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by

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Imperfect Capital Markets and Life-Cycle Consumption:
A Model and Empirical Test for Urban Colombia

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John K. Hill

Abstract

In the conventional life-cycle model, households are assumed to base their current consumption-savings decisions upon lifetime projections for labor incomes and family size. Further, capital markets are assumed to be perfect. It then follows that current consumption will depend upon an arithmetic average of the lifetime income distribution, and that inelastic saving responses to changes in incomes or interest rates necessarily reflect upon the tastes of the consuming unit.

This paper develops a long-range consumption planning model which recognizes a disparity between borrowing and lending rates of interest. Optimal consumption is shown to depend upon current income and higher-order moments of a household's income distribution, as well as an arithmetic average of that distribution. It is also demonstrated that inelastic consumer responses to external shocks which are generally associated with an acceleration in trading activity may be market-induced.

The consumption function which follows from the above planning problem is then estimated using the 1967-68 CEDE budget survey of the four major Colombian cities. Tests are made to determine the sensitivity in the parameter estimates to assumptions about prevailing capital market opportunities. We conclude by estimating, for several income classes, the consumption responses of Colombian households to a transitory income change, a permanent change in income, and changes in borrowing and lending rates of interest.

Imperfect Capital Markets and Life-Cycle Consumption:
A Model and Empirical Test for Urban Colombia*

1. Introduction

Conventional studies of household consumption behavior assume that either (1) effective planning horizons are extremely short and, consequently, that current income is the sole variable which is pertinent to a household's consumption-savings decision, or (2) that families are free to borrow and lend at a single rate of interest and, hence, that the crucial determinant of their behavior is the capitalized value of their labor and property income streams. Individuals do, however, save a portion of windfall income. Assumption (1) cannot be justified on the grounds of an innate disregard for the future. Moreover, some form of intertemporal trading exists in every economy, and a current income model cannot be deduced from such financial environments. These observations are responsible for the emergence of permanent income theories associated with the works of Modigliani, Brumberg, Ando, and (before them) I. Fisher (henceforth referred to as the FMBA model).[1] But an assumption of

*The author wishes to thank Professors Albert Berry, Ronald Soligo, Gordon Smith, and Wayne Thirsk for their helpful comments during the course of this research. The author also wishes to thank the Centre de Estudios de Desarrollo of the Universidad de los Andes, Bogotá, for access to the data used in this study.

[1] I. Fisher, Theory of Interest, 1930, F. Modigliani and R. Brumberg, "Utility Analysis and the Consumption Function: An Interpretation of Cross Section Data," in K. Kurihara (ed.), Post-Keynesian Economics (New Brunswick. Rutgers University Press, 1954); F. Modigliani and A. Ando, "The Life-Cycle Hypothesis of Saving Aggregate Implications and Tests," American Economic Review (1963).

perfect capital markets is patently unrealistic even for developed countries.

In this paper we formulate an intertemporal consumption planning problem whose financial constraints permit an accumulation of debt and assets, but do not impose equality between borrowing and lending rates of interest. An optimal consumption function is deduced from the planning problem, and estimates of several of its parameters are made using a 1967-68 budget survey of Colombian households. We conclude by simulating optimal consumption responses to a variety of policy variables.

Section 2 summarizes the fundamental propositions and results of empirical testing pertaining to the FMBA model, and then presents the implications of a relaxation in the assumption of perfect capital markets. In Section 3, we develop the consumption function which will serve as a theoretical basis for all subsequent analysis. Section 4 is then devoted to an estimation of this function with Colombian household data. Section 5 contains estimates of consumption responses to several policy variables. The paper is summarized and conclusions are drawn in Section 6.

2. Capital Market Opportunities and Consumption Planning

Background: Conventional Analysis a la Fisher, Modigliani, Brumberg, Ando, et al.

The FMBA model is based upon an intertemporal planning problem wherein the individual household maximizes its well-being by selecting an optimum lifetime distribution of consumption expenditures, subject to the financial constraints imposed by its non-property income endowment and the capital market opportunities which are available. We make the

following assumptions.[1]

(a) Consumption in each time period consists of a single, non-durable commodity,

(b) The labor income to be earned in future periods is known with certainty and is exogenous to the household's consumption planning problem;

(c) Households make no bequests and anticipate receiving no inheritances;

(d) Capital markets are perfect. The potential for intertemporal trading is summarized by the existence of one-period loans and investments which bear the same rate of interest. There are no quantitative restrictions on the holding of debt. The household can then select any distribution of consumption which is consistent with nonnegative net worth at the end of its existence,

(e) The current rate of interest is expected to prevail in all future time periods.

The sole motive for saving or incurring debt is to synchronize labor income receipts with a more preferred consumption path. Consumption is nondurable and capital markets are perfect, so that the FMBA model fails to recognize the accumulation of wealth induced by imperfect rental markets and downpayment requirements for durable purchases. There is no

[1]Most of these assumptions can be relaxed without a significant amount of additional complexity. See, for example, W. Somermeyer and R. Bannik, A Consumption-Savings Model and Its Applications (Amsterdam: North Holland, 1972) However, the model is quite sensitive to the assumption of perfect capital markets as we demonstrate in subsequent sections.

uncertainty and hence no precautionary motive for holding assets. But despite its rather limited scope, the model remains a cornerstone in the theory of intertemporal consumer behavior.

With assumptions (a)-(e), the household's planning problem can be formulated as:

$$\text{Maximize } U(c_1,\ldots,c_T) \tag{2.1}$$

$$\text{subject to } PV = \Sigma_{i=1}^{T}(1+r)^{1-i}y_i^L + (1+r)w_0 \geq \Sigma_{i=1}^{T}(1+r)^{1-i}c_i,$$

where c_i denotes consumption in time period i, T the length of the planning period (as measured by the number of remaining years in the life of the household planner), r the rate of interest on saving and debt, y_i^L the labor income expected in time period i, and w_0 the household's initial nonhuman wealth. The household selects a stream of consumption to maximize utility with the constraint that the present value of its consumption not exceed its human and nonhuman wealth.

The major theoretical properties of the model are well-known and are summarized below.

1. Consumption is functionally related to normal income (y_N) rather than current income.[1] An arithmetic average of current and future resources defines the household's constraint set and hence determines its

[1]Normal income is defined as that level of consumption which can be maintained over time, i.e.,

$$y_N = \left[\Sigma_{i=1}^{T}(1+r)^{1-i}\right]^{-1}PV.$$

optimal consumption path. Consumption is invariant to the time distribution of income receipts.

2. If the rate of interest (r) equals the household's rate of impatience evaluated for a constant intertemporal consumption plan (δ), consumption is equalized over time at the level y_N. The optimal saving rate is $(y_1 - y_N)/y_1$, so that, with age-related variations in labor income, saving rates exhibit considerable variation over the life cycle.

3 The marginal propensity to consume out of transitory income is roughly 1/T A large majority of windfall receipts (losses) is saved (dissaved) The propensity to consume transitory income varies directly with the age of the household planner

4 The responsiveness in consumption to a change in interest rates reflects the degree to which households view present and future goods as substitutes A sufficient condition for a positive savings response to an increase in the rate of interest is that the elasticity of substitution (ν) exceed or equal unity Conversely, a zero or negative interest elasticity of saving implies that ν is less than one.

The empirical evidence on life-cycle hypothesis is mixed for developed countries. The model provides a reconciliation of the current income studies from cross section and short- and long-term aggregate data. It also receives support from several empirical studies which indicate a predictive superiority for models embodying long-term income concepts.[1]

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[1]For example, Craig, "Predictive Accuracy of Quarterly and Annual Aggregative Savings Functions," Journal of the American Statistical Association (September, 1970) concludes that "the overall good performance of the Normal Income Model, both with quarterly and annual data, supports the hypothesis that saving, or alternatively consumption, is functionally related to normal rather than measured income" (p. 1137), and that "present income and lagged consumption are the two most important variables to include in savings functions" (p. 1144) Craig's results support the life-

However, the model also implies that a household's marginal consumption response to a windfall income receipt is roughly 1/T, and hence is extremely low for the great majority of the population. While virtually all studies suggest lower propensities to consume income identified as transitory, there is little evidence which corroborates a strict version of this hypothesis.[1] The MPC out of wealth or windfall income should also increase with the age of the household planner. In one of the more supportive tests of this proposition, Landsberger,[2] using data from two Israeli household surveys, can corroborate this tendency for only four out of the eight cases considered.

While the testing of life-cycle models has produced mixed results for developed countries, the evidence from LDCs can be construed as indicating an irrelevance of long-term income variables. There appears to be no pronounced variation in saving rates over the life cycle.[3] Consumption

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cycle model only to the extent that two assumptions are satisfied: (1) that stable aggregate savings relations can be deduced from a life-cycle framework; and (2) that normal income, which is generated by expected future incomes at the micro level, can be measured at the aggregate level as a weighted average of past incomes.

[1] Laumas, "A Test of the Permanent Income Hypothesis," Journal of Political Economy (September-October, 1969), estimates a propensity for nondurable consumption out of transitory income of .368, implying a national planning horizon as short as 2.7 years. These relatively large MPCs are present in cross-section data as well. Cf., R. Bird and R Bodkin, "The National Service Life Insurance Dividends of 1950 and Consumption: A Further Test of the 'Strict' Permanent-Income Hypothesis," Journal of Political Economy (October, 1965).

[2] M. Landsberger, "The Life Cycle Hypothesis: A Reinterpretation and Empirical Test," American Economic Review (March, 1970).

