V.11 LONG-TERM INVESTMENT PLANNING FOR THE EGYPTIAN ELECTRIC POWER SYSTEM

Progress Report

June 1, 1977 - September 1, 1977

The basic objective of this project is to help develop professional skills in applying mathematical economics and operations research to analyze project alternatives for Egyptians concerned with planning electric power projects. The best way to develop autonomous capabilities in this area, as well as the best way to get answers to important investment questions, is to start in right away building simple, practical models, feeding data into them, and analyzing the results. Then, if everything seems to be proceeding smoothly, the models can be made increasingly complex and elaborate, to incorporate more and more features of reality. The emphasis is going to be on developing a hands-on capability for building and analyzing useful, simple, operational models which give answers to some pertinent questions of investment planning.

This basic philosophy has been agreed to by the participants from the Ministry of Electricity, Cairo University and M.I.T. The environment in MOE and Cairo University is favorable for this sort of applied economic analysis because the Egyptian engineers are very able in the technical aspects of their field and are eager to seriously develop their own capabilities in applying operations research and mathematical economics.

In the first weeks of July, several meetings were held in the office of the First Undersecretary of State of the Ministry of Electricity and Energy to discuss implementation of the agreement between M.I.T., Cairo University and the Ministry concerning the study of long-term investment planning for the Egyptian electric power system. The following people attended these meetings: from M.I.T., Prof. Weitzman; from the Ministry of Electricity and Energy, Eng. M. Abaza, Dr. F. Taher, Dr. H. El-Shaer, Dr. E. El-Sharkawi, Dr. M. Sweidan, Eng. El-Gazzar, Eng. K. Yassin, Eng. S. Doss; from Cairo University, Prof. S. Abu-Hassein, Prof. S. El-Sobki, Dr. H. Anis, Eng. H. El-Kolaly, Eng. M. Mahmoud.

At these meetings, a specific simple model of project evaluation was applied to sample data provided by Ministry personnel and some crude sensitivity analysis was performed. Also, we looked at some specific ways of formulating the problem of selecting an optimum combination of steam and gas

turbine generators. Finally, we discussed the general features which would be desirable to include in an expanded model of investment planning.

A research strategy was proposed which consists of the following three projects: (1) a preinvestment survey for investment planning, (2) a study of the optimum mixture of different types of power stations for variable demand, and (3) an expanded model for investment planning.

The first project is concerned with the following set of issues. The Ministry of Electricity is confronted with the task of selecting electricity generating projects from a wide range of options, including hydroelectric, thermal using various fossil fuels, gas turbine, and nuclear. Many of these projects are big relative to the demand being served. In such a situation, it is not clear how to rank projects. Cost per rated kilowatt is not a good measure because there is a fair amount of excess capacity introduced with a large project. We have mathematically formulated the problem of expanding capacity at minimum present discounted cost and, under certain simplifying assumptions, devised a formula for an "equivalent cost per kilowatt" which has a rigorous foundation as the solution to the optimal capacity expansion problem and automatically corrects for the size of a project. This formula can be used to rank various investment alternatives and lends itself very readily to economic analysis whereby the main parameters -- cost of fuel, rate of interest, growth of demand, etc. -- are varied and the effect is noted. We have already applied this methodology to some crude cost figures on various investment alternatives and the results seem to constitute a very useful preinvestment survey. We have agreed on the following execution steps:

- -- data collection for different types of plants;
- -- calculations of the "equivalent cost of capacity" for each type of plant;
- -- ranking the various investment alternatives;
- -- economic analysis (calculating the effects of varying all of the important parameters;
- -- general conclusions and recommendations.

This project is currently underway and it is anticipated that there should be concrete results by November or December. An enclosed paper describes the analytic framework being used. We hope to extend this methodology to a more sophisticated and complicated model. As of September 1st, data collection for six new power plants is almost completed, and preliminary equivalent costs of capacity" have been calculated.

