TEACHERS' GUIDE

MATHEMATICS

PRIMARY ONE

Prepared at the Entebbe Mathematics Workshop

VOLUME ONE

PRELIMINARY EDITION

AFRICAN EDUCATION STUDY
EDUCATIONAL SERVICES INCORPORATED

NOTE

This book is the product of a study and writing workshop conducted during the summer of 1962 at Entebbe, Uganda, with more than fifty mathematicians and mathematics teachers from Africa, the United States, and the United Kingdom in attendance.

In order that it might be tested in African schools as quickly as possible, the book was edited and produced with the utmost speed. As a consequence, both editing and production suffer from defects. More time might have made it possible to eliminate most of these, but it would also have made it impossible to try out this book during the 1962-63 academic year, and it was held that this latter need took priority over the other. The African Education Program can only hope that those who use this book will appreciate the circumstances under which it has been made available, and will be tolerant of its imperfections.

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Arithmetic

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The Entebbe Mathematics Workshop was directed by Professor Walter Prenowitz of Brooklyn College, Brooklyn, N. Y.

Primary One manuscripts were written at Entebbe by the following:

Chairman: Professor Clarence Hardgrove
Northern Illinois University,
De Kalb, Illinois

Editor: Professor E. W. Hamilton
State College of Iowa, Cedar Falls, Iowa

Miss Lucy Addy
Achimota Primary School, Achimota, Ghana

Mr. Marvin Bender
Educational Research Council of Greater
Cleveland, Ohio

Miss Shirley Hill
Institute for Mathematical Studies in Social Sciences,
Stanford University, California

Professor Paul B. Johnson
University of California, Los Angeles, California

Mr. John R. Khasenye,
Makerere College School, Kampala, Uganda

Mr. Theodore B. Max-Peters
Fourah Bay College, Freetown, Sierra Leone

Mr. Noah H. Setidisho
Ministry of Education, Ndola, Northern Rhodesia

Miss Isabella B. Williamson
Ministry of Education, Dar es Salaam, Tanganyika

Manuscripts were prepared for publication at the offices of Educational Services Incorporated, under the supervision of Joan Cummings. Editorial services were provided by Miriam White and Irma B. Cohen, under the direction of Professor Shanks and Mr. Swain; the text was also read critically before publication by Professor B. J. Pettis of the University of North Carolina and Professor Donald Richmond of Williams College. Art work and layout were provided in Entebbe by Mrs. Joan Fleet and in Watertown by Margaret Starr O'Leary, and production was supervised by Benjamin T. Richards.

PREFACE

This Teachers' Guide is one of the products of a writing and study workshop held during the summer of 1962 at Entebbe, Uganda, in which mathematicians and mathematics teachers from all English-speaking Tropical Africa, the United States, and the United Kingdom participated.

The workshop was intended to take the first steps toward the preparation of curricular materials for use in the teaching of mathematics in African schools. Ultimately it is to be hoped that the completed materials will cover the entire range of public schooling, from the earliest years through the Sixth Form, and that they will include audio-visual materials and collateral reading as well as text and teachers' guide. To date only a modest start has been made, represented by half a text and half a teachers' guide for the first year of secondary school; the equivalent for the first year of primary school; a volume of general curriculum recommendations; and a volume of suggestions for the production of non-textual teaching aids.

In making use of the workbook and teachers' guide for Primary I, both teacher and class will best be served if there is some general acquaintance with the nature of the materials so far produced, and with the underlying principles of the total program.

I. Existing Materials Are Preliminary

Materials produced for use in the schools must be considered to be preliminary until they have been tested in the schools, and reviewed in accordance with the results of such tests. This teachers' guide, for example, must be extensively corrected in the light of the experience of teachers. Over the next few years the task of experimentation and correction will be at least as important to the program as any new writing that may be done.

This signifies that the teacher who uses this material will face a twofold task: that of working with materials that are known to be subject to improvement, and that of helping identify the areas where improvement is required. Yet the teacher who shoulders this burden will have the satisfaction of knowing that she is taking part in an experiment which is of great potential value to all of Africa, to all other areas faced with rapidly expanding educational systems, and indeed even to those countries with well-established systems which have failed to keep pace with modern developments.

II. Emphasis is on Mathematical Ideas

In recent years there has been accumulated much evidence that children are far more interested in mathematical ideas than they have usually been given credit for, and that they are far more competent to deal with such ideas than current curricula would suggest. A presentation of mathematics that puts its emphasis on concept rather than on the rules of manipulation is likely to lead to far greater satisfaction on the part of the student, and will also lead to a far greater degree of mathematical competence.

The teacher may be surprised to discover that some mathematics in the Primary I text introduces concepts which are commonly first met only at a far more advanced level. Here again, the experience of recent years has led to the conviction that children are capable of far more than they have usually be given credit for. The teacher should not draw away from the presentation of these concepts, or attempt to dilute them, for when materials are well-prepared, a high degree of confidence in the student is rarely misplaced.

III. The General Approach

The best education is the education that the child creates out of his own direct efforts. The teacher must resist the temptation to tell the class what to do and how to do it. It is indeed a great temptation, for by such means a class will appear to be proceeding at a far more rapid pace. But if the teacher takes the time, and exerts the effort, to get the child himself to generate the right idea, there

is considerably more assurance that the idea will be mastered and retained by this child and will become truly his own.

Here again, the demands upon the teacher are greater. The teacher must be thinking at every moment, for it takes far more insight to lead than to direct. This guide is intended primarily to assist the teacher in the actual conduct of the class, by methods which are chosed to encourage imagination and to generate interest. In addition, each chapter contains mathematical background and explanatory material beyond that given in the workbook.

IV. Relationship to Other Material

The text is designed to prepare the child for further mathematics in either the conventional curriculum or in curricula which will be devised in the next few years to follow upon this text.

It should be repeated that this modified curriculum represents a great opportunity for teacher and child alike. In making use of it, the school participates in a great experiment, out of which there will come an African educational system that will be in the forefront of educational systems throughout the world. It is a massive, joint undertaking, in which the child and the teachers work side by side with mathematicians of international eminence, from their own country and from a dozen others, to create within Africa something that will be of major significance for Africa itself, and in a large measure for all the world.

FIRST YEAR PRIMARY TEACHERS' GUIDE

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FIRST YEAR PRIMARY - TEACHERS' GUIDE

INTRODUCTION

Background. The problems that mankind wants to solve change from one generation to another. The problems of an industrial and commercial society are different from those of a society where small farming and animal husbandry are the principal occupations. Air travel and high speed communication present different problems than do road building and land surveying. Even the methods of doing business and of keeping business records are changing rapidly.

A person with a poor knowledge of mathematics is prevented from following many lines of work or study. He is even unable to understand much of the material in newspapers and magazines for they deal with technical advances and scientific exploits described in a language he does not know. All this gives a new direction to mathematics education in the modern world. It is necessary to bring both books and teachers up to date and to introduce new materials and methods.

The combined efforts of mathematicians, scientists, and teachers are being applied to this task. This Primary I book is the report of the work of such a group whose purpose it was to produce a new kind of teaching material for experimental use in some African schools. The book contains some new ideas and some new ways of dealing with them as well as new ways of presenting many of the ideas you already know.

When you have tried the book with children using the suggested methods as carefully as you can, your opinions about how much the pupils learn, their attitudes toward the learning of mathematics, and about the difficulties you may have in teaching will be sought. Your opinions will be important to groups doing further writing. They will furnish later writers with information about how the book should be

used or revised and about how materials for Primary II - Primary VI should be written.

Some Basic Changes. Have you been satisfied with what you learned about mathematics as a child? Did you enjoy it enough to make you want to learn more? Part of the changes in approach used in this Primary I book is in the use of more exact language. Also, some entirely new notions are employed because they have been found to be useful in producing better concepts of numbers. As you encounter the changes, think about how they differ from the way you were taught.

New materials often require new methods of teaching, and these in turn make some of the material more attractive to young pupils and, hence, allow the use of such material much earlier. Some changes have also been brought about by studies of how people learn. We now rely upon activity which leads to discovery by the child. There is evidence that this is a powerful force for making the pupil want to learn and for helping him remember what he has learned.

Activity should have a purpose. It means searching, questioning, and exploring ideas until a problem is solved. Just doing something to keep busy or following directions or repeating the same task after the thrill of first discovery is gone do not result in much learning. The place of repetition (drill) is in polishing performance after learning has taken place.

Methods. For activity to lead to discovery it is necessary to have problems which seem worthwhile to the pupils. They must want to know the answers. Of course, a problem must be so much a part of their lives that they recognize it and feel the need for solving it.

An advantage of the problem approach is that new questions keep coming out as others are answered, and, thus, more and more learning results. Pupils' questions often lead to ideas far ahead of the usual step-by-step instruction.

You should expect and accept answers that are found in many ways and stated in many forms. Frequently the form of an answer shows the pupil's understanding and gives you a chance to guide his thinking. Some examples of the way forms of answers differ and how they show stages of maturity follow:

Suppose children are planting flowers in a bed with room for 5 rows of 8 plants in a row. A first solution to the question of how many plants to provide might be a diagram showing 5 rows of 8 dots - then simple one-by-one counting of the dots. Some children will find that counting by 2's is a little better. Others find counting by 5's better still. Still others find that adding 8 + 8 + 8 + 8 + 8 would be a solution, and the most mature find that the best way is to find 5 × 8. We cannot expect young children to use the latter way until they have had experience with some or all of the other ways.

Similarly, quite young children can solve such a problem as: How many sweets shall we allow for each of 6 children if we have 24 to share? They can solve it by counting the sweets one by one into 6 piles. A somewhat more mature approach might be to take out 6 at a time, really the same solution, but a stage more advanced. Again, the answer to the question, "What factor used with 6 gives 24?" is the most efficient way, one which young children will not know. They will work in the ways they understand.

The simpler ways of solving problems should be recalled from time to time and used by the pupil to show why he believes his results are right. Such questions as, "Why do you think so?", "Show us how you got that answer", or "Can you think of a better way of doing it?" should be asked so often that the pupil is aware of them all the time and begins to do his thinking with these questions in the back of his mind. The habit of defending answers by drawing pictures, arranging objects, or presenting arguments affects the pupil's whole outlook on the world. He ceases to depend so much upon authority and seeks to verify things which he believes.

The choice of the problem is very important. A good problem invites discovery. You can assist this discovery by asking questions to help pupils understand the sort of answers they should seek, by helping with arrangements and diagrams, by supplying a word now and then when someone has a good idea but can't quite express it, and by suggesting sources of information. An important final step is to help pupils make summaries and draw conclusions from their discoveries.

<u>The Plan of the Book.</u> The Primary I book provides you with sixteen units dealing with such things as comparisons, number, counting

operations, and fractions. You are provided with a general discussion of the mathematics involved in each of these topics. This is followed by a teachers' commentary which treats in great detail the things you will teach.

Each unit in the teachers' commentary begins with a short statement about why it is included and what the unit is expected to do. The new vocabulary is listed, the required materials are noted, and stages in development are outlined. Then comes a paragraph suggesting preparation to make before the lesson. Following this, a number of experiences are treated in detail. Usually, some general experiences with objects in and out of the classroom come first, then blackboard or flannelboard exercises. Finally, the pages in the pupil's book are discussed.

The pupil pages are published separately as the pupils' book. The book does not, of course, suggest all the experiences the child will have. It is only a means of guiding him. He first sees and talks about objects; he then looks at pictures of these objects. Later he will use symbols to stand for the pictures and for other ideas which he may have.

Using the Book. Before you begin to use the Primary I book in in class, study the general mathematics background appropriate to the unit you are to teach. Then read the introduction carefully. Think about how this differs from the way you were taught. Lay out the suggested teaching materials. Think out the illustrations you will use both in and out of the classroom. Look at the pages from the pupils' book. Note any new words. When a new word is first used with the pupils it is underlined. You should pronounce these words carefully, use them in sentences if you can, and help the pupils to learn to pronounce them.

Prepare some questions of your own. You may use those in the book, but you should use many of your own. The suggested questions and answers are meant to be models. Try to follow this style but don't stop after having done merely what the book suggests. After a time your questions will be more appropriate than any that can be suggested to you. You will find that often you are turning even wrong answers into good questions which will bring out better and better answers.

Do not hurry to teach counting. The idea of number should come first. Do not hurry to have children write symbols. The number idea must be present first so that the symbol can stand for something the pupil understands. Talk individually with pupils about how they can use the new ideas that they learn. At the end of a lesson ask a question or drop a hint to leave the pupils thinking about learning something new the next time.

Outcomes. As you encourage pupils to hunt for answers rather than to ask you to tell them, you are preparing them to live in the modern world. There is a satisfaction in finding out something you really want to know. This searching attitude and the feeling of satisfaction it often provides are at the heart of learning and of modern scientific thought. The child can experience this just as the adult can.

The problem and discovery approach result in learning that is not soon forgotten. It is fun for both the teacher and the pupil. Moreover, the stimulation of this spirit of inquiry is the surest weapon against ignorance, lethargy, and low standards. Children who are used to having reasonable answers to important questions will want better schools. They will demand good teachers and good equipment as they get older. Later, they will continue to search for new and better ways of doing things long after they have left school and have entered into the main stream of community life.

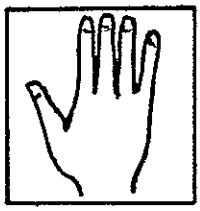
Mathematics for Primary Teachers

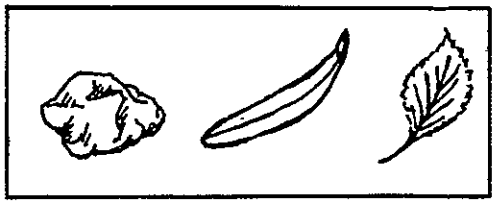
Sets and Numbers

The earliest ideas of number are learned from experiences with physical things. Children learn ideas in this way as well as from adults as they use numbers. The following discussion will describe experiences which develop and emphasize the idea of number. There are many different types of numbers. The only numbers discussed now will be those used for counting and zero. The fractions and other numbers used in more advanced mathematics will be discussed later.

It is convenient in discussing numbers to begin with sets, or groups, of things. Mathematicians, just as you do, use the word "set" to mean any collection of things. Often the things are physical objects like bananas, sticks, stones, pencils, and people. These things are called members of the set.

Below is a picture of a set made up of a boy and a dog.





Another set is the set of fingers on one hand.

Another set is a stone, a banana, and a leaf.

A useful way to compare two sets is to match one-to-one the things in one set with the things in another. For example, the fingers on the right hand can be matched, or paired, with the toes on the right foot. The thumb may be matched, or paired, with the big toe; the little finger with the little toe, and so on. Not all sets can be paired off this way. One cannot match one-to-one the set of fingers of one hand with the set of ears of one child.

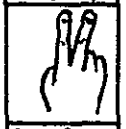
Two sets which can be matched, as the fingers on one hand to the toes on one foot, have a property which is the same for both. They are said to be "equivalent" or "to have the same number of members."

Often the members of a set have something in common. Here is a

picture of a set of children, a boy and a girl.

However, any group of objects whether they have something in common or not may be regarded as a set. For example, a tree and a stone form a set. This set has the same number of members as the set consisting of a boy and a girl.

The names people use for the number of a set differ with the language used. In English the name associated with the set of fingers of one hand is "five;" in Swahili it is "tano." More specifically, some different sets of fingers on the right hand, and the names associated with them are given below. (The outstretched fingers form the set).

