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## TWO THEORETICAL MODELS OF RADIATION HEAT TRANSFER BETWEEN FOREST TREES AND SNOWPACKS

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### ABSTRACT

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Two simple theoretical models have been developed to describe the effect of forest cover on radiation transfer to a snowpack.

Model 1, which describes the effect of varying canopy closure on the net radiation received by an interior snowpack, suggests that the net radiation may increase or decrease monotonically as the canopy closure increases from zero to 100%, or may exhibit a maximum at a non-zero canopy closure depending upon whether or not certain conditions are satisfied. These conditions involve the values of the albedos and radiant fluxes. This perhaps surprising result was due to the combined effect of long-wave radiation emitted from the canopy to the snowpack and multiple reflections of solar radiation between the snowpack and canopy.

Model 2 predicts the spatial variation of the long-wave radiation flux from the bole and crown of an individual tree to the surrounding snowpack. This model may partially explain melt rates which appear to be higher near the boles of trees than at more distant points. Calculations with assumed data showed an intensification of the long-wave radiation flux to the snowpack as the bole is approached. According to this model, a horizontal snow surface receives negligible amounts of long-wave radiation from the tree at distances from the bole that are greater than two to three times the crown radius.

### INTRODUCTION

Forest cover shades a snowpack on the ground from direct beam solar radiation and emits long-wave radiation to the snowpack. These and other radiant interactions between forests and snowpacks may have hydrologic significance since radiation is a major source of energy for snowmelt (United States Army Corps of Engineers, 1956; Federer, 1968). The effect of forest cover on radiation transfer processes varies as a function of such physical characteristics as canopy closure and structure. Since the physical characteristics of forests vary naturally and are also manageable, there is both theoretical and practical interest in developing models that quantitatively describe interrelationships between forest cover, radiation phenomena and snowpacks.

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This paper is an attempt to develop two simple theoretical models that may improve predictions of (1) the effect of forest canopy closure on the areal net radiation of snowpacks under forest cover (Model 1) and (2) the spatial distribution of the long-wave radiation flux emitted by individual trees to an adjacent snowpack (Model 2). Model 1 is concerned with space averages of the radiant energy exchange at snow surfaces under forest canopies and it excludes consideration of areal variations. Model 2 is more narrowly oriented and attempts to explain in part the areal variations in apparent rates of snowmelt around individual trees. Coniferous forest cover is assumed for both models.

## ANALYSIS

### *Model 1*

As the closure of a forest canopy increases, solar radiation impinging on the snowpack decreases while the incident long-wave radiation from the canopy to the snow increases. The snow absorbs only part of the solar radiation, but the long-wave radiation from the canopy is almost entirely absorbed (Reifsnyder and Lull, 1965). In addition, some of the reflected solar radiation is returned to the snow surface in a series of multiple reflections between the canopy and the snow.

It was hypothesized that under some conditions increased long-wave radiation in combination with multiple reflections of solar radiation may more than compensate for a reduced incidence of direct beam solar radiation as the canopy closure increases, giving a maximum net radiation input to the snowpack at some canopy closure greater than zero. If this condition obtains, rates of snowmelt may be higher under some level of forest cover than in the open, neglecting other sources of energy for snowmelt.

A forest is a complex system which is difficult to define geometrically. However, it may be possible to represent this complex system, at least for the description of certain physical processes, by a simplified model. We have chosen the simplest possible model with the hope that this will provide a basis for more sophisticated models.

Consider two large parallel planes which are a fixed distance apart. The top plane represents the forest canopy and the bottom plane represents the snowpack. Representing the snowpack-canopy system in this manner neglects the effect of canopy depth — we are assuming that the canopy is squeezed into a plane. In this analysis, all radiation fluxes are assumed to be isotropic and uniform, that is, we are concerned with average fluxes rather than with the flux distributions over the canopy and snowpack. The solar and long-wave fluxes incident on the canopy,  $E_0^s$  and  $E_0^q$ , respectively, will be partially intercepted by the canopy. If we define the canopy closure,  $x$ , as the ratio of the solid portion of the canopy area projected onto a horizontal plane,  $A_c$ , to the snowpack area,  $A_s$ , the incoming long-wave radiation incident on the snowpack is  $E_0^q(1-x)$ . This assumes that the solid canopy is totally effective in removing radiation from the incoming flux either by reflection or absorption. A fraction  $x$  of the area of the top plane is opaque to incoming radiation of all wavelengths and a fraction  $(1-x)$  is transparent. If  $E_c^q$  is the black body

