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THE METHODOLOGY OF DEVELOPMENT PLANNING:  
A CRITICAL SURVEY

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## FOREWORD

This working paper is part of a larger manuscript which we are preparing as the final report by NPA's Development Planning Project to the Agency for International Development. We eventually intend to publish a book from the material comprised in our final report.

For this reason we wish to give the reader some idea of the broader perspective which encompasses this present working paper. The larger study covers our empirical and theoretical work on the open, dualistic economy. In approaching this larger subject we begin by surveying the present state of the art in growth and development studies. This assists the reader in understanding the evolution of our own analytical framework.

Four chapters (of which the present working paper is one) are devoted to surveying four rather distinctive approaches to the study of growth and development. In addition to the planning approach discussed in this paper, we survey the historical approach, the institutional approach, and the theoretical approach in other chapters now being written.

This working paper, however, goes somewhat beyond merely reviewing and evaluating the planning approach. We attempt to make an original contribution by developing techniques both for investigating planning

methodology and for use in the analysis undertaken in later parts of the larger study mentioned above. These techniques are presented in Sections 2 and 3 of the present paper.

We have attempted to write this paper as a unit in order that we may circulate it to interested students of development economics. Our purpose in this advance circulation is to invite critical comments on any of the large number of issues discussed. We shall be grateful for any reactions the reader may wish to express.

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## 1. BACKGROUND, SCOPE, AND METHOD: GENERAL FEATURES

### 1.1 Background of the Planning School

During the years after World War II an important group of practitioners gradually formed among professional economists. This group, known as the planning school, is dedicated to the application of economic knowledge in the formulation of development plans for less developed countries.<sup>1</sup> The emergence of the planning school on the postwar scene can be attributed to two historical events. On the one hand, less developed countries became aware of their economic backwardness and, more importantly, the belief emerged that their economic problems should be attacked at the national level under the leadership of the central government. On the other hand, a revolution in economic methodology occurred. Professional economists in the Western world became increasingly oriented toward econometrics--broadly interpreted to mean an emphasis upon analytical rigor in the formulation of economic theory (as typified by the use of mathematical models) and empiricism (as typified by the liberal use of statistical data). The planning school is the practitioner's answer to the demand for national planning based on the use of this new econometric methodology.

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<sup>1</sup>Led by Hollis Chenery and Jan Tinbergen, other representatives of this school of economists are Richard Eckaus, S. Chakravarty, H. C. Bos, Alan Manne, Michael Bruno, Alan Strout, Paul Clark, Irma Adelman, and Jan Sandee. This list is by no means exhaustive. Works of several of these writers will be cited.

The planning methods developed in the last two decades may be broadly classified as two major types: partial planning and total planning. Partial planning is concerned with allocation criteria; i.e., criteria for the allocation of investment funds among various industries or investment projects.<sup>2</sup> This approach is referred to as partial planning because it does not postulate a framework for understanding the operation of the whole economy. The major characteristic of total planning, by contrast, is precisely its framework of reference which depicts the operation of the whole economy at a selected level of aggregation. In this chapter we are concerned with reviewing the planning school from the viewpoint of its contribution to the methodology for analyzing economic growth. Since growth analysis inevitably involves a perspective appropriate to investigating the operation of the economy as a whole, our survey is limited to methods developed for total planning.

The circumstances of its origin have given the planning school a unique place among the various schools concerned with economic growth.<sup>3</sup> The planners' approach differs from the other approaches to growth in two major respects: the planners' emphasis upon a direct policy output of their work and in the formalism of their methodology. The policy consciousness of

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<sup>2</sup>This acceptance is similar to Albert Waterston's use of the term, "partial planning." Albert Waterston, Development Planning: Lessons of Experience (Baltimore: The Johns Hopkins Press, 1965), pp. 70-74, 188-189.

<sup>3</sup>The other major schools are reviewed in other chapters. They include the historical, institutional, and theoretical approaches to the study of economic growth.

the planning school reflects the avowed desire to practice the art of economics to affect development through direct and explicit advice. This stands in sharp contrast to the other schools in which policy implications, if any, are more indirect or deduced. The planning school's methodological formality is exemplified by its experimentation with the generous use of quantitative analytical techniques--e.g., dynamics methods, linear programming, and simultaneous equations--to manipulate masses of statistical data. Policy recommendations, therefore, are cast in quantitative terms based on observable and measurable facts. The current influence and prestige of the planning school are primarily explained by this quantitative policy orientation which is indeed conspicuous when compared with the other growth approaches surveyed in these review chapters.

For a critical evaluation of the works of the planning school, it is essential for us to have a working knowledge of its methodology. For it is its methodology which reflects the "professionalism" of this school and, to a large extent, defines its scope; i.e., which guides planners in their selection of what is relevant, or irrelevant, to developmental policies. Furthermore, we stress methodology since this represents the planners' longer run contribution to knowledge. Their specific policy conclusions apropos a particular country, however valuable, are of transient interest. Unfortunately, it is not so easy to discuss the planners' methodology abstractly. Their methodology is intrinsically technical and to some, no doubt, difficult. Furthermore, the planning school does not employ a unique technique. In this context, it is quite important to realize that

planning methodology is an art. A wide variety of quantitative techniques are manipulated in a flexible and experimental way to reach the basic objective of formulating policy recommendations in quantitative terms. In their anxiety to emphasize such policy findings, members of this school have been rather negligent in communicating the methodological content of their work to the general profession.<sup>4</sup>

For these reasons we begin by identifying certain typical features of the planners' methodology. In particular, we discuss the outlines of the two most commonly used model structures, an aggregated model and a disaggregated model. To introduce these models we examine their national income accounting structure in Section 1.2. The notion of accounting consistency, an essential criterion for adequate development planning, is introduced in Section 1.3. The more informal aspects of planning methodology are discussed in Section 1.4. Behavioristic assumptions, which play a crucial role in planning models, are introduced in Section 1.5, while the planning school's approach as a whole is summarized in Section 1.6.

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<sup>4</sup>This remark refers specifically to several contributions of Professor Chenery and his followers, whose works we review later in this chapter. In contrast, Professor Tinbergen has concentrated on certain purely methodological aspects of development planning without investigating the problem of applicability of planning models. See, for example, J. Tinbergen and H. C. Bos, Mathematical Models of Economic Growth (New York: McGraw-Hill, 1962). In our review we show that the purely methodological aspects of planning, based on the use of formal mathematical models, are only a part of planning methodology. We concentrate on planning models which have been actually applied to stress a sense of the totality of this approach, including its theoretical foundations, statistical data requirements, and policy content.

## 1.2 Accounting Systems for Aggregated and Disaggregated Planning Models

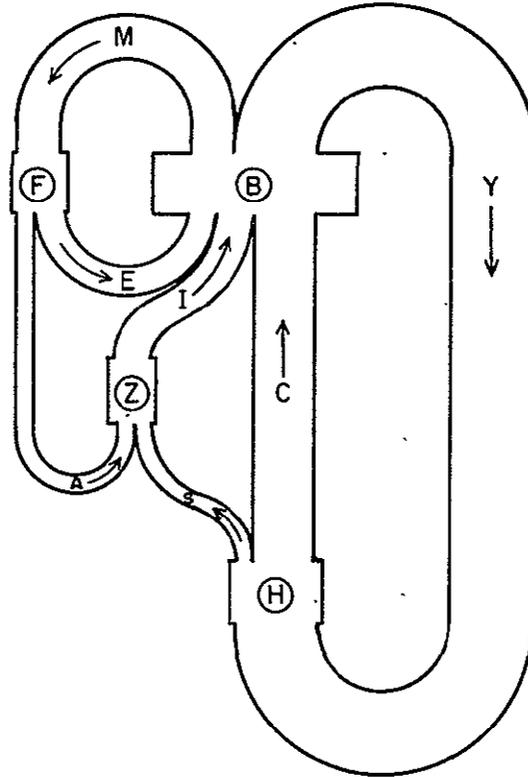
Planners have emphasized two types of models for total planning, the aggregate open model and the disaggregated (n-sector) model, which differ from each other in the degree of aggregation used to portray the whole economy. The outlines of these models can best be described in terms of their national income accounting systems, the foundation of all planning models.

The national income accounting system for the aggregate, open model is represented by the flow chart shown in Diagram 1a or, more succinctly, by the linear graph of Diagram 1b.<sup>5</sup> There are seven variables: Y (national income), E (exports), I (investment), C (consumption), M (imports), S (savings), and A (import surplus or foreign aid) represented by the seven directed flows (or edges) connecting four vertices: (H) (B) (F) (Z). The vertices are sectors of the economy with the following economic interpretations: (B) production sector, (H) household sector, (F) foreign sector, and (Z) finance sector. Conforming to these interpretations, the direction of the flows (edges) indicates the direction of monetary payments between sectors. For any directed edge, the initiating

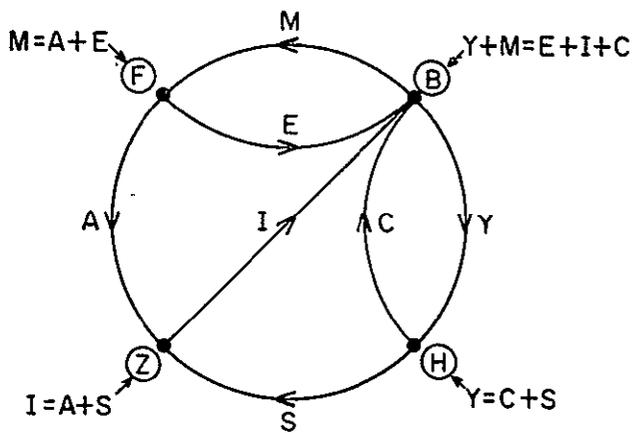
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<sup>5</sup>Diagram 1 will be used repeatedly for reference in the entire text of this chapter. The diagram and the notations should, therefore, be remembered.

# Diagram I: Aggregate National Income Accounting System



a. Flow Diagram



b. Skeleton

(B)	C	E	I
Y	(H)		
M		(F)	
	S	A	(Z)

c. Square Table

vertex stands for the paying sector and the terminal vertex stands for the receiving sector. For example, C, consumption expenditure, is a payment from the household sector (H) to the production sector (B). Here, (H) is the initiating vertex and (B) the terminal vertex. Similarly, every one of the seven planning variables has such an interpretation.

Consistent with the linear graph of Diagram 1 is a system of four income accounting equations--one attached to each vertex. These accounting equations simply state the equality between the total inflows (or total monetary receipts) and total outflows (or total monetary payments) at each vertex (i.e., each sector) with obvious economic interpretations, as follows:

- 1.1.a) At (B) :  $Y + M = I + C + E$  (Total demand for output,  $C + I + E$ , is equal to total payments,  $Y + M$ , from the production sector.)
- b) At (H) :  $Y = C + S$  (Total household income,  $Y$ , equals savings,  $S$ , plus consumption,  $C$ .)
- c) At (F) :  $M = E + A$  (Total imports,  $M$ , are paid for by exports,  $E$ , and foreign aid,  $A$ .)
- d) At (Z) :  $I = A + S$  (Investment,  $I$ , is financed by domestic savings,  $S$ , and foreign aid,  $A$ .)

These four equations represent the four basic resource balances at the aggregate level. They are the total resource balance (1.1.a), the income disposition balance (1.1.b), the foreign trade balance (1.1.c), and the financial balance (1.1.d). Our later analysis demonstrates that the

satisfaction of these four basic resource balances is the primary emphasis in the work of the planning school.

The second model for total planning is the disaggregated (n-sector) model, a direct descendent from input-output methods. According to this tradition, the production sector is disaggregated into a large number (n) of specific commodities or industries. Such a model is pictured in Diagram 2a.<sup>6</sup> Individual industries are represented by the three vertices  $P_1, P_2, P_3$ , in the production sector. At each vertex in this sector, inflows represent monetary demand and outflows represent monetary outpayments for an industry. Following the input-output tradition, there are two types of monetary demand (inflows) for the commodities of each industry in the production sector: demand for intermediate factors of production ( $x_{11}, x_{12}, x_{13}, \dots, x_{33}$ ) which originates within the production sector and final demand (or net output), which originates from without the sector. Final demand includes consumption ( $C_1, C_2, C_3$ ) originating from the household sector (H), investment ( $I_1, I_2, I_3$ ) originating from the finance sector, (Z), and exports ( $E_1, E_2, E_3$ ) originating from the foreign sector (F). For each industry, outpayments consist of payments for intermediate factor costs ( $x_{ij}$  introduced above), payments to the foreign sector (F) for import goods ( $M_1, M_2, M_3$ ), and payments to the household

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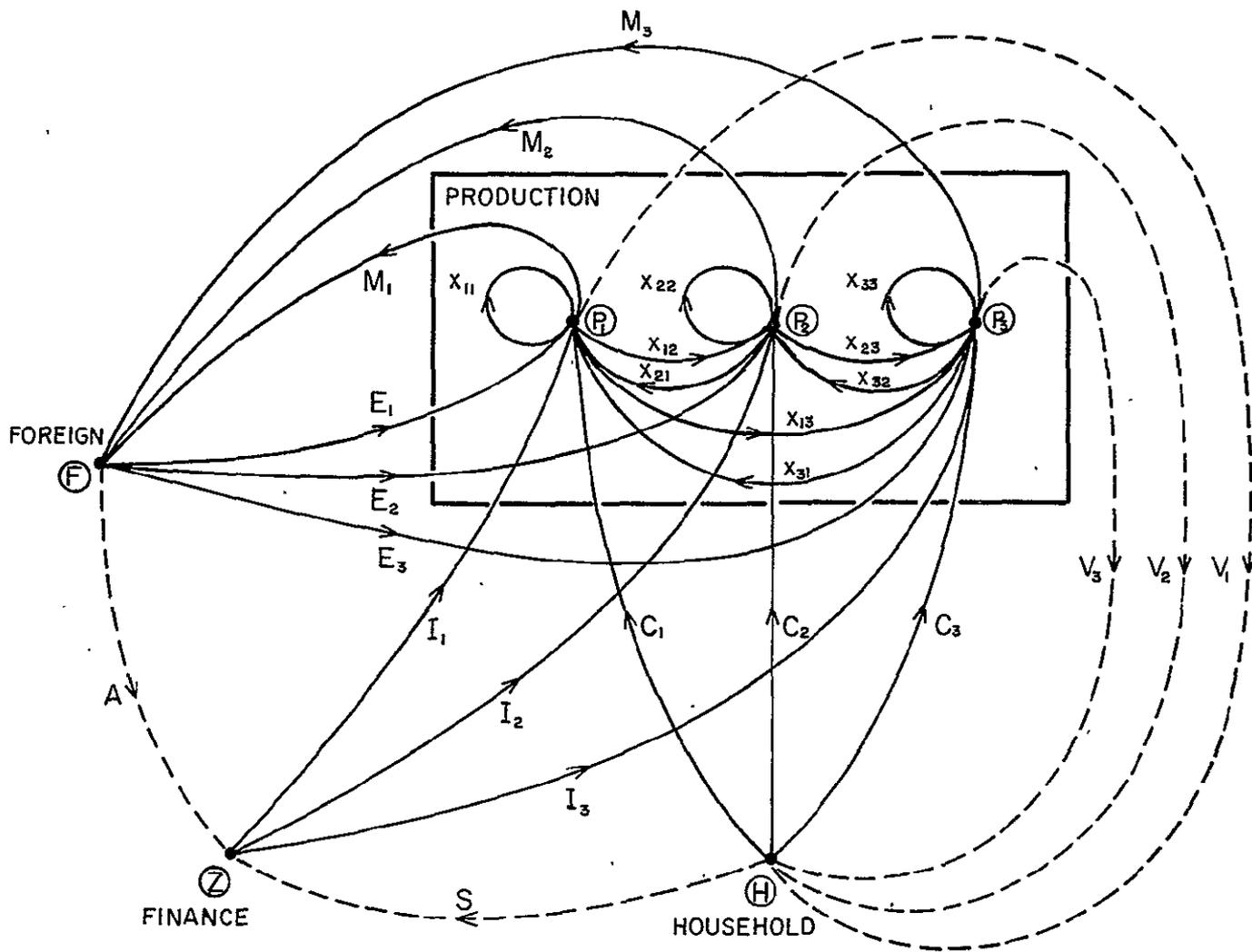
<sup>6</sup>This diagram will be used repeatedly for reference in the entire text of this chapter. The diagram and the notations should, therefore, be remembered.

sector (H) for primary factor costs ( $V_1, V_2, V_3$ ). The income of the household sector is disposed of as consumption expenditures ( $C_1, C_2, C_3$ ) and savings, S. For the foreign sector (F), the difference between total inpayments, or imports ( $M_1, M_2, M_3$ ), and outpayments, or exports ( $E_1, E_2, E_3$ ), is foreign aid, A. Both savings, S, and foreign aid, A, constitute inflows into the finance sector (Z) to finance total investment.

While the disaggregated accounting structure may vary in certain details when used by planners, the above entries are the essential and typical ones. The accounting structure of Diagram 2a emphasizes that the central analytical concern of the n-sector model is a detailed study of the resources aspect of production in respect to inter-industry relationships. In our illustration we use only three production sectors. In actual planning applications, a much larger number of sectors is frequently used. The basic principle, however, remains the same.

In the three-sector disaggregated model just introduced, the 26 variables are bounded by six accounting equations attached to the six vertices. As in the case of the aggregate model, each equation states the equality between inflows and outflows at each vertex. For our three-sector model the six accounting equations are:

# Diagram 2: Disaggregated National Income Accounting System



a. Skeleton

	Production			Household	Foreign	Finance	
	(P <sub>1</sub> )	(P <sub>2</sub> )	(P <sub>3</sub> )	(H)	(F)	(Z)	
(P <sub>1</sub> )	$X_{11}$	$X_{21}$	$X_{31}$	$C_1$	$E_1$	$I_1$	$X_1$
(P <sub>2</sub> )	$X_{12}$	$X_{22}$	$X_{32}$	$C_2$	$E_2$	$I_2$	$X_2$
(P <sub>3</sub> )	$X_{13}$	$X_{23}$	$X_{33}$	$C_3$	$E_3$	$I_3$	$X_3$
(H)	$V_1$	$V_2$	$V_3$				$V$
(F)	$M_1$	$M_2$	$M_3$				$M$
(Z)				$S$	$A$		$S_0$

b. Square Table

1.2.a) At Production Vertex  $(P_1)$  :

$$x_{11} + x_{12} + x_{13} + M_1 + V_1 = x_{11} + x_{21} + x_{31} + I_1 + E_1 + C_1$$

b) At Production Vertex  $(P_2)$  :

$$x_{21} + x_{22} + x_{23} + M_2 + V_2 = x_{12} + x_{22} + x_{32} + I_2 + E_2 + C_2$$

c) At Production Vertex  $(P_3)$  :

$$x_{31} + x_{32} + x_{33} + M_3 + V_3 = x_{13} + x_{23} + x_{33} + I_3 + E_3 + C_3$$

(Allocation of output)

d) At Household Vertex  $(H)$  :

$$V_1 + V_2 + V_3 = C_1 + C_2 + C_3 + S \quad \text{(disposition of income)}$$

e) At Foreign Vertex  $(F)$  :

$$A + E_1 + E_2 + E_3 = M_1 + M_2 + M_3 \quad \text{(financing of imports)}$$

f) At Finance Vertex  $(Z)$  :

$$A + S = I_1 + I_2 + I_3 \quad \text{(financing of investment)}$$

There are certain similarities between the aggregate model and the disaggregated model. We see in these equations that it is only the total resource balance that is disaggregated. As in the aggregate model above, this disaggregated model includes the income disposition balance (2d), the foreign trade balance (2e), and the financial balance (2f). It is the additional detail characterizing the production sector, however, that gives the n-sector model the proliferation in total resource balance. This latter is the special feature of the n-sector model, inherited from input-output economics.

To facilitate our later discussion we introduce the following aggregate accounting variables, definable in terms of the variables introduced above in equation (1.2). Specifically, a total value can be defined at each vertex:

1.3.a) At Production Vertex  $(P_1)$  :

$$X_1 = x_{11} + x_{21} + x_{31} + C_1 + I_1 + E_1 \quad (\text{total output of } P_1)$$

b) At Production Vertex  $(P_2)$  :

$$X_2 = x_{12} + x_{22} + x_{32} + C_2 + I_2 + E_2 \quad (\text{total output of } P_2)$$

c) At Production Vertex  $(P_3)$  :

$$X_3 = x_{13} + x_{23} + x_{33} + C_3 + I_3 + E_3 \quad (\text{total output of } P_3)$$

d) At Household Vertex  $(H)$  :

$$V = v_1 + v_2 + v_3 \quad (\text{national income})$$

e) At Foreign Vertex  $(F)$  :

$$M = M_1 + M_2 + M_3 \quad (\text{total imports})$$

f) At Financial Vertex  $(Z)$  :

$$S_o = S + A \quad (\text{total savings})$$

We have already observed that the planning school, in applying the n-sector type model to the entire economy, frequently covers a large number of sectors in its scope. The complexity resulting from this multiplicity of sectors limits the analytical focus to the symmetrical relationships

involved. This structural symmetry is apparent from the fact that no production sector can be distinguished from other production sectors in respect to intersectoral relationships.

In our review of the historical approach to growth (in an earlier chapter), we observed the evolution of the idea of relationships between major sectors as an aspect of growth. The historians' interpretation of the operational content of the concept of sectors was very different, however, from the one just reviewed for the planning school. The historical school envisaged a small number of large sectors involving asymmetrical patterns of relationships. In contrast, the planners' aggregate models, on the one hand, posit one large production sector, thus suppressing all meaningful analysis of intersectoral relationships. Their disaggregated models, on the other hand, build in a certain rigidity so that only symmetrical relationships can be handled in a formalistic way. As we shall see, this particular treatment of sectors and intersectoral relationships follows from the planners' resource-oriented growth philosophy, sharply distinguishing the planning school from other approaches.

This brief review stresses our earlier observation that the formal postulation of a national income accounting system--containing planning variables and planning equations--is the basic component of the planners' methodology. Given its centrality, the national income accounting system determines the scope of what is included within the purview of this school and reveals clues to its growth philosophy. Development comes to be viewed as basically an allocation phenomenon in the narrow sense of the utilization

and augmentation of resources. More specifically, the approach emphasizes the analysis of the sources and destination (e.g., foreign, domestic, and production sectors) of current and capital resources. In this analysis, stress is placed upon resource balancing in the sense of equality of supply and demand among the sectors identified. While there may be variations relating to the degree of detail involved--as illustrated by the two accounting systems examined above--the central viewpoint remains unchanged. Growth and development are wholly matters of how economic resources are made available and how they are utilized.

The centrality of the national income accounting system in the planners' methodology also imparts a very special flavor to their strong policy orientation. Policy is construed in a very special sense having to do with the application of a national income accounting system for planning purposes. Specifically, policy becomes a matter of choosing particular methods for computing and projecting the numerical values of all planning variables. The operational problems of such choice then revolve around the appropriateness of alternative systems for numerical computation and projection in the context of a particular plan.

In this very special policy focus of the planning school, appropriateness embraces three significant aspects. Two of these are formal and technical in nature; i.e., those concerned with accounting consistency and behavioristic assumptions. The third aspect of appropriateness involves a collection of informal decisions confronting the planner in formulating a particular plan. In the following sections, each of these aspects is

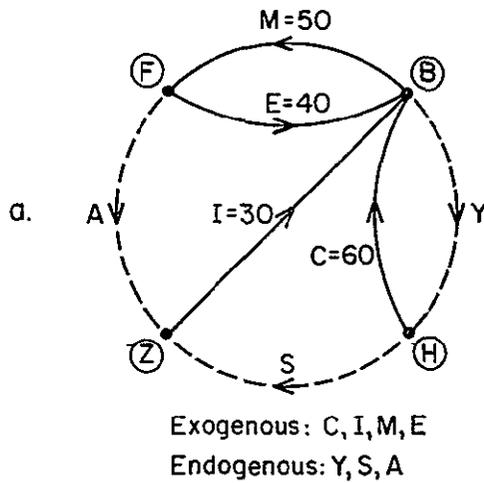
studied. Rather detailed treatment is required for understanding the policy issues posed in regard to the technical aspects of accounting consistency, which we now discuss.