Set	Number Name
	one
	two
	three
	four
	five

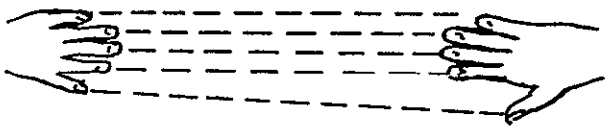
If no fingers are stretched out, there are no members in the set. It is convenient to associate a number name with this set. The name is "zero."

zero

Some Technical Language

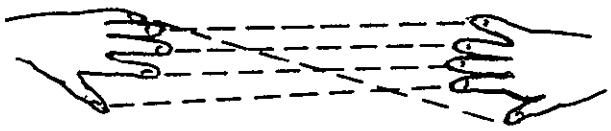
Two sets which are matched are said to be in one-to-one correspondence. The fingers of the right hand can be placed in one-to-one

correspondence with the fingers of the left hand. In the figure below dotted lines indicate which fingers of the right hand may be made to correspond with fingers of the left.



In this correspondence the thumbs match, the little fingers correspond, and so on.

The fingers could be matched in a different way. This new matching is a different one-to-one correspondence. The figure below shows another matching.



A correspondence is a matching of the members of one set with the members of another. The correspondence is one-to-one if each member of one set is matched exactly to one member of the other. Another example of one-to-one correspondence, or matching, is between the set of heads in a room and the set of people in the room. Each person is paired with his own head--or each person corresponds to his own head. Another example of one-to-one correspondence is that between the set of hands and the set of feet in the group of children (assuming that no one has lost a hand or foot.

Not all correspondences of two sets are one-to-one. There is a two-to-one correspondence between the hands in a group and the people in a group. However, we are interested mainly in one-to-one correspondence.

Number, Names, and Numeration

Many different sets may be formed. Since each one of them needs a number name, many number names are needed.

In general, it is not easy to make up and remember many different names. Imagine making up and remembering a new name for every child

in a school. The invention of a method of making names for the number of any set is one of the great successes of mankind. Our method of naming numbers works whether the set has a few or many members.

The method of making names is quite similar to that used by a messenger who reports to his chief how many people he finds in a village. For each person he puts a small stone in a sack. He intends to carry the sack to the chief and say, "Behold! There are as many people as stones." This is a good method for a few stones. However, he finds he has so many stones that he can not lift the sack. So he uses two sizes of stones, with the larger representing, or corresponding to, ten of the smaller and weighing less than ten small stones. He uses ten because this is the number of his fingers. He can easily tell whether a set of small stones corresponds to a large stone by matching them with his fingers. He makes the sack lighter by taking out ten small stones and putting in one large one. He now says to his chief, "Behold! For each large stone there are as many people as fingers on my hands. For each small stone there is one person." The messenger needs no more than nine small stones. If there were ten small stones, or more, ten of them could be replaced by one large stone.

As an example, suppose there are IIIIIIIIII, IIIIIIIIII, IIIIIIIIII, IIII in a sack where each mark "I" stands for a small stone. The sack is made lighter by replacing ten small stones with one large stone. Now in the sack are X, X, X, IIII where "X" stands for a large stone.

Let us now consider some number names. The English names with the corresponding sets are:

Set of Small Stones	Name of the Number
{I}	one
{I I}	two
{I I I}	three
{I I I I}	four
{I I I I I}	five
{I I I I I I}	six
{I I I I I I I}	seven
{I I I I I I I I}	eight
{I I I I I I I I I}	nine

It is also convenient to consider the set of no stones.

{ } zero

With these words statements can be made like: "There are three small stones in the set {III}." "There are eight small stones in the set {IIIIIIII}." "There are two large stones in the set {X X}." "There are three large stones and four small stones in the set {XXX IIII}.

As number ideas grow, it becomes convenient to have a single number name for each of the things the messenger uses which is different from its common name, small stone or large stone. The small stone is called "one;" the large stone is called "ten." Hence we say, "There are three tens, four ones in {XXX IIII}." Also we say, "There are six tens, two ones in {XXXXXX II}."

Some number names are shortened with use. People who use the number names of two or more words together over and over again tend to shorten the sound and run the words together. It is customary to omit the word "ones." Also "tens" is shortened to "-ty" and fastened to the word before it. Hence "six tens two ones" becomes "six ty two," then "sixty-two." Similarly, rather than say, "There are seven tens four ones in {XXXXXXX IIII}" we say, "There are seventy-four in {XXXXXXX IIII}."

The spelling of number names is changed slightly in some cases as the words are run together. For example,

"four tens" becomes "forty" and
"five tens" becomes "fifty"

However, there is no spelling change in some others. For example,

"six tens" becomes "sixty,"
"seven tens" becomes "seventy,"
"eight tens" becomes "eighty" and
"nine tens" becomes "ninety."

There are few other changes from this pattern in the names used. People say "twenty" instead of "two tens" and "thirty" instead of "three tens." Note that "twen" in twenty is close to "twin" in spelling. Perhaps these names came from a word meaning "two" in a

different vernacular than that which used "two."

Some further sets of stones with the names of the numbers they represent are:

{XX III} twenty-three;

{XXX IIIII} thirty-five.

If there are only tens in the set, the name of the tens only is used and "zero" is not used. Some cases of this are:

Set	Name of Number Represented
{XX}	twenty (not twenty-zero)
{XXXX}	forty
{XXXXXXX}	seventy

Some special names for numbers between ten and twenty developed in a different way. The sets and the numbers of small stones each set represents are:

Set	Name of Number Represented
{X}	ten
{XI}	eleven
{XII}	twelve
{XIII}	thirteen
{XIIII}	fourteen
{XIIIII}	fifteen
{XIIIIII}	sixteen
{XIIIIIII}	seventeen
{XIIIIIIII}	eighteen
{XIIIIIIIII}	nineteen

The group of letters "teens" means "and ten." That is, "seventeen" is "seven and ten," and "nineteen" is "nine and ten." Note there is a small change for three and ten. "Thirteen" for "third and ten" is used in place of "threeteen." The special names "eleven" and "twelve" do not follow the pattern.

Often the single word "number" is written instead of "name of number" when the meaning is clear. This reduces the amount of writing needed.

Number Symbols or Numerals

The basic number symbols or numerals we use and their corresponding number names are: 0, zero; 1, one; 2, two; 3, three; 4, four; 5, five; 6, six; 7, seven; 8, eight; 9, nine. These ten symbols are often called "digits" from fingers.

Just as the number names for larger sets are made up of the names one to ten, so the numerals for larger sets are made up of the ten digits 0, 1, 2, 3, 4, 5, 6, 7, 8, 9.

Whether a digit of a numeral stands for the number of ones or the number of tens depends on its place in the numeral. This is the concept of place value.

Some names and numerals are:

forty-five	45
twenty-three	23
seventeen	17
sixty-four	64
eighty	80
thirty	30

The digits 5 in 45, 3 in 23, 7 in 17 indicate the number of ones. This place is called the ones place. The first digit to the left of the ones digit indicates the number of tens. This digit is in the tens place. If the number indicated is only tens with no ones, a zero is placed in the ones place.

Number Names for Numbers Greater than Ninety-Nine

The method of replacing ten small stones with one large stone can be extended to replace ten large stones by a bigger stone, ten bigger stones by a great stone, and so on. New number names are introduced for these sets as follows: ten "tens" are named one hundred; ten "hundreds" are named one thousand; ten "thousands" are named one ten-thousand; ten "ten-thousands" are named one hundred-thousand; ten "hundred-thousands" are named one million.

Let us make a table of sets of stones with the names of the numbers and the numerals for the numbers represented by each set of stones. For a diagram of the set, we have:

"I" standing for a small stone
"X" standing for a large stone
"C" standing for a bigger stone
"M" standing for a great stone

A complete table is endless. Look at the following table of some selected sets.

Set	Name of Number Represented	Numeral for Number
{I}	one	1
{III}	three	3
{XII}	twelve	12
{XIII}	thirteen	13
{XIIII}	fourteen	14
{XIIIIII}	sixteen	16
{XX}	twenty	20
{XXIIIII}	twenty-five	25
{XXXIIIIIIIII}	thirty-nine	39
{XXXXXII}	fifty-two	52
{C}	one hundred	100
{CII}	one hundred and two	102
{CIIIIIII}	one hundred and seven	107
{CXI}	one hundred and eleven	111
{CXXIIIII}	one hundred and twenty-five	125
{CCCII}	three hundred and two	302
{CCCXII}	three hundred and twelve	312
{M}	one thousand	1000
{MII}	one thousand and two	1002
{MXXIII}	one thousand and twenty-three	1023
{MMCCCXI}	two thousand three hundred and eleven	2311

More about Place Value

From the table of number names and symbols observe that numerals are formed from digits as follows. A single digit numeral represents ones. If a numeral has more than one digit, the digit on the right represents the number of ones, and is called the "ones digit." The

first digit to the left of the ones digit represents the number of tens and is called the tens digit. The second digit to the left of the ones digit represents the number of hundreds, and the third digit to the left of the ones digit represents the number of thousands. For example, 2345 is read two thousand three hundred and forty-five. The digit 5 stands for five ones. This is indicated by its place at the right in the numeral. Similarly 4 stands for forty since 4 is the first digit to the left of the ones digit. 3 stands for three hundred since it is the second digit to the left of the ones digit. 2 stands for two thousand because of its place in the numeral, 3 places to the left of the ones digit.

This plan of assigning value to a digit by its place in the numeral is called, "place value notation." The values one, ten, hundred, thousand, and so on which are given to digits which appear in different placed in a numeral, are called <u>place values</u>.

<u>Some Comments on the Unit Set and the Empty Set</u>

The simplest idea of a set is a group of more than one similar members with something in common, like the set of fingers on one hand. People talk about sets using words in sentences like:

"Think of the set of leaves on the tree."
"Think of the set of heads on your body."
"Think of the set of elephants in this room."

The words go together in about the same way in each of these three sentences. Only the first sentence refers to a set in the simplest sense. It is extremely useful, however, to extend or generalize the idea of set so that a set may be used for each of the three situations. This extension leads to the concepts of a set with only one member and the set with no member.

Children have no difficulty making these generalizations or extensions. In fact they find them exciting. The empty set particularly appeals to the "something for nothing" instinct, the "it is here, yet it isn't here" fantasy beloved of childhood. It is perhaps the first opportunity the child has to generalize from physical objects to a purely mental object. This is a thrilling achievement. Hence there is no difficulty in including the empty set.

Ordering Sets and Numbers

People make statements like, "I have two eyes, one head, ten fingers." "I have more fingers than eyes." "I have fewer eyes than toes." "I counted the children in the class." The meaning of these statements is made clear in terms of sets and the natural ordering of sets and numbers.

Sets which can be matched in one-to-one correspondence are said to have "the same number of members."

A system of number names is developed. For each set there is a name called the "name of the number of members," or more briefly "the number of members." For example, two is the name given to the number of members of the set of eyes. This is the meaning in terms of sets of the statement "I have two eyes."

Similarly ten is the name of the number of members in the set of fingers. This is the meaning in terms of sets of "I have ten fingers." The number "two" is used with any set which can be placed in one-to-one correspondence with the set of a person's eyes. We say, "I have two legs, two arms, two ears." Similarly, we say, "I have ten toes," because the toes can be matched one-to-one with the fingers.

Not every pair of sets can be matched exactly one-to-one. If every member of one set is matched one-to-one with a member of a second set, and there are members of the second set unmatched, the first set is said to have fewer members than the second set. The second set has more members than the first. Also the number of members of the first set is smaller than the number of members of the second set. The number of members of the second set is larger than the number of members of the first set.

For example, the number of eyes of a person is larger than the number of his heads. The number of his ears is smaller than the number of his fingers.

Natural Order

When two numbers are listed with the smaller one first, they are said to be listed in natural order. Reading from left to right (3, 5) are in natural order, but (5, 3) are not. Similarly (4, 7), (1, 2), (2, 6) are in natural order, but (5, 2), (6, 3), (3, 1) are not.

There is a natural order for three numbers. (2, 4, 5) are three numbers in natural order. So are (1, 2, 3); (4, 7, 11); (1, 3, 6).

None of the following are in natural order: (8, 6, 3); (5, 1, 2); (1, 4, 2).

The idea of natural order extends to any set of numbers. In every set of numbers there is one number which is smaller than every other number. It is the smallest number, and is listed first. In the set of remaining numbers there is one which is smaller than the others. It is listed next, and so on. For example, suppose you wish to list the set (5, 7, 3, 6, 8) in natural order. Three is smaller than every other number in this set, so 3 is listed first, leaving (5, 7, 6, 8). Of these, 5 is smaller than every other, so (3, 5) is listed with (7, 6, 8) remaining. Of these 6 is smaller than 7 and 8, so 6 is listed giving (3, 5, 6) and (7, 8) remaining. 7 is smaller than 8, so these are listed giving (3, 5, 6, 7, 8) as the natural order of the set (5, 7, 3, 6, 8).

Sets may be ordered by giving them the same natural order as the order of the number of their members. This ordering can also be developed directly without using the number of members in a way similar to the way the natural order of numbers was developed. That is, you define greater than and less than with one-to-one matchings between members of the sets. The sets are listed in natural order when the sets with fewest members are listed first.

Counting

Determining the number of members in a given set is called counting. From the point of view of sets, the general method makes use of counting sets and one-to-one matching.

The counting sets are sets of numbers. The sets are clear from the first few examples, with the number of members.

Set	Number
{1}	1
{1, 2}	2
{1, 2, 3}	3
{1, 2, 3, 4}	4
{1, 2, 3, 4, 5}	5

and so on.

The natural order of all the numbers is the list 0, 1, 2, 3, 4, 5, 6, and so on. In this list each number is one more than the number before it, and one less than the number after it. This pattern is sometimes called a "one-more-than" pattern.

We note that in each set above, the members are listed in natural order, and the number of members in the counting set is the last, and largest, member of the set.

To count the number of members in a given set a counting set which can be placed in one-to-one correspondence with the given set is found. The number of members in the given set is the number of members in the counting set.

For example, to count the marks in the set

$$\{x\ x\ x\ x\ x\ x\ x\ x\}$$

find a counting set which can be placed in one-to-one correspondence with the marks. The desired set is: $\{1, 2, 3, 4, 5, 6, 7, 8\}$. The one-to-one correspondence may be shown by writing each member of the counting set below the corresponding mark of the given set, as follows:

$$\begin{array}{cccccccc} \{x & x & x & x & x & x & x & x\} \\ \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow & \updownarrow \\ \{1 & 2 & 3 & 4 & 5 & 6 & 7 & 8\} \end{array}$$

Since there are 8 members of the counting set as shown by its last member, 8, there are 8 marks in the set of marks.

The numbers without zero are called "counting numbers." The counting numbers in their natural order give us a convenient tool for counting. In this method the elements of a given set are matched one-to-one and one at a time with the counting numbers in their natural order: 1, 2, 3, 4, and so on. When all the members of a given set have been matched, the last number used is the number of members in the given set. Often the matching is done by pointing at the objects in the given set, one after the other, and repeating the names of the numbers in their natural order.

The number of members in the empty set is known immediately (zero) and so too is the number in many small sets. The process of counting is then neither necessary nor used, except possibly as a check.

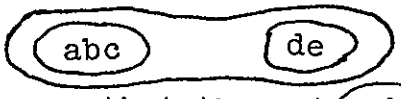
Operations

The basic operations of arithmetic are addition, subtraction,

multiplication, and division. We shall discuss these operations on numbers in terms of the corresponding operations on sets. We shall show how they develop out of the natural operations of sets and the counting of members. In this discussion all numbers considered will be whole numbers. Fractions will be considered later.

Operations on Sets. Set Union or Putting Sets Together.

Consider a set of five letters, a, b, c, d, e. We may regard this set as made of two subsets a, b, c and d, e. The following diagram illustrates this.



Alternatively, we may say that the set a,b,c is put together with the set d,e to form the set a,b,c,d,e . The set a,b,c,d,e is called the union of the sets a,b,c and d,e.

In general, any two sets can be put together. The union of the two sets is the set of all the members of the two sets. The set of children in a class is the union of the set of boys with the set of girls in the class. More than two sets may be put together to form a union if desired.

In this book only sets with no members in common will be put together to form a union. This limitation can be removed later.

