AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D. C. 20523
BIBLIOGRAPHIC INPUT SHEET

FOR AID USE ONLY
Batch 63

1. SUBJECT CLASSIFICATION
A. PRIMARY: TEMPORARY
B. SECONDARY:

2. TITLE AND SUBTITLE
Table functions: TABEL,TABEX,TABUL,TABUX

3. AUTHOR(S)
Abkin,M.H.; Wolf,Christopher; Carroll,T.W.

4. DOCUMENT DATE: 1976
5. NUMBER OF PAGES: 62p.
6. ARC NUMBER: ARC

7. REFERENCE ORGANIZATION NAME AND ADDRESS
Mich.State

8. SUPPLEMENTARY NOTES (Sponsoring Organization, Publishers, Availability)
(In Computer Library for Agr.Systems Simulation. CLASS doc.no.9)

9. ABSTRACT
(LIBRARY & INFO.SCIENCE R&D)

10. CONTROL NUMBER
PN-AAE-014

11. PRICE OF DOCUMENT

12. DESCRIPTORS
Computer programs
Interpolation
Simulation
Statistical analysis
Tables

13. PROJECT NUMBER

14. CONTRACT NUMBER
CSD-2975 Res.

15. TYPE OF DOCUMENT

AID 590-1 (4-74)

TABLE FUNCTIONS:
TABEL, TABEX, TABUL, TABUX

Michael H. Abkin

Chris Wolf

Tom W. Carroll

COMPUTER LIBRARY FOR AGRICULTURAL SYSTEMS SIMULATION

CLASS Document No. 9

Agricultural Sector Analysis and Simulation Projects
Supported under Contract No. AID/csd-2975
U. S. Agency for International Development

Department of Agricultural Economics
Michigan State University
East Lansing, Michigan

July 1976

TABLE OF CONTENTS

Preface vii

Table Function with Equal Intervals and Limited Extrapolation ... TABEL

Table Function with Equal Intervals and Extrapolation ..... TABEX

Table Function with Unequal Intervals and Limited Extrapolation . TABUL

Table Function with Unequal Intervals and Extrapolation ..... TABUX

COMPUTER LIBRARY FOR AGRICULTURAL SYSTEMS SIMULATION

The Computer Library for Agricultural Systems Simulation (CLASS) is one of the four major activities of the Agricultural Sector Analysis and Simulation Projects at Michigan State Unviersity under U. S. Agency for International Development Contract AID/csd-2975. The other three major interrelated project activities include theoretical and methodological research, the Development Analysis Study Program, and field activities, primarily in the Republic of Korea.

The project objective is to develop an approach to institutionalizing an analytical capacity for planning, policy formulation, program development, and project implementation for agricultural sector development within the public decision making structure of developing countries. A major component of the analytical capacity is a series of system simulation models tailored to the needs of the individual country. Much of the experience gained from the field activity and the knowledge gained from the theoretical and methodological research added to the present stock of knowledge about building and maintaining analytical capacities for agricultural sector development can be preserved and extended in the training provided through the Development Analysis Study Program and in the stock of model, component, and utility routine computer software documented in the Computer Library for Agricultural Systems Simulation.

In full operation, the Computer Library for Agricultural Systems Simulation (CLASS) acquires, catalogs, maintains and distributes computer programs and associated documentation. These computer programs are of generalized simulation models, components, and routines designed specifically for the analysis of agricultural development problems and processes. In particular, the library sets standards of admissibility for programs and documentation; catalogs and indexes programs and documentation so as to facilitate their retrieval by users seeking a set of programs to be used in a specific problem analysis; and distributes programs and documentation to users.

To enhance the effectiveness of the library, its functions also include identifying and soliciting needed models; actively bringing programs and documentation up to the library's standards; and providing limited consultation in identifying and implementing appropriate library programs for a particular application. A subsidiary function of the library in conjunction with the identification and solicitation of models is to survey and catalog ongoing research in agricultural systems modeling and simulation.

The CLASS document publication series is the main vehicle for informing potential users of the substance of CLASS holdings and activities.

July, 1976

George E. Rossmiller
Director
Agricultural Sector Analysis and
Simulation Projects

PREFACE

Very frequently in simulation work, particularly with socioeconomic systems, the situation arises that one system variable is to be causally related to another; and either an explicit mathematical function is not known or one cannot be reasonably assumed. Often, however, experimental results, direct observation of the real system, or expert opinion can at least specify corresponding point values of the dependent and independent variables. Examples are birth and death rates as functions of nutrition; projections of production, prices, and other variables as functions of time, i.e., where these variables are treated as inputs to a model rather than as endogenously determined system variables; and labor force participation rates as functions of real wage rates, unemployment rates, education, etc.

The method of interpolation between these specified points depends on the particular problem and the number of observations that are available. Two points may be connected by a straight line or by a curve with a constant growth rate. If there are more than two points, a continuous polynomial may be the most realistic way of connecting them, because the path between two points usually depends on the points beyond those two. However, in many cases a straight line interpolation is quite sufficient.

This publication provides a set of documentations on four table functions: TABEL, TABEX, TABUL, TABUX. In all four routines straight-line interpolation is used between known points values of a dependent

variable. Table 1 summarizes the distinctive features of each routine in terms of (a) the spacing of the point values of the independent variable, and (b) the mode of extrapolation outside the range of known values.

TABEL and TABEX handle cases in which there are equal intervals between the known values of the independent variable; TABUL and TABUX handle cases in which there are unequal intervals. Beyond the specified range of TABEX and TABUX, straight line extrapolation is used, where the slope of the line is the same as that of the last segment within the specified range; beyond the specified range of TABEL and TABUL extrapolation is limited to end point values.

The last three columns of Table 1 show the compile and execution central processor times and core storage required by the table functions on Michigan State University's CDC 6500 computer with its FORTRAN Extended (Version 3.0) compiler and operating under HUSTLER 2, MSU's version of CDC's SCOPE 3.2 operating system.

M. H. A.
C. J. W.
T. W. C.

Table 1

CLASS Table Look-Up Routines

Routines	Limited extrapolation	Straight-line extrapolation	Equal intervals	Unequal intervals	Compile time (central processor)--seconds	Execution time (central processor)--milliseconds (using a table with 7 points)	Core storage--words (decimal)
TABEL	X		X		.285	.25	70
TABEX		X	X		.222	.21	56
TABUL	X			X	.255	.26	59
TABUX		X		X	.187	.22	46

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABEL	Table Function with Equal Intervals and Limited Extrapolation	routine	1A 6/1/75

TABEL Abstract

Component TABEL is a table look-up function which approximates an unknown explicit mathematical function relating two variables by a linear interpolation between known (or assumed) point values of the dependent variable (the output) corresponding to equally spaced values of the independent variable (the input). Extrapolation beyond the specified range is limited to end point values.

Inputs to TABEL include a set of point values for the dependent variable, the smallest value of the independent variable, and the difference between successive values. To use the function, a value for the independent variable must be supplied; and the resulting output is the corresponding value for the dependent variable.

TABEL requires 70 decimal words of core storage and .25 milliseconds to execute on a CDC 6500 computer.

Related routines, which feature limited extrapolation and/or arbitrary spacing of the independent variable, are TABEX, TABUL, and TABUX.

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABEL	Table Function with Equal Intervals and Limited Extrapolation	routine	1A 6/1/75

TABEL Documentation

Contents

I. Process Description 1

II. Technical Description 2

A. Mathematical model 2

B. Sample run 4

III. Program Information 9

A. Program description 9

B. Program implementation 9

C. Program lists 9

1. Of sample run executive 10

2. Of component 11

Bibliographic Reference 12

I. Process Description

Very frequently in simulation work, particularly with socioeconomic systems, the situation arises that one system variable is to be causally related to another; and either an explicit mathematical function is not known or one cannot be reasonably assumed. Often, however, experimental results, direct observation of the real system, or expert opinion can at least specify corresponding point values of the dependent and independent variables. Examples are birth and death rates as functions of nutrition; projections of production, prices, and other variables as functions of time, i.e., where these variables are treated as inputs to a model rather than as endogenously determined system variables; and labor force participation rates as functions of real wage rates, unemployment rates, education, etc.

The method of interpolation between these specified points depends on the particular problem and the number of observations that are available. Two points may be connected by a straight line or by a curve with a constant growth rate. If there are more than two points, a continuous polynomial may be the most realistic way of connecting them, because the path between two points usually depends on the points beyond those two. However, in many cases a straight line interpolation is quite sufficient.

TABEL is a table look-up routine specifically designed for the case where: (1) point values of the dependent variable are known (or assumed) for equally spaced values of the independent variable; (2) straight-line interpolation is used between points; and (3) extrapolation

of the dependent variable beyond the specified range is limited to end point values. TABEL is based on Llewellyn's TABLIE routine [1].

II. Technical Description

A. Mathematical model

Figure 1 illustrates the relationships among the variables and parameters appearing in the equations below. To use a concrete example, assume the curve in Figure 1 gives the growth rate Y of a population as a function of temperature X relative to some base temperature.

