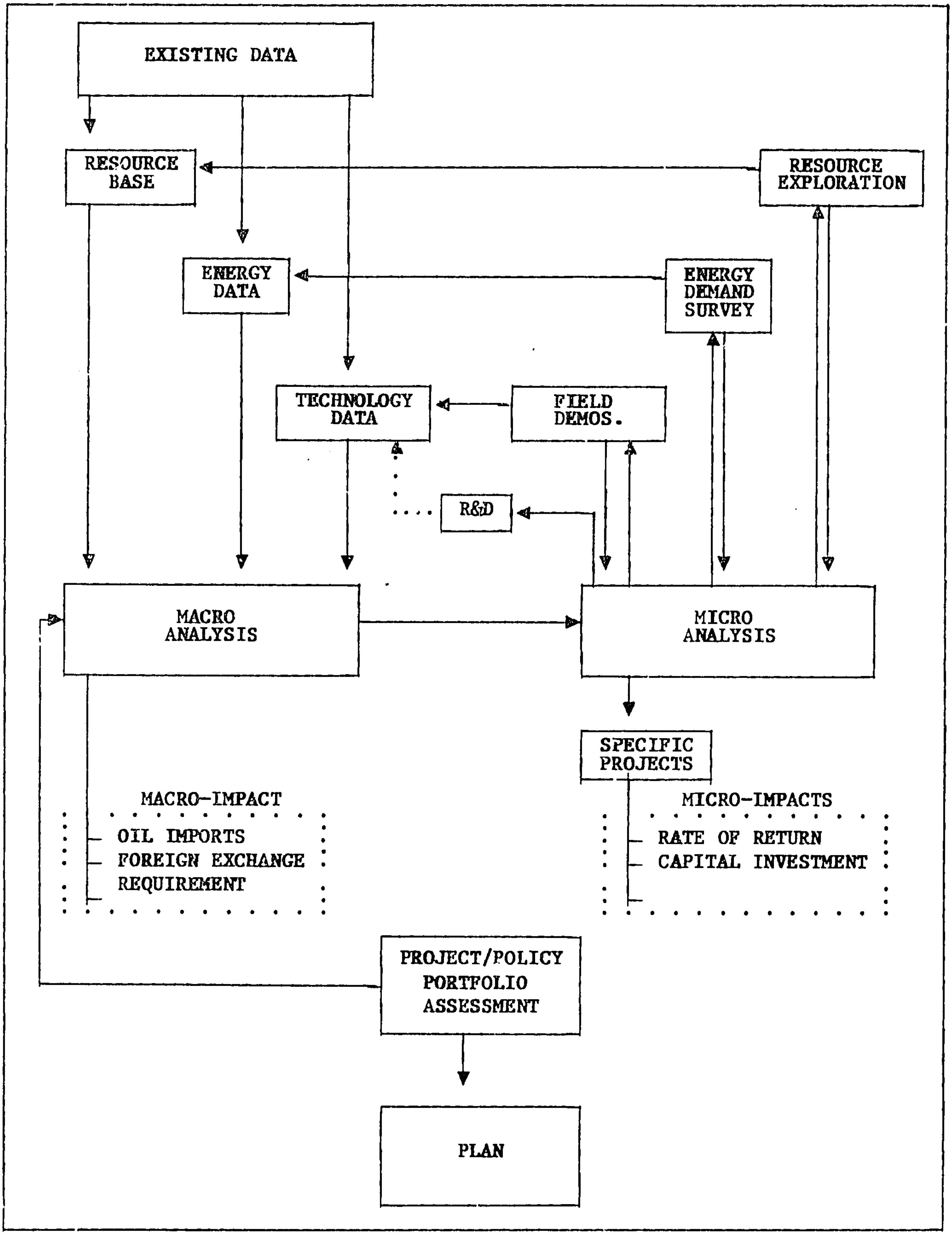


Figure 1.1. A Framework for National Energy Planning

impact of the introduction of solar agricultural crop drying on gas consumption are typical of such first order questions, amenable to RES analysis. More sophisticated techniques provide explicit quantitative links between the energy system and the macroeconomy--such as the models discussed in Chapter 10--and have the ability not just to serve as an accounting scheme amenable to analysis of "what if"-type questions, but have the ability to identify the best strategy. At this level of analysis we might be concerned with Questions such as: given some level and composition of Gross Domestic Product, what is the optimum level of substitution of renewable energy forms. Too much wastes capital, too little misses cost effective opportunities for reducing oil imports.

One of the major problems of macroanalysis is of course data, for which a three way classification proves useful here. First is energy resource data--how much of each type of energy resource do we have, and what is the cost of exploiting it. Ideally, for each domestic resource we would like to have a supply curve--in the case of coal, for example, such a curve may have the form depicted on Figure 1.2--which provides information, for each future year, of the quantity of coal available at what cost. As coal output increases, new mines must be opened, each of which is likely to be more expensive than the next, since the best seams will be mined first. Increasing output generally means mining thinner, deeper seams, of lower quality (more on supply curves in Chapter 4); similar curves exist for hydropower, oil, gas, and so on.

The practical problem faced by the energy analyst, however, is that the information on the size of the resource, and the cost of exploiting it, may be inadequate. The question then becomes one of how much money should be invested in geological surveys and exploration activity to get better estimates of the size, and economic value, of the resource. This question turns out to be a function not just of the cost of surveys, and the probabilities of exploration success, but also of the value of the resource, if it existed, to the economy.

Similar problems beset the other major categories of data. The second category, energy consumption data, is frequently subject to enourmous

[4]Supply curves, then, also have a probabilistic manifestation, since even if, say, a seismic survey considerably narrows the bounds of uncertainty, it may take years for a potential resource to be fully prospected. Thus the planner must work with supply curves subject to certain levels of uncertainty.

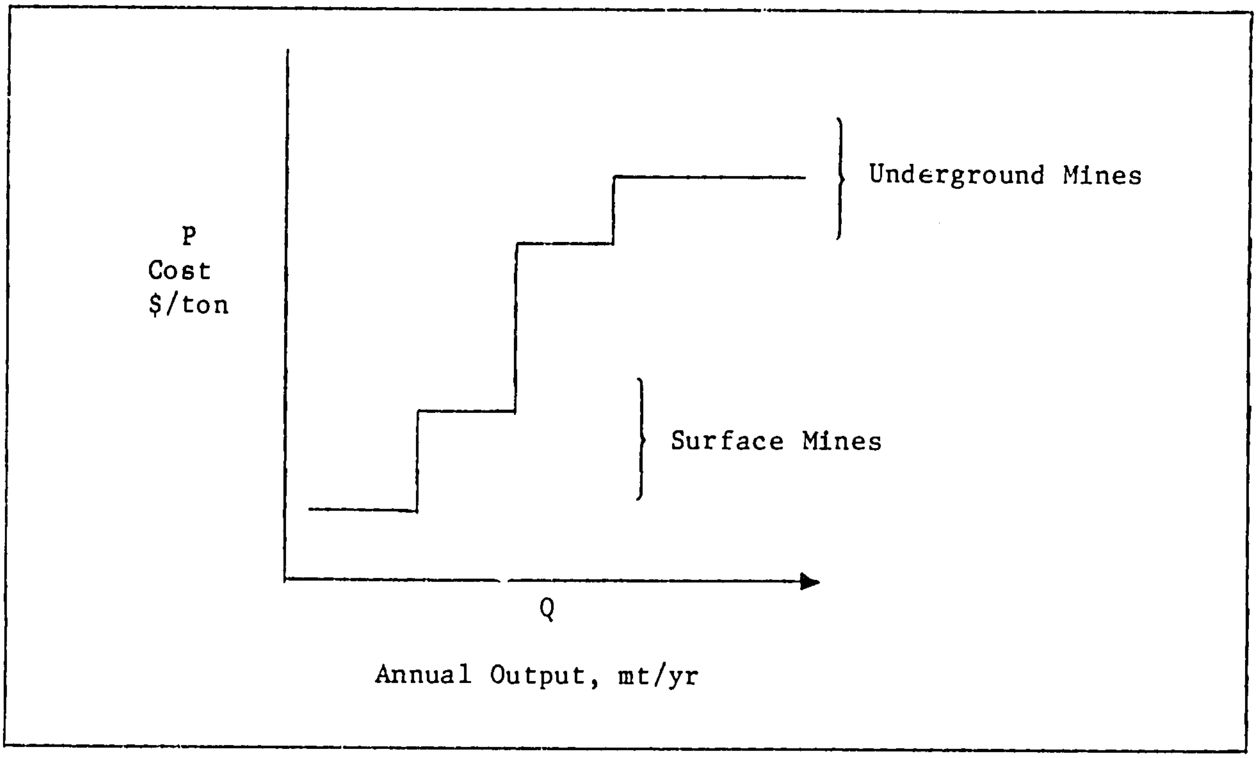


Figure 1.2: A Supply Curve for Coal

uncertainties, especially at the outset of planning activity when surveys of any kind have yet to be made. Whilst data on electricity or petroleum product sales to individual consumer groups may generally be available from the utilities and the local refining/distribution company, in general almost no information exists on end use--that is, the purpose for which the fuel is used--lighting, space heat, appliances -- and without systematic surveys, estimates of rural energy use, and firewood consumption in particular, are totally speculative.[5]

Finally, data on energy technologies may be very sketchy: even in the case of imported technologies, local installation and operation and maintenance costs, and the performance characteristics under local conditions, are

[5]For a complete discussion on data needs, see e.g., P. Palmedo and R. Chatterjee, "Information Needs for Energy Planning,"U.N. Symposium on Energy Planning in Developing Countries, Stockholm, Sweden, October 1981 with respect to rural energy use, Palmedo and Chatterjee note"...almost universally it is found that the weakest area of existing energy information is for households, and particularly for consumption of "non-commercial" fuels such as wood or agricultural waste. The uncertainty of non-commercial fuel use is often at least ± 50 percent, dspite the fact that such use is of great social and often environmental significance."

large uncertainties. And where it is desired to manufacture a technology locally--the manufacture of solar flat plate collectors for water heating, for example, is within the capability of almost every country—costs are even more uncertain.

An important part of the energy planning process, then, is to improve the quality of energy data. This involves, as indicated on Figure 1.1, energy resource surveys, energy consumption surveys, field demonstrations of technologies to establish local cost-performance data, and even research and development activity.[6] Resources and time, however, are rarely unlimited: a major question, therefore, is that of prioritizing data needs. With respect to energy consumption, <u>what</u> items of energy data are to be collected? Given the seasonalities of energy use, <u>when</u> should that data be collected? And how large a sample size is indicated? With respect to field demonstrations, what technologies are to be selected? And with what potential market application in mind? And with respect to resource surveys, which resources are to be surveyed first? It is an almost endless list.

These questions could be answered in an ad hoc fashion, based just on the macroanalysis that identifies the important energy sectors. There is a better way, however, one that we here describe as micro-analysis. This encompases a set of analytical techniques that provide a systematic way of moving from general strategy recommendations to the identification of a specific portfolio of projects for implementation in the national energy plan. As we shall see in Chapter 7, a byproduct of this process is the identification of specific data improvement needs: one may not be able to make an investment decision recommendation if the database is subject to very large uncertainties.

The energy planning process, then, is an iterative one, and several rounds of macro and micro analysis, each based on increasingly better data that are collected as part of the process, are typically required before candiate strategies can be identified, and recommendations made on the details of the plan.

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[6]It may be argued that RµD is best left to the developed countries who are better able to afford the not considerable expense. However, many universities in developing countries have excellent research programs: the challenge to the energy planner is to focus the country's R&D efforts into the most promising areas from the perspective of the local energy system.

1.4 SCOPE OF THIS BOOK.

We begin, in Chapter 2, with a review of the necessary mathematical fundamentals: Matrix algebra, statistical analysis, linear programming and the rudiments of classical optimization techniques. This material can be omitted where the background of the reader is regarded as adequate. In fact, although much of the material in this book looks at first glance to be quite involved mathematically, the only prerequisite assumed of the reader is knowledge of some elementary calculus: all else is either introduced from first principles in this chapter, or is explained in a footnote. Indeed, many explanatory footnotes and digressions have been added as a result of questions raised during the class sessions of the Energy Management training program.

Reference Energy Systems Analysis is introduced in Chapter 3. As in each of the substantitive chapters we present an application of the techniques introduced in the theoretical section to a selected developing country. Most of these examples are drawn from studies conducted at the Institute for Energy Research of the State University of New York at Stony Brook (hereinafter IER) and at Brookhaven National Laboratory (BNL) in the past two or three years; the Dominican Republic is used in this chapter as the illustrative case study.

Chapter 4 is concerned with the techniques of energy demand projection, and introduces the econometric techniques that have become quite widely used in development economics and energy planning in the past decade. A separate section is devoted to the statistical estimation problems inherent in such approaches: the recent literature abounds with estimates of this and that energy elasticity, without any recognition of such problems as multicollinearity, or potential sources of bias in the autoregressive models. We also present here a brief discussion of industrial process models, given their increasing importance for industrial energy projections as a function of technology choices. An analysis of the Egyptian iron and steel industry illustrates the process modelling approach to demand projection.

Energy system optimization models are introduced in Chapter 5. The approach used here is to build directly on the reference energy system framework, and to present the fundamental linear programming formulation in terms of network optimization. The interpretation of shadow prices, and the policy guidance that may be derived therefrom, is introduced here in terms of the linear programming dual. This ties in with the earlier discussion, in Chapter 2, of the Lagrange multipliers of classical optimization. Indeed, given the great importance of energy pricing policy in developing ocuntries, we lay a great deal of stress throughout the book on the uses (and abuses) of shadow prices.

The linkages between energy system models and economic models is introduced in Chapter 6, in which we emphasize the linkages between conventional input-output analysis and energy system models; the basic analytical framework is the Brookhaven extended input-output table, in which energy service and energy product sectors, denominated in appropriate energy related terms, are added to the usual monetary dominated input-output sectors. The case study material of this chapter is based on a recent application of the Brookhaven Energy-Economic Assessment Model (BEEAM) to Portugal.

Microanalysis is introduced in Chapter 7. In addition to the role of microanalysis as part of the planning process, another important application of some of the techniques discussed here is to the specification of the objective functions of the various optimization techniques presented in subsequent chapters. Analysis of solar hot water heating in Tunisia is used as a case study.

Chapters 8 and 9 focus in more detail on the electric and petroleum sectors. The intent here is to present sectoral models that are suited to subsequent integration into comprehensive energy planning models; as such we emphasize linear programming models rather than some of the more sophisticated stochastic simulation models that are in widespread use for detailed sectoral analysis, particularly in the electric sector. An electric capacity expansion planning model for Jordan illustrates the concepts introduced in these chapters.

Chapter 10 integrates all of the material presented in earlier sections into an overall modelling framework for energy-economic analysis for developing countries. The simple LP presented in Chapter 5 is expanded to more realistic detail using the detailed representations of the electric and petroleum sectors introduced in Chapters 8 and 9. By integration of industrial process models into the LP we also free ourselves from some of the more confining assumptions of Input-Output Analysis. The problems of linking energy models with econometrically based macroeconomic models (as opposed to simple accounting identity approaches of Chapter 6) are also explored in this concluding chapter.

2. MATHEMATICAL FUNDAMENTALS

2.1 MATRIX ALGEBRA.[1]

Matrix algebra is one of the essential tools of energy systems analysis. A grasp of even the most fundamental concepts allows much that would otherwise be extremely complex to be reduced to a few simple matrix equations; and the definition of most statistical tools, such as least squares regression, becomes extremely simple when the tedium of scalar algebra is replaced by the elegance of matrix expressions.

A matrix is nothing more than a rectangular array of numbers, called elements of the array (or matrix). The conventional notation uses upper case letters to denote the matrix, and subscipted lower case letters to denote the elements. Thus we write

$$A = \begin{bmatrix} a_{11} & a_{12} & \cdots\cdots & a_{1n} \\ a_{21} & a_{22} & \cdots\cdots & a_{2n} \\ \cdot & & & \\ a_{m1} & a_{m2} & \cdots\cdots & a_{mn} \end{bmatrix} \tag{2.1}$$

Thus the element a_{ij} refers to the number in row i, and column j; thus a_{21} in the matrix above refers to that number in row 2, and column 1. The size of the array is referred to as its dimensions. We shall adopt here the common convention indicating the dimensions of a matrix in parentheses beneath its symbol; thus

$$\underset{(m\times n)}{A} \tag{2.2}$$

indicates an array of m rows and n columns; we say the matrix is of <u>order</u> m x

---

[1] This Chapter is designed solely as a brief refresher of the basic tools of matrix algebra, statistical analyis, and linear programming. We make no claim of comprehensiveness; indeed, the idea here is to present the minimum amount of information necessary for the comprehension of the techniques presented in subsequent chapters.

The transpose of the Matrix A, denoted A^T, is obtained by interchanging the rows and columns of A.[2] Thus if

$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix} \tag{2.3}$$

then the transpose is

$$A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix} \tag{2.4}$$

Thus if A is of order (m x n), it follows that A^T is of order (n x m). Also, if $A = A^T$, then the matrix A (and A^T) is said to be <u>symmetric</u> -- it must obviously be square. If the number of rows equals the number of columns, i.e. if n = m, then the matrix is said to be square. There are a number of special, square matrices that are frequently encountered. A <u>diagonal</u> matrix is a square matrix with all elements equal to zero except those on the principal diagonal, the principal diagonal being defined as those elements for which i = j. If all of the non-zero elements of a diagonal matrix are equal to unity, the matrix is called the <u>identity matrix</u>, denoted by upper case I.

<u>Basic Operations</u>. Matrices can be added and subtracted provided their dimensions are the same. Thus the addition of two (m x n) matrices yields another (m x n) matrix:

$$\underset{(mxn)}{A} + \underset{(mxn)}{B} = \underset{(mxn)}{C} \tag{2.5}$$

where the elements of C are given by

$$c_{ij} = a_{ij} + b_{ij} \tag{2.6}$$

similarly for subtraction, if

$$\underset{(mxn)}{A} - \underset{(mxn)}{B} = \underset{(mxn)}{C} \tag{2.7}$$

---

[2]Transposition is sometimes denoted by use of a prime -- thus A' is the transpose of A.

then

$$c_{ij} = a_{ij} - b_{ij} \tag{2.8}$$

for all of the mx n elements of c. Matrices of unequal dimensions (i.e. that are not of the same order) can neither be added nor subtracted.

If α is some scalar, then the scalar multiplication

$$\alpha \cdot \underset{(mxn)}{A} = \underset{(mxn)}{B} \tag{2.9}$$

is defined by $b_{ij} = \alpha a_{ij}$; that is, we multiply each element of A by the scalar α.

<u>Matrix multiplication</u>. If A is of order m x n and B is of order n x p, then the matrix product AB is defined to be a matrix of order m x p; thus

$$\underset{(mxn)}{A}\ \underset{(nxp)}{B} = \underset{(mxp)}{C} \tag{2.10}$$

where the elements of c are given by

$$c_{ij} = \sum_{k=1}^{n} a_{ik}\, b_{kj} \tag{2.11}$$

For example

$$\underset{\substack{(2x3)\\ A}}{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}} \underset{\substack{(3x2)\\ B}}{\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}} = \underset{\substack{(2x2)\\ C}}{\begin{bmatrix} 2x1 + 3x2 + 4x3 & 2x2 + 3x1 + 4x1 \\ =20 & =11 \\ 1x1 + 2x2 + 1x3 & 1x2 + 2x1 + 1x1 \\ =8 & =5 \end{bmatrix}} \tag{2.12}$$

Thus two matrices are said to be conformable for multiplication if the number of columns in the first matrix equals the number of rows in the second matrix. It follows that, unlike the multiplication of scalars, matrix muliplication is not generally commutative -- thus AB $\neq$ BA. Indeed, in our example of Eq (2.12), the multiplication BA yields

$$\underset{(3x2)}{\overset{B}{\begin{bmatrix} 1 & 2 \\ 2 & 1 \\ 3 & 1 \end{bmatrix}}} \underset{(2x3)}{\overset{A}{\begin{bmatrix} 2 & 3 & 4 \\ 1 & 2 & 1 \end{bmatrix}}} \underset{(3x3)}{\overset{C}{\begin{bmatrix} 1x2 + 2x1 = 4 & 1x3 + 2x2 = 7 & 1x4 + 2x1 = 6 \\ 2x2 + 1x1 = 5 & 2x3 + 1x2 = 8 & 2x4 + 1x1 = 9 \\ 3x2 + 1x1 = 7 & 3x3 + 1x2 = 11 & 3x4 + 1x1 = 13 \end{bmatrix}}} \quad (2.13)$$

In this case, because m = p, the product BA is defined. In general, if A is (m x n), B is (n x p), and $m \neq p$, then the multiplication BA is not defined.

Square matrices can be multiplied by themselves, i.e.

$$\underset{(nxn)}{A} \; \underset{(nxn)}{A} = \underset{(nxn)}{A^2} \quad (2.14)$$

A^n thus denotes raising the square matrix A to the n-th power.

<u>Matrix Inversion</u>. In ordinary scalar algebra, the reciprocal of a number x is defined as that number y, which, when multiplied by x, produces unity, i.e.

$$x \; y = y \; x = 1 \quad (2.15)$$

hence

$$y = \frac{1}{x} = x^{-1} \quad (2.16)$$

In analogy, in matrix algebra we define the reciprocal of a square matrix A to be that matrix B, which, when multiplied by A, yields the identity matrix.

$$A \; B = I \quad (2.17)$$

hence

$$B = A^{-1} \quad (2.18)$$

The reciprocal matrix A^{-1} is generally referred to as the <u>inverse</u> of A. Note that the inverse exists only for a square matrix.

Associated with every square matrix A there is a scalar quantity, called the determinant of A, which is denoted by the symbol [A]. This quantity is determined by the sum of various products of the elements of A--for example, the determinant of a 2x2 matrix is defined as

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = a_{11}\, a_{22} - a_{12}\, a_{21} \tag{2.19}$$

hence if

$$A = \begin{bmatrix} 2 & 1 \\ 3 & 5 \end{bmatrix} \tag{2.20}$$

then [A] is defined as 2x5 - 3x1 = 6.

Notice that if one row (or column) of a matrix is a linear function of another, then the determinant equals zero; for example, if

$$A = \begin{bmatrix} 2 & 1 \\ 4 & 2 \end{bmatrix} \tag{2.21}$$

for which each element of row 2 is given by $a_{2j} = a_{1j} \times 2$, the determinant $|A|$ is seen to be 2x2 - 4x1 = 0. It can be shown that the inverse exists only if $|A|$ is non zero. If no inverse exists, the matrix is said to be <u>singular</u>.

In general, the computation of determinants and the inverse is an extremely tedious operation,[3] best left to computer subroutines and available packages on programmable calculators.[4]

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[3]The general definition of the determinant of an n-th order matrix A is given by as $|A| = \sum \pm a_{1\alpha}\, c_{2\beta} \ldots a_{n\upsilon}$ the sum being taken over all permutations of the second subscripts, with even permutations having positive signs, add permutations a negative sign. A permutation is said to be odd when the number of inversions is odd: an inversion, in turn, is said to occur when of, say, two integer subscripts the larger precedes the former. For further discussion, see e.g. Johnston (1963).

[4]The only reasonably simple method of matrix inversion is by application of the power series expansion

$$(I - A)^{-1} = I + A + A^2 \ldots + A^n$$

which is the analog of the geometric expansion for scalars, i.e.

$$\frac{1}{1 - a} = 1 + a + a^2 + \ldots$$

However, this method is usually applicable only if the sum of the elements in each column of A is less than unity: it can be shown (Digression 6.3) that this technique is applicable to computing the socalled Leontieff inverse of input-output analysis.

2.2 ECONOMETRICS[5]

The Simple Two-Variable Model

We begin by a consideration of the simple, two variable linear model. Suppose we are interested in the relationship between per capita residential energy consumption, and per capita income, for which we have observations from a cross-sectional analysis (taken, say, from electric utility records for a number of cities that exhibit different average income levels). The true relationship might be extremely complex, involving such factors as climate variations, the characteristics of the housing stock, family size, and individual preferences (does the family spend its disposable income on electric appliances or on other, non-electricity consuming outlays). However, suppose here that we have reason to believe that most of the variation in residential electricity consumption can be explained by income variations, and that the relationship is linear (and we shall examine later the issues involved in other, non-linear models). Nevertheless, if we plot just income and consumption, in the manner of Figure 2.1, we would expect to see considerable scatter about the presumed model, attributable to the influence of the other variables. Our model, then, becomes

$$Y = \alpha + \beta X + e \tag{2.22}$$

where

Y = per capita residential electricty consumption (the dependent variable)

X = per capita income (the independent variable)

α, β = parameters to be estimated

e = a random error term.

There are some other reasons why we would formulate our model with a random error term. First, even if there were grounds to believe that X and Y were exactly related in a linear fashion, measurement error would result in a

[5]This, too, is an enourmous subject, to which many textbooks have been dedicated. Our purpose here is simply to review the basics, introduce those techniques most commonly encountered in the statistical analysis of energy economic phenomena, and to lay the groundwork for some of the more complex estimation problems (of energy demand elasticities, and the like), presented in Chapter 4. Any student familiar with this material can proceed to the next section since only fundamentals are discussed here (as they might be treated in any graduate level course on econometrics).

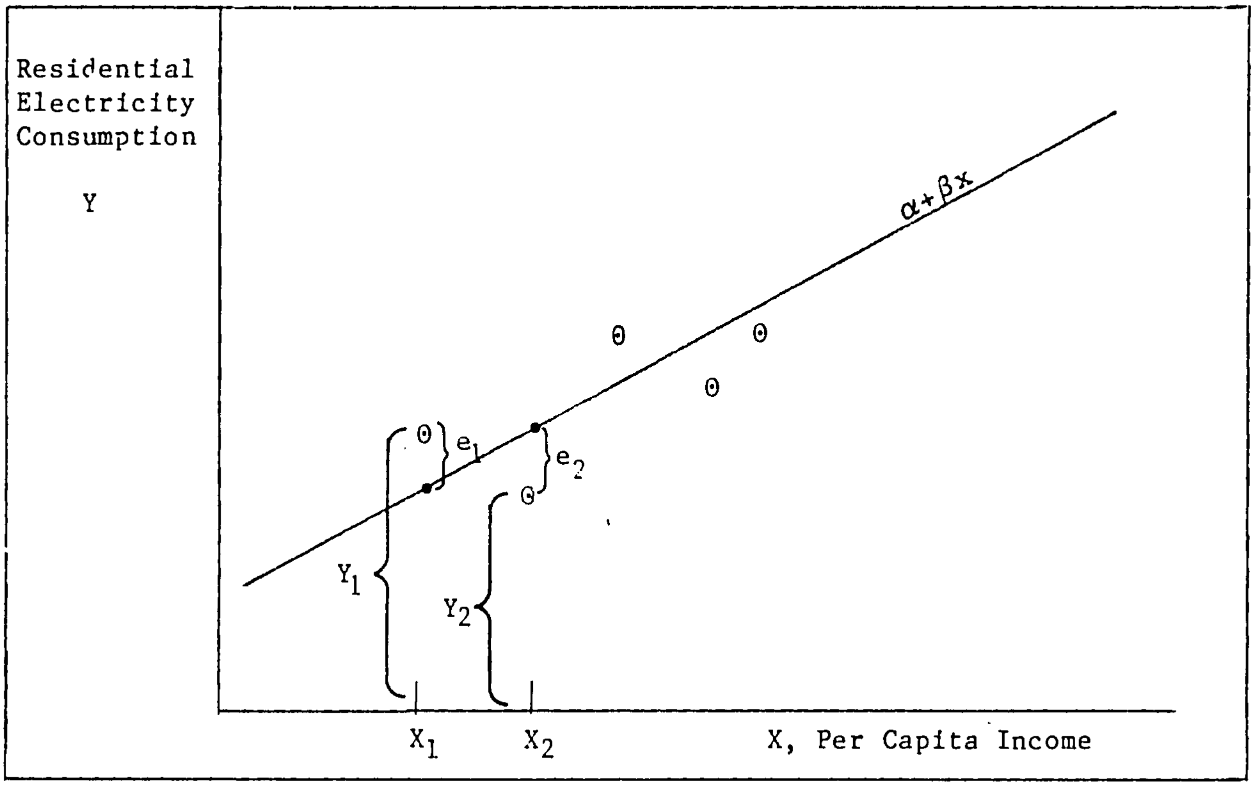

Figure 2.1. The Two Variable Linear Model

situation where the data exhibited considerable scatter. In our case, records might be incomplete or inadequate; non-residential uses might be included together with some residential meters; the number of families per meter may not be known with any great precision. Another reason for the presence of a disturbance term in models of socio-economic phenomena is the assumption that over and above the total effect of all relevant parameters, there is an element of unpredictibility, of randomness in human behavior, that can only be captured quantitatively by the addition of a random variable term.

The objective, then, given our data points, is to estimate the parameters of the linear model. There are clearly a number of different ways in which we might fit the straight line through the observations -- minimize the sum of the absolute value of the residuals, minimize the sum of squares of residuals, or indeed, one might minimize the sum of almost any conceivable function of the residuals. However, for a number of reasons, most of which need not concern us here, minimizing the sum of squares of the residuals is the technique most commonly adopted; it can be shown that least squares analysis is not only computationaly simpler than most other approaches but that the statistical properties associated with such estimates are particularly desirable.[6]

Our least squares criterion, then, can be seen to be the determination of that set of values of α and β that minimizes the expression

$$\sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n} (y_i - \alpha - \beta\, x_i)^2 \tag{2.23}$$

differentiating (2.23) with respect to α and to β one obtains

$$\frac{\partial}{\partial\alpha}\left(\sum e_i^2\right) = -2\sum (y_i - \alpha - \beta x_i) \tag{2.24}$$

$$\frac{\partial}{\partial\beta}\left(\sum e_i^2\right) = -2\sum x_i\,(y_i - \alpha - \beta x_i) \tag{2.25}$$

which, when set equal to zero, yields

$$\sum y_i = n\,\hat{\alpha} + \hat{\beta}\sum x_i \tag{2.26}$$

$$\sum x_i y_i = \sum x_i\,\hat{\alpha} + \beta \sum x_i^2 \tag{2.27}$$

whereby we now use the notation $\hat{\alpha}$ and $\hat{\beta}$ to denote the fact that these particular values of α and β are the so called least squares estimators. These two equations can obviously be solved for $\hat{\alpha}$ and $\hat{\beta}$ to yield.

$$\hat{\beta} = \frac{n\sum x_i y_i - \sum x_i \sum y_i}{n\sum x_i^2 - \left(\sum x_i\right)^2} \tag{2.28}$$

$$\hat{\alpha} = \frac{\sum y_i - \hat{\beta}\sum x_i}{n} \tag{2.29}$$

---

[6]Minimizing the sum of the absolute values of the deviations leads to a linear programming problem (known as MAD estimation, for minimum absolute deviations). For an excellent introduction to this approach, see e.g. Rogers (1968). The MAD approach is sometimes useful in situations in which conventional least squares estimation is beset by serious problems, some of which we shall encounter later in these pages. The MAD technique has perhaps found greatest application in problems of mathematical demography.

[7]This follows from the well known result $\frac{\partial f(g)}{\partial\beta} = \frac{\partial f}{\partial g}\cdot\frac{\partial g}{\partial\beta}$; here, in (2.23), $g = y_i - \alpha - \beta X_i$ and $f(g) = g^2$. Hence $\frac{\partial f}{\partial g} = 2g$ and $\frac{\partial g}{\partial\beta} = X_i$, from which follows (2.25).

Digression 2.1: Least Squares for Variables in Deviation Form.

LS analysis frequently is stated in terms of deviations about the mean.

$$\hat{y}'_i = \hat{y}_i - \bar{y}$$

$$x'_i = x_i - \bar{x}$$

$$y'_i = y_i - \bar{y}$$

$$\hat{y}'_i = \hat{\beta}x'$$

and

$$e_i = y'_i - \hat{y}'_i = y'_i - \beta\hat{x}'_i$$

Thus the sum of squares of the residuals is

$$\sum e_i^2 = \sum (y'_i - \hat{\beta}x'_i)^2$$

which one differentiates with respect to β, i.e.

$$\frac{\partial}{\partial\beta}\sum e_i^2 = -2\sum(\hat{y}'_i - \hat{\beta}x'_i)x'_i$$

hence, setting to zero

$$\sum x'_i y'_i = \hat{\beta}\sum x_i'^2$$

from which $\hat{\beta}$ follows as

$$\hat{\beta} = \frac{\sum x'_i y'_i}{\sum x_i'^2} = \frac{\sum(x_i - \bar{x})(y_i - \bar{y})}{\sum(x_i - \bar{x})^2}$$

and $\hat{\alpha}$ follows from

$$\bar{y} = \hat{\alpha} + \hat{\beta}x$$

whence $\hat{\alpha} = \bar{y} - \hat{\beta}x$

At this point one should make a number of important points concerning the residual. It should be noted that we hypothesize the residual to be characterized by a probability distribution of zero mean and some unknown variance σ_u^2, i.e.,

$$E\left|e_i\right| = 0 \tag{2.30}$$

where the notation $E\left|\ \right|$ denotes expected value of $\left|\ \right|$, and

$$E\left|e_i e_j\right| = \begin{matrix} 0 & \text{for } i \neq j \\ \sigma_u^2 & \text{for } i = j \end{matrix} \quad . \tag{2.31}$$

The latter assumption in particular implies that there is no correlation between successive residual terms. Particularly where time series problems are concerned, this may not be a valid assumption, as we shall see below.

Extensions to the Multivariable Case

In extending this to more than one explanatory variable, it becomes obvious that use of scalar algebra becomes exceedingly tedious. But in matrix algebra, increasing the number of variables does not make things any more complicated. The n equations of our linear model (i.e. one equation for each observation set)

$$Y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \ldots + \beta_k X_{ki} + e_i \tag{2.32}$$

can be written more compactly in matrix notation as

$$\underset{(nx1)}{Y} = \underset{(nxk)}{X} \quad \underset{(kx1)}{\beta} + \underset{(nx1)}{e} \tag{2.33}$$

where

$$Y = \begin{bmatrix} Y_1 \\ Y_2 \\ \cdot \\ Y_n \end{bmatrix} \quad X = \begin{bmatrix} 1 & X_{21} & X_{31} & \cdots & X_{k1} \\ 1 & X_{22} & X_{32} & \cdots & X_{k2} \\ \cdot & \cdot & \cdot & & \\ 1 & X_{2n} & X_{3n} & \cdots & X_{kn} \end{bmatrix} \quad \beta = \begin{bmatrix} \beta_1 \\ \beta_2 \\ \cdot \\ \beta_k \end{bmatrix} \quad u = \begin{bmatrix} u_1 \\ u_2 \\ \cdot \\ u_n \end{bmatrix}$$

Notice that the intercept term, corresponding to β_1, requires the insertion of a column of ones in the matrix of observations; there are thus k - 1 explanatory variables.[8] Now let $\hat{\beta}$ denote the least squares estimate of β. Then we may write

$$Y = X\hat{\beta} + e \tag{2.34}$$

where e is now an nx1 vector of residuals. The least squares estimate of β is that value of β for which the sum of squares of the residuals, $\sum e_i^2$, is minimized. This sum of squares is given by

$$\sum_{i=1}^{n} e_i^2 = e^T e \tag{2.35}$$

$$= (Y - X\hat{\beta})^T (Y - X\hat{\beta})$$

$$= Y^T Y - 2\hat{\beta}^T X^T Y + \hat{\beta}^T X^T X \hat{\beta} \quad . \tag{2.36}$$

Differentiating (2.36) to obtain the value of β that minimizes the sum of squares[9]

$$\frac{\partial}{\partial \beta} (e^T e) = - 2X^T Y + 2X^T X\beta \tag{2.37}$$

which, when set equal to zero, and solving for $\hat{\beta}$, yields

$$0 = - 2X^T Y + 2X^T X\beta \tag{2.38}$$

$$X^T Y = X^T X\beta \quad . \tag{2.39}$$

By multiplying both sides by $(X^T X)^{-1}$, we obtain

$$(X^T X)^{-1} X^T Y = (X^T X)^{-1} X^T X\hat{\beta} \quad . \tag{2.40}$$

But from (2.17), a matrix multiplied by its inverse yields the identity matrix; and any vector multiplied by the identity matrix of conformable dimension yields again the vector; thus

$$\hat{\beta} = (X^T X)^{-1} X^T Y \quad . \tag{2.41}$$

---

[8] We follow here the notational convention used in Johnston (1963)

[9] The rules of matrix and vector differentiation are analogous to that of scalars: compare (2.36) and (2.37) with (2.25) and (2.27), and recall footnote 7, *supra*.

One of the issues in such analysis is that of bias. That is to say we would like the estimate $\hat{\beta}$ to be free from systematic error, to be unbiased; that is, we wish the expected value of $\hat{\beta}$ to be equal to β, the true (but unknown) value. To establish the mean and variance of $\hat{\beta}$, let us substitute (2.33) into (2.41), to yield

$$\hat{\beta} = (X^T X)^{-1} X^T \; [X\beta + e] \tag{2.42}$$

$$= \beta + (X^T X)^{-1} X^T e \quad . \tag{2.43}$$

Taking expectations of both sides

$$E\left\{\hat{\beta}\right\} = \beta + (X^T X)^{-1} X^T \; E\left\{e\right\} \tag{2.44}$$

from which it follows that $E\left\{\hat{\beta}\right\} = \beta$ if the expected value of the residuals is zero. We shall see in later sections that in a number of regression estimators of interest to energy analysis, simple least squares estimates may be biased (which in turn means that results must be interpreted with considerable caution). The variance of $\hat{\beta}$ follows from the definition

$$\mathrm{Var}\left\{\beta\right\} = E\left\{(\beta - \hat{\beta})(\beta - \hat{\beta})^T\right\} \tag{2.45}$$

Since, from (2.42),

$$\begin{aligned}
\beta - \hat{\beta} &= \beta - (X^T X)^{-1} X^T (X\beta + e) \\
&= \beta - (X^T X)^{-1} (X^T X) \; \beta + (X^T X)^{-1} X^T e \\
&= \beta - \beta + (X^T X)^{-1} X^T e \\
&= (X^T X)^{-1} X^T e
\end{aligned} \tag{2.46}$$

then inserting (2.46) into (2.45), one obtains

$$\begin{aligned}
\mathrm{Var}\left\{\hat{\beta}\right\} &= E\left\{(X^T X)^{-1} X^T u u^T X (X^T X)^{-1}\right\} \\
&= (X^T X)^{-1} X^T \; E\left\{ee^T\right\} \; X (X^T X)^{-1} \quad .
\end{aligned} \tag{2.47}$$

Thus if $E\left\{ee^T\right\} = \sigma^2 I$, an assumption noted previously,

$$\mathrm{Var}\left\{\hat{\beta}\right\} = \sigma^2 (X^T X)^{-1} \quad . \tag{2.48}$$

It can also be shown that[10]

$$E\left\{e'e\right\} = (n - k)\sigma^2 \tag{2.49}$$

from which follows that our estimate of σ^2, say S^2, is given by

$$S^2 = \frac{e^T e}{n - k} \tag{2.50}$$

but from (2.36)

$$e^T e = Y^T Y - 2\hat{\beta} X^T Y + \hat{\beta}^T X^T X \hat{\beta}$$

$$= Y^T Y - \hat{\beta} X^T Y \quad . \tag{2.51}$$

<u>Hypothesis Testing</u>: Consider for example, a model of the form[11]

$$Y = \beta_1 + \beta_2 \ln X + \beta_3 \ln N \tag{2.52}$$

where

X = GNP per capita

N = population

Y = Gross Domestic Investment (as a fraction of GDP)

for which we have observations for n different countries. How does one go about testing the significance of the $\hat{\beta}$ estimates? Is, say, the estimated value of $\hat{\beta}_3$ significantly different from zero? Is the value of $\hat{\beta}_2$ statistically different from some value $\hat{\beta}_2^*$ obtained from some theoretical model? How good is the overall predictive performance of the model?

Answers to such questions, based as they are upon statistical information, are known as <u>statistical decisions</u>; in attempting to reach such decisions it is frequently useful to make an assumption about the answer, known as a <u>hypothesis</u>, and then proceed to apply certain rules for the purpose of either rejecting or accepting the hypothesis. Thus a hypothesis might be that the population of a country, as measured by the variable N in Eq. (2.52), does not significantly affect domestic savings as a fraction of GDP--a hypothesis known as a null hypothesis, symbolized as

$$H_o : \hat{\beta}_3 = 0 \quad .$$

---

[10]See e.g. Johnston (1963) p. 112.

[11]This is the basic form of regression model in M. Chenery and M. Syrguin (1975), although we have omitted here the non linear terms and time dummy variables for illustrative purposes.

Any hypothesis which differs from the one under study is termed the alternative hypothesis; perhaps prior theory might lead us to expect that $\hat{\beta}$ should be positive, leading to the alternative

$$H_a : \hat{\beta}_3 > 0 \quad .$$

Notice that this alternative differs to one that simply states that β_3 is non zero, i.e.,

$$H_a^* : \hat{\beta}_3 \neq 0 \quad .$$

If one rejects a hypothesis (the null hypothesis), when in fact it is true, we say that a type I error has been committed. If, on the other hand, we accept the alternative hypothesis, when in fact it should be rejected, then we say that a type II error has been made.[12] The so called level of significance of a statistical test, usually denoted α, is the maximum probability that one wishes to incur of making a Type I error.

Suppose we wish to test the hypothesis that the value of the sample mean $\bar{X}$ is equal to μ, where the distribution of X is believed to be normal of variance σ_x^2. Then the distribution of the standarized variable (generally denoted the z score), given by

$$z = \frac{\bar{X} - \mu}{\sigma_X} \tag{2.53}$$

is the standardized normal distribution (of zero mean and unit variance), shown on Figure 2.2. Where the population variance σ_X^2 is unknown, (as is usually the case) we use the estimate $E\left\{\sigma_X^2\right\} = s^2/n$ where n is the sample size and s^2 the sample variance.

[12]The classic illustration of the difference between a type I and type II error is in terms of the axiom of anglo-saxon law: the presumption being that an accused is innocent (the null hypothesis) unless proven guilty (the alternative). The requirement for unanimity in juries is an effort to minimize type I error; under no circumstances do we wish to reject the null hypothesis if it is in fact true (i.e. we wish to minimize convictions of persons actually innocent) even if it means committing a large type II error (accepting innocence even though the individual may be guilty).

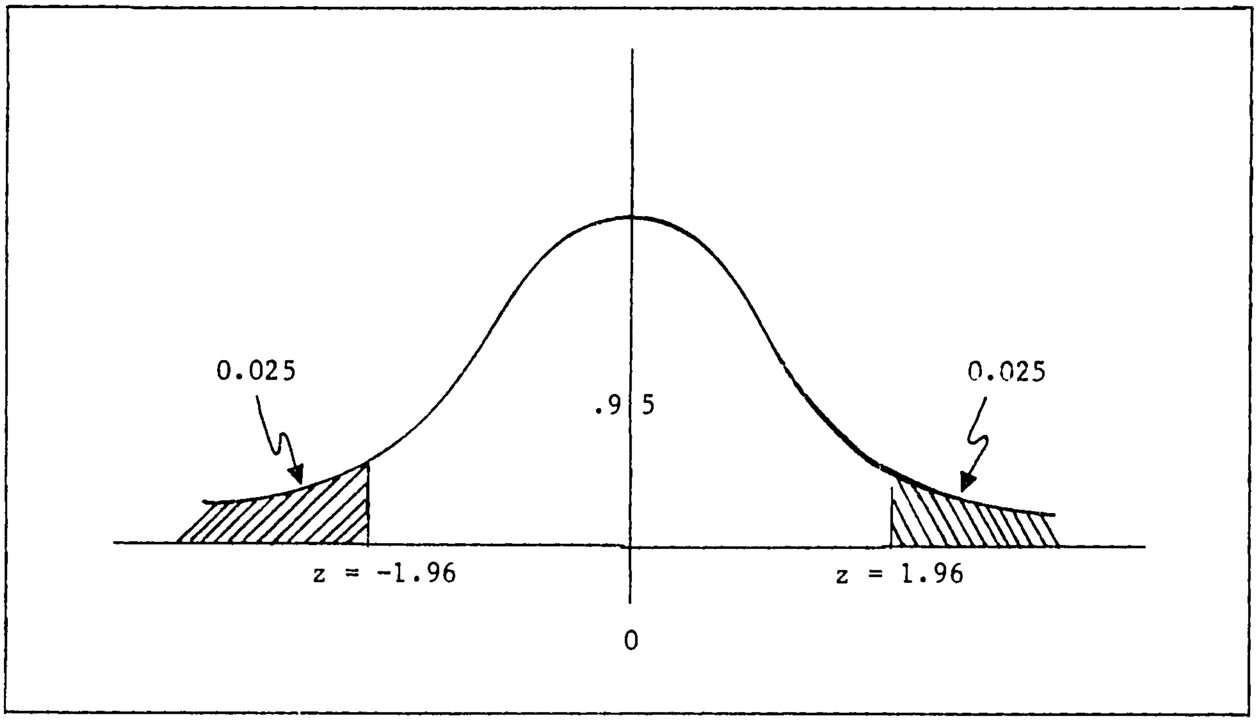

Figure 2.2. Standarized Normal Distribution

Suppose we wish to be 95% confident of making the right decision. That is, we wish to reject the hypothesis, if true, only with probability 5% -- which is the level of significance. Then, from Figure 2.2, we can be 95% sure that if the hypothesis $\bar{X} = \mu$ is true, the Z score of the sample statistic will lie between - 1.96 and 1.96, since 95% of the area under the normal curve lies between these values. Per contra, if the z score lies outside this range, we conclude that this would happen only 5% of the time if the hypothesis is true; at $\alpha = 0.05$ we would then reject the hypothesis if $z > 1.96$ or $z < - 1.96$.

The shaded area, then, corresponds the level of significance of the test; the set of z scores outside the region $\pm$ 1.96 constitutes the so called critical region or the region of rejection of the hypothesis (at $\alpha = 0.05$).[13]

In this discussion we tested the null hypothesis $H_o : X = \mu_o$ against the alternative $X \neq \mu_o$; and therefore we were interested in both "tails" of the distribution. Such a test is known as a "two-tailed" test. Suppose, however, that we were interested in the alternative hypothesis $H_a : X \geq \mu_o$.

[13]Also known sometimes as the region of significance.

In such a case, for α of 0.05, all of the critical region would be on the positive side of the distribution, and we speak of a "one-tailed" test. In such a case, by reference to Table A1,[14] we note that the region of significance would be defined by a z value of 1.645.

Whether in any given situation one uses a one or a two tailed test depends on the particular situation. In general, if the alternative hypothesis is simply that the null hypothesis is not true, and there is no particular reason to believe that the sample value is either lower or higher than the null hypothesis, the two sided test is appropriate. On the other hand, if on theoretical grounds, say, the sample value cannot be less than the null hypothesis, one would be interested in $H_a : X > \mu_0$, which leads to a one tailed test. Figure 2.3 displays the rejection regions for one and two-sided hypotheses at some level of significance : Note that a hypothesis that might be rejected in a one sided test would not be rejected in a two-tailed test (if the z score falls within the range z_1 to z_3 of Figure 2.3).

Let us return, then, to the significance of regression coefficients. Hypotheses concerning individual regression coefficients are evaluated with the help of the t-test.[15] Thus to test the hypothesis $\hat{\beta}_i = \hat{\beta}_i^*$, we compute

$$t_{n-k} = \frac{\hat{\beta}_i - \beta_i^*}{\sqrt{\frac{\sum e_i^2}{n - k}} \cdot \sqrt{a_{ii}}} \tag{2.54}$$

where a_{ii} is the i-th element of the principal diagonal of $(X^TX)^{-1}$, and n - k are the degrees of freedom. As before, if the computed value of t exceeds the tabulated value of t (for some level of significance and n - k degrees of freedom), then the hypothesis $\hat{\beta}_i = \beta_i^*$ can be rejected.

[14] See Appendix A

[15] This test is most commonly used to test for sample means where the sample size is small.

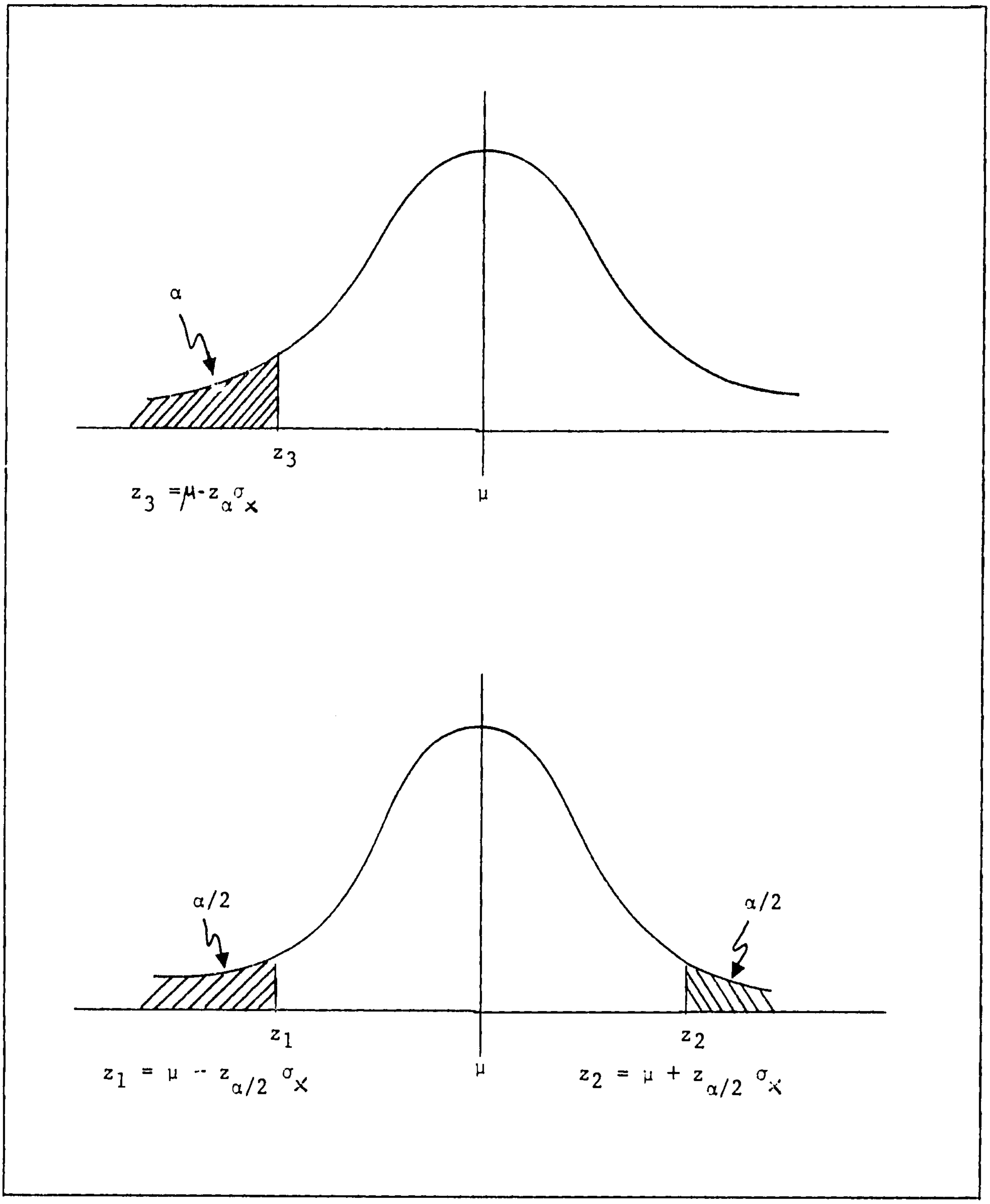

Figure 2.3. One and two-tailed tests.

Example 2.1 Significance of New Data.

There is great uncertainty over the consumption of diesal fuel in the agricultural sector in a certain country. Most diesel fuel is sold in the gas stations operated by the Government Oil Monopoly. An old rural survey indicates a mean per farm consumption of $\mu_1 = 230$ gallons per year. A new survey of 25 gas station purchases of diesel fuel to tractors suggests a mean annual consumption of $\mu_2 = 250$ gallons per farm. In both years $\sigma^2 = 400$, is the difference in mean annual consumption statistically significant?

In this situation the null hypothesis is

$$H_0 : \mu_1 = \mu_2$$

against the alternative

$$H_2 : \mu_2 > \mu_1$$

Hence at $\alpha = 0.05$, the z score computes to

$$z = \frac{250 - 230}{20/\sqrt{25}} = 5$$

Since this exceeds the tabulated value of 1.645, we reject the null hypothesis that consumption has not changed significantly: the increase is statistically significant.

Closely related is the so-called confidence interval for $\hat{\beta}_i$, which at the 100(1 - α) percent confidence level is given by

$$\hat{\beta}_i \pm t_{\alpha/2} \sqrt{\frac{\sum e_i^2}{n - k} \cdot a_{ii}} \quad . \tag{2.55}$$

Multiple Correlation Coefficient. Let us partition the total sum of squares into "explained" sum of squares, which is attributable to the linear influence of the independent variables in the model, and "unexplained," or residual sum of squares. If we use lower case y to denote deviations about the mean, and Y to denote the mean, then the total sum of squares, TSS, about the mean is given by

$$\begin{aligned} \sum_i^n y_i^2 &= \sum_i^n (Y_i - \bar{Y})^2 \\ &= \sum_i (Y_i^2 + \bar{Y}^2 - 2\bar{Y}\, Y_i) \\ &= \sum Y_i^2 + n\bar{Y}^2 - 2\bar{Y} \sum Y_i \end{aligned} \tag{2.56}$$

but since

$$\bar{Y} = \frac{\sum Y_i}{n} \tag{2.57}$$

then substituting (2.57) into (2.56)

$$\begin{aligned} TSS &= \sum Y_i^2 + n \cdot \frac{(\sum Y_i)^2}{n^2} - 2\frac{\sum Y_i}{n} \sum Y_i \\ &= \sum Y_i^2 - \frac{1}{n} (\sum Y_i)^2 \\ &= Y^T Y - \frac{1}{n} (\sum Y_i)^2 \end{aligned} \tag{2.58}$$

The explained sum of squares, ESS, is given by

$$\begin{aligned} ESS &= \sum y_i^2 - \sum e_i^2 \\ &= Y^T Y - \frac{1}{n} (\sum Y_i)^2 - e^T e \end{aligned} \tag{2.59}$$

hence, from (2.36)

$$ESS = \hat{\beta} X^T Y - \frac{1}{n} (\sum Y_i)^2 \tag{2.60}$$

The coefficient of multiple correlation, denoted R, is then given by the ratio of explained to total sum of squares (about the mean), i.e.

$$R^2 = \frac{\hat{\beta} X^T Y - \frac{1}{n} (\sum Y_i)^2}{Y^T Y - \frac{1}{n} (\sum Y_i)^2} = \frac{ESS}{TSS} \tag{2.61}$$

If the variables are in deviation form, i.e. in terms of deviations about the mean, then $\sum Y_i = 0$, and (2.61) reduces simply to

$$R^2 = \frac{\hat{\beta} X^T Y}{Y^T Y} \tag{2.62}$$

Analysis of Variance (ANOVA). The decomposition of sum of squares is frequently presented in tabular form. If the variables are in original form; one constructs the following table:

Source of Variation	Sum of Squares	Degrees of Freedom	Mean Square
X_1	$X'Y \frac{1}{n} (\Sigma Y_i)^2$	$k - 1$	$\left\{X'Y - \frac{1}{n} (\Sigma Y_i)^2\right\}/(k - 1)$
Residual	$e'e$	$n - k$	$e'e/(n - k)$
Total	$Y'Y \frac{1}{n} (\Sigma Y_i^2)$	$n - 1$	

The overall statistical validity of the model ($H_0 : \beta_1 = \beta_2 = \beta_3 = \ldots = 0$) is established with the use of the F-statistic, for which one computes the ratio of mean squares as

$$F = \frac{\left\{\hat{\beta} X^T Y - \frac{1}{n} (\sum Y_i)^2\right\} \cdot \frac{1}{k - 1}}{e'e/(r - k)} \qquad (2.63)$$

which is distributed as the F-statistic with $(k - 1, n - k)$ degrees of freedom. If the computed value of $F(k - 1, n - k)$ exceeds the tabulated value of F for some significance level, then the null hypothesis is rejected.

If the variables are in deviation form, the term $(\sum Y_i)^2/n$ in the ANOVA table and in (2.63) is simply omitted.

Digression 2.3: Weighted Regression

Suppose one wishes to weight each observation in a multivariate regression; let w_i denote the weight of the i-th observation. Then, if

$$W = \begin{bmatrix} w_1 & & & & & 0 \\ & w_2 & & & & \\ & & \cdot & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ 0 & & & & & \cdot \\ & & & & & w_n \end{bmatrix}$$

We can define a new LS criterion as

$$S = \underset{(1xn)}{e^T} \; \underset{(nxn)}{W} \; \underset{(nx1)}{e}$$

from which, in analogy to (2.35) - (2.37) one obtains

$$S = (Y - X\beta)^T W(Y - X\beta)$$

$$= Y^TWY + \beta^TX^TWX\beta - 2\beta X^TWY$$

differentiation with respect to β, and setting equal to zero, yields

$$\frac{\partial S}{\partial \beta} = 2X^TWX\hat{\beta} - 2X^TWy = 0$$

hence

$$\hat{\beta} = (X^TWX)^{-1}X^TWy$$

clearly, if all $w_i = 1$, then W is the identity matrix, and $X^TW = X^TI = X^T$, which is the result for unweighted regression.

2.3 LINEAR PROGRAMMING[16]

Linear programming can best be introduced by example. Consider some hypothetical country, which we shall call Republica,[17] which produces only two goods-agricultural goods, X_1 and machinery, x_2. Assume that three primary inputs are available--labor, energy and capital, and that to produce one unit of x_1 and x_2, the following inputs are required.

	Output	
Input	x_1	x_2
Labor	10	5
Capital	2	5
Energy	2	4

Suppose both units of X_1 and X_2 have the same value, and that the planners in Republica wish to maximize total output in the economy, i.e.

$$\text{Max } S = 1.x_1 + 1\, x_2 \quad . \tag{2.64}$$

Suppose also that the total supply of labor is 25, of capital is 15 and of energy is 10 units. What is the optimum combination of production in Republica?

Obviously, production is contrained by resources; from the above table it follows that

$$\begin{aligned} 10x_1 + 5x_2 &\leq 25 \\ 2x_1 + 5x_2 &\leq 15 \\ 2x_1 + 4x_2 &\leq 10 \end{aligned} \tag{2.65}$$

x_1 and x_2 are also real activities, and must therefore be nonnegative, i.e.

$$x_1,\ x_2 \geq 0 \quad . \tag{2.66}$$

Such an optimization problem is known as a linear programming problem, which can be written in matrix terms as

$$\begin{aligned} \text{Max } S = \quad & cx \\ \text{subject to} \quad & Ax \leq b \\ & x \geq 0 \end{aligned} \tag{2.67}$$

[16]For a more leisurely exposition, but also based on a single numerical abstraction (of "Malawi") see e.g. Todaro (1971), p. 87ff.

[17]This "country" will be used for illustrative purposes throughout this book.

For simple problems that have only 2 variables (or "activities"), this type of problem can be solved graphically. Figure 2.4 shows a graphical portrayal of the constraints (2.65) and (2.66). Combinations of values of x_1 and x_2 that meet all of the constraints are shown by the shaded area on Figure 2.4--this is the socalled "feasible space." Note also that for the particular combination of capital availability and capital inputs, the capital constraint does not help determine the feasible space: in the jargon of linear programming such a constraint is called redundant.

On Figure 2.5 we now plot various values of the objective function. It should be clear that the optimum value of the objective function is given by superimposing Figure 2.5 on to Figure 2.4, and finding that objective function line which is the most to the right, but which is still feasible. As indicated on Figure 2.6, this occurs at one of the corners, or "extreme points" of the feasib.'` region. Indeed, it can be shown that if the feasible region is "convex" (which will always be the case for a set of linear constraints),[18] then the optimum will always occur at an extreme point.[19] Thus the optimum solution for the Republica economy is X_1 = 1.66; X_2 = 1.66 with S = 3.333

Clearly, when there are more than 2 variables (or activities), graphical solution is no longer practical as a solution approach. Indeed, linear programming only became a useful tool with the development by G. Dantzig of the Simplex algorithm, a procedure that enumerates the extreme points of the feasible region in a highly efficient manner, moving from one extreme point to another until the optimum is found. The details of the algorithm need not concern us here: with the availability of high speed digital computers, no

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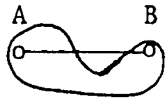
[18]A convex space is one for which it is impossible to find a line drawn between two points on the boundary of the space that intersects another part of the boundary. For example, the space



is not convex, since the line AB intersects the boundary

[19]Subject of course to the exception of having an objective function that has the identical slope to one of the constraints: then the objective function will coincide with that constraint, and any one of the combinations of X_1 and X_2 that lie along the feasible segment of such a constraint will be optimal. For example, if in our example, the objective function were Max S = $4x_1 + 8x_2$, then the optimum would be anywhere along the line YY of Figure 2.6.

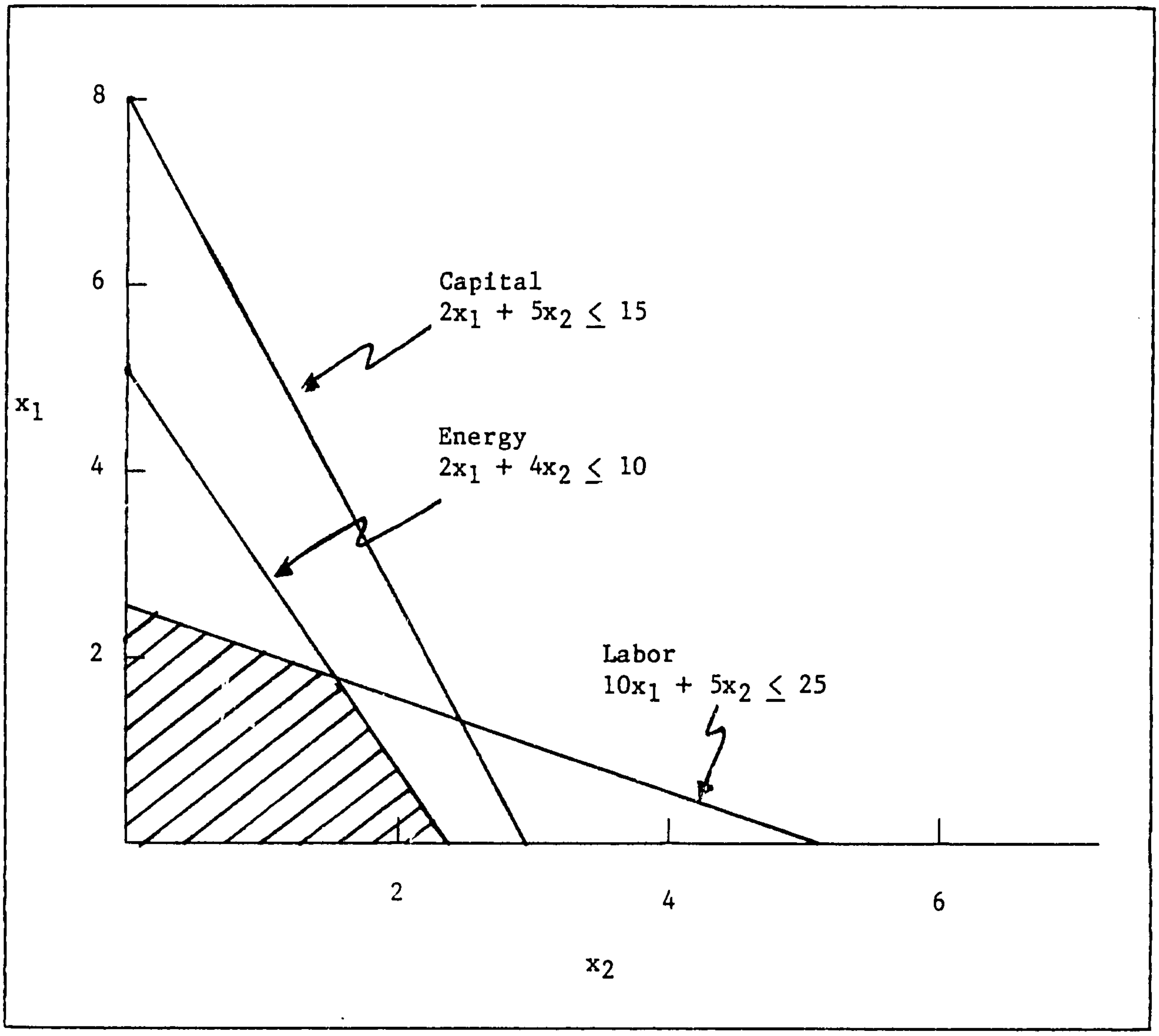


Figure 2.4: The Feasible Space

one still solves LP's by hand (except perhaps graduate students in class exams). In any event, the proprietary algorithms offered by the computer manufacturers for modern machines bear very little resemblance to the traditional sequence of simplex tableaus.

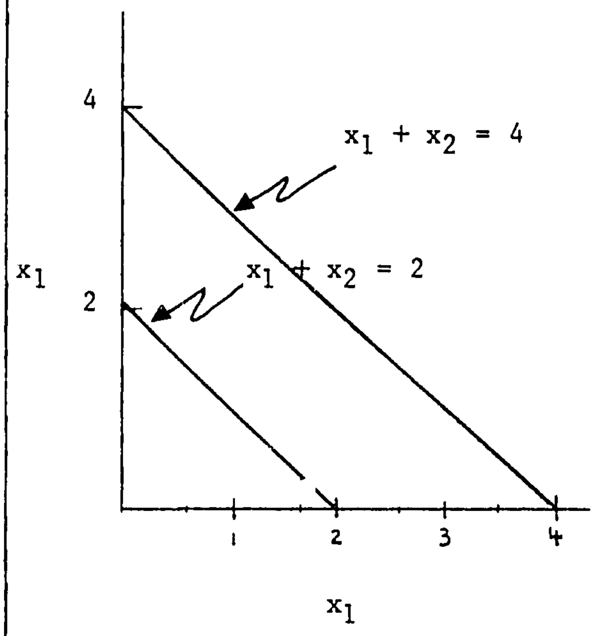


Figure 2.5: Objective Function Values

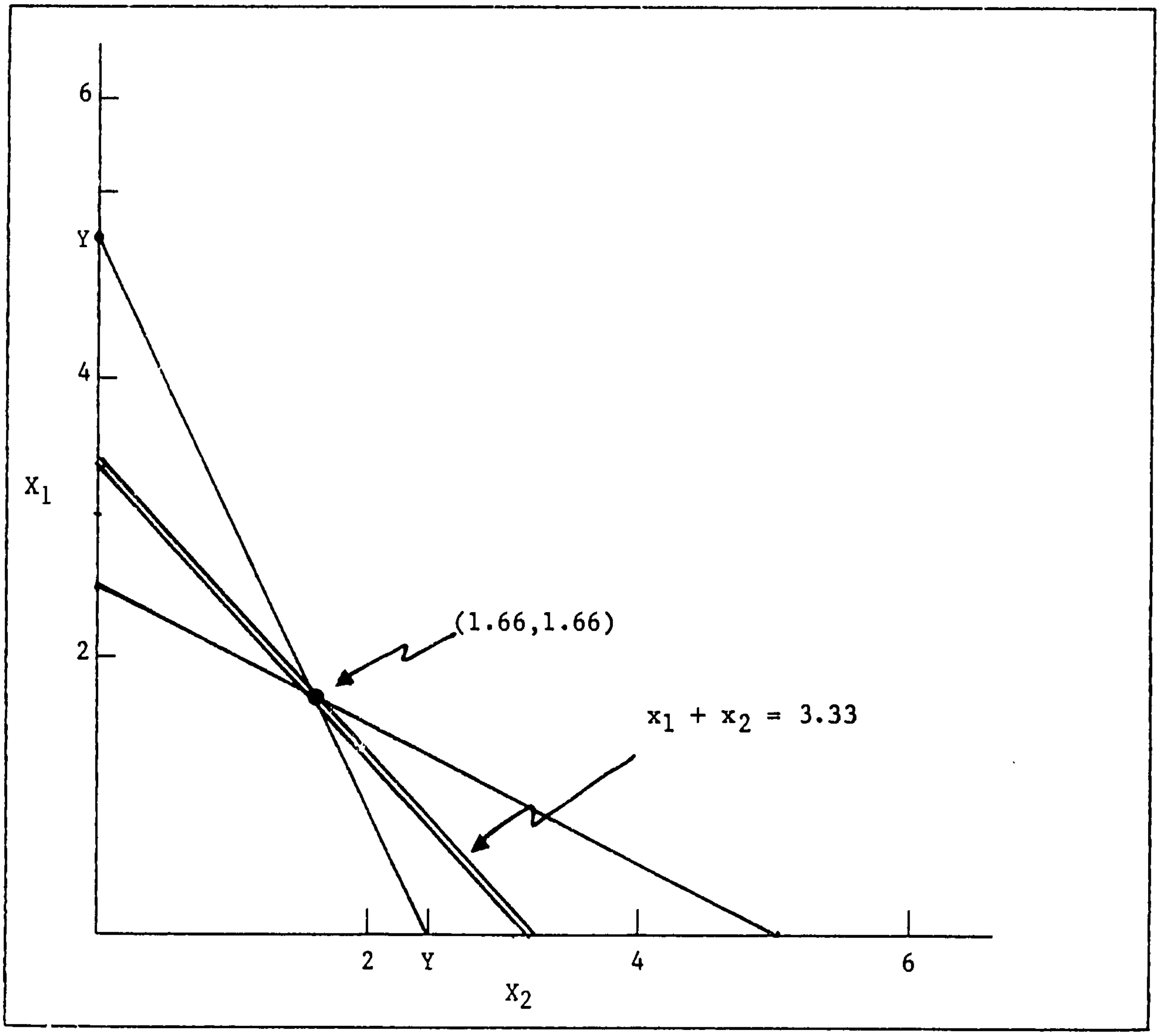


Figure 2.6: Optimum Solution

Example 2.3: Graphical Solution of LP's

A 500 Mw thermal plant of heat rate 10000 Btu/kWh and 0.65 plant factor can use two coal sources: from Mine A, deep mined bituminous coal at \$20/ton, with 12000 Btu/lb; or coal B, stripmined subbituminous coal of 9000 Btu/lb at \$10/ton. Coal Mine A can produce no more than 1 million tons/year: Mine B nor more than 1.3 million tons/yr. A constraint on the railroad system prevents delivery of more than 300 trains/yr. (at 5000 tons/train). Which coal (or mix of coal) should be selected?

Let x_1 be the amount of coal from Mine A, and x_2 the amount from mine B. Then the constraints follow from the problem statement as follows:

1. Production limit, Mine A

$$x_1 \le 1.0$$

2. Production limit, Mine B

$$x_2 \le 1.3$$

3. Railroad capacity limit: 300 trains per year

$$\underset{[\text{trains}]}{300} \times \underset{\left[\frac{\text{tons}}{\text{trains}}\right]}{5000} = \underset{[\text{tons}]}{1.5}$$

$$\therefore \quad x_1 + x_2 \le 1.5$$

4. The Btu requirement for a 500 Mw plant computes to

$$\underset{[\text{Mw}]}{500} \times \underset{\left[\frac{\text{hr}}{\text{yr}}\right]}{8760} \times \underset{[\ ]}{0.65} \times \underset{\left[\frac{\text{Btu}}{\text{kWh}}\right]}{10000} \times \underset{\left[\frac{\text{kWh}}{\text{MWh}}\right]}{10^3} = 28.4 \times 10^{12} \text{ Btu}$$

hence, if coal A is used, we require

$$\underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{28.4 \times 10^{12}} \cdot \underset{\left[\frac{\text{lb}}{\text{Btu}}\right]}{\frac{1}{12000}} \cdot \underset{\left[\frac{\text{ton}}{\text{lb}}\right]}{\frac{1}{2000}} = \underset{[\text{tons}]}{1.18 \times 10^6}$$

Similarly, the tons of coal B compute to

$$\underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{28.4 \times 10^{12}} \ \underset{\left[\frac{\text{lb}}{\text{Btu}}\right]}{\frac{1}{9000}} \ \underset{\left[\frac{\text{ton}}{\text{lb}}\right]}{\frac{1}{2000}} = \underset{[\text{tons}]}{1.57 \times 10^6}$$

We can combine the above two constraints to ensure sufficient coal is delivered to the plant. By noting that if

$$x_1 = 0, \ x_2 = 1.57$$

and

$$x_1 = 1.18, \ x_2 = 0$$

then it follows that the constraint is:

$$x_1 + \frac{1.18}{1.57} x_2 = 1.18$$

5. The objective function is simply

$$\text{Min } y = 20\, x_1 + 10\, x_2$$

We now enter the constraints (1) - (4) onto Figure 2.7. We also draw in a line for the objective function, and then move that function outward until we touch the feasible space. This will occur at one of the extreme points--which in this case can be seen to be the point $x_1 \simeq .0.275$, $x_2 \simeq 1.225$, for which insertion in the objective function yields the optimal value of

$$y = 20 \times .275 + 10 \times 1.255 = 17.75$$

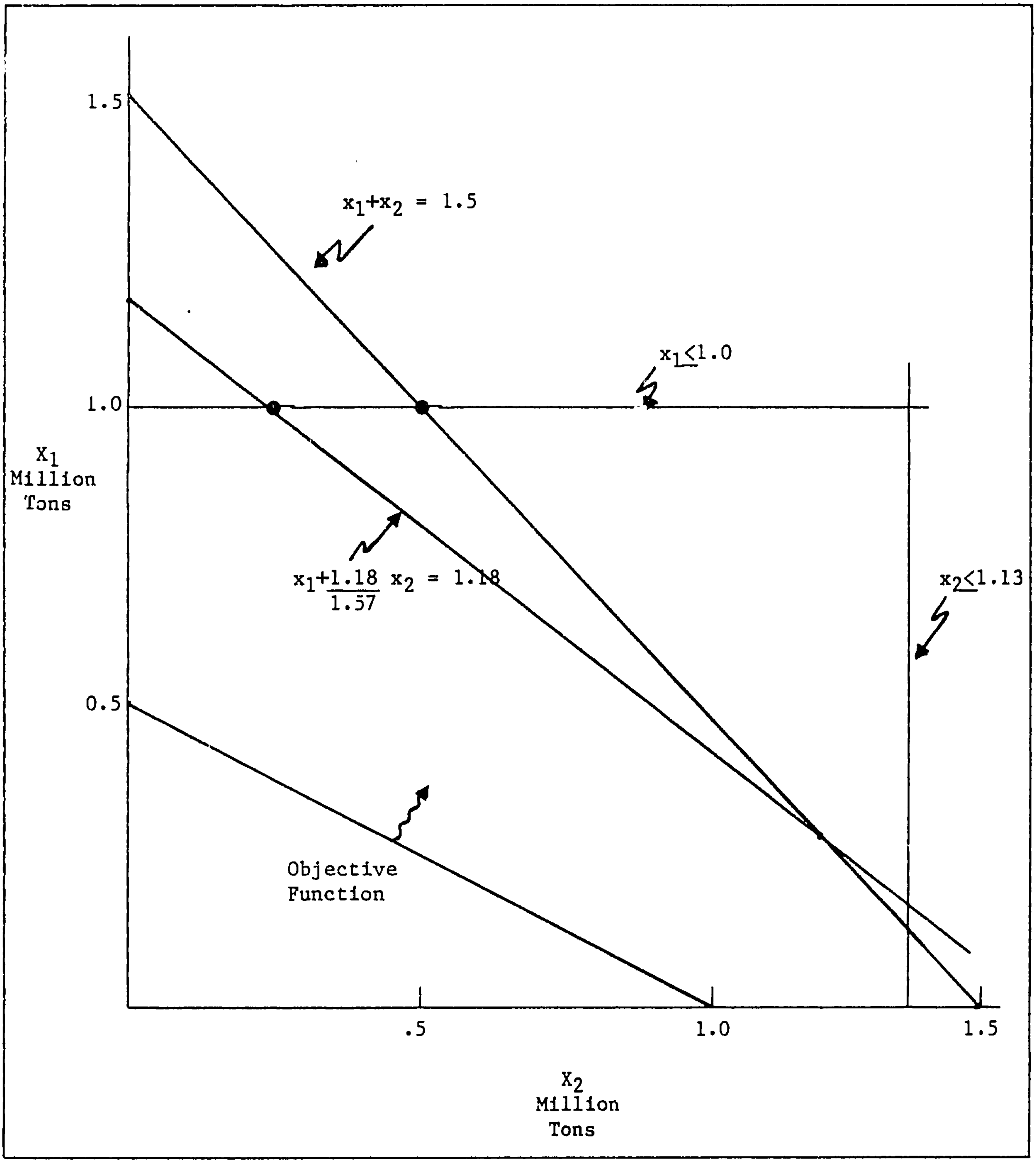


Figure 2.7: Graphical Solution, Example 2.3

Lagrange Multipliers: Suppose we wish to minimize the function

$$f(x_1, x_2) \tag{2.68}$$

subject to

$$g(x_1, x_2) = b \tag{2.69}$$

This can be solved by the method of Lagrange Multipliers, for which we first set up the Lagrangian function

$$\emptyset(x_1, x_2, \lambda) = f(x_1, x_2) + \lambda(g(x_1, x_2) - b) \tag{2.70}$$

where λ is an added variable called the Lagrange Multiplier. We now take the partial derivatives of $\emptyset$ with respect to x_1, x_2 and λ, and set equal to zero, namely

$$\frac{\partial(x_1, x_2, \lambda)}{\partial x_1} = \frac{\partial f(x_1, x_2)}{\partial x_1} + \lambda \cdot \frac{\partial g(x_1, x_2)}{\partial x_1} = 0 \tag{2.71}$$

$$\frac{\partial(x_1, x_2, \lambda)}{\partial x_2} = \frac{\partial f(x_1, x_2)}{\partial x_2} + \lambda \cdot \frac{\partial g(x_1, x_2)}{\partial x_2} = 0 \tag{2.72}$$

$$\frac{\partial(x_1, x_2, \lambda)}{\partial \lambda} = g(x_1, x_2) - b = 0 \tag{2.73}$$

which represents a system of 3 equations in three unknowns. In general, if we have a function of n variables, say

$$f(x_1, x_2 \ldots, x_n) \tag{2.74}$$

subject to m constraints (n > m), say

$$g_i(x_1, x_2 \ldots, x_m); \; i = 1, \ldots m \quad . \tag{2.75}$$

Then we obtain, in general, n + m simultaneous equations

$$\frac{\partial f}{\partial x_j} + \sum_{i=1}^{m} \lambda_i \cdot \frac{\partial g_i}{\partial x_j} = 0 \qquad j = 1, 2, \ldots n \tag{2.76}$$

$$g_i(x_1, x_2 \ldots x_r) = 0 \qquad i = 1, 2, \ldots m \tag{2.77}$$

Inequality Constraints: Suppose we wish to maximize f(x) subject to m inequality constraints $g_i(x) \leq b_i$. By adding a slack variable u_i to each inequality, we obtain

$$g_i(x) + u_i^2 = b_i \tag{2.78}$$

Example 2.4: Method of Lagrange Multipliers

Suppose we are given the problem

$$\text{Min } f(x_1, x_2) = x_1^2 + x_2^2$$

$$\text{s.t. } x_1 + x_2 = 3$$

Graphically, this problem can be interpreted as finding the smallest circle that intersects the li
$x_1 + x_2 = 3$.

The Lagrange Function is

$$\phi(x_1, x_2\ \lambda) = x_1^2 + x_2^2 + \lambda(x_1 + x_2 - 3) \quad .$$

Setting the appropriate partial derivatives equal to zero:

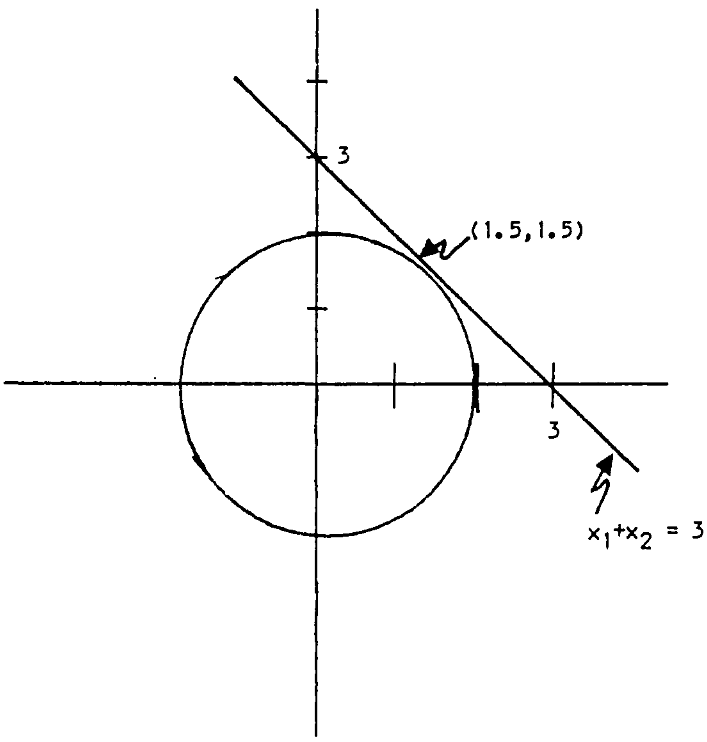
$$\frac{\partial\phi(x_1, x_2, \lambda)}{\partial x_1} = 2x_1 + \lambda = 0$$

$$\frac{\partial\phi(x_1, x_2, \lambda)}{\partial x_2} + 2x_2 + \lambda = 0$$

$$\frac{\partial\phi(x_1, x_2, \lambda)}{\partial \lambda} = x_1 + x_2 - 3 = 0 \quad .$$

For which we have the solution (in this particular case unique)

$$x_1 = x_2 = 1.5 \ ; \ \lambda = -3$$



(the reason for writing the slack as u_i^2 rather than u_i will become apparent). Now we proceed, as before, to write the Lagrangian

$$\phi(x, \lambda, u) = f(x) - \sum_{i=1}^{m} \lambda_i \bar{g}_i(x, u_i) \tag{2.79}$$

where $\bar{g}_i = g_i(x) + u_i^2 - b_i$. The stationary points are thus given by the set of partial derivatives

$$\frac{\partial\phi(x, \lambda, u)}{\partial x_j} = 0 = \frac{\partial f(x)}{\partial x_j} - \sum_{i=1}^{m} \lambda_i \frac{\partial \bar{g}_i(x)}{\partial x_j} \tag{2.80}$$

$$\frac{\partial\phi(x, \lambda, u)}{\partial \lambda_i} = 0 = \bar{g}_i(x, u_i) = g_i(x) + u_i^2 - b_i \tag{2.81}$$

$$\frac{\partial\phi(x, \lambda, u)}{\partial u_i} = 0 = 2\lambda_i u_i \quad . \tag{2.82}$$

If we multiply (2.82) by $u_i/2$, then

$$0 = \lambda_i u_i^2 \quad . \tag{2.83}$$

Furthermore, because (2.78) can be written as

$$u_i^2 = b_i - g_i(x) \tag{2.84}$$

it follows that

$$\lambda_i(b_i - g_i(x)) = 0 \quad . \tag{2.85}$$

Notice that from (2.83), for any i, if

$$\begin{array}{ll} & u_i > 0 \quad \text{then } \lambda_i = 0 \\ \text{and if} & \\ & u_i = 0 \quad \text{then } \lambda_i \neq 0 \quad . \end{array} \tag{2.86}$$

Thus, if an inequality is <u>exactly</u> met, the slack is zero, and the Lagrange multiplier is non-zero. On the other hand, if the inequality is not met exactly, the slack is non-zero, and the Lagrange multiplier is zero. We shall return to this important point in Chapter 5 with respect to the so-called dual of LP problems, and the condition known as complimentary slackness.

Economic Interpretation of Lagrange Multipliers: Suppose we regard b as a variable, and take the partial derivative of the Lagrangian with respect to b;

$$\frac{\partial \phi}{\partial b} = - \lambda \tag{2.87}$$

thus λ is the rate of change of ϕ with respect to b at the optimum point. But at the optimum point the Lagrangian $\phi(x, \lambda) = f(x)$, so that λ is also the rate of change of f(x) in the immediate vicinity of the optimum. In economic problems, where constraints represent resource availabilities, it follows that the λ represent the change in objective function per unit change of the constrained resource: λ are therefore equivalent to shadow prices.

Can we apply the Lagrange multiplier technique to the allocation problem for Republica discussed in Section 2.3? If we omit the redundant capital constraint, we have the problem

$$\begin{aligned} \text{Max} \quad & x_1 + x_2 \\ \text{s.t.} \quad & 10x_1 + 5x_2 \leq 25 \\ & 2x_1 + 4x_2 \leq 10 \\ & x_1, x_2 \geq 0 \quad . \end{aligned} \tag{2.88}$$

From (2.78) the constraints are i.e. written as

$$\begin{aligned} 10x_1 + 5x_2 + u_1^2 &= 25 \\ 2x_1 + 4x_2 + u_2^2 &= 10 \\ -x_1 + u_3^2 &= 0 \\ -x_2 + u_4^2 &= 0 \quad . \end{aligned} \tag{2.89}$$

Hence the Lagrangian can be stated as

$$\begin{aligned} \phi(x, \lambda, u) = x_1 + x_2 - \lambda_1(10x_1 + 5x_2 + u_1^2 - 25) - \\ -\lambda_2(2x_1 + 4x_2 + u_2^2 - 10) - \lambda_3(-x_1 + u_3^2) - \lambda_4(-x_2 + u_4^2) \quad . \end{aligned} \tag{2.90}$$

The partial derivatives follow:

$$\frac{\partial \phi}{\partial x_1} = 1 - 10\lambda_1 - 2\lambda_2 - \lambda_3$$

$$\frac{\partial \phi}{\partial x_2} = 1 - 5\lambda_1 - 4\lambda_2 - \lambda_4$$

$$\frac{\partial \Phi}{\partial \lambda_1} = -(10x_1 + 5x_2 + u_1^2 - 25) \qquad \frac{\partial \Phi}{\partial u_1} = -2\lambda_1 u_1$$

$$\frac{\partial \Phi}{\partial \lambda_2} = -(2x_1 + 4x_2 + u_2^2 - 10) \qquad \frac{\partial \Phi}{\partial u_2} = -2\lambda_2 u_2$$

$$\frac{\partial \Phi}{\partial \lambda_3} = -x_1 + u_3^2 \qquad \frac{\partial \Phi}{\partial u_3} = -2\lambda_3 u_3$$

$$\frac{\partial \Phi}{\partial \lambda_4} = -x_2 + u_4^2 \qquad \frac{\partial \Phi}{\partial u_4} = -2\lambda_4 u_4$$

Which is a system of 10 equations in 10 unknowns. Note that the last 4 are non-linear, which points to the general difficulty of solving such problems. Inspection of these equations does yield the following solutions, the veracity of which is left to the reader to confirm (by substitution into the above equations).

	Solution			
	(1)	(2)	(3)	(4)
x_1	2.5	0	0	1.66
x_2	0	0	2.5	1.66
u_2^2	0	25	12.5	0
u_2^2	5	10	0	0
u_3^2	2.5	0	0	1.66
u_4^2	0	0	2.5	1.66
1	0.1	0	0	0.066
2	0	0	0.25	0.1666
3	0	1	0.5	0
4	0.5	1	0	0

It turns out that these 4 solutions correspond to the extreme points of the feasible space of Figure 2.4. Thus we see that the Lagrange multiplier technique does result in an enumeration of the extreme points: But these are merely the local optima--we seek the global optimum, which means we must identify the global optimum from among the list of extreme points by enumeration, not a very useful procedure for large problems.

Digression 2.4: Least Squares Estimation subject to an equality restriction on the regression coefficients

One sometimes encounters situations in which the regression coefficients in a multivariable estimation are subject to some a priori restriction: for example, theory may require that the sum of the coefficients is unity; in general, a set of ℓ linear restrictions on the regression coefficients can be written:

$$\underset{(\ell \times 1)}{r} = \underset{(\ell \times k)}{R} \; \underset{(k \times 1)}{\beta} \quad .$$

Our least squares problem therefore becomes

$$\text{Min } S = (y - X\beta)^T(y - X\beta)$$

$$\text{s.t.} \quad R\beta - r = 0 \quad .$$

By applying the Lagrange multiplier technique one obtains the new objective function

$$\text{Min } S = (y - X\beta)^T(y - X\beta) - 2\lambda^T(R\beta - r)$$

where λ is an (ℓ x 1) vctor of Lagrange multipliers. Differentiating with respect to β one obtains (omitting the algebra of Eq. (2.36))

$$\frac{\partial S}{\partial \beta} = -2X^TY + 2X^TX\hat{\beta} - 2R^T\lambda$$

hence setting equal to zero, and multiplying all through by $(X^TX)^{-1}$, yields

$$0 = -(X^TX)^{-1}X^Ty + (X^TX)^{-1}(X^TX)\hat{\beta}^* - (X^TX)^{-1}R^T\lambda^*$$

hence

$$\hat{\beta}^* = \hat{\beta} + (X^TX)^{-1}R^T\lambda^* \qquad (1)$$

where the asterisk superscript denotes the restricted estimator. Premultiplying (1) by R yields

$$R\beta^* = Rb + R(X^TX)^{-1}R^T\lambda^*$$

But since $R\beta^* = r$

$$r = Rb + R(X^TX)^{-1}R^T\lambda^*$$

which we can now solve for λ^* by premultiplying all terms by $[R(X^TX)^{-1}R^T]^{-1}$, i.e.

$$[R(X^TX)^{01}R^T]^{-1}(r - Rb) = \lambda^* \qquad (2)$$

hence, inserting (2) back into (1)

$$\hat{\beta}^* = \hat{\beta} + (X^TX)^{-1}R^T[R(X^TX)^{-1}R]^{-1}(r - R\hat{\beta}) \qquad (3)$$

The restricted estimation $\hat{\beta}^*$ differs from the unrestricted estimator $\hat{\beta}$ by a linear function of the original restriction: if the original restriction were met exactly by the unrestricted estimator, r - R. = 0, and $\hat{\beta}^* = \hat{\beta}$. The sampling properties of $\hat{\beta}^*$ are derived in Goldberger (1964).

As an illustration of the matrix algaebric approach to regression analysis, consider the data of Table 2.1, from which we wish to model per capita energy consumption as a function of income.

Suppose we hypothesize the linear model

$$\underset{\begin{bmatrix}\text{Energy/}\\ \text{Capita}\end{bmatrix}}{Y} = \beta_1 + \beta_2 \underset{[\text{Price}]}{x_2} + \beta_3 \cdot \underset{\begin{bmatrix}\text{Income/}\\ \text{Capita}\end{bmatrix}}{x_3} \tag{2.91}$$

where β_1, β_2, β_3 are the regression coefficients to be estimated. The expectation would be that the β_1 coefficient is negative, since increases in price should yield a decrease in energy consumption. Similarly, the initial expectation is that β_2 have positive sign, since increases in income lead to increases in energy consumption. In order to make use of matrix algebra, let us rewrite Eq (2.91) as

$$Y = \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 \tag{2.92}$$

or, in matrix terms

$$\underset{(n\text{x}1)}{Y} = \underset{(n\text{x}m)}{X} \; \underset{(m\text{x}1)}{\beta}$$

Table 2.1
Tunisian Data

		x_1		Y	x_2
	(1)	(2)	(3)	(4)	(5)
Year	GDP 10^6 1972 Dinars	Wholesale Price Index, Energy (1970-100)	Population (10^6)	Energy Consumption kg oil Equiv. per capita	per capita GDP (1)/(3)
1969	766	94	4.8	250	159
1970	824	100	4.9	288	168
1971	914	103.8	5.0	322	182
1972	1077	106	5.20	359	207
1973	1082	105.8	5.33	378	203
1974	1181	111.1	5.46	416	216
1975	1286	138.4	5.61	429	229
1976	1443	145.9	5.77	460	250

where X is given by

$$\underset{(8x3)}{X} = \begin{bmatrix} 1 & 94.0 & 159 \\ 1 & 100.0 & 168 \\ 1 & 103.8 & 182 \\ 1 & 106.0 & 207 \\ 1 & 105.8 & 203 \\ 1 & 111.1 & 216 \\ 1 & 138.4 & 229 \\ 1 & 145.9 & 250 \end{bmatrix}$$

and Y is given by

$$Y = \begin{bmatrix} 250 \\ 288 \\ 322 \\ 259 \\ 378 \\ 416 \\ 429 \\ 460 \end{bmatrix}$$

Hence the matrix product X^TX follows as

$$X^TX = \begin{bmatrix} 8. & 905. & 1614. \\ 905. & 104824. & 186223. \\ 1614. & 186223. & 332284. \end{bmatrix}$$

Before computing the inverse of this matrix, note that the largest elements are 5 orders of magnitude greater than the smallest: such a matrix is said to be "poorly scaled" and accurately computing the inverse may prove difficult.[20] On a high precision scientific computer this may not matter very much, and the inverse readily computes to[21]

$$(X^TX)^{-1} = \begin{bmatrix} 6.25550 & -0.0063 & 0.0269 \\ -0.0063 & 0.0022 & -0.0012 \\ -0.0269 & -0.0012 & 0.0008 \end{bmatrix}$$

Much better, in such cases, is to scale the X matrix, such that each column of X has roughly the same order of magnitude. In our case, dividing the second and third columns of X, and Y, by 100, yields the X^TX matrix

[20] For a full discussion of the numerical problems of computing inverse matrices, see G. Forsythe and C. Moler, "Computer Solution of Linear Algebraic Systems," Prentice-Hall, Englewood Cliffs, NJ, 1967.

[21] But see exercise E4 for a discussion of significant figures.

$$X^TX = \begin{bmatrix} 8.0 & 9.05 & 16.14000 \\ 9.05 & 10.4827 & 18.62230 \\ 16.14 & 18.62232 & 33.22340 \end{bmatrix}$$

and

$$(X^TX)^{-1} = \begin{bmatrix} 6.22500 & -0.62672 & -2.68700 \\ -0.62672 & 21.85528 & -11.94397 \\ -2.68700 & -11.94397 & 8.02905 \end{bmatrix}$$

The X^Ty vector follows as

$$X^Ty = \begin{bmatrix} 29.0200 \\ 33.6475 \\ 60.0882 \end{bmatrix}$$

hence $\hat{\beta}$ follows as the matrix product

$$\hat{\beta} = \begin{bmatrix} 6.22500 & -0.62672 & -2.68700 \\ -0.62672 & 21.85518 & -11.94397 \\ -2.68700 & -11.94397 & 8.02905 \end{bmatrix} \begin{bmatrix} 29.0200 \\ 33.6475 \\ 60.0882 \end{bmatrix} = \begin{bmatrix} -1.0244 \\ -0.5062 \\ 2.5896 \end{bmatrix}$$

The explained sum of squares follows from Eq. (2.59) as

$$= X^Ty - \frac{1}{n}(\sum Y_i)^2$$

$$= [-1.0244 \quad -0.5062 \quad 2.5896] \begin{bmatrix} 29.0200 \\ 33.6475 \\ 60.0882 \end{bmatrix} - \frac{1}{8}[29.02]^2$$

$$= 3.5746$$

whilst the residual sum of squares e^Te follows from Eq. (2.51) as $Y^TY - \beta X^Ty$, i.e.

$$e^Te = 108.959 - 108.84395$$
$$= 0.11505$$

The analysis of Variance Table is

	S.S	D.F	M.S.
Model	3.574	2	1.787
Residual	.115	5	0.023
Total	3.689	7	

0.023 is thus our estimate of σ^2, (recall that $S^2 = e^Te/n-k$ is an unbiased estimate of σ^2). Thus the variance-covariance matrix for $\hat{\beta}$, given by

$$\text{Var}\left\{\beta\right\} = \sigma^2(X^YX)^{-1}$$

$$= 0.023 \begin{bmatrix} 6.225 & -0.62672 & -2.687 \\ -0.62672 & 21.85518 & -11.94397 \\ -2.687 & -11.94397 & 8.02905 \end{bmatrix}$$

$$= \begin{bmatrix} 0.143 & 0.0143 & -0.061 \\ -0.0143 & 0.4999 & -0.213 \\ -0.0615 & -0.273 & 0.1836 \end{bmatrix}$$

How good is our model? From a statistical standpoint, "goodness" is measured by statistical significance, the tests for which were introduced in Section 2.2. First let us examine the appropriate null hypotheses on the regression coefficients; are the values of the regression coefficient significant. For β_2, that measures the influence of price, we have

$$H_o : \beta_2 = 0$$
$$H_a : \beta_2 \leq 0$$

As expected, the sign of the coefficient β_2 is negative (increases in price lead to <u>decreases</u> in consumption); the appropriate test is the one-tailed t-test. Using Eq. (2.54).

$$t = \frac{-0.5062}{\sqrt{0.023 \cdot 21.85518}} = .07139$$

For 5 degrees of freedom and $\alpha = 0.05$, the tabulated value of t is 2.015: therefore the null hypothesis cannot be rejected. β_2 is not statistically significant. This follows also from looking at the confidence interval for β_2; given by Eq. (2.55) as

$$\hat{\beta}_2 \pm t_{\alpha/2}\sqrt{\frac{\sum e_i^2}{n-k} a_{ii}}$$

$$= -0.50623 \pm 2.571 \quad 0.023 \cdot 21.855$$

$$= -0.50623 \pm 1.8228$$

from which follows that the 95% confidence interval is very wide indeed, indicative of the lack of statistical significance. The corresponding interval for β_2, is given by

$$\hat{\beta}_3 = 2.5896 \pm 2.571 \sqrt{0.023 \cdot 8.0290}$$
$$= 2.5896 \pm 1.104$$

with a t-value for the null hypothesis of 6.043, which is very much greater than 2.015, and for which the null hypothesis can be rejected.

If it is true that the β_2 lacks statistical signifiance, then we would expect that the simpler model

$$Y = \alpha + \beta_3 X_3$$

in which we ignore the price variable, would be as good a model as the one that includes both price <u>and</u> income.

In exercise E5, we show that the new estimate of the regression coefficients is given by

$$\hat{\beta} = \begin{bmatrix} -1.039 \\ 2.313 \end{bmatrix}$$

with a 95% confidence limit for β_2 given by

$$\beta_2 = 2.313 \pm 2.571 \sqrt{0.021 \cdot 1.501}$$
$$= 2.313 \pm 0.559$$

This is a much narrower range of uncertainty than in our previous model that <u>included</u> price. Another way of putting this is that the addition of the price variable to the basic model relating energy consumption to income does not add anything from a statistical standpoint. But does this mean that price is <u>insignificant</u>? Because if so, this would appear to run counter to expectations based on economic theory. We shall therefore return to this example in Section 4, following a more detailed discussion of the statistical estimation problems involved, and the underlying theoretical principles of the role of price and income.

E1. Matrix Multiplication.

Compute the matrix equation $A = B \cdot C^T \cdot D$ given

$$B = \begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \qquad C = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \qquad D = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

E2. Matrix Determinants.

Compute the determinant of the matrices

$$A = \begin{bmatrix} 3 & 8 \\ 6 & 16 \end{bmatrix} \qquad B = \begin{bmatrix} 3 & 7 \\ 7 & 2 \end{bmatrix}$$

E3. Graphical Solution to Linear Programs.

Resolve the linear programming probelm of Section 2.3 (the production planning problem for Republica), using the objective function

$$\text{Max } S = X_2 + 8X_2$$

E4. Significant Figures and Decimal Places in Matrix Regression.

Compute the three following matrix products

$$I_1 = \underset{X^{-1}}{\begin{bmatrix} 6.255 & -0.627 & -2.687 \\ -0.627 & 21.855 & -11.944 \\ -2.687 & -11.944 & 8.209 \end{bmatrix}} \underset{X}{\begin{bmatrix} 8.000 & 9.05 & 16.140 \\ 9.050 & 10.482 & 18.622 \\ 16.140 & 18.622 & 33.228 \end{bmatrix}}$$

$$I_2 = \underset{X^{-1}}{\begin{bmatrix} 6.25500 & -0.62672 & -2.68700 \\ -0.62672 & 21.85518 & -11.94397 \\ -2.68700 & -11.94397 & 8.02905 \end{bmatrix}} \underset{X}{\begin{bmatrix} 8.00000 & 9.0500 & 16.14000 \\ 9.0500 & 10.48247 & 18.62232 \\ 16.1400 & 18.62232 & 33.22840 \end{bmatrix}}$$

$$I_3 \underset{X^{-1}}{\begin{bmatrix} 6.255 & -0.6267 & -2.687 \\ -0.6267 & 21.86 & -11.94 \\ -2.687 & -11.94 & 8.029 \end{bmatrix}} \underset{X}{\begin{bmatrix} 8.000 & 9.050 & 1.614 \\ 9.050 & 10.48 & 18.62 \\ 16.14 & 18.62 & 33.22 \end{bmatrix}}$$

What lesson is to be drawn from a comparison of I_1, I_2 and I_3?

E5. Statistical Analysis of Energy Consumption

Using the data of Table 2.1, estimate the parameters of the linear model

$$\underset{\text{[Energy Consumption]}}{Y} = \alpha + \beta \cdot \underset{\text{[GDP/Capita]}}{X}$$

There are possibly hundreds of published textbooks covering the material so briefly presented in this overview. We present here only a sample of texts that in our experience present the material in the most comprehensible way given the likely background of those engaged in energy planning in developing countries.

A. Rogers "Matrix Methods in Urban and Regional Analysis" Holden-Day, San Fransisco, 1971 (pp. 508).

> An outstanding text covering most of the material presented here in much greater detail. Many of the numerical examples given in the text deal with the Yugoslavian Economy in general, and planning problems in the City of Ljubljana. Covers most of the basic techniques of operations research, and statistics; the book is likely to be of interest not just to energy planners but to all interested in the application of quantitative methods to public sector planning, as well as to demographers.

J. Johnston "Econometric Methods" McGraw-Hill Company, London and New York, 1963 (pp. 295).

> Despite many more recent texts that have appeared on this subject in the last few years, this remains, in the judgement of the writer, the best introduction to statistical methods suited to the economist and planner.

R. G. D. Allen "Mathematical Economics" 2nd Edition, 1960, MacMillan, London.

> Whilst some 20 years old at the time of this writing, still the best introduction to mathematical economics around; it is so well written that it must surely rank as one of the few texts that are actually enjoyable as reading matter. The mathematics are presented in a notation and style that is immediately comprehensible, in contrast to many recent offerings of a similar genre. Covers everything from Simple Keynesian models to differential equations, linear algebra, input-output analysis and mathematical game theory.

3. REFERENCE ENERGY SYSTEMS

3.1 FUNDAMENTALS[1]

A Reference Energy System (RES) is a way of representing the activities and relationships of an energy system, depicting estimated energy demands, energy conversion technologies, fuel mixes, and the resources required to satisfy those demands.[2] The pictorial format for the Reference Energy System is a network diagram which indicates energy flows and the associated conversion efficiencies of the technologies employed in various stages of the nergy system. A simplified RES is shown in Figure 3.1. For each energy resource, a complete reference Energy System specifies the technologies employed in the following activities.

1. Extraction
2. Refining and/or conversion
3. Transport of primary energy source
4. Centralized conversion (e.g., electricity generation)
5. Transport or transmission and storage of secondary energy form
6. Decentralized conversion
7. Utilization in an end use device.

Figure 3.2 shows a more complete RES for India. As illustrated in this figure, each path through the energy system network indicates a possible route for the flow of energy from an energy resource to a given demand category. Alternate paths and branches reflect the substitutability of various resources and technologies for one another. The energy flowing through each step or process is shown above the line representing the activity. The numbers in parentheses represent the efficiencies, or relative effectiveness, of the processes. The RES representation permits calculation of the amount of a particular energy resource, for example oil, used to satisfy a particular demand, for example space heating, either through a particular intermediate fuel form, such as electricity, or directly. Energy demands are assumed for each reference year from historical data and projections. Both commercial

[1]This section is adapted from "An Analytical Framework for the Assessment of Energy Resources and Technology Options in Developing Countries" BNL 50800, Brookhaven National Laboratory.

[2]The Reference Energy System approach was developed at Brookhaven National Laboratory in 1971 for energy R&D assessment and has been extended for various analyses since that time. For a complete discussion, see e.g. Beller, (1975.)

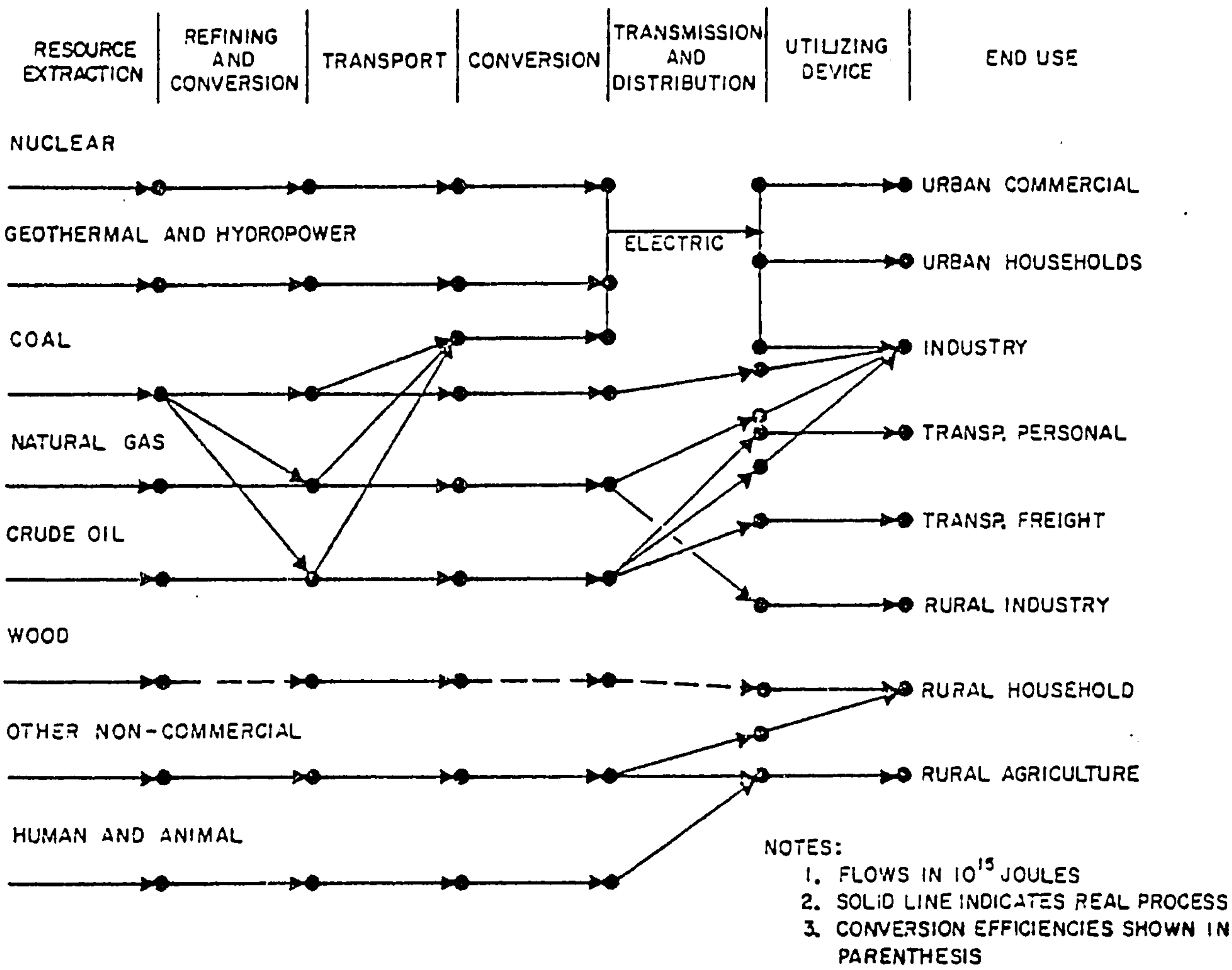


Figure 3.1 LDC Reference Energy System (Illustrative).

and non-commercial[3] fuel forms can be accomodated, and centralized energy systems (such as large scale electricity generation) are distinguished from decentralized systems (such as small scale solar electricity production).

The values on the left side of the RES under the heading "extraction" represent the raw energy input needed to satisfy the basic energy demands.

[3]The term "commercial" energy generally refers to energy forms normally actively traded in developed country markets such as oil, gas, coal and electricity. The primary categories of "noncommercial" energy are wood, agricultural wastes, and animal dung. The term "noncommercial" is in fact a misnomer, since there are monetary markets for these fuels. In some contexts, the term must be extended to include human and animal power. Other terms sometimes used for "noncommercial" are "traditional," "primitive" (vs. "modern") or "nonconventional" (vs. "conventional").

The values on the right side of the RES under the heading "Demand Category" represent the basic energy demands in terms of a specific set of end-use categories, as discussed below. The actual fuel inputs to the end uses are shown under the heading "Utilizing Device."

Demand Analysis: The analytical approach is "driven" by a detailed consideration of energy demand. In this section we describe the approach to demand analysis in some detail in order to provide guidance to data collection.

Three important points must be stressed at the outset. First, data requirements can not be specified in the abstract. They depend strongly upon (1) the options to be analyzed and (2) the availability of information. If stress is to be laid on increased energy efficiency in industry, for example, more detail is required in industrial energy demand than would be the case if only increased supply options were being examined. Similarly, since a current program precludes the collection of new primary data, one cannot disaggregate demand more than permitted by existing information.

The second general point to be stressed is that the demand analysis must be flexible, and will probably require some imagination. Where "necessary" data on demands or efficiencies do not exist, some reasonable surrogates or approximations based on experiences in other countries, if necessary, should be invoked. This leads to a final point: that every number in the analysis be documented with respect to its source or the assumptions made in deriving it. In this way the more uncertain parts of the analysis can be identified or alternative assumptions can be made.

The specification of an energy demand for the RES entails three pieces of information:

1. Measure of Demand Activity: the demand level specified in units of activity, or other determinant of demand, e.g., passenger car kilometers, tons of steel produced, number of rural households (for cooking), number of irrigation pumps in operation, etc.

2. Direct Fuel Consumption: the amount of fuel (in joules) of various kinds ("fuel" includes electricity) delivered to a particular end use category.

3. Relative Effectiveness: the relative efficiency with which various fuels provide energy to satisfy a specific activity. Relative effectiveness is distinguished from efficiency in that it reflects differences in utilization practice as well as device efficiency.[4]

[4]The relative effectiveness of electric space heating, for example, includes the effect of different use patterns, and the assumed insulation levels.

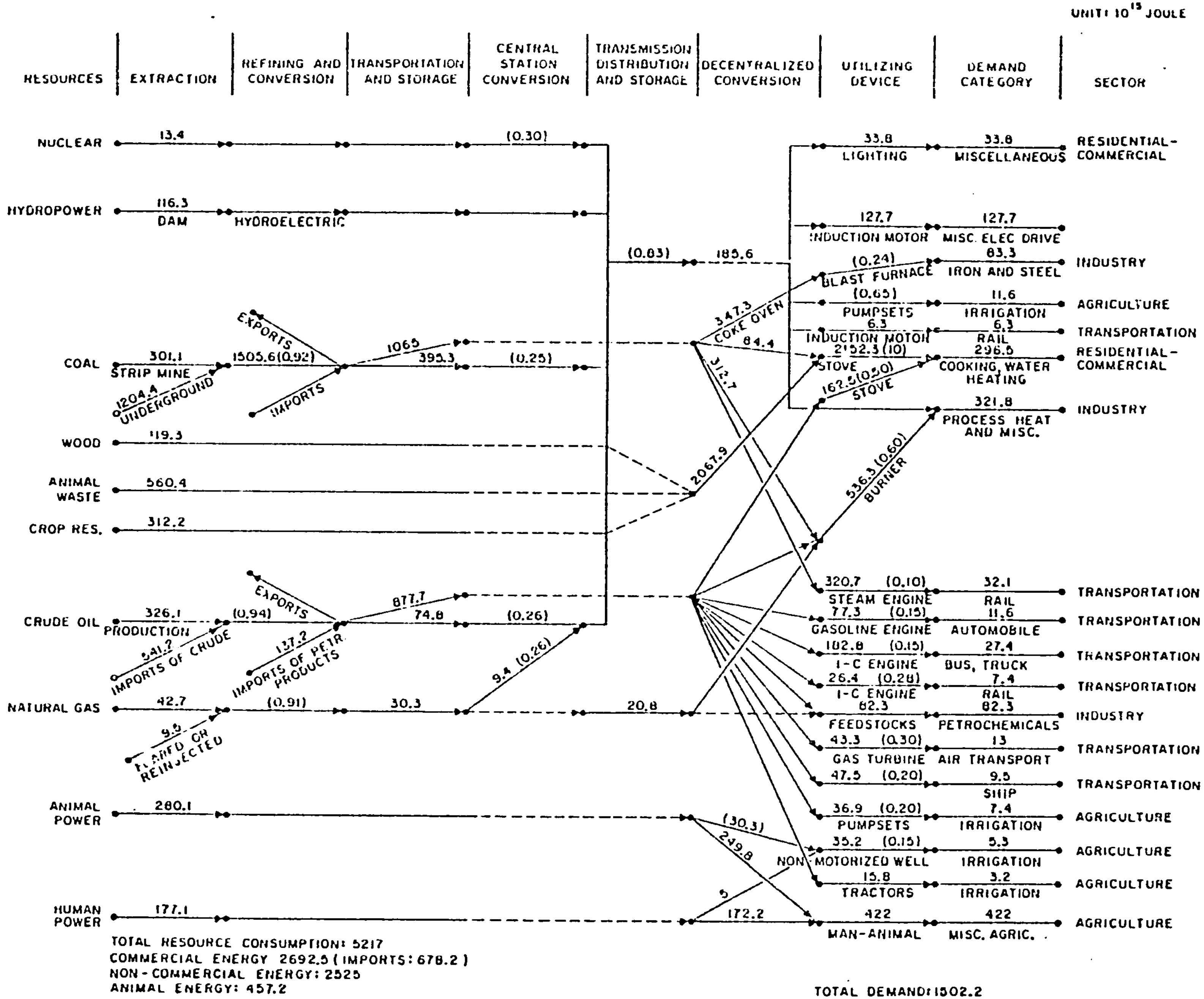


Figure 3.2 Reference Energy System For India, 1972

In addition to these basic quantities it is useful to define a quantity known as Basic Energy Demand. Basic Energy Demand is the amount of energy (in joules) which represents the useful energy required by a particular demand category. This is a derived quantity, independent of fuel type.

Let D_i = amount of fuel i used for a given activity

e_i = relative effectiveness with which fuel i is used for the activity

E = Basic Energy Demand for the activity.

Then

$$E = \sum_i e_i D_i \qquad (3.1)$$

Basic Energy Demands are represented on the rightmost column of the Reference Energy System under "Demand Category." Under "Utilizing Device" are shown the Fuel Consumption and Relative Effectiveness. An example, for cooking rural households, is shown in Figure 3.3. A typical list of demand categories and projection variables is shown in Table 3.1

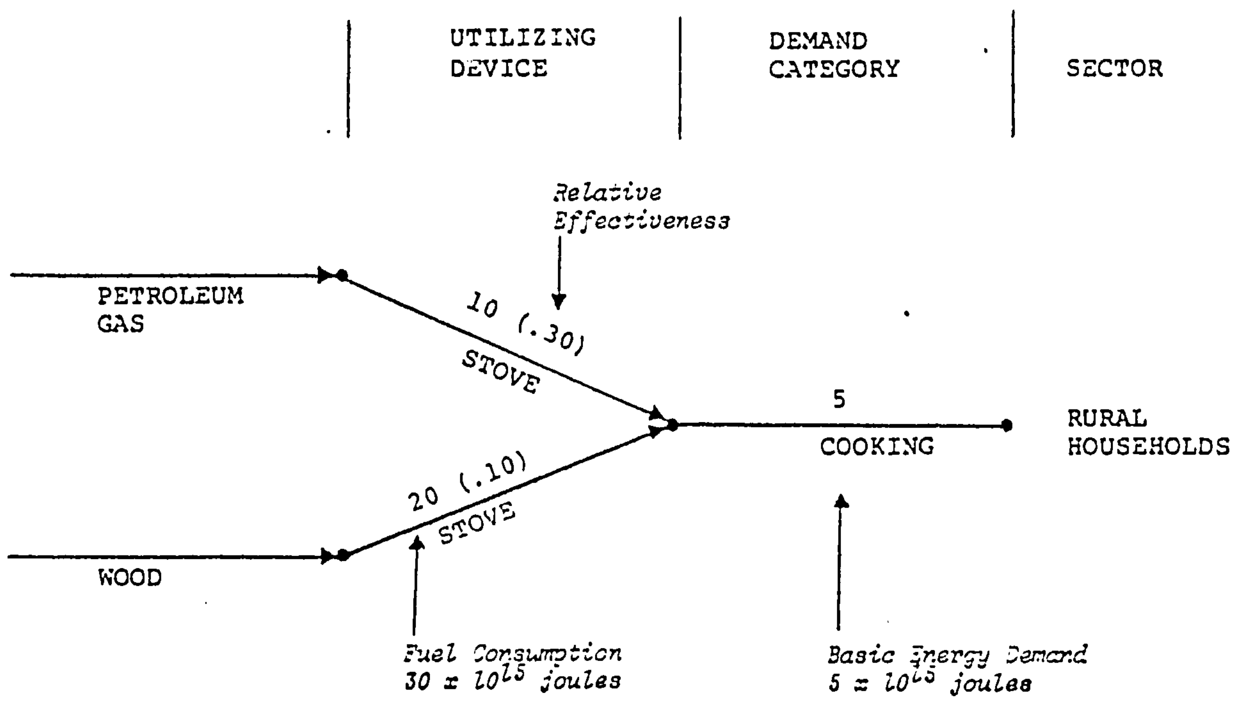


Figure 3.3 RES Demand Element - Rural Cooking.