(long-wave) flux emitted by the canopy, then  $E_c^R A_c$  is the total emitted canopy radiation and  $E_c^R A_c / A_s = E_c^R x$  is the incident flux on the snowpack from the canopy. The snowpack is emitting a black body flux  $E_s^R$ , therefore the net long-wave radiation input to the snowpack,  $Q^R$ , is given by:

$$Q^R = E_0^R (1 - x) + E_c^R x - E_s^R \quad (1)$$

This assumes that all long-wave radiation on the snowpack is totally absorbed.

Incoming solar radiation will be reflected and absorbed by the snowpack. In addition, there will be multiple reflections between snowpack and canopy. The solar radiation transmitted by the canopy is  $E_0^S (1 - x)$ . A fraction  $(1 - \rho_s)$  of this radiation is absorbed by the snow and a fraction  $\rho_s$  is reflected, where  $\rho_s$  is the solar albedo of the snow. A fraction  $x$  of the radiation reflected from the snow will be incident on the canopy and a fraction  $\rho_c$ , where  $\rho_c$  is the canopy solar albedo, will be reflected back to the snowpack. This radiation will be partially reflected and partially absorbed by the snowpack. If this process is continued indefinitely we generate an infinite series for the net solar input to the snowpack:

$$Q^S = E_0^S (1 - x)(1 - \rho_s) + E_0^S (1 - x) \rho_s x \rho_c (1 - \rho_s) + E_0^S (1 - x) (\rho_s x \rho_c)^2 (1 - \rho_s) + \dots$$

which may be readily summed to yield:

$$Q^S = \frac{E_0^S (1 - x)(1 - \rho_s)}{1 - \rho_s \rho_c x} \quad (2)$$

Therefore, the net radiation input to the snowpack is:

$$Q = Q^R + Q^S = (E_0^R - E_s^R) + (E_c^R - E_0^R) x + \frac{E_0^S (1 - \rho_s)(1 - x)}{1 - \rho_s \rho_c x} \quad (3)$$

The conditions for an interior maximum at  $x = x_m$ ,  $0 < x_m < 1$ , are:

$$\frac{\partial Q}{\partial x} = 0, \quad \frac{\partial^2 Q}{\partial x^2} < 0 \text{ at } x = x_m$$

which yields:

$$x_m = \frac{1}{\rho_s \rho_c} \left[ 1 - \left( \frac{E_0^S (1 - \rho_s)(1 - \rho_s \rho_c)}{E_c^R - E_0^R} \right)^{\frac{1}{2}} \right] \quad (4)$$

subject to the condition:

$$\frac{(1 - \rho_s \rho_c)}{(1 - \rho_s)} < \frac{E_0^S}{E_c^R - E_0^R} < \frac{1}{(1 - \rho_s)(1 - \rho_s \rho_c)} \quad (5)$$

Therefore,  $Q$  will be greater under the canopy than in the open provided that eq.5 is satisfied.  $Q$  will be a monotonically increasing function of  $x$  if:

$$\frac{E_0^s}{E_c^s - E_0^s} < \frac{(1 - \rho_s \rho_c)}{(1 - \rho_s)}, E_c^s > E_0^s \quad (6)$$

and a monotonically decreasing function of  $x$  if  $E_c^s < E_0^s$  or if:

$$\frac{E_0^s}{E_c^s - E_0^s} > \frac{1}{(1 - \rho_s)(1 - \rho_s \rho_c)}, E_c^s > E_0^s \quad (7)$$

There are no conditions under which a minimum exists.

If the solid portion of the canopy is capable of transmitting solar radiation, eq.3 can be modified to include the effect of a finite transmission coefficient  $\tau_c$ :

$$Q = (E_0^s - E_s^s) + (E_c^s - E_0^s)x + \frac{E_0^s(1 - \rho_s)(1 - x(1 - \tau_c))}{1 - \rho_s \rho_c x} \quad (8)$$

However, for coniferous forests, it is assumed that  $\tau_c \approx 0$ , although substantiating measurements are difficult to obtain (Gates et al., 1965).