### 1.3 Accounting Consistency in Development Planning

After a national income accounting system is adopted, the first basic requirement of an appropriate plan is that it be a consistent plan. What is meant by consistency here is accounting consistency; namely, the projected planning variables must satisfy all accounting equations in the system, such as the sets (1.1) and (1.2). The importance of accounting consistency, therefore, is that it imposes a discipline of cohesiveness and orderliness in respect to the utilization of resources for the whole economy, specifically referring to the resource balancing requirements mentioned above; i.e., total resources, income disposition, foreign trade, and finance. While an adequate development plan involves much more than accounting consistency, this criterion is nevertheless the most essential requirement and takes precedence over all others.

The postulation of a system of accounting equations and the consistency requirement immediately imply that, in constructing a consistent plan, only a part of the planning variables need be estimated; the values of the other planning variables can be determined with the aid of the accounting equations. For example, in Diagram 3a, which is a reproduction of the aggregate accounting framework of Diagram 1b, the edges (representing flows) are classified into two types: the solid edges (C, I, E, M) are termed

### Diagram 3: Alternative Planning Models

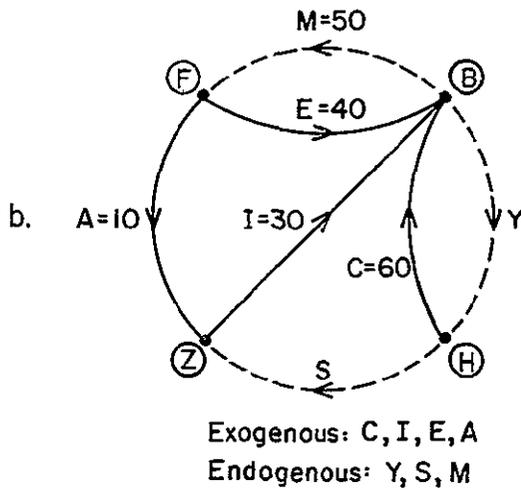


Calculation:

Step 1:  $Y = C + I + E - M = 80$  (B)

Step 2:  $S = Y - C = 20$  (H)

Step 3:  $A = I - S = 10$  (Z)

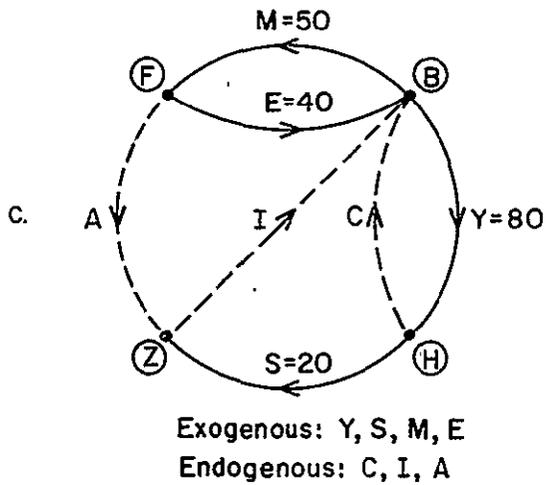


Calculation:

Step 1:  $M = E + A = 50$  (F)

Step 2:  $Y = C + I + E - M = 80$  (B)

Step 3:  $S = Y - C = 20$  (H)



Calculation:

Step 1:  $A = M - E = 10$  (F)

Step 2:  $I = S + A = 30$  (Z)

Step 3:  $C = Y - S = 60$  (H)

exogenous planning variables, and the dotted edges (Y, S, A) are termed endogenous variables. When the values of all the exogenous variables are given independently (e.g.,  $C = 60$ ,  $I = 30$ ,  $E = 40$ ,  $M = 50$ , as they are indicated on the edges representing these variables in Diagram 3a), we can calculate a unique set of values for all the endogenous variables by taking advantage of the necessity for accounting consistency. This calculation is indicated by the three steps below the linear graph of Diagram 3a and proceeds in the following order:

Step one: By requiring the balancing (i.e., equality of total inflows and total outflows) of the production sector (B), we can calculate the value of national income (i.e.,  $Y = 80$ ).

Step two: By requiring the balancing of the household sector (H), we can calculate the value of savings (i.e.,  $S = 20$ ).

Step three: By requiring the balancing of the financial sector (Z), we can calculate the value of foreign aid (i.e.,  $A = 10$ ).

This calculation process makes use of the principle that in each step the value of one endogenous variable is calculated by the requirement of balancing one particular sector (i.e., at a vertex). In this way, the value of all endogenous variables are computed and a consistent plan constructed.

This example shows that inherent in the notion of accounting consistency is the idea that the planning variables may be classified

according to a causal order of determination. In effect, the planner can then concentrate on the projection of the values of a subset of planning variables with higher causal order (i.e., the exogenous variables) with the assurance that the planning variables with a lower causal order (i.e., the endogenous variables) can be determined in a routine fashion. The choice of a particular set of exogenous variables and techniques projecting their values can be thus singled out as the first essential steps in development planning.

There are obviously many alternative ways in which a subset of planning variables can be identified and designated as a set of "exogenous variables." Some alternative possibilities are indicated in Diagram 3a, b, and c. In each case, the solid edges represent the exogenous variables on which arbitrary numerical values are first assigned. The dotted edges are the endogenous variables whose values are then calculated by three steps (shown beside each linear graph) using the same principle mentioned above. The mere existence of these (and many other possible) alternatives (any of which might be chosen as the specific planning procedure) reveals the "artistic" nature of planning methodology, involving a combination of judgment and technique. The judgment aspect arises because there are alternative ways to select the exogenous variables, amounting to alternative ways to begin the planning process and, hence, choice must be exercised. The technical aspect arises from the intrinsically quantitative nature of planning in terms of resource consistency. We discuss these two aspects separately, concentrating first upon the technical aspect.

Having postulated a national income accounting system as a planning framework, the following technical questions are immediately confronted:

- 1) How many variables are included in any set of exogenous (and endogenous) variables?
- 2) What types of sets of variables qualify as a set of exogenous variables?
- 3) How are values computed for all endogenous variables, given the predetermined values for all exogenous variables?

Answers to technical questions of this kind constitute the basic tools of the planner. They are important questions in abstract planning methodology--precisely because they are addressed to the problem of accounting consistency. Indeed, satisfactory answers to these questions are important, not only for the understanding and evaluation of important works of the planning school but also for our own work in later chapters. Therefore, it is important for us to develop the technical background needed to answer these questions.

It is by no means a simple matter to answer the above technical questions. For, to answer them in their full generality it is essential to investigate, abstractly and generally, the art of constructing national income accounting systems. The accounting systems introduced for the aggregate and disaggregated planning models are only special cases, while we are interested in the general case. We shall undertake this task in Section 2. This is followed, in Section 3, by an investigation of the technique of accounting consistency for planning, and where the

three questions posed above will be answered in their full generality. The results obtained will be used both for our further review and evaluation of the planning school in Sections 4 and 5 of this chapter and for our own analysis in later chapters.

#### 1.4 The Informal Aspect of Planning

The informal aspect of development planning relates to a large number of judgments essential to the application of planning methods to a particular country. Decisions which must be made at this point involve assessments of many facets of the country's institutional and economic setting. Essentially, the problem is one of feeding into the more formal planning framework, adopted for a particular country, information about certain critical environmental factors treated more formally by other schools we have reviewed.

The art of development planning cannot be understood without recognizing that these factors are incorporated by planners on the basis of judgment rather than scientific analysis. It is precisely these significant elements of informal choice which mark planning as an art, despite the impression of scientism given by the econometric bent of the planning school. In fact, a major difference between this school and the institutional school lies in the planning school's informal approach to treatment of the environmental factors affecting the economy which, as we have seen, receive increasingly formal analysis by the institutional school.

The environmental factors upon which informal judgments must be made include, inevitably, both domestic conditions and those external to the country. There are many aspects of the domestic situation upon which the planner must make a judgment, but only a few of those most commonly taken into account will be mentioned. There is, first, the problem of evaluating the society's preference system in terms of its orientation toward growth, consumption, equity of income distribution, and other economic and noneconomic goals. A related problem involves an assessment of the political tolerance for government interference; i.e., the degree of market intervention and control of resources a government can undertake in a particular society. This amounts to evaluation of a government's political power to prosecute a plan and has obvious implications for the specification of plan targets. The determination of targets also involves judgments related to the execution of a plan; i.e., its administrative feasibility. Considerations of data capabilities, supplies of technical personnel, and the general efficiency of administrative bureaucracies are relevant to this type of evaluation. Also significant for adjudging implementation potential of alternative plans is the problem of availability of policy instruments. This area involves, for example, appraisal of the country's tax and foreign exchange systems and the society's response to changes in their existing structure. Finally, we mention the most fundamental choice on which judgment must be exercised, that of choosing a particular development strategy, and its associated planning methods, on the basis of a diagnosis of the particular country's central development problems. In practice,

this choice has been reduced to a much simpler one by the search for a country's dominant development bottlenecks (e.g., saving capacity, absorptive capacity, etc.).

While judgments of conditions external to the country are necessarily more limited in range, they, too, may be of critical importance for effective planning. Two examples will suffice, foreign aid and export potential. Virtually all plans include a foreign aid component. Judgments about the international political climate are essential to projecting this foreign aid component. Similarly, in regard to a country's foreign exchange earning capacity, decisions must be made about export potential on the basis of judgments about future world economic growth and other intangible international developments.

The perplexing problem of dealing with these environmental factors in the context of planning has led to attempts by planners to take these factors into account through adapting terminology for classification of planning variables. At the present time, these efforts to formalize judgments constitute little beyond "language." Prominent in this regard are the following descriptions of variables:

(i) Welfare variable (or target variable): a variable whose value is construed as a direct indicator of economic welfare (e.g., per capita consumption, growth rate, unemployment).

(ii) Predetermined variable: a variable whose value is determined by forces external to the economy or intractable to government interference (e.g., export potential, volume of foreign aid).

(iii) Instrumental variable: a variable which can be directly affected by government policy; i.e., independently of the planning process (e.g., tax revenues, foreign exchange allocation).

(iv) Neutral variable: a variable which cannot be assigned to any of the above types.

In a particular planning context, the determination of which variables are assigned to any of these categories is entirely a matter of judgment. For example, foreign aid may be classified as an instrumental variable if it is believed that the government has some leverage to bargain for alternative levels of foreign assistance. If the planner believes that the government has no such option, foreign aid will be classified as a predetermined variable. If, moreover, acceptance of foreign aid is judged to be politically harmful, foreign aid may also be classed as a negative welfare variable. While the level of exports is frequently regarded as a predetermined variable, because their volume is considered to be beyond the planner's control, the level of imports is generally regarded as neutral. Such classification judgments, in the final analysis, however, are makeshift devices to organize for planning the many environmental forces affecting the economy.

To clarify further the operational significance of such classification schemes, we recall our previous distinction between exogenous and endogenous types of variables. Applying this distinction, the general principle is that instrumental and predetermined variables are considered as exogenous, while welfare (target) and neutral variables are considered as

endogenous. The underlying idea is simple. When certain policy measures are specified as instrument variables and when certain environmental factors are specified as predetermined variables, the planner can calculate the values of the welfare variables and the consistent values of the neutral variables. Let us consider one simple example. Suppose the planner envisions (for a particular country) the following values for a target year in the future:

- (i) Availability of foreign aid ( $A = 10$ )--a predetermined variable  
(by international climate)
- (ii) Export potential ( $E = 40$ )--a predetermined variable (projection of  
potential exports, independently estimated)
- (iii) Investment program ( $I = 30$ )--a predetermined variable (based on the  
judgment of absorptive capacity or  
investment ability of the country)
- (iv) Consumption program ( $C = 60$ )--a predetermined variable (based on a  
judgment of a consumption standard which  
can be politically accepted and enforced)

The simple model given in Diagram 3b can then be employed for calculating the consistent program related to the neutral variables--imports,  $M$ ; savings,  $S$ ; and the consistent value of the welfare variable, national income,  $Y$ .

Obviously, there are many alternative patterns of causal order which may be derived by informal classification of variables. Some examples selected from innumerable possibilities are given in the simple national

income accounting systems shown as individual cases in Diagram 3. These cases demonstrate the planning procedure implied in such choice. If, for example, foreign aid, A, is treated as endogenous, then a "needed foreign aid" model is required rather than an "available foreign aid" model. If a particular welfare variable (e.g., a target level of national income) is considered essential and one which cannot be compromised for political reasons, then it will be taken as predetermined (exogenous) rather than endogenous. If an import substitution program is considered feasible, then the volume of imports, M, may be treated as an instrument (exogenous) variable. These examples suffice to demonstrate the flexibility of classification of variables, revealing that judgments play a central role in development planning over a very wide range of fundamental planning issues.<sup>7</sup> The important conclusion is that the operation of planning models is not automatic. Judgment on many complex issues is required before planning models become relevant to the solution of a very narrow range of problems. This narrowing of scope required for application of formal planning methods remains a matter of informal, and sometimes unconscious, choice.

This combination of an informal judgment ingredient and rigorous methods stems from the applied nature of planning and affects all levels of the process. The previous discussion implies that planning relies upon an

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<sup>7</sup>For a more systematic discussion of this problem, essentially one of planning typology, see John C. H. Fei and Gustav Ranis, "A Study of Planning Methodology with Special Reference to Pakistan's Second Five-Year Plan," The Institute of Development Economics, Monographs in the Economics of Development, No. 1 (Karachi, Pakistan: June 1960).

informal area of knowledge which precedes the formal planning process and which is arrived at independently of the planning process. In this sense, it is tantamount to an issue of development strategy since basic guidelines are required to give direction and orientation to the formal planning process. It follows that choice of development strategy determines whether a plan and the methods it employs will be addressed to a society's critical and meaningful problems.

Although of crucial importance, the area of knowledge associated with development strategy has been neglected because it is difficult and elusive by its very nature. In the absence of guiding principles, the improvisations used by planners to cope with the concrete problems of this kind tend to be primitive, fragmentary, and deceptively simple.<sup>8</sup> Given the heterogeneous nature of the sample listing of these problems above, we can readily understand both the necessity for improvisation and the barriers to a more scientific approach. It will be argued in a later section of this book that an escape from this impasse may be found in approaching the selection of development strategy from a much broader perspective than the planner is accustomed to adopt.

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<sup>8</sup>The planners' attempts to deal with development strategy issues are discussed below in Section 4.5.

## 1.5 Behavioristic Assumptions for Planning Models

A national income accounting system, as a basic conceptual tool for a planning model, comprises economic variables and accounting equations. The set of variables identifies the essential content of the model, and the accounting equations describe its structural outline. In addition, a planning model always contains certain behavioristic equations, specified in terms of the model's variables. These behavioristic equations are postulated to describe how economic agents behave, usually in respect to production (i.e., behavior of producers) and income disposition (i.e., behavior of income recipients). The usefulness of a planning model depends, to a large extent, on the conformance of its behavioristic assumptions to economic reality. Behavioristic assumptions used by planners have been mainly inherited from other areas of economics. As planning has been more generally applied in less developed countries, however, planners have begun to formulate behavioristic assumptions from their own experience. Investigation of these assumptions is necessary for understanding the planners' philosophy of growth inasmuch as behavioristic assumptions represent a summary view of the essential behavior of the economic agents relevant to growth. In the present section we merely explore the origins of the behavioral relationships employed by planners. A critical review of behavioristic assumptions, and the planning models in which they are employed, will be undertaken in Section 4.

In the decade or two preceding the rise of the planning school to prominence, the discipline of economics in general was profoundly influenced by three pioneering developments: Keynesian economics (Keynes), input-output models (Leontief), and national income accounting (Kuznets). Each of these had a part in providing an impetus toward a new orientation in economics emphasizing quantitative methodology (e.g., linear programming and econometrics) and economic dynamics (e.g., growth and development models). To an important extent, the formation and methodology of the planning school has been affected by all of these developments. As observed earlier, they are reflected in the formalism of technical methodology and the manipulation of masses of quantitative data, trademarks of the contemporary planner. In addition, Keynesian economics, input-output economics, and dynamic models have contributed significantly to the school by providing many of the behavioristic assumptions employed in the planners' models.

From Keynesian economics are drawn assumptions apropos income disposition behavior. Most frequently, a Keynesian-type savings assumption is employed to estimate savings generated by national income. The input-output tradition is commonly relied upon for positing production behavior in the disaggregated model. The production function from this tradition specifies for individual industries the real cost of production on current account, including intermediate goods costs. Growth models of the Harrod-Domar type provide guidance for production behavior at the aggregate level. The capital-capacity assumption, frequently used, relates additions to capital stock to additional output capacity. Savings generated under the

savings assumption often represent the major growth promotion force in determining the level of investment, and, via the Harrod-Domar capital-capacity assumption, the level of output.

We shall investigate the explicit form of these assumptions in Section 4. We note here that these assumptions from diverse branches of theoretical economics are employed by planners to construct a wide variety of planning models, several of which are examined in Section 4. Whatever particular type of model is chosen, however, these assumptions are used to implement the planner's resource-oriented growth philosophy. Emphasis is always placed upon the augmentation of productive capacity through investment financed from savings resources. In short, despite the heterogeneous intellectual heritage of the planning school, their work focusses upon this special aspect of central planning.

The exigencies of planning have forced planners to devise additional behavioral assumptions. These efforts, stemming from a necessity to make planning models operative, seldom have the force of deductive theorizing or even inductive support behind them. Typically, a behavioral assumption is posited to formalize a wide variety of intangible forces. Because of the very nature of the diverse forces they encompass, the regularity and reliability of the relationships posited by these assumptions have not been, and perhaps cannot be, established by scientific inquiry. Unlike the behavioral assumptions borrowed from other branches of economic theory, it is doubtful that verification is possible for the assumptions improvised by the planner. Once again, we see an informal component in the planner's methods.

Several examples of these quasi-behavioristic assumptions may be given. First, we frequently find a behavioral relationship purporting to represent "absorptive capacity,"<sup>9</sup> a measure of the economy's capacity to "absorb" (i.e., execute) an investment plan of particular size or, alternatively, a collection of projects. It is obvious that such capacity is the product of many qualitatively different factors; for example, the number and quality of entrepreneurs, administrative capacity of the government, receptivity to and ability to introduce new technology, etc. Second, we frequently find an export potentiality assumption positing the future course of exports. Resort may be made to projection of past trends as a simple proxy for describing the diversity of external conditions affecting demand for the country's exports as well as internal conditions affecting their supply. Finally, we mention that frequently growth targets are governed by explicit or implicit behavioristic assumptions. Exogenous growth targets may be imposed on the basis of what is judged to be politically feasible or tolerable, or else posited on the basis of any number of other noneconomic forces. These examples are adequate to demonstrate that behavioral assumptions, which must be supplied by the planner, are critical to the application of planning models but are likely to be the product of human intuition and judgment rather than scientific investigation.

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<sup>9</sup>This concept of absorptive capacity was originally used in the context of a recipient country's capacity to absorb foreign aid.

## 1.6 Summary: General Features of the Planning School

The planning school's approach to growth is marked by its focus upon the problem of resource allocation. The preoccupation with augmentation and utilization of resources as the central issue in economic growth represents a growth philosophy leading naturally to the various aspects of the school's work reviewed in this section.

In the highly practical strain in the planners' work, the relevant policy issues, though of central importance to their performance as practitioners, are specific in scope; policy is equated with criteria to guide resource allocation through projection of a consistent plan.

In pursuing this problem-solving orientation, planners have shown considerable eclecticism and ingenuity in developing their methodology. Their methods embrace both informal and formal tools. Their approach is best described as an art, in which judgment continues to overshadow rigor. Informal methods have been devised to treat a wide variety of environmental factors. In particular, informal devices have been utilized to handle such imponderables as absorptive capacity, foreign aid, exports, and growth targets.

The planners' formal methods and behavioristic assumptions have been largely drawn from a wide variety of areas of recent interest to economists. These more systematic components include the use of national income accounting to organize and interpret masses of empirical data. Planning models are either highly aggregated, suppressing all intersectoral.

relationships, or highly disaggregated, involving only symmetrical relationships among sectors. In applying models of both types, the central issue of concern to planners is resource balances (total, finance, trade, and income).

In the remainder of this chapter we elaborate on two of these methodological issues, the national income accounting framework as a consistency device and the selection of behavioristic assumptions for planning models. In the course of this discussion a sampling of typical planning models are studied from the viewpoint of their methodological content. A brief evaluation of the strengths and weaknesses of the planning school concludes the chapter.

## 2. LINEAR GRAPH THEORY AND NATIONAL INCOME ACCOUNTING

In Section 1 we mentioned the importance of national income accounting in the planners' methodology and the need to study abstractly how to construct national income accounting systems. Without detailed and abstract treatment of national income accounting principles, we cannot fully understand the methodological content of the planning school nor can we adequately grasp the policy issues concerned with consistency. Moreover, national income accounting is an indispensable tool in our own later analysis of the open, dualistic economy. The operation of this type of economy involves intersectoral relationships which can only be neatly portrayed by such an accounting system. Hence, we systematically develop certain basic principles of national income accounting essential to our later work as well as for understanding planning techniques.

It is apparent from the two examples given in Section 1.2 that national income accounting systems are flow diagrams, or, in mathematical language, linear graphs. In Section 2.1, we introduce certain basic concepts of linear graphs. These concepts are interpreted (in Section 2.2) in the more conventional form of square tables. Using these concepts, we formally define national income accounting systems in Section 2.3. Finally, in Section 2.4, we introduce the idea of aggregation of a national income accounting system; i.e., producing a more macroscopic national income accounting system from a detailed, microscopic one. We shall introduce these technical matters at a deliberate pace, and using elementary methods. No

mathematical background is needed to read this and the following sections (Sections 2 and 3).

## 2.1 Linear Graph Concepts

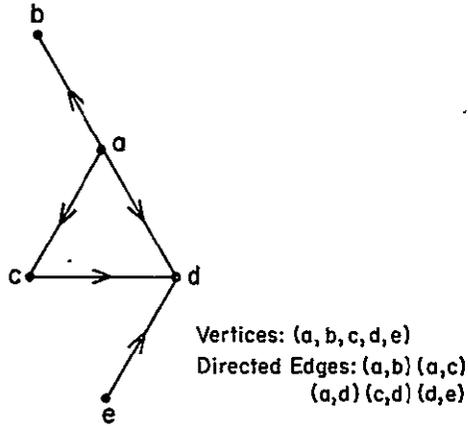
The natural language to describe a general national income accounting system consists of concepts in linear graph theory. To facilitate the later analysis we define two concepts which are basic to the techniques we develop--the concepts of a directed linear graph and a valued linear graph.

Definition: A directed linear graph,  $G$ , is formed of a set of vertices ( $a, b, c, \dots$ ) and a set of directed edges which are ordered pairs of vertices  $(x, y)$ . For the directed edge  $(x, y)$ ,  $x$  is the initial vertex and  $y$  is the terminal vertex.

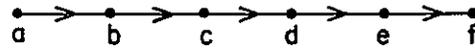
Diagram 4a is a directed linear graph in which a directed edge is represented by an arrow pointing from the initial to the terminal vertex. The formal mathematical definition of the directed linear graph shown is given just below the diagram. Unless otherwise noted, we will be concerned only with directed edges in this paper. Hence, in several places, we dispense with the adjective "directed."

Definition: A valued linear graph,  $A$ , is a directed linear graph,  $G$ , with a number written on each edge. If the number  $x$  is written on edge  $(a, b)$ ,  $x$  is referred to as the value of the edge. The linear graph,  $G$ , is called the skeleton of  $A$ .

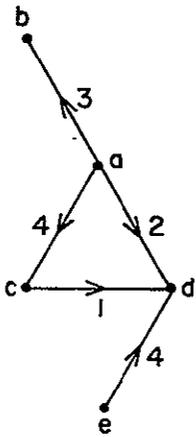
# Diagram 4: Linear Graphs



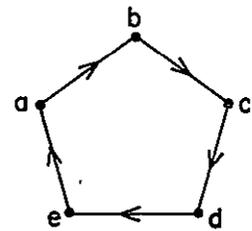
a. Directed Linear Graph



d. Path

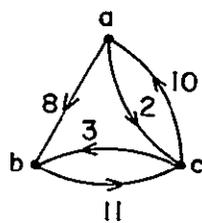


	a	b	c	d	e
a		3	4	2	
b					
c				1	
d					
e				4	



e. Circuit

b. Valued Linear Graph and Table



	a	b	c	
a		8	2	⑩
b			11	⑪
c	10	3		⑬
	⑩	⑪	⑬	

c. Euler Graph and Balanced Table

Diagram 4b is a valued linear graph constructed on the skeleton shown in Diagram 4a. We readily see that the national income accounting systems which we have constructed in Diagrams 1a and 2a can be construed as valued linear graphs, as defined above.