[3] Kelley and Williamson, "Household Saving Behavior in Developing Economies: The Indonesian Case," Economic Development and Cultural Change (April, 1968), find that "the variation in average age-specific household savings predicted by the [FMBA] model is far greater than the actual mean savings level (p. 396)," and that "the predictions for the 20-29 age group are especially poor (p. 397)."

out of windfall income is too high to be consistent with a strict version of the theory. But perhaps the most striking characteristic of the empirical results from LDCs is that there is not a clear-cut superiority of normal or permanent income variables over current income in explaining household consumption.[1]

The empirical anomolies to life-cycle savings behavior have been explained by appealing to a number of realities unrecognized by the conventional FMBA model uncertainty in both real incomes and rates of return, imperfect funds markets, imperfect information, and consumer irrationality. This paper examines the sensitivity of life-cycle properties and empirical tests to a recognition of imperfect capital markets. The next sections summarize the implications of such imperfections for the theory and analysis of household consumption planning.

The Importance of the "Perfect Capital Markets" Assumption

A perfect financial market is one wherein an individual can exchange goods across time at a constant rate.[2] Imperfections in funds markets

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[1] Howe's ("Estimation of the Linear and Quadratic Expenditure Systems: A Cross-Section Case for Colombia," unpublished Ph.D. dissertation, University of Pennsylvania, 1974) estimates of MPCs show only moderate increases when current income is replaced by normal income, and the coefficients are more dispersed according to their higher standard errors. He concludes that "on the basis of frequency of violation of the theoretical structure of the expenditure system, the current income ELES is judged superior to the normal income ELES (p. 209)." Ramanathan, "Estimating the Permanent Income of a Household: An Application to Indian Data," Review of Economics and Statistics (August, 1968), finds little change in both the predictive accuracy and coefficients of his regressions when normal income is measured as the mean income of households in a given education- or occupation-age class rather than a discounted average of these cell means over future periods. The first technique is consistent with short time horizons, so that these results do not establish a particular importance of permanent income variables in household savings decisions.

[2] Variations in rates of interest across households would

consist of those features present in markets for intertemporal trading which truncate a household's opportunity locus between present and future consumption. Consider the simple case of constant, but unequal borrowing and lending rates. Let r^a denote the rate of interest on assets and r^b the rate on debt. The rate at which an individual can transform present into future consumption, or vice versa, depends upon the status of his capital account. If he is holding debt, the rate of transformation equals $(1+r^b)$. If he is holding assets, the rate equals $(1+r^a)$. And when the individual is neither borrowing nor lending, the rate equals $(1+r^b)$ for an increase and $(1+r^a)$ for a decrease in present consumption.

Figure 1 illustrates the optimal time paths of consumption when capital markets are perfect ($\hat{c}$) and when $r^b > \delta > r^a$ (c^*). A relatively high penalty rate discourages the acquisition of debt during the early years. As a consequence, once the debt is retired, the household consumes at a higher level than that selected in a perfect financial environment. A declining consumption plan toward the end of the planner's life would be expected if $\delta > r^a$. Note that the variation in saving rates associated with plan c^* are much less pronounced than those corresponding to $\hat{c}$. The presence of a disparity between r^a and r^b provides one possible explanation for observed invariance in saving rates to age.

When borrowing and lending rates differ, the life cycle becomes segmented into intervals defined by the different phases of an optimal capital account. Consumption within any given segment is responsive to

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constitute a market imperfection as viewed from the aggregate These distortions are very real and they are recognized in the empirical analysis. In this section, however, we focus upon the behavior of an individual household and do not emphasize the effects of fragmented market situations upon aggregate saving.

income variations which occur in neighboring periods, but it is (locally) invariant to income receipts or losses during periods which lie outside the segment. This can be used to explain the high correlation between consumption and current income, and the rather poor performance of long-term income concepts, which typify empirical estimation of consumption and saving relations for LDCs.

To illustrate, consider the consequences of a windfall income receipt for individuals currently borrowing on their future resources. To the extent that income profiles exhibit the kind of structure portrayed in Figure 1, there is a presumption--and frequent empirical verification--that this class contains a relatively large number of young households. Thus, we can associate net debtors with individuals whose current income and, as a result of high borrowing costs, current consumption are below their respective life-time averages. The marginal benefit of having an extra unit of consumption today will then exceed the incremental benefit of increasing future consumption. If a household's most preferred consumption path is relatively stable, it is not unreasonable to expect a high marginal consumption response to windfall income from households whose current income and consumption are below their lifetime norms. A disparity between r^b and r^a accentuates the importance of current income for liquidity-constrained households. Their effective time horizon can be extremely short, and yet this result is consistent with long-range foresight.

The variations in marginal rates of transformation which occur during various phases of the life cycle have implications for the effects on consumption and saving of changes in interest rates. For example,

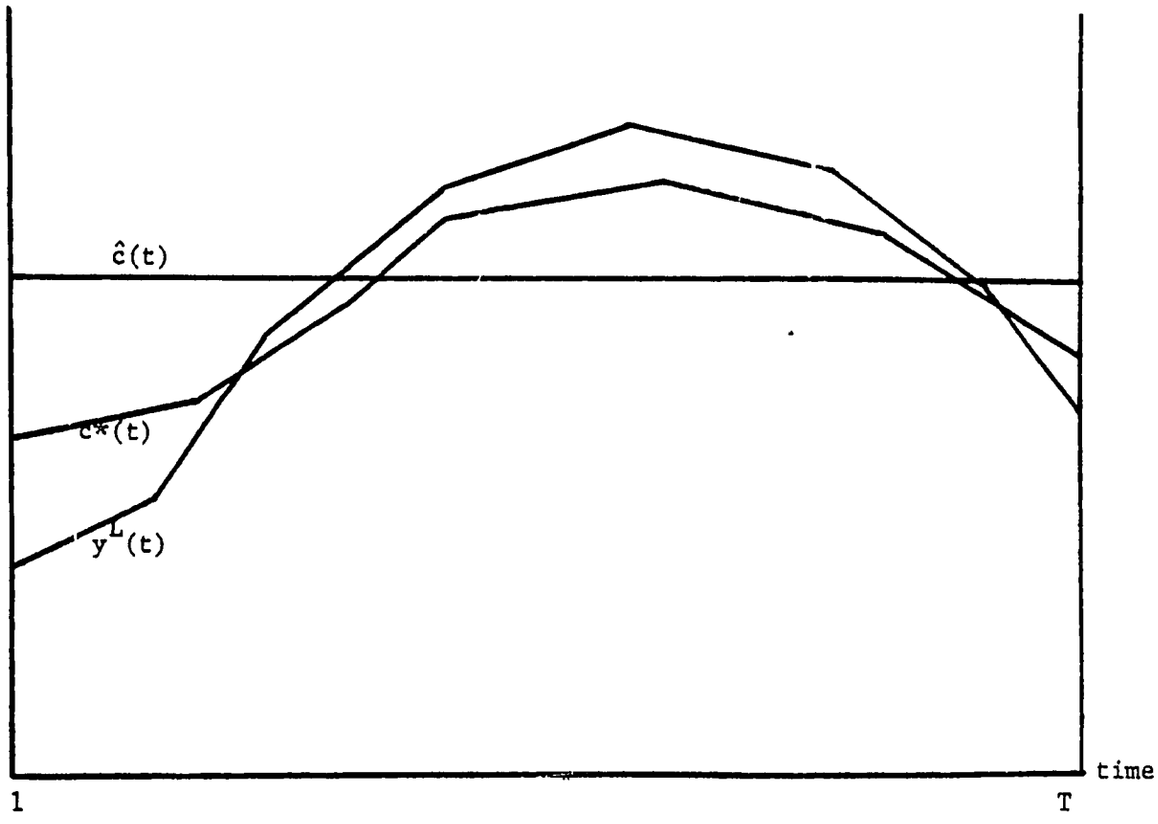

FIGURE 1: Optimal Consumption and
Capital Market Opportunities

in a two-period planning problem, the equilibrium marginal rate of substitution of present for future consumption is a weighted average of $(1+r^a)$ and $(1+r^b)$. The first term receives no weight in households which are borrowing on prospective resources, so that their consumption will be perfectly inelastic with respect to changes in r^a. Similarly, individuals who are saving a portion of their current income will be unresponsive to variations in borrowing rates. Families neither lending nor borrowing may be insensitive to a change in either r^a or r^b. When capital markets are imperfect, inelastic consumption responses to changes in interest rates need not suggest an unwillingness to substitute present and future goods on the part of the population. Indeed, a high elasticity of substitution will ceteris paribus increase the probability of a household finding itself in equilibrium at a kink in its financial constraint set, at which point the household is (locally) insensitive to variations in both r^a and r^b

Consumption Analysis with Market Imperfections

An explicit allowance for differences between borrowing and lending rates, or other forms of capital market imperfections, introduces a significant amount of complexity into long-range consumption modelling. When funds markets are less than perfect, optimal consumption depends upon the distribution of income receipts. There is no single index of permanent income into which the current and prospective resources of a household can be collapsed.

In contrast to the simple linear structure in (2.1), the constraint set exhibits variable marginal rates of transformation. Consider a

single-commodity model where loans and securities are available at constant, but unequal, rates of interest. If the current rates r^a and r^b are regarded as permanent, and if there are no planned bequests, the constraints within which the household must optimize are

$$y_i^L + (1+r^a)A_{i-1} + B_i - (1+r^b)B_{i-1} - A_i - c_i = 0 \qquad i=1,\ldots,T; \tag{2.2}$$

$$(1+r^a)A_T - (1+r^b)B_T = 0; \qquad A_i, B_i \geq 0 \qquad i=1,\ldots,T.$$

where A_i and B_i denote the stocks of assets and debt at the end of time i. The constraint set is a T-dimensional polyhedron, which can be geometrically represented by the intersection of 2^{T-1} half-spaces. Each half-space corresponds to a given sign pattern of the state variables, A_i and B_i. For example, the set of (efficient) consumption vectors consistent with $B_i > 0$ $(i=1,\ldots,J)$ and $A_i > 0$ $(i=J+1,\ldots,T)$ can be derived recursively from (2.2) as[1]

$$w_0^* + \Sigma_{i=1}^{J}(1+r^b)^{1-i}(y_i^L - c_i) + \Sigma_{i=J+1}^{T}(1+r^b)^{1-J}(1+r^a)^{J-i}(y_i^L - c_i) = 0,$$

where $w^*_0 = (1+r^a)A_0 - (1+r^b)B_0$. This equation defines one of a number of (T-1)-dimensional hyper-surfaces which together form the boundary of (2.2).

