Actuarial Science and Mathematical Concepts

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SEADI is a joint project of the U.S. Agency for International Development and the Republic of Indonesia.

Agenda

- My Background
- The Actuarial Profession
- Characteristics of Actuaries
- Why Become an Actuary?
- iPad Insurance Pricing Example

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My Background

- Ph.D. in Insurance and Risk Management, The Wharton School, University of Pennsylvania
- M.S. in Actuarial Science, University of Wisconsin-Madison
- B.A. in Mathematics & Economics, Whitman College

- **University of Wisconsin-Madison**
- **Audencia Nantes School of Management in Nantes, France**

- University of Georgia
- University of North Carolina at Charlotte

- Actuarial Consultant, Employee Benefits
- Expert Witness

UW-Madison Risk Management & Insurance Program

- Undergraduate Insurance Program Rankings

  U.S. News & World Report (2013)

1) University of Pennsylvania
2) **University of Wisconsin-Madison**, University of Georgia
4) Georgia State University
5) University of Texas-Austin
6) New York University, Temple University
8) Florida State University
9) University of South Carolina
10) St. Joseph's University

http://colleges.usnews.rankingsandreviews.com/best-colleges/rankings/business-insurance-risk

UW-Madison Actuarial Science Program

- Center of Actuarial Excellence designated by the Society of Actuaries
- 300+ students in the program; 75 graduates per year
- Broad offering of courses
- Significant advantage being paired with highly rated Risk Management program; both in the School of Business

The Actuarial Profession

Math Related Professions

- Actuary
- Math teacher
- Engineer
- Computer systems analyst
- Economist
- Statistician

What is an Actuary?

- Business professional with strong quantitative skills
- Problem solver with expertise in understanding, measuring and managing (financial) risks
- Specialist who uses historical information and models to help predict the future
- Expert in an area of practice (like a lawyer or doctor)

Actuarial Areas of Expertise

- Property & Casualty (Non-life) – expert in personal property risks and risks associated with businesses
- Life & Health – expert in risks of illness, disability or death
- Pension – expert in developing and analyzing retirement programs

Where Do Actuaries Work?

- Insurance companies
- Consulting firms
- Government
- Others – where quantifying risk is needed (management consultants, investment firms)

Desirable Skills for Actuaries

- Good analytical and problem solving skills
- Strong computer skills
- Good business acumen
- Excellent communication and interpersonal skills

Actuarial Exercise

Automobile insurance: What variables do you think are predictive of insurance claims for :

- Different drivers
- Different cars

Actuarial Exercise

Some of the predictive factors for auto insurance

- Driver : Age, Sex, Driving Record, Credit Report, Grades (students), Residence (area you drive)

- Vehicle: Age, Make/Model, Value

Actuaries identify these predictive factors from a wealth of policy data and quantify the expected impact of each of these factors.

Types of Actuarial Projects

- Pricing hurricane insurance
- Designing and pricing of health insurance programs
- Evaluating price and risks of potential business acquisitions
- Designing and pricing life insurance

Time Value of Money

- Actuaries evaluate the financial impact today of events in the (sometimes distant) future
- What is the value today of the promise to pay future claims if there is an insured loss (house fire, death, etc.)?
- If a loss is reported, how much is required to set aside today for our expected obligations (liability insurance)?
- For a promised pension payment of $1,000 a month, how much money is needed today in order to assure that we can meet our obligations?

Characteristics of Actuaries

Diverse Roles of an Actuary

- Accountant – Financial reporting actuary
- Financial analyst – Investment actuary
- Sales – Marketing actuary
- Management – Executive actuary (CEO, CFO)
- Consultant – Consulting actuary
- Product manager – Product development actuary

Fits a Broad Range of Strengths

WISCONSIN SCHOOL OF BUSINESS

Working with computers ↔ Working with people

Great mathematicians ↔ Great business skills

Working in teams ↔ Working individually

Why Become an Actuary?