The computation of the output Y falls into three categories, depending on the value of the input X:

(1) if $VX_i < X \leq VX_{i+1}$, then

$$Y = VY_i + \frac{VY_{i+1} - VY_i}{VX_{i+1} - VX_i} (X - VX_i)$$

$$= VY_i + \frac{X - VX_i}{DX} (VY_{i+1} - VY_i), \qquad i \varepsilon \{1, 2, \ldots, k-1\}$$

(2) if $X \leq VX_1$, then

$$Y = VY_1$$

(3) if $X > VX_k$, then

$$V = VY_k$$

where:

X = the current value of the independent variable (the input to TABEL)--i.e., relative temperature

Y = the corresponding value of the dependent variable (the output of TABEL)--i.e., the population growth rate

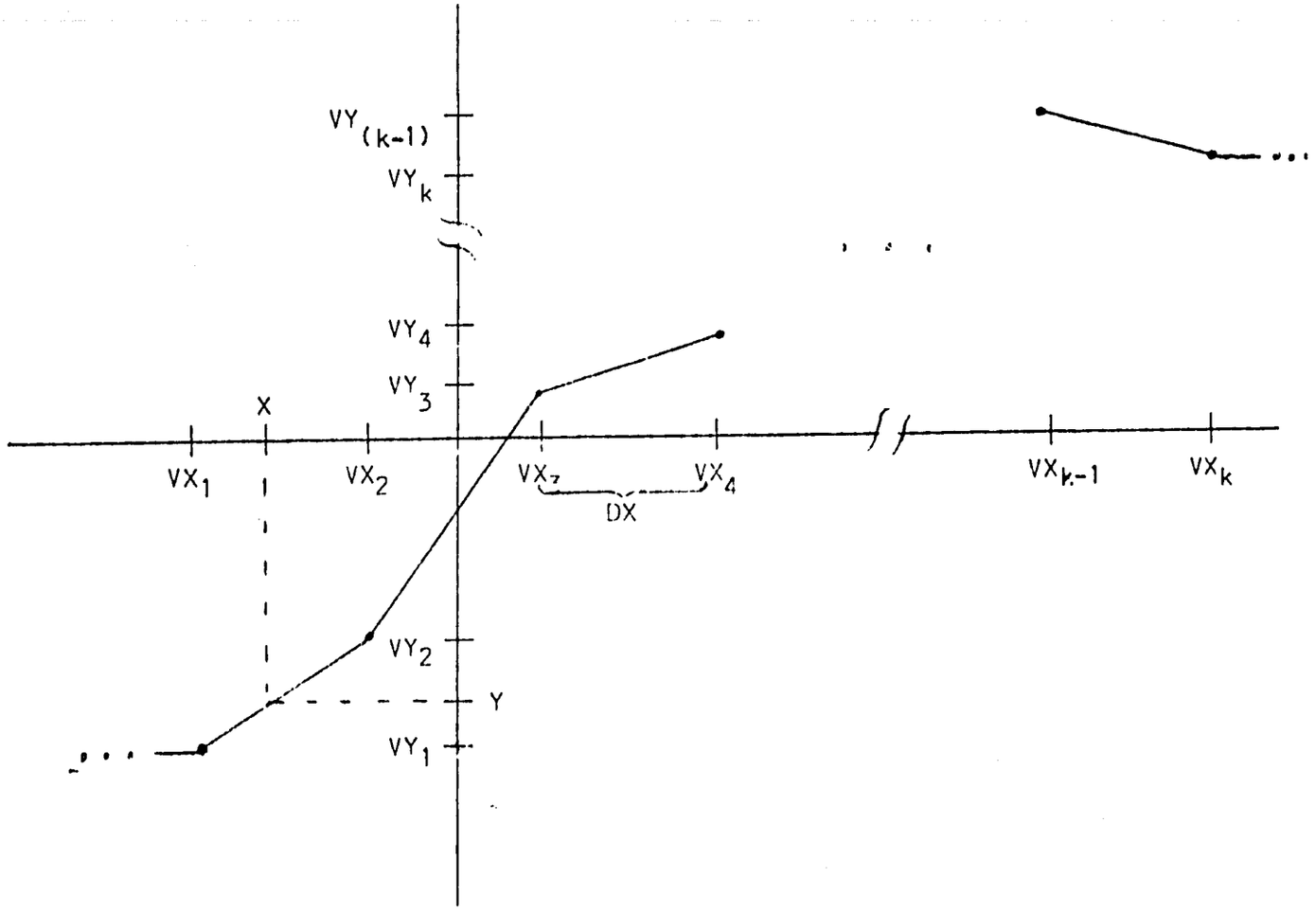

Figure 1

The TABEL Table Look-Up Function

VX_i = the ith specified value of the independent variable*

VY_i = the ith specified value of the dependent variable

DX = the constant dependent variable interval,* $VX_i - VX_{i-1}$, $i = 2, \ldots, k$

i = indexes the data points, $i = 1, \ldots, k$.

Notice that for $X \notin [VX_1, VX_k]$, Y is set equal to the respective end point value, VY_1 or VY_k.

TABEL is one of a set of four table look-up routines (Table 1) which cover situations where straight-line extrapolations are made byeond the specified range and/or unequal intervals of the independent variable are used.

B. Sample run

The output for a sample run of TABEL appears in Figure 3. (TABEL does not write any output itself.) For the sample run, the example followed above is used; that is, the percentage growth rate of a population is assumed to be determined as a function of relative temperature, where relative temperature is defined as the difference between ambient temperature and some base value (Figure 2). Growth rates are assumed known at five-degree temperature intervals, and TABEL linearly interpolates between these points as shown in Figure 2. For example, if temperature is 12.5 degrees, the growth rate is computed to be 4% per unit time. Percentage

---

*The equal interval feature of TABEL allows efficiencies in the computer programming (see the program list and the TABEL Program Documentation). Instead of requiring all k values for VX_i, $i = 1, \ldots, k$, TABEL only requires values for VX_1, DX, and k. VX_i is then found from $VX_i = VX_1 + DX \cdot (i - 1)$, $i = 1, \ldots, k$.

growth rates are determined in the test program by TABEL for temperatures (the first column in Figure 3) at the mid-points of the specified intervals and outside the specified range.

The last three columns of Table 1 show the compile and execution central processor times and core storage required by TABEL (and the other table look-up routines for comparison) on Michigan State University's CDC 6500 computer with its FORTRAN Extended (version 3.0) compiler and operating under HUSTLER 2, MSU's version of CDC's SCOPE 3.2 operating system.

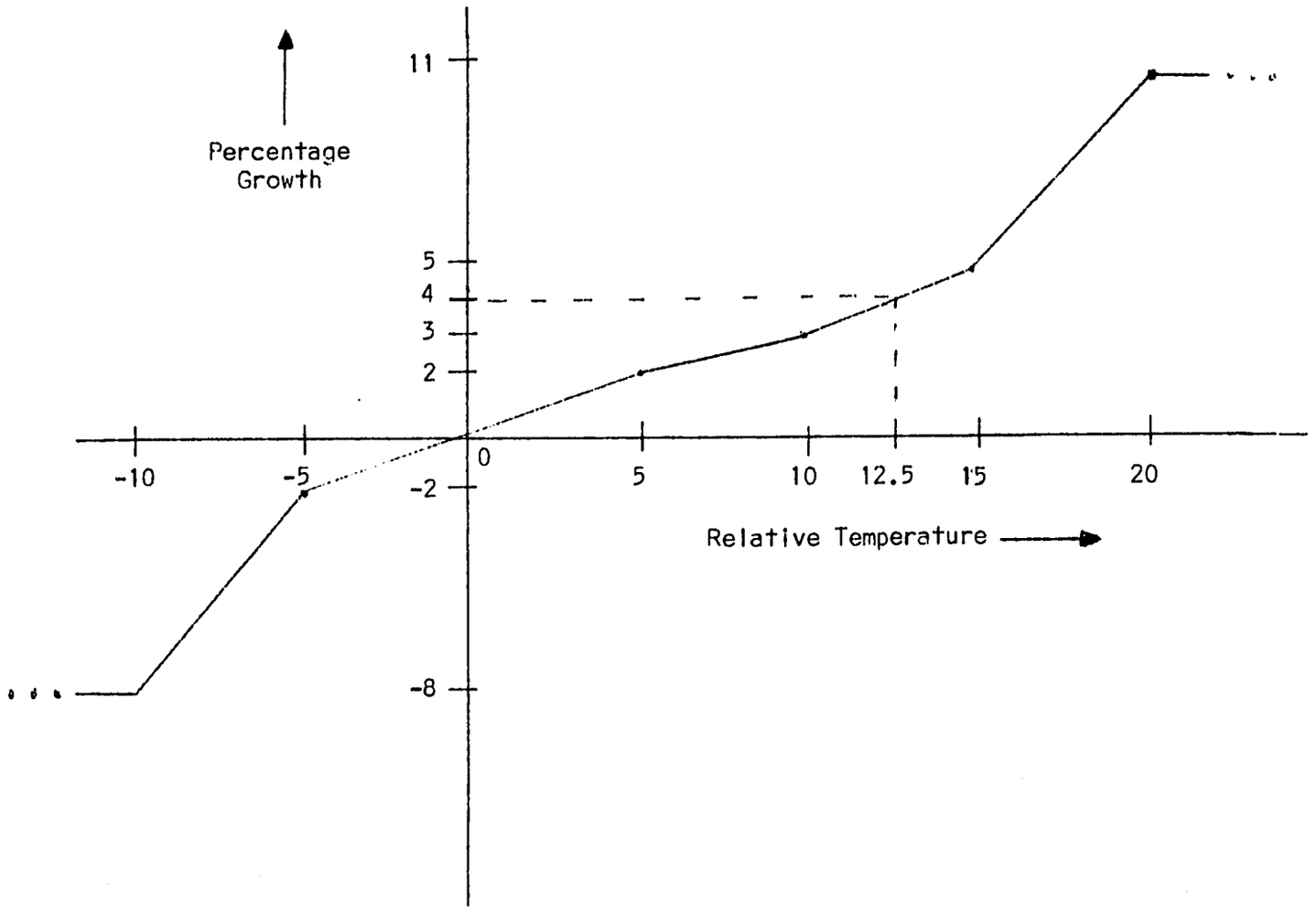

Figure 2

TABEL Sample Run Table Function

RELATIVE TEMPERATURE	GROWTH RATE
-12.5	-8.00
-7.5	-5.00
-2.5	-1.00
2.5	1.00
7.5	2.50
12.5	4.00
17.5	8.00
22.5	11.00

Figure 3

TABEL Sample Run Output

Table 1

CLASS Table Look-Up Routines

Routines	Limited extrapolation	Straight-line extrapolation	Equal intervals	Unequal intervals	Compile time (central processor)--seconds	Execution time (central processor)--milliseconds (using a table with 7 points)	Core storage--words (decimal)
TABEL	X		X		.285	.25	70
TABEX		X	X		.222	.21	56
TABUL	X			X	.255	.26	59
TABUX		X		X	.187	.22	46

III. Program Information

A. Program Description

TABEL was programmed and tested in the following operating environment:

Machine: CDC 6500
Operating system: MSU HUSTLER 2 L239 LSD32--an extension of CDC SCOPE 3.2
Compiler: CDC FTN V3.0-P357
Core requirements: 70 decimal words
CP compile time: .285 seconds
CP execution time: .25 milliseconds
Tapes: none
Direct access files: none
FORTRAN library subprograms used: none

TABEL consists of a single function subprogram which calculates the approximate value of a single mathematical function for a given value of the independent variable by linear interpolation between specified points of the function. No COMMON blocks are used.

