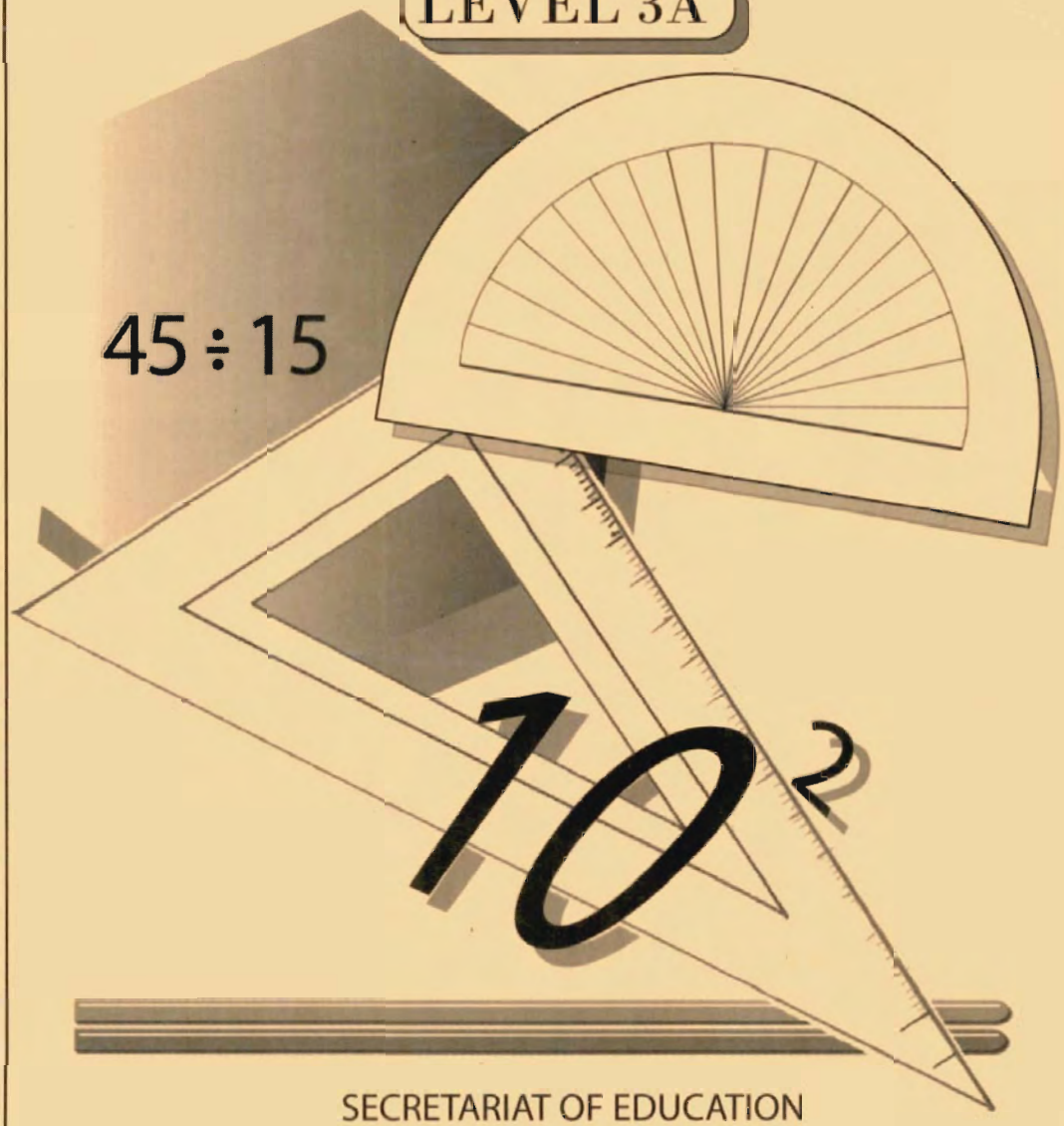
ACCELERATED LEARNING PROGRAM

MATHEMATICS

LEVEL 3A



SECRETARIAT OF EDUCATION
NEW SUDAN

ACCELERATED LEARNING PROGRAM

MATHEMATICS

LEVEL 3A

Secretariat of Education
New Sudan

Published and Printed by
Secretariat of Education - New Sudan

First Edition

ISBN 9966-33-095-X

USAID
FROM THE AMERICAN PEOPLE

These materials have been prepared with the financial assistance of USAID under Sudan Basic Education Program (SBEP) Cooperative Agreement No. 623-A-002-0068

SUDAN BASIC EDUCATION PROGRAM

Sudan Basic Education Program (SBEP) is a consortium of CARE International, American Institutes for Research and the University of Massachusetts.

Printed by: The Creative Printhouse Ltd

CONTENTS

UNIT 1 Operations on Numbers

1.1 Numbers and Place Value

1.1a Identify place value of numbers up to a million

Numbers	Million	Hundred Thousand	Ten Thousand	Thousand	Hundred	Ten	One
76,453			7	6	4	5	3
314,675		3	1	4	6	7	5
1,514,987	1	5	1	4	9	8	7

	DIGIT	PLACE VALUE	TOTAL VALUE
3421658	8	Ones	8
	5	Tens	50
	6	Hundreds	600
	1	Thousands	1000
	2	Ten thousands	20,000
	4	Hundred thousands	400,000
	3	Millions	3,000,000

Exercise 1

A. Fill in the blank spaces

a) 15002 = ----thousands -----hundreds -----tens----ones

b) 23460 =----thousands----hundreds-----tens-----ones

c) 91521 = -----thousands -----hundreds----tens----ones

B. Write the Place Value of the numbers marked in bold print

1) 2**3**40 2) 2736**5** 3) **8**625 4) 23**4**745

5) 2**0**30 6) 841**2**7 7) **5**123006

C. Write the numbers below that have a 5 in the:

(i) Thousands place (ii) Hundreds place (iii) Tens place

(iv) Ten thousands place (v) Millions place

a) 28452 b) 41582 c) 57842 d) 125184

e) 563120 f) 42158 g) 2541 h) 58216

i) 41852 j) 52814 k) 18524 l) 5123004

D. Write the digit in the

i. tens place
ii. thousands place
iii. ten thousands place
iv. millions place

1) 4861 2) 94867 3) 579846

4) 7243091 5) 3120475 6) 40752 7) 29

E. Write the numbers below in expanded form.

Example:

a) 28 = 20 + 8 b) 435679 = 400000 + 30000 + 5000 +600 + 70 +9

a) 72948 b) 1231 c) 1461 d) 18346729

F. Write the expanded forms as single numbers

Example:

500 + 70 + 3 = 573

a) 7000 + 200 +80 + 4 =

b) 3,000,000 + 300,000 + 10,000 +4,000 =

c) 4,000,000 + 200,000 + 10,000 + 2000 + 400 +80+3 =

d) 900,000 + 50,000 + 4000 + 800 + 10 +2 =

1.1 (b) Reading and Writing Numbers

Exercise 2

1. Write the following numbers in words

Examples:

11,653 = eleven thousand six hundred and fifty-three

50,056 = fifty thousand and fifty-six

1) 254 2) 15,634 3) 50,867

4) 8,421,505 5) 9, 999, 999 6) 4,021,567

2. Write the following in symbols

Example:

One million, four hundred and forty two thousand, three hundred and fifty-two
1,442,352

a) seven hundred and five
b) two thousand and seventy
c) eight million, three hundred thousand, two hundred and twenty
d) thirty four thousand, seven hundred and sixty
e) one million, twelve thousand, one hundred and four

1.1 (c) Rounding off Numbers

Example 1

A) 17 to the nearest ten is 20
b) 231 to the nearest ten is 230
c) 579 to the nearest ten is 580
d) 13 to the nearest ten is 10

Exercise 3a

1. Round off the following numbers to the nearest 10 (tens)

1) 5	2) 7	3) 71	4) 81
5) 356	6) 1346	7) 67234	8) 28.3

Example 2

a. 775 to the nearest hundred is 800
b. 1,535 to the nearest hundred is 1,500
c. 5,299 to the nearest hundred is 5,300

Exercise 3b

Round off the numbers below to the nearest hundred

a) 6,234	b) 6,345	c) 12,467	d) 1,024
e) 436,712	f) 83	g) 9,990	

Example 3

1. 5001 to the nearest thousand is 5000
2. 9450 to the nearest thousand is 9000
3. 6800 to the nearest thousand is 7000

Exercise 3c

Round off the numbers below to the nearest 1000 (thousand)

1) 459	2) 4,672	3) 1,234	4) 7,578	5) 13,467
6) 96,890	7) 101,010	8) 20,024	9) 7,349	10) 98,765

Divisibility test for numbers 2, 5, 10

1. All numbers divisible by 2 end with zero or an even number.
2. All numbers divisible by 5 end with zero or 5.
3. All numbers divisible by 10 end with zero. All numbers divisible by 10 are divisible by both 5 and 2.

Examples:

1) 200, 8152, 16, and 1994 are all divisible by 2
2) 35, 440, 50 and 1985 are divisible by 5

Exercise 4

1) Use the divisibility test rule above to find out which of the following numbers are divisible by 2:
   2, 16, 34, 45 , 63, 80, 1990

2) Use the divisibility test rule above to find out which or the following numbers are divisible by 5:
   7, 19, 20, 125, 152, 1943, 200, 2755

3) Use the divisibility test rule above to find out which of the following numbers are divisible by both 2 and 5:
   144, 485, 875, 840, 140, 785, 880, 220, 119, 660, 325, 1990

4) Which of the following numbers are divisible by 10?
   12, 30, 45, 60, 82, 95, 100, 3475, 19980

5) Which of the following numbers are divisible by 2, 5 and 10?
   30, 75, 2410, 4046, 121,980, 15,345, 78,982, 1565, 19,200, 8,000,000, 986,022, 970

6) Write down ten whole numbers which are divisible by 2, 5 and 10.

7) Which of the following statements are true?
   a) 412 is divisible by 2.
   b) 210 and 8245 are divisible by both 2 and 5.
   c) The number 470 is divisible by 2, 5 and 10.

Divisibility test for numbers 3, 4 and 6

Examples:

1) A number is divisible by 3 if the sum of its digits is divisible by 3.
(96 is divisible by 3 because 9 +6 = 15 and 15 is divisible by 3)

2) A number is divisible by 4 if the number formed by the last two digits is divisible by 4.
(312 is divisible by 4 because 12 is divisible by 4)

3) A number is divisible by 6 if it is divisible by both 2 and 3. For example the number 162:

a) 162 is divisible by 2 because the last digit of 162 is even.

b) 162 is divisible by 3 because 1 +6 + 2 = 9 and 9 ÷ 3 = 3 therefore, 162 is divisible by 6

Exercise 5

1. Which of these numbers are divisible by 3?

   9, 28, 30, 39, 247, 306, 5,004, 8,143

2. Which of these numbers are divisible by 4?

   124, 168, 1,046, 14,842, 106,255, 19,390, 22,300

3. Which of the following numbers are divisible by 6?

   12, 79, 600, 1,994, 99,

   304, 342, 81,576, 35,152, 2,551

4. Use the divisibility test rule above to find which of the numbers below are divisible by:
   (i) 2
   (ii) 3
   (iii) 6

   102, 453, 2221, 97, 54, 45, 132, 334,
   504, 222, 216, 162, 666, 84, 702

5. (a) Write a two digit odd number which is divisible by 3.

   (b) Write a three digit even number which is divisible by 3.

1.1 (d) Divisibility tests for numbers 8 and 9

Examples

1)A number is divisible by 8 if its last 3 digits are divisible by 8.

Are the following numbers divisible by 8?
(i) 187432 (ii)27448

Solution

(i) $432 \div 8 = 54$ (ii) $448 \div 8 = 56$
Since 432 and 448 are divisible by 8, then

(i) 187432 and
(ii) 27448 are both divisible by 8

2. A number is divisible by 9 if the sum of its digits is 9 or a a multiple of 9 e.g. 9, 18,27,36

   Are 72 and 72018 divisible by 9?
   72 is divisible by 9 because $7 + 2 = 9$

Exercise 6

1. Which of the following numbers are divisible by 8?

   a) 3,672 b) 1,824 c) 1,024

   d) 9,672 e) 2,603 f) 8,640

2. Which of the following numbers are divisible by 9?

   a) 2,492 b) 7,351 c) 6,528

   d) 71,846 e) 25,450 f) 909,288

3. Use the divisibility test rule above to find out which of the following numbers are divisible by 8:

   a) 46,728 b) 39,872 c) 9,136

d) 693,578 e) 5,475,685 f) 4,567,128

4. Use the divisibility test rule above to find out which of the following numbers are divisible by 9:

a) 24,918 b) 723,912 c) 192,342

d) 693,578 e) 5,475,682 f) 4,567,128

5. From the numbers below identify which numbers cannot be divided by 9:

683, 171, 621, 309, 62,171,

64, 442, 468, 3,016, 153

6. From the numbers below, list the numbers divisible by 6 and 8:

48, 64, 472, 264, 822,

318, 4,440, 4,404, 32

Prime Numbers

A **prime number** is a number which can only be divided by itself or 1. Some of the prime numbers are 2, 3, 7, and 11.

Many years ago, a Greek mathematician discovered a method of separating the prime numbers; he used a table like the one shown here.

The set of prime numbers between 1 and 100 can be found by the following method:

First, copy the numbers 1 to 100 in your exercise books as on this chart:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	42	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Now carry out these instructions:

1. Shade out 1 (which is not a prime number)
2. Shade out all numbers divisible by 2 except 2 itself
3. Shade out all numbers divisible by 3 except 3 itself
4. Shade out all numbers divisible by 5 except 5 itself
3. Shade out all numbers divisible by 7 except 7 itself

(Notice you will have to shade some numbers more than once)

The numbers remaining unshaded are the prime numbers between 1 and 100. Write them out.

You have seen those numbers in different ways already. Note that there is only one even prime. Which number is it?

Examples

1) Write down prime numbers between

   (a) 0 and 20 (b) 20 and 50 (c) 50 and 100

2) Which is the only even prime number?

3) Write 5 as a sum of two primes.

Solution

a) 2, 3, 5, 7, 11, 13, 17, and 19
b) 23, 29, 31, 37, 41, 43, 47
c) 53, 59, 61, 67, 71, 73, 79, 83, 89, 97

2) 2 3) 2 + 3

Exercise 7

1. Which of the following numbers are prime numbers?
   (a) 3 (b) 7 (c) 9 (d) 21 (e) 37

2. Make a list of prime numbers between 10 and 40.

3. Make a list of prime numbers less than 50.

4. Write down the following numbers as the sum of two prime numbers:

   Example: 2 + 5 = 7

   (a) 7 (b) 8 (c) 24 (d) 29 (e) 36 (f) 48

5. Write down the following numbers as a product of two prime numbers.
   Example: 21 = 3 x 7

   (a) 10 (b) 15 (c) 35 (d) 39 (e) 26 (f) 77

1.1 (f) (l) Factors

Look at the examples below:

Number	Factors
4	1, 2, 4
6	1, 2, 3, 6
8	1, 2, 4, 8
15	1, 3, 5, 15
18	1, 2, 3, 6, 9, 18
36	1, 2 ,3 ,4, 6, 9, 18, 36
54	1, 2, 3 ,6, 9, 18, 27, 54

The **factors** (divisors) of a number are all those numbers that can divide into it without any remainder as you see in the table above.

1.1 (f) (ii) Multiples

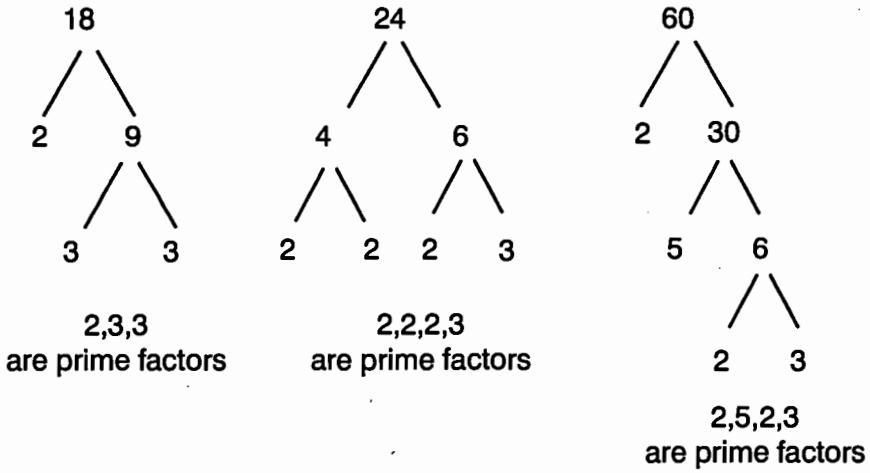
Number	Multiples
2	2, 4, 6, 8, 10, 12 ....
3	3, 6, 9, 12 ,15, 18, 21, 24
5	5, 10, 15, 20, 25, 30, 35, 40
6	6, 12, 18, 24, 30, 36, 42, 48
7	7, 14, 21, 28, 35, 42, 49

The **multiples** of a whole number are those numbers that can be divided exactly by the whole number in question as shown in the table above.