The second project ar e out of a practical problem suggested by Ministry personnel. The demand for electricity varies -- daily, weekly and seasonally. Some types of plants (steam generators) are relatively cheap per kilowatt provided they are used at near full capacity. At low capacity utilization, they are relatively expensive. Other types of plants (gas turbines) are relatively expensive per peak kilowatt, but the output can be easily and cheaply varied, and they are relatively cheaper per kilowatt at low output. The basic question is this: Given the pattern of demand fluctuations, what is the least cost combination of power station types? We have succeeded in mathematically formulating a very simple version of this problem, but a more sophisticated model should be developed, and it should be applied to real data. The execution steps of this task are:

-- development of a good mathematical model;
-- data collection;
-- model solution on a computer;
-- economic analysis (calculating the effects of varying parameters);
-- general conclusions and recommendations.

This project is underway and we hope to have some results by December. At the present time, a model has been created of fairly general applicability, and a complete computer algorithm to determine the minimum cost plant mixture ratio has been prepared.

The third project area is less well defined than the other two. It involves constructing a much more complicated complete programming model which would determine least cost investment combinations for a very general situation, as contrasted with the piecemeal, partial approaches being undertaken in the other two projects. In principle, such a model could encompass regional effects, investment lumpiness, transmission losses, time variable demand, etc. The drawback is that such a model is hard to formulate, very difficult to solve on a computer, and cumbersome to analyze parametrically because of its large size. We hope to work up to more general formulations of the problem of optimal investment strategies by successively relaxing the constraints in the existing simple models (after we thoroughly understand those models) in directions that seem most fruitful. The third project is not completely defined at this stage, because it depends very much on what we learn from the other two models. As we solve the simpler models, however, we will continually be thinking of how we would like to formulate a more general model.

We are hoping the funding of this project can be expanded in the future because th. project is turning out to be of greater interest and use than was originally suspected and the initial budget underestimated project needs. The original proposal funded sixty (60) hours a month of Egyptian research time, and it now looks as if the project should employ triple that amount if it is to realize its full potential. In addition to developing the third project area, we would to incorporate several new topics in the future.

Attachment A: Long-Term Investment Planning for the Egyptian Electric Power System

Investment Criteria for Lumpy Capacity Expansion

M.L. Weitzman

Summary

Suppose that capacity must be expanded over time in some industry or sector. If all capacity expanding projects are small, it is easy to rank them. The next project to be undertaken should have the lowest cost per unit of capacity. When some investments are large, this simple criterion no longer works. A large project might have low unit costs and yet be undesirable because its huge capacity could only be fully utilized at some future time. The standard approach to investment planning based on an individual evaluation of separate projects doesn't normally work when there are indivisibilities. To determine if and when an individual project should be untertaken, the entire investment strategy must be mapped out in unison. This can be a very messy combinatoric problem, especially if repetitive strategies are disallowed because some ways of increasing capacity are uniquely tied to geographic or other conditions.

Surprisingly enough, it turns out that even when investment projects are big and lumpy, the project analysis approach can be made to work. This is a tremendous simplification, since investment priorities can be ranked by the simple expedient of calculating a single cost-like number for each project. The present paper is concerned with explaining this result, which provides a rigorous theory of investment criteria for choosing among large scale projects.

Introduction

The present paper can be viewed as an exercise in trying to strongly characterize an optimal policy for expanding capacity when investments must be

made in discrete or lumpy projects. Although the theory is developed at an abstract level, perhaps the easiest way to convey its flavor is by starting with a specific example.

Suppose planners are considering the best way to expand electricity generation in some country or region. A more or less steady expansion of the system is contemplated, far into the future. Assume that all electricity must be domestically produced (later we consider the possibilities of exports or imports).

Generating capacity is expanded by investing in individual projects. There may be many alternatives of various sizes. Some projects are repeatable - like thermal or perhaps nuclear power plants of various sizes. Other projects, frequently very large ones, are by their very nature once and for all investments because they are tied to a specific location. Hydro-electric installations readily come to mind. Many of the newer exotic technology projects are also non-repeatable - harnessing tidal flows (Fundy, Britainy), diverting water into natural depressions (Quattara), etc.

The basic question is: which project should be undertaken next? This question is more difficult to answer then might be supposed. Among small projects it would suffice to develop next that generating source offering the lowest (present discounted) cost per rated kilowatt of capacity. But when a project is large, initial excess capacity is almost inevitable. Nor is it clear how to correct the "lowest cost per kilowatt" formula for excess capacity. Indeed, there is no reason to expect that one lumpy investment project may be analyzed in isolation from the alternatives by assigning to it a single number which can meaningfully be compared with the other assigned numbers. In a world of lumpy projects, optimal investment strategies can usually only be

analyzed as an entire (mixed integer) programming problem formulated over some large time horizon. The solution typically involves a complicated interdependence[1] among all the projects which defies any simple analysis. Such problems are combinatoric in nature and they are frequently difficult or expensive to solve.

