Technical Report

A Bayesian DSGE Prototype for BSP

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Preface

This report is the result of technical assistance provided by the Economic Modernization through Efficient Reforms and Governance Enhancement (EMERGE) Activity, under contract with the CARANA Corporation, Nathan Associates Inc. and The Peoples Group (TRG) to the United States Agency for International Development, Manila, Philippines (USAID/Philippines) (Contract No. AFP-I-00-03-00020-00, Delivery Order 800). The EMERGE Activity is intended to contribute towards the Government of the Republic of the Philippines (GRP) Medium Term Philippine Development Plan (MTPDP) and USAID/Philippines' Strategic Objective 2, "Investment Climate Less Constrained by Corruption and Poor Governance." The purpose of the activity is to provide technical assistance to support economic policy reforms that will cause sustainable economic growth and enhance the competitiveness of the Philippine economy by augmenting the efforts of Philippine pro-reform partners and stakeholders.

This technical report was written by Paul D. McNelis, Ph.D., Department of Finance, Fordham University, New York, and the staff of the Center for Monetary and Financial Policy, Bangko Sentral ng Pilipinas (BSP), following technical assistance requested by letter dated January 29, 2007, by Maria Cyd N. Tuaño-Amador, Sector-in-Chief, Monetary Stability Sector, on behalf of Diwa C. Guinigundo, Deputy Governor for Monetary Policy of the BSP. It illustrates the methodology of the dynamic stochastic general equilibrium (DSGE) framework with Bayesian estimation within a well-known closed-economy framework. The aim is to show how such models are derived from the first principles of optimizing behavior by households and firms and to illustrate how Bayesian estimation of the deep parameters is carried out, with recent data from the economy of the Philippines.

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Abstract

This paper illustrates the methodology of the dynamic stochastic general equilibrium framework with Bayesian estimation, with a well-known closed-economy framework. The aim is to show hows such models are derived from the first principles of optimizing behavior by households and firms and to illustrate how Bayesian estimation of the deep parameters is carried out, with recent data from the economy of the Philippines.

1 Introduction

This project describes one prototype stochastic dynamic general equilibrium model for Bayesiean estimation and policy analysis by the Center for Monetary and Financial Policy (CMFP) at the Central bank of the Philippines (BSP). This is the Smets-Wouters Euro Area Regional (SWEAR) model.

The aim is to show how the basic structure of the model is derived from the first principles of optimizing behavior of households and firms in this closed economy context. We start with the closed economy simply because it avoids the closure issue all open-economy models have to face. In an open economy, the is no reason why consumers would not borrow up to infinity, and of course there is no steady-state consumption or foreign debt. In the open-economy framework, models of a single country have to be "closed" either through a debt-elastic risk premium, an endogenous discount rate or the construction of a counter-part economy representing the rest of the world.

The other reason for starting with this model is that many (though not all) of the elements of the closed-economy setting carry over to the open-economy framework. So understanding these elements first, in the less-complex but still demanding analytical framework, allows a structured progression in the process of model development.

The SWEAR model is one example of the New Neoclassical Synthesis (NNS) approach to macrodynamic models used by central banks in their inflation targeting program. This approach combines many of the elements of the new classical macroeconomics, based on microfoundations of household and firm optimization, with Keynesian rigidities, particularly in the form of sticky price and wage adjustment, and the assumption of monopolistic competition.

The NNS models follow a simple symmetric strategy for households and firm. Households provided a differentiated labor product, charging a specific individual wage for the differentiated labor. With the device of a competitive bundler, the differentiated labor is transformed, through a CES aggregator, into an aggregate labor supply, priced with the aggregate wage. The household determines its wage under the assumption that it will last over a fixed number of periods. At any given period, only a fraction of the households are setting their wage. The rest are supplying their differentiated labor to the bundler based on a past wage (with partial adjustment for past inflation).

