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HICKS COEFFICIENT TO DEPICT DIRECTION OF MOVEMENTS IN RELATIVE FACTOR SHARES IN AGRICULTURAL PRODUCTION

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I. Introduction

Technological change in agricultural production plays a crucial role in agricultural development [16, Part 2]. Apart from the direct increase in output, the new technologies have two major impacts on agriculture which are of special importance in low-income countries. First is the widening of income disparities between the laboring class on the one hand, and the capital and landowning classes on the other. Basically this has occurred due to biased distribution of increased output among factors involved in production. This bias appears normally in favor of the land and capital-owning classes. Second, the impact on income distribution is coupled with shifts in demand schedules for certain key inputs such as labor and fertilizer.

The above facts have been explored by several researchers. On the income distribution side, the most striking work has been done by Mellor and Lele [18], Rao [20], and by Srivastav and Heady [27] for India; by Kelley et. al. [11] and by Barker [3] for Philippines; and by Lee [12] for Taiwan. With respect to demand for inputs the analysis done by Billings and Singh [4] and by Evenson [5] for the Indian case are very important.

The analytical framework used by the above writers, however, has several drawbacks. Kelley et. al., Rao and Srivastav restrict their analysis to only two inputs (i.e. either capital and labor or land and labor) and hence predict biased results. Barker and Lee do recognize the importance of land as an input along with capital and labor but they use Cobb-Douglas type production functions which normally are inappropriate to analyze changes in income distribution. Although Mellor and Lele incorporate several inputs and analyze their shares in output, they do not link the apparent income distribution with different levels of inputs and with the corresponding technology. Billings and Singh, and Evenson, on the other hand, give emphasis to the interactions of farm labor and mechanization while leaving out the income distribution side.

In the theoretical context also, the recent technological breakthroughs in agriculture have been dealt with in conflicting ways. The dualistic model developed by Lele and Mellor [13] does take into account land-augmenting technological change but it incorporates only land and labor, and thus excludes the capital input. On the

other hand, Kelley et. al. [11] exclude land and confine their model only to capital and labor by saying:

> "We recognize that the quality and character of land in a low-income economy may have an important effect on the pattern of production and on the rate of increase in per capita output. Yet economic theory provides few guidelines explaining the rate of land improvement. --In sum, since we cannot identify a specification of land augmentation or hypotheses relating savings out of land rents that command significant empirical support, we have elected to omit land as an argument in the production function..." [11, pp. 27 and 28].

In surveying the earlier works on the dualistic models Mellor [17] clearly points out the importance of land-augmenting technological change and its impacts directly on employment, income distribution and capital formation in agriculture and indirectly on growth of an economy as a whole.

The above discussion suggests that the issue of analyzing an innovation might be tackled in the following way. First, find an appropriate production function corresponding to the given innovation; second, estimate emerging patterns of demand for all relevant inputs; and finally, link the nature of the apparent income distribution with the innovation, subject to different factor supply conditions.

The first two steps are not difficult to take. Techniques of statistical analysis of production functions have changed substantially over the last several years and with the introduction of several new functional forms it is possible to identify a production function appropriate for an innovation. The third step requires a certain criterion of changes in relative factor shares. In this respect, several criteria are proposed in the literature giving rise to a problem of choice among them. The purpose of this paper is to review those criteria and then to explore the most appropriate criterion of changes in relative factor shares.

After a brief discussion of the analytical framework, the remaining paper is divided into different sections according to the way several criteria of changes in relative factor shares were developed in the past. The two-input case has been well established and hence only a brief summary of the results is reported in Section III. Sections IV, V, VI and VII narrate attempts made by Hicks, Meade, Samuelson and Sato, respectively, to find a criterion of changes in relative factor shares for three (or more) input cases; and those sections conclude with the need for exact criterion. In Section VIII, it is shown that the Hicks coefficient is the most relevant criterion of changes in relative factor shares. Section IX compares the Hicks coefficient with the elasticity of substitution for some specific production functions. Finally, Section X concludes the paper.

II. Analytical Framework

In order to link the relative factor shares with the technological side the following assumptions are made:

(i) There are no fixed, non-augmentable factors of production.[1/]

(ii) The production function in the agricultural sector is subject to constant returns to scale and diminishing marginal rates of substitution.