B. Program Implementation

All input is transmitted through the argument list. The single output value is transmitted through the function name TABEL.

VAL must be dimensioned to at least K in the calling program. The following must also be true:

K > 1

DIFF > 0

C. Program Lists

The function TABEL and the program used for the sample run are listed below:

```
      PROGRAM TSTTB3(OUTPUT)
      DIMENSION XARG(8),VAL1(7)
      DATA VAL1 / -8., -2.,  0.,  2.,  3.,  5., 11. /
      DATA XARG / -12.5, -7.5, -2.5,  2.5,  7.5, 12.5, 17.5, 22.5 /
      DATA SMALL,DIFF / -10., 5. /
      PRINT90
      DO 60 I=1,8
        VAL1 = TABEL(VAL1,7,SMALL,DIFF,XARG(I))
        PRINT91,XARG(I),VAL1
60    CONTINUE
      STOP
90    FORMAT(1H1,19X,8HRELATIVE,8X,6HGROWTH/19X,11HTEMPERATURE,7X,
     +       4HRATE/)
91    FORMAT(20X,F7.1,F15.2)
      END
```

```
FUNCTION TABEL(VAL,K,SMALL,DIFF,XARG)

TITLE - TABLE FUNCTION WITH EQUAL INTERVALS AND LIMITED
        EXTRAPOLATION

VERSION - 1A

DATE - 1 JUNE 1975

PURPOSE - TO CALCULATE THE APPROXIMATE VALUE OF A MATHEMATICAL
          FUNCTION FOR A GIVEN VALUE OF THE INDEPENDENT VARIABLE
          BY LINEAR INTERPOLATION BETWEEN SPECIFIED POINTS OF THE
          FUNCTION.

INPUT REQUIREMENTS

    VALUES TRANSMITTED THROUGH THE ARGUMENT LIST -- DIFF, K,
              SMALL, VAL(I), XARG

    VARIABLES REQUIRING VALUES WHICH WILL NOT CHANGE DURING A RUN
              -- DIFF, K, SMALL, VAL(I)
    VARIABLES REQUIRING VALUES WHICH MAY CHANGE DURING A RUN
              -- XARG

OUTPUT VARIABLES

    THE SINGLE OUTPUT VALUE IS TRANSMITTED THROUGH THE FUNCTION
              NAME, TABEL.

VARIABLE DEFINITIONS

    DIFF - THE COMMON DIFFERENCE, IN UNITS OF THE INDEPENDENT
           ARGUMENT, BETWEEN ADJACENT VALUES OF THE INDEPENDENT
           ARGUMENT FOR WHICH THE FUNCTION VALUE IS SPECIFIED.

    K - THE NUMBER OF SPECIFIED POINTS OF THE FUNCTION.
        UNITLESS.

    SMALL - THE SMALLEST VALUE OF THE INDEPENDENT ARGUMENT, IN
            UNITS OF THE INDEPENDENT ARGUMENT, FOR WHICH THE
            FUNCTION VALUE IS SPECIFIED.

    VAL(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED FUNCTION
             VALUES IN UNITS OF THE DEPENDENT VARIABLE.
             (FUNCTION VALUE)

    XARG - THE VALUE OF THE INDEPENDENT ARGUMENT, IN UNITS OF THE
           INDEPENDENT ARGUMENT, FOR WHICH THE FUNCTION IS TO BE
           EVALUATED.

 DIMENSION VAL(K)
 DUM = XARG - SMALL
 IF(DUM .LT. 0.) DUM = 0.
 IF(DUM .GT. FLOAT(K-1)*DIFF) DUM = FLOAT(K-1)*DIFF
 I = 1. + DUM/DIFF
 IF(I .EQ. K) I = K-1
 TABEL = VAL(I) + ((VAL(I+1) - VAL(I))/DIFF)*(DUM - FLOAT(I-1)*
+        DIFF)
 RETURN
 END
```

Reference

[1] Llewellyn, Robert W., FORDYN: An Industrial Dynamics Simulator (Raleigh, North Carolina, 1965).

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABEX	Table Function with Equal Intervals and Extrapolation	routine	1A

TABEX Abstract

Component TABEX is a table look-up function which approximates an unknown explicit mathematical function relating two variables by a linear interpolation between known (or assumed) point values of the dependent variable (the output) corresponding to equally spaced values of the independent variable (the input). Beyond the specified range, a straight-line extrapolation is used, where the slope of the line is the same as that of the last segment within the specified range.

Inputs to TABEX include a set of point values for the dependent variable, the smallest value of the independent variable, and the difference between successive values. To use the function, a value for the independent variable must be supplied; and the resulting output is the corresponding value for the dependent variable.

TABEX requires 56 decimal words of core storage and .21 milliseconds to execute on a CDC 6500 computer.

Related routines, which feature limited extrapolation and/or arbitrary spacing of the independent variable, are TABEL, TABUL, and TABUX.

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABEX	Table Function with Equal Intervals and Extrapolation	routine	1A

TABEX Documentation

Contents

I. Process Description 1

II. Technical Description 2

A. Mathematical model 2

B. Sample run 4

III. Program Information 9

A. Program description 9

B. Program implementation 9

C. Program lists 9

1. Of sample run executive 10

2. Of component 11

Bibliographic Reference 12

I. Process Description

Very frequently in simulation work, particularly with socioeconomic systems, the situation arises that one system variable is to be causally related to another; and either an explicit mathematical function is not known or one cannot be reasonably assumed. Often, however, experimental results, direct observation of the real system, or expert opinion can at least specify corresponding point values of the dependent and independent variables. Examples are birth and death rates as functions of nutrition; projections of production, prices, and other variables as functions of time, i.e., where these variables are treated as inputs to a model rather than as endogenously determined system variables; and labor force participation rates as functions of real wage rates, unemployment rates, education, etc.

The method of interpolation between these specified points depends on the particular problem and the number of observations that are available. Two points may be connected by a straight line or by a curve with a constant growth rate. If there are more than two points, a continuous polynomial may be the most realistic way of connecting them, because the path between two points usually depends on the points beyond those two. However, in many cases a straight line interpolation is quite sufficient.

TABEX is a table look-up routine specifically designed for the case where: (1) point values of the dependent variable are known (or assumed) for equally spaced values of the independent variable; (2) straight-line interpolation is used between points; and (3) straight-line extrapolation

of the dependent variable is used beyond the specified range. TABEX is based on Llewellyn's TABEXE routine [1].

II. Technical Description

A. Mathematical model

Figure 1 illustrates the relationships among the variables and parameters appearing in the equations below. To use a concrete example, assume the curve in Figure 1 gives the growth rate Y of a population as a function of temperature X relative to some base temperature.

The computation of the output Y falls into three categories, depending on the value of the input X:

(1) if $VX_i < X \leq VX_{i+1}$, then

$$Y = VY_i + \frac{VY_{i+1} - VY_i}{VX_{i+1} - VX_i} (X - VX_i)$$

$$= VY_i + \frac{X - VX_i}{DX} (VY_{i+1} - VY_i), \qquad i \varepsilon \{1, 2, \ldots, k-1\}$$