The firms behave in the same way. They bring their intermediate goods, with a differentiated price, to a competitive bundler, who uses the same CES technology to produce an aggregate product with an economy-wide price. The firm sets its own price in the same way as the households set their wages. Only a fraction, at any given time, set a price over a finite period. The price they set will equates the expected marginal revenue to the expected marginal cost multiplied by a markup factor.

The optimization of the household and firms leads to a series of nonlinear stochastic difference equations, which are the behavioral equations of the model. For implementing these equations for simulation, a standard practice is to log-linearize these equations. In this format, each variable represents a percentage deviation from its steady-state value. This practice drastically simplifies the computational burdens. The log-linear system can be quickly solved. The initial condition of each variable is now zero. Levels can be backed out by multiplying the percentage deviation value with the corresponding steady-state value.

Different general equilibrium models can generate different effects, so it is essential to have a good strategy for developing a good dynamic stochastic general equilibrium (DSGE) model. As McCallum (2001) points out, it is desirable for a model to be consistent with both economic theory and empirical evidence, but this "dual requirement" is only a starting point for consideration of numerous issues. McCallum also points out that "depicting individuals as solving dynamic optimization problems," as is done in general equilibrium settings, is "useful in tending to reduce inconsistencies and forcing the modeler to think about the economy in a disciplined way" [McCallum (2001) p.15]. But adhering to dynamic general equilibrium models still leaves room for enormous differences/

Once the model is set up for computational experiments, the challenge is to calibrate it in such a way to give realism to the results. After all, we would like parameters of the utility and production function, as well as the assumed degrees of price and wage stickiness, to reflect underlying conditions in the country we are investigating. Calibration can go just so far, however, since we have to rely on extraneous information. A more rigorous approach is to specify the values of the parameters as Bayesian priors, with means and standard deviations, as expressions of our certainty (or lack thereof) about the values of these parameters.

In the Bayesian framework, these prior distributions are combined with the likelihood function of the model for matching a given subset of variables with real world data. The posterior distribution of the parameters gives us information about the fundamental parameters of the model.

This Bayesian approach has a number of major advantages over traditional econometrics. First, it can be used when there are small number of observations. Traditionally, econometrcians are "frequentists" interested in laws of convergence in probability. But most commonly we do not have sufficiently large numbers of observations on quarterly macro variables for statisical inference. This is true not just of developing countries but also of the industrialized countries. In the Euro Area, with the unification of Germany and the introduction of the Euro, data prior to the year 1998 are suspect. Similarly, in the United States, data prior to the regime switch at the Federal Reserve are also suspect. These data represent different regimes which should not be combined with more recent data

Secondly, while the model may contain variables like utility or habits, we do not have to match all of the variables in the model with real world observations. We are interested in how sub-sets of key endogenous variables in the model (gdp growth, inflation, investment) match real world observations. Finally, the Bayesian method will help us to identify which parameters (including those stochastic processes for productivity or taste or government spending), really matter and those which do not. This helps us to identify, in a more rigorous way, what are the important sources of volatility in business cycles from those which are not. It also helps us to identify sources of instability coming from monetary or fiscal policy.

Preliminary results from Bayesian estimation of this model, using quarterly data for the Republic of Philippines from 1988-2006,shows the following: (1) the volatility of government spending is much less important than the volatility of productivity and private demand; (2) both productivity and private demand show a high degree of persistence; (3) risk aversion is relatively high while the disutility of labor, affecting the supply of labor, is not much different from estimates reported in other countries; (4) monetary policy puts more weight on inflation than on the output gap; (5) the smoothing parameter on interest adjustment, for the overall period, is much lower than commonly assumed; (6) the captial share in production is about .7, validating studies from direct estimation.

It should be noted that these models should be used in conjunction with econometric models, either structural models, a-theoretic vector autoregressive models (VAR), or structural vector autoregressive (SVAR) models. After all, there is no one true underlying model of any economy, nor are there a set of invariant true underlying parameter estimates of any one model. As Sargent (1999) suggested in his analysis of the conquest of American inflation, we have to acknowledge "model uncertainty" when we undertake economic policy. It is much better to base decisions on a "thick model", with results coming from a variety of models.