1.1 (f) (iii) Prime Factors

If a factor or divisor of a number is a prime number, that factor is called a prime factor. For example, 30 has the factors 2, 3, 5 which are prime factors.

Sometimes we use what we call a factor tree to get the prime factors of a number. For example, to find the prime factors of 18, 24 and 60 you can use a factor tree as follows:



Exercise 8

1. Write down two numbers that are multiples of 2, 3 and 7.

2. Write down all the multiples of 4 which are less than 50.

3. Write down in order, starting from the smallest, all the factors of the following numbers:

   (a) 12 (b) 16 (c) 20 (d) 28

   (e) 28 (f) 32 (g) 64

4. Write in order, starting from the smallest, all the factors of the following numbers:

   (a) 14 (b) 18 (c) 27 (d) 30

   (e) 56 (f) 84

5. Write down the next four multiples of 7 in this series 28, 35, - - - -, - - - - -

6. Write down the missing numbers in the series - - - , - - - -, - - - , 44, 48, 52.

7. Write down the prime factors for each of these numbers: 20, 24, 51, 75, 100

8. Write down the common multiples of 4 and 6 less than 50.

9. Write down the missing numbers in the series 22,33,--, --,66, --, --, 99.

1.1 (g) (i) Highest Common Factor (HFC) (or Greatest Common Divisor)

Example 1: What is the H.C.F. of 16 and 24?

The factors of 16 are 1,2,4,8 and 16
The factors of 24 are 1,2,3,4,6,8,12 and 24
1,2,4 and 8 are common factors of 16 and 24
8 is the common factor which is the highest
Therefore 8 is the Highest Common Factor (H.C.F.), also called The Greatest Common Divisor (G.C.D.), of 16 and 24.

Following is a shorter method:

Example 2: What is the G.C.D. of 18, 12 and 24?

Solution: Start dividing by the smallest prime numbers that divide all the numbers.

2	18	12	24
3	9	6	12
	3	2	4

G.C.D. = 2 x 3 = 6

Example 3: Use another method to find the G.C.D of example 2 above.

Solution: Find the G.C.D or H.C.F of 18,12 and 24 by using the prime factors method:

The prime factors of each number are:
18 = 2 x 3 x 3
12 = 2 x 2 x 3
24 = 2 x 2 x 2 x 3

The G.C.D or H.C.F is the product of all the factors which are common in all the numbers.

Exercise 9

1. Find the Highest Common Factors (H.C.F) from the divisors of the following pairs:

   (a) 6, 9 (b) 12, 48 (c) 21, 56

2. Find the Highest Common Factors (H.C.F) from the divisors of the following numbers:

   (a) 22, 55, 66 (b) 32, 64, 72 (c) 10, 15, 30

3. Find the Highest Common Factors (H.C.F) of the following pairs of numbers using the short method:

(a) 8,12 (b) 75,30 (c) 24,48

(b) 6,36 (c) 24,40 (d) 39,42

4. Find the Highest Common Factors (H.C.F) of the following numbers:

(a) 9, 18, 24 (b) 2, 10, 12 (c) 28, 56, 72

(d) 22, 44, 55 (e) 32, 64, 72 (f) 24, 36, 40

5. Find the H.C.F of the following:

(a) 10, 15 (b) 18, 24 (c) 15, 18

(d) 14, 20, 36 (e) 9, 18, 72 (f) 45, 48, 90

1.1 (g) (ii) The Lowest Common Multiple (L.C.M)

Example 1: What is the L.C.M of 4 and 6?

Solution: Multiples of 4 are 4, 8, 12, 16, 20, 24, 28, 32, 36, ...
Multiples of 6 are 6, 12, 18, 24, 30, 36, 42, ...

Common multiples of 4 and 6 are 12, 24, 36.
Therefore, the Least Common Multiple (L.C.M) of 4 and 6 is 12.

Example 2: Find the L.C.M of 8, 12 and 30 using the short method:

Solution: We start dividing by the smallest prime number that can divide into any one or all of the numbers given. We keep dividing until all the numbers are divided out:

2	8	12	30
2	4	6	15
2	2	3	15
3	1	3	15
5	1	1	5
	1	1	1

L.C.M of 8, 12 and 30 is: $2 \times 2 \times 2 \times 3 \times 5 = 120$
Therefore, the L.C.M is the product of the factors we have used.

Exercise 10

1. Find the Least Common Multiples (L.C.M) of the following numbers:

   (a) 3 and 2 (b) 4 and 5 (c) 2 and 4

   (d) 9 and 12 (e) 12 and 15 (f) 8 and 9

2. Find the L.C.M of the following numbers:

   (a) 6, 8 and 12 (b) 16 and 20

   (c) 10, 25 and 4 (d) 9, 18 and 12

   (e) 14, 9 and 12 (f) 8, 4 and 10

Roman Numerals

Sometimes you may not see Arabic Numerals (1, 2, 3, 4, 5 etc), but you will see the following: I, II, III, IV, V etc. These numbers are called Roman Numerals.

Arabic Numerals	Roman Numerals	Arabic Numerals	Roman Numerals	Arabic Numerals	Roman Numerals
1	I	11	XI	30	XXX
2	II	12	XII	40	XL
3	III	13	XIII	50	L
4	IV	14	XIV	60	LX
5	V	15	XV	70	LXX
6	VI	16	XVI	80	LXXX
7	VII	17	XVII	90	XC
8	VIII	18	XVIII	100	C
9	IX	19	XIX	500	D
10	X	20	XX	1000	M

Exercise 11

1. Write the Roman numbers for the following:

   (a) 2 (b) 6 (c) 12 (d) 19 (e) 20

   (f) 30 (g) 50 (h) 80 (i) 400

2. Write Arabic numbers for the following:

   (a) IV (b) VII (c) IX

   (d) XXX (e) L (f) CL

3. Work out the following and give your answer in Roman numerals:

(a) 3 + 5 (b) 20 + 4 (c) 90 - 14 d)100 - 32

(e) 12 x 2 (f) 50 x 3 (g) 320 ÷ 4 (h) ¼ of 80

4. Work out the following:

(a) XXX + IX (b) CDX + XXIX (c) CMXC + XLVI

(d) XLVI + XIV (e) LI + XX

1.2 Operations on whole numbers

Exercise 12

1.2 (a) (i) Addition (+)

1)	2)	3)	4)
270135	742239	209143	334189
+ 538426	729160	126304	362417
	526314	761343	793781
	+ 425603	+ 485213	+ 465780

Subtraction (-)

1)	2)	3)	4)
1000000	403890	931483	8000000
- 999999	- 20179	- 571450	- 580649

Work out the following:

9. 126147 + 230829 + 1453 + 21156 =

10. 601 + 304406 + 550 - 60771 =

11. 1000000 - 3462 - 126440 =

12. Juba has a population of 350,000. If 165,350 are children, how many adults are there?

13. In 1979 the population of Southern Sudan was estimated at 4,206,000. If the estimated population in 1989 is 6,026,100 what was the increase in population between 1979 and 1989?

14. Riaka has three hundred and fifty head of cattle, Chuol and Tut have one hundred and fifty nine head altogether, Lopiding has four hundred and thirty three head. What is the total number of animals owned by Riaka, Chuol, Tut and Lopiding?

15. A brick maker produced 34896 bricks in 1990, 46723 in 1991, 56729 in 1992 and 42346 in 1993. What was his total brick production in the four years?

16. Deng earns £s 35,000, his wife earns £s 42,000, his son earns £s 32,000 and his daughter earns £s 25,000. What is the total income of Deng's family?

Multiplication (x)

Exercise 13

A. Find the products of the following

(a) 649 x 72	(b) 728 x 56	(c) 2719 x 123	(d) 781 x 137	(e) 781 x 137	(f) 5324 x 241
(g) 2356 x 321	(h) 3146 x 53	(i) 827 x 135	(j) 1426 x 213	(k) 1734 x 39	

B. Find the products of the following

1. 486 x 251 =
2. 235 x 54 =
3. 4769 x 312 =
4. 324 x 221 =
5. 2143 x 31 =
6. 6325 x 76 =
7. 4312 x 85 =
8. 4925 x 100 =
9. 4153 x 105 =
10. 4752 x 21 =

C. Find the products of the following

1. 789 x 12	2. 853 x 25	3. 598 x 21	4. 764 x 84	5. 1718 x 13	6. 2726 x 123

7. 315 cattle keepers each have 85 cows. How many cows do they own altogether?

8. A bag of maize costs £s 1500. What is the total cost of 95 bags of maize?

9. Lopiding has 75 cows. Each gives 10 litres of milk every day. If Lopiding's family uses 50 litres of milk and he sells the rest at £s 10 per litre, how much money does Lopiding's family make per day?

1.2 (c) Division (÷)

Example

Divide 28350 by 70 28350 ÷ 70

Procedure:

Step 1. Consider dividing 28 ÷ 70. It is not possible.

Now consider 283 ÷ 70. It is possible.

To find out how many times the number 70 goes into 283, let us try the following

70 x 1 = 70 70 x 2 = 140 70 x 3 = 210 7 x 4 = 280 70 x 5 = 350

From our work above, the number which is nearest to 283 is 280 (70 x4).

Put 4 above 3 and subtract 280 from 283 as shown below.

$$\begin{array}{r} 4 \\ 70\overline{)28350} \\ \underline{280} \\ 3 \end{array}$$

Step 2. Bring 5 down to make 35 as shown below

$$\begin{array}{r} 4 \\ 70\overline{)28350} \\ \underline{280} \\ 35 \end{array}$$

Now consider 35 ÷70. It goes zero times (70 x 0). Write zero above 5 to make 40 as shown below and subtract zero from 35. Bring zero down to make 350.

$$\begin{array}{r} 405 \\ 70\overline{)28350} \\ \underline{280} \\ 35 \\ \underline{0} \\ 350 \end{array}$$

70 x 5 = 350. Write 5 above 0 to make 405 and subtract 350 from 350 as shown below.

$$\begin{array}{r} 405 \\ 70\overline{)28350} \\ \underline{280} \\ 35 \\ \underline{0} \\ 350 \\ \underline{350} \\ 0 \end{array} \quad = 405$$

Exercise 14:

Work out the following

1. 280 ÷70 =
2. 480 ÷80 =
3. 630 ÷90 =
4. 960÷ 120 =
5. 1080 ÷40 =
6. 1500÷ 50 =

7 Batali has £s 2,500. He shared it equally among his 50 clan members. How much does each member get?

8. A school consumes 30 bags of maize every week. If the school has 540 bags of maize in the store, how many days will the maize last?

9. Deng walked a distance of 560 km. If it took him 40 days of walking an equal distance each day, how many kilometers did he walk each day?

10. There are 12,000 primary teachers in Southern Sudan. If each school has 40 teachers, how many schools are there in Southern Sudan?

11. The cost of building four classroom blocks in Southern Sudan is estimated to be $12.500 U.S.D.
    (a) What is the cost of one classroom block?
    (b) What is the cost of two classroom blocks?

12. 572,911 fencing posts had to be divided among 39 persons. How many remained?

Exercise 15: Work out the following division problems.

1. $12\overline{)1896}$
2. $25\overline{)2325}$
3. $36\overline{)3024}$
4. $39\overline{)1482}$
5. $89\overline{)4183}$
6. $36\overline{)5796}$

7. 14,400 bags of relief food were air dropped at Narus town. These bags were equally distributed among 25 locations. How many bags did each location receive?

8. In a certain county in Sudan, there are 23 locations. In 1970 a total of £s794,650 was collected as tax by the government. If all locations collected equal amounts, how much money did each location collect?

Number Patterns

Numbers can be represented in different ways. For example the number five can be written as:

5 V IIIII

xx
x
xx

and many other ways.

One of the simplest ways of writing numbers is to represent them as crosses in some kind of pattern or shape.

Examples

(a)

xxxxxxx
horizontal
representation

(b)

x
x
x
x
x
vertical
representation

(c)

x
x
x
x
xxxxxx
letter L-shape

1	2	3	4	5	6	7
x	xx	xx	xx	xx	xxx	x x
		x	xx	xx	xxx	xxx
				x		x x

Number patterns with equal rows:

x x x	x x x x	x x x x x	x x x x x x
x x x	x x x x	x x x x x	x x x x x x
x x x	x x x x	x x x x x	x x x x x x

When numbers have rectangular patterns, they are called rectangular numbers:

x x x x	x x	x x x x x x
x x x x	x x	x x x x x x
(8)	x x	x x x x x x
	x x	x x x x x x
	x x	(24)
	(10)	

How to use rectangular patterns to obtain factors

x x x
x x x This is a cross pattern for number 6.

In how many ways can we write 6 as a rectangular number?

x x x
x x x

2 rows 3 columns

x x
x x
x x

3 rows and 2 columns

The cross pattern shows 2 rows and 3 columns or 3 rows and 2 columns that is:
2 x 3 and 3 x 2.

We say that 6 is the product of 2 and 3 and that 2 and 3 are factors of 6.
6 is a multiple of 2 and 3

(b) Square Numbers

When numbers have square patterns of crosses, they are said to be square numbers:

x
1

x x
x x
4

x x x
x x x
x x x
9

x x x x
x x x x
x x x x
x x x x
1 6

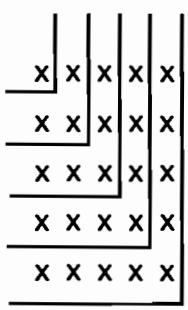
x x x x x
x x x x x
x x x x x
x x x x x
x x x x x
2 5

x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
x x x x x x
36

x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
x x x x x x x
49

Therefore 1, 4, 9, 16, 25, 36, 49 ------- are square numbers.

You can get square numbers without necessarily drawing square patterns of the numbers. Look at the square patterns for the square numbers up to 25:



1 =1

1+ 3 = 4

1 + 3+ 5 =9

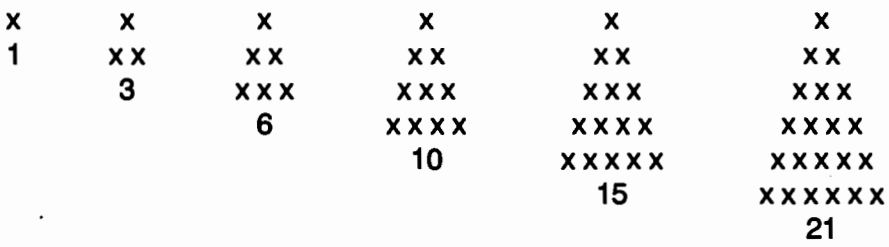
1+3 + 5 + 7 =16

1+ 3 + 5 + 7 + 9 = 25

Do you understand the pattern of getting square numbers on the right?
Get the next three square numbers.

(c) Triangular numbers

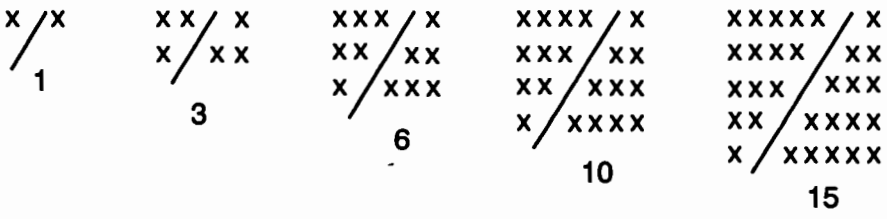
When numbers have triangular patterns of crosses, then they are said to be triangular numbers:



Therefore 1, 3, 6, 10, 15, 21 ------- are triangular numbers.

You can also get triangular numbers without drawing patterns of crosses.

Look at the patterns for getting the triangular numbers:



$\frac{1 \times 2}{2} = 1$ $\frac{2 \times 3}{2} = 3$ $\frac{3 \times 4}{2} = 6$ $\frac{4 \times 5}{2} = 10$ $\frac{5 \times 6}{2} = 15$

The first number shown in each example is the number of rows. We multiply that by the number of crosses in each row. We divide by 2 because 2 parts (or triangles) have been formed by the diagonal line.

Let us look at the second example: The number of rows is 2; the number of crosses in each row is 3; the number of parts is 2. So the sum is 2 times 3 divided by 2.

Do you understand the patterns for getting the triangular numbers without drawing triangular patterns? Get the next three triangular numbers.

(d) The 100 square

By colouring numbers in the 100 square we will see patterns. This will help us to understand how numbers relate to one another.