(iii) Perfect competition in product and factor markets.

(iv) Supply of inputs is exogenously given.

Thus if the production function of a particular crop is as follows:

$$Q = F(X_1, \ldots, X_i, \ldots, X_n) \qquad \ldots (1)$$

with $F_i = \frac{\delta F}{\delta X_i} > 0 > \frac{\delta^2 F}{\delta X_i^2} = F_{ii}$

where,

Q = output

X_i = i^{th} input $\qquad i = 1, \ldots, n$

then the relative share of ith input, S_i, will be

$$S_i = \frac{X_i \frac{\delta F}{\delta X_i}}{Q} \qquad \ldots (2)$$

where,

F_i = marginal product of ith input

= price of ith input

Due to constant returns to scale we have

$$\sum_{i=1}^{n} S_i = 1$$

1/ The classical assumption of a fixed land supply could be improved by measuring land in efficiency units with the assumption that the efficiency of land changes due to technological change.

III. The Two-Input Case

Let us consider a traditional agricultural technology involving only land (X_1) and labor (X_2) as the inputs. Now consider the effect of an increase in the supply of land and labor upon their relative shares. In such a case Hicks [8] has shown that the elasticity of substitution serves as the criterion of changes in the relative factor shares,[2/] and the result can be stated as: the relative share of rapidly growing factor will fall over time if its "elasticity of substitution" is less than unity. This relationship for the relative share of land can be written as follows:

$$\frac{\dot{S}_2}{S_2} = S_1 \left(\frac{1}{\sigma_{12}} - 1\right) \left(\frac{\dot{X}_1}{X_1} - \frac{\dot{X}_2}{X_2}\right) \qquad \ldots (3)^{3/}$$

where, $\sigma_{12} = \frac{F_1 F_2}{F\, F_{12}}$ = elasticity of substitution ... (4)

From (3) we see that if land and labor are relatively less substitutable (i.e. σ_{12} less than one) and if the supply of land grows slower than that of labor then the relative share of land will increase. This result raises the following questions.

1. What do we mean by the concept, "elasticity of substitution"?

2. What can we say about the relative factor shares when more than two inputs are involved?

The first question will be considered in this section while the second question will be discussed in succeeding sections.

Hicks discovered this concept in 1932 and explained it as:

> "The elasticity of substitution is a measure of the ease with which the varying factors can be substituted for others." [8, p. 117]

Then he gave the formula as given in (4). This definition, however, is not precise. In this respect Kahn [10] commented:

> "Here is to be found a symbolic definition of elasticity of substitution which gives appearance of having been dictated by the requirements of algebra rather than of economics, and Dr. Hicks fails to tell us what it really means." [10, p. 72]

2/ See Hicks [8] for discussion on this. For the detailed mathematical proof, see Ferguson [7].

3/ Throughout the paper the dot over a variable refers to change in that variable over time.

Following Hicks' contribution, in 1933 Joan Robinson [21] defined the elasticity of substitution as follows:

> ". . . . the proportionate change in the ratio of the amounts of the factors divided by the proportionate change in the ratio of their prices (marginal physical productivities)." [21, p. 256]

For the two-input case this definition becomes:

$$\sigma_{12}^{(D)} = \frac{\delta \log (X_1/X_2)}{\delta \log (F_2/F_1)} \qquad \ldots (5)$$

It can further be shown that for the two-input production function the elasticity of substitution given by $\sigma_{12}^{(D)}$ is exactly equal to the one given by σ_{12},[4/] and thus is the criterion of changes in the relative factor shares.

The two-input case, however, is not of much importance to changing agriculture where the successful application of the new high yielding seed varieties also requires some capital input (like chemical fertilizers, insecticides and pesticides), and thus production is subject to at least three inputs, land, capital and labor.

IV. Three-Input Case and Hicks

in 1963, while revising the first edition of his book, The Theory of Wages, Hicks did discuss the relevance of the elasticity of substitution given in (4) for the case where production involves more than two inputs.[5/] He commented:

> "There are two basic cases in which two-factor theory continues to apply. One is that in which the proportions in which the other factors are used remain constant; the other in which these proportions vary, but vary in such a way as to keep the relative prices of other factors (i.e. the ratios of their marginal products) constant. In either of these cases the other factors (B, C, D. . .) can be treated as if they formed a single factor; so that it remains true that the condition for an increase in the relative supply of factor A to increase in its relative share is that the elasticity of substitution (of factor A against the complex) should be greater than unity." [8, pp. 339-340] [6/]

4/ See Hicks [8], pp. 373, for proof of this statement.

