AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D. C. 20523

BIBLIOGRAPHIC INPUT SHEET

FOR AID USE ONLY

1. SUBJECT CLASSIFICATION

A. PRIMARY: Agriculture

B. SECONDARY: Irrigation

2. TITLE AND SUBTITLE

Calucation of hydraulic conductivity, a further evaluation of some predictive methods

3. AUTHOR(S)

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4. DOCUMENT DATE	5. NUMBER OF PAGES	6. ARC NUMBER
1971	6 p.	ARC

7. REFERENCE ORGANIZATION NAME AND ADDRESS

University of Hawaii, Department of Agronomy and Soil Science, College of Tropical Agriculture, Honolulu, Hawaii 98622

8. SUPPLEMENTARY NOTES *(Sponsoring Organization, Publishers, Availability)*

(In Soil Science Society of America Proceedings, v. 35, no. 1, p. 3-8)
(In Hawaii Agricultural Experiment Station Journal, Series No. 1147)

9. ABSTRACT

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10. CONTROL NUMBER: PN-AAC-353

11. PRICE OF DOCUMENT:

12. DESCRIPTORS

Filtration
Hydraulic conductivity
Mathematical models
Percolation
Soil water

13. PROJECT NUMBER:

14. CONTRACT NUMBER: CSD-2833 211(d)

15. TYPE OF DOCUMENT:

AID 590-1 (4-74)

Reprinted from the *Soil Science Society of America Proceedings*
Volume 35, Number 1, January-February 1971
677 South Segoe Road, Madison, Wisconsin 53711 USA

CSC-2833 211(A)
PNC-AAC-353

DIVISION S-1—SOIL PHYSICS

Calculation of Hydraulic Conductivity: A Further Evaluation of Some Predictive Methods[1]

R. E. GREEN AND J. C. COREY[2]

ABSTRACT

A computational method based on the pore-interaction model of Marshall was shown to compare favorably with modified methods of Millington and Quirk and of Marshall for prediction of hydraulic conductivity vs. water content on a number of soils and a glass bead system. All methods required matching one point on the calculated hydraulic conductivity curve to an experimentally measured hydraulic conductivity value. The calculation methods adequately predicted the experimentally measured values and provide satisfactory conductivity data for many applications. An advantage of the proposed method is its independence of the value chosen for the exponent on the porosity term in the prediction equation.

Additional Key Words for Indexing: **soil water, unsaturated conductivity, infiltration, computer methods.**

IN RECENT years several investigators have explored the possibility of predicting the hydraulic conductivity of porous materials from pore-size distribution data. Interest in such predictions is warranted inasmuch as the hydraulic conductivity-water content relationship $K(\theta)$ is relatively difficult to measure while the pore-size distribution is characterized easily by the standard measurement of water content vs. pressure. Childs and Collis-George (1950), Marshall (1958), and Millington and Quirk (1959, 1960, 1961) developed equations for this purpose, and Nielsen et al. (1960), Jackson et al. (1965), Sharma (1966), and Kunze et al. (1968) have tested the equations against experimental data. Some of these tests have indicated the superiority of the Millington and Quirk method when a matching factor is used. Widespread use of conductivity calculations has been discouraged somewhat by conflicting published results on the optimum value of the exponent on the porosity term.

This paper (a) describes a revision of the Marshall computation and (b) evaluates the revised method and those of Marshall and of Millington and Quirk (both modified by the use of a matching factor) by comparing computed values with published experimentally obtained hydraulic conductivities.