100 square showing numbers divisible by 2 or multiples of 2

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	98	99	100

Exercise 16

1. Represent the following rectangular numbers with cross-patterns:
   (a) 12 (b) 14 (c) 18 (d) 24

2. Represent the following square numbers with cross-patterns:
   (a) 9 (b) 36 (c) 49 (d) 64

3. When do we call a number a triangular number?

4. Complete the missing numbers:

   (a) $1 \times \frac{2}{?} = 1$ (b) $2 \times \frac{?}{2} = 3$ (c) $? \times \frac{4}{2} = 6$

   (d) $4 \times \frac{5}{?} = 10$ (e) $? \times \frac{6}{2} = 15$ (f) $6 \times \frac{?}{2} = 15$

100 square showing numbers divisible by 3 or multiples of 3:

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	99	99	100

Shade the numbers divisible by both 2 and 3. This gives you the multiples of 2 and 3. (You may use coloured pencils to shade the numbers.)

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50
51	52	53	54	55	56	57	58	59	60
61	62	63	64	65	66	67	68	69	70
71	72	73	74	75	76	77	78	79	80
81	82	83	84	85	86	87	88	89	90
91	92	93	94	95	96	97	99	99	100

5. Write down the missing numbers in the sequence __, __, __, __, 15, 18, 21
6. Write down the missing numbers in the sequence __, __, __, 44, 55, 66
7. Write down the missing numbers in the sequencee 25, 36, __, __, __, __,
8. Complete the pattern shown below

$$1=1$$
$$1+3=4$$
$$1+3+5=9$$
$$1+3+5+7=16$$
$$1+3+5+7+9=25$$
$$1+3+5+7+9+\_=$$
$$1+3+5+7+9+\_+\_=$$
$$1+3+5+7+9+\_+\_+\_=$$

1.2 (e) Squares and square roots

1.2 (e) Squares

The number 1 is a perfect square because it is the square of 1. Similarly, 16 is a perfect square because it is the square of 4.

Definition

A perfect square is a number obtained when a number is multiplied by itself.

Number squared	Perfect square number
$1^2 = (1 \times 1)$	1
$2^2 = (2 \times 2)$	4
$3^2 = (3 \times 3)$	9
$4^2 = (4 \times 4)$	16
$5^2 = (5 \times 5)$	25
$6^2 = (6 \times 6)$	36
$7^2 = (7 \times 7)$	49
$8^2 = (8 \times 8)$	64

Exercise 1: Which of the following are perfect squares?

a) 16 b) 17 c) 25 d) 10 e) 36 f) 400

Square of the Whole Numbers

Number	Squares
0	0
1	1
2	4
3	9
4	16
5	25
6	36
7	49
8	64
9	81
10	100
11	121
12	144
13	169
14	196
15	225
16	256
17	289
18	324
19	361
20	400

Exercise 18

A. Write the squares of the following:

1) 3 2) 12 3) 19 4) 15 5) 41 6) 32

7) 7 8) 13 9) 9 10) 22 11) 14 12) 50

B. What is the area of the square with one side of 17 cm?

1.2 (e) (ii) Square Roots

What is the square of the following: (a) 2 and (b) 3?

The number 2 and 3 are square roots of 4 and 9 because $2 \times 2 = 4$, and $3 \times 3 = 9$

The symbol $\sqrt{\ }$ is used to represent the square root. For example

i) $\sqrt{4} = 2$ ii) $\sqrt{9} = 3$

The square root is the opposite of squaring a number, for example:

$\sqrt{16} = \sqrt{(4 \times 4)} = 4$ $\sqrt{25} = \sqrt{(5 \times 5)} = 5$

Exercise 19

(a). Work out the square roots of the following whole numbers:

1) 4	2) 25	3) 1	4) 9	5) 49	6) 64
7) 144	8) 100	9) 81	10) 169	11) 225	12) 256
13) 400	14) 625	15) 121	16) 0		

b) What is the difference between the squares of 6 and 5?

c) What is the sum of squares of 10 and 14?

d) The sum of two numbers is 49. What is square root of the sum of the two numbers?

1.3. Fractions and Decimals

1.3 (a) (i) Fractions

You have already learnt that a fraction is a part of a whole. Let us remind ourselves of how addition and subtraction of fractions are carried out.

Addition and Subtraction (Revision)

Like Fractions

Like fractions have a common denominator, for instance $\frac{3}{7}$ and $\frac{2}{7}$ are like fractions.

1. $\frac{14}{13} + \frac{16}{13} =$

2. $\frac{5}{x} + \frac{8}{x} =$

3. $\frac{2}{15} + \frac{7}{15} + \frac{7}{15} =$

4. $\frac{8}{9} + \frac{5}{9} =$

5. $\frac{3}{7} + \frac{5}{7} =$

6. $\frac{7}{10} + \frac{13}{10} + \frac{13}{10} =$

Subtract $\frac{12}{15} - \frac{4}{15} =$

(i) Subtract the numerators $\frac{12}{15} - \frac{4}{15} = \frac{12-4}{15}$

(ii) Write the difference over the common denominator $= \frac{8}{15}$

Exercise 20 a

1. $\frac{5}{12} - \frac{1}{12} =$

2. $\frac{6}{17} - \frac{2}{17} - \frac{3}{17} =$

3. $\frac{17}{25} - \frac{11}{25} + \frac{2}{25} =$

4. $\frac{7}{x} - \frac{5}{x} =$

Exercise 20 b

5. $\frac{27}{30} - \frac{7}{30} =$

6. $\frac{6}{7} - \frac{4}{7} - \frac{1}{7} =$

Mixed exercise 20 c

1. $\frac{3}{11} + \frac{5}{11} =$

2. $\frac{4}{17} + \frac{11}{17} =$

3. $\frac{7}{19} + \frac{12}{19} =$

4. $\frac{14}{16} + \frac{1}{16} =$

5. $\frac{2}{5} + \frac{4}{5} =$

6. $\frac{19}{46} - \frac{18}{46} =$

7. $\frac{3}{7} - \frac{2}{7} =$

8. $\frac{7}{13} - \frac{6}{13} =$

9. $\frac{13}{26} + \frac{19}{26} - \frac{4}{26} =$

10. $\frac{5}{19} + \frac{3}{19} + \frac{6}{19} =$

11. $\frac{4}{7} + \frac{5}{7} + \frac{1}{7} =$

12. $\frac{9}{23} + \frac{9}{23} - \frac{6}{23} =$

13. $\frac{27}{30} - \frac{7}{30} - \frac{13}{30} =$

14. $\frac{6}{15} - \frac{11}{15} - \frac{5}{15} =$

1.3 (a) (ii) Addition of Fractions with different Denominators

Often fractions in addition or subtraction problems do not have the same denominators.

Example: Add

$$\frac{1}{4} + \frac{7}{16}$$

First find the L.C.M of the denominators i.e. 4 and 16

2	4	16
2	2	8
2	1	4
2	1	2
	1	1

The L.C.M of 4 and 16 is 2 x2 x 2 x = 16

Rewrite $\frac{1}{4}$ to have denominator 16

That is $\frac{1}{4} = \frac{4}{16}$, now $\frac{4}{16}$ and $\frac{7}{16}$

Have the same denominator.

Next, we carry out the addition as shown earlier.
Therefore,

$$\frac{4}{16} + \frac{7}{16} = \frac{11}{16}$$

Let us introduce another method of addition of fractions of different denominators.

Example:

Add $\frac{1}{4} + \frac{7}{16}$

Solution

We know that L.C.M of 4 and 16 is 16 which becomes the common denominator for the two fractions.

Step 1

$$\frac{1}{4} + \frac{7}{16} = \frac{}{16}$$

Write the fraction as shown above. On the right hand side of the equal sign, draw a line and write the common denorninator below it.

Step 2

$$\frac{1}{4} + \frac{7}{16} = \frac{4+}{16}$$

We say it like this: the common denominator 16 divided into the first fraction goes 4 times. We then say the numerator 1 of the first fraction multiplied by the dividend 4, is 4 and we write it above the common denominator as shown above. Put in the plus sign.

Step 3

$$\frac{1}{4} + \frac{7}{16} = \frac{4+7}{16}$$

Like step 2, we say, the common denominator 16 divided by the denominator 16 of the second fraction goes 1 time. We then say the numerator 7 multiplied by the dividend 1 is 7 and write it as show above.

Step 4

We then carry out addition of the numbers above the common denominator, to get:

$$\frac{1}{4} + \frac{7}{16} = \frac{4+7}{16} = \frac{11}{16}$$

Subtraction of fractions with different denominators

Example

$\frac{5}{6} - \frac{2}{9} =$

First find the L.C.M. of 6 and 9

2	6	9
3	3	9
3	1	3
	1	1

Therefore, L.C.M. of 6 and 9 = 2 x 3x 3 x 3 =18

Rewrite the given fractions with denominator 18

$\frac{5}{6} = \frac{15}{18}, \quad \frac{2}{9} = \frac{4}{18} \quad \therefore \frac{5}{6} - \frac{2}{9} = \frac{15}{18} - \frac{4}{18} = \frac{15-4}{18} = \frac{11}{18}$

Again, let us introduce another method for solving subtraction of fractions of different denominators. The method is similar to that introduced in addition of fractions.

Example:

$\frac{5}{6} - \frac{2}{9} =$

Solution

We know that the L. C. M of 6 and 9 is 18, which becomes the common denominator for the two fractions.

Step 1

$\frac{5}{6} - \frac{2}{9} = \frac{\quad}{18}$

On the right hand side of the equal sign we draw a line and write beneath it the common denominator 18.

Step 2

$\frac{5}{6} - \frac{2}{9} = \frac{15}{18}$

Here we say; the common denominator 18 divided by 6 goes 3 times. The numerator 5 multiplied by 3 is 15 and we write 15 above 18. Put in the minus sign.

Step 3

$$\frac{5}{6} - \frac{2}{9} = \frac{15-4}{18}$$

Like in step 2, we say, 18 divided by 9 goes 2 times. The numerator 2 multiplied by the dividend 2 is 4 and we write it as shown above.

Finally, we carry out the subtraction of the numbers above the common denominator, to get:

$$\frac{15-4}{18} = \frac{11}{18}$$

1.3 (b) Simplification of fractions by cancellation

Equivalent fractions have the same value, for instance,

$$\frac{3}{4} = \frac{6}{8} \qquad \left(\frac{3}{4} = \frac{3 \times 2}{4 \times 2} = \frac{6}{8}\right)$$

We have multiplied the numerator and the denominator by 2 to get $\frac{6}{8}$

Now $\frac{3}{4}$ and $\frac{6}{8}$ are equivalent fractions, which means that if we multiply the numerator and denominator of the fraction by the same number, we do not change the value of the fraction.

Therefore $\frac{3}{4} = \frac{6}{8} = \frac{9}{12} = \frac{12}{16} = \frac{15}{20} = \frac{18}{24}$

i.e. $\frac{3}{4}$; $\frac{6}{8}$; $\frac{9}{12}$; $\frac{12}{16}$; $\frac{15}{20}$; $\frac{18}{24}$ are equivalent fractions.

With which number did we multiply the denominator and the numerator of ¾ to get the above equivalent fractions?

Similarly we may divide the numerator and denominator of a fraction by the same number without changing the value of the fraction. For instance:

a) $\frac{6}{8} = \frac{6 \div 2}{8 \div 2} = \frac{3}{4}$

b) $\frac{4}{10} = \frac{4 \div 2}{10 \div 2} = \frac{2}{5}$

c) $\frac{18}{24} = \frac{18 \div 2}{24 \div 2} = \frac{3}{4}$

d) $\frac{20 \div 20}{40 \div 20} = \frac{1}{2}$

These examples show that fractions can be simplified by dividing both the denominator and the numerator by a common divisor/factor until there is no common factor left, then we say the fraction is in its simplest form.

Examples

$\frac{2}{4} = \frac{2 \div 2}{4 \div 2} = \frac{1}{2}$ Simplest form of $\frac{2}{4} = \frac{1}{2}$

$\frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ Simplest form of $\frac{4}{6} = \frac{2}{3}$

$\frac{24}{36} = \frac{24 \div 12}{36 \div 12} = \frac{2}{3}$ Simplest form of $\frac{24}{36} = \frac{2}{3}$

$\frac{10}{30} = \frac{10 \div 10}{30 \div 10} = \frac{1}{3}$ Simplest form of $\frac{10}{30} = \frac{1}{3}$

$\frac{36}{54} = \frac{36 \div 9}{54 \div 9} = \frac{4}{6} = \frac{4 \div 2}{6 \div 2} = \frac{2}{3}$ Simplest form of $\frac{36}{54} = \frac{2}{3}$

Exercise 21

Simplify the following fractions to their lowest forms

1. $\frac{4}{8}$ **2.** $\frac{24}{48}$ **3.** $\frac{5}{20}$ **4.** $\frac{50}{1000}$ **5.** $\frac{75}{100}$

6. $\frac{33}{66}$ **7.** $\frac{28}{72}$ **8.** $\frac{35}{55}$ **9.** $\frac{6}{9}$ **10.** $\frac{9}{54}$

11. $\frac{125}{225}$ **12.** $\frac{25}{125}$ **13.** $\frac{7}{63}$ **14.** $\frac{8}{56}$ **15.** $\frac{49}{84}$

16. $\frac{2y}{3y}$ **17.** $\frac{3p}{9p}$ **18.** $\frac{45q}{135q}$ **19.** $\frac{104}{169}$ **20.** $\frac{16}{256}$

Introduction to mixed numbers

1.3 (c) Conversion of mixed numbers into improper fractions and vice versa

What is a mixed number?

To answer the above question, let us add $1 + \frac{1}{2}$

Of course, we can write $1 = \frac{1}{1}$, such that $\frac{1}{1} + \frac{1}{2} = \frac{1}{1} + \frac{1}{1} = \frac{2+1}{2} = \frac{3}{2}$

But the numerator 3 is greater than the denominator 2. We know that 3 divided by 2 goes 1 times with remainder 1. The remainder 1 becomes the numerator of the denominator 2. We then write:

$$\frac{3}{2} = 1\frac{1}{2}$$

Note:

i) $1 + \frac{1}{2} = \frac{3}{2} = 1\frac{1}{2}$

ii) $1\frac{1}{2}$ is called a proper fraction or a mixed number

iii) $\frac{3}{2}$ is called an improper fraction.

Example

Write the following as the addition of a whole number and a fraction.

1) $1\frac{3}{2}$ 2) $1\frac{3}{4}$ 3) $3\frac{2}{5}$ 4) $5\frac{7}{8}$

Solution

1) $1\frac{2}{3} = 1 + \frac{2}{3}$ 2) $1\frac{3}{4} = 1 + \frac{3}{4}$

3) $3\frac{2}{5} = 3 + \frac{2}{5}$ 4) $5 + \frac{7}{8}$

Exercise 22a

Write the following as the addition of a whole number and a fraction

1) $3\frac{2}{9}$ 2) $1\frac{4}{7}$ 3) $6\frac{3}{4}$ 4) $8\frac{2}{5}$

Example

Write the following as mixed numbers or proper fractions

1) $3+\frac{3}{7}$ 2) $1+\frac{9}{10}$ 3) $2+\frac{4}{5}$ 4) $4+\frac{7}{8}$

Solution

1) $3+\frac{3}{7}=3\frac{3}{7}$ 2) $1+\frac{9}{10}=1\frac{9}{10}$

3) $2+\frac{4}{5}=2\frac{4}{5}$ 4) $4+\frac{7}{8}=4\frac{7}{8}$

Exercise 22b

Write the following as mixed numbers

1) $1+\frac{4}{11}$ 2) $3+\frac{3}{8}$ 3) $5+\frac{14}{15}$ 4) $8+\frac{7}{9}$

Example

Write the following as improper fractions and then as mixed numbers.