They advantage for policy analysis of using the Bayesian estimation of a DSGE model, as opposed to the econometric estimation of a time-series model, is that we can work with limited data sources, while making use of prior information based on economic theory. In particular, we can use economic theory to impose restrictions on the evolution unobservable variables, such as habits or capital stocks, for which we do not have or need to use observable time series. The prior information on the unobservable variables, of course, has important effects on the rest of the system's variables, which have observable counterparts we can measure. In the DSGE models, we match subsets of the endogenous variables in the model with key characteristics of their observable real world equivalents. The DSGE framework allows us to incorporate fuller a priori information into our policy framework. At the same time, the DSGE framework is up-front and explicit about what constitutes prior information and what does not.

The Bayesian/DSGE approach requires different skill sets from the time-honored time series estimation. As pointed out above, such a general equilibrium framewords tends to reduce inconsistencies and forces the modellers and policy makers to approach the economic in a disciplined way, involving the use of dynamic programming methods.

To make the model data consistent, with parameters representing coefficients not readily apparent in data, such as the coefficient of risk aversion, we have to make use of Bayesian approaches. In this method, we impose priors on the distributions of the unknown parameters, evaluate the likelihood function of the model given these prior assumptions, and using Bayes' theorem, find the posterior densities of the key parameters of the model.

The advantage of this approach is that we can isolate which parameters are more important and which are less important for explaining the key movements and instabilities observed in the data. From a policy perspective, nothing could be more important. If the key parameters are policy related (such as the volatility of goverment spending or the confidence interval for Taylor rule coefficients), then the results give clear signals about monetary and fiscal stabilization policy.

The parameter distributions from Bayesian estimation require computational skills which are different from econometric estimation. In particular, the use of Monte Carlo simulation is needed to calculate the posterior densities of the key parameters, since there are no analytical solutions for these densities.

In the next section, we discuss the structure of the SWEAR model. Then we discuss preliminary results of Bayesian estimation for the Philippines.

2 The SWEAR Model

As mentioned above, the model consists of a household and firm sector. Each engages in optimization under the assumption of stickiness in wage or price setting.

Households maximize a utility function with three arguments (goods, leisure and money) over an infinite life horizon. Consumption appears in the utility function relative to a time-varying external habit variable. Labor is differentiated over households, so that there is some monopoly power over wages. This results in an explicit wage equation with stickiness. Households also rent capital services to firms and decide how much capital to accumulate given certain capital adjustment costs. As the rental price of capital goes up, the capital stock can be used more intensively.

Firms high labor and produce a differentiated product. They set their prices in the same way that households set wages.

The equations of the model are the Euler equations coming from household and firm optimization. We present both the initial nonlinear version in levels, as well as a log-linearized version of this model, in which variables are expressed in percentages rates of change or percentage deviation from their respective steady-state variables. We follow the convention of letting the variable C_t, for example, represent the level of consumption at time t, while $\widehat{C}_t$ is the percentage deviation from the steady-state $\overline{C}$. Variables dated $t+1$ correspond to the expectation of that variable for time $t+1$ given information available at time t.