Of course, if the least cost capacity expansion problem has special features, these may be advantageously utilized in characterizing and finding an optimal solution. Thus, for example, in the Chenery-Manne capacity expansion model,[2] infinite repeatability of all projects and a linearly growing demand limits the strategy space to capacity expansion of one size plant only. An optimal strategy is easily found by locating that size plant which gives lowest present discounted cost when it alone is always expanded to meet new demand. The situation is fundamentally different when non-repetitive projects are considered, and a seemingly messy combinatoric problem immediately emerges.

The basic conclusion of the present paper is that under certain assumptions it is possible to assign each project a "recovery cost" which gives a correct ordinal ranking of when it should be undertaken relative to the other projects. Thus, all the advantages of a single rate of return criterion apply in an unfamiliar context. That project should be next undertaken which has the lowest recovery cost. The formula giving a project's recovery cost is a relatively straightforward adjustment of the cost per kilowatt which takes account of the project's size; it is easy to apply and has an elementary economic interpretation.

While the conclusions of the present paper are based on a simplified model, it is hoped they may be of some use in a wider context. For example,

the notion of recovery cost might prove helpful in providing a rough screening criterion for large projects technically not covered by this paper's formal model.

The Model

We consider a planning situation where the demand for a certain commodity is prescribed at each time and must always be fulfilled. For analytic convenience, the fixed final demand schedule $D(t)$ is taken to be a linear function of time:

$$D(t) = A + Bt . \tag{1}$$

Formula (1) might be defended as a first order approximation holding for at least the near future. It represents a key assumption which greatly simplifies the analysis.

Demand is met by investing in discrete projects which expand capacity. A typical pattern of capacity expansion is depicted in Fig. 1.

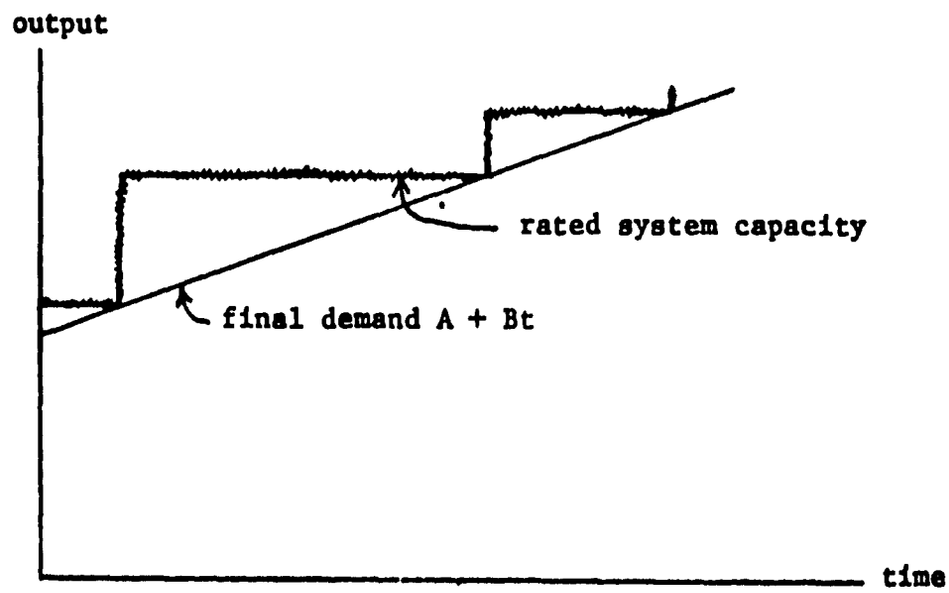


FIGURE 1

Let there be a total of n different projects. The first m of these are unique and non-repeatable. Projects m+1 through n are repeatable and can be invested in over and over again.