Just as two sets can be put together to form a union, a set with more than one member may be separated into two sets. For example, the set of letters p,q,r,s can be separated into two sets p,q and r,s . The set p,q,r,s could also be separated into pairs of sets such as p, q,r,s and p,q,r, s. The set p,q,r,s is the union of any pair of sets into which it may be separated.

Addition, an operation on numbers, is best discussed in terms of the union of sets with no common members. A class is the union of the set of boys with the set of girls in the class. If we know the number of boys in the set of boys and the number of girls in the set of girls, we can find the number of children in the class. The number of children is the sum of the number of boys and the number of girls. The number of boys is called an addend and the number of girls is called an addend. The process of finding the sum is called adding. The whole procedure is called addition. If there are 3 boys and 4 girls, there are 7 children. Seven is called the sum of 3 and 4. Three is an addend, and so is 4. The addends 3 and 4 are added to get 7. "Three added to 4 equals 7" is an example of addition. (In this material sum

will be used only as the result in an addition exercise.)

Another example is: a set of 2 sticks is put together with a set of six more sticks to form a set of 8 sticks. Considering only the numbers, we say "two added to six equals eight."

We may find the sum of any two numbers by considering any pair of sets they apply to. Any two numbers can be added by forming the union of two sets with the given numbers of members, and counting the members of the union set. You can add 7 and 8 as follows. Prepare a set of seven stones and a set of eight other stones. Put them together to form the union of the two sets. Count the members of the union. There are 15 stones in the union. Hence 7 added to 8 is 15.

This set-union-counting method, while fundamental, is too slow and inefficient for anyone other than children during their first steps at learning addition. While it is basic for all methods, we wish them to develop methods to determine the sum of two numbers without reference to any sets. The method used is based on memory of some elementary addition facts, combined with the way the place-value system works. We shall discuss these methods later.

Symbolic Notation for Addition.

You often write an equation to represent addition or an exercise in addition. For example, you may write $4 + 3 = 7$. This may be read "Four plus three equals seven." If you wish to represent the exercise before knowing the sum, you may write $4 + 3 = \square$. You ask "What number should go in the box?" The equation is read, "Four plus three equals the number in the box." Seven is the number which goes in the box. Similarly, for the second example you can write $6 + 2 = 8$. Before you have counted the sticks in the union of the sets, you might write "$6 + 2 = \square$," read "Six plus two equals the number in the box." Eight is the number which goes in the box.

It makes no difference whether a set of four counters is put together with a set of five, or a set of five put together with a set of four, $4 + 5 = 5 + 4$, since in general the order of adding numbers does not affect their sum.

Subtraction

The second operation on numbers, subtraction, is related to addition. Where addition asks, "Given two addends, what is their sum?",

subtraction asks, "Given one addend and a sum, what is the missing addend?"

For example, "Three is one of two addends whose sum is eight. What is the missing addend?" By thinking of the addition facts, you find the missing addend is five.

Another example is, "There are ten boys in a class of 15 children. How many girls are there?" From the knowledge of addition, the number of children is the sum of the number of boys and the number of girls. Here we have the number of boys, the given addend, and the number of children, the sum, and wish to find the number of girls, the missing addend. This is an example of subtraction.

If we let $\square$ represent the missing addend, the number relation in this problem may be written $10 + \square = 15$. To emphasize the role of the missing addend, the equation is also written $\square = 15 - 10$. This is read, "The number in the box equals fifteen minus ten." Note that the sign "-" is read by its mathematical name "minus." This is the usual form for writing an equation for a subtraction exercise.

Before discussing ways of determining what numbers to put in the box, it is helpful to look at some more examples which look a bit different from those above. "John had three bananas and wanted ten. How many more did he need?" "Paul had five beans and Mary had 7 beans. How many more beans did Mary have than Paul?" "Peter had 8 seeds and gave away two. How many did he have left?" In each example, we are given a sum and one addend and wish to find the missing addend.

Look at the examples closely to see how they really represent questions which require the subtraction operation for their answer. "John had three bananas and wanted ten. How many more did he need?" Think of the set of bananas he has and the set of bananas he needs. The second set is imaginary, of course. Putting these two sets together forms their union, which is the set of bananas he wants. The union has ten members, the first set has three, hence $3 + \square = 10$. $\square$ represents the number in the second set, the number of bananas he needs. You may also write $\square = 10 - 3$.

"Paul had 5 beans, Mary had 7 beans. How many more beans had Mary?" You should think of a set of beans which can be put together with the set of Paul's beans so that the union of the two sets has

7 beans, the same number as is in Mary's set. Hence $\square + 5 = 7$ or $\square$ 7 - 5. The number of beans in the imagined set is the answer to "How many more beans did Mary have?" This is a typical comparison problem where the numbers are compared by subtraction. "Peter had eight seeds and took away two. How many did he have left?" We think of the set of seeds taken away (with two members) and the set of seeds left. The union of these two sets is the set Peter had at the beginning, a set of eight members.

There are several ways to determine what number goes in the box. For small numbers the addition facts are learned so thoroughly that you recognize what number goes in the box immediately.

Children may use counters such as seeds, bottle taps, or sticks, and set up the problem in set form. For example, a child looks at $10 + \square = 15$. In terms of sets, he thinks of a set of 15 members which is the union of two sets, one of which has 10 members. Hence to find the number to put in the box, he separates a set of 15 counters into two sets, so that one of them has ten members. The number in the other set is the number to put in the box.

He can find the number to put in the box, $\square = 7 - 5$, in the same way. He separates a set of seven markers into two sets, one of which has five markers. The number of members in the other set is the number to put in the box.

The example of Peter and some seeds was often considered the typical subtraction problem. In solving the problem $8 - 2 = \square$, people tended to say "eight take away two is____," translating the symbol "-" as "take away." We do not do this.

Note that while "take away" applies directly to the seed example, it does not apply so directly to the other examples. Further, "take away" describes something done to objects. Numbers are subtracted, not taken away. Hence we will read "eight minus two equals the number in the box," translating the symbol "-" by its mathematical name "minus."

Ways of solving subtraction problems without using counters will be discussed later. These methods will depend on knowing the addition and subtraction facts for numbers 0-9 thoroughly and on the place-value method of naming numbers.

People like to think of subtraction as being an operation which is the <u>inverse</u> to addition. Subtraction may be regarded as doing the

opposite of addition. For example, adding two to three yields five. Subtracting two from 5 yields three. The subtraction of 2 from 5 was inverse to addition since it had an effect opposite to that of addition.

Since we are considering only whole numbers, the sum must always be greater than or equal to the given addend. Hence with a given number as sum, only a few numbers could be addends.

In a subtraction problem like $7 - 4 = 3$ some authors call 7, the given sum, the "Minuend;" 4, the given addend, "the subtrahend;" and 3, the missing addend, is called "the difference" or "the remainder." It is hard for pupils to remember words like "minuend" and "subtrahend," so these are avoided. However, "difference" and "remainder" for the missing addend will be used.

Multiplication

Multiplication, the third operation of arithmetic to be discussed, may be regarded as repeated addition of equal addends. $5 + 5 + 5$ is regarded as the sum of three fives. In multiplication for it is written 3×5. Similarly 4×2 is the multiplication form of $2 + 2 + 2 + 2$. The multiplication symbol "$\times$" is read "times." "3×5 is read "three times five." Multiplication is extended to situations which might not be thought of as repeated addition by equations such as:

$2 \times 5 = 5 + 5 = 10, \quad 1 \times 5 = 5, \quad 0 \times 5 = 0$

Here are two typical multiplication problems:

(a) Each of six brothers has three children. How many children do they have together?

Here you add six addends of three each, $3 + 3 + 3 + 3 + 3 + 3$. The multiplication form is 6×3.

(b) The students in a class are arranged in four rows with seven students in each row. How many students are in the class?

Here you add four addends of seven each, $7 + 7 + 7 + 7$. The multiplication form is 4×7.

Multiplication can be illustrated with sets of counters or markers. 3×5 is illustrated by arranging three sets of 5 counters in the pattern:

x x x x x
x x x x x
x x x x x

The union of the three sets is a set of 15 members. This pattern of arranging objects can be used to illustrate multiplication of any two numbers. 5×3 can be illustrated by the pattern:

```
x x x
x x x
x x x
x x x
x x x
```

Since the rectangular pattern for 5×3 is the same as the pattern for 3×5, except the position is changed, it is clear that $5 \times 3 = 3 \times 5$.

Similarly the rectangular pattern for 2×4 is:

```
x x x x
x x x x
```

If the pattern is put in the position below:

```
x x
x x
x x
x x
```

it is the pattern for 4×2. Hence, since the same counters are involved, you know that $2 \times 4 = 4 \times 2$. Each is clearly eight. A similar change in the position of patterns shows that $4 \times 7 = 7 \times 4$, $3 \times 6 = 6 \times 3$, and so on.

In a multiplication problem like $4 \times 7 = 28$, the numbers "4", and "7" are called <u>factors</u> and "28" is called the <u>product</u>. You can represent the problem "Find the product of 6 and 3" by the equation "$6 \times 3 =$."

A child can use counters to solve this example, that is, to find the number to put in the box. He makes six sets of three counters in each set. Then he counts to find the number of counters in the union of all the sets together. He may also solve the problem by adding six addends of three each.

Determining the products of two numbers in this fashion is often long and tedious, although correct. A child should learn thoroughly the products of small numbers and use the properties of the place-value system to help find the products of larger numbers. This method will be considered later.

Division

Division, the fourth operation of arithmetic, is related to multiplication. In division you find the missing factor when you are given the product and one of the factors in a multiplication exercise. Suppose fifteen is the product of five and a missing factor. What is the missing factor? Determining this factor is a division exercise.

Other examples which require division are:

(a) A class of forty pupils is lines up in four rows, with the same number of pupils in each row. How many pupils are in each row?

(b) John gives away 18 bananas. He gives three to each of his friends. How many friends has he?

Letting $\square$ represent the missing factor, the number relations in these three examples can be written in the following equations:

$$5 \times \square = 15$$
$$4 \times \square = 40$$
$$\square \times 3 = 18$$

These are read, "Five times the number in the box equals fifteen," "Four times the number in the box equals 40," "The number in the box times three equals 18."

The role of the missing factor, the number in the box, is often emphasized by writing these equations as:

$$\square = 15 \div 5; \qquad \square = 40 \div 4; \qquad \square = 18 \div 3$$

These are read, "The number in the box equals fifteen divided by five," "The number in the box equals forty divided by four." "The number in the box equals eighteen divided by three."

These exercises can be solved, that is, the number to be put in the box can be found, in several ways.

A child may know the multiplication facts thoroughly so that he recognizes what the missing factor is at once. He may use counters as illustrated in the following solutions of two of the above problems:

(1) He thinks of the forty pupils in the four lines and prepares a set of forty counters. He separates the set into four sets with the same number in each set. This can be done by distributing the forty counters into four piles, one counter at a time into one pile after the other in order. The counters may also be arranged in four rows in a

rectangular pattern, like the multiplication pattern. The number in each pile or row is the missing factor to be put in the box.

(2) The child thinks of John giving away 18 bananas, three at a time, and prepares a set of 18 counters. He then separates the set of bananas into as many piles of three counters each as he can. The number of piles is the number to put in the box.

The equation: $5 \times \square = 15$ or $\square = 15 \div 5$ can be solved by the first method as follows: Separate a set of fifteen counters into five piles with the same number in each pile. The number in each pile is the number to be put in the box.

Solving the exercise by the second method, separate the set of fifteen counters into as many piles of five in each pile as you can. The number of piles is the solution, the number to be put in the box.

The use of counters to help solve division problems, while correct, is long and tedious. A child will learn more efficient methods later. These methods are based on knowing thoroughly the multiplication and division facts for small numbers, and using the place-value system.

The multiplication facts show that there is no number which can be multiplied by three to give a product of thirteen. This shows that not every number can be regarded as a product with a given number as a factor.

In a division problem like $18 \div 3 = 6$, 18 the given product, is sometimes called the <u>dividend</u>; 3, the given factor, is sometimes called the <u>divisor</u>, and 6, the missing factor is called the <u>quotient</u>.

People say that division is <u>inverse</u> to <u>multiplication</u>. This is because the effect of division is the opposite of the effect of multiplication. For example, if a given number such as five, is multiplied by three, and the product is then divided by three, the result is the given number.

<u>Operations in General</u>

While addition, subtraction, multiplication, and division are four different operations, they have several properties in common. Think of the following examples:

$$6 + 2 = 8; \quad 6 - 2 = 4; \quad 6 \times 2 = 12; \quad 6 \div 2 = 3.$$

While these are all different, you note that in each case the same two numbers are given (6, 2) in a particular order and an operation is performed which yields a third number. In other words, for two given

numbers in a given order, each operation determines a third number. Any operation which begins with two given numbers in a given order is called a binary operation. The prefix "bi" stands for "two". There are operations which begin with only one number, such as squaring, or finding a factor. These operations are called "unary."

Strictly speaking, a binary operation does not "do" anything. It is a correspondence such that to a given pair of numbers in a given order corresponds a third number. For addition the third number is the sum, for multiplication the third number is the product, for subtraction the third number is the missing addend or difference, and for division the third number is the missing factor, or the quotient.

Special Properties of Zero

Usually when two numbers are added together, the sum is different from either. However the sum of zero and a given number is the given number. For example, $0 + 3 = 3$, and $4 + 0 = 4$. Zero is the only number which has this property for addition. If the sum of two numbers is equal to an addend, the other addend must be zero. Addition of zero is the easiest addition to perform, since the sum is the other addend.

When zero is a factor in a multiplication the product is zero regardless of the other factor. Zero is the only number with this property. The product of zero with another factor does not depend on the other factor.

Special Properties of One

One is a special number because of several special properties. (a) One is the smallest counting number. (b) The difference between two consecutive whole numbers is one. (c) The product in a multiplication exercise with one as a factor is the other factor.

For example, $6 \times 1 = 6$ and $1 \times 5 = 5$.

There is no other number with this property for multiplication. If the product of two numbers is equal to a factor, the other factor must be one. Multiplication by one is particularly easy since the product is the other factor.

Sequential Outline

The First Year Primary Writing Group at the Entebbe Mathematics Workshop developed a sequential outline of thirty units which it believed to be an adequate and modern introduction to elementary mathematics for African children who are instructed in the English language. Because of lack of time, some choices had to be made and it was decided that the essentials of such a presentation consisted of the units which are included in this first pupils' book and teachers' guide.

The outline of the sixteen units chosen for development follows. Numbers in parentheses are the original numbers in the thirty-unit outline.