There are two aspects of the valued linear graph which have significance for national income accounting systems. First, there is the feature of a specific structural image of the economy (the skeleton) which reveals a particular pattern of economic relationships emphasized for a given type of economy. Second, there is the quantitative aspect, an attribute associated with the values assigned to each edge of the skeleton.

The former aspect, the structural skeleton of a system, is of primary significance for the "typology approach" to growth theory. It is the skeleton which describes the totality of economic relationships which exist among the key sectors of an economy of a particular type. With reference to the open dualistic economy, for example, the central features of openness and dualism can be defined mathematically from the characteristics of the skeleton. Basic features of other types of economies should be similarly defined from the skeleton.

The second aspect, values of the edges in the skeleton, allows us to apply the skeleton to a specific economy by assigning concrete numbers to each relationship. It is this aspect which brings to the skeleton the unique numerical substance for individual cases. In combination, therefore, the skeleton and values provide a basis for the study of the qualitative as well as quantitative aspects of the economy. In the

discussion in this and later chapters, we develop techniques for both the analysis of the skeleton and the analysis of values.

## 2.2 The Linear Graph and the Square Table

A valued linear graph can always be cast into the form of a square table under the following rules:

Rule (1): The number of sectors in the square table (i.e., the number of similarly indexed columns and rows) equals the number of vertices in the linear graph. Each column and row must be indexed by a vertex notation.

Rule (2): If the value of the edge (a, b) is x, the value of the cell in the a-th row (i.e., the initiating vertex) and the b-th column (i.e., the terminating vertex) is x.

As an illustration, the square table corresponding to a valued linear graph is shown beside the graph in Diagram 4b. It is apparent that a square table can always be interpreted as a valued linear graph under the above rules. Thus, the idea of a valued linear graph and the idea of a square table are practically the same. We apply this principle to the valued linear graphs in Diagrams 1b and 2a by putting them in the form of square tables (1c and 2b). These latter are the representation of the national income accounting systems, discussed earlier, in table form. The national income accounting system which will be used for the analysis of the open dualistic economy in this book will be based on an extension of the national income accounting system presented in Table I below.

Definition: A square table is a balanced table if the sum of all entries in any row is the same as the sum of all entries in the like-indexed column.

A valued linear graph corresponding to a balanced table is called an Euler graph. In an Euler graph, the value of total inflows into each vertex equals the value of total outflows. Diagram 4c shows an Euler graph and its corresponding balanced table. It can be seen that the table is in balance from the equality between row sums (written at the right-hand margin of the table) and the column sums (written at the bottom of the table). We shall say that a sector of a square table is in balance if the sum of all the entries in a row equals the sum of all entries in the like-numbered column.

The idea of a balanced table (or an Euler graph) corresponds precisely to the idea of accounting consistency in the formulation of a development plan. For example, the aggregate national income accounting structure of Diagram 1c is a balanced table as defined by the accounting equations in (1.1). Similarly, the disaggregated national income accounting system is a balanced table, as defined by the accounting equations in (1.2). Thus, in general, the choice of a national income accounting system amounts to the selection of a skeleton of a linear graph. The construction of a consistent plan (i.e., a plan satisfying the conditions of accounting consistency) is equivalent to the construction of an Euler graph (or a balanced table) on that skeleton.

We now state an elementary theorem to be referred to as the "Balanced Table Theorem":

Theorem (1) : A square table with n sectors is a balanced table when n-1 sectors are in balance.

Graphically, the interpretation of this theorem is that, if a valued linear graph has n vertices and if n-1 vertices are in balance, it is an Euler graph. The reasonableness of the theorem is apparent from this graphic interpretation.

As a simple proof of Theorem (1), suppose we have a 4 x 4 square table;

$x_{11}$	$x_{12}$	$x_{13}$	$x_{14}$
$x_{21}$	$x_{22}$	$x_{23}$	$x_{24}$
$x_{31}$	$x_{32}$	$x_{33}$	$x_{34}$
$x_{41}$	$x_{42}$	$x_{43}$	$x_{44}$

In the case where the first three sectors are in balance, we have:

$$\textcircled{x_{11}} + \textcircled{x_{12}} + \textcircled{x_{13}} + x_{14} = \textcircled{x_{11}} + \textcircled{x_{21}} + \textcircled{x_{31}} + x_{41} \dots \text{(1st sector in balance)}$$

$$\textcircled{x_{21}} + \textcircled{x_{22}} + \textcircled{x_{23}} + x_{24} = \textcircled{x_{12}} + \textcircled{x_{22}} + \textcircled{x_{32}} + x_{42} \dots \text{(2nd sector in balance)}$$

$$\textcircled{x_{31}} + \textcircled{x_{32}} + \textcircled{x_{33}} + x_{34} = \textcircled{x_{13}} + \textcircled{x_{23}} + \textcircled{x_{33}} + x_{43} \dots \text{(3rd sector in balance)}$$

Adding these three equations and cancelling out the encircled terms, we have:

$$x_{14} + x_{24} + x_{34} = x_{41} + x_{42} + x_{43}$$

By adding  $x_{44}$  to both sides of the above equality, we immediately see that the 4th sector is in balance. This proves the theorem in the case where the square table has four sectors. The proof of the theorem for the general (n-sector) case is similar.

An economic application of the Balanced Table Theorem can now be made. Referring to the square table in Diagram 2b which has six sectors, we can specify that the first five sectors are in balance by definition. The balancing of the last sector (i.e., the finance sector) then follows as a logical necessity; i.e.,  $S + A = I_1 + I_2 + I_3$ , which states that total investment is financed by domestic savings (S) and foreign savings (A). Thus we see that the financial balance follows logically from the total resource balance, the income disposition balance, and the foreign trade balance.

### 2.3 The Linear Graph as a National Income Accounting Skeleton

We have seen that an Euler graph may be interpreted as the skeleton of a national income accounting system. This is based on the fact that any meaningful national income accounting system can be put into the form of a balanced table with positive entries. We now investigate what properties a linear graph must have to serve as the skeleton of such a

meaningful national income accounting system. For this purpose we introduce the following definitions:

Definition: A set of edges form a path connecting "a" and "z" if they can be written in the form (a, b) (b, c) (c, d)...(x, y) (y, z) where a, b, c...z are distinct. A path is a circuit if "a" is "z."

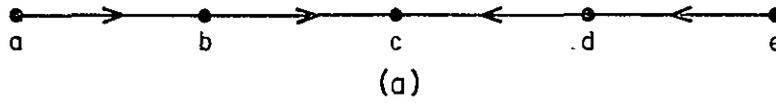
Definition: A set of directed edges form a link between the vertices x and y if they form a path from x to y after the direction of some of these edges is reversed. A link is a circle if x is y.

Diagram 4d is a path with a length of five edges, and Diagram 4e is a circuit of five edges. The linear graph in Diagram 5a is a link. (While a path is a one-way street, a link is a road in which the traffic sign can be neglected.) Diagram 5b is a circle with five edges. When the direction of some edges of a circle is reversed, it becomes a circuit. While all these linear graph concepts will be used in this book, we are immediately concerned with the following definitions:

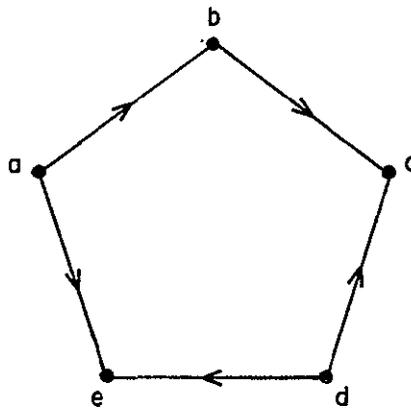
Definition: A linear graph is a cyclic net if for every pair of vertices x, y there is a path from x to y and a path from y to x.

Diagram 5c is a cyclic net. Graphically, if the vertices were construed as cities, then a cyclic net could be interpreted as a very reasonable road system, enabling one to reach any city from any other city. Visual inspection of Diagrams 1b and 2a, the accounting systems for the aggregate and disaggregated planning models, reveals that their skeletons are cyclic nets.

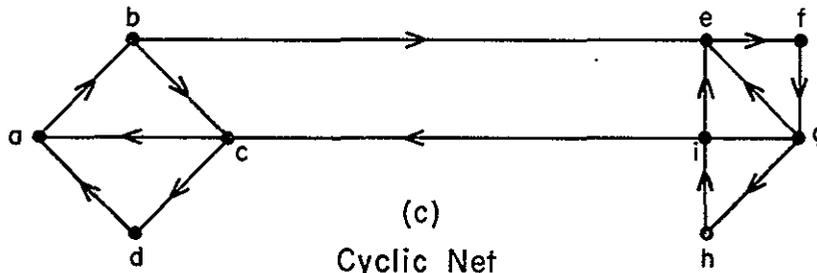
Diagram. 5: Link, Circle and Cyclic Net



Link a to e



(b)  
Circle



(c)  
Cyclic Net

It is readily apparent that a national income accounting system can be put into the form of a balanced table (i.e., an Euler graph). It is also obvious that a national income accounting system can be a positive balanced table; i.e., with entries all positive. Later we shall see that this requirement does not impose undue restrictions for our own analysis. (A negative value  $(-x)$  in a cell, or on an edge  $(x, y)$ , can be replaced by a positive value  $(x)$  to form a positive balanced table.)

Heretofore, we have been using the concept "national income accounting system" as if it were a self-evident, well-defined concept, while actually no formal definition has been given. However, a precise definition is essential for our later work. For reasons indicated in the above paragraph, we may now define a national income accounting structure abstractly as a balanced table containing positive entries. More formally:

**Definition:** A national income accounting structure is a linear graph,  $G$ , on which a strictly positive Euler graph can be defined (i.e., strictly positive numbers can be assigned to every edge of  $G$  to form an Euler graph).

There are certain linear graphs which cannot qualify to represent the skeleton of a national income accounting system according to the above definition. For example, the linear graph of Diagram 4a obviously does not qualify because of the existence of "end edges"  $(d, e)$  and  $(a, b)$ . If a balanced table were constructed, the values assigned to these end edges must be zero; otherwise the end vertex could not be in balance. In other words, strictly positive values cannot be assigned to all edges in the linear

graph of Diagram 4a to form an Euler graph. For this reason, this linear graph cannot represent the skeleton of a national income accounting system according to the above definition.

The above discussion provides some intuitive notion of what types of linear graphs can qualify to represent a national income accounting system; i.e., those without end edges. A cyclic net has precisely this property. For this reason, we can deduce a theorem to characterize a legitimate national income accounting system according to the above definition. First, we define "connected" as follows:

Definition: A linear graph is connected if for every pair of vertices (x, y) there is a link between x and y.

All the linear graphs shown in Diagram 4 are connected linear graphs, which simply means that the linear graph cannot be separated into disjointed parts. With the aid of the above definition, we can now give a characterization of a legitimate national income accounting system:

Theorem (2) : A connected linear graph is a national income accounting structure if and only if it is a cyclic net.

Theorem (2) implies that for analysis of national income accounting systems with positive entries, special attention must be given to cyclic nets. The proof of this theorem, which is required later for development of additional concepts of graph theory, is given in a later chapter.

## 2.4 Aggregation

We have seen that planners employ aggregated and disaggregated models. In the context of the analysis of national income accounting systems, the notion of aggregation refers to a type of operation which condenses (or reduces) a disaggregated national income accounting system to an aggregated one. The aggregated model gives a more macroscopic view of the entire economy than the disaggregated model. Before proceeding to an abstract definition of aggregation, we examine a simple example.

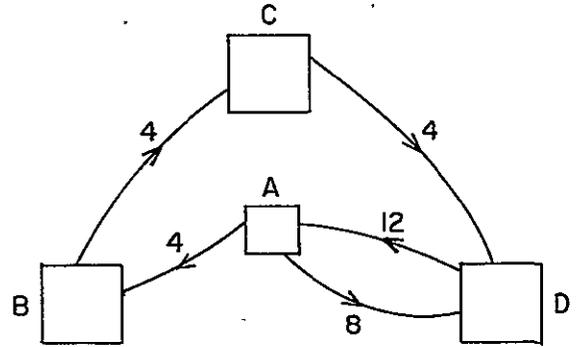
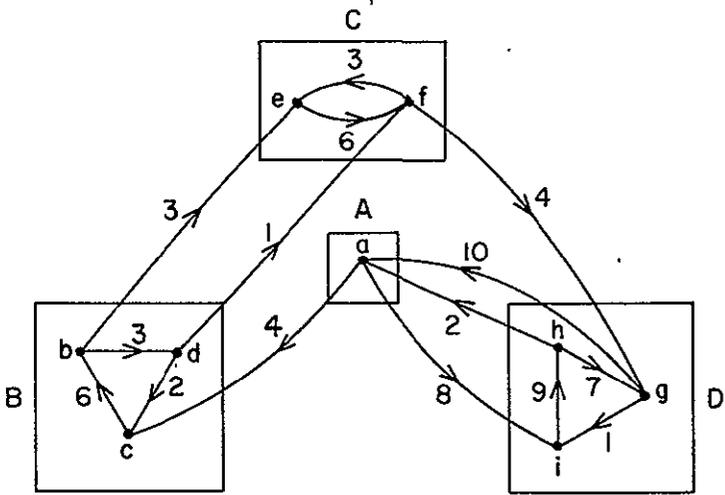
Let the linear graph,  $G$ , of Diagram 6a be given, and let the nine vertices be classified into four subsets (as enclosed by the four squares in the same diagram):

$$A = (a) \quad B = (b,c,d) \quad C = (e,f) \quad D = (g,h,i)$$

such that each vertex belongs to one and only one of these subsets. An aggregated linear graph,  $A(G)$ , based on this vertex classification is a linear graph with vertices  $A, B, C, D$ , and contains an edge  $(X, Y)$  if, and only if, there is an edge  $(u, v)$  in  $G$  such that  $u$  is in  $X$  and  $v$  is in  $Y$ . The aggregation of the valued linear graph shown in Diagram 6a can be seen from the valued edges in Diagram 6b.

These rules of aggregation can be generalized to apply to any linear graph and any valued linear graph as summarized in the following definitions:

# Diagram 6: Aggregation



		A		B		C		D		
		a	b	c	d	e	f	g	h	i
A	a				4					
B	b					3	3			
	c		6							
C	d			2						
	e						6			
D	f					3		4		
	g	10								
D	h	2							7	
	i									9

(a)

		A	B	C	D
A		×	4		8
B			×	4	
C				×	4
D	12				×

(b)

Definition: Let  $G$  be a directed linear graph with vertices  $(a, b, \dots, z)$  and let  $X_1, X_2, \dots, X_r$  be a classification of these vertices (i.e., every vertex of  $G$  belongs to one and only one  $X_i$ ). The aggregated linear graph  $A(G)$  based on this classification is a linear graph containing vertices  $X_1, X_2, \dots, X_r$  and directed edges  $(X_i, X_j)$  if and only if there is an edge  $(u, v)$  in  $G$  such that  $u$  is in  $X_i$  and  $v$  is in  $X_j$ .

Definition: A valued linear graph defined on  $G$  is aggregated into a valued linear graph on  $A(G)$  if the value on the edge  $(X_i, X_j)$  is the sum of the values of all edges  $(u, v)$  where  $u$  is in  $X_i$  and  $v$  is in  $X_j$ .

The operation of aggregation can be represented in table form, as indicated in Diagram 6a and 6b. To perform this operation, let the vertices belonging to the same subset be listed adjacently, as in the table in Diagram 6a. In this diagram the heavier lines mark off the cells in the aggregated table in 6a. The sum of all entries in each cell in the disaggregated table (Diagram 6a) is computed and recorded in the aggregated table, as shown in Diagram 6b. It is apparent from these operations that a balanced table is aggregated into a balanced table. This may be summarized as the following theorem:

Theorem (3): The aggregation of an Euler graph leads to an Euler graph.

The aggregated national income accounting system shown in Diagram 1c can be obtained from the disaggregated system shown in the table of Diagram 2b by such an aggregative operation. The lines in this table mark off 16 cells. By consolidating all entries in each cell and omitting diagonal entries, we obtain the aggregated structure in the table in Diagram 1c.

The economic significance of such aggregation is the simplification of economic relationships into a limited number of aggregates which can be subjected to intensive study. This is essential for understanding the operation of the economy as a whole and the relationship of the parts to the whole system.

## 2.5 Application

In this section we have deduced certain abstract properties of a national income accounting system and certain techniques for studying them. These concepts and techniques will now be applied for the construction of a realistic and concrete national income accounting system which will be used throughout this book.

There are two aspects of a national income accounting system which are important to our work. In the first place, it represents a framework for collection and processing of statistical data. It should be emphasized in this connection that our study of the open, dualistic economy contains a significant empirical component. The data representing our empirical work are organized in a specific national income accounting framework. In the second place, a national income accounting system can be used to describe the structure of an economy, and the system we employ in this book describes the structure of the open, dualistic economy. National income accounting is thus an instrument for both inductive and deductive analysis. As a data framework, a national income accounting system should have sufficient detail for collecting all data relevant to providing adequate coverage of the

entire economy. As an instrument to depict economic structure, it should be selective, concentrating only upon the essential relationships in the economic system. In the remainder of the present section, we investigate the national income accounting system from the first viewpoint; i.e., as a data framework. The use of the system as an analytical device to describe the economy's structure will be investigated in Chapter 6, where the structure of the open, dualistic economy, the major subject of our book, is discussed in detail.

An example of a national income accounting system which meets the requirements for a data framework is provided by the disaggregated national income accounting system used by the planning school, shown in Diagram 2. That system portrays economic flows among the following sectors:

- a) Multiple Production Sector  
( $P_1, P_2, P_3$ )--to record the intersectoral production relations on current account
- b) Household Sector (H)--to record the income disposition activities
- c) Foreign Sector (F)--to record the import and export activities
- d) Finance Sector (Z)--to record the sources of finance for real investment

We see from this outline that this system combines elements from input-output economics (a), Keynesian economics (b and d), and international economics (c).

Complicated as the above national income accounting system might seem, it is still deficient in several major aspects for our later analysis

of the open, dualistic economy. From Diagram 2a, we see that while the production sector is disaggregated, households are consolidated into one sector for the entire economy (i.e., sector  $\textcircled{H}$  ). Consolidation of the household sector is a common national income accounting practice for industrially advanced countries in which, perhaps, no significant economic insights can be gained by grouping households according to industrial origin of their income. This is decidedly not the case, however, for a less developed country. To study the problem of economic development for the latter, it is vitally important to classify households (as consumption and income decision-making units) to conform explicitly with the broad classification made for production entries. Thus, agricultural households, providing services for the agricultural production sector, must be distinguished from industrial households, which provide services for the industrial production sector. This sectoring is based on the premise that economic behavior of the two types of households are qualitatively different (in respect to consumption, saving, and income-earning activities) and that this difference constitutes a significant aspect for the analysis of dualistic economic growth.

The national income accounting system of Diagram 2a is also deficient in that the financial sector is consolidated into one sector  $\textcircled{Z}$  for the nation as a whole. Although the system shows clearly the origin (i.e., the producing sectors) of investment goods ( $I_1, I_2, I_3$ ), the consolidation has, in fact, suppressed one vital type of information; namely, the destination (i.e., the accumulation sector) of capital goods.

For the study of certain crucial issues of development related to capital accumulation, it is important that we know the destination as well as the origin of investment goods. For example, investment goods are usually produced in the industrial sector, while they are allocated to several production sectors (e.g., industry, agriculture, and social overhead). This pattern of allocation is itself a central development issue.

In short, the disaggregated national income accounting system which we have introduced in Diagram 2a as adequate for "planning purposes" must be extended in two directions for our work in this book. These extensions consist of disaggregating the household sector (to show their industrial affiliation) and also disaggregating the finance sector (to show the destinations of investment goods). This extension is illustrated in the square table shown in Table I.

Let us postulate an economy with a large number of economic sectors while, for illustrative purposes, we assume that there are three sectors in the economy. Every economic sector in the realistic world may be conceptually split into three distinct aspects (as represented by three accounting sectors), the production aspect (shown as a, b, c), the income disposition aspect (shown as d, e, f), and the savings-investment aspect (shown as g, h, i). In an open, dualistic economy, for example, the three economic sectors may be industry, agriculture, and government. In each of these economic sectors, decisions must be made in respect to production, income disposition, and savings-investment. In Table I, the industrial sector is then represented by the accounting sectors (a, d, g); the

TABLE I

NATIONAL INCOME ACCOUNTING TABLE

		DOMESTIC									Foreign	Finance	
		Production			Income Disposition			Investment			F	Z	
		1	2	3	1	2	3	1	2	3			Total
		(a)	(b)	(c)	(d)	(e)	(f)	(g)	(h)	(i)	(j)	(k)	
(Industry)	1	(a) $x_{11}$	$x_{12}$	$x_{13}$	$c_{11}$	$c_{12}$	$c_{13}$	$I_{11}$	$I_{12}$	$I_{13}$	$E_1$		$X_1$
(Agric.)	2	(b) $x_{21}$	$x_{22}$	$x_{23}$	$c_{21}$	$c_{22}$	$c_{23}$	$I_{21}$	$I_{22}$	$I_{23}$	$E_2$		$X_2$
(Gov't.)	3	(c) $x_{31}$	$x_{32}$	$x_{33}$	$c_{31}$	$c_{32}$	$c_{33}$	$I_{31}$	$I_{32}$	$I_{33}$	$E_3$		$X_3$
(Industry)	1	(d) $v_{11}$	$v_{12}$	$v_{13}$	$t_{11}$	$t_{12}$	$t_{13}$						$Y_1$
(Agric.)	2	(e) $v_{21}$	$v_{22}$	$v_{23}$	$t_{21}$	$t_{22}$	$t_{23}$						$Y_2$
(Gov't.)	3	(f) $v_{31}$	$v_{32}$	$v_{33}$	$t_{31}$	$t_{32}$	$t_{33}$						$Y_3$
(Industry)	1	(g) $d_1$										$I_1$	
(Agric.)	2	(h)	$d_2$									$I_2$	
(Gov't.)	3	(i)		$d_3$								$I_3$	
(Foreign)	F	(j) $M_1$	$M_2$	$M_3$	$M'_1$	$M'_2$	$M'_3$	$M''_1$	$M''_2$	$M''_3$			$M$
(Finance)	Z	(k)			$S_1$	$S_2$	$S_3$				$A$		
TOTAL								$I'_1$	$I'_2$	$I'_3$			

agricultural sector by (b, e, h); and the government sector by (c, f, i)-- corresponding to the three aspects of economic decisions. In addition, the foreign sector (F) and the finance sector (Z) of Diagram 2 are retained. In Table I, as before, the entries are made so that a paying sector corresponds to a row index and a receiving sector corresponds to a column index. In terms of the principles discussed in this chapter, moreover, Table I is constructed as a balanced table (i.e., an Euler graph) to insure that every sector is in balance. Since there are eleven accounting sectors, we can balance ten by definition. The remaining sector will then be in balance by Theorem (1). Balancing of Table I is based on the following four principles:

Principle (1): Balancing of the Production Sector

The first row (a) represents the demand for the total output ( $X_1$ ) of the first production sector as formed of inter-industry demand ( $x_{1j}$ ), consumption demand ( $c_{1j}$ ), investment demand ( $I_{1j}$ ), and export demand ( $E_1$ ).

The first column (a) represents the disposition of the monetary receipts associated with the production of the total output ( $X_1$ ) into intermediate factor cost ( $x_{i1}$ ), import cost of intermediate goods ( $M_1$ ), primary factor cost, which includes capital consumption allowance ( $d_1$ ) and values added ( $v_{i1}$ ) originating from the various economic sectors.

Thus, the first row and the first column are in balance by definition. The same is true for accounting sectors (b) and (c).