## DISCUSSION

### Model 1

The net radiation input to the snowpack was plotted for several canopy temperatures using representative values for the various physical parameters (Fig.1).  $E_0^s$  is the black body radiation flux for an emitter at 273°K and  $T_c$  is the uniform canopy temperature. It is

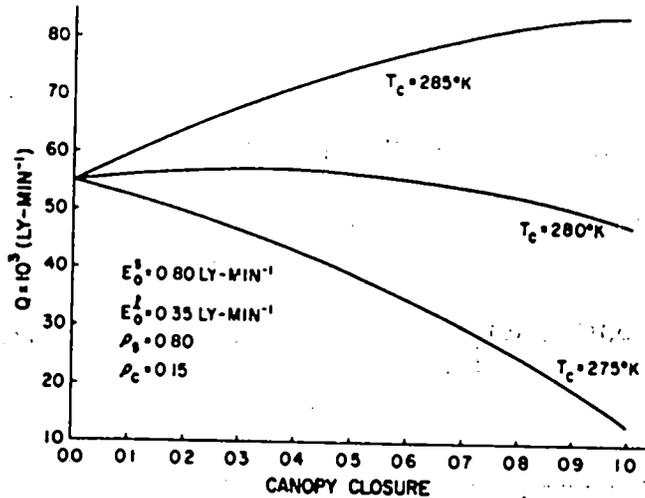


Fig.1. Dependence of net radiation input on canopy closure for canopy temperatures of 275°K, 280°K, and 285°K.

readily seen that canopy temperature influences the dependence of net radiation input on canopy closure. At 275°K,  $Q(x)$  is monotonically decreasing; an increase in canopy temperature of 5°K yields a  $Q(x)$  which exhibits a maximum at a canopy closure of about 0.3; a further increase of 5°K yields a monotonically increasing  $Q(x)$ .

A decrease in snow albedo has a marked effect in destroying the maximal character of the net radiation input (Fig.2). However, a decreased snowpack albedo can be

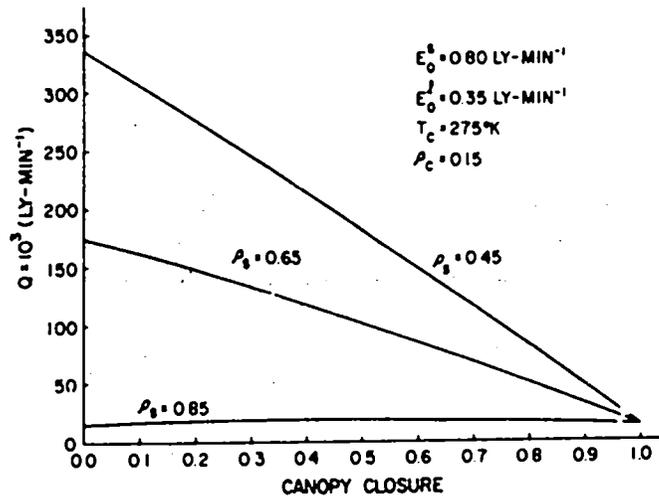


Fig.2. Dependence of net radiation input on canopy closure for snowpack albedos of 0.45, 0.65, and 0.85.

compensated for by an increased canopy temperature to give a maximum net radiation input at a canopy closure of 0.5 (Table I).

Two extreme cases involving a high snowpack albedo (0.85) are shown in Fig.3. For canopy albedos of 0.15 and 0.25, the maximum net radiation input occurs at a canopy closure of approximately 0.5 and is about 25% and 43% higher, respectively, than the input to a snowpack without forest cover. This perhaps surprising result is caused by two

TABLE I

Snow albedo and canopy temperature for a maximum radiation input at a canopy closure of 0.5\*

$\rho_s$	$T_c$ (°K)
0.65	296
0.75	286
0.80	280
0.85	275
0.90	269

\*  $E_0^s = 0.80 \text{ ly}\cdot\text{min}^{-1}$ ;  $E_0^l = 0.35 \text{ ly}\cdot\text{min}^{-1}$ ;  $\rho_c = 0.15$