Table 3.1
REPRESENTATIVE DEMAND PROJECTION VARIABLES

SECTOR/CATEGORY	MEASURE OF ACTIVITY	UTILIZING DEVICE/ACTIVITY	OTHER VARIABLES
Industrial			
Iron and steel	Output (Tonnes)	Blast Furnace (coke) Process Heat	Capacity Projections
Petrochemicals	Output (Tonnes)	Feedstocks Process Heat	Capacity Projections
Cement	Output (Tonnes)	Burners	Construction Projections
Other Large	Output (Tonnes)	Process Heat Electric Drive	Industrial Value Added
Rural Small	Output (Tonnes)	All	Rural Population
Transportation			
Passenger-Auto	Passenger-km	Int. Comb. Engine	Population, disposable income, Road km
Ship	Vessel-km	Diese	
Air	Passenger-km	ICE: Jet	
Truck	Tonne-km	Diesel, ICE	Passenger vs Freight Travel
Railway	Tonne-Passenger-km	Diesel, Electric Drive	
Pipeline	Thruput, Barrels/day	Pump; diesel, electric	Capacity Projections
Agriculture			
Soil Preparation	Tractor-km	Int. Comb. Engine	Number of Tractors
Irrigation	Pump-hours	Diesel Pump Electric Pump	Agricultural Production Number of Pumps
Urban Households			
Cooking	No. of Households	Stoves; gas, oil, electric	
Lighting	No. of Households	Electric Lights	Disposable Income
Misc. Electric	No. of Households; appliance ownership	Motors, electric devices	Disposable Income
Rural Households			
Cooking	No. of Households	Stoves; kerosone & non commercial	
Lighting	No. of Households	Electric & gas lamps	Rural Electrification
Commercial			
Lighting & Appliances	No. of Establishments and/or Floorspace	Vapor Compression	Value Added in Sector
Air Conditioning	No. of Establishments and/or Floorspace & Saturation	Vapor Compression	Value Added in Sector (Including Tourism)
Cooking	No. of Restaurants	Stoves; gas and electric	Value Added in Sector (Including Tourism)
Municipal Services			
Lighting	Urban Population; Saturation	Lamps; florescent & incandescent	Municipal Budgets
Other	Urban Population; Saturation	Pumps, etc.	Municipal Budgets

Since in most countries there already exists a projection of electricity demand and capacity mix prepared by the electric utility, the RES methodology provdes an independent projection of electricity demand. Moreover, the summation of RES end use demands yields a measure of aggregate demand that can be compared to that derived from projections of national economic growth and a specification of income elasticity of energy demand. Such projections can be used to adjust the sectoral RES demands.[5]

Technology Characterization: In order to complete the Reference Energy System the characteristics of each process element or technology must be specified. This applies to the technologies used for final consumption (such as air conditioners and stoves), the conversion technologies (such as for electric generation and refineries), and for supply technologies (such as coal cleaning). The following information should be provided for each technology:

- Primary Efficiency: the ratio of useful energy out of the process to primary energy delivered to it (for example, electricity out to fuel in for electricity production). If significant amounts of ancillary energy of some other form is used it should also be specified. For end use technologies "relative effectiveness" is used rather than primary efficiency (see above).
- Costs: Capital and operating, expressed in constant dollars for a given reference year.

Technology characteristics should be specified for each reference year to reflect likely trends in device efficiency, heat rate, scale economies, and so forth. The most complete data for current and future technologies, as they apply in the member countries of the International Energy Agency (IEA), has been developed in a joint project at BNL and the Kernforschungsanlage Jülich in W. Germany[6]

The resource requirements derived in the analysis can then be compared with projected estimates of resource production to arrive at import require-

---

[5]Obviously there will be problems of intersectoral consistency in such adjustments; much of the later discussion of input-output analysis (Chapter 6) is driven by the need to achieve end-use demand consistency.

[6]Documented in both German and U.S. reports: "EnergyTechnology Data Handbook, Vol I; Conversion Technologies" Kernforschungs Anlage Jülich, JUL-SPEZ-70, January 1980 and "Technology Review Report, IEA Energy Systems Analysis Project, BNL 27074, Brookhaven National Laboratory, December 1979.

ments or export potential. Other characteristics of the system such as capital requirements, and sectoral cost of energy, and environmental impacts can also be calculated on the basis of the complete Reference Energy System.

Option Analysis. Once the Reference Energy System is established for the future years of interest, and the aggregate characteristics of the system are determined, one is ready to perform impact analyses of future options. In some cases this will involve a simple technology substitution, such as substituting solar cookers for wood burning stoves. In other cases it may involve a complex set of resource and technology substitutions, such as increased natural gas production and use in electricity generation, domestic cooking, and industry. In all cases the basic procedure is to:

1. Identify the process or sectors in which the new resource or technology will substitute,
2. Establish feasible rates of introduction of the resource/technology and levels of introduction in the future reference years.
3. Produce a new system description, a Perturbed Energy System, for the appropriate years, and
4. Calculate the change in resource consumption, cost, and other objective functions identified above.

3.2 ENERGY DEMAND AND FUEL MIX ANALYSIS

For the purpose of energy resource and technology assessment it is necessary to develop projections of energy demands in a detailed, or disaggregated, manner. This approach is required in order to evaluate technologies that may apply to very specific end uses. To evaluate the use of solar energy for water heating, for example, the projected growth of this end-use must be exhibited in the reference system

It is recognized that projections made in this disaggregated manner may well underestimate the total energy demand in the future because of unanticipated new uses of energy. Since the technologies employed for such uses obviously cannot be defined, it is not, in general, necessary to reflect these uses in the reference systems. However, such demands are frequently more likely to involve electrical energy than other energy forms; to reflect this impact on the supply systems, several undefined electrical demands can be included in the residential and commercial miscellaneous electric categories (by postulating phantom appliances and demands), and in the demand category for industrial miscellaneous process heat.

The projections of energy demands and the fuel mix for the reference years can be developed on worksheets of the form shown in Table 3.2

1. Fuel Demand, D_i = The quantity of a fuel, i, actually consumed in a specific demand category, such as space heating, automotive transport, or aluminum production.
2. Total Fuel Demand, D = the total fuel required to satisfy the requirements of a specific demand category. Electricity is considered as a fuel in this sense and $D = \sum_i D_i$.
3. Relative Effectiveness, e_i = the relative effectiveness with which fuel, i. is used in a demand category. This parameter depends on the utilization technology employed.
4. Basic Energy Demand, E = the amount of energy that would be required in a specific demand category, assuming a relative effectiveness, e_i of 100% for each fuel employed. Thus, for a given demand category where quantities of fuels, D_i, are consumed with actual Relative Effectiveness, e_i, $E = \sum_i e_i D_i$.
5. Degree of Saturation, S = the fraction of the potential demand for a particular energy use actually being fulfilled at a given time. For example, if 95% of all households have refrigerators, and potentially all houses can have one refrigerator, S = 0.95.

Table 3.2
SAMPLE WORKSHEET

Sector: Residential
End Use: Urban Cooking

Fuel	1977 f_i	1977 e_i	1977 D_i	1990 f_i	1990 e_i	1990 D_i	2000 f_i	2000 e_i	2000 D_i
LPG	0.60	0.6	2.2	0.80	0.6	6.1	0.87	0.6	10.3
Charcoal	0.38	0.3	2.8	0.18	0.3	2.8	0.12	0.3	2.8
Wood	0.02	0.2	0.2	0.01	0.2	0.2	0.01	0.2	0.2
Total Fuel Demand (10^{15}J)			5.2			9.1			13.3
Basis (10^3 Hshlds)	354.4			737			1,148		
Saturation	1.0			1.0			1.0		
Unit Basic Demand: (10^9J)	6.2			6.2			6.2		

Notes: The 1977 fuel consumption estimates are based on an unpublished survey of household expenditures (Banco Central de la Republica Dominicana, Encuesta de Ingresos y Gastos de las Familias, Mayo 1976 - April 1977) and prices provided by the Banco Central. LPG price was adjusted upward to ensure that household consumption did not exceed total sales. Electric cooking, which could not be separated out from other electricity use, is incorporated in electric appliances.

Projections assume constant Unit Basic Demand and constant wood and charcoal consumption in urban areas to the year 2000.

6. Saturated Basic Energy Demand = the Basic Energy Demand that would exist in a category if there was 100% saturation, = E/S.

7. Unit Basic Energy Demand, E_u = the Basic Energy Demand per unit, e.g., per household, per lb of steel produced, etc.

8. Fuel Fraction, f_i = fraction of the Saturated Basic Energy Demand that is satisfied by using the i'th fuel.

$$f_i = \frac{e_i D_i}{E/S} \quad \text{and} \quad \sum_i f_i = S \quad . \tag{3.2}$$

The Basic Energy Demand derived in this manner is independent of the fuels employed to satisfy the demand and is projected into the future on the basis outlined above, including any increased saturation that may be postulated. In categories where a unit basic demand is defined, it may be used as the basis for the projection and, in most cases, is held constant over all reference years. By specifying the Fuel Fractions, f_i, and Relative Effectiveness, e_i, the Fuel Demands, D_i are derived from the basic energy demands for each future reference year.

3.3 MATRIX ALGEBRAIC FORMULATION: APPLICATION TO THE DOMINICAN REPUBLIC

There are a number of computational approaches to the mathematical generation of RES's, especially with the advent of sophisticated computer graphics capabilities and automated graphical displays of both the entire RES network or individual trajectories. Soon after development of the RES concept at Brookhaven in the early 1970's, the Energy Systems Network Simulator (ESNS) was developed, an approach based on network traversal algorithms. A special version of ESNS, designed for developing countries, was developed by R. Malone and A. Reisman in 1978,[7] but this algorithm also suffered from the problem that afflicts so many computer programs: it requires the enourmous computational and storage capability of a CDC 7600, hardware that is not likely to be encountered in many developing countries. A project at the Institute for Energy Research at the State University of New York, under the sponsorship of the Al Diriyyah Institute of Geneva, Switzerland, has developed an RES model that features interactive capability, and display on graphics terminals designed specifically for implementation on mini-computers (and has been implemented in the Dominican Republic). More recently, a new model based on matrix algebra, and that can be implemented on almost any computer including the new generation of micro-computers, has now become the standard tool for computerized RES analysis.[8]

The basic idea behind the matrix algebraic formuation is almost trivial: the Reference Energy System is sliced vertically in the manner indicated on Figure 3.4. Each link in such a slice can be treated as an element of a vector. Individual vectors are linked horizontally, (i.e. from one slice to another), by matrices, whose elements represent the allocations and efficiencies that characterise the links between the nodes of an RES. The Matrix approach to RES construction is best illustrated by example. Figure 3.5 shows a simple RES for the Dominican Republic, constructed in the traditional manner as part of a recent energy assessment for that country.[9] Notice that

[7] R. Malone and A. Reisman "Less Developed Countries Energy System Network Simulator LDC ESNS", BNL 50836, Brookhaven National Laboratory, April 1978.

[8] P. Meier and L. Feinerman "Energy Information for Developing Countries: A Matrix Algaebraic Model for Tunisia," XXVth International Meeting of the Institute for Management Science, Lausanne, Switzerland, 1982.

[9] Energy Development International et al., "Energy Strategies for the Dominican Republic Report of the National Energy Assessment," September 1980.

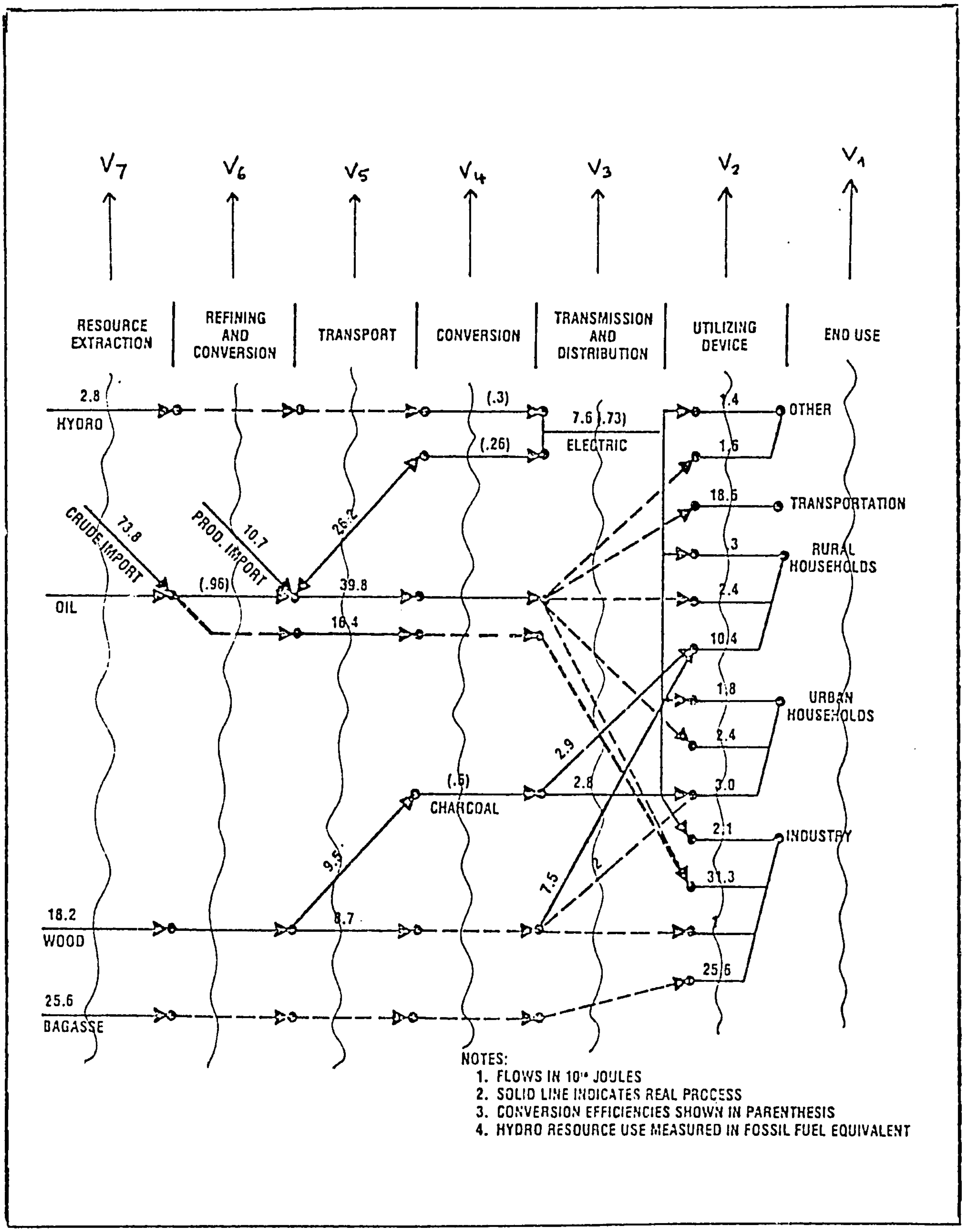


Figure 3.4 Vectors in the RES

the right hand side of the RES is total fuel consumption by major sector; nothing appears here in terms of energy end uses (e.g., differentiating in urban households by the major uses -- cooking, appliances, lighting, personal transportation) - which is not an uncommon situation when an RES is first constructed at the inception of national energy planning efforts, at a time when there is likely no, or only very scanty data on end use.

Let us begin, then by replicating the RES network of Figure 3.5 by a series of matrix equations, moving from right to left. From the extreme right of Figure 3.5 follows the Sectoral Energy Consumption Vector, e,

Mining	974
Govt. Services	122
Commercial	401
Industry	7343
Non-Energy	450
Urban Households	3040
Rural Households	3880
Transportation	4120

Which represents total fuel use in each sector, expressed as thousand tons of oil equivalent (k to e).

The next vector is the Fueld Consumption Vector, V, defined as:

Mining	Central Electricity	0
	Self Generated El.	380
	Oil Products	594
Govt.	Central Electricity	31
	Oil Products	91.4
Commerce	Central Electricity	214
	Self Gener. Electricity	18
	Oil Products	169
Industry	Coal	0
	Oil Products	2070
	Central Electric	363
	Self Gener. Electricity	130
	Bagasse + Ag. Wastes	4600
	Wood	175
Feedstocks		450
Urban Households, Oil	Products	433
	Central Electricity	361
	Charcoal	2230
	Firewood	14
Rural Households, Oil	Products	169
	Charcoal	507
	Wood	3190
Transportation		4120

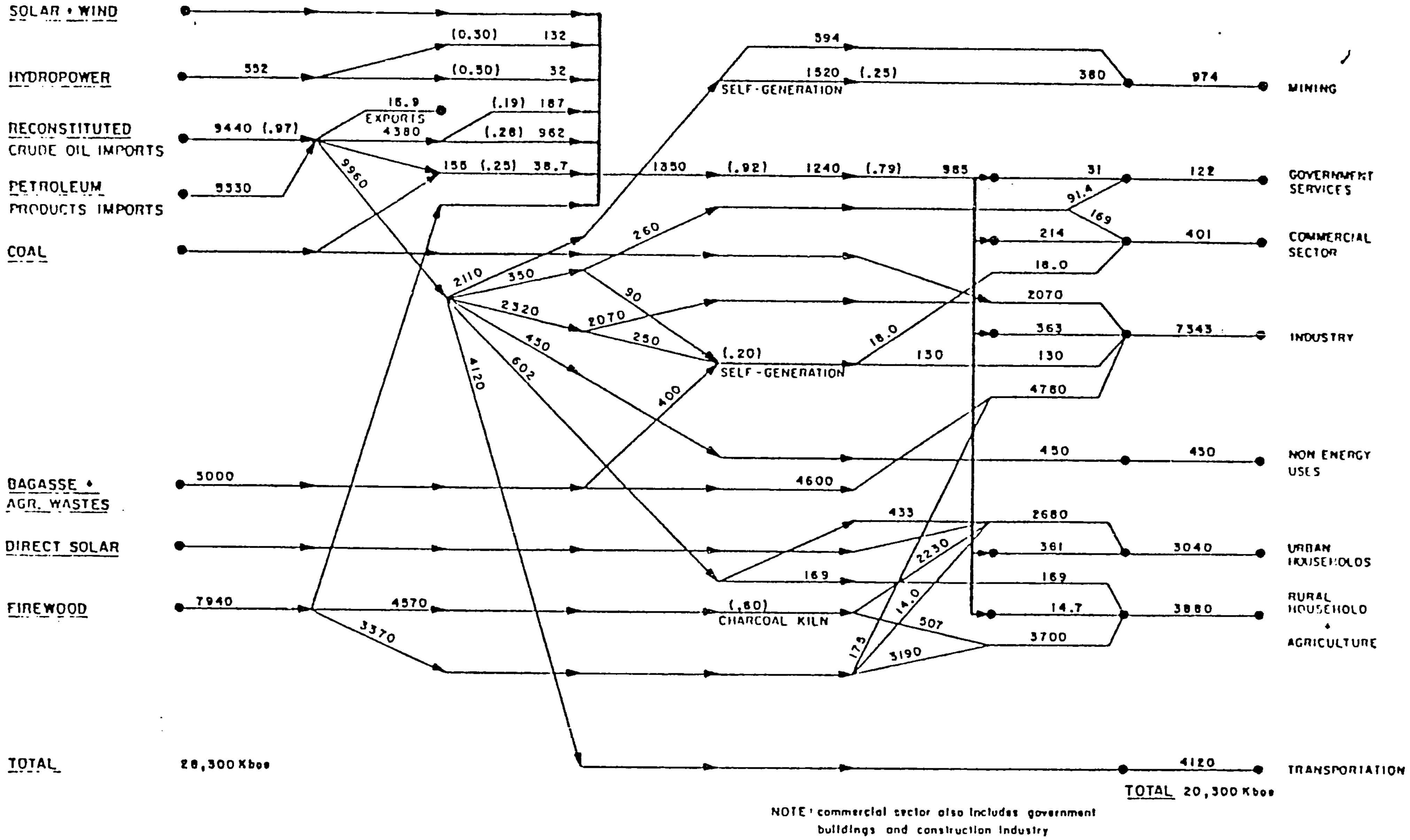


Figure 3.5. 1978 Reference Energy System for the Dominican Republic

hence

$$\underset{(23\text{x}1)}{V} = \underset{(23\text{x}8)}{F} \; \underset{(8\text{x}1)}{e}$$

where F is the (23x8) matrix

$$\begin{bmatrix}
0 & & & & & & & \\
.609 & & & & & & & \\
.391 & & & & & & & \\
 & .254 & & & & & & \\
 & .746 & & & & & & \\
 & & .533 & & & & & \\
 & & .421 & & & & & \\
 & & .046 & & & & & \\
 & & & 0 & & & & \\
 & & & .281 & & & & \\
 & & & .049 & & & & \\
 & & & .017 & & & & \\
 & & & .627 & & & & \\
 & & & .023 & & & & \\
 & & & & 1.0 & & & \\
 & & & & & .142 & & \\
 & & & & & .118 & & \\
 & & & & & .730 & & \\
 & & & & & .010 & & \\
 & & & & & & .043 & \\
 & & & & & & .130 & \\
 & & & & & & .827 & \\
 & & & & & & & 1.0
\end{bmatrix}$$

In practice, the vector V will be the starting point of the analysis, since these are the data that are most readily available -- say from the electricity company, the petroleum corporation, and so on. The sectoral consumption vector e would in practice be calculated from V by the equation

$$\underset{(8\text{x}1)}{e} = \underset{(8\text{x}23)}{G} \; \underset{(23\text{x}1)}{v}$$

where G is an aggregation matrix corresponding to the disaggregation matrix F (and defined by transposing F and setting all nonzero elements of F equal to unity).

The Intermediate Energy Form Vector, I, follows from Figure 3.7

$$
\begin{bmatrix}
\text{Central Electricity} & 1350 \\
\text{Wood} & 3370 \\
\text{Charcoal} & 2735 \\
\text{Bagasse} & 4600 \\
\text{Coal} & 0 \\
\text{Oil Products} & 9960 \\
\text{Solar} & 0
\end{bmatrix}
$$

This computes from the fuel use vector by two transformations

$$
\underset{(7\mathrm{x}1)}{I} = \underset{(7\mathrm{x}7)}{\Omega} \; \underset{(7\mathrm{x}1)}{I'}
$$

$$
\underset{(7\mathrm{x}1)}{I'} = \underset{(7\mathrm{x}23)}{A} \; \underset{(23\mathrm{x}1)}{V}
$$

where Ω is a diagonal matrix that captures losses in electric transmission and distribution, namely

$$
\begin{bmatrix}
1.370 & & & & & & \\
 & 1 & & & & & \\
 & & 1 & & & 0 & \\
 & 0 & & 1 & & & \\
 & & & & 1 & & \\
 & & & & & 1 & \\
 & & & & & & 1
\end{bmatrix}
$$

and A is the Aggregation matrix that assigns each element in the fuel use vector V to an intermediate energy form:

$$
\begin{bmatrix}
1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & \frac{1}{.25} & 1 & 0 & 1 & 0 & \frac{1}{.2} & 1 & 0 & 1 & 0 & \frac{1}{.2} & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
$$

This matrix also takes into account the efficiency of selfgeneration, as indicated by the entries in the sixth row of A.

The next vector, I*, takes into account the electric sector generation mix, and is defined by:

$$\begin{bmatrix} \text{Hydroelectric 1} & 132 \\ \text{Hydroelectric 2} & 32 \\ \text{Oil 1} & 187 \\ \text{Oil 2} & 962 \\ \text{Oil 3} & 38.7 \\ \text{Wood} & 3370 \\ \text{Charcoal} & 2737 \\ \text{Bagasse} & 4600 \\ \text{Coal} & 0 \\ \text{Oil Products} & 9960 \\ \text{Solar} & 0 \end{bmatrix}$$

which computes from the I vector as

$$\underset{(11\text{x}1)}{I^*} = \underset{(11\text{x}7)}{E}\ \underset{(7\text{x}1)}{I}$$

where E is the matrix

$$\begin{bmatrix} 0.097 & & & & & & \\ 0.023 & & & & & & \\ 0.138 & & & & 0 & & \\ 0.713 & & & & & & \\ 0.029 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ 0 & & & & 1 & & \\ & & & & & 1 & \\ & & & & & & 1 \end{bmatrix}$$

which defines the generation mix in the electric sector. The resource consumption vector, r, is then defined as

$$\begin{bmatrix} \text{Hydro 1} & 440 \\ \text{Hydro 2} & 64 \\ \text{Oil 1} & 984 \\ \text{Oil 2} & 3435 \\ \text{Oil 3} & 155 \\ \text{Wood} & 3370 \\ \text{Wood to Charcoal} & 4570 \\ \text{Bagasse} & 5000 \\ \text{Coal} & 0 \\ \text{Oil Products} & 9960 \\ \text{Solar} & 0 \end{bmatrix}$$

which computes from I* as

$$\underset{(11x1)}{r} = \underset{(7x11)}{\lambda} \quad \underset{(11x1)}{I^*}$$

where the matrix λ captures the efficiency of electric generation and of charcoal production, i.e.

$$\begin{bmatrix}
\frac{1}{.3} & & & & & & & & & & \\
& \frac{1}{.5} & & & & & & & & & \\
& & \frac{1}{.19} & & & & & 0 & & & \\
& & & \frac{1}{.20} & & & & & & & \\
& & & & \frac{1}{.25} & & & & & & \\
& & & & & 1 & & & & & \\
& & 0 & & & & \frac{1}{.6} & & & & \\
& & & & & & & 1 & & & \\
& & & & & & & & 1 & & \\
& & & & & & & & & 1 & \\
& & & & & & & & & & 1
\end{bmatrix}$$

The next transformation aggregates the electric generation categories to their respective fuels, resulting in the energy supply vector

$$\begin{bmatrix}
\text{Hydro} & 504 \\
\text{Wood} & 7940 \\
\text{Bagasse} & 5000 \\
\text{Coal} & 0 \\
\text{Oil Products} & 14970 \\
\text{Solar} & 0
\end{bmatrix}$$

which computes from r as

$$\underset{(6x1)}{S} = \underset{(6x11)}{B} \cdot \underset{(11x1)}{r}$$

where B is the matrix

$$\begin{bmatrix}
1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}$$

Finally, the Energy resource vector, a, is given by

$$\begin{bmatrix} \text{Hydropower} & 552 \\ \text{Wood} & 7940 \\ \text{Bagasse} & 5000 \\ \text{Coal} & 0 \\ \text{Imported Petroleum Products} & 5330 \\ \text{Imported Crude} & 9440 \\ \text{Solar} & 0 \end{bmatrix}$$

where a is defined by the transformations

$$\underset{(7\text{x}1)}{a} = \underset{(7\text{x}7)}{\pi} \; \underset{(7\text{x}6)}{Z} \; \underset{(6\text{x}1)}{S}$$

where π is the matrix that accounts for refinery losses, i.e

$$\begin{bmatrix} 1 & & & & & & \\ & 1 & & & & & \\ & & 1 & & & & \\ & & & 1 & & & \\ & & & & \frac{1}{.97} & & \\ & & & & & 1 & \\ & & & & & & 1 \end{bmatrix}$$

and where Z allocates final energy resources to the final resource consumption vector as

$$\begin{bmatrix} 1 & & & & & \\ & 1 & & & & \\ & & 1 & & & \\ & & & 1 & & \\ & & & & .356 & \\ & & & & .643 & \\ & & & & & 1 \end{bmatrix}$$

The entire series of matrix calculations can be collapsed into the single equation

$$a = \pi \; Z \; B \; \lambda \; E \; \Omega \; A \; F \; e \qquad (3.3)$$

Energy Consumption
Fuel Mix
Aggregation (+ Efficiency of Self Generation)
Electric Sector Transmission + Distribution Losses
Electric Sector Generation Mix
Electric Sector Efficiency, Charcoal Production Efficiency
Aggregation
Aggregation
Refinery Losses

3.4 MATRIX NOTATION FOR FUEL MIX TABLES; RESIDENTIAL DEMAND IN THE DOMINICAN REPUBLIC

To the extent that information is available from consumption surveys, the matrix approach can also be used to replace the fuel mix tables introduced in Section 3.2, thus allowing energy end use information to be linked directly to the supply-side matrix model of the previous section.

Consider, as an example, the residential sector of the Dominican Republic, for which urban survey data is summarized on Tables 3.3 to 3.6. This information can be expressed in the form of a network, as indicated on Figure 3.6. As before, e denotes the efficiency of utilization of the end use device, f denotes the fraction of each basic ened use met by a particular fuel, s denotes the saturation level (how many households have a certain device), and $\boxed{Z}$ denotes the corresponding basic energy demand.

We next express this network in matrix terms, beginning with total households, h_t, and disaggregating by degree of urbanization. Hence

$$h_u = \begin{bmatrix} \text{Urban Households} & 479{,}000 \\ \text{Rural Households} & 494{,}000 \end{bmatrix}$$

is given by the matrix equation

$$\underset{(2x1)}{h_u} = \underset{(2x1)}{U} \; \underset{(1x1)}{h_t}$$

where

$$U = \begin{bmatrix} .508 \\ .492 \end{bmatrix}$$

We next introduce a 10x1 vector d of basic energy demands

$$\begin{bmatrix} \text{Air Conditioning, Urban} \\ \text{Lighting, Urban} \\ \text{Cooking, Poor, Large Urban Areas} \\ \text{Cooking, Poor, Small Urban Areas} \\ \text{Cooking, Non-Poor, Large Urban Areas} \\ \text{Cooking, Non-Poor, Small Urban Areas} \\ \text{Water Heat, Urban} \\ \text{Lighting, Rural} \\ \text{Cooking, Rural} \\ \text{Misc. Appl., Rural} \end{bmatrix}$$

given by the matrix equation

$$\underset{(10x1)}{d} = \underset{(10x10)}{\Delta} \; \underset{(10x2)}{D} \; \underset{(2x1)}{h_u}$$

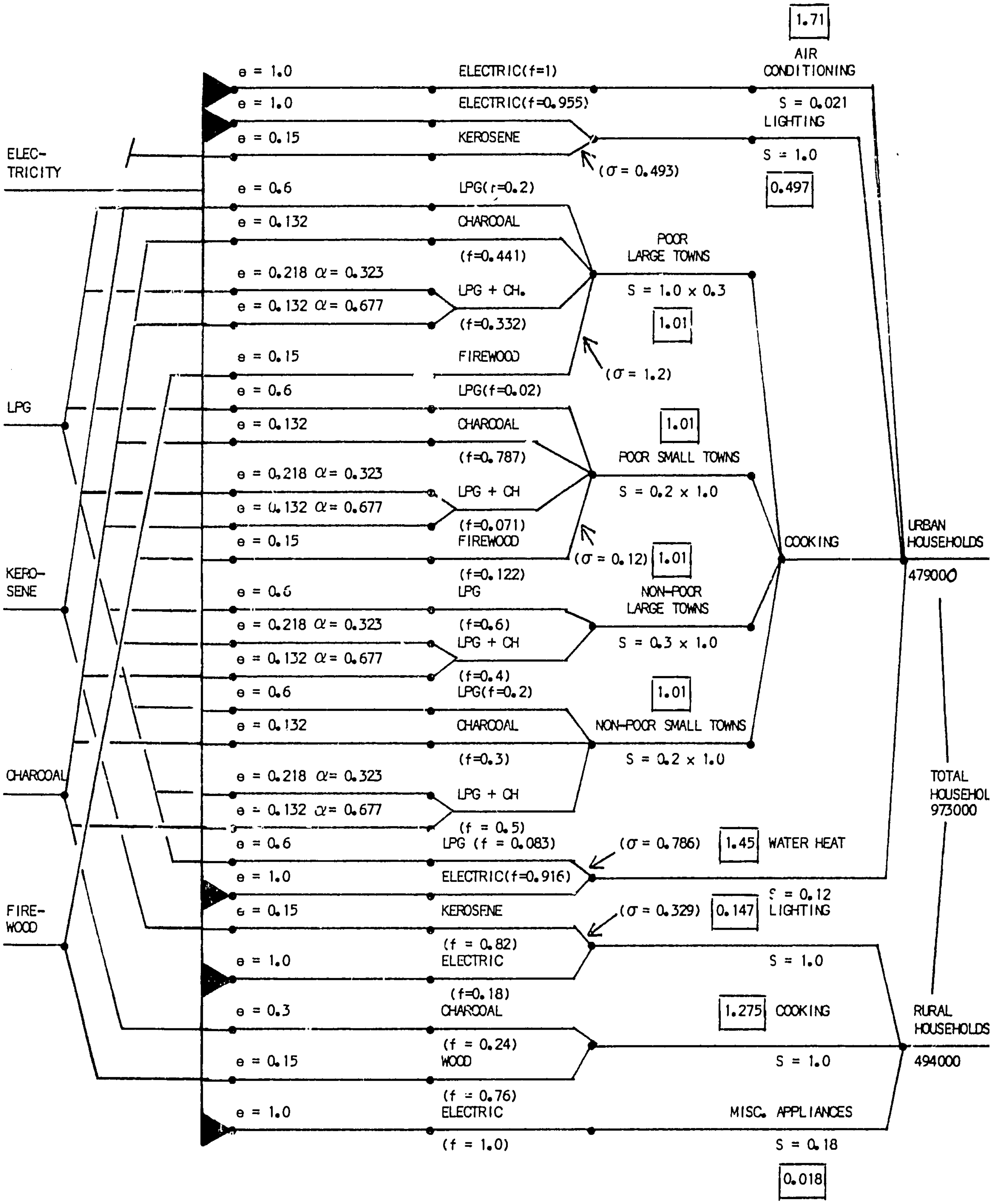


Figure 3.6. Network Representation of Residential Sector Demand for the Dominican Republic

Table 3.3
Urban Households, Summary of Energy Use 1978
(kboe)

	LPG	Kero-sene	Elec-tricity	Char-coal	Fire-wood	Total
Cooking, Total	390.9			2233.2	14.	
Urban-Non Poor	279.7	0	0+	784.1	-	
Urban-Poor	111.2			1449.1	14.	
Water Heating	5.6	-	79.4	NA	NA	
Air Conditioning	-	-	45.9	-	-	
Lighting Appliances Total	0+	36.6	236.1	-	-	
Total	396.5	36.6	361.4	2233.3	14.	3041.7

Table 3.4
Household Energy Use 1978
(in kboe)

	Urban Households	Rural Households	Total Households
LPG	396.5	0	396.5
Kerosene	36.6	78.6	115.2
Electricity	361.4	14.7	376.1
Charcoal	2233.	507.	2740.
Firewood	14.	3186.	3200.
Total Energy	3041.5	3786.3	6827.8

Table 3.5
1978 Energy Use in Water Heating
(kboe)

	Saturation	Total Energy Use
Electric (205 kWh/mo)	0.11	79.4
LPG (70 gals/yr)	0.01	5.6
Solar	0+	N/A
Charcoal (for clothes washing)	Large	N/A

Table 3.6
Fuel Use in Rural Households
(kboe)

Sector	Wood	Charcoal	Kerosene	Electricity
Cooking	3186	507	--	--
Lighting	--	--	78.6	13.1
Misc. Appliances	--	--	--	1.6

Table 3.7
Energy Consumption for Cooking, Urban Households, 1978
(From the Survey of Households taken as
Part of the Dominican Republic Assessment)

	No. of House-holds	Satura-tion	LPB (kboe)	Char-coal (kboe)	Fire-wood (kboe)
URBAN POOR, LARGE URBAN AREAS	143,783				
LPG only		0.200	48.3	-	-
Charcoal only		0.441	-	507.1	
LPG & Charcoal		0.332	71.6	246.7	
Firewood		0.027	-	-	3.4
URBAN POOR, SMALL URBAN AREAS	95,855				
LPG only		0.020	3.2	-	
Charcoal only		0.787	-	657.9	
LPG & Charcoal		0.071	10.2	35.4	
Firewood		0.122	-	-	10.3
URBAN, NON-POOR, LARGE URBAN AREAS	143,783				
LPC only		0.60[1]	144.9	-	
LPG & Charcoal		0.40[1]	86.3	299.1	
URBAN, NON-POOR, SMALL URBAN AREAS	95,855				
LPG only		0.20[1]	32.2	-	
Charcoal only		0.30[1]	-	235.8	
LPG & Charcoal		0.50[1]	71.9	249.2	
TOTAL URBAN HOUSEHOLDS	479,276		468.6	2,233.2	13.7

[1]Estimates

where Δ is a diagonal matrix of basic energy demands (whose elements have dimensions of boe, barrels of oil equivalent) viz

$$\Delta = \begin{bmatrix} 1.71 &&&&&&&&& \\ & 0.497 &&&&&&&& \\ && 1.01 &&&&&&& \\ &&& 1.01 &&&&&& \\ &&&& 1.01 &&&&& \\ &&&&& 1.01 &&&& \\ &&&&&& 1.453 &&& \\ &&&&&&& 0.147 && \\ &&&&&&&& 1.275 & \\ &&&&&&&&& 0.018 \end{bmatrix}$$

and D is a dissagregation matrix that assigns basic energy demand categories to the household categories, i.e.,

$$D = \begin{bmatrix} 0.021 & 0 \\ 1 & 0 \\ 0.3 & 0 \\ 0.2 & 0 \\ 0.3 & 0 \\ 0.2 & 0 \\ 0.12 & 0 \\ 0 & 1 \\ 0 & 1 \\ 0 & 0.18 \end{bmatrix}$$

and whose elements represent the saturation level. Thus the entry 0.021 in row 1 indicates that 0.021 of urban households have air conditioning. In the case of cooking in urban households, we multiply the saturation level (S = 1.0) by the fraction of households that fall into each subcategory (poor in large urban areas, etc.).

There are many problems associated with the calculation of basic energy demand, which is defined for each category of household by the relationship

$$\underset{b}{\begin{bmatrix}\text{Basic}\\ \text{Energy}\\ \text{Demand}\end{bmatrix}} = \frac{\underset{c}{\begin{bmatrix}\text{Fuel}\\ \text{Consumption}\\ \text{from Survey}\end{bmatrix}} \quad \underset{E}{[\text{Efficiency}]}}{\begin{bmatrix}\text{No. of}\\ \text{Households}\end{bmatrix} \quad [\text{Saturation}]} \tag{3.4}$$

For example, for rural lighting we obtain from the survey data (Table 3.6) for kerosene-lighted households

$$0.0291 = \frac{78600 \cdot 0.15}{494000 \cdot 0.82} \text{ (boe)}$$

and for electricity lighted households

$$0.147 = \frac{13100 \cdot 1.0}{494000 \cdot 0.18} \text{ (boe)}$$

which suggests that if the standard values, used here, are correct (i.e., on a btu basis, electricity is four times as efficient as kerosene), then households with access to electricity for lighting have a basic demand for electricity° that is about <u>five times higher</u> than that of households° illuminated with kerosene. One explanation, perhaps, is that the survey data is wrong; but a properly designed survey ought not to be in error by a factor of three. If both types of household had in fact the <u>same</u> basic energy demand for lighting, then one is led to the following equation for calculation of the efficiency of kerosene lighting:

$$e = \frac{0.147 \times 494000 \times 0.82}{78600} = 0.75$$

yielding a value of e that is somewhat unlikely.

In order to maintain accounting accuracy, the expedient adopted here is to use as basic energy demand the largest of the set of values calculated for each sector: thus for rural lighting we use the figure based on electricity (i.e., 0.147 boe/HH/yr). At the same time, we make an adjustment for the other fuels that we call the <u>supressed demand adjustment</u> (denoted on Figure 3.6 by σ); thus as more and more households become electrified, the lighting demand in such homes will be consistent with other electrified homes, rather than with their previous level of lighting.

The only instance where this procedure still leaves unanswered questions is for firewood use in urban cooking, for which the suprresed demand adjustment computes to $\sigma = 0.12$. Even though it is true that firewood users would fall into the lowest income group even among the poor, it seems unlikely that such families would have but 12% of the cooking demand of the other fuel use groups. Since firewood users constituted only 3% of the total sample (about 20 of 600 surveyed households), sample error may be significant. Moreover, of all the fuel categories, precise measurement of wood quantities are the most imprecise, especially in view of the fact that as a non-commercial fuel, cross-checks of sample survey data with sales information is of course not possible.

In the case of urban cooking, if one makes the usual assumptions on efficiency (0.15 for charcoal braziers, 0.6 for LPG stoves) then the basic energy demands compute as follows

LPG only	1.02
Charcoal only	1.23
LPG + Charcoal	1.37

The value for LPG only users is consistent with that of other studies. However, we see no reason why charcoal-only users should have higher basic energy demand; indeed, the evidence, from the survey, on family income, suggests the contrary. Consequently the reasonable thing to do here is to use 1.02 for all categories, and to adjust the efficiency of the charcoal brazier; applying eq. (3.4), to the urban poor in large towns, for example, one obtains

$$e = \frac{1.02 \times 497{,}000 \times 0.3 \times 0.441}{507100}$$

$$= 0.132$$

The data for urban cooking is further complicated by the fact that some families use both LPG and charcoal; it is likely that such families have less efficient, table top LPG burners (as opposed to regular stoves with ovens in the all-LPG households). To calculate the efficiency of such stoves, we again apply Eq. (3.4) to the data for the urban poor in large towns, i.e.,

$$1.01 = \underbrace{\frac{246.7 \times 0.132}{143.7 \times 0.332}}_{[\text{charcoal}]} + \underbrace{\frac{71.6 \times e_{LPG}}{143.7 \times 0.332}}_{[\text{LPG}]}$$

which solves to $e_{LPG} = 0.218$. This equation can also be used to establish the fraction of the total basic cooking energy demand is met by each fuel (a number required below); for example, the fraction met by LPG is

$$\alpha_{LPG} = \frac{\frac{71.6 \times 0.218}{143.7 \times 0.332}}{1.01} = 0.323$$

This discussion serves to highlight one of the difficulties associated with the use of basic energy demands in fuel mix tables (or the matrix and network equivalents): contrary to appearances, such basic energy demands are not constants, but are functions of consumer income level, and the degree of access to certain technologies.

The next vector captures the end use devices, y

$$\begin{bmatrix} \text{Electric Air Conditioners} \\ \text{Electric Lighting} \\ \text{Kerosene Lighting} \\ \text{LPG, Cooking, Poor, Large Towns} \\ \text{Charcoal, Cooking, Poor, Large Towns} \\ \text{LPG + Ch, Cooking, Poor, Large Towns} \\ \text{Firewood, Cooking, Poor, Large Towns} \\ \text{LPG, Cooking, Poor, Small Towns} \\ \text{Charcoal, Cooking, Poor, Small Towns} \\ \text{LPG + Ch, Cooking, Poor, Small Towns} \\ \text{Firewood, Cooking, Poor, SMall Towns} \\ \text{LPG, Cooking, Non-Poor, Large Towns} \\ \text{LPG + Ch, Cooking, Non-Poor, Large Towns} \\ \text{LPG, Cooking, Non-Poor, Small Towns} \\ \text{Charcoal, Cooking, Non-Poor, Small Towns} \\ \text{LPG + Ch, Cooking, Non-Poor, Small Towns} \\ \text{LPG, Water Heat, Urban} \\ \text{Electric, Water Heat, Urban} \\ \text{Kerosene, Rural Lighting} \\ \text{Electric, Rural Lighting} \\ \text{Charcoal, Rural Cooking} \\ \text{Wood, Rural Cooking} \\ \text{Electric, Misc. App.} \end{bmatrix}$$

which is given by the matrix equation

$$\underset{(23x1)}{\dot{y}} = \underset{(23x23)}{S}\ \underset{(23x10)}{F}\ \underset{(10x1)}{d}$$

where S is a diagonal matrix of demand adjustment values

$$S = \begin{bmatrix}
1 \\
& 1 \\
& & 0.493 \\
& & & 1 \\
& & & & 1 \\
& & & & & 1 \\
& & & & & & 0.12 \\
& & & & & & & 1 \\
& & & & & & & & 1 \\
& & & & & & & & & 1 \\
& & & & & & & & & & 0.12 \\
& & & & & & & & & & & 1 \\
& & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & 0.786 \\
& & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & 0.329 \\
& & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & & & 1
\end{bmatrix}$$

and where F is a matrix of end use device allocation fractions, whose non-zero entries correspond to the "f" values of Figure 3.6;

$$\begin{bmatrix}
1.0 \\
& 0.955 \\
& 0.045 \\
& & 0.2 \\
& & 0.432 \\
& & 0.332 \\
& & 0.027 \\
& & & 0.02 \\
& & & 0.837 \\
& & & 0.071 \\
& & & 0.122 \\
& & & & 0.6 \\
& & & & 0.4 \\
& & & & & 0.2 \\
& & & & & 0.3 \\
& & & & & 0.5 \\
& & & & & & 0.083 \\
& & & & & & 0.916 \\
& & & & & & & 0.82 \\
& & & & & & & 0.18 \\
& & & & & & & & 0.24 \\
& & & & & & & & 0.76 \\
& & & & & & & & & 1.0
\end{bmatrix}$$

Note that the column sums equal unity: the f values therefore describe the fraction of each end use that is met by the corresponding end use device. The fact that some urban households use two fuels (and hence two devices) for cooking introduces certain complications. From the urban energy survey, however, we know what fraction is met by each device. Therefore the end use device fuel consumption vector h is given by the 27x1 vector.

$$\begin{bmatrix} \text{Electric Air Conditioners} \\ \text{Electric Lighting} \\ \text{Kerosene Lighting} \\ \text{LPG, Cooking, Poor, Large Towns} \\ \text{Charcoal, Cooking, Poor, Large Towns} \\ \text{LPG in homes of LPG + CH, Cooking, Poor, Large Towns} \\ \text{CH in homes of LPG + CH, Cooking, Poor, Large Towns} \\ \text{Firewood, Cooking, Poor, Large Towns} \\ \text{LPG, Cooking, Poor, Small Towns} \\ \text{Charcoal, Cooking, Poor, Small Towns} \\ \text{LPG in homes of LPG + CH, Cooking, Poor, Small Towns} \\ \text{CH in homes of LPG + CH, Cooking, Poort, Small Towns} \\ \text{Firewood, Cooking, Poor, SMall Towns} \\ \text{LPG ,Cooking, Non-Poor, Large Towns} \\ \text{LPG in homes of LPG + CH, Cooking, Non-Poor, Large Towns} \\ \text{CH in homes of LPG + CH, Cooking, Non-Poor, Large Towns} \\ \text{LPG, Cooking, Non-Poor, Small Towns} \\ \text{Charcoal, Cooking, Non-Poor, Small Towns} \\ \text{LPG in homes of LPG + CH, Cooking, Non-Poor, Small Towns} \\ \text{CH in homes of LPG + Cooking, Non-Poor, Small Towns} \\ \text{LPG, Water Heat, Urban} \\ \text{Electric, Water Heat, Urban} \\ \text{Kerosene, Rural Lighting} \\ \text{Electric, Rural Lighting} \\ \text{Charcoal, Rural Cooking} \\ \text{Wood, Rural Cooking} \\ \text{Electric, Misc. App.} \end{bmatrix}$$

which is obtained by the matrix equation

$$\underset{(27x1)}{h} = \underset{(27x27)}{\tau} \; \underset{(27x23)}{A} \; \underset{(23x1)}{y}$$

where π^{-1} is a diagonal matrix of end use device efficiencies

$$
\pi^{-1} = \begin{bmatrix}
1.0 \\
& 1.0 \\
& & 0.15 \\
& & & 0.6 \\
& & & & 0.132 \\
& & & & & 0.218 \\
& & & & & & 0.132 \\
& & & & & & & 0.15 \\
& & & & & & & & 0.6 \\
& & & & & & & & & 0.132 & & & & & & & & & & 0 \\
& & & & & & & & & & 0.218 \\
& & & & & & & & & & & 0.132 \\
& & & & & & & & & & & & 0.15 \\
& & & & & & & & & & & & & 0.6 \\
& & & & & & & & & & & & & & 0.218 \\
& & & & & & & & & & & & & & & 0.132 \\
& & & & & & & & & & & & & & & & 0.6 \\
& & & & & 0 & & & & & & & & & & & & 0132 \\
& & & & & & & & & & & & & & & & & & 0.6 \\
& & & & & & & & & & & & & & & & & & & 1.0 \\
& & & & & & & & & & & & & & & & & & & & 0.15 \\
& & & & & & & & & & & & & & & & & & & & & 1.0 \\
& & & & & & & & & & & & & & & & & & & & & & 0.3 \\
& & & & & & & & & & & & & & & & & & & & & & & 0.15 \\
& & & & & & & & & & & & & & & & & & & & & & & & 1.0
\end{bmatrix}
$$

and A is the matrix used to adjust for dual appliance use in urban cooking, i.e.,

$$
A = \begin{bmatrix}
1 \\
& 1 \\
& & 1 \\
& & & 1 \\
& & & & 1 \\
& & & & & 0.323 \\
& & & & & 0.677 \\
& & & & & & 1 \\
& & & & & & & 1 \\
& & & & & & & & 1 \\
& & & & & & & & & 0.323 \\
& & & & & & & & & 0.677 \\
& & & & & & & & & & 1 \\
& & & & & & & & & & & 1 \\
& & & & & & & & & & & & 0.23 \\
& & & & & & & & & & & & 0.617 \\
& & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & 0.323 \\
& & & & & & & & & & & & & & & 0.677 \\
& & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & & 1 \\
& & & & & & & & & & & & & & & & & & & & & & 1
\end{bmatrix}
$$

Finally, total fuel consumption is given by the (5xl) vector

$$
\begin{bmatrix}
\text{Electricity} \\
\text{LPG} \\
\text{Kerosene} \\
\text{Charcoal} \\
\text{Firewood}
\end{bmatrix}
$$

defined by the matrix equation

$$
\underset{(5\text{x}1)}{f} = \underset{(5\text{x}27)}{W} \; \underset{(27\text{x}1)}{h}
$$

and where W is the aggregation matrix:

$$
\begin{bmatrix}
1&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&1&0&0&1 \\
0&0&0&1&0&1&0&0&1&0&1&0&0&1&1&0&1&0&1&0&1&0&0&0&0&0&0 \\
0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&0&1&0&0&0&0 \\
0&0&0&0&1&0&1&0&0&1&0&1&0&0&0&1&0&1&0&1&0&0&0&0&1&0&0 \\
0&0&0&0&0&0&0&1&0&0&0&0&1&0&0&0&0&0&0&0&0&0&0&0&0&1&0
\end{bmatrix}
$$

The entire series of matrix equations can again be collapsed into the single expression:

$$f = W \pi A S F \Delta D U h_t$$

Total Households
Urbanization
Basic Energy Demand Assignments
Basic Energy Demands
Device Allocations
Demand Adjustment Equation
Dual Appliance Use Adjustment
End Use Device Efficiencies
Aggregation Matrix

The use of this system for forecasting future energy demands is illustrated in exercise E7.

E6. TRAJECTORY ANALYSIS

(a) The 1972 RES for India (Figure 3.2) depicts all refined petroleum products as a single commodity. Redraw the RES oil trajectory making the distinction between heavy oil (used in the electric sector), middle distillate used for industrial process heat, diesel oil for trucks and irrigation pumping, kerosene for domestic use, jet fuel, gasoline, etc.

(b) Suppose 10% of the irrigation pumps were replaced by solar systems. What is the impact on imports of refined products?

(c) If the refined imports are 100 units of middle distillate and 37.1 of kerosene, what is the fractional yield of the refinery (e.g. bbls of jet/bbl of crude). Does this look like a realistic refinery product mix if there is only atmospheric distillation? Assume that the energy input to refinery (0.94 efficiency) is met from middle distillate oil.

E7. MATRIX REPRESENTATION OF THE RESIDENTIAL DEMAND IN THE DOMINICAN REPUBLIC

(a) Using the network of Figure 3.6 as a reference point, derive a series of matrix equations for the rural sector.

(b) Make a projection of rural fuel consumption for the year 2000, assuming a growth to 798,000 households. Assume that by 2000, (i) kerosene assumes only a 20% share for lighting; (ii) per household appliance use of electricity increases 4 fold; (iii) that the average efficiency of wood stoves for cooking increases to 20%.

4. SECTORAL PROJECTIONS

Fundamentals:* A supply curve depcits the relationship between the supply of a commodity, such as energy, and its price (Figure 4.1), and is generally upward sloping to reflect the circumstance that new sources of supply will be more costly than those already in existence: increasing the supply of coal, for example, mean opening new mines, which will in general be deeper, and exploit narrower seams than those already in production. Deeper and thinner seams means higher prices will be necessary to justify exploitation of that resource. Supply curves can shift upward, and downward (Figure 4.2): an upward shift might be the result of mine safety regulations, that decreases productivity per man-hour. A downward shift would result from some new technology that increases productivity.

Demand curves represent the relationship between the demand (consumption) of a particular commodity, and its price; it is downward sloping to reflect the circumstance that, all other things equal, lower prices will increase demand (and higher prices will tend to lower the demand). Demand curves are also subject to displacement; (Figure 4.3); as an economy grows, demand for a commodity at a given price will increase.

Why should demand curves be downward sloping? The traditional explanation espoused by economists rests on the concept of diminishing marginal utility. Consider the situation of a rural household newly connected to a rural electrification project. The satisfaction derived from the first unit of electricity purchased will be very high; if only a limited amount is available, it will be applied to the use that the consumer values the most--perhaps a TV. The second + subsequent kWh's will be applied to uses less and less valuable; the price that the consumer is willing to pay for the first unit is higher than the price he is willing to pay for the n-th unit. An industrial enterprise is no different: for the first units of electricty, used for purposes for which there is no substitute, the industrialist is willing to pay more than for the last n units, for which substitutes may be readily applied (such as for process heat).[1]

*This section may be omitted by economists and others familiar with the basic concepts.

[1]Although as noted in Samuelson's well known text "Economics: An Introductory Analysis," there are some exceptions, such as the demand for diamonds, which are purchased not for their intrinsic quantities, but for their snob appeal; if their price were suddenly cut, demand may well fall.

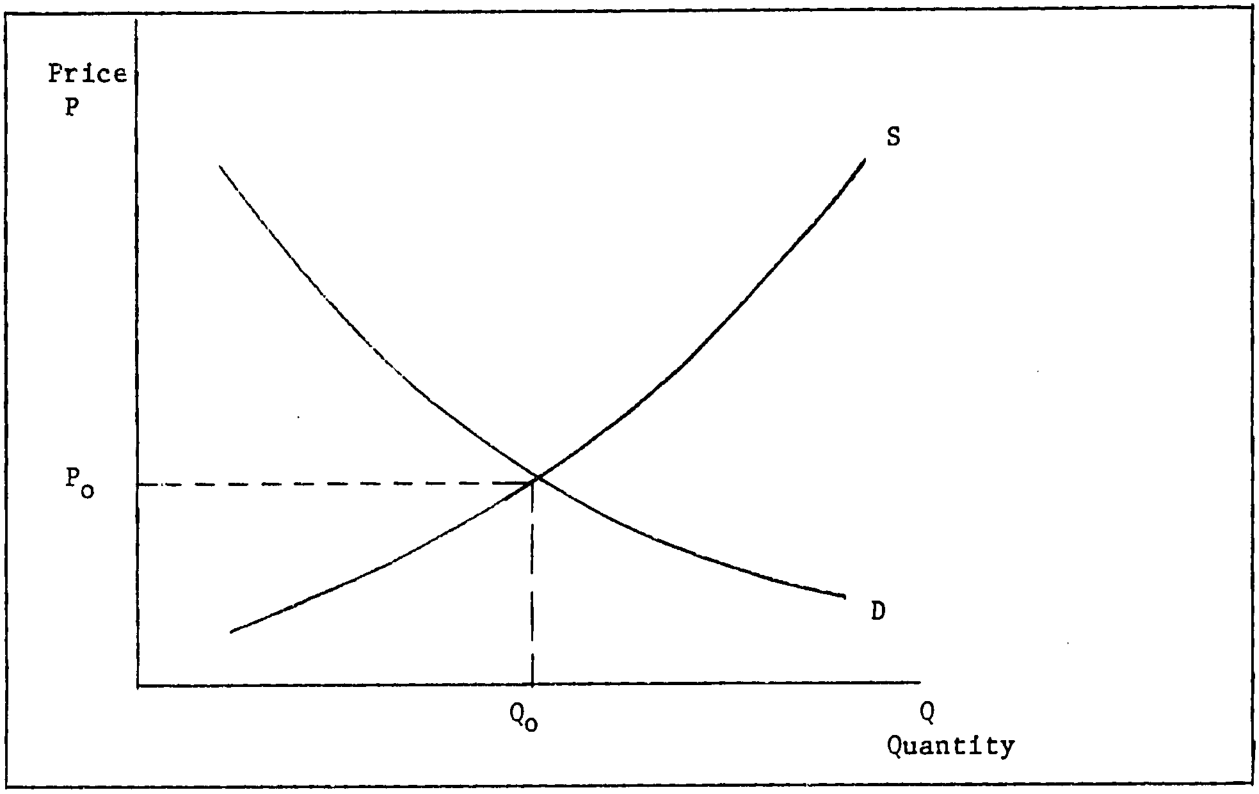
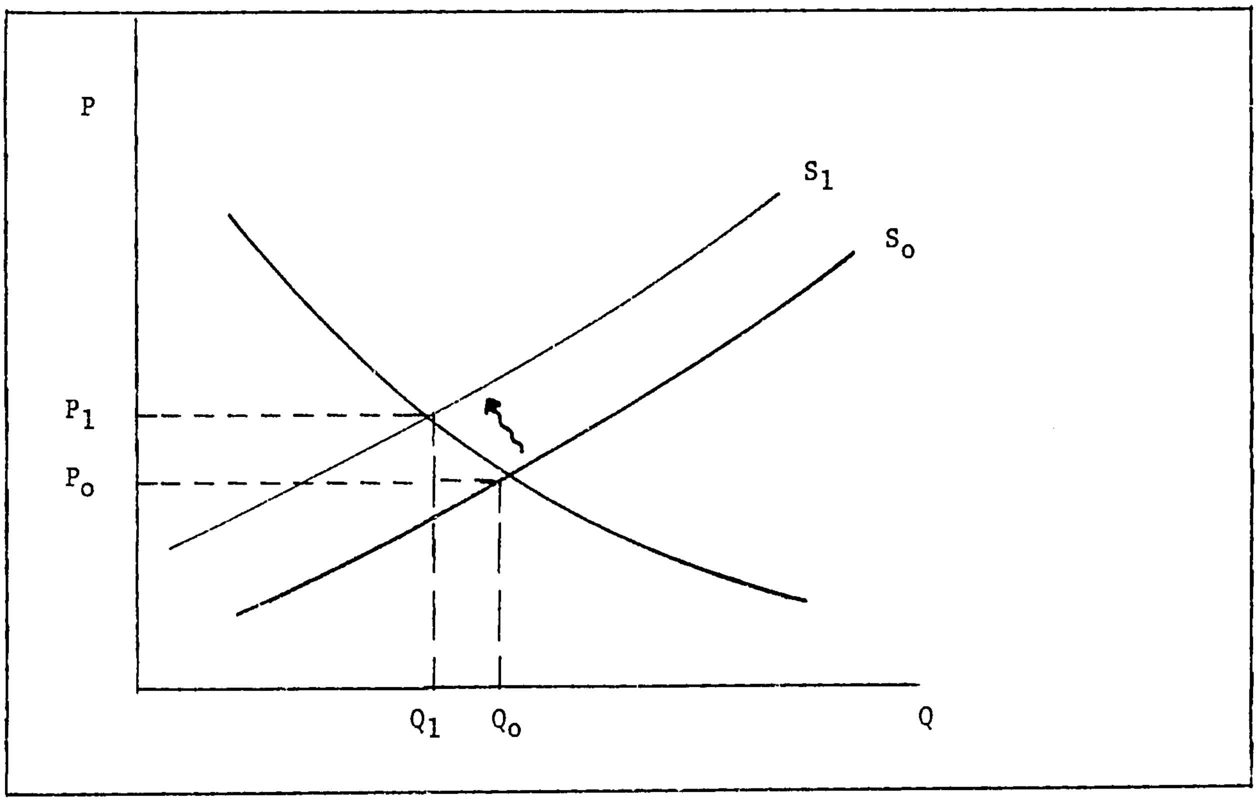


Figure 4.1. Supply and Demand



Figur 4.2. Supply Curve Shifts

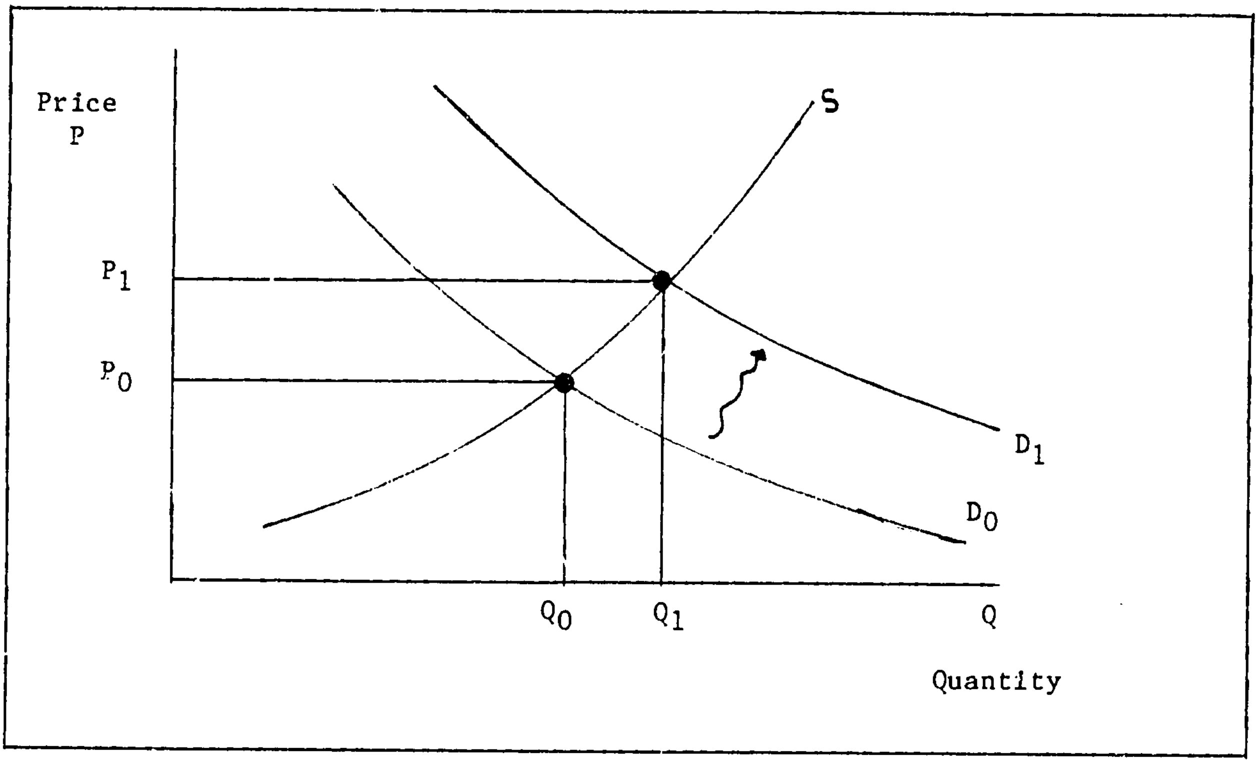


Figure 4.3. Demand Curve Shifts

The point of intersection of the supply curve and the demand curve is the point of equilibrium: the corresponding price is known as the market clearing price, that is, the price for which supply will exactly equal demand. As can be seen from Figures 4.2 and 4.3, shifts in supply and demand curves will result in shifts in market clearing price.

An important concept is that of "surplus." Consider Figure 4.4, in which the market clearing price is P_0. However, there is a consumer at A, who would be willing to pay P_A, but actually experiences only the cost P_0. And the consumer at B, willing to pay P_B, similarly only pays P_0. This gap, integrated over all consumers to the left of consumer C, who is the marginal consumer, is the crosshatched area ZYC, and is referred to as the consumer surplus. Similarly, there is a producer at W, willing to sell for P_W, but who actually receives P_0; the total area YCX is referred to as the Producer's surplus.

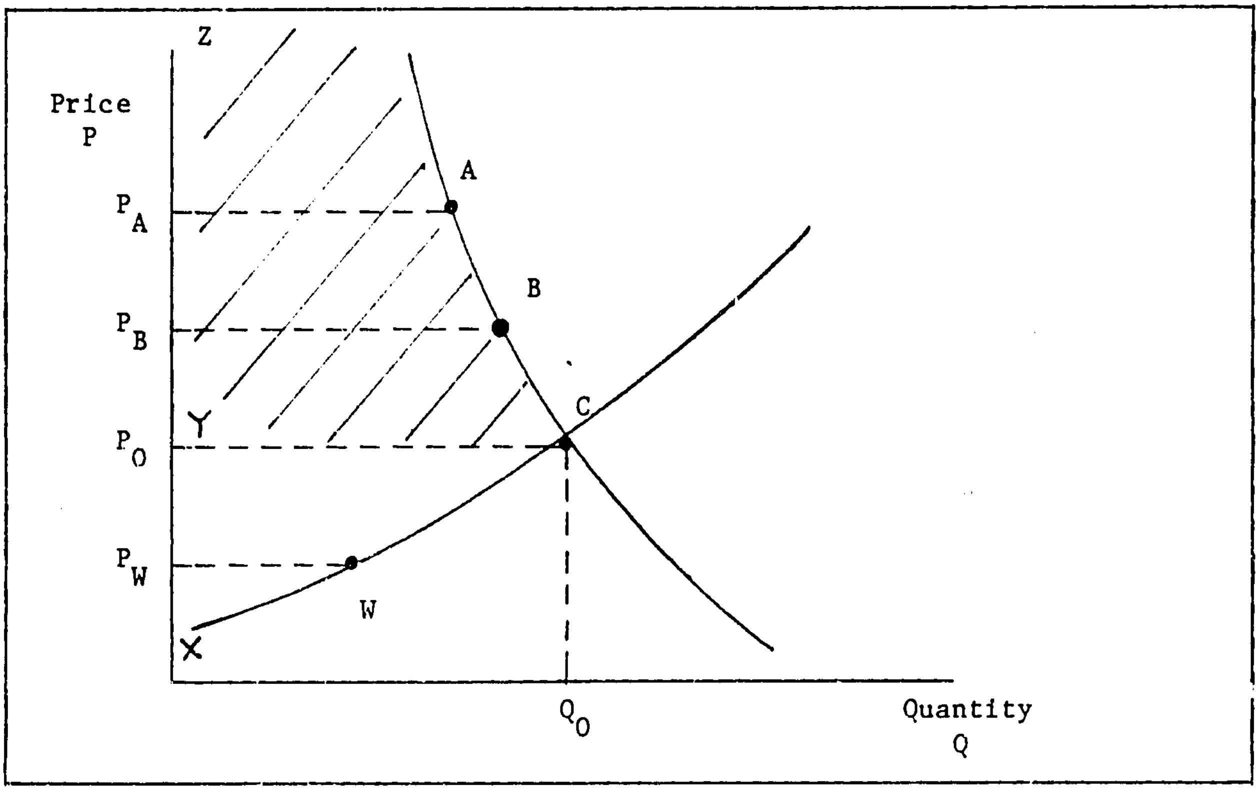


Figure 4.4. Consumer + Producer's Surplus

Elasticity. Elasticity is a numerical measure of the response of supply and demand to price. The so-called elasticity of demand, for example, measures the relative change in quantity demanded per unit of change in price. Mathematically, we define elasticity as:

$$\epsilon = \frac{\frac{dQ}{Q}}{\frac{dP}{P}} = \frac{dQ}{dP} \cdot \frac{P}{Q} \tag{4.1}$$

The demand for a commodity is characterized as "elastic," if $\epsilon > 1$, which means that a given relative change in price induces a much greater change in quantity; if a one percent change in price results in a two percent change in demand, demand would be described as "elastic." Conversely, if a one percent change in price produces only a one-half percent change in quantity, demand would be described an inelastic, which we would in general use to describe situations for which $\epsilon < 1$ (see Figure 4.5).

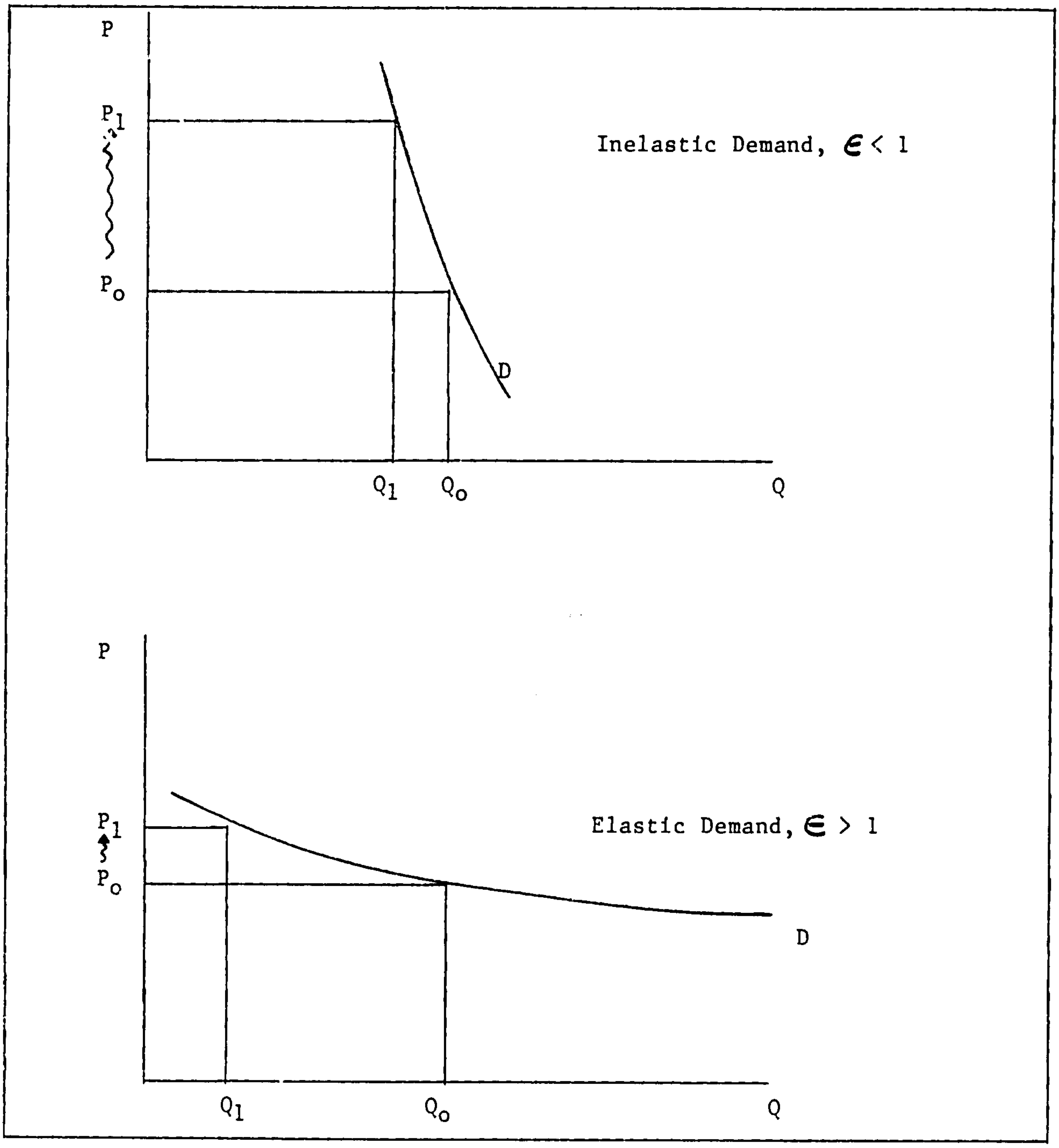


Figure 4.5. Basic Elasticity Concepts

It follows immediately from Eq. (4.1) that the numerical value of the elasticity depends not just on the slope of the demand curve, but also on the values of P and Q about which the unit changes are made. As illustrated on Figure 4.6, if the demand curve is linear, elasticity may vary from less than unity to greater than unity as we move along the demand curve.

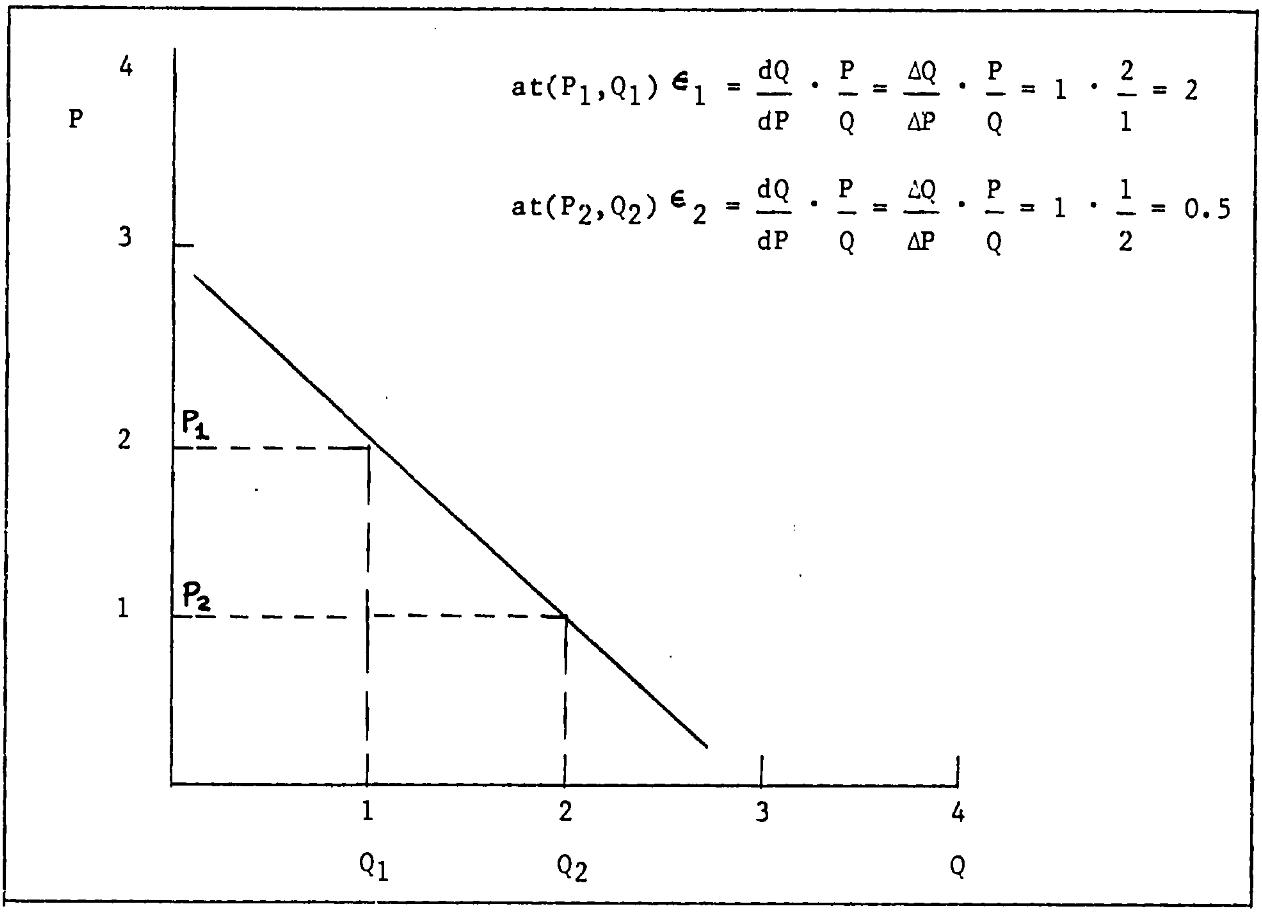


Figure 4.6 Elasticity of a Linear Function

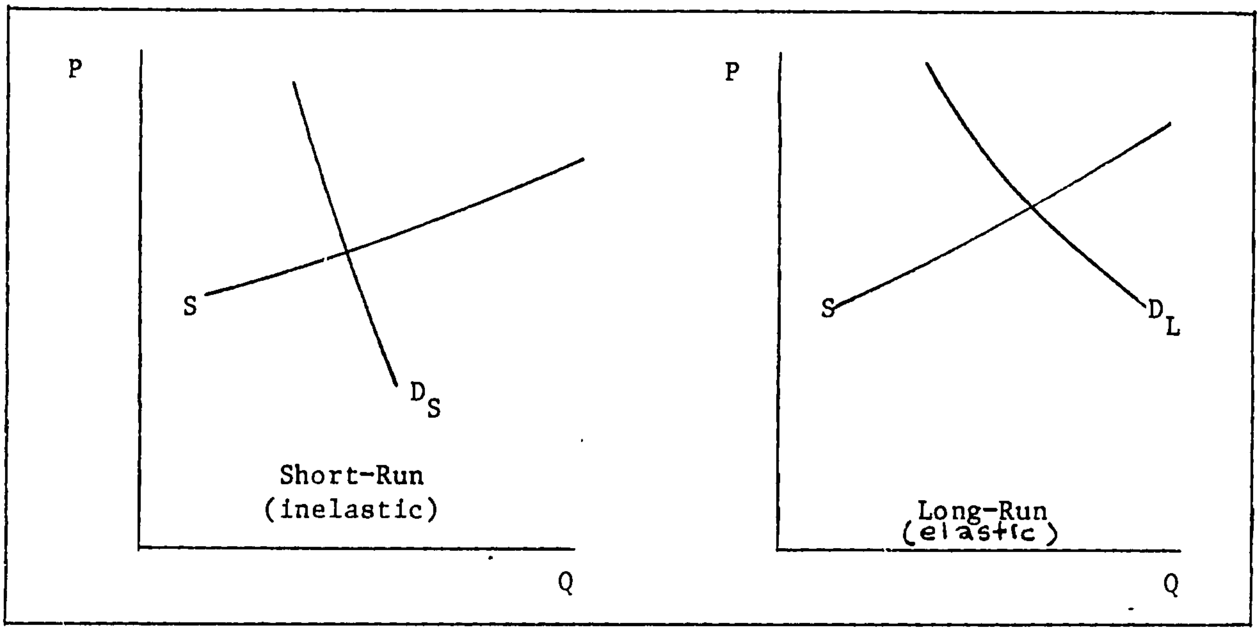


Figure 4.7. Short and Long Run Elasticities

It sould be noted that many demand functions encountered in energy analysis are inelastic in the short-run, but elastic in the long-run. Energy demand changed very little in response to the 1973-74 oil embargo and the steep price increases that followed; in the short-run, energy demand tends to be relatively inelastic, as there are limits to what conservation can accomplish without changes to the capital stock. Over the long-run, however, as consumers replaced old cars and appliances by new, energy efficient models, demand proves to be quite elastic; the current world oil glut at the time of writing (early 1981) is in large large measure a result of oil conservation measures initiated over the past decade (although the recession in the western industrialized countries has of course also contributed to the decrease in demand).

The Impact of Taxes and Subsidies

Consider the situation of Figure 4.8, with initial equilibrium at (P_0, Q_0). Now suppose a tax of t per unit is imposed, which shifts the supply curve from S_0 to $S' = S_0 + t$. The new equilibrium is at (P^*, Q^*). Notice that the increase in price to the consumer, $P^* - P_0$ is less than t; the consumer bears only part of the burden. The other part is suffered by the domestic producer, who now receives only the price P_S, corresponding to the decrease in output. For the slopes of demand and supply curves shown, consumers and producers share the incidence about equally. Consumers surplus decreases by the area ADBFG, producers surplus decreases by FBDCE, which is greater than the revenue to the Government, ABCEFG, corresponding to $(P^* - P_S)Q^* = tQ^*$. This resulting distribution of tax burden is a function of the elasticity of demand (and supply). We leave to exercise E8 the analysis of the incidence of the tax burden in the case of highly elastic demand.

Consider next the situation where there are no domestic producers of an energy form--say, gas oil--the demand for which is met entirely by imports. The supply curve for such a good can be regarded as essentially flat (Figure 4.9). Initially, Q_0 units of gasoil are imported at the border price P_0. The foregin exchange cost is $F = Q_0P_0$. A tax of t per unit results in a new price to the consumer of P^* (which is simply the border price plus the tax. ($P^* = P_0 + t$). The market clears at Q^*, resulting in a foreign exchange saving of $P_0 \cdot (Q_0 - Q^*)$. Government revenue

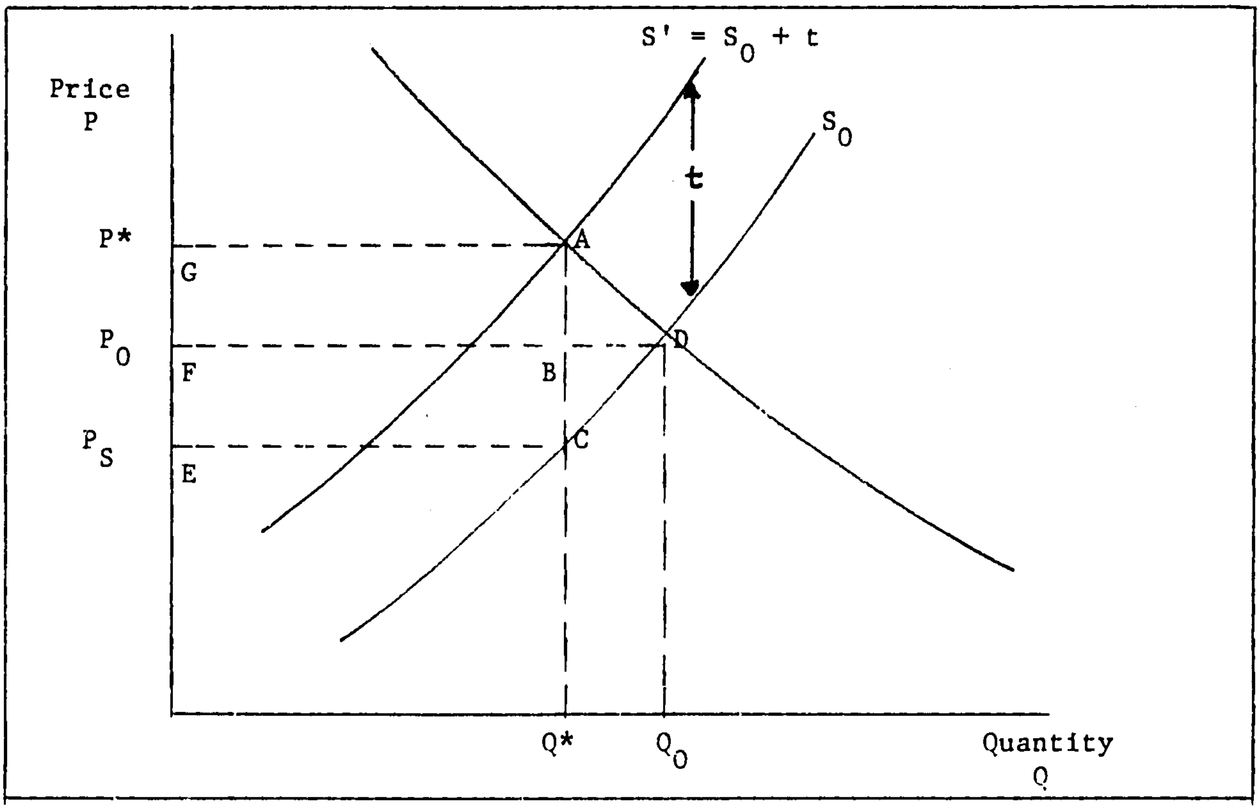


Figure 4.8. Impact of an Energy Tax

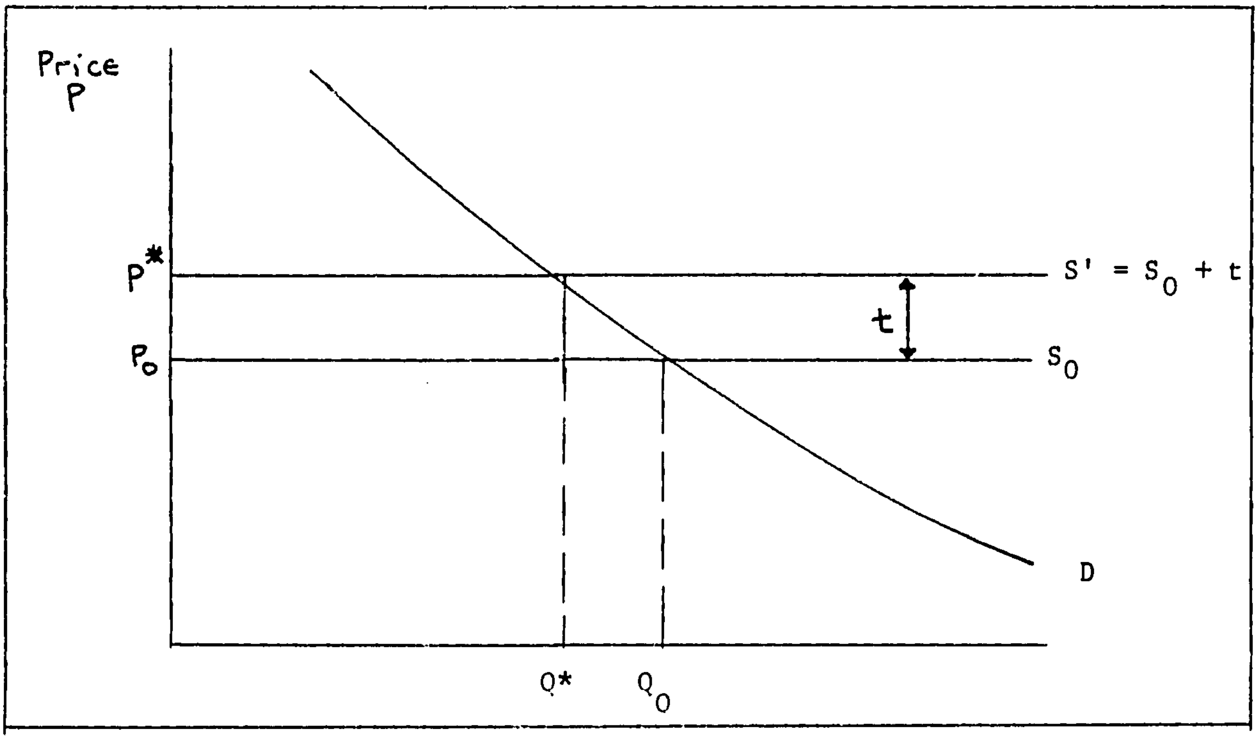


Figure 4.9. Taxation of an Imported Energy Form

increases by $Q^* \cdot (r^* - P_0)$. In contrast to the previous situation, the entire cost of tax is borne by consumers.

Many developing countries provide extensive subsidies on certain energy products--kerosene and fuels used in the agricultural sector frequently enjoy large subsidies. In the case where the subsidized product is completely imported (Figure 4.10), the effect of a subsidy, say α, is to increase foreign exchange cost. In the absence of a subsidy, the market clears at Q_0, P_0, with a foreign exchange cost of P_0Q_0. At the subsidy level α, the new price to the consumer is $P^* = P_0 - \alpha$, which results in higher imports to the level Q^*. Foreign exchange costs increase by $P_0(Q^* - Q_0)$, whilst the cost to the government treasury is αQ^* (and is given by the area ABFE). Consumer surplus, however, increases only by the area FDBCE; the cost to government is greater than the increase in consumer surplus.

Disturbances to Market Equilibrium

Consider a situation for which supply and demand are in equilibirium, at (P_1Q_1). As a result of problems with foreign exchnage, insufficient crude can be imported to meet domestic requirements; supply falls to Q^*. Now if the official price remains at P_1, note that there exist more buyers at this price (namely Q_1) than is available. The available supplies must of course be allocated in some way; if the price is left uncontrolled, it will rise to P^*, as there will be exactly Q^* buyers who are willing to pay P^* (or more). If the price remains controlled, at P_1, then the government must impose some rationing scheme to allocate supplies. But if the rationing does not allocate the supply to only those users who are willing to pay P^*, a black market is inevitable. The individual at point X would in fact be willing to pay more for oil than even the market clearing price P^*; while the individual at Y, who might be a legal recipient, could net a handsome profit by offering to sell his allocation (received at price P_1) to X for a price anywhere between P_Y and P_X; if the black market price is between Py and P_1, then Y would loose by selling, since the value of oil to Y is exactly P_Y. The black market price will depend on two main factors: the size of the shortfall, and the degree to which the official rationing scheme provides petroleum products to those who value them the highest.

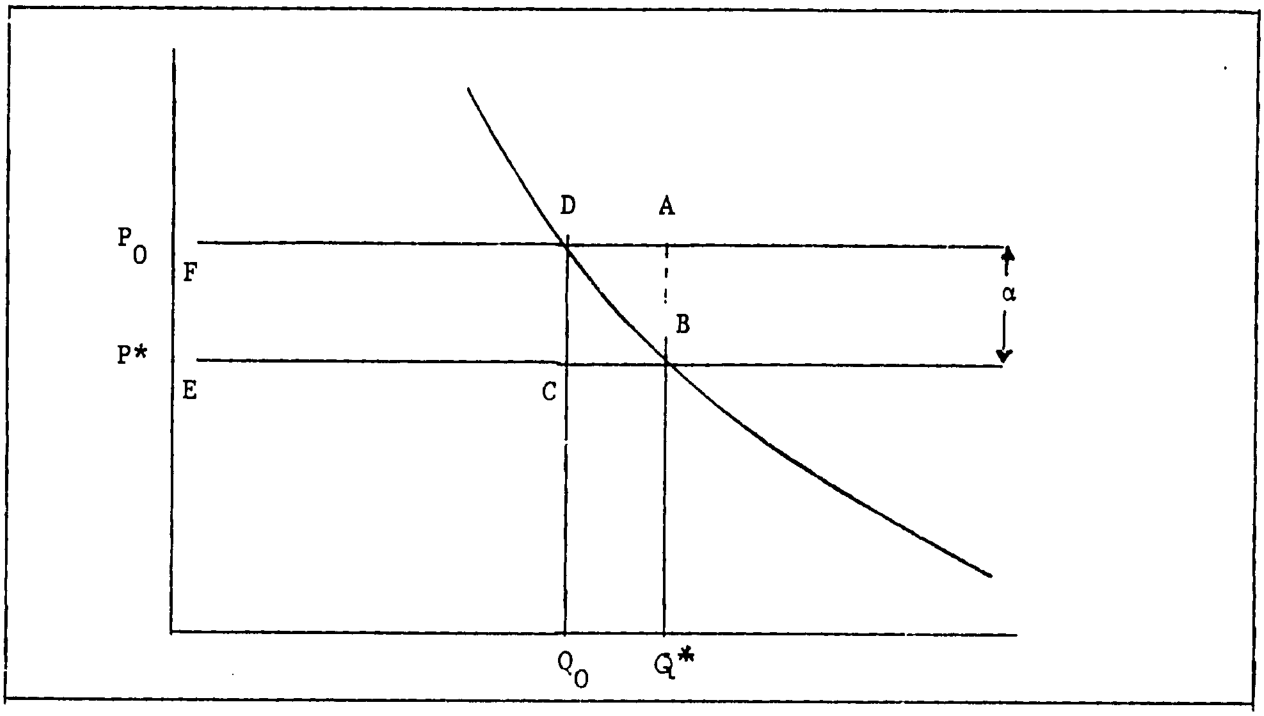


Figure 4.10. Impact of a Kerosene Subsidy

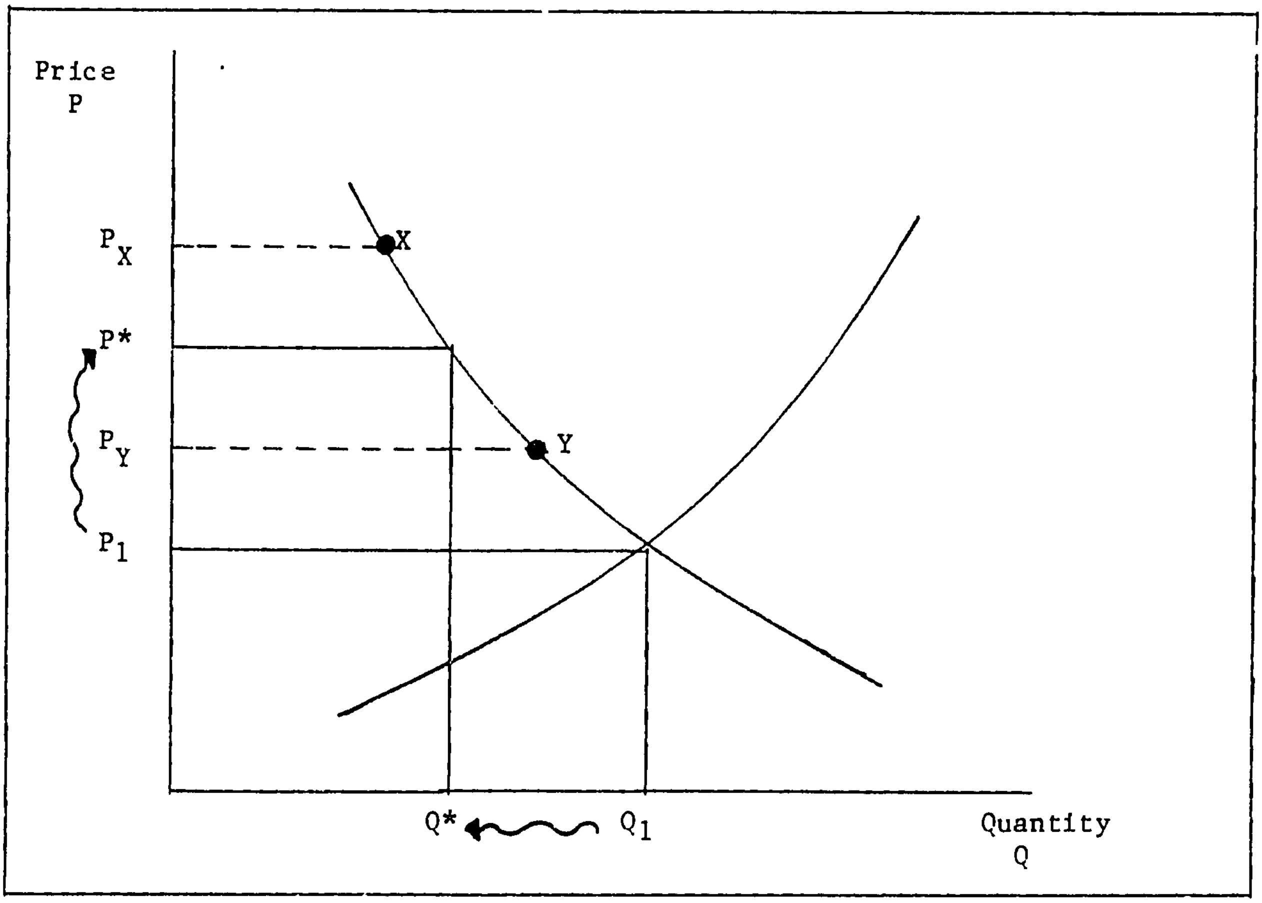


Figure 4.11. Impact of Supply Shortfalls

Price regulation is another intervention that ultimately creates difficulties. Considere Figure 4.12, with initial market equilibrium at P_0, Q_0. Suppose that the government sets the price at P*: this results in consumers capturing part of the producers' surplus (the shaded area on Figure 4.12). However, at the regulated price, demand is Q_E; consumers demand more than the producers are willing to supply (Q_E - Q* is sometimes referred to as excess demand). The United States experience of regulating natural gas prices confirms this: by the mid-1970's, the ceilings imposed by the Federal Power commission on interstate gas supplies resulted in long waiting lists of customers, increased incidence of supply interruptions where contracts permitted, and, in severe winters, very severe demand curtailments to industrial and commercial customers as priority was given to residential requirements for heating.

Subsidies and Import Fees

Consider the situation on Figure 4.13, in which domestic producers of an energy good (supply curve S_d) compete with imports (imported at the border price P_0). Initially the market clears at P_0, Q_0, the point at which the import supply schedule intersects the domestic demand schedule D. At this intial equilibrium, domestic producers produce Q_D units (because up to this production level the domestic supply curve lies below P_0), while imports capture the remaining share of the domestic market, i.e., $Q_0 - Q_D$.

Now suppose a tariff is levied on the imported energy product, in the amount f. This raises the effective supply curve for imports from S_I to S_I'. Now the market clears at Q*, P*, the intersection of the new import price with the demand curve. Domestic production increases from Q_D to Q_D^*, whilst imports decline to $Q^* - Q_D^*$. As can be seen from an inspection of Figure 4.13, not only does the absolute amount of imports decrease (with a concomitant decrease in foreign exchange requirement), but the <u>share</u> of the domestic market captured by imports declines (from about 50% to 30%, for the situation sketched in Figure 4.12). At the same time, government revenue increases by f $(Q_D^* - Q^*)$.

Consumer surplus decreases from ACG to ABH, the hatched area of Figure 4.13. This is offset by an increase in producer surplus (from CDJ to BIJ), and the increase in government revenue (the rectangle IHFE). The difference

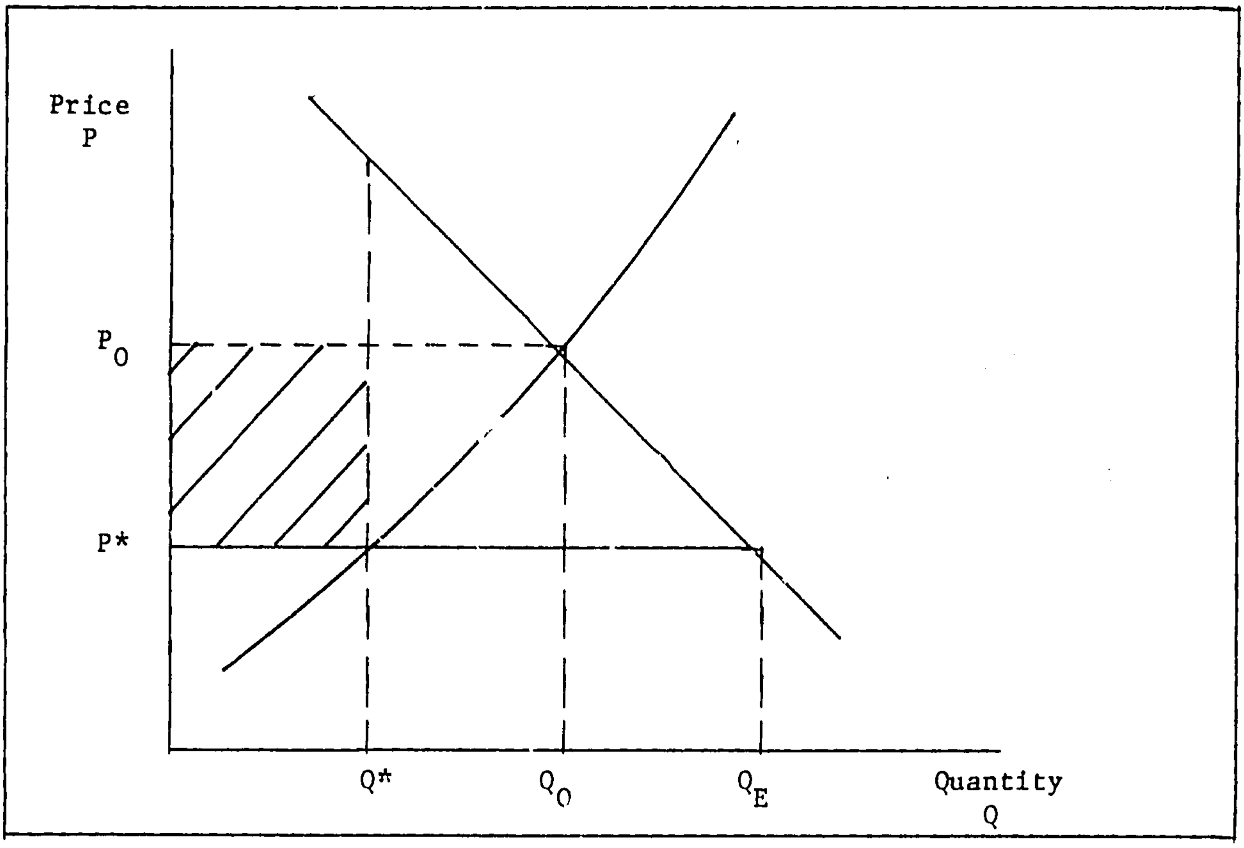


Figure 4.12. Price Regulation

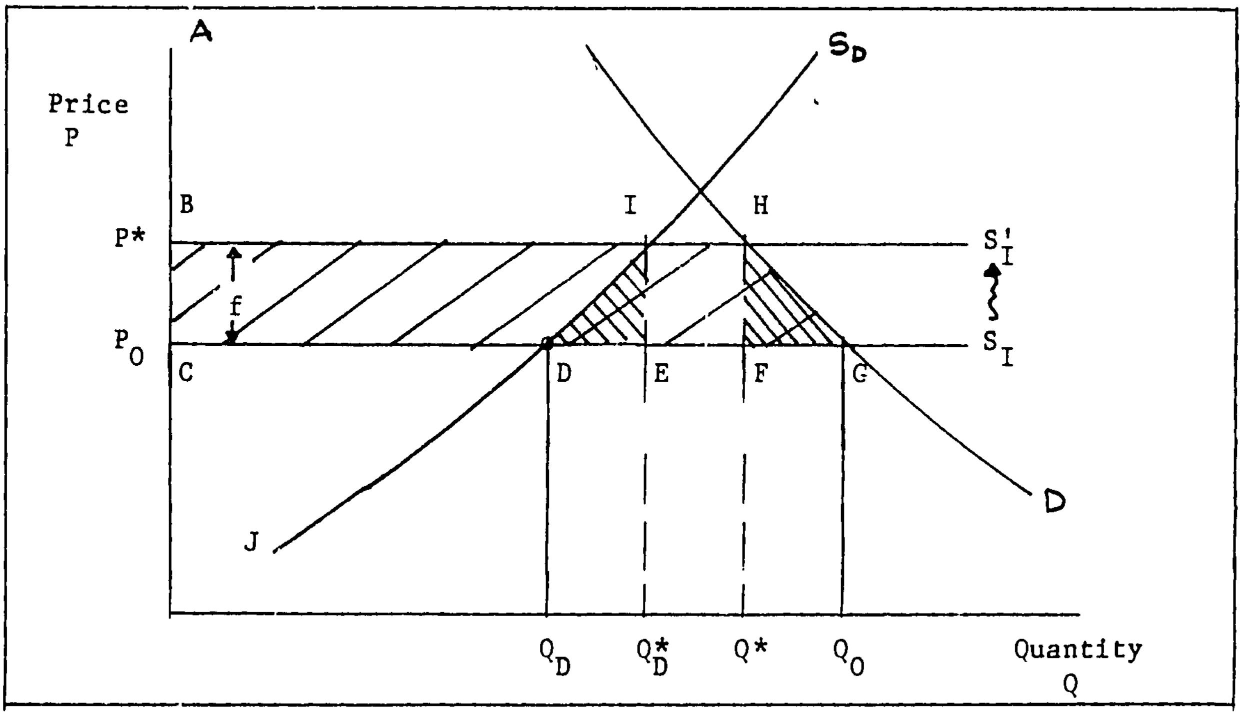


Figure 4.13. Import Competition

is the sum of the areas IED and HFG (the crosshatched area of Figure 4.13) which represents a net loss in welfare, which may or may not be offset by the foreign exchange gain.

Now consider a similar situation (Figure 4.14) for a non-energy industry (say for machinery, for which an import substitution strategy wishes to be persued). The initial domestic supply cruve is S_0, whilst imports enter at the world price P_0. The domestic market clears at (P_0, Q_0): in analogy to the previous example, domestic industry supplies Q_d, imports capture the remaining share of the domestic market $Q_0 - Q_D$. Now suppose that the cost of industrial fuel oil is subsidized, resulting in a downward displacement of the domestic supply curve. The market still clears at the same point, but now the share of the domestic market held by domestic suppliers increases, from Q_D to Q_D^* whilst imports decline to $Q_D - Q_D^*$ (with a concomitant decrease in foreign exchange). Finally domestic producer surplus increases from ACB to ADE. These gains are offset, however, by the cost of the subsidy to the government treasury that amounts to αQ_D^*.

It is important to recognize in this context that any subsidy to a domestic industry (or to domestic agriculture) be it in the form of a subsidized price of energy, or through some other mechanism, represents nothing more than a transfer payment; the amount of the government subsidy has to be found somewhere in the treasury. In many developing countries, the extent of energy subsidies has increased since the price escalations of 1973-74, and again in 1979-80, as domestic price increases have not kept pace with world price increases. And in many cases, these subsidies have been financed by foreign aid, or by assistance from agencies such as the IMF. Obviously there comes a point at which these agencies begin to take a rather hard line, and make further assistance contingent on the gradual elimination of subsidies.

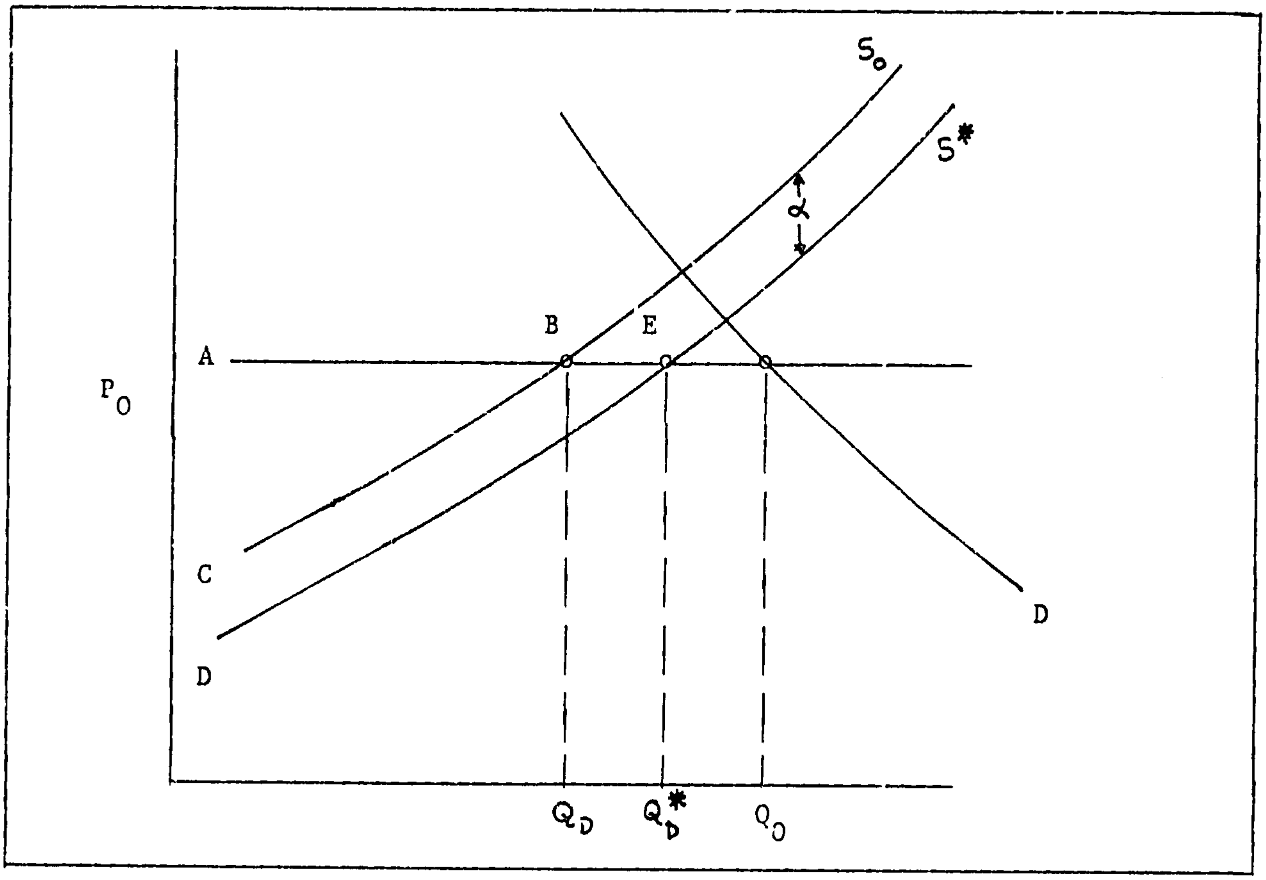


Figure 4.14. Eliminating an Energy Subsidy to a Domestic Industry

Estimating Elasticities: The Cobb-Douglas Production Function

Consider the functional relationship

$$Q = \alpha \cdot P^{\beta} \tag{4.2}$$

where

P = price

Q = demand

Now the elasticity of demand is defined as

$$\epsilon = \frac{\frac{\partial Q}{Q}}{\frac{\partial P}{P}} \quad . \tag{4.3}$$

Therefore, differentiating Equation (4.2)

$$\frac{\partial Q}{\partial P} = \alpha\beta P^{\beta-1}$$

$$= \frac{\alpha\beta P^{\beta}}{P} \qquad (4.4)$$

but from Eq (4.2) $\alpha P^{\beta} = Q$, hence

$$\frac{\partial Q}{\partial P} = \frac{\beta Q}{P} \qquad (4.5)$$

and

$$\frac{\partial Q}{Q} = \beta \cdot \frac{\partial P}{P} \qquad (4.6)$$

from which follows the important result that $\beta = \epsilon$, and that the elasticity of the Cobb-Douglas production function is given by the exponent of the price term.

Suppose that the elasticity is to be estimated from a historical data series by regression analysis. Then by taking logarithms of both sides.

$$\log Q = \alpha + \beta \log P \qquad (4.7)$$

from which follows that the regression coefficient $\hat{\beta}$ is equivalent to the corresponding elasticity.

For example, suppose we have the general production function

$$Y = f\,(K,L,E,M,X) \qquad (4.8)$$

where

Y = Output (value added)

K = Capital stock

L = Labor

E = Energy

M = Raw Material

X = Other .

Then if we specify these functions in the Cobb-Douglas form, i.e.,

$$Y = \alpha \cdot K^{\beta_1} \cdot L^{\beta_2} \cdot E^{\beta_3} \cdot M^{\beta_4} \cdot X^{\beta_5} \qquad (4.9)$$

the corresponding elasticities follow from the log linear regression

$$\log Y = \alpha + \beta_1 \log K + \beta_2 \log L + \beta_3 \log M + \beta_4 \log X + \ldots. \quad (4.10)$$

Such an estimation is given in example 4.2.

<u>Long-Run vs. Short-run Elasticities:</u> Consider the energy consumption function

$$C_t = \beta_o + \beta_1 Y_t \quad (4.11)$$

where C_t is the energy consumption at time t and Y_t is the income level at t. This implies instantaneous adjustment of consumption to income level. For reasons of psychology, or the adjustment in capital stock, one can readily define a more realistic specification involving a single lagged value of the independent variable:

$$C_t = \beta_o + \beta_1 Y_t + \gamma C_{t-1} \quad (4.12)$$

which implies that adjustments to income are not instantaneous (if $\gamma >> \beta_1$, then adjustment can be seen to be very slow).

More generally, (4.12) can be written

$$C_t = \beta_o + \beta_1 Y_t + \gamma_1 C_{t-1} + \gamma_2 C_{t-2} + \ldots \quad . \quad (4.13)$$

In practice, because of multicollinearity problems, one frequently makes the assumption that the coefficients decline according to a geometric series, i.e.

$$C_t = \beta_o + \beta_1 Y_t + \beta_1 \lambda Y_{t-1} + \beta_1 \lambda^2 Y_{t-2} + \ldots + U_t; \; o < \lambda < 1 \quad (4.14)$$

which can be written

$$C_t - \lambda C_{t-1} = \beta_o(1 - \lambda) + \beta_1 Y_t + U_t - \lambda U_{t-1} \quad (4.15)$$

or, for econometric estimation, one would write.

$$C_t = \beta_o(1 - \lambda) + \beta_1 Y_t + \lambda C_{t-1} + (U_t - \lambda U_{t-1}) \quad (4.16)$$

Now if (4.16) has an exponential form

$$C_t = \alpha \, Y_t^{\beta_1} \, C_{t-1}^{\pi} \quad (4.17)$$

then taking logarithms of both sides yields

$$\log C_t = \log \alpha + \beta_1 \log Y_t + \pi \log C_{t-1} \quad (4.18)$$

from which the short run elasticity follows as before, namely β_1; and $\alpha = \beta_o (1 - \lambda)$; $\lambda = \pi$.

Now the long term effect is that represented by the ultimate change in consumption from a once-and-for-all change in income level, so that, by setting all $C_t = \bar{C}$, and $Y_t = Y$, and inserting in (4.15.)

$$C (1 - \lambda) = \beta_o (1 - \lambda) + \beta_1 Y \tag{4.19}$$

hence

$$C = \beta_c + \frac{\beta_1}{1 - \lambda} Y \quad . \tag{4.20}$$

If estimated logarithmically it follows that the long run income elasticity is given by

$$\epsilon_i = \frac{\beta_1}{1 - \pi} \quad . \tag{4.21}$$

Thus, suppose we have the relation

$$Q_t = \alpha \cdot Q_{t-1}^{\pi} \cdot Y_t^{\beta_1} \cdot P_t^{\beta_2} \tag{4.22}$$

where

Q_t = consumption of, say, electricity in the residential sector.

Y_t = income

P_t = price

then the long run price elasticity is given by

$$\epsilon_P = \frac{\beta_2}{1 - \pi} \quad . \tag{4.23}$$

Even though the principles elaborated in section 4.1 appear simple enough, there are all manner of hazards in applying statistical methods to the estimation of elasticities and production functions. Multicolinearity is certainly one of the more serious problems, a condition whereby the assumed independence of the "independent" variables is not actually reflected in the data. So called autocorrelation of residuals, especially when lagged variables are presen in a regression, further confound the analysis. The problem in all of these instances is that of <u>bias</u>: under what conditions is the expected value of a regression coefficient equal to the assumed model parameter? As we shall see, the algebra of least squares does not always guarantee that estimators are unbiased. Before we discuss these problems in turn, first two definitions.[2]

> $\hat{\beta}$ is an unbiased estimator if $E\{\hat{\beta}\} = \beta$
>
> $\hat{\beta}$ is said to be inconsistent if the bias persists for large sample size: a consistent estimator is thus one that is biased only for small sample size.

<u>Multicolinearity</u>. Multicollinearity is a problem frequently encountered in econometrics, arising from the situation in which the "independent" variables are <u>not</u> in fact independent. Suppose, first, that there is an exact relationship between X_1 and X_2 in the (zero intercept) model.

$$Y = \beta_1 X_1 + \beta_2 X_2 + u \tag{4.24}$$

such that $X_{2i} = \alpha X_{1i}$ for all i. Then it can be shown that

$$X^T X = \begin{bmatrix} \sum X_{1i}^2 & \sum X_{1i}^2 \\ \alpha \sum X_{1i} & \alpha^2 \sum X_{1i}^2 \end{bmatrix} \tag{4.25}$$

hence, by reference to (2.19), the determinant computes to

$$\left| X^T X \right| = \sum X_{1i}^2 \cdot \alpha^2 \sum X_{2i}^2 - \alpha \sum X_{1i} \cdot \alpha \sum X_{2i}^2 = 0 \tag{4.26}$$

from which it follows that $X^T X$ is singular, and $(X^T X)^{-1}$, required in the computation for $\hat{\beta}$ (Eq. 2.41), does not exist.

---

[2]For a full discussion of the properties of least squares estimators, see e.g. A. Goldberger "Econometric Theory," John Wiley & Sons, New York, 1964.

Clearly, in real situations one rarely finds exact relationships between the independent variables; but frequently the independent variables do show some degree of correlation: say $X_{2i} = X_{1i} + u_i$, for which

$$X^TX = \begin{bmatrix} \sum X_{1i}^2 & \sum X_{1i}(\alpha X_{1i} + U_i) \\ \sum X_{1i}(\alpha X_{1i} + U_i) & \alpha^2 \sum X_{1i}^2 + 2\alpha \sum X_{1i} \cdot \sum U_i^2 + U_i \end{bmatrix} \quad (4.27)$$

from which the determinant follows as

$$\sum X_{1i}^2 \left| \alpha^2 \sum X_{1i}^2 + 2\alpha \sum X_{1i}U_i + \sum U_i^2 \right| - \left| \sum X_{1i}(\alpha X_{1i} + U_i) \right|^2 \quad (4.28)$$

hence, multiplying out the individual terms

$$\alpha^2(\sum X_{1i}^2)^2 + 2\alpha \sum X_{1i}U_i \cdot \sum X_{1i}^2 + \sum X_{1i}^2 \cdot \sum U_i^2$$
$$-\alpha^2(\sum X_{1i})^2 - (\sum X_{1i}U_i)^2 - 2\sum X_{1i}U \cdot \sum X_{1i}^2\alpha \quad (4.29)$$

yields

$$|X^TX| = \sum U_i^2 \cdot \sum X_{1i}^2 - \sum X_{1i}U_i \quad . \quad (4.30)$$

If one makes (as does Goldberger, 1964, p. 192), the additional assumption that $\sum X_{1i}U_i = 0$, then it follows that the smaller $\sum U_i^2$, the smaller will be $|X^TX|$, hence the larger the variance of $\hat{\beta}$, since

$$\text{Var}\left\{\hat{\beta}\right\} = \sigma^2(X'X)^{-1} \quad (4.31)$$

Thus the presence of multicolinearity may cause the the estimated parameters to have large variances, and their interpretation therefore becomes subject to considerable uncertainty. There is another more perverse problem: suppose that the independent variables were in reality perfectly correlated, but because of measurement errors, the collected data show some less than perfect correlation. In this case one can see that the greater the measurement error, the less the apparent multicollinearity. Multicollinearity is an ever-present potential problem in time series analyses; a model, say, of historical residential energy consumption as a function of household income and energy price would typefy the potential problem, because of the likely correlation of the independent variables. In such situations a frequent expedient is to add cross-sectional data (additional country data, or regional data) to obtain the income coefficient. Indeed, the almost universal prescription in the face of multicollinearity is the aquisition of additional data.[3]

Autocorrelation: One of the critical assumptions in the LS model discussed in Chapter Two is that the variance of the error term can be given as

$$E \left\{ uu' \right\} = \sigma^2 I \tag{4.32}$$

which implies that successive disturbances are drawn independently of previous values, i.e.

$$E \left\{ u_t u_{t+s} \right\} = 0 \text{ for all } t \text{ and all } s \neq 0 \quad . \tag{4.33}$$

Unfortunately, the assumption of serial independence in the error term is often not very plausible, particularly in time series analysis.