2.1 Household Behavior

Each household τ maximizes an intertemporal utility function:

$$E_0 \sum_{t=0}^{\infty} \beta^t U_t^{\tau}$$

The parameter β is the discount rate and the utility function is separable in consumption, labor, and real balances:

$$U_t^{\tau} = \varepsilon_t^b \left(\frac{1}{1-\sigma_c} [C_t^{\tau} - H_t]^{1-\sigma_c} - \frac{\varepsilon_t^l}{1+\sigma_l} [l_t^{\tau}]^{1+\sigma_l} + \frac{\varepsilon_t^m}{1-\sigma_m} \left[\frac{M_t^{\tau}}{P_t} \right]^{1-\sigma_m} \right) \quad (1)$$

Utility depends positively on the level of consumption C_t^{τ} (relative to external habit H_t), and on the level of real balances, $\frac{M_t^{\tau}}{P_t}$, and negatively on the labor supply l_t^{τ}. This function also contains three preference shocks. One is a shock to the discount rate, ε_t^b,affecting the intertemporal substitution of household, ε_t^L is a shock to labor supply, while ε_t^M is a liquidity preference shock. All three follow an autoregressive process in logarithms:

$$\begin{aligned}
\ln(\varepsilon_t^b) &= \rho_b \ln(\varepsilon_{t-1}^b) + \eta_{b,t} \\
\ln(\varepsilon_t^m) &= \rho_m \ln(\varepsilon_{t-1}^m) + \eta_{m,t} \\
\ln(\varepsilon_t^l) &= \rho_l \ln(\varepsilon_{t-1}^l) + \eta_{l,t} \\
& \eta_{b,t} \tilde{} N(0, \sigma_b^2) \\
& \eta_{m,t} \tilde{} N(0, \sigma_m^2) \\
& \eta_{l,t} \tilde{} N(0, \sigma_l^2)
\end{aligned}$$

The habit stock H_t evolves according to the external habit formation law of motion:

$$H_t = hC_{t-1}$$

The budget constraint for the household is given by the following equation:

$$b_t \frac{B_t^\tau}{P_t} + \frac{M_t^\tau}{P_t} = \frac{B_{t-1}^\tau}{P_t} + \frac{M_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau \tag{2}$$

where $\frac{B_t^\tau}{P_t}$ represents a government bond with payoff b_t, Y_t^τ is the income of the household and I_t^τ is investment in capital goods (to be rented to firms).

Household income is defined by the following identity:

$$Y_t^\tau = w_t^\tau l_t^\tau + A_t^\tau + r_t^\tau z_t^\tau K_{t-1}^\tau - \Psi(z_t^\tau) K_{t-1}^\tau + Div_t^\tau \tag{3}$$

The variable z_t is the capacity utilization rate, which in the steady state is unity. The function $\Psi(z_t^\tau)$ is an adjustment cost function for changes in this utilization rate. Labor income at time t is given by $w_t^\tau l_t^\tau$, income from state-contingent assets A_t^τ, and dividends from monopolistic firms is given by Div_t^τ

The relationship between capital and investment provided by the capital is given by the following law of motion:

$$K_t^\tau = (1-\delta) K_{t-1}^\tau + \left[1 - S\left(\frac{\varepsilon_t^I I_t^\tau}{I_{t-1}^\tau}\right)\right] I_t^\tau \tag{4}$$

The function $S\left(\frac{\varepsilon_t^I I_t^\tau}{I_{t-1}^\tau}\right)$ is the adjustment cost function, and the term ε_t^I is the shock to these adjustment costs. At the steady-state, $S = S' = 0$. As discussed in Christiano, Eichenbaum and Evans (2005), specifying adjustment costs as a function of the change in investment rather than its level introduces additional dynamics in the investment equation, which is useful in capturing the observed humped-shaped response of investment to various shocks, including monetary policy shocks.

The household optimizes the following intertemporal Lagrangean function subject to the budget constraint 2, income identity 3 and law of motion for capital 4:

$$\begin{aligned} V_t^\tau &= U_t^\tau(\cdot) + \Lambda_t \left[\frac{B_{t-1}^\tau}{P_t} + \frac{M_{t-1}^\tau}{P_t} + Y_t^\tau - C_t^\tau - I_t^\tau - b_t \frac{B_t^\tau}{P_t} - \frac{M_t^\tau}{P_t}\right] ... \\ &\quad + \Lambda_t Q_t \left[(1-\delta) K_{t-1}^\tau + \left[1 - S\left(\frac{\varepsilon_t^I I_t^\tau}{I_{t-1}^\tau}\right)\right] I_t^\tau - K_t^\tau\right] + \beta V_{t+1}^\tau \end{aligned}$$