Interesting Work

- Projects are diverse and the work is dynamic
- Use a broad range of your problem solving skills: analytical, technical, & creative
- High level of job satisfaction
- Work individually and in teams to accomplish objectives

Strong Job Market

- High demand profession
- Historically, more positions available than there are actuaries
- Easy to get a job after college
- Many actuaries hold senior level executive positions (CEO, CFO, Consulting Partner)

A Highly Rated Profession

Top 5 Professions by Jobsrated.com (2013)

1. **Actuary**
2. Biomedical Engineer
3. Software Engineer
4. Audiologist
5. Financial Planner

- Ranking of 200 professions
- Factors of evaluation: Stress, Work Environment, Physical Demands, Income and Outlook

iPad Insurance Pricing Example

Insurance Policy Pricing

- Critical actuarial function – What is the value today of the expected losses that a policy insures?
- Utilize present value concepts along with probabilities of loss events
- Identify the amount necessary to charge for the policy exposures
- Understand risks associated with this process
  - Underwriting risk: premium risk
  - Market risk
  - Operational risk
  - Financial risk

Insurance Policy Pricing

Example (with simplifying assumptions):

- 3 year iPad insurance
- Covers owner against any iPad failure
- If loss occurs, entire iPad is replaced (not fixed) **at the end of the year of loss** and no salvage value for the original unit
- Policy only covers the initial iPad (not any replacement units)
- Pr(Failure in any year given survival to the beginning of that year) = 5%
- $500 cost to replace iPad
- Current interest rate we can earn is 7%

Insurance Policy Pricing

- **What should the price of this policy be?**

- Present value of loss random variable Z

$$Z = \begin{cases} 500 * (1.07^{-1}) = 467.29 & \text{if fail during 1st yr} \\ 500 * (1.07^{-2}) = 436.72 & \text{if fail during 2}^{\text{nd}}\text{ yr} \\ 500 * (1.07^{-3}) = 408.15 & \text{if fail during 3}^{\text{rd}}\text{ yr} \\ 0 & \text{otherwise} \end{cases}$$

- Since payment is at the end of the year of failure, there are only three possible claim payment times

Insurance Policy Pricing

- Probability of failure in first year = 5%
- Conditional probability of failure in second year given that unit survived first year = 5%
- Conditional probability of failure in third year given that unit survived second year = 5%
- Of initial units sold
  - 5% fail in the first year
  - (95%)*(5%) = 4.75% fail in second year
  - (95%)*(95%)*(5%) = 4.51% fail in third year
  - 1 – 5% - 4.75% - 4.51% = 85.74% survive all 3 years

Insurance Policy Pricing

- E[Z] = PV of iPad insurance losses per unit
  = (0.05) * $467.29 +
  (0.0475) * $436.72 +
  (0.0451) * $408.15 +
  (0.8574) * $0 = $62.52 per unit
- To cover the cost of the insurance, we would have to charge an additional 12.5% (62.52/500) of the purchase price at the time of purchase
- Or the policy could be funded by premiums at the beginning of each of the three years of the policy
- Premium = $22.27 per year

Variance of iPad Insurance Policies

- $Var[Z] = E[Z^2] - E[Z]^2$
- $E[Z^2] =$

  $(0.05) \quad * [\$467.29]^2 +$

  $(0.0475) * [\$436.72]^2 +$

  $(0.0451) * [\$408.15]^2 +$

  $(0.8574) * [\$0]^2 \quad = \$27{,}490.45$
- $Var[Z] = \$27{,}490.45 - (62.52)^2 = \$23{,}581.70$
- Expected Value = \$62.52, Standard Deviation = 153.56,
- Expect wide variation on any one policy
- Look to minimize variation through writing more policies

Variance of iPad Insurance Policies

- 10,000 iPad policies
- $Y = Z_1 + Z_2 + \ldots + Z_{10,000}$
- E[Y] = 10,000 * E[Z] = $625,200
- Assuming that $Z_1, Z_2, \ldots Z_{10,000}$ are independent (not a good assumption for this policy – design flaws, etc.)
- Var[Y] = 10,000 * Var[(Z)] = $235,817,000
- St Dev[Y] = 15,356
- 95% confidence interval for Y = [($625,200 ± (1.96 * 15,356))] = [$595,102 , $655,298]

Thank You

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