Equilibrium may occur in the interior of a (T-1)-dimensional facet or along an edge of lower dimension. In the first instance, a locally accurate solution for optimal consumption follows by substituting the

[1] It can be shown that, for any feasible and efficient solution, $A_i \cdot B_i = 0$. An individual expecting to hold both financial assets and debt can maintain his wealth at the end of period i, and increase consumption in periods beyond i, by retiring the debt.

appropriate discount factors into the constraint equation in (2.1), and then optimizing with a particular utility function The solution routine is simplified even further if the equilibrium occurs in a facet of smaller dimension. Each facet of dimension (T-i) corresponds to a subset of the constraint set where (i-1) of the state variables are zero If the utility function is additive, the household's planning problem can be reformulated as a group of separable sub-problems of lower dimensions.[1]

Thus, given the optimal sign values of the state variables, it is a straightforward exercise to express optimal consumption as a function of its exogenous determinants. However, such a function will be valid only for data in a neighborhood of their initial values Moreover, the optimal sign values can only be ascertained through computer solutions to a full-scale planning problem There appear to be two alternative modes of analysis: (1) Simulate household responses to changes in incomes and interest rates through actual computer solutions to consumption planning problems. Microeconomic simulation has been used by Tobin and

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[1]Suppose, for example, that $A_J = B_J = 0$. Then (2.2) can be written as a collection of two independent constraints

$$y_i^L + (1+r^a)A_{i-1} + B_i - (1+r^b)B_{i-1} - A_i - c_i = 0 \qquad i=1,\ldots,J; \qquad (2.3)$$

$$(1+r^a)A_J - (1+r^b)B_J = 0; \qquad A_i, B_i \geq 0 \qquad i=1, \quad ,J.$$

$$y_i^L + (1+r^a)A_{i-1} + B_i - (1+r^b)B_{i-1} - A_i - c_i = 0 \qquad i=J+1,\ldots,T, \qquad (2.4)$$

$$A_J = B_J = 0, \ (1+r^a)A_T - (1+r^b)B_T = 0; \ A_i, B_i \geq 0 \qquad i=J+1,\ldots,T.$$

Optimal current consumption can be obtained by maximizing a utility function with arguments $(c_1,\ldots,c_J)$ subject to (2.3)

Dolde[1] and Dolde[2] in the context of U.S. financial environments. The primary advantage of this approach is that it is possible to incorporate a high degree of realism into the constraint set. The drawback is that the method is often too expensive to be used for statistical testing and estimation.

(2) Limit the scope for funds markets imperfections to the case of constant, but unequal rates of interest on debt and financial assets. Then approximate the truncated constraint set by a smooth transformation frontier which, when substituted into the planning problem, will yield an analytical solution for the optimal consumption path. This is accomplished in Section 3 of the paper. The approach is useful in a statistical context where a large number of solutions are required and where the ranges in the exogenous data are sufficiently broad to preclude the employment of consumption functions of local validity.

3. Consumer Behavior in Imperfect Financial Environments: An Approximation

General Framework

The constraint set

When capital markets are perfect, a household can exchange goods across time at a constant rate. The marginal rate of transformation between present and future consumption is independent of the status of

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[1] J. Tobin and W. Dolde, "Wealth, Liquidity and Consumption," in Federal Reserve Bank of Boston, Consumer Spending and Monetary Polity: The Linkages, 1971.

[2] W. Dolde, "Capital Markets and the Relevant Horizon for Consumption Planning," unpublished Ph.D. dissertation, Yale University, 1973.

the household's capital account. Imperfections in funds markets, on the other hand, truncate the consumption possibilities frontier about the income endowment point. The constraint set exhibits variable marginal rates of transformation, and it is precisely this feature of capital market imperfections which complicates the household planning problem The severity of the additional complications naturally depends upon the nature of the financial environment modelled through the constraints As argued in Section 2, equations (2 2), where loans and securities are assumed to bear different but constant rates of interest, are too complex to be used in a statistical context.

The strategy adopted here is to introduce, as an approximation to (2 2), a constraint equation which is endowed with a relatively simple (yet meaningful) analytic structure. The following criteria define the class of acceptable formulations

(a) The elasticity of transformation between consumption in any two periods is assumed constant and equal. This facilitates a solution for the optimal consumption path and minimizes the number of parameters to be dealt with in the empirical estimation.

(b) The transformation equation is to be time additive in the c_i's This assumption appears crucial if an optimal consumption function is to be deduced.

(c) The constraint equation is to be normalized about the household's income endowment. That is, in addition to the case of perfect markets, the smoothed constraint should provide an exact approximation to the other polar extreme, where capital markets are nonexistent and where the household's choice set is a cube with $(y_1^L+w_0^*)$ as the first

coordinate and $(y_2^L,\ldots,y_T^L)$ the remaining coordinates of its vertex. Such a restriction supports the interpretation of the elasticity of substitution as an index of the severity of imperfections in funds markets.

These three conditions are sufficient to dictate the following constraint equation:

$$\Sigma_{i=1}^{T}(1+r)^{1-i}(c_i/y_i)^{\sigma}c_i = PV, \tag{3.1}$$

where, for notational convenience, we have defined $(y_1^L + w_0^*)$ as y_1 and y_i^L as y_i for each $i = 2,\ldots,T$.

The household's opportunity set depends upon its endowment vector $(y_1,\ldots,y_T)$, an interest parameter r, and an exponential parameter σ. The interest rate is the marginal rate of transformation between consumption in two successive periods when in a neighborhood of the endowment. The parameter σ reflects the degree of difficulty in trading intertemporally. When σ is positive, the term $(c_i/y_i)^{\sigma}$ penalizes consumption in the ith period when c_i exceeds y_i, while it lowers the marginal price when y_i exceeds c_i. This creates a disparity between the rates of interest on loans and saving. When σ equals $+\infty$, the household's consumption possibilities are restricted to its endowment

The above arguments can be algebraically demonstrated by calculating the marginal rate of transformation between c_{i+1} and c_i.

$$-\frac{\partial c_{i+1}}{\partial c_i} = (1+r)(c_i/y_i)^{\sigma}(c_{i+1}/y_{i+1})^{-\sigma} \tag{3.2}$$

When consumption is evaluated at the endowment, the marginal rate of transformation is (1+r). Hence, if r is assumed constant across households, autarchic consumption possibilities are equivalent up to a

scalar multiple From (3.2) it is also clear that σ denotes the percentage change in the marginal rate of transformation associated with a one percent change in the ratio (c_{i+1}/c_i). It is the reciprocal of the elasticity of substitution around the transformation frontier. When σ equals 0, the constraint set is linear with slope $-(1+r)$. As σ grows positive, the constraint becomes truncated about the endowment. All of these properties of (3.1) are summarized and illustrated in Figure 2.[1]

Utility function

The objective function to be maximized subject to (3.1) is

$$U = -\rho^{-1}\Sigma_{i=1}^{T}(1+\delta)^{1-i}N_i^{\rho+1}c_i^{-\rho} \tag{3.3}$$

where N_i denotes the number of equivalent adults expected to reside within the household during period i, and δ and ρ are parameters of the utility function. Household utility is a discounted sum of utility levels pertaining to present and future time periods.[2] Periodic utility,

[1]One caveat is in order. Equation (3.1) is defined for strictly positive endowment vectors. If a household optimizes within the constraints (2.2), it may accumulate sufficient debt to render y_i negative. While a real theoretical possibility, such a circumstance is not likely to arise in empirical applications. We have ignored any constraints on the net worth of the consuming unit, aside from the terminal condition. Yet, it is extremely difficult, especially in developing countries, for an individual to obtain a net debt position in excess of his current income. The remaining y_i's denote nonproperty incomes, and the assumption that they be strictly positive might seem to rule out retirement. However, social security receipts, an important source of retirement income, should also be included in y_i. They, like future wages, are regarded as exogenous and cannot be costlessly liquidated to finance current consumption. Moreover, for many economic classes in less developed countries, there is no retirement decision. The household head merely works until he dies or becomes an appendage to another primary unit

[2]The additivity assumption is commonplace in a dynamic context. It is restrictive only in that consumption in each time period will be a normal good.