Let project i ($1 \leq i \leq n$) have a full capacity output of K_i. Without significant loss of generality, we allow projects to begin producing only when no excess capacity exists in the system (all previous projects are producing at full capacity). If project i is chosen to increase the rated capacity of the system by coming on line at T (a time when there is zero excess capacity in the system), it is restricted to do so according to the following prearranged production schedule. At time t, $T \leq t \leq T + \frac{K_i}{B}$, the output of project i is $B(t-T)$, leaving excess capacity $K_i - B(t-T)$. At time $T + \frac{K_i}{B}$, the output of project i reaches its full rated capacity level, K_i, and it remains there forever (every capacity expanding project is assumed for simplicity to be infinitely durable). Thus, the output stream of project i coming on line at time T would look as follows:

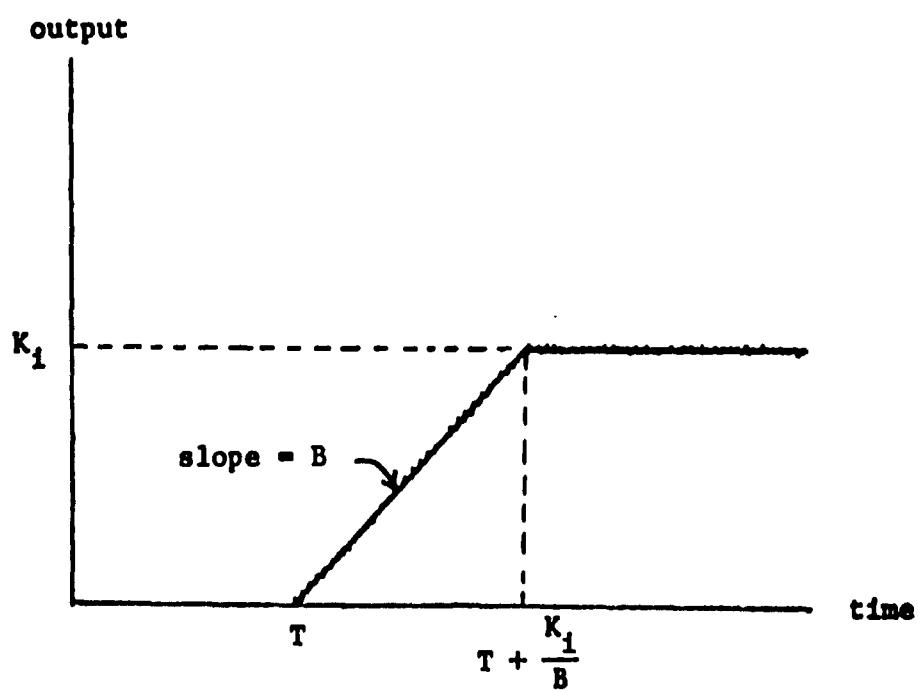


FIGURE 2

If project i is chosen to begin coming on line at T, it forces outlays to be made at time t as given by the project cost function $F_i(t-T)$. When $t<T$, project i is in the primary construction phase. It is during this phase that are incurred the major construction costs associated with creating an installation of size K_i. At times from T to $T + \frac{K_i}{B}$, basic overhead installations have already been completed and secondary capacity is filled in according to the prescribed linear schedule. Finally, for times after $T + \frac{K_i}{B}$, all construction has been completed and the only outlays are variable operating costs.

To use an example from hydro-electric generation, the first phase would be concerned with constructing a dam and creating the basic overhead for a transmission system and other installations. The second phase would involve filling in the existing system with more generators and transmission lines until full capacity is reached. At that time the third phase begins, with full capacity operating costs being the only outlays. For a small thermal plant, the second phase is of minor consequence, the first and third phases being dominant.

Optimal Capacity Expansion

The basic problem is to schedule capacity expanding projects to come on line meeting final demand at minimum present discounted cost. Let the hth plant installed (h=1,2,...) be a project of type j(h) and let it begin operating at time T_h. Mathematically, the problem is to introduce projects in the optimal order j*(1),j*(2),... which minimize

$$\int_0^\infty \sum_{h=1}^{\infty} F_{j(h)}(t-T_h) \quad e^{-rt}\, dt, \tag{2}$$

subject to $$T_1 = \overline{T}_1 \tag{3}$$

$$T_{h+1} = T_h + \frac{K_{j(h)}}{B} \tag{4}$$

for $i=1,\ldots, m$, and $\ell \neq h$, $\quad j(h) = i \Rightarrow j(\ell) \neq i$. (5)

The objective function (2) is the present discounted value of all costs incurred in expanding capacity. Time T_1 ($= \overline{T}_1$, given) is the first time when no excess capacity exists in the system and a new plant must begin operating. Thereafter, as expressed by equation (4), new capacity comes on line at times when the old capacity just exactly covers demand. Condition (5) precludes using any of the non-repeatable projects $i=1,\ldots, m$ more than once. Note that we are implicitely assuming there is enough start up time to complete project $j^*(h)$ before time T_h. So long as there is at least one repeatable project, a solution of (2)-(5) must exist.