Unit 1. (1) Vocabulary of Size and Comparison
- a. Big, little
- b. Bigger, smaller
- c. Near, far
- d. Long, short
- e. Longer, shorter

Unit 2. (2) Sets
- a. Awareness of objects occurring in sets
- b. Description of sets of objects
- c. Empty sets

Unit 3. (3) One-to-One Correspondence
- a. Matching one-to-one the objects of sets
- b. Comparison of sets

Unit 4. (4) Number I
- a. Ordering of sets
- b. Number naming of sets (one-five)
- c. Recognition of number in a set (one-five)
- d. Forming sets of one - five members

Unit 5. (5) Number II
- a. Counting (one-five)
- b. Use of numerals (0-5) to represent number
- c. Recognition of numerals (0-5)

d. Writing of numerals
e. Use of number to answer questions

Unit 6. (8) Addition I
a. Operations on sets of five or fewer: combining sets
b. Adding two numbers with sums of five or fewer
c. Equality and the symbol for equality
d. Writing equations for sums through five
e. Informal use of "not equals" relation
f. Finding the missing addend in a sentence expressing addition

Unit 7. (10) Number III
a. Number of objects in a set through nine
b. Recognition of number of objects (5-9)
c. Naming numbers through nine
d. Reading and writing numerals (6-9)

Unit 8. (12) Addition II
a. Operations on sets of nine or fewer
b. Adding two numbers with sums of nine or less
c. Writing equations for sums of nine or less
d. Renaming numbers as the sum of two or more numbers

Unit 9. (14) Number IV
a. Sets of tens and notation
b. Place value
c. Tens and ones
d. Counting beyond 10
e. Numerals beyond 10

Unit 10. (16) Ordinals

Unit 11. (23) Fractions I

Unit 12. (20) Subtraction
a. Subtracting from numbers less than 10
b. Renaming numbers as the sum or difference of two numbers

Unit 13. (21) Counting
a. Revise counting by ones and tens
b. Counting by twos to twenty and beyond
c. Counting by fives to fifty and beyond

Unit 14. (25) Addition and Subtraction
a. Adding two or more numbers, sums through 18
b. Expanded notation (12 = 10 + 2)

c. Adding multiples of ten
d. Subtracting from numbers 18 or less
e. Subtracting multiples of ten

Unit 15. (27) Fractions II

a. Ideas of fourths (quarters) and symbol
b. Revise halves
c. One half as two fourths

Unit 16. (13) Number Line

a. Less than and greater than relations
b. Symbols

Fourteen units of the original outline were not developed. It is unfortunate that nothing could be done toward an introduction to measurement or toward the presentation of geometric concepts; most of the remaining units could be classified under one or the other of these headings. However, some of the material on recognition of figures, such as squares and circles, will be encountered incidentally, and it appears as though some of the applications in measurement, such as money and time, might better be developed on a local basis, since money units differ from country to country and since the method of reading the clock varies with the language background of the pupils.

The remaining units, numbered as in the original outline, follow:

Original Unit VI Unstandardized Linear Measure (body units, etc.)
(Vocabulary: wide, narrow)

Original Unit VII Geometric Forms
Recognition of shapes and forms (circle, square, triangle)
(Vocabulary: inside, outside, big, little, in front of, behind, over, under)

Original Unit IX Weight
(Vocabulary: heavy, light, more than pound, less than pound)

Original Unit XI Money (Specific to area)

Original Unit XV Money (Specific to area)

Original Unit XVII Measurement of Time

Original Unit XVIII Geometry
Points and lines
1. Connecting 2,3, and 4 points

2. Revise triangles and squares (Vocabulary: long, short, above, below, between)

Original Unit XIX Standard Linear Measurement
Original Unit XXII Clock Measures (hour)
Original Unit XXIV Clock Measures (half hour)
Original Unit XXVI Money (Specific to areas)
Original Unit XXVIII Clock Measures (quarter hour)
Original Unit XXIX Geometry: Recognition of cubes and spheres.
Original Unit XXX Capacity (Specific to area)
Vocabulary: empty, full.

The addition of such material as local teachers can develop to cover these fourteen units should result in a good start in Mathematics for Primary 1 pupils. Experience in developing such material in something the same spirit as the sixteen units now available should be of great value to those who rewrite or extend the present material.

FIRST YEAR PRIMARY TEACHERS' GUIDE

UNIT I

VOCABULARY OF SIZE AND COMPARISON

Introduction

Purpose. The purpose of this section is to help children develop and clarify the idea of comparison and use words of comparison.

Teaching Plan. The plan of teaching consists of (1) introductory experiences which require the use of the vocabulary in describing familiar objects in the classroom and outside the classroom, (2) experiences at the blackboard describing drawings, and (3) a discussion describing the objects pictured on pupils' pages 1 - 3.

These experiences should be followed by use of the words of comparison in all classroom experiences in which the words are needed and appropriate. For example, the words may be used in language, reading, painting, and physical education lessons.

The order of some of the experiences may be changed if the weather does not permit a trip outside. The planned stages may take five or more days depending on the maturity and understanding of the pupils.

Study the plan suggested and adjust it to the children in your class. Have in mind the vocabulary which you wish to develop and have the objects available which you wish to have the children describe. You may need more or different objects than those suggested here. Also plan a sequence of questions to ask to bring out the meaning of the words being introduced. Consider the phrasing of the questions so that you will get the desired answer.

Development

Vocabulary: big, little, large, small; near, far; long, short; bigger, larger, smaller; nearer, farther; longer, shorter. These words are used to compare an object to a similar object of average size. Hence when choosing objects to describe as little or small or large be sure the objects are little or small or large as compared with an object of average size even though only two objects are compared. Observe the same care in selecting objects to describe as long or short.

Materials. Classroom: sticks, stones, seeds, leaves, books, benches, tables, chalk, pencils, pieces of paper, pieces of string, seed pods, people, or whatever is convenient and suitable.

Outside: trees, bushes, houses, cows, dogs, goats, stones, rocks, leaves, seed pods, people, sticks, blades of grass, or whatever is appropriate and convenient.

Experience in use of words big, little, small, large, long, and short to describe objects in the classroom.

Collect objects suggested above or other suitable objects to illustrate the vocabulary to be developed. The objects may be collected with the help of the children and placed on box lids or trays or on a table.

Have the children name many objects in the classroom. You may begin by pointing and saying, "I see a window," and then "I see a pencil. What do you see?" Ask individual children to name what they see. Objects named among others will be those listed in the Materials Section above and also bench, table, blackboard, door, and floor.

After many objects have been named, select two stones of different sizes and say, "I have two stones. One of them is a big stone. This is a big stone. This is a little stone." Have the children in turn select two similar objects and describe one of them as big and one as little or describe them as being the same size.

When the children can use the words big and little, begin similar activities in the use of the words small and large in comparing two

objects, for example, two books. Statements like these may be used: "Here are two books. Are they the same size? This one is a little book. It is also a small book. If this is a small book, then this is a big book. It is also a large book." Continue by comparing other objects.

Now have the children describe which of two objects is long and which is short. Pick up two pencils of different length and hold them side by side. Guide the children to say that one is a long pencil and one is a short pencil by asking, "Are these two pencils the same length? Which one is a short pencil? Which one is a long pencil?" Continue using the words long and short by having the children in turn select two objects which can be compared in length and say "This _____ is long. This _____ is short."

Repeat the series of experiences just described but this time use descriptive sentences like those below using bigger, larger, smaller, longer, and shorter. Take the different objects and say:

This stone is smaller than this stone.
This leaf is larger than this leaf.
This seed is smaller than this seed.
This book is bigger than this book.
This string is longer than this string.
This stick is shorter than this stick.

Experience in use of words big, bigger, little, small, smaller, long, longer, short, shorter, near, nearer, far, and farther to describe objects outside the classroom.

Study the immediate surroundings of the school. Note the objects available for the children to describe. A suggested list is in the Materials Section. Objects available will vary in different areas. Select objects most suitable for the words to be used. For example, one of a pair of houses, trees, or stones may be described as large, big, small, and one of a pair of sticks or seed pods may be described as long or short.

Take the children outside to the spot you have chosen to hold your lesson. Have them point out and name the different objects they see. Ask them to point out a big tree and then a little tree. Get the children to use these words to describe other objects. Find two leaves of different size. Hold them up and say, "Here are two leaves. This is a large leaf. This is a small leaf." Have some of the children find their own pairs of leaves and describe them in this way. Give them practice in describing other pairs of objects using big, little, large, small. When they have done this, introduce the words long and short in the same way, using suitable objects such as sticks and long seed pods.

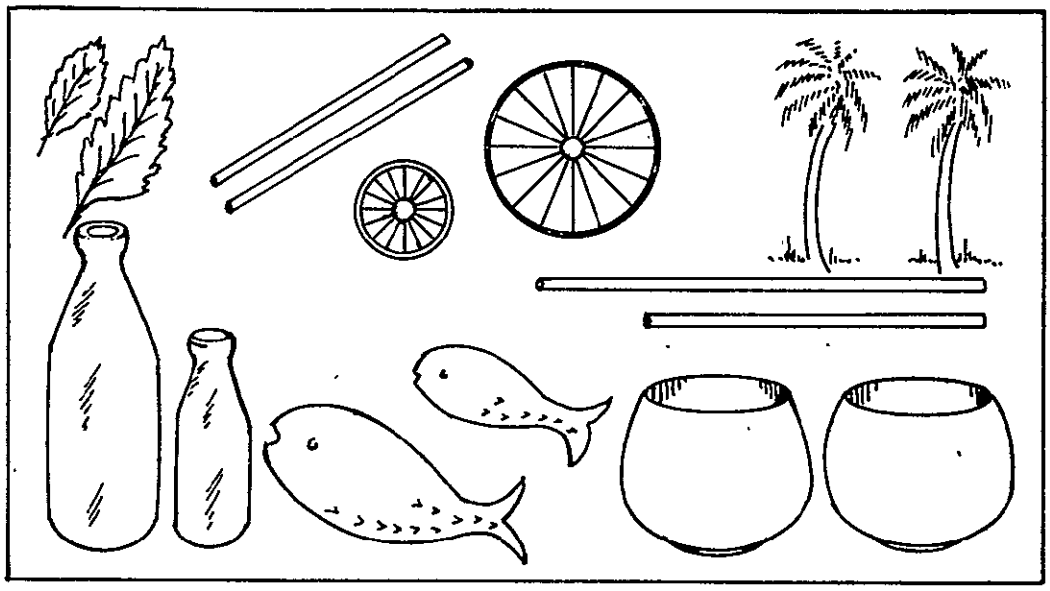
Next introduce the words bigger, larger, smaller, longer, shorter, by again comparing two similar objects of different size. For example, describe two leaves in this way. "This is a big leaf. This is a little leaf. This leaf is bigger than this leaf." Have children choose two similar objects of different size and make sentences of comparison using the words listed above. For example, they may say, "This pod is longer than this pod; this stick is shorter than this stick." Repeat this experience until most of the children use the words correctly.

Introduce the words near and far by naming a child and saying he is near you, and then asking children to name a friend who is standing near them. Next point to and name a child who is standing far from you. For example, say, "John is standing far from me." Have individual children point to and name children who are far from them.

When children understand the difference between near and far, introduce nearer and farther in the following way: Point to a child and say, "John is near me, " and then point to another child who is also near and say, "Mary is near me too; but John is nearer to me than Mary is. " Have individual children repeat this experience using "nearer" and "farther." The words may also be used in sentences describing distance between different objects; for example, "The goats are nearer to the trees than John is," and "The houses are farther from the school than the trees are from the school."

Experience in using the words of comparison to describe objects drawn on the blackboard. Sketch pairs of objects, some of the same

size and some of different sizes so that the words of comparison may be used. Suggestions: two sticks of the same length; two sticks of different lengths; two wheels of different sizes; two bottles of different sizes; two trees of different sizes; two bowls of the same size; trees of different sizes; two fish of different lengths. The blackboard drawings may be similar to the ones below:



Have the children study the pictures on the blackboard and name the objects pictured. When they have named them, ask such questions as, "Are the leaves the same size?"; "Are the sticks the same length?"; "Are the wheels the same size?"; etc.

If the answer is, "They are not the same size," as for example with the leaves, ask a child to come to the blackboard and point to the big leaf. Ask another child to point to the little leaf. After this, point to the big leaf and then to the little leaf and say, "This leaf is bigger than that leaf." Ask a child to point to the leaves as you did and repeat the sentence.

Repeat these steps with the other pairs of objects, using the words "same size," "larger than," "longer than," "shorter than," and "smaller than" when appropriate. In each case point to the pictures of the objects being compared.

"Far and near," "farther and nearer," should be used to discuss the position of the pairs of objects in relation to one another. Using the blackboard example above, point to the fish and ask the children to name groups that are near them. For example, "The bowls

are near the fish," and "The bottles are near the leaves." Similarly, have the children answer questions which require the use of "far from;" for example, "The bowls are far from the leaves." Questions should next be asked about single objects of a group; for example, "Which is nearer the leaves, the big fish or the little fish?" etc. Continue with this experience, encouraging the children to describe the position of as many different objects and pairs of objects as possible.

Experience for Page 1. Study pages 1-3 in the pupil's book and the suggestions which follow. Carefully plan a class discussion of the objects pictured. Try to make the widest use of the vocabulary of comparison.

Have the children look at the picture on the top of page 1. Discuss what is seen in the picture and try to develop a story. The story might be about a man and his children returning home from their fishing boats.

Guide the discussion of the picture by questions and suggestions so that the vocabulary of comparison is used. Some of the resulting statements may be:

The boat in the water is bigger than the other boat.
One house is smaller than the other house.
The trees are the same size.
The father is taller than the boy.
The boy is shorter than the father.
The father is near to the boats.
The boats are far from the trees.
The girl is near to home.
The big boat is near to the big house.
The girl is nearer to home than is the boy.
The smaller child is nearer home than the larger child.
The girl is nearer the big house than the boy.

Encourage the children to point to the objects as they are mentioned. Continue until the picture has been completely described and, if possible, until the words of comparison are used many times.

Have the children look at the picture on the bottom of page 1 and name the objects in the picture. Ask questions and guide a conversation with special emphasis on the words near, far, nearer, and farther.

Questions asked may be: "What is near the house?" "Which person is nearer the house than the other?" "Is the mother or the girl nearer the dog?" "What do you see near the house?" "Is the drum farther from or nearer to the house than the trees?" Continue the discussion with statements such as these:

The dog is nearer to the house than is the mother.
The dog is nearer to the drum than to the trees.
One bird is farther from the house than the other bird.
The dog is near the girl.
The birds are farther from the dog than are the girls.

Experience for Page 2. Have the children look at each of the pictures in turn and make statements comparing the size of the different objects. Statements they may make are:

The white pot is bigger than the black pot.
The large house has a bigger door than the small house.
The little girl is carrying the smaller bucket.
The legs of the stools are shorter than the legs of the table.
The little flower has a shorter stem than the big flower.

Children should be encouraged to make up as many sentences as possible about each picture using the words of comparison. They should point to each object as it is named in the sentence.

Experience for Page 3. Have the children study the picture at the top of the page and name the different objects in the picture. By means of questions, encourage them to make statements regarding the size and position of the objects, using all the words of comparison that have been already learned. Statements made could be similar to the following:

The cows are bigger than the dog.
One boat is larger than the other.
The aeroplanes are far away.
The trees near the little house are the same size.
One house is big and one is small.

Have the children look at the picture at the bottom of the page and name the different objects for sale in the shop, such as, jar, brush, cup, bow, teapot, bottle, bucket, dress, and hat.

Discuss the size of the different objects by comparing one object with another. For example, ask if the bucket and the cups are the same size. The answer is "No, the bucket is bigger than a cup." Continue with statements of this kind using the words "smaller," "larger," "longer," "shorter," "nearer," "farther," comparing as many different pairs of objects as possible.

Revision Experience. The use of the words of comparison may be revised by giving directions for the children to follow using all the words similar to the following:

(a) Choose the two leaves which are near to you. Are the leaves the same length? Place the larger leaf near to the teacher. Place the shorter leaf far from a friend.

(b) Choose two stones which are far away from you. Are the stones the same size? Give the bigger one to a boy near you. Give the smaller one to a friend far from you.

Continue the use of these words both in the mathematics period and at convenient times during other experiences in the days which follow until the meanings of the words are clear.

FIRST YEAR PRIMARY TEACHERS' GUIDE

UNIT 2

SETS

Introduction

Purpose: The purpose of this section is to help children develop the idea of a set. This idea is basic to the development of number ideas and other mathematical ideas.

Background for Teachers: The word used in mathematics for any collection of things is set. Children see and handle many objects every day of their lives. As they approach school age, the idea of a collection is formed. For example, children may see a set of pineapples in the marketplace. Father may lead a set of cows out to pasture. Mother may bring home a bundle of firewood, which is a set of sticks. Children may group an orange, a banana, and a pawpaw into a set. The things in a set are called members of the set.

The idea of set is simple, and leads easily to the idea of numbers and counting. We can take advantage of the simplicity of set ideas to begin arithmetic.

You may be used to thinking about collections with two or more members in them. It is easy and useful to talk also about sets with one or even no members. For example, since we speak of sets of 5 pencils, or 2 pencils, it is also convenient to speak of a set of one pencil.