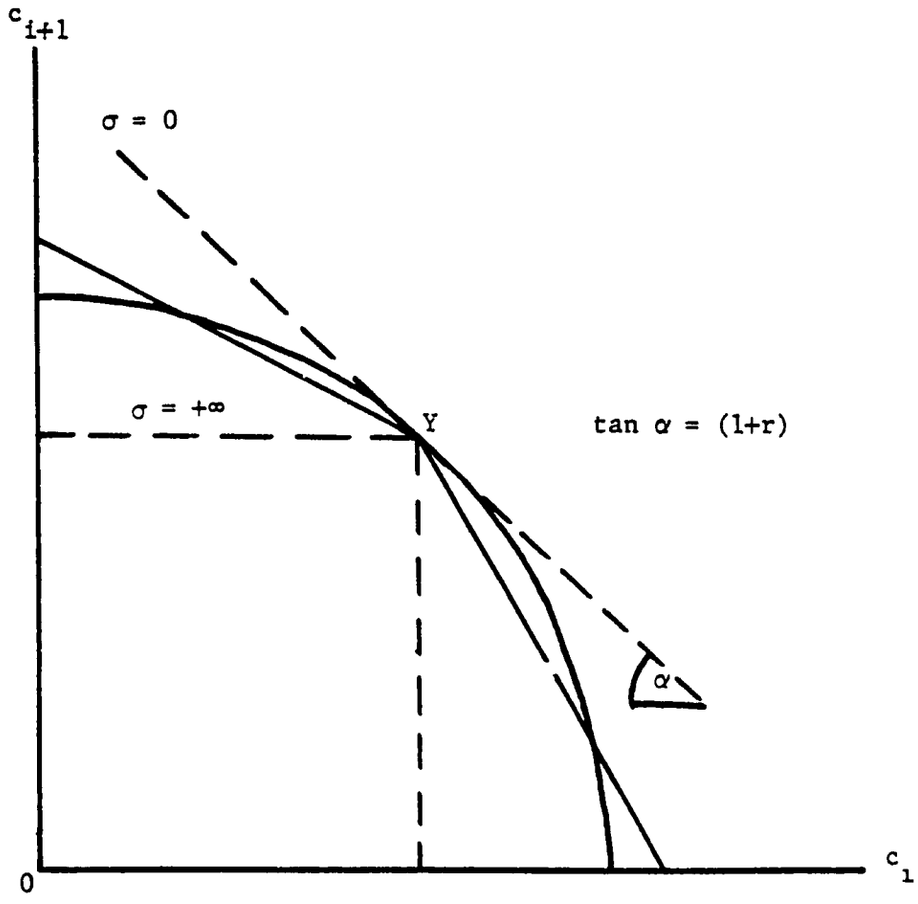

FIGURE 2: Approximation of Capital Market Opportunities

in turn, depends upon total consumption and the size and composition of the family.[1]

The partial elasticity of substitution (ν) between (per capita) consumption in any two time periods is constant and equal to $(1+\rho)^{-1}$. The CES form is consistent with a broad spectrum of indifference curves. The real drawback is that utility is constrained to be homothetic. In a world of perfect capital markets, the household would allocate a fraction of its human and nonhuman wealth to c_1 which is independent of the level of its wealth.[2]

As noted by Landsberger,[3] the time path of optimal consumption is sensitive to the specification of utility when family size and composition are included as arguments. Some allowance must be made for family size, and indeed Landsberger suggests that its conventional omission may explain many nonsupportive tests of the life-cycle model. In (3.3) the number of equivalent adults enters with an exponent of $(\rho+1)$, a procedure which differs from the convention of merely substituting (c_1/N_1) into the utility function. The formulation in (3.3) gives the household an incentive to equalize per capita consumption over time. It is marginal utility which determines the optimal consumption path, and (3.3) yields an expression

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[1]The exponential form of (3.3) has been used extensively in the literature on choice under uncertainty due to its property of constant relative risk aversion. It is also well established in the literature on consumption functions. Cf., D. Heien, "Demographic Effects and the Multiperiod Consumption Function," Journal of Political Economy (January-February, 1972), and W. Weber, "The Effects of Interest Rates on Aggregate Consumption," American Economic Review (September, 1970).

[2]The homotheticity or "proportionality" assumption is also used by M. Friedman, A Theory of the Consumption Function (Princeton: Princeton University Press, 1957), and F. Modigliani and R. Brumberg, op. cit.

[3]M. Landsberger, op. cit.

for marginal utility which is homogeneous of degree zero in family consumption and family size.

Solution and Its Properties

A maximization of (3.3) subject to (3.1) yields the following expression for optimal current consumption.[1]

$$\log c_1 = -(1+\sigma)^{-1}\log\left\{\Sigma_{i=1}^{T}(1+r)^{1-i}\left(\frac{1+\delta}{1+r}\right)^{\gamma(1-i)}\left(\frac{1+\sigma}{1+\sigma\gamma}\right)y_i^{\sigma\left(\frac{\gamma-1}{1+\sigma\gamma}\right)}\left[\frac{N_i}{N_1}\right]^{\left(\frac{1+\sigma}{1+\sigma\gamma}\right)}\right\} \quad (3.4)$$

$$+ \sigma\gamma(1+\sigma\gamma)^{-1}\log y_1 + (1+\sigma)^{-1}\log PV.$$

To facilitate an understanding of this consumption function, we now enumerate and discuss several of its properties.

The extent to which a household participates in external funds markets depends upon its need and its ability to do so. A desire to trade goods across time arises whenever the distribution of family needs (N_i) is poorly synchronized with the distribution of its income receipts (y_i). But the volume of transactions eventually undertaken by the household is also related to the nature of financial markets, here measured by the parameters r and σ. What we wish to emphasize in this section is the relationship between truncations in the household's opportunity set and its optimal consumption path. This is most readily accomplished by ignoring the demographic terms in (3.4). However, before proceeding, we first illustrate the role of the N_i's for the case of perfect financial

[1]See Appendix A for a derivation of (3.4). All appendices cited are available upon request from the Program of Development Studies, Rice University, Houston, Texas.

markets. All of the points now made will hold with only a marginal reinterpretation under more general financial conditions.

The expression for optimal current consumption under perfect markets follows from (3.4) by setting σ to 0.

$$\log c_1 = \log PV - \log\left\{\sum_{i=1}^{T}(1+r)^{1-i}\left(\frac{1+\delta}{1+r}\right)^{\gamma(1-i)}(N_i/N_1)\right\} \tag{3.5}$$

Notice first that what is crucial is the family's current size and composition <u>relative</u> to their future distribution. If, for example, family size is constant over time, optimal consumption is independent of its magnitude. The consumption function is misspecified if current family size is included as an independent variable, but the distribution of N_i is ignored. Secondly, as with the household's monetary resources, it is a discounted sum of the N_i's which is relevant in consumption decisions. However, the discount factors in the demographic terms involve both δ and r. Indeed, if γ equals unity, the household's impatience rate is the appropriate discount factor.

A nonconstant distribution of family size may serve to increase or decrease the need to exchange goods, depending upon the interactions between N_i and y_i Generally speaking, there is a rough correspondence between the two distributions, and this diminishes the need to save or dissave. For example, if both N_i and y_i are expected to grow at the same geometric rate, and if δ equals r, then it follows from (3.5) that the optimal consumption path is the household's resource endowment. Whenever there is a positive correlation between the distributions of N_i and y_i, a failure to recognize family needs which vary over the life cycle

places undue stress on financial markets or household tastes to account for any apparent sluggishness on the part of the household to participate in external markets.

We now examine equation (3.4) to determine the relationships between an optimal consumption path and the parameter σ, an index of financial market imperfections. In comparison to the results of Section 2, where the interest rates r^a and r^b were assumed constant, the properties of (3.4) are qualitatively consistent and differ only by a degree of continuity. Special cases are used only to expedite the communication and they do not fundamentally alter the results.

In a perfect financial environment, normal income y_N is a sufficient statistic for the y_i distribution. An arithmetic average of current and expected consumable resources, with weights $(1+r)^{1-i}\left[\Sigma_{i=1}^{T}(1+r)^{1-i}\right]^{-1}$, defines the financial opportunities of the household and hence determines its optimal consumption path. As argued in Section 2, such is not the case when loans and securities bear different rates of interest. The distribution of y_i, and not merely its first moment, is of potential significance to the planning decision. Equation (3.4) conveys this message. Assuming $\delta=r$, ignoring the demographic variables, and using a first-order Taylor approximation (about $\sigma=0$) of the first logarithmic term in (3.4) we have

$$\log c_1 = \sigma\gamma(1+\sigma\gamma)^{-1}\log y_1 + (1+\sigma)^{-1}\log y_N + \sigma(1-\gamma)(1+\sigma)^{-1}\log y_G, \qquad (3.6)$$

where y_G is the geometric mean of the y_i distribution with weights identical to those in y_N. Even after using the smoothed constraint set, there are a multiplicity of pertinent income concepts. There is no single index of permanent income when capital markets are imperfect.

A household's consumption choice depends inter alia upon its current resources y_1 and first and higher order moments of the y_1 distribution.

The consumption function in (3.4) is easily manageable when $\nu=1$ and $N_1 = N_1$.

$$\log c_1 = (1+\sigma)^{-1}\log\left\{\frac{\Sigma_{i=1}^{T}(1+r)^{1-i}}{\Sigma_{i=1}^{T}(1+\delta)^{1-i}}\right\} + \sigma(1+\sigma)^{-1}\log y_1 + (1+\sigma)^{-1}\log y_N. \qquad (3.7)$$

Aside from a time preference factor, consumption is a geometric average of y_1 and the income concept used in conventional life-cycle studies. The weights in this average depend upon the ease with which goods can be exchanged across time. Figure 3 illustrates optimal consumption paths for various values of σ.

In Section 2 we developed several propositions concerning market-induced rigidities in consumer responses to external shocks which are generally associated with an acceleration in trading activity This was most clear in the case of an equilibrium which occurs at a "kink" in the constraint set and where, consequently, an individual would save a small fraction of windfall income or would exhibit an inelastic response to changes in borrowing and lending rates. These rigidities are attributable to the truncated opportunity set and are also present in the continuous version of the model.

Consider the impact of a transitory income change. From (3.7)

$$\frac{\partial c_1}{\partial y_1} = \sigma(1+\sigma)^{-1}(c_1/y_1) + (1+\sigma)^{-1}(c_1/PV) = \sigma(1+\sigma)^{-1} + (1+\sigma)^{-1}T^{-1} \qquad (3\ 8)$$

If households can borrow and lend at a single interest rate, windfall income is distributed evenly over the remainder of the planning period.