Suppose, then, that we drop the assumption $E \left\{ uu' \right\} = 0$. From (2.44) we recall the expression

$$E \left\{ \hat{\beta} \right\} = \beta + (X^T X)^{-1} X^T E \left\{ u \right\} \tag{4.34}$$

which, since it does not involve $E \left\{ uu' \right\}$, yields, as before,

$$E \left\{ \hat{\beta} \right\} = \beta \tag{4.35}$$

and hence the estimator remains unbiased.

From (2.47) recall the expression for the variance, i.e.

$$\mathrm{Var} \left\{ \hat{\beta} \right\} = (X^T X)^{-1} X^T E \left\{ uu' \right\} X (X^T X)^{-1} \quad . \tag{4.36}$$

It can be shown[4] that (4.36) will yield a higher value of var $\left\{ \hat{\beta} \right\}$ than if $E \left\{ uu' \right\}$ is set to $\sigma^2 I$: this implies that we will underestimate the true variance of $\hat{\beta}$ by application of the usual expression

$$\mathrm{Var} \left\{ \hat{\beta} \right\} = \sigma^2 (X^T X)^{-1} \tag{4.37}$$

The Durbin-Watson d Statistic: is a test used to test for autocorrelated residuals. Let $z_t (t = 1 \ldots n)$ be the residuals from a fitted least squares regression. Then define

---

[3]Johnston (1963), for example, notes in this regard (p. 207) "... If multicollinearity is serious, in the sense that estimated parameters have an unsatisfactorily low degree of precision, we are in the statistical position of not being able to make bricks without straw. The remedy lies essentially in the aquisition, if possible, of new data which will break the multicollinearity deadlock."

[4]See e.g. Johnson (1963), p. 168

$$d = \frac{\sum_{t=2}^{n} (z_t - z_{(t-1)})^2}{\sum_{t=1}^{n} z_t^2} \tag{4.38}$$

a statistic for which Durbin and Watson have tabulated lower and upperbounds d_ℓ and d_u for various values of n and k. In a one-sided test of positive autocorrelation, then if

$d < d_\ell$, reject hypothesis of random disturbances

$d > d_u$, do not reject hypothesis

$d_\ell < d < d_u$, the test is inconclusive.

Lagged Variables: In the previous section we encountered specifications involving lagged values of the independant variable (e.g. in Eq. 4.12). What can we say about application of LS to such a function? Consider the form

$$Y_t = \alpha + \beta Y_{t-1} + \epsilon_t \quad . \tag{4.39}$$

Applying the results of digression 2.1, the least squares estimator of $\hat{\beta}$ is

$$\hat{\beta} = \frac{\sum_{t=2}^{n} (Y_t - \bar{Y})(Y_{t-1} - \bar{Y}')}{\sum_{t=2}^{n} (Y_{t-1} - \bar{Y}')^2} \tag{4.40}$$

where

$$\bar{Y} = \frac{1}{n-1} \sum_{t=2}^{n} Y_t$$

$$\bar{Y}' = \frac{1}{n-1} \sum_{t=1}^{n-1} Y_t \quad . \tag{4.41}$$

But from (4.32), it follows that

$$Y_t - \bar{Y} = \beta(Y_{t-1} - \bar{Y}') + (e_t - \bar{\epsilon}) \tag{4.42}$$

hence, inserting (4.35) into (4.33)

$$\hat{\beta} = \beta + \frac{\sum_{t=2} (Y_{t-1} - \bar{Y}') \cdot \epsilon_t}{\sum_{t=2} (Y_{t-1} - \bar{Y}')^2} \tag{4.43}$$

Taking expectations, we see that

$$E\left\{\hat{\beta}\right\} = \beta + E\left\{\frac{\sum (Y_{t-1} - \bar{Y}') \cdot \epsilon_t}{\sum (Y_{t-1} - \bar{Y}')^2}\right\} \tag{4.44}$$

It can be shown that if the error term is normally disturbed with zero mean and constant variance σ_e^2, then the extimate of $\hat{\beta}$ is biased for small samples, although it has the desirable asymptotic property of consistency.[5]

Let us return to Eq. (4.16);

$$C_t = \beta_o(1 - \lambda) + \beta_1 Y_t + \lambda C_{t-1} + (u_t - \lambda u_{t-1}) \tag{4.45}$$

which we note has very particular form for the error term, namely

$$t = u_t - \lambda u_{t-1} \quad . \tag{4.46}$$

If, and only if, u_t follows this scheme exactly, will ϵ_t be serially independent, for which application of LS will give consistent estimates of the parameter (although biased for finite sample sizes). But is it likely that the error term conveniently follows Eq. (4.46)? For example, if u_t is serially independent, then obviously ϵ_t will be autocorrelated; now the estimates of α and β will not even be consistent. And as demonstrated by the classical sampling experiments of Orcut and Cochrane (1949), the combination of lagged variables and autocorrelation leads to very serious problems of bias.

[5]See e.g. Johnson (1963), p. 214. Orcutt and Cochrane examined the scheme $X_t = 0.4 X_{t-1} + u_t$ where u_t showed positive autocorrelation. The authors showed that the combination of lagged variables and autocorrelation leads to very serious problems of bias. They found that even though lagged variables would lead to negative bias, and autocorrelation to no bias, the combination of the two led to substantial positive bias.

Income and Price Elasticity of Energy Demand in Tunisia: Let us return to the example of Section 2.5. In place of the linear model

$$Y = \alpha + \beta_1 X_1 + \beta_2 X_2$$

we use the log-linear specification

$$Y = \alpha \cdot X_1^{\beta_1} \cdot X_2^{\beta_2} \quad .$$

In accordance with the discussion of Section 4.1, the price elasticity of demand is equal to β_1, the income elasticity of demand is β_2. Taking logarithms,

$$\log Y = \log \alpha + \beta_1 \log X_1 + \beta_2 \log X_2$$

hence, defining Y' = log Y, and X' = log X, the new data matrices become

$$Y' = \begin{bmatrix} 5.521 \\ 5.663 \\ 5.774 \\ 5.883 \\ 5.934 \\ 6.031 \\ 6.131 \\ \end{bmatrix} \qquad X' = \begin{bmatrix} 1 & 4.54 & 5.06 \\ 1 & 4.60 & 5.12 \\ 1 & 4.64 & 5.20 \\ 1 & 4.66 & 5.33 \\ 1 & 4.66 & 5.31 \\ 1 & 4.71 & 5.37 \\ 1 & 4.93 & 5.43 \\ 1 & 4.98 & 5.52 \end{bmatrix}$$

This data leads to the following matrices

$$X^T X = \begin{bmatrix} 8.0000 & 37.7394 & 42.3732 \\ 37.7394 & 178.2033 & 200.0442 \\ 42.3732 & 200.0442 & 224.6040 \end{bmatrix}$$

$$(X^T X)^{-1} = \begin{bmatrix} 167.923 & 2.231 & -33.667 \\ 2.231 & 29.953 & -27.099 \\ -33.667 & -27.099 & 30.491 \end{bmatrix}$$

$$X^T y = \begin{bmatrix} 47.000 \\ 221.9174 \\ 249.1690 \end{bmatrix}$$

and the estimate for β

$$\hat{\beta} = \begin{bmatrix} -1.200 \\ -0.1848 \\ 1.500 \end{bmatrix}$$

From which we obtain a price elasticity of -0.18 and an income elasticity of 1.5. These estimates have a sign consistent with expectations, and are consistent in magnitude with what one encounters in the literature.

However, as in the case of the linear model (recall Section 2.5), a t-test on the price elasticity indicates lack of statistical significance; the computed value of t is

$$t = \frac{-0.18}{\sqrt{\frac{0.0097}{(8-3)}} \cdot \sqrt{29.95}} \cdot = -0.766$$

whereas the tabulated value of t with 5 d.f. is hence the null hypothesis cannot be rejected. Indeed, the 95% confidence limit is

$$\beta_2 = -0.18 \pm t_{5d.f,\ 0.025} \sqrt{\frac{0.0097}{5}} \cdot \sqrt{29.95}$$

whence

$$\leq \beta \leq \quad .$$

Indeed, this almost a classic illustration of multicollinearity: the correlation coefficient between price and income (i.e., between log X_1 and log X_2) computes to 0.999906.

4.3 INDUSTRY PROCESS MODELS

While application of the econometric approach outlined in the previous sections can give a general indication of how energy demand will evolve in response to fuel prices, it should be noted that this response cannot be stated in terms of specific technological changes, even though it is recognized that for a change in energy demand to occur, some change or modification to production technology must take place. Indeed, proponents of the econometric approach would argue that this is one of the advantages of the approach, in that the effect of technologies yet to be developed can be taken into account (without actually defining them). Of course, for this to be valid, one must suppose that the type of technological response induced by the price changes in the historical period that is subject to the econometric estimation will continue into the future.

An alternative to the econometric approach is that of socalled process modelling, in which the slate of technological alternatives is explicitly stated, and for which the model then selects the optimum capital expansion path for that industry under some given fuel price trajectory. Obviously, technologies that have yet to be discovered cannot be captured by the model, which is not too serious for short planning horizons, given the very long gestation periods between discovery of a process, and its ultimate commercialization. Indeed, one might argue that for longer time horizons, the econometric approach is not much better either, given the hazards of projecting industry behaviour on the basis of coefficients estimated from historical time series. As we shall see in later chapters, a combination of the two approaches is often the best strategy.

A process model can be viewed as a tool to derive energy demand curves for specific industries. That is, for some given level of production, (say P tons of finished steel per year), and for a given set of energy and non-energy prices, what quantity of energy will be demanded to sustain that level of output. Clearly that will depend on the specific technologies used in the production process, and in particular on the degree to which capital can substitute for energy. A transition from oil dependent technology to say coal based technology requires some level of investment; the process model will determine the optimal level and type of investment given resource price trajectories and the investment associated with different technologies. Obviously, since one is trading off capital expenditure for future fuel savings, the assumed discount rate becomes important.

Figure 4.15 illustrates the manner in which energy demand curves can be generated. For some production target P, and some energy price λ_1, we compute the quantity of energy Q_1. Keeping production constant, at P_1, but rerunning the model at energy price λ_2 will yield another energy demand value Q_2; by repeating this calculation over a range of energy prices we generate the demand curve D_1. Increasing the production target from P_1 to P_2, and repeating the above sequence, allows one to derive the second demand curve D_2, and so on.

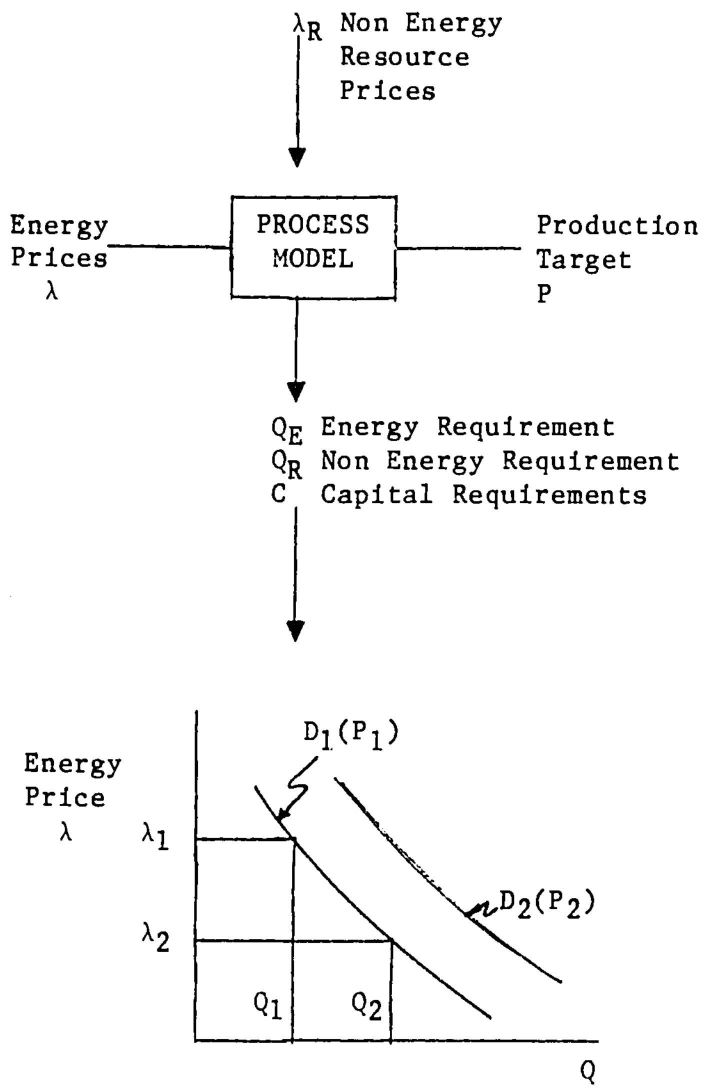


Figure 4.15. Process Model Derivation of Demand Curves

In practice, "energy" is not a very useful quantity; rather one is interested in specific fuels, and specific fuel prices. But at this point it becomes difficult to show the relationships in simple graphical form, since for every fuel one needs a separate axis. The demand for coal, say, will be a function not just of the price of coal, and the costs of coal using capital equipment, but also on the price of oil, and natural gas, and so forth.

Letting the subscripts c, o, and g represent the price of coal, oil and natural gas, the demand functions of interest can be written as

$$
\begin{aligned}
Q_c &= f(P, \lambda_c, \lambda_o, \lambda_g) \\
Q_o &= f(P, \lambda_o, \lambda_c, \lambda_q) \\
Q_g &= f(P, \lambda_g, \lambda_c, \lambda_o)
\end{aligned}
\tag{4.47}
$$

One way of numerically deriving such functions would be to run the process model over ranges of each of the dependent variables: but for three production levels, three gas prices, three oil prices and three coal prices, one would need to run the model $3^4 = 81$ times.[6] Actually this is not as formidable as it appears; if the process model is based on linear programming techniques (as almost all of them are), and if one can store the optimal solution of the first run, and use it as the starting point in the solution algorithm for succeeding runs, the oeverall cost can be quite low, especially if the process model itself is kept rather simple, and does not exceed a few hundred rows and variables in size. The type of process model useful in energy analysis will likely have that necessary simplicity, and be much smaller than the very large models that might be used by the individual industry for detailed sector studies.

The data generated by the model could then be subjected to econometric techniques to estimate the demand schedules of (Eq. 4.47). It should be noted that this approach will yield results quite different than one applying the same estimation process on historical data of P, λ_c, λ_o, λ_g; in the latter case, the demand functions would reflect only historical conditions (i.e., over historical price ranges, and valid only for the technologies of the past). By using the process model generated data, however, the impacts of new technologies are better reflected in the functional representations.

[6]There may actually be better shcemes that rely on random combination of variables that would significantly reduce the number of model runs required without compromising the ability to make statistically valid estimates of the demand functions.

There is another way of integrating the results of process models into energy analysis that avoids the need to estimate demand curves explicitly; if the process model has a linear programming structure, it is always possible to include it into the overall energy system model, assuming that, that model also has an LP structure. A discussion of this approach is deferred to Chapter 10.

What then, is the typical mathematical structure of a process model? consider, as an illustrative example, a very simple model of the steel industry (Figure 4.16). Assume that there are J steelmaking processes (open hearth, electric arc, etc.), and let the amount of steel produced by each technology be denoted x_j. Then if the total production level for domestic steel is x, we require first that

$$\sum X_j \geq X$$

Next, let the existing capacity of each type be C_j. Then it follows that production in each process cannot exceed the capacity of existing plus new mills of that type, i.e.

$$x_j \leq \bar{C}_j + C_j(N) \tag{4.48}$$

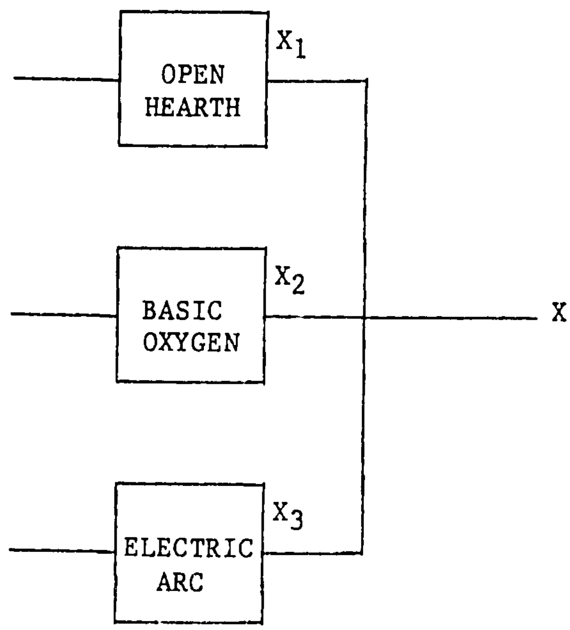


Figure 4.16: A Simple Model of the Steel Industry.

where $C_j(N)$ is the capacity to be added over the planning horizon. In constraint form, in which all endogenous variables appear on the left hand side, (4.48) is written

$$x_j - C_j(N) \leq \bar{C}_j \quad . \tag{4.49}$$

Suppose there are K energy inputs (different fuels) and L non-energy inputs (labor, iron ore, etc.) and suppose that

e_{jk} represents the unit requirement of energy of type k for process j

λk cost per unit of k

fj represents the unit requirement for the non-energy factor of production ℓ for process j.

π_ℓ cost per unit of ℓ

then the total industry wide consumption of fuel k is given by

$$\sum_j x_j \cdot e_{jk} = E_k \tag{4.50}$$

and the industry wide input of factor ℓ is given by

$$\sum_j x_j \cdot f_{j\ell} = F_\ell \quad . \tag{4.51}$$

Hence the cost to be minimized is

$$C = \lambda_k \cdot E_k + \pi_\ell \cdot F_\ell + C_j(N) \cdot \omega_j \cdot \text{CRF}\ (i,n) \tag{4.52}$$

where

ω_j is the capital cost for additions of type j.

CRF(i,n) is the capital recovery factor at discount rate i and planning horizon n.

It should be noted that each technological modification of some basic process (such as oxygen landing in open hearth steelmaking) would be represented in the model by a separate x_j, since sometimes even minor process and conservation modifications will result in drastic changes in the mix of factor (and thus also energy) inputs.

This simple Linear program, then, enables one to predict the future energy consumption in the iron and steel industry as a function of energy and non-energy factor prices, and as a function of the capital costs for different technologies. This is, to be sure, a highly simplified description of the industry, and the models used in practice are considerably more complex; the Brookhaven Iron and Steel Industry Process Model, for example, includes 43 technological options just for the production of process steam and cogeneration of steam and electricity.[7] The objective function can be more complex, too, reflecting industry decision criteria that take into account such items as taxes and depreciation allowances, and may be framed in terms of a minimization of net present value rather than minimization of an equivalent annual cost (see Chapter 7). A dynamic version may also be necessary to be more precise in estimating the capacity expansion path of the industry.[8] However, even the most complex of process models is based on the basic idea outlined in these pages.

[7] F. Sparrow et. al. "The Iron and Steel Industry Process Model," BNL 51073, Brookhaven National Laboratory, Upton, New York, January 1980.

[8] Actually the specification of planning horizon in such studies is not a simple matter, because of end effects. If the model considers, say, time steps of 5 years and the planning horizon is, say, 20 years, then it is usual to consider 6 rather than only 4 time periods in the model, so that the end effects in the last period do not distort the result.

4.4 A PROCESS MODEL OF THE EGYPTIAN IRON AND STEEL INDUSTRY

4.4.1 Introduction

The iron and steel industry is generally one of the most energy intensive industries, and therefore for any country with a domestic iron and steel industry, or a country that plans to build up its own industry in the future, decisions made in this industry with respect to process choice, conservation, and output levels may greatly affect the overall energy balance. However, because of the complexity of the industry, and the many different types of processes and process modifications potentially available, analysis of future industry developments, and industry energy demand, is a formidable task. Figure 4.17 depicts, in highly simplified form, some of the basis unit operations and fuel requirements in the iron and steel industry: there are fundamental choices to be made in how iron ore is to be reduced (with the socalled direct reduction methods becoming increasingly more widespread, displacing the traditional blast furnace route to iron making), and how steel is made (the major choices being the electric arc, open eharth and basic oxygen processes). In addition, there is considerable flexibility in fuel requirements. Figure 4.18 compares the fuel inputs to blast furnace iron-making in the U.S. and Japan: per ton of iron produced, Japanese furnaces use much less coke, but more fuel oil than do U.S. furnaces. Indeed, overall, the Japanese blast furnaces have a net energy requirement about 25% lower than U.S. technology. And, as we shall see, direct reduction technology, suited especially to natural gas utilization, offer seven further scoipe for reductions in energy input.

The intent of this section, is to illustrate the application of process modelling approaches to the analysis of energy consumption in a major industry, using the Egypitan iron and steel industry as a case study. Readers unfamiliar with the basic unit operations of the industry should review Exhibit 4.1 as a first step.

4.4.2 Options for the Egyptian Iron & Steel Industry

The Egyptian Iron and Steel Industry is highly centralized, with the bulk of productive capacity centered at Helwan, just south of Cairo, in a

*This case study is based on a report by Gordian Associates on the Industrial/Agriculture Sectors of Egypt (published as Annex 2, Joint Egypt/U.S. Report on Egyput/U.S. Cooperative Energy Assessment, DOE/IA-0002/03, April 1979) U.S. Department of Energy.

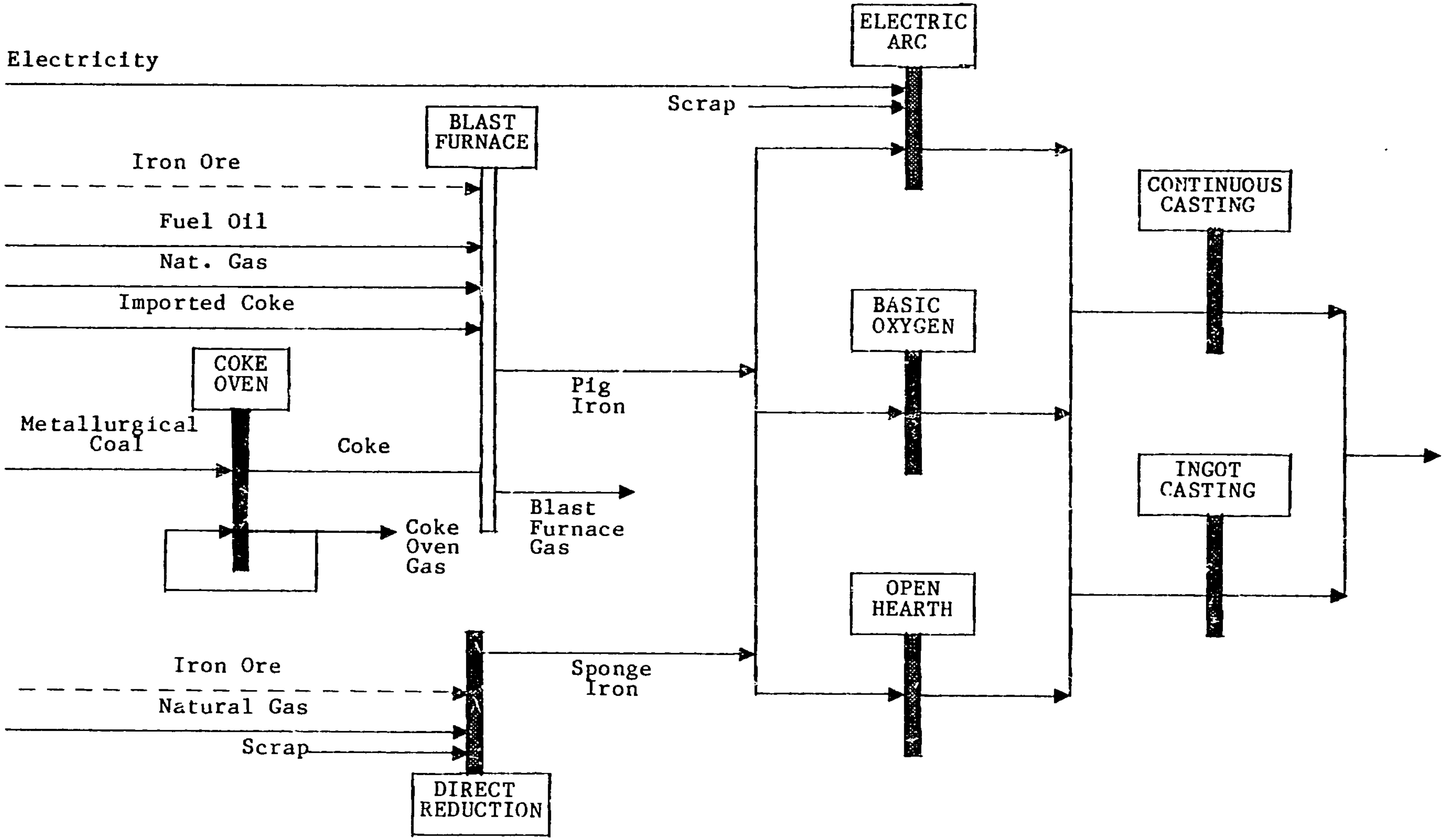
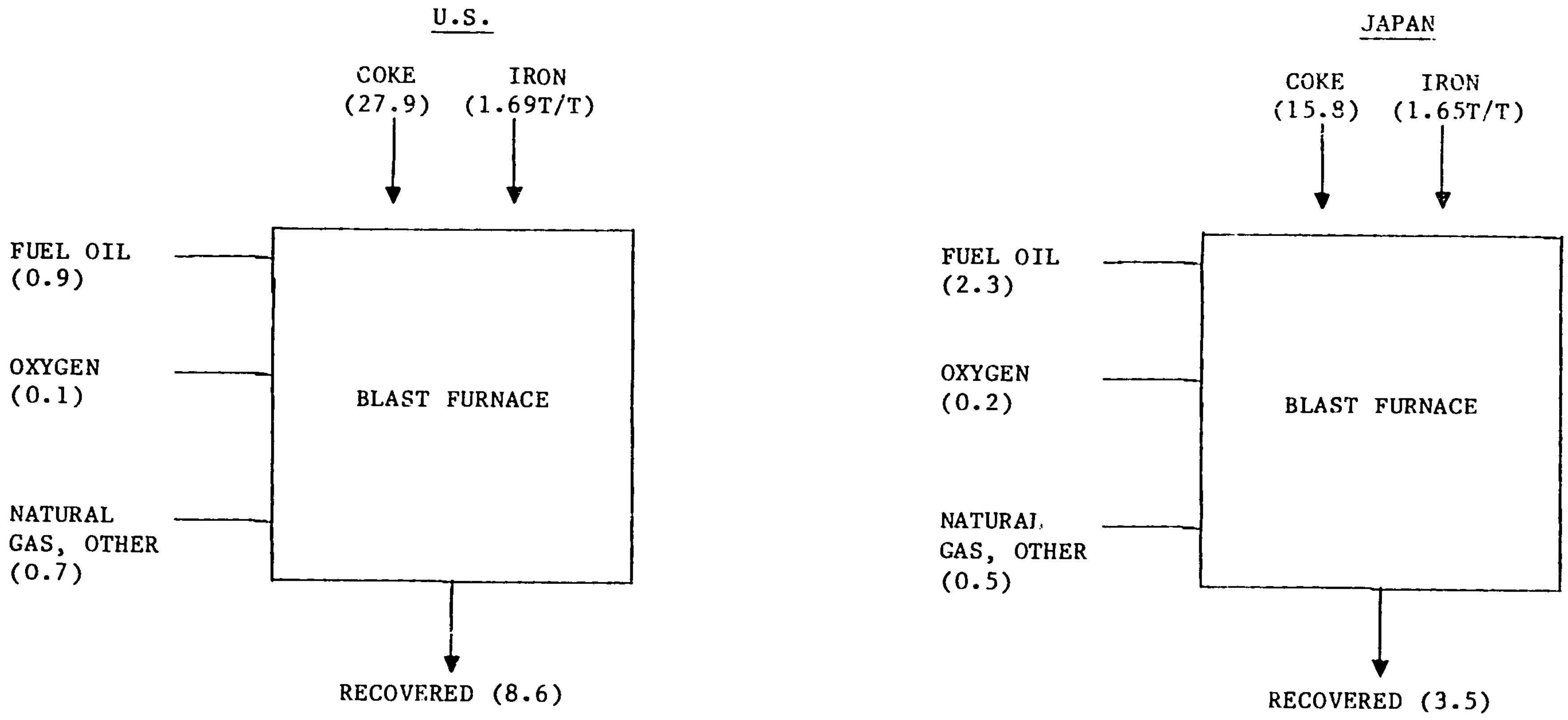


Figure 4.17. Schematic of Iron and Steel Industry Process



TOTAL	=	29.6
− RECOVERED	=	8.6
NET	=	21.0 GJ/TONNE

TOTAL	=	18.8
− RECOVERED	=	3.5
NET	=	15.3 GJ/TONNE

Figure 4.18. Energy Inputs to U.S. and Japanese Ironmaking Technology.

EXHIBIT 4.1: MAJOR OPERATIONS OF IRON AND STEELMAKING*

Basic Definitions

Iron for industrial use generally contains a number of materials besides pure iron. These materials can either be added on purpose (in which case they are called alloys) or be regarded as impurities originating from the raw materials or the processing operations. In most qualities of industrial iron, small amounts of carbon, silicon, and manganese remain. Materials like phosphorus, sulphur, hydrogen, copper, zinc, tin, and oxygen are generally regarded as undesired elements, i.e. impurities. Various types of special steels are produced by adding alloy materials like chromium, nickel, tungsten, molybdenum, cobalt, vanadium, titanium, aluminium, lead, nitrogen, copper, niobium and boron.

Depending upon whether the carbon content is lower or higher than 2 percent, chemical associatons between iron and carbon are classified as steel or cast iron. Steel, in turn, can be classified as either unalloyed steel (carbon steel) or alloyed steel.

Major Operations

The transofrmation of iron ore into steel and various forms of steel products takes place in a series of distinctly different processing steps. The five principal steps are:

1. Preparation of raw materials;
2. Cokemaking;
3. Raw iron manufacturing;
4. Raw steel manufacturing; and
5. Finishing of steel products.

Preparation of Raw Materials

Sintering: In the sintering process, the fine ore particles are agglomerated into a porous mass that is suitable to be charged into the blast furnace. The purpose of the sintering is, apart from an agglomeration of the ore concentrate, to burn some of the sulphur compounds contained by the ore concentrate. The sintering machine is charged with a mixture of iron ore concentrate, coke dust, and limestone which is lit by gas flames. In order to get the high temperature necessary for the agglomeration to take place, air is sucked through the mixture by the help of large fans.

Cokemaking: The coke, which serves as fuel and reactive agent in the raw iron manufacturing process (ie., in the blast furnace), is produced by heating metallurgical coal in the absence of air to a temperature at which the major part of the non-carbon components of the coal (i.e., volatile matter, water and sulphur) are driven off. Since approximately 25 per cent of the coal is carburated during the coking process, large amounts of gas (approximately 4-5 000 Nm^3/ ton or coke) are generated. In the most common method of coke manufacturing, the by-product coking process, the coke oven gas is treated in order to remove by-products like tar, ammonia, sulphur, light oil and phenol. The treated coke oven gas is either used as fuel elsewhere in the plant, sold as town gas or flared. Coke oven gas has a heat content of about 500-550 Btu/cu.ft. (GJ/m^3).

Raw iron manufacturing: Blast Furnace inputs to the process of raw iron manufacturing are sinter or pellets. Lump iron ore is sometimes also used.

*This overview of the basic unit operations of iron and steel-making is taken, in some parts directly, form "Emission Control Costs, Iron and Steel Industry," Organization for Economic Cooperation and Development, Paris 1977, p. 14-33.

Each ton of raw iron requires approximately 1.6 tons of sinter or pellets. Sinter, pellets and iron ore contain iron primarily in the form of iron oxide and in order to get raw iron, the iron oxide must be reduced. This reduction is accomplished under high temperature in the blast furnace. The carbon needed for the reduction is added in the form of coke. In order to attain the high temperature necessary for the reduction of the iron oxide and the melting of the metallic iron hot air is injected under pressure at the bottom of the furnace.

One of the major impurities in the iron ore (and in the coke) is silica (SIO_2). This mineral has a very high melting point and if some method of removing the silica from the furnace were not provided, the furnace would rapidly fill up with ashes and become inoperative. This is prevented by the addition of limestone since, the high temperature of the furnace, the lime combines with the silica to form a low melting material called slag. The molten slag is lighter than the melted iron and, therefore, floats on top of the iron. As the iron is tapped from the bottom of the furnace, the floating slag is skimmed off. Molten iron and slag is drawn off from the furnace 5-6 times each day. Depending upon their size, blast furnaces will produce from a few hundred to over ten thousand tons of raw iron per day.

What happens inside a blast furnace during the manufacturing cycle is that when air (preheated to approximately 1 000αC) is blown through the furnace from the bottom, the coke burns and forms carbon monoxide which passes through the furnace. This carbon monoxide is the active agent in the process since it can combine with oxygen. Part of the carbon monoxide generated combines with the oxygen of the iron oxide and forms carbon dioxide. The gas leaving the top of the blast furnace consists of carbon monoxide (25-30%), carbon dioxide (17-19%), hydrogen (1%) and nitrogen (58%). Since this combustible gas has a high load of particulates, it must be cleaned before it can be used as fuel elsewhere in the plant. Cleaning of the blast furnace gas is therefore considered a normal part of the raw iron manufacturing processes. Blast furnace gas has a typical heat content of about 50 Btu/cu.ft.

<u>Direct Reduction</u>: An alternative way of manufacturing raw iron is to use the so-called direct reduction process. Although many types of direct reduction processes have been proposed, only a few have been commercialized so far. A sizeable percentage of the world's installed direct reduction capacity is accounted for by the shaft furnace, the static bed reduction and fluid bed processes, all of which utilize gaseous reductants.

Direct reduction/electric arc furnace combination plants are becoming increasingly attractive in the future as an alternative route to conventional steel-making, especially in the low capacity ranges where the availability of low-cost natural gas or other fossil fuels might offer some favorable economies. A second factor in favor of direct reduction in many locations is that it does not require coking coal. For these reasons, many developing countries are basing their steel industry expansion or new facilities on direct reduction.

<u>Raw Steel Manufacturing</u>: The molten iron which is tapped from the blast furnace has a high content of carbon (3.0-4.5%) and also contains undesirable amounts of silicon, manganese and phosphorus. The process of steel manufacturing aims at oxidizing these materials and reducing the carbon content to between 1.7 and 0.03%. For the different grades of steel there are specifications for the content of carbon as well as the concentration of varous "impurities." If the production of special steels, various alloys elements that contribute to the desired properties of the steel are added.

The three types of steel furnaces in common use today are:

a) the open hearth furnace;

b) the basic oxygen furnace; and,

c) the electric arc furnace.

The Open Hearth Furnace Process: Steelmaking in an open hearth furnace is a relatively old method of production. The furnace consists of a rather shallow, refractory-linked rectangular room with openings at both ends. These openings are interchangeably used for injection of preheated air and evacuation of exhaust gases. Adjacent to these openings are either fuel oil burners or inlet pipes for combustible gas. Underneath the furnace there are four brick-filled chambers for preheating of air and (in some cases) gas.

During the steelmaking cycle, preheated air together with fuel is blown into the furnace from one end for approximately fifteen minutes. The exhaust gases leave at the other end of the furnace and are led through two of the brick chambers where they heat the bricks. After fifteen minutes the direction of the flow is reverse and the heat of the bricks is now used for preheating the incoming air. These changes of flow direction are continued through the whole steelmaking cycle.

Since the open hearth furnace steelmaking cycle is relatively time-consuming (8-10 hours) compared with other steelmaking process oxygen is often blow into the steel bath with a water-cooled lance for 15-90 minutes in order to reduce the cycle time. The oxygen accomplishes three things, namely: 1. reduces the melting time; 2. increases the temperature; and, 3. raises the rate of oxidization. With the use of oxygen, the tap-to-tap cycle time can be reduced to between 5 and 8 hours.

Depending upon the chemical composition of the material with which the furnace is lined, one talks about the acid or the basic open hearth furnace steelmaking process. Characteristic for both processes is the high proportion of steel scrap in the charge. For the acid process, however, it is necessary to use hgh quality steel scrap (i.e., primarily internal steel scrap) and pig iron with a very low phosphorus and sulphur content. The basic process is somewhat less sensitive to the composition of the charge. As in other basic steelmaking processes, however, limestone has to be added to facilitate slag formation.

The Basic Oxygen Furnace Process: In the basic oxygen furnace process, pure oxygen is injected into a charge of melted pig iron and steel scrap through a water cooled lance. The two types of basic oxygen furnaces of commercial significance are the LD (Linz-Donavitz) and the Kaldo converters. The LD converter, which is the most commonly used type, consists of a cylindrical steel shell closed at one end and lined with basic refractory material. The furnace vessel rotates about a lateral axis for charging and tapping. A tap-hole near the mouth of the furnace permits retaining the slag in the furnace during tapping.

Unlike the open hearth and the electric arc furnace processes, no supplementary heat source is provided during the BOF cycle. The only sources of heat are the heat in the hot metal and the heat generated by the reactions between the oxygen and the metalloids in the hot metal. The Kaldo process, in which the furnace vessel also rotates around its veritcal axis, has a somewhat better heat economy than the LD process and can therefore use a higher proportion of scrap. The Kaldo process can also use raw iron with a relatively high phosphorous content, something which is not possible in the LD process. However, the LD process has a shorter tap-to-tap cycle time than the Kaldo process (approximately on hour vs two hours).

In a typical low-carbon steel cycle, the furnace is first tilted for charging with scrap and hot metal and then brought to a vertical position. When the oxygen injection lance is lowered into the furnace, a visible reaction flame begins to leave the furnace mouth almost immediately. Lime and fluorspar fluxes are then added to the furnace through an overhead chute. The reaction flame decreases quite suddenly when the carbon content of the steel has reached about 0.06 percent. In order to make sure that the carbon content has reached the desired level, temperature readings, slag samples, and refined metal samples are taken for analysis. Finally, the furnace is tilted to tap the steel into a ladle. Carbon, ferro-manganese, ferro-silicon, aluminium, etc., may be added to the ladel as tapping proceeds.

The Electric Arc Furnace Process. In the third method of steel manufacture, the electric arc furnace process, a cold charge of steel scrap is melted and refined with electric power as the source of heat. The genral configuration of the electric arc furnace is a shallow-depth, large-diameter, cylindrical shell with a dished bottom. The shell is covered with a removable roof through which are inserted three graphite electrodes. The entire shell and roof are lined with high-performance refractories.

A furnace heat cycle begins with the loading of apartial charge into the top of the furnace. The roof is then closed, and the elctrodes are lowered so that the heat developed by the elctrical resistance of the metal in combination with the heat radiated from the electric arc metls the charge. After the initial charge is melted, the balance of the charge is put into the furnace, and melting and refining proceed to completion of the heat. In order to reduce the process time (which is around four hours), oxygen can be lanced through the heat during a period of 15 to 30 minutes. When the desired chemical properties of the steel have been attained the steel is tapped into a transfer or teeming ladle and the remaining slag is dumped from the furnace into a slag pit. Refractory patching is then done to make the furnace ready for the next heating.

The elctric arc furnace process is characterized by flexibility of operation and close control of heat chemistry. Therefore, it may be used to produce the full range of carbon and medium-alloy structural steels and specialty alloys, stainless teels, tool steels, and superalloys. In the case of high-alloy and special steels, the steel coming from the steel furnace generally needs a final adjustment (or refining) in order to meet quality specifications. The main refining reaction at this stage is dgassing which is generally carried out under vacuum large integrated complex owned by the Egyptian Iron and Steel Company.[9] There are, in addition, a number of smaller steel-making facilities which produce reinforcing iron using open hearth technology. The principal companies are:

	Capacity Metric Tonnes Per Year[10]	
	Steelmaking	Rolling
Cooper Works, Alexandraia	100,000	100,000
Abu Zaabel, Abu Zaabel	100,000	250,000
Delta Steel Co., Cairo	50,000	80,000

Clearly, both the Abu Zaabel complex and the Delta Steel Company will have to buy steel for rolling from the Helwan complex for the time being until and if they are allowed to raise their own steelmaking capacity. Outside of these few rolling facilities, the existing Egyptian Iron and Steel industry consists entirely of the complex at Helwan.

The planning for the complex started in 1955; blast furnace No. 1 was commissioned in 1958 and was followed by No. 2 in 1960. The German made furnaces were identical, with a nominal capacity of 150,000 tpa each when using a charge of Aswan ore.

Aswan ore is rather low in iron content (42-43%) and, in some instances, high in silicon. It is planned to phase out the use of Aswan ore as the El-Gedida ore deposit at Bahariya Oasis is developed.

The energy demand for the facility was furnished in its early days by a dedicated 60 MWe power station, fuel oil and imported coke. Steelmaking was done in 17 ton Thomas converters, of which four were eventually installed, two with each blast furnace. Thomas converter technology was selected because of the high phosphorous content of the Aswan ore. This phosphorous, which remains in the residue in the converters, has always been sold as fertilizer; however, due to its unsuitability for most Egyptian soil conditions, a large portion of it has had to be exported.

As the use of imported coke represented a heavy drain on foreign exchange resources, the next major development at Helwan was the installation of coking plant. The original coking plant, of Russian design and origin,

[9]For a good general discussion of the problems of the Iron and Steel Industry in Developing Countries, see e.g., The Iron & Steel Industry in the Developing Countries, Report of the 3rd Interregional Symposium on the Iron and Steel Industry, Brasilia, Brazil, 1973, U.N. Report ID/139.

[10]All refrence to tonnages in this section are in metric tonnes (1.1023 short tons).

was a 50-oven battery with an annual capacity of 380,000 tonnes. It was commissioned in 1965 using an imported coke blend, as there was no coal of coking quality in Egypt. Some of the coke oven gas from the cokery was used for firing its own ovens, after which the surplus was piped across the road for use in the steel works. A second battery of coke ovens was installed in 1972, raising the coke capacity to 640,000 tons/yr. This moved the gas balance into surplus and led to the establishment of a fertilizer plant (calcium ammonium nitrate) to utilize this excess coke oven gas.

As regards rolling capacity, facilities were commissioned in 1958 along with the first blast furnace given an ability to roll plate, sheet and heavy sections. In 1969, the rolling capability was furhter extended by the installation of a Russian strip mill with a capacity of 300,000 tpa. This facility had to be started using imported slabs because of steel production shortfalls and for quality reasons.

Naturally, some capacity was needed for handling the steel scrap generated in the complex. This was provided by two 12-ton electric arc furnaces which were commissioned in 1958. They utilized both imported and local scrap to give a production of 40,000 tons/yr.

Early energy conservation was achieved by using the blast furnace gas to drive a Hungarian gas turbine power plant. This power station, which can be run on distillate fuel oil as well as blast furnace gas, was started up in the 1960's.

In 1964, the decision was taken to embark upon a major expansion of the Helwan complex to raise the capacity to 1.5 x 10^6 tpa of steel (1.75 x 10^6 tpa pig iron). This was to be achieved by the installation of two new 1,033 m^3 Russian blast furnaces each with a capacity of 670,000 tons/yr (Figure 4.19). This, together with an expansion of the two existing furnaces to 300,000 tons/yr steel (410,000 tons/yr pig iron), would give the following total capacity:

	Annual Capacity (tonnes)
Blast furnaces 1 & 2	410,000
Blast furnace 3	670,000
Blast furnace 4	670,000
	1,750,000

The increase in the capacity of the old blast furnaces was to be achieved by changing to Bahariya ore and using a 100 percent sintered feed.

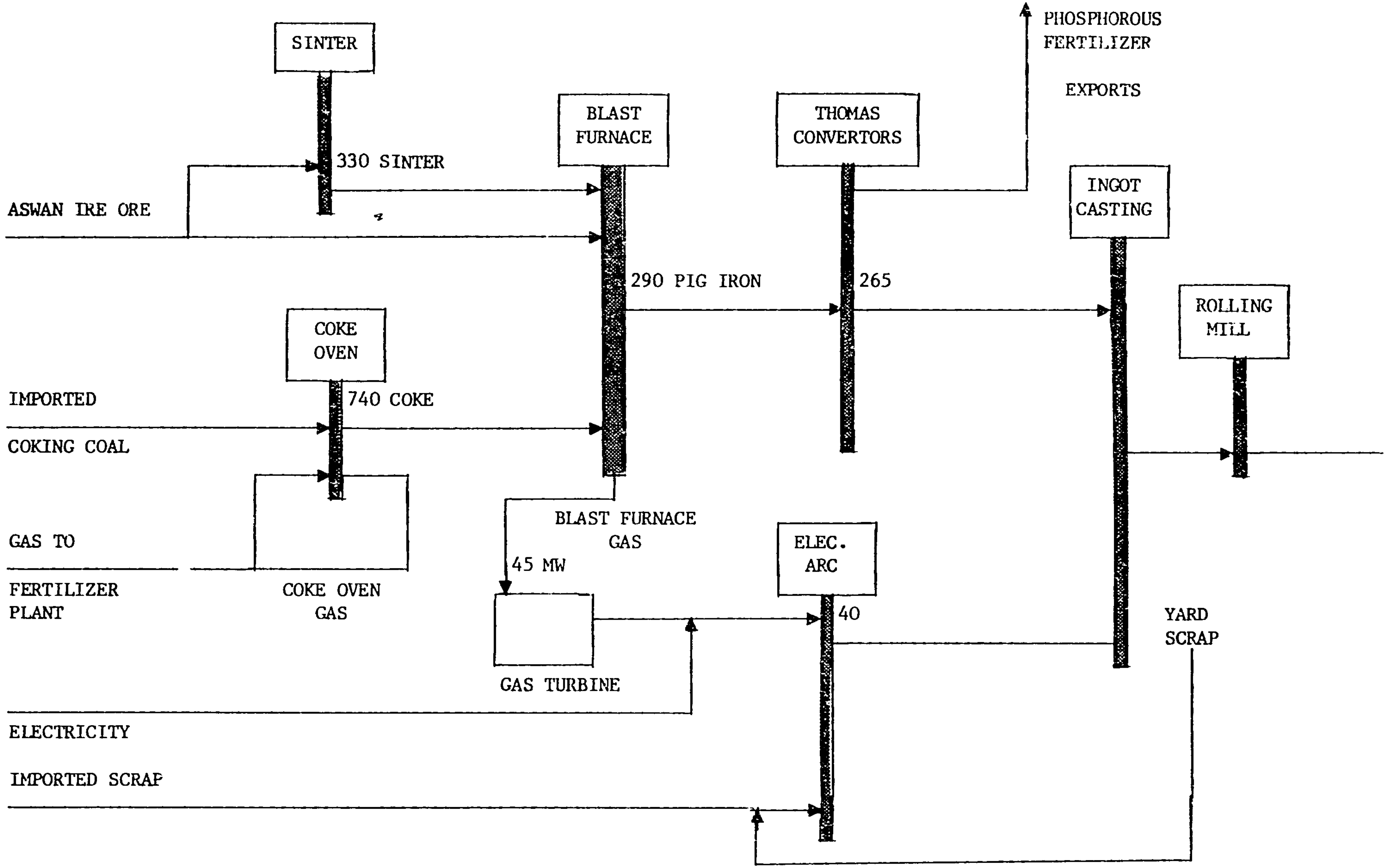


FIGURE 4.19. The Helwan Complex in 1972

The necessary sintering capacity was achieved in 1968 when a Russian sintering plant with a capacity of 30,000 tpa came on stream.

Current planning is aimed at eventually shutting down the Aswan iron ore mine and relying solely upon the better quality Bahariya ore (Fe 52%, Si 8%, lower P). It should be noted tht the iron ore mines but not the cokery are owned by the egyptian Iron and Steel Company. The new mine at El Gedida near the Bahariya Oasis is an open pit operation which started functioning in 1974. It appears that reserves are sufficient for approximately 15 years at current consumption rates. It has a dedicated railway line to the Helwan complex, although it uses normal shunting and marshalling techniques and not the unit train method. The expansion of the iron and steel making capacity is being implemented in two stages, the first of which was completed when No. 3 blast furnace commenced operations in 1974. At this stage, half the new sinter plant (full capacity 3.3 x 10^6 tons/yr), No. 3 blast furnace, and three of the six continuous casters were operational. The second stage which will include the No. 4 blast furnace, is scheduled for start-up in late 1978.

As regards the steelmaking capacity in the expanded plant, this will be provided by three 80-ton basic oxygen furnaces. One of these was started in 1974 along with the No. 3 blast furnace, and the remaining two will be commissioned with the No. 4 blast furnace. Additional coke production will be provided by a new 65-oven battery with a capacity of 550,000 tpa, raising the total coking capacity to 1.25 million tons/yr.

The switch to Bahariya ore has caused some difficulties with the old Thomas converters and it has been found necessary to add phosphate rock to the feed to raise the phosphorous level. A decision on the future of the Thomas converters will be made on receipt of U.S. Steel's study of the whole Helwan complex.

Energy Considerations: In the early days of the Helwan complex, energy consevation was achieved by utilization of blast furnace gas to generate power. At this stage, the only energy sources available to the complex were fuel oil, power and coke.

With the advent of the coking plant, coke oven gas was added to the available energy sources. However, it was found that the supply became somewhat erratic after the ammonium nitrate plant was started up adjacent to the cokery; and since the arrival at the site of natural gas in October 1977, there has been a concerted effort to switch the complex facilities away from coke oven gas. This process is still under way.

The emphasis in the Helwan complex has been more towards reducing the coke rate than on pure energy conservation because of the effect of coke imports on the foreign exchange position. To this end the coking plants were installed, allowing coke imports to be replaced by cheaper coking blends. Although it was stated above that there is no coking coal in Egypt, there is in fact a poor quality deposit in the Sinai. It is a high-ash, high-volatility coal which, it is believed, might be used in a 1:3 mix with high-quality, imported coke blend. Plans under consideration before the 1967 war were to mine this deposit at 300,000 tons/yr. There has been a lot of discussion about the use of local petrocoke in the coking blend, but this is still in early experimental stage. The high-sulphur levels in the coke may well discourage its eventual use.

As mentioned above, the capacity of the old blast furnaces Nos. 1 and 2 were to be raised by injecting fuel oil into the tuyeres. This was, in fact, superceded by the injection of oxygen enriched natural gas. The oxygen concentration is currently 23%, but the target is for 27% as the maximum. No. 3 blast furnace was switched to unenriched natural gas in March 1978, and this has led to an immediate reduction in the coke rate from 750 kg per tonne when injecting fuel oil to 620 kg per tonne with natural gas. Oxygen at Helwan has been provided to date by electric driven air compression and separation unit. The final phase of the expansion will include an additional new oxygen plant, possibly to be based on steam turbine technology.

With the commissioning of the No. 3 blast furnace and the arrival of natural gas in 1977, the complex had access to a diverse range of energy sources, namely:

- coke oven gas
- natural gas
- waste heat from the converters
- blast furnace gas
- fuel oil
- coke
- electric power

They were thus able to -- and in most instances did -- take the opportunity to maximize their energy utilization efficiency. However, all energy considerations at the Helwan complex have to reflect the high foreign exchange component of coke regardless of its relative merits in terms of overall energy efficiency terms.

The two new blast furnaces (Nos. 3 and 4) will be accompanied by six new continuous casters, three for slabs and three for billets. While continuous casting is energy efficient, a utilization level of the three commissioned casters of 25% in 1976 is very low and may reflect an opportunity for improvement.

Assessment of Current Demand and Efficiency: A discussion of energy consumption presently used for the production of iron and steel in Egypt, and the efficiency with which this energy is used, cannot be based on actual operating data. Sufficient data on which to base an analysis is simply not available. In its place, design data for the first- and second-stage developments of the Helwan iron and steel works were used. At the very least, this data will permit an estimation of the kinds of energy sources and the minimum quantities that could be used, provided the plant operates at design conditions. Departures from design conditions and some of their causes will be treated later.

The Helwan iron and steel mill was originally designed to operate on three energy sources: coking coal, fuel oil, and electricity. Actually, fuels such as coke oven gas and blast furnace gas are also used, but these represent fuels derived from coal (or coke). Table 4.1 shows the quantities of each energy source used in the integrated production of iron and steel and the total thermal energy equivalent of each. Total energy input to the mill is seen to be 34.98 million GJ or 39.89 GJ per tonne of steel produced. In addition to the 877,000 tonnes of steel capacity, the energy shown also accounts for 83,000 tonnes of surplus pig iron which is sold.

Conversion fuels, which leave the plant for consumption elsewhere or are used internally to generate electric energy, are shown as credits for the process. The total energy credit of 3.54 GJ reduces the energy consumed to a net total of 31.44 million GJ or 35.85 GJ/tonne steel.

It is difficult to compare this net specific energy consumption with U.S. practice, because energy usage is sensitive to the mix of final products

Table 4.1
Fuels and Energy Sources Used to Produce
Iron and Steel at Helwain in 1978

	Quantity Per Tonne of Steel	Total Quantity	Energy (10^6 GJ)
Input Fuels and Energy Sources			
Coking coal	1.099 t/t	964,000 tonnes	29.34
Fuel oil	0.136 t/t	118,900 tonnes	5.14
Electricity	158.5 kWh/t	139 (10^6) kWh	0.50
Total Consumption			34.98
Fuel Credits			
Blast furnace gas	52.2 Nm^3/t	51(10^6)Nm^3	0.18
Coke oven gas	122.0 N,^{3}/t	107(10^6)Nm^3	1.99
Tars and pitch	0.035 t/t	31,000 tonnes	1.08
Light oil	0.008 t/t	7,000 tonnes	0.29
Total Credits			3.54

$$\text{Total energy consumption} = \frac{34.98 \times 10^6 \text{ GJ}}{877{,}000 \text{ tonnes of steel}} = 39.89 \text{ GJ/tonne}$$

$$\text{Net energy consumption} = \frac{31.44 \times 10^6 \text{ GJ}}{877{,}000 \text{ tonnes of steel}} = 35.85 \text{ GJ/tonne}$$

produced. However, U.S. energy consumption for all stages through to finished products can vary anywhere from a low of 35 GJ/tonne of structural mill products to a high of 49 GJ/tonne of tin-plated steel strip. On this basis the energy consumption shown for Egyptian steel based on design data is quite good.

Actual energy consumption is probably considerably above that calculated from design data. Actual production of steel in 1976 was 700,000 tons compared to almost 900,000 of capacity. Operating a mill below its rated capacity will always lead to less efficient use of energy. Also, the continuous casting equipment, which should serve to produce 600,000 tonnes of BOF steel, is reported to have been used only 25% of the time. The use of the cast ingot route will considerably increase actual energy used relative to design estimates. Apparently there also has been some difficulty with the high salt and silicon content in the El-Gedida ore, which has caused some upsets in the operation of the blast furnaces. Finally, the low phosphorus El-Gedida ore has required that this element be artificially added to the Thomas furnaces in order to produce good quality steel.

Discussion of options: As mentioned previously, tentatively planned new steel capacity has been proposed using two technologies: the traditional blast furnace-BOF steel convertor route and the merging direct reduction-electric furnace route. Although there is general agreement that the direct reduction-electric furnace route does not necessarily reduce total energy consumption in steelmaking, it does have one significant virtue for Egypt's emerging iron and steel industry. Most direct reduction processes for the production of sponge iron substitute a reducing gas--generally reformed natural gas--for the coke used in the blast furnace process. This, of course, is an important consideration due to the lack of metallurgical coke in Egypt and the general availability of natural gas.

It is logical that any alternative to steel production by the presently existing technology should involve exploiting fuels and energy sources readily available in Egypt. These energy sources include natural gas, fuel oil and electricity, although there is some concern about the future availability of electricity. Traditional thinking invariably couples direct reduction iron with steel produced in the electric furnace, because of the view that sponge iron is the equivalent of scrap steel. However, it is entirely possible to produce steel from sponge iron by the open hearth process and thereby replace electric energy used by the electric furnace with fossil fuels. Although a greater amount of total energy per ton of steel produced is required by the open hearth process than by the electric furnace, a reduction in electric energy usage would result if open hearth technology is principally used.

Based on these considerations, there are four options that should be explored with respect to developing the Egyptian steel industry.

1. A major expansion of steel production through the direct reduction-electric furnace route and a minimization of iron produced by blast furnaces. This option will reduce the industry's dependence on coking coal, which must be imported, and require the use of energy resources available, namely, natural gas, fuel oil and electricity.

2. A similar processing sequence could be based on direct reduction followed by the use of an open hearth steel furnace fired by fuel oil and operating in 100% cold charge.

3. A large part of the burden to produce iron can be avoided by importing major quantities of iron and steel scrap. This option will reduce fuel oil and natural gas consumption and

require no more electricity than would be used by electric steel furnaces operating on iron made by direct reduction.

4. The possibility exists of reducing the steel industries dependence on imported coking coal by finding suitable alternatives more readily available. As mentioned before, in the Sinai a poor quality coal exists and could be blended with imported coal for the production of coke. Also, there is the possibility that petroleum coke, if it could be produced in sufficient quantity and of the appropirate quality, could be used to replace or diminish the need for imported coking coal to support blast furnace operations.

In addition, there is a fifth option for improving energy efficiency. That is the promotion of rigorous energy conservation programs in all plants. Among the measures which may prove effective in reducing energy consumption are better combustion operations (minimum excess air, combustion air preheat, etc.), scrap preheating, maximum use of continuous casters, insulation of reheat furnaces and soaking pits, and installation of furnace recuperators. A target for energy efficiency improvement for 9% from 1972 to 1980 has been set for the U.S. primary metals industry, of which 3% has been achieved. It is reasonable to expect that a 5% improvement is achievable by "old" plants in Egypt by 1985, and a further 10% by the year 2000. The capacity of these old plants corresponds to about 300,000 tons/yr steel production. It is not believed likely that major energy efficiency savings can be achieved from the newest plants, and therefore potential energy svaings, based entirely on 300,000 tons/yr steel production, are estimated as follows:

	"Old Plant" Energy Consumptions	Annual Savings for 1985 (5%)	Additional Annual Savings by 2000 (10%)
Coking coal tons/yr	330,000	16,500	33,000
Fuel oil tons/yr	40,800	2,000	4,100
Electricity million kWh	47,500	2,400	4,800

In the projections of energy efficiency for 1985 and 2000 which follow, these savings have not been included.

4.4.3 Formulation as a Linear Program

In addition to the prevIsouly described planned expansion at Helwan, which is expected to be completed by 1982, there is a tentative schedule of new iron and steel facilities in Egypt to the year 2000. Although there is no commitment to locations or the type of facilities, present concepts indicate the rate of growth and the anticipated technology which might be employed.

Year	Site	Technology	Steel Capacity (10^6 tons)
1982	Helwan	Blast furnace, BOF steel	0.8
	El Dekhela, Alexandria	Direct reduction, electric steel	0.8
	El Sadat City*	Direct reduction, electric steel	0.8
1987	Nubiria Canal*	Blast furnace, BOF steel	2.0
1990	El Dekhela*	Direct reduction, electric steel	1.6
	El Sadat City*	Direct reduction, electric steel	1.6
	---	Direct reduction, electric steel	1.8
2000	Helwan*	Blast furnace, BOF steel	1.5
	Nubriria Canal*	Blast furnace, BOF steel	2.0
	---	Direct reduction, electric steel	1.8
		Total increase	14.7×10^6 tons

(* Tentative)

Rather than simply accept the above schedule as given, the purpose of the model is to examine the sensitivity of energy consumption to such process choices in a systematic manner. Indeed, we might even be interested in the energy consequences of the overall year 2000 production target of 14.7 tons. For purposes of the model, we shall use the term "existing" as corresponding to the Helwan configuration as projected for the end of 1982, as illustrated in Figure 4.20.

Total steel capacity: Total steel production capacity by the year 2000 is expected to be X = 14.7 mt/yr. Let the steel production in different processes, X_j, be defined as follows:

	Existing	New
Basic Oxygen	X_1'	X_1
Electric Arc	X_2'	X_2
Open Hearth	X_3'	X_3
Open Hearth with Oxygen Lancing	X_4'	X_4
Thomas Converter	X_5'	X_5

hence

$$\sum_{j=1}^{5} X_j + \sum_{j=1}^{5} X_j' = X$$

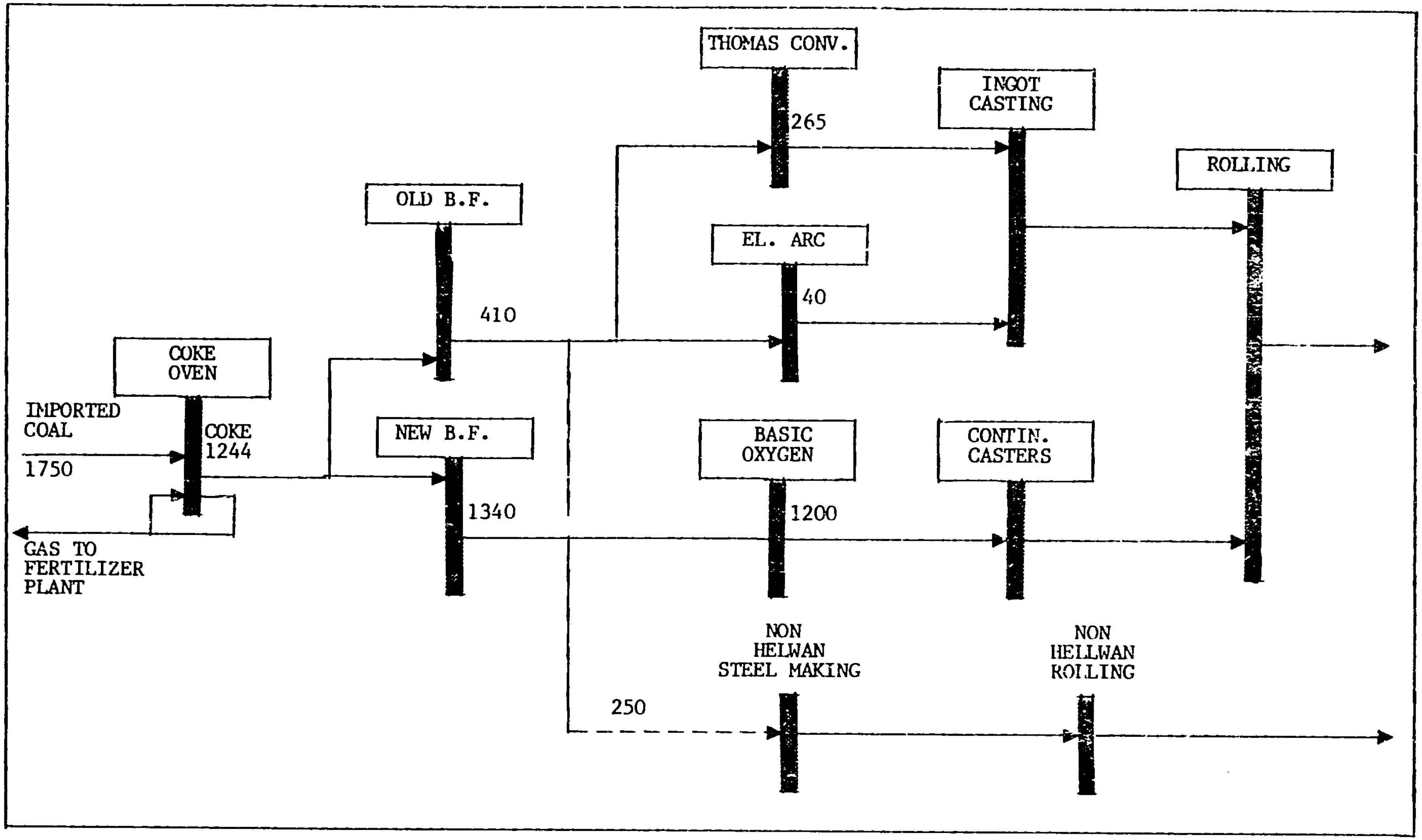


Figure 4.20. The Helwan Complex by 1982

Production in each facility must be less than or equal to capacity: if we denote the capacity variables by $\bar{X}$, then we have for new facilities the constraint set

$$X_j \leq \bar{X}_j \cdot \pi_j(S) \qquad j=1,\ldots 4$$

where $\pi_j(S)$ is the capacity factor for the j-th steel making process. In constraint form we therefore have

$$X_j - \pi_j \bar{X}_j \leq 0 \quad ; \; j=1,\ldots 4 \quad .$$

And for production in existing facilities (from Figure 4.19)

$$\begin{aligned} X_1' &\leq 1200 \\ X_2' &\leq 40 \\ X_3' &= 0 \\ X_4' &= 0 \\ X_5' &= 265 \end{aligned}$$

Iron making: There are three sources of feedstock for the steelmaking facilities: Blast furnace Pig Iron, Direct Reduction Iron, and Scrap. Let us ignore the latter for didactic simplicty. Then let iron production in the different processes, Z_j, be defined as

	Existing	New
Blast Furnace with Natural Gas	Z_1	Z_1'
Blast Furnace with Oxygen Enriched Natural Gas	Z_2	Z_2'
Blast Furnace with Fuel Oil	Z_3	Z_3'
Direct Reduction	Z_4	Z_4'

Then capacity constraints follow in analo to steelmaking as

$$Z_j - \bar{Z}_j \cdot \pi_j(I) \leq 0$$

and for existing plants, from Figure 4.19

$$\begin{aligned} Z_1' &\leq 1340 \\ Z_2' &\leq 410 \\ Z_3' &\leq 0 \\ Z_4' &\leq 0 \end{aligned}$$

Total iron production must equal total input to steelmaking processes, i.e.,

$$\sum Z_j + \sum Z'_j = \sum X_j + \sum X'_j$$

or, if it is assumed that existing steelmaking plants are fed entirely be existing iron making capacity, we have two constraints

$$\sum Z_j = \sum X_j$$
$$\sum Z'_j = \sum X'_j$$

It is also necessary to make sure that direct reduced iron is limited to electric arc and/or the open hearth processes: hence

$$\sum_{j=2}^{4} X'_j + \sum_{j=2}^{4} X_j \geq Z_3 + Z'_3$$

E8. Impact of Energy Taxes

Using graphical analysis, discuss the short-run impacts of imposing a tax on kerosene, given a completely inelastic demand for kerosene, and a flat supply curve (assuming kerosene is imported at the world price).

E9. Import Quotas

Using graphical analysis, discuss the impact of an import quota on imported oil. Under what circumstances would an import quota system lead to a black market?

5. ENERGY SYSTEM OPTIMIZATION MODELS.

Consider the simple Reference Energy System depicted on Figure 5.1. The end use demands shown on the right, D_1, D_2, and D_3 are known; as in our previous discussion of Reference Energy Systems (RES) in Chapter 3, the system is demand driven. Recall that in the RES, intermediate fuel and supply variables are defined by the series of matrix transitions discussed in Section 3.2. However, the coefficients of those transition matrices that represent the market shares of each fuel meeting end use demand categories, or the electric generation mix, require exogenous specification by the analyst-which translates, in practice, to a great deal of judgement on the part of the analyst in an attempt to bring supply and demand into balance. Indeed, in many early developing country energy assessments, such supply-demand balances were derived manually even for future years.

Suppose, however, that we wish to replace "judgement" by a more precise criterion: in particular we wish to determine that set of values of the endogenous variables that minimizes (or maximizes) some mathematically

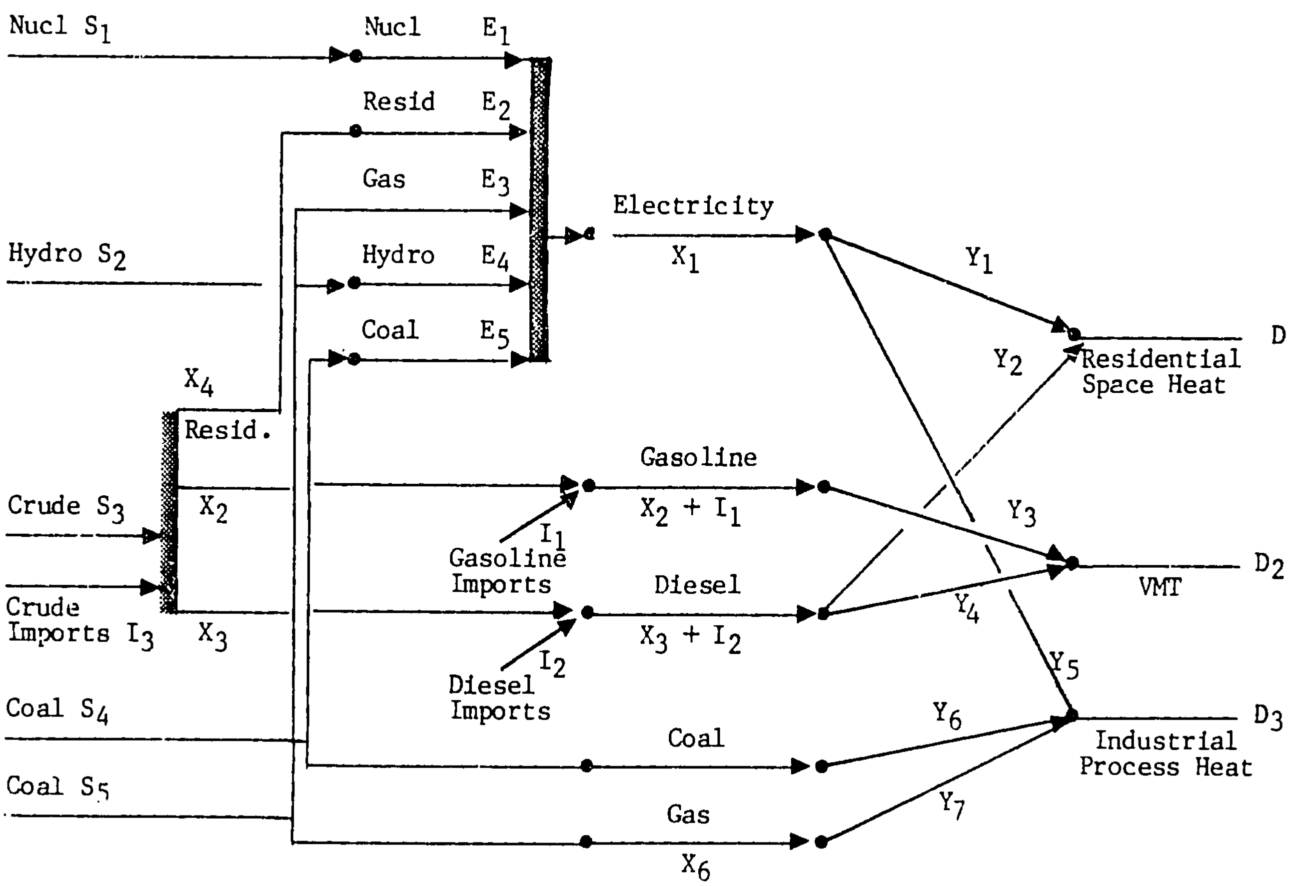


Figure 5.1. A Sample RES

defined objective: minimum total annual cost, minimum capital requirement, minimum net present worth, among other possible criteria. Such an objective can be established as a mathematical function of the mix of technologies and fuels used to satisfy the given end use demand, and is referred to as the "objective function." Obviously this optimization will in general be constrained: since all of the variables in the RES represent real activities, all variables must be constrained to be greater than or equal to zero (the "non-negativity constraint set"). Further, a series of constraints must be imposed to ensure that sufficient energy supplies are brought into the system to satisfy the exogenously specified end use demands. Where these constraints are linear, and where the objective function is linear, we obviously have a linear programming problem of the type introduced in Section 2.4.

Suppose, then, that in our example the applicable objective is the minimization of total system cost. The objective function is therefore given by

$$\text{Min } S = \sum_i c_{xi'} X_i + \sum_j c_{Yj}\, Y + \sum_k c_{sk} S_k + \sum_\ell c_{I\ell} I_\ell \tag{5.1}$$

where the c_{ji} represent the costs per unit of activity. Note that the cost functions are strictly linear, passing through the origin (herein lies one of the more serious problems in LP - the inability to take into account economies of scale, and fixed charges that are independent of the level of activity. We return to this problem later on). We turn now to each of the constraints in the LP.

<u>End Use Demand-End Use Device</u>: Each end use demand can be met by different end use devices, each of which may have a different efficiency (and cost). For each of the three end use demands, energy balance requires the following equalities:

$$\underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{D_1} = \underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{Y_1} \cdot \underset{\left[\frac{\text{Btu}}{\text{Btu}}\right]}{\frac{1}{\epsilon_1}} + \underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{Y_2} \; \underset{\left[\frac{\text{Btu}}{\text{Btu}}\right]}{\frac{1}{\epsilon_2}} \tag{5.2}$$

$$\underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{D_2} = \underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{Y_3} \; \underset{\left[\frac{\text{VM}}{\text{Btu}}\right]}{\frac{1}{\epsilon_3}} + \underset{\left[\frac{\text{Btu}}{\text{yr}}\right]}{Y_4} \; \underset{\left[\frac{\text{VM}}{\text{Btu}}\right]}{\frac{1}{\epsilon_4}} \tag{5.3}$$

$$D_3 = Y_5 \cdot \frac{1}{\epsilon_5} + Y_6 \cdot \frac{1}{\epsilon_6} +$$

$$\left[\frac{\text{lb Steam}}{\text{yr}}\right] \quad \left[\frac{\text{Btu}}{\text{yr}}\right] \left[\frac{\text{lb Steam}}{\text{Btu}}\right] \quad \left[\frac{\text{Btu}}{\text{yr}}\right] \left[\frac{\text{lb Steam}}{\text{Btu}}\right]$$

$$Y_7 \quad \frac{1}{\epsilon_7}$$

$$\left[\frac{\text{Btu}}{\text{yr}}\right] \left[\frac{\text{lb Steam}}{\text{Btu}}\right] \tag{5.4}$$

Intermediate Energy Balances: For each of the 5 nodes in the second node set of Figure 5.1, we have one emergy balance equation

$$X_1 = Y_1 + Y_5 \tag{5.5}$$

$$X_2 + I_1 = Y_3 \tag{5.6}$$

$$X_3 + I_2 = Y_2 + Y_4 \tag{5.7}$$

$$X_5 = Y_6 \tag{5.8}$$

$$X_6 = Y_7 \tag{5.9}$$

The first of these equations represents the balance for electricity: since transmission and distribution losses may be substantial, and differ according to user class (lightly loaded, low voltage rural lines incur higher losses than a large industrial user connected directly to the gird). Thus in place of 5.5 we write

$$(1 - t_G)X_1 = \frac{1}{(1 - t_{D1})} Y_1 + \frac{1}{(1 - t_{D2})} Y_5 \tag{5.10}$$

where

t_G is the loss factor for the high voltage grid (2-3%)

t_{Dj} is the loss factor for distribution to the j-th user class (1-12%)

Note that X_1 denotes the quantity of electricity at the generation busbar; the Y_j denote the quantities of electricity at the respective user meters.

Electric Sector: The total electricity output at the busbar, in Btu, is X_1. This is made up of contributions from the different generation types, i.e.

$$\sum_j E_j = X_1 \quad \frac{1}{3412} \tag{5.11}$$

$$[\text{kwh}] \quad [\text{Btu}] \quad \left[\frac{\text{kwh}}{\text{Btu}}\right]$$

where E_j is the output of generation type j, which is in turn related to fuel input by the appropriate heat rate; for example, for residual oil plants we have

$$E_2 = \frac{1}{h_2} \cdot X_4 \tag{5.12}$$

$$[\text{kwh}] \quad \left[\frac{\text{kwh}}{\text{Btu}}\right] \quad [\text{Btu}]$$

where h_4 is the heat rate in Btu/kwh

Refinery: Here the refined product outputs are related to the crude input by the socalled yield coefficients, which measure the units of crude oil necessary to produce a unit of refined product:

$$S_3 + I_3 = \frac{1}{W_4} \cdot X_4 + \frac{1}{W_2} \quad X_2 \quad + \ldots$$

$$\left[\frac{\text{Btu}}{\text{Crude}}\right] \quad \left[\frac{\text{Btu Crude}}{\text{Btu Resid}}\right] \quad [\text{Btu Resid}] \quad \left[\frac{\text{Btu Crude}}{\text{Btu gas}}\right] \quad [\text{Btu gaso}] \tag{5.13}$$

Of course, as we shall see later on, this is a considerable simplification of a real refinery: we make no distinction for the moment among different crude types, and we impose no special constraints on the mix of refined products that might be available from a particular crude. It might be noted, however, that many of the best known energy system models currently in use do not even make the elementary distinction between different kinds of refined products.[1]

Supply Constraints: The variables representing domestic energy supply are typically constrained according to whatever resource bound may be present. Each energy supply variable S_i may thus be bounded by a limit S_i^*, i.e.

[1]For example, the widely used BESOM model (Brookhaven Energy Systems Optimization Model), and its derivatives DESOM and TESOM (for dynamic and timestepped optimization model, respectively), make only a distinction between crude oil and the summary category "refined products." Only the more recent MARKAL model, developed at Brookhaven in collaboration with the International Energy Agency, and the BEEAM II model (see Chapter 10), make the necessary distinctions between different refined products, and recognize the only limited degree of substitutions among them.

$$S_i \leq S_i^* \tag{5.14}$$

Existing Capital Stock: A final constraint set will account for the existing capital stock. For example we may wish to make sure that existing nuclear plants are fully utilized, thus

$$S_1 \geq S_1' \tag{5.15}$$

where S_1' is the existing generation in nuclear plants, and S_1 is the model forecast for nuclear generation in the scenario year. Such simple capital stock constraints are not without their problems; a more reasonable (but necessarily more complicated) formulation is deferred to section 5.2.

Our LP is now complete: the constraints (5.2) to (5.15) will ensure that the end use demands are in fact met (assuming that the set of supply constraints are not so low that mathematical infeasibility occurs). In a real application a typical energy system LP may have several hundred to over a thousand constraints, and several thousand variables; modern computers process such problems with little difficulty.[2]

[2]The coefficient matrix of such LP problems is typically quite sparse, making them particularly suited to such proprietary algorithms as the Control Data Corporation APEX code, which can devour a thousand row LP with several thousand variables in a matter of seconds. Even on relatively small machines, virtual storage capability of modern computers makes obsolete the cumbersome overlay procedures once necessary. Indeed, it can be stated without much risk that the real limit to solving very large LP's lies not so much in the storage and computational bounds of modern machines, as much as in the analyst's ability to comprehend and interpret the output.

In the previous section we noted that it was desirable to take into consideration the existing capital stock, with the justification that it was probable that, say, an existing nuclear plant would still be in service in the planning year. However, a constraint of the type

$$S_1 \geq S_1'$$

raises a number of objections; what we really wish to do is to make the distinction between the energy variable (the Btu's or Kwh in the RES) and the capacity variable, which has dimensions not of energy but of power (i.e. Mw, rather than Btu). Thus the type of constraint that is required is one that ensures that the energy output of an existing facility does not exceed its capacity. Hence if we let $\bar{E}_1$ be the capacity of existing nuclear power plants then the applicable constraint becomes

$$\underset{\left[\frac{kWh}{yr}\right]}{E_1} \leq \underset{[Mw]}{\bar{E}_1} \quad \underset{\left[\frac{hr}{yr}\right]}{8760} \quad \underset{[\ ]}{PF} \quad \underset{\left[\frac{kW}{MW}\right]}{1000} \tag{5.16}$$

where PF is the plant factor.. Because the future captial stock is likely to have quite different technological and economic characteristics than the existing capital stock, it becomes desirable to make explicit the distinction between old and new, especially on the supply side for the electric and refinery sectors. Taking the electric sector as an example, we thus rewrite Eq. (5.11) as

$$\sum E_j + \sum E_j' = X_1 \frac{1}{3412} \tag{5.17}$$

where E_j is now the electricity generated in new plants of type j, and E_1' the electricity generated in existing plants of type j. In addition we require a capacity constraint for each new plant type, i.e.

$$E_j' \leq \bar{E}_j' \cdot 8760 \cdot PF \quad \text{for all } j \tag{5.18}$$

where $\bar{E}_j'$ is the new capacity of type j; since this is also an endogenous variable, which should appear on the left hand side, the socalled "constraint" form of (5.18) is written

$$E_j' - \bar{E}_j' \, 8760 \; PF \leq 0 \quad . \tag{5.19}$$

Notice that the LP now decides to what extent the existing capacity is utilized; for example, in Eq. (5.16), if $E_1 = 0$, the implication is that the existing plant is retired. If $E_1 > 0$, but the corresponding capacity constraint is not binding, the implication is that the existing facilities are used only partially.

It is now time to consider more carefully the objective function. Thus far we have simply noted that each endogenous variable has associated with it some unit cost (recall Eq. 5.1). Assume for the moment that we wish to minimize the net annual cost of the energy system. It follows that the objective function must have dimensions of \$ per year. Therefore, writing in the dimensions of the terms of Eq. 5.1, we have

$$\min S = \underbrace{\sum_i C_{xi} X_i}_{\text{Intermediate Energy}} + \underbrace{\sum_j C_{Yj} Y_j}_{\text{End Use Devices}} + \underbrace{\sum_k C_{sk} S_k}_{\text{Domestic Supplies}}$$

$$\left[\frac{\$}{\text{yr}}\right] \quad \left[\frac{\$}{\text{Btu}}\right]\left[\frac{\text{Btu}}{\text{yr}}\right] \quad \left[\frac{\$}{\text{Btu}}\right]\left[\frac{\text{Btu}}{\text{yr}}\right] \quad \left[\frac{\$}{\text{Btu}}\right]\left[\frac{\text{Btu}}{\text{yr}}\right]$$

$$+ \underbrace{\sum_\ell C_{I\ell} I_\ell}_{\text{Imports}} + \underbrace{\sum_n C_{En} E_n}_{\text{Electric Generation}} \tag{5.20}$$

$$\left[\frac{\$}{\text{Btu}}\right]\left[\frac{\text{Btu}}{\text{yr}}\right] \quad \left[\frac{\$}{\text{Kwh}}\right][\text{kwh}]$$

The interpretation of the $C_{I\ell}$ is the most straight forward: in the absence of any special bilateral supply arrangements, these are simply the world market prices (plus transportation costs to port of entry).

Before one can specify the remaing cost coefficients one must consider from whose perspective the system is being optimized. As we shall see later on in Chapter 9 in the context of refinery operation, it makes a great deal of difference as to whether the optimization is from the perspective of a private sector enterprenuer in which domestic fuel costs are simply extant market prices, or whether the perspective is that of a central government, for which use of price as an energy system optimization may be quite misleading. For example, the owner of a private coal mining enterprise faces a market clearing price for coal that is related to world market oil prices, and that has little to do with his actual cost. To be sure, in most developing countries mineral resources are in the hands of some nationalized enterprise, precisely in order to ensure that any such market induced rents accrue to society as a whole, rather than to some fortunate capitalist. Nevertheless, the distinction between price and cost is an important one, that is frequently forgotten.[3]

From all this it becomes fairly obvious that the coefficients C_{En}, C_{Yj} : represent the non-fuel operating and maintenance costs for electric plans and end use devices, respectively; and the C_{sk} represent the unit costs of domestic fuel resources, f.o.b. mine. In the case of hydropower the C_{sk} might represent the power sector's share of any upstream regulation or watershed management scheme, and in the case of solar technologies they are simply zero. Fuel markups due to transportation are imposed on the intermediate fuels: thus, for example, the cost coefficient C_{xi} for natural gas represents the non-fuel operating costs of gas pipelines (compressor operation, distribution system maintenance, etc.), or the C_{xi} for coal represents average rail freight cost for delivery to end users.

How are capital costs entered into the objective function? Clearly if the objective function has dimensions of \$/yr, one must annualize the capital costs using the capital recovery factor (see Chapter 7). In general, we would always include capital stock variables for electric, intermediate fuel, and end use sectors; and for certain imports where appropriate (e.g. LNG imports require expensive terminal facilities). Whether one also applies capital costs to domestic fuel sectors depends on the level of information. One option is to simply constrain domestic resources at the level believed sustainable from existing mines, wells etc.; and then examine the shadow price for this constraint (as described in the next section). Alternatively one must set up an explicit supply curve, which in general will be upward sloping to reflect the increasing cost of incremental supply (oil wells get more expensive as one moves further offshore, mines become more expensive as ever deeper, or narrower seams must be exploited). The linearization of supply curves is illustrated in example 5.1. Thus, finally, we add to the objective function the terms

[3]Although, as we shall also see later on, when the optimization is from the perspective, say, of the state owned electricity enterprise, it, too, would use market prices for fuels it must purchase from other nationalized sectors. Therefore, where subsidization of petroleum products is used for some social or developmental objective, great caution must be exercised in selecting the appropriate measures.

$$\mathrm{CRF}\ (i,n) \left\{ \sum_x \alpha_{xi} X_i + \sum_j \alpha_{Yj} Y_j + \sum_k \alpha_{sk} \bar{S}_k + \sum_\ell \alpha_{I\ell} \cdot \bar{I} + \sum_n \alpha_{En} \bar{E}_n \right\} \tag{5.21}$$

where

CRF (i,n) is the capital recovery factor (for interest rate i and amortization period n)

α_{ji} is the capital cost for the corresponding capital stock variable j_i.

The capital requirement is given by the expression

$$\sum \alpha_{xi} X_i + \sum \alpha_{Yj} Y_j + \sum \alpha_{sk} S_k + \sum \alpha_{I\ell} I_\ell + \sum \alpha_{En} E_n \le C \tag{5.22}$$

where we write this as an inequality constraint wherein C is the total capital available for energy sector investment.[4] Where such a capital limitation is not imposed, we keep Eq. (5.22) as a socalled "free" row in the LP, in which (5.22) becomes an unconstrained accounting equality.[5]

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[4] Caution is required here: (5.22) represents the undiscounted capital requirement over the planning horizon. This is unavoidable in a static model applied to some future year: recall that a static model will establish the optimum system in the scenario year, but says nothing about how the system reached that optimum.

[5] Most modern LP solution algorithms allow the use of any free row as the objective function, often by a simple control card change. Energy system LP's are often run in "multiobjective" fashion to establish the trade-offs between different objectives. Employment maximization, and the minimization of investment capital and Foreign Exchange requirements are the most frequently encountered in developing country applications.

Associated with every linear programming problem, the primal, is another LP known as the dual; if the primal is a problem, the resulting dual is a maximization problem. Thus, if the primal is

$$\min z = \sum_j C_j X_j, \text{ or, in vector notation}$$

$$= \underset{(1\mathrm{x}m)}{C^T} \underset{(m\mathrm{x}1)}{X} \tag{5.23}$$

$$\text{s.t} \quad \underset{(n\mathrm{x}m)}{A} \underset{(m\mathrm{x}1)}{X} \geq \underset{(n\mathrm{x}1)}{b}$$

$$X \geq 0$$

then the dual is

$$\max z = \underset{(1\mathrm{x}n)}{b^T} \cdot \underset{(n\mathrm{x}1)}{\pi} \tag{5.24}$$

$$\text{s.t} \quad \underset{(m\mathrm{x}n)}{A^T} \underset{(n\mathrm{x}1)}{\pi} \leq \underset{(m\mathrm{x}1)}{C}$$

$$\pi \geq 0 \quad .$$

Note that there is one dual variable for each row in the primal; and one constraint in the dual for each variable in the primal.[6] Suppose that one of the constraints in the primal is binding: in the context of our energy system LP, suppose that one of the domestic supply constraints of the type

$$S_i \leq S_i^*$$

is binding, which means that all of the available resource is used. One might then ask how the optimal solution would change if this binding constraint were relaxed by 1 unit. Mathematically, one is interested in the partial derivative of the objective function with respect to the binding constraint; the value of this derivative is known as the shadow price,

[6]This has a number of important computational implications: if the primal has more constraints than variables, say 10 constraints and 3 variables, then it is more advantageous to solve the LP using the dual. The number of constraints determines the size of the basis, and a basis of dimensions 3 x 3 obviously is much easier to manipulate than one of dimensions 10 x 10. In most of our energy system applications, however, the number of variables in the primal exceeds the number of constraints, and we cannot make use of this property of the dual.

which is thus defined by

$$\pi(s_i^*) = \frac{\partial z}{\partial(s_i^*)} \quad . \tag{5.25}$$

In a minimzation problem, relaxation of a binding constraint will <u>reduce</u> the objective function value, and thus the shadow prices are negative. But note that the shadow prices have meaning only at the margin: increasing the value of S_i^* by one numerical unit may or may not decrease the objective function by $\pi(S_i^*)$. This is because some other constraint may become binding, and other variables may enter the solution, before S_i^* has been increased by one unit. Whenever a constraint is non-binding, the corresponding shadow price is zero (since a change in the RHS of a non-binding constraint will not, at the margin, affect the value of the objective function).

Given that the shadow prices of the primal are equivalent to the variables of the dual LP, what interpretation does the Dual have? If the primal determines the optimal mix of activities such that the overall system cost is minimized, then the dual determines the price structure that will optimally allocate a resource constrained system to maximize system output.

We can summarize the primal of our Energy System LP as follows

$$\left.\begin{array}{lrl} \text{Min.} & C_1X_1 + C_2X_2 + C_3X_3 & \text{(objective function)} \\ \text{s.t.} & G_1X_1 = D & \text{Demand Constraint} \\ & G_2X_2 \leq S & \text{Supply Constraint} \\ & G_3X_1 + G_4X_3 = 0 & \text{IEF/Demand Constraint} \\ & G_5X_2 + G_6X_3 = 0 & \text{IEF/Supply Constraint} \\ & X_1, X_2, X_3 \geq 0; & \text{non-negativity} \end{array}\right\} \tag{5.26}$$

where

X_1 Demand variables

X_2 Supply variables

X_3 Intermediate Energy Form (IEF) variables

C_j vectors of cost coefficients

G_j coefficient matrices

The demand constraint ensures that energy demands will be met, whilst the supply constrain ensures that supply does not exceed domestic resources. The IEF/supply constraints capture the relationship between supply variable and IEFS (e.g., electric generation); whilst the IEF/demand constraints represent the allocation of fuels (and end-use devices) to the individual demand variables. In defining the dual of this problem, one should note first that any constraint of the type

$$Ax \leq b$$

can be converted to a greater than or equal to inequality by multiplication by minus 1; i.e.,

$$-Ax > -b$$

bringing it to the form of (5.23). Second, it can be shown that if the i-th constraint in the primal is an equality, then the corresponding dual variable λ_i is unrestricted in sign; in a minimzation problem, if the i-th constraint is a (greater than or equal to) inequality, the λ_i must be ≤ 0. Finally, note that even though each constraint, including equality constraints, has associated with it some shadow price (dual variable), those corresponding to constraints with zero on the right hand side will not be part of the objective function of the dual -- since the objective function coefficient for that row of the primal is zero. Since our zero equality constraints represent energy balances, the corresponding shadow prices for these rows have no particular economic interpretation, since the equality is absolute.

Writing down the dual is easier if we first rewrite (5.26) as an ordered matrix equation, i.e.,

$$\begin{array}{llllll}
\text{Min.} & C_1X_1 + C_2X_2 + C_3X_3 & & & & \text{Objective function} \\
\text{s.t.} & G_1X_1 & & & = D & \text{Demand Constraints} \\
 & & G_2X_2 & & \leq S & \text{Supply Constraints} \\
 & G_3X_1 & & + G_4X_3 & = 0 & \text{IEF/Demand Constraint} \\
 & & G_5X_2 & + G_6X_3 & = 0 & \text{IEF/Supply Constraint}
\end{array}$$

Since the constraint set for the dual requires transposition of (5.27), the dual can be stated as

$$
\begin{array}{llllll}
\text{Max.} & D^T\pi_D + S^T\pi_S & & & & \\
\text{s.t.} & G_1^T\pi_D & & + G_3^T\pi' & & \leq C_1 \\
 & & + G_2^T\pi_S & & G_5^T\pi'' & \leq C_2 \\
 & & & G_4^T\pi' & G_6^T\pi'' & \leq C_3
\end{array}
\tag{5.28}
$$

where

π_D dual variables associated with demand constraints

π_S supply constraints

π' IEF/demand constraints

π'' IEF/supply constraints

and where the superscript T represent transposition.

How then are the various shadow prices to be interpreted. The π_D represent the partial derivative of the primal objective function with respect to the magnitude of the end-use demands. Comparison of the magnitudes among the π_D therefore offers guidance as to where conservation will be most cost effective from a total systems viewpoint and may pinpoint the sectors where government action may be the most important.[7] One can also compare the magnitude of the shadow price with the unit cost of the conservation measure, say C_{Di}. Conservation investments are justified in all instances for which

$$\pi_{Di} \gg C_{Di} \quad .$$

Indeed, it can be argued that even in a full market economy, such a condition would warrant a government conservation policy, especially where conservation

[7]Even though the shadow price for an equality is unbounded, in this particular case we can predict that the result of lowering the end use demand will be a decrease in the total system cost. For computational reasons, most of the more sophisticated algorithms will change equality constraints of this type to inequality constraints, since they are computationally less demanding (for example the REDUCE option in the APEX LP code used on CDC equipmentwill automatically make such a change).

benefits might be insufficient to warrant the investment when seen only from the standpoint of the end user.[8]

The π_S represent the value of additional domestic energy resources to the energy system: if we have bounded supplies at the currently sustainable level, then the value of π_S is to be compared to the cost of the next increment of supply (the next step in the supply curve). Where we use the full supply curve step function in the analysis, the optimal level of supply development will be given by the solution itself and unless we run into an absolute upper bound on resources, the corresponding shadow prices will be zero. Even more important policy guidance emerges from a comparison of the π_D and π_S; whether priority should be given to supply enhancements, or to conservation investments, emerges directly from a ranking of the respective shadow prices.[9]

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[8] The shadow price associated with each end use demand constraint does not differentiate among fuels (i.e. reflects the value of a reduction of space heat, rather than oil for space heat, or electricity for space heat); Moreover, if system cost is being used as the objective function, then the end use demand shadow prices cannot be used to differentiate between conservation strategies targetted at reducing oil imports as opposed to strategies that would simply reduce more abundant domestic resource. Use of minimization of oil imports as an objective function, however, would be required to yield such guidance from end use demand shadow prices.

[9] For a discussion of the interpretation of shadow prices in a dynamic energy systems LP, see Abilock et. al., (1976). In a dynamic model there is the additional category of shadow prices corresponding to capacity growth equations.

Example 5.1: Shadow Prices for Republica

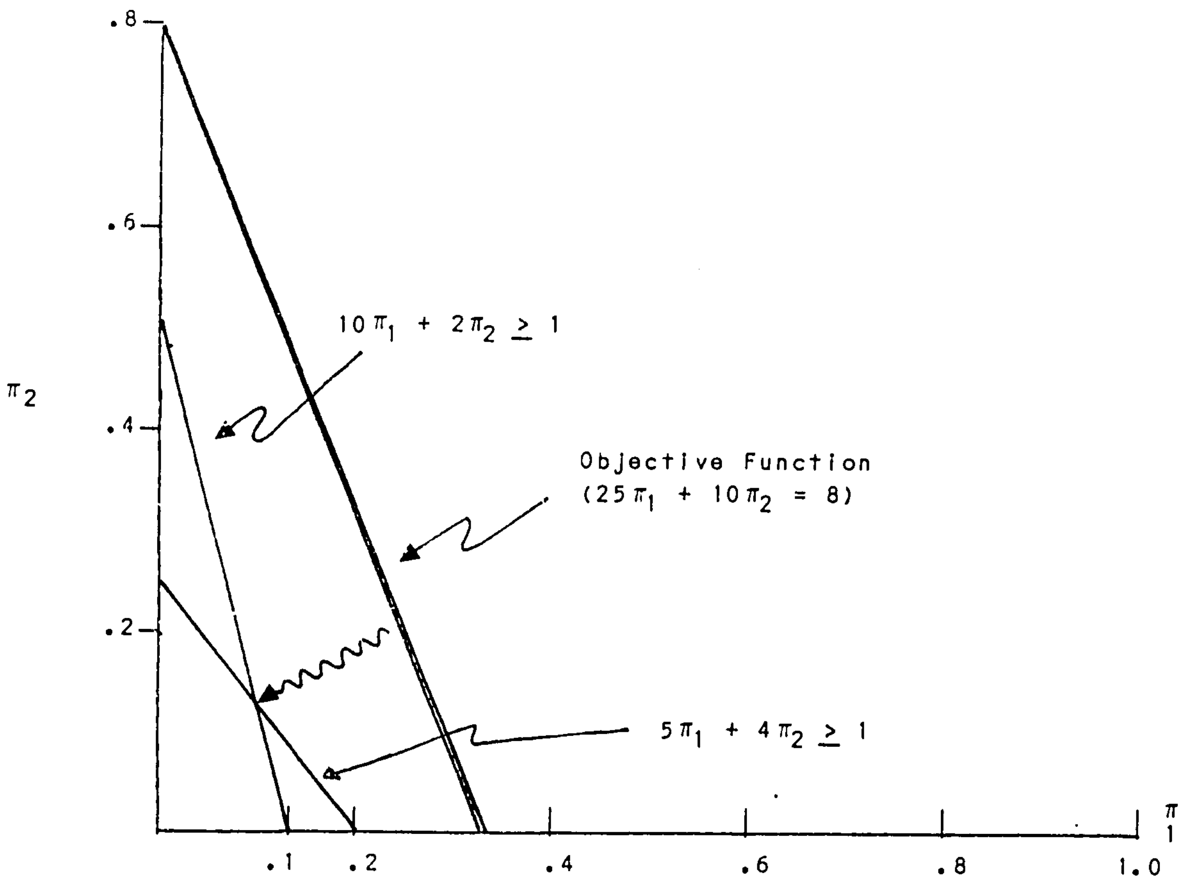
Let us return to our hypothetical country Republica, introduced in Section 2.3. There we considered the problem

$$\begin{aligned} \text{Max} \quad & x_1 + x_2 \\ \text{s.t.} \quad & 10x_1 + 5x_2 \le 25 \text{ (labor)} \\ & 2x_1 + 4x_2 \le 10 \text{ (energy)} \\ & x_1,\ x_2 \ge 0 \end{aligned}$$

Therefore the dual can be written

$$\begin{aligned} \text{Min} \quad & 25\,\pi_1 + 10\,\pi_2 \\ \text{s.t.} \quad & 10\,\pi_1 + 2\,\pi_2 \ge 1 \\ & 5\,\pi_1 + 4\,\pi_2 \ge 1 \\ & \pi_1,\ \pi_2 \ge 0 \end{aligned}$$

This can be solved graphically:



From this figure, the optimum solution is $\pi_1 = 0.0666$, and $_2 = 0.1666$. Therefore, an increase of one unit of energy would increase the optimal solution of the primal by 0.1666 units. Let us confirm this. The optimal value of the old primal was S = 3.33, at the point $x_1 = 1.66$, $x_2 = 1.66$. Therefore, if we change the energy resource constraint by 1 unit, we expect an objective function value of S = 3.33 + 0.166 = 3.5. We leave it to the reader to confirm that the solution of the LP

$$\begin{aligned} \text{Max} \quad & x_1 + x_2 \\ \text{s.t.} \quad & 10x_1 + 5x_2 \le 25 \\ & 2x_1 + 4x_2 \le 11 \\ & x_1,\ x_2 \ge 0 \end{aligned}$$

is $X_1 = 1.5$, $X_2 = 2$ with S = 3.5, as expected.

Finally, note that the values of π_1 and π_2 correspond to the values of the Lagrange multipliers obtained in Section 2.4 for the same problem. This confirms that the dual variables of the LP are equivalent to the Lagrange multipliers.

A. Kydes "The Brookhaven Energy System Optimization Model - Its Variants and Uses" BNL 50873, Brookhaven National Laboratory, May 1978.

This brief paper describes the family of energy system optimization models developed at Brookhaven National Laboratory starting with BESOM, the original energy systems optimization model (of which a FORTRAN version is available in addition to the more advanced programming language versions suitable only for the biggest machines), DESOM, a dynamic version of BESOM, TESOM, the time-stepped version, and the most recent MARKAL, for market allocation model which was developed in collaboration with the members of the International Energy Agency for collaborative R&D systems analysis.

G. Hadley "Linear Programming" Addison-Wesley Publishing, Inc., Palo Alto and London, numerous editions and printings.

Probably the most widely used linear programming text in the U.S., familiar to most who have ever taken a graduate level course in LP. Although this requires a thorough understanding of matrix algebra and advanced calculus, it has few rivals as a reference text, including discussion of many applications.

E10. Addition of Capacity Variables and Capital Constraints

Add to the LP of Eq (5.26) and equation to represent capacity variables in the energy supply and energy conversion sectors, and rewrite, and interpret, the dual of this expanded LP.

6. ENERGY INPUT-OUTPUT ANALYSIS

6.1 FUNDAMENTALS OF INPUT-OUTPUT ANALYSIS[1]

Consider an economy that consists of three sectors: agriculture, machinery, and construction. Let the domestic output of each of these three sectors be denoted x_1, x_2, and x_3, respectively. Let the final demand in each sector be denoted y_i: final demand is typically broken down to private consumption, government consumption (say for defense), investment and foreign trade (exports less imports). For given final demand, say machines, we write

$$\underset{\left[\frac{\$\text{ mach}}{\$\text{ Agr.}}\right]}{a_{21}}\ \underset{[\$\text{ Agr.}]}{x_1} + \underset{\left[\frac{\$\text{ mach}}{\$\text{ mach}}\right]}{a_{22}}\ \underset{[\$\text{ mach}]}{x_2} + \underset{\left[\frac{\$\text{ mach.}}{\$\text{ Const}}\right]}{a_{23}}\ \underset{[\$\text{ Const}]}{x_3} + \underset{\left[\begin{array}{c}\text{Final}\\ \text{Demand}\\ \text{for}\\ \text{machines}\end{array}\right]}{y_2} = \underset{\left[\begin{array}{c}\text{Gross}\\ \text{output}\\ \text{of machine}\\ \text{sector}\end{array}\right]}{x_2} \quad . \qquad (6.1)$$

Thus part of machine output goes to the agricultural and construction sectors--the coefficient a_{21} captures the requirement for machinery per unit output of agriculture, and so on; part of the output is consumed by the sector itself, and part of the output goes to final demand (exports, purchases by government, etc.). The complete three sector economy is thus captured by the three equations

$$\begin{aligned} a_{11}x_1 + a_{12}x_2 + a_{13}x_3 + f_1 &= x_1 \\ a_{21}x_1 + a_{22}x_2 + a_{23}x_3 + f_2 &= x_2 \\ a_{31}x_1 + a_{32}x_2 + a_{33}x_3 + f_3 &= x_3 \end{aligned}$$

which, as usual, can be expressed more compactly in matrix form as[2]

$$\underset{(n\mathrm{x}n)}{A}\ \underset{(n\mathrm{x}1)}{x} + \underset{(n\mathrm{x}1)}{f} = \underset{(n\mathrm{x}1)}{x} \qquad (6.2)$$

for an n-sector economy.

---

[1] Those familiar with conventional input-output analysis as used in the general context of development planning may proceed directly to Section 6.2

[2] This input-output model is said to be "open" in the sense that final demand is exogenously specified. It is possible to "close" and I/O model by assuming that final demands can be determined in the same way as interindustry demands (see Digression 6.3). However, it is never possible to completely close a static system, since investment necessarily requires exogenous specification.

We begin with the usual identity for total supply being equal to total demand:

Supply Demand

$$X + M^{c} = C + G + E + I + \Delta S \qquad (1)$$

where

X = Domestic output
M^{c} = Competitive imports
C = Private consumption
G = Government
E = Exports
I = Gross fixed capital formation
ΔS = Stock changes

In this discussion we make the important distinction between competitive and non-competitive imports. To use the example given by Taylor (1979), wheat imports into a country growing that crop would be classified as competitive; jet aircraft imports into almost every developing country would be classified as non-competitive (or complementary). Competitive imports are treated as being equivalent to domestic output, and are therefore added to supply in Eq. (1).

The second identity relates total inputs to domestic output, i.e.

$$X = wL + vN + rK + T^{IND} + M^{NC}$$

where

M^{NC} = Non-competitive imports
W = Wage rate
L = Employment
V = Rate of return on non-incorporated enterprise
N = Capital in non-incorporated enterprise
r = Rate of return on capital (rent, dividends, interest)
K = Capital in incorporate enterprise (or Nationalized industries)
T^{IND} = Indirect Taxes (value added, import tarrifs)

The total return to non-incorporated enterprise, vN, may be quite substantial in developing countries, given the typical preponderance of peasant farmers and small independent trades people. Indeed, this distinction between incorporated and incorporated enterprise will prove useful in subsequent discussions of commercial and non-commercial energy forms.

The rate of return on capital decomposes into two parts: depreciation, and after-depreciation return on capital (e.g. dividends to stockholders). The non-competive imports M^{NC} shown in (2) are just those necessary for domestic production (e.g. refined products for a small country without a refinery): other non-competitive imports may go to final demand (e.g. such as imported luxury goods to households).

Gross Domestic Product (GDP) is defined as

$$GDP = wL + vN + rK + T^{IND} \qquad (3)$$

i.e. as sume of payments to primary inputs plus business taxes. Of course, by substitution of (1) and (2) into (3) GDP is also equal to

$$GDP = C + G + E + I + S - M^{c} - M^{NC} \quad . \qquad (4)$$

Finally, value added at factor cost, Y, is defined by

$$Y = wL + vN + rK \qquad (5)$$

which is the sum of payments to participants in the production process. Disposable income, Y^{DISP} accrues to persons and follows from (5) as

$$Y^{DISP} = Y - T^{DIR} - S^{CORP} \qquad (6)$$

where

T^{DIR} = Direct taxes (income taxes, corporate profit taxes)
S^{CORP} = Corporate retained earnings and deprecation allowances.

Disposable income then is used either for consumption or for savings, i.e.

$$Y^{DISP} = C + S^{PRIV} \quad . \qquad (7)$$

One can also view foreigners as an actor in a national economy: whenever imports exceed exports, then the rest of the world makes up the difference. This current account deficit is made up by foreign aid, direct investment, and commercial lending, and such payments obviously contribute to the potential savings flow. Thus

$$S^{FOR} = M^{c} + M^{NC} - E \quad . \qquad (8)$$

Finally, government receipts come from direct and indirect taxes, whilst its expenditures fall under the rubric government consumption. Thus

$$S^{GOVT} = T^{IND} + T^{DIR} - G \qquad (9)$$

We leave it as an exercise for the reader to show by substitution of these identities the validity of the savings-investment identity

$$S^{PRIV} + S^{CORP} + S^{FOR} + S^{GOV} = I + \Delta S \qquad (10)$$

[1]This discussion, and notation is based on Chapter 2 of Taylor (1979).

Suppose that the vector of final demand is given; the problem then becomes one of solving (6.2) for X. If we rewrite (6.2) as

$$X - Ax = f \tag{6.3}$$

or

$$(I - A)X = f \tag{6.4}$$

then by premultiplying both sides by $(I - A)^{-1}$ we obtain

$$X = (I - A)^{-1}f \quad . \tag{6.5}$$

The (I - A) matrix is sometimes referred to as the "Leontief" matrix, in honor of W. Leontief, the father of modern input-output analysis (and for which he received the Nobel prize in economics).[3]

The final demand vector f decomposes in the input-output framework as follows:

$$f = C + G + E + I + S \tag{6.6}$$

where

C is the vector of private consumption

G is the vector of government consumption

I is the vector of investment

E is the vector of exports

S is the vector of stock changes (which is usually ignored in the simple treatments of I/O).

If one makes the additional assumptions that capital and labor are linear functions of output in each sector, then the input-output framework provides a basis for linking capital and labor requirements to particular

[3]Input-output can be traced back to Francois Quesnay and his Tableau Economique presented first in 1758. This was designed to show how goods and services circulated among the four socioeconomic classes of prerevolutionary France-land owners, farmers, traders and manufacturers. Whilst Quesnay's work established the fundamental interdependence of economic activities, it was Leon Walras who established the necessary mathematical framework in terms of a set of simultaneous linear equations. In his 1874 work "Elements d'Economie Politique Pure." It remained for Wassily Leontief, however, to publish the first input-output tables for the U.S. economy, and to bring input-output to the state of being a tool of applied rather than merely theoretical economics. "Quantitative Input-Output Relations in the Economic System of the U.S." Review of Economics and Statistics, 18, p. 105-25 (1936).

levels of final demand. Thus if a_L and a_k are the vectors of sectoral labor-output and capital output ratios, then the total labor and capital requirements in the economy compute to

$$\underset{(1x1)}{L} = \underset{(1xr)}{X^T} \underset{(nx1)}{a_L} \tag{6.7}$$

$$K = X^T a_k \quad . \tag{6.8}$$

Let us return to Republica, introduced in Chapter 2.3. Table 6.1 shows an input-output table for Republica, in which we have added construction, electric power and petroleum refining sectors to agriculture and manufacturing. Such an input-output table, also known as "transactions" table, is denominated in monetary units, and shows the transactions -- purchases and sales--between sectors. For example, the table indicates that the power sector purchases 5 units from the construction sector, and 15 units (of oil) from the refining sector. There are 4 quadrants to Table 6.1, as follows

Interindustry Transactions I	Final Use II
Value Added III	Direct Factor Purchases IV

The total inputs to the Republica power industry, for example, thus consist of purchases from other industries (quadrant I), plus imports, plus payments to labor, government (for taxes) and payments for use of capital, which together represent "value added," which is the difference between total output (or sales) to final demand and purchases from other industries. Note that the distinction between "investment," in the final demand column, and "capital" in the primary input row; the capital row represents that part of the capital stock that is depreciated during the year plus payments for use of capital, whereas the investment column represents the flow of new investments during the year.

In the direct factor purchases quadrant (IV) are shown those inputs that are used by final users -- governments and households. Civil servants

Table 6.1
Input-Output Table for Republica

	Agriculture	Machinery	Construction	Electric Power	Petroleum Refining	Total Intermediate Demand	Households	Government	Exports	Investment	Total Final Demand	Gross Output
1. Agriculture	15	3	5	0	0	23	26	14	20	12	72	95
2. Machinery	10	10	15	5	5	45	10	5	5	15	35	80
3. Construction	5	7	10	5	5	32	5	10	0	5	20	52
4. Electric Power	5	3	2	0	2	12	3	20	0	0	23	35
5. Petroleum Refining	7	4	3	15	0	29	15	15	0	3	33	62
TOTAL DOMESTIC PURCHASES	42	27	35	25	12	141	59	64	25	35	183	324
6. Imports	8	3	0	0	37	48	5	0	0	0	5	53
7. Labor	20	30	10	5	3	68	1	12	0	0	13	81
8. Capital	5	10	3	3	5	26	0	0	0	0	0	26
9. Taxes	20	10	4	2	5	41	35	0	2	0	37	78
TOTAL VALUE ADDED	45	50	17	10	13	135	36	12	2	0	50	185
TOTAL INPUTS	95	80	52	35	62	324	100	76	27	35	238	562

employed by government exemplify the intersection of the labor row and government consumption column; the intersection of the labor row with the column household consumption would represent payments to domestic servants. Payments of taxes by households and by the export sector (export taxes and license fees), are also shown in this quadrant. Republica can be seen to have a poor trade balance,with imports exceeding exports by 53 - 27 = 26 units,[4] with oil imports (37) accounting for the entire payments deficit: the magnitude of this deficit is set for reasons of pedagogic illustration rather than beingrepresentative of any particular, real situation.

Dividing each column in the intermediate demand sector by the sectoral output, the A-matrix of technological coefficients computes as follows

$$\begin{bmatrix} \frac{15}{95} & \frac{3}{80} & \frac{5}{52} & 0 & 0 \\ \frac{10}{95} & \frac{10}{80} & \frac{15}{52} & \frac{5}{35} & \frac{5}{62} \\ \frac{5}{95} & \frac{7}{80} & \frac{10}{52} & \frac{5}{35} & \frac{5}{62} \\ \frac{5}{95} & \frac{3}{80} & \frac{2}{52} & 0 & \frac{2}{62} \\ \frac{7}{95} & \frac{4}{80} & \frac{3}{52} & \frac{15}{35} & 0 \end{bmatrix} = \begin{bmatrix} 0.157 & 0.037 & 0.096 & 0 & 0 \\ 0.105 & 0.125 & 0.288 & 0.142 & 0.081 \\ 0.052 & 0.0875 & 0.192 & 0.142 & 0.081 \\ 0.052 & 0.0375 & 0.038 & 0 & 0.032 \\ 0.073 & 0.05 & 0.057 & 0.428 & 0 \end{bmatrix} \quad (6.9)$$

whence

$$(I - A) = \begin{bmatrix} 0.842 & -0.037 & -0.096 & 0 & 0 \\ -0.105 & -0.875 & -0.288 & -0.142 & -0.081 \\ -0.052 & -0.0875 & 0.807 & -0.142 & -0.081 \\ -0.052 & -0.0375 & -0.038 & 1.0 & -0.032 \\ -0.073 & -0.05 & -0.057 & -0.428 & 1.0 \end{bmatrix}$$

and the inverse follows as

$$(I - A)^{-1} = \begin{bmatrix} 1.212 & 0.072 & 0.173 & 0.042 & 0.021 \\ 0.205 & 1.22 & 0.487 & 0.307 & 0.147 \\ 0.130 & 0.157 & 1.33 & 0.267 & 0.128 \\ 0.0815 & 0.059 & 0.084 & 1.03 & 0.045 \\ 0.142 & 0.100 & 0.149 & 0.479 & 1.03 \end{bmatrix} . \quad (6.10)$$

[4]Notice that imports could also be treated as negative exports, rather than treating them in the primary inputs sector. The column "Exports" would therefore become "net exports" (= Exports - Imports). For example, if imports to the agricultural sector were removed from the input row, inputs to agriculture reduce to 87. This would require that net exports show 20 - 8 = 12. This reduces final demand to 64, and total output becomes, again, equal to 64 + 23 =87.

We leave it to the reader to show that multiplication of inverse matrix by the vector of final demands yields the gross output vector shown on Table 6.1

It is also useful to account for the savings-investment balance for Republica. Suppose that of the 26 units paid as return on capital, 4 is in the form of corporate savings, and 22 is in the form of payments to individuals. Then from the definitions in Box 6.1

$$Y^{DISP} = Y - S^{CORP} - T^{DIR}$$
$$= (81 + 26) - 4 - 37 = 66$$

but this equals $C + S^{PRIV}$, hence

$$S^{PRIV} = 66 - 65 = 1 \quad .$$

Government savings compute to

$$S^{GOVT} = T^{IND} + T^{DIR} - G$$
$$= 41 + 37 - 76$$
$$= 2 \quad .$$

And foreign savings are given by

$$S^{FOR} = \text{IMPORTS} - \text{EXPORTS}$$
$$= 53 - 25 = 28 \quad .$$

Now the sum of the savings terms equal investment, i.e.

$$I = 4 + 1 + 2 + 28 = 35$$

which equals the value shown under the column heading "investment" on Table 6.1.

A number of critical assumptions must be clearly understood if the input-output technique is to have any operational utility. The most important, concerns the constancy of input coefficients: each additional unit of new output is produced by an unchanging proportional combination of inputs from other sectors, an assumption that also implies constant returns to scale. Thus the input-output model does not allow the possibility of substitution among inputs, an assumption that becomes increasingly questionable with increasing length of planning horizon. There are a number of ways to compensate for this problem, possiblities discussed exhaustively elsewhere in the general context of the use of input-output for development planning.[5] We shall, however, return to this subject later in this chapter in the context of reducing energy inputs over time (which of course is one of the major objectives of any comprehensive energy policy).

[5]Todaro (1971) has an elementary discussion of this subject, with some simple numerical examples to illustrate the implications of such assumptions. For a more advanced treatment, see e.g. L. Taylor "Theoretical Foundations and Technical Implications," in Blitzer et. al., (1975).

The model discussed in Section 6.1 would be termed "static" in the vocabulary of input-output, in that it does not rest on any explicit theory of investment. In the most widely used "dynamic" I/O model, an accelerator-type investment relationship is used in which current demands for investment goods depend on the expectation of future output growth. Thus

$$D(t) = k[X(t \pm 1) - X(t)] \tag{1}$$

where

X(t) is the vector of output at time t

D(t) is the vector of investment demands for new capital formation

k is a diagonal matrix of incremental capital-output ratios

Further, let R(t) be a vector of replacement capital demands, given by

$$R(t) = \delta k(t) \tag{2}$$

where

δ is a diagonal matrix of depreciation coefficients,

where

k(t) is the vector of capital stocks at time t.

Finally, if B is a matrix of capital coefficients, whose element b_{ij} describes the amount of output of sector i to produce a unit amount of investment in sector j. Typically many rows of the B matrix will be zero, as many sectors produce only for consumption or intermediate demands. And again we have the usual computational assumption of constant coefficients, which violates the reality that changes in the prices of capital goods would change the mix of capital goods used for investmant. Thus we replace the investment term I in the static model by the expression.

$$I(t) = B([kX(t + 1) - X(t)] + R(t)) \tag{3}$$

the dynamic version becomes

$$Y(t) = C(t) + G(T) + E(t) + B([kX(t + 1) - X(t)] + R(t)) \tag{4}$$

If $Y'(t) = C(t) + G(t) + E(t)$, i.e. final demand exclusive of investment consumption, then the dynamic version of Eq. (6.2) is

$$A\,X(t) + Y'(t) + Bk[X(t + 1) - X(t)] + B\delta kX(t) = X(t) \tag{5}$$

which can be rewritten as

$$(I - A - B\delta k - Bk)X(t) + BkX(t + 1) = Y'(t) \tag{6}$$

The problem now is to solve for X(t + 1) given X(t = 0). Writing $\bar{B}$ for Bk, then premultiplication of all terms of (6) by $\bar{B}^{-1}$ yields

$$X(t + 1) = \bar{B}^{-1}Y'(t) + [I + \bar{B}^{-1}(I - A - B\delta K)]X(t) \tag{7}$$

The question now is whether or not the inverse $\bar{B}^{-1}$ exists. As alreay noted, many rows of B will be zero, since some producing sectors do not meet investment demands: thus B is singular, and B^{-1} does not exist. Consequently some special decomposition procedures must be invoked in order to solve (7), a subject treated in some detail elsewhere in the Literature. The point to be made here is simply to show some of the problems involved in more "realistic" versions of the I/O framework.