Another important set is the set which has no members. This set is called the empty set. Later we shall see that this idea results in a need for the number zero, or naught. We can describe a set, even though it has no existing members. For example, we can talk about the

set of brothers of each child in the class. But there may be a child who has no brothers. The set of his brothers has no members. It is the empty set. It is sometimes convenient to be able to talk about a set which has no members.

Sets are further discussed in this book in Section 2, Mathematics for Primary Teachers, pages 6-19.

Teaching Plan: The plan of teaching consists of the introduction of the idea of set and the term "set" through experiences with collections of objects. The objects which are members of sets are displayed in the classroom, drawn on the blackboard, or pictured in the pupils' books. Experiences for the children involve the recognition and description of the sets pictured. The children locate and identify the sets by describing the set members.

Development

Vocabulary: set, member, few, many.

Materials: stones, seeds, sticks, nuts, beans, shells, leaves, feathers, clay-balls, bamboo or papyrus seeds, beads, books, pencils, chalk pieces, stamps, buttons, match-sticks, bottle tops, chairs, table, or whatever objects are appropriate and convenient.

Sets as collections of physical objects.

Preparation. Have collections of many different objects such as those listed above available in the classroom. Some of them should be small things which can be picked up and others should be large objects which can be pointed to and described. Think of sets of things which may be seen outside the classroom and around the school.

Experience in Describing Sets. Show the children a handful of beans and tell them, "This is a set of beans." Show them a bunch of sticks and ask them if they would call this a set of sticks. Show them a few pieces of chalk and ask them what they would call this (a set of pieces of chalk). Continue this activity with several other kinds of objects. Be sure to include a set of children, such as the set of all boys in the room or all children in one row, or the set consisting of Kofi, Kweku, and Ama. (Use names of children in your

class for the last example.)

Have the children describe some sets in the classroom and outside, for example, a set of trees in the compound. Try to have as many children as possible take part in this discussion of sets. They should describe a set by telling what its members are. For example, a child may say, "That is a set of pencils." or "The members of this set are chairs."

Among the sets you display, there should be some with a single member. For example, hold up a pencil and say, "This is a set. The pencil is a member of the set." Then point to a table. "This is a set. The table is the only member of this set." Ask the children to name the members of the set of all teachers in the room. (Usually this set will have only one member, but there may be a visitor.)

Introduce the idea of <u>many</u> and <u>few</u> by showing or describing sets with many members and sets with few members. Hold in your hands many small objects and say, "Here is a set which has many members." Then hold up two or three of the objects and say, "Here is a set with few members." Next hold up one of the objects and say, "Here is a set with one member." Discuss sets with many members such as the set of all children in the school, the set of all people in a particular village, and the set of all birds. Describe sets with few members such as the set of blackboards in the room and a set of several books on the table. Ask the children to name other sets which have many members and to name sets which have few members.

<u>Experience in Describing Sets as Having Few or Many Members</u>. Ask each child to bring to class a set of things having many or few members. Use the list of materials suggested previously for examples, but do not tell any child exactly what things to bring.

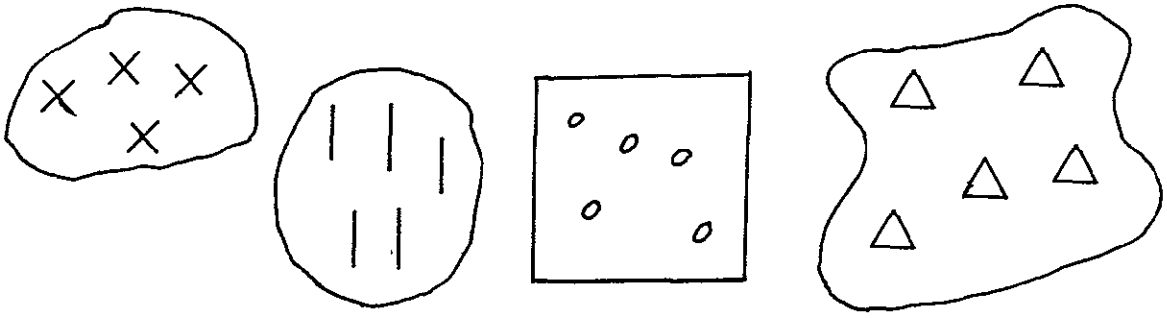
Have each child show the members of his set and describe his set. For example, a child may say "My set has few members,""My set has many members," "My set is a set of many big beans," or "My set is a set of few small beans." Avoid discussing or mentioning numbers of things at this time. The purpose here is to develop the idea of set, so that the idea of number may be developed later.

As a further example, take a duster, a few beans, and a stick and place them all together. Ask a child to name the members of this set and then ask him if it has many or few members. Point out that a set can have several different kinds of members in it. Have the children describe some other sets which have different kinds of members. When sufficient time has been spent in the suggested activities, go on to the further activities described below.

Experience in Recognizing and Describing Sets which are Pictured.

Show the children a set of things which can then be pictured on the blackboard. For example, hold up several books and say, "Here is a set of books. I can put a picture of this set on the blackboard. I can draw the members of the set. What are the members of the set?" The answer should be, "The members are books." When the children have answered, draw a picture of that same set by drawing exactly the same number of books. Then say, "Here is a picture of this set of books. It is a picture of the set of books I am holding." To show that all the members pictured are members of a set, you can draw a ring around all the members. You need not emphasize the ring but simply say, "All the things pictured inside are members of the set."

Continue by drawing simple pictures of sets of cows, trees, books, circles, squares, or triangles, etc. Ask children questions about these sets, for example, "What are the members of this set?" "What can you tell about this set?" Have several children come to the blackboard and draw pictures of sets. These may be very simple, consisting of strokes, x's, or dots. Have other children describe the sets by telling what the members of the set are.



Picture sets with few members, many members, and some with a single member. Represent a wide variety of things as members of a set.

At first, in drawing pictures of sets, it is helpful to always draw a ring or square around the set members. This stresses the idea of the set as a collection of things. It gives a point of reference so that you can say, "Here is a set pictured inside the ring. What things are members of the set?" Occasionally, picture a set without the ring so that the children are aware that it is convenient but not necessary to enclose the pictures of members of a set.

Continue the experiences as needed.

Experience for Page 4. Ask the children to look at the pictures on page 4. Allow a few moments for them to consider all the sets represented. Begin by saying that one picture shows a set of drums. Ask the children to find the set of drums and point to it. Use and emphasize the word "set" and be certain that the children understand that the set may be the entire collection of drums pictured.

Have the children put their hands around the set pictured in order to emphasize the idea of the set as the collection of drums. Remind the children that they can describe or talk about the set by explaining that the drums pictured are all members of the set.

Ask the children if they can find another picture of a set. Some of the children may name or tell about the sets they have located. For example, a child might say, "Here is a set of fish," or "I have found a set of pencils," or "The member of this set is an elephant." Encourage the children to use the word "set" and to refer to the things in the set as "members." Do this be asking such questions as, "What are the members of the set which you have found?"

(Added note: The idea of number will be developed later from the idea of set. Now, we are concerned with the set as a collection and not specifically with the exact number of its members. Thus at this point in the development, you should avoid references to number in discussing the members of the set. Do not encourage the counting of set members or the description of sets by telling the number of its members.

For the first several pages, discuss "sets of drums," "sets of pencils," and the "set of an elephant," as collections and not "the number of pencils," "three drums," or "the set of two pineapples.")

Experience for Page 5. The kinds of activities suggested for page 4 of the pupils' book may be continued for page 5. Ask the children to find pictures of sets and discuss the members of the sets. Begin by saying, "I am looking at a set. Its members are oranges. Find the picture of that set in your book and point to it."

Ask a child to find some set pictured on the page and describe it to the class by telling about the members of that set. The other children can then find on their pages the picture of the set which was described. For example, a child might say, "The set I have found is a set of goats" or "The member of my set is a chicken." Tell the other children to put their hands around the picture of the set of goats or to point to the set of a chicken. Go on in this way until each of the sets on the page has been identified and described.

In each of the last two sets represented on page 5 the set members are different kinds of objects. There is, for example, a set consisting of bowls and pots. It should be described as "a set of bowls and pots" or as "a set which has bowls and pots as members." Since we are not yet concerned with number, you should avoid reference to the number of pots or bowls. If a child suggests a number, just accept it and go on.

In the last set, the members are different kinds of fruit. The child might refer to the set as "a set of fruit," "a set of an orange, a pineapple, and a pawpaw," or perhaps as "a set of several pieces of fruit."

Encourage the children to see the whole collection of things in each picture as a set by moving your fingers across all the objects pictured when pointing to a particular set pictured in the book. Ask the children occasionally to locate a set described by putting their hands around the picture of the whole set or by tracing a ring around the set picture with their fingers.

Experience for Page 6 . Direct the attention of the children to the first picture and have them describe some of the sets of things they see . As a set is discussed, ask the children to trace a ring around the pictures of the members of the set with their fingers. If none of the examples offered by the children include sets with unlike objects, you should mention some.

The same kinds of activities are appropriate to the second picture. Ask the children to find a set of things pictured and tell the class what the set members are. A child might respond, "I find a set of boys." Then ask the children to point to or put their hands around the picture of the set of boys. Another child might say, "There is a set of baskets," and the other children can trace a circle around the picture of the set of baskets with their fingers.

Continue until many sets of like objects have been described. If they are not suggested, you should describe some sets which combine different kinds of things and have the children locate them in the picture. For example, ask them to find "the set of women and children," "the set of bottles and baskets," etc. If no child gives an example of a set of one, ask the children to find a set pictured which has just one member. Aside from this, make no reference to the number of members in a set. The idea of number will be developed in the sections to follow.

At this stage, the children should be thoroughly familiar with the term "set." They should use it readily in referring to collections of things they see about them and in pictures. Have the children use the term "set" in reference to collections of things or persons they cannot see or point to immediately. For example, they can imagine and describe the set of all children absent from school, even though they cannot point to the set members themselves. They can think about the set of all fish in the river, the trees in the forest, all the leaves on the trees, etc.

Experience with the Empty Set. Have the children describe several sets in the classroom such as the set of doors and the sets of teachers. Then ask them to describe the set of elephants in the classroom. Some children may laugh at this, but they will agree that this set has no

members. Explain that any set which has no members is the empty set.

Ask some of the children to name the members of the set of their sisters. For example, a child might say, "My sisters are Marjuma and Sara" or "The set of my sisters has one member. It is Myasa." Try to find a child who has no sisters. Ask him to tell about the members of the set of sisters he has. If he says he has none, explain that the set of his sisters is the empty set since it has no members.

Other examples can be given. For example, in a boys' class, you can ask for a description of the set of all girls in the class. Ask for a description of the set of men who have two heads. Then get the children to describe the empty set in several ways. Make sure the sets they describe really have no members. Some possible examples are:

Set of men with four hands,
Set of giraffes in the classroom,
Set of green elephants, and
Set of cows which can fly.

Draw a picture of the empty set on the blackboard in the following way:

Say to the children, "The set pictured inside this ring is the empty set. The things in the ring are members of the set. The set has no members at all."

Experience for Page - 7. Have the children find sets pictured on page 7 and describe them. A child may say, "I have found a picture of a set of chairs" or "The members of the set I see are chairs." Then tell the others to point to or enclose with their hands the picture of the set of chairs. Sets should be described by simply telling what its members are with no mention of the number of members.

Notice that one of the rectangles contains no objects at all. If the children do not raise the point, you should call their attention

to this empty space. Ask them to tell about the set pictured in that space. They may say that there are no things in that space. Remind them that we may speak of the set which has no members. This set is the empty set. We may say, then, that we have a picture of the empty set. If the idea of the empty set needs more discussion, you may give more examples of sets which have no members. Explain that the empty space on page 7 is a picture of those examples. The children should learn to use the expression "the empty set" and should be able to tell you that it is "the set which has no members."

FIRST YEAR PRIMARY TEACHERS GUIDE

UNIT 3

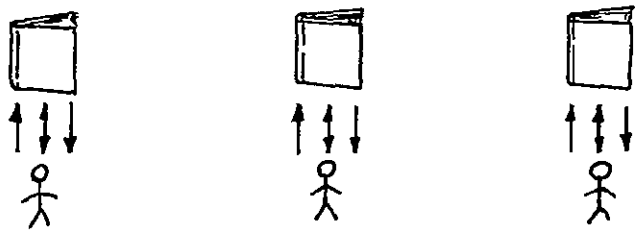
ONE-TO-ONE CORRESPONDENCE

Introduction

Purpose: The purposes of this section are to help children develop the idea of one-to-one correspondence of sets and to develop the ability to determine whether one set has more members than another by matching one-to-one.

Background for Teachers: It is possible to compare the number of members in two sets without counting. For example, we can tell whether there are more members in the set of children or in the set of books by seeing whether or not every child has a book. By matching each child with a book, we can see whether there are more children than books, more books than children, or just as many books as children. This is an example of one-to-one matching.

The principle of one-to-one matching is that two sets are matched in such a way that a member of one set is matched against exactly one member of the other set and vice-versa. For example, in the case above, two people do not use one book, and one person does not use two books, but one person uses only one book. See the illustration below.



By matching one-to-one, we can compare two sets and tell which has more members or whether one set has just as many members as the other. If one set has exactly as many members as the other, we know that they have the same number of members. We shall use expressions

such as, "as many members," "fewer members," and "more members" and will not refer to specific numbers. It is possible then to tell that there are as many members in one set as another or that there are more members in one set than another without knowing the exact number in either set. A child who cannot count, can match books with children in the class and find that there are as many books as children, or more members in one set than in the other.

Teaching Plan: The introductory activities consist of matching sets of objects one-to-one to determine whether a set has fewer, as many, or more members than another set. Moving to a more abstract level, two sets are represented on the blackboard, and the members are matched by drawing lines from a picture in one set to a picture in another set. Then the children match sets represented in their books by marking each member of one set and matching this by marking a member of a second set.

Development

Vocabulary: matching sets, one-to-one matching, more than, fewer than, as many as.

Materials: Objects of many kinds which can be used to form sets and which are easily manipulated, such as stones, bottle caps, beans, pencils, books, buttons, and beads; a large number of objects such as beans, seeds, buttons, and kernels of maize which can be used as markers for the pages of the pupils' books. Each child should have at least ten objects to use as markers. If possible, half should be of one type or color and half of another type or contrasting color such as red beans and white beans, or beans and maize seeds.

Experience in matching members of sets of physical objects one-to-one. Give each child a pencil and a book to place on his desk. Put one book on a table. Remind the children that all the pencils on all the desks make up a set of pencils. All the children in the class also make up a set. "We have a set of pencils placed on the desks. We have a set of children in the room. Do we have as many members in the set of pencils as there are in the set of children? Are there as many pencils as children?"

The children should tell you that there are just as many pencils as children. Ask them to tell how they know this. Guide the children in this explanation, "Since each child has exactly one pencil we know that there are just as many pencils as there are children. The set of pencils has just as many members as the set of children." Explain that they have compared the sets by matching one member of one set with exactly one member of the other. One pencil can be matched with exactly one child.

Ask the children to look at the set of books on their desk and on the table and to compare the set of books (including the extra book on the table) with the set of children in the class. Ask, "Are there fewer books than children, just as many books as there are children, or more books than children?" Guide them to the conclusion that there are more books than children because each child has a book, and there is also one book which is not matched with any child. They may say, "When we match one book with one child we find one extra book which is not matched with a child. Therefore, we know that the set of books has more members than does the set of children."

Pick up the pencils from several desks and ask the children to match the set of pencils still on desks with the set of children in the class. Since there are some children who do not have pencils on their desks and no child has more than one pencil, the children should be able to conclude that the set of pencils has fewer members than does the set of children. There are fewer pencils than children. Explain that we match one child and one pencil until all the pencils have been matched. We then find that there are still children left. Therefore, there are more members in the set of children.