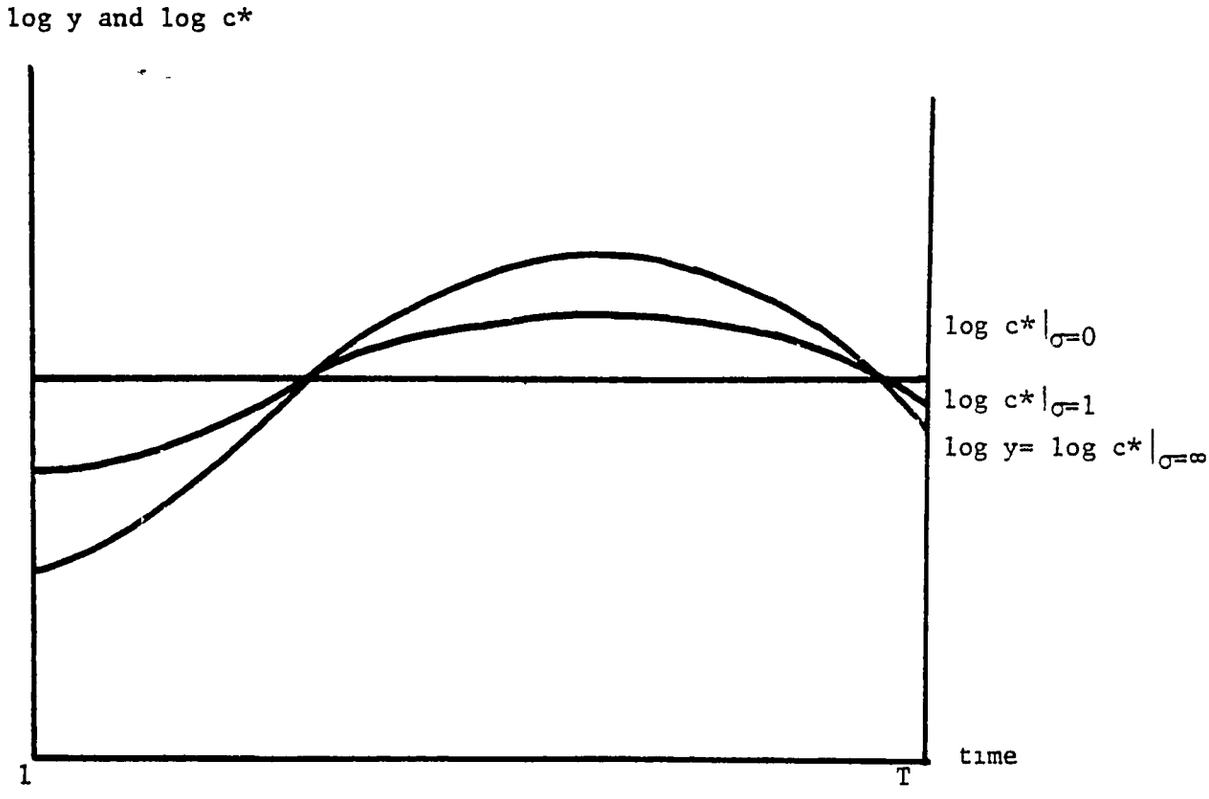

FIGURE 3: Optimal Consumption Paths
for Selected Values of σ:Eq. (3.7)

The presence of impediments to intertemporal trading, however, can lead to a marginal propensity to consume out of transitory income as large as unity.

The nature of the financial environment also influences an individual's responsiveness to changes in interest rates. Let η_{cr} denote the interest elasticity of consumption computed from (3.7) and η^*_{cr} the elasticity which would obtain under perfect markets Then it follows from (3.7) that

$$\eta_{cr} = (1+\sigma)^{-1}\eta^*_{cr}. \tag{3.9}$$

The response of an individual to a change in interest rates varies inversely with the index of imperfections in funds markets. This result echoes the proposition developed in Section 2 that a low interest elasticity of consumption need not reflect upon the willingness with which individuals would indifferently trade across time, but that it may be a consequence of prevailing financial environments.

A final property of (3.4) which should be noted is that the consumption function exhibits an elasticity of unity with respect to permanent income. That is, if the resources expected in all time periods increase by the same proportion dlog y_p, then

$$\frac{\text{dlog } c_1}{\text{dlog } y_p} = 1. \tag{3.10}$$

This property is due to the homotheticity assumed for both the utility function and the constraint equation. It holds irrespective of the value of σ.

4. Econometric Results

Measurement of Variables and Parameter Identification

The consumption function in (3.4) requires several types of information: current household consumption, the length of the planning period T, income and demographic profiles, two constraint parameters r and σ, and the two taste parameters γ and δ In this section we summarize the measurement of exogenous data and raise questions concerning the appropriate designation of parameters in (3.4).[1]

Dependent variable

Current household consumption was measured as all nondurable expenditures plus imputed or paid rent plus educational outlays plus (1/2)(home furnishings and appliances plus vehicle purchases). For a limited number of sensitivity experiments, variations in the definition of c_1 produced relatively small changes in the parameter estimates.

Planning period

The planning horizon was measured, for all households in the CEDE sample, as the difference between 71 and the age of the primary earner. Differences in life expectancy across age and socioeconomic classes were ignored.

Expected labor incomes

The variables $y_i^L (i=2,\ldots,T)$ denote the labor and labor-related receipts expected by the household in (i-1) periods. They were quantified using a procedure analogous to the "cell mean" approach suggested by

[1] A more complete account of this section is contained in Appendix F.

Watts.[1] Households were first grouped according to the educational and occupational characteristics of their primary earners. An average income profile ($\hat{y}_i^L$) was then obtained by regressing, within job groups, household labor income (y_i^L) on a quadratic function of the age of the primary earner (see Appendix D). Income expected in (i-1) periods was measured by

$$y_i^L = (y_1^L/\hat{y}_1^L)\hat{y}_i^L(1+g)^{i-1},$$

where g is the rate of labor-augmenting technological progress. Differences in earnings between households of the same age bracket are attributed to permanent characteristics and are assumed to persist over the life cycle A growth rate of 3 percent in real wages was used for all income groups.

Current resources

The first coordinate of the household's endowment (y_1) consists of its current labor income net of personal taxes and certain transitory elements (y_1^L), transitory nonproperty income (y_1^T), initial net worth ($A_0 - B_0$), and current capital income. The first two components were measured as

y_1^L = salaried and self-employed earnings less personal taxes plus social security receipts less social security payments,

y_1^T = inheritances plus income in kind plus transfer payments (exclusive of social security) plus nonclassified income.

[1] H. Watts, "An Objective Permanent Income Concept for the Household," Cowles Foundation Discussion Paper No. 99 (November, 1960).

The stock-related components proved more difficult to quantify.

Data on the capital accounts of Colombian households is limited to the estimated market value of owned dwellings and remaining mortgage payments. Estimates of net worth were made by treating as perpetuities observed capital income (${}^{+}y_1^K$) and interest and principal payments on outstanding debt, and converting them to their respective stock counterparts through two nominal interest rates RA and RB. Wealth in the form of owner-occupied housing was distributed evenly over the income endowment.[1] Household endowments were measured as

$$y_1 = y_1^L + y_1^T + y_1^R + [(1+RA)(RA)^{-1}({}^{+}y_1^K)] - [(1+RB)(RB)^{-1}({}^{-}y_1^K)]$$

and

$$y_i = y_i^L + y_i^R \qquad i = 2,\ldots,T$$

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[1]The treatment of imputed rent on owner-occupied housing is very much ad hoc, but it has the positive qualities of reducing asymmetries in wealth measurement, of deemphasizing the role of durables in the estimation, and of recognizing imperfections in housing rental markets. First, due to a predominance of imputed rent in observed capital incomes, estimates of the utility parameters are overly sensitive to the interest factor RA when imputed rent is included in ${}^{+}y_1^K$ and wealth in the form of owner-occupied housing is captured solely in y_1. Families with owned dwellings are endowed with large amounts of wealth simply because this portion of their wealth is in an observable form. Secondly, the model developed in Section 3 assumes that consumption in each period consists of a single, nondurable commodity. It is disturbing that the presence of a durable good (such as housing) should significantly influence the testing of a model which is not well-suited to an explanation of durable acquisitions. Equity in housing may be the result of imperfect rental markets for housing services, and it need not signal an expected decline in future income as would be suggested by a life-cycle model. Finally, it can be argued that, because of imperfections in housing rental markets, consumption possibilities are more accurately represented by viewing imputed rent as an exogenous stream of services to be enjoyed in future (as well as present) time periods and not to be liquidated to finance current expenditures. It is often more efficient to borrow at a rate r^b rather than liquidate the dwelling and rent housing services in present and future periods.

where $^{+}y_1^K$ = imputed rent on owned housing (other than the occupied home) less property taxes plus interest and dividends plus capital gains or losses plus other capital income.

$^{-}y_1^K$ = interest payments on outstanding loans (excluding mortgages).[1]

y_1^R = imputed rent on owner-occupied housing.

Demographic profiles

Measurement of the N_1 distribution poses two problems the weighting of family members in computing equivalent adults, and a method of evaluating expected family size and composition. Howe[2] in an application of the linear expenditures system to the CEDE data set, provides a set of weights for each city and for family members of ages 0-7, 8-17, and 18 or over The intercity estimates varied considerably and, consequently, were averaged according to the fraction of the total sample represented by the relevant city The household's current number of equivalent adults, irrespective of city of residence, was computed as

N_1 = (33) x (# members of age 0-7) plus
(59) x (# members of age 8-17) plus
(1.0) x (# members 18 or over).

To measure expected family size and composition, we regressed, within education-occupation groups, N_1 on a quadratic function of the age of the

[1] A household's outstanding home mortgage payments are accounted for in $^{+}y_1^K$ and y^R where imputed rent applies to the family's net equity in housing.