5/ See the second edition of The Theory of Wages, pp. 339-340 and 378-381.

6/ Proof of this proposition is given in Section IX of this paper.

For example, let production for a crop involve the following three inputs.

$$X_1 = \text{Land}$$

$$X_2 = \text{Labor}$$

$$X_3 = \text{Fertilizer}$$

In such a case the elasticity of substitution given in (4) can be generalized as follows:

$$\sigma_{ij} = \frac{F_i F_j}{F_{ij} F}, \quad i \neq j \text{ and } i, j = 1, 2, 3 \qquad \ldots (4)'$$

Now, let quantity of fertilizer be proportional to amount of labor. Next, let the elasticity of substitution between land and labor (fertilizer) be less than unity. Furthermore, empirically, one would expect the land-labor (fertilizer) ratio to decline over time. Then the Hicks' proposition implies that distribution of the increased output of the crop would be favorable to the landowning classes.

The assumptions stated above regarding the composite input are too stringent, however, and empirically there is no reason to believe that the proportions among either some of the factors or their prices should remain constant over time. Hence, the Hicks' proposition does not resolve the nature of the issue with which this paper is concerned. Let us examine further work done in this area following Hicks' formulations.

V. Three-Input Case and Meade

The next intuitive alternative, analogous to the two-input case, would be the use of the generalizations of the elasticity of substitution for more than two-input production functions. The elasticity of substitution defined by Hicks was further generalized by Allen [1] in 1938, and is known as the partial elasticity of substitution. The generalization for the elasticity of substitution defined by Joan Robinson was later called the direct elasticity of substitution by McFadden [14].

Consider the production function (1) for a crop involving only land (X_1), labor (X_2) and fertilizer (X_3) as the three inputs. Then the above two concepts are defined as follows:

1. The partial elasticity of substitution between two inputs X_i and X_j is given by:

$$\sigma_{ij}^{(P)} = \frac{Q}{X_i X_j} \frac{D_{ij}}{D} \quad \text{for } i \neq j \text{ and } i,j = 1, 2, 3 \qquad \ldots (6)$$

where D is the bordered Hessian determinant of the production function and D_{ij} is the cofactor of the $(i,j)^{th}$ element in D.

2. The generalization of the <u>direct elasticity of substitution</u> given in (5) is as follows:

$$\sigma_{ij}^{(D)} = \frac{\delta \log (X_i / X_j)}{\delta \log (F_j / F_i)} \quad \text{for } i \neq j \qquad \ldots (7)$$

where $i,j = 1, 2, 3$ and

$dQ = ds = o, \quad s \neq i \neq j, \quad s = 1, 2, 3$

In other words, the direct elasticity of substitution between two inputs is defined as: the proportionate change in the ratio of amounts of the factors divided by the proportionate change in the ratio of their marginal physical productivity where output as well as the other inputs are held constant.

Meade [15] uses the <u>direct elasticity of substitution</u> as the criterion of changes in the relative factor shares in the following special cases and then derives the results given below:

<u>Case (i)</u>: Suppose that all three factors receive the same proportion of the output, i.e. their relative factor shares are equal, implying $S_1 = S_2 = S_3 = 1/3$. Then we have:

$$\frac{\dot{S}_i}{S_i} = \sum_{\substack{j=1 \\ j \neq i}}^{3} \left(\frac{\dot{X}_i}{X_i} - \frac{\dot{X}_j}{X_j}\right) \left[1 - \frac{1}{3}\left(\frac{5}{\sigma_{ij}^{(D)}} - \frac{1}{\sigma_{is}^{(D)}} - \frac{1}{\sigma_{js}^{(D)}}\right)\right] \qquad \ldots (8)$$

where $j \neq s \neq i$ and $s, j, i = 1, 2, 3$

This result is too clumsy and the <u>direct elasticities of substitution</u> do not reflect anything definite about changes in the relative factor shares unless we know the specific magnitude of all the variables involved in the above expression.