1) $2 + \frac{1}{4}$ 2) $3 + \frac{5}{6}$ 3) $4 + \frac{7}{9}$ 4) $5 + \frac{10}{11}$

Solution

To change the addition of a whole number and fraction into an improper fraction, we simply multiply the whole number by the denominator of the fraction and add the numerator to it. That is:

1) $2 + \frac{1}{4} = \frac{9}{4} = 2\frac{1}{4}$ 2) $3 + \frac{5}{6} = \frac{23}{6} = 3\frac{5}{6}$

3) $4 + \frac{7}{9} = \frac{43}{9} = 4\frac{7}{9}$ 4) $5 + \frac{10}{11} = \frac{63}{11} = 5\frac{10}{11}$

Exercise 22c

Write the following as improper fractions and then as mixed numbers

1) $8 + \frac{4}{5}$ 2) $3 + \frac{8}{9}$ 3) $10 + \frac{13}{14}$ 4) $4 + \frac{9}{10}$

Addition and subtraction of mixed numbers

Addition (+)

i) Example:

$$6\frac{4}{7} + 4\frac{1}{7}$$

Solution

First, we break the mixed numbers into the addition of whole numbers and fractions. Second, we add the whole numbers and fractions separately and finally add the result of the whole numbers to that of the addition of the fractions. That is:

$$6\frac{4}{7} + 4\frac{1}{7} = 6 + \frac{4}{7} + 4 + \frac{1}{7}$$

$$= (6 + 4) + \left(\frac{4}{7} + \frac{1}{7}\right)$$

$$= 10 + \frac{5}{7} = 10\frac{5}{7}$$

ii) Example: Add

$$4\frac{3}{4} + 2\frac{2}{9} = (4 + 2) + \left(\frac{3}{4} + \frac{2}{9}\right) = 6 + \left(\frac{27 + 8}{36}\right) = 6$$

Exercise 23

Add the following mixed fractions:

1. $2\frac{1}{2} + 3\frac{1}{2} =$ **2.** $3\frac{1}{4} + 4\frac{1}{8} =$ **3.** $5\frac{1}{7} + 6\frac{1}{5} =$ **4.** $8\frac{1}{9} + 2\frac{2}{3} =$

5. $9\frac{1}{10} + 3\frac{2}{3} =$ **6.** $1\frac{1}{7} + 5\frac{2}{3} =$ **7.** $8\frac{1}{4} + 5\frac{5}{16} =$ **8.** $6\frac{1}{2} + 6\frac{2}{16} =$

9. $9\frac{1}{9} + 9\frac{1}{8} =$ **10.** $7\frac{1}{7} + 7\frac{1}{21} =$ **11.** $5\frac{1}{8} + 3\frac{1}{4} =$ **12.** $3\frac{5}{16} + 2\frac{7}{24} =$

13. $2\frac{3}{4} + 3\frac{4}{5} =$ **14.** $3 + 3\frac{2}{3} + 1\frac{1}{8} =$

Subtraction of mixed numbers

Examples

1) $3\frac{2}{5} - 1\frac{3}{4}$ 2) $2\frac{8}{9} - 1\frac{4}{5}$

Solution

$$3\frac{2}{5} - 1\frac{3}{4} = (3 - 1) + \left(\frac{2}{5} + \frac{3}{4}\right) = 2 + \frac{8 - 15}{20}$$

Since 15 is greater than 8, we borrow 1 whole number from 2 and add it to the fraction

$$= 1 + \frac{20 + 8 - 15}{20} = 1 + \frac{28 - 15}{20}$$

$$= 1 + \frac{13}{20} = 1\frac{13}{20}$$

Or we can solve it by first changing the mixed fractions into improper fractions and then carrying out the subtraction. That is

$$3\frac{2}{5} - 1\frac{3}{4} = \frac{17}{5} - \frac{7}{4} + \frac{68 - 35}{20} = \frac{33}{20} = 1\frac{13}{20}$$

Whichever method is easier to understand go with it.

2) $2\frac{8}{9} - 1\frac{4}{5} = (2 - 1) + \left(\frac{8}{9} - \frac{4}{5}\right)$

$$= 1 + \frac{40 - 36}{45} = 1 + \frac{4}{45} = 1\frac{4}{45}$$

Exercise 24

Subtract the following mixed fractions:

1. $4\frac{1}{4} - 2\frac{1}{8} =$
2. $3\frac{1}{3} - 1\frac{1}{6} =$
3. $8\frac{4}{5} - 6\frac{1}{10} =$
4. $9\frac{5}{6} - 1\frac{2}{3} =$
5. $7\frac{3}{7} - 2\frac{1}{3} =$
6. $5\frac{1}{5} - 3\frac{5}{9} =$
7. $2\frac{7}{9} - 1\frac{5}{12} =$
8. $4\frac{3}{4} - 4\frac{11}{24} =$
9. $6\frac{2}{5} - 3\frac{3}{4} =$
10. $5\frac{3}{8} - 2\frac{5}{6} =$
11. $5\frac{1}{5} - 1\frac{5}{12} =$

Example of mixed subtraction and addition

$8\frac{1}{2} - 4\frac{5}{6} + 2\frac{1}{8} =$

Solution

$= (8 - 4 + 2) + (\frac{1}{2} - \frac{5}{6} + \frac{1}{8})$

$= 6 + (\frac{1}{2} - \frac{5}{6} + \frac{1}{8})$

$= 6 + (\frac{1}{2} - \frac{5}{6} + \frac{1}{8})$

Mixed Exercises

Exercise 25

1. $1\frac{1}{2} + 2\frac{1}{4} + 3\frac{1}{3} =$
2. $4\frac{1}{2} + 2\frac{1}{4} - 3\frac{1}{3} =$
3. $7\frac{3}{5} - 2\frac{1}{5} =$
4. $10\frac{1}{3} + 4\frac{1}{4} - 6\frac{1}{12} =$
5. $25\frac{7}{15} - 10\frac{2}{15} + \frac{23}{30} =$
6. $5\frac{1}{8} + 3\frac{1}{4} + 6\frac{1}{2} =$

7. $5 - 2\frac{1}{2} + 1\frac{1}{4} =$

8. $20 - 10\frac{1}{4} =$

9. $3\frac{5}{16} - 2\frac{1}{4} + 5 =$

10. $3\frac{1}{2} - 4\frac{5}{6} + 2\frac{1}{8} =$

11. $25 - 15\frac{1}{2} + 2\frac{1}{4} =$

12. $6\frac{1}{3} - 2\frac{1}{4} + 1\frac{1}{2} =$

13. $7 - 2\frac{3}{8} - 2\frac{1}{8} =$

14. $\frac{3}{2} + \frac{4}{3} - \frac{2}{5} =$

15. $\frac{15}{7} - \frac{8}{7} + 3 =$

16. $8\frac{5}{16} - 3\frac{1}{8} + \frac{17}{16} =$

17. $\frac{16}{9} - 1\frac{1}{9} =$

18. $2\frac{1}{4} - 3\frac{1}{2} + 6\frac{3}{4} + 3\frac{1}{8} =$

19. Eliaza Tiliyani spent $4\frac{1}{3}$ hours cycling, $\frac{1}{2}$ hour walking, and $\frac{1}{4}$ hour resting when going on a journey. How many hours did it take to cover the journey?

20. Violet's dress material needs 3½ m cotton, ¼m of silk and $\frac{1}{8}$ m of lining. How many metres of material does Violet's dress take?

21. Amina spent 3½ hours working and then 2½ hours reading. How many hours did she spend working and reading?

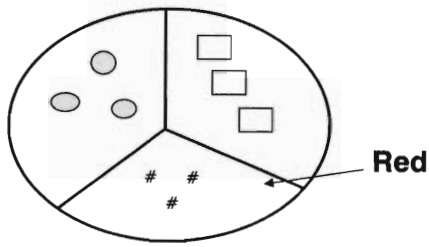
22. Add $\frac{7}{12}$ to the difference between $4\frac{2}{9}$ and $3\frac{5}{6}$

23. The area of a wall is $8\frac{1}{9}$ square metres. Harun begins to paint the wall. After painting $5\frac{1}{12}$ square metres he runs out of paint. How many square metres of wall remain unpainted?

24. I subtracted $1\frac{1}{9}$ from $6\frac{1}{6}$ and then added $\frac{5}{9}$. The teacher marked my answer right. What was my answer?

25. If I added 1½ litres of water to $3\frac{1}{6}$ of milk and give $2\frac{1}{6}$ litres of mixture to my uncle, how many litres of mixture am I left with?

1.3 (d) Multiplication of fractions by fractions



How many parts has the circle been divided into?

How many shapes are there in each portion?

How many shapes are there altogether?

How many shapes are red?

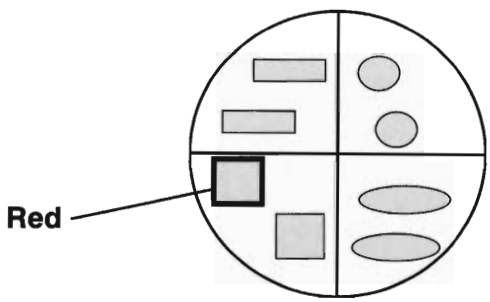
What fraction of the shapes are squares?

What fraction of the squares are red?

Exercise 26

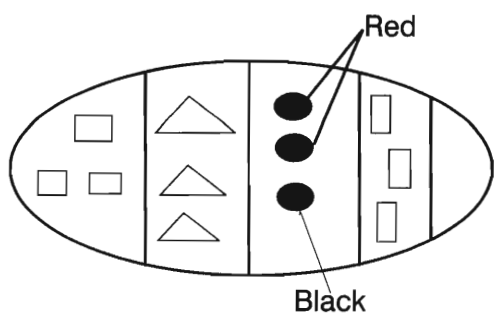
Use the diagrams below to answer the following questions.

i)



1. What fraction of the shapes are red?
2. What fraction of the shapes are rectangles?
3. What fraction of the squares are red?
4. What is ½ of ¼ ?

ii)



1. What fraction of the shapes are red?
2. What fraction of the shapes are rectangles?
3. What fraction of the squares are red?
4. What is $\frac{2}{3}$ of $\frac{1}{5}$?

iii) Work out the following:

1. $\frac{3}{4}$ of $\frac{1}{4}$ = **2.** $\frac{3}{8}$ of $\frac{1}{2}$ = **3.** $\frac{2}{3}$ of $\frac{1}{5}$ =

4. $\frac{3}{4}$ of $\frac{3}{4}$ = **5.** $\frac{1}{8}$ of $\frac{1}{4}$ = **6.** $\frac{1}{8}$ of $\frac{1}{8}$ =

7. $\frac{4}{7}$ of $\frac{2}{9}$ = **8.** $\frac{2}{7}$ of $\frac{1}{5}$ = **9.** $\frac{3}{10}$ of $\frac{1}{4}$ =

10. $\frac{5}{6}$ of $\frac{1}{3}$ =

Example
Deng bought ¾ m of cotton material. He used ½ of it to finish a dress he was making. What fraction of a metre of cotton material did he use?

Solution
The fraction of cotton material used was $\frac{1}{2}$ of $\frac{3}{4}$ metres.

$$\frac{1}{2} \text{ of } \frac{3}{4} = \frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4}$$

Cotton material used = $\frac{3}{8}$ metres

Exercise 27

Work out the following:

1. $\frac{7}{10} \times \frac{3}{8} =$ **2.** $\frac{3}{4} \times \frac{2}{5} =$ **3.** $\frac{3}{4} \times \frac{2}{7} =$

4. $\frac{7}{9} \times \frac{1}{3} =$ **5.** $\frac{4}{7} \times \frac{2}{5} =$ **6.** $\frac{2}{7} \times \frac{1}{9} =$

Find the products of the following pairs of numbers.

7. $\frac{2}{5}, \frac{3}{7}$ **8.** $\frac{3}{11}, \frac{2}{9}$ **9.** $\frac{5}{7}, \frac{2}{3}$ **10.** $\frac{8}{9}, \frac{1}{5}$

11. Akot had ¾ of a cake. The children ate ½ of it. What fraction of the cake did the children eat?

12. Kenyi bought $\frac{3}{8}$kg of fruit. He gave Lado $\frac{1}{3}$kg of what he bought. What fraction of the fruit did he give to Lado?

Multiplication of mixed numbers

The process is similar to the multiplication of proper fractions. First, you need to change mixed fractions into improper fractions.

For instance:

(i) $4\frac{1}{2} \times 2\frac{1}{4} = \frac{9}{2} \times \frac{9}{4} = \frac{81}{8} = 10\frac{1}{8}$

(ii) $1\frac{1}{3} \times 5\frac{2}{7} = \frac{4}{3} \times \frac{37}{7} = \frac{148}{21} = 7\frac{1}{2}$

Exercise 28

A. Work out the following:

1. $2\frac{1}{3} \times 3\frac{1}{2} =$
2. $6\frac{7}{8} \times 2\frac{2}{3} =$
3. $10\frac{5}{12} \times \frac{14}{15} =$
4. $1\frac{2}{3} \times 4\frac{1}{2} =$
5. $8\frac{2}{3} \times 4\frac{1}{2} =$
6. $\frac{5}{6} \times 5\frac{1}{4} =$
7. $5\frac{1}{4} \times 6\frac{3}{4} =$
8. $9\frac{1}{3} \times 4\frac{3}{4} =$
9. $10\frac{1}{5} \times 11\frac{2}{5} =$
10. $5\frac{2}{9} \times 7\frac{2}{3} =$
11. $8\frac{1}{2} \times 9\frac{1}{3} =$

Exercise 29

1. A rectangle measures $5\frac{1}{2}$cm by $7\frac{1}{7}$ cm. What is its area in cm^2?
2. A lorry uses $18\frac{2}{3}$ litres of diesel an hour. How many litres does it use in $\frac{3}{4}$hour?
3. Alfred lives $1\frac{2}{3}$km from school. How many km does he travel in 22 days?
4. If an orange weighs $\frac{1}{4}$of kg, how many kilogrammes will 40 similar oranges weigh?
5. Each radio manufactured by a firm weighs $2\frac{1}{4}$kg. What is the total weight of 150 radios manufactured by the firm?
6. What is the area of a floor of a house that measures $5\frac{1}{2}$m by $7\frac{1}{4}$m?
7. A rectangular field measures $3\frac{1}{3}$ha, by $2\frac{1}{4}$ha. What is its area in hectares?

Work out:

8. $\frac{3}{8} \times 24$
9. $\frac{1}{7} \times 56$
10. $1\frac{1}{2} \times 12$
11. $3\frac{3}{4} \times 128$
12. $1\frac{3}{8} \times 168$
13. $\frac{7}{9} \times 81$

1.3 (e) Division of fractions

Reciprocals

The reciprocal of a fraction is the interchanging of the numerator to become the denominator and the denominator to become the numerator.

For example:

The reciprocal of:

a) $\frac{1}{2} = \frac{2}{1}$ OR 2 b) $\frac{2}{3} = \frac{3}{2}$ OR $1\frac{1}{2}$

Division of a fraction by a fraction

Example 1

$\frac{1}{2} \div \frac{1}{4} =$

$\frac{1}{2} \times \frac{4}{1} =$ Therefore: $\frac{1}{2} \div \frac{1}{4} = 2$

Example 2

$\frac{7}{9} \div \frac{2}{3} = \frac{7}{9} \times \frac{3}{2} = \frac{7}{6} = 1\frac{1}{6}$

$\frac{3}{4} \div \frac{1}{4} = \frac{3}{4} \times \frac{4}{1} = 3$

Exercise 30a

1. $\frac{2}{3} \div \frac{1}{3} =$ 2. $\frac{1}{5} \div \frac{1}{6} =$ 3. $\frac{1}{6} \div \frac{1}{18} =$

4. $\frac{9}{11} \div \frac{3}{44} =$ 5. $\frac{17}{21} \div \frac{34}{63} =$ 6. $\frac{22}{39} \div \frac{11}{13} =$

7. $\frac{7}{12} \div \frac{7}{24} =$ 8. $\frac{22}{45} \div \frac{44}{15} =$ 9. $\frac{50}{23} \div \frac{25}{46} =$

10. $\frac{a}{b} \div \frac{a}{2b} =$ 11. $\frac{27}{56} \div \frac{9}{8} =$ 12. $\frac{8}{17} \div \frac{1}{34} =$

13. The length of a rope is $40\frac{7}{8}$ m long. It was cut into lengths of $\frac{1}{8}$m each. How many pieces of rope were cut?

14. A barrel contains $11\frac{2}{5}$ litres of alcohol. It was put into bottles holding $\frac{3}{5}$of a litre each. How many bottles were filled?

15. A bag of maize weighed $55\frac{1}{2}$kgs. It was to be divided into $1\frac{1}{2}$kilogramme containers. How many containers were used?

1.3 (e) (ii) Division involving mixed fractions

Division of mixed fractions is similar to the division of fractions that we have already discussed. The mixed fractions should be changed to improper fractions first before dividing them.

Example 1

$$5\frac{1}{3} \div 1\frac{1}{3} = \frac{16}{3} \div \frac{5}{3} = \frac{16}{3} \times \frac{3}{5} = \frac{16}{5} = 3\frac{1}{5}$$

Example 2

$$2\frac{3}{4} \div 1\frac{2}{9} = \frac{11}{4} \div \frac{11}{9} = \frac{11}{4} \times \frac{9}{11} = \frac{9}{4} = 2\frac{1}{4}$$

Exercise 30b

Find the values of the following:

1. $1\frac{1}{2} \div 2\frac{1}{2} =$ **2.** $2\frac{1}{4} \div 3\frac{1}{3} =$ **3.** $2\frac{3}{4} \div 1\frac{2}{9} =$

4. $10\frac{1}{9} \div 2\frac{1}{3} =$ **5.** $5\frac{4}{7} \div 4\frac{1}{3} =$ **6.** $8\frac{1}{6} \div 1\frac{1}{20} =$

7. $3\frac{3}{5} \div 2\frac{2}{5} =$ **8.** $14\frac{2}{5} \div 1\frac{1}{8} =$ **9.** $7\frac{1}{7} \div \frac{5}{7} =$

10. $5\frac{2}{5} \div 10\frac{1}{5} =$ **11.** $15\frac{1}{2} \div 3\frac{1}{3} =$ **12.** $9\frac{1}{3} \div 3\frac{1}{6} =$

13. A shopkeeper had $10\frac{4}{5}$ kg of salt. How many customers could be served with $1\frac{1}{5}$ kg of salt each?

14. How many bottles of $1\frac{1}{3}$ litres each can be filled from a jerrycan containing $13\frac{1}{2}$ litres of oil?