[2] H. Howe, op. cit., p. 290.

the household planner (see Appendix E). Differences between N_1 and the value predicted by the regression were assumed to reflect a equi-proportionate difference within all age brackets. It was felt that the data set partition into education-occupation groups would partially control for phase differences in the family cycle. Equation (3.4) is invariant to a multiplicative shift of the N_1 distribution, so that the predicted profile was used for N_i $(i=1,\ldots,T)$.[1]

Evaluation of constraint parameters

There are two ways of evaluating the constraint parameters. First, r and σ could be estimated simultaneously, together with the taste parameters δ and γ, at values which minimize the sum of squared errors obtained from (3.4) and the observed Colombian consumption decisions. A methodological defense of this procedure could be made by appealing to data deficiencies in observed financial parameters and to the inherent error involved in using (2 1), (2.2), or (3.1) to approximate a more complex set of financial restrictions. Second, r and σ could be determined independently of the CEDE data by using "observed" lending and borrowing rates to construct the constraint locus described by (2.2), and then searching over r and σ to minimize the discrepancy between (2.2) and (3.1). Observations on r^a and r^b are imperfect and multiple, but it is information which could improve the estimates of δ and γ.

[1]Estimates of γ and δ were also made under the assumption that N_i is constant over time. For households with secondary education and white collar occupations, the average changes (in absolute values) in γ and δ were 8% and 19%, respectively, when the demographic variables were removed from the regressions.

The above arguments, of course, are academic if the analytical structure of (3.4) and/or the Colombian data base do not permit an identification of all four parameters. In a nonlinear context, a set of parameters is identified if the problem of minimizing the sum of squared errors by choice of these parameters has a unique global solution. A sufficient condition for identification is then that the residual function be strictly convex in its parameter arguments. Unfortunately, the Hessian of (3.4) is data dependent and too complex to be globally tested for positive definiteness.

To guage the sensitivity in parameter estimates to the procedure used to determine the constraint parameters, a three dimensional search for (σ, γ, δ) was made for households with secondary education and white collar occupations. The parameter r was evaluated at 14, a value which best approximates the constraints (2.2) for a real rate of return on assets equal to 0.0 and a real rate of interest on debt of .30. The most striking results were that the estimates of σ were well above unity for all regressions, in contrast to a best approximating of .14.

There are several explanations for the high estimates of σ. First, we have ignored any uncertainty a household may face in its future labor income, in (real) rates of interest it receives on investments, or in its life expectancy. And yet each form of randomness, depending upon the decision maker's attitude toward risk, can induce household behavior which is consistent with short effective planning horizons. As demonstrated in Section 3, this phenomenon may be reconciled with the structure of our model through severe truncations in a family's consumption possibilities set.

Second, the values for r and σ which minimize the approximation error are valid only if the error is measured over all possible consumption choices, i.e., all non-negative and financially feasible points When the error is measured over a region defined by the intersection of the above constraint and $\{c_1 | c_1 \geq \alpha y_1\}$, the optimal values (r^*, α^*) are, respectively (.14, .34) and (.14, .70) for $\alpha = .5$ and $\alpha = .75$. It can therefore be argued that the high estimates of σ partly reflect the fact that households eventually trade within a neighborhood of their endowments, and that a higher σ provides a better approximation over a subset of the constraint set.

However, it is also suspected that the cross-section data base may not permit a separation of consumption characteristics which are attributable to the tastes of the household from those which are due to its financial environment. There did not appear to be multiple optima in any of the regressions. But the parameter behavior between iterations was very similar to a problem reported by D. Smallwood[1] for a highly nonlinear, vintage model of production. Therefore, r and σ were extraneously determined in the final regressions, leaving only the utility parameters γ and δ to be estimated with the Colombian data.

The best approximating values (r^*, σ^*) depend upon the time distribution of a household's resources. As lifetime resources become more concentrated in current (future) periods, the optimal solutions for r and σ tend toward r^a and 0 (r^b and 0). While it is impractical to solve a multidimensional approximation problem for each sampled y_1 distribution, some

[1] D. Smallwood, "Problems of Indeterminacy with the Fixed Coefficients, Vintage Model," Yale Economic Essays (Fall, 1970).

adjustment must be made in (r^*, σ^*) for the timing of income receipts. To this end, five sets of financial parameters were computed for every income group. A household was assigned a parameter set in accordance with its ratio of current to average future resources.

It can be argued that those subsets of the constraint set which correspond to extreme ratios of current to future consumption are irrelevant for most households, and that the area over which the set difference between (2.2) and (3.1) is measured should be appropriately restricted. All approximation errors were, therefore, computed over a restricted area of the constraint set

Financial data

To compute (r^*, σ^*) and household wealth, the capital market opportunities available to Colombian familes during the 1967-68 period must be summarized by two real rates of interest, r^a and r^b. The interest rates used are reported in Table 1 by (permanent) income class Lending rates were computed by weighting the individual interest rates which applied to financial assets, productive assets, and housing. The weights correspond to the shares in total assets represented by each type of asset. Borrowing rates were computed as a weighted average of the rates on consumption and home mortgage loans. These weights reflect the shares in total consumption of nondurable and home-related consumption.

TABLE 1. Real Rates of Interest*

Income Class	Borrowing Rate	Lending Rate
Upper	195	.124
Middle	370	.042
Lower	.395	.024

*Computations assume an inflation rate of 8% which is uniform across income classes.

Regressions

The consumption function in (3.4) was independently estimated for six groups of Colombian households defined by the educational and occupational characteristics of their primary earners.[1] To facilitate an interpretation of the results, the group characteristics are reproduced in Table 2.[2] The regression results are presented in Table 3 [3] The R-statistics are simple correlation coefficients between observed and predicted consumption. Figures in parentheses are asymptotic standard errors calculated from the inverse of the information matrix.

TABLE 2 Group Characteristics

Group	Income Class	Education	Occupation
1-2	Upper	Secondary, university, or post-graduate	Professionals
3	Middle	Secondary	White collar
4	Middle	Secondary	Blue collar
5	Lower	Primary	White collar
6	Lower	Primary	Blue collar
7	Lower	Primary	Other

[1]This data set partition was motivated by two considerations: (1) Families within groups are likely to be more homogeneous and will presumably exhibit a more stable set of taste parameters; (2) Due to the complex algebraic structure of (3.4), computational considerations favor the use of relatively small numbers of observations per regression.

[2]Households in groups 1 and 2 were pooled to increase sample size. For more detail on the grouping variables, see Appendix C.

[3]All parameter estimates were made with the assistance of two computer programs designed for unconstrained extremum problems: FLETCH and GRADX (on file at the University of British Colombia). Gradients of the likelihood function were approximated through finite differencing.

TABLE 3 Final Regressions

Group	# Observations	Gamma	Delta	R-Statistic
1-2	368	.336 (.171)	-.085 (.113)	.709
3	469	238 (.034)	-.126 (.038)	.847
4	243	.289 (.050)	-.028 (.036)	.836
5	300	.204 (.045)	- 103 (.055)	.844
6	629	.334 (.034)	.046 (.015)	.839
7	256	.173 (.067)	-.036 (.078)	.838

The regressions in Table 3 presuppose a specific functional relation between family size, education, occupation, age, wealth, prevailing capital markets, and household consumption The only parameters which are estimated with the CEDE data set are the utility parameters γ and δ Viewed from this perspective, the consumption function in (3.4) provides a relatively accurate "explanation" of household consumption. All correlation coefficients are greater than .7, and for five out of the six cases, the R-statistics exceed .8.

The elasticity of substitution γ measures the degree to which a household will indifferently substitute between present and future consumption. A low γ indicates that the family will maintain a constant

per capita consumption plan, irrespective of prevailing capital markets and the distribution of its monetary resources. Other things being equal, the lower the elasticity of substitution, the smaller will be a household's consumption response to a change in the rates of interest on assets and/or debt. The point estimates of γ in Table 3 range from .17 to .34 with a mean value of .26. Although none of the estimates are statistically distinguishable at a 90 percent level of confidence, the point estimates of γ do tend to increase with the permanent income class of the sampled units The mean values are .34 for the upper income group (1-2), .26 for the middle income groups (3 and 4), and .24 for the poorest groups (5,6, and 7).

Unfortunately, there is no research on consumption and saving in less developed countries which provides estimates of utility functions comparable to (3.3). Four independent estimates of the elasticity of substitution derived from U.S. data are summarized in Table 4.

TABLE 4: Estimates of γ for U.S. Data

Author	Estimate of γ
Fellner	.67
Friend	.40
Heien	1.26
Weber	.13-.41[a]

[a]Range in point estimates for five interest rate series.

SOURCES: W. Fellner, "Operational Utility: The Theoretical Background and a Measurement," in W. Fellner (ed.), Ten Economic Essays in the Tradition of Irving Fisher, 1967. I. Friend, "Rates of Return on Bonds and Stocks, the Market Price of Risk and the Cost of Capital," Conference on Risk and Return, Vail, Colorado, 1973. D. Heien, op. cit. W. Weber, op. cit.

With the exception of the study by Heien, the above results place γ between .4 and .5 for U.S. households The estimates reported in Table 3 are broadly consistent, although somewhat smaller. The inter-country differences conform to the tendency revealed in the Colombian regressions for γ to increase with the standard of living enjoyed by the sampled units.

The impatience rate δ denotes the excess over one unit of future consumption which a household requires if it is to forego one unit of present consumption and yet remain at the level of utility defined by an initial lifetime distribution of constant per capita consumption. It was the contention of the classical capital theorists (Bohm-Bawerk, Wicksell, Fisher, et al) that δ is positive and that it varies inversely with the real income of the consuming unit. Given these a priori standards as to what constitutes a plausible rate of impatience, the estimates in Table 3 are rather disappointing The estimates of δ are highly dispersed, both between and within groups of households The range is from -.13 to 05, and an average of the absolute values of the asymptotic t-ratios equals 1 71--a relatively small number in view of the large sample size. Only one of the estimates has an upper confidence limit which is negative at a 95 percent level However, the point estimates of δ are negative in five out of the six cases, and their mean value over all regressions equals - 055 There is a tendency for δ to decrease with permanent income. The mean values are -.09 for the upper income group, -.08 for the middle income groups, and -.03 for the poorest groups. However, given the inaccuracy with which each estimate is measured, no firm conclusion can be reached regarding the relationship between δ and permanent income.