<u>Case (ii)</u>: Suppose all the <u>direct elasticities of substitution</u> are equal, i.e.

$$\sigma_{12}^{(D)} = \sigma_{13}^{(D)} = \sigma_{23}^{(D)} = \sigma \text{ (say)} \qquad \ldots (9)$$

Then Meade shows that we get the following relationship,

$$\frac{\dot{S}_i}{S_i} = \left(\frac{1-\sigma}{\sigma}\right) \sum_{\substack{j=1 \\ i \neq j}}^{3} S_j \left(\frac{\dot{X}_j}{X_j} - \frac{\dot{X}_i}{X_i}\right) \quad \text{for } i = 1, 2, 3 \qquad \ldots (10)$$

The above expression is easy to interpret. Casual empiricism leads to the belief that land-labor and land-fertilizer ratios would decline over time. Further, assume that the fertilizer-labor ratio is constant over time. Then from (10) we see that the relative shares of land and fertilizer will rise while that of labor will fall over time if the direct elasticity of substitution is less than unity.

Case (iii): If $\sigma = 1$. Then from (10) it can be seen that changes in the supplies of the factors will have no effect upon the proportion of output paid to different factors.

Case (iv): Suppose all three inputs are infinitely substitutable for each other, i.e.

$$\sigma \longrightarrow \infty$$

Then from (10) we get,

$$\frac{\dot{S}_i}{S_i} = -\sum_{\substack{j=1 \\ i \neq j}}^{3} S_j \left(\frac{\dot{X}_j}{X_j} - \frac{\dot{X}_i}{X_i}\right) \quad \text{for } i = 1, 2, 3 \qquad \ldots (11)$$

In this case the conclusions will be similar to those in case (ii).

Case (v): Finally, if $\hat{\sigma}$'s approach zero, then either the distribution of income was indeterminate or one or two of the factors will be paid nothing.

Thus, in the above special cases the direct elasticity of substitution does represent the criterion of changes in the relative factor shares. However in the general context, we still have not solved the problem. In fact, in Section IX it will be shown that in the above special cases the direct elasticity of substitution is nothing but the Hicks Coefficient.

VI. Three-Input Case and Samuelson:

In fact, complications arise if one attempts to find a relationship between changes in the relative factor shares and in the input quantities through either direct or partial elasticity of substitution, when more than two inputs are involved. In this regard, Samuelson [22] summarizes the earlier attempts and their drawbacks as follows:

> "When more than two inputs are involved, the problems of generalizing the Hicks σ Coefficient becomes complex. Champernowne, Pigou, and others in pursuit of the question of how the change in a factor price ... affects the amount demanded of a different factor ..., have moved in the direction of defining a square matrix of partial-elasticities of substitution.... Aside from being complex, this approach has led to little in the way of definite results and has moved away from the task of providing a criterion of changes in relative factor shares." [22, p. 467]

In this context Samuelson suggests a new criterion as follows:

$$\sigma_i^{(S)} = \frac{F_i}{X_i F_{ii}} \left(1 - \frac{F_i X_i}{F}\right) \quad \ldots (12)$$

Then it can be shown that after a rise in the quantity of one input, say i^{th} input (other inputs remaining constant) the relative share of the input in the total output would increase, remain constant or decline depending upon

$$\sigma_i^{(S)} \gtreqless 1 \quad \ldots (13)$$

The above criterion is very simple to handle. But what happens if one is interested in observing changes in the relative factor shares over time or otherwise after simultaneous change in the amount in two or more inputs? One has to seek some criterion other than the one suggested by Samuelson.

VII. Three-Input Case and Sato-Koizumi:

Thus we see that although the issue of finding the criterion of changes in the relative factor shares has been recognized long ago, no one arrived at the exact criterion for the multi-input production case. The problem is solved, however, partly by the recent important contribution of Sato and Koizumi [25]. They found that the partial elasticity of complementarity discovered by Hicks [9] serves as the criterion of change in the relative share of one factor with respect to change in the supply of any other factor. The partial elasticity of complementarity between any two inputs X_i and X_j is defined as

$$c_{ij} = \frac{FF_{ij}}{F_i F_j}, \quad \ldots (14)$$

and the result can be stated as:

> "The relative share of one factor increases or decreases as the quantity of another factor increases depending on whether the partial elasticity of complementarity between the two factors in question is greater or smaller than unity ..." [25, p. 486].