Equation [1] is a modified form of the Marshall and Millington and Quirk equations in which the factor K_s/K_{sc} is introduced to match the calculated and observed conductivity at saturation. This form of the equation is similar to that given by Kunze et al. (1968).

$$K(\theta)_i = \frac{K_s}{K_{sc}} \cdot \frac{30\,\gamma^2}{\rho\, g\, \eta} \cdot \frac{\epsilon^p}{n^2} \sum_{j=1}^{m} \left[(2j + 1 - 2i)h_j^{-2}\right] \qquad i = 1, 2, \ldots, m \qquad [1]$$

where

- $K(\theta)_i$ is the calculated conductivity for a specified water content or pressure (cm/min),
- θ is the water content (cm³/cm³),
- i denotes the last water content class on the wet end, e.g., $i = 1$ identifies the pore class corresponding to the saturated water content, and $i = m$ identifies the pore class corresponding to the lowest water content for which conductivity is calculated,
- K_s/K_{sc} is the matching factor (measured saturated conductivity/calculated saturated conductivity),
- γ is the surface tension of water (dynes/cm),
- ρ is the density of water (g/cm³),
- g is the gravitational constant (cm/sec²),
- η is the viscosity of water (g/cm sec⁻¹),
- ϵ is the porosity (cm³/cm³), defined in various ways depending on the method of calculation,
- p is a parameter that accounts for interaction of pore classes,
- n is the total number of pore classes between $\theta = 0$ and θ_s, the saturated water content, $n \geq m$ (see text); n may or may not vary with θ depending on the calculation method, and
- h_j is the pressure for a given class of water-filled pores (cm of H_2O).

VARIATIONS IN CALCULATION METHODS

The manner in which ϵ and n are varied or held constant in equation [1] leads to three methods (I, II, and III) of calculating $K(\theta)$. The difference between methods I, II, and III are summarized in Table 1. All three methods involve the use of a matching factor, as implied by equation [1]. Although Marshall and Millington and Quirk did not use matching factors, later workers (Jackson et al., 1965; Kunze et al., 1968) have modified the Millington and Quirk calculation to improve prediction ability by using a matching factor. We have incorporated this improvement to the original equation in this study.

In Method I, ϵ is the water-filled porosity at each water content θ_i for which the conductivity $K(\theta)_i$ is calculated; n is the number of pore classes in the water content interval from zero to θ_i. Thus, both ϵ and n decrease as the water content decreases. For example, if the total number of pore classes at saturation ($\theta_s = 0.50$ cm³/cm³) is 20, then $n = 20$ and $\epsilon = 0.50$ cm³/cm³; at 50% of saturation $n = 10$ and $\epsilon = 0.25$ cm³/cm³.

[1] Joint contribution from the Savannah River Lab., E. I. du Pont de Nemours & Co., Aiken, S.C. 29801 and the Univ. of Hawaii, Honolulu 96822. Approved for publication as Hawaii Agr. Exp. Sta. Journal Series no. 1147. The information contained in this article was developed during the course of work under Contract AT(07-2)-1 with the US Atomic Energy Commission. Received Oct. 6, 1969. Approved Sept. 29, 1970.

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Table 1—Variations in the conductivity equation [1] for three methods of computation

Method	Definition of ϵ	Definition of n	Value of p in equation [1] in previous work	Reference
I (Modified Marshall, with matching factor)	$\epsilon = \theta_i$ variable	Number of water-filled pore classes corresponding to θ_i variable.	$p = 2$	Marshall (1958) Nielsen et al. (1960) Jackson et al. (1965)
II (Modified Millington and Quirk, with matching factor)	$\epsilon = \theta_i$ variable	Total number of pore-classes at saturation; constant.	$p = 1.0$ $p = 1.33$ $p = 1.33$ $p = 2.0$	Kunze et al. (1968) Millington and Quirk (1959) Jackson et al. (1965) Sharma (1966)
III (this paper)	$\epsilon = \theta_s$ constant.	Total number of pore-classes at saturation; constant.		