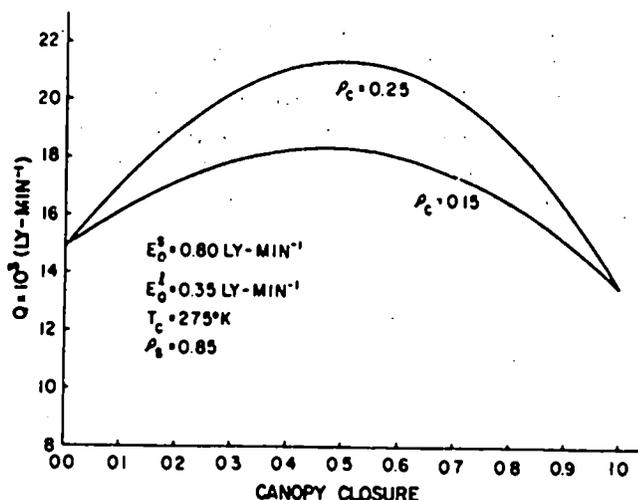


Fig. 3. Dependence of net radiation input on canopy closure for canopy albedos of 0.15 and 0.25.

physical effects: (1) the canopy emits long-wave radiation which is totally absorbed by the snowpack but removes solar radiation which is only partially absorbed by the snowpack; (2) the canopy is effective in returning some solar radiation to the snowpack by multiple reflections. The importance of multiple reflections in producing an interior maximum is evident from eq. 3. Without multiple reflections,  $\rho_c = 0$ , and the net radiation will be a monotonically increasing or decreasing function of  $x$ .

The maximum net radiation under forest cover predicted by Model 1 is associated with a high snowpack albedo, 0.85 or so, assuming canopy temperatures near the freezing point. A melting snowpack is more likely to have an albedo between 0.45 and 0.65 (United States Army Corps of Engineers, 1956) where our model predicts a monotonically decreasing net radiation as canopy closure increases. In this latter case, we would expect higher melt rates in the open than under any level of forest cover, accepting the conditions and limitations of Model 1, and assuming other sources of energy for melting are constant.

However, the initial rates of melt metamorphism may be somewhat higher in the forest than in the open after a fresh snowfall when snowpack albedos may be 0.80 or higher, assuming the dominance of net radiation in the melt process. Eventually this initial difference in radiant energy input between forest and open could reverse, due to a decrease in snowpack albedo which is normal as melt metamorphism proceeds, and as the snowpack ages (United States Army Corps of Engineers, 1956).

Our results differ from other work in some respects. For example, the United States Army Corps of Engineers (1953) reported that "... allwave radiant heat transfer toward the snow is maximum in the open, drops to a minimum under moderate forest cover, and then increases with denser cover". On the other hand, Swanson and Stevenson (1971) reported higher ablation rates under leafless aspen than in the open; their explanation of

this result is similar to ours although they did not construct a model of the radiant energy transfer process. Reconciling such differences is difficult without additional information and controlled field observations, but it can be seen from eq.5 that the existence of a maximum net radiation under forest cover requires special conditions. Furthermore, relatively small changes in the physical parameters of the snowpack-canopy system can change the relationship of net radiation input to canopy closure.

Model 1 requires additional development and field testing. A more complex model that considers structural variations in a three-dimensional canopy and the directional nature of the incident solar radiation flux is feasible. Such a model should be an improvement over Model 1 for predicting net radiation as a function of forest density characteristics.

## ANALYSIS

### *Model 2*

Observations in forests of Arizona (Jaenicke and Foerster, 1915) and elsewhere (Sartz and Trimble, 1956) suggest that energy interactions between trees and the snowpack may cause heterogeneous melt rates in the forest. Snowmelt seems to occur at higher rates in the immediate vicinity of trunks and foliage than at points more distant. The pattern of melt around trees suggests that long-wave radiation emitted from bark and foliage, which are heated partly by solar radiation, may contribute to the accelerated melt. This pattern is particularly apparent in the open-grown ponderosa pine forests of Arizona, where clear skies and high solar radiation intensities are common during the snowmelt season.