Because projects are indivisible, problem (2)-(5) possesses a complicated combinatoric structure which seemingly defies easy solution. Surprisingly, it turns out that the model as it is formulated has enough structure to induce a very simple characterization of an optimal solution. A "recovery cost" is assigned to each project. That project is next undertaken which has the lowest recovery cost among all remaining alternatives. Thus, we have the unexpected result that lumpy projects can be meaningfully ranked as investment priorities.

Many of the underlying assumptions of formulation (2)-(5) -- arithmetic growth of demand, infinite project lifetime, etc. -- may appear to be un-

realistic. Yet the model as a whole captures enough essential aspects of reality that its solution is likely to be relatively robust over the near future. What we are really interested in knowing is not so much the complete solution to an infinite horizon problem, so much as the identification of the next project, or the next few, to be built (after that, the data will change anyway). Identifying the next few projects to be built is easily and quickly done for the model (2)-(5). The optimality of this choice is unlikely to be much sensitive to the shortcomings of the present formulation. In other words, my feeling is that if a more complicated and realistic model were constructed, the investment program for the near future obtained by numerically solving it would not differ appreciably from the present model's recommendations.

Even for complicated investment situations, the project ranking which emerges as a solution to the simplified model (2)-(5) might serve as a rough screening device or as a starting point for more sophisticated analyses, like integer programming.[3] The fact that it is possible to sharply characterize an optimal solution makes problem (2)-(5) a natural preliminary to any more general analysis. And the present model may even be a reasonable description of some situations.

Solving the Problem

Define the recovery cost of a project to be the hypothetical payment per unit of output which would make the project just break even. The recovery cost for project i, C_i, is defined as the solution to:

$$\int_0^{K_i/B} C_i\, Bs\, e^{-rs}\, ds + \int_{K_i/B}^{\infty} C_i\, K_i\, e^{-rs} ds = \int_{-\infty}^{\infty} F_i(s)\, e^{-rs}\, ds. \quad (6)$$

The right hand side of equation (6) is the total cost of project i, discounted back to the time when it first starts coming on line. The left hand side is the hypothetical discounted revenue accumulated by charging a price of C_i per unit of output (the first integral covers the expansion phase and the second full capacity operation).

Integrating out the left hand side of (6) yields the formula

$$C_i = \frac{r^2 \int_{-\infty}^{\infty} F_i(s)\, e^{-rs}\, ds}{B(1-e^{-rK_i/B})} . \tag{7}$$

Although the notion of recovery cost has intuitive appeal, the following theorem gives it a rigorous basis as a criterion for selecting among discrete projects of differing sizes.

Theorem: A necessary and sufficient condition for an optimal solution of problem (2)-(5) is the rule:

<u>always select next the project with lowest recovery cost.</u>

At time T_1, that project i^* (=$j^*(1)$) should come on line which satisfies:

$$C_{i^*} = \min_{i=1,\ldots,n} C_i . \tag{8}$$

If i^* is a non-repeatable project ($1 \le i^* \le m$), it is deleted off the list from which the next project $j^*(2)$ is selected (by the criterion of lowest recovery cost) to come on line at time $T_2 = T_1 + \frac{K_{i^*}}{B}$. Note that once a repeatable project ($i=m+1,\ldots,n$) is selected by the criterion of minimum recovery cost, it will continue to be selected by that same criterion (so

long as a project with lower recovery cost is not added to the list of available projects).

Because it is so easy to calculate recovery costs, sensitivity analysis is made especially simple. The effects on project ranking of changing such parameters as the interest rate, growth of demand, plant capacity or cost estimates is easily determined. It is also easy to say how an optimal investment strategy changes when certain projects are added to or deleted from the list of prospective candidates.