In Method II, ϵ is the water-filled porosity at each water content and n is the total number of pore classes at $\epsilon = \theta_s$ (i.e., n is a constant even though the cross sectional area available for flow decreases with unsaturation). This procedure was developed by Millington and Quirk to describe a larger reduction in conductivity at low water contents than was predicted by Marshall's equation. In their 1961 paper, Millington and Quirk state that ". . . for the computation of unsaturated permeability the value of ϵ taken is that of the liquid-filled pore space and the r^2 series is commenced at the appropriate pore class, that is the largest pore containing liquid." They do not state explicitly that the term m^{-2} in their equation [1] (which corresponds to n^{-2} in equation [1] of this paper) remains constant as the value of ϵ is reduced with unsaturation, but the constancy of their m is inferred in their equations for relative permeability of materials with very narrow pore-size distributions. In an earlier paper (1959), Millington and Quirk explain their rationale for allowing ϵ to vary with n constant: "The basis for the variation of ϵ is due to changes in the water velocity which are not accounted for in the r^2 term . . .".

In Method III, proposed here, ϵ is the water-saturated porosity (i.e., $\epsilon = \theta_s$) and n is the total number of pore classes at $\epsilon = \theta_s$. In this method the values of both ϵ and n remain constant when $K(\theta)$ is computed for different values of θ. Our definitions for ϵ and n arise from Marshall's derivation. Marshall's model of pore interaction results in a series of r_i^2 terms each of which has the coefficient $(\epsilon/n^2)(2i - 1)$, where $i = 1, 2, \ldots, n$. At each level of unsaturation in Method I, ϵ takes on a new value ($\epsilon = \theta$) and n is adjusted to correspond to the number of classes of *water filled* pores. On the other hand, in Method III, ϵ/n^2 is constant; ϵ refers to the total porosity and n to the *total* number of interacting pore classes, regardless of the degree of unsaturation. The two approaches are consistent in concept, but lead to the same results only when $p = 2$. The ϵ/n^2 term in Marshall's equation is the product of ϵ in Poiseuille's equation (when applied to porous media) and ϵ/n^2 which results from the pore interaction model.

In equation [1] we have used ϵ^p/n^2 to make the equation applicable to the three methods of calculation performed here. The exponent p in ϵ^p has been assigned values of 2.0 (Marshall), 1.33 (Millington and Quirk), and 1.0 (Kunze et al.). The derivation of Marshall could lead to $1 < p < 2$, depending on the nature of pore interaction described by the physical model. Because both derivations are based largely on Poiseuille's Law, which has recognized limitations in porous media (Philip, 1958), the value chosen for p appears arbitrary within the above defined limits.

In all three methods of computation, m is a constant at all water contents, and the value of m establishes the number of pore classes for which h_i^{-2} terms are included in the calculation at saturation.

A value of $m = 20$ was chosen to calculate the results given in this report as a result of an evaluation of the effect of changes in m (5, 10, 20, and 50) on the relation between predicted and measured $K(\theta)$ curves. The principal consideration in choosing m for the calculations was accurate representation of the actual water content-pressure curve, which determines the pressure values assigned to each pore class. Accurate representation of the water content-pressure curve is often difficult with m less than 10. Calculations showed inconsistencies in the direction of the effect of m on conductivities at low water contents; this inconsistency was apparently due to the interaction between the effect of m on the assignment of pressure classes and the influence of m on the behavior of ϵ/n^2.

The value of n in equation [1] is not generally the same as m; there are n water content classes between saturation and zero water content, but the calculation is accomplished with only the m classes which cover the experimental water content-pressure curve. Thus with $m = 20$ in the present study, the 20 classes covered the water content range from saturation down to the lowest water content for which experimental data were available. If θ_L is the lowest water content on the experimental moisture characteristic curve and θ_s is the water content at saturation, then the water content range is $\theta_s - \theta_L$. Each pressure class corresponds to a standard water content increment I, which is calculated as follows: $I = (\theta_s - \theta_L)/20$. The parameter n in equation [1] is then calculated, $n = (\theta_s - 0.0)/I$. Thus, n is the number of classes (each having the magnitude of the standard increment) into which the total water content range from saturation to zero can be divided. In summary, $n \geq m$ except for those cases in which $\theta_L = 0$. The relationship between n and m is summarized in equation [2]:

$$n = m\left[\frac{\theta_s}{\theta_s - \theta_L}\right] \qquad [2]$$

The distinction between n and m was recognized implicitly in the example shown by Marshall (1958, his Fig. 1 and Table 1) in which $n = 14$ and $m = 12$ by our definitions. Later papers on the subject have not explicitly stated this distinction. A word of caution here is warranted in that large errors in the calculated curves obtained by Method I will result if n is given the value of m when $\theta_L = 0$. Method II and III results are unaffected by erroneously equating n and m.

COMPARISON OF CALCULATED AND EXPERIMENTAL DATA

We calculated hydraulic conductivities for glass beads and a number of soils with Methods I, II, and III and equation [1]. The calculations were made with a computer program that is a revision of one provided by Dr. R. J. Kunze. Method I calculations were made with $p = 2.0$. Method II calculations were made with $p = 1.33$ and $p = 1.0$ in view of the previous use of $p = 1.33$ by Millington and Quirk (1959) and $p = 1.0$ by Kunze et al. (1968). Method III calculations are independent of the value of p, but the matching factor changes with p; all Method III calculations were made with $p = 2$.

The first comparison of the three methods was made with data on a Guelph loam (Elrick and Bowman, 1964). These same data had been used in a previous test of vari-

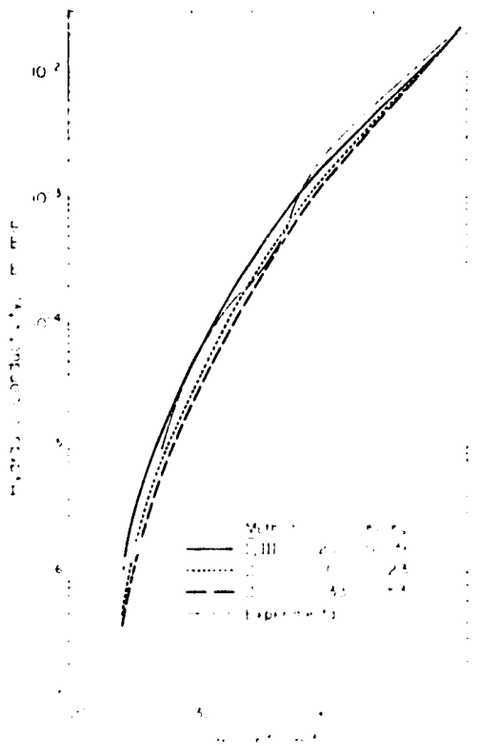

Fig. 1—Calculated hydraulic conductivities for Guelph loam using three methods of computation compared with experimental data from Ehrick and Bowman (1964, their Fig. 3). Calculated and experimental data were matched at θ_s.

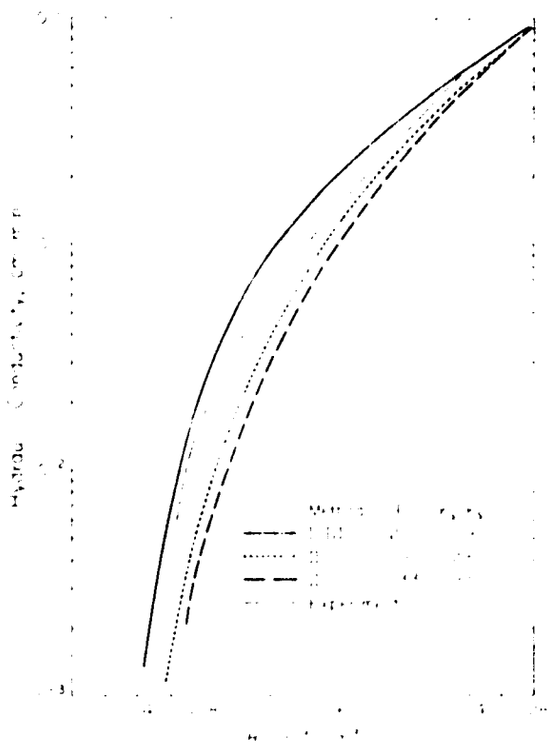

Fig. 2—Calculated hydraulic conductivities for monodispersed glass beads using three methods of computation compared with experimental data from Topp and Miller (1966, their Fig. 2c). Calculated and experimental data were matched at θ_s.