One question still remains: "What is the criterion of changes in the relative factor shares when <u>all</u> the three inputs change?"

VIII. Three-Input Case and Hicks Coefficient

I shall define the Hicks Coefficient between any two inputs X_i and X_j as follows:

$$\sigma_{ij}^{(H)} = \frac{F_i F_j}{F F_{ij}} \qquad \ldots (15)$$

This is similar to the elasticity of substitution defined by Hicks in 1932 (see equation (4)').

A general relationship will now be derived between relative factor shares, changes in the amount of inputs and the Hicks coefficients for different pairs of inputs.

Let us assume that agricultural production involves three inputs such as land (X_1), capital (X_2) and labor (X_3). Since the production function is assumed to be subject to constant returns to scale, from (1) we get the following:

$$Q = X_1 F_1 + X_2 F_2 + X_3 F_3 \qquad \ldots (16)$$

and

$$q = f(x_1, x_2) \qquad \ldots (17)$$

where

$$q = \frac{Q}{X_3} = \text{output per labor,}$$

$$x_1 = \frac{X_1}{X_3} = \text{land per labor,}$$

$$x_2 = \frac{X_2}{X_3} = \text{capital per labor.}$$

Differentiating (16) partially with respect to X_3 gives

$$X_1 F_{13} + X_2 F_{23} + X_3 F_{33} = 0 \qquad \ldots (18)$$

Since we are assuming perfect competition in the factor markets the wage rate (W) with be

$$W = F_3 \qquad \ldots (19)$$

and the relative share of labor will be

$$S_3 = \frac{WX_3}{Q} = \frac{W}{q} \qquad \ldots (20)$$

Differentiating (17), (19) and (20), logarithmically, with respect to time we shall get the rates of growth of output per labor, wage rate and relative share of labor as follows:

$$\frac{\dot{q}}{q} = \frac{X_1F_1}{F}\frac{\dot{x}_1}{x_1} + \frac{X_2F_2}{F}\frac{\dot{x}_2}{x_2} = S_1\frac{\dot{x}_1}{x_1} + S_2\frac{\dot{x}_2}{x_2} \qquad \ldots (21)$$

$$\frac{\dot{W}}{W} = \frac{X_1F_{13}}{F_3}\frac{\dot{X}_1}{X_1} + \frac{X_2F_{23}}{F_3}\frac{\dot{X}_2}{X_2} + \frac{X_3F_{33}}{F_3}\frac{\dot{X}_3}{X_3} \qquad \ldots (22)$$

$$\frac{\dot{S}_3}{S_3} = \frac{\dot{W}}{W} - \frac{\dot{q}}{q} \qquad \ldots (23)$$

Substituting the value for $X_3 F_{33}$ in (22) from (18) and rearranging the terms, we get,

$$\frac{\dot{W}}{W} = S_1\frac{F_{13}F}{F_1F_3}\frac{\dot{x}_1}{x_1} + S_2\frac{F_{23}F}{F_2F_3}\frac{\dot{x}_2}{x_2} \qquad \ldots (24)$$

Substituting (15) in (24) we get,

$$\frac{\dot{W}}{W} = S_1\frac{1}{\sigma_{13}^{(H)}}\frac{\dot{x}_1}{x_1} + S_2\frac{1}{\sigma_{23}^{(H)}}\frac{\dot{x}_2}{x_2} \qquad \ldots (25)$$

Finally, substituting (21) and (25) in (23) gives us the required relationship.

$$\frac{\dot{S}_3}{S_3} = S_1\left(\frac{1-\sigma_{13}^{(H)}}{\sigma_{13}^{(H)}}\right)\frac{\dot{x}_1}{x_1} + S_2\left(\frac{1-\sigma_{23}^{(H)}}{\sigma_{23}^{(H)}}\right)\frac{\dot{x}_2}{x_2} \qquad \ldots (26)$$

By symmetry the general relationship between any relative factor share and changes in the supply of all inputs will be

$$\frac{\dot{S}_i}{S_i} = \sum_{\substack{j=1\\ i\neq j}}^{n} S_j\left(\frac{1-\sigma_{ij}^{(H)}}{\sigma_{ij}^{(H)}}\right)\left(\frac{\dot{X}_j}{X_j} - \frac{\dot{X}_i}{X_i}\right) \text{ for } i = 1, \ldots, n \qquad \ldots (27)$$

From (26) and (27) it is evident that our basic purpose of finding the criterion that would depict movements of the relative factor shares is satisfied by the Hicks coefficient. The result for the relative share of labor can be stated from (26) as follows.