Digression 6.3: Computing the Leontieff Inverse Without a Computer

We observed earlier that one method of computing the inverse of the matrix A was to make use of the series

$$(I - A)^{-1} = I + A + A^2 + \dots A^n \quad .$$

However,[1] this method is applicable only if the sums of the elements in each column A is less than unity. Fortunately, this is true of most matrices of technological coefficients -- see, for example, the matrix (6.9) for Republica. Let us apply this inversion formula to this situation.

$$I = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$I + A = \begin{bmatrix} 1.157 & 0.037 & 0.096 & 0 & 0 \\ 0.105 & 1.125 & 0.288 & 0.142 & 0.081 \\ 0.052 & 0.0875 & 1.192 & 0.142 & 0.081 \\ 0.052 & 0.0375 & 0.038 & 1.0 & 0.032 \\ 0.073 & 0.05 & 0.057 & 0.428 & 1.0 \end{bmatrix}$$

at this point, even without adding any of the higher order terms, I + A looks not all that unlike the inverse $(I - A)^{-1}$ shown in Eq. (6.10)!
Let us compute A^2:

$$\begin{bmatrix} 0.157 & 0.037 & 0.096 & 0 & 0 \\ 0.105 & 0.125 & 0.288 & 0.142 & 0.081 \\ 0.052 & 0.0875 & 0.192 & 0.142 & 0.081 \\ 0.052 & 0.0375 & 0.038 & 0 & 0.032 \\ 0.073 & 0.05 & 0.057 & 0.428 & 0 \end{bmatrix} \times \begin{bmatrix} 0.157 & 0.037 & 0.096 & 0 & 0 \\ 0.105 & 0.125 & 0.288 & 0.142 & 0.081 \\ 0.052 & 0.0875 & 0.192 & 0.142 & 0.081 \\ 0.052 & 0.0375 & 0.038 & 0 & 0.032 \\ 0.073 & 0.05 & 0.057 & 0.428 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0.0355 & 0.0188 & 0.0441 & 0.0188 & 0.01077 \\ 0.0578 & 0.054 & 0.1113 & 0.0933 & 0.0379 \\ 0.004 & 0.0039 & 0.077 & 0.074 & 0.0027 \\ 0.0016 & 0.0015 & 0.0249 & 0.0244 & 0.0006 \\ 0.041 & 0.0029 & 0.0486 & 0.0151 & 0.0223 \end{bmatrix}$$

at this point, then $I + A + A^2$ equals

$$\begin{bmatrix} 1.19 & 0.0558 & 0.140 & 0.018 & 0.010 \\ 0.162 & 1.179 & 0.399 & 0.235 & 0.118 \\ 0.092 & 0.126 & 1.269 & 0.216 & 0.108 \\ 0.068 & 0.049 & 0.063 & 1.024 & 0.004 \\ 0.114 & 0.079 & 0.105 & 0.443 & 1.022 \end{bmatrix}$$

which should again be compared to (6.10).

After two more iterations, we have $I + A + A^2 + A^3 + A^4$, which computes to

$$\begin{bmatrix} 1.206 & 0.0684 & 0.166 & 0.0387 & 0.019 \\ 0.203 & 1.21 & 0.468 & 0.292 & 0.142 \\ 0.122 & 0.151 & 1.318 & 0.256 & 0.125 \\ 0.078 & 0.0568 & 0.0787 & 1.036 & 0.043 \\ 0.136 & 0.0968 & 0.139 & 0.471 & 1.032 \end{bmatrix}$$

The percentage error for each element at this point is given by comparison to (6.10), and shows

$$\begin{bmatrix} 0.04\% & 5\% & 4\% & 7.8\% & 9\% \\ 0.95\% & 0.8\% & 3.9\% & 5\% & 3.4\% \\ 6.15\% & 3.8\% & 0.9\% & 4.1\% & 2.3\% \\ 4.2\% & 3.7\% & 5\% & 0.5\% & 4.4\% \\ 4.2\% & 3.2\% & 6.7\% & 1.6\% & 0.19\% \end{bmatrix}$$

which indicates both the convergence of the method, and the convenience of the series expansion as a method for computing the Leontieff inverse.

[1]This condition can be weakened if the matrix is indecomposable, in which case the limit $n \to \infty$, $A^n = 0$ if the sum of elements in each column is less than or equal to unity, and if at least one of the columns sum is less than unity.

The key to solving the interfuel substitution dilemna is the recognition that is is not fuel forms per se that are required by a production activity (or to satisfy personal consumption demands), but energy services--process heat, motive power, space heat, and so on. Indeed, there is no reason why one cannot add additional rows to our three sector economy that depict energy service and energy supply. Suppose that the only type of energy demanded in our economy were industrial process heat (IPH) and residential space heat, that can each be satisfied only by either coal or oil as a fuel. We can thus write for industrial process heat the equation

$$\underset{\left[\frac{\text{Btu IPH}}{\text{\$ Agr}}\right]}{g_{I1}} \quad \underset{[\text{\$ Agr}]}{x_1} \quad + \quad \underset{\left[\frac{\text{Btu IPH}}{\text{\$ Mach}}\right]}{g_{I2}} \quad \underset{[\text{\$ Mach}]}{x_2} \quad + \quad \underset{\left[\frac{\text{Btu IPH}}{\text{\$ Constr}}\right]}{g_{I3}} \quad \underset{[\text{\$ Constr}]}{x_3} = h_I \tag{6.11}$$

where the g_{Ij} represent the unit energy service demand (in this case industrial process heat) per \$ of output in Sector j. Industrial process heat can be supplied by the refinery sector (in the form of refined oil), or by the coal mining sector (as coal). Thus for the coal mining sector we may write

$$\underset{\left[\frac{\text{Btu Coal}}{\text{Btu Coal}}\right]}{b_{1c}} \quad \underset{[\text{Btu Coal}]}{x_c} \quad + \quad \underset{\left[\frac{\text{Btu Coal}}{\text{Btu IPH}}\right]}{b_{1I}} \quad \underset{[\text{Btu IPH}]}{h_I} \quad + \quad \underset{\left[\begin{array}{c}\text{final}\\ \text{demand}\\ \text{for coal,}\\ \text{Btu}\end{array}\right]}{f_c} \quad = \quad \underset{\left[\begin{array}{c}\text{Gross}\\ \text{output}\\ \text{of coal}\end{array}\right]}{x_c} \tag{6.12}$$

where f_c represents imports/exports of coal. The coefficient b_{1I} represents the fraction of IPH that is met by coal, and the coefficient b_{1c} represents the Btu of coal that must be mined to produce 1 Btu of coal delivered to users--which would take into account the Btu losses of coal cleaning. Similarly for the refining sector, we must, as usual, differentiate between crude and refined products. For refined products (of which in our hypothetical model there is only one, a middle distillate!) we have

$$\underset{\left[\frac{\text{Btu RR}}{\text{Btu IPH}}\right]}{b_{2I}} \quad \underset{[\text{Btu IPH}]}{h_I} \quad + \quad \underset{\left[\begin{array}{c}\text{final}\\ \text{demand}\end{array}\right]}{f_D} \quad = \quad \underset{\left[\begin{array}{c}\text{Gross}\\ \text{Output}\end{array}\right]}{x_D} \tag{6.13}$$

Thus b_{2I} represents the fraction of IPH met by refined oil, again adjusted for the efficiency of conversion. Final demand for refined oil again depicts exports minus imports.

Finally, for the crude oil sector, one has the equation

$$\underset{\left[\frac{\text{Btu Crude}}{\text{Btu Dist.}}\right]}{b_{3D}} \cdot \underset{[\text{Btu Dist.}]}{x_D} + \underset{[\text{Btu Crude}]}{f_R} = \underset{[\text{Btu Crude}]}{x_R} \quad . \tag{6.14}$$

Here b_{3D} represents the Btu of crude oil required to produce a Btu of refined product, the final demand f_R represents crude imports (or exports) and x_R represents total domestic crude output.

We are not quite finished, however. In general, some level of energy services will also be required in the production of energy supplies (Refineries need process heat, coal mines require electricity). Thus we must add to (6.11) a term for each energy supply sector; thus (6.15) becomes

$$\underset{\left[\frac{\text{Btu IPH}}{\text{Btu Coal}}\right]}{g_{Ic}} \underset{[\text{Btu Coal}]}{x_c} + \underset{\left[\frac{\text{Btu IPH}}{\text{Btu Dist}}\right]}{g_{ID}} \underset{[\text{Btu Dist}]}{x_D} + \underset{\left[\frac{\text{Btu IPH}}{\text{Btu Crude}}\right]}{g_{IR}} \underset{[\text{Btu Crude}]}{x_R}$$

$$+ g_{I1}x_1 + g_{I2}x_2 + g_{I3}x_3 = h_I \quad . \tag{6.15}$$

Finally, some sectoral output is required per unit of energy supply provided: for example, coal mining requires some contribution from the machinery sector for equipment. Whence, for the machiner sector,

$$\underset{\left[\frac{\text{\$ Mach.}}{\text{Btu Coal}}\right]}{a_{2c}} \underset{[\text{Btu Coal}]}{x_c} + \underset{\left[\frac{\text{\$ Mach.}}{\text{Btu Dist.}}\right]}{a_{2D}} \underset{[\text{Btu Dist}]}{x_D} + \underset{\left[\frac{\text{\$ Mach.}}{\text{Btu Crude}}\right]}{a_{2R}} \underset{[\text{Btu Crude}]}{x_R}$$

$$+ a_{21}x_1 + a_{22}x_2 + a_{13}x_3 + f_2 = x_2 \tag{6.16}$$

with analogous expressions for the agricultural and construction sectors.

Let us now write these equations in the order coal, crude, refined oil, IPH, and the sectoral outputs agriculture, machinery, and construction. in the manner of Table 6.2

Table 6.2
Hypothetical Energy I/O

Coal	Crude	Ref. Prod.	IPH	Agr.	Mach.	Constr.	Final Demand		Gross Output
$+b_{1c}x_c$			$+b_{1I}h_I$				$+ f_c$	$=$	x_c
		$+b_{3D}x_D$					$+ f_R$	$=$	x_R
			$+b_{2I}h_I$				$+ f_D$	$=$	x_D
$g_{Ic}x_c$	$+g_{IR}x_R$	$+g_{ID}x_D$		$+g_{I1}x_1$	$+g_{I2}x_2$	$+x_{I3}x_3$		$=$	h_I
$a_{1c}x_c$	$+a_{1R}x_R$	$+a_{1D}x_D$		$+a_{11}x_1$	$+a_{12}x_2$	$+a_{13}x_3$	$+ f_1$	$=$	x_1
$a_{2c}x_c$	$+a_{2R}x_R$	$+a_{2D}x_D$		$+a_{21}x_1$	$+a_{22}x_2$	$+a_{23}x_3$	$+ f_2$	$=$	x_2
$a_{3c}x_c$	$+a_{3R}x_R$	$+a_{3D}x_D$		$+a_{31}x_1$	$+a_{32}x_2$	$+a_{33}x_3$	$+ f_3$	$=$	x_3

If one partitions the matrix into three by three blocks, each block has coefficients demomiated in the following units (or are zero)

	Energy Supply	Energy (Service Product)	Non-Energy
Energy Supply	$\frac{\text{Btu}}{\text{Btu}}$	$\frac{\text{Btu}}{\text{Btu}}$	Zero
Energy (Service or product)	$\frac{\text{Btu}}{\text{Btu}}$	zero	$\frac{\text{Btu}}{\$}$
Non-Energy	$\frac{\$}{\text{Btu}}$	zero	$\frac{\$}{\$}$

It now becomes convenient to adopt a simpler matrix notation, as follows

x_S = output vector for energy supply (x_c, x_R, x_D)

x_P = output vector for energy-products (h_I)

x_I = output vector for non energy sectors (x_1, x_2, x_3)

f_S = final demand for energy supply (f_c, f_R, f_D)

f_P = final demand for energy products (f_P)

f_I = final demand for non energy sectors (f_1, f_2, f_3)

from which follows immediately the matrix equations

$$\begin{aligned} A_{SS}X_S + A_{SP}X_P \qquad\qquad &+ f_S = X_S \\ A_{PS}X_S + \qquad\qquad + A_{PI}X_I &+ f_P = X_P \\ A_{IS}X_S \qquad\qquad + A_{II}X_I & \quad f_I = X_I \end{aligned} \tag{6.17}$$

where the A matrix elements are defined on Figure 6.1 and Table 6.3

	Energy Supply	Energy Product	Non-Energy
Energy Supply	A_{SS} Btu/Btu	A_{SP} Btu/Btu	0
Energy Product	A_{PS} Btu/Btu	0	A_{PI} Btu/$
Non-Energy	A_{IS} $/Btu	0	A_{II} $/$

Figure 6.1 A-MATRIX SHOWING ENERGY SECTORS AND UNITS IN ADMISSIBLE SECTORS

TABLE 6.3
A-MATRIX COEFFICIENTS

A_{SS}	= input-output coefficients describing sales of the output of one energy/supply conversion sector to another energy conversion sector and conversion losses incurred in producing or distributing energy. Conversion losses may be excluded if all coefficients are calculated on the basis of delivered energy.
A_{SP}	= input-output coefficients describing how distributed energy products are converted to end use forms.
A_{SI}	= 0 implying that energy supplies are not used by non-energy producing sectors; energy is distributed to the non-energy-producing sectors via energy product sectors.
A_{PS}	= input-output coefficients describing how energy products - final energy forms - are used by the energy-supplying industries. For example, electricity use for lighting a refinery would be included here.
A_{PP}	= 0 implying that energy products are not used to produce energy products.
A_{PI}	= input-output coefficients describing how energy products - final energy forms - are used by non-energy sectors. Examples are industrial process heat or electric drive.
A_{IS}	= input-output coefficients describing the uses of non-energy materials and services by the energy industry. An example of this would be requirements for machinery for oil drilling or coal mining.
A_{IP}	= 0 implying that energy product-sectors equipment require no material or service inputs. This is because they are pseudosectors and not real producing sectors.
A_{II}	= input-output coefficients describing how nonenergy products are used in the non-energy producing sectors. Coefficients in this submatrix are enumerated in purely monetary terms.

Relationship of the Extended I/O to the RES: It should be obvious that the information contained in the A_{SS}, A_{SP} and A_{PI} matrices can also be stated in terms of the Reference Energy System. For example, the coefficients of the A_{SP} matrix represent the mix of fuels (or intermediate energy forms) that meet individual end use demands for energy services: in terms of the computational structure introduced in Section 2.2, the A_{SP} matrix corresponds to the F-matrix. The general relationships between the extended I/O table and the RES are depicted on Figure 6.2.

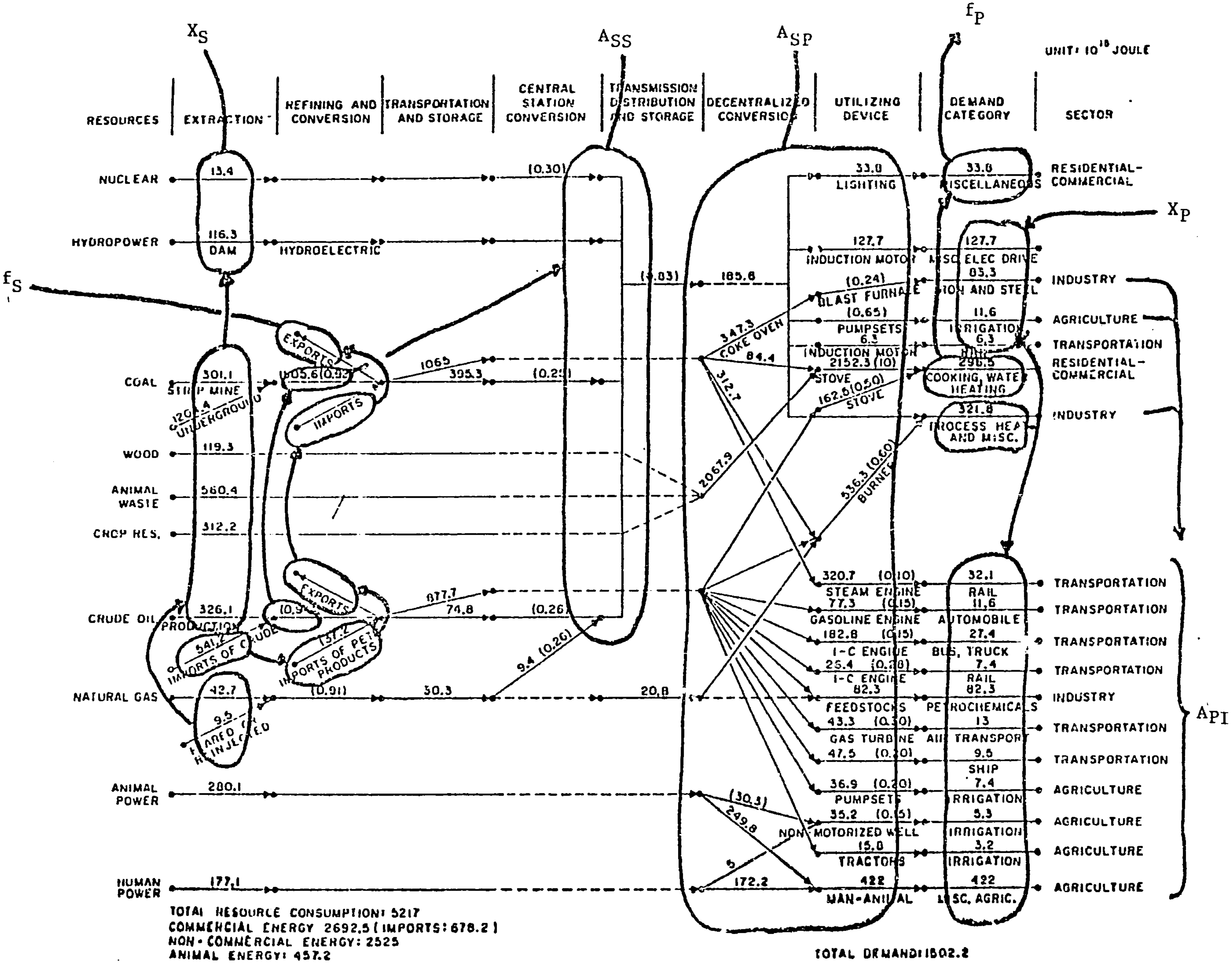


Figure 6.2. General Relationships between the I/O and the RES

At this point let us return once again to Republica, and create an extended I/O table as shown on Table 6.4. In accordance with our general scheme, we move the electric power and the refining industry into the first two rows, and define two energy product sectors--electricity and process heat. All of the transactions between the power and refining industry shown on Table 6.1 in fact represent sales/purchases of energy: the transactions, in each case, can be assumed to be equal to price times quantity (defined as price/10^6 Btu, and 10^6 Btu). Given the following set of energy prices,[6] one can readily enter the elements for the A_{PI} and A_{PS} sectors:

Refined oil : 1 unit/10^6 Btu

Electricity : 2 units/10^6 Btu.

Thus, for example, the 20 units shown on the transactions table as sales of electricity to Government implies a sale of 10 x 10^6 Btu of electricity; or the 4 units of sales from refining to machinery imply a sale of 4 x 10^6 Btu of refined oil for process heat. As noted above, the information in the energy sectors of the extended I/O has a one-to-one correspondance with the reference energy system, which is shown on Figure 6.3. We leave it as an exercise to the reader to compare each RES entry with its counterpart on the extended transactions table. In this instance, since all of the crude oil is imported, we treat imports as negative exports, and thus the intersection of the crude oil row, and the export column, indicates the oil imports as 10^6 Btu (i.e. equal to -107.3). Indeed, by comparison of the RES with the extended transactions table, it becomes clear that associated with each link in the RES is a price and a monetary transaction that provides the relationship to the transactions table.

[6]In this analysis we assume that sales of electricity and oil from the electric utility and the refining company are at constant price across all sectors. Of course in practice, this would rarely be the case, since electric rates, for example, would vary by customer class, with the residential consumers paying more per kWh than a major industry. Whilst this may seem at first glance just a matter of algebra, in practice, a full reconciliation between price, energy and total monetary transaction among sectors may be difficult to establish.

Table 6.4
Extended Transactions Table for Republica

	Crude Oil	Electric Power	Refining	Electricity	Process Heat	Agriculture	Machinery	Construction	Households	Government	Exports	Investment	Total Final Demand	Gross Output
Crude	0	0	107.3	–	–	–	–	–	–	–	-107.3	–	-107.3	0
Electric Power	0	0	0	17.5	0	–	–	–	–	–	–	–	0	17.5
Refining	0	52.5	0	0	44	–	–	–	–	–	–	–	–	96.5
Electricity	0	0	1.0	–	–	2.5	15	1	1.5	10	0	0	11.5	17.5
Process Heat	0	0	0	–	–	7	4	3	15	15	0	0	30	44
Agriculture	0	0	0	–	–	15	3	5	26	14	20	12	72	95
Machinery	0	5	5	–	–	10	10	15	10	5	5	15	35	80
Construction	0	5	5	–	–	5	7	10	5	10	0	5	20	52

From the transactions table we recompute the matrix of technical coefficients as follows

$$\begin{bmatrix} 0 & 0 & \frac{107.3}{96.5} & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{17.5}{17.5} & 0 & 0 & 0 & 0 \\ 0 & \frac{52.5}{17.5} & 0 & 0 & \frac{44}{44} & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{96.5} & 0 & 0 & \frac{2.5}{95} & \frac{15}{80} & \frac{1}{52} \\ 0 & 0 & 0 & 0 & 0 & \frac{7}{95} & \frac{4}{80} & \frac{3}{52} \\ 0 & 0 & 0 & 0 & 0 & \frac{15}{95} & \frac{3}{80} & \frac{5}{52} \\ 0 & \frac{5}{17.5} & \frac{5}{96.5} & 0 & 0 & \frac{10}{95} & \frac{10}{80} & \frac{15}{52} \\ 0 & \frac{5}{17/5} & \frac{5}{96.5} & 0 & 0 & \frac{5}{95} & \frac{7}{80} & \frac{10}{52} \end{bmatrix}$$

From which the inverse $(I - A)^{-1}$ follows as

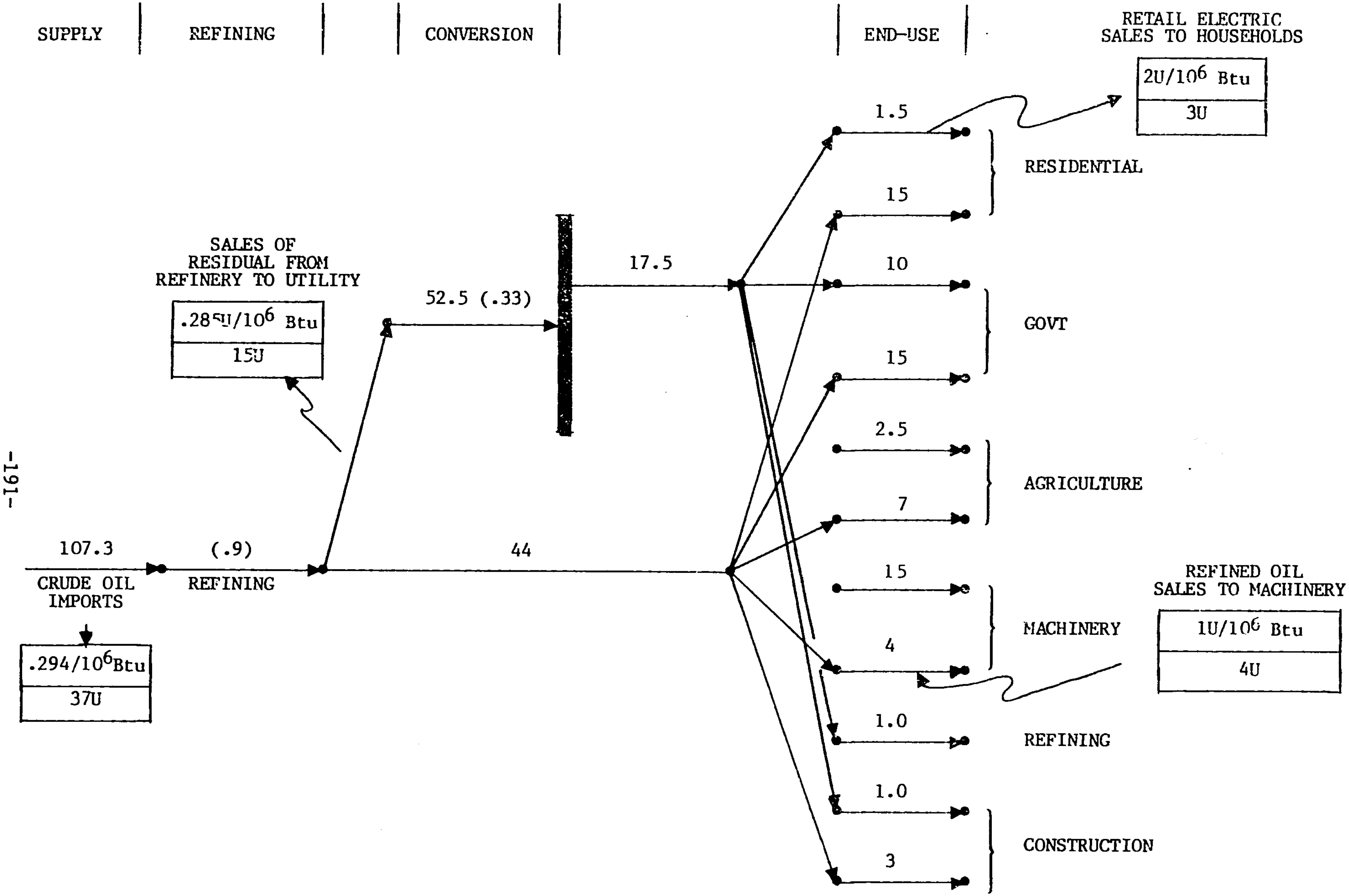


FIGURE 6.3 RES For Republica

$$
\begin{bmatrix}
1. & 3.64 & 1.17 & 3.647 & 1.17 & 0.256 & 0.182 & 0.266 \\
0. & 1.06 & 0.0147 & 1.06 & 0.0147 & 0.0408 & 0.0295 & 0.0418 \\
0. & 3.28 & 1.054 & 3.28 & 1.05 & 0.23 & 0.164 & 0.239 \\
0. & 0.064 & 0.0147 & 1.06 & 0.0147 & 0.0408 & 0.0295 & 0.0418 \\
0. & 0.0863 & 0.0107 & 0.086 & 1.01 & 0.107 & 0.0756 & 0.114 \\
0. & 0.112 & 0.0139 & 0.112 & 0.0139 & 1.21 & 0.0725 & 0.173 \\
0. & 0.779 & 0.0967 & 0.779 & 0.0967 & 0.216 & 1.22 & 0.488 \\
0. & 0.678 & 0.084 & 0.678 & 0.084 & 0.1316 & 0.158 & 1.33
\end{bmatrix} \qquad (6.18)
$$

We are now in a position to use the extended I/O for analysis of planning scenarios. For example, given some new vector of final demand, we may wish to determine a new vector of gross output, and the associated energy requirements and the level of oil imports. Suppose, then, that for the year 1990, we anticipate the following final demand vector, which reflects a 20% final demand increase in non-energy sector output and a 25% increase in final demand for both electricity and process heat requirements of households and governments. Since the level of crude oil imports is one of the unknowns, we obviously cannot specify it in the vector of final demand; however, by setting the corresponding entry to zero, crude oil requirements will emerge in the first element of the gross output vector--we simply have to remember that in this instance the entire crude oil "output" is in fact imported. The new vector of final demands, then, computes as follows:

$$
f = \begin{bmatrix} 0 \\ 0 \\ 0 \\ \hline 14.375 \\ 37.5 \\ \hline 86.4 \\ 42 \\ 24 \end{bmatrix} \qquad (6.19)
$$

[7] We ignore, for the moment, how such an increase in final energy demand might be established: clearly it will depend on changes in household income and changes in price.

Table 6.5
Revised Transactions Table for Republica

	Crude Oil	Electric Power	Refining	Electricity	Process Heat	Agriculture	Machinery	Construction	Final Demand	Total Output
Crude	0.	0	132	0	0	0	0	0	0	132
Electric Power	0.	0	0	21.6	0	0	0	0	0	21.6
Refining	0.	64.9	0	0	54.3	0	0	0	0	119.2
Electricity	0.	0	1.23	0	0	3.	1.8	1.2	14.3	21.6
Process Heat	0.	0	0	0	0	8.4	4.8	3.6	37.5	54.3
Agriculture	0.	0	0	0	0	18.	3.6	6.0	86.4	114.0
Machinery	0.	6.18	6.17	0	0	12.	12.0	18.1	42.	96.5
Construction	0.	6.18	6.17	0	0	6.	8.45	12.1	24.	62.9

which, when multiplied by the Leontieff matrix (6.18), results in the new gross output vector

$$\begin{bmatrix} 132.0 \\ 21.6 \\ 119.2 \\ 21.6 \\ 54.3 \\ 114.0 \\ 96.5 \\ 62.9 \end{bmatrix} \tag{6.20}$$

from which oil exports follow as 132, an increase of 23%.

Given this new gross output vector X', one can readily reconstruct a corresponding transactions Table; we simply multiply each column of the A matrix by the corresponding gross output. The revised transactions table is shown on Table 6.5

6.4 INTERFUEL SUBSTITUTIONS IN THE I/O FRAMEWORK.

Suppose we wish now to analyse the possibilities for substituting coal for oil in electric generation, and coal for oil in industrial process heat applications. This implies that inputs to the process heat column will not only be met by the refining industry, but also by the new coal mining industry. Similarly, inputs to the electric power industry will consist not just of refined oil products, but also of coal.

These changes result in the new transactions Table 6.6. We assume that of the total process heat requirement of 44, 20 units are supplied by coal; and of the total fuel supply to electric power, 20 units of oil are replaced by coal (which reduces the input to electric power from refining from 52.5 to 32.5). Thus for the base year, oil imports compute to 62.8, shown as negative exports to preserve the arithmetic consistency of the table.

Assume now the same increases in final demand as considered previously with, the f-vector of Eq. 6.19 replacing that shown on Figure 6.5. Repeating the I/O analysis then yields the following gross output vector denoted (A), which is compared with the output of the previous, non-coal scenario (B):

	(A) Coal Scenario	(B) Non-Coal Scenario	(A)-(B)
Coal	49.4	0	49.4
Coal Mining	49.4	0	49.4
Crude Oil	77.5	132.0	-54.5
Electric Power	21.6	21.6	0
Refining	69.7	119.2	-49.4
Electricity	21.6	21.6	0
Process Heat	54.3	54.3	0
Agriculture	114.1	114.1	0
Machinery	100.3	96.5	3.8
Construction	65.3	62.9	2.4

As before, crude oil imports appear as positive quantities in the gross output vector; to avoid this, the algebra of I/O needs some modification, as presented in the next section. Arithmetically, however, the results stay the same; in any event, our example shows the expected declines in crude oil imports, and refinery industry output for the level of coal substitution assumed. What is not possible in this simple I/O framework is question of how much coal substitution must. Unfortunately, the more interesting problem of determining the level of coal substitution that is necessary to achieve a given level of oil imports is not possible in this simple I/O framework; only trial and error will yield the desired answer.

Table 6.6
Introducing Coal to Republica

	Coal	Coal Mining	Crude	Electric Power	Refining	Electricity	Process Heat	Agriculture	Machinery	Construction	Households	Government	Exports	Investment	Final Demand	Output
Coal	0	40	0	0	0	0	0				0	0	0	0	0	40
Coal Mining	0	0	0	20	0	0	20				0	0	0	0	0	40
Crude	0	0	0	0	62.8	0	0		0		0	0	-62.8	0	-62.8	0
Electric Power	0	0	0	0	0	17.5	0				0	0	0	0	0	17.5
Refining	0	0	0	32.5	0	0	24				0	0	0	0	0	56.5
Electricity	0	0	0	0	1	0		2.5	1.5	1	1.5	10	0	0	11.5	17.5
Process Heat	0	0	0	0	0			7	4	3	15	15	0	0	30	44
Agriculture	0	0	0	0	0			15	3	5	26	14	20	12	72	95
Machinery	0	3	0	5	5	0		10	10	15	10	5	5	15	35	83
Construction	0	2	0	5	5			5	7	10	5	10	0	5	20	54

EXHIBIT 6.4 Hybrid Operation of the I/O Model

The usual framework for the I/O analysis presupposes that the vector of final demands is exogenously specified, and that the interest focusses on the appropriate gross output vector that meets this set of final demands. In some developing countries, however, such final demand projections may not be available. Rather what is available is a set of sectoral output projections, i.e., the x_I vector. Within what we shall term the hybrid extended I/O model, then, the quantities f_S, f_P, and x_I are exogenously specified, and the problem becomes one of solving for f_I, x_S, and x_P.

Some algebra quickly yields the requisite modifications. Recall the extended I/O equations (6.17):

$$\begin{array}{lllllll} A_{SS}X_S & + A_{SP}X_P & & & + f_S & = X_S \\ A_{PS}X_S & & + A_{PI}X_I & & + f_P & = X_P \\ A_{IS}X_S & & + A_{II}X_I & & + f_I & = X_I \end{array} \tag{6.21}$$

hence, taking the unknowns to the left-hand side, and now the known quantities to the right-hand side

$$\begin{array}{ll} (A_{SS} - I)X_S + A_{SP}X_P & = -f_S \\ A_{PS}X_S - IX_P & = f_P - A_{PI}X_I \\ A_{IS}X_S + f_I & = X_I - A_{II}X_I \end{array} \tag{6.22}$$

which can be solved by the system

$$\begin{bmatrix} X_S \\ X_P \\ f_I \end{bmatrix} = \begin{bmatrix} A_{SS} - I & A_{SP} & 0 \\ A_{PS} & -I & 0 \\ A_{IS} & 0 & I \end{bmatrix}^{-1} \cdot \begin{bmatrix} -f_S \\ -f_P - A_{PI}X_I \\ X_I - A_{II}X_I \end{bmatrix} \tag{6.23}$$

6.5 AN ENERGY INPUT-OUTPUT MODEL FOR PORTUGAL

To illustrate the application of an energy denominated I/O model, we shall analyze some of the problems in setting up such a model for Portugal, extracted from a recent BNL study.* Portugal has no oil resources of its own, and imports 70% of its total energy requirement. The specific question to be addressed was the degree to which conservation strategies could alleviate the severe oil import burdens expected for the future. In theory, this is a quite straight-forward procedure: for a given level of final demand, determine gross output for given levels of conservation, the latter achieved by manipulating the coefficients in the A_{PI} sub-matrix that defines the basic energy demand requirement per unit of sectoral output. In practice, however, there are numerous data problems to be resolved in setting up the energy I/O, problems that are the focus of this section.

Background

By 1977, the Portuguese economy had begun to stabilize following the dislocations of the 1974 Revolution and the return of an estimated 500,000 former colonists. The period from 1974 through 1976 was characterized by increasing private consumption (both in absolute terms and relative to other expenditures), decreasing savings, expanding government spending, rapid inflation, and record-breaking trade and current account deficits. The large balance of payments deficit allowed Portugal to maintain investment at about 20% of GDP. By the end of 1977, the rapid growth in private consumption had subsided, but the consumer price index still recorded its greatest increase since 1974 and the balances of trade and current account reached new lows. Table 6.7 reveals the composition of GDP by expenditure in 1976 prices for 1976 and 1977 as reported in the OECD 1979 Portugal Survey. As the table indicates, Portugal is dependent upon international trade for many goods. Imports amounted to 33.5% of GDP in current prices and the trade balance to 15.5% of GDP.

Portugal has been able to run chronic trade deficits in the past because of the high value of remittances by Portuguese citizens working abroad, and,

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*This section is based on the detailed and much longer report; S. Rogers et al., "Application of the Brookhaven Energy-Economic Assessment Model in the Portugal-U.S. Cooperative Assessment," BNL 51424, Brookhaven National Laboratory, Upton, NY, June 1981.

Table 6.7 (10^6 1976 contos)		
	1976	1977
Private Consumption	367	370
Government Consumption	66	72
Gross Fixed Capital Formation	79	88
Stock Changes	8	8
Exports (Goods and Services)	79	84
Imports (Goods and Services)	142	156
Foreign Balance	-63	72
GDP at Market Prices	457	466

Source: National Statistics Institute and OECD Portugal Survey, July 1979.

to a lesser extent, tourism. As Table 6.8 shows, Portugal's current account has fared better than its trade balance and even recorded a surplus in the second half of 1978. The contribution of worker remittances and tourism are significant, outweighing the trade deficit in the second half of 1978.

The improvement of the current account since the first half of 1977 is also attributable to the authorities' monetary and foreign exchange policies. The escudo depreciated by more than 50% since 1973, having depreciated by 20% against the dollar between December 1977 and April 1979. Because of the large import content of domestically produced goods (particularly manufactured goods), this policy contributed to both the raising of domestic prices and the redistribution of income growth away from wages toward profits, exporters in particular, as well as shifting the terms of trade in favor of Portugal.

The tightening of monetary policy has been employed as a means to counter both inflation and the balance of payments deficit. By raising most interest rates and imposing controls on credit, the monetary authorities intended to constrict private credit and thus private consumption, easing domestic inflationary pressures caused by excess demand and reducing consumption on all goods and services, including imports. Consumer spending increased 0.8% in 1977 and 0.5% in 1978 compared to an average of 4.3% from 1973 to 1976.

Recent plans have included the revaluation of the escudo, the relaxation of import duties, and a somewhat more liberal monetary policy. Portugal's recent economic performance has significantly improved its international

Table 6.8 (10^6 Contos)					
	1975	1976	1977	1978	1978II
Trade Balance	-42.8	-63.7	-96.4	-105.2	-46.5
Services (NET)	-4.7	-3.4	-4.9	-4.4	3.0
of which Tourism	2.6	5.5	10.3	19.6	13.9
Workers Remittances	26.6	29.2	43.6	74.3	46.5
Current Balance	-20.9	-37.9	-57.7	-35.3	3.0

Source: Bank of Portugal, converted from US$ at current exchange rate.

credit rating. The authorities plan to run external deficits for the next five years or so as a means of ameliorating the immediate burden of investment financing placed upon the general consuming public since 1977. The ultimate success of this financing strategy can only be determined when Portugal's loans come due.

Energy plays an important role in Portugal's trade balance. Nearly 70% of its energy was imported in 1977, nearly all in the form of petroleum and petroleum products. Total expenditures of foreign exchange on petroleum rose from 9% of the trade balance in 1971-1973 to nearly 30% in 1977 (in current prices). In this same year, imports of petroleum and petroleum products accounted for 16.3% of the total import bill, up from 6.7% in 1973.

Table 6.9 reveals the level and configuration of energy resource consumption and imports for 1977.

Table 6.9
1977 Energy Resource Consumption (10^3 TEP)

Resource	Consumption	Imports
Coal	443	347
Oil	6,521	6,521
Hydropower*	2,324	--
Natural Gas	--	--
Nuclear*	--	--
Wind*	--	--
Biomass	600	--
Solar*	--	--
TOTAL	9,888	6,868

*In terms of fossil fuel equivalent.

Sector Definition

One begins with a definition of the sectors to be used in the analysis. As indicated on Table 6.10, three different sector types must be defined: energy supply sectors, energy product sectors, and non-energy sectors. These should be defined in such a way that the total matrix has no more than 50 sectors, in order to keep the anlaysis tractable. Ideally, the energy product sectors should be in the form of basic energy demands--lighting,

Table 6.10
Sector Definitions for Portugal

Energy Supply Sectors	1) Coal
	2) Crude Oil
	3) Refined Oil Products
	4) Thermal Electric
	5) Hydroelectric
	6) City Gas
	7) Natural Gas
	8) Nuclear
	9) Wind
	10) Biomass
	11) Solar
Energy Product Sectors	12) Coal-Based Feedstocks
	13) Oil-Based Feedstocks
	14) Motive Power
	15) Industrial Process Heat
	16) Electric Power
	17) Household Nonelectric
	18) Household Electric
Nonenergy Sectors	19) Agriculture and Fishing
	20) Mining
	21) Food, Beverage, and Tobacco
	22) Textiles
	23) Clothing, Leather, Wood, and Furniture
	24) Paper and Printing
	25) Chemicals, Rubber, and Plastic
	26) Glass
	27) Cement
	28) Other Nonmetallic Minerals
	29) Iron and Steel
	30) Nonferrous Metals
	31) Metal Products, Machinery, and Misc. Manufacturers
	32) Construction and Public Works
	33) Water, Sanitation, Commerce, and Services
	34) Road and Rail Transportation
	35) Water Transportation
	36) Air Transportation

motive power, cooking, and so on. Unfortunately, in some case--as in our example--such data is not available. Thus for the household, sector, for example, one can only distinguish between electric and non-electric, since there were no data available to disaggregate electric consumption into end uses--lighting, cooking, air conditioning, and misc. applicance use.

In order to utilize the I/O technique at all, one must of course start with an existing I/O table. In almost all cases, where such tables do exist they are too detailed, and to make the analysis tractable one must aggregate the I/O table to between 10 to 20 sectors. The Portuguese I/O table has 60 sectors: and Table 6.11 shows how these 60 sectors were aggregated in such a way as to highlight the more energy-intensive consuming sectors. The manner in which the I-O could be aggregated was constrained by the similarity or dissimilarity of sectors. For instance, one would not combine cement with agriculture since the two sectors have very little in common in terms of either their production processes or energy consumption patterns. Some sectors which are currently insignificant in terms of their contribution to GDP were, nonetheless, kept separate because they are particularly energy intensive and are likely to be quite important in the future; the most notable of these are the cement, chemicals and glass, and transportation sectors.

Definition of the Final Demand Vector

The next problem is that the year of the I/O table is not the same as the base year to be utilized for the energy study. The base year in the Portuguese case was to be 1977, but the I/O table was for 1974. There are two options here: either one modifies the entire I/O table of 1974 to 1977 prices (and then use 1977 prices throughout the subsequent analysis), or one adjusts the known 1977 final demand matrix back to 1974 (and then use 1974 prices in the analysis). In general, the second option is somewhat simpler.

Three different reference points were employed in this adjustment of the disaggregated final demand matrix: (1) a deflated 1977 final demand vector, (2) sectoral value-added growth rates (in real terms), and (3) 1977 energy resource consumption. It was necessary to use these three reference points as checks against one another because of inconsistencies between the various Portuguese accounts. Differential sectoral inflation rates (food inflated at nearly 3.5 times the rate of manufactured goods based on historic wholesale

Table 6.11
Aggregation of the 1974 I/O Table

Original Sectors 60 x 60		Sectors in BEEAM	
01	Agriculture	19	Agriculture and Fishing
02	Forestry		
03	Cattle		
04	Fishing & Fish Conservation		
05	Petroleum Extraction Coal Mining & Mfg.	20	Mining
06	Extracted Nonmetallic Minerals		
07	Meat and Meat Products	21	Food, Beverage, Tobacco
08	Milk		
09	Fruit Products		
10	Edible Oil		
11	Animal Feed		
12	Other Edibles		
13	Beverage		
14	Tobacco		
15	Textiles and Mixtures	22	Textiles
16	Textiles, Cotton & Mixtures		
17	Hand Fiber Textiles		
18	Clothing	23	Clothing, Leather, Wood, and Furniture
19	Footwear		
20	Tanning and Shearing		
21	Wood		
22	Cork		
23	Furniture and Bedding		
24	Pulp for Paper	24	Paper and Printing
25	Paper and Cardboard Products		
26	Printing		
27	Rubber and Rubber Products	25	Chemicals, Rubber, & Plastic
28	Plastic Products		
29	Chemical Based Products		
30	Resins		
31	Nonedible Oils		
32	Paints, Varnish, and Lacquer		
33	Other Chemical Products		
34	Petroleum and Coal-Derived Products	01	Coal
		02	Crude Oil
		03	Oil Refining
35	Glass and Glass Products	26	Glass
36	Cement	27	Cement
37	Other Nonmetallic Minerals	28	Other Nonmetallic Minerals

Table 6.11 (cont.)
Aggregation of the 1974 I/O Table

Original Sectors 60 x 60		Sectors in BEEAM	
38	Ferrous Metals	29	Iron and Steel
39	Nonferrous Metals	30	Nonferrous Metals
40	Metal Products	31	Metal Products, Machinery, and Misc. Manufacturers
41	Electric Machinery		
42	Electric Materials & Machines		
43	Naval Construction & Repairs		
44	Transportation Materials		
45	Other Factories		
46	Civil & Public Works Construction	32	Construction & Public Works
47	Electric	04	Thermal Electric
		05	Hydroelectric
48	Gas	06	City Gas
49	Water and Sanitation	33	Water, Sanitation, Commerce, and Services
50	Commerce		
54	Communications		
55	Horeca (unable to translate)		
56	Educational Services		
57	Housing Services		
58	Other Services		
59	Banking		
51	Highway & Railroad Transportation	34	Road & Rail Transportation
52	Ocean & Inland Transportation	35	Water Transportation
53	Air Transportation	36	Air Transportation
60	Government		Government is included in final demand

prices), different sector definitions, and the devaluation of the escudo by nearly 50% against the dollar between 1974 and 1977 made it unlikely that the Portuguese accounts would be in accordance with one another. It was also impossible to distinguish between the effects of domestic inflation, world inflation, and exchange rate devaluations on the domestic prices index, a significant issue considering that imports (of primarily intermediate goods) constitute nearly 40% of the value added in manufacturing. This problem was circumvented by applying growth rates in exports and imports developed by the Ministry of Commerce and Tourism.

Private Consumption: The OECD 1979 Portugal Survey, based on statistics from INE and the Central Department of Planning (CDP), reported 1977 Private Consumption as 370 x 10^6 1976 contos. The distribution of private expenditure between food and nonfood items was based on government estimates which indicate that in 1977 approximately 50% of private expenditure was spent on food and beverages. The disaggregation of private consumption into food (including beverages and agricultural products) and nonfood products was performed in 1976 prices and then deflated. Private expenditure on nonfood products was disaggregated into the same relative proportions as exhibited in the 1974 I/O table. Private expenditure on food and beverage and agricultural products was distributed between the two sectors 66 to 34% respectively. The 1974 I/O revealed a distribution of 56 to 44% respectively. The redistribution was performed to bring the corresponding value added in the two sectors more in line with estimates of these parameters. The definitional distinction between food and beverage on the one hand and agricultural products on the other was not made explicit in either source. Separate deflators were applied to food and nonfood products: 1.633 and 1.164 (the Lisbon wholesale food and manufactures deflators respectively, from INE). Private consumption of energy products (motive power, household nonelectric, and household electric) were derived from historical data series.

Government Consumption of Goods and Services: The OECD survey, again based on INE & CDP statistics, reports 1977 government consumption as 72.3 x 10^6 1976 contos. Government consumption was disaggregated according to the 1974 I/O table which records no government consumption of food products. As a result the manufacturers index was used. This is not conceptually the ideal index to use since approximately 68% of government expenditures in 1974 and 75% in 1977 were on wages. However, a government wage index was not available. Government consumption of non-energy products was disaggregated according to the 1974 I/O proportions.

Gross Fixed Capital Formation: Gross Fixed Capital Formation (GFCF) in 1977 amounted to 78.8 x 10^6 contos according to INE and CDP statistics. Since approximately 90% of GFCF originates from the manufacturing and construction industries, the manufacturing index was used. Because no disaggregated account of GFCF by source for 1977 was available, the 1974 disaggregation was used.

Exports and Imports: A fairly disaggregated account of exports and imports for 1977 in 1977 prices was available, as were sectored growth rates for the same. Since it was not possible to disassociate domestic inflation, world inflation, and the price effects of devaluation, no satisfactory import-export price indices could be developed. As a result, volume growth rates from the Ministry of Commerce and Tourism were applied to the level and configuration of trade in the 1974 I/O. The value of energy imports was calculated as a percentage of total imports based on Bank of Portugal and IMF statistics. Because of the different sector definitions, the probable difference in accounting methods, domestic and world inflation, multiple currency devaluation, and import and export controls it is difficult to compare the trade balance implied by the given growth rates and that which is given by both sources in 1977 prices. The 72.8 x 10^6 conto deficit translates into $1,893 million if the 1977 exchange rate is used and $2,839 million if the 1974 exchange rate is used. The trade balance, in 1977 prices, reported by the World Bank based on Bank of Portugal and IMF statistics is $2,531 million. The 1974 and 1977 trade balances are shown on Table 6.12, and Table 6.13 shows the results of the adjustment process.

Table 6.12
Trade Balances
(1974 10^3 contos)

Nonenergy Sectors	1974	1977
19. Agriculture & Fishing	-16,490	-15,570
20. Mining	-2,131	-3,240
21. Food	-3,379	-1,774
22. Textiles	5,113	3,711
23. Wood	11,172	9,747
24. Paper	1,752	1,282
25. Chemicals	-8,937	-14,716
26. Glass	126	-107
27. Cement	32	19
28. Non-Metal Mining	112	-135
29. Iron & Steel	-6,530	-7,263
30. Non-Ferrous Metals	-3,139	-3,448
31. Metal Products	-22,443	-19,523
32. Construction	0	0
33. Misc. Services	170	162
34. Road, Rail Transport	0	0
35. Water Transport	3,524	3,425
36. Air Transport	28	24
Nonenergy Balance	-41,020	-47,406
Energy Balance	-11,568	-19,273
Taxes	102	128
Net Trade Balance	-52,486	-66,551

Table 6.13
Final Demand & Value Added
(10^{12} Joules and 10^3 1974 contos)*

	1977 Final Demamd	1977 Value Added by Sector	
Coal	-14,762	-	
Crude Oil	-247,467	0	
Refined Oil Products	-13,420	833	
Thermal Electric	0		
Hydroelectric		6154	
City Gas	0		
Natural Gas	0	73	
Nuclear Gas	0	0	
Nuclear	0	0	
Wind	0	0	
Biomass	0	0	10^{12}
Solar	0	0	Joules
Coal-Based Feedstocks	0	0	
Oil-Based Feedstocks	72	0	
Motive Power	7,144	0	
Industrial Process Heat	227	0	
Electric Power	898	0	
Household Nonelectric	12,098	0	
Household Electric	9,959	0	
Agriculture and Fishing	7,542	47,082	
Mining	-103	2,269	
Food, Beverage, and Tobacco	61,326	15,586	
Textiles	10,026	11,292	
Clothing, Leather, Wood, & Furniture	26,444	11,099	
Paper and Printing	5,176	5,952	
Chemicals, Rubber, & Plastic	4,878	9,497	
Glass	667	1,081	10^3
Cement	33	449	1974
Other Nonmetallic Minerals	3,145	5,524	Contos
Iron & Steel	-6,521	1,901	
Nonferrous Metals	-3,284	450	
Metal Products, Machinery	47,496	36,195	
Construction	37,112	20,078	
Misc. Services	73,400	94,314	
Road & Rail Transportation	8,904	6,418	
Water Transportation	5,079	3,578	
Air Transportation	5,929	3,055	

Completing the Base Year Transactions Table: Referring back to Table 6.1, complete specification of the extended table of technical coefficients (the A-matrix) requires information on 6 submatrices. Let us use T to represent the corresponding transactions tables--thus T_{II} corresponds to the previously defined A_{II}. Obviously, both T_{II} and T_{IS} can be derived directly from the existing I/O data, once the aggregation scheme has been settled.

The elements of the T_{PI} matrix have dimensions of BTU (or joules): T_{PI} must therefore be constructed from energy sales data. THus, for example, we need to know how many Btu of, say, industrial process heat were consumed in the Cement Industry. Given knowledge of the sales of petroleum products to that industry, and some assumptions about the efficiency of the end use devices involved, Btu (Joule) estimates can be derived. Table 6.14 shows a typical work sheet, as used in the Brookhaven assessment. In explanation of this Table, the sales data from refinery and utility sources are listed in the far right-hand column. These entries are then moved to the appropriate end-use: LPG, for example, is assumed to be used for process heat. When multiplied by the assumed efficiency, a basic energy demand of (46.7)(0.68) = 31.8 is obtained. This is repeated for each fuel: in most cases the end-use category follows directly from the fuel type: gasoline, for example, is not likely to be used in anything but motive power.

It should be noted at this point that it may be desirable to be much more specific about energy end-use categories. Because different industries have different tempeature requirements, "industrial process heat" may be much too general a category. For example, to assess the degree to which solar industrial process heat has any application the low temperature forms of process heat (hot water, hot air) should be distinguished. Indeed, one may wish to maintain an industry identity in the process heat categories, and talk about cement kilns, food industry driers and so forth. Some of the more recent Brookhaven models include this type of sophistication, and the interested reader is directed to the appropriate reports.[8]

[8]See e.g., P. Meier and V. Mubayi, "Modelling Energy-Economic Interactions in Developing Countries: A Linear Programming Approach," BNL 29747, June 1981.

Table 6.14
Worksheet for the T_{PI} Matrix
Non-Ferrous Metals (Sector 32)

	Coal Based Feedstocks			Oil Based Feedstocks			Motive Power			Process Heat			Electricity			Total		
	f_i	e_i	D_i	f_i	e_i	D_i	f_i	e_i	D_i	f_i	e_i	D_i	f_i	e_i	D_i	f_i	e_i	D_i
DIRECT FUEL USE																		
LPG										31.8	0.68	46.7						46.7
PETROL							0.3	0.2	1.5									1.5
GAS/DIESEL OIL							1.5	0.2	7.3									7.3
FUEL OIL										127.8	0.68	202.7						202.7
GASOLINE																		
NON-ENERGY PROD.				0.4	1.0	0.4												0.4
COAL																		
OTHER																		
ELECTRICITY													177.1	1.0	177.1			177.1
TOTAL FUEL DEMAND, D,						0.4			8.8			249.4			177.1			435.7
																This column from utility & refinery sales data.		
BASIC ENERGY DEMAND, E,				0.4			1.8			169.6			177.1					

E = basic energy dmeand
e_i = efficiency
D_i = fuel demand

Finally there are the T_{SS}, T_{PS}, and T_{SP}. These submatrices contain the information that is contained in the Reference energy system; exercise 6.3 illustrates the details.

Having then specified the complete Transactions matrix, the corresponding A matrix follows quickly by division of the corresponding base year gross output vector, in the manner discussed in an earlier section. And at this point one is ready to use the model for practical analyis.

EXERCISES, CHAPTER 6

E11. Savings-Investment Identity

Using the definitions of Digression 6.1 for savings by government, corporations, foreigners and households, show that total savings equals total investment.

E12. Republica Gross Outputs.

Suppose that exports of agricultural goods increase from 20 to 30 units. Using the Inverse Matrix (6.9) (see Table 6.1), determine the impact on gross output.

E13. Interpretation of the A_{SS} Matrix/A_{SP} Matrices

Suppose we have an A_{SS} matrix with 7 sectors:

	S_1 Crude	S_2 Coal	S_3 Coal/ Elect.	S_4 Resid.	S_5 Oil Fired Electr.	S_6 Gaso- line	S_7 Electric Power
S_1 Crude				r_1		r_2	
S_2 Coal		t_c	$\frac{1}{\eta_c}$				
S_3 Coal/ Elect.			t_r				f_c
S_4 Resid.					$\frac{1}{\eta_o}$		
S_5 Oil fired Elect.					t_r		f_o
S_6 Gasoline							
S_7 Electric Power							

(a) Explain each of the non-zero entries shown above.

(b) If only gasoline and electric power appear in the vector of final demand, say Y_6 and Y_7, compute the gross output vector X. (Hint: Use the relationship $A_{SS}\ X + Y = X$)

(c) Partition the above matrix into the A_{SS}, A_{SP}, and A_{PS}, submatrices.

7. MICRO ANALYSIS

7.1 THE ANALYTICAL PROCESS

This Chapter turns to the techniques of the area of analysis defined in Chapter 1 as micro analysis. The distinction between our use of the term micro analysis, and project investment analysis, is perhaps a fine one: many of the analytical tools -- such as rate of return calculations -- are the same. The major point of distinction is one of degree of project specific detail. Micro analysis as part of the overall planning process deals with the evaluation of particular technologies to particular markets, using generalized data, perhaps based on a "typical" facility. For example in assessing the applicability of solar water heating for application to hotels or hospitals, calculations at the planning stage might be made for a typical 100 room facility, rather than some specific facility.

Indeed, while a great deal of attention has been focussed on the tools of what we here term macro analysis (as is the concern of most of this book) relatively little attention has been given, either in the research literature or in practice, to the process of translating general energy strategies into investment programs involving the application of specific technologies to specific markets. Thus, while manipulation of an energy supply/demand model may quickly allow the conclusion that industrial conservation or use of solar hot water heating in the commercial sector will reduce oil imports to some more desirable level, it is an altogether more difficult process to set forth a financially feasible investment plan containing a portfolio of specific projects recommended for implementation.[1] Figure 7.1 illustrates the relationship of national energy planning to energy sector investment. The feedback loop indicated has two dimensions - a real time dimension, insofar as an energy plan drawn up at some time depends on the investment achieved <u>up to</u>

[1] This is of course not a problem confined solely to energy sector investment. For example, in Tunisia for the period of the fourth Development Plan (1973-1976), investment was the only macro-economic objective not met (or exceeded). The World Bank, in its review of the prospects for the 5th Plan, noted that the otherwise excellent economic performance was offset by investment, "...which was 10 percent short of target because Government investment remained 30 percent below target. Investment performance during the Fourth Plan showed that the Government's and public enterprises' capacity to identify, prepare and implement projects still needs considerable strengthening. The measures taken by the Government since the beginning of the decade to shift greater responsibility to the private sector, and supporting it through a generous incentive system, were reflected in the excellent investment performance of this sector that exceeded its target."

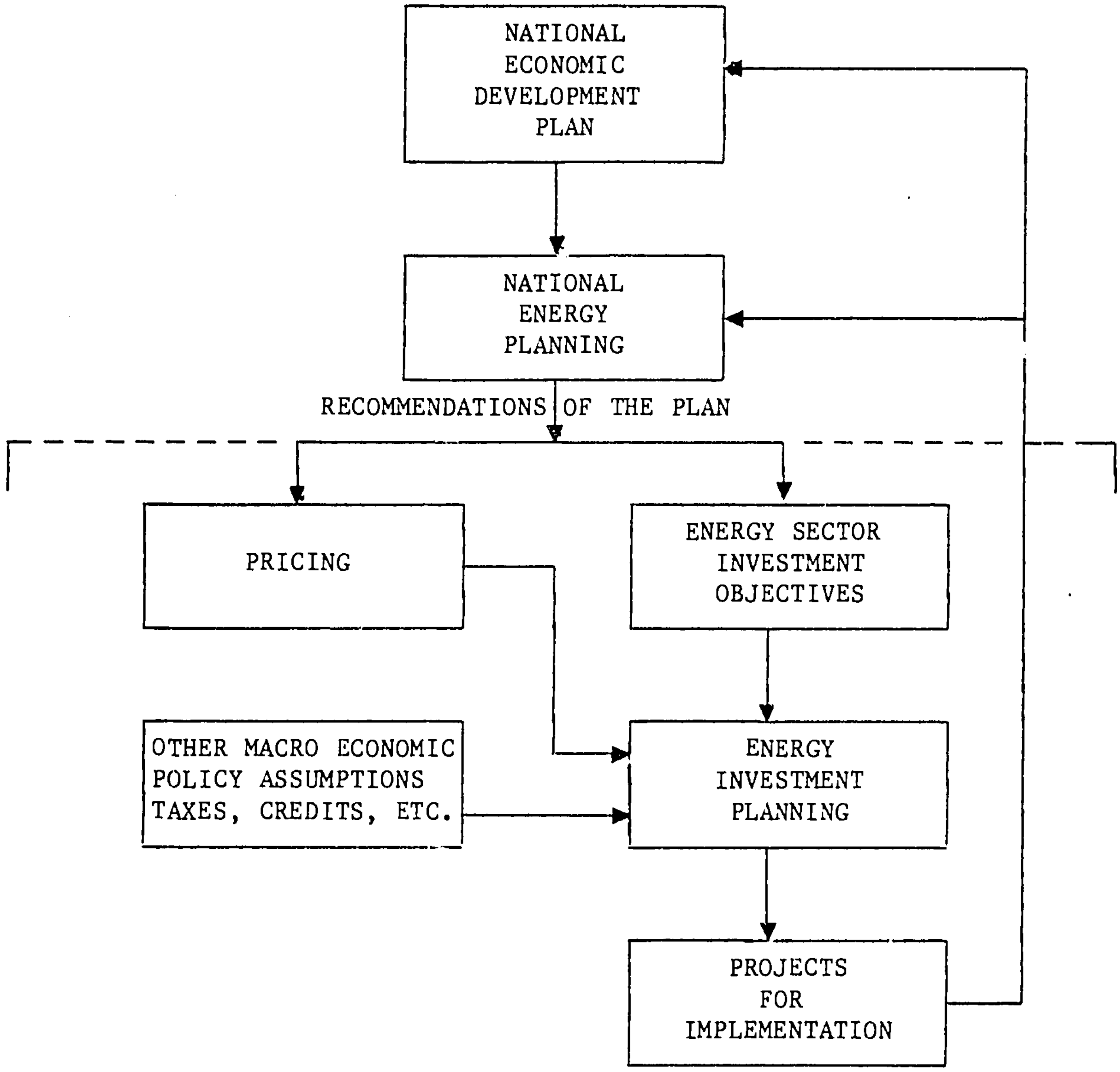

Figure 7.1. The Planning Process.

that time, and an analytical dimension: a plan that gives little attention to what can in fact be financed in the way of investment is likely to be a failure. Thus energy analysis must include a first order consideration of investment criteria - not as a substitute for detailed investment planning in preparation for a loan application, but as a means for improving the precision of the energy plan. This is a major goal of what is here termed micro analysis.

Field Demonstration and Survey Decisions

One of the most important factors in chaping a successful national energy planning effort is the degree to which its various component activities can be coordinated. Given the reality, in most developing countries, of restricted budgetary resources for the energy planning activity, an effective allocation of resources is critical to ultimate success. Thus, unless energy consumption surveys and field demonstrations are viewed as activities intimately linked with the planning process, and given structure and prioritization by the needs of analysis, resources are likely to be wasted.[2] Figure 7.2 depicts the overall process of micro analysis. The basic notion here is that of uncertainty. The point of the analytical process is to reduce the variability of an initial rate of return calculation (step 3) to the point at which the probability of an unsound investment decision, as it may be imposed by a financing institution, is reduced to acceptable levels.[3] Suppose, for example that the macro analysis points to solar hot water heating as a desirable strategy (1).[4] The initial step is to identify a series of specific domestic hot water heating in detached houses, flat plate collector systems for hotels, etc., and to perform a first rate of return calculation using whatever data is available. This may require, for example, the use of U.S. or European cost estimates for flat plate collectors even where local manufacture is ultimately envisaged. This first computation also requires some other assumptions (4) - such as the price of competing fuel oil and or electricity the level of investment tax credits, if any, and so forth.[5]

---

[2]Indeed, the failure of the U.S. Department of Energy is in large measure attributable to the lack of coherence among its constituent entities (even despite the ever increasing amounts of money made available to the agency) - the Energy Information Administration collecting data largely for its own sake; the various technology offices each pursuing its own favorite projects; and the Planning and Analysis arm becoming bogged down in disputes over models, assumptions and goals.

[3]For example, the World Bank now requires that an applicant demonstrate the sensitivity of the rate of return to major assumptions and applicants are encouraged to use more formal methods of probability analysis.

[4]The figures in parentheses refer to the steps shown in Figure 7.2.

[5]For a full discussion of the role of such variables in rate of return calculations for renewable technologies see e.g., V. Mubayi et al., "Harnessing the Sun for Development: Actions for Consideration by the International Community at the U.N. Conference on New and Renewable Sources of Energy," Nairobi, Kenya. August 1981.

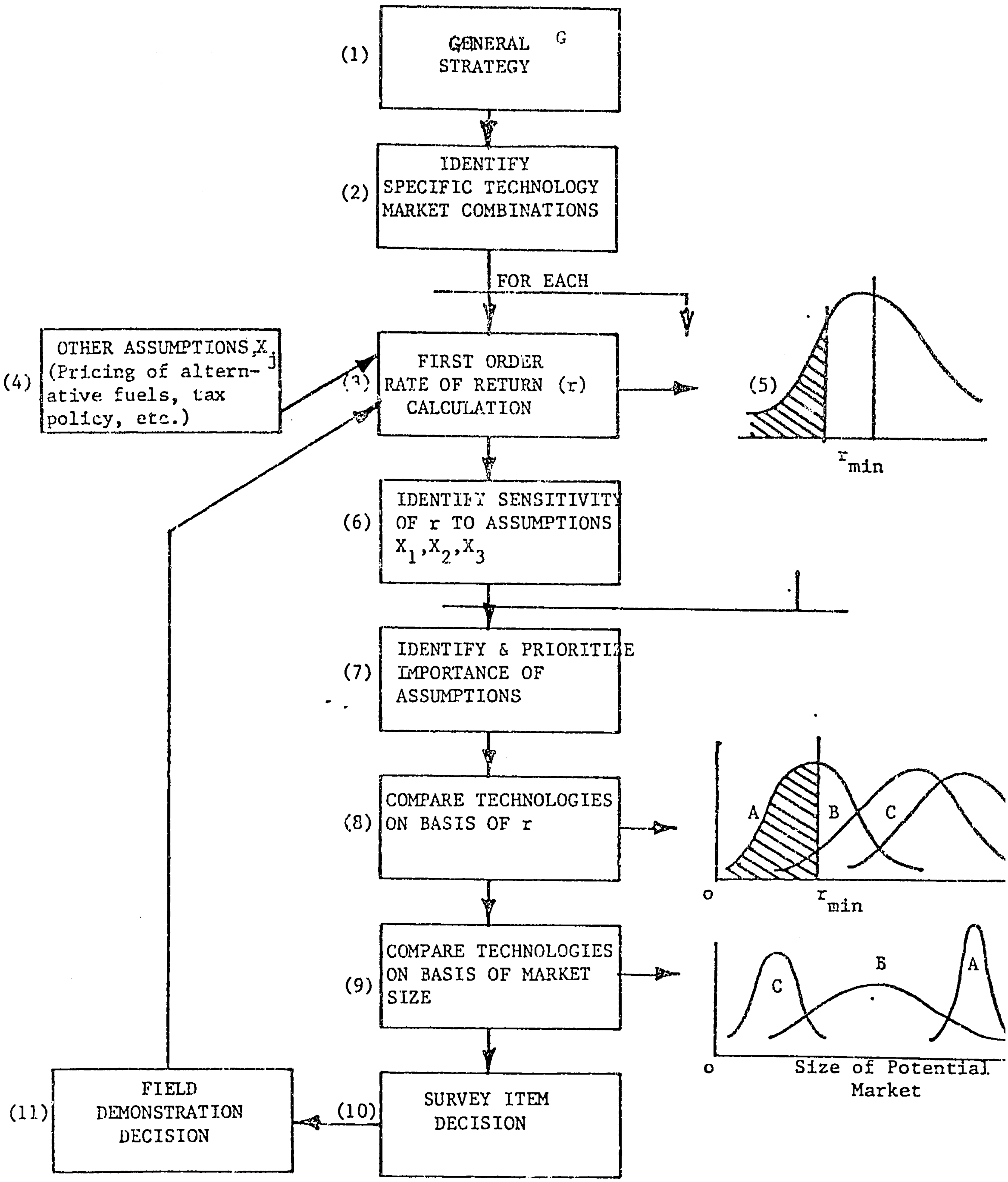

Figure 7.2. The General Framework.

Because of the many uncertainties, the resulting rate of return can be viewed as a probabilistic function, of mean r (5). If we denote r_{min} as the minimum acceptable rate of return on investment (more of which below), then the shaded area represents the probability of making an unsound investment. The idea of the analytical process is to reduce the variance of this probability distribution, hence minimize the probability of making an unsound decision. Obviously there are some variables over which the analyst and planner has no control - such as the world price of oil. However, because many petroleum products and electricity are presently still subsidized in most developing countries, selective elimination of the existing subsidies provides a point of policy leverage which should be included in the analysis. By repeating the rate of return calculation for ranges of assumptions one can generate the desired probability distribution for r. Whenever possible, the sensitivity to each assumption should be established (6).

The next step is to prioritize the importance of each assumption according to its relative influence on the rate of return calculations (7), and to rank the technology/market pairs on the basis of candidate rate of return (8) and market size (9). Given the existing information, there may also be considerable uncertainty over the potential market, since particular technologies may be suited only to very limited housing types (or specific industrial processes), whose occurence may or may not be well established.

At this point the results of (8) and (9) must be combined. A technology may have a very high r, but small potential market (e.g. technology C of Figure 7.2), and may not deserve any further attention. Technology B of Figure 7.2, however, may merit both a field demonstration to establish local costs, and a survey to identify market size more carefully.

Figure 7.3 demonstrates this in a more specific way, for illustrative purposes using the example of solar hot water heating for luxury hotel application. Assume that the two most important parameters subject to uncertainty is the collector cost C (TD/m^2 of locally manufactured flat plate collector), and the hot water consumption (in liters/room/day).[6] The first analysis might be done with U.S. data for flat plate collector costs, say $C_1 \pm \gamma_1$ and

[6]In actuality total consumption will not be the only important parameter - to properly size the solar units requires a breakdown between kitchen, laundry and guest use.

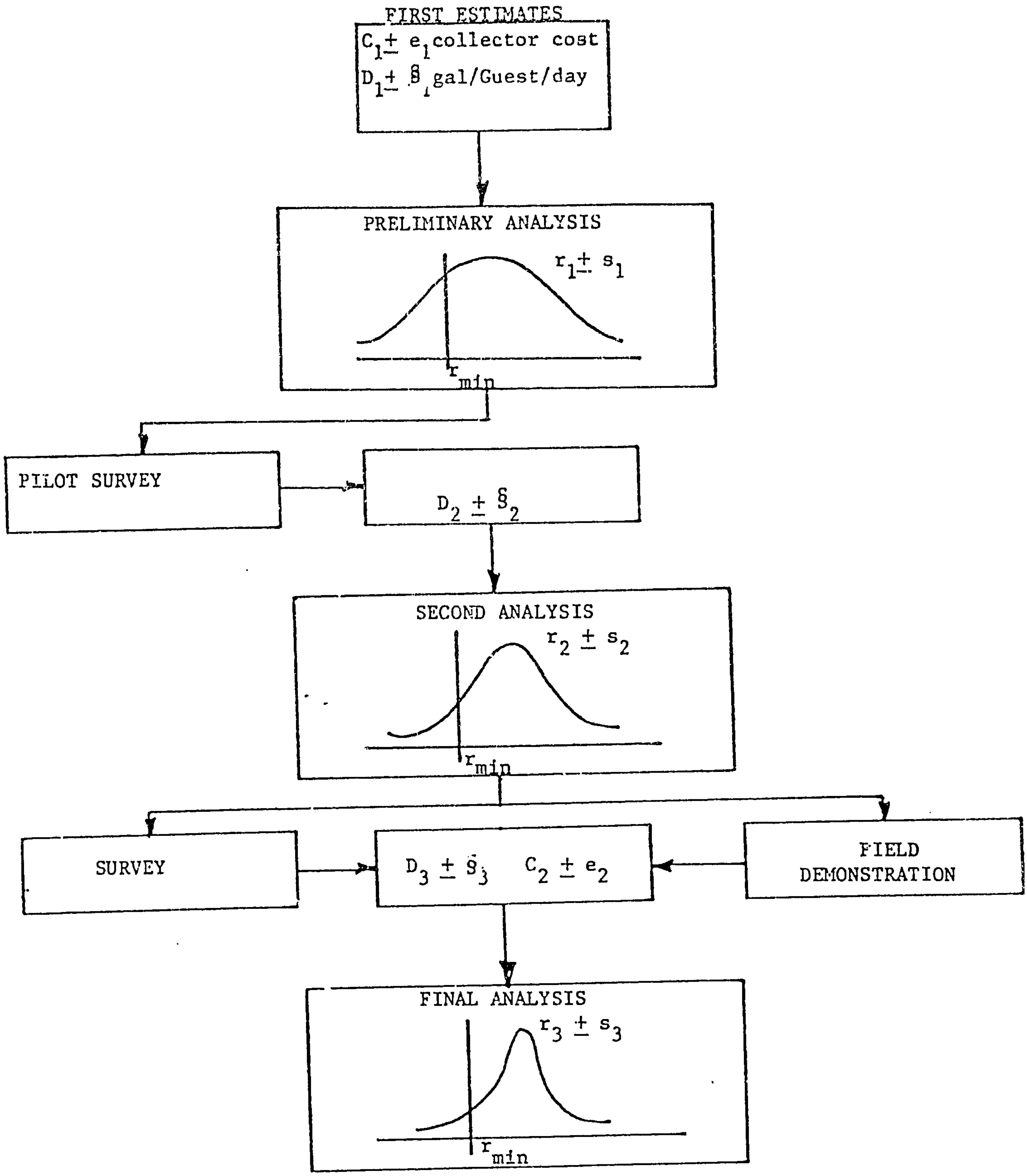

Figure 7.3. Field Testing and Survey Decisions.

some very sparse hotel data from a cursory initial survey, say $D_1 \pm \delta_1$. The preliminary analysis might yield a rate of return $r_1 \pm \sigma_1$, where r_1 is less than r_{min}. This would be sufficient to warrant a pilot survey (say of the 10 major hotels) to establish hot water consumption with greater precision - say $D_2 \pm \delta_2$, where δ_2 is less than δ_1. A second analysis may yield $r_2 \pm \sigma_2$ (σ_2 less than σ_1), upon which one may now make a decision to field test (to reduce the variance of collector costs, γ_1), as well as a further data survey (the next 20 hotels, with more details on consumption patterns). The final analysis reduces the uncertainty to the point at which the probability of an unsound decision is considerably reduced.

Dealing With Uncertainty

In addition to the technological and data uncertainties sketched in the previous section, the planning process must deal with other types of uncertainty - future conditions of the world energy market, oil and gas production in the country itself, overall national economic growth and its sectoral composition. The wrong way of dealing with such questions is to attempt to predict the future, and then design the energy plan to fit that future. The correct way is to accept the impossibility of predicting the future, and to identify policies that are flexible and robust. A useful framework here is that of mathematical game theory, that pits the energy planner "against" the rest of the world in a so-called two person zero-sum game.[7] Assume that the world oil price is the principal dimension beyond the control of the national government - which may take on the values shown. The national government itself may elect one of three strategies-say A, B, or C. The so called pay-off matrix (Table 7.1) displays the results of all combinations of government strategy and oil price in terms of some national objective - e.g. GDP growth or employment. In the simple example shown, policy B dominates policy C (i.e. is better under all conditions of future oil price), whereas the choice of A or B depends on the level of risk aversion of the decision-maker. If all three oil price outcomes are regarded as equally likely, then the conservative choice is strategy A, since the worst GDP impact is 3. If the

[7]The illustrative example used here assumed that the choices of the parties are independent - that is, the World Oil Price is unaffected by the choice of the National Government. Such a model would not be appropriate to very large exporters (such as Saudi Arabia), very large importers (such as the U.S.) or smaller countries that may strongly influence regional markets (such as Indonesia in South East Asia).

Table 7.1
The Payoff Matrix

		External Conditions Beyond Control of the National Government (e.g. World Oil Price in 1985)		
		$32/bbl	$40/bbl	$60/bbl
Conditions	A	3	4	6
Controlable				
by the	B	8	4	2
National				
Government	C	2	3	2

decision-maker is indifferent to risk, then the optimum strategy is B[8]. Where the decision-maker (or the analyst) views the different scenarios as being of differing likelihood, application of the appropriate probabilities will of course modify the optimum choice for a given level of risk aversion.

This discussion underscores two further points of note. The first concerns the proper use, and limitations of modelling: the current state of the art is such that energy-economic models cannot be used to identify optimum strategies: at best they can be used in a simulation mode to compute the elements of the pay off matrix given some particular set of policy parameters. The second is that the choice of energy strategy is necessarily linked to the overall development goal; what matters is not so much the absolute level of oil imports but the effects of the implied foreign currency burden on the development of non-energy sectors.[9]

[8] If all three oil price cases are equally likely, then strategy A has an expected value of E(A) = 0.33 x 3 + 0.33 x 4 + 0.33 x 6 = 4.29. E(B), however, computes to 4.62.

[9] Many of the most successful economies in South and East Asia (Japan, Singapore, among others) illustrate this point.

7.2 FUNDAMENTALS*

The essential concept of engineering economics is the time value of money. One dollar today is not equivalent to one dollar 1 year from hence. If today's one dollar were invested at interest rate i, then that dollar would be worth 1(1 + i) in one year's time. If the interest is reinvested, then after two years we have 1(1 + i)(1 + i). Generalizing, if the amount P(o) is left in a savings account for n periods, then the amount in the bank at the end of the n-th period is

$$P(n) = P(o)(1 + i)^n \tag{7.1}$$

This is, of course, the definition of compound interest. We say that the dollar amount P(n) at the end of time period n is equivalent to the amount P(o) at time zero.

To eliminate the tedium of having to compute the quantity $(1 + i)^n$, this is tabulated in so called annuity tables as the Compound Amount Factor (or CAF(i, n))[1]--although with the advent of cheap, programmable electronic pocket calculators, the convenience of annuity tables is perhaps less obvious.

There is, of course, the reciprocal problem. Given some amount of money P(n) at some future time n, what is the equivalent worth of that amount at time 0 -- a quantity termed the present value. By rearranging (7.1), we quickly obtain

$$P(o) = P(n) \cdot \frac{1}{(1 + i)^n} \tag{7.2}$$

The quantity $1/(1+i)^n$ is known as the present worth factor PWF(i, n), which is the reciprocal of the compound amount factor, and is also tabulated in annuity tables.

In many situation one deals with situations that generate a whole series of disbursements and/or receipts. Suppose, for example, we invest 1$ at the end of each of 4 years in a savings account paying interest i, and are interested in the amount of money at the end of the 4th year. For this we draw the following cash flow diagram

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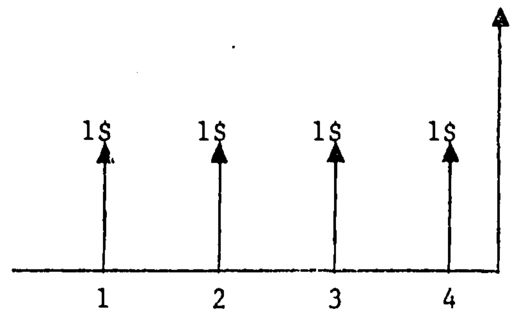
*This section can of course be omitted by readers familiar with engineering economics.

[1] Annuity tables are almost always arranged in such a way that one page relates to a paticular interest rate, and the rows of the tables identify the number of time periods.



Clearly one could compute X from first principles, as follows:

$$
\begin{array}{lr}
1 \times CAF(i = 6,\ n = 3) & = 1.1910 \\
+\ 1 \times CAF(i = 6,\ n = 2) & = 1.1236 \\
+\ 1 \times CAF(i = 6,\ n = 1) & = 1.0600 \\
+\ 1 & = \underline{1.0000} \\
 & 4.3746
\end{array}
$$

To eliminate the tedium of this arithmetic, we introduce the so called series compound amount factor (SCAF(i,n)), which, when multiplied by the amount of a uniform series of n payments, yields the equivalent amount at the end of the n-th time period. Thus SCAF (6%, n = 4) = 4.3746. Algaebraically, we have the definition

$$SCAF(i,n) = \sum_{t=1}^{n-1} (1 + i)^t + 1$$

which can be shown to equal

$$SCAF(i,n) = \frac{(1+i)^{n-1}}{i}$$

Again there is the reciprocal problem. Suppose we wish to accumulate some amount Y by the end of the n-th time period, by setting aside n equal payments, Z. For this situation we introduce the socalled sinking fund factor (SFF(i,n)), such that

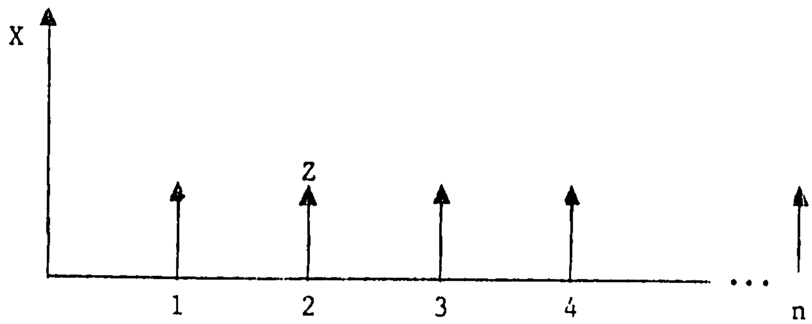
$$Z = Y \ .\ SFF(i,n)$$

The definition of the series present worth factor, SPWF(i,n), follows in a similar fashion. Suppose we have the cash flow situation:

Table 7.2
Summary of Formulae

n = Time period (in years, unless otherwise stated)
i = Interest Rate (per year, unless otherwise stated)

$CAF\ (i,n)$	$= (1+i)^n$	$= \dfrac{1}{PWF\ (i,n)}$	Compound Amount Factor
$PWF\ (i,n)$	$= \dfrac{1}{(1+i)^n}$	$= \dfrac{1}{CAF\ (i,n)}$	Present Worth Factor
$SCAF\ (i,n)$	$= \dfrac{(1+i)^n-1}{i}$	$= \dfrac{1}{SFF\ (i,n)}$	Series Compound Amount Factor
$SFF\ (i,n)$	$= \dfrac{i}{(1+i)^n-1}$	$= \dfrac{1}{SCAF\ (i,n)}$	Sinking Fund Factor
$CRF\ (i,n)$	$= \dfrac{i\ (1+i)^n}{(i+1)^n-1}$	$= \dfrac{1}{SPWF\ (i,n)}$	Capital Recovery Factory
$SPWF\ (i,n)$	$= \dfrac{(1+i)^n-1}{i(1+i)^n}$	$= \dfrac{1}{CRF\ (i,n)}$	Series Present Worth Factor
$CRF\ (i,n)$	$= SFF\ (i,n) + i$		



From first principles, by applying the (single payment) present worth factor to each payment

$$X = \sum_{t=1}^{n} Z \frac{1}{(1+i)^t}$$

$$= Z \cdot \sum_{t=1}^{n} \frac{1}{(1+i)^t}$$

$$= Z \cdot SPWF(i,n)$$

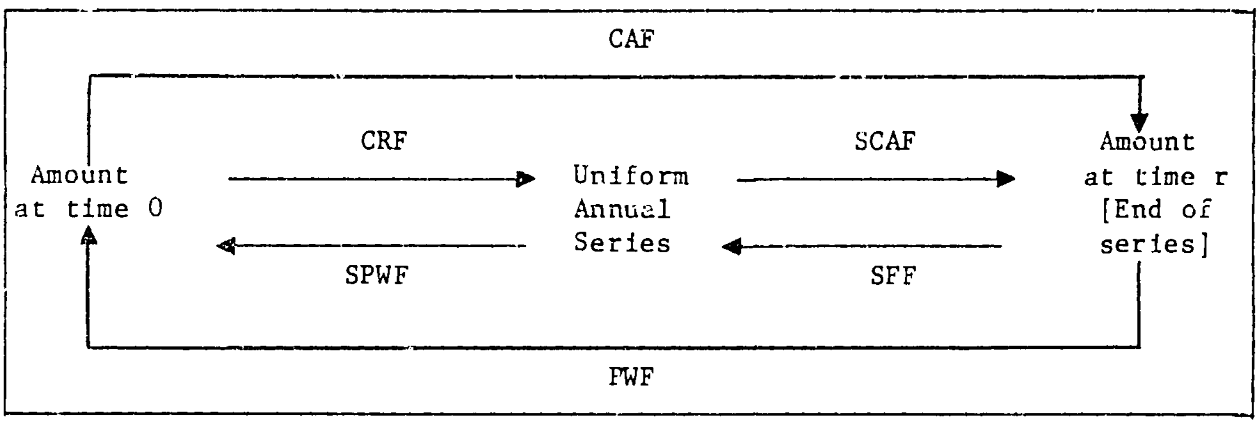
which as again to be found in the annuity tables. The reciprocal problem is known as the Capital Recovery problem: given a loan of amount Y at t=0, what is the annual repayment Z, if repaid over n time periods at interest rate i. In anology to the previous derivations, it is readily shown that

$$Z = Y \cdot CRF(i,n)$$

where CRF(i,n) is the so-called Capital Recovery factor.

At this point one might emphasize the following points:

1. Unless otherwise stated, it is to be assumed that all disbursements and receipts occur at the <u>end</u> of each time period. The annuity tables for discrete compounding always make this assumption unless explicitly stated to the contrary.
2. To assist selection of the correct annuity factor, the following chart has proven useful:



   For example, if the value of the series is known, and the quantity of interest is at the end of the series, apply SCAF.

3. Never apply arithmetic operations to money amounts that do not have the same time reference.

<u>Continuous Compounding</u>

The discussion thus far assumes that interest is accrued at the end of each time period: Interest of 6% on 1 dollar held for 1 year is added at the end of that year, to yield $1.06. Suppose, however, that interest is compounded quarterly for which we speak of a nominal annual rate of i, and a quarterly rate of $\frac{i}{4}$. Then the <u>effective</u> interest rate is defined as

$$r = \left(1 + \frac{i}{4}\right)^4 - 1$$

which, for a nominal annual rate of 12%, is

$$r = \left(1 + \frac{0.12}{4}\right)^4 - 1$$

$$= .1255$$

note that the effective rate is higher than the nominal rate.

If interest is compounded m times per year, then the above generalizes to

$$r = \left(1 + \frac{i}{m}\right)^m - 1$$

Return to the compound amount factor CAF(i,n), defined as

$$= (1 + i)^n$$

If interest is compounded m times per year, this can be written

$$= \left(1 + \frac{i}{m}\right)^m$$

If we let $k = \frac{m}{i}$, then

$$= \left(1 + \frac{1}{k}\right)^{ki}$$

It can be shown that as $m \to \infty$, $k \to \infty$, and that

$$\lim_{k \to \infty} \left(1 + \frac{1}{k}\right)^k = e \ (= 2.7182)$$

Hence the compound amount factor for continuous compounding is given by

$$CAF^*(i,n) = e^{in}$$

By applying this result to the other expressions of Table 7.2, the following continuously compounded annuity factors are readily derived:

$$CAF^*(i,n) = e^{in}$$

$$PWF^*(i,r) = e^{-in}$$

$$SCAF^*(i,r) = \frac{e^{in} - 1}{e^{i} - 1}$$

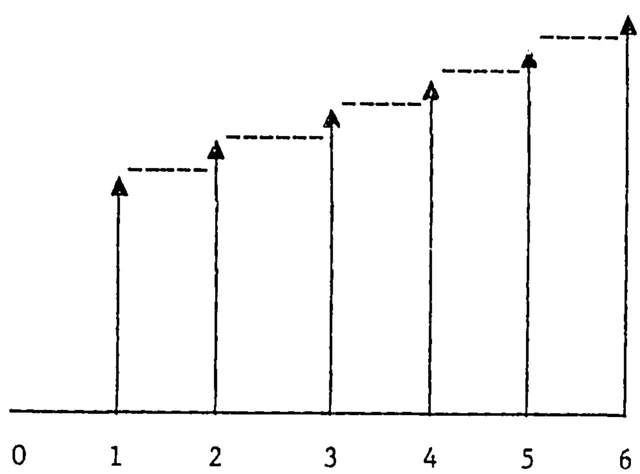
$$SFF^*(i,r) = \frac{e^{i} - 1}{e^{in} - 1}$$

$$CRF^*(i,n) = \frac{e^{i} - 1}{1 - e^{-in}}$$

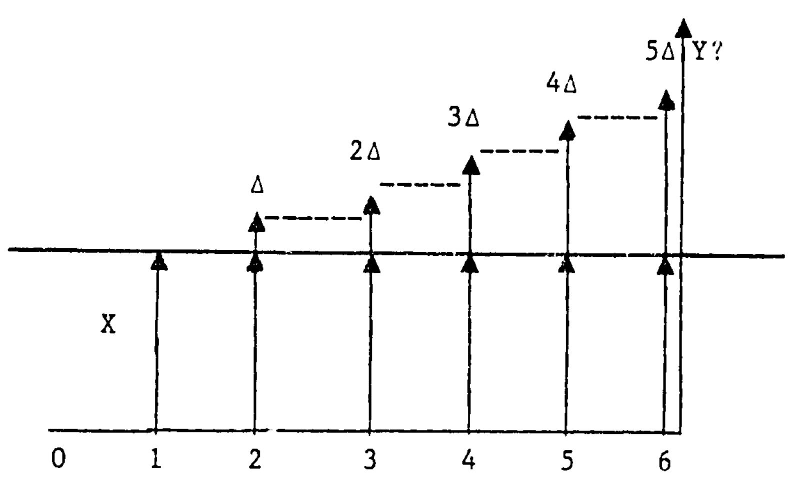
$$SPWF^*(i,n) = \frac{1 - e^{in}}{e^{i} - 1}$$

Gradient Series

In many situations a series of receipts or disbursements is not uniform, but increasing over time with some constant increment :



Suppose one wished to determine the equivalent amount at the end of the 6-th time period. Note first that the above cash flows can be broken in two -- a uniform series in the amount X, and a so-called gradient series:



Inspection of this diagram readily confirms that

$$Y = \Delta \cdot SCAF9i,5) + \Delta \cdot SCAF(i,4) + \Delta \cdot SCAF(i,3) + \Delta \cdot SCAF(i,2) + \Delta \cdot SCAF(i,1)$$

which, by reference to Table 7.2, can be written as

$$= \left\{ \frac{(1+i)^5 -1}{i} + \frac{(1+i)^4 -1}{i} + \frac{(1+i)^3 -1}{i} + \frac{(1+i)^2 -1}{i} + \frac{(1+i)^1 -1}{i} \right\} \Delta$$

hence

$$= \frac{\Delta}{i} \left\{ (1+i)^5 -1 + (1+i)^4 -1 + (1+i)^3 -1 + (1+i)^2 -1 + (1+i)^1 -1 \right\}$$

$$= \frac{\Delta}{i} \left\{ (1+i)^5 + (1+i)^4 + (1+i)^3 + (1+i)^2 + (1+i)^1 +1 \right\} - \frac{6\Delta}{i}$$

Now the quantity in brackets is readily seen to be equal to the series compound amount factor: thus

$$Y = \frac{\Delta}{i} SCAF(i,6) - \frac{6\Delta}{i}$$

$$= \Delta \ \frac{SCAF(i,6) - 6}{i}$$

The quantity in brackets is known as the Gradient compound amount factor (GCAF), whose general definition is

$$GCAF(i,n) = \frac{SCAF(i,n) - n}{i}$$

To convert to a uniform series of the amount Z, one simply multiplies by the sinking fund factor SFF(i,n)

$$\Delta \ \frac{SCAF\ (i,n) - n}{i} \cdot SFF\ (i,n)$$

hence

$$Z = \frac{\Delta}{i} \left\{ \frac{(1+i)^n -1}{i} \cdot \frac{i}{(1+i)^n - 1} - n \cdot \frac{i}{(1+i)^n - 1} \right\}$$

$$= \frac{\Delta}{i} - \frac{n\Delta}{i} SFF(i,n)$$

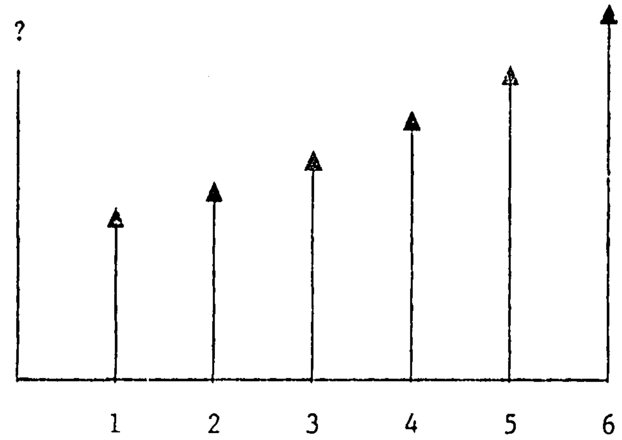
$$= \Delta \left\{ \frac{1 - n \cdot SFF(i,n)}{i} \right\}$$

The quantity in brackets is called the Gradient to Uniform Series Factor (GUSF), which is sometimes to be found, together with the GCAF, in annuity tables.

Compound Growth Series

In the case where the gradient is defined as a compounded growth rate, present worth must be calculated from first principles. For example, in the following diagram, compound growth at a rate g is given by

$$x_j = x_1(1+g)^{j-1}$$



The present worth quickly follows as

$$PW = \sum_j \frac{x_1(1 + g)^{j-1}}{(1 + i)^j}$$

which is not amenable to simplification except in the special case of $i = g$.

Interest Rates

Prevailing interest rates tend to have a number of components -- the real time value of money (i_{real}), the inflation rate (i_{infl}), and risk (i_{risk}). Thus

$$(1 + i) = (1 + i_{real})(1 + i_{infl})(1 + i_{risk})$$

In the U.S., the real interest rate or time value of money has historically been some 2-3%, as measured by relatively risk free long term debt -- such as U.S. Treasury Bills, corporate bonds rated in the highest category ("AAA")

The normal business risk premium typically runs at some 5-10% above the real rate plus inflation. For example, recent prime interest rates, charged by the U.S. banks to their best corporate customes, have run to about 20% at a time when inflation rates have been 10-12%. It should be noted that in the context of project financing, normal business risk includes two quite distinct components: technical risk, which relates to the ability of a project to produce a product at some given cost, over the duration of the

Exhibit 7.1: Financing Sources and Terms

Financing terms for typical developing country investment programs can vary greatly. Below is shown the sources and terms of financing considered in a recent investment planning study of the Dominican Republic. The Venezuela and Venezuela/Mexican credits were established by the Agreement of San Jose (also known as the oil facility); they carry extremely favorable conditions (2% interest), and are available for the domestic portion of energy investments.

	Financing Terms			
Source	Grace period (years)	Loan period (years)	Interest (%)	Commission (%)
Export Cedits	construction period + 1/2 yr	construction period + 8 yrs	7.75	1.125
Commercial Banks	construction period + 1 yr	construction + 5 yrs	12.5	1.5
Venezuelen Credits	construction period	construction period + 20 yrs	2	0
Inter-American Development Bank	5	15	8.5	1.25
World Bank	4	13	10.6	0.75
Venezuelan Mexican Credits	5	15	2.0	0
Suppliers Credits	2	10	15	0.5
Banco Exterior de Espana	5	12	8	0.375
OPEC	5	15	8	0
Bilateral	4	15	7.5	0.5

*"Planning for National Energy Investment in the Domincan Republic, Phase I," prepared for La Comision Nacional de Politica Energetica de la Republica Dominica, by Energy Development International, May 1982.

project life, and commercial risk, which relates to the ability to sell a product at a given price over the life of the project.

Depreciation

Depreciation is a measure of the loss of value of an asset. In accounting terms, depreciation is used to recover the initial value of the investment. There are many different methods of depreciation, and because of the tax significance, tax regulations usually determine the choice of method used. In general, a business wishes to depreciate its investment as fast as possible (thereby minimizing short-term tax liability): whereas Government has an interest in slower depreciation rates (thereby maximizing short run tax revenue). In addition, to secure the interest of stockholders where private corporations are concerned, it is desirable that the book value of an asset (i.e., first cost less cumulative depreciation) corresponds reasonably well to market value of that asset: if depreciation is too slow, the book value will overstate the value of the asset (as measured in the marketplace), and hence overstates the value of the corporation itself. Especially in an inflationary world, corporations are interested in rapid depreciation, since the benefit of a given future depreciation deduction becomes increasingly less when inflation is taken into account.

The simplest method of depreciation is the so-called straight line method of depreciation, in which the (constant) annual depreciation, d, is given by the relationship

$$d = \frac{P-L}{n}$$

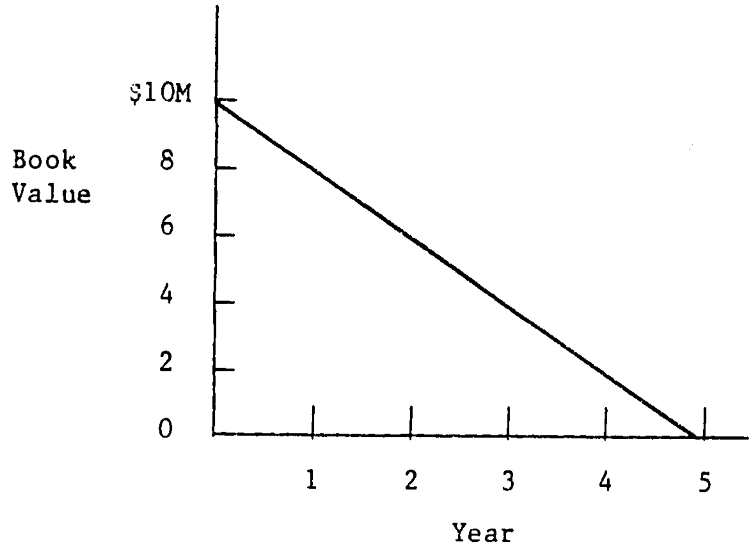
where

n = asset life

L = salvage value

P = purchase price

For example, a \$10M asset depreciated over 5 years by the straight line method results in the following book value over time.



Thus the annual depreciation expense is $2 million and the NPV of depreciation deductions over the asset life at interest rate i = 12% is

$$\$2M \cdot SPWF(12\%) \cdot \tau$$

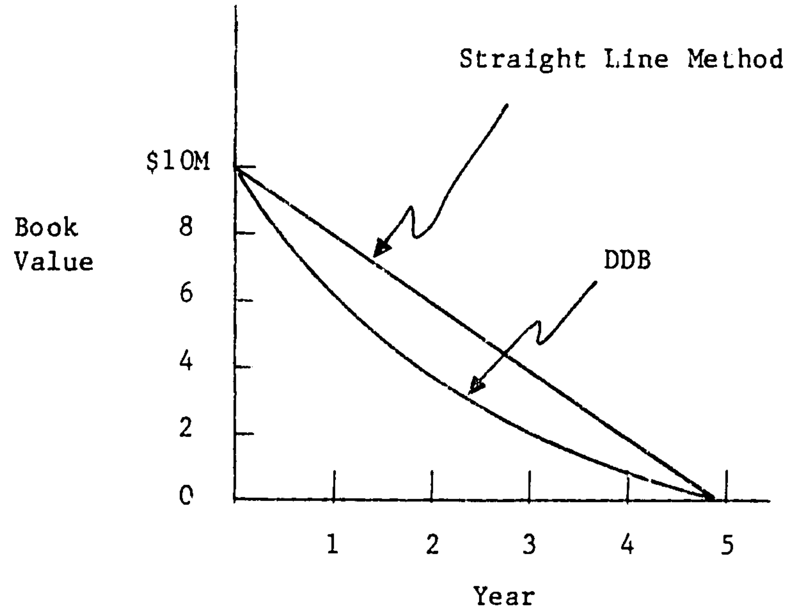
where τ is the applicable rate of income tax. For τ = 40%, this evaluates to

$$NPV_{SL} = \$2 \text{ million} \cdot 2.605 \cdot 0.4 = \$2.884 \text{ million}$$

More widely used are the socalled <u>declining balance</u> methods of depreciation. Here salvage value is ignored (although tax rules usually do not allow an asset to be depreciated below salvage value): the starting point is the choice of <u>rate</u>, usually expressed as a multiple of the straight line rate. Thus the double declining balance (DDB), or 200% declining balance methods are based on a depreciation rate that is twice that of the straight line method (which is the reciprocal of the asset life). Return to our numerical example. The schedule of depreciation follows from the following table, where each year's depreciation charge is computed by applying the depreciation on rate to the current (remaining) book value. Here the DDB method requires a rate of 0.4 (= 2x 1/5)

Year	Book Value at Start	Depreciation D_i	Book Value at End
1	10	0.4 x 10 = 4	6
2	6	0.4 x 6 = 2.4	3.6
3	3.6	0.4 x 3.6 = 1.44	2.16
4	2.16	0.4 x 2.16 = .864	1.29
5	1.29	1.29	0

As inidcated on the diagram below, this method yields a much faster rate of depreciation in the early years



Here the NPV of depreciation deductions is given by

$$NPV_{DDB} = \tau \cdot \sum_{j-1}^{N} \frac{D_j}{(1+i)^j}$$

$$= 0.4 \quad 4 \times .893 + 2.4 \times .797 + 1.44 \times .712 + .864 \times .636 + 1.29 \times .567$$

$$= \$3.116 \text{ million}$$

a gain of some $0.232 million.

7.3 INVESTMENT CRITERIA

Investment Criteria quantify the merit of a givcn investment project or set of projects; and in many cases the measure of merit of the proposed investment(s) is evaluated with respect to some baseline. The matter is far from simple, however, since many different rankings can be used; and because different criteria may yield different rankings of projects, the choice of method is often subject to considerable uncertainty. Rate of return on capital investment, net present value, and benefit/cost ratios are among the more commonly encountered criteria. Great care must be exercised in the selection of the appropriate method: and many of the necessary assumptions (such as the correct value of discount rate or the cost of capital) may significantly affect the outcome.

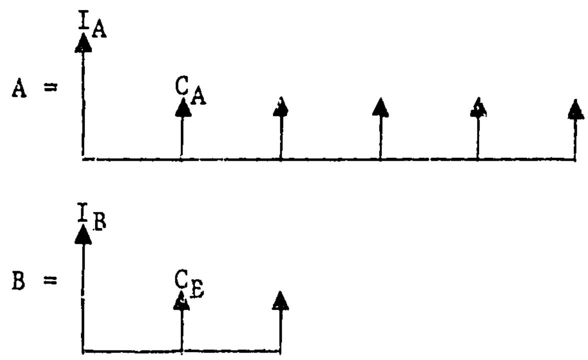
Net Present Value(NPV).

The NPV criterion is perhaps the simplest of all investment criteria: by bringing all disbursements and revenues associated with a given project to time t=0 (by using, say, the PWF and SPWF factors discussed above), the NPV is readily computed. If C_j represents the net cash flow in year j (receipts less disbursements), then

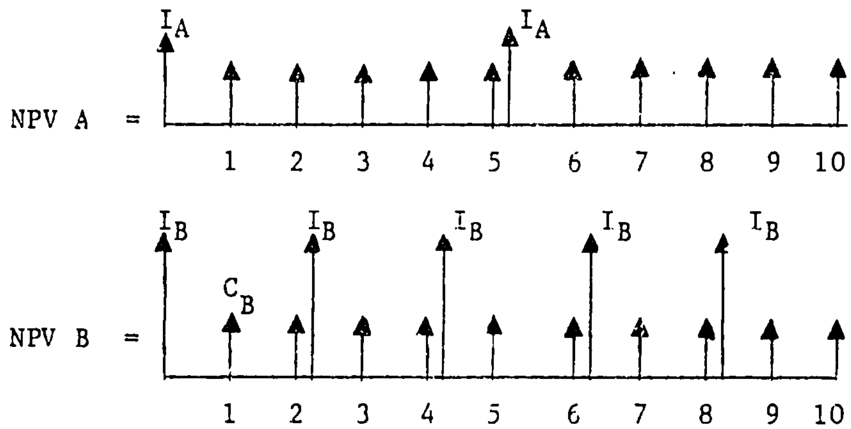
$$NPV = C_o + \sum_{j=1}^{n} \frac{C_j}{(1+i)^j} \tag{7.4}$$

where C_o represents the initial outlay (and therefore usually has a negative sign). Given a set of alternative projects, that with the highest NPV is the one to be preferred: projects with negative NPV would not normally be undertaken. Even a project with positive NPV should sometimes not be persued, if the NPV of doing nothing (say simply leaving the available capital in high yield securities) is higher.

The NPV criterion should not normally be used to evaluate project alternatives of different project life. To compare, say, a two year project with a 5 year project, it is necessary to bring both to a common project life; thus if



then the correct NPV comparison is



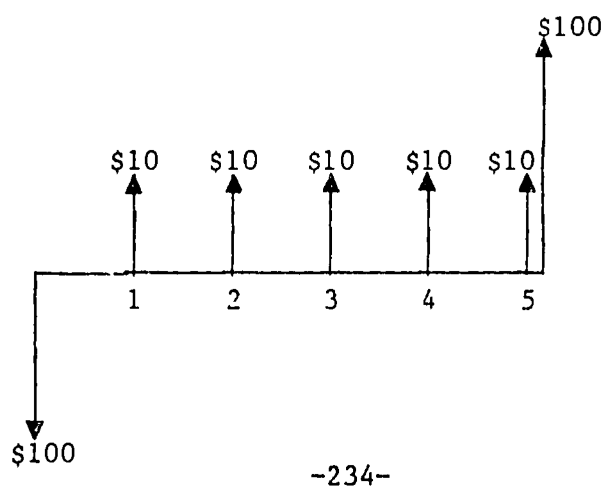
Discounted Cash Flow Rate of Return(DCFRR)

The DCFRR, or "internal" rate of return, r, is defined by that value of discount rate for which the net present value of cash flows is equal to zero, i.e. that value of r for which

$$C_o + \sum_j \frac{C_j}{(1+r)^j} = 0 \tag{7.5}$$

where C_i is the net annual cash flow.

Consider, for example, a purchase of a fixed interest security in the value of \$100, yielding 10% per year. If the security is held for 6 years, what is the DCFRR. The cash flow can be represented as



Applying the identity

$$0 = \sum_{j=1}^{n} \frac{C_i}{(1+i)^j} + C_o$$

to the above cash flow, one obtains

$$0 = -100 + \frac{10}{(1+r)} + \frac{10}{(1+r)^2} + \frac{10}{(1+r)^3} + \frac{10}{(1+r)^4} + \frac{100}{(1+r)^5} + \frac{10}{(1+r)^5}$$

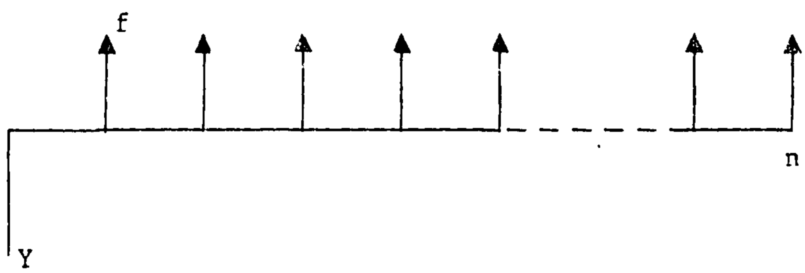
to be solved by trial and error. Let us try r = 0.1 (!). Then substituting into the above equation one obtains

$$0 = -100 + \frac{10}{(1.1)} + \frac{10}{(1.21)} + \frac{10}{(1.331)} + \frac{10}{(1.464)} + \frac{110}{(1.610)}$$

$$0 = -100 + 9.09 + 8.26 + 7.551 + 6.83 + 68.32$$

which is met exactly. Thus for the very special case of a fixed interest security, the internal rate of return is equal to the interest rate.

A more common situation is that of a purchase of equipment of zero (or almost zero) salvage value; e.g. the installation of a solar hot water heater with, say, an n-year life, costing Y, and with annual fuel savings of f:



Applying the DCFRR identity, we have

$$0 = Y + \sum_{j=1}^{n} \frac{f}{(1+r)^j}$$

hence

$$Y = f \cdot SPWF(r,n)$$

Thus the DCFRR is defined by that value of r for which

$$SPWFI(r,n) = \frac{Y}{f} = \lambda \qquad (7.6)$$

Figure 7.4 plots the DCFRR as a function of λ for a number of time periods.

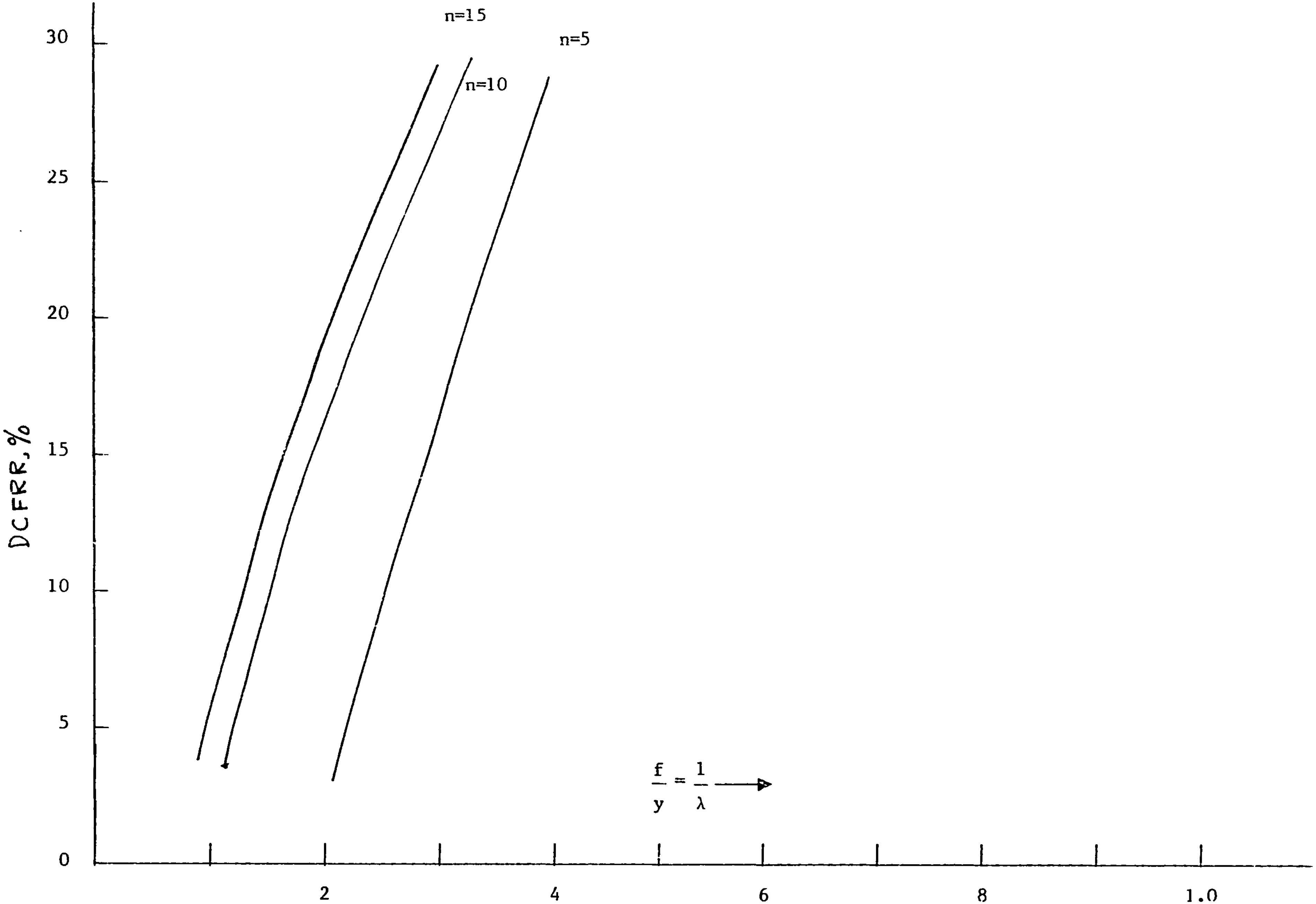
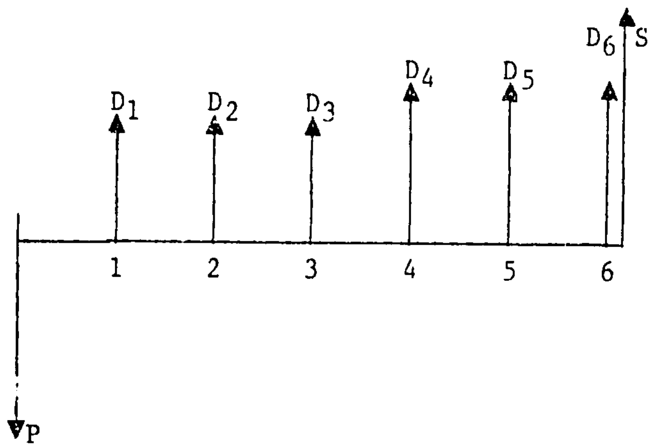


Figure 7.4 DCFRR v. λ

Exhibit 7.2: Discounted Cash Flow Rate of Return of a Stock Purchase

If the purchase price of a stock is P \$/share, and the anticipated selling price 6 years hence is S (S > P), and the annual dividend is D_j, define the equation for computing the after tax DCFRR if the tax rate on capital gains is τ_c, and the tax rate on dividends is τ_d. The before-tax cash flow can be represented as follows:



hence, setting net present value equal to zero

$$= -P \quad + \sum_{i=1}^{6} \frac{(D_i(1-\tau_d)}{(1+r)^i} + \frac{S}{(1+r)^6} - \frac{(S-P)\tau_c}{(1+r)^6}$$

Purchase Price	Dividends	Selling Price	Capital Gains Tax

Benefit Cost Ratio

Introduce the following notation:

B_j = Annual Benefits

E_j = Annual Expenses

K = Fixed investment (at time t=0)

Then the B/C ratio is defined as

$$B/C = \frac{\text{NPV benefits}}{\text{NPV costs}}$$

$$\boxed{B/C = \frac{\sum_{j=1}^{n} \frac{B_j}{(1+i)^j}}{\sum_{j=1}^{n} \frac{E_j}{(1+i)^j} + K}} \tag{7.7}$$

For the special case where the E_j and B_j are constant, then (7.7) reduces to

$$B/C = \frac{SPWF(i,n)\ B}{SPWF(i,n)E + K} \qquad \frac{SPWF(i,n)\ B}{SPWF(i,n)E + K}$$

$$= \frac{B}{E + K\ CRF(i,n)} \tag{7.8}$$

Now the rate of return, r, is given by

$$0 = -K + \sum_j \frac{B - E}{(1+r)^j}$$

hence

$$K = \sum_j \frac{B - E}{(1+r)^j}$$

thus

$$CRF(r,n) = \frac{B - E}{K}$$

Solving (7.8) for B, and inserting into (7.7) yields

$$B/C = \frac{CRF(r,n)K + E}{CRF(i,n)K + E} \tag{7.8}$$

from which follows that if r = i, B/C = 1. As we shall see later on, the B/C criterion tends to favor the most capital intensive projects, which is perhaps one reason why it is the investment criterion chosen by agencies charged with building water resource projects -- big dam projects yield high B/C ratios !

Revenue Requirements Method

The revenue requirements method starts out at the opposite end of the DCFRR method. The DCFRR method starts with a specification of all known cash flows, and then searches for the discount rate that is equivalent to the opportunity cost of capital invested in the venture. The RR method, however, starts with a required rate of return on capital, and then computes the minimum revenue that must be obtained to cover all of the costs. One encounters two variants of the method: the present worth of revenue requirements (PWRR) method (which should be self-explanatory), and the levelized annual revenue requirements method (LARR), frequently used by electric utilities in the evaluation of alternative generation expansion plans.

In the simplest case of debt capital, annual revenue must be sufficient to cover the cost of capital recovery and operating costs: thus

$$RR_j = K \cdot CRF(i_j,n) + E_j \tag{7.9}$$

where i_j is the cost of debt capital, K is the initial investment, and E_j are annual expenses. If E_j is a constant, then RR_j is constant, and equivalent to the levellized annual revenue requirement. If E_j varies from year to year, then

$$PWRR = K + \sum_{j=1}^{N} \frac{E_j}{(1+i_d)} \tag{7.10}$$

from which levelized annual costs follow as

$$LARR = CRF(i_j,n) \cdot K + \sum_{j=1}^{r} \frac{E_j}{(1+i_d)} \tag{7.11}$$

Most investments are based on a mix of debt and equity capital. Let

f_d = fraction financed by debt

f_e = fraction financed by equity (=$1 - f_d$)

i_d = cost of debt capital

i_e = cost of equity capital.

D_j = Depreciation in the j=th year

V_j = Book value in the j-th year

T_j = Taxes in year j

Where the book value V_j is the remaining, undepreciated capital, given by

$$V_j = K - \sum_{1=1}^{j-1} D_j \tag{7.12}$$

The after tax cost of capital r, (or, in the case of a regulated utility, the allowed rate of return), is defined by

$$r = f_d \cdot i_d + f_e \cdot i_e \tag{7.13}$$

from which follows the annual revenue requirement

$$RR_j = E_j + D_j + r \cdot V_j + T_j \tag{7.14}$$

where taxes are given by

$$T_i = \tau(RR_j - E_j - D_j - V_j f_d i_d) \tag{7.15}$$

assuming a tax rate τ, and that interest can be deducted as an expense for tax purposes. Substituting (7.15) into (7.14)

$$RR_j = E_j + D_j + r \cdot V_j + \tau(RR_j - E_j - D_j - V_j f_d i_d)$$

hence[1]

$$\boxed{RR_j = E_j + D_j + \frac{V_j(r - f_d i_d)}{(1-\tau)}} \tag{7.16}$$

[1]For a full discussion of the RR method, see D. Phung "Cost Comparison of Energy Products: Discounted Cash Flow and Revenue Requirement Methods." Energy, 5, p. 1053-1072, 1980.

The Choice of Investment Criterion

To illustrate the dependency of decision on choice of criterion, consider first the three projects of Table 7.3. Projects A and B differ only in the magnitude of revenues and operating costs -- both have the same net annual cash flow of \$3 million, and both require the same investment cost of \$10 million. Yet the B/C ratio differs considerably, whilst both NPV and DCRFF show identical values. Project C requires only \$9 million in first cost, but generates the same \$3 million in net annual revenue. As one might expect, both NPV and DCFRR increase: but the B/C ratio is only 1.27, less than the B/C ratio of a project that generates the same net annual revenue, but that costs \$1 million less. Thus it can be seen that the B/C ratio favors capital intensive projects.

Table 7.3
Project Comparisons (in \$ million)

	A	B	C	D
Initial Investment K	10	10	9	20
Annual Cost E_j	0.5	3.5	3.5	4.0
Annual Revenue B_j	3.5	6.5	6.5	10.0
Annual Cash Flow C_j (= B_j - E_j)	3	3	3	6
NPV (at i = 12%)	6.95	6.95	7.95	13.9
DCFRR	27.4%	27.3%	30%	27.3%
B/C	1.54	1.23	1.27	1.32

Now compare project D with projects A and B: D is defined as the <u>sum</u> of A + B. Note that DCFRR remains unchanged at 27.3%, whilst NPV doubles. Thus one may state that for projects of constant annual cash flow over the lifetime of the project, DCFRR is a function only of the <u>ratio</u> of annual cash flow to investment, whereas NPV is also dependent on the <u>absolute</u> magnitude of the project.