ous conductivity equations by Jackson et al. (1965). All three methods gave reliable predictions of the experimentally measured values (Fig. 1) when matched to the experimentally measured saturated conductivity.

The second comparison was made with data on glass beads (Topp and Miller, 1966). These data had been used in a test of conductivity equations by Kunze et al. (1968). Glass beads provide an extreme case for a porous media because the pore-size distribution is unusually narrow (nearly all the water was removed between pressures of −40 and −50 cm of water). The calculated curves were matched at the saturated conductivity point and adequately represent the data. Method II with $p = 1.0$ and $p = 1.33$ predicted lower hydraulic conductivities than observed; Methods I and III with $p = 2$ predicted values that were higher than those observed.

The range in hydraulic conductivity obtained during desorption and absorption at different positions in a packed soil column subjected to repeated wetting and drying is considerable (Topp, 1969). Computed curves were obtained by the three methods for the desorption case (Fig. 3). These lie within the "envelope" formed by the experimental values except at the lowest water contents. The calculations were made with the desorption main branch of the water content-pressure curve (Topp's Fig. 3, 1969). All of the computed curves for the absorption case lie beneath the measured hydraulic conductivities (Fig. 4). These computed curves were based on the absorption main branch (Topp's Fig. 3, 1969). These results were anticipated in that equation [1] by which the computations were made, requires an accurate description of pore-size distribution. The sequence of pore sizes is more adequately described by the desorption curve than by the absorption curve.

The computed values for the $K(\theta)$ relationship by Methods I, II, and III for an Iowa loess soil, Ida silt loam, are all higher than those experimentally measured by Green et al. (1964). There is generally a greater difference between calculated and measured hydraulic conductivity values than between the three methods of calculating the hydraulic conductivity.

The calculated hydraulic conductivity values by Methods I, II, and III for three depths, 15.2–30.5, 30.5–45.7, and 45.7–60.9 cm (6–12 in., 12–18 in., and 18–24 in.), of a Panoche clay loam are compared (Fig. 6, 7, and 8) with the values measured by Nielsen et al. (1964). In contrast with the earlier figure where the calculated and measured conductivities were matched at saturation, the conductivity at a water content of 0.35 cm^3/cm^3 was used because the saturated conductivity was not available. In these figures only the calculated curves in Fig. 6 deviate seriously in shape from the measured data.

DISCUSSION

The experimental data chosen for comparison with computed conductivity curves represent a wide range of porous materials. Measurement methods were not the same in all

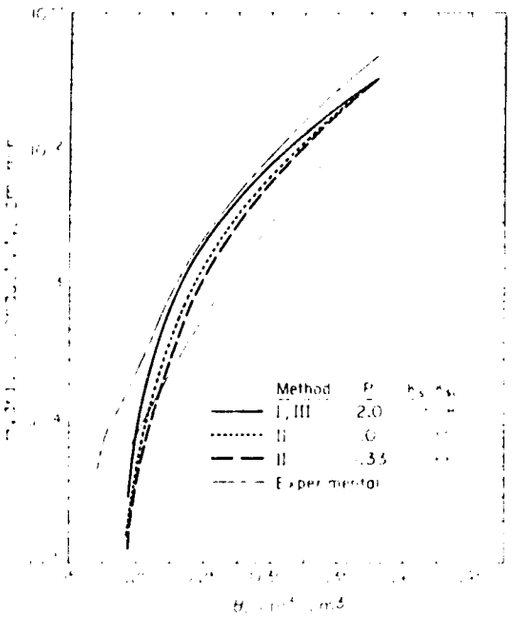

Fig. 3—Calculated hydraulic conductivities for Rubicon sandy loam (desorption characteristic) using three methods of computation compared with experimental data from Topp (1969, his Fig. 4). Calculated and experimental data were matched at θ_s. The experimental data are the upper and lower limits obtained along a soil column during desorption studies.