Notice that the balancing of the production sector now specifies that there may be nontrivial intersectoral relations, not only in respect to the production area (i.e., of the familiar input-output variety  $x_{1j}$ ) but (unlike Diagram 2) also in respect to intersectoral consumption demand ( $c_{1j}$ ). This is done to provide a more realistic data framework for studying the open, dualistic economy. For example, in the growth process, the agricultural sector produces not only cotton ( $x_{21}$ ) to provide raw material for the industrial sector but also food ( $c_{21}$ ) to feed laborers in the industrial sector. The accounting system also depicts certain details in regard to the intersectoral allocation of investment goods. In the growth process, the industrial sector will provide investment goods not only for itself ( $I_{11}$ ) but also for the agricultural sector ( $I_{12}$ ), etc.

Principle (2): Balancing of the Income Disposition Sector

The (d)-th row represents the sources of total income ( $Y_1$ ) received by the first sector as productive (or primary factor) income ( $v_{1j}$ ) and transfer income ( $t_{1j}$ ). (Transfer income is paid out by the household sector while productive income is paid out by the production sector.)

The (d)-th column represents disposition of total income ( $Y_1$ ) by the income recipients of the first sector as consumption expenditure ( $c_{11}$ ), transfer expenditure ( $t_{11}$ ), savings ( $S_1$ ), and expenditure on imported consumer goods ( $M^c_1$ ).

Thus, the (d)-th sector is in balance by definition. The same is true for sectors (e) and (f).

Notice that in the above description some prominence is given to the possibility of the emergence of sectoral savings from three sources; i.e., the two private production sectors ( $S_1$  and  $S_2$ ) and the government sector ( $S_3$ ). It is commonly believed that the sectoral origin of savings is an essential notion for the development of a dualistic economy. Intersectoral transfers are also important development concepts, for example, as tax payments by the private sectors to the government sector ( $t_{31}$  and  $t_{32}$ ).

### Principle (3): Balancing of the Saving-Investment Sector

The (g)-th column represents the total gross investment by the first sector ( $I^g_1$ ), divided into expenditures on domestically produced investment goods ( $I_{11}$ ) and expenditures on imported capital goods ( $M_{11}$ ).

The (g)-th row represents the capital consumption allowances ( $d_1$ ) and the net investment ( $I_1$ ) of the first sector.

The (g)-th sector is in balance by the very definition of net investment; i.e., as gross investment ( $I'_1$ ) minus capital consumption allowances ( $d_1$ ). Similarly, sectors h and i are in balance.

Principle (4): Balancing of the Foreign Sector

The (j)-th row represents total imports (M) and their composition as intermediate goods for production ( $M_1$ ), consumption goods ( $M'_1$ ), and capital goods ( $M''_1$ ).

The (j)-th column represents sectoral exports ( $E_1$ ) and capital imports (A).

The j sector is in balance because capital inflow (A) is defined as total imports minus total exports.

Disaggregation of both imports and exports is essential to understanding the open, dualistic economy. It permits investigation of the changing composition on both sides of the foreign trade accounts as an aspect of growth.

Ten of the eleven sectors are in balance according to these principles. We know that the last sector (k) will also be in balance. The balancing of this sector specifies a financial balance ( $I_1 + I_2 + I_3 = S_1 + S_2 + S_3 + A$ ) which states that the total net investment expenditure is financed by the savings of three domestic sources and capital imports.

This national income accounting system, as a data framework, gives considerable detail in its description of the economy. Although we have shown only three economic sectors in Table I, a large number of economic sectors can be incorporated in the system based on the four principles just presented. The table exhibits a symmetrical system, characterized by the simplicity and uniformity of the accounting principles applied to production, income disposition, investment, foreign trade, and financial relationships. This general system is designed for several applications in our book. It covers all the essential economic flow data that will be used in the later analytical chapters. It is also the framework employed for collection of data from the Philippines, Thailand, and Taiwan--the empirical focus in later chapters.

As we have pointed out earlier, the national income accounting system will also be used as a basic conceptual tool to describe the operation of the particular type of economy under study, the open, dualistic economy. For this purpose, the accounting system must be more selective than the general system of Table I so that we may concentrate on a particular group of essential economic relationships. The adjustments required for this purpose will be discussed in detail in Chapter 6. It will suffice to mention here that the open, dualistic economy system requires the omission of certain entries from Table I, so that those which remain constitute the essential entries conforming to our preconceived theoretical notion of the operation of such an economy. The entries to be deleted from the general system for our later specific purpose are encircled in Table I. We discuss

the meaning of the deletions for the government sector to give a preliminary example of this process.

In Table I, Sectors (c), (f), and (i) correspond to the production, income, and investment-disposition aspects of the government sector. Much of the simplification indicated by the deleted government sector entries conforms to conventional treatment. Thus, in Column (c), as a producer, the government may purchase intermediate factors ( $x_{13}$  and  $x_{23}$ ) from the two private production sectors (i.e., the industrial and agricultural sectors), as well as primary factors ( $v_{13}$  and  $v_{23}$ ) from the two private household sectors. In Row (c) the only entry remaining after deletion is  $c_{33}$ . The balancing of the (c) sector (i.e.,  $c_{33} = x_{13} + x_{23} + v_{13} + v_{23}$ ) implies that  $c_{33}$  is the conventional valuation of government output as total factor cost. From Row (f) we hypothesize that government revenues are mainly in the nature of transfer income ( $t_{31}$ ,  $t_{32}$ ); i.e., tax payments by industrial and agricultural households. Column (f), after deletion, contains only  $c_{33}$  and  $S_3$ . The balancing of the f sector ( $c_{33} + S_3 = t_{31} + t_{32}$ ) implies that government revenue is disposed of either as expenditures on publicly consumed goods and services (i.e.,  $c_{33}$ ) or government savings ( $S_3$ ). Finally, all entries in Column (i) and Row (i) are deleted which signifies that, in the conventional treatment of the government sector, capital accounting for the government is omitted. In the formulation of the national income accounting system for the open, dualistic economy in Chapter 6, simplification for treatment of the government sector will follow these conventions.

### 3. THE METHODOLOGY FOR ACCOUNTING CONSISTENCY

#### 3.1 Exogenous and Endogenous Variables

In the previous section we investigated certain basic definitions related to the construction of legitimate national income accounting systems. We concluded, on the one hand, that a linear graph, used as a skeleton, is the natural language for an abstract description of a national income accounting system. On the other hand, we conceive a concrete national income accounting system to be an Euler graph (balanced table) constructed on the skeleton. In applying this distinction, we think of the skeleton as a framework for development planning. The ultimate objective of the planning process is a concrete one, the projection of a consistent plan, which is an Euler graph, or, in using the language of Section 1, a set of values which satisfies all the accounting equations implied by the framework.

We have seen (in Section 1.3) that once a skeleton is selected as a framework, the planner begins by making projections for a part of the planning variables (i.e., those designated as exogenous) and then proceeds by employing the accounting equations to compute the values of the remaining planning variables (i.e., the endogenous variables). We raised three technical questions in regard to this computational procedure: (1) how many variables are there in a set of exogenous variables; (2) what types of sets constitute exogenous or endogenous variables; and (3) how are the values of the endogenous variables computed when the values of all exogenous variables are arbitrarily preassigned?

In Section 1 we explored the economic significance of these central questions in planning methodology. We are now in a position to answer them rigorously. The purpose of developing a generalized concept of national income accounting systems in the previous sections was precisely to enable us to provide general answers to these questions. We begin by formally defining exogenous and endogenous variables, heretofore accepted in a heuristic sense. Their very definitions presuppose the postulation of an abstract national income accounting system,  $G$ .

**Definition:** A set of variables (edges) of a linear graph,  $G$ , form a set of exogenous variables and the other edges form a set of endogenous variables if, after arbitrary values are assigned to all exogenous variables, unique values can be assigned to all endogenous variables to form an Euler graph.

To illustrate these definitions we again refer to Diagram 3. For each case shown, the solid edges form a set of exogenous variables while the dotted edges form a set of endogenous variables. We see that these cases conform to the above definitions. First, the national income accounting skeleton,  $G$ , is given, as required. Moreover, the values of all endogenous variables can be uniquely determined after the values of all exogenous variables are arbitrarily preassigned, also conforming to the definition. We now investigate the mathematical (or graphic) characteristics of sets of variables of both types, exogenous and endogenous. The first characteristic is the number of variables in each set.

### 3.2 The Cyclomatic Number

The first question above involves the number of variables in a set of exogenous variables. Referring to the Diagram 3 example, just cited, we see that in every case the number of exogenous variables is four. It is not difficult to show why this number must be four. In the skeleton for all cases of Diagram 3, there are seven planning variables (C, I, E, Y, M, S, A), bounded by four accounting equations (one defined for each vertex). We know that one accounting equation is a dependent equation and can be deduced from the other accounting equations for an Euler graph (see Theorem (1) of Section 2.2). Thus, there are only three independent accounting equations; i.e., one less than the number of vertices. Hence, the number of exogenous variables is four, which is the difference between the number of variables (seven) and the number of independent accounting equations. The above reasoning leads us directly to posit the definition of a cyclomatic number for any connected linear graph.

Definition: The cyclomatic number of a connected linear graph is  $u = E - (V - 1)$  where  $E$  is the number of edges and  $V$  is the number of vertices.

Applying the definition to Diagram 3, we see that the skeleton is connected and that the cyclomatic number is 4. This seems to confirm our intuition that the number of variables in any set of exogenous variables is the same as the cyclomatic number. Furthermore, the number of endogenous variables is  $E - u = V - 1$ ; i.e., the number of endogenous variables is the

same as the number of independent equations (or one less than the number of vertices). We shall rigorously investigate these assertions later.

We have emphasized the importance of determining the values of endogenous variables once the values of exogenous variables are given. For this purpose we introduce the notion of the causal order of the variables involved.

### 3.3 Causal Order

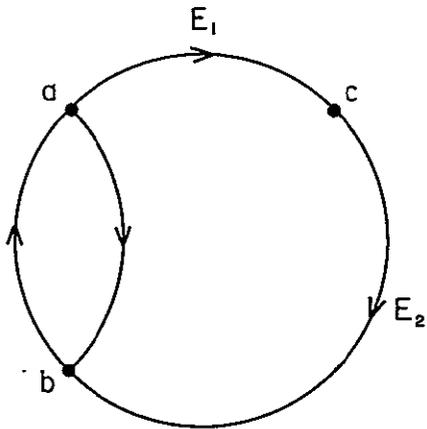
Assuming that a set of exogenous variables always contains  $u$ -edges (the cyclomatic number), it is not true that any set of  $u$ -edges can be arbitrarily chosen to form a set of exogenous variables. For example, in Diagram 7a where  $u = 2$ , the set of edges  $(E_1, E_2)$  cannot be a set of exogenous variables because the value of  $E_1$  is always equal to the value of  $E_2$  in an Euler graph. Hence, two numbers cannot be assigned to them arbitrarily. Similarly, in Diagram 7b,  $E_1$  and  $E_2$  cannot both be exogenous variables at the same time (see below). Thus, only certain sets of  $u$ -edges, to be called a basic edge set, can qualify as a set of exogenous variables.

We introduce the following definitions with the aid of the linear graph concept, circle, defined earlier (see Diagram 5b):

Definition: A linear graph is circle-free if it has no circle.  
A linear graph is a tree if it is circle-free and connected.

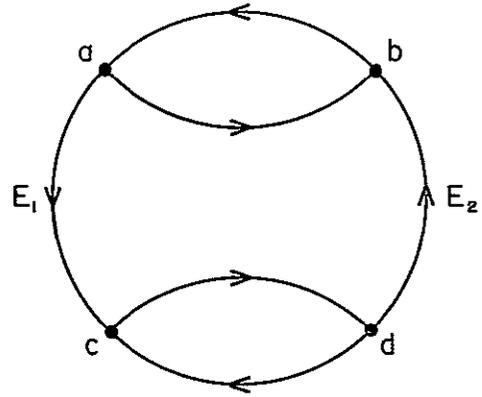
Diagram 8a is an example of a directed linear graph which is a tree; i.e., it is circle-free and connected. From this example, it is not

# Diagram. 7: Edge Set Examples



$$\mu = 2$$

(a)



$$\mu = 3$$

(b)

Diagram 8: Determination of Endogenous Variables by Causal Order

Tree	<p>(a)</p>	<p>(b)</p>	<p>(c)</p>	<p>(d)</p>
End Vertices	a, b, c, d	e, f, g	h, i	j, k
End Edges	$T_a^1, T_b^1, T_c^1, T_d^1$	$T_e^2, T_f^2, T_g^2$	$T_h^3, T_i^3$	$T_j^4$
Maximum Tree (solid) and Basic Edge Set (dotted)	<p>(a')</p> <p><math>M = 18 - 11 + 1 = 8</math></p>	<p>(b')</p>	<p>(c')</p>	<p>(d')</p>
Edge Value Determined	$T_a^1(-2)$ $T_b^1(2)$ $T_c^1(-13)$ $T_d^1(1)$	$T_e^2(6)$ $T_f^2(6)$ $T_g^2(1)$	$T_h^3(4)$ $T_i^3(-10)$	$T_j^4(4)$

difficult to see that a tree always has an end edge; e.g., edges such as those denoted by  $T_a^1, T_b^1, T_c^1, T_d^1$ . An end edge is an edge which touches a vertex (e.g., a, b, c, and d in Diagram 8a) which is touched by only one edge. Rigorously we define:

**Definition:** A vertex x of a directed linear graph is an end vertex if there is only one edge (i.e., in the form of  $(x, a)$  or  $(a, x)$ ) touching it. The edge that touches an end vertex is an end edge.

It is easy to see that a tree has at least one end edge. Suppose this is not true (i.e., suppose a tree has no end edge for the sake of argument, by reductio ad absurdum). Starting from any edge of a tree, one can first "go in" to a vertex, x, and if x is not an end vertex, we can "go out" of the vertex x through another edge to a vertex y. Now if y is not an end vertex, we can go out of y and reach another vertex, z. Repeating this argument, we can then construct a link with ever-increasing length, going through x, y, z... Now if there is only a finite number of vertices in the linear graph, a circle will sooner or later be formed, contradicting the fact that there is no circle in the tree. Thus, a tree must have at least one end edge. Next, we see that when an end edge of a tree is deleted from the tree, the remaining edges must again form a tree (i.e., the remaining linear graphs are still connected and circle free) and the tree retains the same cyclomatic number. These elementary facts may be summarized as:

Lemma 1a: A tree has at least one end edge.

b: If an edge of a tree is deleted, the remaining edge will again form a tree with the same cyclomatic number as the original tree.

Notice that in Lemma 1b, the shorter tree remaining after deleting one end edge obviously has the same cyclomatic number as the original tree since both the number of edges and the number of vertices are reduced by one.

In Diagram 8a, b, c, d, we illustrate the repeated application of the above lemma in four steps. In each step, all the end edges are first identified and then deleted to form a new tree. In Diagram 8a, the end vertices and end edges are, respectively, a, b, c, d, and  $T_a^1, T_b^1, T_c^1, T_d^1$ . When these are removed, we have the tree shown in Diagram 8b. The end vertices and end edges of this tree are, respectively, e, f, g, and  $T_e^2, T_f^2, T_g^2$ , which, when removed, leave the tree shown in Diagram 8c. This process can be repeated until finally only one edge (or one vertex) is left.

Symbolically, the end edges removed successively may be assigned a causal order, as follows:

Causal Order 1:  $T_a^1, T_b^1, T_c^1, T_d^1$  (end edges removed in first step)

Causal Order 2:  $T_e^2, T_f^2, T_g^2$  (end edges removed in second step)

Causal Order 3:  $T_h^3, T_i^3$  (end edges removed in third step)

Causal Order 4:  $T_j^4$  (end edge removed in fourth step)

When any tree is given, a causal order can be assigned to every edge in this way. Notice from the above deletion process that as the end

edges are successively deleted, the number of remaining edges gradually decreases. The process will sooner or later come to a halt when finally one single vertex is left. Now for a linear graph containing a single vertex, the cyclomatic number is obviously zero (i.e.,  $u = E - (V - 1) = 0 - (1 - 1) = 0$ ). Since the cyclomatic number is not affected when an end edge is deleted, we see that the cyclomatic number of any tree is zero. This may be summarized as:

Lemma 2: The Cyclomatic Number of a tree is zero.

This lemma can be verified for all the trees in Diagram 8a, b, c, d; i.e., the cyclomatic number of all these trees is zero.

### 3.4 Basic Edge Set

A connected linear graph which is not a tree will contain some sub-graphs which are trees. The following definition emphasizes this phenomenon.

Definition: A subset of edges, T, of a linear graph, G, is a maximum tree if T is a tree which touches every vertex of G. The edges of G not in T form a basic edge set.

In Diagram 8a' in the lower deck, the solid edges form a maximum tree (in fact, the same tree shown in Diagram 8a, in the upper deck, for which a causal order analysis has been made above). All vertices, a to k, are touched by edges in this tree. The remaining edges (dotted), therefore, by definition form a basic edge set. We observe that the cyclomatic number

of this linear graph,  $u = 8$ , is equal to the number of edges in the basic edge set (i.e., the dotted edges). This is an example of the following theorem:

Theorem (4) : If a connected linear graph,  $G$ , has  $V$  vertices and a cyclomatic number  $u$ , every maximum tree of  $G$  has  $V-1$  edges and, hence, every basic edge set has  $u$  edges.

To prove this theorem, let  $T$  be a maximum tree and let  $B$  be the set of edges  $(E_1, E_2, \dots, E_r)$  of a linear graph,  $G$ , not in  $T$ . (We want to show that  $r = u$ .) From Lemma 2 we know that the cyclomatic number of  $T$  is zero. We may successively add the edges  $E_1, E_2, \dots, E_r$  to  $T$  until the linear graph becomes  $G$ . Notice that when an edge is added in this way, the cyclomatic number increases by one because the number of edges increases by one while the number of vertices remains unchanged (because  $T$  is assumed to touch all vertices). This proves  $r = u$ .

### 3.5 Accounting Consistency: General Solutions

We have now developed sufficient linear graph concepts to provide satisfactory general answers to all three aspects of the problem of accounting consistency in planning (see Section 3.1). We again assume that a linear graph,  $G$ , is given as the skeleton of a national income accounting system. The following theorem can be stated:

Theorem (5) : Let a connected linear graph G be given. A set of edges of G form a set of exogenous variables if, and only if, they form a basic edge set (i.e., if and only if the endogenous variables form a maximum tree of G).

The validity of this theorem can be seen from the examples in Diagram 3. In each case we see that the endogenous variables (dotted edges) form a maximum tree. By assigning arbitrary values to all exogenous variables, we can uniquely determine the values of the endogenous variables. Together with Theorem (4) we see that every set of exogenous variables must have exactly  $u$ -variables ( $u$  being the cyclomatic number) and every set of endogenous variables must have  $(V - 1)$  variables ( $V$  being the number of vertices).

We discuss the sufficient condition of this theorem by using an example, the principles of which can be easily generalized. The necessary condition is proved in the mathematical appendix.

The dotted edges in Diagram 8a' are a set of basic edges, i.e., the exogenous variables. Let values be assigned to these variables as indicated by the encircled numbers. We now seek to determine the values of the endogenous variables (the solid edges which form a maximum tree), according to the causal order shown in Diagram 8a, b, c, d.

We begin with edge  $T_a^1$  in Causal Order 1.  $T_a^1$  is the only endogenous variable touching the end vertex "a." By requiring that vertex "a" be in balance, the value of  $T_a^1$  is uniquely determined as  $-2 (= 3 - 5)$ . In this way, the values of all edges in Causal Order 1 ( $T_a^1, T_b^1, T_c^1, T_d^1$ ) can be determined, as indicated at the bottom of Diagram 8a'.

We then proceed to Causal Order 2 (shown in Diagram 8b). All endogenous variables in this causal order ( $T_e^2, T_f^2, T_g^2$ ) are determined by the same principle; i.e., of all the endogenous variables of Causal Order 2 or higher, each  $T_i^2$  is the only endogenous variable touching the i-th end vertex. Proceeding through each causal order in this way, we can determine uniquely the values of all endogenous variables in r steps, where r is the maximum causal order. We can be certain that an Euler graph is formed since the endogenous variables form a maximum tree and, hence, every vertex is in balance. Hence, the sufficient condition of Theorem (5) is proved.

In Section 1 we posed the problem of accounting consistency as a fundamental issue in planning methodology. To formulate the problem in general terms and to provide satisfactory analysis of this issue, we undertook development of the requisite techniques in Sections 2 and 3.

Our particular focus is still addressed to the three questions raised (in Section 1.3) in regard to accounting consistency:

1. How many variables are there in a set of exogenous variables?
2. What types of sets form sets of endogenous or exogenous variables?
3. Given pre-assigned values for exogenous variables, how can the values of endogenous variables be uniquely determined?

The answer to the first question is that every set of exogenous variables equals u, where u is the cyclomatic number. The answer to the second question is that a set of endogenous variables must form a maximum tree or, equivalently, a set of exogenous variables must form a basic edge set. It is now apparent that only certain

types of sets qualify to represent a set of exogenous or endogenous variables. Referring to Diagram 7a, we now see that the set of  $(E_1, E_2)$  edges (though equal to the cyclomatic number) is not a set of exogenous variables because the other edges do not form a maximum tree. Similarly in Diagram 7b, the two edges  $(E_1, E_2)$  cannot belong to any set of exogenous variables since the remaining variables would be disconnected. Finally, we have seen that, given pre-assigned values for exogenous variables, unique values of endogenous variables are logically determined according to a particular causal order emerging from the successive removal of end edges from the maximum tree representing the set of endogenous variables. This answers the third question.

In Section 2 we gave an abstract formulation of national income accounting systems. That formulation provided the background for a general solution, in this present section, to the problem of accounting consistency for planning. The analysis in this and the previous sections (2 and 3) will be useful in evaluating the work of the planning school in the next section. We will also find the principles developed to be indispensable for some parts of our later analysis in this study. We return to applications of these principles in our own work in Chapter 6.

We are now ready to return to the main theme of this chapter, the evaluation of the planning school's methodology from the viewpoint of growth theory. Having discussed the first pillar in the planner's framework, the requirement of accounting consistency, we shift our

attention in the next section from accounting relationships to behavioral relationships. The use of behavioristic assumptions is the second basic ingredient in planning methodology, and investigation of the nature and operational significance of these assumptions is essential to our evaluation.

#### 4. PLANNING MODELS

##### 4.1 Types of Planning Models

In Section 1 we made a preliminary and general survey of the planning school, emphasizing major features of this approach, including its resource-oriented growth philosophy, the formalism and judgment aspects of its methodology, its data consciousness, and its policy sensitivity. In Sections 2 and 3 we concentrated upon two important conceptual tools, the national income accounting system and accounting consistency. We are now ready to discuss the more formal part of planning methodology as reflected in actual planning models. In this present section, we review various types of planning models, introducing several classification criteria. In this review, particular stress is placed upon behavioristic assumptions which play a crucial role in many of the planners' models.

Classification is needed because of the proliferation of planning models in the recent past, as well as to provide a system for discussing "sample models" of each important class. We do not attempt an exhaustive coverage. Our review is selective in nature, and it is designed to illustrate the various ways by which planners have attempted to apply their resource-oriented growth philosophy in planning for the economy as a whole.

We begin with the distinction, introduced in Section 1, between aggregate models and disaggregated models. This distinction is based on the national income accounting structure of models, as shown in Diagrams 1 and 2. We have seen that the disaggregated, or n-sector, model is

appropriate for investigating the resource consistency problem at the interindustry level. For this disaggregated type of model, we adopt a further distinction between those which are descriptive and those which are prescriptive in nature. The descriptive n-sector model, a direct descendant from input-output economics, is concerned with the consistency of resource allocation when final demands are specified. The prescriptive n-sector model, by contrast, is an optimizing model in which the objective of economic welfare receives explicit formulation. We present a descriptive n-sector model in Section 4.2 and a prescriptive n-sector model in Section 4.6.

Aggregate planning models are based upon an aggregative national income accounting framework such as the one shown in Diagram 1. Since that framework postulates foreign planning variables--imports (M), exports (E), and foreign aid (A)--as well as domestic planning variables--national income (Y), savings (S), investment (I), and consumption (C)--it may be considered as a model of an open economy. To use this model for planning, a set of behavioristic assumptions is needed. We have briefly discussed in Section 1.6 the intellectual origins of typical assumptions used by planners. These will be formally presented in Section 4.3. We distinguish two types of aggregate planning models which have been built with the aid of these assumptions: formal planning models and strategy models. Ideally, strategy models should precede the use of formal planning models. Strategy models are designed to identify, in a relatively informal way, major development bottlenecks confronting a particular economy. The

development strategy thus formulated may then be used in the choice of the appropriate formal planning model for actual planning operations.