15. A patient requires $5\frac{1}{2}$ c.c. of malaria injection which is to be given $\frac{11}{18}$ c.c. at a time. How many times will the patient be injected? (c.c means cubic centimetre).

1.3 (f) Decimals:

You learned in level 2 that when adding decimals you must make sure the decimal points are vertically in line with each other.

Operations on decimals

1.3 (f) (i) Addition

Example 1

4.8+2.45 + 25.15

Solution

$$\begin{array}{r} 4.8 \\ 2.45 \\ +25.15 \\ \hline 32.40 \end{array}$$

Example 2

6.104 + 4.423 + 5.06 =

Solution

$$\begin{array}{r} 6.104 \\ 4.423 \\ +\ 5.060 \\ \hline 15.587 \end{array}$$

Exercise 31a

1. a) 4.38 + 2.62 b) 5.14 + 8.32 c) 4.71 + 2.83 d) 5.24 + 6.35

2. a)
$$\begin{array}{r} 32.41 \\ +8.03 \\ \hline \end{array}$$
b)
$$\begin{array}{r} 341.45 \\ +57.69 \\ \hline \end{array}$$
c)
$$\begin{array}{r} 63.201 \\ 21.685 \\ +3.832 \\ \hline \end{array}$$
d)
$$\begin{array}{r} 3246.1051 \\ 1762.4327 \\ +2105.1361 \\ \hline \end{array}$$

3. a) 3.621 + 4.01 + 13.32 =

b) 70.413 + 9.25 + 491.26 + 142.102 =

c) 5104.32 + 217321 + 1321.34 + 91.310 =

1.3 (f) (ii) Subtraction

Examples

i) 98.67 - 56.31 = 42.36

ii)
$$\begin{array}{r} 123.952 \\ -76.046 \\ \hline 47.905 \end{array}$$

iii)
$$\begin{array}{r} 236.291 \\ -24.720 \\ \hline 211.571 \end{array}$$

Exercise 31b

1. i) 8.6 - 6.7 ii) 6.9 - 3.5 iii) 2.1 - 1.2 iv) 75.34 - 50.12

2. i)
$$\begin{array}{r} 100.56 \\ -85.34 \\ \hline \\ \hline \end{array}$$

ii)
$$\begin{array}{r} 5.314 \\ -4.275 \\ \hline \\ \hline \end{array}$$

iii)
$$\begin{array}{r} 11.670 \\ -4.328 \\ \hline \\ \hline \end{array}$$

iv)
$$\begin{array}{r} 213.940 \\ -37.132 \\ \hline \\ \hline \end{array}$$

3. Lado is 1.8 m tall, Deng is 2.2m tall, Kenyi is 1.9 m tall and Brigit is 1.5 m tall. Find their total height.

4. A bar of soap measures 45.56 cm. If Kyila cuts a piece of length 10.37 cm for washing, what length of bar of soap remains?

5 Otwari's home is 3.5 km from school. How far does he have to travel every day from home to school and back?

1.3 (f)(iii) Multiplication of decimals

Multiplication by whole numbers

Example

i) $0.7 \times 8 = 5.6$ $\quad 0.7 = \frac{7}{10}$

$$\therefore \frac{7}{10} \times 8 = \frac{56}{10} = 5.6$$

ii) 4.51×7 $\quad 4.51 = \frac{451}{100}$

$$\frac{451}{100} \times 7 = \frac{3157}{100}$$

$$= \underline{31.57}$$

iii) 2.38×42 $\quad 2.38 = \frac{238}{100}$

$$\frac{238}{100} \times 42 = \frac{9996}{100}$$

$$= \underline{99.96}$$

Multiplication of decimals by 10

Multiplication by multiples of 10 is easy. You need only to move the decimal point to the right, according to the number of zeros in the multiples of 10.
Look at examples (i), (ii), (iii).

i) $0.9 \times 10 = 9$	Decimal point moves 1 place to the right.
ii) $21.34 \times 100 = 2134$	Decimal point moves 2 place to the right.
iii) $35.642 \times 1000 = 35642$	Decimal point moves 3 place to the right.

Exercise 32a

1. a) $\begin{array}{r} 0.8 \\ \times\ 10 \\ \hline \end{array}$ b) $\begin{array}{r} 0.7 \\ \times\ 100 \\ \hline \end{array}$ c) $\begin{array}{r} 0.25 \\ \times\ 10 \\ \hline \end{array}$ d) $\begin{array}{r} 0.634 \\ \times\ 100 \\ \hline \end{array}$

2. a) 0.200 x 100 = b) 2.575 x 1000 c) 12.5 x 10

d) 9.69 x 100 = e) 0.75 x 1000 f) 0.528 x 100

Multiplication of a decimal by a decimal

Example

i) 3.2 x 4.2

$$\begin{array}{r} 3.2 \\ \times\ 4.2 \\ \hline 64 \\ 1280 \\ \hline 13.44 \end{array}$$

ii) 1.62 x 5.34

$$\begin{array}{r} 1.62 \\ \times\ 5.34 \\ \hline 648 \\ 4860 \\ 81000 \\ \hline 8.6508 \end{array}$$

Exercise 32b

1. Work out the following.

a) 1.1 x 1.2 b) 2.3 x 6.1 c) 3.6 x 8.5

d) 72.1 x 3.14 e) 18.13 x 15.61 f) 14.1 x 1.6

g) 9.615 x 4.8 h) 0.0024 x 1.2

2. a) $\begin{array}{r} 156.75 \\ \times\ 2.42 \\ \hline \end{array}$ b) $\begin{array}{r} 276.204 \\ \times\ 6.1 \\ \hline \end{array}$ c) $\begin{array}{r} 178.23 \\ \times\ 0.8 \\ \hline \end{array}$ d) $\begin{array}{r} 0.052 \\ \times\ 0.2 \\ \hline \end{array}$

3. Which one is greater?

a) 3.12 x 1.2 **or** 12.14 x 1.6?

b) 12.1 x 0.5 **or** 13.3 x 0.4?

c) 25.2 x 0.9 **or** 16.25 x 0.7?

d) 25.2 x 0.3 **or** 28.1 x 0.2?

4. Nako's cow produces 10.25 litres of milk every day. How many litres of milk will the cow produce in one week?

5. John's family consumes 1.5kg of meat every day. How many kilograms of meat will they consume in 35 days?

1.3 (g) (iv) Division of decimals

Division by whole numbers

Division by whole numbers is done just like division, which you are already familiar with. Remember to keep the decimal point in place.

Examples

(i) Divide 0.9 by 5

Solution

$$5\overline{)0.9} = 0.18$$

5
40
40

Ans. 0.18

(ii) Divide 13.50 by 3

$$3\overline{)13.50} = 4.50$$

12
15
15
00

Ans. 4.50

(ii) 17.642 ÷ 2

$$2\overline{)17.642} = 8.821$$

16
16
16
4
4
2
2
0

Ans. 8.821

Exercise 33a

1. Work out the following:

(a) 7.15 ÷ 5

(b) 11.42 ÷ 2

(c) 135.6455 ÷ 3

(d) 649.14 ÷ 6

(e) 284.13 ÷ 9

(d) 654.29 ÷ 13

(g) 74.120 ÷ 8

(h) 22.41 ÷ 3

Division by Multiples of 10

Examples

(a) 135.64 ÷ 10 = 13.564	Decimal point moves 1 place to the left.
(b) 284.3 ÷ 100 = 2.843	Decimal point moves 2 place to the left.
(c) 1134.64 ÷ 1000 = 1.13464	Decimal point moves 3 place to the left, and so on.

Exercise 33b

Work out the following:

a) 1.5 ÷ 10
b) 30.6 ÷ 10
c) 113.14 ÷100
d) 0.445 ÷100
e) 124.56 ÷ 1000
f) 0.4567 ÷1000

Division of Decimals by Decimals

Examples

a) 1.54 ÷ 1.1 Change the divisor into a whole number by multiplying it by 10.
1.1 x 10 = 11

Similarly, multiply the number to be divided by 10.
1.54 x 10 = 15.4 15.4 ÷ 11

Next, we carry out the division.

$$
\begin{array}{r}
1.4 \\
11\overline{)15.4} \\
\underline{11} \\
44 \\
\underline{44}
\end{array}
\qquad = \underline{1.4}
$$

b) 0.625 ÷ 0.25 Change divisor into a whole number. 0.25 x 100 =25

Multiply the number to be divided by 100 as well 0.625 x 100 = 62.5

62.5 ÷ 25

$$
\begin{array}{r}
2.5 \\
25\overline{)62.5} \\
\underline{50} \\
25 \\
\underline{25}
\end{array}
\qquad = \underline{2.5}
$$

Exercise 34

Work out the following:

(a) $2.5 \div 0.5 =$ (b) $7.5 \div 2.5 =$ (c) $1.25 \div 0.5 =$

(d) $0.625 \div 0.25 =$ (e) $0.50 \div 0.20 =$ (f) $2.46 \div 0.123 =$

(g) $0.002 \div 0.01$ (h) $3.80 \div 1.9 =$ (i) $2.9 \div 1.8 =$

Mixed exercise 35

1. Write as decimals:

a) $\frac{7}{10}$ b) $\frac{7}{100}$ c) $\frac{7}{1000}$ d) $\frac{64}{100}$ e) $\frac{125}{1000}$

e) $\frac{227}{100}$ f) $\frac{27}{10}$ g) $\frac{99}{10}$ h) $\frac{491}{100}$

2. Write each of the following as fractions:

a) 0.9 b) 0.09 c) 2.3 d) 0.125

e) 2.25 f) 0.001 g) 5.07 h) 30.65

3. Write each of the following as decimals:

a) $\frac{2}{5}$ b) $\frac{1}{4}$ c) $\frac{1}{8}$ d) $\frac{4}{5}$ e) $\frac{12}{25}$ f) $\frac{3}{8}$

g) $\frac{3}{4}$ h) $\frac{1}{2}$ i) $\frac{49}{50}$ j) $2\frac{2}{5}$ k) $3\frac{3}{4}$

Introduction of the signs greater (>) and less than (<)and equal to (=)

Examples

(i) $\frac{1}{2} > \frac{1}{4}$ (ii) $0.6 < 0.9$ (iii) $\frac{1}{3} > \frac{1}{5}$ (iv) $\frac{1}{4} = 0.25$

4. Make the following statements true by using >; < or =

a) $\frac{1}{2}$ ___0.5 b) $\frac{3}{4}$ ___0.25 c) $\frac{81}{100}$ ___0.25 d) $\frac{909}{100}$ ___9.09

e) $\frac{155}{10}$ ___1.5 f) $\frac{22}{100}$ ___0.22 g) $\frac{45}{10}$ ___0.46

5. Find the values of the following:

(a) 25.21 x 10 (b) 6.25 x 100 (c) 1.25 x 8

(d) 6.71 x 9 (e) 25.12 x 5.1 (f) 7.31 x 4.15

(g) 0.0625 x 3.14 (h) 34.71 x 5.18

6. a) 24.18 x 2 b) 18.36 x 3 c) 39.13 x 13

d) 10.25 x 5 e) 72.1 x 10 f) $\frac{125.26}{100}$

g) $\frac{45.46}{1000}$ h) $\frac{62.36}{100}$ l) $\frac{467}{10000}$

j) 1.25 x 0.5 k) 7.5 x 0.75 l) 0.80 x 0.01 m) 6.6 x 3.3

7. A driver bought 4.09 litres of diesel and 0.91 litres of engine oil. How much did he buy altogether?

8. Taban has to travel a total of 25.08 kilometres. If he cycled 15.25km, how far did he have to walk?

9. Meat costs £s 125.5 per kg. How many kilogrammes will you buy with £s 50.20?

10. The length of an exercise book is 11.2cm. What is the total length of 150 exercise books?

Rounding off decimals

Examples

1. Rounding to 1 Decimal Place

I) Rounding off 2.76 to 1 decimal place becomes 2.8. i.e The second decimal place is 6, which is greater than 5. So we increase the first decimal by 1 to make 8, hence 2.76 = 2.8 (1dp)

ii) Rounding to 2 decimal places (2dp)

3.387 to 2 decimal places becomes 3.39. i.e The third decimal place is 7, which is greater than 5. We increase the second decimal by 1 to make 9.

Hence 3.387 = 3.39 (2 d.p)

iii) Rounding off to 3 decimal places (3dp)

8.6508 becomes 8.651 (3 d.p)

3.7854 become 3.785 (3 d.p)

Exercise 36

i) Write to one decimal place

a) 98.67 b) 56.31 c) 63.16 d) 27.24 e) 143.53

ii) Write to two decimal places

a) 63.201 b) 21.687 c) 3.832

d) 76.046 e) 1762. 439 f) 143 .8675

1.4 (a) The percentage of Numbers

Definition:

(a) A percentage is a fraction with the denominator 100.

$\frac{35}{100}$ is written as 35% and read as thirty-five percent.

Some fractions are easily written as percentages by changing them to equivalent fractions with 100.

$\frac{4}{5} = \frac{80}{100} = 80\%$

(b) The symbol % means "out of 100". It is read as "per cent". If we want to express 20 out of a hundred, we write 20%. An expression such as 25% is called a percentage.

1.4 (b) Conversion of fractions and decimals into percentage and vice versa

Examples:

(i) $30\% = \frac{30}{100}$

(ii) $45\% = \frac{45}{100}$

(iii) $\frac{1}{4} = \frac{25}{100} = 25\%$

(vi) $\frac{3}{4} = \frac{75}{100} = 75\%$

(v) $0.6 = \frac{6}{10} = \frac{60}{100} = 60\%$

Exercise 37

1. Express each of these fractions as percentage:

(a) $\frac{1}{2}$ (b) $\frac{14}{50}$ (c) $\frac{3}{4}$ (d) $\frac{9}{10}$

(e) $\frac{4}{5}$ (f) $\frac{3}{100}$ (g) $\frac{17}{20}$ (h) $\frac{24}{25}$

2. For each of the following, write the first number as a percentage of the second number e.g.

Examples

a) $\frac{2}{4} \times 100\% = \frac{200}{4} = 50\%$

b) $\frac{20}{60} \times 100\% = \frac{200}{6} = 33\frac{1}{3}\%$

(c) 25/50 (d) 60/30 (e) 100/250

(f) 40/25 (g) 25/75 (h) 79/100

3. Express the following percentages as fractions (Do not simplify):

(a) 70% (b) 45% (c) 5% (d) 22%

(e) 33% (f) 22% (g) 75% (h) 91%

4. Express the following decimals as percentages:

(a) 0.27 (b) 0.69 (c) 0.3 (d) 0.55

(e) 0.5 (f) 0.02 (g) 0.15 (h) 0.22

(b) Problems in Percentages

Example: There are 48 cows on a farm. 16 are white and the rest are black. What percentage of the cows are black?

Solution: No. of white cows: 16
No. of black cows: 32

The fraction of the cows which are black $\frac{32}{48}$

Write the fraction as a percentage:

$$\frac{32}{48} \times 100 = \frac{\cancel{32}^{\,2}}{\cancel{48}_{\,12}} \times \frac{\cancel{100}^{\,25}}{1} = \frac{800}{12} = 66\frac{2}{3}\%$$

Exercise 38

1. In a class of 40 pupils, 75 percent passed an examination.
   (a) What percentage of the pupils failed?
   (b) How many pupils failed?
   (c) How many pupils passed?

2. In a school, 30 percent of the pupils had measles. The school had 600 pupils.
   (a) What percentage did not have measles?
   (b) How many pupils did not have measles?
   (c) How many pupils had measles?

3. 49 percent of the babies born in 1989 were girls. What percentage were boys?

4. In a school, 36 percent of the pupils eat school meals. What percentage do not eat school meals?

5. 64 percent of the pupils in a mixed school are boys. What percentage are girls?

6. 400 pupils took an examination. 240 passed.
   (a) What percentage passed?
   (b) What percentage failed?

7. Garang had 200 sheep on his farm. 40 of them died.
   (a) What percentage died?
   (b) What percentage did not die?

8. 40 percent of the passengers in a train were women. The train carried 200 passengers. 20 of them were children.