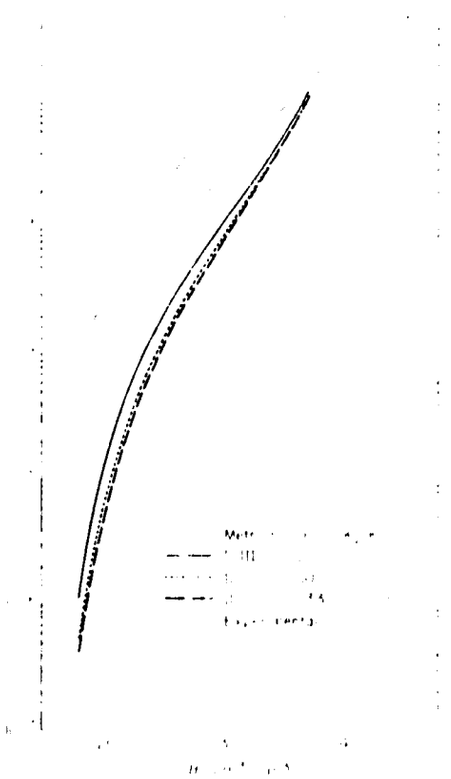

Fig. 4—Calculated hydraulic conductivities for Rubicon sandy loam (absorption characteristic) using three methods of computation compared with experimental data from Topp (1969, his Fig. 4). Calculated and experimental data were matched at θ_s. The experimental data are the upper and lower limits obtained along a soil column during absorption studies.

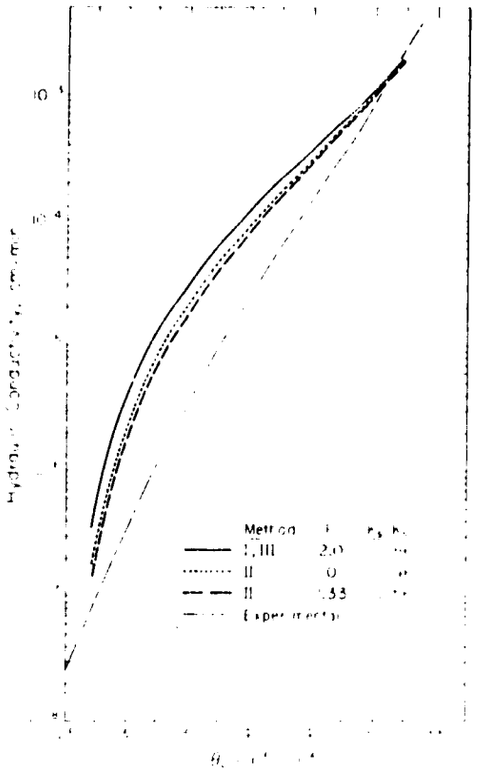

Fig. 5 - Calculated hydraulic conductivities for Ida silt loam using three methods of computation compared with experimental data from Green et al. (1964, their Fig. 3, 0-15 cm depth). Calculated and experimental data were matched at θ_s.

experiments. The most accurate conductivity values were probably determined on packed laboratory columns in which the conductivity was determined by careful measurements of water flux, pressure gradient, and water content as in the experiments of Elrick and Bowman (1964), Topp and Miller (1966), and Topp (1966). The transient outflow method used by Green et al. (1964) is probably less reliable due to undefined hysteresis effects. Field measurements of hydraulic conductivity accomplished by Nielsen et al. (1964) appeared to accurately describe water movement through the various horizons. This field method should be more reliable than conductivity measurements on cores removed from the profile. The water content-pressure data given by Nielsen et al., however, were determined on cores. Thus, in the analysis with data of Nielsen et al. we have computed conductivity values from water retention data on "undisturbed" cores and compared computed conductivities with field-measured conductivity values. This comparison does not provide a reliable test of the calculation methods, but was included to determine how closely the calculated conductivities matched field-measured values.