Implicit in this approach, combining strategy and formal planning models, is the notion that particular planning models are appropriate to certain types of economies, the latter revealed by the strategy model's identification of the economy's bottleneck configuration. In this way, a typology approach to development planning is envisaged. We discuss this very special planning approach to typology in Section 4.4 while strategy models are reviewed in Section 4.5.

We find it useful to classify planning models in yet another way, based on the formality given to the treatment of "time." A dynamic model is one in which the time dimension is non-trivial and formally recognized. In contrast, when the time dimension is treated informally, the planning model is considered to be a projection model used for projective planning. This distinction between dynamic and projective can be applied to all planning models. For example, the descriptive type of disaggregated model presented below in Section 4.2 is a projective model while the disaggregated optimizing model discussed in Section 4.6 is a dynamic one. Among aggregated models the formal planning models reviewed in Section 4.4 are dynamic, while the strategy model in Section 4.5 is projective because of its more informal nature.

## 4.2 Disaggregated Models

The first important class of planning models to be considered is the disaggregated type which has grown out of the input-output tradition. Their model structure is described by the national income accounting system of Diagram 2, which clearly shows that the analytical emphasis is on the interindustry relationships among a large number of relatively small production sectors. As adopted by the planners, the model is mainly employed as a framework for consistent planning of resource allocation among production sectors of the economy.

We consider a typical planning problem of this kind. Let a future year be designated as a target year, for example, the last year of a five-year planning horizon. A consistent projection is to be made for this target year for all planning variables contained within the framework of Diagram 2. We distinguish the demand for net output of the production sector from the other variables of the system, as is common in the input-output approach. The causal order of planning then proceeds by the following steps:

Step 1: The demand for net output--investment ( $I_1, I_2, I_3$ ), consumption ( $C_1, C_2, C_3$ ), and exports ( $E_1, E_2, E_3$ )-- is planned independently, usually by a projection device.

Step 2: The other variables are planned by using the accounting and production assumptions of the input-output model.

To illustrate the applicability of the techniques developed in Section 3 to this planning problem, we observe from Diagram 2a that the five variables,  $V_1$ ,  $V_2$ ,  $V_3$ ,  $A$ , and  $S$  (i.e., the value added of the three production sectors, foreign aid, and domestic savings), form a maximum tree. This is true since the six vertices,  $(P_1)$ ,  $(P_2)$ ,  $(P_3)$ ,  $(H)$ ,  $(F)$ , and  $(Z)$ , are connected by these variables, and there is no circle involved. The set of remaining variables is then a set of exogenous variables.

We know from our previous discussion that if we can determine the values of the exogenous variables, we can determine the values of all the remaining (endogenous) variables. Let us classify the exogenous variables into two types:

Step 1: Final demand: consumption ( $C_i$ ), investment ( $I_i$ ), and export ( $E_i$ ) (totalling nine variables).

Step 2: Interindustry flows ( $x_{ij}$ ) and imports ( $M_i$ ) (totalling twelve variables).

When the values of these 21 exogenous variables are estimated (projected) for the target year, the values of the endogenous variables ( $V_1$ ,  $V_2$ ,  $V_3$ ,  $A$ , and  $S$ ) can be routinely calculated for that target year.

In the first step, the three components of final demand are projected for the target year. Export demand ( $E_i$ ) is obtained by a simple extrapolation of the past trend as representing the estimated export potential. To project consumption ( $C_i$ ) we may, for example, first calculate

the value of aggregate income (i.e., national income) for the target year, by imposing a politically determined growth target for the intervening years. The individual consumption components ( $C_i$ ) can then be determined under the assumption, for example, that the present consumption coefficients will also prevail in the target year. Projection of investment ( $I_i$ ) may also be done by a simple extrapolation from the past trend interpreted as representing the growth of "absorptive capacity." Note that in each of these projection devices there is a large amount of judgment and even, perhaps, expediency. This exemplifies the point we made in Section 1.4 that a heavy judgment ingredient is always found in the planning school's practical work. The second step is estimation of the interindustry flows ( $x_{ij}$ ) and import demand ( $M_j$ ) for the target year. Estimation of these twelve variables is based on the production assumptions highlighted in the input-output tradition. With the concept of total output ( $X_1, X_2, X_3$ ) defined as in equation (3a),<sup>10</sup> demand for intermediate factors of production and demand for imports are based on simple proportionality assumptions:

$$4.1.a) \quad x_{11} = a_{11}X_1, \quad x_{12} = a_{12}X_2, \quad x_{13} = a_{13}X_3$$

$$x_{21} = a_{21}X_1, \quad x_{22} = a_{22}X_2, \quad x_{23} = a_{23}X_3$$

$$x_{31} = a_{31}X_1, \quad x_{32} = a_{32}X_2, \quad x_{33} = a_{33}X_3$$

$$b) \quad M_1 = m_1X_1, \quad M_2 = m_2X_2, \quad M_3 = m_3X_3$$

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<sup>10</sup>Notice that the introduction of  $X_1, X_2, X_3$ , and the accounting assumptions in equation (3a) add an equal number of variables and accounting equations and, hence, will not affect our previous reasoning.

These production behavior assumptions are taken directly from the input-output tradition. Both input coefficients ( $a_{ij}$ ) and import coefficients ( $m_i$ ) simply state that the essential characteristic of production is that any particular input is always proportional to total output ( $X_i$ ).

These 12 production behavior assumptions are used to determine the 12 interindustry flows ( $x_{ij}$ ) and imports ( $M_i$ ). Specifically, these assumptions and the projected values of final demand ( $\bar{I}_i$ ,  $\bar{C}_i$ , and  $\bar{E}_i$ ) are substituted in equation (3a) to obtain the following system of three linear equations in three unknowns,  $X_1$ ,  $X_2$ ,  $X_3$ :

$$4.2) \left\{ \begin{array}{l} (1 - a_{11})X_1 - a_{12}X_2 + (-a_{13}X_3) = \bar{I}_1 + \bar{C}_1 + \bar{E}_1 \\ (-a_{21}X_1) + (1 - a_{22})X_2 - a_{23}X_3 = \bar{I}_2 + \bar{C}_2 + \bar{E}_2 \\ (-a_{31}X_1) - a_{32}X_2 + (1 - a_{33})X_3 = \bar{I}_3 + \bar{C}_3 + \bar{E}_3 \end{array} \right.$$

When the values of total output,  $X_1$ ,  $X_2$ ,  $X_3$ , are determined by solving this system of equations, the values of the interindustry flows ( $x_{ij}$ ) and imports ( $M_i$ ) are then determined by the proportionality assumptions of equation 4.1. In this way all the 21 exogenous variables are determined. The values of the five endogenous variables (foreign aid, A, domestic savings, S, and the three components of value added,  $V_1$ ,  $V_2$ ,  $V_3$ ) are determined residually.

The input-output model just described may be regarded as the basic disaggregated model for development planning. There are many possible

ways to refine and extend the basic model structure, and some of these adaptations are discussed in Section 4.6.<sup>11</sup> The basic model presented suffices, however, to illustrate the essential characteristics of this type of planning approach, which we now summarize briefly.

Input-output analysis, as a branch of economic theory, is a general equilibrium approach, characterized by numerical strength and its emphasis on interindustry production relationships. Compared to other general equilibrium models (e.g., the Neo-Classical model, a la Hicks, or the more recent activity analysis variety),<sup>12</sup> the special feature of input-output models is that they can be implemented statistically with empirical data. In fact, their very emphasis on interindustry relationships requires extensive empirical work involving masses of statistical data. This numerical strength is achieved, however, at a price. Important aspects of the general equilibrium system--notably the consumption aspect and the income distribution aspect--cannot be adequately handled. The numerical strength of the input-output approach and its disadvantages are also apparent when this type of model is adopted for planning purposes.

Given the special numerical strength of input-output models, it is easy to understand their popularity among planners. Such models provide

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<sup>11</sup>For an example of an imaginative application of this approach, see Michael Bruno, Interdependence, Resource Use and Structural Change in Israel, Bank of Israel Research Department, Special Studies No. 2 (Jerusalem: Jerusalem Post Press, 1962).

<sup>12</sup>See, for example, J. R. Hicks, Value and Capital (Oxford: Clarendon Press, 1946) and Gerard Debreu, Theory of Value (New York: John Wiley and Sons, Inc., 1959).

numerical answers to the planners' central problem of resource consistency, though only at the interindustry level (and only on this particular problem). This feature conforms to two biases of the planning school, their penchant for policy results in numerical terms and their philosophy that production (rather than consumption or income distribution) lies at the heart of economic development. Thus, as inherited by planners, the special characteristic of the input-output approach is its emphasis on resource calculation supported by a massive empirical effort to generate data.

#### 4.3 Behavioral Hypotheses in Aggregate Models

Aggregate planning models are distinguished from the disaggregated type by the underlying assumption that there is only one aggregate production sector. Thus, in contrast to disaggregated models, aggregate models postulate no differentiation of production conditions within the production sector. The national income accounting structure of the most important aggregate planning model is shown in Diagram 1a. The skeleton of this model in Diagram 1b shows seven planning variables (I, C, E, Y, M, S, A) connected by four accounting equations (i.e., the four vertices). The model's cyclomatic number is  $u = 7 - (4 - 1) = 4$ . Hence, every possible set of exogenous variables contains four variables and every set of endogenous variables (i.e., every maximum tree) has three variables ( $v - 1 = 4 - 1 = 3$ ). These basic technical properties of this model should be kept in mind.

In Section 1.6 we discussed the behavioristic assumptions for aggregate planning models in general terms. We now examine these assumptions more systematically. Behavioristic assumptions commonly used by planners in connection with the aggregate model described in the above paragraph may be classified into five types. These five types represent "behavior" in respect to income disposition, production, investment, political factors, and economic geography.

Using the notation  $\dot{x}$  to denote the "rate of growth of the time variable x" (i.e.,  $\dot{x} = (dx/dt)/x$ ), we list these as:

#### 4.3 INCOME DISPOSITION BEHAVIOR

- a) Savings function:  $S = s^I + sY$  (s is the marginal saving ratio)

#### PRODUCTION BEHAVIOR

- b) Capital requirements:  $K = k^I + kY$  (k is the marginal capital-output ratio)
- c) Import requirements:  $M = m^I + mY$  (m is the marginal import coefficient)

#### INVESTMENT BEHAVIOR

- d) Absorptive capacity:  $I = I_0 e^{it}$  or  $\dot{I} = i$   
(i is the constant learning rate)

### POLITICAL BEHAVIOR

- e) Target growth rate:  $Y = Y_0 e^{vt}$  or  $\eta_Y = v$   
(v is the constant target growth rate for GNP)
- f) Growth rate of aid:  $A = A_0 e^{at}$  or  $\eta_A = a$   
(a is the constant growth rate of foreign aid)

### ECONOMIC GEOGRAPHY

- g) Export growth rate:  $E = E_0 e^{ut}$  or  $\eta_E = u$   
(u is the constant export growth rate)
- h) Population growth rate:  $L = L_0 e^{rt}$  or  $\eta_L = r$   
(r is the constant population growth rate)

In an earlier section we observed that several of the planners' behavioristic assumptions originate from related branches of economics, while others have been devised by planners themselves. The first assumption (a), the savings function, inherited from the Keynesian tradition, describes the availability of savings from income. The second assumption (b), capital productivity, is inherited from the Harrod-Domar type growth model and is often used in conjunction with (a) to formulate dynamic planning models. (See Section 4.4.) The third assumption (c), the import function, is inherited from the input-output tradition discussed in the previous section. The fourth assumption (d), absorptive capacity (i.e., the assumption that capacity to plan and execute investment projects is growing at a constant

rate) is more or less an invention of the planning school. The last four assumptions (e, f, g, h) are proxies for behavioristic assumptions, the realism and stability of which are taken for granted, with little or no inductive or theoretical support. Constant growth rates for these proxy variables are often postulated as a matter of expediency, in the absence of definitely contradictory evidence. The political behavior assumptions roughly summarize certain internal (e) and external (f) political forces affecting, respectively, the acceptability of the growth target and the availability of aid. The economic geography assumptions (g and h, respectively) summarize a wide variety of forces which determine the population growth rate and the economy's export potential.

It is apparent from the list of equations (4.3) that a large number of distinct planning models can be constructed from our example of the aggregate model structure (Diagram 1). Bearing in mind that only four (the cyclomatic number) behavioristic assumptions are needed to determine the system completely, the listing obviously allows many possibilities for particular models. In fact, planning models constructed by altering the behavioristic assumptions used may be interpreted as the planners' own brand of typology for development planning. Implicit in this procedure is the belief that there are many growth types, each requiring a specific set of behavioristic assumptions. We investigate growth models based on this notion of typology in the next section.

In the context of the aggregate planning model, the planners' data orientation takes the form of estimating the parameters of these behavioristic

equations (s, m, k, i, v, a, u, r) to determine their numerical value. Frequently, estimation is based on cross-section data (i.e., data from many less developed countries at a point in time), based on the assumption that the same behavioristic hypothesis is valid for all countries. Although the number of economic variables in the aggregate model is small, compared to those in the disaggregated model, this emphasis on parameter estimation and the cross-section procedure employed gives the aggregate model approach a mass data focus, also observed for disaggregated models.

A further observation applies to behavioristic assumptions for both aggregate models (listed in equations 4.3) and disaggregated models, given in equation 4.1. Behavioristic relationships for both are of the simple linear type (i.e., showing proportionality of incremental values). This simplicity is not accidental. Rather, it stems from the planners' insistence that behavioristic assumptions must be readily statistically implementable. This insistence, we believe, stems from the planners' overriding preoccupation with numerical policy results.

This penchant for ready numerical results frequently leads to naive methods for estimating the parameters involved in behavioristic assumptions. A "direct" approach to estimating each parameter, exemplified in the use of cross-section data, is customarily used. Such a direct approach naively ignores the statistical problem of identification. More precisely, this means that the behavioristic assumptions are assumed to be valid individually rather than valid within the context of a particular planning model structure.

The difficult problem of statistical identification, which arises when several behavioristic relationships are implicitly assumed to occur simultaneously in a model context, is seldom faced squarely.<sup>13</sup>

The various behavioristic assumptions reviewed in this section are the most important building blocks used by planners for constructing planning models. In accepting the validity of the behavioristic assumptions individually, the planners have borrowed or inherited what may be called "mechanisms" (and only these "mechanisms") from diversified areas of the economics discipline. For example, the savings function (4.3.a) was invented by Keynes and was intended to be used, together with other theoretical ingredients in Keynesian economics, for the purpose of studying a distinct social problem, the problem of unemployment in industrially-mature economies. In borrowing the Keynesian savings function, planners, however, have taken over only the explicit mechanism of the savings function while rejecting the context of Keynesian economics. The other theoretical ingredients and the sense of unemployment as the dominant social problem are considered irrelevant for planning. In a similar way, all the planners' behavioristic assumptions taken from other branches of economics are treated as individual mechanisms and removed from their original context. Thus, the heterogeneous intellectual origin of the planning school is a matter of its selective borrowing of these unrelated mechanisms.

As employed by planners, these mechanisms assume a position of dominant importance. They are accepted as immutable and appropriate to all.

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<sup>13</sup>We return to discussion of this problem in Section 4.5.

contemporary less developed countries. Less developed countries may differ, for example, in respect to cultural background and geographic characteristics, or they may represent different types of economies (large, small, open, closed, dualistic, monomorphic), or they may be at different stages of long-run development, as emphasized by the historical school. To planners, however, these differences are irrelevant to the validity of behavioristic mechanisms. The immutability and universality of these mechanisms must be assumed to enable the planner to use cross-section data covering all countries. The only relevant consideration is the availability of data to estimate the needed numerical values of the parameters in the behavioristic assumptions. In accepting behavioristic assumptions as fundamental mechanisms, the planning school is seen to embrace a mechanistic philosophy of economic growth. Growth is interpreted in terms of particular sets of these mechanisms. The operational implications are clear. Planning for growth consists of skillful manipulation of these mechanical relationships, giving their policy work a distinctly technocratic flavor. The combination of a mechanistic growth philosophy and a technocratic operational orientation is an outgrowth of the approach just described. This approach is based upon eclectic use of mechanisms from heterogeneous intellectual origins, an insistence on estimation procedures which produce numbers for these mechanisms and the related assumption of their universal validity. This mechanistic strain distinguishes the planning school's approach to development, marking it as both anti-historical and

anti-typological in nature. In discussing the planners' interest in planning typology in the next section, therefore, we stress that the planners' particular brand of typology conforms to their mechanistic view of economic growth.

#### 4.4 Planning Typology

We have seen in our review of the historical school that contemporary development economists increasingly recognize economic growth not as a unique experience but rather as a phenomenon comprising many different types. In this evolving typology approach, each growth type is considered to be subject to unique growth promotion forces and characterized by particular rules of growth. Planners have been influenced by this concern with typology, and their special application of the typology concept will be briefly examined in this section.

Disaggregated planning models, discussed in Section 4.2, are inappropriate for a typology approach to planning. In view of the large number of production sectors involved, these models become unwieldy unless intersectoral relationships portrayed are regular and symmetrical. This is clearly apparent in the input-output model, in which the production structure of an industry cannot be structurally distinguished from any other sector in the model. Growth typology, however, rests upon identifying asymmetrical relationships between a small number of large economic

sectors.<sup>14</sup> Hence, planning typology has been used exclusively in connection with aggregate planning models, in which the number of sectors is small.

To study planning typology, therefore, we accept the aggregate national income accounting framework of Diagram 1a. In addition, a number of behavioristic assumptions must be selected from a list of alternatives such as that given in equations (4.3) in the previous section. This selectivity permits the construction of a large number of models to depict various growth types. We illustrate some alternative growth types by presenting several planning models which have been actually put to use by planners:

- (i) The Harrod-Domar growth model
- (ii) The skill-limit growth model
- (iii) The saving-limit growth model
- (iv) The trade-limit growth model

The familiar Harrod-Domar model, originally constructed to study the problem of the stability of growth in industrially mature economies, has been borrowed by planners and has had profound effects upon their works. The other three models have been developed by representatives of the planning school.<sup>15</sup>

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<sup>14</sup>See our discussion of growth types (epochs) in Chapter 2 where the historical approach to growth is shown to involve such sectoring for typology.

<sup>15</sup>See, for example, the use of all three "limit models" in Hollis B. Chenery and Alan M. Strout, "Foreign Assistance and Economic Development," American Economic Review, Vol. LVI, No. 4, Part I (September 1966), pp. 679-733.

To facilitate our discussion of these models, we indicate their structure in the four rows, a, b, c, d, of Diagram 9. In each row, a particular model structure consists of a selected set of behavioristic assumptions selected from equations (4.3) and the skeleton of the aggregate national income accounting framework (Diagram 1b), common to all of these aggregate models. For each model shown in Diagram 9, the solid edges form a basic edge set, representing exogenous variables, and the dotted edges (forming a maximum tree) represent a set of endogenous variables. This classification immediately gives us a preliminary idea of the causal order employed by these types of planning models. We proceed to identify the growth promotion forces and to describe briefly the rules of growth for each model. We then discuss the methodological issues involved in application of these models for planning and the implications for the planners' view of typology.

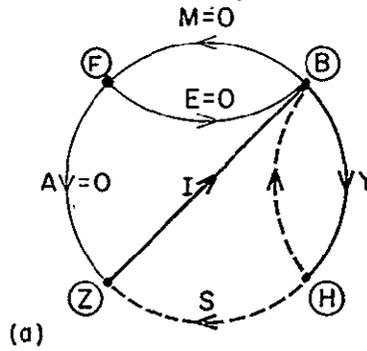
#### Harrod-Domar Model

The familiar Harrod-Domar model is a growth model for the closed economy. (Thus, imports, exports, and foreign aid are all set to be zero in Diagram 9a.) The two most essential assumptions of this model are given by the following pair of equations which are commonly used in combination to describe a stock-flow relationship:

$$4.4.a) \quad dK/dt = I$$

$$b) \quad K = kY$$

# Diagram 9: Examples: Planning School Growth Typology

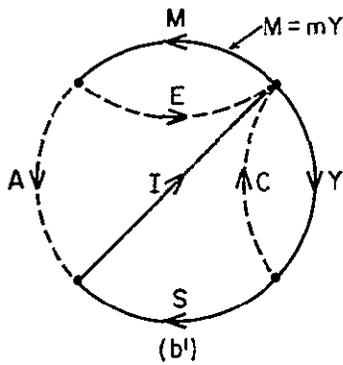


Major growth promotion force:

$$\frac{dk}{dt} = I, K = kY$$

$$S = sY$$

(a) a. Harrod-Domar Model



Major growth promotion force:

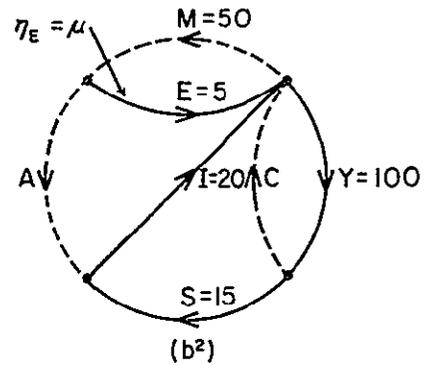
$$\eta_t = i$$

$$\frac{dk}{dt} = I, K = kY$$

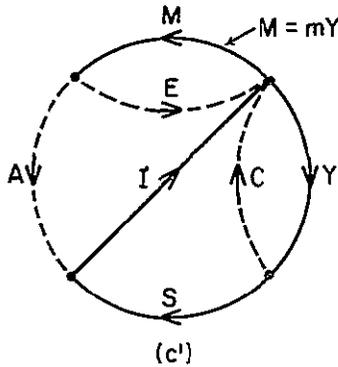
$$S = sY$$

(b')

b. Skill-Limit Model



(b²)



Major growth promotion force:

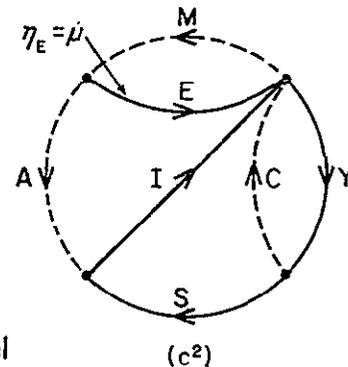
$$\eta_Y = v$$

$$\frac{dk}{dt} = I, K = kY$$

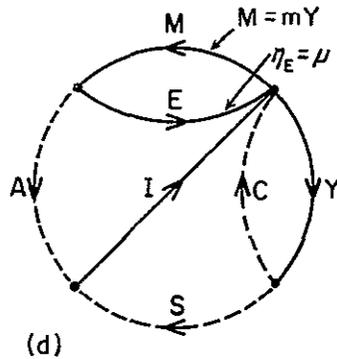
$$S = sY$$

(c')

c. Saving-Limit Model



(c²)



(d)

d. Trade Limit Model

Major growth promotion force:

$$\eta_Y = v$$

$$\frac{dk}{dt} = I, K = kY$$

The first equation (4a) is a dynamic accounting equation stating that investment (I) constitutes (i.e., is equal in magnitude to) the increment of capital stock (K) between two adjacent time points. Thus, the size of capital stock at any point in time is the sum of the accumulated value of investment. For this reason, we can deduce quantitatively the time path of investment and the time path of capital stock from each other. The second equation (4b) states that capital stock (K) and capacity output (Y) are proportional, the proportionality factor being the capital output ratio (k). This pair of equations mean that the time path of output (Y) and the time path of investment (I) mutually determine each other. It is for this reason that the two edges, Y and I, are selected as the exogenous variables in the skeleton shown in Diagram 9a.