Whilst the discussion thus far has emphasized the evluation of single projects, we now turn to the problem of choosing combinations of projects from among a larger set of acceptable projects, a choice dictated by a constraint on total capital investment. Recall, for example, the alternatives of Table 7.3. Suppose the $30 million dollars was the maximum possible capital outlay. Which projects would we select? In this simple example, the choice is not too difficult -- One builds project D and project C, since D and C yields a higher NPV than any other combination.

In general, however, the problem is not so simple, especially where NPV and DCFRR yield different rankings, and where the number of possible projects is quite large. Consider, for example, the set of projects listed on Table 7.4.

Table 7.4
Project Portfolio

	Project			
	1	2	3	4
Investment, K_i	10	20	11	18
Net Cash Flow C_i	2.8	5.5	3.5	4.5
NPV_i (at i = 12%)	5.8	11	8.7	7.4
DCFRR	25%	23.7%	28.2%	21.2%

Then for some particular total capital constraint K_{max}, the choice of project is given by

$$\max \sum_{i}^{n} X_i \quad (NPV_i)$$

$$\text{s.t.} \sum_{i}^{n} K_i X_i \leq K_{max} \qquad (7.17)$$

$$X_i = 0 \text{ or } 1 \text{ for all } i$$

where K_i is the capital requirement for the i-th project, and the X_i are the decision variables to be identified.

which is a so-called binary programming problem, since the decision variables take on only the values zero or one -- corrsponding to the build no-build decision for each project. For our numerical example, if we assume a capital limatation of \$40 million, this yields the problem:

$$\begin{aligned} \max\ & 5.8X_1 + 11X_2 + 8.7X_3 + 7.4X_4 \\ \text{s.t. } & 10X_1 + 20X_2 + 11X_3 + 18X_4 \leq 40. \\ & X_1, X_2, X_3, X_4 = 0 \text{ or } 1 \end{aligned}$$

Such binary programming problems, unfortunately, are very difficult to solve, even on a high speed computer: in constrast to standard linear programming techniques, for which a solution can be guaranteed within some well defined number of iterations, large binary programming problems may require inordinate amounts of computer time.

For small problems, a graphical solution is possible, by drawing out a complete decision tree, and evaluating the value of each branch in the manner shown on Figure 7.5. The solution is given by searching the NPV column for the maximum value that still meets the \$40 million capital limitation -- which is seen to be build projects 1,3 and 4, yielding a total NPV of \$21.9 million, with the capital requirement of \$39 million.

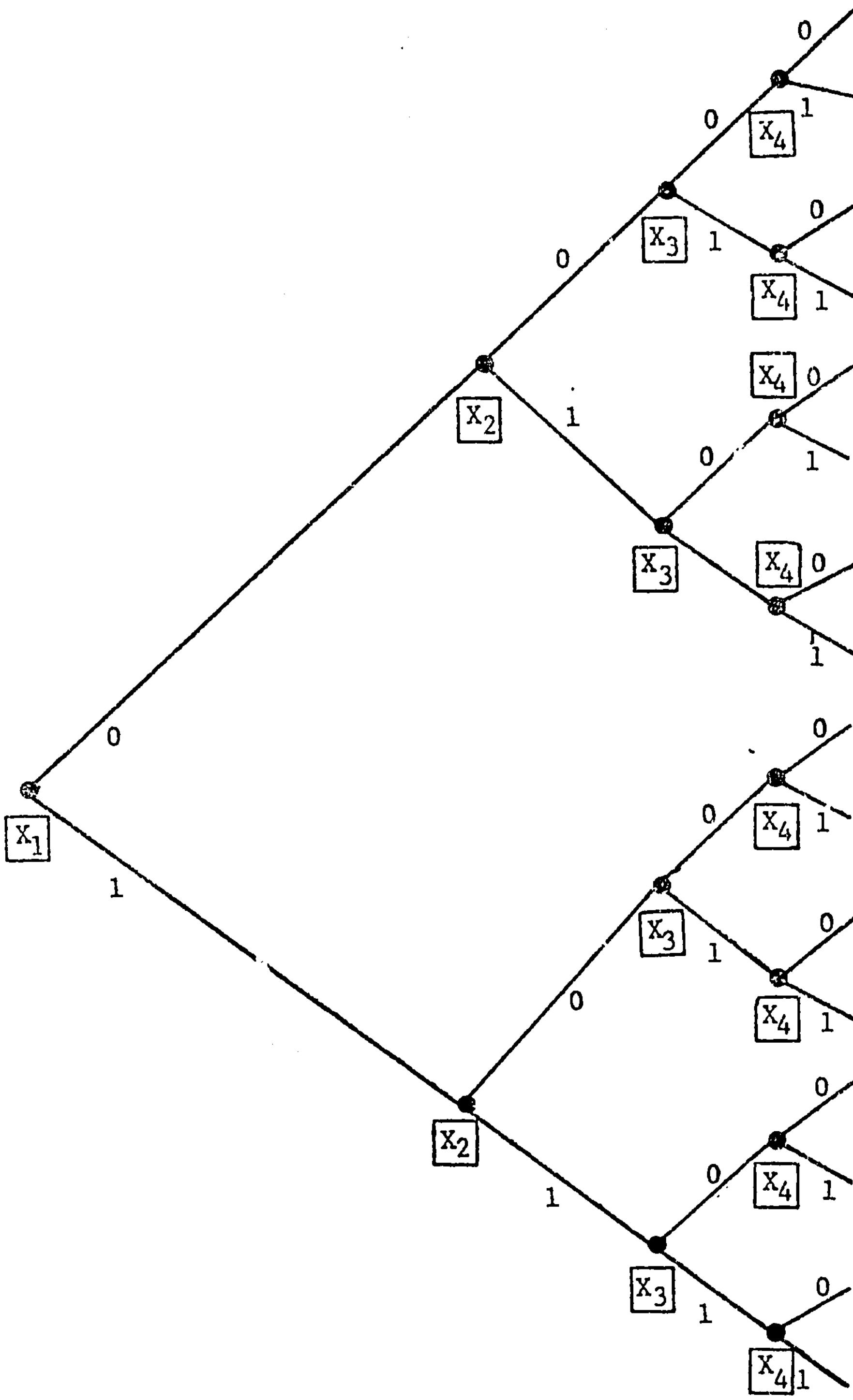
One might compare this solution with that yielded by simple rankings: if one orders the alternatives by NPV, one obtains

Project	NPV	Cumulative Capital
2	11	20
3	8.7	31
4	7.4	49
1	5.8	59

on the basis of which would build projects 2 + 3. On the other hand, if one ranks by DCFRR

Project	NPV	Cumulative Capital
3	28.2	11
1	25.0	21
2	23.7	41
4	21.2	59

one would build projects 1 and 3.



ACTION	CAPITAL COST	NPV
Do Nothing	0	0
Build 4	18	7.4
Build 3	11	8.7
Build 3 + 4	29	16.1
Build 2	20	11
Build 2 + 4	38	18.4
Build 2 + 3	31	19.7
Build 2, 3 + 4	49	27.1
Build 1	10	5.8
Build 1 + 4	28	13.2
Build 1 + 3	21	14.5
Build 1, 3, + 4	39	21.9
Build 1 + 2	30	16.9
Build 1, 2 + 4	48	24.2
Build 1, 2 + 3	41	25.5
Build 1, 2, 3 + 4	59	32.9

Figure 7.5: Decision Tree Solution to the Capital Budgeting Problem

How does one reconcile the NPV and DCFRR approaches? Return to the project portfolio of Table 7.4; here if NPV is the criterion, one would select project 2; if DCFRR is the criterion, one would select project 3. We have asserted that the correct decision criterion is the maximization of NPV. This appears to run against the conventional wisdom that maximization of rate of return is the criterion of choice. In fact, by applying the DCFRR method correctly, one obtains the same result as by maximizing NPV. The key is to recognize that one must evaluate the rate of return on each increment of investment in order to assure that each increment of capital justified its expenditure. Thus if we require that each increment be higher than some minimum acceptable rate (usually the borrower's cost of capital), the results will be consistent.

Return again to the project portfolio of Table 7.4; since we used $i = 12\%$ for the NPV calculation, we require that the rate of return on each increment of capital be greater than 12%. Since all 4 projects have a DCFRR greater than 12%, all projects are initially acceptable. If any project showed a DCFRR < 12%, it should be eliminated from further consideration.

We next evaluate the DCFRR for each increment of capital. Comparing project 1 to doing nothing gives a rate of return of 25% versus 12% -- and at this point one would select project 1. Next we compare project 3 and project 1; building project 3 requires an additional \$1M of capital; for projects of zero salvage value and constant return, Eq. 7.6 defined DCRFF as that value of r for which

$$\mathrm{SPWF}(r,n) = \frac{Y}{f}$$

where Y is the initial outlay and f the annual return. Thus the incremental rate of return is given by

$$\frac{\$11M - \$10M}{\$3.5M - \$2.8M} = 1.42 = \mathrm{SPWF}(r,10)$$

which yields, by inspection of the appropriate annuity table,

$$r \simeq 60\%$$

Therefore, at this point, we build project 3, since the first \$10M yields 25% (as if we had built only project 1), but the additional capital expenditure of \$1M yields 60%, which is higher than 12%.

If we were to build project 4, rather than 3, we require an additional \$7M; thus the incremental rate of return is given by

$$\frac{\$18M - \$11M}{\$4.5M - \$3.5M} = 7.0 = SPWF(r,10)$$

for which r = 7.5%. This is less than 12%, and thus project 4 need not be considered further.

Finally, if we were to build project 2 (against building project 3), the incremental capital is \$9M: hence

$$\frac{\$20M - \$11M}{\$5.5M - \$3.5M} = 4.5 = SPWF(r,10)$$

for which r = 20%. Since this is greater than 12% the increment is justified, and thus project 2 should be built. This is the same result as that yielded by a maximization of NPV.

Thus far in this discussion we have assumed perfect knowledge of all disbursements and receipts, and their timing. In reality, almost every variable necessary for the computation of investment criteria is subject to uncertainty, and therefore the determination of the robustness of the conclusions of a ranking of projects with respect to uncertainty of assumptions, by whatever investment criterion is chosen plays an important role in investment planning.

Fundamentals

Consider, first, some fundamentals. If X is a random variable of mean μ_x and variance σ_x^2, and if k is a constant, then the mean and variance of the product

$$Z = k \cdot X$$

are given by

$$\begin{aligned} E\left\{kX\right\} &= \mu_Z = k \cdot \mu_x \\ \mathrm{Var}\left\{kX\right\} &= \sigma_z^2 = k^2\ \sigma_x^2 \quad . \end{aligned} \tag{7.18}$$

Suppose that the net cash flow in year t is a r.v. with mean μ_t and variance σ_t^2. Then the net present value is also a r.v. with expected value and variance

$$\begin{aligned} E\left\{NPV\right\} &= \sum_{t=1}^{n} \frac{\mu_t}{(1+i)^t} \\ \mathrm{Var}\left\{NPV\right\} &= \sum_{t=1}^{n} \frac{\sigma_t^2}{(1+i)^{t\ 2}} \end{aligned} \tag{7.19}$$

The above is readily generalized. Consider for example, the function

$$y = (x_1, x_2 \ldots x_n)$$

where x_i are independantly distributed random variables (r.v.'s) of variance σ_i^2. Obviously, if the x_i are random variables, then y_i is also a random variable: and it can readily be shown that the variance of y, σ_y^2, is given by

$$\sigma_y^2 = \left(\frac{\partial y}{\partial x_1}\right)^2 \sigma_1^2 + \left(\frac{\partial y}{\partial x_2}\right)^2 \sigma_2^2 + \ldots. + \left(\frac{\partial y}{\partial x_n}\right)^2 \sigma_n^2 \quad .$$

For example, if y is the linear function

$$y = k_1x_1 + k_2x_2 + \ldots. + k_nx_n$$

then if the x_i are independently distributed variables of variance σ_i^2, the variance of y is given by

$$\sigma_y^2 = k_1^2\sigma_1^2 + k_2^2\sigma_2^2 + \ldots. \; k_n^2\sigma_n^2 \quad . \tag{7.20}$$

In the special case where the x_i are normally distributed (denoted $x_i = N(\bar{x}_i, \sigma_i^2)$), then y is also normally distributed as

$$y = N(k_1\bar{x}_1 + k_2\bar{x}_2 + \ldots. \; k_n\bar{x}_n, \; k_1^2\sigma_1^2 + k_2^2\sigma_2^2 + \ldots. \; k_n^2\sigma_n^2) \quad . \tag{7.21}$$

Consider for example, the NPV criterion. Suppose the net annual cash flow c_j is defined as

$$c_j = R_j - O_j$$

where R_j are the revenues in the j-th year, and O_j are the operating expenses, each of which is subject to uncertainty, distributed with means $\mu(R_j)$, $\mu(O_j)$, and variances $\sigma^2(R_j)$, $\sigma^2(O_j)$. From the definition of NPV

$$E \; NPV\} = \sum_{t=1}^{n} \frac{\mu(R_j) - \mu(O_j)}{(1+i)^t}$$

$$= \sum_{t=1}^{n} \frac{\mu(R_j)}{(1+i)^t} - \sum_{t=1}^{n} \frac{\mu(O_j)}{(1+i)^t}$$

Applying the result of (7.20), the variance of the NPV is

$$\text{Var} \; \{NPV\} = \sum_{t=1}^{n} \frac{\sigma^2(R_j)}{(1+i)^{2t}} + \sum_{t=1}^{n} \frac{\sigma^2(O_j)}{(1+i)^{2t}}$$

note that the variances are additive !

For multiplicative functional forms, say of the type

$$y = X_1X_2$$

the variance of y computes to

$$\sigma_y^2 = x_2^2\sigma_1^2 + x_1^2\,\sigma^2 \quad .$$

Thus, suppose a particular revenue stream is a function of price plus yield, both subject to uncertainty, say

$$R_t = P_t \cdot Y_t$$

then, if P_t and Y_t are independant (which might be a reasonable assumption if P_t is a world market price,. and the contribution made by Y_t to the market is extremely small), then

$$\sigma_{R_t}^2 = Y_t^2\sigma_{P_t}^2 + P_t^2\sigma_{Y_t}^2 \quad .$$

Unfortunately, if P_t, Y_t are <u>normally</u> distributed, the distribution of R_t is complex (a modified Bessel function of the third kind). But if the <u>logarithms</u> of P_t and Y_t are distributed normally, (in which case we say that P_t and Y_t have "lognormal" distribution), i.e.,

$$\log P_t = N\left(P_t, \sigma_{P_t}^2\right)$$

Then it follows from the definition of logarithms that if

$$R_t = P_t Y_t$$

then taking logarithms of both sides yields

$$\log B_t = \log P_t + \log Y_t$$

hence from (7.20) log B_t is also normally distributed.

<u>Time Uncertainty</u>

A fundamental result in probablity theory concerns the expected value of a discrete random variable (r.v.). If the values that this r.v. can take on are X_i, with probability P_i, then the expected value of X, denoted $E\{X\}$ is given by

$$E\{X\} = \sum_{i=1}^{n} P_i X_i \quad ; \quad \sum_{i=1}^{n} P_i = 1 \tag{7.22}$$

Suppose, then, that there is uncertainty in the timing of some disbursement: for example in a wind electric project, it is unclear when batteries must be replaced -- which might occur, say in year 5 with probability 0.2, in year 6 with probability 0.5, in year 7 with probability 0.3. If the cost of the battery pack is \$5000, what is the present value of battery replacement. From (7.22) we have

$$E\{NPV\} = P_5 \cdot NPV\{X_5\} + P_6 NPV\{X_6\} + P_7 NPV\{X_7\}$$
$$= 0.2 \cdot \frac{5000}{(1+i)^5} + 0.5 \frac{5000}{(1+i)^6} + 0.3 \frac{5000}{(1+i)^7}$$

which, for i=12%, yields

$$= 1000 \cdot PWF(5,12\%) + 2500 \cdot PWF(6,12\%) + 1500 \cdot PWF(7,12\%)$$
$$= 2512$$

In general, the NPV of a random variable occurring in year j with probability P_j at a value X_j is given by

$$NPV = \sum_{j=1}^{N} P_j X_j \frac{1}{(1+i)^j} \qquad (7.23)$$

Stochastic Project Life

The above discussion is readly extended to stochastic project life. Especially when dealing with new technologies, for which there is insufficient operating experience, project life may be very difficult to establish with precision. Consider a project that may last i years with probability P_i. Then it follows immediately from (7.22) that

$$E\left\{\sum_{i=1}^{N} X_i\right\} = \sum_{i=1}^{n} P_i \sum_{j=1}^{i} E\{(X_j)\}$$

To put this in a discounting framework, we simply define

$$X_j = \frac{C_j}{(1+r)^j}$$

from which

$$E\{NPV\} = E\left\{\sum_{i=1}^{N} X_i\right\}$$
$$= \sum_{i=1}^{n} P_i \sum_{j=1}^{i} E\{X_j\} \qquad (7.24)$$

Consider the following numerical example:

j	$E\ C_j$	P_i	$E\ X_j$ (at r = 12%)
1	10	0.0	8.929
2	10	0.0	7.7972
3	10	0.333	7.118
4	10	0.333	6.355
5	10	0.333	5.567

Applying (7.24) yields

$$E\{NPV\} = 0.333 \cdot \sum_{j=1}^{3} E\{X_j\} + 0.333 \cdot \sum_{j=1}^{4} E\{X_j\} + 0.333 \sum_{j=1}^{5} E\{X_j\}$$

$$= 0.333 \cdot 23.84 + 0.333 \cdot 30.199 + 0.333 \cdot 35.766$$

$$E\{NPV\} = 29.90$$

Notice that if we had ignored the variablility of N, and simply used the series present worth Factor using n = 4 (corresponding to $E\{N\} = 4$), then

$$E\{NPV\} = 10 \cdot SPWF(i,4)$$
$$= 30.37$$

It follows that the assumption of fixed project life, when the project life is in fact stochastic, may result in an overestimate of Net Present Value. In the simple numerical example given here, the overestimate is only some 1.5%; depending on the discount rate and probability distribution of N the error could be much greater.

The bias in the estimate of variance is more serious, however. From the basic definition of variance

$$Var\{W\} = E\{W^2\} - (E\{W\})^2$$

hence if

$$W = \sum_{i=1}^{N} X_i ,$$

then in our discounting framework

$$Var\{NPV\} = E\left\{\left(\sum_{i=1}^{N} X_i\right)^2\right\} - \left(E\left\{\sum_{i=1}^{N} X_i\right\}\right)^2$$

which, in analogy to the above discussion, can be written

$$= \sum_{i=1}^{n} p_i \cdot E\left\{\left(\sum_{j=1}^{i} X_j\right)^2\right\} - \left(\sum p_i \, E\left\{\sum_{j=1}^{i} X_j\right\}\right)^2 \qquad (7.25)$$

Following Bey[1] (1981), introduce the substitution

$$Z_i = \sum_{j=1}^{i} X_j$$

Again from the definition of variance

$$\mathrm{Var}\{Z_i\} = E\{Z_i^2\} - \left(E\{Z_i\}\right)^2$$

hence

$$E\{Z_i^2\} = \mathrm{Var}\{Z_i\} + (E\{Z_i\})^2$$

$$= \sum_{j=1}^{i} \sigma^2(X_j) + \left(\sum_{j=1}^{i} E\{X_j\}\right)^2 \qquad (7.26)$$

substituting (7.26) into (7.25), and recalling that

$$E\left\{\sum X\right\} = \sum E\{X\}$$

Then (7.25) can be written[1]

$$\mathrm{Var\ NPV} = \sum_{i=1}^{n} p_i \left[\sum_{j=1}^{i} \sigma^2(X_j) + \left(\sum_{j=1}^{i} E\{X_j\}\right)^2\right]$$

$$- \left(\sum p_i \sum E\{X_j\}\right)^2 \qquad (7.27)$$

Return to the previous numerical example, but now also taking into account the <u>variance</u> of the cash flows:

[1]For extensions to the case where the X_j are <u>not</u> independent, see Bey (1981).

j	E C_j	P_i	Var C_j	E X_j	^{2} X_j
1	10	0.0	4	8.929	3.18
2	10	0.0	4	7.797	2.53
3	10	0.333	4	7.118	2.02
4	10	0.333	4	6.355	1.61
5	10	0.333	4	5.567	1.28

Let us first compute Var$\{$NPV$\}$ assuming N=4. From (7.21)

$$\mathrm{Var}\{NPV\} = \sum_{t=1}^{n} \frac{\sigma^2\{C_j\}}{[(1+i)^t]^2}$$

which in the case of constant $\sigma^2\{C_j\}$, reduces to

$$= \mathrm{SPWF}(i,2n) \cdot \sigma^2\{C_j\}$$
$$= 22.6$$

If we take into account the uncertainty in project life, and use (7.27), note first that the last term of (7.27) is equal to (E$\{$NPV$\}$)2, already evaluated at 29.90. The first term of (7.27) evaluates to:

$$= 0.33 \quad [7.73 + 23.84^2] + 0.333\ [9.34 + 30.199^2] + 0.333\ [10.62 + 35.76^2$$
$$= 928.3$$

hence Var $\{$NPV$\}$ = 928.3 - $(29.90)^2$ = 34.29, some 50% higher than where the uncertainty in project life is not taken into account.

7.6 SOLAR HOT WATER HEATING IN TUNISA: APPLICATION TO HOTELS

As an illustration of the type of first order analysis of the application of a specific technology to a specific market, we shall analyze the application of flat plate solar collectors for meeting hot water demands in Tunisian hotels. The tourist industry is very important to the Tunisian economy, and these are numerous large hotels that currently rely on fossil fuels, and electricity generated by fossil fuels, for meeting hot water needs.

The calculations to be shown here are typical in terms of data inadequacies encountered in developing countries. To begin with, there is no data on hot water consumption in the tunisian hotels -- neither on quantity, nor temperature, nor end-use. This data must be inferred from what we do know about fuel use one or two major hotels that were included in the first energy assessment conducted in Tunisia a few years ago. Because there are grounds for believing that such solar hot water heaters may be cost effective, the Tunisian energy planners have expressed an interest in this application provided that the collectors can be manufactured locally. Indeed, there are several working prototypes of such collectors, constructed by manufacturing subsidiaries of STEG, the Tunisian electric utility.

However, there is no information on the likely cost range of such locally manufactured collectrs. Given these uncertainties, we formulate the problem as follows: given that a rate of return of investment of 15% is required, what is the maximum allowable cost of locally manufactured collectors for some level of solar system efficiency.[1]

If A is the area of collectors to be installed, and x is the allowable cost in \$/sq.ft., and $F_j(A)$ is the fuel saving in the j-th year, then it follows that for the investment to be profitable at a 15% discount rate, the present worth of future fuel savings must equal the investment cost, that is;

$$A \cdot x = \sum_{j=1}^{n} \frac{F_j(A) \cdot \lambda_j}{(1 + 0.15)^j}$$

from which x solves to

[1]Obviously, the more efficient the solar system, the higher is the affordable cost, all other things equal.

$$x = \frac{1}{A} \cdot \sum_{j=1} \frac{F_j(A) \cdot \lambda_j}{(1 + 0.15)^j} \quad .$$

In the special case where annual fuel savings are a constant (equivalent to the assumption of constant fuel cost), this reduces to

$$x = \frac{1}{A} \, SPWF(15\%,n) \; F(A)\lambda$$

where SPWF(15%,n) is the series present worth factor.

If we make the more reasonable assumption of some constant annual escalation of fuel costs, say Δ \$/bbl/yr, then from the earlier discussion of gradient annuity factors (Section 7.2), x computes to:

$$x = \frac{1}{A} \cdot SPWF(15\%,n) \cdot \lambda_1 \cdot F(A) + \frac{1}{A} \cdot \Delta \cdot F(A) \; GCAF(i,n) \; PWF(i,n)$$

where GCAF(i,n) is the gradient to annual series factor and PWF(i,n) is the present worth factor.

In order to solve this equation we need to quantify the relationship between the area of flat plate collector to be installed, and the concomitant fuel saving. Obviously the starting point must be solar insolation, for which data is fortunately available, (Table 7.5). To give the calculations some more specific context, we shall assume the hotel in question is the Tunis Hilton, for which the insolation data for Sidi Bou said, just north of Tunis, is suitable.

Suppose the monthly insolation is I, the overall efficiency of the solar system is ϵ_s, the efficiency of the existing fossil fueled boiler is ϵ_f, and the total monthly hot water demand is D. Then for some area of solar collectors, A the Btu of useful energy delivered by the solar system, is α_s is

$$\begin{array}{ccccc} \alpha & = & I & \epsilon_s & A \\ [Btu_s] & & \left[\dfrac{Btu_I}{sq.ft.}\right] & \left[\dfrac{Btu_s}{Btu_I}\right] & [sq.ft.] \end{array}$$

hence the fossil fuel saving is

$$F(A) = \frac{\alpha_s}{\epsilon_f}$$

However, α_s is always subject to the upperbound D.

Table 7.5
Solar Insolation on a Horizontal Surface (1978)

	Calories per cm^2		
	Sidi Bou Said	Kairouan	Gafsa
January	6107	8916	8372
February	7049	9470	9364
March	11962	15588	15040
April	12893	16327	16905
May	16278	19591	19746
June	19523	20784	19838
July	20197	20165	20007
August	17210	20489	17073
September	13969	15920	-
October	9846	11882	-
November	6817	9492	-
December	6413	9093	7961
Total	148264	177717	(170000) approx.

The specificationof D is more difficult, since no direct data exists. An existing assessment does include fuel consumption for the Tunis Hilton: and we here make the assumption that 60% of the total fuel consumption of 600,000 liters/yr goes toward hot water whose temperature requirements are in the range that could be provided by flat plate collectors. Obviously this assumption needs to be subjected to some close scrutiny: a sensitivity analysis reveals however that the results are not greatly affected by the value of this number. Thus the total annual useful energy demand is given by

$$\begin{array}{ccccccccccccc} D_T & = & 0.6 & & 600{,}000 & \cdot & 0.2642 & \cdot & 150{,}000 & \cdot & 0.5 & = & 7.13 \times 10^9 \\ & & [\;\;] & & \left[\frac{\ell}{yr}\right] & & \left[\frac{gal}{\ell}\right] & & \left[\frac{Btu}{gal}\right] & & [\;\;] & & \left[\frac{Btu}{yr}\right] \end{array}$$

assuming an 0.5 overall system efficiency. This has next to be disaggregated by month: we make here an arbitrary disaggregation that reflects a summer peak of room occupancy:

Month	Fraction Total
Jan.	0.06
Feb.	0.08
March	0.08
April	0.09
May	0.07
June	0.10
July	0.12
Aug.	0.10
Sept.	0.08
Oct.	0.08
Nov.	0.08
Dec.	0.06

To do these computations by hand is extremely tedious, since they must be repeated for each month over a range of installed collector surface area. Moreover, since one of the objectives of the exercise is to determine the sensitivity of the result to input assumptions, a small computer program seems indicated. A sample output of such a program is shown on Table 7.6.

For the parameter values listed at the top of the page, the program establishes the fraction of hot water needs met by the solar system, as a function of the collector surface area: obviously the more collector area installed, the higher the fraction of hot water demand is met. Even without considering any scale economies,[2] the relationship between collector surface area and solar fraction is linear only up to the point at which the system is of sufficient size to meet the total demand of at least one month (in the example of the sample table, this occurs between 25,000 and 30,000 sq.ft. of collector area). The row "solar cost, $/sq.ft." indicates what one could afford to pay (if 15% rate of return on investment is required): as the cost of solar collectors goes down, so does the cost-effective collector area, and the solar fraction, go up. Thus at 18$/sq.ft., one would only install some 2000 - 25000 sq.ft.; whereas at 13$/sq.ft., 40000 sq.ft. of collector area would be justified.

The bottom three rows show what the fossil fuel cost would have to be for given collector cost. For example, if it turns out that local manufacture of collectors costs 20$/sq.ft., then to justify <u>any</u> solar installation (at 15% rate of return) requires an oil price of $40/bbl. The

[2]Although the program used to generate the data of this table does not consider scale economies, it would be no great problem to incorporate these into the equations.

Table 7.6
Sample Output

SOLAR HOT WATER HEATING, APPLICATION TO HOTELS IN TUNISIA

.40=SOLAR SYSTEM EFFICIENCY
15=SOLAR SYSTEM LIFE,YEARS
.50=FOSSIL FUEL BOILER EFFICIENCY
35.00=FUEL COST,$/BBL,INCREASING AT 2.00$/YR
1.00=HOT WATER DEMAND ADJUSTMENT
.15=DISCOUNT RATE

11.11=$VALUE PER 10.6 BTU DELIVERED BY SOLAR SYSTEM

	MONTHLY INSOLATION BTU/SQ.FT.	MONTHLY HOT WATER DEMAND 10..6BTU	BTU SOLAR ENERGY DELIVERED FOR COLLECTOR SURFACE AREA OF							
			5000.	10000.	15000.	20000.	25000.	30000.	35000.	40000.
JAN	22400.	516.	44.8	89.6	134.4	179.2	224.0	268.8	313.6	358.4
FEB	25900.	591.	51.8	103.6	155.4	207.2	259.0	310.8	362.6	414.4
MAR	44000.	558.	88.0	176.0	264.0	352.0	440.0	528.0	558.0	558.0
APR	47400.	602.	94.8	189.6	284.4	379.2	474.0	568.8	602.0	602.0
MAY	59900.	471.	119.8	239.6	359.4	471.0	471.0	471.0	471.0	471.0
JUN	71800.	651.	143.6	287.2	430.8	574.4	651.0	651.0	651.0	651.0
JUL	74300.	771.	148.6	297.2	445.8	594.4	743.0	771.0	771.0	771.0
AUG	63300.	657.	126.6	253.2	379.8	506.4	633.0	657.0	657.0	657.0
SEP	51400.	550.	102.8	205.6	308.4	411.2	514.0	550.0	550.0	550.0
OCT	36200.	569.	72.4	144.8	217.2	289.6	362.0	434.4	506.8	569.0
NOV	25100.	630.	50.2	100.4	150.6	200.8	251.0	301.2	351.4	401.6
DEC	23600.	553.	47.2	94.4	141.6	188.8	236.0	283.2	330.4	377.6
TOTAL			1090.6	2181.2	3271.8	4354.2	5258.0	5795.2	6124.8	6381.0
FIRST YEAR SAVING			12.1	24.2	36.3	48.4	58.4	64.4	68.0	70.9
SOLAR FRACTION			.15	.31	.46	.61	.74	.81	.86	.90
SOLAR COST,$/SQ.FT.			17.9	17.9	17.9	17.8	17.2	15.8	14.3	13.1
FUEL COST AT 10. $/SQ.FT			16.	16.	16.	16.	16.	19.	22.	25.
FUEL COST AT 20. $/SQ.FT			40.	40.	40.	40.	42.	47.	52.	58.
FUEL COST AT 30. $/SQ.FT			65.	65.	65.	65.	68.	75.	83.	92.

higher the oil price, the more solar panel area becomes cost effective for fixed panel cost: thus at \$52/bbl, and 20\$/sq.ft., one would install as much as 35000 sq.ft.

In fact, because solar panel costs currently fall in the 20-40\$/sq.ft. range (fully installed, U.S. prices), the range of 13 to 18\$/sq.ft. for local manufacture in Tunisia seems attainable. The conclusion from this kind of analysis, then, would be that the solar hot water heating in Tunisia should certainly be examined further; on the other hand, had the costs been, say, in the 50 - 1090\$/sq.ft. range, for the current level of oil prices, one could then reject the technology as a feasible option for the near future.

E14. Rate of Return

A company is considering the installation of a waste heat recuperator for \$12,000, to be depreciated over 5 years using the double declining balance method. The equipment qualifies for a 3 1/3% investment tax credit. Maintenance and operation costs are estimated at \$500 per year. Salvage value after 5 years is estimated at \$2000. The equipment is expected to reduce energy costs by \$4500 per year. What is the after-tax return on investment? Corporate Income Tax is 48%.

E15. Net Present Value Comparisons

Two plans are proposed as follows:

	A	B
First Year Investment	\$1,800,000	\$800,000
Annual O M	40,000	\$20,000, rising by \$1000/yr.
Special Maintenance at 5 year intevals	100,000	-
Life	25 years	25 years
10th year Investment	-	\$800,000

If the cost of money is 12%, compare the net present value of the two options.

8. THE ELECTRIC SECTOR

8.1 INTRODUCTION

Analysis of investment decisions in the electric sector is a subject with a long history of scholarship, with a rich literature, and extensive applications in practice. Many complex mathematical models are used by electric utilities for planning purposes, some of which, such as the WASP model, have found application in a number of developing countries.[1] But given the existence of a number of excellent literature reviews, such as that of Anderson (1972), we shall not attempt to provide any detailed review here. Rather, the emphasis will be on an elaboration of the fundamental concepts involved, and an exposition of modelling approaches that lend themselves to integration with other energy sectors, or to integration with energy system-wide and economy-wide models. Thus, while many of the finer points of reliability analysis, and transmission line planning, are not taken up in any great detail here, we pay a great deal of attention to the interaction between investment requirements in the electric sector to investment flows throughout the economy -- a subject taken up in some detail in Chapter 10.[2] In this chapter, then, we develop the fundamental concepts; the linearization of load duration curves, formulation of the investment decision as a mathematical programming problem, the complications arising from the addition of a spatial dimension, and some environmental considerations.

[1]WASP (for Wien Automatic System Planning Package) is a series of six computer codes originally developed for a study of developing countries by the International Atomic Energy Agency (IAEA), and is designed to find the optimum power system expansion plan. By minimizing the discounted cash flow of all capital and operating expenses over the study period Dynamic programming techniques lie at the heart of the system. See IAEA (1973), Appendix A for details.

[2]For an excellent presentation of the details of electric sector planning in the context of developing countries, see Munasinghe (1979).

Basic Concepts: The starting point for analysis of the electric sector investment decision is the load duration curve, which characterizes the fraction of time that the electrical load in a given system is greater than or equal to a particular output level. Figure 8.1 depicts an annual load duration curve, plotting systemwide demand (in MW) against the number of hours per year for which that demand is attained. Generation capacity must be sufficient to meet this demand: the basic question being what types of capacity should be built. Clearly, if we only built one type, say, oil fired plants, then for much of the year those plants would be sitting idle. The more expensive the initial capital cost, the less desirable is it to have a plant sitting idle: and hence we might wish to meet some parts of the demand curve with capacity that is less expensive. We would even be willing to settle for less efficient equipment to serve the peak hours, since such equipment would not be in operaton very frequently. The basic trade-off, then, is between capital intensive, generally fuel efficient plants (exemplified by nuclear

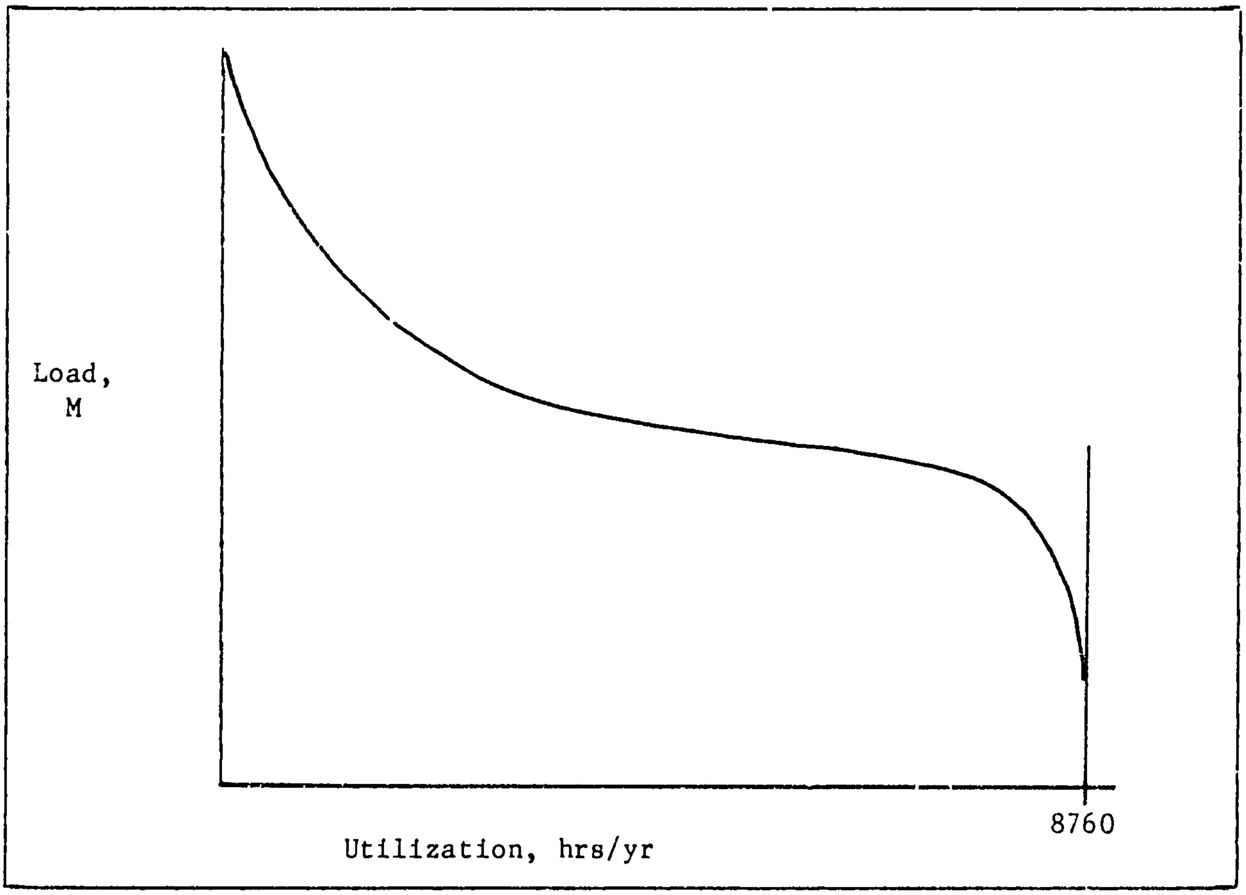


Figure 8.1. An Annual Load Duration Curve

or large fossil fueled steam electric plants) and relatively cheaper, but usually less efficient capacity to serve the peaks, or "peaking plants" (exemplified by combustion turbines).[3]

Consider, then, the average annual cost of production for the k-th type of generating capacity, expressed in cents per kWh., which can be stated as[4]

$$Ac = \frac{\pi_k \text{ CRF} \cdot 100}{U} + \frac{q_k \cdot \sigma_k}{10^6} \tag{8.1}$$

where

π_k is the capital cost, in \$/kw

U is the utilization rate, in hours/yr

CRF is the capital recovery factor

q_k is the heat rate, in Btu/kWh

σ_k is the fuel cost, in cents/10^6 Btu.

Suppose there are three units varying inversely in a capital costs and operating costs (which would be the case, say, for nuclear, oil steam electric, and combustion turbines). The average cost per kWh produced as a function of utilization of these plants is shown graphically in Figure 8.2. This figure indicates that for any plant used more than U_B hours, capacity of type k = 1 would be used: for any plant operated less than U_P hours, capacity of type k = 3 is indicated. The crossover points U_B and U_P follow directly by equating the AC expression for each pair, i.e.,

$$U_B = \frac{100\ \pi_1 \text{ CRF} - 100\ \pi_2 \text{ CRF}}{\dfrac{q_2\ \sigma_2 - q_1\ \sigma_1}{10^6}} \tag{8.2}$$

$$U_B = \frac{100\ \pi_2 \text{ CRF} - 100\ \pi_3 \text{ CRF}}{\dfrac{q_3\ \sigma_3 - q_2\ \sigma_2}{10^6}} \tag{8.3}$$

[3]Hydroelectric capacity, whether conventional or pumped storage, is less amenable to such generalizations, as we shall see in later sections.

[4]When we ignore, for the sake of simplicity of notation, non-fuel operating costs (that may be a function both of installed capacity, or operating level, or both).

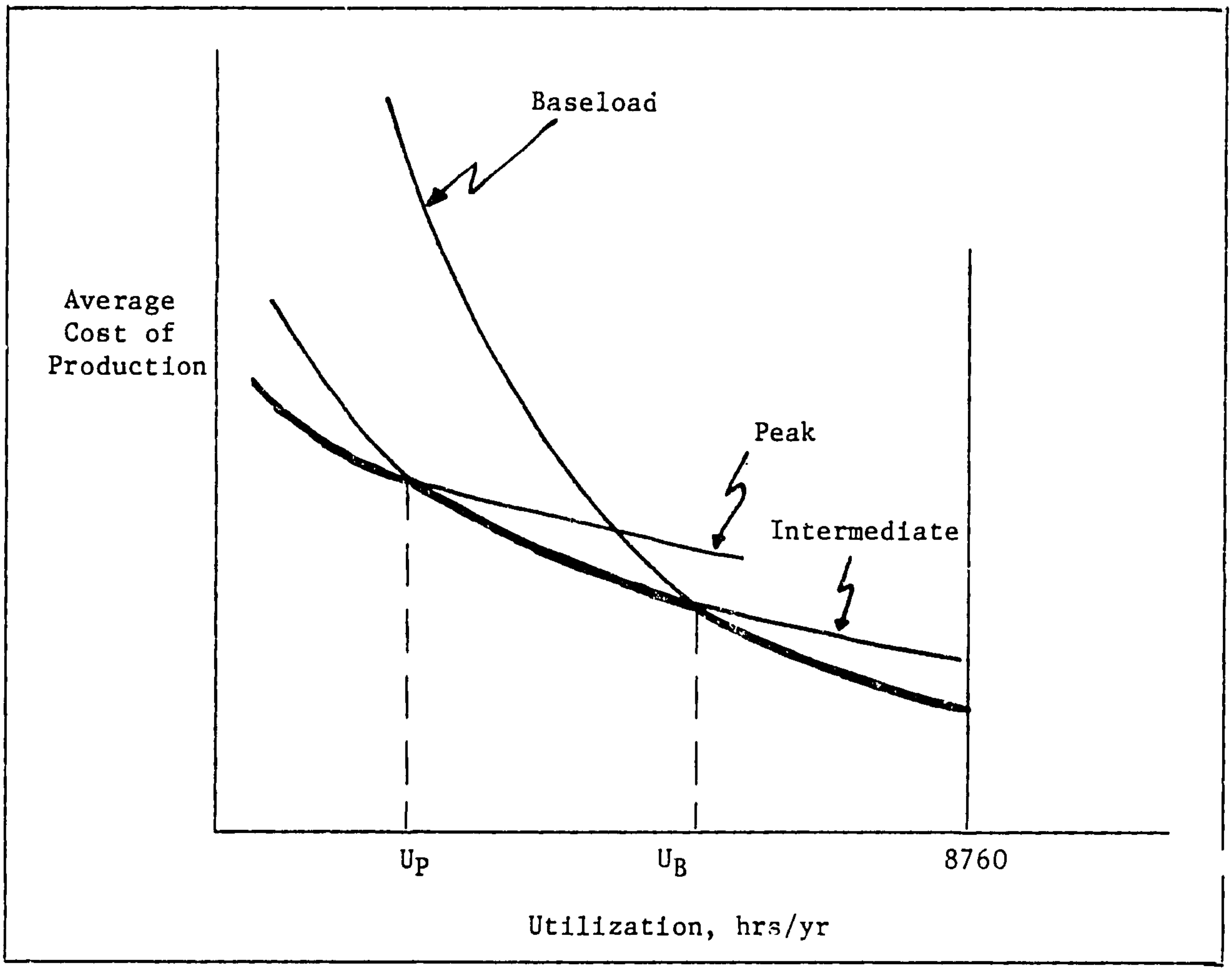


Figure 8.2 Average Production Costs

It follows immediately that the levels of capacity that provide the optimum mix is given by the intersection of U_B and U_P with the annual load duration curve, as shown on Figure 8.3. Thus, for our three plant example, if one had no existing plants, one would build a system with the indicated levels of each of the three types of capacity. In practice, however, one constructs only increments corresponding to the difference between desired and existing capacity, subject to retirements. We leave to the reader, in Exercise 8.1, to show that a drastic change in fuel cost, even with no load growth, may require the retirement of an existing unit to be replaced by a more modern unit, using either the same fuel more efficiently, or some other fuel.

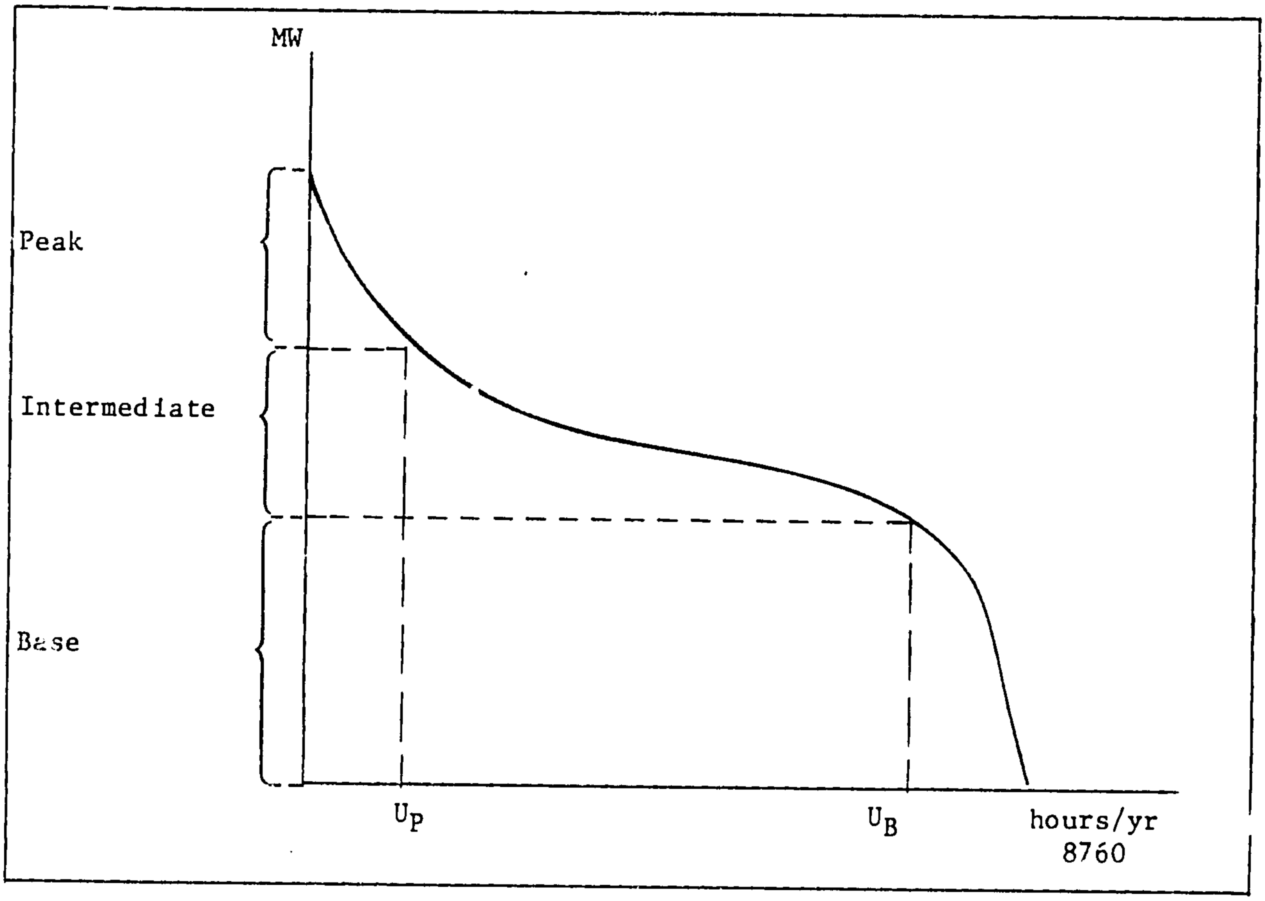


Figure 8.3. Optimal Generation Mix.

Linearization of the Load Curve: For application to linear programming models one must linearize the load duration curve in the manner shown on Figure 8.5. Generally for the type of indicative planning that might use LP approaches, four or five segments suffice.[5] In this pedagogic exposition, we limit ourselves to three sectors, identified as base, intermediate and peak modes. Note that the segmentation is horizontal, rather than vertical.

A Linear Programming Formulation: Consider first a static model, in which the problem is one of identifying the optimum system for some future point, say n years hence, given some level of existing capacity. In such a model, we are not concerned with the timing of investments *within* the n-year period, which is the focus of *dynamic* models: and since we are concerned just with a single snapshot of the future system, we can frame the analysis

[5] Anderson and Thanart (1972), for example, in their study of Turkey, used four segments, using a nine segment trial run only to identify the degree of error introduced by use of the four-segment scheme.

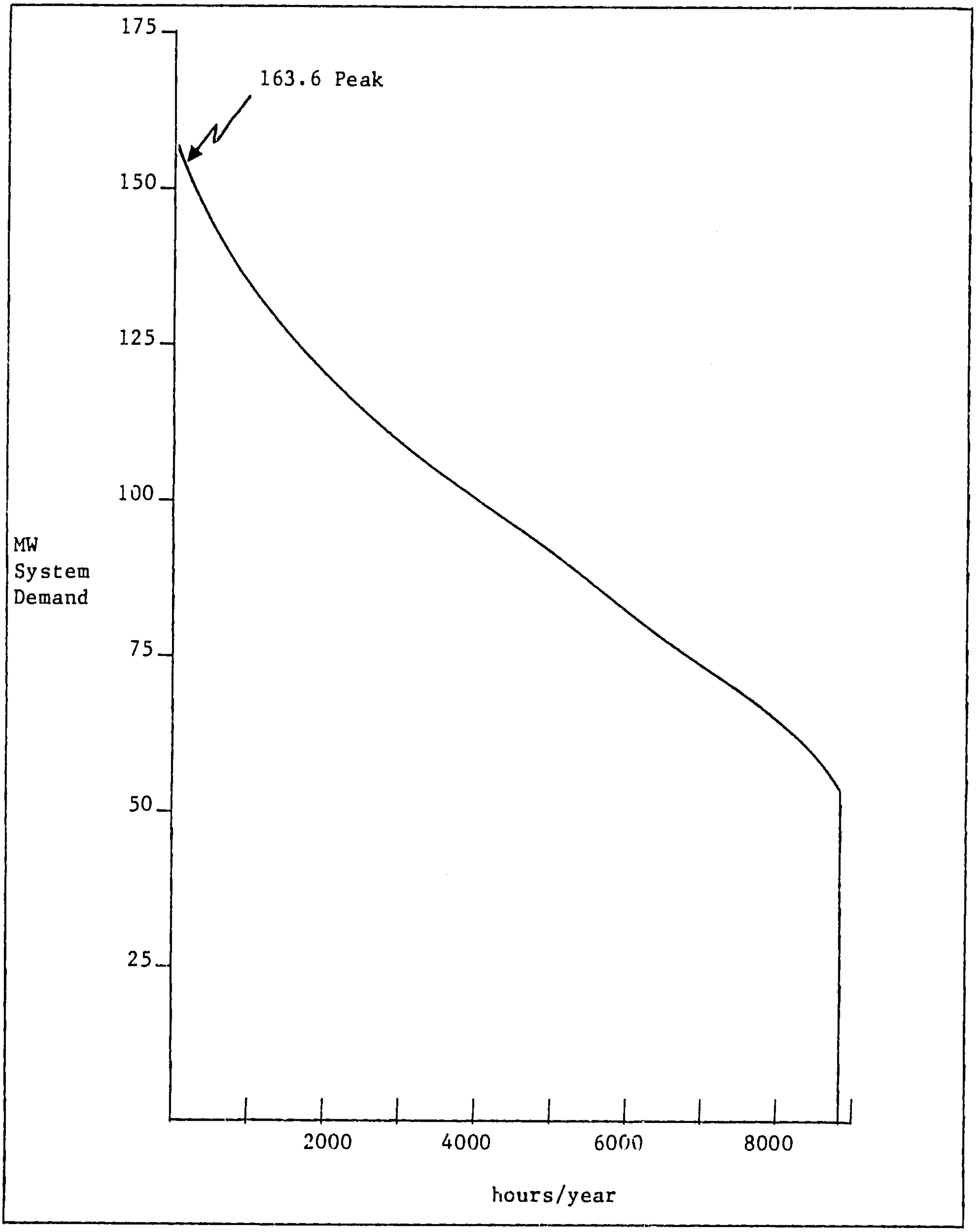


Figure 8.4. 1980 Load Duration Curve for Jordan

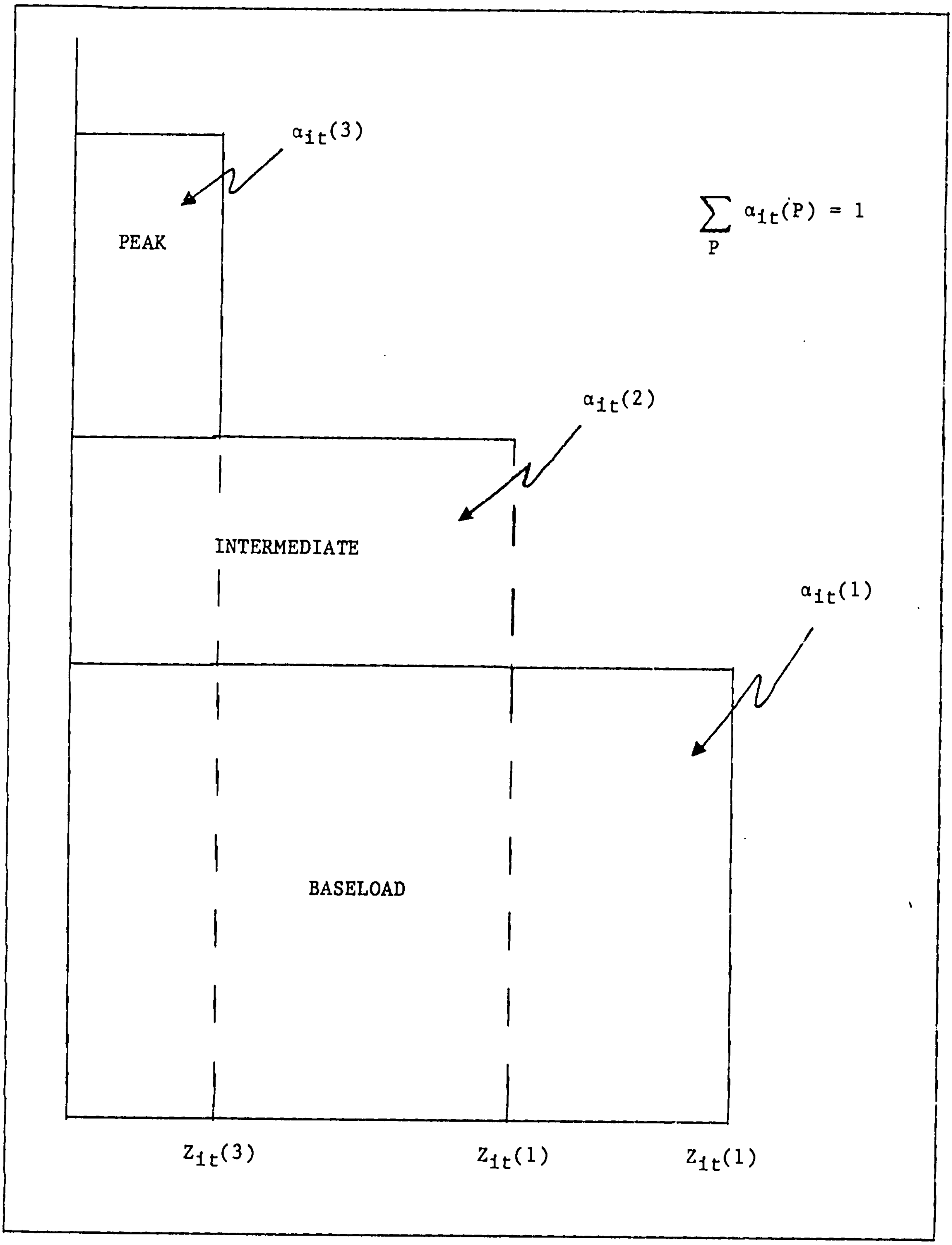


Figure 8.5 Linearizing the Load Curve

Example 8.1: Optimal Generation Mix

Suppose we have the following data

Capacity Type		π_k \$/kW	q_k Btu/kWh	σ_k ¢/10^6Btu
Nuclear	1	1000	10200	200
Oil	2	600	9900	500 (about 30\$/bbl)
Combustion Turbine	3	200	12000	600

If the applicable capital recovery factor is 0.1, and assuming no existing or hydro capacity, determine the optimal generation mix for the annual load duration curve shown on Figure 8.4?

Using (8.2) and (8.3), we compute the crossover points as follows:

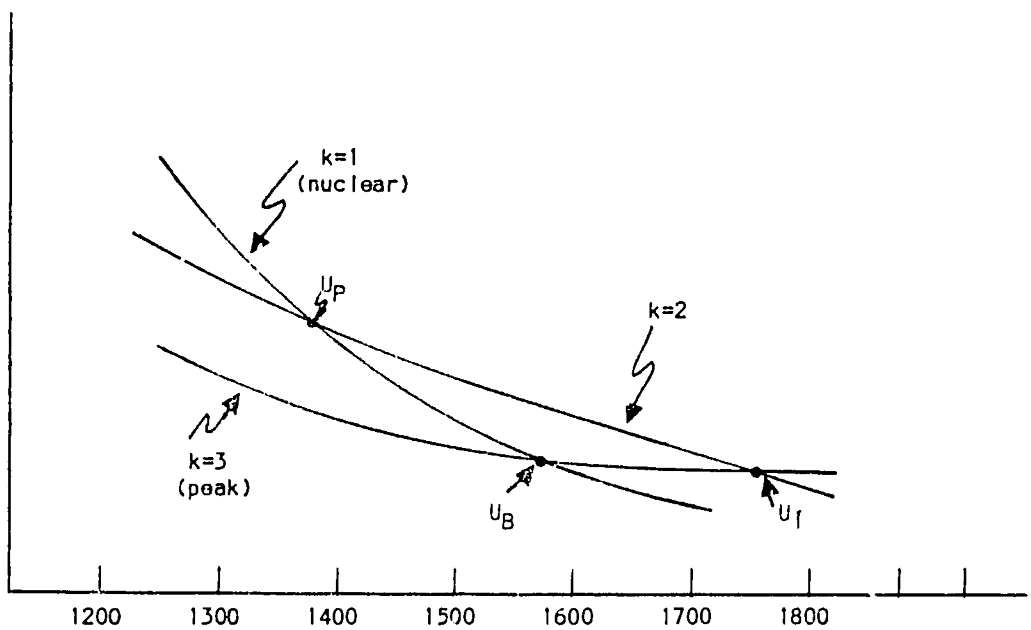
$$U_B = \frac{100 \times 1000 \times 0.1 - 100 \times 600 \times 0.1}{\frac{900 \times 500 - 10200 \times 200}{10^6}} = 1374$$

$$U_I = \frac{100 \times 600 \times 0.1 - 100 \times 200 \times 0.1}{\frac{12000 \times 600 - 9900 \times 500}{10^6}} = 1777$$

Since U_2 lies to the right of U_1, no part of the oil capacity curve lies below either the nuclear or combustion turbine curve. Thus we compute the intersection of k = 1 and k = 3 as

$$U_P = \frac{100 \times 1000 \times 0.1 - 100 \times 200 \times 0.1}{\frac{1200 \times 600 - 10200 \times 200}{10^6}} = 1550$$

Plotting these points on the load duration curve of Figure 8.4, the optimal mix is Mw combustion turbine, and Mw of nuclear.



in terms of identifying that system which, at the given time point, minimizes its total annual cost, including both appropriately amortized capital costs as well as operating costs.

The following notation is required:

$x^k(P)$	is the generation in plants of type k to meet demand in the p-th model [GWh/yr]
$d(P)$	is the energy requirement in the p-th mode, [GWh/yr]
X^k	is the capacity in plants of type k, in [MW]
$z(P)$	is the number of hours in the p-th mode
a_{pk}	is the plant factor of generation type k in the p-th mode.
π_k	is the capital cost of generation type k, in [\$/kW]
σ_k	is the fuel cost, for generation type k, in [¢/10^6 Btu]
β_k	is the heat rate for generation k, in [Btu/kWh]
CRF	is the capital recovery factor.

The first requirement is that we generate sufficient energy to meet each portion of the load curve: for each mode p we thus require that

$$\sum_k x^k(P) \geq d(p); \; p = 1, \ldots P \quad . \tag{8.4}$$

The socalled capacity constraint requires that generation cannot exceed the physical limitations of installed capacity, i.e., for each plant type k,

$$\sum_P \frac{1}{a_{pk}} \cdot \frac{1}{z_p} \cdot x^k(P) \leq X^k \qquad k = 1, \ldots K \tag{8.5}$$

or, in constraint form, and noting the dimensions of each term

$$1000 \cdot \sum_P \frac{1}{ap_k} \cdot \frac{1}{z_p} \; x^k(P) - X^k \leq 0 \quad . \tag{8.6}$$

$$\left[\frac{MW}{GW}\right] \qquad \left[\;\right] \; \left[\frac{1}{hr}\right] \; \left[\frac{GWh}{yr}\right] \quad [MW]$$

Several points are important here. First, note that X^k is specified in MW, $x^k(p)$ in GWh/yr. If we left $x^k(p)$ as MWh/yr, then we see that the coefficient for $x^k(p)$ would, for the baseload portion, compute to

$$\frac{1}{a_{pk}} \cdot \frac{1}{z_p} = \frac{1}{0.65} \cdot \frac{1}{8760} = 0.0001756 \quad . \tag{8.7}$$

Since the coefficient for X^k is unity, for this constraint, the coefficients would vary by 4 orders of magnitude. In large LP's, this is a very undesirable state of affairs, and great efforts are typically made to dimension each variable such that the resulting constraints are well "scaled," which means variation by at most over 2 orders of magnitude.[6] Thus the use of GWh/yr for the $x^k(P)$, which makes the coefficient of Eq. (8.7) equal to 0.1756.

Second, note the presence of the α_{pk} factors in Eq. (8.6). These take into account the fact that any particular plant, even when operated exclusively in a baseload mode, does not operate 8760 hours per year: with scheduled maintenance and forced outages even the most efficient unit may operate only 60-80% of the possible number of hours - hence α_{pk} for baseload plants would be in the 0.6 to 0.8 range.[7]

The objective function is written as

$$\text{Min } S = \sum_k \underset{\left[\frac{\text{kW}}{\text{MW}}\right]}{1000} \cdot \underset{\left[\frac{\$/\text{yr}}{\$}\right]}{\text{CRF}} \cdot \underset{\left[\frac{\$}{\text{kW}}\right]}{\pi_k} \quad \underset{[\text{MW}]}{X^k} +$$

$$\sum_k \sum_p \underset{\left[\frac{¢}{10^6\text{Btu}}\right]}{\sigma_k} \quad \underset{\left[\frac{\text{Btu}}{\text{kWh}}\right]}{\beta_k} \quad \underset{\left[\frac{\$}{¢}\right]}{\frac{1}{100}} \quad \underset{\left[\frac{10^6\text{Btu}}{\text{Btu}}\right]}{\frac{1}{10^6}} \quad \cdot \underset{\left[\frac{\text{kWh}}{\text{GWh}}\right]}{10^6} \quad \underset{\left[\frac{\text{GWh}}{\text{yr}}\right]}{x^k(P)} \tag{8.8}$$

---

[6] This is only one of several issues in computational optimality in very large linear programs, which, when both time and space dimensions are added, can quickly reach several thousand constraints and variables. Modern LP algorithms, mostly proprietary packages developed by the computer hardward manufacturers (such as the Control Data Corporation APEX-III code, or the IBM MPSX package) function best when the coefficient matrix is tridiagonal in structure (i.e., with as many zeros above the diagonal as possible) and with as few equality constraints as possible. Tridiagonality, as in the case of scaling, requires thought by the analyst (by rearranging the order of appearance of rows and columns in the matrix generator). Substitution of equality constraints by inequality constraints should also be done by the analyst whenever possible, although some automated options, such as the REDUCE option in the APEX code, automatically make such conversions (in addition to eliminating any redundant constraints and variables).

[7] The controversy surrounding the discrepancy between <u>ex ante</u> expectations and actual reliability are well known, particularly for the larger baseload units whose economic depend critically on plant factors. The prudent analyst does a series of runs with different values of the plant factor in such situations, to determine the robustness of the predicted investment decision as a function of such assumptions.

8.3 ADDING A SPATIAL DIMENSION: The Siting Problem

The simple LP presented in the preceeding section would not be a very useful tool in practice. First, with respect to application in developing countries, we have not considered hydroelectric generation, which, because of its unique dependence on hydrological factors, obviously requires additional information. Second, the LP has not taken into account the inventory of existing plants, not all of which would typically be retired for the particular year of analysis. Third, even if we analyze only the single year of a static model, as soon as we introduce hydroelectric generation into the system it becomes necessary to consider seasonal effects. At a minimum we should differentiate among wet and dry seasons, although an even finer differentiation may be required for practical application. Finally, we have yet to consider the spatial dimension, which in turn is important for many reasons. First, as soon as hydroelectric projects are introduced into the slate of generation alternatives, one must recognize that each hydro project is unique to a particular location (which is not true, in general, for thermal plants). And particular locations imply the need to consider more carefully the question of transmission losses; a project remote from the load centers may be less desirable than one that is slightly more costly in terms of reservoir construction, but that lies close to the load center.

Introduction of hydroelectric generation, and explicit representation of existing plants, require relatively minor changes to the LP of Section 8.2; it is mainly a matter of notation, and the understanding of the unique characteristics of conventional and pumped storage hydro plants. However, adding the spatial dimension requires a fundamental modification, to allow the movement of goods (in this case electricity) from one place to another; which leads to a programming problem typically categorized as transhipment problems; or, more generally, as multi-regional or spatial programming problems.

Transhipment Problems:[8] The basic idea of the transhipment problem is to extend the mass (or energy) balance equations to allow imports and exports at each location in the problem. For example, suppose we have two adjacent,

[8]The use of such transhipment model formulations for power system analysis appears to have been first used in the USSR-the earliest such reference in the literature known to the author is Makarova et. al, (1966)

interconnected, electric utilities. Then, adding a subscript to denote location, if the regions were not interconnected, Eq (8.4) to both regions would result in

$$\sum_k x_1^k(P) \geq d_1(p) \tag{8.9}$$

$$\sum_k x_2^k(P) \geq d_2(P) \tag{8.10}$$

As soon as an interconnection is present, then for each region, and each mode, the following equality must hold[9]

Generation + Imports - Exports = Demand

hence

$$\sum_k x_1^k(P) + y_{21}(P) - y_{12}(P) \geq d_1(P) \tag{8.11}$$

$$\sum_k x_2^k(P) + y_{12}(P) - y_{21}(P) \geq d_2(P) \tag{8.12}$$

where $Y_{ij}(P)$ denotes the quantity of energy shipped from i to j in mode p. In the case of electricity one must also take into account transmission losses: if $y_{12}(P)$ is sent out from region 1 to region 2, only $(1 - \Omega)y_{12}(P)$ actually is received by region 2, where Ω is the fraction lost in transmission. Thus, generalizing (8.11) and (8.12), one obtains

$$\sum_k x_i^k(P) + y_{ji}(P)\ (1 - \Omega) - y_{ij}(P) \geq d_i(P) \tag{8.13}$$

Further generalizations to an arbitrary number of adjoining utilities (or subregions), and the manner in which the transmission flows y_{ij} are linked to the capacity of transmission lines, are discussed in later sections.

The Siting Problem: Let us now turn to a complete exposition of a capacity expansion model that includes consideration of space (represented by the index subscript i), of hydroelectric and existing plants, and of season-

[9]The assumption here is that baseload generated in one region is used in the baseload portion of the curve the other region, and so on. In the presence of diversity, this need not necessarily be so, and baseload generation in one region might serve intermediate load in the other. Indeed, diversity among adjacent systems is one of the reasons for interconnection -- if the peaks are not coincident, then the peak of the interconnected system is less than the sum of the individual peaks, which lowers the total generation capacity required in the interconnected system to meet the same reliability criterion.

ality; for ease of exposition we shall use but two seasons in this discussion (represented by the index t); a wet season ($t = 1$) and a dry season ($t = 2$). The results are generalized without difficulty to any number of seasons in a single year. For hydro reservoirs we thus assume at most an annual cycle.[10]

Energy Balance: The first requirement, obviously, is that sufficient capacity be operated to meet the requirements of demand. Thus, for each season t, the energy generated in each mode p is equal to the demand in that mode, subject to adjustments for imports and exports. Thus, for the i-th location,

$$\underbrace{\sum_{k}^{K} x_{it}^{k}(p)}_{\text{generation in new plants}} + \underbrace{\sum_{e\in i} \bar{x}_{et}(p)}_{\text{generation in existing plants}} + \underbrace{\sum_{j\in A_i} y_{jit}(p)\left(1 - \Omega_{ji}d_{ji}\right)}_{\text{imports}}$$

$$- \underbrace{\sum_{j\in A_i} y_{ijt}(p)}_{\text{exports}} = \underbrace{\alpha_{it}(p)D_i(t)}_{\text{demand}} \tag{8.14}$$

where

$\alpha_{it}(p)$	is the fraction of the total demand in the t-th season in i that occurs in the p-th mode (see Figure 8.5)
$x_{it}^{k}(p)$	is the generation in plants of type k in season t in mode p at i, in GWh
$y_{ijt}(p)$	is the energy input into the line from i to j in season t in mode p
$\bar{x}_{et}(p)$	is the energy generated in the e-th existing facility in season t in mode p
Ω_{ji}	is the transmission loss factor (see Section 8.4 below)
d_{ji}	is the transmission distance between i and j
$D_i(t)$	is the seasonal energy demand in location i.

All of the terms in this constraint have dimensions of energy, i.e., GWh (per season).

[10]Even though this formulation includes a time dimension to capture seasonalities, such a model would not normally be classified as "dynamic." Dynamic is a categorization generally applied only to models that attach a time dimension to the investment decision path -- which the model being described here does not do.

Build Limits: Generation in new plants cannot exceed the capability of capacity actually provided. Thus

$$\sum_{p}^{P} \frac{1}{a_{pk}} \frac{1}{z_{it}(p)} x_{it}^{k}(p) \leq X_{i}^{k} \tag{8.15}$$

$$[\;] \quad \left[\frac{1}{h}\right] \quad [\mathrm{GWh}] \quad [\mathrm{Gw}]$$

where X_i^k is the new capacity of type k at i. In constraint form this becomes

$$\sum_{p}^{P} \frac{1}{a_{pk}} \frac{1}{z_{it}(p)} x_{it}^{k}(p) - X_{i}^{k} \leq 0 \quad \text{for} \left\{ \begin{array}{l} t = 1, \ldots, T \\ i = 1, \ldots, n \\ k = 1, \ldots, K \end{array} \right\} \tag{8.16}$$

where

a_{pk} = capacity factor of generation of type k in the p-th mode

$z_{it}(p)$ = hours in the p-th load curve segment in season t in region i

Clearly, for each i and k, at least one of the T constants in (8.16) will be binding.

Existing Capacity Limits: In analogy to the previous section, generation in existing plants is also bounded by the available capacity, i.e.,

$$\sum_{p}^{P} \frac{1}{a_{e}} \frac{1}{z_{it}(p)} \bar{x}_{et}(p) \leq \bar{X}_{e} \quad 0 \text{ for} \left\{ \begin{array}{l} t = 1, \ldots, T \\ \text{all } e \end{array} \right. \tag{8.17}$$

where a_e is the plant factor, and $\bar{X}_e$ the installed capacity of the e-th existing facility.

Reserve Margin Requirements: The reserve margin requirement can be stated as

$$\underbrace{\sum_{e} \bar{X}_{e}}_{\text{existing capacity}} + \underbrace{\sum_{i}^{n} \sum_{k}^{K} X_{i}^{k}}_{\text{new capacity}} \geq (1 + \mu)\, s_{D} \underbrace{\sum_{i}^{n} P_{i}(\max)}_{\text{peak demand}} \tag{8.18}$$

where

$P_i(\max)$ is the peak annual demand at i

s_D is a diversity factor that adjusts for noncoincidence of demand peaks[11]

μ is the desired reserve margin.

Adjustment for Pumped Storage Hydro: It is assumed that the pumping energy for pumped hydro generation always occurs in base loaded plants[12] (see Figure 8.6). Thus, assuming the index k = 6 corresponds to this mode, the energy balance equation for p = 1 requires the additional term

$$-\frac{1}{e_p e_t} \cdot \sum_{p=2}^{P} \left\{ x_{it}^{6}(p) + \sum_{e \in 6,i} \bar{x}_{et}(p) \right\} \tag{8.19}$$

where

e_t is the turbine efficiency

e_p is the pumping efficiency

$e \in 6,i$ denotes the set of existing pumped storage plants at location i.

This term is of course negative, representing the pumping energy required for given peak generation.

Hydro Limits: Because of the inherent site specific engineering features of hydro projects, special treatment of hydro capacity is necessary. In particular, unlike thermal plants, there will typically be very wide variations in construction costs ($/kW), and thus rather than using a homogeneous hydro build variable (as is done for nuclear and coal), we use a separate variable for each potential project in the region. Thus in the energy balance Equation (8.14)

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[11] Borenstein (1974) has explored a more sophisticated method of incorporating reliability issues using chance-constraints, but concludes that an adequate treatment within an LP framework is intractable. Almost all the electric sector spatial programming models in the literature use the formulation (8.18), or a close equivalent.

[12] Pumped storage projects are assuming increasing importance in many developing countries: in India, several large projects in a number of Southern States are now underway including the Kadamparai project in Tamil Nadu, and the Nagarjunasagar facility in Andrha Pradesh--see Ahamed (1975) and Subrahmanyam and Singh (1975).

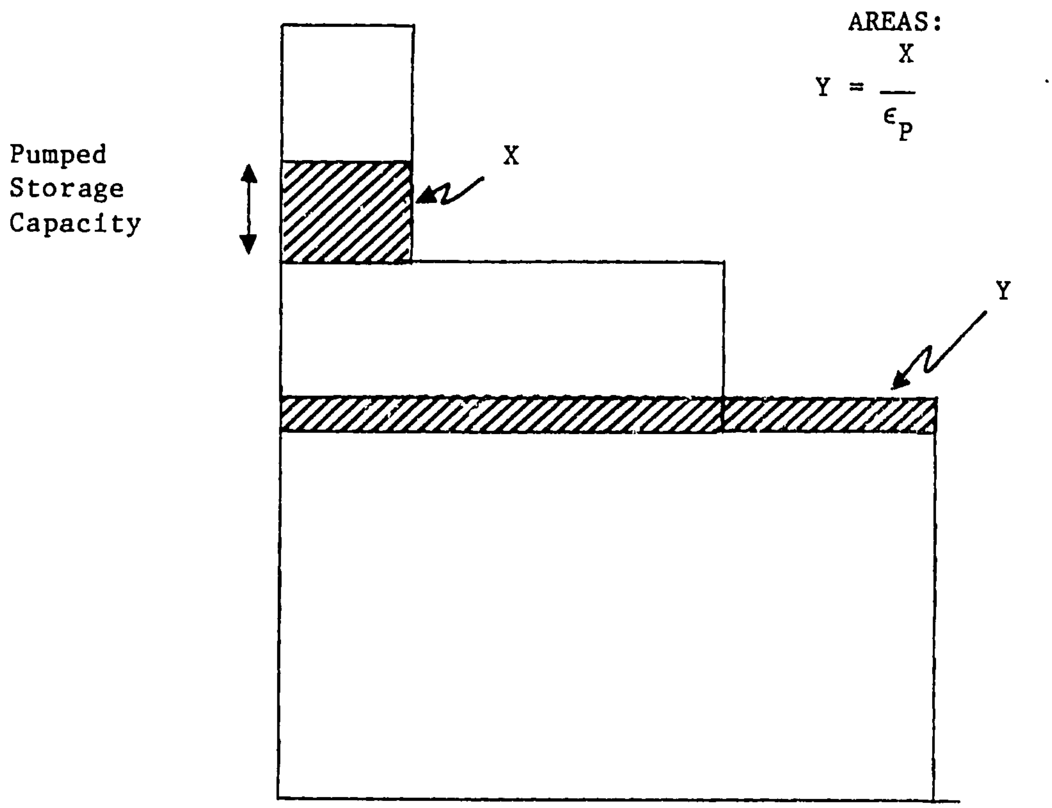


Figure 8.6: Assumed Adjustments for Pumped Hydro

we delete the term $x^5_{it}(p)$, representing generation in new hydroplants, and replace it by

$$\sum_{h\epsilon i}^{H} x^5_{th}(p) \tag{8.20}$$

where

$h\epsilon i$	is the set of potential hydro projects in Subregion i
$x^5_{th}(p)$	is the hydrogeneration in season t in mode p in the h-th potential hydroproject.

In the simplest treatment of hydroelectric generation, we may assume that a table of potential projects is available, specifying for each project the seasonal generation limit. Table 8.1, for example, shows the table used by Gately (1971) in his study of Southern India. If the maximum energy capacity of the h-th project in season t is denoted $x^5_{th}(max)$, then the generation variables $x^5_{th}(p)$ are subject to the constraint

$$\sum^{P} x^5_{th}(p) \leq x^5_{th}(max) \tag{8.21}$$

Table 8.1
Typical Hydro Power Projects Constraints (In Madras, India[a])

Project	Power Capacity (in MW) wet season	Power Capacity (in MW) dry season	Energy Capacity (in 10^3 MW-hrs) wet season x^5_{1h}(max)	Energy Capacity (in 10^3 MW-hrs) dry season x^5_{2h}(max)	Construction Cost (in Million Rupees)
Pandiar-Punnapuzha	100	75	250	165	150
Cholathipuzha	60	45	135	90	65
Kadamparai	35	27	72	48	62
Paralayar	35	27	63	42	41
Suruliyar	35	27	70	46	40
Coonoor-Kallar	50	38	60	40	84
Lower Moyar	70	52	94	64	118
Upper Manimuthar	90	45	162	80	123
Upper Amaravathy	70	35	173	85	141
Upper Thambarapani	200	100	195	95	250

[a]Adapted from Gately, (1971), p. 45.

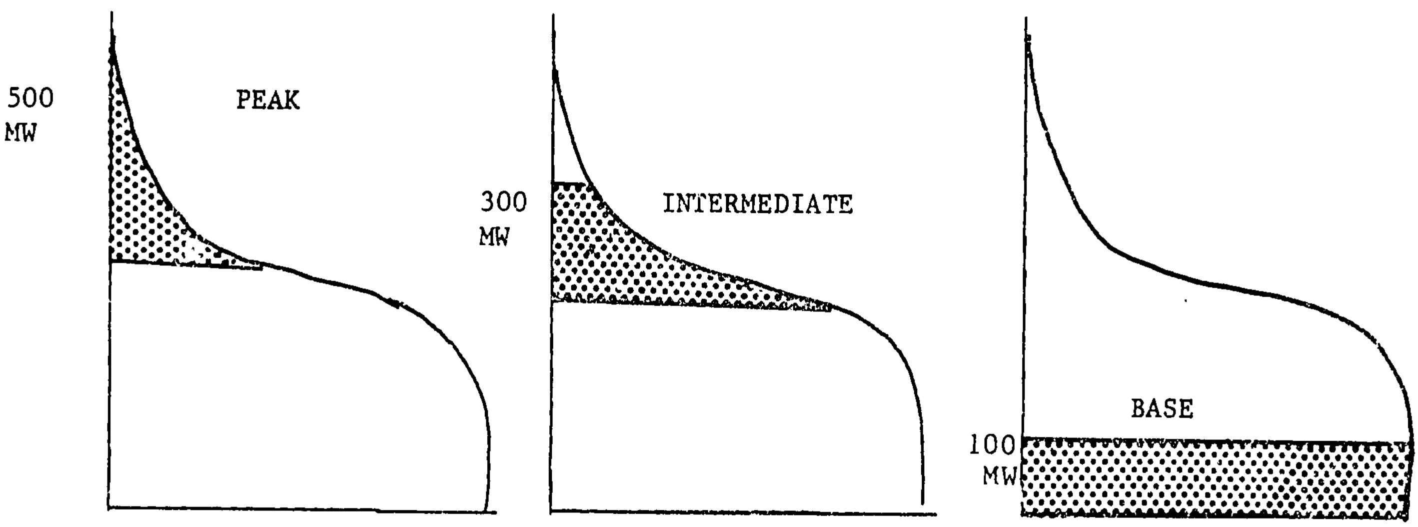


Figure 8.7: Capacity as a Function of Plant Mode.

As to the power output (in Gw rather than GWh of energy) it must of course be noted that this will depend on which part of the load curve the plants serves. Figure 8.7 should illustrate this point: a hydro project of constant energy output might yield 500 MW as a strictly peaking unit, but only 250 MW as a base load plant. The shaded area on Figure 8.7, representing the energy output, is constant for all three cases shown. The capital cost of the hydro plant will be strongly influenced by the power output, since this determines the size of the turbine generator sets required. Thus, for new hydro projects, the build limits are given by

$$\sum_{p}^{P} \frac{1}{a_{ph}} \frac{1}{z_{it}(p)} x^5_{th}(P) \leq X^5_h \qquad (8.22)$$

where X^5_h is the installed capacity of the h-th hydro project (in GW). This installed capacity is subject to the project specific limit $X^5_h(\max)$, as also specified, for example, on Table 8.1 in the case of Gately's study of Southern India. Thus

$$X^5_h \leq X^5_h(\max) \qquad (8.23)$$

It should be noted that use of this kind of data table externalizes a number of important decision variables. In particular, specific bounds for seasonal energy capacity does not allow the model to build more, or less, interseasonal storage than the fixed amount assumed in the table. We shall formulate a more complete representation of such issues in Section 8.4.

Transmission Limits: The energy transmitted from place to place cannot exceed the line capacity provided. Thus, for each season t,

$$\underbrace{\sum_p \frac{1}{z_{jt}(p)} y_{ijt}(p)}_{\text{power flow from i to j}} + \underbrace{\sum_p \frac{1}{z_{it}(p)} y_{jit}(p)}_{\text{power flow from j to i}} \leq \underbrace{Y_{ij}}_{\text{line capacity from i to j}} \qquad (8.24)$$

hence, in constraint form;

$$\sum_p \frac{1}{z_{jt}(p)} y_{ijt}(p) + \sum_p \frac{1}{z_{it}(p)} y_{jit}(p) - Y_{ij} \leq 0 \qquad (8.25)$$

In the event that there is an existing line between i and j, (8.25) becomes

$$\sum_p \frac{1}{z_{jt}(p)} y_{ijt}(p) + \sum_p \frac{1}{z_{it}(p)} y_{jit}(p) - Y_{ij} \leq \bar{Y}_{ij} . \qquad (8.26)$$

Note also that the O M costs for generation include fuel costs for nuclear and oil fired plants, but exclude fuel costs for coal and lignite fired plants, since these are treated directly by the model (see below).

The Objective Function: We are now in a position to state the objective function for the basic energy transhipment model presented thus far, namely

$$\min s = \underbrace{\sum_i \sum_k X_i^k \pi_k CRF}_{\substack{\text{capital costs of} \\ \text{new generation plants}}} + \underbrace{\sum_t \sum_p \sum_k \sum_i x_{it}^k (p) \sigma_k 8.76}_{\substack{\text{operating costs of} \\ \text{new generation plants}}}$$

$$+ \underbrace{\sum_t \sum_p \sum_e \bar{x}_{et}(p) \sigma_k 8.76}_{\substack{\text{operating costs of} \\ \text{existing facilities}}} + \underbrace{\sum_{(i,j)} Y_{ij} \sigma_y d_{ij} CRF}_{\substack{\text{capital costs of} \\ \text{transmission line} \\ \text{construction}}}$$

$$+ \underbrace{\sum_{(i,j)} Y_{ij} d_{ij} \sigma_o}_{\substack{\text{operating costs of} \\ \text{new transmission} \\ \text{lines}}} + \underbrace{\sum_{(i,j)} \bar{Y}_{ij} d_{ij} \sigma_o}_{\substack{\text{operating costs of} \\ \text{existing transmission} \\ \text{lines}}} \qquad (8.27)$$

where

- π_k is the capital cost of the k-th generation type, in \$/kW
- σ_k is the O M cost of the k-th generation type, in ¢/kWh
- σ_y is the capital cost of transmission line construction, in \$/MW/mi
- σ_o is the O M cost of transmission lines, in \$/MW/mi
- CRF is the appropriate capital recovery factor.

It should be noted that the operating costs of transmission lines include only such items as right-of-way maintenance: the energy losses are accounted for by upward adjustment of the amount of generation required to meet a specific demand level.

The question of appropriate cost expressions has received some attention in the recent literature, particularly in light of the fact that market prices may not be suitable for an evaluation of alternative investments in a developing country. Where a substantial part of construction costs are labor costs, as is generally the case for hydroelectric, social costs may be much lower than market costs. Conversely, the critical issue for nonlabor costs may be the foreign exchange requirement. A number of approaches have been suggested to capture these considerations. Lahiri (1979) and Bhattia (1974), in particular, have used adjusted cost coefficients to derive a social cost objective function, whose solution is then compared to that obtained using a market cost objective function. Another might be to drive the model in an explicit multiobjective mode, using foreign exchange minimization, employment maximization, and market cost minimization objective functions. An employment maximization objective might be particularly useful in situations where the determination of social cost of labor is imprecise, since employment requirements can more readily be determined.

The major disadvantage of using linear programming is the conflict with the indivisibilities of transmission line construction. In particular, one must make *a priori* decisions as to the voltage levels of particular links, and then use cost functions and loss coefficients corresponding to that voltage level. The LP solution, however, then yields transmission levels in terms of GW of power transmitted, which may or may not be appropriate to the assumed voltage level. One expedient is to augment the node-to-node transmission variables (which would correspond to the voltage level required to connect likely new facilities to the grid) with an assumed network of a much higher voltage accross greater distances--corresponding to a super grid intended for regional reliability and bulk power transmission to serve major load centers. On Figure 8.8, for example, we connect A to B, and B to C with transmission variables using, say 230 kV for loss and cost coefficient determination: we connect A and C, however, with a transmission variable corresponding to 400 kV. It is, of course, an inherent feature of LP that choice between alternative voltage levels between the same set of nodes (on Figure 8.8, between the 115 and 230 kV lines between A and B) will always be in favor of that showing the least unit cost. Gately, in his analysis of the southern India Grid, did experiment with integer variables to denote possible transmission links of different voltage: however the computational effort involved, and the requirement for mixed integer programming in such an approach, appear to be serious disadvantages.

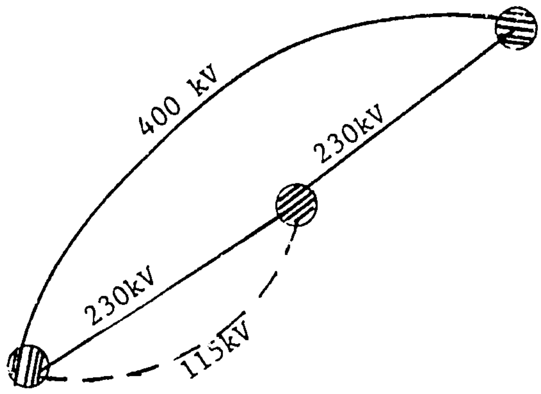


Figure 8.8: Transmission Line Definition.

An obvious expedient is to insepct the LP solution for unreasonable power transfers. Where excessively large transfers are indicated for the voltage level assumed, a second iteration might superimpose some additional, higher voltage links between appropriate points; and similarly where unreasonably low transfers are indicated, for the assumed voltage lower cost lines of lower voltage (albeit of higher unit losses) might be inserted.

Coal Supply: Suppose $k^* K$ is the subset of generation types that burn coal. Suppose further that each type of plant can only burn coal of one type (a reasonable assumption, since, for example, a lignite burning plant has different technological characteristics to, say, a bituminous coal plant), and let the plants that burn coal of type ℓ be denoted K^*_ℓ. Then it follows th the total coal requirements of coal type ℓ in season t at location i, $B_{i\ell t}$, is given by

$$B_{i\ell t} = \sum_{\substack{e \in K^*_\ell \\ e \in i}} \sum_{p}^{P} \gamma_\ell \beta_e x_{et}(p) + \sum_{k \in K^*_\ell} \sum_{p}^{P} \gamma_\gamma \beta_k x^k_{it}(p) \tag{8.28}$$

where

γ_ℓ is the heat value of the ℓ-th coal, in ton/Btu

β_ϵ is the heat rate of the e-th existing plant, in Btu/kWh

$B_{i\ell t}$ is the coal requirement of type ℓ at i in season t, in 10^6 tons/year

β_K is the heat rate of new plants of type k

$k \epsilon K^*_\ell$ is the set of plants that burn coal of type ℓ.

Define next the following notation:

$M_{i\ell t}$	coal of type ℓ mined in i in period t, in 10^6 tons
$S'_{i\ell t}$	minemouth stockpile of coal of type ℓ at i at the end of the t-th period, in 10^6 tons
$C_{ij\ell t}$	coal shipments from i to j in period t in 10^6 tons
$S''_{i\ell t}$	power plant stockpile of coal of type ℓ at i at end of period t, in 10^6 tons.
$I_{i\ell t}$	industrial use of coal of type ℓ at i in season t, in 10^6 tons.
$S'''_{i\ell t}$	industrial coal stockpile of coal type ℓ at i at end of t-th season, in 10^6 tons.

Then the maintenance of mass balance at each node i for each coal type ℓ, and for each season t, requires that

$$\underbrace{M_{i\ell t}}_{\text{coal mined}} + \underbrace{S'_{i\ell(t-1)} - S'_{i\ell t}}_{\text{coal from mine stockpile}} + \underbrace{\sum_{j\in A_i} C_{ji\ell t}}_{\text{coal shipments to i}} - \underbrace{\sum_{j\in A_i} C_{ij\ell t}}_{\text{coal shipments from i}}$$

$$+ \underbrace{S''_{i\ell(t-1)} - S''_{i\ell t}}_{\text{coal from power plant stockpile}} - \underbrace{B_{i\ell t}}_{\text{coal burnt}}$$

$$+ \underbrace{S'''_{i\ell(t-1)} - S'''_{i\ell t}}_{\text{coal from industrial coal stockpile}} - \underbrace{I_{i\ell t}}_{\text{industrial coal use}} = 0. \tag{8.29}$$

However, the three types of coal stockpile are indistinguishable for any given location, and thus we utilitize the single storage variable $S_{i\ell}(t)$ to denote total coal of type ℓ in storage at i at the end of the t-th season.

Since industrial coal use will, in general, be specified exogenously, the $I_{i\ell t}$ will occur on the right hand side of the corresponding constraint form. Also, since optimal mining production requires constant output, the $M_{i\ell t}$ variables can be defined as

$$M_{i\ell t} = \frac{d_t}{365}\bar{M}_i \tag{8.30}$$

where $\bar{M}_{i\ell}$ is the annual production of coal type ℓ in i and d_t the number of days in the t-th season. Thus the constraint form of (8.29) becomes

$$\frac{d_t}{365}\bar{M}_{i\ell} + S_{i\ell(t-1)} - S_{i\ell t} + \sum_{j\in A_i} C_{ji\ell t} - \sum_{j\in A_i} C_{ij\ell t} - B_{i\ell t} = I_{i\ell t}. \quad (8.31)$$

The quantity of coal ℓ mined at i, $\bar{M}_{i\ell}$, is, in turn, made up of the segments of a location specific supply curve, each segment of which reflects an increasing cost per ton. Suppose there are three steps to the curve, with increments $m_{i\ell j}$, j = 1, ... ,3. Each will have a different price, of course. They are also bounded, as indicated on Figure 8.9, by

$$\begin{aligned} 0 \le m_{i\ell 1} \le m^*_{i\ell 1} \\ 0 \le m_{i\ell 2} \le m^*_{i\ell 2} \end{aligned} \quad (8.32)$$

hence $\bar{M}_i$ is given by the constraint

$$\bar{M}_{i\ell} - \sum_{j=1}^{3} m_{i\ell j} = 0 . \quad (8.33)$$

The objective function requires a term corresponding to the price of each supply function step, viz.,

$$\sum_i \sum_j m_{i\ell j} \cdot p_{i\ell j} \quad (8.34)$$

where $p_{i\ j}$ is the price, in \$/ton, of coal type ℓ at step j in region I.

Some cost will also be associated with the storage variables. For this we need the additional objective function term

$$\sum_t \sum_\ell \sum_i S_{i\ell t}\sigma \quad (8.35)$$

where σ is the cost of storage in \$/ton (land, site preparation, handling equipment, etc.).

Railroad Capacity Constraints: From Eq. [8.34] we see that the total coal to be shipped across any link (i,j) in the t-th season is

$$\sum_{\ell=1}^{L} \left\{ C_{ij\ell t} + C_{ji\ell t} \right\} \quad (8.36)$$

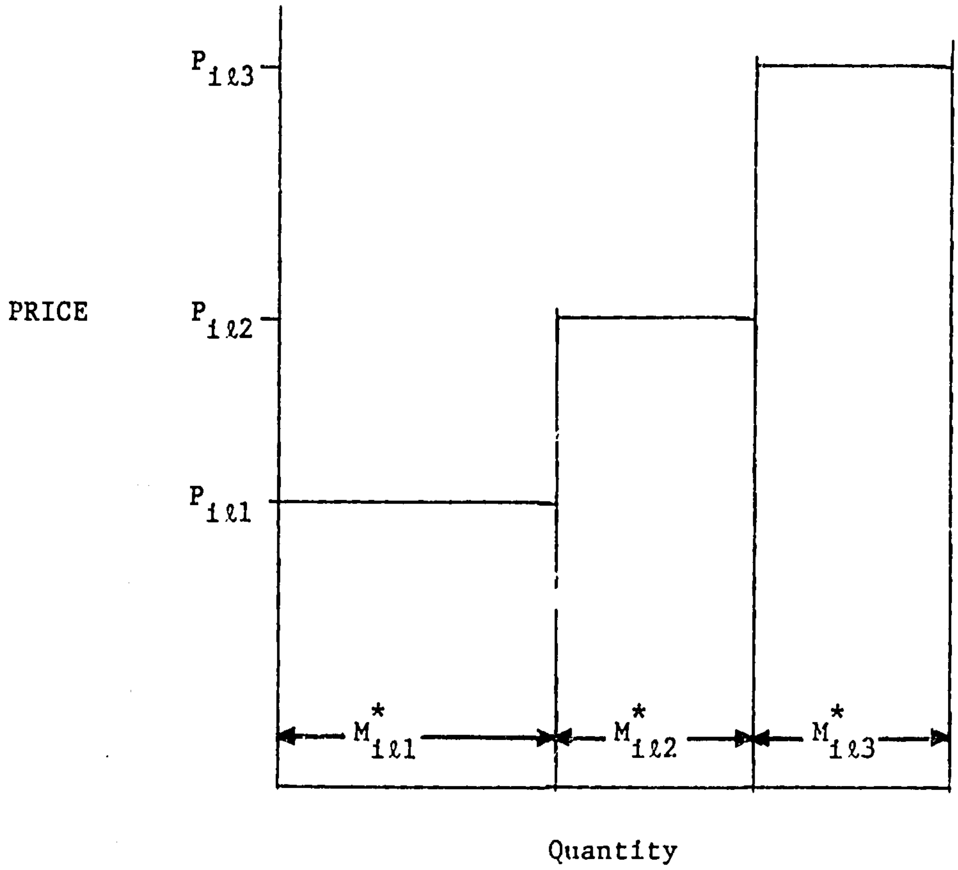


Figure 8.9. Representation of Coal Supply Curves

This is constrained by the capacity of the rail line in question, say $\bar{R}_{ij}$. Because of important, seasonally dependent non-coal rail freight traffic, however, the available capacity for coal shipments is a time dependent function; thus the actual capacity limit is $\bar{f}_R(t) \cdot R_{ij}$. Thus for each season t, and each link (i,j), we have the equation

$$\sum_{\ell=1}^{L} \left\{ C_{ij\ell t} + C_{ji\ell t} \right\} \leq f_R(t)\bar{R}_{ij} \ . \tag{8.37}$$

The total rail capacity $\bar{R}_{ij}$ can be determined from the expression

$$\bar{R}_{ij} = Vn_1(i,j) + Vf'n_2(i,j) \tag{8.38}$$

where

$n_1(i,j)$ is the number of double track lines from i to j

V is the capacity of a double track line, in 10^6 t/year

$n_2(i,j)$ is the number of single track lines from i to j

f' is the capacity of a single track line as a fraction of the capacity of a double track line (and is in the neighborhood of .33). Note that the capacity of two single track lines is less than that of one double track line.

Finally, if the cost of coal transportation is g_t \$/ton/mi, the objective function requires the term

$$\sum_{\ell} \sum_{t} \sum_{(i,j)} d_{ij} g_t \left\{ Cd_{ij\ell t} + C_{ji\ell t} \right\} . \tag{8.39}$$

In actuality, however, railroad transportation rates are rarely expressed in terms of cost per ton per mile: even in a country such as the U.S., where most of the freight carrying railroads are privately owned, extensive governmental regulation of freight tariffs creates a fairly non-systematic rate structure. In developing countries, rates for nationalized railroads are most frequently set by central planning authorities, with a rate structure that is generally some form of step function with respect to major distance and freight volume classes. Such a structure can be accommodated in our modelling framework by applying the transportation cost term not to coal movements through individual railroad links [as given by (8.39)], but to the $B_{i\ell t}$ terms, which represent the amount of coal of type ℓ burnt in season t at location i. If we require that each coal type be associated with a unique coalfield, then the distance implied by each $B_{i\ell t}$ term is readily computed, and the appropriate cost coefficient applied. Thus the objective function term for coal transportation is

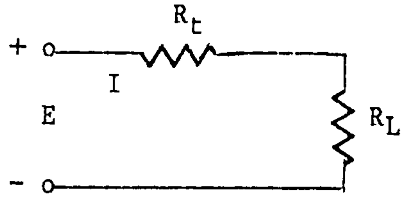
$$\sum_{t}^{T} \sum_{i}^{N} \sum_{\ell}^{L} B_{i\ell t} \cdot k_c(i,\ell) \tag{8.40}$$

where $k_c(i,\ell)$ is the rate that applies to coal movement from the ℓ-th coalfield to location i. However there are two problems with this formulation. First is that the requirement that each coal type be associated with its own coalfield may significantly increase the problem size, since in the pure transhipment version of the problem it would be possible to have multiple coal fields all of the same coal type. Thus, whereas in the pure transhipment model, the number of constraints in (8.28) is a function of the number of coal types, in the version suggested here the number of constraints is a function of the number of distinct coalfields. Our initial evaluation indicates, however, that for most developing country applications this is not a severe limitation, as there will tend to be a very limited number of coalfields that represent potential fuel supplies, each of whose coal characteristics are so different that they would in any case require separate coal types.

More serious, however, is the point that if there is no objective function coefficient associated with coal shipments through individual links, there is no guarantee that an optimal path through the network has been determined--the only thing that is assured is that the capacity constraints not be violated. The way out is to bring back an objective function term of the type (8.39), but to make the g_t term extremely small, say on the order of 0.1% of the k_c values. This will ensure that an optimal path is indeed found, but will not significantly distort the actual cost of rail haul.

8.4 THE TREATMENT OF ELECTRIC TRANSMISSION IN SPATIAL PROGRAMMING MODELS.

The energy balance equation (8.14) accounts for transmission losses by assuming that some fraction Ω_{ij} of the power Y_{ij} entering a line from i to j is lost, with the receiving node therefore receiving only $Y_{ij}(1 - \Omega d_{ij})$. The rationale for such a formulation is as follows. Consider the following simple circuit:



where

R_t is the resistance of the transmission line,

R_L is the resistance of the load,

E is the applied voltage.

The total power transmitted, P_t, is given by

$$P_t = \frac{E^2}{R_t + R_L} , \qquad (8.41)$$

and the power loss in transmission, P_{Loss}, is given by the well known formula

$$P_{Loss} = I^2 R_t = \left\{ \frac{E}{R_t + R_L} \right\}^2 R_t , \qquad (8.42)$$

from which follows

$$P_{Loss} = P_t \left\{ \frac{R_t}{R_t + R_L} \right\} . \qquad (8.43)$$

Thus we see that the power losses in transmission are equal to the total power transmitted times the ratio of line resistance to total system resistance. In the notation of the model, the loss in any particular line is given by the expression $Y_{ij} \Omega d_{ij}$, which is clearly analogous to (8.43), since Y_{ij} represents the total power transmitted from i to j. Ω thus represents the loss per unitof input per mile; one should note, however, that Ω is itself a function of line length, as we shall see below. Although (8.43) suggests that the losses can be determined exactly by knowledge of R_t and R_L, in a model of the type considered in here, in which a multitude of lines of different characteristics make up the capacity of transmission between two adjacent nodes, Ω is clearly not a parameter that can be easily determined, even if it, too, were indexed to correspond to the line length d_{ij}.

One possibility might be to let Ω be an endogenous model parameter. Overall system losses in our model compute to

$$L_T = \sum_{i=1}^{n} \sum_{\substack{j=1 \\ j\neq i}}^{n} 8760 Y_{ij}\,\Omega\, d_{ij}. \tag{8.44}$$

But since good overall system design would in general hold transmission losses to about 2 to 3% of generation, we require that, say,

$$L_T \leq G = 0.025 \left\{ \sum_p \sum_t \sum_e x_{et}(p) + \sum_i \sum_p \sum_t \sum_k x_{it}^{k}(p) \right\} \tag{8.45}$$

where G represents total generation, and thus the problem would be to select that value of Ω for which Eq. (8.45) is satisfied.

There are several problems here. First, if one were to combine Eq. (8.44) and Eq. (8.45) into a constraint, namely,

$$\sum_i^n \sum_j^n 8760 Y_{ij}\Omega d_{ij} - 0.025\ G = 0 \tag{8.46}$$

Ω appears as a multiplicative term with other problem variables (the Y_{ij}), and hence cannot be solved endogenously in a linear programming framework. Thus one could only use a trial and error method, specifying values of Ω exogenously, and having the model compute the resulting transmission losses. More serious, however, is the fact that Ω is not the constant implied by (8.46), since, as we noted above, Ω is itself a function of distance. In fact, the loading of transmission lines is generally dependent on two considerations: thermal capability and system stability, factors that merit some discussion here.

Thermal capability is essentially self-explanatory. The choice of conductor size is based upon its mechanical behavior under conditions of elevated temperatures where large currents are being transmitted. Wind speed, radiation, and convection, as well as the incidence of solar heating and ambient temperature, are included in the judgment, and published conductor tables contain suggested maximum currents for average operating conditions. Selection of a given conductor for a desired current level, therefore, specifies the resistance per length of line (usually in ohms per mile), and the corresponding line losses may then be determined for a range of currents (loadings) and line lengths. However, this simple procedure becomes complicated by other factors when the system operating voltage level is increased. Under such conditions, it is necessary to decrease the effects of corona and associated electromagnetic radiation and audible noise by effectively increasing the surface area of the conductor so as to reduce the voltage gradient at the surface of the conductor. The maximum single-conductor size used for this purpose is about 1.5 inches in outside diameter and is applied to transmission lines operating at the 230- to 275-kV level. For higher voltages, so-called "bundled" conductors are used, typically with two subconductors per phase at 345 kV, three subconductors per phase at 500 kV, and four subconductors per phase at 765 kV. The net result for all these configurations is that the actual thermal capability is often well above the maximum capacity of the associated system. Also, the resistance per mile and the subsequent line losses are lower than for single-conductor lines of the same current capability.

System stability margin is related to the maximum transmission-line loadability that can be tolerated without danger that the alternators of the system will pull out of synchronization following a severe fault or a sudden major change in load requirements. The standard surge impedance loading (SIL) curves, also known as "St. Clair curves," are based upon this consideration. These curves specify line loadability as a function of length of line in terms of the so-called "natural power," or surge-impedance loading, defined as

$$\mathrm{SIL} = \frac{E_R^2}{Z_0}\ \mathrm{MW}, \tag{8.47}$$

where

SIL = surge impedance loading,

E_R = line-to-line receiver-end voltage in kV,

Z_0 = surge impedance in ohms.

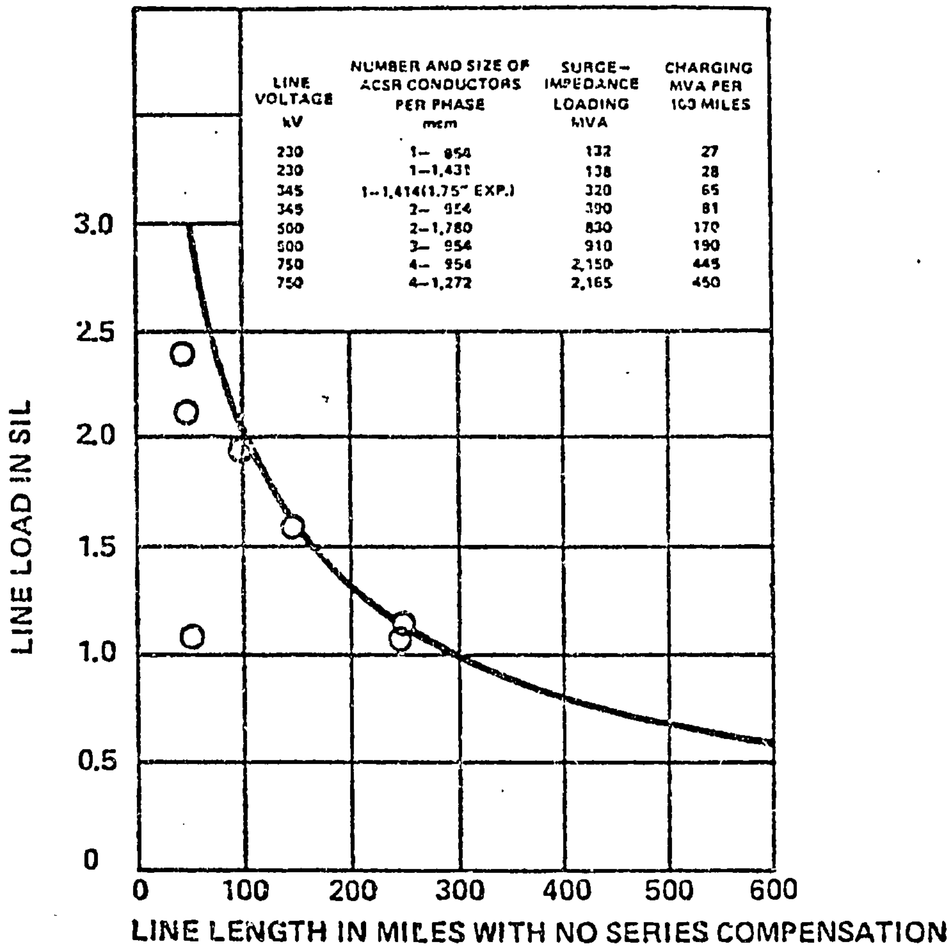


Figure 8.10. Transmission Line Capability

In general, the St. Clair curves are not used for distances less than 50 miles, so that line loadability up to that distance is limited by the thermal capability. Safe transmission MW levels beyond 50 miles are limited by SIL. Line losses are therefore dependent upon the square of the line current for the prescribed load level and the resistance of the corresponding line length, with the resistance of the line per unit length being the only constant factor. Figure 8.10 shows a typical SIL curve.

The use of these St. Clair curves to determine Ω values is described in detail in the next section. Typical values emerging from such calculations are given in Table 8.2.

For existing transmission lines, this table can be used to determine both the approximate current line loadability (i.e., the $\bar{Y}_{ij}$), as well as the ap-

Table 8.2
Summary of Typical Ω Computations

Voltage, kV	Line length, mi	Typical line loadability, MW	Ω,mi^{-1}
765	50	4878	0.00027
	100	3686	0.00020
	200	2710	0.00015
345	50	864	0.00039
	100	653	0.00030
	200	480	0.00021
230	50	306	0.00038
	100	170	0.00029
	200	136	0.00022
138	50	127	0.00048
	100	95	0.00034

propriate value of Ω to be used. For new transmission lines, i.e., those corresponding to the Y_{ij} variables, it is necessary to assume a particular voltage, but this can generally be established with the advice of the electric utility.

Calculation of the Ω Parameter: Consider the American Electric Power Company (AEP) 765-kV transmission system. The conductor used throughout the system is known as "Drake," a 795,000 circular mil (CM) aluminum-conductor-steel-reinforced (ACSR) cable having a resistance of 0.1288 ohm per mile. The rated current of this conductor is 910 amperes. Four Drake subconductors, spaced 18 in. apart, make up one phase of this three-phase line.

Thus, the actual thermal rating of this line is 4 X 910 = 3640 amperes, and the resistance per mile of four conductors in parallel = 0.1288/4 = 0.0322 ohm per mile. Assuming unity-power-factor operation, the thermal capability of this line is

$$P_{Th} = \sqrt{3} \times 3640 \times 765 \times 10^3 = 4823 \text{ MW}.$$

From Gabrielle et al. (1964), the Z_0 of this line = 270 ohms. The surge-impedance loading of this line is then:

$$SIL = \frac{(765)^2}{270} = 2168 \text{ MW}.$$

Table 8.3

765kV, 4-bundle configuration

Voltage Level = 765 kV (AEP 4-bundle configuration)
Conductor: "Drake": 795,000 CM ACSR, R = 0.0322 ohm/mile
Rated Current = 910 amperes/subconductor, 3640 amperes/phase
Z_o = 270 ohms[a] SIL = 2168 MW

Line length, miles	Line loadability,[b] MW	Line losses MW/100 miles	MW	%	Ω loss/input/mile
50	4878	132	66	1.34	0.00027
100	3686	75	75	2.04	0.00020
200	2710	40	81	3.0	0.00015
300	2168	39	78	3.6	0.00012

[a]From Gabrielle et al. (1964)
[b]From St. Clair curves at normal loading without series compensation.

From the St. Clair curves, this amount of power may be transmitted for 300 miles without exceeding stability limits. Also, from the curves at 50 miles (for normal loading):

$$P_{50} = 2.25 \times SIL = 2.25 \times 2168 = 4878 \text{ MW}.$$

In this case, the loadability of the line at 50 miles is essentially equal to the thermal capability. Thus, the 3-phase power loss for 50 miles of line is

$$P_{loss} = 3I^2 \times 50R = 3I^2 \times 50(0.0322).$$

But at 50-mile loading:

$$I = \frac{4878 \times 10^6}{3 \times 765 \times 10^3} = 3682 \text{ amperes.}$$

$$P_{loss} = 3 \times (3.682)^2 \times 10^6 \times 50 \times 0.0322 = 65.5 \text{ MW}.$$

Percent loss in terms of power received is

$$\% \text{ loss} = \frac{P_{loss}}{P_{Load}} \times 100 = \frac{65.5}{4878} \times 100 = 1.34\%;$$

Table 8.4

138 kV, Single Line Configuration

Voltage Level = 138 kV (Single-conductor, single three-phase line configuration)
Conductor: "Hawk," 477,000 CM ACSR, R = 0.212 ohm/mile
Rated Current = 670 amperes/phase
Z_0 = 400 ohms[a]   SIL = 50 MW

Line length, miles (loading condition)	Line loadability,[b] MW	Line losses MW/100 miles	MW	%	Ω loss/input/mile
50 (Heavy)	150	26	13	8.7	0.00164
50 (Normal)	113	14	7	6.2	0.00117
100 (Normal)	85	8	8	9.4	0.00086
150 (Normal)	70	5	8	11.4	0.00068

[a]From Gabrielle et al. (1964)
[b]From St. Clair curves at loading indicated without series compensation.

and Ω for the 50-mile line is

$$= \frac{P_{loss}}{(P_{Load} + P_{loss})50} = \frac{65.5}{(4878 + 65.5)50} . \tag{8.48}$$

$$= 0.000265 \text{ loss/input/mile}$$

Table 8.3 shows these results for a series of line lengths at the 765kV voltage level.

In order to cast some light on the behavior of subtransmission components in the system, similar calculations can be made for the 138-kV level, resulting in the data shown in Table 8.4. It is clear, however, that this configuration involves very large line losses. If the conductor is changed to 954,000 CM ("Rail") with a resistance of 0.1 ohm/mile, the losses would be halved for the same level of power transmission. A further reduction of power loss by another factor of 2 would be accomplished by using a double-circuit configuration composed of the "Rail" conductors. Such arrangements are

Table 8.5

138 kV, double Line Configuration

Voltage Level = 138 kV (Single-conductor, double three-phase line configuration)
Conductor: "Rail," 954,000 CM ACSR, R = 0.108 ohm/mile
Rated Current = 1300 amperes/phase
Z_o 340 ohms SIL = 56 MW

Line length, miles (loading condition)	Line loadability,[a] MW	Line losses MW/100 miles	MW	%	Ω loss/input/mile
50 (Heavy)	168	8.2	4.1	2.4	0.00048
50 (Normal)	127	4.4	2.2	1.7	0.00034
100 (Normal)	95	2.9	2.9	3.1	0.00029
150 (Normal)	78	1.7	2.5	3.2	0.00021

[a]From St. Clair curves at loading indicated without series compensation.

popular with electric utilities at the 138-kV level. The surge impedance, Z_o, for these changes remains approximately the same so that almost the same SIL may be used. Results are shown in Table 8.5.

Again these results are compatible with those of the bulk transmission system. The overall conclusion, with respect to total average loss at both bulk and subtransmission levels, is that typical line designs cause the percent line losses to be more or less equally divided between the two levels at an average of about 2.5 percent.[13]

[13]For further discussion of the degree to which approximate methods accurately simulate real power flows in a transmission grid, see Crevier (1972).

8.5 INCORPORATING WATER RESOURCE PLANNING CONSIDERATIONS

There exists an of course an enormous literature in the water resources engineering field that deals with the design, operation and overall optimization of a water resources system, in which hydropower generation is an important component. However, there is very little indeed on the more general subject of integration of water resource projects and entire power systems (i.e., more than just hydropower production), although individual aspects of the system have been addressed.[14]

Basic Continuity Relationships: The basic continuity constraint about each potential reservoir site, for some season t, follows from Figure 8.11 as:

$$S_{i(t-1)} + Q_{it} - R_{it} - I_{it} - E_{it} - Q^*_{ijt} = S_{it} \qquad (8.49)$$

where
- S_{it} is the storage at the end of the t-th season
- Q_{it} is the inflow in season t
- R_{it} is the downstream release in season t
- E_{it} is the evaporation and seepage loss in season t
- Q^*_{ijt} is the diversion from i to j in season t, where j is some location in a different basin.
- I_{it} is the irrigation release in season t.

In turn, the inflow is given by

$$Q_{it} = R_{(i-1)t} + Q^{\ell}_{it} + Q^*_{jit}. \qquad (8.50)$$

Inflow at i	Release at i - 1	Natural flow increment in i-th reach	Inflow from other river basins

To reduce the number of constraint sets, we substitute (8.50) into (8.49), and rearrange in constraint form, to yield

$$S_{i(t-1)} + R_{(i-1)t} + Q^{\ell}_{it} + Q^*_{jit} - I_{it} - E_{it} - Q^*_{ijt} - S_{it} - R_{it} = 0\ . \qquad (8.51)$$

A minimum flow requirement in any stream reach is captured by the bound set

$$R_{it} \geq R_{it}(\min)\ . \qquad (8.52)$$

[14] Some of the relevant discussion include Reger μ Schwarts (1973) or Windsor (1975). For a good discussion of water resource screening models, that follows a general philosophy of approach similar to that discussed here, see Cohon, (1972). O'Laoghaire and Himmelblau, (1974), is another excellent treatment.

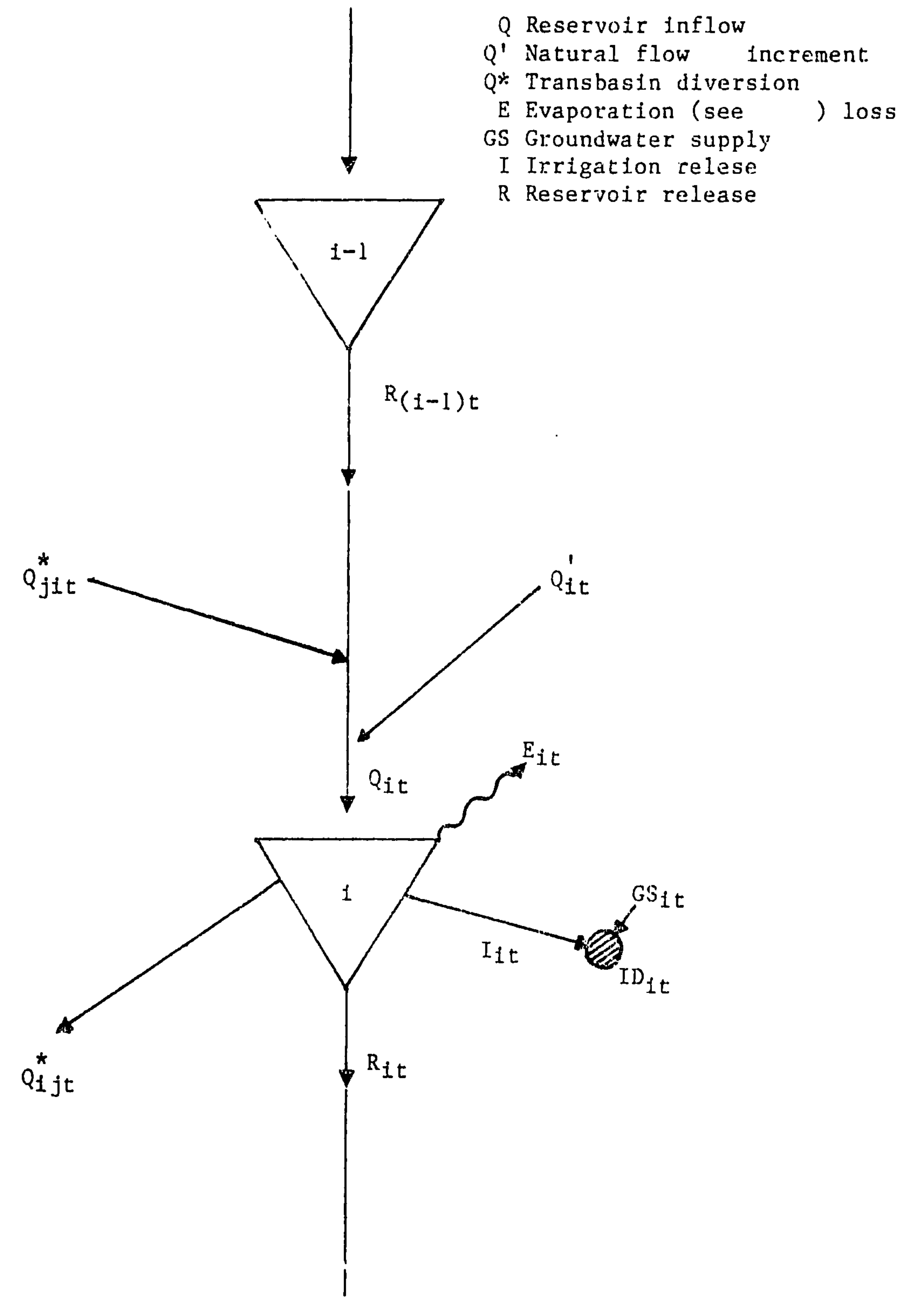


Figure 8.11: Generalized Water Project Configuration.