(2) if $X \leq VX_1$, then

$$Y = VY_1 + \frac{VY_2 - VY_1}{VX_2 - VX_1} (X - VX_1)$$

$$= VY_1 + \frac{X - VX_1}{DX} (VY_2 - VY_1)$$

(3) if $X > VX_k$, then

$$Y = VY_k + \frac{VY_k - VY_{k-1}}{VX_k - VX_{k-1}} (X - VX_k)$$

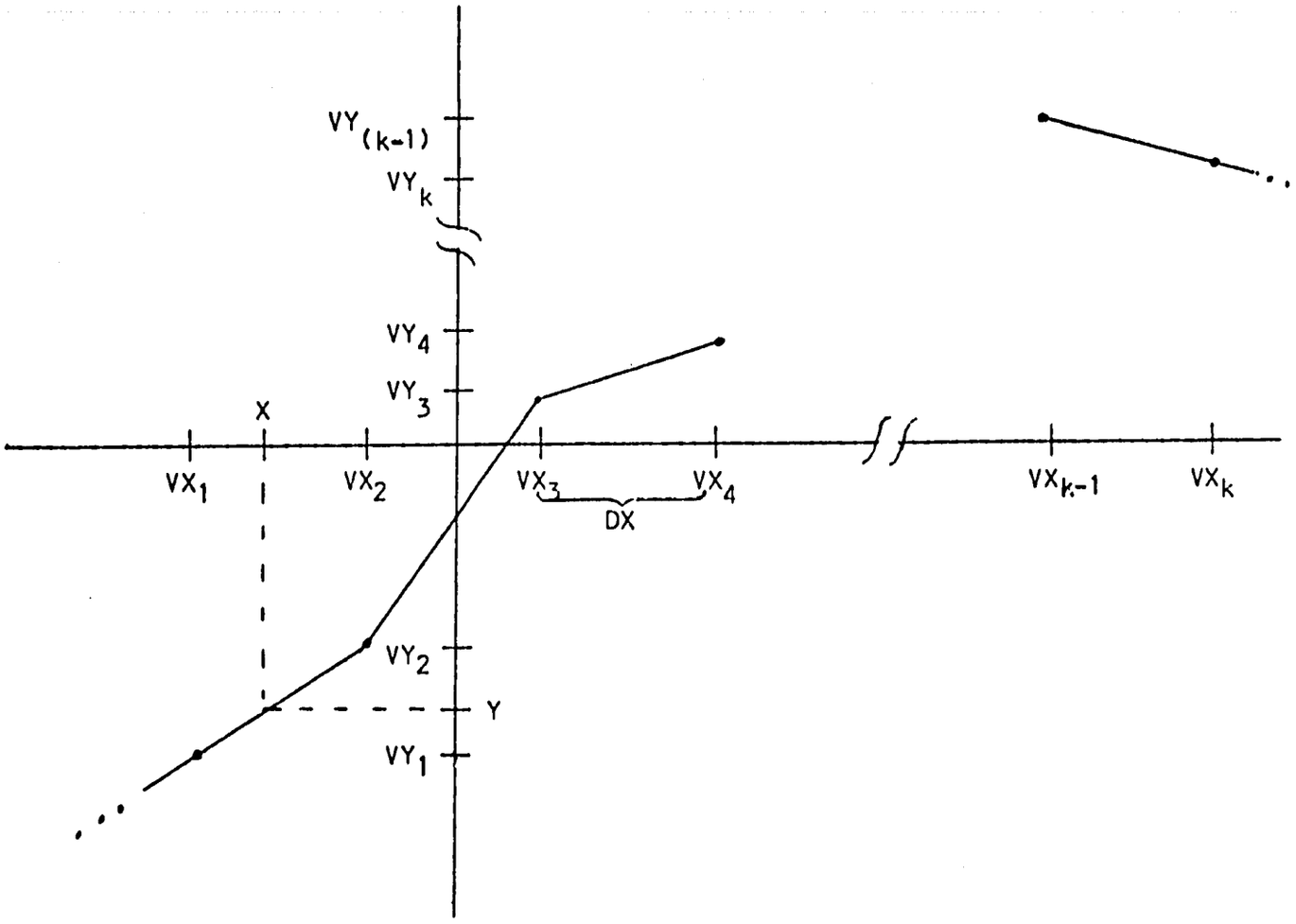

Figure 1

The TABEX Table Look-Up Function

$$= VY_k + \frac{X - VX_k}{DX} (VY_k - VY_{k-1})$$

where:

X = the current value of the independent variable (the input to TABEX)--i.e., relative temperature

Y = the corresponding value of the dependent variable (the output of TABEX)--i.e., the population growth rate

VX_i = the ith specified value of the independent variable*

VY_i = the ith specified value of the dependent variable

DX = the constant dependent variable interval,* $VX_i - VX_{i-1}$, $i = 2, \ldots, k$

i = indexes the data points, $i = 1, \ldots, k$.

Notice that for $X \notin [VX_1, VX_k]$, Y is extrapolated beyond the range using the slope of the last segment within the range.

TABEX is one of a set of four table look-up routines (Table 1) which cover situations where extrapolations are limited to end point values and/or unequal intervals of the independent variable are used.

B. Sample run

The output for a sample run of TABEX appears in Figure 3. (TABEX does not write any output itself.) For the sample run, the example followed above is used; that is, the percentage growth rate of a population is assumed to be determined as a function of relative temperature, where relative temperature is defined as the difference between ambient temperature

---

*The equal interval feature of TABEX allows efficiencies in the computer programming (see the program list and the TABEX Program Documentation). Instead of requiring all k values for VX_i, $i = 1, \ldots, k$, TABEX only requires values for VX_1, DX, and k. VX_i is then found from $VX_i = VX_1 + DX \cdot (i - 1)$, $i = 1, \ldots, k$.

and some base value (Figure 2). Growth rates are assumed known at five-degree temperature intervals, and TABEX linearly interpolates between these points as shown in Figure 2. For example, if temperature is 12.5 degrees, the growth rate is computed to be 4% per unit time. Percentage growth rates are determined in the test program by TABEX for temperatures (the first column in Figure 3) at the mid-points of the specified intervals and outside the specified range.

The last three columns of Table 1 show the compile and execution central processor times and core storage required by TABEX (and the other table look-up routines for comparison) on Michigan State University's CDC 6500 computer with its FORTRAN Extended (version 3.0) compiler and operating under HUSTLER 2, MSU's version of CDC's SCOPE 3.2 operating system.

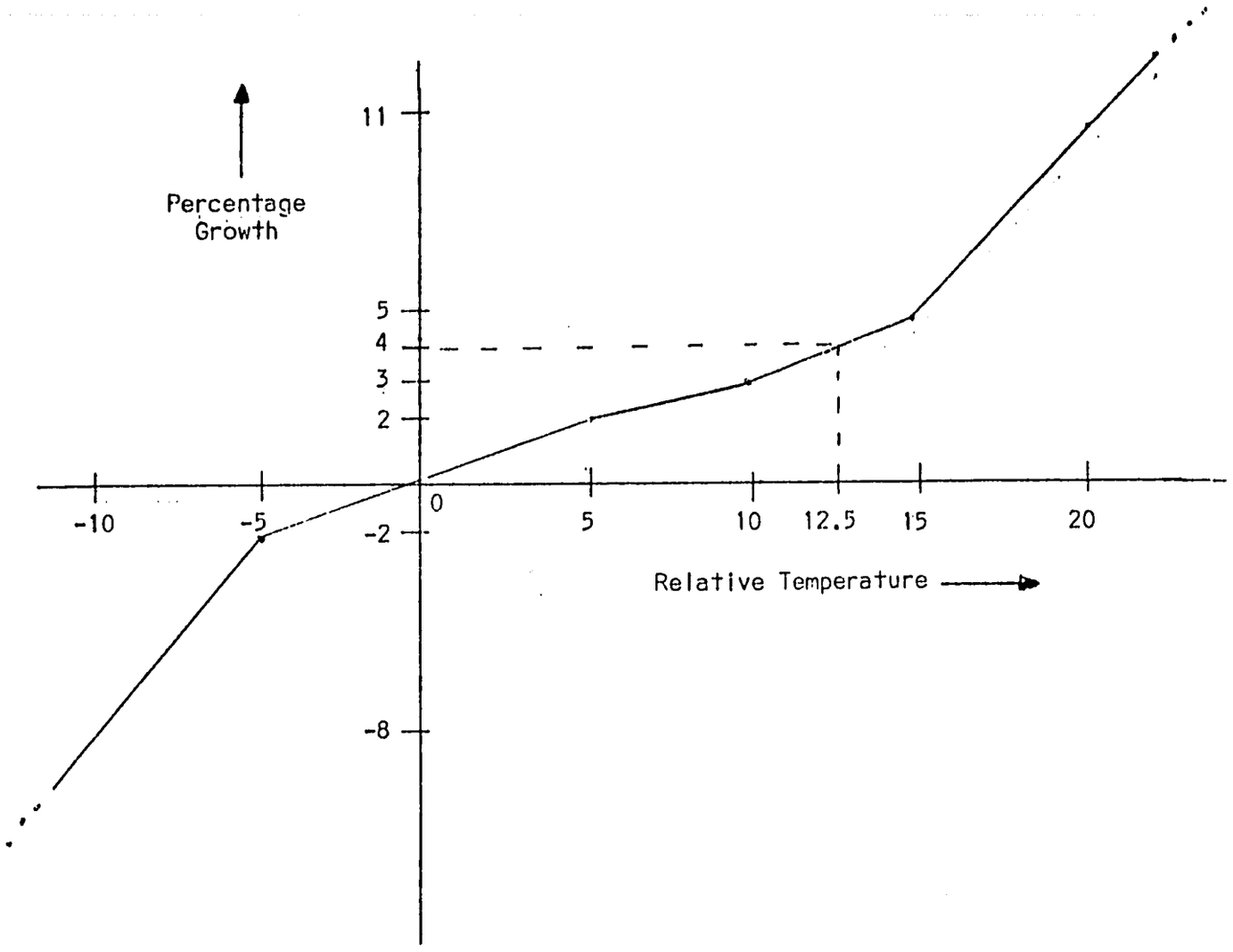

Figure 2

TABEX Sample Run Table Function

```
EXEC BEGIN.15.44.22.

                    RELATIVE          GROWTH
                   TEMPERATURE         RATE

                      -12.5           -11.00
                       -7.5            -5.00
                       -2.5            -1.00
                        2.5             1.00
                        7.5             2.50
                       12.5             4.00
                       17.5             8.00
                       22.5            14.00
STOP
TK-
```

Figure 3

TABEX Sample Run Output

Table 1

CLASS Table Look-Up Routines

Routines	Limited extrapolation	Straight-line extrapolation	Equal intervals	Unequal intervals	Compile time (central processor)--seconds	Execution time (central processor)--milliseconds (using a table with 7 points)	Core storage--words (decimal)
TABEL	X		X		.285	.25	70
TABEX		X	X		.222	.21	56
TABUL	X			X	.255	.26	59
TABUX		X		X	.187	.22	46

III. Program Information

A. Program Description

TABEX was programmed and tested in the following operating environment:

Machine: CDC 6500
Operating system: MSU HUSTLER 2 L239 LSD32--an extension of CDC SCOPE 3.2
Compiler: CDC FTN V3.0-P357
Core requirements: 56 decimal words
CP compile time: .222 seconds
CP execution time: .21 milliseconds
Tapes: none
Direct access files: none
FORTRAN library subprograms used: none

TABEX consists of a single function subprogram which calculates the approximate value of a single mathematical function for a given value of the independent variable by linear interpolation between specified points of the function. No COMMON blocks are used.

B. Program Implementation

All input is transmitted through the argument list. The single output value is transmitted through the function name TABEX.