The multiplier Λ_t represents the marginal utility of income while Q_t is the shadow price of capital. Embedding the income definition in V_t^τ yields the following equation:

$$
\begin{aligned}
V_t^\tau &= U_t^\tau(\cdot) + \\
& \Lambda_t \left[\begin{array}{c} \frac{B_{t-1}^\tau}{P_t} + \frac{M_{t-1}^\tau}{P_t} + \left(W_t^\tau l_t^\tau + A_t^\tau + r_t^\tau z_t^\tau K_{t-1}^\tau + \Psi(z_t^\tau) K_{t-1}^\tau + Div_t^\tau\right) \ldots \\ -C_t^\tau - I_t^\tau - b_t \frac{B_t^\tau}{P_t} - \frac{M_t^\tau}{P_t} \end{array} \right] \quad (5) \\
& + \Lambda_t Q_t \left[(1-\delta) K_{t-1}^\tau + \left[1 - S\left(\frac{\varepsilon_t^I I_t^\tau}{I_{t-1}^\tau} \right) \right] I_t^\tau - K_t^\tau \right] + \beta V_{t+1}^\tau \quad (6)
\end{aligned}
$$

The household optimizes equation (**??**) with respect to the choice of $C_t^\tau, B_t^\tau, M_t^\tau$, as well as $K_t^\tau, I_t^\tau, z_t^\tau$, and its wage, W_t^τ. We take up wage-setting behavior first, then optimization with respect to $C_t^\tau, B_t^\tau, M_t^\tau$, and finally $K_t^\tau, I_t^\tau, z_t^\tau$.

2.1.1 Wage-Setting Behavior

Optimal Bundling and Labor Demand For some households, wages are pre-set at time t. The wage for the household that cannot re-optimize at time t is given by:

$$
W_t^\tau = \left(\frac{P_{t-1}}{P_{t-2}} \right)^{\gamma_w} W_{t-1}^\tau
$$

The solution for the demand for l_t^τ comes from the optimal labor-bundling problem. Aggregate labor is bundled from individual labor supply in the following way:

$$
L_t = \left[\int_0^1 \left(l_t^t\right)^{1/(1+\lambda_{w,t})} d\tau \right]^{(1+\lambda_{w,t})}
$$

The bundling technology is a CES aggregator function. The optimal competitive bundler minimizes costs when purchasing labor from household τ, at wage W_t^τ, given the bundling technology by the CES aggregator:

$$
Min \int W^\tau l^\tau d\tau
$$

$$
\text{st:} \qquad L_t = \left[\int_0^1 \left(l_t^t\right)^{1/(1+\lambda_{w,t})} d\tau \right]^{(1+\lambda_{w,t})}
$$

Under assumption of perfect competition, the Lagrangean for this problem is the optimal aggregate wage set by the bundler. The demand for labor is given by the following equation:

1. $$
l_t^\tau = \left(\frac{W_t^\tau}{W_t} \right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t
$$

The aggregate wage $W^{\tau}$is given by:

$$W_t = \left[\int_0^1 \left(W_t^t\right)^{-1/\lambda_{w,t}} d\tau \right]^{-\lambda_{w,t}}$$