Accelerated melt around trees may contribute to the development of a patchy, discontinuous snowpack. The hydrologic significance of a discontinuous snowpack has not been clearly defined, but some evidence suggests that the quantity of snowmelt converted to runoff may be lessened once snowpacks become patchy, at least in some localities in the western United States (Rantz, 1964; Rothacher, 1965; Leaf, 1971). Perhaps a higher rate of evaporation from exposed, wet ground as contrasted to evaporation from snowpacks (Hutchison, 1966) results in more snowmelt being converted to vapor and less to streamflow. By implication, if conditions favored more uniform melting possibly a greater proportion of the snowmelt could be realized as runoff.

The occurrence and distribution of discontinuous snowpacks are partly determined by variations in topography on watersheds (United States Army Corps of Engineers, 1956; Leaf, 1971) but the radiating characteristics of forest vegetation may also be contributory. Model 2 is suggested as a simple approximation of the contribution that long-wave radiation emitted from trees may contribute to the development of discontinuous snowpacks.

If we have an emitting surface with area  $A_1$  which obeys Lambert's Law, then the

total radiant energy,  $G_2(x_2, y_2)$ , received from  $A_1$  in unit time by a unit area at a point with coordinates  $(x_2, y_2)$  on a surface with area  $A_2$  is:

$$G_2 = E_1 \int_{A_1} \frac{\cos \theta_1 \cos \theta_2 dA_1}{\pi r^2} \quad (9)$$

where  $r$  is the distance between  $(x_1, y_1)$  and  $(x_2, y_2)$ ,  $\theta_1$  is the angle between the outward normal to  $A_1$  and the vector from  $(x_1, y_1)$  to  $(x_2, y_2)$ ,  $\theta_2$  is the angle between the outward normal to  $A_2$  and the vector from  $(x_2, y_2)$  to  $(x_1, y_1)$ , and  $E_1$  is the uniform flux emitted by  $A_1$ . Integrating  $G_2$  over  $A_2$  yields:

$$\int_{A_2} G_2 dA_2 = E_1 A_1 F_{1,2} \quad (10)$$

where the view factor (Reifsnyder, 1967)  $F_{1,2}$  is defined by:

$$F_{1,2} = \frac{1}{A_1} \int_{A_1} \int_{A_2} \frac{\cos \theta_1 \cos \theta_2 dA_1 dA_2}{\pi r^2}$$

and is equal to the fraction of the total radiation emitted by  $A_1$  which is received by  $A_2$ .

View factors satisfy the identity:

$$F_{1,4} = F_{1,2} + F_{1,3} \quad (11)$$

where  $A_4 = A_3 + A_2$ . Eq. 10 can be extended to any number of surfaces uniformly emitting radiation which is received by  $A_2$ :

$$\int_{A_2} G_2 dA_2 = E_1 A_1 F_{1,2} + E_3 A_3 F_{3,2} + \dots \quad (12)$$

The view factor  $F_{j,j}$  will be zero unless  $A_j$  is a concave surface. A more detailed discussion of radiant exchange between surfaces may be found in standard heat transfer works (Wiebelt, 1966; Sparrow and Cess, 1970) and in Reifsnyder (1967).

An isolated tree surrounded by snowpack is a source of long-wave radiation to the snowpack. If the tree is assumed to uniformly emit radiation in accordance with Lambert's Law, the above analysis should be applicable to the problem of determining the long-wave input to the snowpack from the tree. However, a tree is a complex system which is difficult to describe geometrically. Therefore, a simplified model must be chosen which hopefully retains the essential features of the problem while reducing the complexity.