VAL must be dimensioned to at least K in the calling program. The following must also be true:

K > 1

DIFF > 0

C. Program Lists

The function TABEX and the program used for the sample run are listed below:

```
      PROGRAM TSTIPS(OUTPUT)
      DIMENSION XARG(8),VAL1(7)
      DATA VAL1 / -8.,  -2.,   0.,   2.,   3.,   5.,  11. /
      DATA XARG / -12.5, -7.5, -2.5,  2.5,  7.5, 12.5, 17.5, 22.5 /
      DATA SMALL,DIFF / -10., 5. /
      PRINT90
      DO 60 I=1,8
        YVAL1 = TABSX(VAL1,7,SMALL,DIFF,XARG(I))
        PRINT91,XARG(I),YVAL1
60    CONTINUE
      STOP
90    FORMAT(1H1,19X,8HRELATIVE,3X,5HGROWTH/19X,11HTEMPERATURE,7X,
     +       4HRATE/)
91    FORMAT(20X,F7.1,F15.2)
      END
```

```
FUNCTION TABEX(VAL,K,SMALL,DIFF,XARG)
```

TITLE - TABLE FUNCTION WITH EQUAL INTERVALS AND EXTRAPOLATION

DATE - 1 DEC 1973

PURPOSE - TO CALCULATE THE APPROXIMATE VALUE OF A MATHEMATICAL FUNCTION FOR A GIVEN VALUE OF THE INDEPENDENT VARIABLE BY LINEAR INTERPOLATION BETWEEN SPECIFIED POINTS OF THE FUNCTION.

INPUT REQUIREMENTS

VALUES TRANSMITTED THROUGH THE ARGUMENT LIST -- DIFF, K, SMALL, VAL(I), XARG

VARIABLES REQUIRING VALUES WHICH WILL NOT CHANGE DURING A RUN -- DIFF, K, SMALL, VAL(I)

VARIABLES REQUIRING VALUES WHICH MAY CHANGE DURING A RUN -- XARG

OUTPUT VARIABLES

THE SINGLE OUTPUT VALUE IS TRANSMITTED THROUGH THE FUNCTION NAME, TABEX.

VARIABLE DEFINITIONS

DIFF - THE COMMON DIFFERENCE, IN UNITS OF THE INDEPENDENT ARGUMENT, BETWEEN ADJACENT VALUES OF THE INDEPENDENT ARGUMENT FOR WHICH THE FUNCTION VALUE IS SPECIFIED.

K - THE NUMBER OF SPECIFIED POINTS OF THE FUNCTION. UNITLESS.

SMALL - THE SMALLEST VALUE OF THE INDEPENDENT ARGUMENT, IN UNITS OF THE INDEPENDENT ARGUMENT, FOR WHICH THE FUNCTION VALUE IS SPECIFIED.

VAL(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED FUNCTION VALUES IN UNITS OF THE DEPENDENT VARIABLE. (FUNCTION VALUE)

XARG - THE VALUE OF THE INDEPENDENT ARGUMENT, IN UNITS OF THE INDEPENDENT ARGUMENT, FOR WHICH THE FUNCTION IS TO BE EVALUATED.

```
DIMENSION VAL(K)
DUM = XARG - SMALL
I = 1. + DUM/DIFF
IF(I .LT. 1) I = 1
IF(I .GT. K-1) I = K-1
TABEX = VAL(I) + ((VAL(I+1) - VAL(I))/DIFF)*(DUM - FLOAT(I-1)*
+          DIFF)
RETURN
END
```

Reference

[1] Llewellyn, Robert W., FORDYN: An Industrial Dynamics Simulator (Raleigh, North Carolina, 1965).

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABUL	Table Function with Unequal Intervals and Extrapolation	routine	1A 6/1/75

TABUL Abstract

Component TABUL is a table look-up function which approximates an unknown explicit mathematical function relating two variables by a linear interpolation between known (or assumed) point values of the dependent variable (the output) corresponding to arbitrarily spaced values of the independent variable (the input). Extrapolation beyond the specified range is limited to end point values.

Inputs to TABUL include a set of point values for the dependent variable and a corresponding set of values for the independent variable. To use the function, a value for the independent variable must be supplied; and the resulting output is the corresponding value for the dependent variable.

TABUL requires 59 decimal words of core storage and .26 milliseconds to execute on a CDC 6500 computer.

Related routines, which feature extrapolation and/or equal spacing of the independent variable, are TABEL, TABEX, and TABUX.

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABUL	Table Function with Unequal Intervals and Limited Extrapolation	routine	1A 6/1/75

TABUL Documentation

Contents

I. Process Description 1

II. Technical Description 2

A. Mathematical model 2

B. Sample run 4

III. Program Information 8

A. Program description 8

B. Program implementation 8

C. Program lists 8

1. Of sample run executive 9

2. Of component 10

Bibliographic Reference 11

I. Process Description

Very frequently in simulation work, particularly with socioeconomic systems, the situation arises that one system variable is to be causally related to another; and either an explicit mathematical function is not known or one cannot be reasonably assumed. Often, however, experimental results, direct observation of the real system, or expert opinion can at least specify corresponding point values of the dependent and independent variables. Examples are birth and death rates as functions of nutrition; projections of production, prices, and other variables as functions of time, i.e., where these variables are treated as inputs to a model rather than as endogenously determined system variables; and labor force participation rates as functions of real wage rates, unemployment rates, education, etc.

The method of interpolation between these specified points depends on the particular problem and the number of observations that are available. Two points may be connected by a straight line or by a curve with a constant growth rate. If there are more than two points, a continuous polynomial may be the most realistic way of connecting them, because the path between two points usually depends on the points beyond those two. However, in many cases a straight line interpolation is quite sufficient.

TABUL is a table look-up routine specifically designed for the case where: (1) point values of the dependent variable are known (or assumed) for arbitrarily spaced values of the independent variable; (2) straight-line interpolation is used between points; and (3) extrapolation is limited to end point values. TABUL is based on Llewellyn's TABEL routine [1].

II. Technical Description

A. Mathematical model

Figure 1 illustrates the relationships among the variables and parameters appearing in the equations below. To use a concrete example, assume the curve in Figure 1 gives the growth rate Y of a population as a function of temperature X relative to some base temperature.

The computation of the output Y falls into three categories, depending on the value of the input X:

(1) if $VX_i < X \leq VX_{i+1}$, then

$$Y = VY_i + \frac{VY_{i+1} - VY_i}{VX_{i+1} - VX_i} (X - VX_i) \qquad i \varepsilon \{1, 2, \ldots, k-1\}$$

(2) if $X \leq VX_1$, then

$$Y = VY_1$$

(3) if $X > VX_k$, then

$$Y = VY_k$$

where:

X = the current value of the independent variable (the input to TABUL)--i.e., relative temperature

Y = the corresponding value of the dependent variable (the output of TABUL)--i.e., the population growth rate

VX_i = the ith specified value of the independent variable

VY_i = the ith specified value of the dependent variable

i = indexes the data points, i - 1, ..., k.

Notice that for $X \not\varepsilon [VX_1, VX_k]$, Y is set equal to the respective end point value, VY_1 or VY_k.

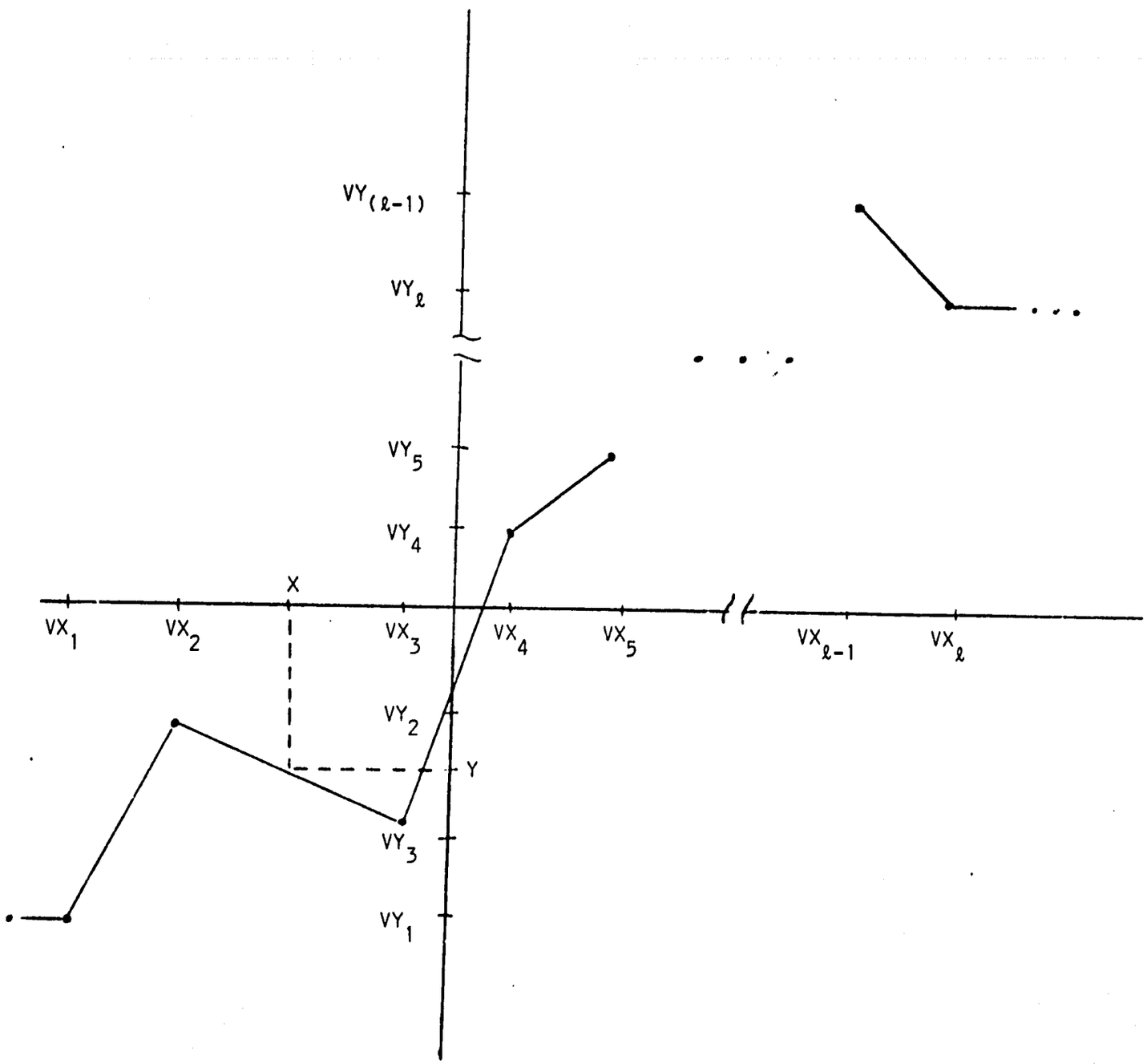

Figure 1

The TABUL Table Look-Up Function

TABUL is one of a set of four table look-up routines (Table 1) which cover situations where straight-line extrapolations are made beyond the specified range and/or equal intervals of the independent variable are used.