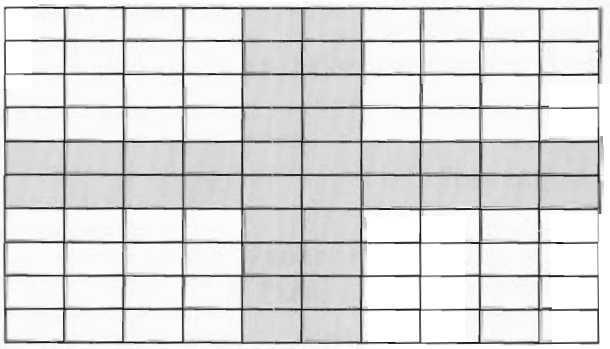
(a) How many men were there?
(b) What percentage of the passengers were men?
(c) How many women were there?

9. Express 21% + 32% + 22% as a fraction.

10. Find
(a) 30% of 30 (b) 73% of 100 (c) 50% of 50
(d) 0.25 of 60 (e) 2% of 1km (f) 70% of 800 (g) 25% of 72m

11. A company paid 30% of its profits in taxes. How much profit did the company make if it paid L.s.21,000 in taxes?

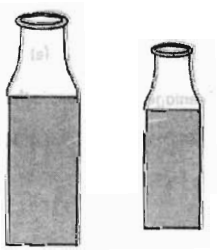
12. What percentage of the squares is:
(a) shaded (b) unshaded



1.5 Ratio and Proportion

1.5 (a) Ratio as way of comparing quantities

Example 1:
Below are two containers of milk.



The smaller container contains 6 litres, the bigger container contains 9 litres. What is the ratio of the smaller container to the bigger one?

The ratio of the smaller container to the big one is 6 litres to 9 litres.
In short it is written as 6:9

To write 6:9 in its simplest form we divide 6 and 9 by their greatest common divisor. The greatest common divisor for 6 and 9 is 3.

Therefore when we divide 6:9 by 3 we get 2:3.

Exercise 39: Express the following ratios in their simplest forms.

a) 24:4 b) 15:45 c) 9:54 d) 30:6 e)75:45

f) 12:36 g) 125:25 h) 100:120 i) 240:00 j) 22:242

1.5 (b) Writing Ratio in the form a : b

Example 1

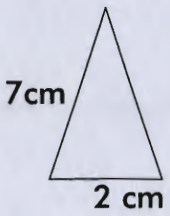
In a class there are 15 girls and 18 boys. What is the ratio of girls to boys? The comparison is 15 girls to 18 boys:

15 : 18 = 5:6

Example 2

The length of a triangle is 7cm and the width is 2cm. What is the ratio of the width to the length?

The comparison is 2cm width to 7cm length.
This can be written as 2:7.



Use the table to calculate the simplest ratio of:

	tall	short	fat	thin
Boys	4	6	3	5
Girls	4	8	6	3

a) The number of short boys to the number of short girls.
b) The number of tall girls to the number of thin boys.
c) The number of fat girls to the number of tall boys.
d) The number of thin girls to the number of fat boys.
e) The number of short girls to the number of tall boys.
f) The number of thin boys to the number of short girls.
h) The number of short girls to the number of thin boys.

1.5 (c) Ratio expressed as fractions, decimals or percentage or a whole

Examples

In a school the ratio of the number of girls to the number of boys is 2:3.

(i) Express

a) This ratio as a fraction $= \frac{2}{3}$

b) The ratio of the number of girls to the total, 2:3 adds up to a total of 5.

$\therefore$ The ratio of the number of girls to the total $= \frac{2}{5}$

c) The ratio of the number of boys to the total $= \frac{3}{5}$

(ii) Express

a) The number of girls as a decimal of the total $= \frac{2}{5} = 0.4$

b) The number of boys as a decimal of the total $= \frac{3}{5} = 0.6$

(Iii) Express

a) The number of girls as a percentage of the total.

$\frac{2}{5} \times 100 = 40\%$

b) The number of boys as a percentage of the total.

$\frac{3}{5} \times 100 = 60\%$

Exercise 40

1. In a medical team that was sent out from Yambio to the villages to vaccinate children against polio, the ratio of the doctors to the nurses was 1:4.

   (i) Express
   a) This ratio as a fraction.
   b) The ratio of the number of doctors to the total.
   c) The ratio of the number of nurses to the total.

   (ii) Express
   a) This ratio of the number of doctors as a decimal to the total.
   b) The ratio of the number of nurses as a decimal to the total.

   (iii) Express
   a) The ratio of the number of doctors as a percentage to the total.
   b) The ratio of the number of nurses as a percentage to the total.

1.5 (d) Sharing a number of items in a given ratio

Example:

Rauf has 560 animals, both goats and cows. If the ratio of the number of goats to the number of cows is 2:5,
a) How many goats does Rauf own?
b) How many cows does he have?

Solution

a) The total number of animals is 560. The ratio of the number of goats to the total is $\frac{2}{7}$

$\therefore$ Number of goats $= \frac{2}{7} \times 560 = 160$ goats

b) The ratio of the number of the cows to the total $= \frac{5}{7}$

$\therefore$ Number of cows $= \frac{5}{7} \times 560 = 400$ cows

Solve the following problems:

1. Deng, Nyuon and Chud inherit 1800 of their father's cattle. If the father told them to divide the animals in the ratio 1:2:3, how many animals does each of the brothers receive?

2. £s2500 is divided between Alice and Rebecca in the ratio of 2/3. How much does each get?

3. The gear ratio on a bike is 3:1. How many times must the pedals turn in order to turn the wheels 207 times?

4. The ratio of spectators to police at League football matches is 400:1. How many police are required for a gate of 32,000 spectators?

5. Army rules require that on exercises the ratio of officers to men must be 1 to 20. How many officers are there if there are 420 personnel altogether on an exercise?

1.5 (e) Proportions

A proportion is a mathematical statement of an equation which tells us that two ratios are equal.

Example 1

i) $\frac{2}{4} = \frac{4}{10}$ ii) $\frac{7}{10} = \frac{70}{100}$ iii) $\frac{1}{7} = \frac{2}{14}$

Exercise 41

For the following proportions write true or false

1. $\frac{3}{5} = \frac{9}{15}$ 2. $\frac{1}{2} = \frac{4}{10}$ 3. $\frac{1}{3} = \frac{12}{36}$ 4. $\frac{5}{4} = \frac{20}{15}$

5. $\frac{5}{6} = \frac{10}{12}$ 6. $\frac{3}{2} = \frac{3}{4}$ 7. $\frac{4}{3} = \frac{2}{5}$ 8. $\frac{3}{100} = \frac{2}{75}$

9. $\frac{8}{12} = \frac{3}{12}$ 10. $\frac{3}{2} = \frac{3}{4}$ 11. $\frac{4}{3} = \frac{2}{5}$ 12. $\frac{3}{100} = \frac{2}{75}$

13. $\frac{8}{12} = \frac{3}{4}$ 14. $\frac{4}{5} = \frac{20}{25}$ 15. $\frac{11}{12} = \frac{7}{8}$ 16. $\frac{3}{18} = \frac{2}{12}$

1.5 (f) Solving problems involving direct and indirect proportions using the unitary method

Examples

i) A lorry covers a distance of 215km in 3 hours. How long will it take the same lorry to cover 344km?

Solution

Let x = the number of hours it takes to travel 344km.

$$\frac{\text{Lesser number of hours}}{\text{Greater number of hours}} = \frac{\text{Lesser distance}}{\text{Greater distance}}$$

$$\frac{3}{x} = \frac{215}{344} = x \times 215 = 3 \times 344 \quad 215x = 1{,}032$$

$$\frac{215x}{215} = \frac{1032}{215}$$ Divide each side by 215 in order to remain with x.

$$x = \frac{1032}{215}$$ To get the value of x divide

$$\begin{array}{r} 4.8 \\ 215 \overline{)\,1032} \\ \underline{860} \\ 1720 \\ \underline{1720} \end{array}$$

$\therefore$ x = 4.8 hrs = 4 hrs 48 mins

ii) A piece of rope a hundred metres long has a mass of eight kilograms. What is the mass of forty metres of rope?

Let the mass of 40kg = $x$kg

$$\frac{\text{Lesser Length}}{\text{Greater mass}} = \frac{\text{Lesser mass}}{\text{Greater mass}}$$

$$\frac{40}{100} \times \frac{x}{8} = 100 \times x = 40 \times 8 = \frac{100x}{100} = \frac{320}{100}$$

$$x = \frac{320}{100} = x = 3.2\text{kg}$$

Exercise 42

Write the equations of the following proportions and solve them.

1. A car travels 100 kilometres in 3 hrs. How many hours will it take to cover 500km?
2. Violet Api earns £s 150 a week. How many weeks will she take to earn £s 36,000?
3. Two chapters of a book contain 60 pages. How many pages will 24 chapters of a similar book contain?
4. A farmer digs 3 acres in 5 days. How many acres will he dig in 25 days if he works at the same rate?
5. Karama takes 8 hours to earn 16 dollars. How many hours will she take to earn 40 dollars?
6. If it takes 10 hours to wash 8 cars, how many hours will it take to wash 40 cars of the same make?

Proportion and Percentage

When a fraction has its denominator equal to 100, it is called a percentage e.g.

(i) $\frac{25}{100} = 25\%$

(ii) $\frac{4}{100} = 4\%$

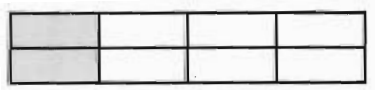
(iii) $\frac{21}{100} = 21\%$

(vi) $\frac{3}{100} = 3\%$

Proportions can also be written as percentages. For instance:

Example 1

The shaded region



$\frac{2}{\cancel{8}_{4}} \times \cancel{100} = 25\%$

Example 2

$$\frac{\cancel{4}^{2}}{\cancel{8}_{2}} \times \frac{50}{\cancel{100}} = 50\%$$

Example 3

In a recent election of school prefects 75% of the pupils at Pageri Primary School voted. There are 1200 pupils in the school. How many pupils voted?

Solution

100% of pupils = 1200 pupils

1% of pupils = x 75

Then 75% of pupils who voted = x 75 = 900 pupils

Exercise 43

B. Write a proportion, then work it out.

1. 25% of 40 = x
2. 21 is 30% of b
3. 15 is 20% of a
4. 7 is 0.5% of 3
5. 8% of 7200 is p
6. r% of 92 = 69
7. 40% of 35 = y
8. 250 is y% of 100
9. x is 25% of 36
10. 112 is $87\frac{1}{2}$% of x

Revision Exercise

1. Write the following figures in words

   (a) 3704345, (b) 348,095 (c) 9053103

2. Write in figures
   (a) two million and twenty
   (b) ten million, five hundred and forty two thousand and three.

3. Add or Subtract

   (a) 4271389 (b) 9648324
   +2348615 -2732618

(4) (a) 121 x 212 =

   (b) 6729 ÷ 32 =

(5) Okello needs to build a new chicken house. He has 3840 birds and each house need 256 birds. How many can he build?

(6) i) Write in Arabic numerals

   (a) XX (b) CCXCV (c) CDL

   ii) Write in Roman numerals

   (a) 322 (b) 444 (c) 50

(7) Work out

   (a) 0.6 x 0.7 (b) 0.07 x 0.2 (c) 9.12 ÷ 0.4

(8) List the multiples of 4 which are between 10 and 42.

(9) Aroma digs 1 of her garden each day. What portion will she dig in

   (a) three days (b) five days

(10) Keji had 210 oranges in her basket and sold 40 % of them. How many oranges did she sell?

UNIT 2 Measurement

2.1 Length

2.1 (a) Conversion of mm into cm, cm into m, m into km and vice versa

Example

Change the following meters into centimeters:

i) 5m ii) 10 m iii) 1005 m

Solution

We know that 1 m = 100cm. Therefore,

i) 5 m = 5x100 cm = 5000 cm

ii) 10 m = 10 x 100cm = 1,000 cm

iii) 1005 m = 1005 m x 100cm = 100,500 cm

Exercise 1a

Convert the following metres into centimetres:

(i) 502.3m (ii) 1003m (iii) 10.5m

(iv) 703m (v) 2500m (vi) 165.9m

(vii) 0.82m (viii) 2.85m

To change cm to m, we divide the cm by 100 as follows:

We know that,

1) 100 cm = 1 m

$1 \text{ cm} = \frac{1}{100} \text{ m}$

Therefore

$200\text{cm} = 200 \times \frac{1}{100}\text{m} = 2 \text{ m}.$

2) We know that;
100 cm = 1 m
Therefore

$$500 \text{ cm} = 500 \times \frac{1}{100} \text{ m} = 5\text{m}$$

3) We know that,
100 cm = 1 m,
Therefore

$$1000 \text{ cm} = 1000 \times \frac{1}{100} \text{m} = 10\text{m}$$

There is another method of converting centimeters into metres. That is, we know that 100 cm = 1 m; therefore, to find how many metres there are in a given number of centimeters we divide the centimeters by 100.

Example

$$200 \text{ cm} = \frac{200}{100} = 2 \text{ m}$$

Exercise 1b

b) Convert the following cm into metres:

i) 200 cm ii) 500 cm iii) 1000 cm iv) 2,000 cm

2.1 (a) (i) Identifying millimeters as a unit of length

A millimeter is the smallest unit of measuring length. For example, a ruler is marked in millimeters and centimeters. There are 10 divisions between zero and 1 cm. The 10 divisions are millimeters. That is 10 millimeters = one centimeter.

Example

Convert the following millimeters into centimeters:

1) 100 mm 2) 750 mm 3) 130 mm

Solution

1) We know that

$10\ mm = 1\ cm$, then $1\ mm = \frac{1}{10}\ cm$

Therefore $100\ mm = 100 \times \frac{1}{10}\ cm = 10\ cm$

2) We know that $1\ mm = \frac{1}{10}\ cm$

Then $750\ mm = 750 \times \frac{1}{10}\ cm = 75\ cm$

3) We know that $1\ mm = \frac{1}{10}\ cm$

Then $130\ mm = 130 \times \frac{1}{10}\ cm = 13\ cm$

Exercise 1c

Convert the following mm into cm.

1) 150 mm 2) 850 mm 3) 200mm 4) 60 mm 5) 120mm

Example

Convert the following centimeters into millimeter

1) 7 cm 2) 14 cm 3) 25 cm

Solution

1) We know that;
$1\ cm = 10\ mm$ Then $7\ cm = 7 \times 10\ mm = 70\ mm$

2) We know that $1\ cm = 10\ mm$ then $14\ cm = 14 \times 10\ mm = 140\ mm$

3) We know that $1\ cm = 10\ mm$, then $25 \times 10\ mm = 250\ mm$

Exercise 1b

Convert the following cm into millimeters

1) 60 cm 2) 23 cm 3) 4 cm 4) 18 cm 5) 9cm

2.1 (a) (ii) Recognizing km as unit for measuring distances

A km is a unit of measuring distance that is greater than a metre. It can be used to measure longer distances. For example the distance between Wau and Tonj is measured in kilometers. The distance from Wau to Tonj is about 90 km.

Relationship:

1 kilometer (km) = 1,000 metre (m)
1 kilometer (km) = 100,000 centimeter (cm)
1 kilometer (km) = 1,000,000 millimeter (mm)

Example

Converting kilometres into metres and vice versa

a) Convert the following kilometers into metres

1) 2 km 2) 5 km 3) ¾ km

Solution:

To change kilometers into metres, we multiply the kilometers by 1000 as shown below.

1. Since 1 km = 1,000 m then
   2 km = 1,000 x 2 m = 2,000 m
   5 km = 5 x 1000 m = 5000m

2. 5km = 5 x 1000 m = 5000m

3. ¾ km = ¾ x 1,000m = 3 x 250 m = 750 m

b) Convert the following metres into kilometers:

1) 3000 m 2) 4000 m 3) 500 m

Solution

Convert metres by 1000 as shown below.

1. Since 1000 m = 1 kilometre, then $1m = \frac{1}{1000}$ km

   Therefore 3000 m $= \frac{3000}{1000}$ km = 3 km

2. Since 1000 m = 1 km, Then 1 m $= \frac{1}{1000}$ km

   Therefore 4000 = 4000 x $\frac{4000}{1000}$ km = 4 km

3. Since 1000 m = 1 km Then 1 m $= \frac{1}{1000}$ km

   Therefore 500m $= \frac{1}{1000}$ x 500 km = 0.5 km

c) Convert the following metres into centimeters:

I) 3m 2) 5m 3) 8m

Solution

To convert metres to centimeters, we multiply the metres by 100 as shown below.