The continuity relationship for irrigation demands are given by[15]

$$I_{it}(1 - \lambda) + GS_{it} \geq ID_{it} \tag{8.53}$$

Irrigation releases	Groundwater supply	Irrigation demand

where λ is the loss factor in the surface canal system.

Capacity Relationships: The quantity of water in storage, S_{it}, cannot exceed the storage capacity of the reservoir, V_i, i.e.

$$S_{it} \leq V_i \quad \{\text{for } t = 1, \ldots, T\} \tag{8.54}$$

or, in constraint form, since V_i is also a model variable,

$$S_{it} - V_i \leq 0 \quad \{\text{for } t = 1, \ldots, T\} . \tag{8.55}$$

Unfortunately, the capital costs of reservoir construction are not, in general, a linear function of storage volume, typically following an S curve as shown on Figure 8.12. A linear approximation subject to an upper bound of maximum storage capacity that could reasonably be expected of a specific site, however, would appear to suffice for our purposes; hence we need the bound set

$$V_i \leq V_i(\max) \tag{8.56}$$

and the corresponding objective function term is

$$\sum_i a_i V_i \tag{8.57}$$

where a_i is the unit cost of reservoir construction in \$/Acreft.

Similarly, the irrigation releases cannot exceed the capacity of the irrigation canal system, IC_i

$$I_{it} - IC_i \leq 0 \quad \{t = 1, \ldots, T\} \tag{8.58}$$

with the corresponding objective function term

$$\sum_i a_i' IC_i \quad . \tag{8.59}$$

where a_i' is the unit cost of canal system construction.

[15] The subject of conjunctive use of groundwater and surface water is another that has received extensive attention in the literature, with highly complex optimization models addressed to the details of system operation, see e.g. Yu and Haimes (1974).

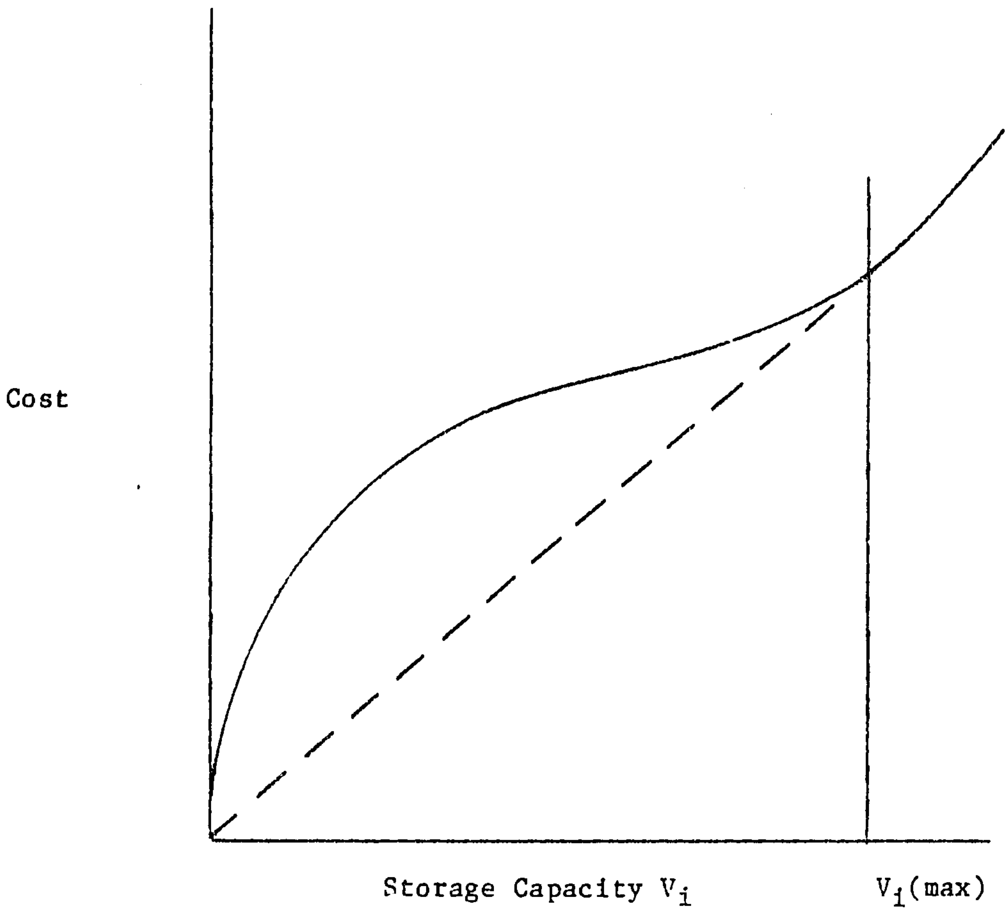


Figure 8.12: Generalized Storage/Cost Relationship
Adapted from O'Laoghaire + Himmelblau(1974)

Hydro Energy Production: Suppose Q_{pit} is the flow through a power turbine in season t, and if $(h_{it} + h_{i(t-1)})/2$ is the average head in season t, then the energy generation in season t is given by

$$\sum^{P} x^{5}_{ti}(p) = 1.024\ Q_{pit}\left\{\frac{h_{it} + h_{i(t-1)}}{2}\right\} 10^{-6}\ e\ . \qquad (8.60)$$

$$[\mathrm{GWh}] \qquad \left[\frac{\mathrm{kWh}}{\mathrm{AF.ft}}\right] [\mathrm{AF}] \qquad [\mathrm{ft}] \qquad \left[\frac{\mathrm{GWh}}{\mathrm{kWh}}\right] [\ ]$$

Since not all releases need necessarily flow through the turbine, this is better expressed as an inequality, and rearranging in constraint form, one obtains

$$\sum^{P} x^{5}_{ti}(p) - 1.024\ e\ 10^{-6}\left\{\frac{h_{it} + h_{i(t-1)}}{2}\right\} Q_{pit} \leq 0\ . \qquad (8.61)$$

The inequality simply ensures that releases take place even if a hydroproject is not built at a particular location.

There is, however, a more serious problem to Eq. (8.61), in that h_{it} and Q_{pit} appear as multiplicative terms, and therefore (8.61) does not meet linearity requirements. This is a problem long recognized in LP formulations of multipurpose reservoir projects.[16]

There appear to be two approaches to the linearization of (8.61). One is to assume a value for Q_{pit}, leaving the head h_{it} as a variable related to storage by a linear function of constant k, namely

$$S_{it} - kh_{it} = 0 \; . \tag{8.62}$$

The problem here is that a priori assumptions about the flow Q_{pit} run counter to the objective of optimization, quite aside from the fact that the relationship between storage and head is not linear - although piecewise linearization may be possible.

The other is to assume a value for h_{it}, which would certainly be reasonable for high head projects, but which for most variable head plants presents some problems. In particular, as noted by Cohon, the major problem here is that Q_{pit}, and hence, from (8.61), $x^5_{it}(p)$ may be positive even if S_{it} is zero, an unlikely situation. However, when using the first approach, and (8.62), h_{it} can be nonzero only if S_{it} is nonzero, and hence, from (8.61), energy can only be produced if a reservoir is in fact constructed.

Finally, it should be noted that (8.61) replaces the original hydroenergy constraint (8.21); thus

$$x^5_{th}(\max) = 1.024 \text{ e } 10^{-6} \left\{ \frac{h_{it} + h_{i(t-1)}}{2} \right\} Q_{pit} \tag{8.63}$$

and implies that data tabulations of the type illustrated on Table 8.1 rest on a priori specification of head and discharge.

Evaporative Losses. Evaporative losses from reservoirs may constitute a significant fraction of the available resource, and must be explicitly considered in any modelling framework. Obviously, the quantity evaporated will be a complex function of temperature, humidity and wind movement, and is

[16] For further discussion, see e.g. Loucks (1969) or Cohon (1972).

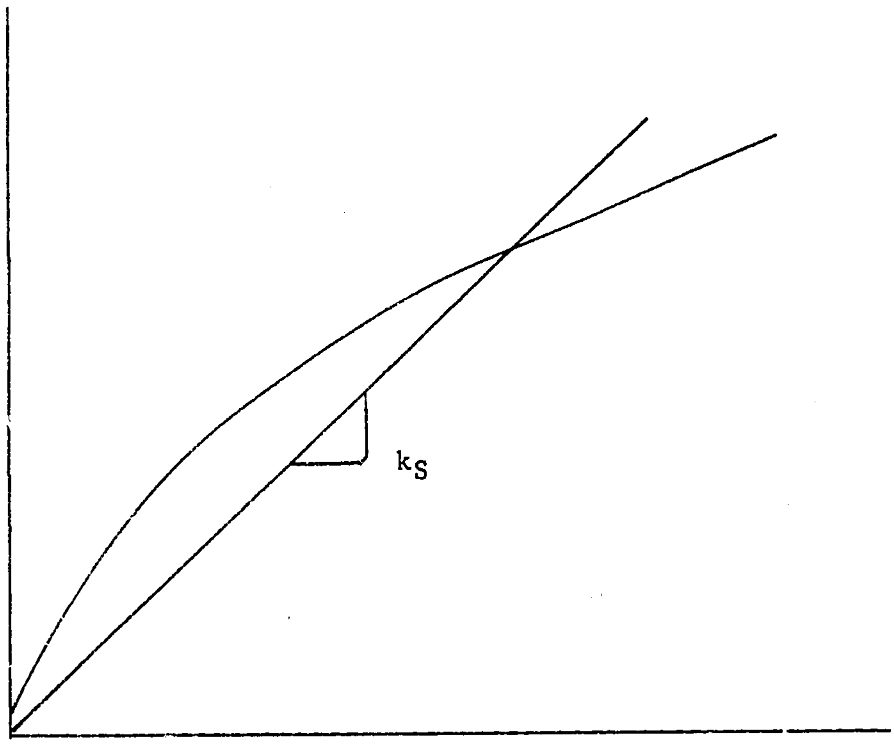


Figure 8.13: Surface Area - Storage Relationship

captured as an "evaporation coefficient," say $\epsilon_i(t)$, that typically is quoted in dimensions of ft/month.[17] Thus

$$E_{it} = \epsilon_i(t) \cdot m_t \cdot \left\{\frac{A_{it} + A_{i(t-1)}}{2}\right\} \tag{8.64}$$

$$\left[\frac{AF}{season}\right] \quad \left[\frac{ft}{month}\right] \quad \left[\frac{months}{season}\right] \quad [Acres]$$

where A_{it} is the surface area of the reservoir in the t-th season. The surface area can of course be defined in terms of the volume in storage, S_{it}; unfortunately this relationship, determined by the topography of the site, proves usually to be nonlinear in character, as shown on Figure 8.13. For the purposes of this generalized model, however, iterative adjustment of an assumed linear relationship between surface area and storage volume probably suffices (since the iterative adjustment is necessary anyway for reasons of

[17]The $\epsilon_i(t)$ coefficient is thus, of course, a function of season. For example, O'Laoghaire and Himmelblau (1974), notes a value of evaporation coefficient for the Highland Lakes area of Texas of 0.55 for the summer months May-October, falling to 0.29 for the rest of the year.

transmission line and hydropower generation approximations). Thus, for any particular run we rewrite (8.64) as

$$E_{it} = \epsilon_i(t) m_t \hat{k}_s' \left\{ \frac{S_{it} + S_{i(t-1)}}{2} \right\} \qquad (8.65)$$

where $\hat{k}_s'$ is the first estimate of the slope of the surface area-volume relationship. Then, for a computed value of S_{it}, we adjust in the second iteration to $\hat{k}_s''$.

Water Requirements for Thermal Generation: The outflow in each section must be adequate to support the withdrawal needs for thermal generation; if the set H represents thermal generation types, and Ω_k the withdrawal water requirement for the k-th generation type, then

$$\sum_{k \in H} x_{it}^k(p) \;\; \Omega_k \;\; - f_w(t) \; R_{it} \leq 0 \qquad (8.66)$$

$$[\text{GWh}] \; \left[\frac{\text{AF}}{\text{GWh}}\right] \quad [\;\;] \quad [\text{AF}]$$

where $f_w(t)$ is the fraction of total streamflow that can be withdrawn for thermal power stations in season t.

Even once through cooled plants will involve some degree of evaporative water loss.[18] Such losses are generally expressed as cfs per GW of installed capacity, say γ_k'. Hence the consumptive loss Z_{it} is given by

$$Z_{it} = \sum_{k \in H} \sum_{p} x_{it}^k(p) \; 3600 \;\; \gamma_k' \; \frac{1}{43560} \qquad (8.67)$$

$$[\text{AG}] \qquad [\text{GWh}] \left[\frac{\text{Sec}}{\text{hr}}\right] \left[\frac{\text{cfs}}{\text{Gw}}\right] \left[\frac{\text{AF}}{\text{cu.ft}}\right]$$

or, in constraint form

$$Z_{it} - \sum_{k \in H} \sum_{p} x_{it}^k(p) 3600 \; \gamma_k' \; \frac{1}{43560} \; \{t = 1, \; \ldots \; ,T\} = 0 \qquad (8.68)$$

Typical values for γ_k' would be 5 cfs/GW for fossil plants, 12 cfs for nuclear.

---

[18]Evaporatively cooled thermal plants, say using mechanical and natural draft cooling towers, would of course imply an even greater consumptive use, but such systems are unlikely to be in use in most developing countries for the forseeable future. In any event, all one needs to accomodate such technology is to use a higher value for the γ_k' coefficient in Eq. (8.68).

Whilst environmental considerations have not heretofore received as much attention in developing countries as they have in the developed countries there can be little doubt that as development proceeds, environmental issues will become increasingly important.[19] Because fossil fueled electric power generation is one of the major sources of pollutants, particularly sulfur dioxide and particulates, it is inevitable that these, together with refineries, will be among the first sources subjected to closer environmental scrutiny.[20]

Incorporation of emission limitations on new facilities (ELNF) proves to be relatively straightforward. For the case of sulfur dioxide (SO_2), if we define

ψ_t = SO_2 removal performance of subtechnology t,
β_ℓ = heat content of the ℓ-th coal, in tons/10^{15} Btu,
σ_ℓ = sulfur content of the ℓ-th coal in lb/lb coal,

then SO_2 emissions at node i, in lb of SO_2 per year, from plants that are subject to ELNF, are given by

$$\underset{[10^{15}\ \text{Btu}]}{(C_{i\ell t} - C^*_{i\ell t})} \quad \underset{\left[\frac{\text{ton}}{10^{15}\ \text{Btu}}\right]}{\beta_\ell} \quad \underset{\left[\frac{\text{lb S}}{\text{lb}}\right]}{\sigma_\ell} \cdot \underset{\left[\frac{\text{lb}}{\text{ton}}\right]}{2000} \cdot \underset{\left[\frac{\text{lb } SO_2}{\text{lb S}}\right]}{1.998} \quad (1 - \psi_t), \tag{8.69}$$

where $C^*_{i\ell t}$ are the quads of coal of type ℓ burnt at node i in technology of type t <u>not</u> subject to ELNF (i.e., accounts for existing capacity). But the emissions of Eq. (8.69) are limited by the ELNF for that node, say ξ_i lb $SO_2/10^6$ Btu. Hence

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[19] For example, the fact that such considerations are important in the Indian context is evidenced by the public concern over the emissions from the refinery under construction at Mathura, which could cause significant damage to famous monuments such as the Taj Mahal. In Bombay, public concern over environmental questions is reflected in controversies over industrial siting in the Bombay Metropolitan area (see, for example, numerous reports in the Times of India and other major Indian newspapers over the period September 1977 to June 1978 on the controversy over the proposed siting of a natural-gas based fertilizer plant utilizing the newly discovered gas from the Bombay High fields. The Bombay Bachas Committee, an environmental group, was at the focal point of this controversy).

[20] For a comprehensive discussion of Environmental Issues in Developing Countries, see e.g., Bowonder (1980).

$$\frac{3996 \sum^{T} \sum^{L} (C_{i\ell t} - C^*_{i\ell t}) \beta_\ell \sigma_\ell (1 - \psi_t)}{\sum^{T} \sum^{L} (C_{i\ell t} - C^*_{i\ell t})} \leq \xi_i \cdot 10^9 , \tag{8.70}$$

which can be written in the form of a linear constraint as

$$\sum^{T} \sum^{L} C_{i\ell t} \left\{ 3996 \beta_\ell \sigma_\ell (1 - \psi_t) - \xi_i \cdot 10^9 \right\}$$
$$\leq \sum^{T} \sum^{L} C^*_{i\ell t} \left\{ 3996 \beta_\ell \sigma_\ell (1 - \psi_t) - \xi_i \cdot 10^9 \right\}. \tag{8.71}$$

Ambient Air Quality Constraints: The incorporation of ambient air quality restrictions is more complex and requires prior knowledge of atmospheric dispersion and transportation characteristics, and explicit quantitative knowledge of a transformation matrix, say T, the element T_{ij} of which describes the ground-level pollutant concentration at location j attributable to a unit emission in location i. *Ceteris paribus*, T_{ij} will be a function of stack height; the higher the stack, the greater the geographical dispersion of the resultant emissions. Recall the expression for total SO_2 emissions in county i, namely,

$$\sum_{t=1}^{T} \sum_{\ell=1}^{L} C_{i\ell t} \beta_\ell \sigma_\ell \cdot 2000 \cdot 1.998(1 - \psi_t), \tag{8.72}$$

or

$$\sum_{i=1}^{T} \sum_{\ell=1}^{L} C_{i\ell t} \phi_t , \tag{8.73}$$

where $\phi_{\ell t} = \beta_\ell \sigma_\ell (1 - \psi_t) \cdot 3996$. Then if $T_{ij}(SO_2)$ captures the SO_2 concentration in j attributable to a unit emission in i, total SO_2 in j, to be constrained by the ambient SO_2 standard in j, say AQS_j (SO_2), is given by

$$\sum_{i=1}^{n} \left\{ \sum_{t=1}^{T} \sum_{\ell=1}^{L} C_{i\ell t} \phi_{\ell t} \right\} T_{ij}(SO_2) \leq AQS_j(SO_2) - A_j(SO_2), \tag{8.74}$$

where $A_j(SO_2)$ is the measured average ambient SO_2 concentration in j.

Unfortunately, the determination of the transfer matrices T_{ij} is not a minor matter, since it requires application of a trajectory-diffusion model to each county in the problem area. Because the computation of sulfate concentrations depends on complex atmospheric chemistry, and because such diffusion trajectory models require the solution of three-dimensional partial differential equations over long time periods (to simulate the weather of a typical month or season), the computational effort is considerable. However, once these calculations have been completed, and the transformation matrices for a unit emission determined, the matrices can be stored on tape and accessed by the matrix generator for the LP as needed.

The above constraint set is of course most suited to annual average concentrations, given the nature of long-range transport that is captured by the transition matrix T, the corresponding ambient standards being those for annual arithmetic mean. There is, however, very good evidence that the maximum 24-hour concentration may be the most critical from the standpoint of standards compliance. In order to capture such impacts, ideally one would have available the results of detailed short-range dispersion modeling studies for typical sites in each of the counties for which coal-fired capacity was allowable. If one makes the assumption that the maximum ground-level SO_2 concentration from a new source would occur within the immediate region in which that new source is located, then for each location one can require that

$$\sum^{T} \sum^{L} (C_{i\ell t} - C^{*}_{i\ell t})\phi_{\ell t} \cdot \frac{1}{8760\delta^{t}_{k}} D_i \leq AQS^{*}_{i}(SO_2) - A^{*}_{i}(SO_2), \qquad (8.75)$$

$$\left[\frac{10^{15}\ \text{Btu}}{\text{yr}}\right]\left[\frac{\text{lb } SO_2}{10^{15}\ \text{Btu}}\right]\left[\frac{\text{yr}}{\text{hr}}\right]\left[\frac{\text{ppm } SO_2}{\text{lb } SO_2/\text{hr}}\right] \qquad [\text{ppm } SO_2]$$

where

$AQS^{*}_{i}(SO_2)$ is the 24-hr average ambient air quality standard in county i;

$A^{*}_{i}(SO_2)$ is the measured annual maximum 24-hr average SO_2 concentration in county i;

D_i is the dispersion factor relating emissions to maximum 24-hr average ground-level SO_2 concentration.

One should note, of course, that any locations for which the expression $AQS^{*}_{i}(SO_2) - A^{*}_{i}(SO_2)$ is negative implies violations of an ambient standard which in turn would force the $C_{i\ell t}$ term in Eq. (10.8) to zero, implying that the model would site no new coal capacity in a nonattainment area.

8.7 AN LP OF THE JORDANIAN ELECTRIC SYSTEM*

The purpose of this section is to illustrate the basic modelling concepts of this Chapter by formulating the capacity expansion problem of an actual electric utility system--that of the Jordian Electricity Authority (JEA).

The problems faced by JEA are typical of those in many developing economies; very rapid growth in electrical demand (in 1980, electric consumption in Jordan rose by 21% over 1979), rapidly growing systems in which new capacity additions may be an order of magnitude larger than those currently in existance, and a transition from a heavy dependence on Diseal generation to steam cycle thermal units. Table 8.6 lists the installed capacity of JEA and current plans for capacity additions. Figure 8.14 shows a map of the Jordanian National Grid.

Table 8.6
JEA Installed Capacity

Hussein Thermal Power Station	Steam	3x33	99
	Combustion Turbine	1x14	14
	Combustion Turbine	1x18	18
Marka	Combustion Turbine	2x20	40
	Diesel	45	45
Old Aqaba	Diesel	5.8	5.8
Aqaba Central	Diesel	2x3.5	7
Karak	Diesel	3x1.5	4.5
Ma'an	Diesel	1.87	1.87
Tafila	Diesel	1.3	1.3
Total Existing Capacity (Dec. 1980)			236.47
Hussein Thermal Power Station	Steam	3x66	198
Marka	Combustion Turbine	2x20	40
Aqaba Central	Diesel	3x5	15
Karak	Combustion Turbine	1x20	20
Total Under Construction			273.00

Source: JEA Annual Report, 1980

*The writer acknowledges the assistance of Niazi Musa and Ziad Khamis, of the Jordan Electricity Authority (JEA), Amman, Jordan, for providing the essential data. However, the responsibility for the accuracy and content of this discussion is solely that of the writer, and does not necessarily reflect the views of either JEA or messers Musa and Khamis.

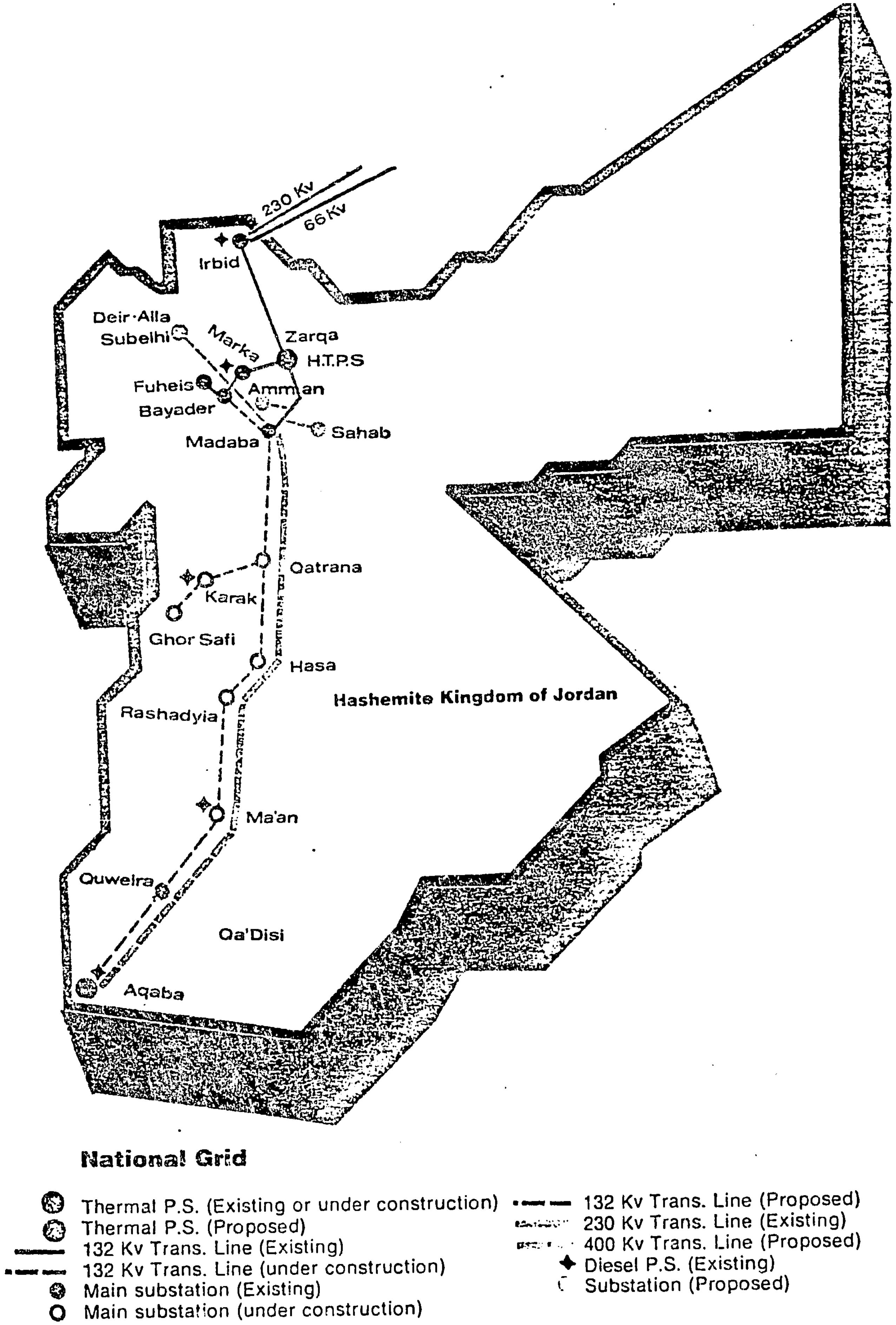


Figure 8.14: The Jordanian National Grid

To set up the problem as an LP, one begins with a linearization of the load-duration curve, as indicated on Figure 8.15. For illustrative purposes we have divided the curve into 6 segments, somewhat arbitrarily named. We assume that the load curve will maintain its shape (and therefore its system load factor) into the planning year. Using the identical load curve shape for a projected system demand of 7000 GWh/yr results in the load curve of Figure 8.16. If there are N existing plants (including those under construction), and M types of additional capacity that could be built to meet the demand, then

$$\sum_{i=1}^{N+M} x_i(p) = d(p) \quad ; \; p = 1, \ldots 6 \tag{8.76}$$

where $x_i(p)$ is the generation at plant i that contributes to the p-th demand block, and where the d(p) are given by Figure 8.16, and for which

$$\sum_{i=1}^{6} d(p) = 7000 \text{ GWh} \quad .$$

This constraint will ensure that sufficient power is generated to meet the anticipated demand.

Generation at each plant, however, may not exceed its capacity. For existing plants

$$\sum_{p=1}^{6} \underset{\left[\frac{MW}{GW}\right]}{1000} \cdot \underset{[\;\;]}{\frac{1}{Z_p}} \cdot \underset{\left[\frac{yr}{hr}\right]}{\frac{1}{a_{pi}}} \cdot \underset{\left[\frac{GWh}{yr}\right]}{x_i(p)} \le \underset{[MW]}{X_i} \quad : \; i=1, \ldots N \tag{8.77}$$

where

Z_p is the number of hours in the p-th block,

X_i is the installed capacity

a_{pi} is the availability of the i-th plant in the p-th block.

For example, assume that for technical reasons the new 66 MW Hussein Thermal Power Station units would only be dispatched into the base load portions of the load curve (p=1,2). Thus, applying (8.77) yields:

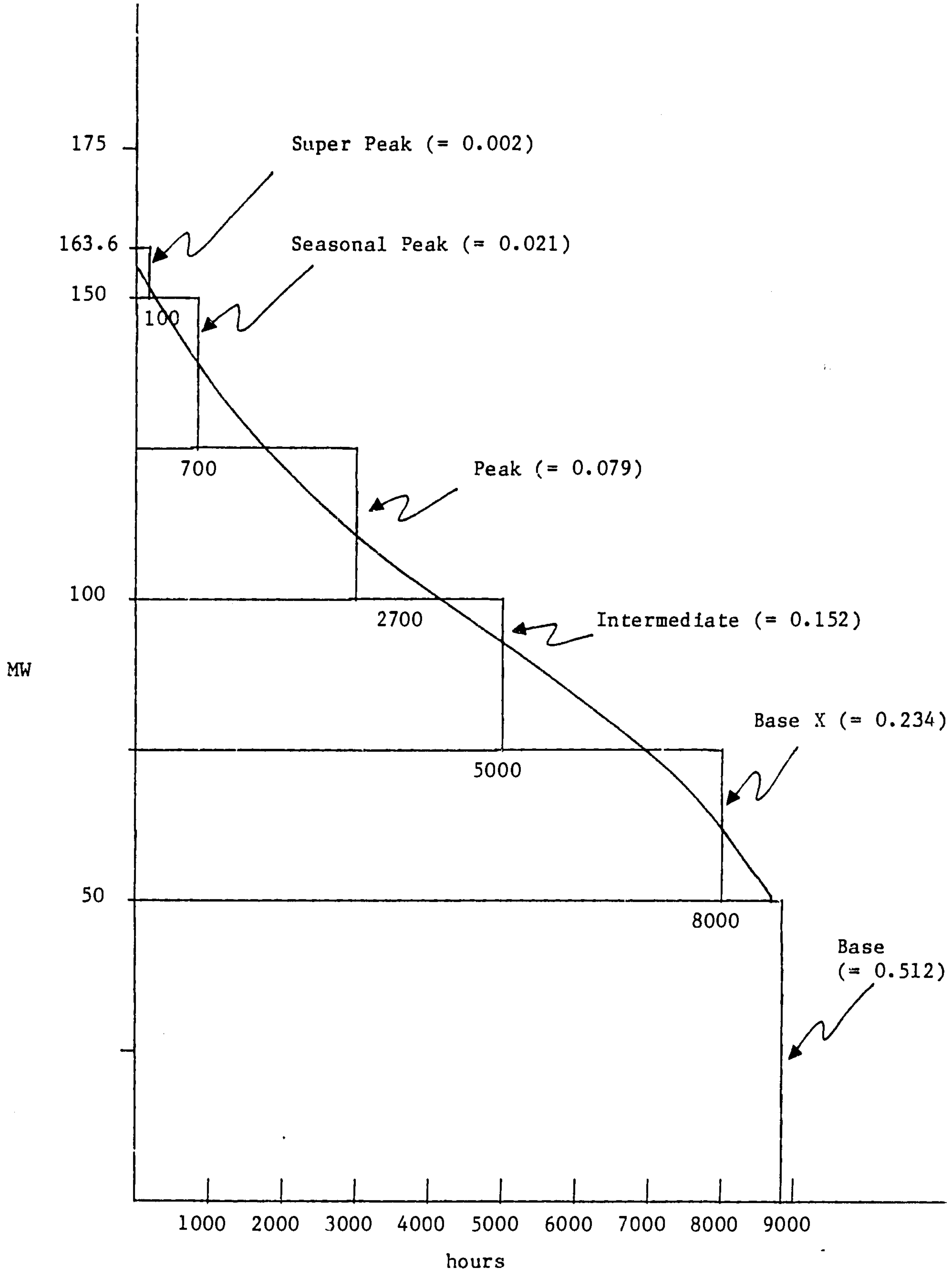


Figure 8.15: 1980 System Load Curve (from 1980 Annual Report of the Jordanian Electricity Authority)

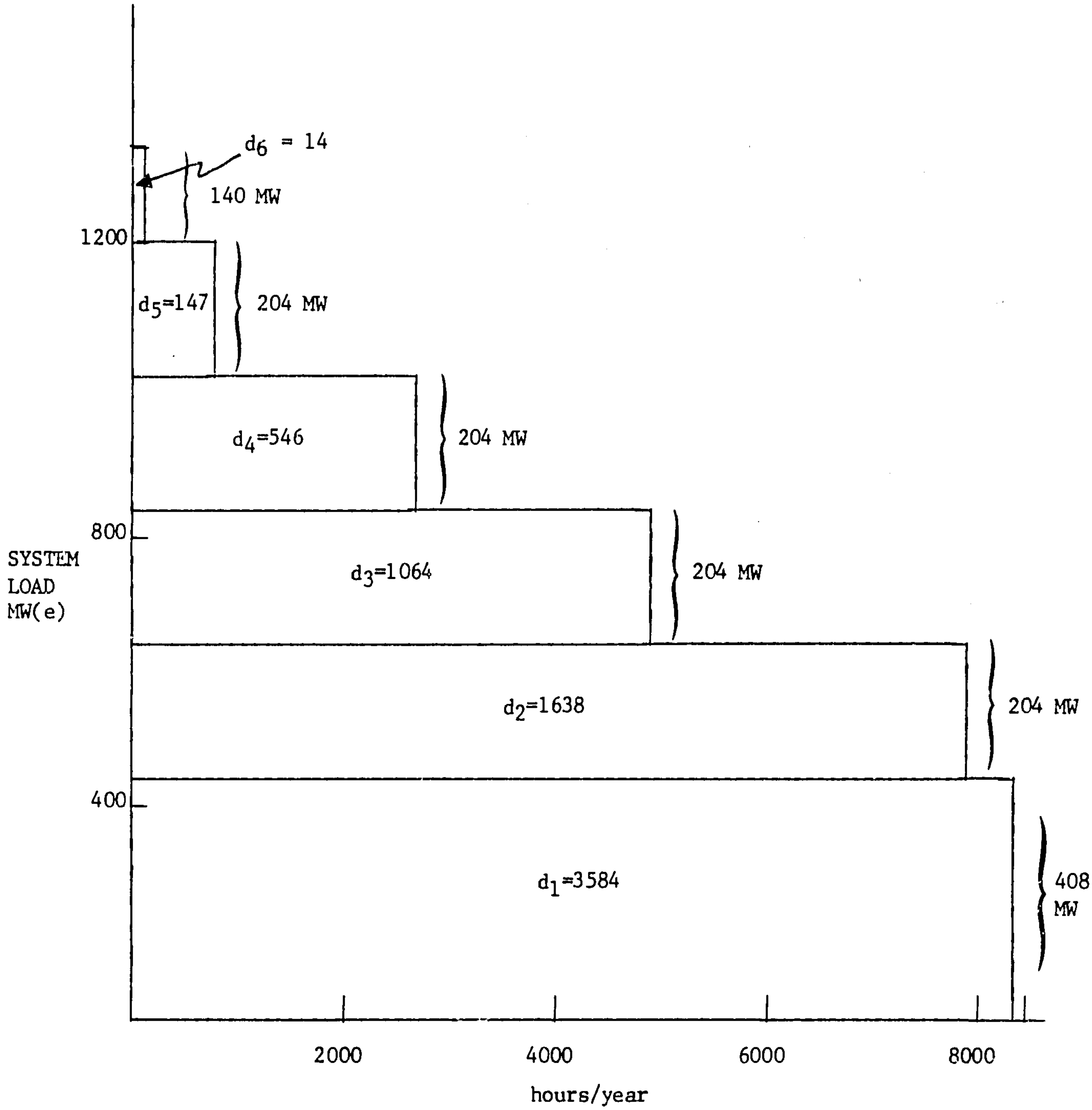


Figure 8.16: System Annual Load Duration Curve
for 7000 GWh Demand

$$1000 \cdot \frac{1}{0.65} \cdot \frac{1}{8760} \cdot x(1) + 1000 \frac{1}{0.711} \frac{1}{8000} (2) \leq 198 \quad .$$

There is one such constraint for each existing facility. For $p = 1$, $a_{pi} = 0.65$, which corresponds to the assumed annual plant factor (PF). In actuality, a_{pi} represents the probability that the unit will not be available to meet a given load curve portion. Thus for the peak segments, a_{pi} is a function of the forced outage rate (FOR);

$$a_{pi} = (1 - \text{FOR}) \quad .$$

For intermediate segments, a_{pi} will lie in the range

$$(1 - \text{FOR}) < a_{pi} < \text{PF}$$

and only for $p = 1$ does a_{pi} equal the PF. The rule used here for the intermediate segments is

$$a_{pi} = \text{PF} \cdot \frac{8760}{Z_p}$$

hence

$$a_{21} = 0.65 \frac{8760}{8000} = 0.711$$

which explains the second term of the above capacity constraint.

For each new generation type we have an analogous capacity constraint

$$\sum_{p=1}^{6} 1000 \frac{1}{Z_p} \cdot \frac{1}{a_{pj}} \quad {}_j(p) - X_j(N) \leq \quad ; \; j = 1, \ldots M \tag{8.78}$$

where $X_j(N)$ is the MW of additional capacity of type j added by the model.

For each fuel j we have a fuel balance equation of the type

$$\sum_{i \in F_j} \sum_{p=1}^{6} \underset{\left[\frac{\text{GWh}}{\text{yr}}\right]}{x_i(p)} \quad \underset{\left[\frac{\text{kWh}}{\text{GWh}}\right]}{10^6} \quad \underset{\left[\frac{\text{Btu}}{\text{kWh}}\right]}{\beta_i} \quad \underset{\left[\frac{10^6\text{Btu}}{\text{Btu}}\right]}{\frac{1}{10^6}} \quad - \underset{[10^6\text{Btu}]}{F_j} = 0 \tag{8.79}$$

where

β_i is the heat rate of the i-th plant

F_j is the total fuel consumption of type j

$i \in F_j$ is the set of plants consuming fuel j.

Unfortunately no information on plant specific heat rates was available: for purposes of this illustrative analysis we therefore made the following assumptions:

	Existing	New
Oil-Steam	11000	10500
Coal-Steam	-	10000
Diesel	13700	13700
Shale Oil Steam	-	10500

Finally, the objective function is the minimization of annual cost, namely

$$\text{Min } S = \sum F_j \cdot \lambda_j + \sum_i X_i(N) \cdot \pi_i \cdot CRF(r,N) \tag{8.80}$$

where λ_j is the fuel cost, in $/10^6 Btu, π_i is the capital cost of capacity of type i in $/MW, and CRF(r,N) is the appropriate capital recovery factor for discount rate r over N years. The following assumptions were made for the cost coefficients:

Fuel Costs		
Coal	85 $/ton,	3.86 $/10^6 Btu
Diesel	43 $/bbl,	7.11 $/10^6 Btu
Heavy Fuel Oil	26 $/bbl,	4.32 $/10^6 Btu

Capital Cost, $/kW	
Coal	450
Oil	325
Shale Oil	350
Diesel	120

The cost of coal in Jordan is based on a 1981 quotation for Austrialian coal delivered to the Israeli port of Eilat.

The solution is shown on Figure 8.17. New coal capacity serves to meet the base load portion of the load, whilst the existing Hussein Thermal Power Station, assumed to be less efficient, is relegated to intermediate duty. The seasonal peak is mostly met by new diesel capacity, whilst the existing Diesels serve only to meet the extreme peak. This is of course an anticipated result, predictable on the basis of heat rates (the worse the heat rate, the fewer hours per year will such a unit be dispatched).

Indeed, the assumptions made concerning heat rates will affect the solution. To illustrate the sort of sensitivity analysis that would be conducted in an actual planning study, suppose that heat rates for new diesel capacity

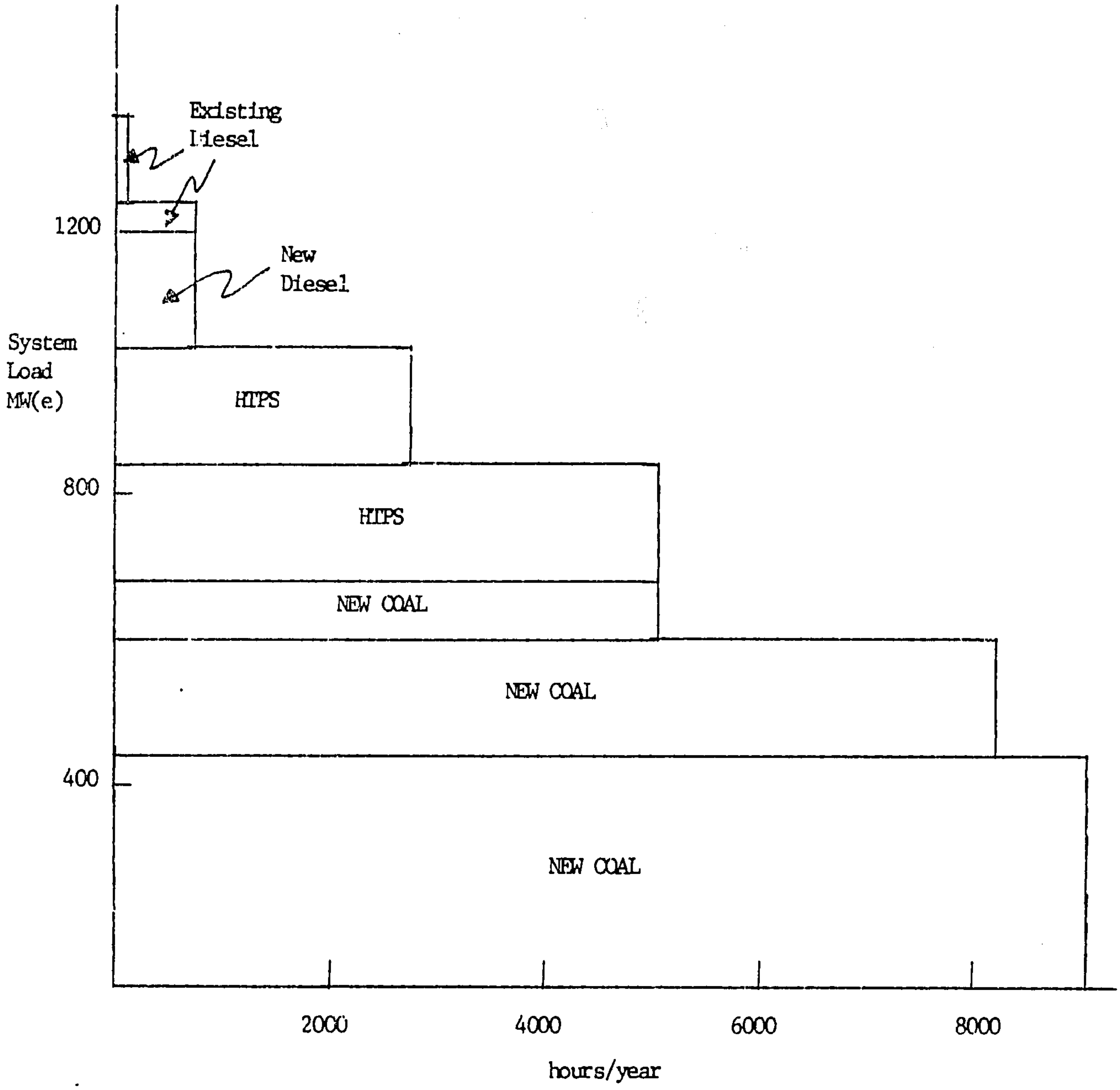


Figure 8.17: Optimal Solution

could be improved from 13700 Btu/kWh to 12330 Btu/kWh, a 10% improvement. Such an improvement is quite consistent with the increasing use of low speed marine diesel technology in electric utility applications. As indicated on Figure 8.18, the fuel savings associated with such increased efficiency justify the capital expediture of new diesel capacity, and retiring the old units; all of the peak portions of the load curve are satisfied by new diesel capacity, with an improvement in objective function value of some $3 million per year.

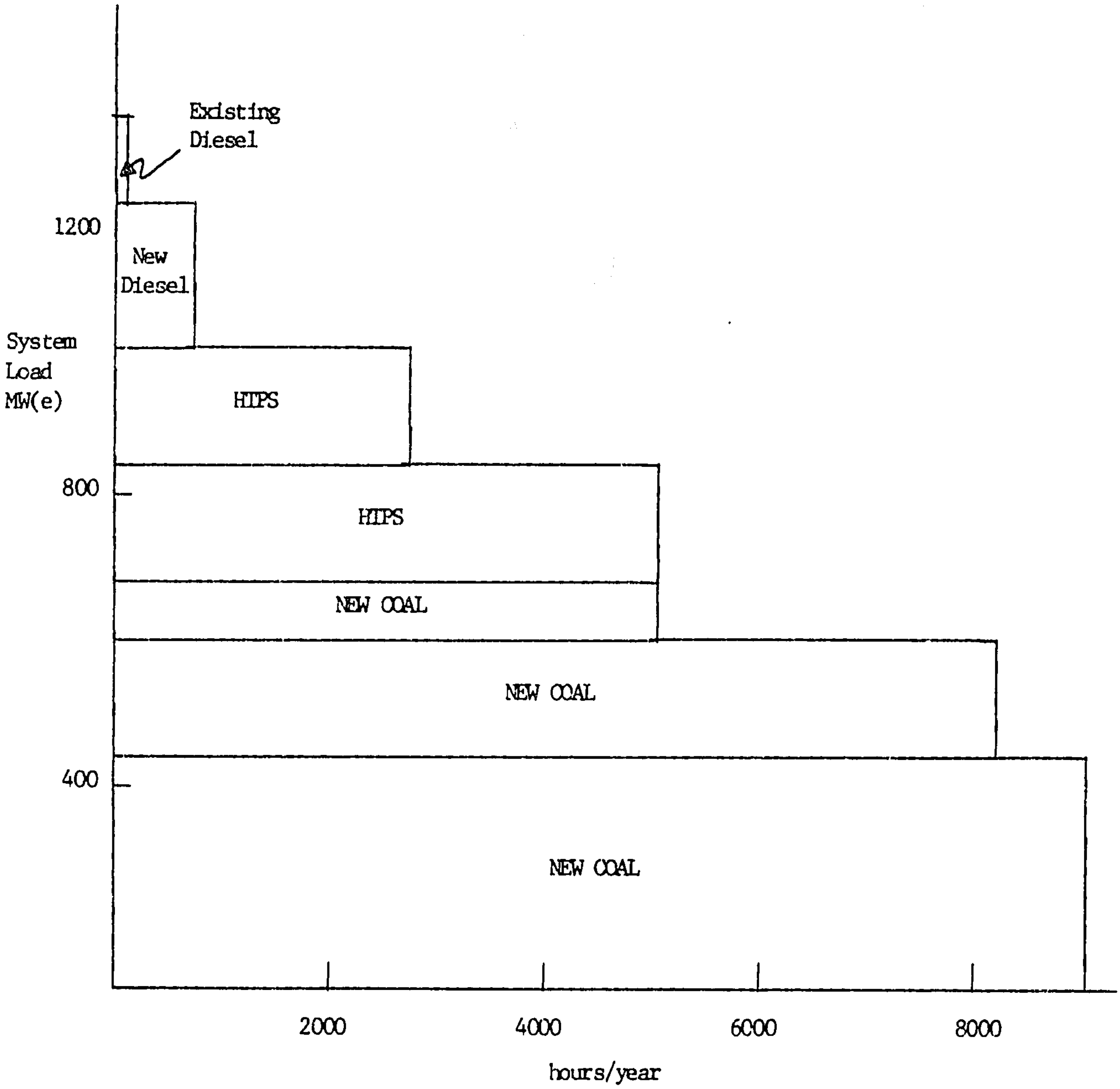
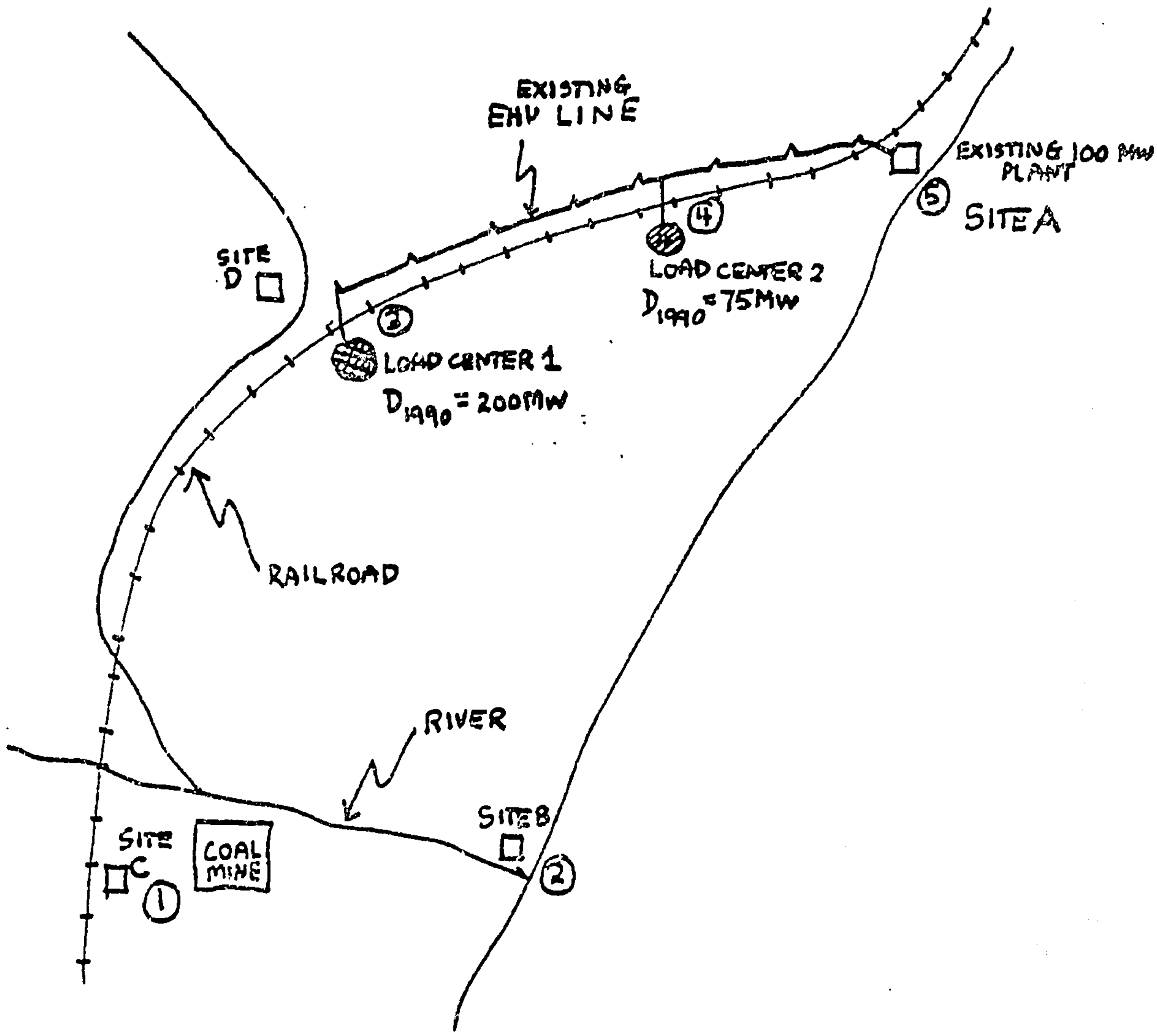


Figure 8.18: Solution Under Improved Diesel Heat Rates

E16. CAPACITY RETIREMENTS

Suppose, in example 8.1, that the system had been built at a time when residual oil was \$5/bbl, with distillate price of 110¢/10^6 Btu, and residual price of 105¢/Btu. What was the optimal mix of the original system? At what residual oil price does it become economic to retire the oil steam electric plant and replace it by a nuclear plant?

Consider the following geographical situation



A new site must be chosen for the necessary 175 MW to meet the 1990 load. Formulate the siting problem as an LP.

BACKGROUND READING

S. Lahiri "Investment Planning for the Power Industry in Northern India: A Spatial Programming Approach" Indian Economic Review, Vol XII (New Series), April 1977, p. 43-72.

Analyses the investment planning problem for the Northern Region of India as a spatial programming problem; based on the techniques described in section 8.3. Comes to some important conclusions regarding the trade-offs between mine-mouth and load center siting, and the distortions introduced by the tariff policy of the railraods.

D. Anderson "Optimum Development of the Electric Power Sector in Turkey: A Case Study Using Linear Programming," World Bank, February 1972.

One of the first comprehensive investment planning studies for the electric sector using a linear programming formulation of the capacity expansion problems. Based on the technique described in Section 8.1

D. Anderson "Models for determining Least Cost Investments in Electric Supply" Bell Journal of Economics and Management Science, Vol 3, No. 1, p. 267-300, Spring 1972.

A comprehensive survey of the literature of capacity expansion models. Although some 9 years old at the time of writing, it remains an important first source for any serious student of this topic.

APPENDIX

COMPUTER PROGRAM OUTPUT

The APEX-III linear programming package (the LP package used on CDC hardware) produces three output files: an equation listing, a listing of constraints (indicating shadow prices associated with each constraint at the optimal solution), and a listing of the optimal vlaues of the variables. The shadow prices are listed under the "MARGINAL" column of the constraint listing. The output reproduced here is for the base case computation of the text (with heat rate, of 13700 Btu/kWh) for new diesel capacity.

GLOSSARY OF CONSTRAINTS

Constraint Name	Text Equation	Explanation
ECAPExxxx	(8.76)	Capacity constraint, existing facility xxxx.
ECAPNkkkk	(8.78)	Capacity constraint, new capacity of type k.
EDEMpp	(8.77)	Demand constraint for the p-th load curve segment.
EFUELzzzz	(8.79)	Fuel consumption equation for fuel zzzz.
EANCST	(8.80)	Objective function.