B. Sample run

The output for a sample run of TABUL appears in Figure 3. (TABUL does not write any output itself.) For the sample run, the example followed above is used; that is, the percentage growth rate of a population is assumed to be determined as a function of relative temperature, where relative temperature is defined as the difference between ambient temperature and some base value (Figure 2). Growth rates are assumed known at the plotted points and TABUL linearly interpolates between these points as shown in Figure 2. For example, if temperature is 6.5 degrees, the growth rate is computed to be 3% per unit time. Percentage growth rates are determined in the test program by TABUL for temperatures (the first column in Figure 3) at the mid-points of the specified intervals and outside the specified range.

The last three columns of Table 1 show the compile and execution central processor times and core storage required by TABUL (and the other table look-up routines for comparison) on Michigan State University's CDC 6500 computer with its FORTRAN Extended (version 3.0) compiler and operating under HUSTLER 2, MSU's version of CDC's SCOPE 3.2 operating system.

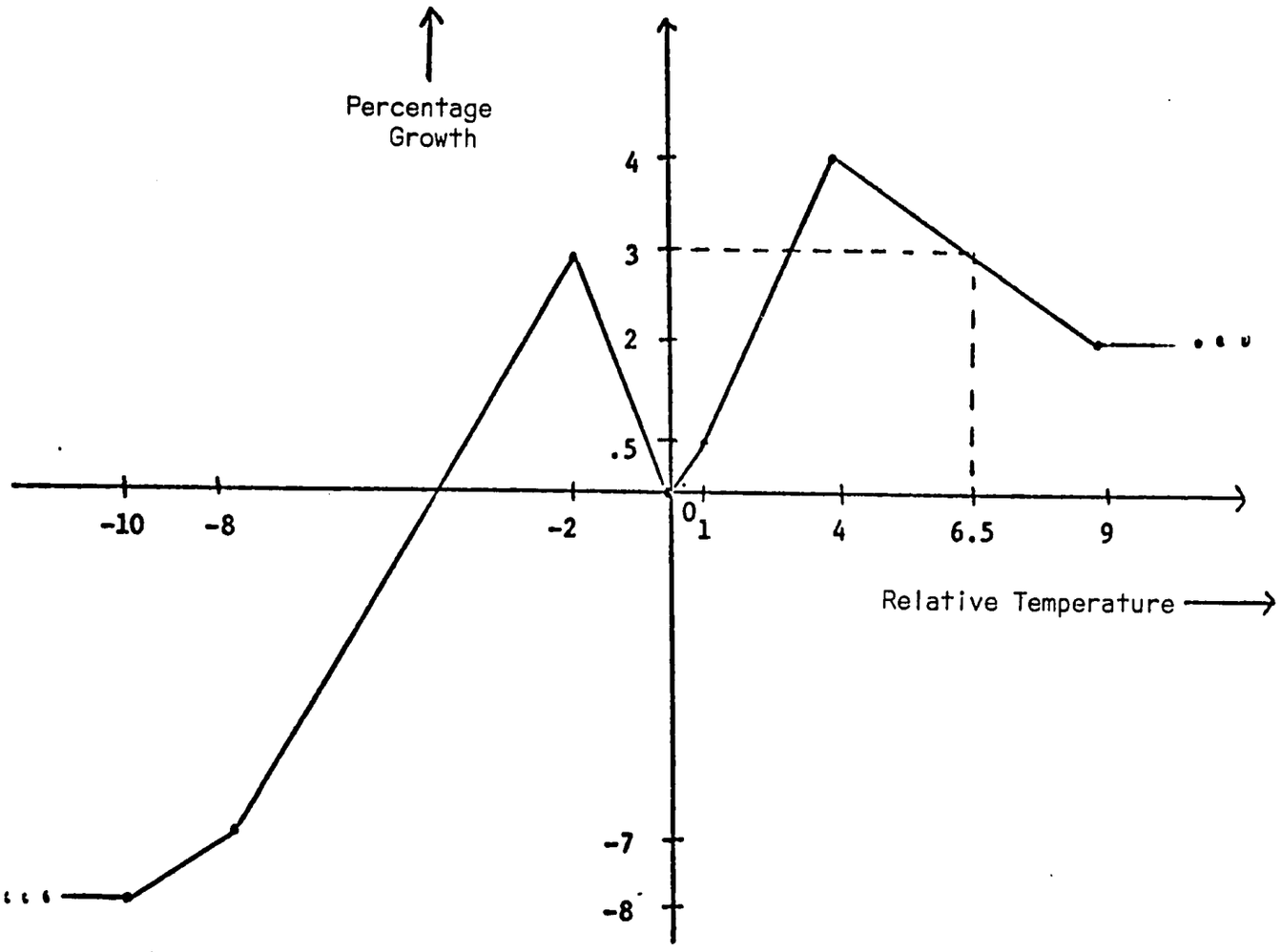

Figure 2

TABUL Sample Run Table Function

RELATIVE TEMPERATURE	GROWTH RATE
-12.0	-8.00
-9.0	-7.50
-5.0	-2.00
-1.0	1.50
.5	.25
2.5	2.25
6.5	3.00
11.5	2.00

Table 1

CLASS Table Look-Up Routines

Routines	Limited extrapolation	Straight-line extrapolation	Equal intervals	Unequal intervals	Compile time (central processor)--seconds	Execution time (central processor)--milliseconds (using a table with 7 points)	Core storage--words (decimal)
TABEL	X		X		.285	.25	70
TABEX		X	X		.222	.21	56
TABUL	X			X	.255	.26	59
TABUX		X		X	.187	.22	46

III. Program Information

A. Program Description

TABUL was programmed and tested in the following operating environment:

Machine: CDC 6500
Operating system: MSU HUSTLER 2 L239 LSD32--an extension of CDC SCOPE 3.2
Compiler: CDC FTN V3.0-P357
Core requirements: 59 decimal words
CP compile time: .255 seconds
CP execution time: .26 milliseconds
Tapes: none
Direct access files: none
FORTRAN library subprograms used: none

TABUL consists of a single function subprogram which calculates the approximate value of a single mathematical function for a given value of the independent variable by linear interpolation between specified points of the function. No COMMON blocks are used.

B. Program Implementation

All input is transmitted through the argument list. The single output value is transmitted through the function name TABUL.

K must be greater than 1. ARG and VAL must be dimensioned to at least K in the calling program. ARG must be arranged in ascending order, i.e., each element must be greater than the preceding one.

C. Program Lists

The function TABUL and the program used for the sample run are listed below:

```
      PROGRAM T3TTB3(OUTPUT)
      DIMENSION ARG(7),VAL1(7),XARG(8)
      DATA ARG /  -10.,  -8.,  -2.,   0.,   1.,   4.,   9. /
      DATA VAL1 /  -8.,  -7.,   3.,   0.,   .5,   4.,   2. /
      DATA XARG / -12.,  -9.,  -5.,  -1.,   .5,  2.5,  6.5, 11.5 /
      PRINT90
      DO 60 I=1,8
        YVAL1 = TABUL(VAL1,7,ARG,XARG(I))
        PRINT91,XARG(I),YVAL1
60    CONTINUE
      STOP
90    FORMAT(1H1,19X,8HRELATIVE,8X,6HGROWTH/19X,11HTEMPERATURE,7X,
     +       4HRATE/)
91    FORMAT(20X,F7.1,F15.2)
      END
```

```
      FUNCTION TABUL(VAL,K,ARG,XARG)

      TITLE - TABLE FUNCTION WITH UNEQUAL INTERVALS AND LIMITED
              EXTRAPOLATION

      VERSION - 1A

      DATE - 1 JUNE 1975

      PURPOSE - TO CALCULATE THE APPROXIMATE VALUE OF A MATHEMATICAL
                FUNCTION FOR A GIVEN VALUE OF THE INDEPENDENT VARIABLE
                BY LINEAR INTERPOLATION BETWEEN SPECIFIED POINTS OF THE
                FUNCTION.

      INPUT REQUIREMENTS

          VALUES TRANSMITTED THROUGH THE ARGUMENT LIST -- ARG(I), K,
                    VAL(I), XARG

          VARIABLES REQUIRING VALUES WHICH WILL NOT CHANGE DURING A RUN
                    -- ARG(I), K, VAL(I)
          VARIABLES REQUIRING VALUES WHICH MAY CHANGE DURING A RUN
                    -- XARG

      OUTPUT VARIABLES

          THE SINGLE OUTPUT VALUE IS TRANSMITTED THROUGH THE FUNCTION
                    NAME, TABUL.

      VARIABLE DEFINITIONS

          ARG(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED VALUES OF
                   THE INDEPENDENT ARGUMENT, IN UNITS OF THE INDEPEN-
                   DENT ARGUMENT. (ARGUMENT VALUE).

          K - THE NUMBER OF SPECIFIED POINTS OF THE FUNCTION.
              UNITLESS.

          VAL(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED FUNCTION
                   VALUES IN UNITS OF THE DEPENDENT VARIABLE.
                   (FUNCTION VALUE)

          XARG - THE VALUE OF THE INDEPENDENT ARGUMENT, IN UNITS OF THE
                 INDEPENDENT ARGUMENT, FOR WHICH THE FUNCTION IS TO BE
                 EVALUATED.

      DIMENSION VAL(K),ARG(K)
      DUM = XARG
      IF(DUM .GT. ARG(K)) DUM = ARG(K)
      IF(DUM .LT. ARG(1)) DUM = ARG(1)
      DO 10 I=2,K
        IF (DUM .GT. ARG(I))GO TO 10
        TABUL = VAL(I-1) + ((VAL(I) - VAL(I-1))/(ARG(I) - ARG(I-1)))*
     +          (DUM - ARG(I-1))
        RETURN
10    CONTINUE
      END
```

Reference

[1] Llewellyn, Robert W., FORDYN: An Industrial Dynamics Simulator (Raleigh, North Carolina, 1965).