Similar difficulties have been encountered in studies attempting to measure U.S. impatience rates. Depending upon which interest rates were assumed to characterize U.S. financial markets, Weber's[1] estimates of δ range from -.03 to .46 Heien's[2] estimate equals -.08 with an asymptotic t ratio of -.47. Precise and plausible estimates of household impatience rates have continued to elude researchers using data from both developed and underdeveloped countries.

The negative estimates of δ suggest that life-cycle motives for saving are statistically imperceptible within the Colombian data base. Early accumulation, as a precautionary fund, to meet downpayment requirements on future home purchases, to acquire materials for self-constructed dwellings, to finance outlays on household durables, or for purposes of social status, can be erroneously interpreted as a negative rate of impatience. However, this is not to say that life-cycle motives are unimportant to Colombian families. Because of imperfections in capital markets, the need to save or borrow for purposes of smoothing per capita consumption may have been largely circumvented through adjustments in work effort and/or family planning.

To gauge the sensitivity in the parameter estimates to the underlying financial data, regressions were also made for groups 4 and 6 using a real lending rate of 9 percent. The tests indicate that a one percentage point decrease in the lending rate leads to a change in the estimates of the elasticity of substitution and impatience rate of -4.5 percent and

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[1] W. Weber, op. cit.

[2] D. Heien, op. cit.

-30 percent, respectively. Estimates of household taste parameters are sensitive to assumptions about prevailing capital markets.

5 Consumption Responses to Policy Variables Own Estimates and Inter-study Comparisons

The primary focus of this research is not upon the estimates of the taste parameters *per se*, but rather upon household responses to changes in policy variables which follow from our modelling of the consumer's economic problem. In this section we provide estimates of the consumption responses of Colombian households to a purely transitory income receipt, a permanent income change, and changes in borrowing and lending rates of interest

Two sets of estimates were made for each experiment The first set was derived from the taste parameters presented in Table 3 and from the constraint parameters used in their estimation. The second set was derived from a more plausible set of taste parameters. The values used for each group are shown in Table 5. Small, but positive rates of impatience were assumed for each group.

TABLE 5· Second Set of Taste Parameters

Group	Gamma	Delta
1-2	.35	.02
3	.25	.03
4	.30	.03
5	.20	.05
6	.30	05
7	.20	.05

All estimates were originally computed for 24 age-income cohorts. The cohorts correspond to the six education-occupation groups used previously and to families with primary earners of ages 21, 30, 40, and 55. The estimates were then weighted and averaged for presentation. The demographic and labor income profiles used are those described in Appendices D and E. The wealth variables assigned to households of ages 21, 30, 40, and 55 are averages over all families in the relevant group and in age brackets 16-25, 26-35, 36-50, and 51-65, respectively.

Income effects

One source of variation in a household's consumption decision is the income endowment vector. In Table 6 we present, by age group, estimates of marginal consumption responses to a transitory income receipt and, by income class, marginal propensities to consume out of permanent income Each MPC was computed through finite differencing of the optimal consumption choices which pertain to the initial and perturbed income endowments. A perturbation factor of 100 was used in all computations.[1]

MPC out of transitory income: In a life-cycle model with perfect capital markets, a household's marginal propensity to consume transitory income is roughly equal to the reciprocal of the number of years remaining in the life of the decision maker. Hence, for a population with a median age of 40 and a life expectancy of 70 years, the aggregate MPC out of windfall income should equal approximately .03. However, in

[1]Because the income changes are non-infinitessimal, the above propensities to consume represent averages of the MPCs which apply to the initial and new consumption points. However, experimentation with the perturbation factor did not significantly alter the propensities.

TABLE 6: Marginal Propensities to Consume

Out of Transitory Income

Age	MPC for Parameter Set I	MPC for Parameter Set II	Age	MPC for Parameter Set I	MPC for Parameter Set II
21	.13	.16	40	.14	.16
30	.13	.15	55	.15	.17

Out of Permanent Income

Income Class	Financial Regime	MPC for Parameter Set I	MPC for Parameter Set II	Income Class	Financial Regime	MPC for Parameter Set I	MPC for Parameter Set II
upper	no reform	.51	.70	upper	reform	.51	.70
middle	no reform	.66	.78	middle	reform	.64	.78
lower	no reform	.76	.84	lower	reform	.74	.84

Section 2 it was argued that larger MPCs could be expected from a population operating in imperfect financial markets. The results in Table 6 are consistent with this remark. The MPCs in the first two rows range from .13 to .17 with a mean value of .14 for the first parameter set and a mean of .16 for the second set. These values are similar to the propensities to consume windfall income reported by Crockett and Friend in a consumption study of the CEDE data. Their estimates range from .11 to .19 with a mean of .14.[1]

A second proposition which follows from the life-cycle model is that, in a world of perfect markets, a household's marginal propensity to consume windfall income should vary directly with the age of the planner. The MPCs reported in Table 6 are age-insensitive. Differences between households of ages 21 and 55 are no greater than .02. This result, however, is consistent with a life-cycle theory tailored to imperfect financial environments. Households with the longest planning horizons often possess insufficient wealth to insulate themselves from randomness in their labor incomes. In the face of high borrowing rates, they are prevented from maintaining consumption given a temporary reduction in their incomes. And households experiencing a windfall gain often find it optimal to allocate a disproportionate share of the gain to present consumption.

The effectiveness of countercyclical fiscal policies depends _inter alia_ upon the distribution of liquidity-constrained and liquidity-unconstrained households within the economy, and hence upon the monetary policy in effect at the time of the fiscal action. When capital markets

[1] J. Crockett and I. Friend, "Consumption and Saving in Economic Development," presented at the International Conference on Income, Consumption and Prices, October, 1973.

are perfect, each peso temporarily injected into the system will lead to an increase in consumption of roughly .03 pesos.[1] However, under moderate assumptions about prevailing market imperfections, our results predict an MPC out of transitory income of .13-.17.[2] Finally, the age distribution of the population appears irrelevant to the outcome of countercyclical fiscal policies. The MPCs varied more significantly across income rather than age classes.[3]

MPC out of permanent income In the absence of capital market reforms, the MPCs computed for a permanent income change and for the first set of taste parameters range from .51 to .76 with a mean value of .64 The MPCs derived from a more plausible set of parameters, with positive rates of impatience for all cohorts, are consistently higher These estimates range from .70 to .84 with a mean of .77

The propensities to consume permanent income reported in Table 6 are very similar to the normal income coefficients obtained by Crockett and Friend.[4] Their estimates, for three definitions of household consumption, range from .66 to .79 with a mean of .73. The propensities to consume normal or permanent income estimated by Howe[5] and

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[1]The above discussion understates the potency of fiscal policies in that the MPCs reported refer to propensities to consume rather than propensities to spend. An inclusion of expenditures on housing and other durables typically increases the MPCs by .2-.3.

[2]More pessimistic assumptions about the severity of capital market imperfections (σ=.5) further increased the MPCs by roughly .1.

[3]The mean differences in the MPCs between lower and upper income groups were .08 for the first set of taste parameters and .06 for the second set.

[4]J. Crockett and I Friend, op. cit.

[5]H. Howe, op. cit.

Musgrove[1] are somewhat larger. Howe's estimates, for the four cities surveyed, range from .78 to .90 with a mean value of .85. The MPC reported by Musgrove equals .88.

A question which has drawn considerable attention in the development literature is whether or not policies directed toward improving the income distribution will have adverse effects on aggregate saving. The results in Table 6 suggest that an increment in aggregate output of one peso allocated to lower income groups rather than upper income groups implies a reduction in aggregate saving of .14- 25 pesos The MPCs computed from the first set of taste parameters are .51, .66, and .76 for households in upper, middle, and lower income classes. The estimates derived from the second parameter set are respectively .70, .78, and .84.[2]

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[1]P. Musgrove, "Determination and Distribution of Permanent Household Income in Urban South America," unpublished Ph.D. dissertation, Massachusetts Institute of Technology, 1974.

[2]Unfortunately, there is no research involving the CEDE data which offers further evidence on the relation between income class and the MPC out of permanent income. There are several studies which delineate household MPCs by age, education, and socioeconomic group. Cf., P. Musgrove et al., Income and Spending of Urban Families in Latin America (ECIEL Consumption Study), Brookings Institution, 1974; H. Howe and P. Musgrove, "The Extended Linear Expenditure System: Estimates from ECIEL Household Budget Data for Bogotá, Caracas, Guayaquil, and Lima," in Lluch, Powell and Williams (eds.), Household Demand and Saving in Economic Development (Brookings Institution, 1976). However, these propensities are all derived from current income models of saving behavior. Current income is composed of both a permanent and a transitory component. Evidence pertaining to the effects of age, education, etc., upon the propensity to consume current income does not necessarily suggest a corresponding relationship between these variables and a household's marginal response to a permanent income change.

Differences in the MPCs across income classes reflect differences in capital market opportunities, in demographic characteristics, in the distributions of expected labor incomes, and in the taste parameters δ and γ. It is of interest to filter out the partial influence of each of these variables Differential savings behavior which results from differences in family planning and/or attitudes toward the future is difficult to eliminate. If, on the other hand, behavioral differences are market-induced, a redistribution of income coupled with capital market reforms will not impinge upon the growth possibilities of the economy In columns three and four of Table 6 we present, for both sets of taste parameters, MPCs computed under the assumption that the capital market opportunities enjoyed by the upper income group are available to all Colombian households The results indicate that the interclass differences in the propensities are generated by differences in tastes and family size rather than capital market opportunities. The spread in the MPCs across income classes is reduced by only 02 for the first parameter set and remains unchanged for the second set.