The impact of the Harrod-Domar model on the thinking of the planning school is apparent from the fact that this pair of equations (4.4) is postulated for all four models in Diagram 9. (To emphasize this point, the pair of equations from (4.4) is marked off by a "box" in all four cases.) This reveals the planners' strong conviction that capital accumulation is a dominant growth promotion force, for in all four models the size of capital stock alone determines capacity output. Because the time path  $I(t)$  and  $Y(t)$  mutually imply each other, we can distinguish two types of planning models, representing different approaches:

Type 1: Needed Investment Approach: When  $Y(t)$  causally determines the needed investment stream,  $I(t)$ , to build up the required capital stock.

Type 2: Implied Capacity Approach: When  $I(t)$  causally determines capacity output,  $Y(t)$ , implied by the capital capacity built up by  $I(t)$ .

The significance of this distinction will be emphasized in our discussion of the other models in Diagram 9.

The second conspicuous aspect of the Harrod-Domar model is the postulation of a savings function ( $S = sY$ ) of the Keynesian type and the implied savings-pushed characteristic of growth. Under this assumption, capacity output ( $Y$ ), at each point in time, determines the amount of savings ( $S$ ) which, for the closed model, equals investment ( $I$ ). Hence, the size of the capital stock in the next period will be determined. This, in turn, determines the capacity output in the next period and in this way the growth path is dynamically determined. Thus, the pushing force exercised by saving is the crucial and, in fact, the only growth promotion force. It is apparent that this model implies that the economy possesses adequate entrepreneurial capacity to execute all of the investment projects needed to absorb the full capacity savings. It was recognized, however, that this may not be the case in a less developed country and that the ability to invest (i.e., absorptive capacity) rather than savings may be the crucial bottleneck. Hence, the skill-limit model was evolved by planners as an investment-pull growth type (in contrast to the Harrod-Domar savings-push type).

### The Skill-Limit Model

The skill-limit model recognizes that entrepreneurial capacities to undertake investment projects may be the bottleneck factor, and that overcoming this bottleneck through acquisition of skills by learning is a time-consuming process. Accordingly, this model assumes that investment,  $I$ , (construed as a proxy variable to measure investment ability) is growing at a constant rate,  $i$ , constrained as it is by limitations on learning capacity. The model, therefore, represents an implied capacity approach since the national income stream,  $Y(t)$ , is causally determined by investment,  $I(t)$  (through the stock-flow relationship in equation 4.4).

A second behavioristic assumption in this model is the Keynesian savings function ( $S = sY$ ), which in the next step causally determines the amount of savings through time,  $S(t)$ . However, the savings function has an entirely different growth significance here than in the Harrod-Domar model-- a point which might be easily missed by the casual reader. In the present case, the savings function has no growth significance at all! It is postulated to enable the planner to calculate the needed foreign aid stream,  $A(t)$ , as the difference between investment and savings; i.e.,  $A(t) = I(t) - S(t)$ . Thus, the ultimate objective of this model is to estimate needed foreign aid, calculated as an investment-saving gap in an open economy.

Two subcases ( $b_1$  and  $b_2$  in Diagram 9) are shown for the skill-limit (or investment ability-pull) model. In the first ( $b_1$ ), the import

function ( $M = mY$ ) is chosen as the next behavioristic assumption to determine the value of imports. This gives four pre-determined variables (I, Y, S, M), forming a basic edge set, and the model is dynamically closed. In this subcase, the endogenous variables, determined residually, are needed foreign aid, A, exports, E, and consumption, C. In the second ( $b_2$ ), the time path of exports,  $E(t)$ , is determined by projection. This closes the model as the four variables (I, Y, S, E) form a basic edge set, and the three endogenous variables (A, E, C) are determined residually.

These two subcases represent another aspect of an economy's openness; i.e., that aspect associated with trade rather than aid. The first sub-model assumes that the country has some capacity for export promotion; i.e., that it is capable of exporting the volume of goods implied by the endogenous variable, E. The second sub-model assumes some degree of import substitution capacity to enable the country to live with the volume of imports, determined as an endogenous variable. In both cases, however, the investment pull is the basic growth-promotion force, while a finer distinction is made between a country's relative strength to promote exports or to engage in import substitution.

#### The Saving-Limit Model

The saving-limit model shown in Diagram 9c gives some prominence to political and institutional forces in less developed countries by permitting specification of planned targets, usually in terms of target rates of growth of national income ( $\dot{Y} = v$ ). Since a target growth path of

national income,  $Y(t)$ , is implied, we may think of this as a growth target model. It also implies a needed investment approach; i.e., the investment stream is determined by the stock-flow relationship in equation (4.4). Employing the Keynesian savings function ( $S = sY$ ), needed foreign aid is then determined as in the skill-limit model. Two subcases are also shown for the saving-limit model, depending upon whether an import function (Diagram 9c<sub>1</sub>) or an export projection (Diagram 9c<sub>2</sub>) is used to close the model. Both operate similarly to the two subcases discussed for the skill-limit model.

The skill-limit model and the saving-limit model differ from each other only by reversing the causal order of determination between  $Y(t)$  and  $I(t)$ . As a result of this difference, the first is characterized by an investment-pull growth promotion force, while in the latter the politically-determined target is the growth promotion force.<sup>16</sup> (In terms of our distinction apropos the Harrod-Domar model, the skill-limit model is an implied capacity approach and the saving-limit model, a needed investment approach.) They are similar in that both determine foreign aid as a saving-investment gap and both rely upon either the presence of export promotion or import substitution capacity. In other words, there is an underlying assumption that the country already possesses the ability to deal with balance of payments problems (should they arise) either through increasing exports or decreasing imports. If, however, neither of these

<sup>16</sup>Thus, Chenery's saving-limit characterization of this latter model is somewhat a misnomer.

abilities is present, the planner will then view foreign exchange as the critical bottleneck. For this type of situation the trade-limit model was developed.

### The Trade-Limit Model

In the trade-limit model, the planner recognizes a country's limitations in respect to both import substitution and export promotion, as well as the country's need (or desire) to grow by a planned target. As in the skill-limit model, a targeted rate of growth of national income,  $Y(t)$ , is given, determining the investment stream,  $I(t)$ , by the stock-flow relationship. The model then postulates both an import function ( $M = m' + mY$ ), with the aid of which  $M(t)$  is determined, and the projection of export streams,  $E(t)$ . The endogenous variables are thus seen to be foreign aid, A, savings, S, and consumption, C.

The basic difference between the skill-limit or saving-limit model and the trade-limit model is that foreign aid is determined as a saving-investment gap in the former but as an import-export gap in the latter. Related to this difference is the implicit understanding that foreign exchange will not be a critical bottleneck in countries where the skill-limit or saving-limit model is appropriate because of export promotion and/or import substitution capabilities. Similarly, for countries where the trade-limit model is appropriate, it is implicitly assumed that both investment and savings capabilities are adequate. The essence of the "limit

model" approach, therefore, is that planners concentrate planning efforts upon what are deemed to be the bottleneck factors, while factors judged to be non-binding are neglected.

#### The Discretionary Use of Behavioral Assumptions

In the application of planning models, the use of behavioral assumptions, of the types described in Section 4.3, involves a large input of discretionary judgment on the part of the planner. This aspect of the aggregate planning technique can be best appreciated by applying the technical results of our previous analysis developed in Section 3.

We have seen from that analysis that the distinction between exogenous and endogenous variables amounts to a causal order distinction. With exogenous variables identified as a set of basic edges, it has been shown that they are of a higher causal order than the set of endogenous variables defined as a maximum tree. Once values are specified for the set of exogenous variables, values of endogenous variables are determined routinely as residuals.

The discretion exercised by planners in choosing among the various models we have just described is essentially a matter of such a distinction between exogenous and endogenous variables. In their approach, the procedure is reflected in their making subtle distinctions among behavioristic assumptions such as those listed in Section 4.3. Each variable in the planning model is depicted as governed by a specific behavioristic force. In utilizing these behavioral relationships in the

context of a particular planning model, however, certain are selected as relatively unyielding or "hard," and these are conceived of as the bottleneck factors. Others are judged to be relatively flexible or "soft." Planning effort is directed mainly to the "hard" bottleneck factors.

In our interpretation, these distinctions represent a choice between exogenous and endogenous variables. Those variables controlled by relatively rigid behavioral forces are exogenous; they are of the highest causal order and, as such, impose limits on the values of the endogenous variables in the system. In this sense, the exogenous variables are construed to be bottleneck factors. Endogenous variables are those controlled by less rigid behavioral forces, and the planner assumes that their ex post values can be brought into line with their projected values, determined residually, because the behavioral forces controlling them are relatively flexible.

We demonstrate these observations by use of an example. In Model 2b of Diagram 9, the skill-limit model, imports (M) are considered an endogenous variable. To illustrate the conceptual treatment of this variable, let us assume the following numerical values for the parameters of the behavioristic equations governing the exogenous variables.

$s = .15$   
(saving rate)

$k = 3$   
(capital-output  
ratio)

$i = 2\%$   
( $\eta_I$ )

$u = 5\%$   
( $\eta_E$ )

Knowing the current (initial) values of investment ( $I_0$ ) and exports ( $E_0$ ), we can calculate, for any future year, the values of exports (e.g.,  $E = 5$ ) with the aid of  $u = 5\%$  and  $E_0$ . We can also compute values of capacity output (e.g.,  $Y = 100$ ) with the aid of accumulated capital capacity (using  $k = 3$ ,  $i = 2\%$ , and  $I_0$ ). Thus, the major growth promotion force (i.e., the investment capacity pull) allows us to determine the value of  $Y$ . We also know the value of investment (e.g.,  $I = 20$ ), with the aid of  $i = 2\%$ ; and savings (e.g.,  $S = 15$ ) with the aid of  $s = .15$ . The values of the four exogenous variables ( $Y = 100$ ,  $I = 20$ ,  $E = 5$ , and  $S = 15$ ) are thus first determined by formally taking into consideration the "hard" or unyielding behavioristic forces involving them.

Notice that the endogenous variables ( $A$ ,  $M$ ,  $C$ ) now form a maximum tree. Their values can now be calculated according to the procedure described in Section 3.5, according to their causal order, in a residual and routine manner. In this way we determine  $C = 85$  (i.e.,  $100 - 15$ );  $A = 5$  (i.e.,  $20 - 15$ ); and  $M = 10$  (i.e.,  $5 + 5 = 10$ ). We observe that none of the behavioristic assumptions listed in 4.3 are formally used in the computation of these endogenous variables. This implies that those behavioristic forces are judged to be flexible or not effectively binding. For example, suppose that the import function is  $M = .08Y$  (i.e., the average propensity to import is eight per cent, then needed imports, from the ex ante behavioristic viewpoint, equal 8, which is adequately covered by the projected value (10), and, hence, the value of 8 is not binding. Conversely, suppose  $M = .12Y$ ; then needed imports from

ex ante behavioristic considerations is 12, exceeding the projective value of 10. However, now, in the judgment of the planner the deficiency can be corrected by an independent set of policy devices (e.g., import controls, import substitution, etc.) to bring about a change in the behavioristic force (in this case, the propensity to import) so that the ex post value for M can be brought into line with the projected value.<sup>17</sup> The same principle applies to C and A, the other endogenous variables. In short, in the judgment of the planner, these endogenous variables do not constitute serious development bottlenecks.

The notion that some behavioristic assumptions are hard and unyielding (and thus more serious) while others are soft or weak (and hence more tractable) is basic to certain recent trends in the evolution of planning methodology. The distinction follows readily from the planners' mechanistic view of growth based upon indiscriminate acceptance of a wide variety of mechanisms (such as the list presented in (4.3) above), all regarded as more or less relevant to the operation of the economy at a given point in time. This distinction among behavioristic assumptions, however, need not arise (and would, in fact, be meaningless) if the more familiar method for construction of unambiguously determined models were employed; i.e., accepting an equal number of equations and unknowns.<sup>18</sup>

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<sup>17</sup>The distinction between ex ante and ex post values of planning variables is the essence of the "two gap approach," popularized in recent years. For an evaluation of this approach from another viewpoint, see the article by John C. H. Fei and Gustav Ranis in American Economic Review, September, 1968.

<sup>18</sup>Equivalently, this condition is that the number of behavioristic assumptions must be the same as the cyclomatic number,  $u$ .

The departure leading to a distinction stems from the planners' mechanistic inclination leading to respect for all mechanisms (acceptance of all behavioristic assumptions) as a general principle. This orientation implies a view of a "mechanism space," the set of all possible mechanisms. The choice of a particular model to describe growth reality must be argued in the context of that total "mechanism space" to provide confidence in the result.

The acceptance of such a mechanism space containing more equations than are needed for determination purposes leads once again to an inevitable and important aspect of judgment so characteristic of the planners' approach. The choice between rigid and flexible behavioristic forces, of necessity a very complex matter, precedes and determines the entire planning procedure. It would appear that there is, as yet, no scientific method to guide such choice;<sup>19</sup> and while knowledge of innumerable characteristics of a country may be helpful, a large input of discretion will always be present.

#### The Operational Meaning of the Planners' Typology

The idea of growth typology, as found in the intellectual and practical work of planners, recognizes differences in growth promotion forces among countries and for different stages in a country's development. These differences are judged to be traceable to variations in societies'

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<sup>19</sup>See, however, Section 4.5 below.

capacities in regard to several economic and political components of development. The choice of a particular model depicts the operation of the system as driven by the major growth promotion force; for example, "the savings-push" nature of the Harrod-Domar model; the investment ability or "the investment-pull" nature of the skill-limit model; and the politically-determined "target-push" of the other two models, the saving-limit and the trade-limit model. We note that, in the "limit model" approach, these primary growth promotion forces are always conceived of as domestic growth forces affecting the domestic variables. A second set of forces, some domestic and some foreign, affect the foreign sector variables (foreign aid, exports, and imports). Differences here focus upon abilities to promote exports, substitute imports, and attract foreign aid. The superimposition of these two types of forces are then used to produce a wide variety of growth situations. The operational implications for development planning are that, first, the bottleneck factor(s) must be selected and, second, that the bottleneck factor must be emphasized (or given a higher causal order) in planning. The models employed then give concrete numerical answers to the key policy issues implied by this choice; for example, the prominent emphasis on needed foreign aid in all the open models examined above.

Given the wide choice of behavioristic forces postulated in equation (4.3), many aggregate models can be built for any open economy. Each possibility represents a particular growth type, determined by a special set of growth characteristics. In addition to the "needed foreign aid" type, considered above, an "available foreign aid" type is possible.

More systematically, we can distinguish a "savings-push" type, an "investment-pull" type, and a "politically-induced" type as the three major types of growth originating from domestic origin. In addition to these types, discussed above, three alternative types can be envisaged for an open economy, depending on whether growth is "export-pushed," "import substitution-led," or "foreign aid-dominated." We see that a model space would be an apt description of the set of all possible models generated in this way. At the present time, a systematic planning taxonomy, involving identification and classification of all significant cases in the model space, has not been explored by the planners to any significant extent.<sup>20</sup>

Although a large number of alternative models can be constructed in this way, planners have a definite preference for the particular type of model which is addressed to a key policy issue. For example, planners have tended to adopt a "needed foreign aid model" rather than an "available foreign aid model," the difference being that the latter postulates the availability of foreign aid as a hard "inflexible" condition. Thus, the needed foreign aid approach relates directly to the moot development issue in postwar assistance strategy and politics of deciding how much assistance a country will need. This orientation toward policy with a strong pragmatic

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<sup>20</sup> A first effort has been made in John C. H. Fei and Gustav Ranis, "A Study of Planning Methodology with Special Reference to Pakistan's Second Five-Year Plan," loc. cit.

undertone has a sharp anti-historical bias. Policy models, as illustrated by those examined in this section, tend to be forward looking in order to provide practical advice. They are not designed to explain historical events.

In view of this problem of a large (and actually unknown) variety of growth models that may be adopted for planning, the need for guidance is pressing. Guidance is needed to identify precisely what growth model is, in fact, relevant to planning for a particular period; e.g., the 3-5 years immediately ahead. Unless a method is developed to help the planner to select the relevant model, effective planning cannot begin. What is needed, therefore, is another type of planning, which may be described as planning for development strategy. Such strategy planning should, ideally, precede actual planning operations so that the relevant planning model may be selected before plans are formulated. Growth typology must obviously be a major component in this kind of advance planning since growth bottlenecks likely to be confronted in the near future must be identified.

Despite the obvious importance of strategy planning, this area of knowledge has been largely ignored by development economists. Satisfactory methods have not been developed to solve--or even to examine--this problem systematically. One major obstacle to the study of growth types is statistical identification, a problem associated with the estimation of the parameter values for the set of all behavioristic assumptions employed; e.g., those listed in the equations in 4.3 ( $k, s, e, u, i, r, v...$ etc.).

Presumably, these parameter values must be estimated from the observed values of economic variables exhibited by the economy in recent

years (perhaps 5-10 years preceding the initiation of planning). The identification problem lies in uncertainty that the true ex ante values of all the behavioristic forces can be revealed by the observed ex post data. This impasse stems from the very notion of the "flexibility" of the behavioristic forces involved; i.e., in the possibility of divergence between the ex ante and ex post values of these parameters. We have seen that such flexibility is an essential part of the planners' thinking. In the models discussed above (with a cyclomatic number of four), four, and only four, behavioristic forces are, in effect, relevant. This means that the other behavioristic relations posited, though potentially relevant, are not effectively binding; hence, the time series data do not in fact reveal the ex ante forces involved but show their ex post adjustment to the four dominant behavioristic forces. However, if the planner knew what the four dominant behavioristic forces were in the past, he could estimate these four parameters. In fact, there is no basis for such judgment, so none can be estimated on the basis of identification.

Barred by these difficulties from formally solving the planning strategy problem, planners have resorted to more informal methods. One example is the hypothesis that a country may pass through a specific life cycle, in which successive stages of growth will regularly occur for a large number of contemporary less developed countries. Through inductive

analysis of statistical data from many countries, it is hoped that such a stage of growth sequence can be eventually deduced. In other words, a historical orientation has gradually crept into the planners' methodology in the form of this stages of the growth notion in regard to strategy planning. We turn to this topic in the next section.

#### 4.5 Models for Strategy Planning

One essential notion of development planning is its forward-looking characteristic, its inclination to envisage the future performance of the economy in a predictive or prescriptive sense. This is reflected in accepting as the planning horizon a finite number of future time periods (1, 2....n), where "1" is the initial year and "n" the terminal or target year. In a five-year plan, for example,  $n = 5$ . Planning models can be classified into two types on the basis of this time dimension; namely, dynamic and projective. The dynamic type is the more ambitious. A development plan, built with a dynamic model, plans not only for economic achievements at the target year but also for the time path for reaching the target year through the intervening years. A projective model aims merely at planning for the target year, ignoring (at least formally) the process by which the target objective is reached. The models reviewed in the last section are dynamic models, while the n-sector model, considered in Section 4.2, is a projective model since the time dimension is not explicitly specified.

We have seen that the function of strategy planning is to provide guidance for selecting dynamic models used for planning operations. By suppressing the time dimension, projective models become more manageable, avoiding the difficulties which plague dynamic models in such strategy applications. It is for this reason that projective models are suitable for strategy planning.

Our previous discussion of behavioristic assumptions suggests that the essence of strategy planning is the identification of those behavioristic assumptions that will be relevant and effectively in force during the planning period. In considering the list of behavioristic assumptions in Section 4.3, we noted that, in the context of a given model, only a part of these growth forces will be relevant. In the case of the aggregate planning model used in our example, only four are relevant since the cyclomatic number is  $u = 4$ . The remaining behavioristic forces, though potentially relevant, are not effective.

To investigate the nature of this problem, we can systematically list all the parameters in the behavioristic equations for our example:  $m, m', k, k', s, s', i, v, a, u, r, \dots$ . In order to evaluate which behavioristic equations are relevant or irrelevant, we must investigate the comparative magnitudes of all the parameters. Thus, the problem is one of examining the relationships among the values of these parameters. Heretofore, we have assumed that the values of the parameters are constant. To investigate the present problem we must allow the possibility that parameters may take on alternative values to define different types of inter-parameter relations. For example, if there are ten parameters, we may think of a ten-dimensional parameter space. As a point moves in this parameter space, the relative magnitude of all parameters changes. The problem of determining which behavioristic equations are effective and which are not depends on investigation of such relative magnitudes. A planning model which formally admits that the parameters of a system of behavioristic equations may

change is called parametric programming. This type of model can be (and has been) used in projective planning to identify the relevant planning strategy.

Because it is informal by its very nature, strategy planning is an area of knowledge for which we can say very little by way of generalization. For this reason, we review only one strategy planning model of the parametric programming type, actually employed for planning,<sup>21</sup> as an illustration of the essential notions involved in this method. The model to be considered is of special interest to us since it is concerned with the open economy, giving us the views of the planner on strategy for development of the type of economy studied in our book. We introduce this model in the following steps: (1) the model structure, (2) the method of parametric programming, (3) the application, and (4) summary and evaluation.

### The Model Structure

Let us accept the aggregate national income accounting structure of the open economy (Diagram 1a) containing seven planning variables (C, I, E, Y, M, S, and A) and having a cyclomatic number,  $u = 4$ . We make the following three behavioristic assumptions:

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<sup>21</sup> See Hollis B. Chenery and Michael Bruno, "Development Alternatives in an Open Economy: The Case of Israel," Economic Journal (March 1962), pp. 79-103.

4.5.a) Saving function:  $S = s' + sY$

b) Investment function:  $I = d' + dY$

c) Import function:  $M = u_e E + u_c C + u_i I$

Equation 4.5.a is the familiar Keynesian saving function.

Equation 4.5.b states that investment demand is linearly related to national income (Y) as a description of investment demand behavior (i.e., absorptive capacity). Equation 4.5.c is a general type of import function, stating that demand for imports (M) is sensitive to the three demand components of national income (E, C, I). Since only three behavioristic assumptions are postulated, the system is, as yet, not determined (the cyclomatic number is  $u = 4$ ).

In this model, once we know Y (national income), we can immediately calculate foreign aid (A) needed to plug the gap between domestic savings and investment, calculated by the saving function and the investment function of equations 4.5.ab. Thus:

$$A = I - S = (d' + dY) - (s' + s)Y = (d' - s') + (d - s)Y$$

Alternatively, we can compute the needed foreign aid as a foreign exchange gap by requiring that it fill the gap between exports and import demand:

$$A = M - E = (u_e - 1)E + u_c C + u_i I \dots\dots \text{by (4.5.c)}$$

$$= (u_e - 1)E + u_c(Y + A - I) + u_i I \dots \text{by } C = Y + A - I \text{ or}$$

$$C + I + E = Y + A + E = Y + M$$

$$= (u_e - 1)E + u_c(Y + A) + (u_i - u_c)(d' + dY) \dots \text{by (4.5.c)}$$

From the above reasoning, we see that the needed foreign aid can be calculated in two ways; i.e., either as a saving-investment gap or a foreign exchange gap, by the following pair of equations:

4.6.a)  $A = (d' - s') + (d - s)Y \dots\dots$  saving-investment gap

b)  $A = \frac{d'(u_i - u_c)}{1 - u_c} + \frac{u_c + du_i - du_c}{1 - u_c} Y + \frac{u_e - 1}{1 - u_c} E \dots$  foreign exchange gap

If foreign aid is to satisfy both the savings-investment gap and the foreign exchange gap, then both of this pair of equations must be satisfied. Furthermore, when the values of any one of the variables (A, Y, E) is assigned, we can determine the value of the two other variables and, hence, determine the entire system.<sup>22</sup> Thus, when only three conditions are postulated in 4.5, there is a possibility of a multiplicity of solutions

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<sup>22</sup>In other words, we have in 4.6 two equations to determine two unknowns. Notice that since the cyclomatic number is 4, determination of the system requires the postulation of one condition additional to those in 4.5a, b, c.

for the system since a particular variable, for example, E, can take on many alternative values. The possibility of a multiplicity of solutions is a typical situation encountered in parametric programming.<sup>23</sup> We first study an elementary method of this type abstractly.