CLASS Document

1. *Symbolic Name*	2. *Name*	3. *Type*	4. *Version No. and Date*
TABUX	Table Function with Unequal Intervals and Extrapolation	routine	1A 6/1/75

TABUX Abstract

Component TABUX is a table look-up function which approximates an unknown explicit mathematical function relating two variables by a linear interpolation between known (or assumed) point values of the dependent variable (the output) corresponding to arbitrarily spaced values of the independent variable (the input). Beyond the specified range, a straight-line extrapolation is used, where the slope of the line is the same as that of the last segment within the specified range.

Inputs to TABUX include a set of point values for the dependent variable and a corresponding set of values for the independent variable. To use the function, a value for the independent variable must be supplied; and the resulting output is the corresponding value for the dependent variable.

TABUX requires 46 decimal words of core storage and .22 milliseconds to execute on a CDC 6500 computer.

Related routines, which feature limited extrapolation and/or equal spacing of the independent variable, are TABEL, TABEX, and TABUL.

CLASS Document

1. Symbolic Name	2. Name	3. Type	4. Version No. and Date
TABUX	Table Function with Unequal Intervals and Extrapolation	routine	1A 6/1/75

TABUX Documentation

Contents

I. Process Description 1

II. Technical Description 2

A. Mathematical model 2

B. Sample run 4

III. Program Information 8

A. Program description 8

B. Program implementation 8

C. Program lists 8

1. Of sample run executive 9

2. Of component 10

Bibliographic Reference 11

I. Process Description

Very frequently in simulation work, particularly with socioeconomic systems, the situation arises that one system variable is to be causally related to another; and either an explicit mathematical function is not known or one cannot be reasonably assumed. Often, however, experimental results, direct observation of the real system, or expert opinion can at least specify corresponding point values of the dependent and independent variables. Examples are birth and death rates as functions of nutrition; projections of production, prices, and other variables as functions of time, i.e., where these variables are treated as inputs to a model rather than as endogenously determined system variables; and labor force participation rates as functions of real wage rates, unemployment rates, education, etc.

The method of interpolation between these specified points depends on the particular problem and the number of observations that are available. Two points may be connected by a straight line or by a curve with a constant growth rate. If there are more than two points, a continuous polynomial may be the most realistic way of connecting them, because the path between two points usually depends on the points beyond those two. However, in many cases a straight line interpolation is quite sufficient.

TABUX is a table look-up routine specifically designed for the case where: (1) point values of the dependent variable are known (or assumed) for arbitrarily spaced values of the independent variable; (2) straight-line interpolation is used between points, and (3) straight-line extrapolation

of the dependent variable is used beyond the specified range. TABUX is based on Llewellyn's TABEX routine [1].

II. Technical Description

A. Mathematical model

Figure 1 illustrates the relationships among the variables and parameters appearing in the equations below. To use a concrete example, assume the curve in Figure 1 gives the growth rate Y of a population as a function of temperature X relative to some base temperature.

The computation of the output Y falls into three categories, depending on the value of the input X:

(1) if $VX_i < X \leq VX_{i+1}$, then

$$Y = VY_i + \frac{VY_{i+1} - VY_i}{VX_{i+1} - VX_i} (X - VX_i) \qquad i \varepsilon \{1, 2, \ldots, k-1\}$$

(2) if $X \leq VX_1$, then

$$Y = VY_1 + \frac{VY_2 - VY_1}{VX_2 - VX_1} (X - VX_1)$$

(3) if $X > VX_k$, then

$$Y = VY_k + \frac{VY_k - VY_{k-1}}{VX_k - VX_{k-1}} (X - VX_k)$$

where:

X = the current value of the independent variable (the input to TABUX)--i.e., relative temperature

Y = the corresponding value of the dependent variable (the output of TABUX)--i.e., the population growth rate

VX_i = the ith specified value of the independent variable

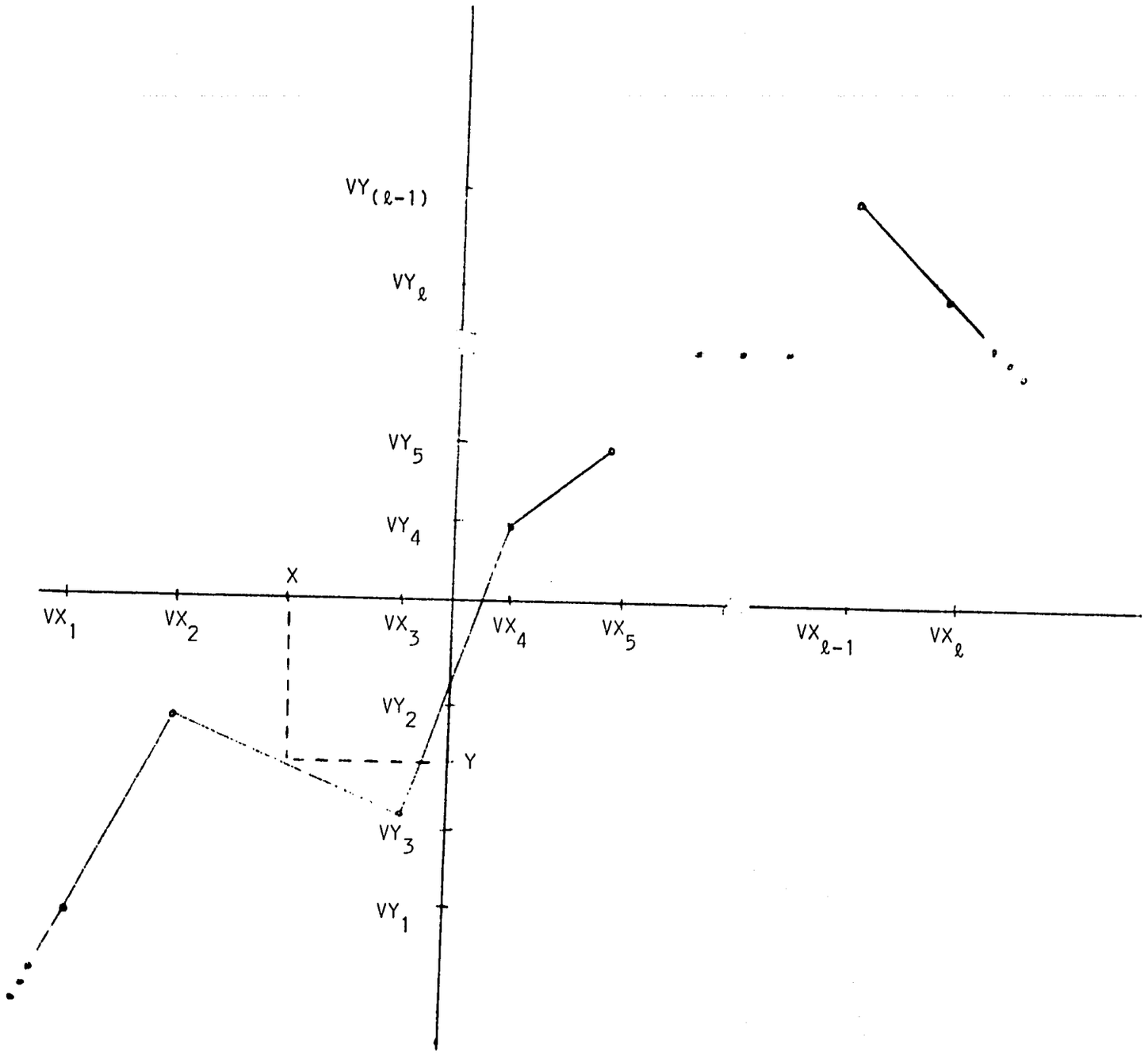

Figure 1

The TABUX Table Look-Up Function

VY_i = the ith specified value of the dependent variable

i = indexes the data points, $i = 1, \ldots, k$.

Notice that for $X \notin [VX_1, VX_k]$, Y is extrapolated beyond the range using the slope of the last segment within the range.

TABUX is one of a set of four table look-up routines (Table 1) which cover situations where extrapolations are limited to end point values and/or equal intervals of the independent variable are used.

B. Sample run

The output for a sample run of TABUX appears in Figure 3. (TABUX does not write any output itself.) For the sample run, the example followed above is used; that is, the percentage growth rate of a population is assumed to be determined as a function of relative temperature, where relative temperature is defined as the difference between ambient temperature and some base value (Figure 2). Growth rates are assumed known at the plotted points and TABUX linearly interpolates between these points as shown in Figure 2. For example, if temperature is 6.5 degrees, the growth rate is computed to be 3% per unit time. Percentage growth rates are determined in the test program by TABUX for temperatures (the first column in Figure 3) at the mid-points of the specified intervals and outside the specified range.

The last three columns of Table 1 show the compile and execution central processor times and core storage required by TABUX (and the other table look-up routines for comparison) on Michigan State University's CDC 6500 computer with its FORTRAN Extended (version 3.0) compiler and operating under HUSTLER 2, MSU's version of CDC's SCOPE 3.2 operating system.