Let the Hicks coefficient between capital and labor be greater than unity, and between land and labor be less than unity. Further, one would expect land-labor and labor-capital ratios to fall over time. Then from (26) we can see that the relative share of labor would fall over time.

From (26) we can find the effect on the relative share of labor due to change in the amount of some other factor, for example, capital:

$$\frac{\dot{S}_3}{S_3} = S_2 \left(\frac{1-\sigma_{23}^{(H)}}{\sigma_{23}^{(H)}}\right) \frac{\dot{X}_2}{X_2} \qquad \ldots (28)$$

Thus if the Hicks coefficient between capital and labor is greater than unity then increase in the amount of capital alone would lead to decline in the relative share of labor.

Next, consider the effect of change in the amount of one input on its own relative share in production. From (26) we can devine the desired relationship for the labor input as follows:

$$\frac{\dot{S}_3}{S_3} = \left(S_1 \left(\frac{\sigma_{13}^{(H)}-1}{\sigma_{13}^{(H)}}\right) + S_2 \left(\frac{\sigma_{23}^{(H)}-1}{\sigma_{23}^{(H)}}\right)\right) \frac{\dot{X}_3}{X_3} \qquad \ldots (29)$$

The result can be stated as: the relative share of one factor increases or decreases as the quantity of the same factor increases depending upon whether the Hicks coefficients between that factor and all the remaining factors are all greater or smaller than unity.

Thus we see that the Hicks coefficient can take care of the cases considered by Samuelson and Sato-Koizumi as well as the general case where all inputs change.

Before concluding this section we must note one thing. Compare the Hicks coefficient defined in (15) with the <u>partial elasticity of complementarity</u> as a criterion suggested by Sato and Koizumi:

$$\sigma_{ij}^{(H)} = \frac{1}{c_{ij}} \qquad \ldots (30)$$

This relation shows that the <u>partial elasticity of complementarity</u> equally serves as the exact criterion of changes in relative factor shares.

IX. Hicks Coefficient and Specific Production Functions:

The new technologies in agriculture have had impacts on income distribution through the biased changes in the relative shares of factors in production, and they have also been affecting rural-urban employment and the demand for capital inputs through the nature of internal substitution and complementarity among the inputs used in agricultural production [19]. This latter side of the phenomenon occurring in the changing agriculture could be explained by determining elasticities of substitution among the inputs involved. Although from the earlier discussion we saw that the Hicks Coefficient is the criterion of changes in the relative factor shares, it would be equally important to know its correspondence with the elasticity of substitution, if any; so that the both-way impacts of the changing agricultural production could be analyzed at the same time. In order to do so, let us consider only the Constant Elasticity of Substitution (CES) Production Functions which have been widely used in empirical work.

Assume that agricultural production involves the following four inputs:

X_1 = land,

X_2 = capital,

X_3 = family labor,

X_4 = hired labor.

Then the CES production functions can be written as follows:

1. Cobb-Douglas Production Function:

In this case all direct and partial elasticities of substitution are equal to unity.

$$Q = \alpha X_1^{\beta_1} X_2^{\beta_2} X_3^{\beta_3} X_4^{\beta_4} \qquad \ldots (31)$$

where α, $\beta_1, \ldots, \beta_4$ and all positive constants and the sum of β's is equal to unity.

2. Uzawa Type 1:

Here all the partial elasticities of substitution are constant and identical.[7]

[7] This production function was developed by Uzawa [28] as a direct generalization of the CES production function for the two-input case discovered by Arrow, Chenery, Minhas and Solow [2].

$$Q = (\alpha_1 X_1^{-\beta} + \ldots + \alpha_4 X_4^{-\beta})^{-\frac{1}{\beta}} \qquad \ldots (32)$$

where $\alpha_1, \ldots, \alpha_4$ are all positive constants and β is a constant greater than -1.