### Parametric Programming

Suppose we have a pair of equations in two variables,  $x_1$  and  $x_2$ , and two parameters,  $\theta_1$  and  $\theta_2$ , such that one parameter appears in only one equation:

$$4.7.a) \quad \phi_1(x_1, x_2, \theta_1) = 0$$

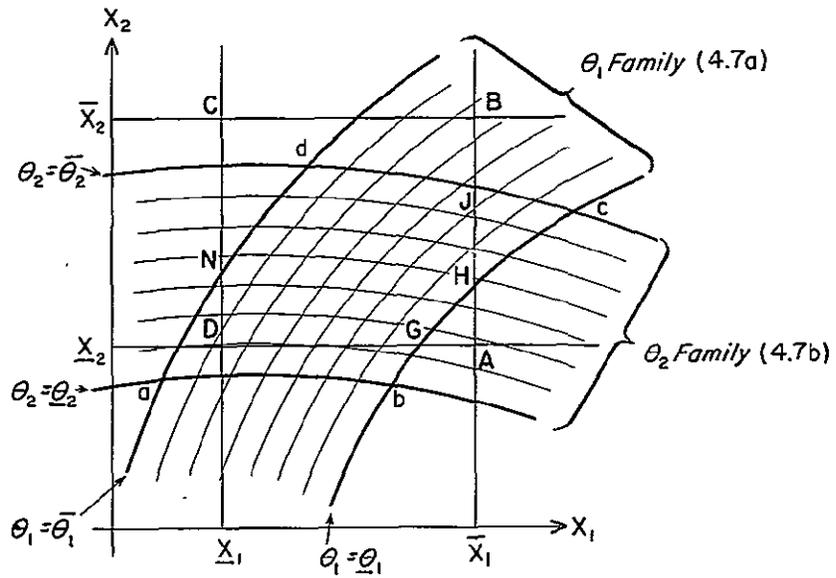
$$b) \quad \phi_2(x_1, x_2, \theta_2) = 0$$

In the  $x_1 - x_2$  plane of Diagram 10a, we let  $\theta_1$  take on alternative values so that a family of curves is generated by equation 4.7a, each curve indexed by a particular value of  $\theta_1$ . Similarly, the variations of the values of  $\theta_2$  generate another family of curves corresponding to equation 4.7b. If both equations must be satisfied by  $x_1$  and  $x_2$ , the solutions to (4.7) are represented by all the points of intersection of these curves. Each intersection is relative to a particular set of parameter values.

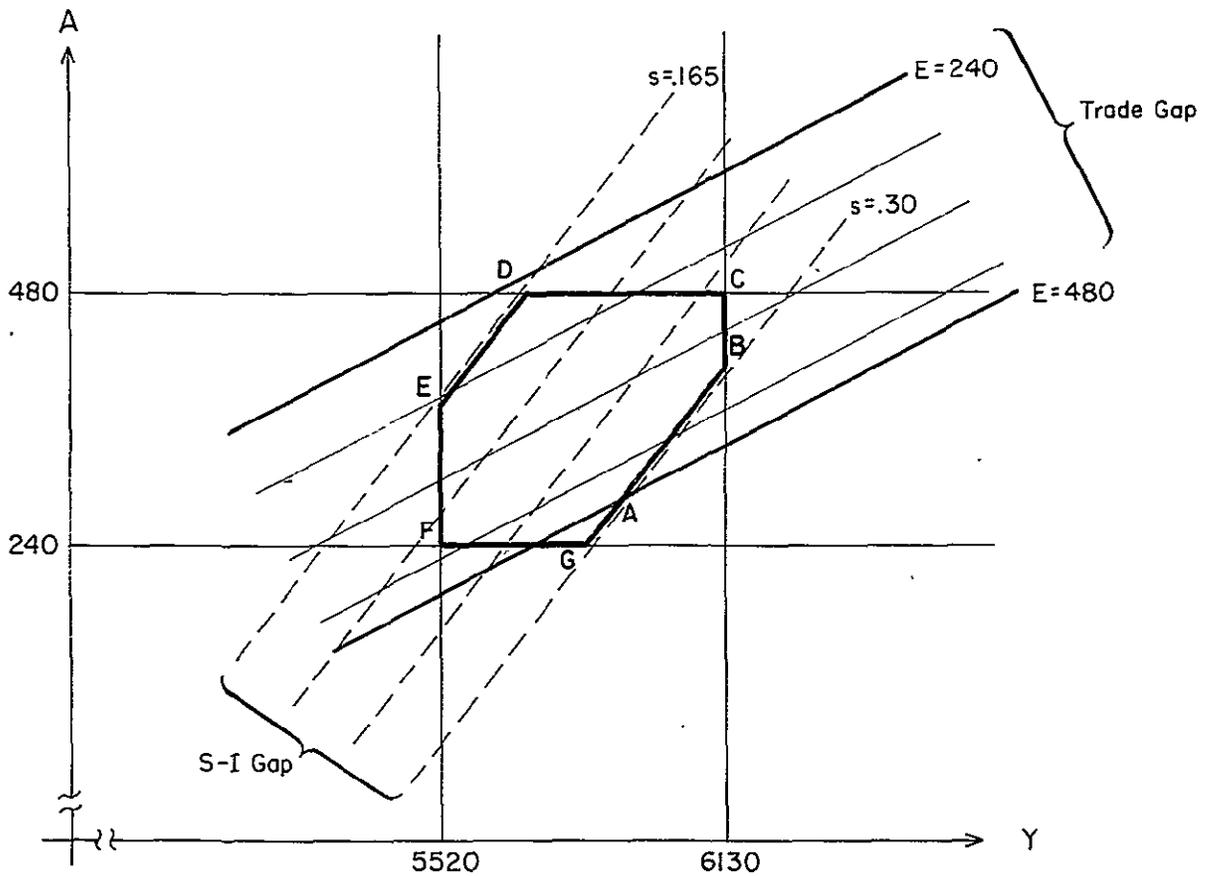
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<sup>23</sup>A particular trait of the planning school lies in its manipulation of underdeterminacy and overdeterminacy. We have seen an overdetermined model in the last section where more behavioral assumptions were specified than used for determination and here we see the opposite case of underdeterminacy.

# Diagram IO: Optimization Techniques



(a)



(b)

Next, let us assume that the variables  $x_1$ ,  $x_2$ , and the parameters,  $\theta_1$ ,  $\theta_2$ , are all constrained within given intervals; i.e., lying between a ceiling value and a floor value:

$$4.8.a) \quad \underline{\theta}_1 \leq \theta_1 \leq \bar{\theta}_1 \quad ; \quad \underline{\theta}_2 \leq \theta_2 \leq \bar{\theta}_2$$

$$b) \quad \underline{x}_1 \leq x_1 \leq \bar{x}_1 \quad ; \quad \underline{x}_2 \leq x_2 \leq \bar{x}_2$$

Now, if we let  $\theta_1$  vary between the two extreme values, the family of curves varies between two extreme members. This is true also for  $\theta_2$ . Thus, the solution must lie in the boundary enclosed by abcd in Diagram 10a if equation 4.8a is to be satisfied. Next, we indicate the extreme values of the variables in equation 4.8b on the vertical and horizontal axis and obtain the rectangle ABCD in Diagram 10a which encloses all values for  $x_1$  and  $x_2$  if equation 4.8b is to be satisfied. The overlapping part of abcd and ABCD then indicates solutions for equation 4.7 when all interval constraints in equation 4.8 are satisfied. The feasible solution is now seen to be bounded by GHJdND.

If, in addition, we postulate that  $x_1$  and/or  $x_2$  have certain economic welfare implications, then the optimum solutions must lie on the frontier (rather than in the interior) of GHJdND. For example, if  $x_1$  is a positive welfare indicator and  $x_2$  is a negative welfare indicator, then the optimum feasible solution lies on the line segment GH which constitutes the southeast frontier of the feasible solution set. Moreover, by moving along this frontier, we can determine at any point on this segment which

parameters and which variables have taken on their extreme values (indicated in equation 4.8). In case a variable has not taken on any of its extreme values, it is then not effectively binding the solution.<sup>24</sup>

In summary, we see that this method of parametric programming, as abstractly stated, has three distinct elements. The first is the multiplicity of feasible solutions as parameters vary. The second is the feasible solution boundary given by interval constraints imposed on parameters and variables. Relative to the extreme values of the interval constraint, a notion of an effective or ineffective constraint is introduced. Finally, by adding certain welfare considerations, the optimum feasible solutions are restricted to certain segments of the boundary.

#### Application of Parametric Programming

In order to apply the method just described, let us rewrite the pair of equations (4.6) (i.e., the saving-investment gap and the foreign exchange gap) in the following form:

$$4.9.a) \quad \phi_1(Y, A, s, s', d, d') = 0 \dots\dots\dots \text{saving-investment gap}$$

$$b) \quad \phi_2(Y, A, E, u_i, u_e, u_c, d, d') = 0 \dots\dots \text{foreign exchange gap}$$

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<sup>24</sup>As we shall see, this is the abstract formulation of the planner's notion that a behavioristic assumption (i.e., a mechanism) may not be binding.

In order to apply the method introduced in equation 4.7, let us choose  $(Y, A)$  to be the pair of economic variables corresponding to  $x_1$  and  $x_2$  in equation 4.7 and  $(s, E)$  to be the pair of parameters corresponding to  $\theta_1, \theta_2$ . All the other parameters are assumed to take on constant values. Thus, in planning terms, the economic problem is to investigate the possibility of projection when a country's export potential and/or its saving capacity varies. Notice that the first parameter "s" appears only in the first equation and that the second parameter "E" appears only in the second equation so that the diagrammatic method can be applied. We introduce the following interval constraints for  $(Y, A)$  and  $(s, E)$ . These numerical values selected are purely illustrative:

$$4.10.a) \quad 5520 = Y \leq 6130 \quad ; \quad 240 \leq A \leq 280$$

$$b) \quad .165 = s \leq .30 \quad ; \quad 240 \leq E \leq 480$$

In employing interval constraints, the planner has some specific economic justifications in mind. In our example, the parameters, s and E, are regarded as policy or instrumental variables subject, within certain limits, to public policy. The propensity to save, s, for example, may be subject to some degree of government control through tax policy and/or moral suasion, while exports, E, may be significantly influenced by government export promotion policies (e.g., manipulation of the foreign exchange rate). The variables, Y and A, are constrained by the planners' estimates or judgment of their likely value. For example, national income,

Y may be estimated from judgments about political considerations such as tolerable growth targets, and foreign aid, A, may be based on guesses about the generosity of potential donors. The two extreme values of the interval constraints represent the most optimistic or pessimistic values. For example, the upper bound of  $s$ ,  $E$ , and  $Y$  and the lower bound of  $A$  constitute the optimistic extreme. The variables  $Y$  and  $A$  are assumed to connote certain economic welfare implications; national income,  $Y$ , is construed to be a positive welfare indicator and foreign aid,  $A$ , a negative welfare indicator.<sup>25</sup>

In Diagram 10b let national income,  $Y$ , be plotted on the horizontal axis, and let foreign aid,  $A$ , be plotted on the vertical axis. Let the extreme values of these variables, given in (4.10a), be marked off. In this diagram the dotted family of curves is drawn to represent equation (4.7a), the alternative saving-investment gaps associated with varying values of the saving propensity,  $s$ . The two extreme curves are indexed by the extreme values of the propensity to save given in 4.10b, and all the other curves lie between the two extreme curves. Similarly, the solid family of curves represents the alternative foreign exchange gaps associated with alternative values of export potential,  $E$ . The extreme curves, corresponding to 4.10b, are indicated. In this case, the feasible solution set has a boundary, with seven sides, indicated by ABCDEFG. Notice that each family of curves

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<sup>25</sup>It is assumed here that foreign aid has a negative political cost in the less developed country context.

is drawn on the principle that increasing values of the saving propensity,  $s$ , or export potential,  $E$ , are associated with shifts of these curves toward the southeast directions. This conforms to our expectation that a favorable change in the values of these parameters ( $s$  and  $E$ ) would lead to higher national income and/or reduced need for foreign aid. Since  $Y$  is a positive welfare indicator and  $A$  is a negative welfare indicator, the optimum feasible solution is the southeast frontier of ABCDEFG; i.e., the line segments GA and AB.

Certain policy implications of the model are now apparent. Suppose that through time the country gradually increases its national income from the floor value (5520) to the ceiling value (6130). Assuming that the country moves along the welfare frontier FGAB (i.e., that the country desires to develop with a minimum foreign aid), the country will then move through three phases of growth, each phase marked by an effective limiting factor:

(i) In the first phase, along FG, foreign aid is the limiting factor.

(ii) In the second phase, along GA, export potential ( $E = 480$ ) is the effective limiting factor. Growth is now in the "trade-limit phase," and foreign aid fills the foreign exchange, or trade, gap.

(iii) In the third phase, along AB, the savings-investment gap is the effective limiting factor. The country is in the "saving-limit phase" of growth and foreign aid fills the saving-investment gap.

In addition to identifying phases of growth, the model can also be used to deduce certain quantitative conclusions about the substitutability between national income,  $Y$ , and foreign aid,  $A$ , as welfare objectives during the growth process. The slopes of the line segments of  $GA$  and  $AB$  indicate how much one welfare objective must be sacrificed if the other welfare objective is to be raised.

### Summary and Evaluation

The above example shows how parametric programming may be applied to guide the formulation of broadly defined development strategies. The guidance consists mainly of identification of expected growth phases, their sequential order, and the factors that are anticipated to limit growth in each phase. With such guidance from projective planning, in which the time dimension is suppressed, the planner would, ideally, select appropriate dynamic models (discussed in the previous section) to plan the time path of development. Thus, we would expect that the choice would vary for each growth phase. More precisely, in each stage of growth for which a particular limiting factor has been identified, the rules of growth of the dynamic model would be formulated to give special emphasis to that bottleneck factor. Conversely this means that those behavioristic constraints not effectively limiting growth during a particular phase may be neglected; i.e., omitted from formal model consideration. We see, therefore, that a strategy planning model, such as the one discussed in this section, is considered to be complementary to the dynamic models discussed in the previous section.

A number of important issues should be raised about parametric programming as a method for guiding the formulation of development strategy. In the broad perspective, a planning model should be judged in terms of its contribution to planning methodology. We have pointed out in regard to the abstract model that this method involves a combination of techniques whose use requires both special assumptions and a generous amount of judgment. It is not clear, therefore, to what extent this method of planning can be generalized. The model has serious limitations as an instrument of deductive reasoning, independent of statistical data. This may be readily seen from the nature of the pair of equations (4.6). Conclusions cannot be drawn from abstract knowledge of the model's structure and its behavioristic assumption before parameters are estimated from actual statistical data. For a model of this type, therefore, dependence on statistical data is of overriding importance, placing a very heavy burden on the reliability of data, a serious problem indeed in the less developed countries for which the model is designed.

Beyond this data problem, extra-model judgment is required for certain numerical components such as the interval estimates for the availability of foreign aid and the feasible target (see equation 4.10). At the present time, neither economists nor other social scientists have developed theoretical foundations and techniques to make these judgments about social, political, and institution forces on a sound basis. Despite this very shaky basis for obtaining the numbers to put into the model, the conclusions depend in a very crucial way upon numerical inputs. For example,

it can be seen from Diagram 10b that a slight variation of the extreme value of exports,  $E = 480$ , would cause the curve to shift in such a way that the sequential order of the phases of growth would be changed. In view of the parameter estimation difficulties, we believe that strategy planning of this kind requires, at a minimum, sensitivity analysis to test the reliability of the conclusions.

The problem of statistical identification of the parameters of the behavioristic equations is a very basic methodological problem. Indeed, it is not clear from the model structure postulated what is the correct procedure for identifying the parameters. For example, if the economy in the recent past has not grown according to the welfare principles of maximizing income and minimizing foreign aid, how can the observed data be interpreted for parameter estimation?

The relevance of the parametric model to the dynamic planning models, discussed in the previous section, is rather vague. For example, it is not clear to what extent and by what method the welfare considerations (e.g., the minimization of foreign aid) can be actually formulated as analytical conditions for the dynamic models. In fact, there is no clear indication that these considerations are even relevant for dynamic planning.

The model discussed in this section closely resembles a prominent model developed by Chenery and Bruno.<sup>26</sup> In that model, a long list of

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<sup>26</sup>Chenery and Bruno, "Development Alternatives in an Open Economy: The Case of Israel," loc. cit.

behavioristic assumptions is postulated, related to many facets of the economy (e.g., labor unemployment, unutilized capital stock, investment and replacement, government current expenditures, population growth, labor productivity change, savings, capital accumulation, absorptive capacity, effective exchange rate, foreign capital inflow, public and private consumption, export prices, ability to plan, etc.) in order to simulate economic reality by an aggregate model. We have attempted to condense these assumptions to reveal the essential methodological content of such a model, summarizing the innumerable relationships into the three behavioristic assumptions in equation 4.5. Thus, the model which we have examined is consistent with a framework capable of including in its scope a much larger range of economic phenomena as intended by the originators of this method.

We see that this type of model is designed to reflect a multiplicity of forces influencing the growth of developing economies. The resulting complexity is the epitome of the planner's effort to approximate reality.<sup>27</sup> This preoccupation with positivism as an approach to development is not a costless virtue. As the complexity of the model increases, the demand for data, the exercise of human judgment, and the need for improvisation rise a fortiori. These additional strains further

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<sup>27</sup>In fact, this school of planners frequently pride themselves in the philosophy that it is their basic purpose to make their models look more and more like the real world rather than making the real world look like models, as they believe other economists are wont to do.

jeopardize the reliability of policy conclusions in an art that has not been noted for its prescriptive powers.

Let us assume for the moment that these problems do not exist, and let us accept as a premise that perfect data, perfect judgment, and successful improvisation always bless planning efforts. The attempt to be realistic through adding complexity involves an even more basic problem. The most serious methodological problem of this positivistic approach lies in its negation of the analytical method. All analysis requires selectivity, determination as to what is relevant or irrelevant. This sense of narrowing down the scope of the problem under analysis is missing in the simulation approach. Given a wide conglomeration of factors involved in an economy's growth process, one is unlikely to distill conclusions apropos the selection of development strategy from such an approach. Without the refining cause and effect ordering provided by analysis, one cannot isolate influences affecting the outcome. From simulation results based upon data for Israel, for example, we cannot distill generalizations appropriate to another country, say, Pakistan, which would be studied from another, quite different, mass of facts. In short, scientific quality of the conclusions is in doubt because there is no basis to assess transferability of the knowledge from simulating one country.

Yet development strategy issues are, by their very nature, based upon comparative experience of different types of countries. Without classification and analysis of different sets of rules of growth, we are unlikely to be able to design strategies of development appropriate to

differing growth types. It is true, indeed, that less developed countries are the only realistic laboratories for developing such knowledge. Lacking an analytical scheme to differentiate among growth types, however, it is unlikely that comparative empirical studies will yield transferable knowledge relevant to the choice of appropriate development strategies for particular countries.

Models of the type reviewed in this section have two major characteristics. First, there is an explicit welfare-optimization orientation; and second, they have a simulation tendency, shown by their attempt to reflect "reality" in its full complexity. These two tendencies are reflected in their fuller evolution in the optimization models examined in the next sections.

#### 4.6 Optimizing Planning Models

In addition to our earlier distinctions, used to discuss different types of planning, we can distinguish between descriptive models and optimizing models. A descriptive model is constructed to describe historical reality for the purpose of forecasting what is likely to occur in the future. The model must be constructed to describe the behavior of economic agents in the real world. An optimizing model, by contrast, is designed to show what should be done. Thus, rather than being descriptive or predictive of economic reality, an optimizing model is prescriptive or normative in nature. If, of course, a country pursues the course of action prescribed by an optimizing model successfully, the optimizing model may also be predictive

in this narrow sense. The optimizing models are considered as a separate category of planning models because they emphasize an explicit formulation of the idea of optimization to show what should be done. In such an approach, economic welfare and the maximization of economic welfare constitute the main analytical content.

Although optimizing models can be constructed at any level of aggregation, in their empirical applications these models have been relatively ambitious, involving disaggregated and dynamic formulations. Thus, the optimization model is usually a large scale model involving a large number (usually hundreds) of variables and equations. Furthermore, because of their size, these models take on a more technical character, requiring special quantitative techniques and frequently using machines and computers for their solution. It follows that they also involve massive data inputs, much greater than for any of the models discussed. Many planning models with these characteristics have been constructed. They all share the typical features of the model we now present.<sup>28</sup>

Ordinarily, a national income accounting framework of the disaggregated type (see Diagram 2) is postulated. In addition, a finite number of consecutive time periods is postulated as the planning horizon.

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<sup>28</sup> Representative models of this type are found in Richard S. Ekhaus, "Appendix on Development Planning," in Charles Kindleberger, Economic Development (2nd edition; New York: McGraw-Hill Book Company, 1965), pp. 400-410; Jan Sandee, A Demonstration Planning Model for India (New York: Asia Publishing House, 1960); and in several of the contributions in Irma Adelman and Erik Thorbecke (eds.), The Theory and Design of Economic Development (Baltimore: Johns Hopkins University Press, 1966).

Thus, for a planning horizon of five years, a typical planning variable,  $x$ , can take on the form  $x^1, x^2, x^3, x^4, x^5$ , where the superscript stands for the dating of the variable. Typically, an optimizing model postulates, in addition, five types of constraints: exogenous, foreign aid, production, investment, and capacity constraints. Let us consider an example involving three planning periods and a model with two production sectors. The five types of constraints in such a model are indicated by the five rows in Table II, which will now be briefly explained.

Exogenous Constraint (Row 1)

The time variables for consumption ( $C_1^t, C_2^t$ ), export ( $E_1^t, E_2^t$ ) and foreign aid ( $A^t$ ) are specified to be constrained in each period by certain exogenously determined magnitudes indicated by a super bar (e.g.,  $\bar{x}$ ). In each step, the inequality signs specify certain extreme optimistic assumptions which the planner can reasonably expect to prevail in the planning horizon. Thus, the planner specifies that consumption must not drop below certain floor values and that foreign aid and exports must not exceed certain (most optimistic) ceiling values. The use of inequality signs in this fashion is a distinctive feature of these models.

TABLE II  
CONSTRAINTS FOR AN OPTIMIZING MODEL

Time Constraint	First Year	Second Year	Third Year
Exogenous Constraint (1)	$C_1^1 \geq C_1^{-1} ; C_2^1 \geq C_2^{-1}$ $E_1^1 \leq E_1^{-1} ; E_2^1 < E_2^{-1}$ $A^1 \leq A^{-1}$	$C_1^2 \geq C_1^{-2} ; C_2^2 \geq C_2^{-2}$ $E_1^2 \leq E_1^{-2} ; E_2^2 \leq E_2^{-2}$ $A^2 \leq A^{-2}$	$C_1^3 \geq C_1^{-3} ; C_2^3 \geq C_2^{-3}$ $E_1^3 \leq E_1^{-3} ; E_2^3 \leq E_2^{-3}$ $A^3 \leq A^{-3}$
Foreign Constraint (2)	$A^1 \geq M_1^1 + M_2^1 - (E_1^1 + E_2^1)$ $M_1^1 = m_1 X_1^1$ $M_2^1 = m_2 X_2^1$	$A^2 \geq M_1^2 + M_2^2 - (E_1^2 + E_2^2)$ $M_1^2 = m_1 X_1^2$ $M_2^2 = m_2 X_2^2$	$A^3 \geq M_1^3 + M_2^3 - (E_1^3 + E_2^3)$ $M_1^3 = m_1 X_1^3$ $M_2^3 = m_2 X_2^3$
Production Constraint (3)	$X_1^1 \geq a_{11}^1 X_1^1 + a_{12}^1 X_2^1 + E_1^1$ $+ C_1^1 + I_{11}^1 + I_{12}^1$ $X_2^1 \geq a_{21}^1 X_1^1 + a_{22}^1 X_2^1 + E_2^1$ $+ C_2^1 + I_{21}^1 + I_{22}^1$	$X_1^2 \geq a_{11}^2 X_1^2 + a_{12}^2 X_2^2 + E_1^2$ $+ C_1^2 + I_{11}^2 + I_{12}^2$ $X_2^2 \geq a_{21}^2 X_1^2 + a_{22}^2 X_2^2 + E_2^2$ $+ C_2^2 + I_{21}^2 + I_{22}^2$	$X_1^3 \geq a_{11}^3 X_1^3 + a_{12}^3 X_2^3 + E_1^3$ $+ C_1^3 + I_{11}^3 + I_{12}^3$ $X_2^3 \geq a_{21}^3 X_1^3 + a_{22}^3 X_2^3 + E_2^3$ $+ C_2^3 + I_{21}^3 + I_{22}^3$

(Continued)

TABLE II (Continued)

Time Constraint	First Year	Second Year	Third Year
Investment Constraint (4)	$I_{11}^1 \geq b_{11}^1 J_1^1 ; I_{12}^1 \geq b_{12}^1 J_2^1$ $I_{21}^1 \geq b_{21}^1 J_1^1 ; I_{22}^1 \geq b_{22}^1 J_2^1$	$I_{11}^2 \geq b_{11}^2 J_1^2 ; I_{12}^2 \geq b_{12}^2 J_2^2$ $I_{21}^2 \geq b_{21}^2 J_1^2 ; I_{22}^2 \geq b_{22}^2 J_2^2$	$I_{11}^3 \geq b_{11}^3 J_1^3 ; I_{12}^3 \geq b_{12}^3 J_2^3$ $I_{21}^3 \geq b_{21}^3 J_1^3 ; I_{22}^3 \geq b_{22}^3 J_2^3$
Capacity Constraint (5)	$\bar{X}_1^1 \geq X_1^1 , \bar{X}_2^1 \geq X_1^2$	$X_1^2 + J_1^1 \geq X_1^2$ $X_2^2 + J_2^1 \geq X_2^2$	$X_1^3 + J_1^2 + J_1^3 \geq X_1^3$ $X_2^3 + J_2^2 + J_2^3 \geq X_2^3$
Planning Variables	$X_1^1, C_1^1, E_1^1, I_{11}^1, I_{12}^1, M_1^1, J_1^1$ $X_2^1, C_2^1, E_2^1, I_{21}^1, I_{22}^1, M_1^2, J_1^2$ $A^1$	$X_1^2, C_1^2, E_1^2, I_{11}^2, I_{12}^2, M_1^2, J_1^2$ $X_2^2, C_2^2, E_2^2, I_{21}^2, I_{22}^2, M_1^2, J_2^2$ $A_2$	$X_1^3, C_1^3, E_1^3, I_{11}^3, I_{12}^3, M_1^3, J_1^3$ $X_2^3, C_2^3, E_2^3, I_{21}^3, I_{22}^3, M_2^3, J_2^3$ $A_3$

### Foreign Aid Constraint (Row 2)

In each period, the total value of exports is  $(E_1^t + E_2^t)$ , while the total value of imports is  $(M_1^t + M_2^t)$ . Their difference-- $(M_1^t + M_2^t) - (E_1^t + E_2^t)$ --is to be covered by foreign aid. Thus, the foreign aid constraint in Row 2 specifies that these constraints must be met. In the same row, we specify that the value of imports is determined by a constant import coefficient assumption of the type used earlier (Equation 4.1b) and is, therefore, proportional to total output  $(X_i^t)$  of a particular commodity at a particular date.