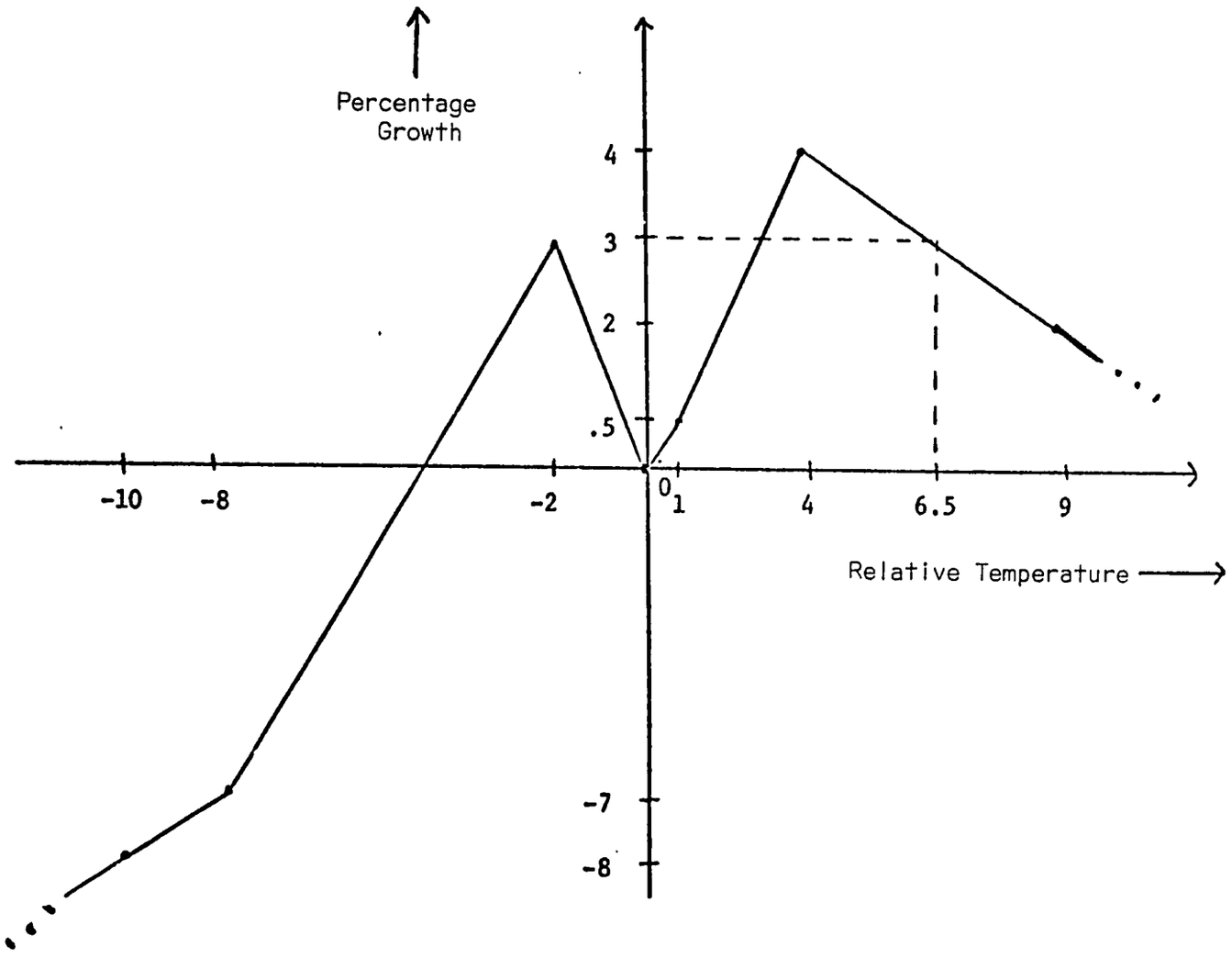

Figure 2

TABUX Sample Run Table Function

RELATIVE TEMPERATURE	GROWTH RATE
-12.0	-9.00
-9.0	-7.50
-5.0	-2.00
-1.0	1.50
.5	.25
2.5	2.25
6.5	3.00
11.5	1.00

Figure 3

TABUX Sample Run Output

Table 1

CLASS Table Look-Up Routines

Routines	Limited extrapolation	Straight-line extrapolation	Equal intervals	Unequal intervals	Compile time (central processor)--seconds	Execution time (central processor)--milliseconds (using a table with 7 points)	Core storage--words (decimal)
TABEL	X		X		.285	.25	70
TABEX		X	X		.222	.21	56
TABUL	X			X	.255	.26	59
TABUX		X		X	.187	.22	46

III. Program Information

A. Program Description

TABUX was programmed and tested in the following operating environment:

Machine: CDC 6500
Operating system: MSU HUSTLER 2 L239 LSD32--an extension of CDC SCOPE 3.2
Compiler: CDC FTN V3.0-P357
Core requirements: 46 decimal words
CP compile time: .187 seconds
CP execution time: .22 milliseconds
Tapes: none
Direct access files: none
FORTRAN library subprograms used: none

TABUX consists of a single function subprogram which calculates the approximate value of a single mathematical function for a given value of the independent variable by linear interpolation between specified points of the function. No COMMON blocks are used.

B. Program Implementation

All input is transmitted through the argument list. The single output value is transmitted through the function name TABUX.

K must be greater than 1. ARG and VAL must be dimensioned to at least K in the calling program. ARG must be arranged in ascending order, i.e., each element must be greater than the preceding one.

C. Program Lists

The function TABUX and the program used for the sample run are listed below:

```
      PROGRAM TSTTBS(OUTPUT)
      DIMENSION ARG(7),VAL1(7),XARG(8)
      DATA ARG /  -10.,  -8.,  -2.,   0.,   1.,   4.,   9. /
      DATA VAL1 /  -8.,  -7.,   3.,   0.,   .5,   4.,   2. /
      DATA XARG / -12.,  -9.,  -5.,  -1.,   .5,  2.5,  6.5, 11.5 /
      PRINT90
      DO 60 I=1,8
        YVAL1 = TABUX(VAL1,7,ARG,XARG(I))
        PRINT91,XARG(I),YVAL1
60    CONTINUE
      STOP
90    FORMAT(1H1,19X,8HRELATIVE,8X,6HGROWTH/19X,11HTEMPERATURE,7X,
     +       4HRATE/)
91    FORMAT(20X,F7.1,F15.2)
      END
```

```
      FUNCTION TABUX(VAL,K,ARG,XARG)

      TITLE - TABLE FUNCTION WITH UNEQUAL INTERVALS AND EXTRAPOLATION

      VERSION - 1A

      DATE - 1 JUNE 1975

      PURPOSE - TO CALCULATE THE APPROXIMATE VALUE OF A MATHEMATICAL
                FUNCTION FOR A GIVEN VALUE OF THE INDEPENDENT VARIABLE
                BY LINEAR INTERPOLATION BETWEEN SPECIFIED POINTS OF THE
                FUNCTION.

      INPUT REQUIREMENTS

          VALUES TRANSMITTED THROUGH THE ARGUMENT LIST -- ARG(I), K,
                    VAL(I), XARG

          VARIABLES REQUIRING VALUES WHICH WILL NOT CHANGE DURING A RUN
                    -- ARG(I), K, VAL(I)
          VARIABLES REQUIRING VALUES WHICH MAY CHANGE DURING A RUN
                    -- XARG

      OUTPUT VARIABLES

          THE SINGLE OUTPUT VALUE IS TRANSMITTED THROUGH THE FUNCTION
                    NAME, TABUX.

      VARIABLE DEFINITIONS

          ARG(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED VALUES OF
                   THE INDEPENDENT ARGUMENT, IN UNITS OF THE INDEPEN-
                   DENT ARGUMENT. (ARGUMENT VALUE).

          K - THE NUMBER OF SPECIFIED POINTS OF THE FUNCTION.
              UNITLESS.

          VAL(I) - AN ORDERED ARRAY CONTAINING THE SPECIFIED FUNCTION
                   VALUES IN UNITS OF THE DEPENDENT VARIABLE.
                   (FUNCTION VALUE)

          XARG - THE VALUE OF THE INDEPENDENT ARGUMENT, IN UNITS OF THE
                 INDEPENDENT ARGUMENT, FOR WHICH THE FUNCTION IS TO BE
                 EVALUATED.

      DIMENSION VAL(K),ARG(K)
      DO 10 J=2,K
        IF(XARG .LE. ARG(J)) GO TO 20
10    CONTINUE
      J = K
20    TABUX = VAL(J-1) + ((VAL(J) - VAL(J-1))/(ARG(J) - ARG(J-1)))*
     +        (XARG - ARG(J-1))
      RETURN
      END
```

Reference

[1] Llewellyn, Robert W., FORDYN: An Industrial Dynamics Simulator (Raleigh, North Carolina, 1965).

CLASS Documents

Number	Title
1	"Computer Library for Agricultural Systems Simulation: A Progress Report," Michael H. Abkin and Tom W. Carroll. (July 1976)
2	"Software Standards Manual." (July 1976)
3	"Policy Analysis Language, Version 2.3, Programmer's Guide for CDC Cyber Computers," Claudia S. Winer and Chris Wolf. (July 1976)
4	"Policy Analysis Language, Version 2.3, Reference Manual," Claudia S. Winer anc Chris Wolf. (July 1976).
5	"Policy Analysis Language, Version 2.3, Guide to Using a PAL Program," Claudia S. Winer and Chris Wolf. (July 1976)
6	"DEMOGC: Demography with Distributed Age Cohorts," Michael H. Abkin and Chris Wolf. (July 1976)
7	"DEMOGD: Demography with Discrete Age Cohorts," Michael H. Abkin and Chris Wolf. (July 1976)
8	"Distributed Delay Routines: DEL, DELS, DELF, DELLF, DELVF, DELLVF," Michael H. Abkin and Chris Wolf. (July 1976)
9	"Table Functions: TABEL, TABEX, TABUL, TABUX," Michael H. Abkin, Chris Wolf, and Tom W. Carroll. (July 1976)
10	"AGACC: Accounting Routine for the Agricultural Sector," Dennis Pervis and Chris Wolf. (July 1976)
11	"User's Guide for the Beef Cattle Enterprise Simulation Model," Michael R. Jaske. (July 1976)
12	"User's Guide to SYSOPT: An Interactive System Optimization Computer Program," Marcus Buchner. (August 1976)