The geometric configuration which represents the tree is shown in Fig. 4. This is the simplest model which allows an analysis to be performed without unwieldy calculations. The bole of the tree is represented by a line source which emits an amount of energy  $E_b^0 2\pi R_b$  per unit length and time, where  $R_b$  is the radius of the bole and  $E_b^0$  is the black body flux corresponding to a bole temperature  $T_b$ ; we have replaced the cylindrical bole with a line of length  $L$  which emits the same amount of radiation as the bole. The

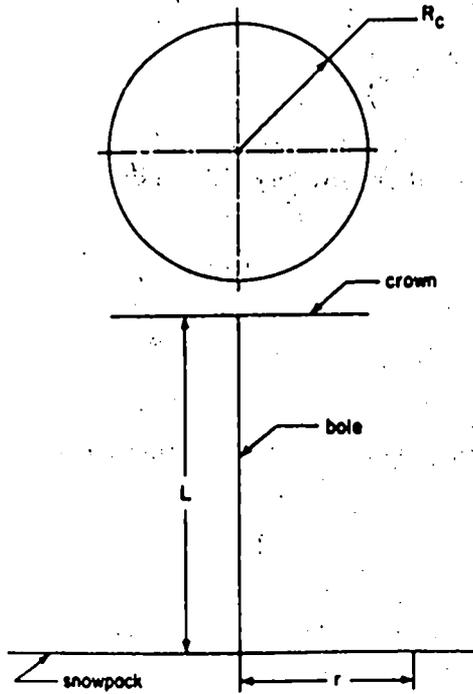


Fig.4. Simplified tree model. The cylindrical bole is replaced by a line source which emits the same amount of radiation.

crown is represented by a disk of radius  $R_c$  which is emitting black body radiation at a temperature  $T_c$ . Because of the cylindrical symmetry,  $G_s^0$ , the amount of energy received per unit time and unit area of snowpack, will depend only on  $r$ , the radial distance from the center of the bole. The view factor between a line source and a disk of radius  $r$  which is perpendicular to the line is given by:

$$F_{b,r} = \frac{1}{\pi} \tan^{-1} (r/L) \tag{13}$$

where  $L$  is the length of the line. Derivation of eq.13 is tedious but straightforward and the details will be omitted here. For two parallel disks separated by a distance  $L$  with radii  $R_c$  and  $r$ , the view factor is given by (Hamilton and Morgan, 1952):

$$F_{c,r} = \frac{1}{2} (r/R_c)^2 \left\{ 1 + (L^2 + R_c^2)/r^2 - [(1 + (L^2 + R_c^2)/r^2)^2 - 4R_c^2/r^2]^{1/2} \right\} \tag{14}$$

Consider an annular ring of snowpack located between  $r$  and  $r + \Delta r$ . Then from eq.11 and 12:

$$G_s^0(r) [2\pi r \Delta r + \pi(\Delta r)^2] = (E_b^0 2\pi R_b) L [F_{b,r+\Delta r} - F_{b,r}] + E_c^0 \pi R_c^2 [F_{c,r+\Delta r} - F_{c,r}] \tag{15}$$

$r < r < r + \Delta r$

Dividing eq.15 by  $\Delta r$  and taking the limit as  $\Delta r \rightarrow 0$  yields:

$$G_s^q(r) = \frac{1}{2\pi r} \left\{ E_b^q 2\pi R_b L \frac{d}{dr} F_{b,r} + E_c^q \pi R_c^2 \frac{d}{dr} F_{c,r} \right\}$$

Performing the indicated differentiations, using eq. 13 and 14, and introducing the dimensionless parameters  $y = r/R_c$ ,  $a = R_c/L$ ,  $b = R_b/R_c$  gives the final result:

$$\begin{aligned} G_s^q(y) &= G_{s,b}^q(y) + G_{s,c}^q(y) \\ G_{s,b}^q(y) &= (E_b^q/\pi)(b/y)(1+a^2y^2)^{-1} \\ G_{s,c}^q(y) &= (E_c^q/2) \left\{ 1 - \frac{1+(ay)^{-2}-y^{-2}}{[(1+(ay)^{-2}+y^{-2})^2-4y^{-2}]^{1/2}} \right\} \\ b &\leq y < \infty \end{aligned} \quad (16)$$

$G_{s,b}^q(y)$  is the contribution from the bole and  $G_{s,c}^q(y)$  is the contribution from the crown.

#### DISCUSSION

##### Model 2

The bole radiation  $G_{s,b}^q(y)$ , crown radiation  $G_{s,c}^q(y)$ , and the sum of bole and crown radiation  $G_s^q(y)$  were plotted for several tree geometries (Fig.5-7). The selected tree parameters  $R_b/R_c$ ,  $T_b$ , and  $T_c$  are assumed but realistic. The general conclusions which follow from the model are not greatly dependent on specific values for  $T_b$  and  $T_c$ . For example, if the bole and crown temperatures are increased by  $10^\circ\text{C}$  the bole and crown radiations will shift uniformly upward by only 15%.