That such an elementary decision rule as (8) is optimal depends more crucially than might be supposed on the simplifying assumptions of the model. There does not seem to be available a sharp characterization of an optimal solution when, for example, demand varies non-linearly with time, projects depreciate, or the discount rate is not constant. About all that might be said of a general character in such situations is that a limiting argument could be used to show the results presented here are valid as an approximation when the stipulated preconditions are close to being met.

Proof of Optimality

For completeness, we prove the optimality of decision rule (8) from first principles. An alternative approach would be to demonstrate that (2)-(5) is a particular example of a resource pool problem and then to apply the general theory developed in Weitzman [1976] to this special case.

The following monotonicity property is important:

In an optimal policy,

$$C_{j(h)} \leq C_{j(h+1)} \tag{9}$$

for all h=1,2,...

proof: Consider an alternative policy that reverses the order of investment in projects j(h) and j(h+1) but leaves unchanged the rest of the program. That is, at time T_h project j(h+1) comes on line and at time $T_h + \frac{K_{j(h+1)}}{B}$ project j(h) comes on line - otherwise everything else remains the same. The difference in present discounted cost between the old policy and the new variation is

$$e^{-rT_h} Y_{j(h)} + e^{-r(T_h+K_{j(h)}/B)} Y_{j(h+1)}$$

$$- (e^{-rT_h} Y_{j(h+1)} + e^{-r(T_h+K_{j(h+1)}/B)} Y_{j(h)})$$

$$= e^{-rT_h}[Y_{j(h)}(1-e^{-rK_{j(h+1)}/B}) - Y_{j(h+1)} (1-e^{-rK_{j(h)}/B})], \tag{10}$$

where

$$Y_i \equiv \int_{-\infty}^{\infty} F_i(s)\, e^{-rs}\, ds. \tag{11}$$

Inequality (9) must hold because otherwise, from formulas (7) and (11), expression (10) is positive and the original policy is non-optimal.

The main body of proof now proceeds by contradiction with the selection rule based on (8). Suppose that it is optimal to choose as the first investment some project j(1)(≠i*) satisfying

$$C_{j(1)} > C_{i*} \tag{12}$$

Let the present discounted cost of this allegedly optimal policy be

$$\alpha \equiv \sum_{h=1}^{\infty} Y_{j(h)} e^{-rT_h}. \tag{13}$$

Equation (13) can be rewritten as

$$\alpha = \sum_{h=1}^{\infty} \frac{Y_{j(h)}}{1-e^{-rK_{j(h)}/B}} ((1-e^{-rK_{j(h)}/B}) e^{-rT_h}), \tag{14}$$

where, making use of (4),

$$\sum_{h=1}^{\infty} (1-e^{-rK_{j(h)}/B}) e^{-rT_h} = \sum_{h=1}^{\infty} e^{-rT_h} - \sum_{h=2}^{\infty} e^{-rT_h} = e^{-rT_1}. \tag{15}$$

It follows from the monotonicity property (9) that since project i* satisfying (12) does not come on line at time T_1 in the allegedly optimal policy, it never does. Consider an alternative policy of commencing with project i* and postponing to a starting date of $T_1 + \frac{K_{i*}}{B}$ the allegedly optional policy which had previously begun at T_1. In other words, what was policy over $[T_1,\infty)$ now becomes policy over $[T_1 + \frac{K_{i*}}{B},\infty)$ and project i* now begins at T_1. The present discounted cost of this alternative policy is

$$\alpha' \equiv e^{-rT_1} Y_{i*} + e^{-rK_{i*}/B} \alpha. \tag{16}$$

From (12), (9), (11), and (7),

$$\frac{Y_{i*}}{1-e^{-rK_{i*}/B}} < \frac{Y_{j(h)}}{1-e^{-rK_{j(h)}/B}} \qquad h=1,2,\ldots \tag{17}$$