GLOSSARY OF VARIABLES

Variable Name	Text Notation	Explanation
ECAPVkkkk	X_k	MW capacity additions of type k.
EOPppkkkk	$_k(p)$	GWh generated in new plants of type k dispatched into the p-th segment.
EOPppxxxx	$_i(p)$	GWh generated in existing plant x dispated into segment p.
ETOTzzzz	j	Consumption of fuel z.

```
MAP NAME =                    **********   E Q U A T I O N   L I S T I N G   **********

                              KEY TO COLUMN TYPES ( SEE SELECTION FOR EFFECTIVE VECTORS )

                    X : FIXED VARIABLE        L : PLUS VARIABLE WITH BOUND(S)        B : BIVALENT VARIABLE
                    P : PLUS VARIABLE         R : MINUS VARIABLE WITH BOUND(S)       I : INTEGER VARIABLE
                    M : MINUS VARIABLE        1 : VARIABLE IS SOS-1 SET MEMBER
                    F : FREE VARIABLE         2 : VARIABLE IS SOS-2 SET MEMBER

ECAPE1D-MA     LE        1    .571000000 P EOPBA1D-MA    .625000000 P EOPBX1D-MA   1.000000000 P EOPIN1D-MA   1.852000000 P EOPPK1D-MA
  RHS LO:   -INF             1.786000000 P EOPSP1D-MA   12.50000000 P EOPSS1D-MA
  RHS UP:    40.00000000

ECAPE2D-MA     LE        2    .571000000 P EOPBA2D-MA    .625000000 P EOPBX2D-MA   1.000000000 P EOPIN2D-MA   1.852000000 P EOPPK2D-MA
  RHS LO:   -INF             1.786000000 P EOPSP2D-MA   12.50000000 P EOPSS2D-MA
  RHS UP:    45.00000000

ECAPE1S-HT     LE        3    .176000000 P EOPBA1S-HT    .192000000 P EOPBX1S-HT    .222000000 P EOPIN1S-HT    .412000000 P EOPPK1S-HT
  RHS LO:   -INF
  RHS UP:    99.00000000

ECAPE2D-HT     LE        4    .571000000 P EOPBA2D-HT    .625000000 P EOPBX2D-HT   1.000000000 P EOPIN2D-HT   1.852000000 P EOPPK2D-HT
  RHS LO:   -INF             1.786000000 P EOPSP2D-HT   12.50000000 P EOPSS2D-HT
  RHS UP:    32.00000000

ECAPE1D-AQ     LE        5    .571000000 P EOPBA1D-AQ    .625000000 P EOPBX1D-AQ   1.000000000 P EOPIN1D-AQ   1.852000000 P EOPPK1D-AQ
  RHS LO:   -INF             1.786000000 P EOPSP1D-AQ   12.50000000 P EOPSS1D-AQ
  RHS UP:    5.800000000

ECAPE2D-AQ     LE        6    .571000000 P EOPBA2D-AQ    .625000000 P EOPBX2D-AQ   1.000000000 P EOPIN2D-AQ   1.852000000 P EOPPK2D-AQ
  RHS LO:   -INF             1.786000000 P EOPSP2D-AQ   12.50000000 P EOPSS2D-AQ
  RHS UP:    7.000000000

ECAPE1D-KA     LE        7    .571000000 P EOPBA1D-KA    .625000000 P EOPBX1D-KA   1.000000000 P EOPIN1D-KA   1.852000000 P EOPPK1D-KA
  RHS LO:   -INF             1.786000000 P EOPSP1D-KA   12.50000000 P EOPSS1D-KA
  RHS UP:    4.500000000

ECAPEMAAN      LE        8    .571000000 P EOPBAMAAN     .625000000 P EOPBXMAAN    1.000000000 P EOPINMAAN    1.852000000 P EOPPKMAAN
  RHS LO:   -INF             1.786000000 P EOPSPMAAN    12.50000000 P EOPSSMAAN
  RHS UP:    1.870000000

ECAPETAFIL     LE        9    .571000000 P EOPBATAFIL    .625000000 P EOPBXTAFIL   1.000000000 P EOPINTAFIL   1.852000000 P EOPPKTAFIL
  RHS LO:   -INF             1.786000000 P EOPSPTAFIL   12.50000000 P EOPSSTAFIL
  RHS UP:    1.300000000

ECAPE3S-HT     LE       10    .176000000 P EOPBA3S-HT    .192000000 P EOPBX3S-HT    .222000000 P EOPIN3S-HT    .412000000 P EOPPK3S-HT
  RHS LO:   -INF
  RHS UP:    198.0000000

ECAPE3D-MA     LE       11    .571000000 P EOPBA3D-MA    .625000000 P EOPBX3D-MA   1.000000000 P EOPIN3D-MA   1.852000000 P EOPPK3D-MA
  RHS LO:   -INF             1.786000000 P EOPSP3D-MA   12.50000000 P EOPSS3D-MA
  RHS UP:    40.00000000

ECAPE3D-AQ     LE       12    .571000000 P EOPBA3D-AQ    .625000000 P EOPBX3D-AQ   1.000000000 P EOPIN3D-AQ   1.852000000 P EOPPK3D-AQ
  RHS LO:   -INF             1.786000000 P EOPSP3D-AQ   12.50000000 P EOPSS3D-AQ
  RHS UP:    15.00000000

ECAPE2D-KA     LE       13    .571000000 P EOPBA2D-KA    .625000000 P EOPBX2D-KA   1.000000000 P EOPIN2D-KA   1.852000000 P EOPPK2D-KA
  RHS LO:   -INF             1.786000000 P EOPSP2D-KA   12.50000000 P EOPSS2D-KA
  RHS UP:    20.00000000
```

```
ECAPPAS-HT     LE      14     .170000000 P EOP...            ...
  RHS LO:  -INF
  RHS UP:   66.00000000

ECAPNCOAL      LE      15  -1.000000000 P ECAPVCOAL      .176000000 P EOPBACOAL      .192000000 P EOPBXCOAL      .222000000 P EOPINCOAL
  RHS LO:  -INF
  RHS UP:   0.000000000

ECAPNS-OIL     LE      16  -1.000000000 P ECAPVS-OIL     .176000000 P EOPBAS-OIL     .192000000 P EOPBXS-OIL     .227000000 P EOPINS-OIL
  RHS LO:  -INF               .421000000 P EOPPKS-OIL    1.623000000 P EOPSPS-OIL   11.36400000 P EOPSSS-OIL
  RHS UP:   0.000000000

ECAPNOILTH     LE      17  -1.000000000 P ECAPVOILTH     .163000000 P EOPBAOILTH     .179000000 P EOPBXOILTH     .217000000 P EOPINOILTH
  RHS LO:  -INF
  RHS UP:   0.000000000

ECAPNDIESE     LE      18  -1.000000000 P ECAPVDIESE     .571000000 P EOPBADIESE     .625000000 P EOPBXDIESE    1.000000000 P EOPINDIESE
  RHS LO:  -INF              1.852000000 P EOPPKDIESE   1.786000000 P EOPSPDIESE   12.50000000 P EOPSSDIESE
  RHS UP:   0.000000000

ENETPV         FR      19    .450000000 P ECAPVCOAL      .350000000 P ECAPVS-OIL     .330000000 P ECAPVOILTH     .120000000 P ECAPVDIESE
  RHS LO:  -INF
  RHS UP:  +INF

ECAPRO         FR      20    .450000000 P ECAPVCOAL      .350000000 P ECAPVS-OIL     .330000000 P ECAPVOILTH     .120000000 P ECAPVDIESE
  RHS LO:  -INF
  RHS UP:  +INF

EFORGC         FR      21    .200000000 P ECAPVCOAL      .130000000 P ECAPVS-OIL     .130000000 P ECAPVOILTH     .040000000 P ECAPVDIESE
  RHS LO:  -INF
  RHS UP:  +INF

EANCST         FR      22    .060000000 P ECAPVCOAL      .040000000 P ECAPVS-OIL     .040000000 P ECAPVOILTH     .010000000 P ECAPVDIESE
  RHS LO:  -INF              3.864000000 P ETOTCOAL     7.110000000 P ETOTDIES      4.329000000 P ETOTRESD      6.803000000 P ETOTSOIL
  RHS UP:  +INF

RESERV         GE      23   1.000000000 P ECAPVCOAL     1.000000000 P ECAPVS-OIL    1.000000000 P ECAPVOILTH    1.000000000 P ECAPVDIESE
  RHS LO:   1654.690000
  RHS UP:  +INF

EDEMBA         EQ      24   1.000000000 P EOPBACOAL     1.000000000 P EOPBAS-OIL    1.000000000 P EOPBAOILTH    1.000000000 P EOPBADIESE
  RHS LO:   3584.000000     1.000000000 P EOPBA1D-MA    1.000000000 P EOPBA2D-MA    1.000000000 P EOPBA1S-HT    1.000000000 P EOPBA2D-HT
  RHS UP:   3584.000000     1.000000000 P EOPBA1D-AQ    1.000000000 P EOPBA2D-AQ    1.000000000 P EOPBA1D-KA    1.000000000 P EOPBAMAAN
                            1.000000000 P EOPBATAFIL    1.000000000 P EOPBA3S-HT    1.000000000 P EOPBA3D-MA    1.000000000 P EOPBA3D-AQ
                            1.000000000 P EOPBA2D-KA    1.000000000 P EOPBA4S-HT

EDEMBX         EQ      25   1.000000000 P EOPBXCOAL     1.000000000 P EOPBXS-OIL    1.000000000 P EOPBXOILTH    1.000000000 P EOPBXDIESE
  RHS LO:   1638.000000     1.000000000 P EOPBX1D-MA    1.000000000 P EOPBX2D-MA    1.000000000 P EOPBX1S-HT    1.000000000 P EOPBX2D-HT
  RHS UP:   1638.000000     1.000000000 P EOPBX1D-AQ    1.000000000 P EOPBX2D-AQ    1.000000000 P EOPBX1D-KA    1.000000000 P EOPBXMAAN
                            1.000000000 P EOPBXTAFIL    1.000000000 P EOPBX3S-HT    1.000000000 P EOPBX3D-MA    1.000000000 P EOPBX3D-AQ
                            1.000000000 P EOPBX2D-KA    1.000000000 P EOPBX4S-HT

EDEMIN         EQ      26   1.000000000 P EOPINCOAL     1.000000000 P EOPINS-OIL    1.000000000 P EOPINOILTH    1.000000000 P EOPINDIESE
  RHS LO:   1064.000000     1.000000000 P EOPIN1D-MA    1.000000000 P EOPIN2D-MA    1.000000000 P EOPIN1S-HT    1.000000000 P EOPIN2D-HT
  RHS UP:   1064.000000     1.000000000 P EOPIN1D-AQ    1.000000000 P EOPIN2D-AQ    1.000000000 P EOPIN1D-KA    1.000000000 P EOPINMAAN
                            1.000000000 P EOPINTAFIL    1.000000000 P EOPIN3S-HT    1.000000000 P EOPIN3D-MA    1.000000000 P EOPIN3D-AQ
                            1.000000000 P EOPIN2D-KA    1.000000000 P EOPIN4S-HT

EDEMPK         EQ      27   1.000000000 P EOPPKS-OIL    1.000000000 P EOPPKDIESE    1.000000000 P EOPPK1D-MA    1.000000000 P EOPPK2D-MA
  RHS LO:   546.0000000     1.000000000 P EOPPK1S-HT    1.000000000 P EOPPK2D-HT    1.000000000 P EOPPK1D-AQ    1.000000000 P EOPPK2D-AQ
  RHS UP:   546.0000000     1.000000000 P EOPPK1D-KA    1.000000000 P EOPPKMAAN     1.000000000 P EOPPKTAFIL    1.000000000 P EOPPK3S-HT
                            1.000000000 P EOPPK3D-MA    1.000000000 P EOPPK3D-AQ    1.000000000 P EOPPK2D-KA    1.000000000 P EOPPK4S-HT

EDEMSP         EQ      28   1.000000000 P EOPSPS-OIL    1.000000000 P EOPSPDIESE    1.000000000 P EOPSP1D-MA    1.000000000 P EOPSP2D-MA
  RHS LO:   147.0000000     1.000000000 P EOPSP2D-HT    1.000000000 P EOPSP1D-AQ    1.000000000 P EOPSP2D-AQ    1.000000000 P EOPSP1D-KA
  RHS UP:   147.0000000     1.000000000 P EOPSPMAAN     1.000000000 P EOPSPTAFIL    1.000000000 P EOPSP3D-MA    1.000000000 P EOPSP3D-AQ
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EOPMSS        EQ      29   1.000000000 P EOPSSS-OIL   1.000000000 P EOPSSDIESE   1.000000000 P EOPSS1D-MA   1.000000000 P EOPSS2D-MA
  RHS LO:   14.000000000   1.000000000 P EOPSS2D-HT   1.000000000 P EOPSS1D-AQ   1.000000000 P EOPSS2D-AQ   1.000000000 P EOPSS1D-KA
  RHS UP:   14.000000000   1.000000000 P EOPSSMAAN    1.000000000 P EOPSSTAFIL   1.000000000 P EOPSS3D-MA   1.000000000 P EOPSS3D-AQ
                           1.000000000 P EOPSS2D-KA

EFUELPCOAL    EQ      30    .010000000 P EOPPACOAL     .010000000 P EOPPXCOAL     .010000000 P EOPINCOAL   -1.000000000 P ETOTCOAL
  RHS LO:    0.000000000
  RHS UP:    0.000000000

EFUELBDIES    EQ      31    .014000000 P EOPPADIESE    .014000000 P EOPPXDIESE    .014000000 P EOPINDIESE    .014000000 P EOPPKDIESE
  RHS LO:    0.000000000    .014000000 P EOPSPDIESE    .014000000 P EOPSSDIESE    .014000000 P EOPPA1D-MA    .014000000 P EOPPX1D-MA
  RHS UP:    0.000000000    .014000000 P EOPIN1D-MA    .014000000 P EOPPK1D-MA    .014000000 P EOPSP1D-MA    .014000000 P EOPSS1D-MA
                            .014000000 P EOPPA2D-MA    .014000000 P EOPPX2D-MA    .014000000 P EOPIN2D-MA    .014000000 P EOPPK2D-MA
                            .014000000 P EOPSP2D-MA    .014000000 P EOPSS2D-MA    .014000000 P EOPPA2D-HT    .014000000 P EOPPX2D-HT
                            .014000000 P EOPIN2D-HT    .014000000 P EOPPK2D-HT    .014000000 P EOPSP2D-HT    .014000000 P EOPSS2D-HT
                            .014000000 P EOPPA1D-AQ    .014000000 P EOPPX1D-AQ    .014000000 P EOPIN1D-AQ    .014000000 P EOPPK1D-AQ
                            .014000000 P EOPSP1D-AQ    .014000000 P EOPSS1D-AQ    .014000000 P EOPPA2D-AQ    .014000000 P EOPPX2D-AQ
                            .014000000 P EOPIN2D-AQ    .014000000 P EOPPK2D-AQ    .014000000 P EOPSP2D-AQ    .014000000 P EOPSS2D-AQ
                            .014000000 P EOPPA1D-KA    .014000000 P EOPPX1D-KA    .014000000 P EOPIN1D-KA    .014000000 P EOPPK1D-KA
                            .014000000 P EOPSP1D-KA    .014000000 P EOPSS1D-KA    .014000000 P EOPPAMAAN     .014000000 P EOPPXMAAN
                            .014000000 P EOPINMAAN     .014000000 P EOPPKMAAN     .014000000 P EOPSPMAAN     .014000000 P EOPSSMAAN
                            .014000000 P EOPPATAFIL    .014000000 P EOPPXTAFIL    .014000000 P EOPINTAFIL    .014000000 P EOPPKTAFIL
                            .014000000 P EOPSPTAFIL    .014000000 P EOPSSTAFIL    .014000000 P EOPPA3D-MA    .014000000 P EOPPX3D-MA
                            .014000000 P EOPIN3D-MA    .014000000 P EOPPK3D-MA    .014000000 P EOPSP3D-MA    .014000000 P EOPSS3D-MA
                            .014000000 P EOPPA3D-AQ    .014000000 P EOPPX3D-AQ    .014000000 P EOPIN3D-AQ    .014000000 P EOPPK3D-AQ
                            .014000000 P EOPSP3D-AQ    .014000000 P EOPSS3D-AQ    .014000000 P EOPPA2D-KA    .014000000 P EOPPX2D-KA
                            .014000000 P EOPIN2D-KA    .014000000 P EOPPK2D-KA    .014000000 P EOPSP2D-KA    .014000000 P EOPSS2D-KA
                          -1.000000000 P ETOTDIES

EFUELBRESD    EQ      32    .011000000 P EOPPAOILTH    .011000000 P EOPPXOILTH    .011000000 P EOPINOILTH    .011000000 P EOPPA1S-HT
  RHS LO:    0.000000000    .011000000 P EOPPX1S-HT    .011000000 P EOPIN1S-HT    .011000000 P EOPPK1S-HT    .011000000 P EOPPA3S-HT
  RHS UP:    0.000000000    .011000000 P EOPPX3S-HT    .011000000 P EOPIN3S-HT    .011000000 P EOPPK3S-HT    .011000000 P EOPPA4S-HT
                            .011000000 P EOPPX4S-HT    .011000000 P EOPIN4S-HT    .011000000 P EOPPK4S-HT  -1.000000000 P ETOTRESD

EFUELPSOIL    EQ      33    .010000000 P EOPPAS-OIL    .010000000 P EOPPXS-OIL    .010000000 P EOPINS-OIL    .010000000 P EOPPKS-OIL
  RHS LO:    0.000000000    .010000000 P EOPSPS-OIL    .010000000 P EOPSSS-OIL  -1.000000000 P ETOTSOIL
  RHS UP:    0.000000000

TOTANCSE      ER      34    .060000000 P ECAPVCOAL     .040000000 P ECAPVS-OIL    .040000000 P ECAPVOILTH    .010000000 P ECAPVDIESE
  RHS LO:   -INF
  RHS UP:   +INF

TOTCAPRG      ER      35    .450000000 P ECAPVCOAL     .350000000 P ECAPVS-OIL    .350000000 P ECAPVOILTH    .120000000 P ECAPVDIESE
  RHS LO:   -INF
  RHS UP:   +INF

TOTFORSC      ER      36    .200000000 P ECAPVCOAL     .130000000 P ECAPVS-OIL    .130000000 P ECAPVOILTH    .090000000 P ECAPVDIESE
  RHS LO:   -INF
  RHS UP:   +INF

        END EQUATION LISTING
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DATE 01/11/82 TIME 14.58.40 C O N S T R A I N T S APEX-III 1.100 PAGE 1

PRINT OPTION = PARTIAL MAP NAME = SPECIAL/POS = NO VALUE OF OBJECTIVE = 359.21934
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DIR = MINIMIZE COBJ = CRHS = RNG = RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	ROW ACTIVITY	SLACK	RHS LOWER	RHS UPPER	MARGINAL
1	ECAPE1D-MA	LE	** BINDING	40.00000	.	-INF	40.00000	.
2	ECAPE2D-MA	LE	** BINDING	45.00000	.	-INF	45.00000	.
3	ECAPE1S-HT	LE	BINDING	99.00000	.	-INF	99.00000	.00955
4	ECAPE2D-HT	LE	** BINDING	32.00000	.	-INF	32.00000	.
5	ECAPE1D-AQ	LE	** BINDING	5.80000	.	-INF	5.80000	.
6	ECAPE2D-AQ	LE	** BINDING	7.00000	.	-INF	7.00000	.
7	ECAPE1D-KA	LE	** BINDING	4.50000	.	-INF	4.50000	.
8	ECAPEMAAN	LE	** BINDING	1.87000	.	-INF	1.87000	.
9	ECAPETAFIL	LE	** BINDING	1.30000	.	-INF	1.30000	.
10	ECAPE3S-HT	LE	BINDING	198.00000	.	-INF	198.00000	.00955
11	ECAPE3D-MA	LE	** BINDING	40.00000	.	-INF	40.00000	.
12	ECAPE3D-AQ	LE	** BINDING	15.00000	.	-INF	15.00000	.
13	ECAPE2D-KA	LE	** BINDING	20.00000	.	-INF	20.00000	.
14	ECAPE4S-HT	LE	BINDING	66.00000	.	-INF	66.00000	.00955
15	ECAPNCOAL	LE	BINDING	.	.	-INF	.	.05000
16	ECAPNS-OIL	LE	BINDING	.	.	-INF	.	.03000
17	ECAPNOILTH	LE	BINDING	.	.	-INF	.	.03900
18	ECAPNDIESE	LE	SLACK	-386.17800	386.17800	-INF	.	.
19	ENETPV	FR	SLACK	542.89800	-542.89800	-INF	+INF	.
20	ECAPRO	FR	SLACK	542.89800	-542.89800	-INF	+INF	.
21	EFORGC	FR	SLACK	233.13800	-233.13800	-INF	+INF	.
22	EANCST	FR	SLACK	359.21934	-359.21934	-INF	+INF	.
23	RESERV	GE	BINDING	1654.69000	.	1654.69000	+INF	-.01000
24	EDEMBA	EQ	BINDING	3584.00000	.	3584.00000	3584.00000	-.04744
25	EDEMMX	EQ	BINDING	1638.00000	.	1638.00000	1638.00000	-.04824
26	EDEMIN	EQ	BINDING	1064.00000	.	1064.00000	1064.00000	-.04974
27	EDEMPK	EQ	BINDING	546.00000	.	546.00000	546.00000	-.05156
28	EDEMSP	EQ	BINDING	147.00000	.	147.00000	147.00000	-.09954
29	EDEMSS	EQ	BINDING	14.00000	.	14.00000	14.00000	-.09954
30	EFUELBCOAL	EQ	BINDING	.	.	.	.	3.86400
31	EFUELBDIES	EQ	BINDING	.	.	.	.	7.11000
32	EFUELBRESD	EQ	BINDING	.	.	.	.	4.32300
33	EFUELBSOIL	EQ	BINDING	.	.	.	.	6.80300
34	TOTANCST	FR	SLACK	68.71890	-68.71890	-INF	+INF	.
35	TOTCAPRO	FR	SLACK	542.89800	-542.89800	-INF	+INF	.
36	TOTFORGC	FR	SLACK	233.13800	-233.13800	-INF	+INF	.

DATE 01/11/82 TIME 14.[illegible] C O L U M N S APEX-III 1.100 PAGE 1

PRINT OPTION = PARTIAL MAP NAME = SPECIALZPDS = NO VALUE OF OBJECTIVE = 259.21734
NAME = EDCLP OBJ = EANCST RHS = RHS1 BND = RESOBJ = 1.0000 RESRHS = 1.0000
DIR = MINIMIZE COBJ = CRHS = RNG = RPCHOBJ = 0.0000 RPCHRHS = 0.0000

NUMBER	NAME	TYPE	STATUS	COL ACTIVITY	OBJ COEF	BND LOWER	BND UPPER	MARGINAL
1	ECAPVCOAL	PL	ACTIVE	1043.44000	.06000	.	+INF	.
2	ECAPVS-OIL	PL	ACTIVE	.	.04000	.	+INF	.
3	ECAPVOILTH	PL	ACTIVE	.	.04000	.	+INF	.
4	ECAPVDIESE	PL	ACTIVE	611.25000	.01000	.	+INF	.
5	EOPBACOAL	PL	ACTIVE	3554.00000	.	.	+INF	.
6	EOPBXCOAL	PL	ACTIVE	1656.00000	.	.	+INF	.
7	EOPINCOAL	PL	ACTIVE	442.16216	.	.	+INF	.
21	EOPSPDIESE	PL	ACTIVE	126.02016	.	.	+INF	.
27	EOPSP1D-MA	PL	ACTIVE	2.33463	.	.	+INF	.
29	EOPSS1D-MA	PL	ACTIVE	2.86640	.	.	+INF	.
34	EOPSS2D-MA	PL	ACTIVE	3.60000	.	.	+INF	.
38	EOPPK1S-HT	PL	ACTIVE	240.29126	.	.	+INF	.
43	EOPSP2D-HT	PL	ACTIVE	17.91713	.	.	+INF	.
50	EOPSS1D-AO	PL	ACTIVE	.46400	.	.	+INF	.
56	EOPSS2D-AO	PL	ACTIVE	.56000	.	.	+INF	.
62	EOPSS1D-KA	PL	ACTIVE	.36000	.	.	+INF	.
69	EOPSSMAAN	PL	ACTIVE	.14960	.	.	+INF	.
73	EOPSPTAFIL	PL	ACTIVE	.72798	.	.	+INF	.
77	EOPIN3S-HT	PL	ACTIVE	324.54054	.	.	+INF	.
78	EOPPK3S-HT	PL	ACTIVE	305.70874	.	.	+INF	.
84	EOPSS3D-MA	PL	ACTIVE	3.20000	.	.	+INF	.
90	EOPSS3D-AO	PL	ACTIVE	1.20000	.	.	+INF	.
96	EOPSS2D-KA	PL	ACTIVE	1.60000	.	.	+INF	.
99	EOPIN4S-HT	PL	ACTIVE	297.29730	.	.	+INF	.
101	ETOTCOAL	PL	ACTIVE	56.64162	3.86400	.	+INF	.
102	ETOTDIES	PL	ACTIVE	2.25400	7.11000	.	+INF	.
103	ETOTRESD	PL	ACTIVE	12.84622	4.32900	.	+INF	.
104	ETOTSOIL	PL	ACTIVE	.	6.80300	.	+INF	.

9. SECTORAL MODELS: PETROLEUM REFINING

9.1 TECHNICAL ASPECTS OF REFINERIES[1]

Given the remarkable lack of refinery sector detail in many energy models it would seem that a brief discussion of the basic technical aspects of refineries and their operations is necessary before we discuss the approaches to modelling refinery operation and capacity expansion.[2]

Crude Oil: Crude oil is a mixture of thousands of different hydrocarbon compounds as well as compounds containing sulfur, nitrogen, and various metals. The proportions of the compounds comprising a given crude oil determine its properties. It should come as no surprise, then, that almost no two crude oils are alike:[3] Table 9.1 lists the most frequently encounted crude oils traded in the International Market.

The specific gravity or density (measured as mass per unit volume) of crude petroleum is a rough indication of its composition. API gravity, expressed in degrees API, is mathematically related to specific gravity in such a way that it usually indicates the relative gasoline and kerosene contents of the crude petroleum. Light crudes with API gravity between 30 and 50 generally contain high levels of gasoline and kerosene, while heavy crudes, which have lower numerical API gravity than light crudes, contain low levels of these products and large amount of asphalt and residuum.

Sulfur compounds which exist in crude oil are corrosive to handling and processing equipment in the refinery. A very low sulfur level in the feedstock to catalytic reforming, cracking, and hydrocracking is critical for these refining processes to perform normally. These processes rely on catalysts to convert a large portion of crude oil into valuable high-octane gasoline. However, the expensive catalysts can be "poisoned" by coming in contact with sulfur which makes them less efficient and sometimes totally useless in performing as originally designed. In order that these processes function properly to meet demand for high-octane gasoline, the refiner must desulfurize feedstocks containing high amounts of sulfur. Other important reasons for desulfurization are that sulfur in gasoline has an offensive odor, and that it can destroy engine parts in automobiles. Finally, fuels should be low in sulfur from the standpoint of environmental considerations.

The heavy metals that make up ash in crude oils tend to build up in the heavier streams such as the residuum and heavy gas oils. When these must be processed further, metals such as vanadium, nickel, and iron can poison catalysts with the same effects as sulfur. In some cases heavy metal poisoning is irreversible, while sulfur poisoning can be remedied by simply burning off the sulfur.

The most important constituents of crude oil are hydrocarbons. Hydrocarbons by definition are organic compounds consisting entirely of atoms of

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[1]This discussion is taken, in some parts verbatim, from F. Sebulsky (presented in the 1978 EMTP) and J. D'Acierno and A. Hermelee "Physical Aspects of the U.S. Oil and Gas Systems" BNL 51076, Brookhaven National Laboratory, November 1979.

[2]As usual, though, this section can be omitted by those already familiar with the rudiments of refinery operation.

Table 9.1
World Crude Oil Exports Streams

Most Commong Designation of Crude Stream	Producing Country	Gravity °API	Sulfur Wt%	Pour Point	Shipping Point	Operators, Participants or Producers §
Amna	Libya	36.1	0.15	+ 75° F	Ras Lanuf, Libya (SBM)	Mobil/Gelsen./LNOC
Anguille*	Gabon	32.0	0.74	+ 3° C		Elf
Arabian heavy*	Saudi Arabia	28.2	2.84	- 30° F	Ras Tanura, Saudi Arabia	Aramco/SaudiArabia
Arabian light-Berri	Saudi Arabia	38.8	1.10	- 30° F	Ras Tanura, Saudi Arabia	Aramco/Saudi Arabia
Arabian light	Saudi Arabia	33.4	1.80	- 30° F	Ras Tanura, Saudi Arabia Juaymah, Saudi Arabia (SBM) Sidon, Lebannon	Aramco/Saudi Arabia
Arabian medium	Saudia Arabia	30.8	2.40	+ 5°F	Ras Tanura, Saudi Arabia	Aramco/Saudi Arabia
Arabian medium-Zuluf*	Saudia Arabia	20.7	2.51	- 40° F	Zuluf field (SBM)	Aramco/Saudi Arabia
Arjuna*	Indonesia, Java	37.7	0.12	+ 80° F	Arjuna field (SBM)	ARCO
Arzew blend	Algeria	44.3	0.10	- 21° C	Arzew, Algeria	Sonatrach
Attaka*	Indonesia, East Kalimantan	43.2	0.07	- 30° F	Santan Term., E. Kali	Union Oil
Bachequero, 16.8°	Venezuela	16.8	2.40	- 10° F	La Salina, Venezuela Bachequero, Ven. (13° API) Puerto Miranda, Ven.	Venezuela
Bai Hanssan Jambur	Iraq	34.1	2.40	- 18° C	Tripoli, Lebanon	INOC
Bu Attifel	Libya	40.6	0.10	+ 39° C	Zueitina, Libya	AGIP/LNOC
Barsrah	Iraq	33.9	2.08	+ 15° C	K. r al Amaya, Iraq (SBM)	INOC
Bekapai*	Indonesia, East	41.1	0.08	-32.5°C	Field (SBM)	Total
Beryl*†	U.K.	39.5	0.36	<-65° F	Beryl field (SBM)	Mobil et al
Bonny light	Nigeria	37.6	0.13	+ 36° F	Bonny, Nigeria (SBM)	BP/Shell
Bonny Medium	Nigeria	26.0	0.23	<- 5° F	Bonny, Nigeria (SBM)	BP/Shell
Boscan	Venezuela	10.3	5.4	+ 50° F	Bajo Grande, Ven.	Venezeuela
Brass River	Nigeria	43.0	0.08	- 5° F	Mouth of Brass (SBM)	AGIP/Phillips/NNOC
Brega	Libya	40.4	0.21	+ 30° F	Marsa el Brega, Libya (SBM)	Esso Libya/LNOC
Bunju	Indonesia, E.	32.2	0.08	+ 17° F	Balikpapan, E. Kali	Pertamina
Burgan (Wafra)	Neutral Zone	23.3	3.37	- 5° F	Mina Saud, Neutral	Getty
Cabinda*	Angola (Cabinda)	32.9	0.15	+ 65° F	Molongo field (SBM)	Gulf
Cinta*	Indonesia, Sumatra	32.0	0.08	+ 95° F	Field (SBM)	Iiapco
Cyrus*	Iran	19.0	3.48	- 10° F	Field (SBM)	Amoco/NIOC
Darius*	Iran	33.9	2.45	0° F	Kharg Island, Iran	Amoco/NIOC
Dubai*	Dubai	32.5	1.68	+ 5° F	Field (SBM)	Dubai Pet. Co.
Duri	Indonesia, Sumatra	20.6	0.21	+ 57° F	Dumai, Sumatra	PT. Caltex
Ecuador crude Oriente)	Ecuador	30.4	0.67	+ 20° F	Puerto Balao/Exmeraldas, Ecua.	Gulf/Texaco
Ekhabinskaya	U.S.S.R.	30.7	0.37	-17.5°F	Okha, Sakhalin, U.S.S.R.	U.S.S.R.
Ekofisk*	Norway	35.8	0.18	+ 15° F	North Tees, U.K.	Phillips
El Bunduq*	Abu Dhabi	38.5	1.12	- 10° C	————	
Emeraude	Congo(Brazzaville)	23.6	0.50	- 36° C	Djeno, Congo (SBM)	Elf
Eocene	Neutral Zone	18.6	4.55	- 20° F	Mina Saud, N.Z.	Getty

Table 9.1. cont.
World Crude Oil Exports Streams

Most Common Designation of Crude Stream	Producing Country	Gravity °API	Sulfur Wt%	Pour Point	Shipping Point	Operators, Participants or Producers §
Escravos*	Nigeria	36.2	0.16	+ 50° F	Escravos River, Nigeria (SBM)	Gulf
Es Sider	Libya	37.0	0.45	- 1° C	Sidra, Libya	Oasis
Fereidoon blend*	Iran	31.0	2.60	- 10° F	Kharg Island, Iran	Amoco/NIOC
Forcados blend	Nigeria	30.5	0.18	+ 5° F	Forcados, Nigeria (SBM)	BP/Shell
Forties*	U.K.	36.6	0.28	+ 30° F	Firth of Forth, U.K.	BP
Gamba	Gabon	31.8	0.11	+ 23° C	Gamba (SBM)	Shell
Gulf of Suez blend*	Egypt	31.5	1.40	+ 40° F	Ras Shukheir, Egypt	Amoco/E.G.P.C.
Handil*	Indonesia, E. Kalimantan	30.8	0.09	+ 35° C	Field (SBM)	Tota
Hassi Messaoud	Algeria	44.0	0.14	- 24° C	Bougie, Algeria	Sonatrach
Hout	Neutral Zone	34.1	1.7	0° F	Ras Khafi, N.Z.	Arabian Oil Co.
Iranian heavy	Iran	30.8	1.6	- 5° F	Kharg Island, Iran	NIOC & former IOP
Iranian light	Iran	33.5	1.4	- 20° F	Kharg Island, Iran	NIOC & former IOP
Isthmus (see Reforma)						
Jatibarang	Indonesia, Java	28.9	0.11	+110° F	SBM	Pertamina
Kerindingam*†	Indonesia, E. Kalimantan	21.6	0.30	< 10° F	Santan Term., E. Kali	Union Oil
Khafji	Neutral Zone	28.7	2.88	- 35° C	Ras Khafji, N.Z.	Arabian Oil Co.
Kirkuk	Iraq	35.9	1.95	- 36° C	Banias, Syria, Tripoli, Lebanon	INOC
Klamono	Indonesia, Irian Jaya	18.7	0.97	+ 40° C	————	Pertamina
Kuwait	Kuwait	31.2	2.50	0° F	Mina al Ahmadi, Kuwait	Kuwait/Gulf/BP
Labuan light* (Samarang)	Malasi, Sabah	36.0	0.07	+ 60° F	Labuan, Sabah (SBM)	Shell
Lagomedio	Venezuela	32.0	1.3	- 15° F	Puerta de Palmas, Venezuela	Venezuela
Mandji blend	Gabon	29.0	1.26		Cap Lopez, Gabon	ERAP
Melahin*	Indonesia, E. Kalimantan	24.7	0.27	< 10° F	Santan Term. E. Kali	Union Oil
Minas (Sumatran light)	Indonesia, Sumatra	35.2	0.09	+ 90° F	Dumai, Sumatra	P. T. Caltex
Montrose*†	U.K.	41.9	0.23	+ 20° F	————	Amoco et al
Mubarras*	Abu Dhabi	38.1	0.93	- 30° F	Field SBM	ADOC
Murban	Abu Dhabi	39.4	0.74	- 15° C	Jebel Dhanna, Abu Dhabi	ADPC
Ninian*†	U.K.	35.1	0.41	+ 45° F	————	Chevron, et al
North Rumalia	Iraq	34.3	1.98	- 19° C	Fao/Khor al Amaya, Iraq	INOC
North Slope†	U.S.A.	26.8	1.04	- 5° F	Valdez, Alaska	ARCO, Exxon, et al
Oman	Oman	34.7	0.97	- 24° C	Mina al Fahal, Oman	Pet Dev. Ltd. Oman
Pennington*	Nigeria	37.7	0.08	+ 37° F	Apoi (offshore)	Texaco/Chevron
Poleng*	Indonesia, Java	43.2	0.19	+ 15° F	Surabaja	P.T. Cities, Ashland
Piper*†	U.K.	.08467 (S.G.)	0.92	- 9° C	Kirkwall, Orkney Is.	Occidental et al

Table 9.1. cont.
World Crude Oil Exports Streams

Most Commong Designation of Crude Stream	Producing Country	Gravity °API	Sulfur Wt%	Pour Point	Shipping Point	Operators, Participants or Producers §
Qatar land (Dukhan)	Qatar	40.9	1.29	- 5° F	Umm Said, Qatar (SBM)	QPC
Qatar marine*	Qatar	37.0	1.50	+ 25° F	Halul Island, Qatar (SBM)	QPC
Qua Iboe*	Nigeria	37.4	0.106	+ 50° F	Qua Iboe, Nigeria (SBM)	Mobil
Ratawi	Neutral Zone	23.5	4.07	+ 15° F	Mina Saud, N.Z.	Getty
Reforma (Cactus Reforma)	Mexico	33.0	1.56	-5to10°F	Pajaritos, Mexico	Memex
Ramashkinskaya	U.S.S.R.	32.6	1.61	- 20° F	Ventspils (Baltic) Odessa, U.S.S.R	U.S.S.R.
Rostam*	Iran	35.9	1.55	-22.5°C	Lavan Island, Iran	Iminico/NIOC
Sarir	Libya	36.5	0.14	+ 26° C	Marsa el Hariga, Libya	AGEC
Sassan*	Iran	33.9	1.91	- 5° F	Lavan Island, Iran	Lavan Pet. Co.
Sepinggan*†	Indonesia, E. Kalimantan	37.9	0.10	+ 15° F	Lawi-Lawi Term, E. Kali.	Union Oil
Seria light*	Brunei	38.8	0.05	+ 60° F	Field SBM	Shell
Statfjord*†	Norway	38.2	0.27	+ 20° F	————	Mobil/Statoil
Sirip blend 27.1° API	Iran	27.1	2.45	- 33° C	Ras Bahrgan, Iran (SBM)	Sirip/NIOC
Taching	China (PRC)	33.0	0.04	95° F	Dairen, China	P.R.C.
Tarakan (Pamusian)	Indonesia, E. Kalimantan	19.5	0.14	- 45° F	Tarakan Island	Tesoro
Tembungo*	Malaysia, Sabah	37.4	0.04	+ 25° F	Field (SBM)	Exxon
Thistle*†	U. K.	37.4	0.31	+ 40° F	————	Conoco/Burmah et al
Trinidad blend	Trinidad	33.6	0.23	+ 57° F	Point Galeota, Trinidad (SBM)	Amoco
Tyumen	U.S.S.R.	34.0	0.97	- 20° C	————	U.S.S.R.
Umm Shaif*	Abu Dhabi	37.6	1.38	+ 5° F	Das Island, Abu	Abu Dhabi Marine
Walio Export Mix	Indonesia, W. Irian	35.4	0.68	+ 20° F	Kasim Term., W. Irian	Petromer Trend
Zakum*	Abu Dhabi	40.1	0.98	+ 5° F	Das Island, Abu Dhabi	ADMA
Zarzaitine	Algeria	42.0	0.08	- 9° C	La Skhirra, Tunisia	Sonatrach
Zueitina	Libya	39.6	0.23	+ 55° F.	Zueitina, Liby (SBM)	Occidental/LNOC

*Wholly or partially offshore † Production not stabilized, values could change
†Field not in productin. Development expected
§Participation or ownership in foreign crude-oil production is in a state of flux in many areas and agreements are often too complext to describe in detail in this limited space. For more information and clarification of abbreviations see 1975 and forthcoming 1976 editions of the International Petroleum Encyclopedia published by the Oil & Gas Journals' parent company, Petroleum Publishing Co., Tulsa, Okla.

carbon and atoms of hydrogen. Figure 9.1 shows some examples of the types of hydrocarbons found in crude oil and in refined petroelum products.

One of the important characteristics of the element carbon is that each atom of carbon can form chemical bonds with four other atoms. This property allows carbon to link up with itself to form simple or complex chains or even rings. The simplest kind of hydrocarbon occuring in crude oil is a straight-chain paraffin. The straight-chain might be anywhere from two carbon atoms in length to as high as thirty or more carbon atoms. Straight-chain paraffins in the gasoline boiling range have very low octane number and are undesirable components of gasoline. However, straight-chain paraffins in the diesel range have very high cetane numbers and in general are desirable components of diesel and furnace oils. Regularity of the structure of straight-chain paraffins allows them to crystallize readily, so crude oils and products containing large amounts of straight-chain paraffins tend to have very high pour points and are waxy. On the other hand, branched-chain paraffins have high octanes and lower cetane numbers. Consequently, branched-chain paraffins are very desirable components of gasoline. Their irregular structure makes it more difficult for them to crystallize.

Aromatic hydrocarbons have the carbon atom arranged in six-membered rings. They contain a deficiency of hydrogen which is represented by alternate double bonds between carbon atoms. In general, aromatics contribute very high octane numbers to gasoline and very low cetane numbers to diesel fuel. They have poor burning characteristics because of the deficiency of hydrogen and in general have very poor lubricating properties.

The carbon atoms are also arranged in rings in cycloparaffins, or naphthenes as they are commonly called in the petroleum industry. Both five-member and six-member rings saturated are found in petroleum. In contrast to the aromatics, naphthene rings are saturated with hydrogen. They burn more cleanly than aromatics do and tend to be intermediate in octane and cetane values. Naphthene rings contribute very low pour points and can be desirable components of lubricating oils.

Olefins are the final type of hydrocarbon to be discussed here. Olefins do not occur naturally in petroleum but are formed from the hydrocarbons in petroleum by some refining processes. In general, olefins contribute very high octane numbers, very low cetane number and have poor burning characteristics.

Figure 9.2 shows critical properties of four different crude oils from around the world: Merey Heavy Crude from Venezuela; Arabian Light which is known worldwide as a marker crude; Taching, a crude oil from China; and a North Sea Crude from the Forties Field of the United Kingdom. The bar charts indicate the relative amounts of light and heavy products that can be made from these crudes. The Merey Crude is a low gravity, high sulfur material with very high contents of nickel and vanadium. Note that it only has about 5% by volume of material boiling in the gasoline range, and nearly half of the Merey Crude will remain as a residue even under vacuum distillation. It has a fairly low pour point and, in areas of the world where sulfur pollution is not a serious problem, would be an attractive crude oil for making heavy

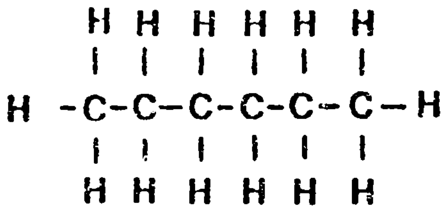
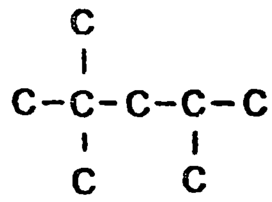
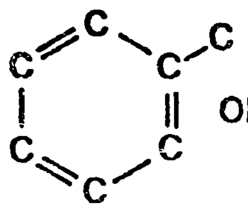
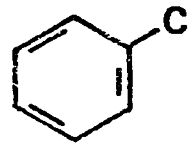
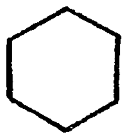
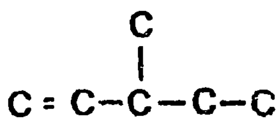
HYDROCARBONS	EXAMPLES	EFFECT ON PRODUCT PROPERTIES
PARAFFINS (STRAIGHT CHAIN)		VERY LOW OCTANE NUMBERS VERY HIGH CETANE NUMBERS HIGH POUR POINTS - WAX CLEAN BURNING
PARAFFINS (BRANCHES)		HIGH OCTANE NUMBERS LOWER CETANE NUMBERS
AROMATICS	 OR 	VERY HIGH OCTANE VERY LOW CETANE POOR BURNING CHARACTERISTICS - SMOKY POOR LUBRICATING PROPERTIES
CYCLOPARAFFINS OR "NAPHTHENES"	 	INTERMEDIATE OCTANE, CETANE LOW POUR POINT
OLEFINS (NOT IN PETROLEUM NATURALLY)		HIGH OCTANE POOR CETANE POOR BURNING CHARACTERISTICS

Figure 9.1. Hydrocarbons

	MEREY	ARABIAN LIGHT	TACHING	FORTIES (U.K.)
GRAVITY, API (SP.GR.)	16.0 (0.959)	33.4 (0.858)	33.0 (0.860)	36.6 (0.842)
SULFUR, WT %	2.59	1.80	0.08	0.28
POUR POINT, F (°C)	0 (−18)	−30 (−34)	+95 (+33)	30 (−1)
NI, PPM	59	3	~0	1
V, PPM	247	13	~0	3

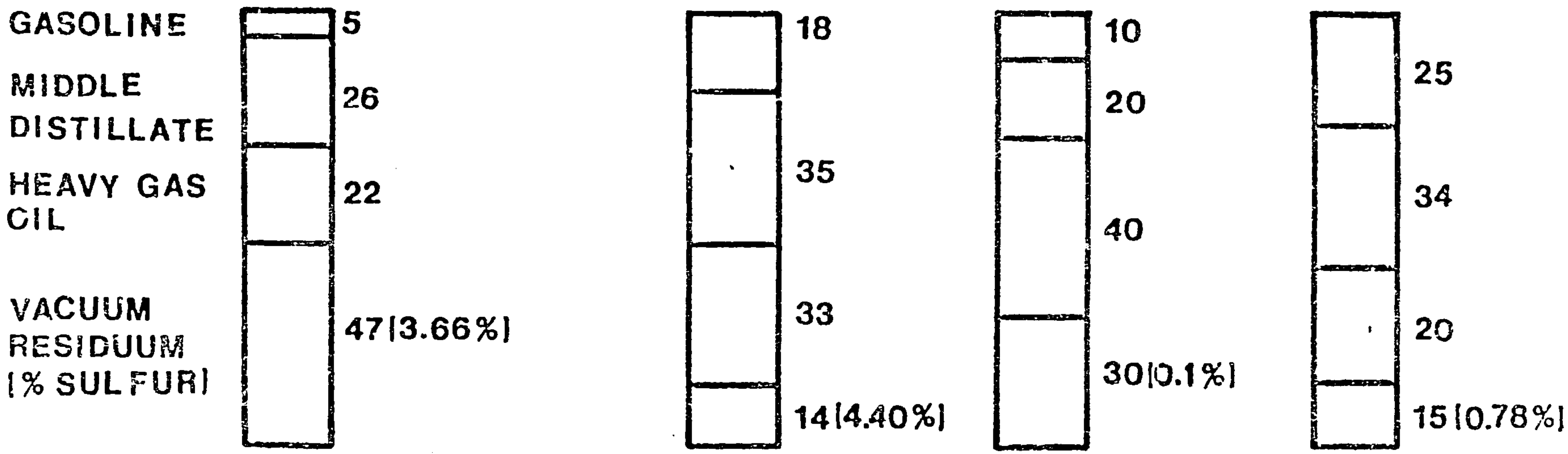


Figure 9.2. Four World Crude Oils

industrial fuel. However, it would not be an attractive crude oil to meet the light product demands of the United States or western Europe.

Arabian Light is a far more attractive crude for these markets. It contains 18% gasoline, has an attractively low pour point, and contains only 14% vacuum tower bottoms. These bottoms, however, are high sulfur and would require desulfurization or dilution with lower sulfur material before they could be burned in many areas of the world.

The Taching field in China produces a very interesting crude. It is very low in sulfur and reasonably light in its product distribution. Note, however, its very high pour point. This crude will still be solid on a hot summer day. Most of the heavier products from this crude would be so waxy that they could not meet most pour point specifications. However, the low metals and the high wax content to this crude would make it very attractive as a feedstock to a refinery that had appropriately designed conversion facilities. It would be expected that the heavier fractions in Taching would be very readily converted to the more desirable gasoline and middle distillate products in a refinery that was appropriately designed to handle it.

The Forties Crude probably presents fewer problems to a refiner than any of the other crudes shown. It is attractively light, containing 25% gasoline and only 15% vacuum tower bottoms; it is very low in sulfur, although not so low as the Taching; and has a moderately low pour point, though not so low as the Arabian Light. The vacuum residual fraction contains only about 0.8% sulfur and would generally be acceptable as a low sulfur industrial fuel without further conversion.

Refinery Products: There is an enormous diversity of petroleum products made to individual specifications: Table 9.2 shows the products made by petroleum refineries and petrochemical plants in the U.S. in some 17 basic classes of refined products.

In general, the products which dictate refinery design are relatively few in number, and the basic refinery processes are based on production of the large-quantity products such as gasoline, and middle distillates. Storage and waste disposal are expensive, and it is necessary to sell or use all of the items produced from crude oil even if some of the materials must be sold at prices lower than the cost of crude oil. This will have some relevance to the economics not only of the refining operation itself, but to product pricing policy, especially where the industry is nationalized.

Refineries: The basic refinery problem, then, is to convert crude oil, or a blend of crude oils, to a mix of refined products. Three basic types of processes occur in refineries: Separation, primarily by fractional distillation, Conversion, in which hydrocarbon structure is altered, and Treating, especially for sulfur removal. A simple refinery may consist only of a distillation tower plus a few simple treatment processes (Figure 9.3); at the other extreme complex refineries, such as that depicted on Figure 9.4), may contain numerous major conversion processes and extensive blending and storage facilities to meet the wide range of product specifications. Appendix B lists the refineries in the developing countries of Africa, Latin America and Asia: even in many small refineries catalytic reforming, and hydrotreating are frequently encountered.

Table 9.2
Products Made by the U.S. Petroleum Industry

Class	No. of Products in Class
Fuel gas	1
Liquefied gas	13
Gasolines	40
Motor	19
Aviation	9
Other (tractor, marine, etc.)	12
Gas turbine (jet) fuels	5
Kerosenes	10
Distillates (diesel fuels and light fuel oils)	27
Residual fuel oils	16
Lubricating oils	1,156
White oils	100
Rust preventitives	65
Transformer and cable oils	12
Greases	271
Waxes	113
Asphalts	209
Cokes	4
Carbon blacks	5
Chemicals, solvents, miscellaneous	300
Total	2,347

Source: <u>Petroelum Refining Technology and Economics</u>, Gary and Handwerk, Marcel Dekker, NY, 1975.

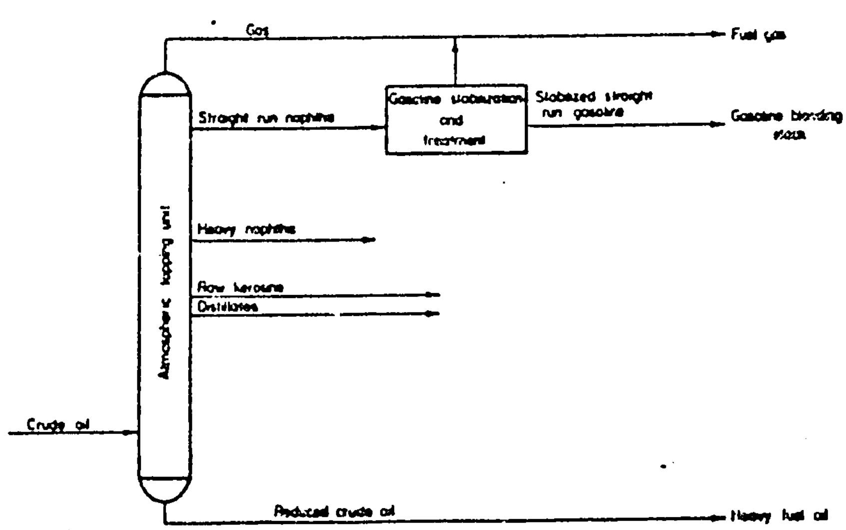


Figure 9.3. A Simple Refinery

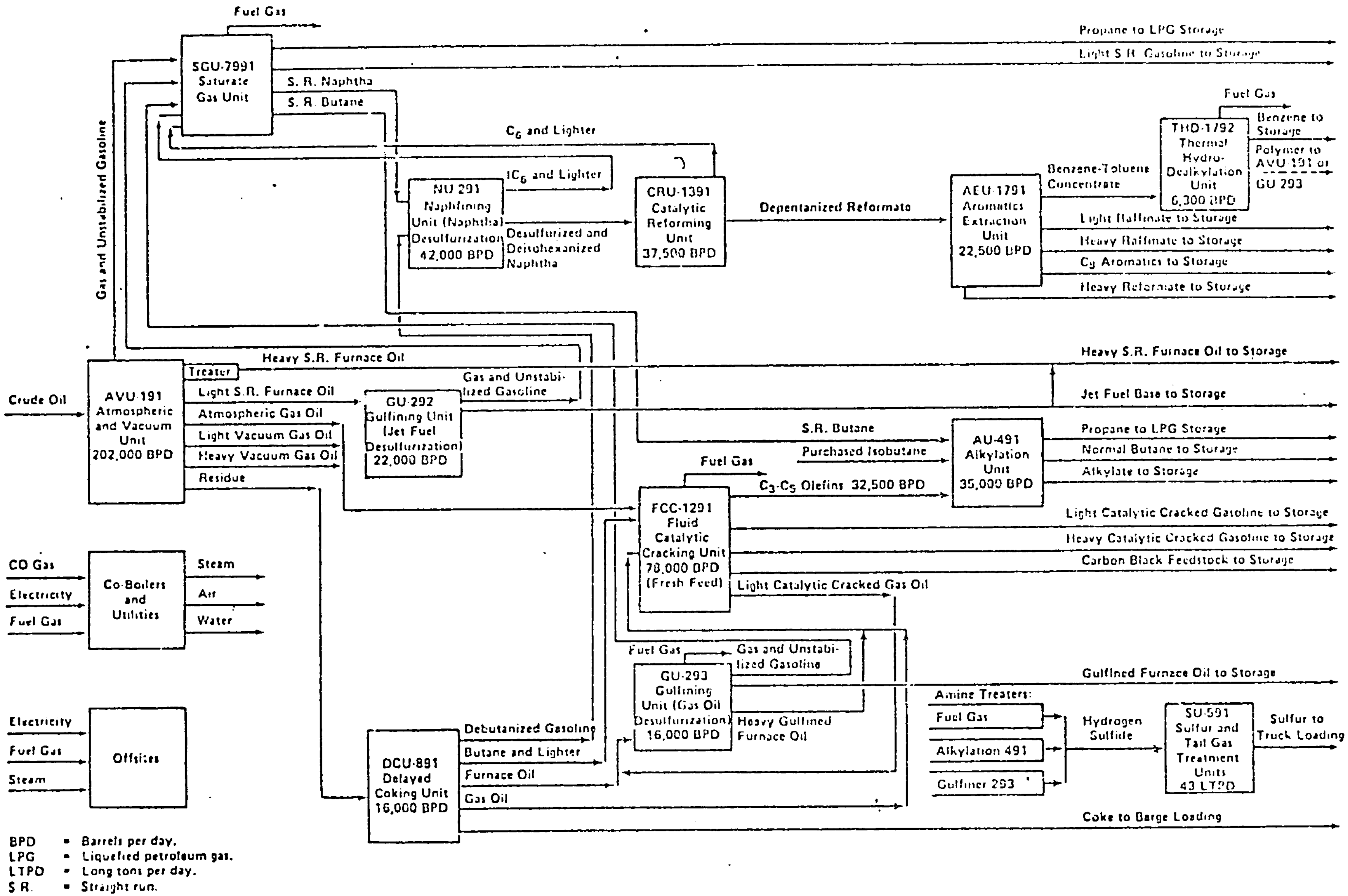


Figure 9.4. Alliance Refinery Flow Diagram
(from Gulf, *op. cit*, Figure 2.2)

Crude Distillation: Distillation is the single most important process used in a refinery. In distillation, the hydrocarbons in petroleum are separated according to their boiling points. Those hydrocarbons with low boiling points are evaporated from the mixture first, condensed and collected. As the distillation proceeds, higher and higher boiling hydrocarbons are evaporated and recovered in a sequential fashion.

Figure 9.5 shows a boiling point curve for Kuwait Crude. Temperature is plotted versus the volume distilled at that temperature. Gasoline, which comprises about 23 volume percent of this crude, comes off first. This is followed by kerosene, and a crude oil fraction known as gas oil, or more properly, atmospheric gas oil. About 50% of Kuwait crude oil, can be distilled overhead at atmospheric pressure. When the temperature gets over 650° F, however, some decomposition of the hydrocarbons begins to occur. In order to distill more hydrocarbons overhead, it is the usual practice to use vacuum distillation. This allows the actual temperature to be reduced, although by convention temperatures corrected to atmospheric pressure are used in plots like this. The next fraction to come over is called vacuum gas oil. This material is useful as industrial fuel; it is also the fraction from which lubricating oils are manufactured. In many refineries, the vacuum gas oil is cracked in some refinery process to make gasoline and kerosene, which have higher values in most markets. Hydrocarbons boiling much over 950° F generally cannot be distilled even under a vacuum; so what is left above 950° F is called vacuum residum. Its main use is as a heavy fuel oil, although some amount can be disposed of as asphalt or bitumen used for paving roads or waterproofing.

Vacuum Distillation: The heavier fractions of crude oil that make up topped crude cannot be further distilled by atmospheric distillation without the occurrence of thermal cracking (product breakdown at high temperatures). However, this material can be distilled under vacuum because the boiling temperature decreases with a lowering of pressure.[3]

There are six main products from a typical crude distillation unit: fuel gas, wet gas, virgin naphthas, gas oils, and residuum (in order of increasing boiling points). Fuel gas consists mainly of methane, and this stream is also referred to as dry gas. In some refineries propane may be used as a fuel gas. Wet gas contains propane and butanes as well as methane and ethane. Propane and butane are separated to be used for LPG separately as a petrochemical plant feedstock. Butane is also used for gasoline blending. Naphtha or heavy straight-run (HSR) gasoline is fed to the catalytic

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[3]Vacuum distillation towers separate topped crude into its components. Low pressure is achieved in the tower by a series of steam ejectors located on the vapor outlet stream at the top of the tower. To improve vaporization and enhance the overall performance, steam is mixed with the feed. The effect of steam is the lowering of actual pressure exerted on the hydrocarbons below the total vacuum pressure in the column. The basic principle of fractionation and overall operation is the same as for atmospheric distillation. Two major differences are the use of steam and larger column diameter. The lower operating pressures cause significant increases in volume of vapor per barrel of liquid vaporized and, as a result, the vacuum distillation columns are much larger in diameter than atmospheric towers.

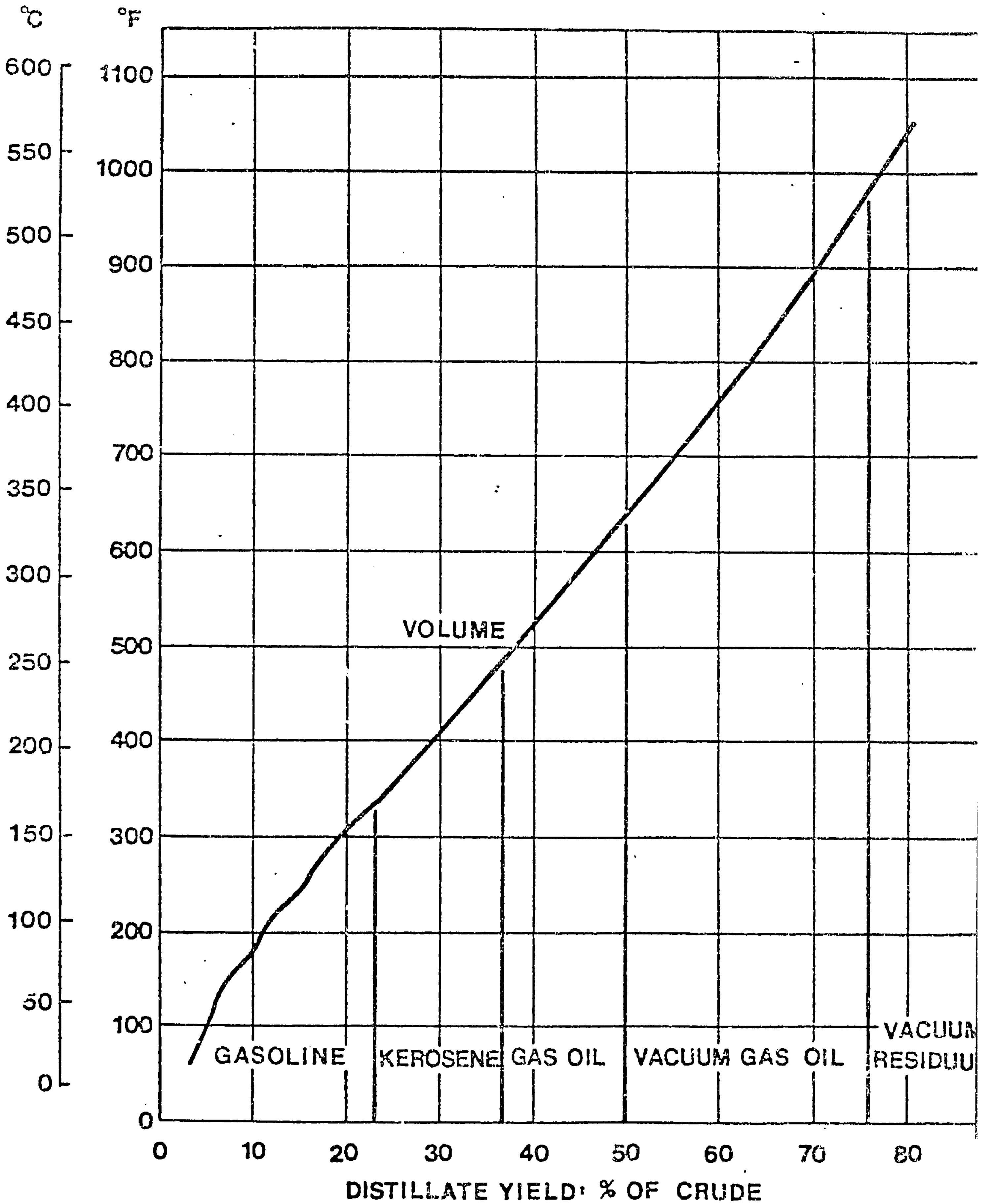

Figure 9.5. True Boiling Point Distillation Curve for 31.4 API Kuwait Crude Oil

reformer to prduce high-octane reformate for gasoline blending. Gas oils (light, atmospheric, and vacuum gas oils) are processed in a catalytic cracker or hydrocracker to produce gasoline, jet, and diesel fuels. Vacuum residuum is the still bottoms from vacuum distillation and can be processed into heavy fuel oil or cracking stocks. For asphaltic crudes, the residuum can be processed further to produce asphalts.

Catalytic Reforming: After distillation, probably the most widely used refining process is catalytic reforming of naphthas. Table 9.3 shows the Research octane number and the Motor octane number of three different naphthas. These octane numbers run from as low as about 37 for Kuwait to as high as 69 for Indonesian crude oil known as Handil. However, none of these naphthas is suitable to make specification gasoline as it is distilled from the crude oil. Catalytic reforming is used to increase the octane number of these naphthas so that they can be used as gasoline or as components in a gasoline pool.

Figure 9.6 shows the chemical reactions that take place in catalytic reforming to accomplish this goal. During catalytic reforming, some straight-chain paraffins (very poor in octane property) will be cyclized and converted to aromatics, while others are cracked to form lighter paraffins and are thereby removed from the gasoline pool.[4]

Obviously, the refiner pays a price for this increased octane content. Part of this price is the cost of the large amounts of energy that are required to operate a catalytic reformer. Figure 9.7 shows another part of the price. The volumetric yield of high octane reformate from reforming a low octane naphthas is relatively low. For example, raising the octane of a Kuwait naphtha to 96 results in a loss of 26% of the volume of the naphtha charged. For every 100 barrels of Kuwait naphtha, only 74 barrels of 96 octane reformate are obtained. Since petroleum products are sold by volume, this represents a real loss of salable gasoline. The quality of the naphtha as reforming charge stock is one determinant of the value of any particular crude oil to a refiner, although this can be offset by the value of the rest of the crude barrel for making other products.

Catalytic Cracking: In the United States, and to an increasing extent worldwide, the demand pattern is for more light products, particularly transportation fuels and petrochemical feedstock, than typically occur naturally in crude oil. The process that is most used to satisfy this demand is catalytic cracking. The main purpose of catalytic cracking is to convert heavy fuels in the gas oil and vacuum gas oil boiling ranges into gasoline boiling range materials. Figure 9.8 shows some of the chemical reactions which take place during catalytic cracking. For example, side chains can be cracked off heavy aromatic compounds to form light high octane aromatics and olefins which also have high octane numbers. Heavy paraffins, whether straight- or

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[4]During catalytic reforming, C_6 naphthene rings in the naphtha are dehydrogenated to yield aromatic molecules. As noted earlier, aromatics have very high octane numbers. Note that as a by-product of this reaction, hydrogen molecules are also manufactured. C_5 naphthene rings cannot be directly dehydrogenated to make aromatics. If, however, there are one or more side chains on a C_5 ring, isomerization to a C_6 ring can take place. This C_6 ring, then, can be sequentially dehydrogenated to produce an aromatic ring.

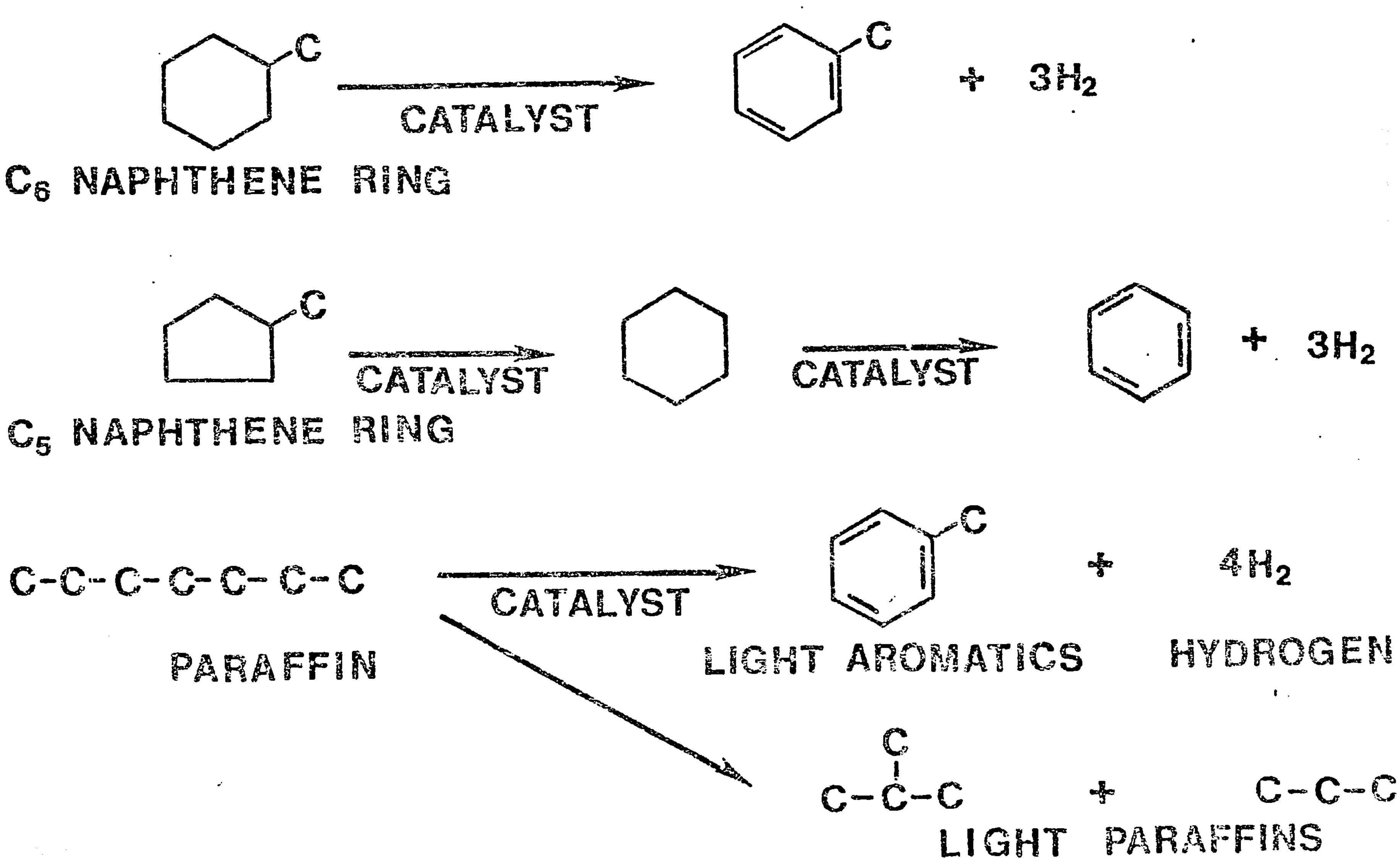


Figure 9.6. Reactions in Catalytic Reforming

Table 9.3
Octane Numbers of Naphthas

Naphtha	Research Octane	Motor Octane
Mid-East (Kuwait)	37.1	40.0
United States (South Louisiana)	51.2	47.0
Indonesian (Handil)	69.1	64.0
Specification Gasoline	91-99	83-91

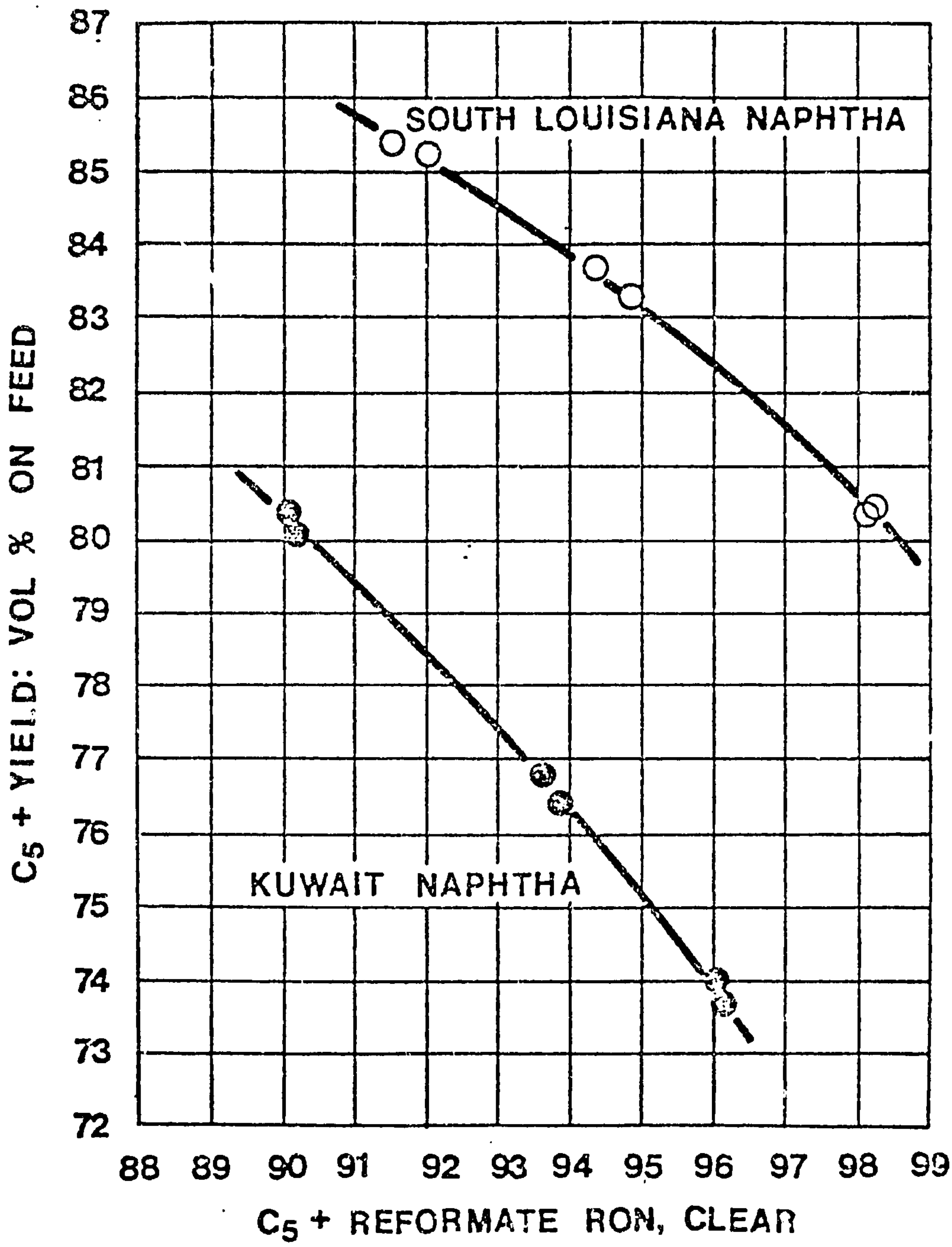


Figure 9.7. Reforming Yields

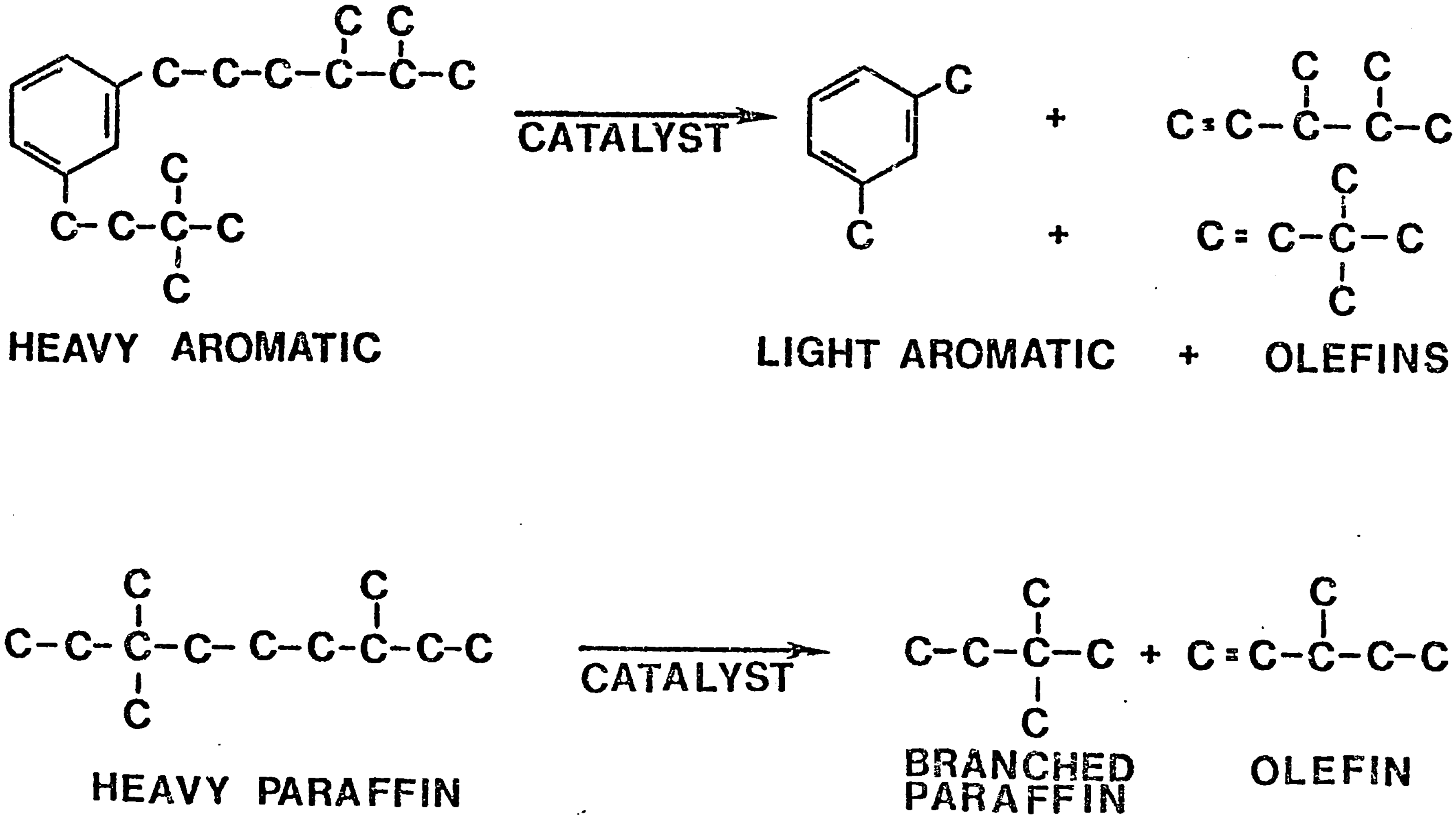


Figure 9.8. Chemical Reactions in Catalytic Cracking

Table 9.4
Catalytic Cracking
Typical Yields

Feed	Products
100 Barrels	
Gas Oil (600-1000°F Boiling Range)	2.0 Barrels Light Gases
	10.0 Barrels Propane & Propylene (C_3)
	18.0 Barrels Butanes & Butylenes (C_4)
	65.0 Barrels Gasoline (92.5 Octane)
	14.0 Barrels Furnace Oil (Low Quality)
	5.0 Barrels Heavy Fuel Oil
	114.0 Barrels of Products
	Plus 5% Coke on Catalyst

branched-chain, are catalytically cracked to a mixture of lighter branched paraffins and olefins. The chemical mechanism of catalytic cracking tends to favor the formation of branched products which are particularly attractive from the standpoint of octane number.

Typical yields from catalytic cracking a vacuum gas oil are shown in Table 9.4. From 100 barrels of gas oil charge, about 65 barrels of high quality gasoline are made, along with 14 barrels of furnace oil. About 5 barrels of heavy fuel oil are recoverd, mostly representing material that is too difficult to crack. Along with these liquid products, the equivalent of barrels of fuel oil in light gases is obtained. These are used as refinery fuel. In addition, 10 barrels of C_3 hydrocarbons and 18 barrels of C_4 hydrocarbons are produced. The saturated C_3 and C_4 hydrocarbons can be sold as liquefied petroleum gas (LPG). The propylenes and butylenes can either be used as petrochemical feedstocks or converted back into the gasoline range by other processes. Note that in catalytic cracking, a total of 114 barrels of products is made from only 100 barrels of feed. In addition, the 5% coke which is deposited on the cracking catalyst when burned in the regenerator supplies nearly all of the energy required to operate the process. Catalytic cracking is a very attractive refining process and indeed represents the heart of many modern refineries.

Hydrogen Treating: The hydrogen made as a by-product in catalytic reforming finds a number of applications throughout the refinery. All hydrogen treating processes have some characteristics in common; these are described in Figure 9.9. The purpose of hydrogen treating is to remove sulfur, nitrogen and oxygen compounds or to hydrogenate either olefinic or aromatic double bonds in certain refinery streams. Removal of sulfur, nitrogen and oxygen from petroleum products reduces the corrosiveness of these products and improves their storage stability. As Figure 9.9 shows, sulfur is removed ashydrogen sulfide, nitrogen is removed as ammonia, and oxygen is removed as water. Unsaturated hydrocarbons can also be hydrogenated in a treating

PURPOSES: REMOVE SULFUR, NITROGEN, OXYGEN COMPOUNDS
REDUCE CORROSIVENESS
IMPROVE STORAGE STABILITY
HYDROGENATE SOME AROMATICS AND OLEFINS TO IMPROVE BURNING PROPERTIES

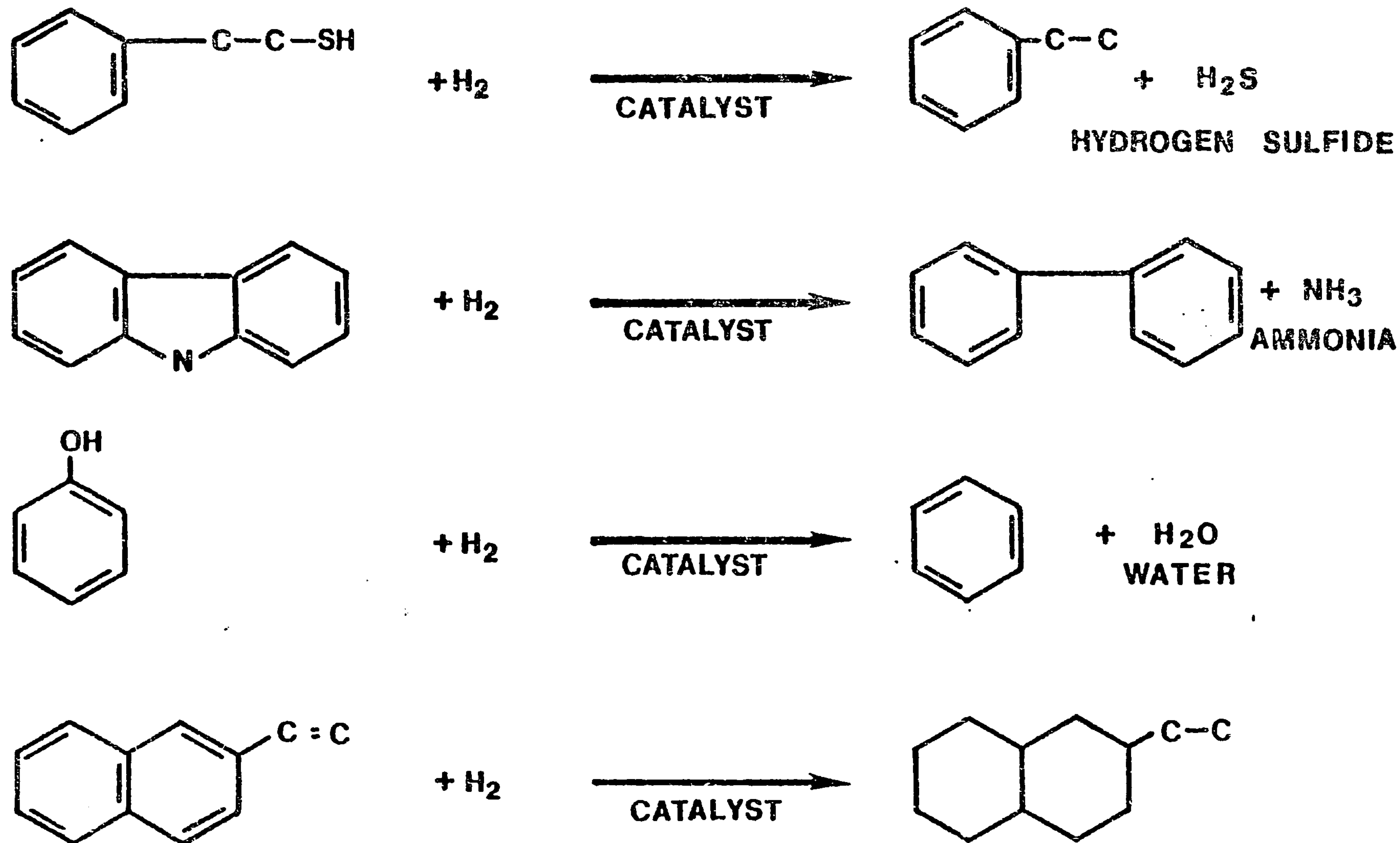


Figure 9.9. Hydrogen Treating

Table 9.5
Other Refinery Processes.

Process	Objective
Coking	Minimize yield of residual fuel oil by severe thermal cracking of vacuum residuals.
Visbreaking	Milder thermal process, reduce viscosity of bottoms, and to make more modest amounts of lighter products than coking
Polymerization Alkylation	Convert light molecules back into gasoline or fuel oil boiling range.
Residual Desulfrization	Reduce sulfur content.
Paraffin Isomerization	Convert low octane, straight-chain paraffins to high octane branched-chain paraffins.

process. The last chemical equation in Figure 9.9 shows both aromatic rings and an olefinic side chain being hydrogenated to a dicycloparaffin with a paraffinic side chain. Actually, the olefinic bond can be saturated with considerable ease, but the saturation of the aromatic double bonds shown is realtively difficult and would require a very active hydrogenation catalyst and fairly severe processing conditions.

Other Important Refinery Processes: Include Alkylation, Isomerization, Coking and Visbreaking, and are summarized on Table 9.5.

Refinery optimization is one of the classic applications of linear programming, and is in wide use by the major oil companies. For pedagogic reasons, however, we shall develop the mathematical programming statement of refinery operation and capacity expansion in terms of the simple, hypothetical refinery illustrated on Figure 9.10. But once the fundamental principles of formulating process balances and blending operations as linear constraints are understood, extension of the model to even the most complex process configurations (as shown, for example, on Figure 9.4, which depicts a real refinery) is merely a matter of additional algebra, and assembly of the necessary supporting data.

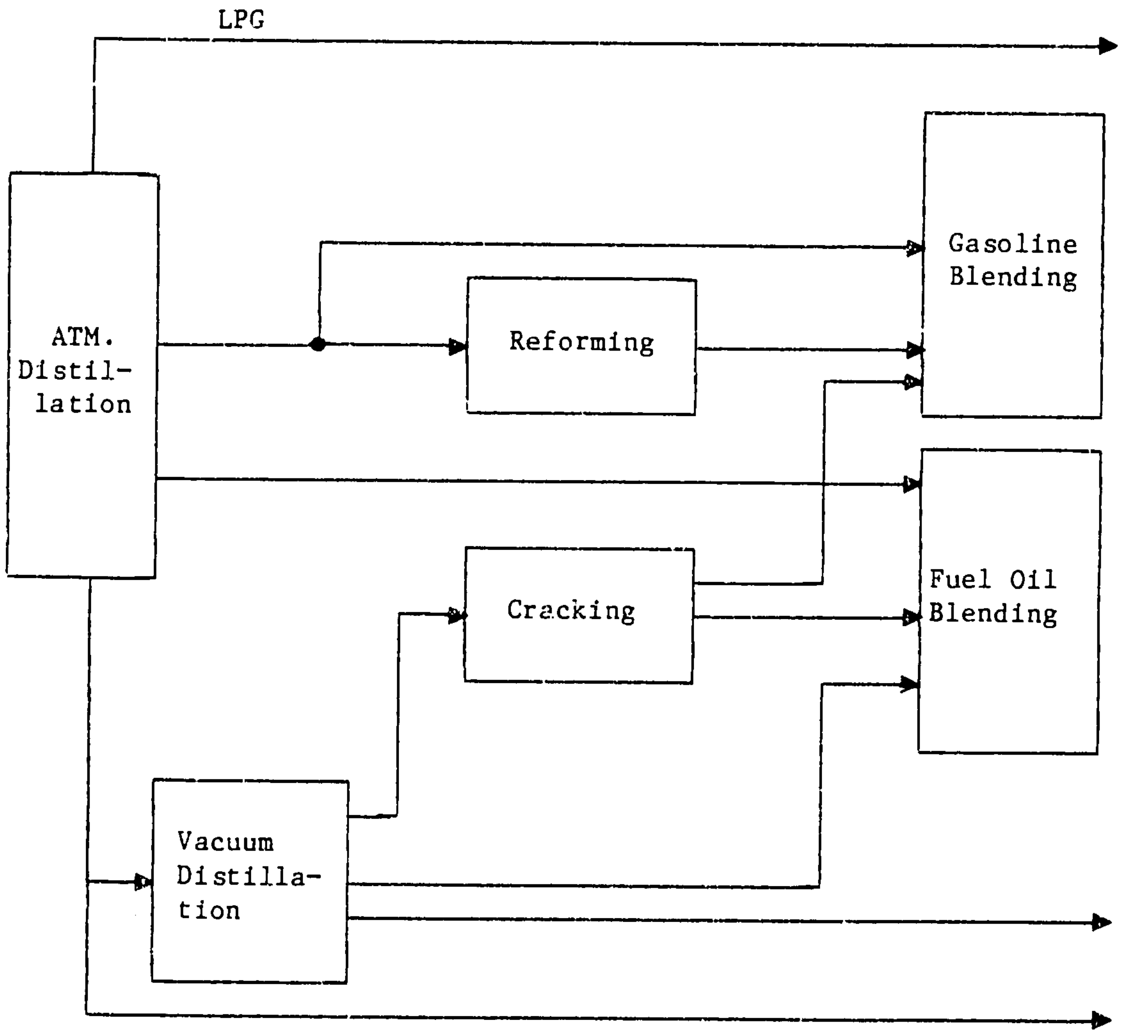


Figure 9.10. Hypothetical Refinery

We begin this discussion with a description of the constraint structure that captures the physical and engineering relationships of the refinery: mass balances around each process unit, conversion efficiencies, stream splitting and blending. A series of linear equations, then, models all possible flows through the refinery, from the input crudes to the refined product outputs. Whether or not the model is driven by some given set of refined product demands depends on the application of the model. If the refinery model depicts the national aggregate of refinery operation, say as part of a larger energy system model, then indeed one might view the optimization as demand driven, with the objective of identifying the optimal capacity expansion path and operational conditions that meets the given set of product demands. In this context optimality would be taken to imply minimization of a suitable specified cost function. On the other hand, if we take the perspective of a single refinery, the appropriate criterion will be profit maximization, driven not by any requirement to meet a particular mix of refined products, but likely to be constrained by investment capital requirements (assuming the refinery to be profitable to begin with). Objective functions will be discussed further in the next section.

In general, around each major unit operation we require a capacity constraint, that links throughout to the installed capacity; and a mass balance equation, that relates possible outputs of a process to possible inputs.

Atmospheric Distillation: The capacity constraint requires that the sum of crude inputs be less than or equal to the existing capacity of the unit.

$$\sum_{i}^{N_c} x_i \leq \bar{C}(A) \tag{9.1}$$

where

x_i is the quantity of crude type i.
N_c is the number of crude oil types available.
$\bar{C}(A)$ is the capacity of the atmospheric distillation unit.

As illustrated on Figure 9.4, in a real refinery there may be a great number of product streams from atmospheric distillation: for our purposes let us assume four streams y_1 through y_4, corresponding, say, to Naphtha, Middle distillate, Gasoil and residuum, and two possible crudes, x_1 and x_2. Then

$$y_1 \text{ [bbls Naphtha 1]} = a_{11} \left[\frac{\text{bbls/Naphtha 1}}{\text{bbl Crude 1}}\right] x_1 \text{ [bbl Crude 1]} +$$

$$a_{12} \left[\frac{\text{bbl Naphtha 1}}{\text{bbl Crude 2}}\right] x_2 \text{ [bbl Crude 2]} \tag{9.2}$$

where the aij are the socalled yield coefficients. Similarly, for the other naphthas we have

$$\begin{aligned} y_2 &= a_{22}\ x_1 + a_{22}\ x_2 \\ y_3 &= a_{31}\ x_1 + a_{32}\ x_2 \end{aligned} \tag{9.3}$$

and hence the generalized constraint set is

$$y_j = \sum_{i=1}^{N_A} a_{ji} \cdot x_i;\ j = 1, \ldots N_N \tag{9.4}$$

where N_A is the total number of product streams from atmóspheric distillation, which includes middle distillate streams and heavy ends. Table 9.6 shows some typical yield coefficients, as used in Bhatia's refinery modelling study of the Indian subcontinent.[1]

As indicated on Figure 9.10, each of the output streams from the distillation unit can be routed either to some further processing unit, such as reforming or catalytic cracking. Thus we split each stream as

$$y_i(G) + y_i(R) = y_i \quad i \in R \tag{9.5}$$

where $y_i(G)$ is the straight-run fraction of type i routed directly to gasoline blending, $y_i(R)$ is the straight-run fraction of type i routed to the reforming unit, and i R is the set of fractions that are candidates for the reforming unit. Similarly, for the Middle distillates streams that are candidates for the cracking units (say the set $i \in C$),

$$y_i(C) + y_i(F) = y_i \quad i \in C \tag{9.6}$$

[1]R.K. Bhatia "A Spatial Programming Model for India's Petroleum and Petrochemical Industries" Thesis submitted to the University of Dehli, Dept. of Economics, 1974. A short summary paper was published in Indian Economic Review.

Table 9.6
Yield Coefficients for Straight-Run Fractions of Typical Crudes (a_{ji})

j \ i	Assam (Lakwa-Rudra-sagar 24° API)	Gujarat (Ankle-shwar 46.9° API)	Agha Jari 34° API	Rostam 35° API	Arabian Light 33° API	Darius 34° API
Gas	.029	.041	.034	.015	.03	.017
Naphtha	.097	.112	.165	.152	.130	.178
Kerosene	.169	.148	.140	.197	.190	.178
Diesel	.275	.170	.20	.235	.186	.196
Residue	.430	.529	.463	.40	.46	.43

Source: Bhatia, op. cit., p. 241

where $y_i(F)$ represents the heavier straight-run fractions that might be routed directly to the fuel oil blending operation.

Reforming: The capacity constraint for the reforming unit is given by

$$\sum_{i \in R} y_i(R) \leq \bar{C}(R) \tag{9.7}$$

where $\bar{C}(R)$ represents the capacity of the unit. The mass balance equation follows in analogy to (9.2) - (9.4); if we suppose only one output stream ("reformed gasoline"), then

$$z_1 = b_{11}\, y_1(R) + b_{12}\, y_2(R) + b_{13}\, y_3(R) \tag{9.8}$$

where the b_{ij} represent the yield coefficients (bbl of reformed gasoline of type i per bbl of input naphtha j), and z_1 is the output of reformed gasoline.

Cracking: In direct analogy to the reforming equations we have

$$\sum_{i \in C} y_i(C) \leq \bar{C}(C) \tag{9.9}$$

$$\begin{aligned} v_1 &= d_{14}\, y_4(C) + d_{15}\, y_5(C) \\ v_2 &= d_{24}\, y_4(C) + d_{25}\, y_5(C) \end{aligned} \tag{9.10}$$

where v_1 is the cracked gasoline, and v_2 is cracked oil; and $\bar{C}(C)$ is the capacity of the catalytic cracking unit.

Strictly speaking one should carry through the subsequent treatment processes the original source of crude: as shown on Figure 9.7 in the case of reforming, reformate yields from naphtha vary considerably across crude source.[2] This requires in practice, one of two steps; either one carries through an additional index in all of the yield coefficients and distillation unit output streams to identify the original source of crude, or one defines the set of distillation outputs in such a way that different output streams are unique to specific crudes (i.e., one differenriates between "Kuwait naphtha," "Arabian Light naphtha," etc., with the straight-run yield coefficients adjusted such that the yield of Kuwait naphtha from Arabian Light is zero!).

Blending:[3] Each of the refinery products must meet certain quality specifications--premium gasoline, for example, must meet a certain octance number. The refinery meets such product specifications by blending the various process streams, since each process stream typically is of different quality. Table 9.7 illustrates typical product quality requirements.

Table 9.7
Typical Product Quality Specifications

Regular gasoline	RVP $\leq$ 9 psi RON $\geq$ 90.5 200°F $\leq$ ASTM 50% $\leq$ 230°F
Premium gasoline	RVP $\leq$ 9. psi RON $\geq$ 100.3 200°F $\leq$ ASTM 50% $\leq$ 230°F
Jet fuel (JP4)	2 psi $\leq$ RVP $\leq$ 3 psi
Fuel oil	SFS $\leq$ 175 at 122°F

RVP = Reid vapor pressure
RON = Research octane number
SFS = Viscosity Constraint
ASTM50% = Temperature at which 50% of the mixture is vaporized.

The algaebra of the blending operation can best be illustrated using gasoline blending as an example. Suppose there are two gasoline products, regular and premium gasoline, whose octane number are Ω_R ; Ω_p . Also, let the inputs to the blending process, and their octane numbers, be given by

[2]Of course another way of saying this is that "naphtha" is not a very precise label, as the chemical speciation of a product in the naphtha boiling range can vary quite widely.

[3]This discussion is adapted from Murtagh (1981).

Naphthas	$y_1(G)$	w_1
	$y_2(G)$	w_2
	$y_3(G)$	w_3
Reformed Gasoline	z_1	w_4
Cracked Gasoline	v_1	w^5

Then the material balances are

$$\begin{aligned} y_1(G) &= y_1(G)(R) + y_1(G)(P) \\ y_2(G) &= y_2(G)(R) + y_2(G)(P) \\ y_3(G) &= y_3(G)(R) + y_2(G)(P) \\ z_1 &= z_1(R) + z_1(P) \\ v_1 &= v_1(R) + v_1(P) \end{aligned} \tag{9.11}$$

where $g(P)$ indicates the fraction of input stream g blended to premium gasolines, and $g(R)$ the fraction blended into regular grade gasoline. Assume octane numbers blend linearly by volume: then the octane constraint for regular gasoline is

$$w_1 \cdot y_1(G)(R) + w_2 \cdot y_2(G)(R) + w_3\, y_3(G)(R) + w_4\, z_1(R) \tag{9.12}$$

$$+ w_5\, v_1 \geq \Omega_R \cdot P_1$$

where P_G is the total volume of regular gasoline produced, i.e.,

$$P_G = \sum_{i=1}^{3} y_i(G)(R) + z_1(R) + v_1(R) \tag{9.13}$$

Eq. (9.12) and (9.13) can be combined into the single constraint

$$(\Omega_R - w_1)\, y_1(G)(R) + (\Omega_R - w_2)\, y_2(G)(R) + (\Omega_R - w_3)\, y_3(G)(R)$$

$$+ (\Omega_R - w_4)\, z_1(R) + (\Omega_R - w_5)\, v_1(R) \leq 0 \tag{9.14}$$

with an analogous expression for premium grade gasoline.

The above example assumed that the product quality specification itself blended linearly by volume. Obviously, for such specifications as viscosity, that requirement is far from met. To get around this problem, refinery engineers have developed so called "linear blending numbers" for each specification. For example, suppose fuel oil is to be blended from a number of process streams in such a way as to meet the 175 SFS constraint (Table 9.7). This specification corresponds to a linear blending number of 10.1. Then if the available process streams, and their linear blending numbers are as follows:

x_1	10.1	(from crude unit)
x_2	12.63	(from crude unit)
x_3	8.05	(vacuum gasoil)
x_4	6.90	(atmospheric gasoil)
x_5	8.05	(from fluid catalytic cracker)
x_6	4.40	(from fluid catalytic cracker)

then the product quality constraint follows quickly as

$$10.1x_1 + 12.63x_2 + 8.05x_3 + 6.9x_4 + 8.05x_5 + 4.4x_6 \leq 10.1z \quad (9.15)$$

where

$$z = \sum_j x_j$$

<u>Tetra-ethyl lead additions: non-linear blending response</u>. The addition of tetra-ethyl lead (TEL) to gasoline, in order to raise the octane number, resists the expedient of linear blending numbers: the response is clearly non-linear, of the type illustrated on Figure 9.10. Let x_{TEL} be the quantity of TEL that is added. Let p be the required octane number of the product gasoline, and suppose first that there is only one gasoline blend stream to which TEL is to be added. Suppose further that the TEL response curve can be adequately linearized by the simple linear expression

$$P_A \cdot x + g_A x_{TEL} = P \cdot (x + x_{TEL}) \quad (9.16)$$

from which we see that if no TEL is added, the resultant RON of the "blend" is the RON of the blend stock, P_A. If there is more than one blend stock, then

$$\sum_i P_{Ai} \cdot x + g'_A x_{TEL} = p x_B \quad (9.17)$$

where

$$x_B = \sum x_i$$

and g'_A is the "average" responsiveness

$$g'_A = \frac{\sum g_A x_i}{\sum x_i} \ .$$

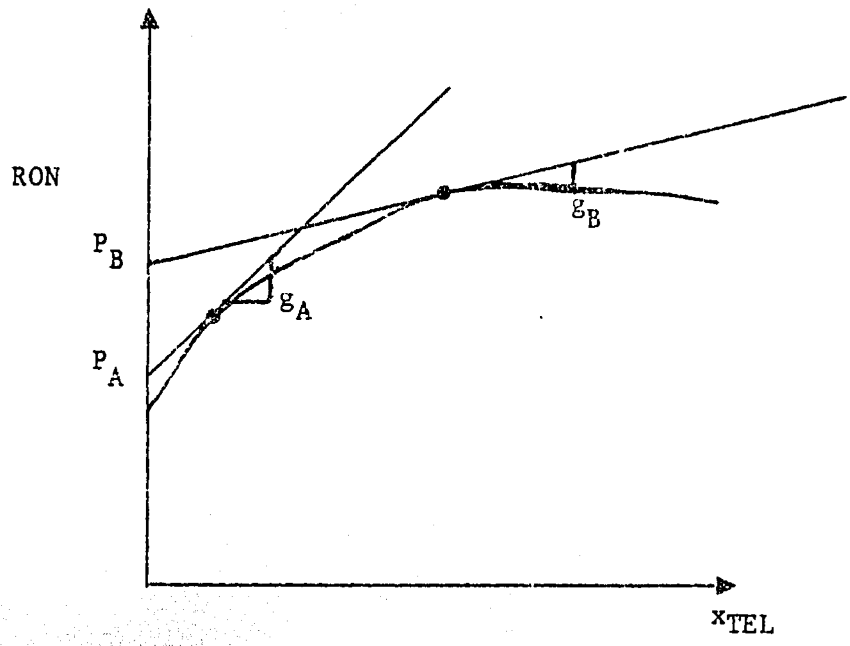


Figure 9.10 Nonelinear blending response of TEL to a gasoline blend stream.

Finally, if one requires two linear segments to characterize the response, then since both must be satisfied, the constraint set can be written

$$\sum_i P_{Ai} x_i + g'_A x_{TEL} = p x_B$$

$$\sum_i P_{Bi} x_i + g'_B x_{TEL} = p x_B \tag{9.18}$$

$$x_B - \sum x_i = 0$$

Typical values for g'_A are in the range 4.3 - 4.9, and for g'_B 1.65 - 2.08.

Capacity Expansions: In all of the capacity constraints require that throughput cannot exceed installed capacity. In the event that we wish to allow expansion of any unit operation, we modify these constraints by addition of a variable to represent added capacity, i.e.

$$\sum_i^{Nc} x_i - C(A) \leq \bar{C}(A) \tag{9.19}$$

and in exact analogy for reforming and cracking units,

$$\sum_{i \in R} y_i(R) - C(R) \leq \bar{C}(R) \tag{9.20}$$

$$\sum_{i \in C} y_i(C) - C(C) \leq \bar{C}(C) \tag{9.21}$$

where $C(A)$, $C(R)$, $C(C)$ represent the requisite additional capacity of atmospheric distillation, reforming and cracking, respectively.

9.3 OBJECTIVE FUNCTION SPECIFICATIONS

There exist a number of quite different forms of objective function, each reflecting different situations of ownership, market conditions, size and the objective at hand. Clearly a privately (or multinationally) owned refinery, operated to maximize profit (or returns to investment) will show differences to that of a state owned enterprise where minimization of, say, foreign exchange requirement for petroleum products may be the most important criterion. For the purposes of this discussion, let us introduce the following notation

γ_i = aquisition cost of the i-th crude, including all freight charges to the refinery gate (in \$bbl)

λ_j = revenue to the refinery operator, \$/bbl of product

Ω_k = operating cost of the k-th refinery unit, \$/bbl of feed

α_k = capital cost of the k-th refinery process, \$bbl/day of throughput.

Case A: Private Sector, Single Refinery, No Expansions. In this case we assume that the refinery is operated by an independent refiner, whose share of the market is such that he faces a given set of prices per bbl of product, and cost per bbl of crude, that is determined by others (in the world or national market). Here, then, the objective is to maximize profit, given simply by

$$z = \max \underbrace{\sum_j \lambda_j P_j}_{\text{Revenue}} - \underbrace{\sum_i \gamma_i x_i}_{\text{Crude Cost}} - \underbrace{\sum_k \Omega_k \cdot C(k)}_{\text{Operating Cost}} \quad . \tag{9.22}$$

If we are concerned only with the optimization of an existing refinery, we constrain the set of expansion capacity variables to zero, i.e.

$$C(k) = 0 \quad \text{for all } k \quad .$$

Note that by setting to zero, rather than simply deleting the variable, the reduced cost (shadow price) associated with these zero constraints can be compared to the cost of capital to the company concerned, which gives an indication of the likely rate of return of expansion.

Case B: Private Sector, Expansion Possibilities, Crude Limiations. Frequently it is not possible to use the optional mix of crude stocks: changing market conditions, or supply disruptions, may require the addition of crude oil constraints of the type

$$x_i \leq t_i \tag{9.23}$$

where t_i is the upperbound to the import of the i-th crude. Another situation might require a constraint of the type

$$\sum_{i \in I} x_i \leq t^*$$

where i I is the set of imported crude types, and t^* is an upper limit for oil imports, perhaps given as a government imposed import quota. Finally, many small developing countries have encountered situations in which they are forced to accept shipment of a certain quantity of heavier crude for each shipment of premium grade crude. In this case we have a constraint of the form

$$\sum_i h_i x_i = 0 \quad . \tag{9.25}$$

where the h_i are the fractions of each type of crude. Finally, if, α_k is the unit capital cost for the k-th process, the objective function is

$$Z = \max \Bigg\{ \underbrace{\sum_j \lambda_j p_j}_{\text{Revenue}} - \underbrace{\sum_i \lambda_i x_i}_{\text{Crude Cost}} - \underbrace{\sum_k {}_k C(k)}_{\text{Operating Cost}}$$

$$- \underbrace{\sum_k \alpha_k C(k) CRF(i,n)}_{\text{Capital Cost}} \Bigg\} \tag{9.26}$$

where CRF(i,n) is the appropriate capacity recovery factor at discount rate i and amortization period n.

Case C: Public Sector Ownership; Fixed Product Demand. This is a situation that is typical of developing countries, where an important objective is the minimize the foreign exchange requirement to meet the country's overall petroleum product demand. In Tunisia, for example, up until very recently much of the production of indigenous high quality crude was exported on the European Spot Market, with the Tunisian refinery at Bizerta importing lower grade crudes (at a considerable profit to the overall transaction). We need the following additional notation:

$\bar{d}_j$ = indigenous demand for product j, bbl/yr.

β_j = export selling price fo refined product j, \$/bbl (net of any transportation costs)

σ_j = import price fo refined product j, \$/bbl, (landed at a principal port)

E_j = exports of refined product j, bbl/yr

I_j = imports of refined product j, bbl/yr.

First we need to add a constraint that allows imports and exports of refined products whilst meeting the balance of demand by the indigenous refinery, i.e.

$$P_j - E_j + I_j = \bar{d}_j \quad . \tag{9.27}$$

Then it follows that the objective of minimization of foreign exchange is given by

$$Z = \min \underbrace{\sum_{i \in I} \lambda_i x_i}_{\text{Crude Aquisition}} + \underbrace{\sum \beta_i I_j}_{\text{Imports of Refined Products}} - \underbrace{\sum \sigma_j E_j}_{\text{Exports of Refined Products}} \quad . \tag{9.28}$$

Such an objective function would also require some form of capital constraint, to prevent the problem from becoming unbounded (since if refining is profitable, maximization of exports will minimize Z, and hence unlimited capacity would be added if left unconstrained). Thus one would require that

$$\sum_k \alpha_k \cdot C(k) \leq D \tag{9.29}$$

where D is the assumed capital bound. Alternatively, one could minimize cost subject to foreign exchange and capital limitations, i.e.

$$\min Z = \underbrace{\sum_i \lambda_i x_i}_{\text{Crude Cost}} + \underbrace{\sum_k \Omega_k C_k}_{\text{Operating Cost}} + \underbrace{\sum \alpha_k C(k) CRF(i,r)}_{\text{Capital Costs}}$$

$$+ \underbrace{\sum \beta_j I_j}_{\text{Cost of Imported Refined Products}} - \underbrace{\sum \sigma_j E_j}_{\text{Export Revenue}} \tag{9.30}$$

with the additional constraints

$$\begin{aligned} &\sum \alpha_k C(k) \leq D \\ &\sum_{i \in I} \lambda_i x_i + \sum \beta_i I_j - \sum \sigma_j E_j \leq F \end{aligned} \tag{9.31}$$

9.4 EVALUATING REFINERY OPTIONS IN THE SUDAN

Background

The Sudan is currently experiencing very severe economic difficulties, brought about by a severe shortage of foreign exchange that has in turn created shortages of petroleum products (as the Sudanese Oil Company, the General Petroleum Corporation, GPC, is unable to open the necessary letters of credit through the Bank of Sudan). Fortunately, for some years now a number of multinational oil companies have been very active in oil and natural gas exploration in the Sudan, and Chevron is now at the point of negotiating terms for the exploitation of the Unity Oil field, with production level by the mid-1980s expected at around 25,000 bbl/day. But as indicated in Figure 9.11, the Unity Field lies some 650 km south of the Khartoum area, is far to the south of the main centers of economic activity, and some 550 km from the proposed refinery site at Kosti, on the White Nile.

How then to exploit these oil resources? There are two basic choices at the 25,000 bbl/day production level: (i) refine the crude oil to produce petroleum products for domestic consumption (and thereby eliminate the need for the importation of refined products through Port Sudan), or (ii) export the crude on overseas markets. Whilst (i) has the advantage of assured supply to the domestic economy, the current economics of refinery construction and operation are quite unfavorable. On the other hand, option (ii) would require the construction of a pipeline of some 1500 km in length, to bring crude to export at Port Sudan.

Because of the critical position of foreign exchange, a key criterion in terms of analyzing the potential options must be the impact on foreign exchange flows, Table 9.8 and 9.9 summarize the capital and operating cost estimates for the crude export, and Kosti refinery option, respectively.

Foreign Exchange Impact

In evaluating the foreign exchange impact, we assume that both the refinery operation or crude export would be assigned to the White Nile Corporation, a body that has in fact been created to explore and promote financing options for the refinery. We shall assume that 50% of the crude is provided free (the Government of Sudan share in the production sharing

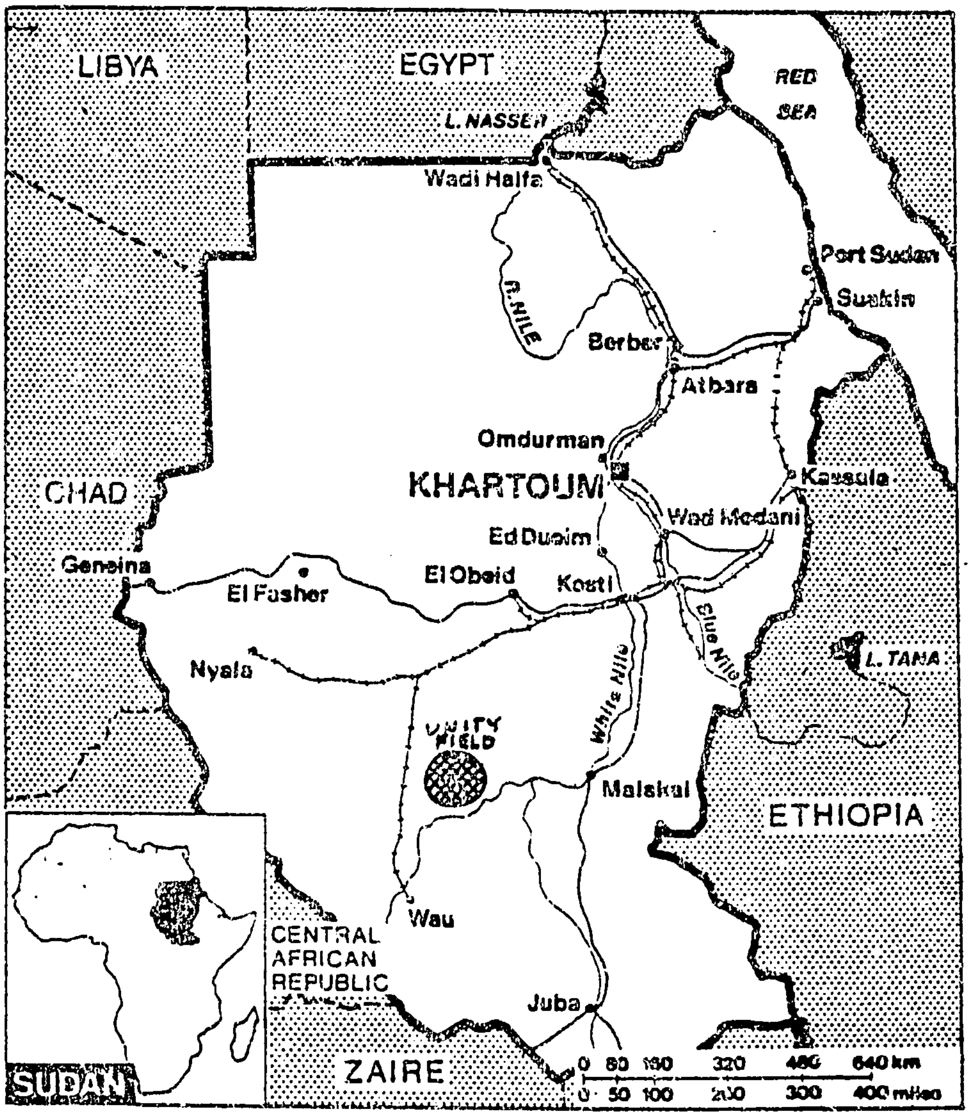


Figure 9.11 The Democratic Republic of the Sudan

agreement with Chevron), the other 50% provided to the White Nile Corporation at an assumed wellhead price of \$27/bbl.[4] Then from these assumptions, and the cost data of Tables 9.8 and 9.9, one can readily derive income statements for the White Nile Corporation, Chevron Sudan, and the Government of Sudan (operating through the General Petroleum Corporation, GPC), presented on Table 9.10 for the crude export option, on Table 9.11 for the refinery option. The results are shown for two possible export prices for Sudanese

[4]At the time of writing, negiotiations between the Government of Sudan and Chevron regarding the wellhead price were still underway. \$27/bbl is the author's estimate of what Chevron would require to yield a 15% return on investment.

Table 9.8
Cost Estimates, Crude Export Option

	Capital costs, 10^6\$			Operating costs, 10^6\$/yr		
	Total	Foreign currency	Local	Total	Foreign currency	Local
Pipeline[1]	620	575[2]	45[2]	15	10	5
Pt. Sudan Facilities (storage, marine terminal)	100	93[2]	7[2]	5	3	2
Total	720	668	42	20	13	7

[1]Assumed distance = 15 km (550 km Unity Field-Kosti, 130 Kosti-Khartoum, 320 km Khartoum-Atbara-Pt. Sudan). Cost is 3x Bechtel estimate of \$206 M for the Unity Field-Kosti section).
[2]Applying 7% local cost share estimated by Bechtel for Unity Field-Kosti Pipeline.

Table 9.9
Cost Estimates, Kosti Refinery Option

	Capital costs, 10^6\$			Operating costs, 10^6\$/yr		
	Total	Foreign currency	Local	Total	Foreign currency	Local
Pipeline	206	193.4	14.5	5	3	2
Kosti Refinery	695	625.5	69.5	54	40.5	13.5
Total	901	819	82	59	43.5	15.5

crude, at \$26/bbl (a pessimistic case), and \$31/bbl (a more optimistic case). Given the rather unique characteristics of the crude from the Unity Field, (very low sulfur, very high pour point), the price f.o.b. Pt. Sudan is subject to some uncertainty.

For the refinery option evaluation, we consider two cases; case A assumes that the refinery output will be sold to the local market at prices roughly equivalent to present levels (which in turn are based on petroleum product imports from Kuwait, at price levels based on the official OPEC price). Case B, on the other hand, assumes that the refinery output is valued on the basis of current spot market prices (considerbly below those involved in contracts with OPEC country governments). This would be a reasonable perspective for evaluating the economics of the Kosti refinery if

Table 9.10
Income Statements, Crude Export by Pipeline

Assumptions: White Nile Corporation is 50% Gov't of Sudan, 50% Chevron; wellhead price $27/bbl.

	$26/bbl	$31/bbl
Income Statement of White Nile Corporation		
Gross Revenues (25,000 bbl/yr)	237	282
Operating expenses	(20)	(20)
Cost of crude (12,500 bbl/yr x $27.4)	(125)	(125)
Gross income	92	137
Income Statement of Chevron-Sudan		
Income	46	68.5
Depreciation[1]	(18)	(18)
Taxable income	28	50.5
Tax at 50%	(14)	(25.25)
After tax income	32	43.25
(Rate of return[2])	~6%	~10.5%
(Rate of return at zero tax rate)	~11%	~18.5%
Income Statement of Govt. of Sudan/GPC		
Income	46	68.5
Debt service (2% interest, 20 yr soft loan)[3]	19.4	19.4
Net return	26.6	49.1

[1]Assume straight line depreciation over 20 years.
[2]Discounted cash flow rate of return, r, given by SPWF (20,r) = Investment/After Tax Cash Flow.
[3]Applied to foreign share of the Sudan Equity = 360 - 42 = 318.

the Sudan were in fact to purchase its petroleum products on the spot market.[5] (See Table 9.12)

To examine the impact on foreign exchange flows, schematics of the type shown in Figure 9.12 will be used. For the crude export option, let us assume that the White Nile Corporation keeps its books in U.S. $, given that the bulk of revenues and expenditures are in U.S. $ rather than Sudanese Pounds (LS). Note from Figure 9.12 that the White Nile Corporation goes to the Bank of Sudan to purchase LS to meet local operating expenses. We leave to the reader to follow each individual cash flow shown on this figure in terms of

[5]The main reason why the General Petroleum Corporation does not presently utilize the spot market to a greater extent is institutional: because of the severe shortages of foreign currency, the 30 day credit terms offered by the Kuwaiti's as part of the long-term contract appear very favorable to the Bank of Sudan, which must open the necessary letter of credit. Spot market purchases, of course, require immediate payment.

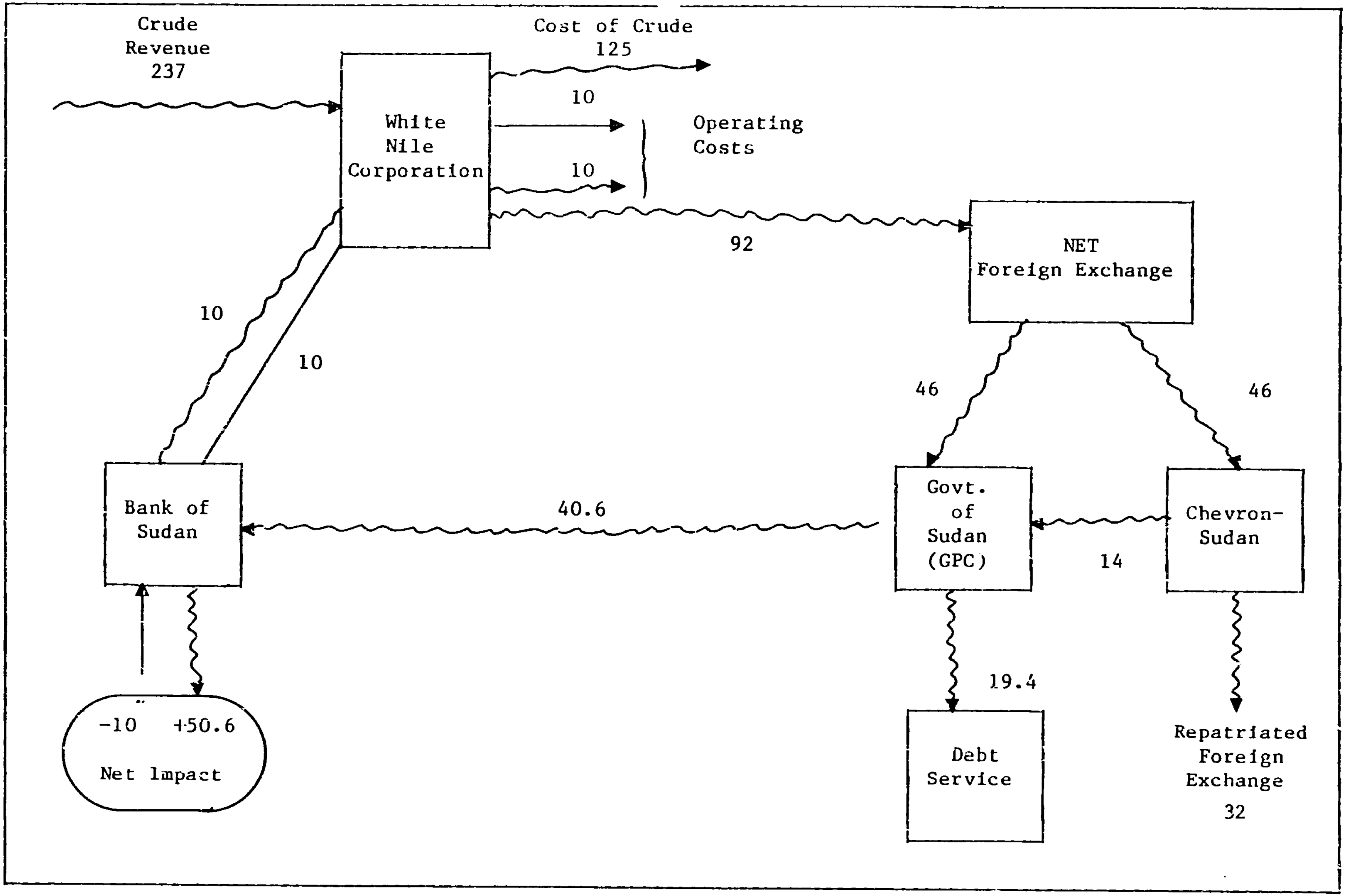


Figure 9.12. Cash Flows, Crude Exports at $26/bbl, 50/50 Sudan/Chevron Equity Shares.

Table 9.11
Income Statement, Refinery Option

Assumptions: White Nile Corporation is 50% Govt. of Sudan, 50% Chevron; wellhead price $27/bbl.

Income Statement of White Nile Corporation	Case A (current costs)	Case B (spot market)
Gross revenue	340	234
Operating expense, local	(9)	(9)
Operating expense, foreign	(50)	(50)
Cost of crude	(125)	(125)
Gross income	156	50
Income Statement of Chevron-Sudan		
Income	78	25
Depreciation	(22.5)	(22.5)
	55.5	2.5
Tax at 50%	(27.75)	(1.5)
After tax income	50.25	23.5
(Rate of return)	9.5%	<1.0%
(Rate of return at zero tax)	16.5%	~1.0%
Income Statement of Govt. of Sudan		
Gross Income	78	25
Debt Service	22	22
Net income	56	3

Table 9.12
Official and Spot Market Prices (March 1982)

	Current cost of Kuwaiti petroleum C.I.F. Pt. Sudan ($/ton)	Mediterranean (Italy) spot market price ($/ton)
Gasoil	329	263
Jet A1	378	305
Gasoline (regular)	358	271
(premium)	382	281

the computations presented in the above tables. The net result is a foreign exchange inflow of $50.6 million, offset by a LS 10.0 million debit (assuming, for simplicity, a 1:1 exchange rate).

In the case of the refinery options, it is assumed (Figures 9.13 and 9.14) that the White Nile Corporation maintains its books in Sudanese Pounds. It must therefore go to the Bank of Sudan to obtain the necessary foreign currency to meet the foreign currency component of operating costs

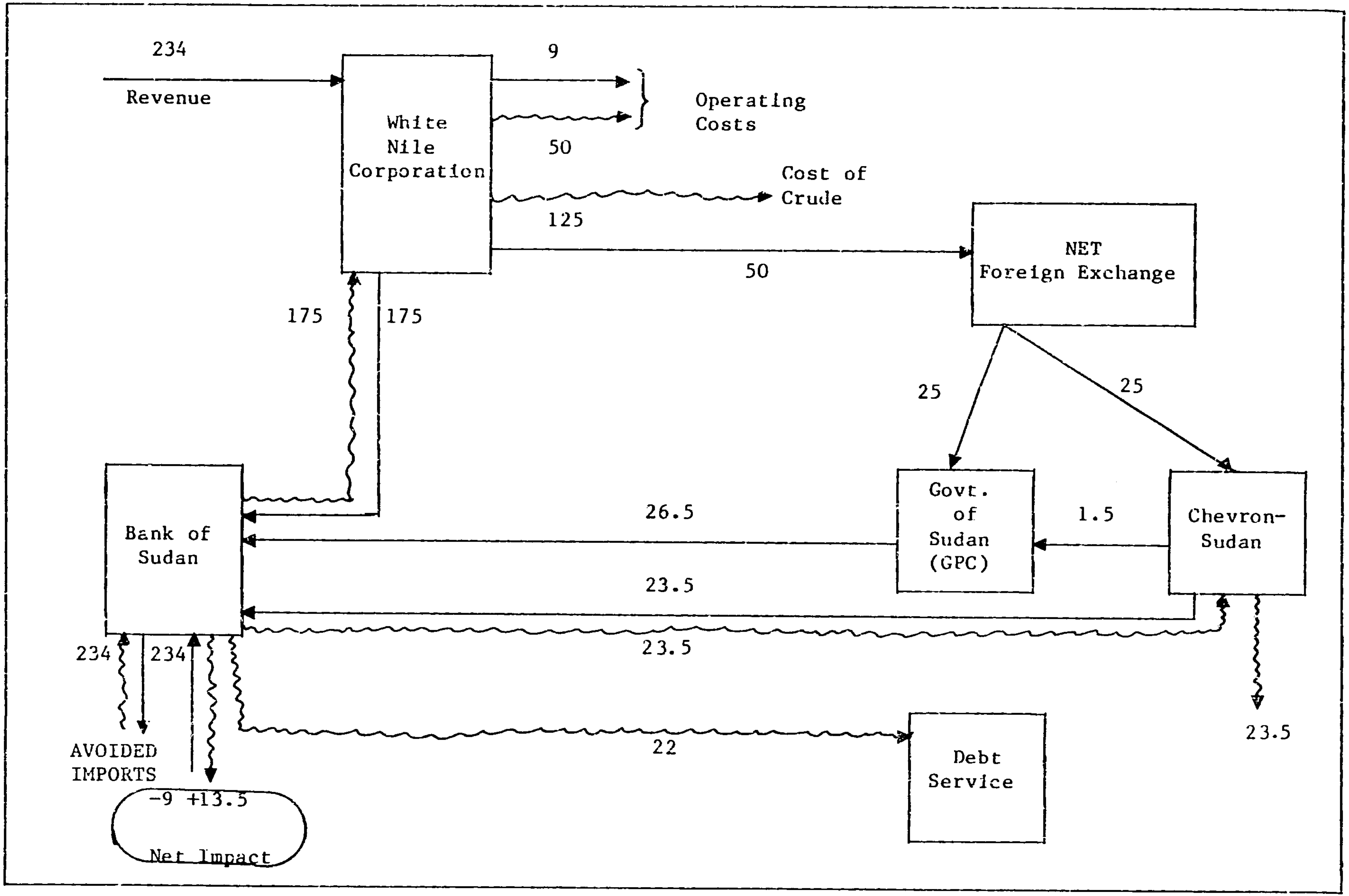


Figure 9.13. Cash Flows, Kosti Refinery, 50/50 Sudan/Chevron Equity Share, Case B (Petroleum Products Valued at Spot Market Prices)

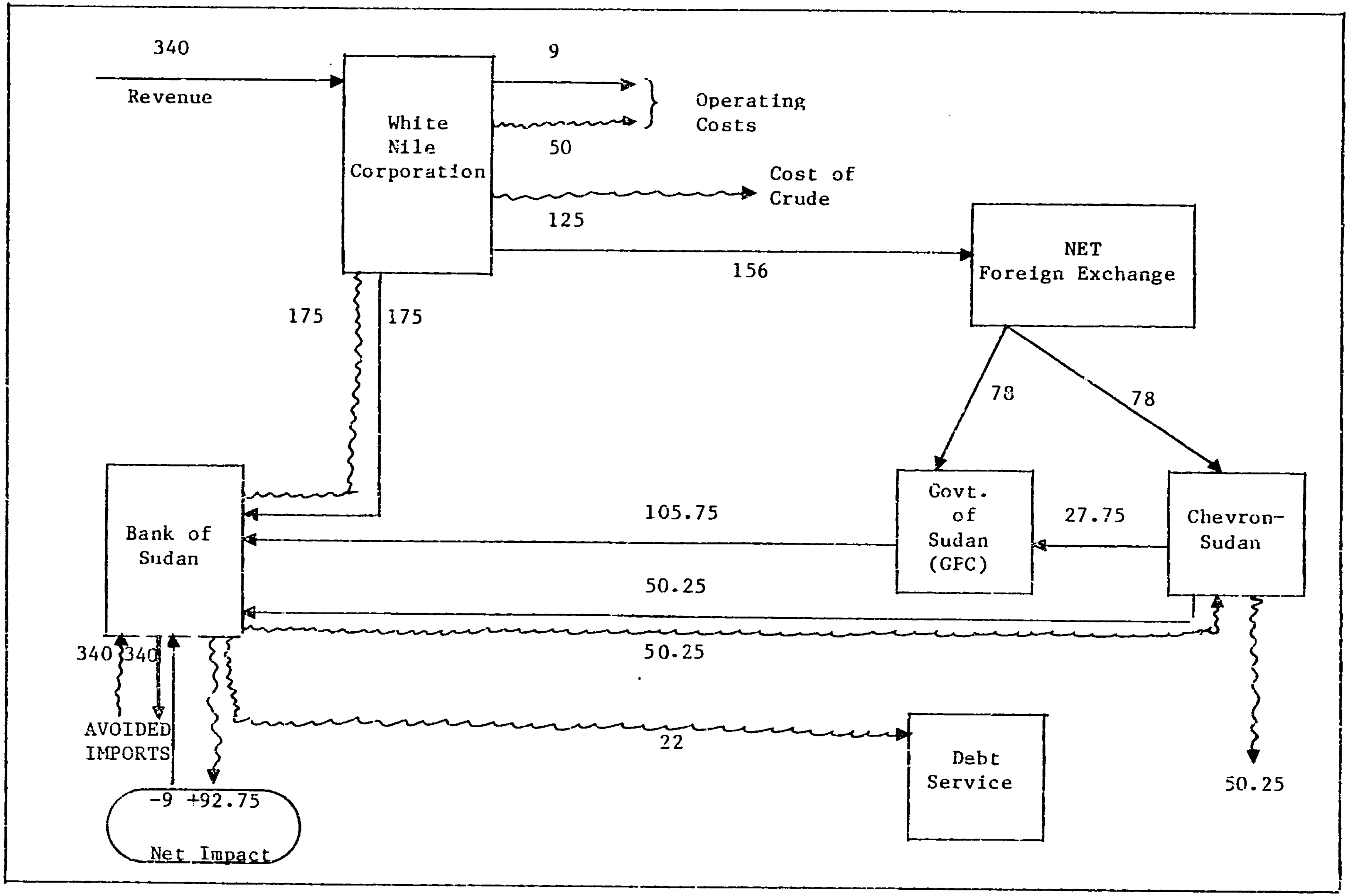


Figure 9.14. Cash Flows, Kosti Refinery, 50/50 Sudan/Chevron Equity Share, Case A (Petroleum Products Value at Current Prices)

(for such items as catalysts and refinery spares that must be purchased overseas, and for foreign labor). Crude costs are of course also in U.S. $, based on the wellhead price.

As one might expect, when one evaluates the refinery option in terms of current spot market prices, the economics look quite unfavorable, as evidenced by rates of return that are barely positive (Case B, Table 9.11). This implies that it would be very difficult to attract an equity partner into the White Nile Corporation and given the fact that over the past year many refineries have been closed down, or are running at significant losses, it should come as no surprise that the economics of a new refinery (especially one that requires a 500km pipeline to bring crude to the plant gate) appear unfavorable. To be sure, when evaluated against official OPEC prices, the refinery option suggests about a 16% rate of return on the foreign equity partner's share, and significant benefits to the Bank of Sudan's foreign exchange position. To the extent that a worldwide economic recovery begins to eliminate the current oil glut that has led to depressed spot market prices, at some point in the future the Refinery option may become attractive. However, under such conditions, it would also be reasonable to assume that Sudanese Crude could also be exported at the more optimistic price of $31/bbl (or even more), an option that has comparable foreign exchange benefits to the refinery option.

10. ENERGY-ECONOMIC LINKAGES

10.1 INTRODUCTION

Some of the difficulties of using even the extended, energy denominated input-output framework to analyse energy policy issues were alluded to in the closing sections of Chapter 6. In our analysis of Republica, for example, we saw the need to use an iterative approach if one were interested in determining that combination of economic activities that would result in oil imports of a given level. One way around such situations is to combine the input-output model with some kind of optimization framework, that in particular allows the solution to constrained optimization problems (whose constraint set might include, for example, upper bounds on energy imports, or lower bounds on particular levels of renewables). Moreover, given that we introduce an element of choice in the energy system, there is no reason why one cannot relax some of the more confining assumptions of conventional input-output analysis (only one industry producing one product using a single technology) to allow choice also in terms of the modalities of the productive sector: and include for example the choice between say, electric arc, open hearth, or sponge iron processes in steelmaking as part of the analysis. Indeed, there is no reason why industrial process models cannot be directly integrated into the energy system LP.

Given all of the previous emphasis on linear programming, it should come as no surprise that we turn once again to this technique. Because of the ease with which linear programming problems can now be solved, LP is a particularly appropriate tool in such situations, since it allows one to make many model runs at relatively low cost. This is important insofar as in almost every planning application, one would want to exercise such a model using a variety of objective functions (minimization of system cost, minimization of capital or foreign exchange requirements, maximization of some macroeconomic indicator, and so forth), and under a variety of different assumptions (or, in the jargon of the trade, for a number of scenarios), particularly varying those factors over which the planner has no control (such as the world oil price). Indeed, analyzing the robustness of decisions that emerge with respect to the underlying assumptions may be one of the most important reasons for a modelling exercise, and certainly more important than making "forecasts" of what might happen (which usually turn out to be wrong anyway).

Another reason for using LP, as opposed to mere heuristics or simulation models, is the economic interpretation of the dual. As noted in Chapter 5, shadow prices can be extremely valuable as guidance for policy making.

In passing one may also note some optimization approaches that are clearly inappropriate to developing countries, particularly the modelling route followed by the Dept. of Energy in the United States that rests on the so-called market clearing price equilibrium models. These models are typically iterative Linear Programs (such as the PIES model)[1] or non-linear network traversal algorithms (such as the DFI model).[2] Our objections are twofold. First, the huge data requirements preclude meaningful analysis: even if it were possible, say, to estimate non-linear cost functions (one of the putative benefits of network traversal algorithms being the ability to handle almost any functional specification) the degree of precision may be entirely spurious. Second, and more importantly, for most developing countries (and especially oil importers) the notion of energy supply curves is far-fetched, since energy prices are set by planning authority, not established by "market clearing equilibrium." To be sure, such models will yield what equilibrium prices should be: but these can just as easily be established from the shadow prices of a more simple linear program.

[1]The Project Independence Energy System, developed originally for the Federal Energy Administration, is now in use by the U.S. DOE for mid-term forecasting.

[2]Originally called the Gulf-SRI model, with furhter refinements added by Decision Focus Incorporated (DFI), it has been adapted by the U.S. Department of Energy for long range planning (where it is known as LEAP for Long Range Energy Analysis Program).

In exercise E10 we derive a full statement of the energy system LP as

$$
\begin{array}{llllllll}
\text{Min} & C_1X_1 + C_2X_2 + C_3X_3 + CRF\lambda_sW_s + CRF\ \lambda_cW_c & & & & & & \\
\text{s.t.} & G_1X_1 & & & & & = D & \text{Demand constraint} \\
 & & G_2X_2 & & & & \le S & \text{Supply constraint} \\
 & G_3X_1 & & G_4X_4 & & & = 0 & \text{IEF/Demand constraint} \\
 & & G_5X_2 & G_6X_3 & & & = 0 & \text{IEF/Supply constraint} \\
 & & H_1X_2 & & -IW_s & & \le \bar{W}_s & \text{Capacity constraint for energy supply} \\
 & & & H_2X_3 & & -IW_c & \le \bar{W}_c & \text{Capacity constraint for energy conversion} \\
 & & & & \lambda_sW_s & \lambda_cW_c & \le k & \text{Capital constraint}
\end{array}
\qquad (10.1)
$$

where

X_1 = demand variables

X_2 = Supply variables

X_3 = Intermediate Energy Form (IEF)

G_j = Coefficient matrices associated with supply/demand balances

C_j = Cost vectors

I = Identity matrix

λ_s = Cost per unit of new capacity of energy supply

λ_c = Cost per unit of new capacity of energy conversion facilities

W_s = New energy supply capacity

W_c = new energy conversion capacity

$\bar{W}_s$ = vector of existing capacity of energy supply facilities

$\bar{W}_c$ = vector of existing capacity of energy conversion facilities

H_j = coefficient matrices associated with capacity constraints

CRF = capital recovery factor

k = capital expenditure limitation

Recall also the I/O model of Eq (6.17), viz.

$$
\begin{array}{llll}
A_{SS}X_S + A_{SP}X_P & & + f_S & = X_S \\
A_{PS}X_S + & + A_{PI}X_I & + f_P & = X_P \\
A_{IS}X_S & + A_{II}X_I & + f_I & = X_I
\end{array}
$$

Solving the I/O model yields X_P and X_S, the gross output of energy services, and energy supply, respectively. Recall also that the first equation of 6.17 is nothing more than a mathematical representation of a reference energy system: the matrices A_{SS} and A_{SP} represent the allocation of energy services to different intermediate energy forms and the conversion of energy supplies to intermediate energy forms.

The I/O and LP can be linked in two different ways. The first method, which was the original linkage developed at Brookhaven National Laboratory, rests on an iterative scheme in which I/O and LP are solved sequentially. For some fixed A_{SP} and A_{SS}, the I/O is solved to yield gross output X_P. This X_P is then used as input to the LP to obtain an optimal configuration of the energy system. This in turn redefines the A_{SP} and A_{SS} submatrices, which are reinserted into the I/O, for a second calculation of X_P. This process continues until convergence is obtained: many years of experience at BNL, and a recent analytical proof by Lee (1982), confirms the convergence property. This iterative scheme is sketched on Figure 10.1.

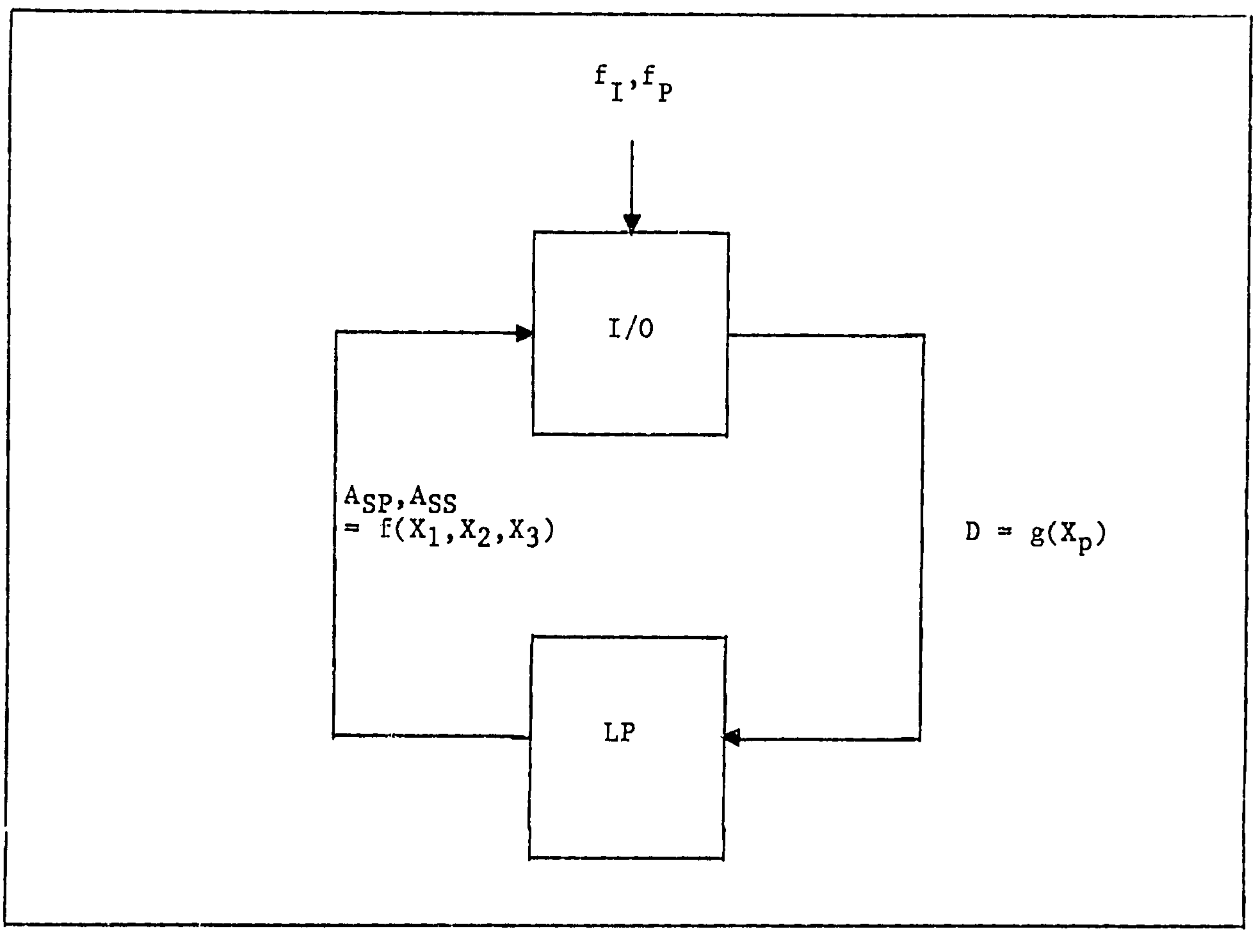


Figure 10.1. Iterative Solution of LP and I/O

A more direct approach by [Meier and Mubayi (1981)] directly integrates the I/O into the LP; the I/O equations are simply added as additional constraints. The equation for energy demands in the I/O model Eq. (6.17) is

$$A_{PS}X_S + A_{PI}X_I + f_P = X_P \quad .$$

Now in the notation of the LP, $X_p = D$ (the energy demand vector), and $X_S = X_2$ (the vector of energy supply variables). Thus (10.2), in LP notation, becomes

$$A_{PS}X_2 + A_{PI}X_I + f_P = D \qquad (10.2)$$

where

X_I = gross output of non energy sectors

f_P = final demand for energy

A_{PS} = coefficient matrix, energy demand per unit of energy supply supply output (see Table 6.3)

A_{PI} = coefficient matrix, energy demand per unit of non-energy sector output (see Table 6.3)

making D one of the LP variables, and recognizing that f_P, the final demand of energy in households, government is exogenous, (10.3) becomes

$$A_{PS}X_2 + A_{PI}X_I - D = -f_P \quad . \qquad (10.3)$$

The non-energy outputs, X_I, are now also in the LP, and are given by the third equation of (6.17), viz.

$$A_{IS}X_S + A_{II}X_I + f_I = X_I \qquad (10.4)$$

which, in LP notation, and taking f_I, the final demand for the non-energy sector, to the right hand side, yields

$$A_{IS}X_2 + (A_{II} - I) \cdot X_I = -f_I \quad . \qquad (10.5)$$

Thus the combined LP/IO can be written

$$
\begin{array}{lllllllcl}
 & C_1X_1 + & C_2X_2 + & C_3X_3 + & CRF\lambda_sW_s + & CRF\lambda_cW_c & & & \\
\text{s.t.} & G_1X_1 & & & & & - D & = & 0 \\
 & & G_2X_2 & & & & & \leq & S \\
 & G_3X_1 & & G_4X_3 & & & & = & 0 \\
 & & G_5X_2 & G_6X_3 & & & & = & 0 \\
 & & H_1X_2 & & - IW_s & & & \leq & W_s \\
 & & & H_2X_3 & & -IW_c & & \leq & W_c \\
 & & & & \lambda_sW_s & \lambda_cW_c & & \leq & k \\
 & & A_{PS}X_2 & & & & A_{PI}X_I - D & = & -f_P \\
 & & A_{IS}X_2 & & & & (A_{II}-I)X_I & = & -f_I
\end{array}
\qquad (10.6)
$$

This system is driven, as is the iteratively linked system, by f_P and f_I, which now appear on the right hand side of the LP constraint matrix.

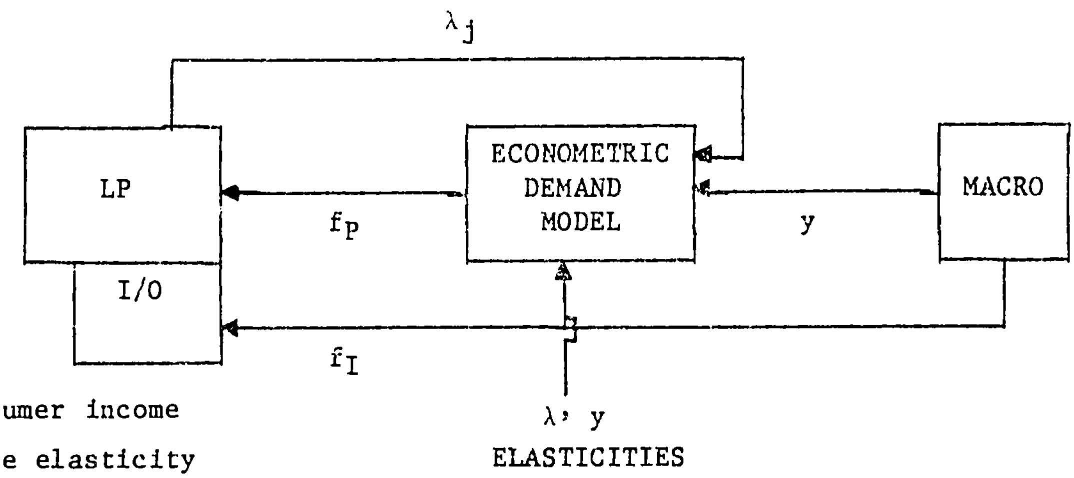
Two at first glance quite different approaches -- econometric and technological -- have been suggested for attacking the problem of energy consumption in final demand (i.e., by households and government). Conventional econometric estimation of price and income elasticities for energy consumption (differentiated perhaps by fuel type -- electricity, oil, etc.) is typically the prescription. There are several problems here: elasticities estimated from past time series (or even from cross-section analyses, for that matter) may have little relevance to likely future conditions, especially under a price regime that is quite different from past experience. Moreover, technological changes and end-use device and fuel substitutions are not considered explicitly, but simply implied.

From a conceptual point of view, however, there is no reason why the econometric regressions cannot be in terms of basic energy demands rather than fuel type. For example, one can just as well regress automobiles, or vehicle miles travelled, against personal income and gasoline and diesel fuel prices, rather than the more conventional regressions of fuel consumption on income and price. The objection may of course be that data on vehicle miles travelled (VMT), or on the characteristics of the vehicle fleet, may not be available. However, even in such a situation it may be better to use an indirect estimate of VMT from total gasoline sales by making some assumption on fleet gas mileage. It is extremely difficult to estimate the effect of improvements in automobile mileage, or a mass transit policy, if standard fuel price elasticities are all that is available.

How does one proceed computationally? Suppose that an estimate of consumer income (Y) is available from a macromodel (or specified by assumption), and that price and income elasticities (ϵ_λ, ϵ_y) are available from country data or simply exogenously assumed. Then, given some initial estimate of the price vector λ_o, f_p follows from the econometrically estimated functions

$$f_p = g_p\,(\lambda_o,\ Y,\ \epsilon_\lambda,\ \epsilon_y) \tag{10.7}$$

Assume also that the f_I, the vector of non-energy final demands, is available from the macromodel.[3] Then f_I and f_P are used to drive the combined IO/LP. This yields a set of costs and shadow prices, from which, given some assumptions concerning pricing policy, one can determine a price vector λ_1 which is inserted in (10.7) in place of λ_0, and the process is repeated. Two or three iterations may be necessary to bring energy prices, the LP/IO solution, and an assumed set of income and price elasticities into balance. These relationships are illustrated on Figure 10.2. Unfortunately, in a completely rigorous system, one would need a consistently specified set of consumer expenditures, with all the intellectual baggage of Engel elasticities, cross price elasticities, and Cournot Aggregations.[4] We are unaware of such a scheme being implemented for an energy assessment[5]:



y = consumer income
ϵ_λ = price elasticity
ϵ_y = income elasticity
λ_j = energy prices

Figure 10.2: Linking LP/IO to the macro-model

[3]One of the problems here is the fact that the sectoral decomposition necessary for non-energy sector demands may not correspond with the sectoral definition in the macro-model, which is typically highly aggregated. For example, all industry may be lumped together as "manufacturing" yet the I/O requires disaggregation into specific major industry groups if the results are to have any utility. Guidance is usually available, however, from individual sectoral plans which probably do provide the necessary disaggregation levels, or some more explicit mathematical scheme may be used to disaggregate the macromodel final demand categories.

[4]For a good discussion of such systems (and the Linear and Direct Addilog Consumer Demand Systems in particular), see e.g., Taylor, (1979).

probably there is considerable merit in keeping to simple functional forms for (10.7).

A somewhat different scheme, based on the technological approach, can also be used to bring prices and quantities into balance. One begins, again, with some basic assumptions of macroeconomic growth: population, households and so on. From these, using other planning data, one derives a detailed picture of the technological structure of end-use demand: the spatial distribution of housing (urban, rural), the type of housing (single family, multi family), vehicle and appliance ownership, and so on, from which one can directly derive the magnitudes of the elements of the f_p vector (at the moment without any reference to price). Non-energy sector final demands (f_I) are estimated as before -- either directly from the macro-model or with an appropriate disaggregation.

The LP/I-O solution proceeds, as before, to yield some price vector. The part of total consumer disposable income allocated to energy purchases, Y_E, is then given by the vector product

$$y_E = \underset{(1xn)}{\lambda_p} \cdot \underset{(nx1)}{f_p} \tag{10.8}$$

This can then be compared with the total consumer disposable income projection y for reasonableness, or compared against historical data and current trends. Likely as not, some changes will have to be made in assumptions (probably downward revisions of VMT, appliance saturations, and so forth), and the scheme proceeds iteratively until balance is reached between consumer energy expenditures, total consumer expenditure, a set of technological assumptions, energy prices, and the LP/I-O solution. Graphically these interactions can be portrayed as indicated on Figure 10.3.

[5]Although such schemes have been attempted for general development planning models - as noted in Taylor (1979).

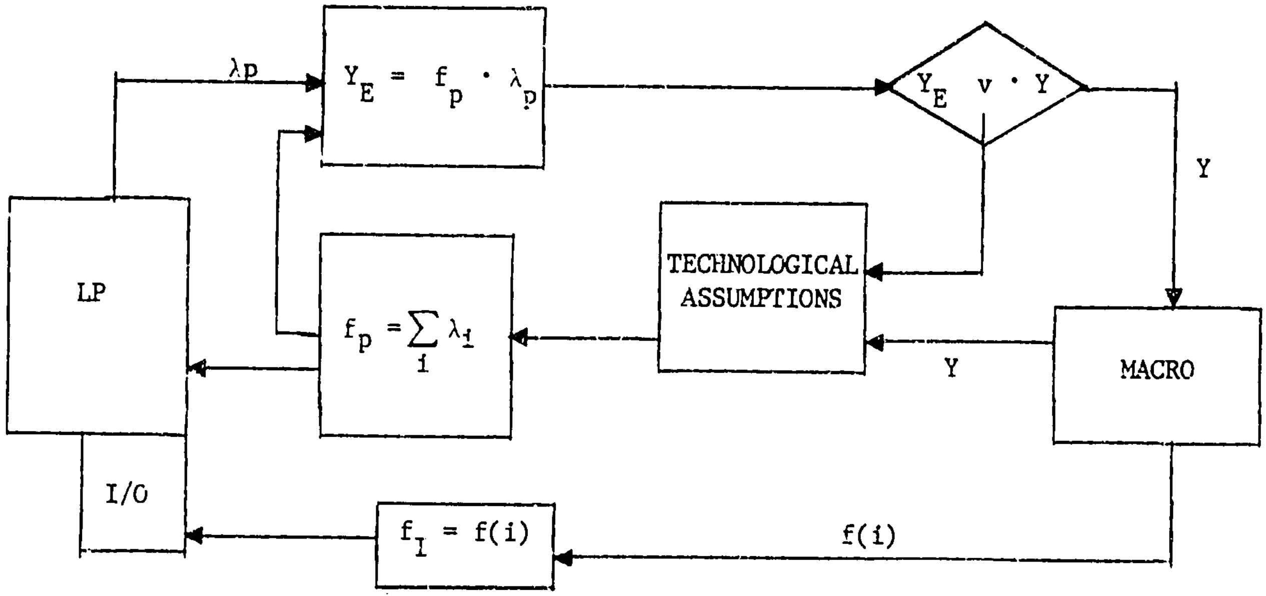


Figure 10.3. LP/IO-Macro Linkages

10.4 PROCESS MODEL LINKAGES

The question of technological constancy is a matter of great concern when projecting interindustry tables over the long-term. In input-output terms, the matrix of technical coefficients represents the technology of the economy, and the assumption that each element on the matrix does not change over time implies that technology used in the future will be identical to that today. It also implies that the economy will exhibit constant returns to scale in expanding output.

Clearly this is not an assumption that will adequately represent reality. It is likely that technical advances and improvements in efficiency will be observed in most economies over time. This is of particular importance in the context of energy systems analysis, as one of he most important policy targets is the level and type of technology used. Therefore, it is likely that some modification or time trending of the coefficients will be necessary, both to construct an adequate reference case and to explore the ramifications of additional technological change in policy cases to be compared with the reference case.

Modification of technical coefficients for future years requires a great deal of careful judgment on the part of the user. In many cases, no information will be available on which to base a modification and in these cases no changes will be made. However, there are various criteria by which future technological configurations can be estimated. For example, a given industry will generally include various processes of various vintages all producing the same product. Thus, the coefficient contained in the base year matrix represents an average for all of the productive units making up the sector. A reasonable assumption is that over time the overall performance of the industry will approach that of the "best practice" available in the base year, either domestically or in some more technologically advanced industrialized country.

Conceptually more satisfying is the explicit incorporation of industry process models into the extended LP/I-O: since process models themselves are generally in a mathematical programming form, their direct integration into a larger LP poses few difficulties.

Process models have the general form

$$
\begin{aligned}
&\min S = \alpha_k E_k + \omega_\ell F_\ell + V_k \Omega_j CRF(i,n) \\
&\text{s.t.}\quad z_j - V_j \le \dot{V}_j : j = 1,\dots,J \\
&\sum_J z_j \ge z \\
&\sum_J z_j e_{jk} - E_k \le 0 \\
&\sum_J z_j f_{j\ell} - F_\ell \le 0, \text{ or } F^* \text{ if resource constraints apply}
\end{aligned}
\tag{10.9}
$$

where

- z is the total industry output
- z_j is the output of the j-th process
- $\bar{V}_j$ is the existing capacity of process j
- V_j is the capacity expansion of process j
- Ω_j is the capital cost for expansion of process j
- CRF(i,n) is the capital recovery factor at discount rate i and planning horizon n
- e_{jk} is the unit requirement of energy of type k for process j
- α_k cost per unit of k
- $f_{j\ell}$ is the unit requirement for the non-energy factor of production ℓ for process j
- ω_ℓ cost per unit of non energy factor ℓ
- E_k total requirement of energy resource k