Calculated curves for all methods of computation gave nearly the same results on a given porous material and there was little difference between results for $p = 1.0$ and $p = 1.33$ with Method II. Method I and III curves tend to diverge upwards at intermediate water contents while Method II results tend to diverge downward from experi-

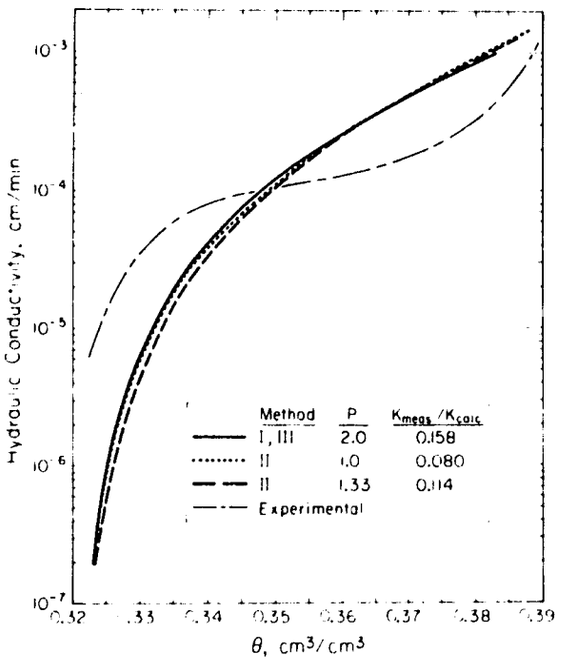

Fig. 6—Calculated hydraulic conductivities for Panoche clay loam ("6 to 12-inch depth") using three methods of computation compared with experimental data from Nielsen et al. (1964, their Fig. 11). Calculated and experimental data were matched at $\theta = 0.35$ cm³/cm³.

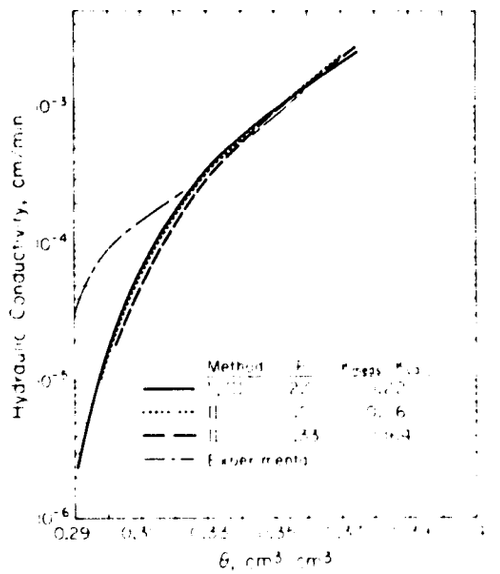

Fig. 7—Calculated hydraulic conductivities for Panoche clay loam ("12 to 18-inch depth") using three methods of computation compared with experimental data from Nielsen et al. (1964, their Fig. 11). Calculated and experimental data were matched at $\theta = 0.35$ cm³/cm³.