Have each of two children identify sets they can see in the room. Have them match the two sets one-to-one and tell whether the set they choose has as many members, more members, or fewer members than the set the other child chose. For example, if Ama chooses the set of pots on the table and Aba chooses the set of chairs, they will place a pot on a chair until they have matched all the pots to the chairs; Ama will say, "The set of pots has fewer members than the set of chairs"; Aba will say, "The set of chairs has more members than the set of pots."

Experience in matching sets-a game. Let one child be a shepherd and let the rest be sheep. Tell the children it is early morning and the sheep are to leave the room one at a time to find their food. The shepherd drops a stone in a box for each sheep as it goes out. (Have the sheep pass by a box as the shepherd drops a counter in.) The teacher asks, "Are there more stones in the box than sheep, more sheep than stones, or as many stones as sheep?" The answer is, of course, "as many as."

Ask what sets are matched. (The set of sheep is matched with the set of stones.) Ask if one set has as many members as the other or whether one set has more members than another.

Now have the children pretend it is evening and the sheep come back home one by one. As each sheep comes in, the shepherd takes one stone out of the box. (Have the sheep again pass the box by the shepherd.) After the last sheep returns, ask if any stones are left in the box. The answer should be, "No." Ask why there are no stones left in the box. The answer may be, "There are just as many sheep as stones. There is one stone for each sheep and one sheep for each stone." Ask what two sets are matched and what set has more members. (The set of sheep and the set of stones are matched. One has just as many members as the other.) (Note: The sets should be compared by matching rather than by counting.)

Remind the children that they have been comparing sets. Ask how they have been compared and guide the children to agree that the sets have been matched. Ask how many members of one set are matched with a single member of the other set. (One). Tell the children we call this kind of matching "one-to-one matching." What can we find out about two sets by one-to-one matching? (We find out whether one of the sets has more members than the other, or as many members as the other.)

Experience in matching sets. Have several sets of objects on a table. Ask children in turn to come to the front of the classroom and match two sets. The children should understand that members of one set are matched with members of the other set, one at a time. Begin with pairs of sets which have the same number of members. Then use some pairs with one set having more members than the other. Be

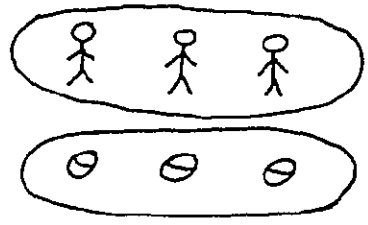
sure to include a few sets having only one member. After each example ask, "Which two sets are you comparing?" and "Which set has more members?"

Include at least one example using the empty set. For example, hold out one hand with several beans in it, the other with nothing. Ask the children to match the two sets one-to-one. They should agree that the set of beans has more members than the empty set, since the empty set has no beans in it and the other set has some beans in it. The matching is immediately apparent in this case.

Now have the children make sets of objects at their desks and match the sets one-to-one. They may work alone, for example, matching a set of pencils with a set of beads, and a set of bottle caps with a set of stones.

A set-matching game may be played as follows: one child arranges a set of beads on his desk keeping it hidden by covering it with his hands. His neighbor arranges a set of shells in the same way. Then they uncover their sets and match them one-to-one by each taking a member from his set at a time. They will then discover whether the sets have just as many members as each other, or which has more members. This game will be special fun when one child has a set of only one member, or even the empty set! It will also be fun to find that sometimes the sets do have just as many elements as each other.

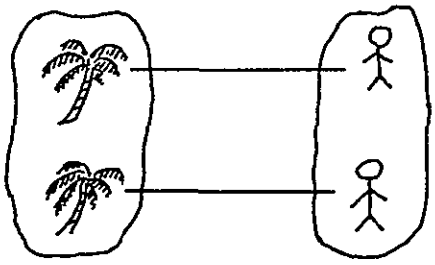
<u>Experience in one-to-one matching of members of sets pictured on the blackboard</u>. Sketch the members of several pairs of sets which can be matched one-to-one. Put the pictures in the sets to be matched one above another so that comparison will be made easily. For example, one sketch could be the following:



Have the children match the pairs of sets one-to-one by drawing a line from one member of one set to exactly one member of the other set. Match many different sets as represented on the blackboard, for example, a set of four oranges and a set of four pineapples.

Vary the example so that some sets match exactly one-to-one while some sets have more members than another. Include some examples of the empty set.

Also sketch set members one after another, and include some examples which can be matched in the following way:



Give as many children as possible the chance to match sets at the blackboard. Include other children by asking them to check on the matching just concluded. Try to get everyone in the class involved in the activity.

As further blackboard experience, have the children sketch the members of a pair of sets and let others do the matching by drawing lines from one member of a set to one member in another set. Encourage them to tell, for each example, whether there are as many members in one set as another or whether there are more members in one set than another.

Experience for page 7 in matching sets. Provide each child with at least ten small objects (beans, seeds, bottle caps, etc.) as markers. Each should have five objects of one kind and five of another, or five of one color and five of another. It will be helpful if one half the objects can be easily distinguished from the other half.

Page 7 has already been used in identifying and describing sets. It is used now for one-to-one matching of sets. Any pair of sets may be compared. Some will match exactly one-to-one while in other instances one set has more members than the other.

Have the children match the sets pictured in the first two rows on page 7 by using their ten small objects as markers. Markers of two different kinds, such as red beans and white beans, are most effective. In order to match the set of chickens pictured with the set of groundnuts pictured, a child places a red bean on the picture of a chicken, then a white bean on the picture of a groundnut. Each time

he puts down a red bean he follows by putting down a white bean for a member of the other set. He may put just one bean on any one picture. If every time a marker of one color is placed, a marker of the other color is also placed, and if all the pictures in both sets are covered, then the sets are matched one-to-one. Without counting or knowing the names of counting numbers, the child shows that one set has the same number of members as another. He should say, "The set of chickens has as many members as the set of groundnuts."

Have the children continue in a similar manner as they match the set of chairs to the set of tables and the set of children to the set of books.

Have the children now use the marker beans to match each of the sets of the last row of page 7 with the set just above it. The children proceed with the matching of the tomatoes and pawpaws in this way: A red bean is placed on a picture of a tomato, then a white bean is placed on a picture of a pawpaw. Next a red bean is placed on the picture of the other tomato. Here they find that they cannot follow by placing a white bean because there is no pawpaw on which to place it. The sets are not in a one-to-one correspondence, and they should say there are more members in the set of tomatoes than in the set of pawpaws.

Guide the children carefully as they match the set of fruit with the empty set which has no members. The fact that these sets do not match one-to-one can be determined readily. If the child places a marker of one color on the picture of a piece of fruit, he cannot place a marker of the second color because the set above has no members.

(Note: In each of the comparisons in the last two rows, a child can show that one of the pairs of sets has more members than the other by matching, with no reference to counting or to the number of members in the sets. He should not yet be asked to count, and the teacher should avoid referring to the exact number of members of any set at this point.)

Experience for page 8. Ask the children if they think the set of butterflies and the set of matches can be matched exactly one-to-one without using the markers. It may be possible to do this by tracing a mark with the finger from each pictured object in turn to a member

pictured in the other set. This activity corresponds to the drawing of marks between pictures on the blackboard as in previous activities. Some of the children will probably be able to determine the matching by simply looking from one picture to the other.

After they have suggested whether they think the members of the sets can be matched exactly one-to-one, they should be asked to check their answers by using markers as they did with page 7. Again, any kind of small object may be used,and it is helpful if each child has two colors or two kinds of markers.

Have the children compare each set represented on the left side of the page with the set represented on its right. They should match the members of the sets one-to-one and then tell whether one set has just as many, more, or fewer members than another.

In addition to matching sets directly across from one another, the children may also match other pairs of sets; for example, they can match the set of books with the members of a set pictured beneath or above. In fact, any pair of sets on the page may be compared. By means of markers, the set of matches pictured at the top could be compared with the set of trees at the bottom. For each example of matching, the children first place a marker of one kind on a member pictured in one set, then place a marker of another kind on one member pictured in the other set. They should be able to tell whether the pair of sets matches exactly one-to-one or whether one has more members than does the other.

Use page 8, also to review some of the activities introduced in the beginning of the book. Ask children to describe a particular set by telling what its members are. Continue to give the children the opportunity to match sets pictured on the blackboard by drawing lines from one picture to another. For example, ask one child to draw a picture of a set of simple objects and another to draw a set which may be matched one-to-one with the first set.

Experience for page 9. Have the children look at the pictures of sets on page 9. Ask if someone can tell about one of the sets he sees. He can describe the set by telling what its members are. For example, a child might describe "a set of flowers" or "a set of books and a pencil" or "a set whose members are a boat and a tree." If, after all other sets pictured have been described, no one has mentioned the

empty set, you should point out that there is still another set pictured which has not been discussed. You might say, "Can anyone find the picture of the other set and tell about it?" If a child answers that it is the empty set, ask him what its members are. If a child answers that it is a set with no members, ask him what we call that set. Encourage the use of the term "empty set" as you emphasize the idea of the set which has no members. If no child finds the picture of the empty set, you should hold up the page, point to the picture of the set with no members, and say, "Here is a picture of a set. Tell us about it. What are its members?"

Ask the children to look at the sets carefully and decide if any one set can be matched exactly one-to-one with another. Allow time for them to consider all the possible matchings. Ask someone to describe the sets which can be matched exactly one-to-one.

When a one-to-one correspondence of two sets has been suggested by a child, ask all the children to check this by using their markers as they did for page 8. Even if a child suggests two sets which cannot be matched exactly one-to-one, let the children decide if the statement is correct by performing the matching activity with the markers. For instance, if someone says that the set of bicycles may be matched exactly one-to-one with the set of books and a pencil, say, "We can find out if it does match exactly by using the markers. Try the matching and see." If each child puts a red bean on a bicycle and a white bean on a book, he should find that the pencil is left and cannot be matched with any member of the set of bicycles. In this way, each child can determine that the set of books and a pencil has one more member than does the set of bicycles. The same procedure may be followed if a child suggests sets which <u>do</u> match exactly one-to-one. You need not say whether they do or do not match. Ask the children to match the sets using markers to find out for themselves. It is useful to do the matching with markers to determine that a particular set has more members than does another as well as to find sets which match exactly one-to-one.

FIRST YEAR PRIMARY TEACHERS' GUIDE

UNIT 4

NUMBER I

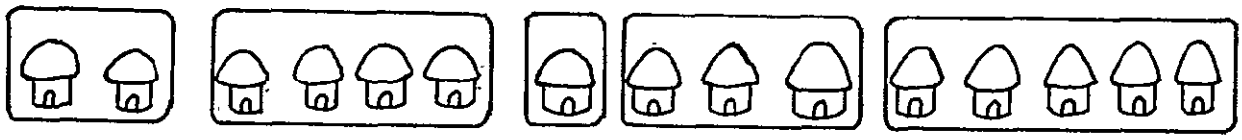
Introduction

Purpose: The purposes of this unit are (1) to develop the natural ordering of sets; (2) to develop the idea of number as a property of sets; (3) to develop the use of numbers as answers to the question, "How many members has a set?"; (4) to help children learn to recognize and name the number of members in a set (0-5); (5) to help children form sets with a given number (0-5) of members; (6) to develop the "one-more-than" pattern for numbers, that is, the idea that two is one more than one, three is one more than two, and so on; and (7) to define whole numbers and counting numbers.

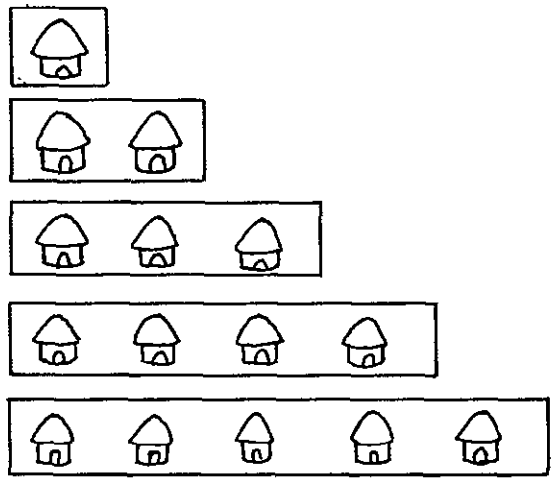
Background for Teachers:

Ordering of Sets: The set $\{x, x\}$ cannot be matched exactly one-to-one with the set $\{y, y, y\}$, because there is one more member in the second set than in the first. That is, if every member in the first set is matched with a member in the second set, there is one member in the second set not matched. This one-more-than relation serves as a pattern for ordering sets.

To explain how this works, start with the set of huts:

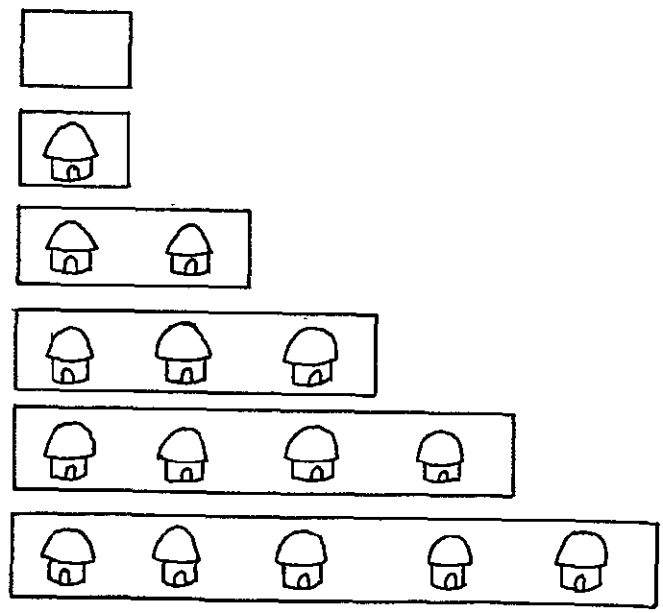


No set of huts matches any other set exactly one-to-one. The sets can be arranged in this order:

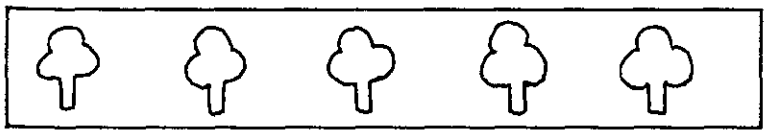


The sets of huts are arranged so that each set has one more member than the preceding one. This list could be continued as far as we like. When sets are arranged in this order, on a one-more-than pattern, we say they are in natural order.

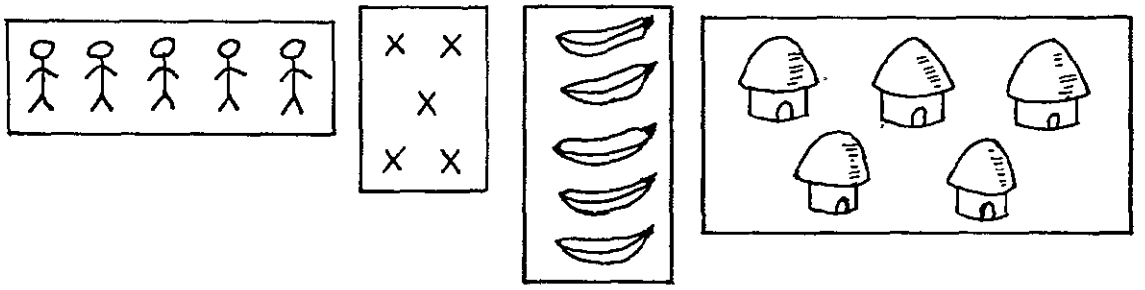
Where does the empty set fit into this pattern? Since the empty set has no members, a set of one member has one more member than does the empty set. So it must be that the empty set comes first in the list, preceding the set of one member as arranged below.