Household Wage Setting Substituting the labor demand function into the intertemporal household optimization problem, we obtain the following expression:

$$\begin{aligned}
V_t^{\tau} &= U_t^{\tau}\left(\frac{1}{1-\sigma_c}[C_t^{\tau} - H_t]^{1-\sigma_c}\right. \\
&\quad -\frac{\varepsilon_t^L}{1+\sigma_l}\left[\left(\frac{W_t^{\tau}}{W_t}\right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t\right]^{1+\sigma_l} && (7)\\
&\quad \left.+\frac{\varepsilon_t^M}{1-\sigma_m}\left[\frac{M_t^{\tau}}{P_t}\right]^{1-\sigma_m}\right)... && (8)\\
&\quad +\Lambda_t \left[\begin{array}{c} \frac{B_{t-1}^{\tau}}{P_t} + \frac{M_{t-1}^{\tau}}{P_t} + \left(\begin{array}{c} W_t^{\tau}\left(\frac{W_t^{\tau}}{W_t}\right)^{-(1+\lambda_{w,t})/\lambda_{w,t}} L_t \\ +A_t^{\tau} + r_t^{\tau} z_t^{\tau} K_{t-1}^{\tau} - \Psi(z_t^{\tau})K_{t-1}^{\tau} + Div_t^{\tau} \end{array}\right)... \\ -C_t^{\tau} - I_t^{\tau} - b_t \frac{B_t^{\tau}}{P_t} - \frac{M_t^{\tau}}{P_t} \end{array}\right]... \\
&\quad +\Lambda_t Q_t \left[(1-\delta)K_{t-1}^{\tau} + \left[1 - S\left(\frac{\varepsilon_t^I I_t^{\tau}}{I_{t-1}^{\tau}}\right)\right] I_t^{\tau} - K_t^{\tau}\right] + \beta V_{t+1}^{\tau} && (9)
\end{aligned}$$

The household optimizes its expected utility,with respect to the choice of W_t^{τ}, with probability ξ_W that it this wage will continue to be in effect in future period i. We obtain the following Euler equation.

$$\left(\frac{\widetilde{W}_t^{\tau}}{P_t}\right) E_t \sum_{i=0}^{\infty} \beta^i \xi_W^i \left[\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+i}/P_{t+i-1}}\right] \frac{l_{t+i}^{\tau}\Lambda_{t+i}}{1+\lambda_{w,t+i}} = E_t \sum_{i=0}^{\infty} \beta^i \xi_W^i \; l_{t+i}^{\tau} U_{l^{\tau},t+i}^{\tau}$$

1. The sum of the left hand side if of course equal to the sum of the right had side expressions, giving us the pricing formula for the wage $\left(\frac{\widetilde{W}_t}{P_t}\right)$. This equation tells us that the expected present value of product of total labor income and the marginal utility of income, should be equal to the present value of the product of the marginal disutility of labor and the total amount of labor effort. Rewriting the above expression, we obtain:

$$\frac{\widetilde{W}_t}{P_t} = \frac{E_t \sum_{i=0}^{\infty} \beta^i \xi_W^i \; l_{t+i}^t U_{t+i}^l}{E_t \sum_{i=0}^{\infty} \beta^i \xi_W^i \left[\frac{(P_t/P_{t-1})^{\gamma_w}}{P_{t+i}/P_{t+i-1}}\right] \frac{l_{t+i}^t U_{t+i}^C}{1+\lambda_{w,t+i}}} \tag{10}$$

Upon substitution and forward recursion, we have the following expression for the log-linearized system:

$$\widehat{\widetilde{w}}_t = (1-\beta\xi_w)\left[\frac{\sigma_L(1+\lambda_w)}{\lambda_w}\widehat{w}_t - \sigma_L\widehat{L}_t - \widehat{\varepsilon}_t^L - \frac{\sigma_c}{1-h_c}(\widehat{C}_t - h_c\widehat{C}_{t-1})\right] + \beta\widehat{\widetilde{w}}_{t+1}$$