Combining (17) with (14), (15),

$$\frac{Y_{i*}e^{-rT_1}}{1-e^{-rK_{i*}/B}} < \alpha,$$

which can be rewritten as

$$e^{-rT_1} Y_{i*} + e^{-rK_{i*}/B} \alpha < \alpha .$$

By (16), this implies

$$\alpha' < \alpha.$$

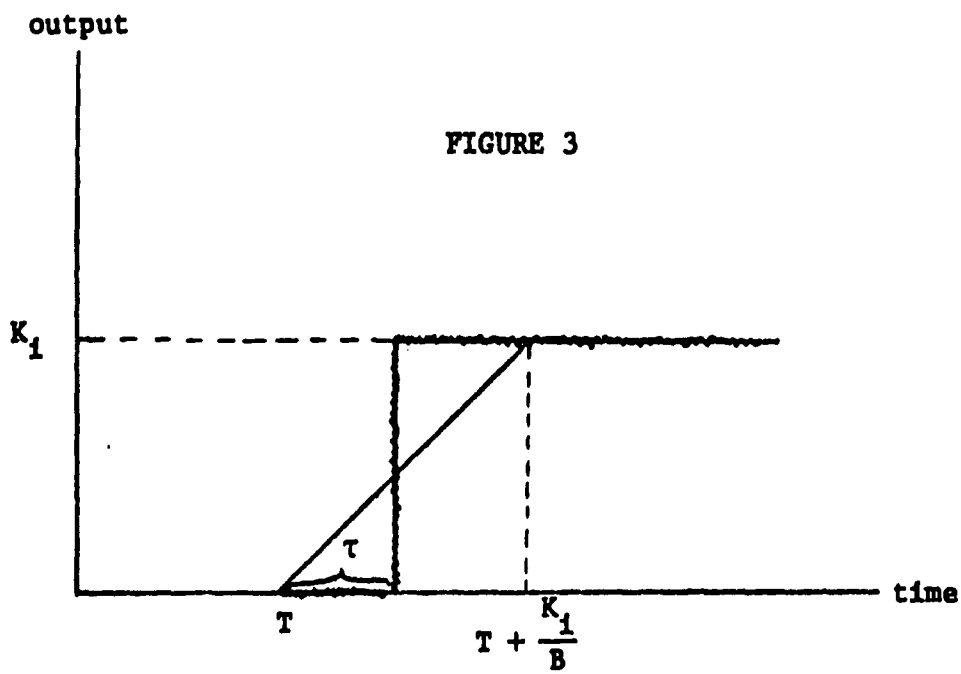
The alternative policy has lower present discounted cost than the proposed optimum, a contradiction. Hence, the supposition (12) is false. The selection rule based on (8) is a necessary condition for an optimum.

This concludes our proof of the form of an optimal policy. Strictly speaking, we have proved the necessity of the selection rule based on (8) for a project coming on line at T_1. The extension to T_h for all h is immediate. The selection rule (8) specifies a unique choice of j*(h) for each h (except in the case of ties, for which it is easy to show that how the tie is broken makes no difference to the value of the objective function). Thus, since an optimum exists, sufficiency of the selection rule based on (8) has also been demonstrated.

A Few Brief Extensions

It is easy to extend the model to cover the case where imports are available at price P and exports can be sold at price p(<P). To make the problem interesting, p should be higher than variable production costs (otherwise there is no exporting) and P should be high enough to ward off a stategy of importing everything and producing nothing domestically.

When imports and exports were disallowed, the output stream of project i coming on line at time T was as depicted in Fig. 2. Now the corresponding output stream would be as depicted in Fig. 3.



For τ periods of time, the commodity is imported to help meet final demand. Then, when plant i comes on line at time $T + \tau$, it immediately produces at full capacity, exporting the excess production over domestic demand. At time $T + \frac{K_i}{B}$, there is no excess capacity.

The time τ is selected to minimize net present discounted project costs, taking into account import costs, export revenues, and all production costs for project i coming on line with full capacity at time τ. Except for these new net present discounted costs replacing the previous $\{Y_i\}$ in formula (7)

and elsewhere, the analysis based on equation (8) remains the same. The effect of having import or export possibilities is generally to increase the relative desirability of large scale projects.

With a slightly different interpretation, the same analysis covers the case of a piecewise linear benefit function kinked at time t around the target demand level A + Bt. In this interpretation, the import price is the welfare loss of falling short a unit from target and the export price is the welfare gain of delivering an extra unit above target.

Some other generalizations of the model are possible, but it would be tedious to go on listing them.

Footnotes

1. See, for example, Chenery [1959].

2. See Chenery [1952] or Manne [1967].

3. See, for example, Westphal [1971].

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