Since 1 m = 100 cm then

1) 3 m = 3 x 100 cm = 300 cm

2) 5m = 5 x 100cm = 500cm

3) 8 m = 8 x 100 cm = 800 cm

d) Convert the following centimeters into metres:

1) 300 cm 2) 600 cm 3) 1000 cm

Solution

To convert centimeters into metres we divide the centimeters by 100 as shown below.

Since 100 cm = 1 m, then 1 cm $= \frac{1}{100}$ m

Therefore

1) 300 cm $= \frac{1}{100} \times 300$ m = 3m

2) 600 cm $= \frac{1}{100} \times 600$m = 6 m

3) 1000 cm $= \frac{1}{100} \times 1000$ m = 10 m

Exercise 2

Convert the following:

1. Kilometers into metres

   i) 8 km ii) 150m iii) 9 km iv) 4 km v) 95,000 km.

2. Metres into centimeters

   i) 2m ii) 5m iii) 13 m iv) 50 m v) 100 m.

3. Centimeters into metres

   i) 600 cm ii) 10, 000 cm iii) 50 cm iv) 700cm v) 8,000 cm

10 mm = 1 cm	1 km = 1,000 m
100 cm = 1 m	1 km = 100,000 cm
1000 m = 1 km	1 km = 1,000,000 mm

2.1 (b) Operations Involving Centimetres, Metres and Kilometers

a) Addition

To carry addition of numbers involving the units metres and centimeters, we put the numbers with the units metres under m and those with units centimetres under cm as follows.

Example

Add 5m 30cm to 15m 85cm

Solution

Record 15cm in the column and carry 1m to m column.

m	*cm*
5	30
+15	85
21	15

When adding numbers involving the units km, m and cm, we start adding the numbers under centimeters. If the addition produces a number greater than 100, then we carry the hundredth place value number over to the column. This is because 100cm = 1m.

Example (ii): Add

2km	400m	55cm
+ 5km	200m	15cm
1km	500m	65cm

Arrange the addition as shown here:

km	m	cm
2	400	55
5	200	15
+1	500	65
9	101	35

55cm + 65cm + 15cm = 135cm = 1m 35cm
Write 35cm in the cm column and carry 1 meter to the meter column.
400m + 500m + 200m + 1m = 1101m = 1km 101m. Write 101m in meter column and carry 1km to km column.
1km *(that was carried)* + 2km + 5km + 1km = 9km

(b) **Subtraction:** To carry out subtraction, the numbers are arranged as follows

a.

	km	m	cm
	6	89	67
-	3	10	32
	3	79	35

b.

	km	m	cm
	8	678	39
-	5	593	12
	3	85	27

c.

	km	m	cm
	7	839	63
-	5	949	52
	1	890	11

d.

	km	m	cm
	14	8	2
-	9	15	7
	4	993	9

Borrowing meters to centimeters, you convert meters to centimeters and add. Then carry out the subtraction. Similarly, when borrowing kilometers to meters, convert the kilometers to meters and carry out the subtraction.

Exercise 3a

(a) 15km 30m 26cm + 20km 46m 12cm

(b) 19km 45m 7cm + 22km 34m 35cm

(c)

	km	m	cm
	39	42	53
-	18	44	32

(d)

	km	m	cm
	48	469	88
+	35	672	62

(e)

	km	m	cm
	41	340	65
+	82	280	34
	55	740	70

(f)

	km	m	cm
	10	47	18
–	7	58	24

(g)

	km	m	cm
	25	575	87
–	14	847	94

(h)

	km	m	cm
	8	726	42
–	5	937	68

(i)

	km	m	cm
	4	436	54
–	3	548	75

(j)

	km	m	cm
	7	834	32
–	5	958	45

(c) Multiplication

Example:
Work out the following:

(i) (6km 320m) x 5 (ii) (16km 8m 35cm) x 4

Solution:

(i) Arrange the numbers as shown below and then carry out multiplication as with whole numbers.
Note that 320 x 5 = 1600m = 1km 600m.
We write 600m under the meter column. Add the 1km to 6km x 5 to get (6km x 5 +1) = 31km.
Write 31 under the kilometer column.

km	m
6	320
x	5
31	600

(ii) Arrange the numbers as shown below and carry out the multiplication.

The procedure is as above

km	m	cm
16	8	35
x		4
64	33	30

Exercise 3b
Work out the following:

1. 3m 42cm x 8
2. 4m 32cm x 12
3. 28m 63cm x 9
4. 9km 14m 34cm x 5
5. 18km 42m 53cm x 6
6. 25km 415m 65cm x 8

7.

m	cm
25	57
x	8

8.

km	m	cm
49	15	340
x		8

(d) Division of numbers involving km, m and cm.

Example:

(i) 6m 50cm divide by 2
(ii) 8km 32m 68cm divide by 4
(iii) 36km 960m 73cm divide by 6
(iv) 45km 810m 63cm divide by 9

Solution:

Division of numbers involving km, m and cm is carried out in the same way as that with whole numbers as follows.

(i)
```
    3m 25 cm
  ---------
2 ) 6m 50cm
    6m
    0  50cm
       50cm
       00          = 3m 25 cm
```

Divide 6m by 2m, it goes 3m and no remainder.
Divide 50cm by 2 and it goes 25 and no remainder.

(ii)
```
     2km  8m 17cm
  ---------------
4 ) 8km 32m 68cm
    8k
    0
        32m
        32m
        00
            68cm
            68cm
            00        = 2km 8m 17cm
```

Divide the 8km by 4 and it goes 2km and no remainder.
Then continue dividing the rest as in example (i) above.

(iii)
```
      6km  160m 12cm
  -------------------
6 ) 36km  960m 72cm
    36krn
    00
          960m
          960m
          000
               72cm
               72cm
               00              = 6km 160m 12cm
```

(iv)
```
      5km   90m   7cm
   ┌──────────────────
 9 ) 45km  810m  63cm
     45km
     00
           810m
           810m
           000
                 63cm
                 63cm
                 00        = 5km 90m 7cm
```

Exercise 3c

Divide the following:

(i) 33m 78cm divide by 3

(ii) 96m 64cm divide by 8

(iii) 56km 700m 63cm divide by 7

(iv) 65km 915m 85cm divide by 5

(v) 108km 540m 72cm divide by 9

(vi) 96km 720m 48cm divide by 12

Word Problems

Example

Lakiko has 15 children, 8 girls and 7 boys. He needs to buy uniforms for them. The trader told Lokiko that a complete uniform for a girl will be 2 metres; Each boy will need 3 metres. How many metres does Lakiko need to buy for his children?

Solution

Length of cloth for the 8 girls = 8 x 2m = 16 m
Length of cloth for the 7 boys = 7x 3 m = 21 m

Total length of cloth Lokiko bought = 16m + 21 m = 37 m

Exercise 4

1. A contractor will supply 25 girl's uniforms and 16 boy's uniforms. A uniform for a girl takes 2 m and that for a boy 3m. How many metres of cloth are needed?

2 450 people are employed to clean the road linking two towns. If each person cleans 1 km and 550m, how far apart are the two towns?

3. A man planted mango trees in a row. Each tree is 15 m apart. If there are 631 mango trees, what is the distance between the first mango and the last mango tree in the row?

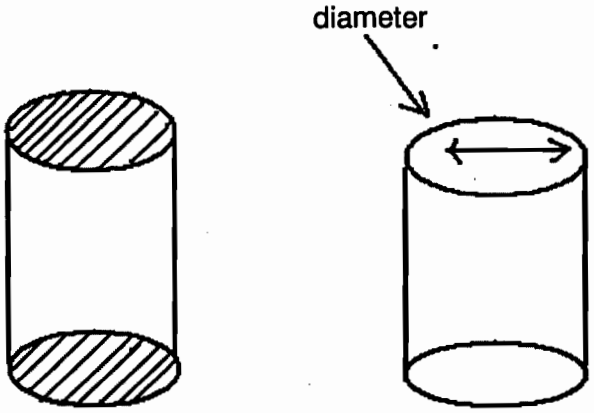
4. The department of power and lighting wants to supply a town with electricity. Electric poles are put 25 m apart. If the town is 62500 m away. how many poles are required?

2.1 (c) Measuring circumference, diameter and radius of a circle

The distance around a circular object is called its circumference.

To measure the circumference of a tin, follow the steps below.

1) Tie a string around the tin as shown below
2) Mark the tying points with a colored mark or chalk
3) Using a ruler, find the distance between the two marked points



The distance across a circular object is called the diameter.

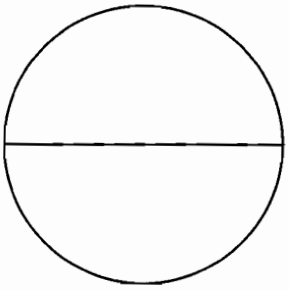
You can find the diameter of a circular tin by measuring with a ruler across the tin's top.

Activity

Using strings and rulers, find the circumference of the following objects.

Item	Circumference	Diameter
shoe polish container		
circular tin		
oil container		
tin lids		

Diameter and Radius of a Circle

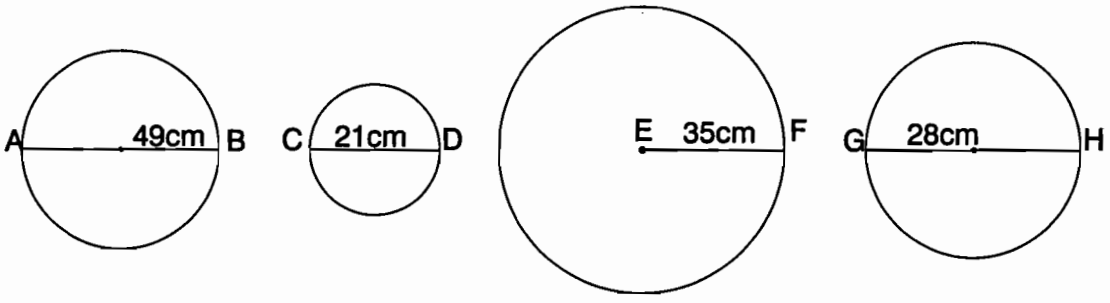


The diameter divides a circle into two equal parts. It passes through the centre of a circle. The radius is half of the diameter.

You can measure the diameter and radius of these circles using a ruler.

Exercise 4a

1. Find the diameter of the following circles in cm.



2. Find the radii of the above circles in cm.

Measuring Circumference

We use a number called pi ($\prod$) to measure the circumference of a circle.

$$\prod = 3.14 \text{ or } \frac{22}{7}$$

Circumference = $\prod$ x diameter

Circumference = $\prod$ x 2 radius

$C = \prod D$

$C = 2r$

Example:

Find the circumference of a circle whose diameter and radius are

1) diameter 7cm 2) radius 1.4m

Solution

1) Circumference = $\prod D$

$C = \frac{22}{7} \times 7 = 22\text{cm}$

2) Circumference = $\prod 2R$

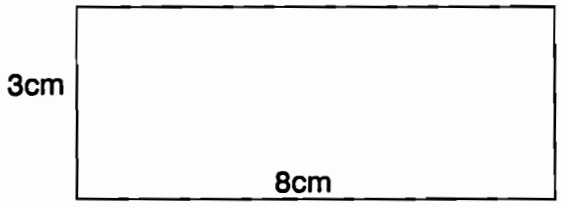
$C = \frac{22}{7} \times 14 \times 2 = \frac{22}{7} \times 28 = 88\text{m}$

Exercise 4b

1. Taking $\prod = \frac{22}{7}$ calculate the circumference of the circles in exercise 4(a).
2. Taking = 3.14 calculate the circumference of the circles whose radii are;
   a) 10 cm b) 40 cm c) 3 cm d) 2.2 cm
3. Calculate the diameter, if the circumference is
   a) 22 cm b) 66 cm Take $\prod = \frac{22}{7}$
4. Calculate the radii, if the circumference of a circle is
   a) 88 cm b) 121 cm Take $\prod = \frac{22}{7}$
5. The diameter of Nadi's bicycle wheel is 42 cm. Find the circumference of the wheel. Take $\prod = \frac{22}{7}$

2.2 Area

Example 1: Find the area of the rectangle below.

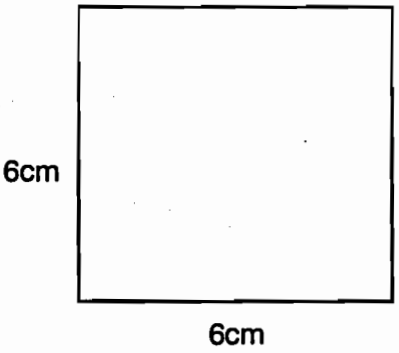


Area of a rectangle = Length x width

length = 8cm
width = 3cm

area = length x width
= 8cm x 3cm
= $24cm^2$

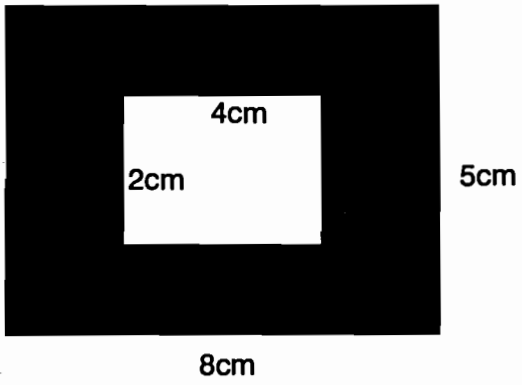
Example 2: Find the area of the square below.



Area of a square = side x side (or $side^2$)

sides = 6cm
area = 6cm x 6cm
= $36cm^2$

Example 3: What is the area of the shaded part of the figure below?



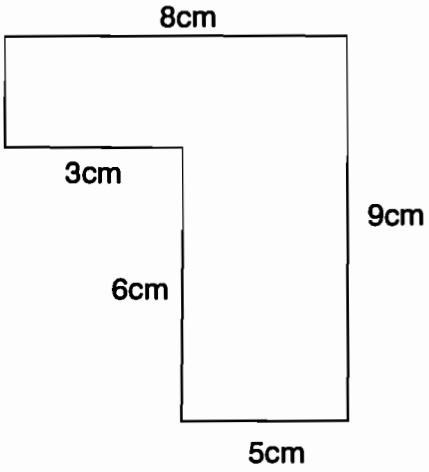
Area of a rectangle = length x width

(a) Area of big rectangle = 8cm x 5cm = $40cm^2$

(c) Area of the small rectangle = 4cm x 2cm = $8cm^2$

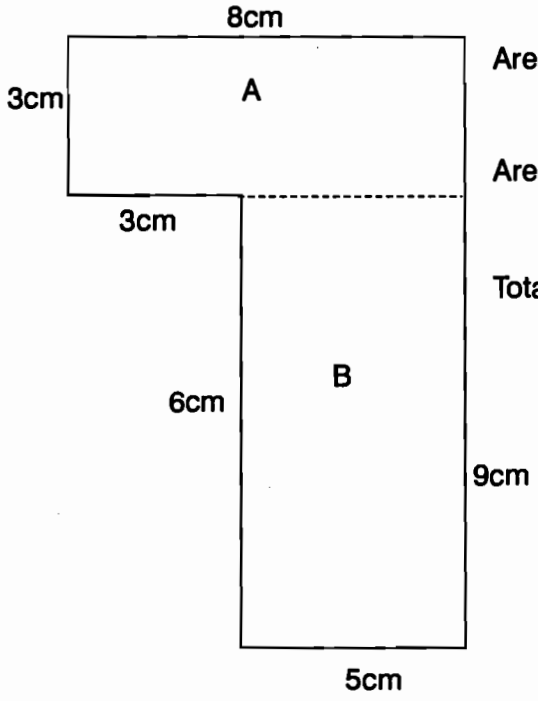
(c) Area of shaded part = area of big rectangle -
Area of small rectangle = $40cm^2 - 8cm^2$
Area of shaded part = $32\ cm^2$

Example 4: Find the area of the figure below:



There are two ways you can do this:

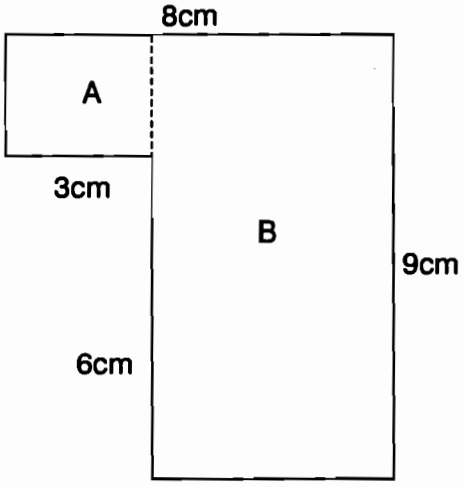
1. Divide the figure into two rectangles A and B as shown below:



Area of A = 8cm x 3cm
= 24cm^2

Area of B = 6cm x 5cm
= 30cm^2

Total Area=Area of A - B
= 24cm^2 + 30cm^2
= 54cm^2



2. Divide the figure into two: square A and rectangle B

Area of square A = 3cm x 3cm = 9cm^2
Area of rectangle B = 9cm x 5cm = 45cm^2
Total area of the figure = Area of A + Area of B
= 9cm^2 + 45cm^2
= 54cm^2

Exercise 5

1. Measure the length and the width of your classroom in metres and centimetres. Find the area.

2. Measure the length and the width of the blackboard in centimetres and find the area.

3. Measure the top of your desk and find the area.

4. Measure the length and the width of your Maths Book and find the area in cm^2.

5. Measure the length and the width of your ruler and find the area.

6. Measure the top of your teacher's table in cm and find the area.

7. Find in your village any garden which is about rectangular in shape. Measure the length and width. Find the area.

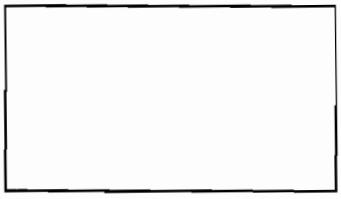
8. Measure the length and the width of an empty sack in cm and find the area.