### Production Constraints (Row 3)

The production constraints are deduced from input-output accounting equations (see Equation 4.2) and specify that, for each industry and in each period, the demand for output capacity  $(X_i^t)$  must be covered by the supply of output capacity. Items which enter into the calculation of demand are (1) demand for intermediary factors  $(a_{ij} X_j^t)$ , (2) consumption  $(C_i^t)$ , exports  $(E_i^t)$ , and investment  $(I_{ij}^t)$ . The use of the notations for investment  $(I_{ij}^t)$  are consistent with those used earlier (Section 2, Table I).

### Investment Constraints (Row 4)

This set of constraints states the relationship between the above investment goods  $(I_{ij}^t)$  and the additional productive capacity that they build up for the next period. More specifically, a 2 x 2 matrix of capital coefficients is postulated:

$$\begin{array}{cc} b_{11} & b_{12} \\ b_{21} & b_{22} \end{array}$$

where the first column ( $b_{11}$  and  $b_{21}$ ) specifies the amounts of investment goods of each type which must be "stockpiled" if the output capacity of the first industry is to be increased by one unit. Thus, if the output capacity of the first industry is to be increased by  $J_1^1$  units in the first period, demands for the investment goods are, respectively, at least  $b_{11}J_1^1$  from the first industry and  $b_{21}J_1^1$  from the second industry. This leads to  $I_{11}^1 = b_{11}J_1^1$  and  $I_{21}^2 = b_{21}J_1^1$ , etc.

#### Capacity Constraints (Row 5)

These constraints state that, in each period, the demand for capacity--calculated in Row 3--must not exceed the supply of capacity for each industry. Notice that in the first period the productive capacities of the two industries ( $\bar{X}_1^1$ ,  $\bar{X}_2^2$ ) are assumed to be given, as a part of the initial conditions. The productive capacities in each succeeding period are seen to be obtained by adding to these initial capacities the successive investments,  $J_i$ , in each period as calculated from Row 4.

In this planning model, there are 48 inequality constraints and 15 planning variables in each period (a total of 45 variables in three periods). The 45 variables are listed at the bottom of Table II. We see that even for our small two-industry three-period model, therefore, the number of variables is large. A feasible plan is a choice of values for

these 45 planning variables<sup>29</sup> which satisfies all the constraints indicated in Table II. If no set of 45 values for the variables can be constructed to satisfy all the constraints, the plan has no feasible solution.

Practically, this means that the constraints have been specified in such a way that they are too ambitious in relation to the initial capacity of the economy. A feasible plan is a dynamic plan, indicating the process by which the economy can move from the initial conditions to the terminal conditions specified for the end of the planning horizon.

It may happen, of course, that there are many alternative feasible plans. This situation typically confronts the planner, especially when the constraints specified are not overly ambitious. In this situation, optimization is required. For optimization, an economic welfare criterion must be specified to enable the planner to select, from the set of all feasible plans, one particular plan (known as the optimum solution), best definable in terms of the welfare criterion. There are many ways to specify an economic welfare criterion. The only technical requirement in its formulation is that the one welfare variable chosen must be definable in terms of one or more of the planning variables. The following are examples of typical welfare criteria:

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<sup>29</sup> Projected values for all variables must obviously be non-negative if the result is to be meaningful.

- (1) Minimization of total foreign exchange.
- (2) Maximization of consumption of a particular commodity for the last planning period.
- (3) Maximization of the level of consumption of all commodities, assuming that a constant proportion is maintained among all commodities consumed.
- (4) Maximization of the productive capacity at the end of the planning horizon, under proportionality assumptions similar to (3).
- (5) Maximization of the level of investment at the end of the planning horizon, under proportionality assumptions similar to (3).

The art of optimization planning has not been developed adequately to evaluate which of these welfare criteria are the most appropriate for a particular country. Thus, in this model, as in all of the others, we see that judgment enters at the most critical point.

Given the specification of all the variables, constraints, and an unambiguous welfare criterion, the technical aspect of the optimizing model is either to find an optimum solution or to arrive at the conclusion that such a solution does not exist. When the welfare function is specified in a linear way, this technical aspect amounts to a typical linear programming problem which, when stated abstractly, is a problem of the following type:

To find non-negative values for the variables  $x_1$ ,  $x_2$ ,  $x_3$ , which satisfy the following linear inequalities:

$$a_{11}x_1 + a_{12}x_2 + a_{13}x_3 \leq b_1$$

$$a_{21}x_1 + a_{22}x_2 + a_{23}x_3 \leq b_2$$

$$a_{31}x_1 + a_{32}x_2 + a_{33}x_3 \leq b_3$$

$$a_{41}x_1 + a_{42}x_2 + a_{43}x_3 \leq b_4$$

and which maximize the following "welfare" variable:

$$y = c_1x_1 + c_2x_2 + c_3x_3$$

Problems of this type have been studied exhaustively in the recent years by economists of the activity-analysis tradition, as well as by mathematicians.<sup>30</sup>

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<sup>30</sup>See, for example, the collection of essays edited by Tjalling C. Koopmans, Activity Analysis of Production and Allocation (New York: John Wiley and Sons, Inc., 1951) and R. Dorfman, P. A. Samuelson, and R. M. Solow, Linear Programming and Economic Analysis (New York: McGraw-Hill Book Co., 1958).

## Evaluation

Optimizing planning models of the type presented descend from the dynamic input-output tradition, which has emphasized a dynamic general equilibrium system with numerical strength.<sup>31</sup> Such models have had growing appeal to practitioners of development planning, including officials in planning organizations, for several reasons: (1) their comprehensiveness in covering all industries and all time points in the planning horizon; (2) their production policy results in numerical terms; (3) their emphasis upon resource consistency; and (4) their explicit formulation of welfare criteria. In short, the appeal of optimizing planning models arises from the fact that they are interpreted as conforming to popular ideas of the essential ingredients of central planning and methods appropriate to an idealized centrally directed economy.

We observed in the last section (Section 3.5) a tendency in the evolution of planning methodology toward growing positivism; i.e., an attempt to incorporate into the analytical framework as much of empirical reality as possible. Optimizing models of the type reviewed in the present section constitute an additional step in this direction. The march to positivism in planning methodology, however, has not ended with these optimizing models.

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<sup>31</sup>See Wassily Leontief and others, Studies in the Structure of the American Economy (New York: Oxford University Press, 1953).

This tendency has reached its fullest flowering in the simulation approach. Simulation models represent a giant step toward greater "realism" and complexity,<sup>32</sup> so much so that the optimizing models just discussed look relatively simple and remote from reality, by comparison.

The simulation approach attempts to construct models which, to the maximum degree, mirror all empirical reality at the most microscopic level, usually covering the economic behavior of all individuals and all firms in a society. The models purport to be a duplication of the realistic world, with the only (and basic) difference from reality being that the real world is duplicated (simulated) in the laboratories of planners. With the model simulating the operation of a real economy, "exogenous" conditions can be fed in by the planner. With the aid of machines and computers, certain observable results may then be identified as the operational consequences of the introduction of the exogenous factors. From these results, policy conclusions are attempted. Despite the complexity of the simulation approach, we see that it reflects the same basic methodology characteristic of the planning school as a whole.

An analogy from biological sciences may help to identify the inherent weakness of the simulation approach. In biological experiments, the operation of a particular organism is studied inductively by testing

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<sup>32</sup>In some examples of simulation models, equations and variables are literally counted by the millions.

reactions of the particular organism to outside stimuli. The equivalent of a simulation approach in this case would be an attempt to construct a mechanical organism. We can readily perceive the folly of constructing a mechanical squirrel, for example, to investigate the problem of a squirrel's growth and development. The mechanical squirrel would reflect the mechanical ingenuity of its creators, but it is doubtful, indeed, that the inductive evidence presented by its operation would be even remotely relevant to the behavior of a real-life squirrel. In our view, the operation of an entire economic system can be no better simulated by a mechanical replica than in the case of a biologist's mechanical animal.

Quite apart from this basic problem of mechanical simulation, there are other difficulties of a more epistemological nature. Even assuming that the realistic work could be duplicated in a planning laboratory, we would still possess only inductive knowledge. Without theorizing, inductive evidence does not further our understanding of the operation of the system under study. Understanding of cause and effect in growth and development requires theories to interpret the functioning of the entire system.

These basic difficulties go far to explain the lack of success marking the application of both the simulation approach and large-scale

planning models as instruments for managing economic affairs of entire nations. There are also the more practical reasons, cited at the end of Section 4.5. These include reliance upon masses of frequently inadequate data; the heavy input of judgment and improvisation; and the great demands of such cumbersome models upon a developing society's limited resources of time and money.

## 5. EVALUATION OF THE PLANNING APPROACH

In this chapter we have surveyed several of the important works by representatives of the planning school. In this survey we have stressed certain special characteristics of this school, such as its accounting framework, methodology, data consciousness, policy orientation, dependence on discretionary judgment, and growth philosophy. Having familiarized ourselves with the substantive content of the planning school's work, we are now in a position to adopt a more synoptic view to evaluate the place of this school in the broader context of our total knowledge about growth and development of less-developed countries. Our purpose here is to assess the planning school's unique contribution as compared and contrasted to the other approaches (historical, institutional, and theoretical) reviewed in other chapters of our book. In this evaluation, we discuss four aspects of the planning approach: 1) its nature as an area of knowledge; 2) its strength; 3) its weakness; and 4) its relevance to our present study.

### 5.1 Nature of the Planning Approach

A. C. Pigou once distinguished two branches of economics, light-bearing and fruit-bearing.<sup>33</sup> Pigou's distinction was meant to emphasize the difference between analytical and applied work in the field

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<sup>33</sup>A. C. Pigou, Economics of Welfare, Fourth Edition, (London: Macmillan and Co., 1952), pp. 3-5.

of economics, and to suggest that without light-bearing analysis, fruit-bearing applications are unlikely to be productive. The planning school clearly represents an area of applied economics, i.e., the application of analytical methods to produce development plans. In this evaluation, our central focus is the Pigovian type question of whether the planning school has only applied received knowledge or whether it has also borne light by adding to our knowledge about the process of economic growth.

To investigate this issue we are concerned with three related problems: 1) what is the planning school's growth philosophy; 2) what is the origin of the planners' growth philosophy; and, 3) how does the planning school's growth philosophy differ from the other major approaches to growth and development?

The preceding survey of the planning school's work leads us to describe their growth philosophy as mechanistic. Economic growth is viewed as a matter of simultaneous operation of many mechanical parts of an economy, with each part having a particular function. Growth occurs as a result of the proper functioning of each part of the economic mechanism. Associated with this mechanistic view of growth is the strong belief in the immutability and universality of the mechanical principles related to the operation of discrete parts. Different growth types are distinguished on the basis of the presence or absence of particular mechanical parts--as we have seen in our review of the various models examined in Section 4.4. Thus, we found the planners' typology to be limited to the narrow confines of mechanistic models. Just as an engineer applies a limited number of basic and immutable principles

(wheel, pulley, internal combustion, jet propulsion, etc.) to construct a large variety of useful machines, so too, the planner constructs from his own mechanical principles a large number of machines (models) for specific uses.

We have seen that this growth philosophy leads to an epistemology which views additions to knowledge as building more parts into the models so that the machine more fully simulates economic reality. This approach reaches its epitome in the optimizing and simulation models (discussed in Section 4.6) where the machines include literally hundreds, or even thousands, of parts. We have noted that these models are not designed to analyze reality through reasoning about rules of growth. They do not provide a selective viewpoint to assist in classifying essential and non-essential forces affecting the operation of the economy. Such models are not, therefore, instruments for analysis; they become meaningful only when data are collected and fed into the "machine." They are intended to be meaningful only in terms of reproducing "reality" in the laboratory in order to produce policy results. It is for precisely this reason that the planning school is conspicuous in its preoccupation with masses of statistical data.

The operational implications of this mechanistic growth philosophy and its associated view of knowledge are clear. The conception of growth as a mechanical functioning of a large number of parts leads directly to a

technocratic approach to social policy.<sup>34</sup> The function of the expert becomes a matter of selecting, assembling, and properly operating the machine (model) which will guide the growth of the economy.

We now raise the question of how planners evolved such a mechanistic growth philosophy. The answer, we believe, lies in the planners' total and indiscriminate acceptance of behavioristic assumptions from virtually all other branches of economics. We have mentioned earlier that planners have drawn their behavioristic assumptions from such diverse branches as input-output economics, Keynesian economics, and economic dynamics. This eclecticism served to provide the mechanistic core of principles essential to their mechanistic growth philosophy, while this view of the growth process was invented by planners themselves. This paradox arises from the fact that the behavioristic assumptions used by planners were borrowed from branches of economics which have little relevance to growth. Hence planners have not taken over the social problems for which the behavioristic assumptions were originally designed, but merely the mechanical relationships. Given the multiplicity of cultural origins of these pieces, it is inevitable that the individual mechanical parts, rather than the whole, become the nub of the growth process in the planners' view.

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<sup>34</sup>Websters New International Dictionary, Second Edition, Unabridged (Springfield, Mass.: G. and C. Merriam Co., 1958) defines technocracy as "government or management of the whole of society by technical experts or in accordance with principles established by technicians."

This is tantamount to arguing that the planners' mechanistic view of economic growth is a new departure, quite alien to the traditional approaches adopted by other economists. In its undue emphasis on the mechanical parts, the planning school's growth philosophy contrasts sharply with the holistic view of the historical approach to growth. The historian's emphasis upon the significance of a total cultural system in which the economy is imbedded, and its evolution through historical time, are irrelevant in the planning school's formal analysis. The conception of contemporary economic development, for example, as a unique epoch characterized by a particular set of growth forces, has no place in the planners' approach. We now see, therefore, that the emphasis upon quantifiable, mechanistic relationships leads to the planning school's exclusive focus upon resource utilization. Avoiding a holistic view of the development process, planners concentrate upon the resources aspect of growth by formalistic manipulation of quantitative techniques. We now briefly evaluate the relative strengths and weaknesses of this unique approach to growth and development.

## 5.2 Strengths in the Planning Approach

The methodology developed by the planning school reflecting the particular growth outlook just described, has been widely accepted for planning in less-developed countries. The current popularity of this approach is based upon three elements which represent positive contributions to development economics. The first and major strength in the planning

approach lies in emphasis upon a general equilibrium framework for the economy as a whole. This emphasis assures a professional economics viewpoint since the essence of professionalism in economics is an approach embracing the economy as a whole rather than preoccupation with its discrete parts. The planners' general equilibrium emphasis is clearly shown in the national income accounting systems (presented in Section 1) which we have used to examine their methodology. We have seen that both the aggregated and disaggregated models involve the entire economy. It is this general social scope which constitutes the basic merit of the planning school's approach to development.

While the general equilibrium theory has been of interest to economists for several years, the second strength in the planning approach, empiricism with numerical emphasis, is more recent in origin, and is associated with the rise of input-output economics. Planners have played an important part in advancing acceptance of this new empiricism in less-developed countries. Our survey of planning methodology has shown a very strong emphasis on factual data of the type subject to numerical measurement. Systematic and large scale statistical work is frequently essential to the very usefulness of the planning model. Almost without exception, the basic criterion in judging the suitability of a planning model is its capacity for statistical implementation.

The third element of strength is found in the planning school's policy consciousness. This stress on policy relevance takes a very pragmatic form in planning models. Policy advice is provided in numerical

terms, as, for example, in the formulation of a development plan whose projections for the future are cast in actual dollar and cents terms. Thus the planner can provide very concrete answers to the decisions confronting policy makers. We have seen in our survey that planners frequently carry this numerical policy orientation to the extreme, seeking to obtain numerical answers at all costs.

In summary, the strength of the planning school lies in its emphasis upon providing numerical policy advice concerning resource utilization problems on an economy-wide basis. The future promise of this school and its potential long-run significance, however, is found in the experimental spirit characterizing its methodology and in the accumulation of factual knowledge through emphasis upon statistical data. It is only through refinement of such an approach that economists can learn more about empirical reality.

### 5.3 Weaknesses in the Planning Approach

The limitations of the planning approach all stem from the highly mechanistic growth outlook of this school. The most apparent and serious weakness is that this mechanistic view of the growth process is devoid of historical connotation. The absence of a holistic perspective and the preoccupation with mechanical parts of the economic system is inimical to understanding the process of growth and development. Understanding of this process requires the historian's perspective of the economy--as a living and evolving whole rather than as a simulated mechanism composed of individual mechanical parts.

The mechanistic growth philosophy of this school is also reflected in the absence of growth and development theory in the planners' works. It is in this sense that we have earlier described the planning approach as positivistic. Despite the weight of traditional economic concern with generalization and theory, the planning school has been content to merely select individual components of received theories to provide the mechanisms needed to apply their models. In concentrating on the pragmatic aspects of special cases, planners ignore the accumulation and transferability of knowledge about economic growth.

In practical terms, their growth philosophy restricts the scope of planners to the narrow aspect of resource allocation. The corollary procedure of neglecting phenomena not amenable to quantitative manipulation rules out many important growth forces. This narrowness precludes investigation of a whole complex of factors related to the central issue of "how growth comes about." For insights into these broader issues, therefore, we must rely upon the much wider range of human experience covered by the institutional school and the time perspective offered by the historical approach. We shall see in the next chapter that in both classical and contemporary growth theory significant efforts have been made to grapple with the much wider range of phenomena bearing upon economic growth and development.

This aversion to wide-ranging theoretical inquiry gives planning methodology a rather naive and simplistic character. We have shown that behavioristic assumptions are taken from other branches of economics and

used out of context, with excessive faith in their immutability and universality. While general equilibrium theory is the most difficult of economic theories, and particularly intractable to satisfactory statistical implementation, planners attempt to accomplish this difficult task by rather uncritical application of these diverse behavioristic assumptions. We have seen that heavy doses of improvisation are required for this purpose and that little restraint is exercised in the planner's preoccupation to make a general equilibrium model operational.

Despite the empirical bent and data consciousness marking this school, we have noted an ironic aversion to deal with historical reality. We have found this feature of the planning approach to be associated with the planners' penchant to produce concrete policy results. This overriding preoccupation gives the planners' work a forward-looking character, an emphasis upon what should be done in the future while largely ignoring what has transpired in the past and why. In Section 4 we observed that planning models increasingly reflect this policy orientation, being designed to yield policy conclusions rather than to assist in understanding reality. This prescriptive emphasis is particularly evident in the case of the optimization models reviewed and in the recent evolution of simulation techniques.

#### 5.4 Relevance to Our Work

Several aspects of the planning school's approach to growth and development, which stand out when compared to the other approaches

reviewed in this book are significant for the design of our present study. First, we are sympathetic with the general equilibrium perspective of the economy which has been advanced by the work of this school. Second, we follow the planning school in appreciating the notion of growth typology, which, we believe, lies very much at an important frontier in growth theory. Third, we heartily endorse the empiricism highlighted by the planning approach. The impetus to development research in an inductive spirit has had an important influence upon our own work. We also embrace the planners' related emphasis upon a quantitative approach to development, and much of our work is devoted to an attempt to make statistical data relevant to our theoretical analysis. Finally, we see considerable value in the planners' stress upon policy results. Thus, despite the critical tone of some parts of the survey of this school, we find much that is useful and relevant for the study of economic growth.

Our major difference from the planning approach is our rejection of the planners mechanistic view of the growth process in its manifestations as a philosophy of growth as well as its epistemological implications. We also find the forward-looking bias of this school irrelevant to the study of growth as a historical phenomenon. Our study, therefore, draws upon the historical and institutional approaches (discussed in earlier chapters) to provide us with a perspective for investigating the problem of how growth came about and proceeded during the post-war time perspective. We return to a discussion of our own analytical framework in Chapter 6.

APPENDIX TO SECTION 1.3

It is our purpose to prove the necessary condition of Theorem (5) in the text. The theorem to be proved may be restated as follows:

Theorem: Let  $G$  be a connected linear graph and let  $X$  be a subset of edges of  $G$  forming a set of exogenous variables. Then  $X$  is a basic edge set.

If we denote the set of edges of  $G$  which is not in  $X$  by  $G-X$ , it is our purpose to prove that  $G-X$  is a maximum tree. Suppose the theorem is false; i.e., suppose  $G-X$  is not a maximum tree of  $G$ . Then either  $G-X$  is not circle-free or  $G-X$  is circle-free. These two cases may be investigated separately.

Case 1:  $G-X$  is not circle-free. In this case, the endogenous variables contain a circle  $C$ . According to the definition of a "circle," the edges of  $C$  can be classified into two classes  $C^+$  and  $C^-$  (one may be empty) such that a circuit can be formed when the direction of all edges in  $C^-$  is reversed. Letting  $k \neq 0$  be any number, we can construct a square table  $B$  by:

- i) assigning the value " $k$ " to every cell of  $B$  corresponding to an edge of  $C$ .
- ii) assigning the value " $-k$ " to every cell of  $B$  corresponding to an edge of  $C^-$ .

Then B is a balanced table. Since X contains a set of exogenous variables, we can assign a particular set of arbitrary values to all variables in X and determine a set of values for the endogenous variables in G-X. These values then form a balanced table (i.e., Euler graph) J, by definition. Notice that B + J is also a balanced table in which the values of the exogenous variables (X) are the same as those in J. This proves that the values of the endogenous variables cannot be determined uniquely by the accounting equation (i.e., by the requirement that an Euler graph be formed).

Case 2: G-X is circle-free. This implies that G-X is either not connected or not maximum in G. In this case, it is easy to see that we can take some (at least one) edge(s) from X which, when added to G-X, would have converted the latter into a maximum tree of G. Thus, according to the sufficient condition of Theorem (5) (proved in the text), a proper subset of X constitutes a set of exogenous variables. This implies that arbitrary values cannot be assigned to all values of X to form an Euler graph. QED.