Alternatively:

$$\begin{aligned}\widehat{w}_t = \tfrac{\beta}{1+\beta}\widehat{w}_{t+1} + \tfrac{1}{1+\beta}\widehat{w}_{t-1} + \tfrac{\beta}{1+\beta}\widehat{\Pi}_{t+1} - \tfrac{1+\beta\gamma_w}{1+\beta}\widehat{\Pi}_t + \tfrac{\gamma_w}{1+\beta}\widehat{\Pi}_{t-1} \\ -\tfrac{1}{1+\beta}\tfrac{(1-\beta\xi_w)(1-\xi_w)}{(1+\frac{(1+\lambda_w)\sigma_L}{\lambda_w})\xi_w}\left[\widehat{w}_t - \sigma_L\widehat{L}_t - \tfrac{\sigma_c}{1-h_c}(\widehat{C}_t - h_c\widehat{C}_{t-1}) - \widehat{\varepsilon}_t^L - \widehat{\eta}_t^w\right]\end{aligned} \quad (11)$$

2.1.2 Consumption, Bonds and Money

The first order conditions for V_t^τ with respect to C_t^τ, B_t^τ, and M_t^τ are given by the following equations:

$$\varepsilon_t^b[C_t^\tau - hC_{t-1}^\tau]^{-\sigma_c} = \Lambda_t \quad (12)$$

$$1 = \beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{(1+R_t)P_t}{P_{t+1}} \quad (13)$$

$$= \beta\frac{\Lambda_{t+1}}{\Lambda_t}\frac{(1+R_t)}{(1+\Pi_{t+1})}$$

$$1+R_t = 1/b_t$$

$$1+\Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

$$\varepsilon_t^M\left(\frac{M_t^\tau}{P_t}\right)^{-\sigma_m} = \left[\frac{1}{1+R_t}\cdot\Lambda_t\right] \quad (14)$$

We can log-linearize the consumption Euler equation 13 by taking logarithms and then reordering the equation. The log-linearized *consumption equation* with external habit formation is given by the following expression:

$$\widehat{C}_t^\tau = \frac{h_c}{1+h_c}\widehat{C}_{t-1}^\tau + \frac{1}{1+h_c}\widehat{C}_{t+1}^\tau - \frac{1-h_c}{(1+h_c)\sigma_c}(\widehat{R}_t - \widehat{\Pi}_{t+1}) + \frac{1-h_c}{(1+h_c)\sigma_c}(\widehat{\varepsilon}_t^b - \widehat{\varepsilon}_{t+1}^b) \quad (15)$$

where $\widehat{C}_t$ is consumption, $\widehat{R}_t$ is the nominal short-term interest rate, $\widehat{\Pi}_t$ is inflation and $\widehat{\varepsilon}_t^b$ is a temporary, but persistent shock to the consumer's discount rate. All of these variables represent percentage deviations from their steady state values, The parameter σ_c is the inverse of the intertemporal degree of substitution. This expression relies on the fact that:

$$\widehat{\Lambda}_t = \varepsilon_t^b + \frac{1}{1-h}\left(\widehat{C}_t^\tau - h\widehat{C}_{t-1}^\tau\right)$$

The money demand is a negative function of the interest rate and a positive function of current consumption:

$$\left(\frac{M_t^\tau}{P_t}\right) = \left[\frac{1}{1+R_t}\cdot\Lambda_t\right]^{-\frac{1}{\sigma_m}}\left(\frac{1}{\varepsilon_t^M}\right)^{-\frac{1}{\sigma_m}}$$

The log-linearized money demand equation (or interest-rate equation when money is exogenous) is given by the following relation:

$$\widehat{M}_t^\tau - \widehat{P}_t = \frac{\sigma_c}{(1-h)\sigma_m}\left(\widehat{C}_t^\tau - h\widehat{C}_{t-1}^\tau\right) - \frac{1}{\sigma_m}\widehat{R}_t + \frac{1}{\sigma_m}\left(\widehat{\varepsilon}_t^m - \widehat{\varepsilon}_t^b\right)$$