Naming of Numbers: In matching two sets one-to-one, it is found that one of the sets has more members than the other or that one set has exactly as many members as the other. Suppose we start with some set, for example, the set pictured here:

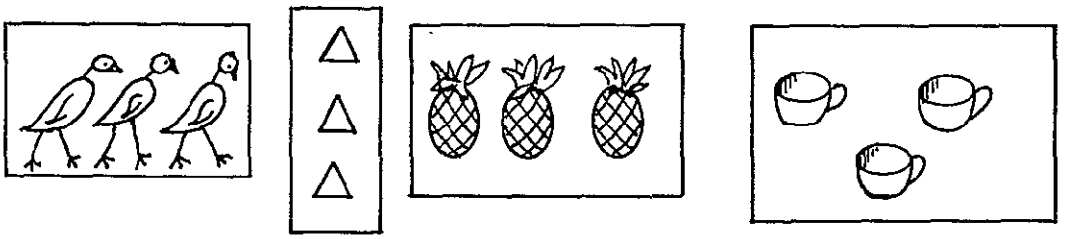


Not every set can be matched exactly one-to-one with this set. Think about the sets which do match the set above exactly one-to-one. Here are the pictures of a few such sets:



What is it that all these sets have in common? Any one has just as many members as has any other. We say that all these sets have the same number of members.

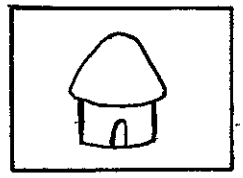
Now consider the sets below. All can be matched exactly one-to-one. Think about the sets which can be matched exactly one-to-one with these sets.



We say that all these sets have the same number of members.

In this way, the sets can be divided into classes by one-to-one

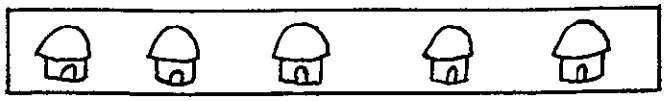
matching. All sets which match one-to-one the set of trees in the first example above are in one class. All sets which match the set of birds above are in another class. In still another class are all sets which match the set below.



In another class, we put all sets which match this set:

In one class will be the empty set since the empty set cannot be matched one-to-one with any other set.

All the sets in any class have the same number of <u>members</u>. It is convenient to give a name to the number. For example, names given to number of members in the sets of huts are given below.

Set of Huts	Number of Members
	Zero
	One
	Two
	Three
	Four
	Five

In assigning names to the number in various sets, it is convenient to list the sets in natural order first. In this way the names of the numbers are in natural order also.

Just as the list of sets is in a one-more-than pattern, we say the list of numbers is in a one-more-than pattern. That is, we say "two is one more than one, three is one more than two, etc."

Both the list of sets and the list of numbers can be extended indefinitely. See Section 2, "Mathematics for Primary Teachers" for ways of extending the list of number names.

The numbers considered in this unit: zero, one, two, three, four, five, and so on, are called whole numbers. The numbers one, two, three, four, five and so on are called counting numbers. We will find that these numbers are used in counting. Counting begins with one. Zero, the number of members in the empty set, is not used in counting, and is not a counting number, although it is a whole number. The other numbers we consider here, one, two, three, etc., are both whole numbers and counting numbers.

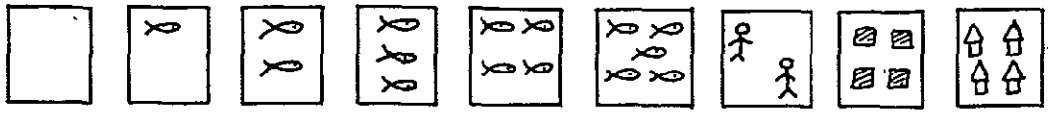
Teaching Plan: The plan is to build upon the prior experiences in one-to-one matching of sets by ordering six sets in natural order and introducing the names of the numbers of their members. Thus the numbers introduced, corresponding to the order of the sets, are zero, one, two, three, four, and five. The plan continues by moving from concrete experiences with actual objects to more abstract examples at the blackboard and in the pupil's book.

Development

Vocabulary: order, number, one more, one, two, three, four, five.
Materials: Objects which can be used to form sets and which can be used as markers and counters. (The same objects used in previous units are convenient. Examples of these are beans, seeds, stones and bottle caps); a set of six 5" by 7" cards which picture sets of 0 to 5 specific objects such as fish on each card(See the figure below); several similar sets of 2" by 3" picture cards for use by the children at their desks; several 3" by 20" strips of cloth on which are glued

or coloured 10 discs (See the figure below).

Cards: 5" by 7" and 2" by 3"



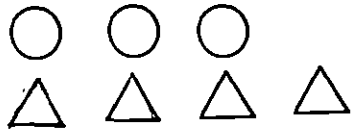
Strip of cloth: 3" by 20"

STAGE 1: Ordering Sets.

Experience in ordering sets of physical objects. Have many objects of different kinds on a table so that different sets of 1 - 5 may be formed. Therefore, for some examples of sets you need at least fifteen objects; for others use five of one object, four of another, three of another, two of another, and one of another.

Introduce the idea of ordering sets informally by having the children match certain pairs of sets. Display five different sets of things on a table so that one set has one member, one set has two members, one set has three members, one set has four members, and one set has five members. (Note: It may be necessary to work with small groups of children at a time on this activity.)

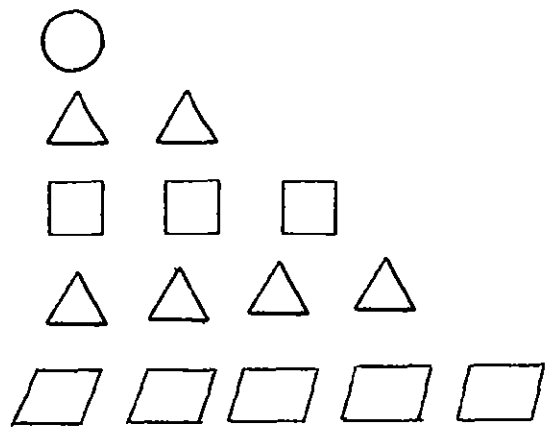
None of these sets will be in one-to-one correspondence with another. But the children can try to match different pairs of sets to find out if any match exactly one-to-one. Have them do this by putting the members of one set in a row and placing the members of the other set in a row just under the first. Just beneath a member of the first set is a member of the second set, etc. A matching of two of the sets would look like this:



Each member in the set arranged above is matched with a member in the set arranged beneath. The child should note at once and say, "The bottom set has more members. It has a triangle which is not matched. The triangle has no circle just above it." Further, he can say, "The bottom set of triangles has one more member than the set of circles."

Give the children time to try matching many of the possible combinations of sets. When they have tried all the pairs or combinations, ask, "Have you found any two sets which can be matched exactly one-to-one?" They should conclude that no two sets can be matched exactly one-to-one. Explain, "We can say that no one of the sets has the same number of members as another," or that, "Each of the sets has a different number of members."

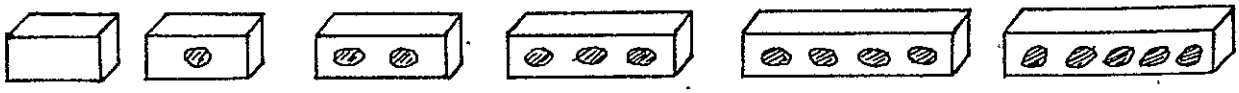
Ask the children to find the set with the fewest members. They may say, "All the other sets have more members." Have each of them place the set with fewest members along the top of his desk and find and place below that set another set with one more member. Continue until the sets are arranged in this way:



Tell the children that the sets are arranged in order. Each set has one more member than the one above it. They should point to sets on their desks and say, "This set has one more member than that one."

Have the children look at the arrangement of sets on their neighbors' desks to see if the sets are arranged in order.

Experience in ordering sets. Have the children work in small groups with each group having six empty match boxes or an egg carton with six compartments. Each group needs at least fifteen small objects. These may be objects such as beans, stones, or matchsticks. Put the match boxes in a row and have the children rearrange the boxes so the sets are in order, again beginning with the set with fewest members (the empty set). The match boxes and contents should look like this when the activity is concluded:

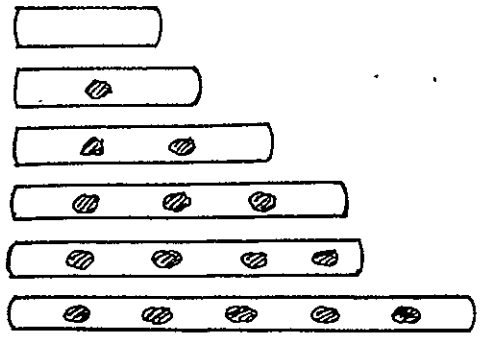


The children should order the sets by beginning with the empty set, then showing the set with one more member, then the set with one more, etc. In each case the child matches the new set one-to-one with the set just completed and then adds one more member to form the next set in the order.

Exchange the boxes among the group. Try to give many opportunities for each child to order different sets of one, two, three, four, and five members. (This is still done without reference to the number names or counting.)

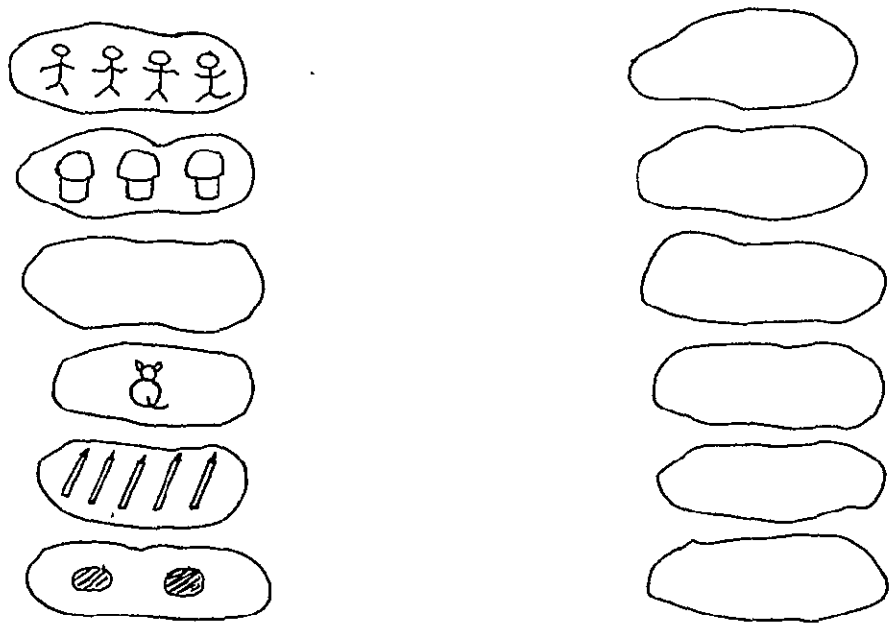
Experience in ordering sets drawn on the blackboard. Place sets of 0 - 5 bottle caps on a table. Have each set of bottle caps on a cardboard so that the sets are easy to identify. Ask a child to choose the set with the fewest members. Ask another child to begin with that set and order all the sets of bottle caps.

Sketch the sets on the blackboard in this way:

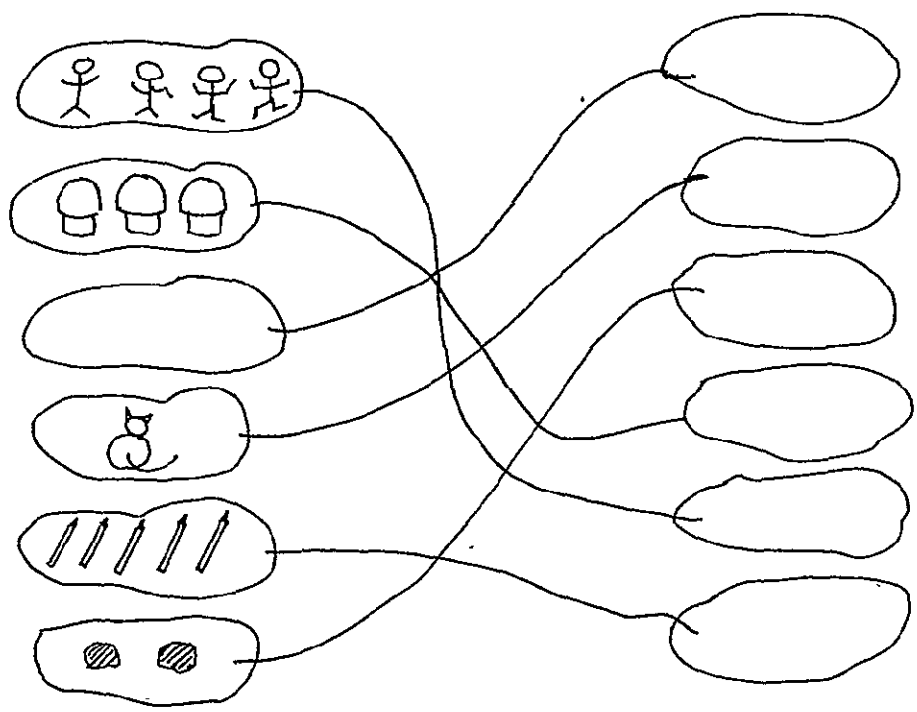


Discuss the relation of each set to the one above it. Have the children describe how to order sets.

Put pictures of six sets, each with a different number of members, on the blackboard in random order. Across from these pictures draw six large rings. For example:



Ask a child to come to the blackboard and show the ordering of sets from that with fewest members to that with most members. He can do this by drawing a line from a set on the left to its position in the order on the right. For example:



Have a child draw the set with the fewest members he can think of. Ask another child to draw a set which has one more member; ask still another to draw a set with still one more member. Continue until a set of five members is drawn.

Ask a child to draw any set he wishes. (It should be of five objects or less.) Have another child complete the next set in the ordering if there is a next set. Have him show that his set is next by matching his set with the previous set and showing that his set has one more member than the other. If a child begins with a set of five members ask the next child to draw the set which comes <u>before</u> it in the ordering, the set with one fewer members, etc.

Give many examples of the kinds suggested so that many children have a chance to participate. To get all children involved in each activity, ask children to check the ordering performed by other children to see if they agree with the answers given.

Continue the blackboard experience as needed.

<u>Experience for Page 10</u>. Ask the children to look at the sets on page 10 and describe the sets they see. The answer should be, "sets of buckets." Ask the children to point to the set with no bucket in it. Have the children describe this set as the empty set. Have the children look at the next set. Ask them if it is the same as the first set. The answer is "No." They should also be able to tell that this is because there is a bucket in it. Say, "Yes, it is a set with one bucket. It has one member,and the empty set has no members. This set has one more member than does the empty set."

Get children to look at the next set and ask whether it is the same as the set just <u>above</u> it. Guide them in concluding that it is not the same; it has one more member. This can be determined by trying to match the sets one-to-one. The one additional member in each set is shown in dotted outline to make the "one-more" pattern very clear. Point out that each set matches the one just beneath it except that there is a member added, as shown by the dotted outline.

Discuss each set on the page in this way. Compare each set in turn with the preceding set. Emphasize the fact that each set has one more member than the one before and that the sets are in order beginning with the set with fewest members. Each pair of sets should be compared

by one-to-one matching which shows that a set has exactly one more member than the one just before it.

Experience for Page 11. The use of page 11 should be similar to that just discussed for page 10. Explain that to order sets we build from the empty set, adding one more member to each set in turn.

Give each child 15 counters or beans of a size similar to the pictures on page 11. Ask children to arrange the counters in sets on their slates or desks in order beginning with the set with fewest members. Check each child's arrangement of sets. Ask them to tell what set is the first in the order. If they say it is the set with one member, remind them that there is a set with fewer members. They should be able to tell that the empty set is first in the order. If any child has missed the empty set, ask him to rearrange the counters to allow space for the empty set.

With children's help draw ordered sets on the blackboard. Call children's attention to the position of each object of the set. Use coloured chalk to bring out the idea of "one more." As a written exercise ask the children to draw pictures of sets on slates or paper as was done on the blackboard. Be certain that the sets of objects are in the right position to easily match the members.