Interest rate effects

A household's optimal consumption path depends not only upon its income endowment, but also upon the interest rates available on assets and debt. In Table 8 we present, by income class, estimates of interest elasticities relating changes in consumption to a change in the rate of of return on saving and a change in the borrowing rate of interest. The estimates were made by first calculating new values for r and σ which correspond to the perturbed values for r^a and r^b. The interest changes used in all experiments are shown in Table 7. Given new values for the

TABLE 7: Changes in Interest Rates

Income Class	Change in r^a	Change in r^b
upper	.05	-.05
middle	.05	-.15
lower	.05	-.15

TABLE 8: Interest Elasticities

Income Class	ETA(r^a) for Parameters I	ETA(r^a) for Parameters II	ETA(r^b) for Parameters I	ETA(r^b) for Parameters II
upper	-.04	-.14	.04	.17
middle	-.01	-.04	.06	.18
lower	-.01	-.03	.12	.18

constraint parameters, interest elasticities were computed by taking the ratio of the percentage change in optimal household consumption to the percentage change in r^a or r^b.

Changes in lending rates The effects of changes in interest rates are traditionally decomposed into two parts. a substitution effect and an income effect. For an increase in r^a, the substitution effect discourages present consumption, which has become relatively more expensive for households currently holding assets and for young debtors who expect to hold assets in the future The income effect favors present consumption, as real incomes have increased. The elasticities in columns one and two of Table 8 indicate a predominance of the former over the latter. The means across all age-income cohorts equals -.02 for the first set of taste parameters and -.07 for the second set The elasticities vary directly (in absolute value) with the income class of the representative household. The mean estimates over both parameter sets equal - 09, - 03, and - 02, respectively, for upper, middle. and lower income conorts.

The only other evidence on the interest elasticity of Colombian saving or consumption is contained in a study by Hanson.[1] Hanson's estimates are derived from the 1968 rise in the yield on BCH cedula bonds. He obtains an elasticity of financial saving with respect to the rate of return on the aggregate Colombian portfolio of 8.8. After adjusting for the rise in the yields on government bonds which followed in 1968 and 1969, his estimate is still high at 7.1.

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[1] J. Hanson, "Estimacion de ahorro familiar en Colombia" (Bogotá· National Planning Department, 1972).

The estimates in Table 8 are significantly lower than those reported by Hanson. Multiplying each entry by -10 (for a rough conversion from a consumption to a saving elasticity), our results place the interest elasticity of Colombian saving between .2 and .7. Even the upper income groups, which hold the majority of aggregate wealth, exhibit an elasticity of only 1.4 when averaged over the second parameter set. Our estimates are more in line with those obtained by Wright[1] and Weber[2] for U.S. data. Wright finds that a one percent rise in interest rates will stimulate an increase in private saving of .19-.24 of one percent, while Weber reports a negative savings response to a higher interest rate.

Changes in borrowing rates: The largest elasticities in Table 8 are those computed for changes in interest rates on debt The average increase in household consumption associated with a one percent decrease in r^b equals .07 of one percent for the first parameter set and .18 for the second. The higher elasticities in the fourth column are due to the higher rate of impatience used in their computation. The estimates are closely related to the age of the household planner (not shown). A household with a primary earner of age 21 increases its consumption by 16-.28 of one percent in response to a one percent decrease in the borrowing rate. A household with a head 55 years of age, on the other hand, exhibits an interest elasticity of .02-.04. The higher elasticities for young households reflect the greater illiquidity and greater relevance of borrowing opportunities associated with the early stages of a life cycle.

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[1] C. Wright, "Some Evidence on the Interest Elasticity of Consumption," American Economic Review (September, 1967).

[2] W. Weber, op. cit.

6. Summary and Conclusions

The assumptions which are most fundamental to the conventional life-cycle model are (1) that a household bases its current consumption and savings decision upon lifetime projections for labor incomes and family size, i.e., planning horizons are extremely long, and (2) that a household is free to synchronize its income and preferred consumption profiles by borrowing and lending at a single rate of interest, i.e., capital markets are perfect. If both assumptions are valid, then effective planning horizons will also be long. Current consumption will depend upon the human and nonhuman wealth of the consuming unit, and not upon its current income Windfall income will be consumed over the entire lifespan of the household. Inelastic responses to changes in interest rates necessarily suggest a lack of substitutability between present and future consumption

Empirical testing of the model, however, has produced evidence, especially for developing countries, which is consistent with very short effective planning horizons. Current income is often as powerful in predicting consumption expenditures as are long-term income concepts. Marginal propensities to consume windfall income are not particularly sensitive to age, and they are too high to corroborate a strict version of the life-cycle model.

These empirical anomolies do not necessarily invalidate the model's first assumption. The origin of behavior which is often misconstrued as indicating short, innate planning horizons may be the imperfections which pervade many financial markets. Expected family needs and incomes may be

largely irrelevant to households facing high penalty rates on borrowing, quantitative restrictions on debt, and imperfect home rental markets. Current income can take on a significant role of its own, even for families which are very conscious of the future.

To empirically test the usefulness of an intertemporal planning problem as a theoretical basis for household consumption analysis, the conventional life-cycle model must be suitably tailored when used in the context of an imperfect financial environment. In Section 3 we formulated a constrained utility maximization problem whose constraints permit an accumulation of debt or assets, but do not impose equality between borrowing and lending rates of interest. The scope for intertemporal trading was summarized by a transformation frontier which depends upon the household's labor income endowment, an interest rate defining the marginal rate of transformation at the endowment, and a parameter σ designed to index the severity of imperfections in capital markets.

An optimal consumption function was then deduced from the above planning problem. Its qualitative properties are consistent with the propositions outlined in Section 2 concerning consumer behavior in imperfect financial environments. The importance of current income as a determinant of current consumption varies directly (and normal or life-cycle income inversely) with the parameter σ.

The consumption function was estimated using the CEDE budget data collected from the four principal Colombian cities. A number of regressions were made in which the constraint parameter σ was endogenous to the statistical search. An assumption of perfect capital markets consistently and significantly <u>reduced</u> the explanatory power of the consumption function.

In the final regressions, both constraint parameters were extraneously determined at values for which the smoothed constraint set best approximates a constraint set constructed from observed borrowing and lending rates of interest. The sensitivity tests performed indicate that a one percentage point decrease in the lending rate leads to a change in the estimates of the elasticity of substitution and impatience rate of -4.5% and -30%, respectively. Estimates of household taste parameters are sensitive to assumptions about prevailing capital markets.

Separate regressions were made for six groups of households with various educational backgrounds and occupations The mean estimate of the elasticity of substitution γ was 26, a value which is broadly consistent with (although smaller than) estimates which have been made for U.S. households. Although none of our estimates were statistically distinguishable between groups for a 90 percent confidence level, they did tend to increase with the permanent income class of the sampled unit. The mean values were .34 for the upper income group, .26 for the middle income groups, and .24 for the poorest groups.

Estimates of the impatience rate δ were less satisfactory. Only one had an upper confidence limit which was negative at a 95 percent level. However, the point estimates were negative in five out of the six cases, and their mean value was - 055. The negative estimates of δ suggest that life-cycle motives for saving are statistically imperceptible within the Colombian data base. Early accumulation, as a precautionary fund, to meet downpayment requirements on future home purchases, to acquire materials for self-constructed dwellings, to finance outlays on household durables, or for purposes of social status, can be erroneously interpreted as a negative rate of impatience. However, the results do not necessarily

imply that life-cycle motives are unimportant to Colombians. Because of imperfections in capital markets, the need to save or borrow for purposes of smoothing per capita consumption may have been largely circumvented through adjustments in work effort and/or family planning.

In Section 5 estimates were presented on the consumption responses of Colombian households to a purely transitory income change, a permanent income change, and changes in borrowing and lending rates of interest. The principal findings are summarized below.

(1) A relaxation in the assumption of perfect capital markets increases by .15-.25 the MPC out of transitory income derived from a life-cycle model of consumption and saving. The effectiveness of counter-cyclical fiscal policies depends inter alia upon the distribution of liquidity-constrained and liquidity-unconstrained households within the economy, and hence upon the monetary policy in effect at the time of the fiscal action. Our results also indicate that, in the absence of capital market reforms, the MPC out of windfall income is invariant to the age of the household planner.

(2) An average Colombian family exhibits a marginal propensity to consume permanent income of .64-.77. The mean MPCs computed for upper, middle, and lower income classes were .61, .72, and .80, respectively. The results indicate that, even when all households are endowed with the capital market opportunities available to upper income Colombians, a redistribution of income from the rich to the poor will decrease aggregate household saving.

(3) The interest elasticities computed depict a more pessimistic outlook for mobilizing domestic saving than that suggested by Hanson.

Simulations of household responses to changes in lending rates place the interest elasticity of Colombian saving between .2 and .7. These estimates are significantly smaller than Hanson's estimate of 7.1, and they are much more in line with those obtained by Weber and Wright for U.S. data. Upper income groups are most responsive to changes in the lending rate Their elasticity is 1.4 based upon the second set of utility parameters.

(4) The largest responses to changes in interest rates were found for changes in borrowing rates. The average increase in household consumption associated with a one percent change in r^b was 13 of one percent. The highest elasticities were obtained for young families, reflecting the greater illiquidity and greater relevance of borrowing opportunities during the early stages of a life cycle.

7. List of Symbols

A	stock of assets
B	stock of debt
RA	nominal interest rate on assets
RB	nominal interest rate on debt
r^a	real interest rate on assets
r^b	real interest rate on debt
w_0	initial net worth
w_0^*	initial net worth plus interest receipts (payments)
PV	human and non-human wealth
c	household consumption
N	number of equivalent adults
y^L	labor income
y^T	transitory income
y^R	imputed rent on owner-occupied housing
$+y^K$	current capital income
$-y^K$	current interest and principal payments
y_1	current resources
y_N	normal income
y_p	permanent income
T	length of planning horizon
g	rate of labor-augmenting technical progress
γ, δ	utility parameters
r, σ	constraint parameters

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