- F_ℓ total requirement of non-energy factor ℓ.

This must now be integrated into the extended scheme of (10.6). The z of (10.9) represents one of the elements of the gross output vector X_I. If one makes the not unreasonable assumption that the output of the different processes considered within an industry process model are indistinguishable (the assumption that steel from a basic oxygen furnace is comparable to the steel produced by an open hearth furnace in the steel industry, for example, is a lot less tenuous than many other types of assumptions considered in these pages), then one need be concerned only with the input coefficients. Moreover, in first approximation, let us ignore the interindustry effects on the input side -- we assume that the inputs to steel, for example, stated in the original I/O table, continue to apply to all of the technologies in the steel industry process model: this is a much more tenuous assumption, since clearly the requirements for construction and machinery for a basic oxygen furnace will be quite different to those of an electric arc furnace.[6]

The integration of this scheme, then, proceeds as follows. First, we modify the demand equation of (10.6), namely

$$A_{PS}X_2 + A_{PI}X_I = f_I \qquad (10.10)$$

by setting to zero the column of A_{PI} corresponding to the steel industry, the revised matrix denoted A'_{PI}. We then add to LP the set of variables Z_j, representing output of the different steel industry technologies, whose unit energy demands are given by the matrix $A_{PI}(z)$. Thus (10.10) becomes

$$A_{PS}X_2 + A_{PI}X'_I + A^*_{PI}(z) = f_I \qquad (10.11)$$

one also needs to ensure that the sum of the steel industry outputs equals gross steel industry output, i.e.,

$$\sum Z_j - X_I(S) = 0 \qquad (10.12)$$

where $X_I(s)$ is that element of X_I corresponding to the steel industry.

At this point the LP would make the choice among steel technologies solely on the basis of minimization of energy costs, a somewhat questionable proposition. In practice one would also need to include the capacity constraints and add capital and non-energy terms into the objective function: whilst there is no mathematical problem in so doing, one must remember that one is now optimizing more than just the energy system, and that any shadow prices are in terms not just of energy system objectives, but in terms of somewhat wider criteria.

---

[6]Of course to the extent that interindustry input requirements are available by process, rather than by industry, there is no problem. Actually such data may well be available from industrialized countries, which for countries that are likely to import the technology, would not be unreasonable.

The full model can therefore be written

$$
\begin{array}{llllllllllll}
\text{Min:} & C_1X_1 & + C_2X_2 & + C_3X_3 & + CRF\lambda_sW_s & + CRF\lambda_cW_c & + & & CRF\ z & & + \alpha F & \\
 & G_1X_1 & & & & & & & & -D & = & 0 \\
 & & G_2X_2 & & & & & & & & \leq & S \\
 & G_5X_1 & & G_4X_3 & & & & & & & = & 0 \\
 & & G_5X_2 & G_6X_3 & & & & & & & = & 0 \\
 & & H_1X_2 & & IW_s & & & & & & = & \bar{W}_s \\
 & & & H_2X_3 & & IW_c & & & & & = & \bar{W}_c \\
 & & & & \lambda_5W_s & \lambda_cW_c & & & & & = & k \\
 & & & A_{PS}X_2 & & & A_{PI}'X_I & A_{PI}^*z & & & = & f_P \\
 & & & A_{IS}X_2 & & & (A_{II}-I)X_I & & & -D & = & f_I \\
 & & & & & & X_I(s) & z & & & = & 0 \\
 & & & & & & & z & -V & & \leq & \bar{V} \\
 & & & & & & & f_z & & -F & \leq & F^*
\end{array}
$$

In the entire discussion thus far of I/O we have not made any rigorous distinction between physcial output and price: clearly each entry in the transactions table represents the product of the unit price of sectoral output times physical volume of output. Prices follow from the output side from the equation:

$$\underset{\substack{\text{Total}\\\text{Sales}}}{P_i X_i} = \underset{\substack{\text{Sales to}\\\text{Inter-}\\\text{mediate}\\\text{Demand}}}{\sum_{j=1}^{n} P_i X_{ij}} + \underbrace{P_i C_i + P_i G_i + P_i E_i + P_i I_i}_{\substack{\text{Sales to}\\\text{Final Demand}}} \tag{10.13}$$

where P_i is the output price in sector i, (in \$/physical units), and X_{ij} are the purchases of commodity i by sector j. In the I/O framework, these are assumed to be given by a constant coefficient times output in purchasing sectors, i.e.

$$X_{ij} = a_{ij} \cdot X_j \tag{10.14}$$

whence (10.14) becomes

$$P_i X_i = \sum_{j=1}^{n} P_i a_{ij} X_j + P_i F_i \tag{10.15}$$

writing out (10.15) for a two sector economy

$$\begin{aligned} P_1X_1 &= P_1a_{11}X_1 + P_1a_{12}X_2 + P_1F_1 \\ P_2X_2 &= P_2a_{21}X_1 + P_2a_{11}X_2 + P_2F_2 \end{aligned} \tag{10.16}$$

hence, in matrix form

$$\underset{P}{\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}} \underset{X}{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}} = \underset{A^*}{\begin{bmatrix} P_1a_{11} & P_1a_{12} \\ P_2a_{21} & P_2a_{22} \end{bmatrix}} \underset{X}{\begin{bmatrix} X_1 \\ X_2 \end{bmatrix}} + \underset{P}{\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix}} \underset{F}{\begin{bmatrix} F_1 \\ F_2 \end{bmatrix}} \tag{10.17}$$

but the matrix A* can be written

$$\begin{bmatrix} P_1 & 0 \\ 0 & P_2 \end{bmatrix} \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} = \begin{bmatrix} P_1a_{11} \\ P_1a_{21} \end{bmatrix} \begin{bmatrix} P_2a_{12} \\ P_2a_{22} \end{bmatrix} \tag{10.18}$$

hence

$$PX = PAX + PF \quad . \tag{10.19}$$

Obviously, if prices stay constant, one can premultiply (10.19) by P^{-1} to get

$$X = AX + F \tag{10.20}$$

which is identical to the formulation in Chapter 6, but where X now represents physical volume rather than monetary sales.

Prices also follow from the input side. If V_i is the value added per (physical) unit of sales sector i, then

$$\underset{\left[\frac{\$}{\text{unit of } j}\right]}{P_i} = \underset{\left[\frac{\$(\text{value added})}{\text{units of } i}\right]}{V_i} + \sum_{j=1}^{n} \underset{\left[\frac{\text{units of } j}{\text{units of } i}\right]}{a_{ji}} \cdot \underset{\left[\frac{\$}{\text{unit of } j}\right]}{P_j} \tag{10.21}$$

hence, in matrix notation

$$\underset{(1xn)}{P^T} = \underset{(1xn)}{V^T} + \underset{(1xn)}{P^T} \cdot \underset{(nxn)}{A} \tag{10.22}$$

which can be solved for P^T in the usual way:

$$P^T (I - A) = V^T$$

$$P^T = V^T(I - A)^{-1} \tag{10.23}$$

If we decompose the value added sector into labor, capital, non-competitive (NC) imports and energy products

$$
\begin{aligned}
\underset{\left[\frac{\$}{\text{output of } i}\right]}{V_i} &= \underset{\left[\frac{\text{employee}}{\text{output of } i}\right]}{a_{Li}} \underset{\left[\frac{\$}{\text{employee}}\right]}{w_i} + \underset{\left[\frac{\$\ \text{capital}}{\text{output of } i}\right]}{a_{Ki}} \underset{\left[\frac{\$}{\$\ \text{capital}}\right]}{k_i} \\
&\quad + \underset{\left[\frac{\text{NC imports}}{\text{output of } i}\right]}{a_{NCi}} \cdot \underset{\left[\frac{\$}{\text{NC imports}}\right]}{P_i^{NC}} \\
&\quad + \sum_j \underset{\left[\frac{\text{Btu energy product } j}{\text{output } i}\right]}{a_{PIij}} \cdot \underset{\left[\frac{\$\ \text{unit energy product}}{\text{Btu energy product } j}\right]}{\lambda_j}
\end{aligned}
\tag{10.24}
$$

where λ_j is the price for energy service j. Leaving aside for the moment how we determine the vector of energy prices, it should be clear how the above equations can be used to determine the economy wide price impacts of a perturbation of one of the unit energy, labor or capital requirements in the value added vector. Of course, this framework says nothing about how quickly and price changes would propagate through the economy, nor indeed how consumers or other industries might react to such changes. As a first order measure of the impact of energy sector price changes, however, the price interpretation of the I/O framework appears to have some value.

How then do we determine the λ vector? Computationally the most straight-forward is the use of average price, which can be directly calculated from the LP variables. Suppose x* be the total quantity of some energy service, say electricity, which will be one of the variables of the LP. Then average price is simply

$$
(X^*) = \frac{\sum_{i \in i^*} c_i X_i}{X^*} \tag{10.25}
$$

where i i^* is the set of activities that are required to produce a unit of electricity: the expression $_{i\ i}$ $C_i X_i$ can be included as a free row in the LP for computational convenience. Indeed, this term might be called the total revenue requirement for electricity production, since it captures the costs that must be recovered from consumers (and from the government in the form of subsidies) to meet the stated production level. How these revenue requirements are to be distributed among customer classes is of course a matter of policy: one point of guidance would be the shadow prices associated with the corresponding end-use demand constraints. Indeed, for advocates of pure marginal cost pricing, the LP shadow prices themselves can be used as the basis for pricing.

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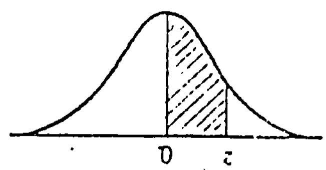
APPENDIX A

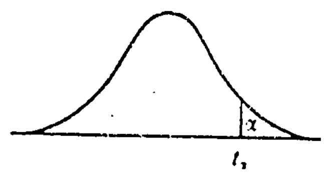
STATISTICAL TABLES

Table A1
Areas Under the Normal Curve



z	.00	.01	.02	.03	.04	.05	.06	.07	.08	.09
0.0	.0000	.0040	.0080	.0120	.0160	.0199	.0239	.0279	.0319	.0359
0.1	.0398	.0438	.0478	.0517	.0557	.0596	.0636	.0675	.0714	.0753
0.2	.0793	.0832	.0871	.0910	.0948	.0987	1026	.1064	.1103	.1141
0.3	.1179	.1217	.1255	.1293	.1331	.1368	.1406	.1443	.1480	.1517
0.4	.1554	.1591	.1628	.1664	.1700	.1736	.1772	.1808	.1844	.1879
0.5	.1915	.1950	.1985	.2019	.2054	.2088	.2123	.2157	.2190	.2224
0.6	.2257	.2291	.2324	.2357	.2389	.2422	.2454	.2486	.2517	.2549
0.7	.2580	.2611	.2642	.2673	.2704	.2734	.2764	.2794	.2823	.2852
0.8	.2881	.2910	.2939	.2967	.2995	.3023	.3051	.3078	.3106	.3133
0.9	.3159	.3186	.3212	.3238	.3264	.3289	.3315	.3340	.3365	.3389
1.0	.3413	.3438	.3461	.3485	.3508	.3531	.3554	.3577	.3599	.3621
1.1	.3643	.3665	.3686	.3708	.3729	.3749	.3770	.3790	.3810	.3830
1.2	.3849	.3869	.3888	.3907	.3925	.3944	.3962	.3980	.3997	.4015
1.3	.4032	.4049	.4066	.4082	.4099	.4115	.4131	.4147	.4162	.4177
1.4	.4192	.4207	.4222	.4236	.4251	.4265	.4279	.4292	.4306	.4319
1.5	.4332	4345	.4357	.4370	.4382	.4394	.4406	.4418	.4429	.4441
1.6	.4452	.4463	.4474	.4484	.4495	.4505	.4515	.4525	.4535	.4545
1.7	.4554	.4564	.4573	.4582	.4591	.4599	.4608	.4616	.4625	.4633
1.8	.4641	.4649	.4656	.4664	.4671	.4678	.4686	.4693	.4699	.4706
1.9	.4713	.4719	.4726	.4732	.4738	.4744	.4750	.4756	.4761	.4767
2.0	.4772	.4778	.4783	.4788	.4793	.4798	.4803	.4808	.4812	.4817
2.1	4821	.4826	.4830	.4834	.4838	.4842	.4846	.4850	.4854	.4857
2.2	.4861	.4864	.4868	.4871	.4875	.4878	.4881	.4884	.4887	.4890
2.3	.4893	.4896	.4898	.4901	.4904	.4906	.4909	.4911	.4913	.4916
2.4	.4918	.4920	.4922	.4925	.4927	.4929	.4931	.4932	.4934	.4936
2.5	.4938	.4940	.4941	.4943	.4945	.4946	.4948	.4949	.4951	.4952
2.6	.4953	.4955	.4956	.4957	.4959	.4960	.4961	.4962	.4963	.4964
2.7	.4965	.4966	.4967	.4968	.4969	.4970	.4971	.4972	.4973	.4974
2.8	.4974	.4975	.4976	.4977	.4977	.4978	.4979	.4979	.4980	.4981
2.9	.4981	.4982	.4982	4983	.4984	.4984	.4985	.4985	.4986	.4986
3.0	.4987	.4987	.4987	.4988	.4988	.4989	.4989	.4989	.4990	.4990

Table A2
Critical Values of t



n	$t_{.100}$	$t_{.050}$	$t_{.025}$	$t_{.010}$	$t_{.005}$	$d.f.$
2	3.078	6.314	12.706	31.821	63.657	1
3	1.886	2.920	4.303	6.965	9.925	2
4	1.638	2.353	3.182	4.541	5.841	3
5	1.533	2.132	2.776	3.747	4.604	4
6	1.476	2.015	2.571	3.365	4.032	5
7	1.440	1.943	2.447	3.143	3.707	6
8	1.415	1.895	2.365	2.998	3.499	7
9	1.397	1.860	2.306	2.896	3.355	8
10	1.383	1.833	2.262	2.821	3.250	9
11	1.372	1.812	2.228	2.764	3.169	10
12	1.363	1.796	2.201	2.718	3.106	11
13	1.356	1.782	2.179	2.681	3.055	12
14	1.350	1.771	2.160	2.650	3.012	13
15	1.345	1.761	2.145	2.624	2.977	14
16	1.341	1.753	2.131	2.602	2.947	15
17	1.337	1.746	2.120	2.583	2.921	16
18	1.333	1.740	2.110	2.567	2.898	17
19	1.330	1.734	2.101	2.552	2.878	18
20	1.328	1.729	2.093	2.539	2.861	19
21	1.325	1.725	2.086	2.528	2.845	20
22	1.323	1.721	2.080	2.518	2.831	21
23	1.321	1.717	2.074	2.508	2.819	22
24	1.319	1.714	2.069	2.500	2.807	23
25	1.318	1.711	2.064	2.492	2.797	24
26	1.316	1.708	2.060	2.485	2.787	25
27	1.315	1.706	2.056	2.479	2.779	26
28	1.314	1.703	2.052	2.473	2.771	27
29	1.313	1.701	2.048	2.467	2.763	28
30	1.311	1.699	2.045	2.462	2.756	29
inf.	1.282	1.645	1.960	2.326	2.576	inf.

APPENDIX B

REFINERIES IN DEVELOPING COUNTRIES

Table B1
Refineries in Africa, Latin America, and Asia

	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
ABU DHABI	Abu Dhabi National Oil Co., Umm Al-Nar	15,000		2,800	5,300 HDT	
ALGERIA	Sonatrach:					
	Arzew	60,000		8,600	8,600 HDT	6,000 V, 1,000 L, 1,100 A
	Maison Carree	60,000			15,000	
	Hassi Messaoud	2,400				
	Total	122,400		23,6000	8,600	
ANGOLA	Compania de Petroleos de Angola, Luanda	32,100		1,900	3,800 HDT	1,900 V, 950 A
					2,800 HDS	
	Total	32,100		1,900	5,600	
ARGENTINA	Astrasur, Refinerias Patagonicas de Petroleo S.A., Comodoro Rivadavia	6,300	2,200 TCC			4,400 V, 200 A
	Destileria Argentina de Petroleo S.A., Lomas de Zamora	2,000				1,500 V, 600 L
	Esso SAPA: Campana	92,000	17,600 FCC	8,000		49,700 V, 1,300 L, 500 C
	Galvan	17,000				
	Ragor SAIC, Quilmes	600				
	Refineria de Petroleo la Isaura S.A., Bahia Blanca	14,00				
	Shell Compania Argentina de Petroleo S.A., Buenos Aires	115,000	21,000 FCC	9,000	25,000 HDT	
	Yacimientos Petroliferos Fiscales: Campo Duran	28,304				
	Dock Sud	6,289				
	La Plata	251,588				1,572 V, 943 Alky, 600 C
	Lujan de Cuyo	113,215	17,611 FCC	9,435	20,756 HC	59,752 V, 1,000 C
	Plaza Huincul	23,485		2,410		2,4170 HDS
	San Lorenzo	33,335				
	Total	703,116	58,411	28,845	45,756	
BAHAMAS	Bahamas Oil Refining Co., Freeport	500,000			60,000 HDT	94,000 V
BAHRAIN	Bahrain Petroleum Co. Ltd., Awali	250,000	34,200 FCC	15,200	15,200 HDT	144,000 V, 1,300 P
					71,000 HDS	3,000 A
	Total	250,000	34,200	15,200	86,200	
BANGLADESH	Eastern Refinery Ltd, Chittagong	31,200		1,720	1,930 HDT	
					2,520	
	Total	31,200		1,720	4,450	
BARBADOS	Mobil Oil Barbados Ltd., Bridgetown	3,000				400 A
BOLIVIA	Yacimientos Petroliferos Fiscales Bolivianos:					
	Cochabamba	23,500				600 V, 200 L
	Santa Cruz	9,000				
	Sucre	3,000				
	Total	37,100				

Cracking processes:
- FCC-Fluid cat cracking
- TCC-Thermofor cat cracking
- HD-Houdriflow cat cracking

Hydroprocessing:
- DHC-Distillate hydrocracking
- RHC-Residual hydrocracking
- HDT-Catalytic hydrotreating
- HDS-Catalytic hydrodesulfurization

Other processing:
- A-Asphalt
- Alky-Alkylation
- C-Coking (t/d)
- L-Lube Manufacture
- P-Polymerization
- V-Vacuum distillation
- W-Waxes

Table B1
Refineries in Africa, Latin America, and Asia

	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
BRAZIL	Petroleo Brasilerio S.A.:					
	Araucaria, Parana	120,600	37,000 FCC			58,500 V, 2,600 A
	Betim, Minas Gerais	72,400	12,500 FCC			30,160 V, 1,000 A
	Canoas, Rio Grade do Sul	72,400	12,500 FCC			35,000 V, 1,000 A
	Cubatao, Sao Paulo	171,300	43,200 FCC	10,200		93,500 V, 5.900 A, 550 C
	Duque de Caxias-Rio de Janerio	212,900	34,100 FCC	11,400	4,700 HDT	89,200 V, 4,400 L, 4,300 A
	Manaus, Amazonas	9,600	1,900 FCC			2,800 V, 400A
	Mataripe, Bahia	79,600	20,800 FCC		4,500 HDT	38,300 V, 3,000 L, 2,200 A
	Maua, Sao Paulo	33,800	14,400 TCC			9,600 V
	Paulinia, Sao Paulo	325,700	32,400 FCC			88,700 V, 4,800 A
	Refinaria de Petroleo Ipiranga SA, Rio Grande do Sul	9,300	2,500 FCC			4,000 V, 1,200 A
	Refinaria de Petroleos de Manguinhos SA, Rio de Janeiro	10,000				
	Total	1,117,600	211,300	21,600	9,200	
BURMA	Myanma Oil Corp., Chauk	6,800				2,700 V, 50(t/d) W
	Syriam	21,500	2,000 TCC			2,300 V, 700L, 50 C
	Total	28,300	2,000			
CHILE	Empresa Nacionai del Petroleo:					
	Concepcion	72,000	14,500 FCC	4,000		38,000 V
	Concon	66,000	20,000 FCC	6,000		28,000 V, 1,100 alky
	Magallanes	1,500				
	Total	139,500	34,500	10,000		
CHINA, PEOPLES REPUBLIC	Government owned					
	Anshan	90,000				
	Chin-Hsi	130,000				
	Dairen	110,000				
	Fushun	100,000				
	Hangchow	26,000				
	Karamia-Tushantzu	75,000				
	Lanchow	132,000				
	Lenghu	22,000				
	Maoming	85,000				
	Nanchung	16,000				
	Nanking	60,000				
	Peking	100,000				
	Shanghai	100,000				
	Shengli	70,000				
	Taching	128,000				
	Takang	30,000				
	Tientsin	26,000				
	Yumen	80,000				
	Others	18,000				
	Total	1,398,000				
COLOMBIA	Empresa Colombiana de Petroleos:					
	Barrancabermeja	106,000	35,000 FCC	6,000	6,000 HDT	52,000 V, 3,000 Alky, 1,000 L, 3,000 A, 320 C
	Cartagena	48,000	14,000 FCC			28,000 V, 3,400 Poly 120 C

Cracking processes:
FCC-Fluid cat cracking
TCC-Thermofor cat cracking
HD-Houdriflow cat cracking

Hydroprocessing:
DHC-Distillate hydrocracking
RHC-Residual hydrocracking
HDT-Catalytic hydrotreating
HDS-Catalytic hydrodesulfurization

Other processing:
A-Asphalt
Alky-Alkylation
C-Coking (t/d)
L-Lube Manufacture
P-Polymerization
V-Vacuum distillation
W-Waxes

Table B1
Refineries in Africa, Latin America, and Asia

	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
	Bitu	2,500				
	International Petroleum (Colombia) Ltd.,					
	LaDorada	5,000				
	Texas Petroleum Co., El Guamo	2,500				
	Orito	1,000				
	Total	165,000	49,000	6,000	6,000	
COSTA RICA	Refinadora Costarrincense de Petroleo SA					
	Limon	8,000		1,500	1,500 HDT 2,500 HDS	600 V, 450 A
	Total	165,000	49,000	6,000	6,000	
CUBA	Instituto Cubano del Petroleo:					
	Cabaiguan	4,970				
	Havana	75,000	14,570 FCC	8,100	12,600 HDS	22,140 V, 2,675 P, 6,000 A
	Santiago de Cuba	42,000		3,600	6,000 HDS	
	Total	121,970	14,570	11,700	18,600	
CYPRUS	Cyprus Petroleum Refinery Ltd.,					
	Larnaca	15,000		2,000	5,000 HDT	
DOMINICAN REPUBLIC	Falconbridge Dominicana C por A.,					
	Bonao	16,500			6,2000 HDS	
	Refineria Dominicana de Petroleo SA					
	Nigua	30,000		9,000	16,000 HDT	
	Total	46,500		9,000	22,200	
ECUADOR	CEPE:					
	Anglo-Ecuadorian Oilfields Ltd.,					
	La Libertad	32,300				
	Texaco-Gulf, Lago Agrio	1,000				
	Petroleos Gulf del Ecuador CA,					
	La Libertad	7,600				
	Total	40,900				
EGYPT	Alexandria Petroleum Co.,					
	Alexandria	60,000				400 A
	El-Nasr Petroleum Co.:					
	Alexandria	32,000				1,000 L, 2,100 A
	Suez	32,000				2,100 A
	Suez Process Petroleum Co.:					
	Mostord	37,300				
	Suez	37,400				1,000 L, 4,110 C
	Tanta	37,300				
	Total	236,000				
EL SALVADOR	Refineria Petrolera Acajutia SA,					
	Acajutia	15,700		2,800	9,800 HDT	1,300 V, 550 A
ETHIOPIA	Ethiopian Petroleum SC, Assab	14,430		1,890	2,330 HDT	3,020 V, 697 A
GABON	Societe Gabonaise de					
	Raffinage, Port Gentil	20,000		1,520	2,120 HDT 5,500 HDS	
	Total	20,000		1,520	7,620	

Cracking processes:
FCC-Fluid cat cracking
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HD-Houdriflow cat cracking

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DHC-Distillate hydrocracking
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HDS-Catalytic hydrodesulfurization

Other processing:
A-Asphalt
Alky-Alkylation
C-Coking (t/d)
L-Lube Manufacture

P-Polymerization
V-Vacuum distillation
W-Waxes

Table B1
Refineries in Africa, Latin America, and Asia

	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
GHANA	Ghananian-Italian Petroleum Co., Tema	26,500		6,200	6,200 HDT	
GUAM	Guam Oil & Refining Co., Apra Hts	29,500				400 A
GUATEMALA	Texas Petroleum Co., Escuintla	14,000		3,000	5,000 HDT	
HONDURAS	Refineria Texaco de Honduras SA Puerto Cortes	14,000		1,800	2,700 HDT 2,100 HDS	
	Total	14,000		1,800	4,800	
INDIA	Assam Oil Co. Letd., Digboi	11,500				1,200 L, 511 A, 40 C
	Bharat Refineries Ltd., Mahul Bombay	120,000	17,000 FCC	7,000		33,000 V, 10,000 A
	Caltex Oil Refining (India) Ltd., Visakhapatam	32,000	9,100 FCC			15,200 V, 2,000 A
	Cochin Refineries Ltd., Ambalamugla, Kerala	70,000		6,227	33,043 HDS	3,020 A
	Hindustan Petroleum Corp. Ltd., Mahul, Bombay	70,000	6,000 FCC			19,440 V, 3,235 L, 5,071 A
	Indian Oil Corp. Ltd:					
	Barauni	60,000	12,800			15,000 V, 910 L, 285 C
	Baroda	82,000		7,000		
	Gauhati	16,000	6,000			125 C
	Haldia	50,000	8,500	4,500	4,500 HDT 13,000 HDS	20,000 V, 2,700 L 2,008 A
	Madras Refineries Ltd., Manali	56,000	2,158 FCC	2,158	31,820 HDT	22,300 V, 4,700 L, 4,270 A
	Total	567,500	61,558	26,885	82,363	
INDONESIA	Lemigas, Cepu, Central Java	4,000				
	Pertamina:					
	Balikpapan, Kalimantan	75,000				13,000 V
	Dumai, Central Sumatra	100,000		7,000		
	Pangakalan Brandan, N. Sumatra	4,500				200 L, 300 A
	Plaju, S. Sumatra	111,200		16,000		24,000 V, 1,100 Alky, 173 P, 1,200 A
	Sungei Gerong, S. Sumatra	79,000	19,500 FCC			87,000 V, 650Alky, 1,200 P
	Sungei Pakning, Central Sumatra	50,000				
	Wonokroma, E. Java	4,000				1,500 A
	Total	427,700	19,500	23,000		
IRAN	National Iranian Oil Co., Abadan	456,500	36,000 FCC	23,500	23,500 HDS	96,000 V, 9,000 Alky, 1,400 L, 10,500 A
	Kermanshah	18,000		3,100	6,200	
	Shiraz	40,000		6,215	9,280 DHC 8,345 HDS	18,400 V, 17,000 A
	Tehran	200,000		27,500	29,400 HDC 23,000 HDS	97,800 V, 2,400 L, 6,775 A
	Oil Service Co., of Iran, Masjid-I Sulaiman	66,000				
	Total	780,500	36,000	60,315	99,725	
IRAQ	Oil REfineries Administration:					
	Basra	70,000				
	Daura	71,000		5,000	13,000 HDT	8,160 L, 1,815 A

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	K3-Haditha	7,000				
	Khanagin	12,000				
	Mufthia	4,500				
	Qalyarah, Mosul	2,000				920 A
	Iraqi Company for Oil Operations, Kirkuk	2,000				
	Total	168,500		5,000	13,000	
ISRAEL	Oil Refineries Ltd., Ashdod	66,000		10,000	11,000 HDT 25,000 HDS	36,000 V
	Haifa	135,000	10,000 TCC	14,500	15,500 HDT 25,000 HDS	28,000 V, 800 L, 3,600 A
	Total	201,000	10,000	24,500	76,500	
IVORY COAST	Societe Ivorienne de Raffinage, Abidjan	74,575		4,950	2,970 HDT	
JAMAICA	Esso W. Indies Ltd., Kingston	32,600		3,000	18,900	1,600 V, 760 A
JORDAN	Jordan Petroleum Refinery Co., Ltd, Zarka	24,377	4,410 FCC	870	2,000 HDT	6,610 V, 1,220 A
KENYA	E. African Oil Refineries Ltd Mombasa	86,000		9,000	26,000 HDT	
KHMER REPUBLIC	*Societe Khmere de Raffinage de Petrole, Krung Kompong Som	12,540		1,800	5,850 HDT	
	*Figures estimated.					
KOREA	Honam Oil Refinery Co. Ltd., Yosu	160,000		6,600	9,500 HDT	1,400 A
	Korea Oil Corp., Ulsan	204,250		15,400	21,140 HDT 3,620 HDS	3,300 V, 1,900 A
	Kyung In Enegery Co. Ltd, Inchon	60,000		2,200	6,770 HDT	
	Total	424,250		24,240	41,030	
KUWAIT	American Independent Oil Co., Mena Abdulla	132,000			32,000 HDS	112,000 V
	Arabian Oil Co. Ltd. (Japan) Ras Al Khafji	30,000				
	Getty Oil Co., Mina Saud	50,000				
	Kuwait National Petroleum Co., Shuabia	172,800		14,200	19,600 HDC 45,900 RHC 103,900 HDS	88,300 V
	Kuwait Oil Co., Mina Al-Ahmadi	300,000		5,600		1,300 A
	Total	684,800		19,800	201,400	
LEBANON	*Iraq Petroleum Co. Ltd., Tripoli	36,000	7,250 FCC	4,400	6,730 HDT	12,730 V, 900 A
	*Mediterranean Refing Co. Sidon	17,500		2,900	2,900 HDT	
	Total	53,500	7,250	7,300	9,630	
*Refineries not in operation at present						
LIBERIA	Liberia Refining Co.., Monrovia	15,000		1,800	2,400 HDT 1,400 HDS	1,00 V, 300 A
	Total	15,000		1,800	3,800	
LIBYA	Esso Standard Libya, Marsa El-Brega	8,000		1,500	3,900 HDT	
	National Oil Co.:					
	Amal	1,800				
	Intisar	1,800				
	Zavia	120,000		13,000	35,440 HDS	

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Refineries in Africa, Latin America, and Asia

	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
	Zeutina	3,000				
	Total	134,600		14,500	39,340	
MALAGASY	Societe Malagache de Raffinage, Tamatave	16,350		2,300	5,500 HDS	
MALAYSIA	Esso Malaysia Berhad, Port Dickson	33,000		4,000	20,000 HDT	3,000 V, 1,100 A
	Sarawak Shell Berhad, Lutong	45,000				
	Shell Refining Co., Berhad, Port Dickson	90,000		4,000	11,000 HDT 5,000 HDS	
	Total	168,800		8,000	36,000	
MARTINIQUE	Societe Anonyme Raffinerie des Antilles, Fort de France	10,400		2,500	3,500 HDT 3,300 HDS	
	Total	10,400		2,500	6,800	
MEXICO	Petroleos Mexicanos:					
	Azcapotzalco	100,000	23,000 FCC		26,000 HDT	48,000 V, 3,000 alky, 2,500 P
	Ciudad Madero	169,000	23,000 FCC	15,000	33,000 HDS	80,000 V, 3,000 alky, 2,000 P, 100,000 A, 400 C
	Minatitlan	258,500	24,000 FCC 21,000 TCC	12,00	50,000 HDS	56,000 V, 2,100 P, 3,000 L
	Poza Rica	27,000			5,000 HDT	
	Reynosa	20,500				
	Salamanca	210,000	18,000 TCC	8,000	45,600 HDS 18,000 RHC	96,200 V, 9,650 L, 3,000 A
	Tula, Hidalgo	150,000	40,000 FCC	30,000	86,000 HDS	
	Total	935,000	149,000	65,000	263,600	
MOROCCO	Government:					
	Sidi-Kacem	17,800	4,000 TCC	3,000		7,300 V
	Mohammedia	50,000		6,500	6,500 HDT 2,200 HDS	
	Total	67,800	4,000	9,500	8,700	
NETHERLANDS ANTILLES	Lago Oil & Transport Co., Ltd., Aruba	440,000			278,000 HDS	230,000 V, 1,000 alky
	Shell Curacao NV, Emmastad	370,000	39,000	17,000	100,000 HDT 25,000 HDS	
	Total	810,000	39,000	17,000	403,000	
NICARAGUA	Esso Stand. Oil SA Ltd., Managua	14,900		2,800	10,200 HDS	1,900 V, 800 A
NIGERIA	Nigerian Petroleum Refining Co. Ltd., Alesa-Eleme	60,000		6,500		
PAKISTAN	Attock Oil Co. Ltd, Rawalpindi	11,000				2,000 V, 200 L, 155 A
	National Refinery Ltd., Korangi, Karachi	12,888			1,585 HDT	4,897 V, 1,585 L, 1,105 A
	Pakistan Refinery Ltd., Korangi, Karachi	50,000		2,500	21,000 HDT	
	Total	73,888		2,500	22,585	
PANAMA	Refineria Parama SA Las Minas, Colon	100,000		7,500	30,000 HDT	14,000 V, 5,000 A

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	Company and Refinery Location	Crude	Catalytic Cracking	Catalytic Reforming	Hydro-Processing	Other Processing
PERU	Petroleos del Peru:					
	Conchan	8,500				3,500 V, 1,800 A
	Iquitos (Loreto)	1,500				
	La Pampilla (Callao)	37,000	6,000 FCC	1,750	2,760 HDT	
	Marsella (Loreto)	1,200				
	Pucalipa (Loreto)	2,700				
	Talara (Piura)	65,000	16,600 FCC			20,000 V, 200 L, 700 A
	Total	115,900	22,600	1,760	2,700	
	Note: Marsella refinery operated by Occidental Petroleum under contract.					
PHILLI-PPINES	Battan Refining Corp., Limay	112,000	10,350 TCC	22,000	47,900 HDT 17,100 HDS	18,450 V, 4,600 A
	Caltex (Phillippines Inc. Batangas	71,000	11,900 FCC	8,600	11,400 HDT 15,200 HDS	18,100 V, 900 P
	Pilipinas Shell Petroleum Corp., Tabangao	68,000		8,000	12,000 HDT 6,000 HDS	
	Total	251,000	22,250	38,600	109,600	
QATAR	National Oil Dist. Co. Umm Said	9,150		1,300	3,455 HDT	
RHODESIA	Central African Petroleum Refineries (Pvt.) Ltd., Umtali	20,000	(Not in operation, not included in total)			
SAUDI ARABIA	Arabian American Oil Co. Ras Tanura	547,500		13,500	21,100 HDS	122,700 V, 12,100 A
	Jeddah Oil Refinery Co., Jeddah	40,600	7,800 FCC	2,300 8,500 HDS	2,300 HDT	12,100 V, 1,800 A
	Riyadh Oil Refinery, Riyadh	15,100		5,300	6,300 DHC 3,100 HDT	5,800 V, 2,500 A
	Total	603,200	7,800	21,100	41,300	
SENEGAL	Societe Africaine de Raffinage, Dakar	18,800		2,100		
SIERRA LEONE	Sierra Leone Petroleum Refining Co. Ltd. Freetown	10,000				
SINGAPORE	BP Refinery Singapore Pte. Ltd., Pasir Panjang	265,650				
	Esso Singapore Pvt. Ltd., Pulau Ayer Chawan	192,000		6,000	42,000 HDT	48,000 V, 5,000 L, 3,600 A
	Mobile Oil Singapore Pet. Ltd/. Jurong	175,000		4,000	4,000 HDT 32,000 HDS	
	Shell Eastern Petroleum Ltd., Pulau Bukom	460,000		10,000	160,000 HDT 22,000 HDS	
	Singapore Petroleum Co. Pte. Ltd., Pulau Merlimau	65,000			16,000 DHC 20,000 HDS	29,000 V, 2,000 A
	Total	917,650		20,000	296,000	
SOUTH YEMEN	BP Refinery (Aden) Ltd., Little Aden	169,100		9,000	2,700 HDS	
SRI LANKA	Ceylon Pet. Corp. Sapugaskande	38,000		3,750	15,300 HDT 2,100 HDS	1,900 V, 950 A
	Total	38,000		3,750	17,400	

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SUDAN	Shell & BP Sudan Ltd. Port Sudan	23,750		2,000	9,000 HDT	
SYRIA	Homs Petroleum Refining Co., Homs	116,728		3,120	25016 HDT	3,465 V, 2,490 A, 684 C
TANZANIA	Tanzanian and Italian Petroleum Refining Co. Ltd, Dar es Salaam	12,200		3,500	5,000 HDT	
THAILAND	Esso Standard Thailand Ltd., Sriracha	35,000		5,000	17,000 HDT	6,000 V, 2,600 A
	Summit Industrial Corp. (Panama), Bangkok	59,657		9,178	5,000 HDT 14,684 HDS	2,000 V, 1,500 A
	Fang, Chieng-Mai	1,000				300 L
	Thai Oil Refinery Co. Ltd., Sriracha, Cholburi	65,000	7,000 FCC	9,000	32,000 HDT 10,000 HDS	14,500 V, 1,400 A
	Total	160,657	7,000	23,178	78,684	
TRINIDAD	Texaco Trinidad Inc. Pointe-a-Pierre	361,000	26,500 FCC	20,000	45,000 HDT 80,000 HDS	187,000 V 2,200 alky, 350 P, 4,000 L
	Trinidad and Tobago Oil Co. Ltd., Point Fortin	100,000		7,000	22,000 HDT	7,000 V, 1,600 A
	Total	461,000	26,500	27,000	147,000	
TUNISIA	Societe Tunisienne des Industries de Raffinage, Bizerte	22,000		3,300		
TURKEY	Anadoli Tasfiyehanesi AS, Mersin	90,000		10,500	22,000 HDT 11,500 HDS	
	Istanbul Petrol Rafinerisi AS, Izmit	150,000	24,000 FCC	10,000	13,300 HDT 24,000 HDS	80,000 V, 10,000 A
	Turkish Petroleum Co.:					
	Aliaga	62,700		5,760	6,660 HDT	36,270 V, 2,700 L
	Batman	22,800	3,600 TCC	1,170	1,170 HDT	2,970 A
	Total	325,500	27,600	27,430	78,630	
URUGUAY	Administracion Nacional de Combustibles, Alcohol y Portland, La Teja, Montevideo	45,900	5,000 FCC	3,000	5,000 HDS	10,000 V, 3,500 A
VENEZUELA	Bariven:					
	El Chaure	40,000				
	El Toreno	5,400				
	Boscanven, Bajo Grande	45,000				15,00 V, 12,500 A
	Corporacion Venezolana del Petroleo, Moron	20,000		1,400	15,000 HDS	15,000 V
	Deltaven, Tucupita	10,000				
	Lagoven:					
	Amuay Bay	630,000		10,000	230,000 HDS	350,000 V, 1,400 L, 40,000 A
	Caripito	70,000				
	Llanoven, El Palito	105,000		7,500	7,500 HDT	
	Maraven:					
	Cardon	328,800	38,400 FCC		38,400 HDT 32,800 HDS	80,600 V, 4,370 hf alky

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	San Lorenzo	26,900				
	Menevan, Puerto la Cruz	160,000	12,000 FCC			15,000 V, 2,000 hf alky
	Roqueven, San Roque	5,200				
	Total	1,446,300	50,400	18,900	323,700	
ZAIRE	Sozir-Societe Zairo-Italienne de Raffinage, Muanda	17,000		3,500	5,000 HDS	
ZAMBIA	Indeni Petroleum Refinery Co. Ltd., Ndola	24,500		5,600	9,000 HDS	2,400 V

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APPENDIX C

ANNUITY TABLES

ANNUITY TABLES FOR I= 1.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.010	.9901	1.000	1.0000	1.0100	.9901	-.0000	-.000
2	1.020	.9803	2.010	.4975	.5075	1.9704	.4975	1.000
3	1.030	.9706	3.030	.3300	.3400	2.9410	.9934	3.010
4	1.041	.9610	4.060	.2463	.2563	3.9020	1.4876	6.040
5	1.051	.9515	5.101	.1960	.2060	4.8534	1.9801	10.101
6	1.062	.9420	6.152	.1625	.1725	5.7955	2.4710	15.202
7	1.072	.9327	7.214	.1386	.1486	6.7282	2.9602	21.354
8	1.083	.9235	8.286	.1207	.1307	7.6517	3.4478	28.567
9	1.094	.9143	9.369	.1067	.1167	8.5660	3.9337	36.853
10	1.105	.9053	10.462	.0956	.1056	9.4713	4.4179	46.221
11	1.116	.8963	11.567	.0865	.0965	10.3676	4.9005	56.683
12	1.127	.8874	12.683	.0788	.0888	11.2551	5.3815	68.250
13	1.138	.8787	13.809	.0724	.0824	12.1337	5.8607	80.933
14	1.149	.8700	14.947	.0669	.0769	13.0037	6.3384	94.742
15	1.161	.8613	16.097	.0621	.0721	13.8651	6.8143	109.690
16	1.173	.8528	17.258	.0579	.0679	14.7179	7.2886	125.786
17	1.184	.8444	18.430	.0543	.0643	15.5623	7.7613	143.044
18	1.196	.8360	19.615	.0510	.0610	16.3983	8.2323	161.475
19	1.208	.8277	20.811	.0481	.0581	17.2260	8.7017	181.090
20	1.220	.8195	22.019	.0454	.0554	18.0456	9.1694	201.900
21	1.232	.8114	23.239	.0430	.0530	18.8570	9.6354	223.919
22	1.245	.8034	24.472	.0409	.0509	19.6604	10.0998	247.159
23	1.257	.7954	25.716	.0389	.0489	20.4558	10.5626	271.630
24	1.270	.7876	26.973	.0371	.0471	21.2434	11.0237	297.346
25	1.282	.7798	28.243	.0354	.0454	22.0232	11.4831	324.320
26	1.295	.7720	29.526	.0339	.0439	22.7952	11.9409	352.563
27	1.308	.7644	30.821	.0324	.0424	23.5596	12.3971	382.089
28	1.321	.7568	32.129	.0311	.0411	24.3164	12.8516	412.910
29	1.335	.7493	33.450	.0299	.0399	25.0658	13.3044	445.039
30	1.348	.7419	34.785	.0287	.0387	25.8077	13.7557	478.489
31	1.361	.7346	36.133	.0277	.0377	26.5423	14.2052	513.274
32	1.375	.7273	37.494	.0267	.0367	27.2696	14.6532	549.407
33	1.389	.7201	38.869	.0257	.0357	27.9897	15.0995	586.901
34	1.403	.7130	40.258	.0248	.0348	28.7027	15.5441	625.770
35	1.417	.7059	41.660	.0240	.0340	29.4086	15.9871	666.028
36	1.431	.6989	43.077	.0232	.0332	30.1075	16.4285	707.688
37	1.445	.6920	44.508	.0225	.0325	30.7995	16.8682	750.765
38	1.460	.6852	45.953	.0218	.0318	31.4847	17.3063	795.272
39	1.474	.6784	47.412	.0211	.0311	32.1630	17.7428	841.225
40	1.489	.6717	48.886	.0205	.0305	32.8347	18.1776	888.637
41	1.504	.6650	50.375	.0199	.0299	33.4997	18.6108	937.524
42	1.519	.6584	51.879	.0193	.0293	34.1581	19.0424	987.899
43	1.534	.6519	53.398	.0187	.0287	34.8100	19.4723	1039.778
44	1.549	.6454	54.932	.0182	.0282	35.4555	19.9006	1093.176
45	1.565	.6391	56.481	.0177	.0277	36.0945	20.3273	1148.107
46	1.580	.6327	58.046	.0172	.0272	36.7272	20.7524	1204.589
47	1.596	.6265	59.626	.0168	.0268	37.3537	21.1758	1262.634
48	1.612	.6203	61.223	.0163	.0263	37.9740	21.5976	1322.261
49	1.628	.6141	62.835	.0159	.0259	38.5881	22.0178	1383.483
50	1.645	.6080	64.463	.0155	.0255	39.1961	22.4363	1446.318

ANNUITY TABLES FOR I= 2.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.020	.9804	1.000	1.0000	1.0200	.9804	-.0000	-.000
2	1.040	.9612	2.020	.4950	.5150	1.9416	.4950	1.000
3	1.061	.9423	3.060	.3268	.3468	2.8839	.9868	3.020
4	1.082	.9238	4.122	.2426	.2626	3.8077	1.4752	6.080
5	1.104	.9057	5.204	.1922	.2122	4.7135	1.9604	10.202
6	1.126	.8880	6.308	.1585	.1785	5.6014	2.4423	15.406
7	1.149	.8706	7.434	.1345	.1545	6.4720	2.9208	21.714
8	1.172	.8535	8.583	.1165	.1365	7.3255	3.3961	29.148
9	1.195	.8368	9.755	.1025	.1225	8.1622	3.8681	37.731
10	1.219	.8203	10.950	.0913	.1113	8.9826	4.3367	47.486
11	1.243	.8043	12.169	.0822	.1022	9.7868	4.8021	58.436
12	1.268	.7885	13.412	.0746	.0946	10.5753	5.2642	70.604
13	1.294	.7730	14.680	.0681	.0881	11.3484	5.7231	84.017
14	1.319	.7579	15.974	.0626	.0826	12.1062	6.1786	98.697
15	1.346	.7430	17.293	.0578	.0778	12.8493	6.6309	114.671
16	1.373	.7284	18.639	.0537	.0737	13.5777	7.0799	131.964
17	1.400	.7142	20.012	.0500	.0700	14.2919	7.5256	150.604
18	1.428	.7002	21.412	.0467	.0667	14.9920	7.9681	170.616
19	1.457	.6864	22.841	.0438	.0638	15.6785	8.4073	192.028
20	1.486	.6730	24.297	.0412	.0612	16.3514	8.8433	214.868
21	1.516	.6598	25.783	.0388	.0588	17.0112	9.2760	239.166
22	1.546	.6468	27.299	.0366	.0566	17.6580	9.7055	264.949
23	1.577	.6342	28.845	.0347	.0547	18.2922	10.1317	292.248
24	1.608	.6217	30.422	.0329	.0529	18.9139	10.5547	321.093
25	1.641	.6095	32.030	.0312	.0512	19.5235	10.9745	351.515
26	1.673	.5976	33.671	.0297	.0497	20.1210	11.3910	383.545
27	1.707	.5859	35.344	.0283	.0483	20.7069	11.8043	417.216
28	1.741	.5744	37.051	.0270	.0470	21.2813	12.2145	452.561
29	1.776	.5631	38.792	.0258	.0458	21.8444	12.6214	489.612
30	1.811	.5521	40.568	.0246	.0446	22.3965	13.0251	528.404
31	1.848	.5412	42.379	.0236	.0436	22.9377	13.4257	568.972
32	1.885	.5306	44.227	.0226	.0426	23.4683	13.8230	611.351
33	1.922	.5202	46.112	.0217	.0417	23.9886	14.2172	655.579
34	1.961	.5100	48.034	.0208	.0408	24.4986	14.6083	701.690
35	2.000	.5000	49.994	.0200	.0400	24.9986	14.9961	749.724
36	2.040	.4902	51.994	.0192	.0392	25.4888	15.3809	799.718
37	2.081	.4806	54.034	.0185	.0385	25.9695	15.7625	851.713
38	2.122	.4712	56.115	.0178	.0378	26.4406	16.1409	905.747
39	2.165	.4619	58.237	.0172	.0372	26.9026	16.5163	961.862
40	2.208	.4529	60.402	.0166	.0366	27.3555	16.8885	1020.099
41	2.252	.4440	62.610	.0160	.0360	27.7995	17.2576	1080.501
42	2.297	.4353	64.862	.0154	.0354	28.2348	17.6237	1143.111
43	2.343	.4268	67.159	.0149	.0349	28.6616	17.9866	1207.973
44	2.390	.4184	69.503	.0144	.0344	29.0800	18.3465	1275.133
45	2.438	.4102	71.893	.0139	.0339	29.4902	18.7034	1344.636
46	2.487	.4022	74.331	.0135	.0335	29.8923	19.0571	1416.528
47	2.536	.3943	76.817	.0130	.0330	30.2866	19.4079	1490.859
48	2.587	.3865	79.354	.0126	.0326	30.6731	19.7556	1567.676
49	2.639	.3790	81.941	.0122	.0322	31.0521	20.1003	1647.029
50	2.692	.3715	84.579	.0118	.0318	31.4236	20.4420	1728.970

ANNUITY TABLES FOR I= 3.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.030	.9709	1.000	1.0000	1.0300	.9709	-.0000	-.000
2	1.061	.9426	2.030	.4926	.5226	1.9135	.4926	1.000
3	1.093	.9151	3.091	.3235	.3535	2.8286	.9803	3.030
4	1.126	.8885	4.184	.2390	.2690	3.7171	1.4631	6.121
5	1.159	.8626	5.309	.1884	.2184	4.5797	1.9409	10.305
6	1.194	.8375	6.468	.1546	.1846	5.4172	2.4138	15.614
7	1.230	.8131	7.662	.1305	.1605	6.2303	2.8819	22.082
8	1.267	.7894	8.892	.1125	.1425	7.0197	3.3450	29.745
9	1.305	.7664	10.159	.0984	.1284	7.7861	3.8032	38.637
10	1.344	.7441	11.464	.0872	.1172	8.5302	4.2565	48.796
11	1.384	.7224	12.808	.0781	.1081	9.2526	4.7049	60.260
12	1.426	.7014	14.192	.0705	.1005	9.9540	5.1485	73.068
13	1.469	.6810	15.618	.0640	.0940	10.6350	5.5872	87.260
14	1.513	.6611	17.086	.0585	.0885	11.2961	6.0210	102.877
15	1.558	.6419	18.599	.0538	.0838	11.9379	6.4500	119.964
16	1.605	.6232	20.157	.0496	.0796	12.5611	6.8742	138.563
17	1.653	.6050	21.762	.0460	.0760	13.1661	7.2936	158.720
18	1.702	.5874	23.414	.0427	.0727	13.7535	7.7081	180.481
19	1.754	.5703	25.117	.0398	.0698	14.3238	8.1179	203.896
20	1.806	.5537	26.870	.0372	.0672	14.8775	8.5229	229.012
21	1.860	.5375	28.676	.0349	.0649	15.4150	8.9231	255.883
22	1.916	.5219	30.537	.0327	.0627	15.9369	9.3186	284.559
23	1.974	.5067	32.453	.0308	.0608	16.4436	9.7093	315.096
24	2.033	.4919	34.426	.0290	.0590	16.9355	10.0954	347.54
25	2.094	.4776	36.459	.0274	.0574	17.4131	10.4768	381.97
26	2.157	.4637	38.553	.0259	.0559	17.8768	10.8535	418.43
27	2.221	.4502	40.710	.0246	.0546	18.3270	11.2255	456.98
28	2.288	.4371	42.931	.0233	.0533	18.7641	11.5930	497.69
29	2.357	.4243	45.219	.0221	.0521	19.1885	11.9558	540.62
30	2.427	.4120	47.575	.0210	.0510	19.6004	12.3141	585.84
31	2.500	.4000	50.003	.0200	.0500	20.0004	12.6678	633.42
32	2.575	.3883	52.503	.0190	.0490	20.3888	13.0169	683.42
33	2.652	.3770	55.078	.0182	.0482	20.7658	13.3616	735.92
34	2.732	.3660	57.730	.0173	.0473	21.1318	13.7018	791.00
35	2.814	.3554	60.462	.0165	.0465	21.4872	14.0375	848.73
36	2.898	.3450	63.276	.0158	.0458	21.8323	14.3688	909.19
37	2.985	.3350	66.174	.0151	.0451	22.1672	14.6957	972.47
38	3.075	.3252	69.159	.0145	.0445	22.4925	15.0182	1038.64
39	3.167	.3158	72.234	.0138	.0438	22.8082	15.3363	1107.80
40	3.262	.3066	75.401	.0133	.0433	23.1148	15.6502	1180.04
41	3.360	.2976	78.663	.0127	.0427	23.4124	15.9597	1255.44
42	3.461	.2890	82.023	.0122	.0422	23.7014	16.2650	1334.10
43	3.565	.2805	85.484	.0117	.0417	23.9819	16.5660	1416.13
44	3.671	.2724	89.048	.0112	.0412	24.2543	16.8629	1501.61
45	3.782	.2644	92.720	.0108	.0408	24.5187	17.1556	1590.66
46	3.895	.2567	96.501	.0104	.0404	24.7754	17.4441	1683.38
47	4.012	.2493	100.397	.0100	.0400	25.0247	17.7285	1779.88
48	4.132	.2420	104.408	.0096	.0396	25.2667	18.0089	1880.28
49	4.256	.2350	108.541	.0092	.0392	25.5017	18.2852	1984.68
50	4.384	.2281	112.797	.0089	.0389	25.7298	18.5575	2093.22

ANNUITY TABLES FOR I= 4.0 PERCENT

CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1.040	.9615	1.000	1.0000	1.0400	.9615	-.0000	-.000
1.082	.9246	2.040	.4902	.5302	1.8861	.4902	1.000
1.125	.8890	3.122	.3203	.3603	2.7751	.9739	3.040
1.170	.8548	4.246	.2355	.2755	3.6299	1.4510	6.162
1.217	.8219	5.416	.1846	.2246	4.4518	1.9216	10.408
1.265	.7903	6.633	.1508	.1908	5.2421	2.3857	15.824
1.316	.7599	7.898	.1266	.1666	6.0021	2.8433	22.457
1.369	.7307	9.214	.1085	.1485	6.7327	3.2944	30.356
1.423	.7026	10.583	.0945	.1345	7.4353	3.7391	39.570
1.480	.6756	12.006	.0833	.1233	8.1109	4.1773	50.153
1.539	.6496	13.486	.0741	.1141	8.7605	4.6090	62.159
1.601	.6246	15.026	.0666	.1066	9.3851	5.0343	75.645
1.665	.6006	16.627	.0601	.1001	9.9856	5.4533	90.671
1.732	.5775	18.292	.0547	.0947	10.5631	5.8659	107.298
1.801	.5553	20.024	.0499	.0899	11.1184	6.2721	125.590
1.873	.5339	21.825	.0458	.0858	11.6523	6.6720	145.613
1.948	.5134	23.698	.0422	.0822	12.1657	7.0656	167.438
2.026	.4936	25.645	.0390	.0790	12.6593	7.4530	191.135
2.107	.4746	27.671	.0361	.0761	13.1339	7.8342	216.781
2.191	.4564	29.778	.0336	.0736	13.5903	8.2091	244.452
2.279	.4388	31.969	.0313	.0713	14.0292	8.5779	274.230
2.370	.4220	34.248	.0292	.0692	14.4511	8.9407	306.199
2.465	.4057	36.618	.0273	.0673	14.8568	9.2973	340.447
2.563	.3901	39.083	.0256	.0656	15.2470	9.6479	377.065
2.666	.3751	41.646	.0240	.0640	15.6221	9.9925	416.148
2.772	.3607	44.312	.0226	.0626	15.9828	10.3312	457.794
2.883	.3468	47.084	.0212	.0612	16.3296	10.6640	502.105
2.999	.3335	49.968	.0200	.0600	16.6631	10.9909	549.190
3.119	.3207	52.966	.0189	.0589	16.9837	11.3120	599.157
3.243	.3083	56.085	.0178	.0578	17.2920	11.6274	652.123
3.373	.2965	59.328	.0169	.0569	17.5885	11.9371	708.208
3.508	.2851	62.701	.0159	.0559	17.8736	12.2411	767.537
3.648	.2741	66.210	.0151	.0551	18.1476	12.5396	830.238
3.794	.2636	69.858	.0143	.0543	18.4112	12.8324	896.448
3.946	.2534	73.652	.0136	.0536	18.6646	13.1198	966.306
4.104	.2437	77.598	.0129	.0529	18.9083	13.4018	1039.958
4.268	.2343	81.702	.0122	.0522	19.1426	13.6784	1117.556
4.439	.2253	85.970	.0116	.0516	19.3679	13.9497	1199.258
4.616	.2166	90.409	.0111	.0511	19.5845	14.2157	1285.229
4.801	.2083	95.026	.0105	.0505	19.7928	14.4765	1375.638
4.993	.2003	99.827	.0100	.0500	19.9931	14.7322	1470.663
5.193	.1926	104.820	.0095	.0495	20.1856	14.9828	1570.490
5.400	.1852	110.012	.0091	.0491	20.3708	15.2284	1675.310
5.617	.1780	115.413	.0087	.0487	20.5488	15.4690	1785.322
5.841	.1712	121.029	.0083	.0483	20.7200	15.7047	1900.735
6.075	.1646	126.871	.0079	.0479	20.8847	15.9356	2021.764
6.318	.1583	132.945	.0075	.0475	21.0429	16.1618	2148.635
6.571	.1522	139.263	.0072	.0472	21.1951	16.3832	2281.580
6.833	.1463	145.834	.0069	.0469	21.3415	16.6000	2420.843
7.107	.1407	152.667	.0066	.0466	21.4822	16.8122	2566.677

ANNUITY TABLES FOR I= 5.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.050	.9524	1.000	1.0000	1.0500	.9524	-.0000	-.00
2	1.103	.9070	2.050	.4878	.5378	1.8594	.4878	1.00
3	1.158	.8638	3.152	.3172	.3672	2.7232	.9675	3.05
4	1.216	.8227	4.310	.2320	.2820	3.5460	1.4391	6.20
5	1.276	.7835	5.526	.1810	.2310	4.3295	1.9025	10.51
6	1.340	.7462	6.802	.1470	.1970	5.0757	2.3579	16.03
7	1.407	.7107	8.142	.1228	.1728	5.7864	2.8052	22.84
8	1.477	.6768	9.549	.1047	.1547	6.4632	3.2445	30.98
9	1.551	.6446	11.027	.0907	.1407	7.1078	3.6758	40.53
10	1.629	.6139	12.578	.0795	.1295	7.7217	4.0991	51.55
11	1.710	.5847	14.207	.0704	.1204	8.3064	4.5144	64.13
12	1.796	.5568	15.917	.0628	.1128	8.8633	4.9219	78.34
13	1.886	.5303	17.713	.0565	.1065	9.3936	5.3215	94.26
14	1.980	.5051	19.599	.0510	.1010	9.8986	5.7133	111.97
15	2.079	.4810	21.579	.0463	.0963	10.3797	6.0973	131.57
16	2.183	.4581	23.657	.0423	.0923	10.8378	6.4736	153.15
17	2.292	.4363	25.840	.0387	.0887	11.2741	6.8423	176.80
18	2.407	.4155	28.132	.0355	.0855	11.6896	7.2034	202.64
19	2.527	.3957	30.539	.0327	.0827	12.0853	7.5569	230.78
20	2.653	.3769	33.066	.0302	.0802	12.4622	7.9030	261.31
21	2.786	.3589	35.719	.0280	.0780	12.8212	8.2416	294.38
22	2.925	.3418	38.505	.0260	.0760	13.1630	8.5730	330.10
23	3.072	.3256	41.430	.0241	.0741	13.4886	8.8971	368.61
24	3.225	.3101	44.502	.0225	.0725	13.7986	9.2140	410.04
25	3.386	.2953	47.727	.0210	.0710	14.0939	9.5238	454.54
26	3.556	.2812	51.113	.0196	.0696	14.3752	9.8266	502.26
27	3.733	.2678	54.669	.0183	.0683	14.6430	10.1224	553.38
28	3.920	.2551	58.403	.0171	.0671	14.8981	10.4114	608.05
29	4.116	.2429	62.323	.0160	.0660	15.1411	10.6936	666.45
30	4.322	.2314	66.439	.0151	.0651	15.3725	10.9691	728.77
31	4.538	.2204	70.761	.0141	.0641	15.5928	11.2381	795.21
32	4.765	.2099	75.299	.0133	.0633	15.8027	11.5005	865.97
33	5.003	.1999	80.064	.0125	.0625	16.0025	11.7566	941.27
34	5.253	.1904	85.067	.0118	.0618	16.1929	12.0063	1021.33
35	5.516	.1813	90.320	.0111	.0611	16.3742	12.2498	1106.40
36	5.792	.1727	95.836	.0104	.0604	16.5469	12.4872	1196.72
37	6.081	.1644	101.628	.0098	.0598	16.7113	12.7186	1292.56
38	6.385	.1566	107.710	.0093	.0593	16.8679	12.9440	1394.19
39	6.705	.1491	114.095	.0088	.0588	17.0170	13.1636	1501.90
40	7.040	.1420	120.800	.0083	.0583	17.1591	13.3775	1615.99
41	7.392	.1353	127.840	.0078	.0578	17.2944	13.5857	1736.79
42	7.762	.1288	135.232	.0074	.0574	17.4232	13.7884	1864.63
43	8.150	.1227	142.993	.0070	.0570	17.5459	13.9857	1999.86
44	8.557	.1169	151.143	.0066	.0566	17.6628	14.1777	2142.86
45	8.985	.1113	159.700	.0063	.0563	17.7741	14.3644	2294.00
46	9.434	.1060	168.685	.0059	.0559	17.8801	14.5461	2453.70
47	9.906	.1009	178.119	.0056	.0556	17.9810	14.7226	2622.38
48	10.401	.0961	188.025	.0053	.0553	18.0772	14.8943	2800.50
49	10.921	.0916	198.427	.0050	.0550	18.1687	15.0611	2988.53
50	11.467	.0872	209.348	.0048	.0548	18.2559	15.2233	3186.96

ANNUITY TABLES FOR I= 6.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.060	.9434	1.000	1.0000	1.0600	.9434	-.0000	-.000
2	1.124	.8900	2.060	.4854	.5454	1.8334	.4854	1.000
3	1.191	.8396	3.184	.3141	.3741	2.6730	.9612	3.060
4	1.262	.7921	4.375	.2286	.2886	3.4651	1.4272	6.244
5	1.338	.7473	5.637	.1774	.2374	4.2124	1.8836	10.618
6	1.419	.7050	6.975	.1434	.2034	4.9173	2.3304	16.255
7	1.504	.6651	8.394	.1191	.1791	5.5824	2.7676	23.231
8	1.594	.6274	9.897	.1010	.1610	6.2098	3.1952	31.624
9	1.689	.5919	11.491	.0870	.1470	6.8017	3.6133	41.522
0	1.791	.5584	13.181	.0759	.1359	7.3601	4.0220	53.013
1	1.898	.5268	14.972	.0668	.1268	7.8869	4.4213	66.194
2	2.012	.4970	16.870	.0593	.1193	8.3838	4.8113	81.166
3	2.133	.4688	18.882	.0530	.1130	8.8527	5.1920	98.036
4	2.261	.4423	21.015	.0476	.1076	9.2950	5.5635	116.918
5	2.397	.4173	23.276	.0430	.1030	9.7122	5.9260	137.933
6	2.540	.3936	25.673	.0390	.0990	10.1059	6.2794	161.209
7	2.693	.3714	28.213	.0354	.0954	10.4773	6.6240	186.881
8	2.854	.3503	30.906	.0324	.0924	10.8276	6.9597	215.094
9	3.026	.3305	33.760	.0296	.0896	11.1581	7.2867	246.000
0	3.207	.3118	36.786	.0272	.0872	11.4699	7.6051	279.760
1	3.400	.2942	39.993	.0250	.0850	11.7641	7.9151	316.545
2	3.604	.2775	43.392	.0230	.0830	12.0416	8.2166	356.538
3	3.820	.2618	46.996	.0213	.0813	12.3034	8.5099	399.930
4	4.049	.2470	50.816	.0197	.0797	12.5504	8.7951	446.926
5	4.292	.2330	54.865	.0182	.0782	12.7834	9.0722	497.742
6	4.549	.2198	59.156	.0169	.0769	13.0032	9.3414	552.606
7	4.822	.2074	63.706	.0157	.0757	13.2105	9.6029	611.763
8	5.112	.1956	68.528	.0146	.0746	13.4062	9.8568	675.469
9	5.418	.1846	73.640	.0136	.0736	13.5907	10.1032	743.997
0	5.743	.1741	79.058	.0126	.0726	13.7648	10.3422	817.636
1	6.088	.1643	84.802	.0118	.0718	13.9291	10.5740	896.695
2	6.453	.1550	90.890	.0110	.0710	14.0840	10.7988	981.496
3	6.841	.1462	97.343	.0103	.0703	14.2302	11.0166	1072.386
4	7.251	.1379	104.184	.0096	.0696	14.3681	11.2276	1169.729
5	7.686	.1301	111.435	.0090	.0690	14.4982	11.4319	1273.913
6	8.147	.1227	119.121	.0084	.0684	14.6210	11.6298	1385.348
7	8.636	.1158	127.268	.0079	.0679	14.7368	11.8213	1504.469
8	9.154	.1092	135.904	.0074	.0674	14.8460	12.0065	1631.737
9	9.704	.1031	145.058	.0069	.0669	14.9491	12.1857	1767.641
0	10.286	.0972	154.762	.0065	.0665	15.0463	12.3590	1912.699
1	10.903	.0917	165.048	.0061	.0661	15.1380	12.5264	2067.461
2	11.557	.0865	175.951	.0057	.0657	15.2245	12.6883	2232.509
3	12.250	.0816	187.508	.0053	.0653	15.3062	12.8446	2408.460
4	12.985	.0770	199.758	.0050	.0650	15.3832	12.9956	2595.967
5	13.765	.0727	212.744	.0047	.0647	15.4558	13.1413	2795.725
6	14.590	.0685	226.508	.0044	.0644	15.5244	13.2819	3008.469
7	15.466	.0647	241.099	.0041	.0641	15.5890	13.4177	3234.977
8	16.394	.0610	256.565	.0039	.0639	15.6500	13.5485	3476.075
9	17.378	.0575	272.958	.0037	.0637	15.7076	13.6748	3732.640
0	18.420	.0543	290.336	.0034	.0634	15.7619	13.7964	4005.598

ANNUITY TABLES FOR I= 7.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.070	.9346	1.000	1.0000	1.0700	.9346	-.0000	-.00
2	1.145	.8734	2.070	.4831	.5531	1.8080	.4831	1.00
3	1.225	.8163	3.215	.3111	.3811	2.6243	.9549	3.07
4	1.311	.7629	4.440	.2252	.2952	3.3872	1.4155	6.28
5	1.403	.7130	5.751	.1739	.2439	4.1002	1.8650	10.72
6	1.501	.6663	7.153	.1398	.2098	4.7665	2.3032	16.47
7	1.606	.6227	8.654	.1156	.1856	5.3893	2.7304	23.62
8	1.718	.5820	10.260	.0975	.1675	5.9713	3.1465	32.28
9	1.838	.5439	11.978	.0835	.1535	6.5152	3.5517	42.54
10	1.967	.5083	13.816	.0724	.1424	7.0236	3.9461	54.52
11	2.105	.4751	15.784	.0634	.1334	7.4987	4.3296	68.33
12	2.252	.4440	17.888	.0559	.1259	7.9427	4.7025	84.12
13	2.410	.4150	20.141	.0497	.1197	8.3577	5.0648	102.00
14	2.579	.3878	22.550	.0443	.1143	8.7455	5.4167	122.15
15	2.759	.3624	25.129	.0398	.1098	9.1079	5.7583	144.70
16	2.952	.3387	27.888	.0359	.1059	9.4466	6.0897	169.82
17	3.159	.3166	30.840	.0324	.1024	9.7632	6.4110	197.71
18	3.380	.2959	33.999	.0294	.0994	10.0591	6.7225	228.55
19	3.617	.2765	37.379	.0268	.0968	10.3356	7.0242	262.55
20	3.870	.2584	40.995	.0244	.0944	10.5940	7.3163	299.93
21	4.141	.2415	44.865	.0223	.0923	10.8355	7.5990	340.93
22	4.430	.2257	49.006	.0204	.0904	11.0612	7.8725	385.79
23	4.741	.2109	53.436	.0187	.0887	11.2722	8.1369	434.80
24	5.072	.1971	58.177	.0172	.0872	11.4693	8.3923	488.23
25	5.427	.1842	63.249	.0158	.0858	11.6536	8.6391	546.41
26	5.807	.1722	68.676	.0146	.0846	11.8258	8.8773	609.66
27	6.214	.1609	74.484	.0134	.0834	11.9867	9.1072	678.34
28	6.649	.1504	80.698	.0124	.0824	12.1371	9.3289	752.82
29	7.114	.1406	87.347	.0114	.0814	12.2777	9.5427	833.52
30	7.612	.1314	94.461	.0106	.0806	12.4090	9.7487	920.86
31	8.145	.1228	102.073	.0098	.0798	12.5318	9.9471	1015.32
32	8.715	.1147	110.218	.0091	.0791	12.6466	10.1381	1117.40
33	9.325	.1072	118.933	.0084	.0784	12.7538	10.3219	1227.62
34	9.978	.1002	128.259	.0078	.0778	12.8540	10.4987	1346.55
35	10.677	.0937	138.237	.0072	.0772	12.9477	10.6687	1474.81
36	11.424	.0875	148.913	.0067	.0767	13.0352	10.8321	1613.04
37	12.224	.0818	160.337	.0062	.0762	13.1170	10.9891	1761.96
38	13.079	.0765	172.561	.0058	.0758	13.1935	11.1398	1922.30
39	13.995	.0715	185.640	.0054	.0754	13.2649	11.2845	2094.86
40	14.974	.0668	199.635	.0050	.0750	13.3317	11.4233	2280.50
41	16.023	.0624	214.610	.0047	.0747	13.3941	11.5565	2480.13
42	17.144	.0583	230.632	.0043	.0743	13.4524	11.6842	2694.74
43	18.344	.0545	247.776	.0040	.0740	13.5070	11.8065	2925.37
44	19.628	.0509	266.121	.0038	.0738	13.5579	11.9237	3173.15
45	21.002	.0476	285.749	.0035	.0735	13.6055	12.0360	3439.27
46	22.473	.0445	306.752	.0033	.0733	13.6500	12.1435	3725.02
47	24.046	.0416	329.224	.0030	.0730	13.6916	12.2463	4031.77
48	25.729	.0389	353.270	.0028	.0728	13.7305	12.3447	4361.00
49	27.530	.0363	378.999	.0026	.0726	13.7668	12.4387	4714.27
50	29.457	.0339	406.529	.0025	.0725	13.8007	12.5287	5093.27

ANNUITY TABLES FOR I= 8.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.080	.9259	1.000	1.0000	1.0800	.9259	-.0000	-.000
2	1.166	.8573	2.080	.4808	.5608	1.7833	.4808	1.000
3	1.260	.7938	3.246	.3080	.3880	2.5771	.9487	3.080
4	1.360	.7350	4.506	.2219	.3019	3.3121	1.4040	6.326
5	1.469	.6806	5.867	.1705	.2505	3.9927	1.8465	10.833
6	1.587	.6302	7.336	.1363	.2163	4.6229	2.2763	16.699
7	1.714	.5835	8.923	.1121	.1921	5.2064	2.6937	24.035
8	1.851	.5403	10.637	.0940	.1740	5.7466	3.0985	32.958
9	1.999	.5002	12.488	.0801	.1601	6.2469	3.4910	43.594
10	2.159	.4632	14.487	.0690	.1490	6.7101	3.8713	56.082
11	2.332	.4289	16.645	.0601	.1401	7.1390	4.2395	70.569
12	2.518	.3971	18.977	.0527	.1327	7.5361	4.5957	87.214
13	2.720	.3677	21.495	.0465	.1265	[illegible].9038	4.9402	106.191
14	2.937	.3405	24.215	.0413	.1213	8.2442	5.2731	127.687
15	3.172	.3152	27.152	.0368	.1168	8.5595	5.5945	151.901
16	3.426	.2919	30.324	.0330	.1130	8.8514	5.9046	179.054
17	3.700	.2703	33.750	.0296	.1096	9.1216	6.2037	209.378
18	3.996	.2502	37.450	.0267	.1067	9.3719	6.4920	243.128
19	4.316	.2317	41.446	.0241	.1041	9.6036	6.7697	280.578
20	4.661	.2145	45.762	.0219	.1019	9.8181	7.0369	322.025
21	5.034	.1987	50.423	.0198	.0998	10.0168	7.2940	367.787
22	5.437	.1839	55.457	.0180	.0980	10.2007	7.5412	418.209
23	5.871	.1703	60.893	.0164	.0964	10.3711	7.7786	473.666
24	6.341	.1577	66.765	.0150	.0950	10.5288	8.0066	534.559
25	6.848	.1460	73.106	.0137	.0937	10.6748	8.2254	601.324
26	7.396	.1352	79.954	.0125	.0925	10.8100	8.4352	674.430
27	7.988	.1252	87.351	.0114	.0914	10.9352	8.6363	754.385
28	8.627	.1159	95.339	.0105	.0905	11.0511	8.8289	841.735
29	9.317	.1073	103.966	.0096	.0896	11.1584	9.0133	937.074
30	10.063	.0994	113.283	.0088	.0888	11.2578	9.1897	1041.040
31	10.868	.0920	123.346	.0081	.0881	11.3498	9.3584	1154.323
32	11.737	.0852	134.214	.0075	.0875	11.4350	9.5197	1277.669
33	12.676	.0789	145.951	.0069	.0869	11.5139	9.6737	1411.883
34	13.690	.0730	158.627	.0063	.0863	11.5869	9.8208	1557.833
35	14.785	.0676	172.317	.0058	.0858	11.6546	9.9611	1716.460
36	15.968	.0626	187.102	.0053	.0853	11.7172	10.0949	1888.777
37	17.246	.0580	203.070	.0049	.0849	11.7752	10.2225	2075.879
38	18.625	.0537	220.316	.0045	.0845	11.8289	10.3440	2278.949
39	20.115	.0497	238.941	.0042	.0842	11.8786	10.4597	2499.265
40	21.725	.0460	259.057	.0039	.0839	11.9246	10.5699	2738.206
41	23.462	.0426	280.781	.0036	.0836	11.9672	10.6747	2997.263
42	25.339	.0395	304.244	.0033	.0833	12.0067	10.7744	3278.044
43	27.367	.0365	329.583	.0030	.0830	12.0432	10.8692	3582.288
44	29.556	.0338	356.950	.0028	.0828	12.0771	10.9592	3911.871
45	31.920	.0313	386.506	.0026	.0826	12.1084	11.0447	4268.820
46	34.474	.0290	418.426	.0024	.0824	12.1374	11.1258	4655.326
47	37.232	.0269	452.900	.0022	.0822	12.1643	11.2028	5073.752
48	40.211	.0249	490.132	.0020	.0820	12.1891	11.2758	5526.652
49	43.427	.0230	530.343	.0019	.0819	12.2122	11.3451	6016.784
50	46.902	.0213	573.770	.0017	.0817	12.2335	11.4107	6547.127

ANNUITY TABLES FOR I= 9.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.090	.9174	1.000	1.0000	1.0900	.9174	-.0000	-.00
2	1.188	.8417	2.090	.4785	.5685	1.7591	.4785	1.00
3	1.295	.7722	3.278	.3051	.3951	2.5313	.9426	3.09
4	1.412	.7084	4.573	.2187	.3087	3.2397	1.3925	6.36
5	1.539	.6499	5.985	.1671	.2571	3.8897	1.8282	10.94
6	1.677	.5963	7.523	.1329	.2229	4.4859	2.2498	16.92
7	1.828	.5470	9.200	.1087	.1987	5.0330	2.6574	24.44
8	1.993	.5019	11.028	.0907	.1807	5.5348	3.0512	33.65
9	2.172	.4604	13.021	.0768	.1668	5.9952	3.4312	44.67
10	2.367	.4224	15.193	.0658	.1558	6.4177	3.7978	57.69
11	2.580	.3875	17.560	.0569	.1469	6.8052	4.1510	72.89
12	2.813	.3555	20.141	.0497	.1397	7.1607	4.4910	90.45
13	3.066	.3262	22.953	.0436	.1336	7.4869	4.9182	110.59
14	3.342	.2992	26.019	.0384	.1284	7.7862	5.1326	133.54
15	3.642	.2745	29.361	.0341	.1241	8.0607	5.4346	159.56
16	3.970	.2519	33.003	.0303	.1203	8.3126	5.7245	188.92
17	4.328	.2311	36.974	.0270	.1170	8.5436	6.0024	221.[illegible]3
18	4.717	.2120	41.301	.0242	.1142	8.7556	6.2687	258.90
19	5.142	.1945	46.018	.0217	.1117	8.9501	6.5236	300.20
20	5.604	.1784	51.160	.0195	.1095	9.1285	6.7674	346.22
21	6.109	.1637	56.765	.0176	.1076	9.2922	7.0006	397.38
22	6.659	.1502	62.873	.0159	.1059	9.4424	7.2232	454.14
23	7.258	.1378	69.532	.0144	.1044	9.5802	7.4357	517.02
24	7.911	.1264	76.790	.0130	.1030	9.7066	7.6384	586.55
25	8.623	.1160	84.701	.0118	.1018	9.8226	7.8316	663.34
26	9.399	.1064	93.324	.0107	.1007	9.9290	8.0156	748.04
27	10.245	.0976	102.723	.0097	.0997	10.0266	8.1906	841.36
28	11.167	.0895	112.968	.0089	.0989	10.1161	8.3571	944.09
29	12.172	.0822	124.135	.0081	.0981	10.1983	8.5154	1057.06
30	13.268	.0754	136.308	.0073	.0973	10.2737	8.6657	1181.19
31	14.462	.0691	149.575	.0067	.0967	10.3428	8.8083	1317.50
32	15.763	.0634	164.037	.0061	.0961	10.4062	8.9436	1467.07
33	17.182	.0582	179.800	.0056	.0956	10.4644	9.0718	1631.11
34	18.728	.0534	196.982	.0051	.0951	10.5178	9.1933	1810.91
35	20.414	[illegible]490	215.711	.0046	.0946	10.5668	9.3083	2007.89
36	22.251	.0449	236.125	.0042	.0942	10.6118	9.4171	2223.60
37	24.254	.0412	258.376	.0039	.0939	10.6530	9.5200	2459.73
38	26.437	.0378	282.630	.0035	.0935	10.6908	9.6172	2718.10
39	28.816	.0347	309.066	.0032	.0932	10.7255	9.7090	3000.73
40	31.409	.0318	337.882	.0030	.0930	10.7574	9.7957	3309.80
41	34.236	.0292	369.292	.0027	.0927	10.7866	9.8775	3647.68
42	37.318	.0268	403.528	.0025	.0925	10.8134	9.9546	4016.97
43	40.676	.0246	440.846	.0023	.0923	10.8380	10.0273	4420.50
44	44.337	.0226	481.522	.0021	.0921	10.8605	10.0958	4861.35
45	48.327	.0207	525.859	.0019	.0919	10.8812	10.1603	5342.87
46	52.677	.0190	574.186	.0017	.0917	10.9002	10.2210	5868.73
47	57.418	.0174	626.863	.0016	.0916	10.9176	10.2780	6442.92
48	62.585	.0160	684.280	.0015	.0915	10.9336	10.3317	7069.78
49	68.218	.0147	746.866	.0013	.0913	10.9482	10.3821	7754.06
50	74.358	.0134	815.084	.0012	.0912	10.9617	10.4295	8500.92

ANNUITY TABLES FOR I=10.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.100	.9091	1.000	1.0000	1.1000	.9091	-.0000	-.000
2	1.210	.8264	2.100	.4762	.5762	1.7355	.4762	1.000
3	1.331	.7513	3.310	.3021	.4021	2.4869	.9366	3.100
4	1.464	.6830	4.641	.2155	.3155	3.1699	1.3812	6.410
5	1.611	.6209	6.105	.1638	.2638	3.7908	1.8101	11.051
6	1.772	.5645	7.716	.1296	.2296	4.3553	2.2236	17.156
7	1.949	.5132	9.487	.1054	.2054	4.8684	2.6216	24.872
8	2.144	.4665	11.436	.0874	.1874	5.3349	3.0045	34.359
9	2.358	.4241	13.579	.0736	.1736	5.7590	3.3724	45.795
10	2.594	.3855	15.937	.0627	.1627	6.1446	3.7255	59.374
11	2.853	.3505	18.531	.0540	.1540	6.4951	4.0641	75.312
12	3.138	.3186	21.384	.0468	.1468	6.8137	4.3884	93.843
13	3.452	.2897	24.523	.0408	.1408	7.1034	4.6988	115.227
14	3.797	.2633	27.975	.0357	.1357	7.3667	4.9955	139.750
15	4.177	.2394	31.772	.0315	.1315	7.6061	5.2789	167.725
16	4.595	.2176	35.950	.0278	.1278	7.8237	5.5493	199.497
17	5.054	.1978	40.545	.0247	.1247	8.0216	5.8071	235.447
18	5.560	.1799	45.599	.0219	.1219	8.2014	6.0526	275.992
19	6.116	.1635	51.159	.0195	.1195	8.3649	6.2861	321.591
20	6.727	.1486	57.275	.0175	.1175	8.5136	6.5081	372.750
21	7.400	.1351	64.002	.0156	.1156	8.6487	6.7189	430.025
22	8.140	.1228	71.403	.0140	.1140	8.7715	6.9189	494.027
23	8.954	.1117	79.543	.0126	.1126	8.8832	7.1085	565.430
24	9.850	.1015	88.497	.0113	.1113	8.9847	7.2881	644.973
25	10.835	.0923	98.347	.0102	.1102	9.0770	7.4580	733.471
26	11.918	.0839	109.182	.0092	.1092	9.1609	7.6186	831.818
27	13.110	.0763	121.100	.0083	.1083	9.2372	7.7704	940.999
28	14.421	.0693	134.210	.0075	.1075	9.3066	7.9137	1062.099
29	15.863	.0630	148.631	.0067	.1067	9.3696	8.0489	1196.309
30	17.449	.0573	164.494	.0061	.1061	9.4269	8.1762	1344.940
31	19.194	.0521	181.943	.0055	.1055	9.4790	8.2962	1509.434
32	21.114	.0474	201.138	.0050	.1050	9.5264	8.4091	1691.378
33	23.225	.0431	222.252	.0045	.1045	9.5694	8.5152	1892.515
34	25.548	.0391	245.477	.0041	.1041	9.6086	8.6149	2114.767
35	28.102	.0356	271.024	.0037	.1037	9.6442	8.7086	2360.244
36	30.913	.0323	299.127	.0033	.1033	9.6765	8.7965	2631.268
37	34.004	.0294	330.039	.0030	.1030	9.7059	8.8789	2930.395
38	37.404	.0267	364.043	.0027	.1027	9.7327	8.9562	3260.434
39	41.145	.0243	401.448	.0025	.1025	9.7570	9.0285	3624.478
40	45.259	.0221	442.593	.0023	.1023	9.7791	9.0962	4025.926
41	49.785	.0201	487.852	.0020	.1020	9.7991	9.1596	4468.518
42	54.764	.0183	537.637	.0019	.1019	9.8174	9.2188	4956.370
43	60.240	.0166	592.401	.0017	.1017	9.8340	9.2741	5494.007
44	66.264	.0151	652.641	.0015	.1015	9.8491	9.3258	6086.408
45	72.890	.0137	718.905	.0014	.1014	9.8628	9.3740	6739.048
46	80.180	.0125	791.795	.0013	.1013	9.8753	9.4190	7457.953
47	88.197	.0113	871.975	.0011	.1011	9.8866	9.4610	8249.749
48	97.017	.0103	960.172	.0010	.1010	9.8969	9.5001	9121.723
49	106.719	.0094	1057.190	.0009	.1009	9.9063	9.5365	10081.896
50	117.391	.0085	1163.909	.0009	.1009	9.9148	9.5704	11139.085

ANNUITY TABLES FOR I=11.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.110	.9009	1.000	1.0000	1.1100	.9009	-.0000	-.000
2	1.232	.8116	2.110	.4739	.5839	1.7125	.4739	1.000
3	1.368	.7312	3.342	.2992	.4092	2.4437	.9306	3.110
4	1.518	.6587	4.710	.2123	.3223	3.1024	1.3700	6.452
5	1.685	.5935	6.228	.1606	.2706	3.6959	1.7923	11.162
6	1.870	.5346	7.913	.1264	.2364	4.2305	2.1976	17.390
7	2.076	.4817	9.783	.1022	.2122	4.7122	2.5863	25.302
8	2.305	.4339	11.859	.0843	.1943	5.1461	2.9585	35.086
9	2.558	.3909	14.164	.0706	.1806	5.5370	3.3144	46.945
10	2.839	.3522	16.722	.0598	.1698	5.8892	3.6544	61.109
11	3.152	.3173	19.561	.0511	.1611	6.2065	3.9788	77.831
12	3.498	.2858	22.713	.0440	.1540	6.4924	4.2879	97.393
13	3.883	.2575	26.212	.0382	.1482	6.7499	4.5822	120.106
14	4.310	.2320	30.095	.0332	.1432	6.9819	4.8619	146.317
15	4.785	.2090	34.405	.0291	.1391	7.1909	5.1275	176.412
16	5.311	.1883	39.190	.0255	.1355	7.3792	5.3794	210.818
17	5.895	.1696	44.501	.0225	.1325	7.5488	5.6180	250.008
18	6.544	.1528	50.396	.0198	.1298	7.7016	5.8439	294.509
19	7.263	.1377	56.939	.0176	.1276	7.8393	6.0574	344.904
20	8.062	.1240	64.203	.0156	.1256	7.9633	6.2590	401.844
21	8.949	.1117	72.265	.0138	.1238	8.0751	6.4491	466.047
22	9.934	.1007	81.214	.0123	.1223	8.1757	6.6283	538.312
23	11.026	.0907	91.148	.0110	.1210	8.2664	6.7969	619.526
24	12.239	.0817	102.174	.0098	.1198	8.3481	6.9555	710.674
25	13.585	.0736	114.413	.0087	.1187	8.4217	7.1045	812.848
26	15.080	.0663	127.999	.0078	.1178	8.4881	7.2443	927.262
27	16.739	.0597	143.079	.0070	.1170	8.5478	7.3754	1055.26
28	18.580	.0538	159.817	.0063	.1163	8.6016	7.4982	1198.33
29	20.624	.0485	178.397	.0056	.1156	8.6501	7.6131	1358.15
30	22.892	.0437	199.021	.0050	.1150	8.6938	7.7206	1536.55
31	25.410	.0394	221.913	.0045	.1145	8.7331	7.8210	1735.57
32	28.206	.0355	247.324	.0040	.1140	8.7686	7.9147	1957.48
33	31.308	.0319	275.529	.0036	.1136	8.8005	8.0021	2204.81
34	34.752	.0288	306.837	.0033	.1133	8.8293	8.0836	2480.34
35	38.575	.0259	341.590	.0029	.1129	8.8552	8.1594	2787.17
36	42.818	.0234	380.164	.0026	.1126	8.8786	8.2300	3128.76
37	47.528	.0210	422.982	.0024	.1124	8.8996	8.2957	3508.93
38	52.756	.0190	470.511	.0021	.1121	8.9186	8.3567	3931.91
39	58.559	.0171	523.267	.0019	.1119	8.9357	8.4133	4402.42
40	65.001	.0154	581.826	.0017	.1117	8.9511	8.4659	4925.69
41	72.151	.0139	646.827	.0015	.1115	8.9649	8.5147	5507.51
42	80.088	.0125	718.978	.0014	.1114	8.9774	8.5599	6154.34
43	88.897	.0112	799.065	.0013	.1113	8.9886	8.6017	6873.32
44	98.676	.0101	887.963	.0011	.1111	8.9988	8.6404	7672.38
45	109.530	.0091	986.639	.0010	.1110	9.0079	8.6763	8560.35
46	121.579	.0082	1096.169	.0009	.1109	9.0161	8.7094	9546.98
47	134.952	.0074	1217.747	.0008	.1108	9.0235	8.7400	10643.15
48	149.797	.0067	1352.700	.0007	.1107	9.0302	8.7683	11860.90
49	166.275	.0060	1502.497	.0007	.1107	9.0362	8.7944	13213.60
50	184.565	.0054	1668.771	.0006	.1106	9.0417	8.8195	14716.10

ANNUITY TABLES FOR I=12.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.120	.8929	1.000	1.0000	1.1200	.8929	-.0000	-.000
2	1.254	.7972	2.120	.4717	.5917	1.6901	.4717	1.000
3	1.405	.7118	3.374	.2963	.4163	2.4018	.9246	3.120
4	1.574	.6355	4.779	.2092	.3292	3.0373	1.3589	6.494
5	1.762	.5674	6.353	.1574	.2774	3.6048	1.7746	11.274
6	1.974	.5066	8.115	.1232	.2432	4.1114	2.1720	17.627
7	2.211	.4523	10.089	.0991	.2191	4.5638	2.5515	25.742
8	2.476	.4039	12.300	.0813	.2013	4.9676	2.9131	35.831
9	2.773	.3606	14.776	.0677	.1877	5.3282	3.2574	48.130
0	3.106	.3220	17.549	.0570	.1770	5.6502	3.5847	62.906
1	3.479	.2875	20.655	.0484	.1684	5.9377	3.8953	80.455
2	3.896	.2567	24.133	.0414	.1614	6.1944	4.1897	101.109
3	4.363	.2292	28.029	.0357	.1557	6.4235	4.4683	125.243
4	4.887	.2046	32.393	.0309	.1509	6.6282	4.7317	153.272
5	5.474	.1827	37.280	.0268	.1468	6.8109	4.9803	185.664
6	6.130	.1631	42.753	.0234	.1434	6.9740	5.2147	222.944
7	6.866	.1456	48.884	.0205	.1405	7.1196	5.4353	265.697
8	7.690	.1300	55.750	.0179	.1379	7.2497	5.6427	314.581
9	8.613	.1161	63.440	.0158	.1358	7.3658	5.8375	370.331
0	9.646	.1037	72.052	.0139	.1339	7.4694	6.0202	433.770
1	10.804	.0926	81.699	.0122	.1322	7.5620	6.1913	505.823
2	12.100	.0826	92.503	.0108	.1308	7.6446	6.3514	587.522
3	13.552	.0738	104.603	.0096	.1296	7.7184	6.5010	680.024
4	15.179	.0659	118.155	.0085	.1285	7.7843	6.6406	784.627
5	17.000	.0588	133.334	.0075	.1275	7.8431	6.7708	902.782
6	19.040	.0525	150.334	.0067	.1267	7.8957	6.8921	1036.116
7	21.325	.0469	169.374	.0059	.1259	7.9426	7.0049	1186.450
8	23.884	.0419	190.699	.0052	.1252	7.9844	7.1098	1355.824
9	26.750	.0374	214.583	.0047	.1247	8.0218	7.2071	1546.523
0	29.960	.0334	241.333	.0041	.1241	8.0552	7.2974	1761.106
1	33.555	.0298	271.293	.0037	.1237	8.0850	7.3811	2002.438
2	37.582	.0266	304.848	.0033	.1233	8.1116	7.4586	2273.731
3	42.092	.0238	342.429	.0029	.1229	8.1354	7.5302	2578.579
4	47.143	.0212	384.521	.0026	.1226	8.1566	7.5965	2921.008
5	52.800	.0189	431.663	.0023	.1223	8.1755	7.6577	3305.529
6	59.136	.0169	484.463	.0021	.1221	8.1924	7.7141	3737.193
7	66.232	.0151	543.599	.0018	.1218	8.2075	7.7661	4221.656
8	74.180	.0135	609.831	.0016	.1216	8.2210	7.8141	4765.254
9	83.081	.0120	684.010	.0015	.1215	8.2330	7.8582	5375.085
0	93.051	.0107	767.091	.0013	.1213	8.2438	7.8988	6059.095
1	104.217	.0096	860.142	.0012	.1212	8.2534	7.9361	6826.187
2	116.723	.0086	964.359	.0010	.1210	8.2619	7.9704	7686.329
3	130.730	.0076	1081.083	.0009	.1209	8.2696	8.0019	8650.688
4	146.418	.0068	1211.813	.0008	.1208	8.2764	8.0308	9731.771
5	163.988	.0061	1358.230	.0007	.1207	8.2825	8.0572	10943.584
6	183.666	.0054	1522.218	.0007	.1207	8.2880	8.0815	12301.814
7	205.706	.0049	1705.884	.0006	.1206	8.2928	8.1037	13824.031
8	230.391	.0043	1911.590	.0005	.1205	8.2972	8.1241	15529.915
9	258.038	.0039	2141.981	.0005	.1205	8.3010	8.1427	17441.505
0	289.002	.0035	2400.018	.0004	.1204	8.3045	8.1597	19583.485

ANNUITY TABLES FOR I=13.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.130	.8850	1.000	1.0000	1.1300	.8850	-.0000	-.00
2	1.277	.7831	2.130	.4695	.5995	1.6681	.4695	1.00
3	1.443	.6931	3.407	.2935	.4235	2.3612	.9187	3.13
4	1.630	.6133	4.850	.2062	.3362	2.9745	1.3479	6.53
5	1.842	.5428	6.480	.1543	.2843	3.5172	1.7571	11.38
6	2.082	.4803	8.323	.1202	.2502	3.9975	2.1468	17.86
7	2.353	.4251	10.405	.0961	.2261	4.4226	2.5171	26.19
8	2.658	.3762	12.757	.0784	.2084	4.7988	2.8695	36.59
9	3.004	.3329	15.416	.0649	.1949	5.1317	3.2014	49.35
10	3.395	.2946	18.420	.0543	.1843	5.4262	3.5162	64.76
11	3.836	.2607	21.814	.0458	.1758	5.6869	3.8134	83.18
12	4.335	.2307	25.650	.0390	.1690	5.9176	4.0936	105.00
13	4.898	.2042	29.985	.0334	.1634	6.1218	4.3573	130.65
14	5.535	.1807	34.883	.0287	.1587	6.3025	4.6050	160.63
15	6.254	.1599	40.417	.0247	.1547	6.4624	4.8375	195.51
16	7.067	.1415	46.672	.0214	.1514	6.6039	5.0552	235.93
17	7.986	.1252	53.739	.0186	.1486	6.7291	5.2589	282.60
18	9.024	.1108	61.725	.0162	.1462	6.8399	5.4491	336.34
19	10.197	.0981	70.749	.0141	.1441	6.9380	5.6265	398.07
20	11.523	.0868	80.947	.0124	.1424	7.0248	5.7917	468.82
21	13.021	.0768	92.470	.0108	.1408	7.1016	5.9454	549.76
22	14.714	.0680	105.491	.0095	.1395	7.1695	6.0881	642.23
23	16.627	.0601	120.205	.0083	.1383	7.2297	6.2205	747.73
24	18.788	.0532	136.831	.0073	.1373	7.2829	6.3431	867.93
25	21.231	.0471	155.620	.0064	.1364	7.3300	6.4566	1004.76
26	23.991	.0417	176.850	.0057	.1357	7.3717	6.5614	1160.38
27	27.109	.0369	200.841	.0050	.1350	7.4086	6.6582	1337.23
28	30.633	.0326	227.950	.0044	.1344	7.4412	6.7474	1538.07
29	34.616	.0289	258.583	.0039	.1339	7.4701	6.8296	1766.02
30	39.116	.0256	293.199	.0034	.1334	7.4957	6.9052	2024.60
31	44.201	.0226	332.315	.0030	.1330	7.5183	6.9747	2317.80
32	49.947	.0200	376.516	.0027	.1327	7.5383	7.0385	2650.12
33	56.440	.0177	426.463	.0023	.1323	7.5560	7.0971	3026.64
34	63.777	.0157	482.903	.0021	.1321	7.5717	7.1507	3453.10
35	72.069	.0139	546.681	.0018	.1318	7.5856	7.1998	3936.00
36	81.437	.0123	618.749	.0016	.1316	7.5979	7.2448	4482.68
37	92.024	.0109	700.187	.0014	.1314	7.6087	7.2858	5101.43
38	103.987	.0096	792.211	.0013	.1313	7.6183	7.3233	5801.62
39	117.506	.0085	896.198	.0011	.1311	7.6268	7.3576	6593.83
40	132.782	.0075	1013.704	.0010	.1310	7.6344	7.3888	7490.03
41	150.043	.0067	1146.486	.0009	.1309	7.6410	7.4172	8503.73
42	169.549	.0059	1296.529	.0008	.1308	7.6469	7.4431	9650.22
43	191.590	.0052	1466.078	.0007	.1307	7.6522	7.4667	10946.75
44	216.497	.0046	1657.668	.0006	.1306	7.6568	7.4881	12412.82
45	244.641	.0041	1874.165	.0005	.1305	7.6609	7.5076	14070.49
46	276.445	.0036	2118.806	.0005	.1305	7.6645	7.5253	15944.66
47	312.383	.0032	2395.251	.0004	.1304	7.6677	7.5414	18063.46
48	352.992	.0028	2707.633	.0004	.1304	7.6705	7.5559	20458.71
49	398.881	.0025	3060.626	.0003	.1303	7.6730	7.5692	23166.35
50	450.736	.0022	3459.507	.0003	.1303	7.6752	7.5811	26226.97

ANNUITY TABLES FOR I=14.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.140	.8772	1.000	1.0000	1.1400	.8772	-.0000	-.000
2	1.300	.7695	2.140	.4673	.6073	1.6467	.4673	1.000
3	1.482	.6750	3.440	.2907	.4307	2.3216	.9129	3.140
4	1.689	.5921	4.921	.2032	.3432	2.9137	1.3370	6.580
5	1.925	.5194	6.610	.1513	.2913	3.4331	1.7399	11.501
6	2.195	.4556	8.536	.1172	.2572	3.8887	2.1218	18.111
7	2.502	.3996	10.730	.0932	.2332	4.2883	2.4832	26.646
8	2.853	.3506	13.233	.0756	.2156	4.6389	2.8246	37.377
9	3.252	.3075	16.085	.0622	.2022	4.9464	3.1463	50.610
0	3.707	.2697	19.337	.0517	.1917	5.2161	3.4490	66.695
1	4.226	.2366	23.045	.0434	.1834	5.4527	3.7333	86.032
2	4.818	.2076	27.271	.0367	.1767	5.6603	3.9998	109.077
3	5.492	.1821	32.089	.0312	.1712	5.8424	4.2491	136.348
4	6.261	.1597	37.581	.0266	.1666	6.0021	4.4819	168.436
5	7.138	.1401	43.842	.0228	.1628	6.1422	4.6990	206.017
6	8.137	.1229	50.980	.0196	.1596	6.2651	4.9011	249.860
7	9.276	.1078	59.118	.0169	.1569	6.3729	5.0888	300.840
8	10.575	.0946	68.394	.0146	.1546	6.4674	5.2630	359.958
9	12.056	.0829	78.969	.0127	.1527	6.5504	5.4243	428.352
0	13.743	.0728	91.025	.0110	.1510	6.6231	5.5734	507.321
1	15.668	.0638	104.768	.0095	.1495	6.6870	5.7111	598.346
2	17.861	.0560	120.436	.0083	.1483	6.7429	5.8381	703.114
3	20.362	.0491	138.297	.0072	.1472	6.7921	5.9549	823.550
4	23.212	.0431	158.659	.0063	.1463	6.8351	6.0624	961.847
5	26.462	.0378	181.871	.0055	.1455	6.8729	6.1610	1120.506
6	30.167	.0331	208.333	.0048	.1448	6.9061	6.2514	1302.377
7	34.390	.0291	238.499	.0042	.1442	6.9352	6.3342	1510.709
8	39.204	.0255	272.889	.0037	.1437	6.9607	6.4100	1749.209
9	44.693	.0224	312.094	.0032	.1432	6.9830	6.4791	2022.098
0	50.950	.0196	356.787	.0028	.1428	7.0027	6.5423	2334.192
1	58.083	.0172	407.737	.0025	.1425	7.0199	6.5998	2690.979
2	66.215	.0151	465.820	.0021	.1421	7.0350	6.6522	3098.716
3	75.485	.0132	532.035	.0019	.1419	7.0482	6.6998	3564.536
4	86.053	.0116	607.520	.0016	.1416	7.0599	6.7431	4096.571
5	98.100	.0102	693.573	.0014	.1414	7.0700	6.7824	4704.091
6	111.834	.0089	791.673	.0013	.1413	7.0790	6.8180	5397.663
7	127.491	.0078	903.507	.0011	.1411	7.0868	6.8503	6189.336
8	145.340	.0069	1030.998	.0010	.1410	7.0937	6.8796	7092.843
9	165.687	.0060	1176.338	.0009	.1409	7.0997	6.9060	8123.841
0	188.884	.0053	1342.025	.0007	.1407	7.1050	6.9300	9300.179
1	215.327	.0046	1530.909	.0007	.1407	7.1097	6.9516	10642.204
2	245.473	.0041	1746.236	.0006	.1406	7.1138	6.9711	12173.113
3	279.839	.0036	1991.709	.0005	.1405	7.1173	6.9986	13919.349
4	319.017	.0031	2271.548	.0004	.1404	7.1205	7.0045	15911.058
5	363.679	.0027	2590.565	.0004	.1404	7.1232	7.0188	18182.606
6	414.594	.0024	2954.244	.0003	.1403	7.1256	7.0316	20773.171
7	472.637	.0021	3368.838	.0003	.1403	7.1277	7.0432	23727.414
8	538.807	.0019	3841.475	.0003	.1403	7.1296	7.0536	27096.252
9	614.239	.0016	4380.282	.0002	.1402	7.1312	7.0630	30937.728
0	700.233	.0014	4994.521	.0002	.1402	7.1327	7.0714	35318.010

ANNUITY TABLES FOR I=15.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.150	.8696	1.000	1.0000	1.1500	.8696	-.0000	-.000
2	1.323	.7561	2.150	.4651	.6151	1.6257	.4651	1.000
3	1.521	.6575	3.473	.2880	.4380	2.2832	.9071	3.150
4	1.749	.5718	4.993	.2003	.3503	2.8550	1.3263	6.622
5	2.011	.4972	6.742	.1483	.2983	3.3522	1.7228	11.616
6	2.313	.4323	8.754	.1142	.2642	3.7845	2.0972	18.358
7	2.660	.3759	11.067	.0904	.2404	4.1604	2.4498	27.112
8	3.059	.3269	13.727	.0729	.2229	4.4873	2.7813	38.179
9	3.518	.2843	16.786	.0596	.2096	4.7716	3.0922	51.906
10	4.046	.2472	20.304	.0493	.1993	5.0188	3.3832	68.691
11	4.652	.2149	24.349	.0411	.1911	5.2337	3.6549	88.995
12	5.350	.1869	29.002	.0345	.1845	5.4206	3.9082	113.344
13	6.153	.1625	34.352	.0291	.1791	5.5831	4.1438	142.346
14	7.076	.1413	40.505	.0247	.1747	5.7245	4.3624	176.698
15	8.137	.1229	47.580	.0210	.1710	5.8474	4.5650	217.203
16	9.358	.1069	55.717	.0179	.1679	5.9542	4.7522	264.783
17	10.761	.0929	65.075	.0154	.1654	6.0472	4.9251	320.501
18	12.375	.0808	75.836	.0132	.1632	6.1280	5.0843	385.576
19	14.232	.0703	88.212	.0113	.1613	6.1982	5.2307	461.412
20	16.367	.0611	102.444	.0098	.1598	6.2593	5.3651	549.624
21	18.822	.0531	118.810	.0084	.1584	6.3125	5.4893	652.067
22	21.645	.0462	137.632	.0073	.1573	6.3587	5.6010	770.878
23	24.891	.0402	159.276	.0063	.1563	6.3988	5.7040	908.509
24	28.625	.0349	184.168	.0054	.1554	6.4338	5.7979	1067.786
25	32.919	.0304	212.793	.0047	.1547	6.4641	5.8834	1251.953
26	37.857	.0264	245.712	.0041	.1541	6.4906	5.9612	1464.746
27	43.535	.0230	283.569	.0035	.1535	6.5135	6.0319	1710.458
28	50.066	.0200	327.104	.0031	.1531	6.5335	6.0960	1994.027
29	57.575	.0174	377.170	.0027	.1527	6.5509	6.1541	2321.131
30	66.212	.0151	434.745	.0023	.1523	6.5660	6.2066	2698.301

ANNUITY TABLES FOR I=16.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.160	.8621	1.000	1.0000	1.1600	.8621	-.0000	-.000
2	1.346	.7432	2.160	.4630	.6230	1.6052	.4630	1.000
3	1.561	.6407	3.506	.2853	.4453	2.2459	.9014	3.160
4	1.811	.5523	5.066	.1974	.3574	2.7982	1.3156	6.666
5	2.100	.4761	6.877	.1454	.3054	3.2743	1.7060	11.732
6	2.436	.4104	8.977	.1114	.2714	3.6847	2.0729	18.609
7	2.826	.3538	11.414	.0876	.2476	4.0386	2.4169	27.587
8	3.278	.3050	14.240	.0702	.2302	4.3436	2.7388	39.001
9	3.803	.2630	17.519	.0571	.2171	4.6065	3.0391	53.241
0	4.411	.2267	21.321	.0469	.2069	4.8332	3.3187	70.759
1	5.117	.1954	25.733	.0389	.1989	5.0286	3.5783	92.081
2	5.936	.1685	30.850	.0324	.1924	5.1971	3.8189	117.814
3	6.886	.1452	36.786	.0272	.1872	5.3423	4.0413	148.664
4	7.988	.1252	43.672	.0229	.1829	5.4675	4.2464	185.450
5	9.266	.1079	51.660	.0194	.1794	5.5755	4.4352	229.122
6	10.748	.0930	60.925	.0164	.1764	5.6685	4.6086	280.781
7	12.468	.0802	71.673	.0140	.1740	5.7487	4.7676	341.706
8	14.463	.0691	84.141	.0119	.1719	5.8178	4.9130	413.379
9	16.777	.0596	98.603	.0101	.1701	5.8775	5.0457	497.520
0	19.461	.0514	115.380	.0087	.1687	5.9288	5.1666	596.123
1	22.574	.0443	134.841	.0074	.1674	5.9731	5.2766	711.503
2	26.186	.0382	157.415	.0064	.1664	6.0113	5.3765	846.344
3	30.376	.0329	183.601	.0054	.1654	6.0442	5.4671	1003.759
4	35.236	.0284	213.978	.0047	.1647	6.0726	5.5490	1187.360
5	40.874	.0245	249.214	.0040	.1640	6.0971	5.6230	1401.338
6	47.414	.0211	290.088	.0034	.1634	6.1182	5.6898	1650.552
7	55.000	.0182	337.502	.0030	.1630	6.1364	5.7500	1940.640
8	63.800	.0157	392.503	.0025	.1625	6.1520	5.8041	2278.142
9	74.009	.0135	456.303	.0022	.1622	6.1656	5.8528	2670.645
0	85.850	.0116	530.312	.0019	.1619	6.1772	5.8964	3126.948

ANNUITY TABLES FOR I=17.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.170	.8547	1.000	1.0000	1.1700	.8547	-.0000	-.00
2	1.369	.7305	2.170	.4608	.6308	1.5852	.4608	1.00
3	1.602	.6244	3.539	.2826	.4526	2.2096	.8958	3.17
4	1.874	.5337	5.141	.1945	.3645	2.7432	1.3051	6.70
5	2.192	.4561	7.014	.1426	.3126	3.1993	1.6893	11.84
6	2.565	.3898	9.207	.1086	.2786	3.5892	2.0489	18.86
7	3.001	.3332	11.772	.0849	.2549	3.9224	2.3845	28.07
8	3.511	.2848	14.773	.0677	.2377	4.2072	2.6969	39.84
9	4.108	.2434	18.285	.0547	.2247	4.4506	2.9870	54.61
10	4.807	.2080	22.393	.0447	.2147	4.6586	3.2555	72.90
11	5.624	.1778	27.200	.0368	.2068	4.8364	3.5035	95.29
12	6.580	.1520	32.824	.0305	.2005	4.9884	3.7318	122.49
13	7.699	.1299	39.404	.0254	.1954	5.1183	3.9417	155.31
14	9.007	.1110	47.103	.0212	.1912	5.2293	4.1340	194.72
15	10.539	.0949	56.110	.0178	.1878	5.3242	4.3098	241.82
16	12.330	.0811	66.649	.0150	.1850	5.4053	4.4702	297.93
17	14.426	.0693	78.979	.0127	.1827	5.4746	4.6162	364.58
18	16.879	.0592	93.406	.0107	.1807	5.5339	4.7488	443.56
19	19.748	.0506	110.285	.0091	.1791	5.5845	4.8689	536.96
20	23.106	.0433	130.033	.0077	.1777	5.6278	4.9776	647.25
21	27.034	.0370	153.139	.0065	.1765	5.6648	5.0757	777.28
22	31.629	.0316	180.172	.0056	.1756	5.6964	5.1641	930.42
23	37.006	.0270	211.801	.0047	.1747	5.7234	5.2436	1110.59
24	43.297	.0231	248.808	.0040	.1740	5.7465	5.3149	1322.39
25	50.658	.0197	292.105	.0034	.1734	5.7662	5.3789	1571.20
26	59.270	.0169	342.763	.0029	.1729	5.7831	5.4362	1863.31
27	69.345	.0144	402.032	.0025	.1725	5.7975	5.4873	2206.07
28	81.134	.0123	471.378	.0021	.1721	5.8099	5.5329	2608.10
29	94.927	.0105	552.512	.0018	.1718	5.8204	5.5736	3079.48
30	111.065	.0090	647.439	.0015	.1715	5.8294	5.6098	3631.99

ANNUITY TABLES FOR I=18.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.180	.8475	1.000	1.0000	1.1800	.8475	0.0000	0.000
2	1.392	.7182	2.180	.4587	.6387	1.5656	.4587	1.000
3	1.643	.6086	3.572	.2799	.4599	2.1743	.8902	3.180
4	1.939	.5158	5.215	.1917	.3717	2.6901	1.2947	6.752
5	2.288	.4371	7.154	.1398	.3198	3.1272	1.6728	11.968
6	2.700	.3704	9.442	.1059	.2859	3.4976	2.0252	19.122
7	3.185	.3139	12.142	.0824	.2624	3.8115	2.3526	28.564
8	3.759	.2660	15.327	.0652	.2452	4.0776	2.6558	40.706
9	4.435	.2255	19.086	.0524	.2324	4.3030	2.9358	56.033
0	5.234	.1911	23.521	.0425	.2225	4.4941	3.1936	75.118
1	6.176	.1619	28.755	.0348	.2148	4.6560	3.4303	98.640
2	7.288	.1372	34.931	.0286	.2086	4.7932	3.6470	127.395
3	8.599	.1163	42.219	.0237	.2037	4.9095	3.8449	162.326
4	10.147	.0985	50.818	.0197	.1997	5.0081	4.0[illegible]	204.545
5	11.974	.0835	60.965	.0164	.1964	5.0916	4.[illegible]	255.363
6	14.129	.0708	72.939	.0137	.1937	5.1624	4.3369	316.328
7	16.672	.0600	87.068	.0115	.1915	5.2223	4.4708	389.267
8	19.673	.0508	103.740	.0096	.1896	5.2732	4.5916	476.335
9	23.214	.0431	123.414	.0081	.1881	5.3162	4.7003	580.075
0	27.393	.0365	146.628	.0068	.1868	5.3527	4.7978	703.489
1	32.324	.0309	174.021	.0057	.1857	5.3837	4.8851	850.117
2	38.142	.0262	206.345	.0048	.1848	5.4099	4.9632	1024.138
3	45.008	.0222	244.487	.0041	.1841	5.4321	5.0329	1230.482
4	53.109	.0188	289.494	.0035	.1835	5.4509	5.0950	1474.969
5	62.669	.0160	342.603	.0029	.1829	5.4669	5.1502	1764.464
6	73.949	.0135	405.272	.0025	.1825	5.4804	5.1991	2107.067
7	87.260	.0115	479.221	.0021	.1821	5.4919	5.2425	2512.339
8	102.967	.0097	566.481	.0018	.1818	5.5016	5.2810	2991.561
9	121.501	.0082	669.447	.0015	.1815	5.5098	5.3149	3558.041
0	143.371	.0070	790.948	.0013	.1813	5.5168	5.3448	4227.489

ANNUITY TABLES FOR I=19.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.190	.8403	1.000	1.0000	1.1900	.8403	-.0000	-.00
2	1.416	.7062	2.190	.4566	.6466	1.5465	.4566	1.00
3	1.685	.5934	3.606	.2773	.4673	2.1399	.8846	3.19
4	2.005	.4987	5.291	.1890	.3790	2.6386	1.2844	6.79
5	2.386	.4190	7.297	.1371	.3271	3.0576	1.6566	12.08
6	2.840	.3521	9.683	.1033	.2933	3.4098	2.0019	19.38
7	3.379	.2959	12.523	.0799	.2699	3.7057	2.3211	29.06
8	4.021	.2487	15.902	.0629	.2529	3.9544	2.6154	41.59
9	4.785	.2090	19.923	.0502	.2402	4.1633	2.8856	57.49
10	5.695	.1756	24.709	.0405	.2305	4.3389	3.1331	77.41
11	6.777	.1476	30.404	.0329	.2229	4.4865	3.3589	102.12
12	8.064	.1240	37.180	.0269	.2169	4.6105	3.5645	132.52
13	9.596	.1042	45.244	.0221	.2121	4.7147	3.7509	169.70
14	11.420	.0876	54.841	.0182	.2082	4.8023	3.9196	214.95
15	13.590	.0736	66.261	.0151	.2051	4.8759	4.0717	269.79
16	16.172	.0618	79.850	.0125	.2025	4.9377	4.2086	336.05
17	19.244	.0520	96.022	.0104	.2004	4.9897	4.3314	415.90
18	22.901	.0437	115.266	.0087	.1987	5.0333	4.4413	511.92
19	27.252	.0367	138.166	.0072	.1972	5.0700	4.5394	627.19
20	32.429	.0308	165.418	.0060	.1960	5.1009	4.6268	765.35
21	38.591	.0259	197.847	.0051	.1951	5.1268	4.7045	930.77
22	45.923	.0218	236.438	.0042	.1942	5.1486	4.7734	1128.62
23	54.649	.0183	282.362	.0035	.1935	5.1668	4.8344	1365.06
24	65.032	.0154	337.010	.0030	.1930	5.1822	4.8883	1647.42
25	77.388	.0129	402.042	.0025	.1925	5.1951	4.9359	1984.43
26	92.092	.0109	479.431	.0021	.1921	5.2060	4.9777	2386.47
27	109.589	.0091	571.522	.0017	.1917	5.2151	5.0145	2865.90
28	130.411	.0077	681.112	.0015	.1915	5.2228	5.0468	3437.43
29	155.189	.0064	811.523	.0012	.1912	5.2292	5.0751	4118.54
30	184.675	.0054	966.712	.0010	.1910	5.2347	5.0998	4930.06

ANNUITY TABLES FOR I=20.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.200	.8333	1.000	1.0000	1.2000	.8333	-.0000	-.000
2	1.440	.6944	2.200	.4545	.6545	1.5278	.4545	1.000
3	1.728	.5787	3.640	.2747	.4747	2.1065	.8791	3.200
4	2.074	.4823	5.368	.1863	.3863	2.5887	1.2742	6.840
5	2.488	.4019	7.442	.1344	.3344	2.9906	1.6405	12.208
6	2.986	.3349	9.930	.1007	.3007	3.3255	1.9788	19.650
7	3.583	.2791	12.916	.0774	.2774	3.6046	2.2902	29.580
8	4.300	.2326	16.499	.0606	.2606	3.8372	2.5756	42.495
9	5.160	.1938	20.799	.0481	.2481	4.0310	2.8364	58.995
0	6.192	.1615	25.959	.0385	.2385	4.1925	3.0739	79.793
1	7.430	.1346	32.150	.0311	.2311	4.3271	3.2893	105.752
2	8.916	.1122	39.581	.0253	.2253	4.4392	3.4841	137.903
3	10.699	.0935	48.497	.0206	.2206	4.5327	3.6597	177.483
4	12.839	.0779	59.196	.0169	.2169	4.6106	3.8175	225.980
5	15.407	.0649	72.035	.0139	.2139	4.6755	3.9588	285.176
6	18.488	.0541	87.442	.0114	.2114	4.7296	4.0851	357.211
7	22.186	.0451	105.931	.0094	.2094	4.7746	4.1976	444.653
8	26.623	.0376	128.117	.0078	.2078	4.8122	4.2975	550.583
9	31.948	.0313	154.740	.0065	.2065	4.8435	4.3861	678.700
0	38.338	.0261	186.688	.0054	.2054	4.8696	4.4643	833.440

ANNUITY TABLES FOR I=25.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.250	.8000	1.000	1.0000	1.2500	.8000	0.0000	0.000
2	1.563	.6400	2.250	.4444	.6944	1.4400	.4444	1.000
3	1.953	.5120	3.813	.2623	.5123	1.9520	.8525	3.250
4	2.441	.4096	5.766	.1734	.4234	2.3616	1.2249	7.063
5	3.052	.3277	8.207	.1218	.3718	2.6893	1.5631	12.828
6	3.815	.2621	11.259	.0888	.3388	2.9514	1.8683	21.035
7	4.768	.2097	15.073	.0663	.3163	3.1611	2.1424	32.294
8	5.960	.1678	19.842	.0504	.3004	3.3289	2.3872	47.367
9	7.451	.1342	25.802	.0388	.2888	3.4631	2.6048	67.209
10	9.313	.1074	33.253	.0301	.2801	3.5705	2.7971	93.012
11	11.642	.0859	42.566	.0235	.2735	3.6564	2.9663	126.265
12	14.552	.0687	54.208	.0184	.2684	3.7251	3.1145	168.831
13	18.190	.0550	68.760	.0145	.2645	3.7801	3.2437	223.038
14	22.737	.0440	86.949	.0115	.2615	3.8241	3.3559	291.798
15	28.422	.0352	109.687	.0091	.2591	3.8593	3.4530	378.747
16	35.527	.0281	138.109	.0072	.2572	3.8874	3.5366	488.434
17	44.409	.0225	173.636	.0058	.2558	3.9099	3.6084	626.543
18	55.511	.0180	218.045	.0046	.2546	3.9279	3.6698	800.178
19	69.389	.0144	273.556	.0037	.2537	3.9424	3.7222	1018.223
20	86.736	.0115	342.945	.0029	.2529	3.9539	3.7667	1291.779

ANNUITY TABLES FOR I=30.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.300	.7692	1.000	1.0000	1.3000	.7692	-.0000	-.000
2	1.690	.5917	2.300	.4348	.7348	1.3609	.4348	1.000
3	2.197	.4552	3.990	.2506	.5506	1.8161	.8271	3.300
4	2.856	.3501	6.187	.1616	.4616	2.1662	1.1783	7.290
5	3.713	.2693	9.043	.1106	.4106	2.4356	1.4903	13.477
6	4.827	.2072	12.756	.0784	.3784	2.6427	1.7654	22.520
7	6.275	.1594	17.583	.0569	.3569	2.8021	2.0063	35.276
8	8.157	.1226	23.858	.0419	.3419	2.9247	2.2156	52.859
9	10.604	.0943	32.015	.0312	.3312	3.0190	2.3963	76.717
0	13.786	.0725	42.619	.0235	.3235	3.0915	2.5512	108.732
1	17.922	.0558	56.405	.0177	.3177	3.1473	2.6833	151.351
2	23.298	.0429	74.327	.0135	.3135	3.1903	2.7952	207.757
3	30.288	.0330	97.625	.0102	.3102	3.2233	2.8895	282.083
4	39.374	.0254	127.913	.0078	.3078	3.2487	2.9685	379.708
5	51.186	.0195	167.286	.0060	.3060	3.2682	3.0344	507.621
6	66.542	.0150	218.472	.0046	.3046	3.2832	3.0892	674.907
7	86.504	.0116	285.014	.0035	.3035	3.2948	3.1345	893.380
8	112.455	.0089	371.518	.0027	.3027	3.3037	3.1718	1178.393
9	146.192	.0068	483.973	.0021	.3021	3.3105	3.2025	1549.911
0	190.050	.0053	630.165	.0016	.3016	3.3158	3.2275	2033.885

ANNUITY TABLES FOR I=40.0 PERCENT

N	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.400	.7143	1.000	1.0000	1.4000	.7143	0.0000	0.00
2	1.960	.5102	2.400	.4167	.8167	1.2245	.4167	1.00
3	2.744	.3644	4.360	.2294	.6294	1.5889	.7798	3.40
4	3.842	.2603	7.104	.1408	.5408	1.8492	1.0923	7.76
5	5.378	.1859	10.946	.0914	.4914	2.0352	1.3580	14.86
6	7.530	.1328	16.324	.0613	.4613	2.1680	1.5811	25.81
7	10.541	.0949	23.853	.0419	.4419	2.2628	1.7664	42.13
8	14.758	.0678	34.395	.0291	.4291	2.3306	1.9185	65.98
9	20.661	.0484	49.153	.0203	.4203	2.3790	2.0422	100.38
10	28.925	.0346	69.814	.0143	.4143	2.4136	2.1419	149.53
11	40.496	.0247	98.739	.0101	.4101	2.4383	2.2215	219.34
12	56.694	.0176	139.235	.0072	.4072	2.4559	2.2845	318.06
13	79.371	.0126	195.929	.0051	.4051	2.4685	2.3341	457.32
14	111.120	.0090	275.300	.0036	.4036	2.4775	2.3729	653.25
15	155.568	.0064	386.420	.0026	.4026	2.4839	2.4030	928.55
16	217.795	.0046	541.988	.0018	.4018	2.4885	2.4262	1314.97
17	304.913	.0033	759.784	.0013	.4013	2.4918	2.4441	1856.95
18	426.879	.0023	1064.697	.0009	.4009	2.4941	2.4577	2616.74
19	597.630	.0017	1491.576	.0007	.4007	2.4958	2.4682	3681.44
20	836.683	.0012	2089.206	.0005	.4005	2.4970	2.4761	5173.01

ANNUITY TABLES FOR I=50.0 PERCENT

	CAF	PWF	SCAF	SFF	CRF	SPWF	GUSF	GCAF
1	1.500	.6667	1.000	1.0000	1.5000	.6667	0.0000	0.000
2	2.250	.4444	2.500	.4000	.9000	1.1111	.4000	1.000
3	3.375	.2963	4.750	.2105	.7105	1.4074	.7368	3.500
4	5.063	.1975	8.125	.1231	.6231	1.6049	1.0154	8.250
5	7.594	.1317	13.188	.0758	.5758	1.7366	1.2417	16.375
6	11.391	.0878	20.781	.0481	.5481	1.8244	1.4226	29.563
7	17.086	.0585	32.172	.0311	.5311	1.8829	1.5648	50.344
8	25.629	.0390	49.258	.0203	.5203	1.9220	1.6752	82.516
9	38.443	.0260	74.887	.0134	.5134	1.9480	1.7596	131.773
0	57.665	.0173	113.330	.0088	.5088	1.9653	1.8235	206.660
1	86.498	.0116	170.995	.0058	.5058	1.9769	1.8713	319.990
2	129.746	.0077	257.493	.0039	.5039	1.9846	1.9068	490.985
3	194.620	.0051	387.239	.0026	.5026	1.9897	1.9329	748.478
4	291.929	.0034	581.859	.0017	.5017	1.9931	1.9519	1135.717
5	437.894	.0023	873.788	.0011	.5011	1.9954	1.9657	1717.576
6	656.841	.0015	1311.682	.0008	.5008	1.9970	1.9756	2591.363
7	985.261	.0010	1968.523	.0005	.5005	1.9980	1.9827	3903.045
8	1477.892	.0007	2953.784	.0003	.5003	1.9986	1.9878	5871.568
9	2216.838	.0005	4431.676	.0002	.5002	1.9991	1.9914	8825.351
0	3325.257	.0003	6648.513	.0002	.5002	1.9994	1.9940	13257.027

APPENDIX D

SOLUTIONS TO EXERCISES

D1. MATRIX MULTIPLICATION

First compute the produce C^T, viz

$$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 4 & 4 & 1 & 4 \\ 2 & 5 & 5 & 2 & 5 \\ 3 & 6 & 6 & 3 & 6 \end{bmatrix}$$

$$(3x2) \qquad (2x5) \qquad (3x5)$$

followed by the multiplication

$$\begin{bmatrix} 2 & 3 & 4 \\ 5 & 6 & 7 \end{bmatrix} \begin{bmatrix} 1 & 4 & 4 & 1 & 4 \\ 2 & 5 & 5 & 2 & 5 \\ 3 & 6 & 6 & 3 & 6 \end{bmatrix} \quad \begin{bmatrix} 2+6+12 & 8+15+24 & 8+15+24 & 2+6+12 & 8+15+24 \\ 5+12+21 & 20+30+42 & 20+30+42 & 5+12+21 & 20+30+42 \end{bmatrix}$$

$$\underset{B}{(2x3)} \qquad \underset{C^T}{(3x5)} \qquad = \qquad \underset{A}{(2x5)}$$

$$= \begin{bmatrix} 20 & 47 & 47 & 20 & 47 \\ 38 & 92 & 92 & 38 & 92 \end{bmatrix}$$

D2. MATRIX DETERMINANTS

$$A = \begin{bmatrix} 3 & 8 \\ 6 & 16 \end{bmatrix} \quad |A| = 3x16 - 6x8 = 0$$ Matrix is singular,

row 2 equals twice row 1

$$A = \begin{bmatrix} 3 & 7 \\ 7 & 2 \end{bmatrix} \quad |A| = 3x2 - 7x7 = -43$$

D7. MATRIX REPRESENTATION OF RESIDENTIAL DEMAND IN THE DOMINICAN REPUBLIC

Using the same notation as in Section 3.4, the Basic Energy demand assignment matrix D is the 3x1 matrix

$$\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} = \begin{bmatrix} \text{Lighting} \\ \text{Cooking} \\ \text{Misc. Appliance} \end{bmatrix}$$

The Basic Energy demand matrix Δ is

$$\begin{bmatrix} 0.155 & 0 & 0 \\ 0 & 7.8 & 0 \\ 0 & 0 & 0.052 \end{bmatrix}$$

Since appliance use increases 4-fold. Thus the vector of basic energy demands is

$$\underset{(3\text{x}1)}{d} = \underset{\substack{(3\text{x}3) \\ \left[\frac{10^9 J}{HH}\right]}}{\begin{bmatrix} 0.155 & & \\ & 7.8 & \\ & & 0.052 \end{bmatrix}} \underset{\substack{(3\text{x}1) \\ [\ ]}}{\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}} \underset{\substack{(1\text{x}1) \\ [10^3 H]}}{[798]} = \underset{[10^{12} J]}{\begin{bmatrix} 123.69 \\ 6224. \\ 41.49 \end{bmatrix}}$$

Basic energy demand by end-use device, y, is given by

$$\underset{(5\text{x}1)}{y} = \underset{(5\text{x}3)}{\begin{bmatrix} 0.2 & & \\ 0.8 & & \\ & 0.24 & \\ & 0.76 & \\ & & 1.0 \end{bmatrix}} \underset{(3\text{x}1)}{\begin{bmatrix} 123.69 \\ 6224. \\ 41.49 \end{bmatrix}} = \underset{(5\text{x}1)}{\begin{bmatrix} 24.7 \\ 99. \\ 1493 \\ 4728 \\ 41.5 \end{bmatrix}}$$

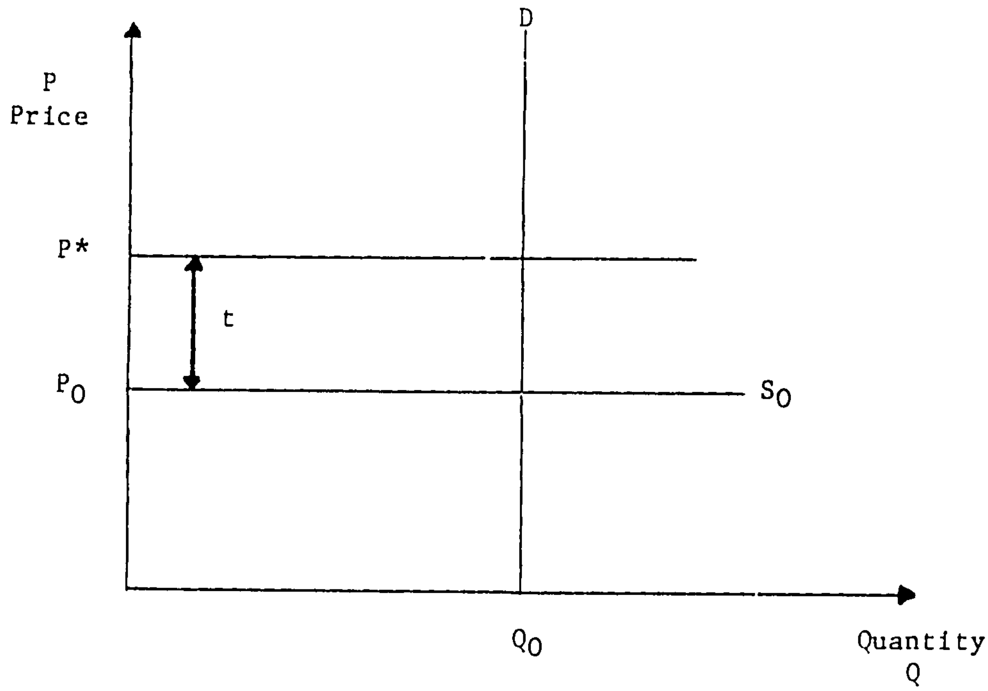
In this case, no dual appliance use occurs, so Z is the identity matrix (and can be ignored). Fuel consumption f is then given by

$$
\underset{(4\times1)}{f} = \underset{(4\times5)}{w}\ \underset{(5\times5)}{\pi}\ \underset{(5\times1)}{y}
$$

$$
\begin{bmatrix} 1&0&0&0&0\\ 0&1&0&0&1\\ 0&0&1&0&0\\ 0&0&0&1&0 \end{bmatrix}
\begin{bmatrix} \frac{1}{0.25} &&&& \\ & 1.0 &&& \\ && \frac{1}{0.3} && \\ &&& \frac{1}{0.2} & \\ &&&& 1.0 \end{bmatrix}
\begin{bmatrix} 24.7\\ 99.\\ 1493.\\ 4728.\\ 41.5 \end{bmatrix}
=
\begin{bmatrix} 98.8\\ 140.5\\ 4797.6\\ 23640 \end{bmatrix}
=
\begin{bmatrix} \text{Kerosene}\\ \text{Electricity}\\ \text{Charcoal}\\ \text{Firewood} \end{bmatrix}
$$

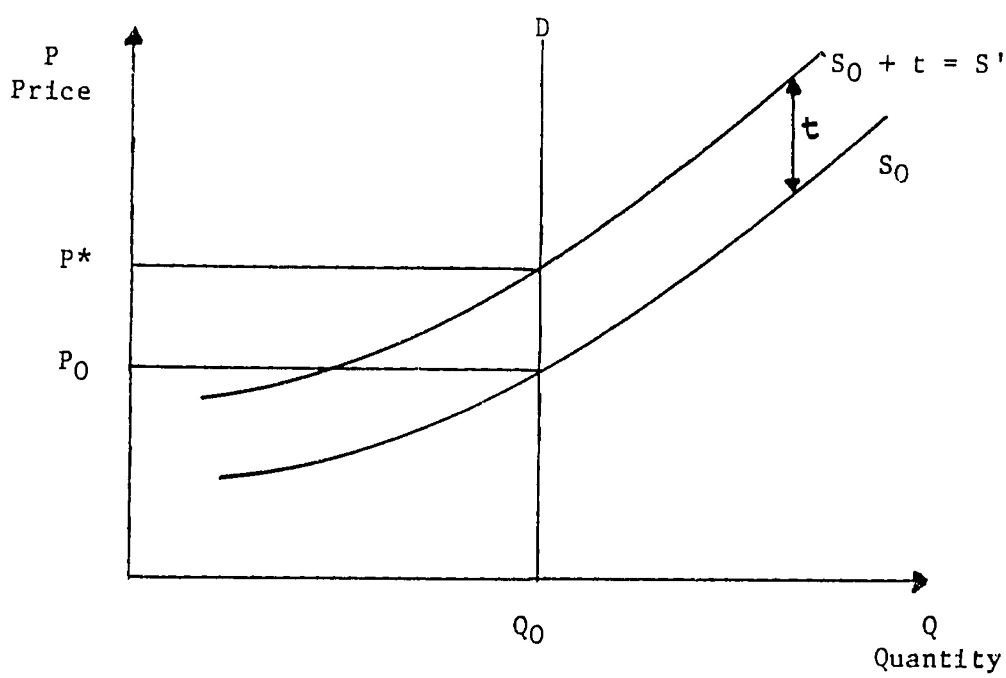
D8. IMPACT OF ENERGY TAXES

If the supply cruve is flat, and the demand for kerosene is completely inelastic, then a tax of size t has the following impact



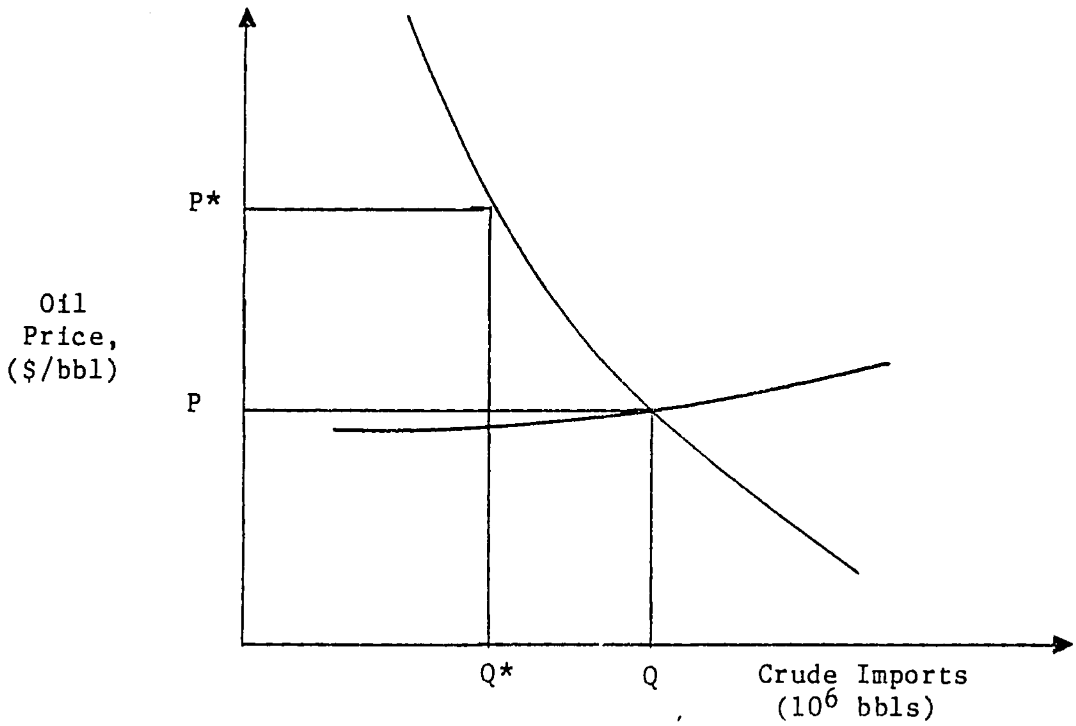
At the new price P*, the quantity of kerosene consumed stays unchanged. Government revenue is $(P^* - P_0) \cdot t$, which is borne entirely by consumers. Note that because consumption remains unchanged, foreign exchange outlay also remains unchanged.

This result holds true even if the supply curve has an upward slope:



D9. Import Quotas

The supply and demand curves might be drawn as follows:



Prior to the import quota, equilibrium exists at point P, Q. We show the supply curve as essentially flat, on the assumption that world oil prices are independant of consumption in our hypothetical country. If we restrict oil imports to Q*, consumers would be willing to pay as much as P*. If prices were not allowed to rise to this level, and remained at P, a black market in oil would occur.

D10. ADDITION OF CAPACITY VARIABLES TO THE ENERGY SYSTEM LP

To the formulation of (5.27);

$$
\begin{array}{llllll}
\text{Min} & C_1X + C_2X_2 + C_3X_3 & & & & \\
\text{S.t.} & G_1X_1 & & & = & D \\
 & & G_2X_2 & & \leq & S \\
 & G_3X_1 & & G_4X_3 & = & 0 \\
 & & G_5X_2 & G_6X_3 & = & 0
\end{array}
$$

we add the acpacity constraint for energy supply and energy conversion as

$$H_1X_2 - W_s \leq \bar{W}_s$$
$$H_2X_3 - W_c \leq \bar{W}_c$$

and add to the objective function the term

$$(\lambda_s W_s + \lambda_c W_c) \text{ CRF}$$

where

λ_s = cost per unit of new capacity of energy supply

λ_c = cost per unit of new capacity of energy conversion facilities

W_s = new energy supply capacity

W_c = new energy conversion capacity

$\bar{W}_s$ = vector of existing capacity of energy supply facilities

$\bar{W}_c$ = vector of existing capacity of energy conversion facilities

H_j = coefficient matrices

CRF = capital recovery factor

Capital expenditures are subject to an overall limitation of λ, then the capital constraint is

$$\lambda_s W_s + \lambda_c W_c \leq k \quad .$$

The complete primal problem now reads as follows

$$
\begin{array}{lllllll}
\text{Min } C_1X_1 + C_2X_2 + C_3X_3 + \text{CRF} \cdot \lambda_s W_s + \text{CRF}\lambda_c W_c & & & & & & \\
\text{s.t } G_1X_1 & & & & & = & D \\
 & G_2X_2 & & & & \leq & S \\
G_3X_1 & & G_4X_3 & & & = & 0 \\
 & G_5X_2 & G_6X_3 & & & = & 0 \\
 & H_1X_3 & & -IW_c & & \leq & \bar{W}_c \\
 & & H_2X_3 & & -IW_c & \leq & \bar{W}_c \\
 & & & \lambda_s W_s & \lambda_c W_c & \leq & k
\end{array}
$$

from which the dual follows immediately as

$$
\begin{array}{llllllll}
\text{Max} & D^T\pi_D + S^T\pi_S & & & + \bar{W}_S\pi_W + & \bar{W}_C\pi_C & + k\pi_k & \\
\text{s.t} & G_1^T\pi_D & G_3^T\pi' & & & & & \leq C_1 \\
& G_2^T\pi_S & & G_5^T\pi'' & H_1^T\pi_W & & & \leq C_2 \\
& & G_4^T\pi' & G_6^T\pi'' & & H_2^T\pi_C & & \leq C_3 \\
& & & & I\pi_W & & \lambda_S^T\pi k & \leq CRF \cdot \lambda_S \\
& & & & & I\pi_c & \lambda_c^T\pi_k & \leq CRF \cdot \lambda_C
\end{array}
$$

π_k represents the dual associated with the capital constraint, and therefore indicates the value to the energy system of an additional unit of capacity. π_W and π_C represent the value to the energy system of an additional unit of existing capacity.

D13. Interpretation of the A_{SS} Matrix

The non-zero elements in the A_{SS} matrix

t_r = transmission loss factor

t_c = Btu loss in coal cleaning

$_o$ = efficiency of oil fired electric plants

$_c$ = efficiency of coal fired electric plants

r_1 = Btu of crude required for 1 Btu of residual

r_2 = Btu of crude required for 1 but of gasoline

f_j = generation mix coefficients

$$
\begin{bmatrix}
0 & 0 & 0 & r_1 & 0 & r_2 & 0 \\
0 & t_c & \frac{1}{\eta_c} & 0 & 0 & 0 & 0 \\
0 & 0 & t_r & 0 & 0 & 0 & f_c \\
0 & 0 & 0 & 0 & \frac{1}{\eta_o} & 0 & 0 \\
0 & 0 & 0 & 0 & t_r & 0 & f_o \\
0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0
\end{bmatrix}
\cdot
\begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \\ x_7 \end{bmatrix}
+
\begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ Y_6 \\ Y_7 \end{bmatrix}
=
\begin{bmatrix}
r_1x_4 + r_2x_6 + 0 = x_1 \\
x_2t_c + \frac{1}{\eta_c}x_3 + 0 = x_2 \\
t_rx_3 + f_cx_7 + 0 = x_3 \\
\frac{1}{\eta_o}x_5 + 0 = x_4 \\
t_rx_5 + f_ox_7 + 0 = x_5 \\
0 + Y_6 = x_6 \\
0 + Y_7 = x_7
\end{bmatrix}
\begin{matrix} (1) \\ (2) \\ (3) \\ (4) \\ (5) \\ (6) \\ (7) \end{matrix}
$$

since from (t), $x_7 = Y_7$, substituting in (5) we have

$$t_rx_5 + f_oY_7 = x_5$$

hence

$$x_5 = \frac{f_oY_7}{(1 - t_r)} \, .$$

Substituting x_5 into (4)

$$x_4 = \frac{1}{\eta_o} \cdot \frac{f_oY_7}{(1 - t_r)}$$

and substituting x_4 into (3)

$$t_Tx_3 + f_cY_7 = x_3$$

hence

$$x_3 = \frac{f_c Y_7}{(1 - t_r)} \quad .$$

substituting x_3 into (2)

$$x_2 t_c + \frac{1}{\eta_c} \cdot \frac{f_c Y_7}{(1 - t_r)} = x_2$$

which, after rearrangement, yields

$$x_2 = \frac{1}{\eta_c} \cdot \frac{f_c Y_7}{(1 - t_r)} \cdot \frac{1}{(1 - t_c)}$$

finally, substituting in (1)

$$x_1 \quad = \quad r_1 \frac{1}{\eta_o} \frac{f_o Y_7}{(1 - t_r)} + r_2 Y_6$$

D14. RATE OF RETURN

Solution Approach: Determine annual depreciation expense. Then determine net annual cash flow, calculate tax payable, then determine after tax DCFRR.

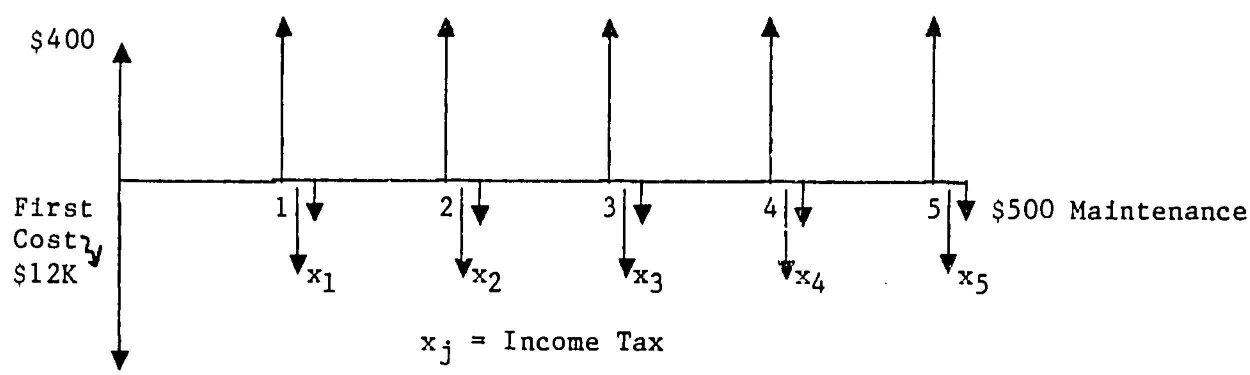
i) Depreciation Schedule: Dpreciation rate = 2 x straight-line rate = 2 x 0.2 = 0.4

Yr	Initial Value	Depreciation	Book Value
1	12000	12000 x 0.4 = 4800	7200
2	7200	7200 x 0.4 = 2880	4320
3	4320	4320 x 0.4 = 1728	2592
4	2592	= 592*	2000
5			2000

*Depreciation determined on the basis of IRS regulations that prevent depreciation to below salvage value.

ii) Annual Cash Flow

(3 1/3% of $12,000) $4500 Labor Savings



x_j = Income Tax

iii)

Tax Calculation:	1	2	3	4	5
1. Gross	4500	4500	4500	4500	4500
2. (O+M)	(500)	(500)	(500)	(500)	(500)
3. (Depreciation)	(4800)	(2880)	(1728)	(592)	0
4. Taxable Income	(-800)	1120	(2272	3408	4000
5. Tax at 48%	(-384)	537	1090	1635	1920
After Tax Cash Flow (1)-(2)-(5)	4384	3463	2910	2365	2080 2000

iv) DCFRR is given by that discount rate for which

$$11600 = \frac{4384}{(1+r)} + \frac{3463}{(1+r)^2} + \frac{2910}{(1+r)^3} + \frac{2965}{(1+r)^4} + \frac{2080}{(1+r)^5}$$

First Cost	\$800,000
10th Year Cost 800,000 x PWF(10,12%) ← 0.3219	257,520
O+M 200,000 x SPWF(25,12%) ← 7.329	146,580
$1000 \times \dfrac{\overset{133.33}{\mathrm{SCAF}(25,12\%)}-25}{0.12} \times \overset{0.0588}{\mathrm{PWF}(25,12\%)}$	53,081
[Gradient to Present Worth Factor]	\$1,257,181

Option A has the higher net present value; hence B is recommended (assuming both meet the same objective).