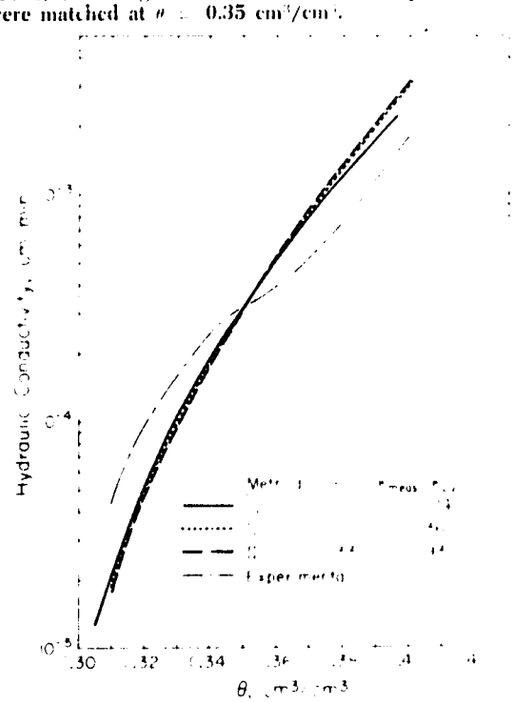

Fig. 8—Calculated hydraulic conductivities for Panoche clay loam ("18 to 24-inch depth") using three methods of computation compared with experimental data from Nielsen et al. (1964, their Fig. 11). Calculated and experimental data were matched at $\theta = 0.35$ cm³/cm³.

mental data at low water contents. All methods of computation appear sufficiently reliable to be used routinely for many field applications. Considering the variation of soils in the field, both horizontally and vertically, a high degree of accuracy in conductivity calculations is not essential for meaningful application of computed results to field problems. It may, in fact, be more important to characterize the variation in the pressure-water content relation of soil cores from many sites in a field location than to accurately measure the $K(\theta)$ relation on a very limited number of cores or field sites.

Another important requirement for meaningful calculated conductivities is a measured saturated conductivity. Hydraulic conductivity results on water-saturated field cores are usually very nonuniform for different cores from the same field site. This is due to variations among cores in the nature of large channels and core boundaries. Cores which show large differences in conductivity at saturation may be quite uniform with respect to the moisture characteristic. For the A horizon, the best alternative to measuring saturated conductivity on "undisturbed" cores may be to measure the "equilibrium" infiltration rate in the field with a suitable double-ring infiltrometer. For greater soil depths, the field method used by Nielsen et al. (1964) can be used to determine water conductivity at one or more water contents for the purpose of matching calculated conductivities. Matching calculated curves to accurately measured saturated conductivity values has a distinct advantage over matching at lower water contents. Inaccuracies in calculated curves can be tolerated at low water contents, but not at high water contents when the $K(\theta)$ results are to be used in subsequent calculations of infiltration or evaporation (Hanks and Bowers, 1963).

Calculations of K versus θ were made but not reported for three Hawaiian soils using Method III. "Equilibrium" infiltration rates were used for matching, and the average moisture characteristics obtained from duplicate or tripli-

cate cores provided the description of pore-size distribution. Matching factors for the three soils were 0.80, 0.66, and 0.50. The nearness of these values to unity indicates that the calculations provided fairly reliable estimates of saturated hydraulic conductivity. Comparisons in Fig. 1 to 8 show, however, that the nearness of the matching factor (K_s/K_{sc} or K_{meas}/K_{calc}) to unity is no index of the accuracy of the entire calculated $K(\theta)$ curve. For example, the relatively low matching factors for Guelph loam (Fig. 1) correspond to reasonably good predictions of the $K(\theta)$ curve.

CONCLUSION

The revised method of calculating K versus θ (Method III), based on Marshall's equation, was found to offer no particular advantage over either the Marshall equation with matching factor (Method I) or the Millington and Quirk equation with matching factor (Method II). All three methods gave results which compared favorably with published experimental results in a number of cases. These results and those of Jackson et al. (1965) and Kunze et al. (1968) should encourage the use of calculation methods, especially for prediction of water relations of field soils.

The computer program used for the calculations is available from either author.

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