3. Uzawa [28] Type 2:

In this case the inputs are grouped in such a way that the partial elasticities of substitution for a pair of inputs within a group are identical, the partial elasticities of substitution for two inputs between any two groups are all unity and all the partial elasticities of substitution are constant. Grouping land and capital, and family and hired labor into two different groups we can write the functional form as follows:

$$Q = (\alpha_1 X_1^{-\beta_1} + \alpha_2 X_2^{-\beta_1})^{-\frac{\rho_1}{\beta_1}} (\alpha_3 X_3^{-\beta_2} + \alpha_4 X_4^{-\beta_2})^{-\frac{\rho_2}{\beta_2}} \qquad \ldots (33)$$

where $\alpha_1, \ldots, \alpha_4$ are all positive constants; β_1 and β_2 are constants but both greater than -1, and ρ_1 and ρ_2 are both positive such that

$$\rho_1 + \rho_2 = 1 \qquad \ldots (34)$$

4. McFadden [14] Production Function

In this case, the inputs are grouped in different groups in such a way that the direct elasticities of substitution for a pair of inputs within a group are identical, the direct elasticities of substitution for two inputs between any two groups are all unity and all the direct elasticities of substitution are constant. Suppose the four inputs were grouped in the same way as they were grouped in the Uzawa Type 2 production function, we can then write the functional form as follows:

$$Q = [\alpha_1 (X_1 X_2)^{-\rho} + \alpha_2 (X_3 X_4)^{-\rho}]^{-\frac{1}{2\rho}} \qquad \ldots (35)$$

where $\alpha_1, \ldots, \alpha_4$ are all positive constants and ρ less than -1/2.

Now let us compare the Hicks coefficient with the direct and partial elasticities of substitution for the above four production functions. The results are given in the following table.[8/]

8/ See Scheper [26] for the analysis of production functions similar to Uzawa Type 1 and Type 2.

Table 1. Hicks Coefficient and Elasticities of Substitution*/

Criterion	Production Functions: Cobb Douglas	Uzawa Type 1	Uzawa Type 2: Within group	Uzawa Type 2: Between group	McFadden: Within group	McFadden: Between group
Partial ($\sigma^{(P)}$)	1	$\frac{1}{1+\beta}$	$\frac{\sigma_s - 1}{\rho_s} + 1$	1	$\frac{1}{\theta^{(s)}}(1-\sigma) + \sigma$	σ
Direct ($\sigma^{(D)}$)	1	$\frac{1}{1+\beta}$	σ_s	1	σ	1
Hicks Coefficient ($\sigma^{(H)}$)	1	$\frac{1}{1+\beta}$	$\frac{\rho_s}{\beta_s+\rho_s}$	1	$\frac{\theta^{(s)}}{(1+\rho)\theta^{(s)}-\rho}$	σ

*/ $\sigma_s = \frac{1}{1+\beta_s}$ where $s = 1, 2$; $\sigma = \frac{1}{1+\rho}$ and

$\theta^{(s)}$ = relative share of all the factors together in the sth group, where $s = 1, 2$.

From the above table we derive the following statements:

A. The Cobb-Douglas production function, Hicks coefficient and the direct and partial elasticities of substitution are all identical and equal to unity. Thus, from (27) it is evident that in this case the relative factor shares will not change regardless of the amount of change of one or all inputs. Thus the Cobb-Douglas production function is inadequate for analyzing the new agricultural technologies.

B. For the Uzawa Type 2 production function, all the three criteria are identical, and thus the relation between the Hicks coefficient and the changes in relative factor shares would also indicate, simultaneously, the extent of substitutability among the factors of production and vice-versa. In this case, the relation (27) reduces to the relation (10) derived by Meade. Next, from (10) we see that if the substitutability among the factors is very low, then the Hicks coefficient will be less than unity and the relative share of the factor that grows the slowest (say land) would rise over time. On the other hand, the factor growing faster (say capital) than the rest would sustain a fall in its relative share. From (10) we can also deduce the change in the ratio of any two factors X_i and X_j as follows:

$$\frac{(\dot{S}_i/S_i)}{(\dot{S}_j/S_j)} = \frac{1-\sigma}{\sigma}\left(\frac{\dot{X}_j}{X_j} - \frac{\dot{X}_i}{X_i}\right) \qquad \ldots (36)$$

The result can be stated as follows: If the Hicks coefficient between two inputs is less than unity then the ratio of the relative shares of those factors increases or decreases depending upon the decrease or increase in the ratios of their amounts.