9. Find any object at home or school which is square and find its area.

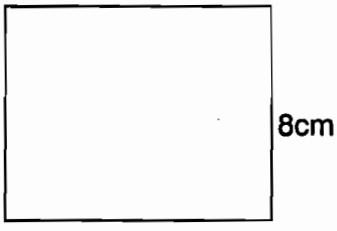
10. Find any object at home or school which is about rectangular in shape
    (i) Find the length
    (ii) Find the width
    (iii) Find the area.

11. Find the areas of the following figures:

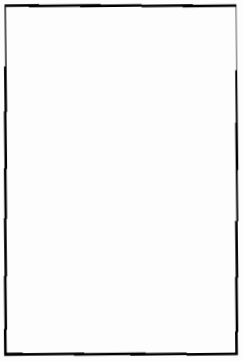
(a)



(b)



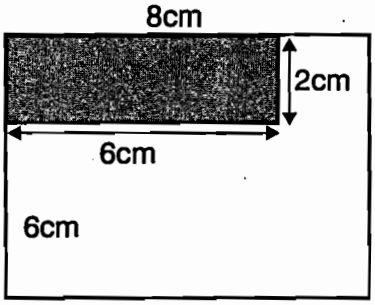
(c)

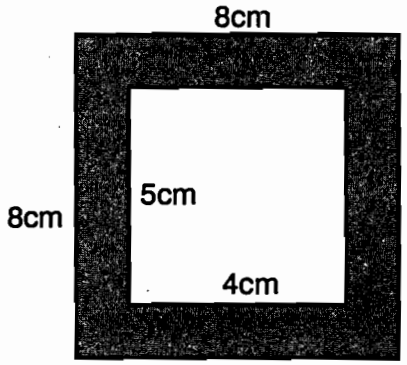


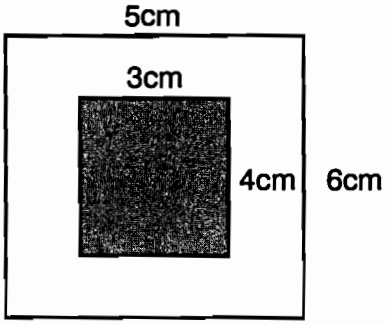
12. In each of the figures below:

(i) find the areas of the shaded parts

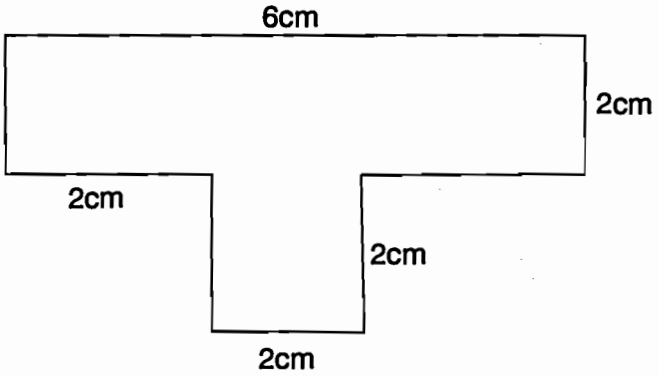
(ii) find the areas of th unshaded parts.

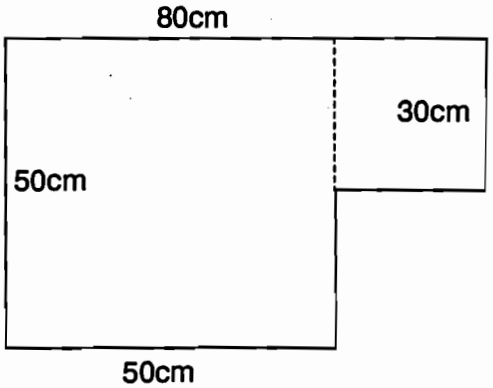


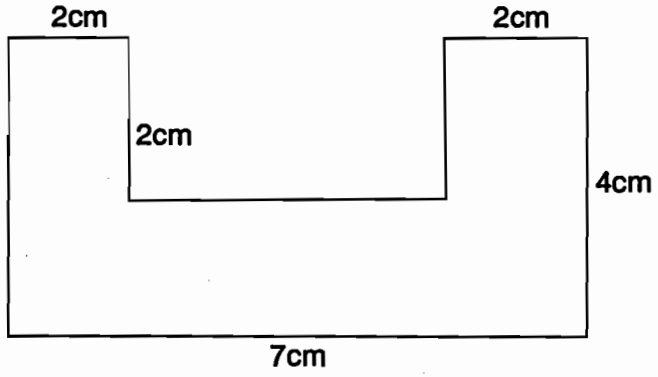


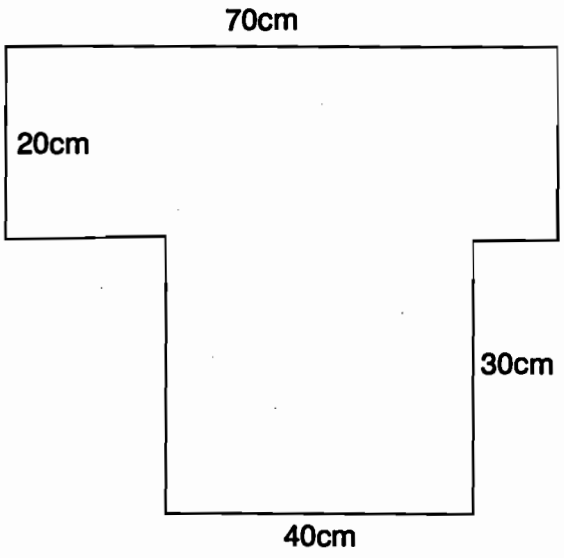


13. Find the areas of the following figures:









14. The area of a rectangular sheet of paper is 42cm^2. If the length is 7cm, what is the width of the sheet of paper?

15. A rectangular sheet of zinc is 3m long and 2.5m wide. What is the area of the zinc sheet in square metres?

16. The distance around (perimeter) a rectangular farm is 260m. If the width of the farm is 60m. What is the area of the farm?

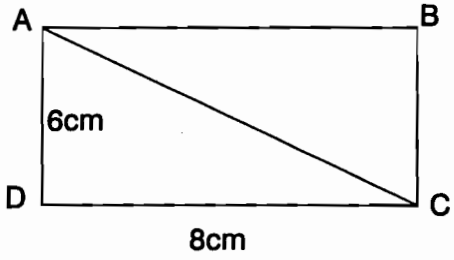
17. A room is 10 metres long, 8 metres wide and 5 metres high. What is the total surface area of the inner walls?

18. How many 30cm by 30cm files would be required to cover a surface 9m by 6m?

(b) Area of a right angled Triangle

You have learnt how to find the area of squares and rectangles. We are now going to find out about the area of a right angled triangle.

Example 1



ABCD is a rectangle. AC is a diagonal which divides the rectangle into two equal right triangles ABC and ACD.

Area of rectangle ABCD = Length x width

= 8cm x 6cm

= $48cm^2$

Area of right angled triangle = $\frac{1}{2}$ area of rectangle

The area of triangle ABC = area of rectangle $\frac{ABCD}{2}$

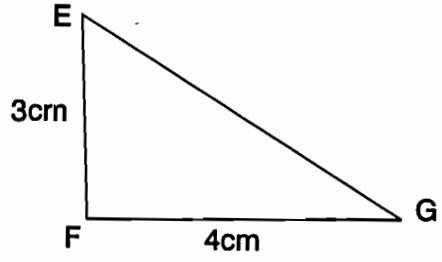
$= \frac{48cm^2}{2}$

$= 24cm^2$

Area of right angled triangle = $\frac{1}{2}$ length x width

Example 2

Find the area of right-angled triangle EFG



Area = $\frac{1}{2}$ x EF x FG

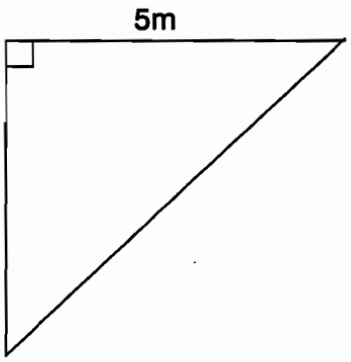
= $\frac{1}{2}$ x 3 x 4 = $\frac{12}{2}$ cm^2

Area of EFG = $6cm^2$

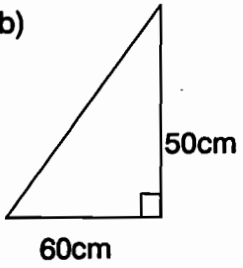
Exercise 6

1. Find the area of the following triangles:

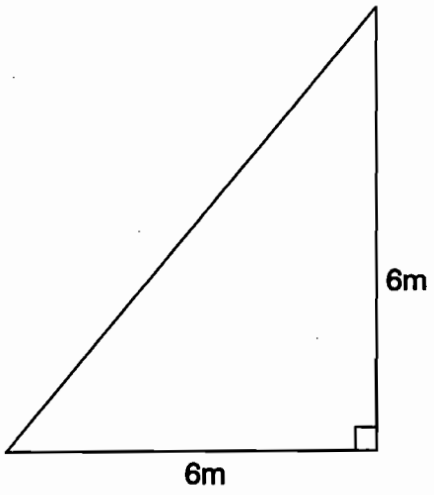
(a)



(b)



(c)



2. Calculate the area of these right angled triangles from the given sides:

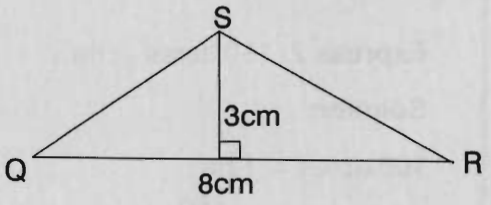
(a) 35cm and 40cm
(b) 7m and 6m
(c) 120cm and 100cm
(e) 3.5cm and 4cm
(d) $\frac{1}{4}$ m and $\frac{2}{5}$m

(c) Area of an Ordinary Triangle

Example:

Area of any triangle $= \frac{1}{2}$ base (or length) x height

Find the area of triangle QRS



Area $= \frac{1}{2}$(base x height)

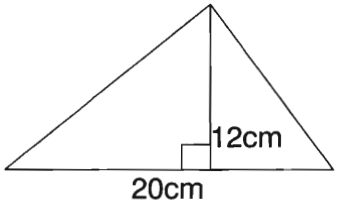
$= \frac{1}{2}$(8cm x3cm) $= \frac{1}{2} \times 24 = \frac{24}{2}$

Area of QRS = $12cm^2$

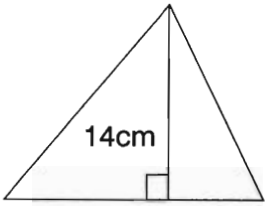
Exercise 7

1. Find the area of the following triangles:

(a)



(b)



2. The area of a triangle is $72cm^2$. If the height of the triangle is 8cm, what is the length of the base?

3. Find the area of the following triangles:
   (a) 4m base and 3m height
   (b) 35cm base and 60cm height
   (c) 7.5m base and 6m height
   (d) 7.5m base and 4m height

2.2 (b) Hectares and Acres as units of area

1 ha = 10,000 m^2

1 are = 100 m^2

1 ha = 100 acres

Express 1: 2.5 ha in acres

Solution

1ha = 100 acres

$\therefore$ 2.5ha = 100 x 2.5

= 250 acres

Express 2: 150 acres in ha

Solution

100 acres = 1 ha

$\therefore$ 150 acres $= \frac{150}{100}$

= 1.5ha

Exercise 8

1. Change the following hectares into acres.

i) $1\frac{1}{2}$ ha ii) 0.25 ha iii) 7 ha

iv) 3.55 ha v) 0.005 ha vi) $1\frac{3}{4}$ ha

2. Change the following into hectares.

i) 375 acres ii) 475 acres iii) $102\frac{1}{2}$ acres

iv) 225 acres v) 50 acres vi) 75 acres

2.2 (c) Expressing area in acres and hectares

The Hectare

Farmers and agriculturists usually measure the area of their farmlands in hectares, which is a metric unit. However, in Sudan farmers prefer to measure the area of their farmlands in feddans, which is a non-metric unit. It is not used widely in the world.

$1m^2 = 100cm \times 100cm = 10000cm^2$

1 hectares (ha) = $100m \times 100m = 10000m^2$

Example 1

A school compound measures 200m by 150m. What is the area in hectares?

Area in $m^2 = 200m \times 150m = 30{,}000m^2$

Area in ha = $\frac{30000m^2}{10000m^2} = 3$ hectares

Exercise 9

1. Wani owns a plot which measures 500 m by 300 m. What is the area in hectare?

2. A building site is divided into 20 rectangular plots, each measuring 50 m by 60 m. What is the area of the building site in hectares?

3. A building site is divided into 40 rectangular plots, each measuring 30 m by 40 m. In addition, $2000m^2$ of the site is taken up by weeds. What is the area of the whole site in hectares?

4. Fifty rectangular pieces, each measuring 30cm by 40cm were cut from a large sheet of cardboard. What was the area of the sheet in square metres?

The Feddan

1 feddan = $70m \times 60m = 4200m^2$

Example 2

A farm measures 140 metres by 120 metres. What is the area of the farm in feddans?

Area in $m^2 = 140m \times 120m = 16800m^2$

Area in feddans = $\frac{16800m^2}{4200m^2} = 4$ feddans

1. A grazing farm measures 630m by 240m. Find the area of the farm in feddans.

2. A garden for groundnuts measures 360m by 350m. Find the area in feddans.

3. In a housing scheme, each plot measures 24m by 35m. Find the area in feddans.

4. A sesame field measures 360m by 140m. Find the area in feddans.

5. A maize field measures 3,000m by 490m. Find the area of the field in feddans.

6. Write the following areas in feddans:

   (a) $8400m^2$ (b) $2100m^2$ (c) $14000m^2$ (d) $26200m2^2$

Example 3

The area of a rectangle is 20 cm^2. If the length of the rectangle is 5 cm, find its width.

Solution

Area of a rectangle = length x width
Therefore, to find the width, we divide the area by the length.

$$W = \frac{\text{Area}}{\text{Length}} = \frac{20}{5} \text{ cm}^2 = 4 \text{ cm}$$

Example 4

A square has an area of 49 cm^2. If one side is 7 cm, find the length of the other side.

The area of a square is equal to one side multiplied by itself; therefore, to find one side of a square, we divide the area by the given side.

$$\text{That is } \frac{49}{7} \text{ cm}^2 = 7 \text{ cm}$$

Exercise 9

1. Fill the blank spaces in the following table with the correct numbers.

Rectangle	Length	Width	Area
a)	6 cm	5 cm	
b)	11 cm		44 cm^2
c)		7 cm	35 cm^2
d)	13 cm	4 cm	
e)		7 cm	56 cm^2

2. Fill in the blank spaces in the table below with the correct numbers.

Square	Length	Width	Area
a)	5 cm	5 cm	
b)		6cm	36 cm^2
c)	11cm	11cm	
d)		12 cm	144 cm^2

3. Find the area of rectangles whose sides are:

a) 4 cm by 3 cm

b) 15 cm by 9cm

c) 8 m by 6 m

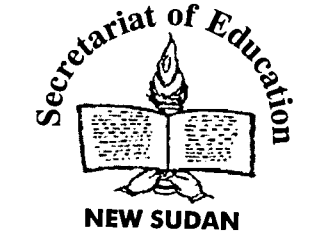
d) 12 m by 7 m

e) 11m by 21m

4. A rectangular plot has an area of 810 m^2. If its length is 30 metres, find the width of the plot.

5 A square playground has a side of 35 metres. What is its area?

6. A rectangular farm is 320m long and 200m wide. It is sub-divided into plots of 400 m^2 each. How many plots are there?



ISBN NO. 9966-33-095-X