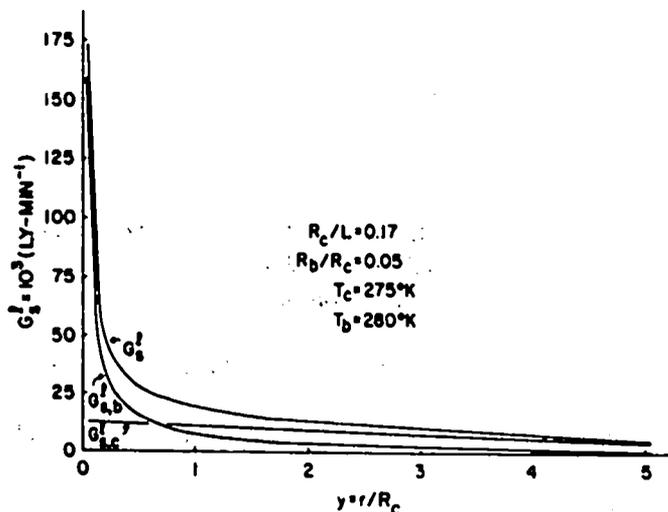


Fig.5. Long-wave radiation incident on a snowpack from an isolated tree for  $R_c/L = 0.17$ .  $G_{s,b}^q$  and  $G_{s,c}^q$  are the contributions from the bole and crown, respectively.

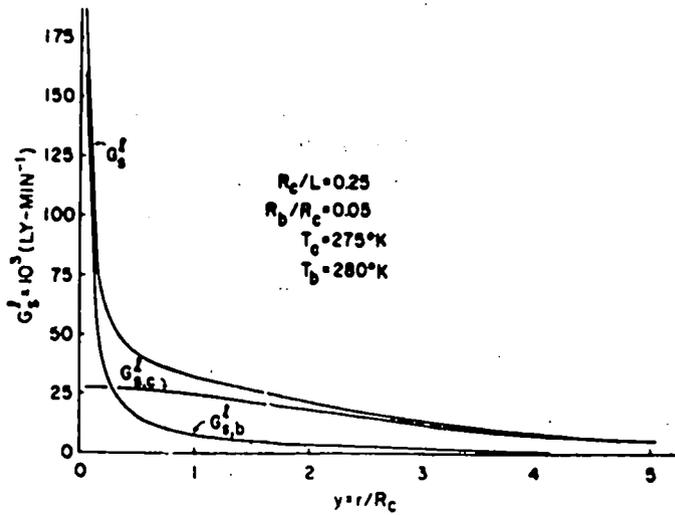


Fig. 6. Long-wave radiation incident on a snowpack from an isolated tree for  $R_c/L = 0.25$ .  $G_{s,b}^l$  and  $G_{s,c}^l$  are the contributions from the bole and crown, respectively.

The spatial distribution and magnitude of bole radiation are relatively insensitive to  $R_c/L$  as indicated by the nearly identical curves for the three trees (Fig. 5-7). As expected, the intensity of bole radiation is high in the immediate vicinity of the bole, exceeding the intensity of crown radiation in this location. However, when the tree has a large crown radius relative to its height (Fig. 7), the relative dominance of bole radiation over crown radiation is less pronounced near the bole. For all three trees the

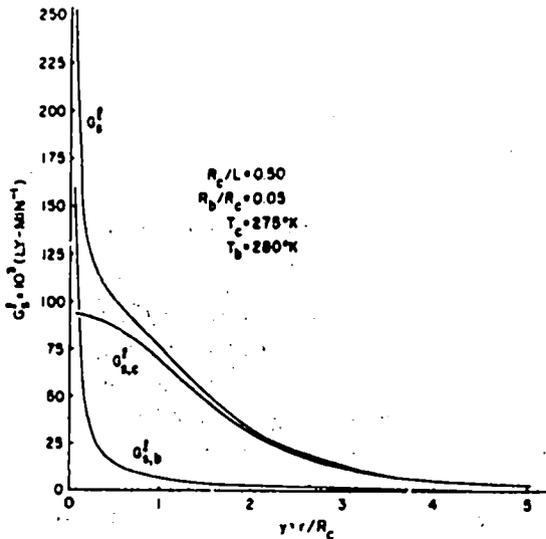


Fig. 7. Long-wave radiation incident on a snowpack from an isolated tree for  $R_c/L = 0.50$ .  $G_{s,b}^l$  and  $G_{s,c}^l$  are the contributions from the bole and crown respectively.