C. For the Uzawa Type 2 production function we see that the between-group Hicks coefficient and the direct and partial elasticities of substitution are all identical and equal to unity; while for inputs within a group they differ. Let us assume that land and labor form one group while in the second group capital is the only input. In this case the pariwise Hicks coefficient between land-capital and labor-capital (i.e. between-group) would be equal to unity, so also the corresponding elasticities of substitution, and hence, the relative share of capital would remain unchanged irrespective of any change in the supplies of the inputs involved. On the other hand, for the grouped inputs, the relative share of the rapidly growing factor (labor) would fall over time if the within-group Hicks coefficient is less than unity.

Let us compare the elasticities of substitution with the Hicks coefficient from the viewpoint of substitutability and complementarity among inputs. From the above table we notice that if the within-group Hicks coefficient is less than unity then the direct as well as the partial elasticities of substitution will also not exceed one, which would, in turn, imply that for the inputs within the group substitution is low. Additionally, in the case of partial elasticities of substitution within a group, the complementarity among the inputs is indicated when the partial elasticities are negative [6, 24]. Thus, when partial elasticity of substitution lies between zero and one or when it is negative, the Hicks coefficients will be between zero and one. When the within-group inputs are either relatively less substitutable or complementary, the Hicks coefficient will be less than unity, and hence the relative share of rapidly growing factor will fall over time. On the other hand, if the within-group inputs are highly substitutable, the partial and the direct elasticities of substitution and the Hicks coefficient will also be greater than unity and as a consequence the share of rapidly growing factor in agricultural production will increase over time.

D. For the McFadden production function, using (27), we see that if the between-group Hicks coefficient is less than unity, implying that the parameter ρ is positive, then the relative shares of rapidly growing factors (like working capital and labor) would fall over time. In this case, the partial elasticity of substitution between groups

would also be less than unity implying that between-group factor substitution is low. On the other hand, by the nature of the form of the production function, the partial elasticity of substitution within a group would be greater than unity implying that substitutability among the factors within a group is very high. In this case, however, nothing definitely, nor of necessity, can be said about the within-group Hicks coefficients. But, when the between-group Hicks coefficient is greater than one, the within-group Hicks coefficient would be less than unity, implying that the relative share of rapidly growing factor in the agricultural production would increase over time. In the latter case, the within-group inputs will be relatively less substitutable compared with the between-group input substitution.

The last point to be made is the implication for the composite-input case suggested by Hicks which we discussed in Section IV. Let us assume that agricultural production involves inputs such as land, capital and labor. Next, let us assume that the production function is such that the capital input is proportional to the amount of labor used. This implies that the capital-labor ratio is constant over time. In this case it can also be shown that the pariwise Hicks coefficients between land and labor, and land and capital are equal. Then using (26) it can be easily seen that we arrive at Hicks's proposition that the relative share of the factor having the lowest growth rate would rise over time if the Hicks coefficient between the composite-input and the remaining factor is less than unity.

X. Summary and Conclusions

The Hicks coefficient is the criterion of changes in income distribution when the agricultural production is subject to constant returns to scale and when the factor-inputs are paid their marginal physical productivities. Meade did arrive near to the solution, however his results using the direct elasticity of substitution are not easy to interpret. Next, the criterion that Samuelson suggested, while criticizing others, is indeed easy to handle. However, Samuelson's criterion is not useful in the case of the multipurpose analysis; for example, if one is interested in linking simultaneous changes in the supplies of more than one input with the relative shares of the factors involved in the agricultural production. The work of Sato and Koizumi moved a step beyond the earlier attempts, however they did not explore the usefulness of the partial elasticity of complementarity as a criterion in the most general case where the supply of all inputs changes. Actually, the Hicks coefficient and the partial elasticity of complementarity are very similar except for the fact that one is reciprocal to the other. Thus, this

paper establishes that the partial elasticity of complementarity is also the criterion of changes in relative factor shares. Our analysis on the CES production functions showed that except for the Uzawa Type 1 case there may not always be one to one correspondence between the Hicks coefficient and the partial and direct elasticies of substitution.

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