Have the children look at the sets on page 11. Ask if the sets are ordered and if they begin with the set with fewest members. Some children may need to do some matching since the sets are not arranged with members of one set directly over the members of another set.

In the space provided on the right side of the page, have the children form six sets corresponding to the sets pictured on the page. Have them order the sets beginning with the empty set. In order to do this, they need only duplicate the ordering represented by the pictures. Remind them also that the ordering may be done with reference to the pictures by beginning with the set with fewest members, the empty set, and building each succeeding set by matching the set before and adding one more bean or counter. Discuss again the fact that each set in the order has more members than the one before.

Experience for Page 12. Each child should have 15 counters or beans of the kind used for page 11 activities. Have the children look at page 11 and describe the order of the sets. The set with no beans is at the top, and the next set has one bean in it, the next set has one more, etc. Have them look at page 12 and ask if the sets of beans there are ordered. The answer will be, "No." Ask the children to rearrange the sets on page 12 so that they are in the same order as the sets on page 11. Explain that spaces have been left on the right of page 12 for the sets to be rearranged.

Ask the children to look at the sets of beans and point to the set with no beans in it. Have them point to the space on the right where the empty set should be. They should agree that the space is left for the empty set. Ask the children to point to the set with one bean more in it than the empty set. Ask where this set should go in the spaces on the right. Have them put one counter in the proper space (below the empty set). Continue in this way until the children have arranged the sets in order beginning with the empty set.

Experience for Page 13. Arrange a variety of concrete objects in sets of one through five objects on a table, a desk, or the floor. Draw a circle around each set with a piece of white chalk. Draw one circle with no objects in it. Ask the children to tell which set has fewest members and therefore should come first in the order. They should say, "It is the empty set." Point to the set with one member and ask children to tell the difference between that set and the first set. It has one member more than the first set.

Point to the set with one member more than the preceding set and ask children what the difference is between them. Continue in this way until all the sets have been described. Here children begin with the empty set and point to the sets in order.

Ask the children to study the sets on page 13. Ask if any two sets can be matched exactly one-to-one. Give them sufficient time to consider this. They should conclude that no one set has the same number of members as any other. They all have a different number of members.

Tell the children that we can put the sets in order beginning with the set which has fewest members. Ask them to point to that set.

Ask them to find the set which comes next in order and to point to it. Ask them to tell how it differs from the one preceding. It has one more member. Ask them to point to the next set in the order. "What set is it?" The answer may be the set of maize or even the set of three. They should be guided to tell that it has one more member than does the set of a single bean. Go on until all sets are included in the ordering. To emphasize the ordering you can duplicate the pictured sets from page 13 on the blackboard in the proper order.

Experience for Page 14. Each of the rows on page 14 represents a separate exercise. In each there are pictures of several sets. The sets may be ordered by the children beginning with the set with fewest members.

Call attention to the first row on page 14. Ask the children to tell what sets are pictured there. They should identify the sets as sets consisting of a hut, the empty set, a set of pencils, and a set of pots. Explain that we wish to point out the order in which the sets could go. Tell the children to point to the set in the first row which has the fewest members. Ask them to tell what set it is. The answer should be, "The empty set." Tell them to point to the set which comes next in order, the set which has one more member. Ask which set it is. The answer is the set consisting of a hut. Then tell them to find and point to the set which comes next (has one more member). This is the set of pencils. Finally, ask them to point to the set with still one more member and tell what it is. It is the set of pots.

For the second, third, and last rows the procedure is the same.

The children can if necessary determine which set has one more member than the one preceding it by trying to match the sets one-to-one. Remind the children who need help to determine order by matching sets. Encourage those who need it to use counters to match the sets.

Experience for page 15. Each of the rows on page 15 represents a separate exercise. In some of the rows the sets are in natural order, and in some they are not. The children are asked to identify the rows which have the sets in the "one-more" order beginning with the set with fewest members.

Have the children look at the row which shows several sets of boats. Ask them to look at all the sets in the row and tell whether they are in the "one-more" order beginning with the set with fewest members. Make sure that the children read from left to right.

Hold the page up, point to the first set of boats, then to the next, then to the next, etc. Say, for example, "In this row all sets are boats. Here is the first set, here is the next, here is the next, and here is the next. Are these sets in order? Is the first set the set with fewest members? Does the next set have one more boat?" etc. After you have gone over each set of boats in the row so that the children are clear that we move from left to right, least to greatest, ask them, "Are all the sets in the row in order?" Then do the same for the next row.

Examine the next row of sets, the sets of stick men in a similar way. Have the children answer the question "Are the sets of stick men in order?"

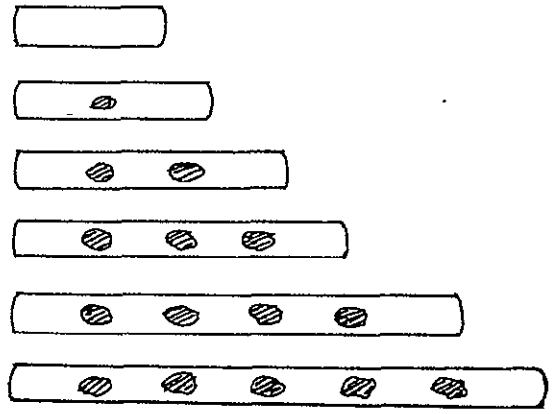
After discussing two rows have the children decide independently whether the sets in the other rows are in a natural order or not. You should go over each set in the other rows as suggested above only if the children continue to require help.

Notice that the rows which include the ordered sets are the row with sets of boats, the row with sets of fish, and the row with sets of pots. The sets of stick figures and the sets of balls are not in natural order.

Stage 2: Number of objects in a set.

Experience in assigning number names zero to five inclusive. As soon as the children are able to order sets in the "one-more" pattern, names can be given to the number of members in sets. The English names, zero, one, two, three, four, five will be introduced in relation to six sets which are ordered. First have the children arrange sets of objects in order. The empty set should be included.

Show the order on the blackboard:



Ask the children if any of the sets are the same. Ask also if any two sets can be matched one-to-one. The answer is "no" because each set in order has one more member than the previous one. Each set has a different number of members from any other.

Give the names for the numbers in each set pictured. For example, point to the empty set and say, "The number of members in this set is zero." For the next set, say "The number of members in this set is one," etc.

Show sets of many different things and have the children tell the number of members. This can be done with physical objects and also by picturing sets on the blackboard. Display and picture several sets of two things (two sticks, two books, two chairs). Explain that "two" names the number of things in each. "Two is the number of things in this set of books, two is the number of chairs in this set and that set," etc. Follow this same procedure for one, three, four and five. Represent the empty set on the board by a ring with no members pictured within and explain that the number of members in the set is zero. Give several descriptions of the empty set such as,"The number of members in the set of cats on this table is zero. The number of members in the set of cows which fly is zero. This is the empty set, and the number of its members is zero."

After many examples ask the children to tell the number of members in sets of different things. Give them considerable practice

in recognizing the number of things in sets and in saying the appropriate number names.

Experience in using the number names zero-five. Have ready a set of six cards about 5" by 7" showing sets of objects (fish) with 0-5 members. Begin by holding up the card with no members and asking the number of fish pictured on the card. The children should say, "zero." Then hold up the card having a set of one fish and ask a child the number of members in the set. The answer is "one." Continue with the other cards with sets of "two", "three", "four", "five" members. You should hold the card up long enough for the children to have a good look at it but not long enough for them to be able to count the objects.

If the children do not recognize the number of members in the sets, revise the previous experience.

After holding up the cards in order once or twice and having the children name the number in the sets, mix the cards so that they are not in order. For example, show the cards in the order "two", "zero", "five", etc. As the children become quicker at recognizing the number in the sets hold the cards up for a shorter time.

If it is possible, prepare other sets of cards; one set having pictures of books, fish, one of trees, one of discs etc. Mix the cards up and have the children identify the number in the sets.

Lay the cards with sets of objects pictured on them face downwards on the table. Ask a child to choose a card, look at it, hold it up for the class to see. He should then tell the class the number of members it has.

Sets of small cards about 2" by 3" similar to those described may now be used by children in groups of two or three. One of the children acts as leader and holds up a card for the others to see and tell the number of objects. Several sets of small cards may be mixed up and set face downward on the table. Each child in turn lifts a card, turns it over, lays it down, and tells the number of members in the set pictured on it.

Experience in making sets of 0-5 members. Have available several strips of cloth about 3" by 20" on which have been glued or painted colored discs as described in the figure under the Materials section

of this unit.

Ask a child to show you how to fold the strip of cloth to show a set of three. Ask the other children if there is a single correct way to fold the strip to show a set of three. Have other children form a set of three discs.

Follow a similar plan for forming sets of five discs, then sets of four discs, no discs, two discs, three discs, and one disc.

Have each child prepare a paper strip on which he colours five discs. Ask the children to fold their strips to form sets of zero to five discs. Children may now work in pairs with one telling the number of a set to form and the second folding his strip to show the set.

Change the activity by having one of the children fold the paper to form a set and the other tell how many discs are in the set.

Experiences in reproducing sets. Place a collection of objects on a table which can be easily seen and handled by the children. The objects may be books, sticks, empty match boxes, cigarette boxes, or any objects available. Have a collection of about ten bottle caps or other counters on each child's desk.

(Note: This is planned as a lesson in which the children select sets of a given number (without counting) and do something with the set.)

Give directions which require a child to select a set of things and do something with it. Suggestions follow. Make the activity move fast so that there is no opportunity for counting even if the children have that ability.

Give the following type of directions for forming sets from material on the table. Have the class decide if the directions are carried out properly:

Move a set of three books from the table to the floor.

Make a pile of three match boxes; make a pile of two match boxes; put the set of three match boxes and the set of two match boxes into one set.

Hold a set of three sticks in one hand; hold a set of two sticks in the other hand; put a set of five sticks together on the table.

Put a set of three match boxes on a set of one book.

Put a set of one stick on a set of two books. Put a set of no books on one table. (This illustrates the empty set and the response of the child is to put nothing at all on the table.)

Continue until the children have had an opportunity to observe the formation of sets of from 0-5 members and to form the sets.

See that each child has some bottle caps on his desk. Give the following type of directions to the children for forming sets from the bottle caps: (Each child should follow the directions as you observe and guide. Often question: "Is this a set of five?" or "Is this a set of one bottle cap?", etc.)

Put two fingers on your head.

Hold up two hands.

Put four bottle caps on the floor.

Pick up three bottle caps from the floor; pick up one bottle cap from the floor.

Put four bottle caps in your right hand; put one bottle cap in your left hand.

Hold five bottle caps in both your hands.

Put a set of three bottle caps in your right hand; put a set of no bottle caps in your left hand; and put a set of three bottle caps in each hand.

Put a set of four bottle caps on the floor; put another set of four bottle caps on the floor; put all the other bottle caps on the floor.

Experience for Page 17 and Pages 4 and 5. Give the children at least 15 counters each. Have them look at the sets of leaves on page 17. Ask if there is the same number of leaves in each set. Ask how many leaves are in the set at the top of the page. Tell the children to put one counter in the empty space beside the leaf. Have them look at the next set. Ask how many leaves are in this set. The answers may be, "None" or "Zero." Ask how many counters should be put in the space beside this set. The answer is again, "None" or "Zero."

Continue with each set until all numbers have been named, first asking how many leaves are in the set and then how many counters should be put in the appropriate space.

Have the children look at the sets pictured on page 4. Ask them to tell the number of members of each set. For example, ask, "How

many drums are in this set?" "How many pineapples are in this set?" "I see a set of fish. How many members has it?", etc.

The same kinds of activities are appropriate for page 5. Ask, for example, "How many pieces of fruit are in the last set?" and "How many pineapples are there in the set of fruit?"

Experience for Pages 7-16. These pages may also be used in the recognition of the number of objects in a set as needed. Ask the children to look at each page in turn. Then ask them how many members are in a particular set. Include all sets which are pictured on any page used. Each answer should be a number name.

Questions appropriate to page 7, for example, are, "How many chickens are in the set of chickens? " "How many members are in the set of chairs? " "Can you find a set of three members? " etc.

The empty set is represented on page 7 by the rectangle which includes no members. Ask, "How many things are in this set?" (Point to the empty rectangle.) "How many members are in the empty set?" The answer to these questions should be "zero." Continue asking questions of this kind for pages 8-16 as needed.

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Cambridge, Massachusetts

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Boston, Massachusetts

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Abbiw-Jackson, D., Kwame Nkrumah University of Science and Technology, Kumasi, Ghana
Addy, Miss L. L., Achimota Primary School, Achimota, Accra, Ghana
Armar, T. Q., West Africa Secondary School, Accra Ghana
Awokoya, L. T. M. A., Technical Institute, Yaba, Nigeria
Bender, M. L., Educational Research Council of Greater Cleveland, Cleveland, Ohio
Blackwell, D., University of California, Berkeley, California
Bradley, H. P., Ministry of Education, Nairobi, Kenya
Budd, C. J., Kyambogo Teacher Training College, Kampala, Uganda
Collard, Mrs. K., University College, Ibadan, Nigeria
Cronje, J. A., West African Examinations Council, Accra, Ghana
Crowe, D. W., University of Wisconsin, Madison, Wisconsin
Crowhurst, Miss M., Ministry of Education, Nairobi, Kenya
Eilenberg, S., Columbia University, New York, New York
Ellis, W., Oberlin College, Oberlin, Ohio
Finney, R. L., Massachusetts Institute of Technology, Cambridge, Mass.
Garstens, Mrs. H., University of Maryland, College Park, Maryland
Hamilton, E. W., State College of Iowa, Cedar Falls, Iowa
Harbottle, Mrs. M. C., Institute of Education, Makerere College, Kampala, Uganda
Hardgrove, Mrs. C. E., Northern Illinois University, DeKalb, Illinois
Harding, F. B. D., Freetown Training College for Teachers, Freetown, Sierra Leone
Hill, Miss S. A., Institute for Mathematical Studies in Social Sciences, Stanford University
Holland, Miss J. M. Ministry of Education, Nairobi, Kenya
Jackson, J. S., University of Ghana, Legon, Accra, Ghana
Johnson, P. B., University of California, Los Angeles, California
Jonah, J. E., Prince of Wales School, Freetown, Sierra Leone
Khasenye, J. R., Makerere College School, Kampala, Uganda
Khonje, S. G., Blantyre Secondary School, Blantyre, Nyasaland
Kimball, R., Kampala Technical Institute, Kampala, Uganda
Lewis, Miss E., University of Oklahoma, Norman, Oklahoma
Max-Peters, T. B., Fourah Bay College, Freetown, Sierra Leone
McDermott, Miss M., Mt. Diablo Unified School District, Concord, California
McLane, Miss L., Makerere College School, Kampala, Uganda

Modu, C. C., West African Examinations Council, Yaba, Nigeria
Myers, S. S., Educational Testing Service, Princeton, New Jersey
Ngalombi, E. M. K., Kampala Technical Institute, Kampala, Uganda
Odunukwe, C., Federal Emergency Science School, Lagos, Nigeria
Offurum, R., Kings College, Lagos, Nigeria
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Sambolah, E. S., College of West Africa, Monrovia, Liberia
Setidisho, N. O. H., Ministry of Education, Ndola, Northern Rhodesia
Shanks, M. E., Purdue University, Lafayette, Indiana
Smith, E. M. R., Fourah Bay College, Freetown, Sierra Leone
Sterling, N. J., Jr., Madison Project, Syracuse, New York
Swain, H., New Trier Township High School, Winnetka, Illinois
Taylor-Pearce, J. M., Secondary Technical School, Ministry of Education, Freetown, Sierra Leone
Ukeje, O., University of Nigeria, Nsukka, Nigeria
van Uden, Father C. S., H. Ghost Teacher Training College, Morogoro, Tanganyika
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Deakin, J., Registrar, West African Examinations Council, Accra, Ghana
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