intensity of bole radiation declines rapidly with distance from the bole and when  $y > 2$  the contribution to the snowpack is inconsequential.

The spatial distribution and magnitude of crown radiation are similar for  $R_c/L$  values of 0.17 (Fig.5) and 0.25 (Fig.6). Where  $R_c/L = 0.50$  (Fig.7), the crown radiation shows more variation with  $y$  than in the other two cases. For all three trees, the crown radiation begins to exceed the bole radiation for values of  $y$  in the range 0.1 to 0.7. The intensity of crown radiation input to the snowpack is about an order of magnitude greater than that of the bole in the range  $0.25 < y < 3$  for  $R_c/L = 0.50$ .

All the curves for the total long-wave radiation input from the tree to the snowpack show the same general functional dependence on  $y$  (Fig.8); radiation inputs are highest near the bole, but decline rapidly to less than 10% of the value near the bole,  $G_s^0(R_b/R_c)$ , when  $y$  exceeds 3. However, the total radiation declines more rapidly for the two smaller values of  $R_c/L$  than for the largest. For  $y > 3$ , the total radiation is approximately the same for all three trees.

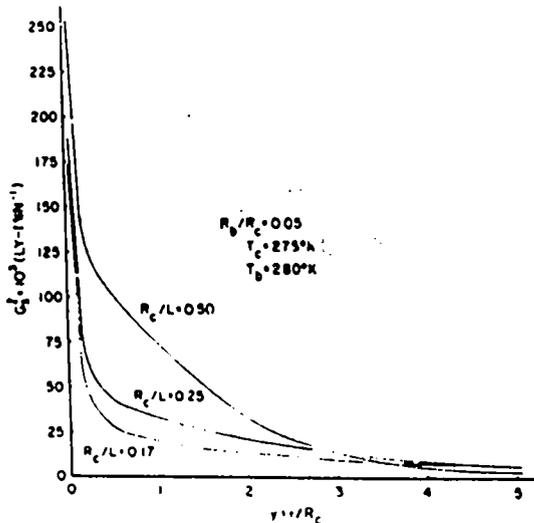


Fig.8. Long-wave radiation incident on a snowpack from an isolated tree.  $R_c/L = 0.17$ ,  $R_c/L = 0.25$ ,  $R_c/L = 0.50$ .

In summary, Model 2 predicts that a horizontal snow surface receives negligible amounts of long-wave radiation from the tree at distances from the bole that are greater than two to three times the crown radius. Closer to the bole the long-wave radiation input to the snowpack is much higher. At  $1/2$  the crown radius, the estimated long-wave flux from the tree is as high as  $0.1 \text{ ly min}^{-1}$  ( $R_c/L = 0.50$ ), assuming crown and bole temperatures of  $0^\circ$  and  $5^\circ\text{C}$ , respectively. Conceivably, long-wave radiation intensities of this magnitude in the vicinity of trees could cause spatially variable melt rates, particularly in open grown forest where foliage and bark in view of the snowpack are heated by solar radiation.

The analysis for Model 2 may be extended to more complex representations of a tree such as those using cylinders and cones. View factors are tabulated for these shapes (Buschman and Pittman, 1961) but not, in general, in analytical form and therefore a computer is necessary for these cases. Furthermore, Model 2 considers only a single tree surrounded by a snowpack. For a group of trees in a forest, the distribution of long-wave radiation input to the snowpack from the trees may be computed by superposition of the contributions from the individual trees, taking into account mutual shading. This computation in combination with other analyses may help explain the development of discontinuous snowpacks during the snowmelt season.

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