2.1.3 Capital, Investment, and Capacity Utilization

The household also maximizes equation (9) with respect to the choice of K_t, I_t and z_t. We obtain the following first order conditions:

$$\begin{aligned}
Q_t &= \beta\frac{\Lambda_{t+1}}{\Lambda_t}\left[r_{t+1}^\tau z_{t+1}^\tau - \Psi(z_{t+1}^\tau) + Q_{t+1}(1-\delta)\right] \\
Q_t\left[1 - S\left(\frac{\epsilon_t^i I_t}{I_{t-1}}\right)\right] &= Q_t S'\left(\frac{\epsilon_t^i I_t}{I_{t-1}}\right)\frac{\epsilon_t^i I_t}{I_{t-1}} \\
&\quad -\beta Q_{t+1}\frac{\Lambda_{t+1}}{\Lambda_t}S'\left(\frac{\epsilon_{t+1}^i I_{t+1}}{I_t}\right)\frac{\epsilon_{t+1}^i I_{t+1}}{I_t}\frac{I_{t+1}}{I_t} + 1 \\
r_t^\tau &= \Psi'(z_t^\tau)
\end{aligned}$$

Log-linearizing the above equations, we obtain the following approximations for Q, investment, and capacity utilization:

$$\widehat{Q}_t = -\left(\widehat{R}_t - \widehat{\Pi}_t\right) + \frac{1-\delta}{1-\delta+\overline{r}}\widehat{Q}_{t+1} + \frac{\overline{r}}{1-\delta+\overline{r}}\widehat{r}_{t+1} + \eta_t^Q \tag{16}$$

$$\widehat{I}_t = \frac{}{1+\beta}\widehat{I}_{t-1} + \frac{\beta}{1+\beta}\widehat{I}_{t+1} + \frac{\varphi}{1+\beta}\widehat{Q}_t + \frac{\beta\widehat{\varepsilon}_{t+1}^I - \widehat{\varepsilon}_t^I}{1+\beta} \tag{17}$$

The parameter φ is the inverse elasticity of the cost function of changing investment. A negative shock to the adjustment cost function, ε_t^I , (also denoted as a positive investment shock) temporarily increases investment. In the steady state, $\beta = 1/(1-\delta+\bar{r}^k)$.The current value of the capital stock depends negatively on the ex-ante real interest rate, and positively on its expected future value and the expected rental rate. The introduction of a shock to the required rate of return on equity investment, η_t^Q , is meant as a shortcut to capture changes in the cost of capital that may be due to stochastic variations in the external finance premium.

The log-linearized *capital accumulation equation* is standard:

$$\widehat{K}_t = (1-\delta)\widehat{K}_{t-1} + \delta\widehat{I}_{t-1} \tag{18}$$

2.2 Firm Behavior

Like the household, each firm brings a differentiated product y_t^j to a competitive bundler, selling at price p_t^j :

$$Y_t = \left[\int_0^1 \left(y_t^j \right)^{1/(1+\lambda_{p,t})} dj \right]^{(1+\lambda_{p,t})}$$

Cost minimization gives the following demand for good y_t^j :

$$y_t^j = \left(\frac{P_t^j}{P_t} \right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t$$

The overall price index is obtained in the same way the overall wage is determined:

$$P_t = \left[\int_0^1 \left(P_t^j \right)^{-1/\lambda_{p,t}} dj \right]^{-\lambda_{p,t}}$$

Each intermediate good-producing firm has the following production function:

$$\begin{aligned} y_t^j &= \epsilon_t^\alpha \widetilde{K}_{j,t}^\alpha L_{j,t}^{1-\alpha} \\ \widetilde{K}_{j,t} &= z_t K_{j,t-1} \\ \ln(\epsilon_t^\alpha) &= \rho_a \ln(\epsilon_{t-1}^\alpha) + \eta_{a,t} \\ & \eta_{a,t} \tilde{} N(0, \sigma_a^2) \end{aligned}$$

The variable $\widetilde{K}_{j,t}^\alpha$ is the effective utilization of the capital stock at time t. Cost minimization implies the following expression for the Marginal Cost at time t :

$$MC_t = \frac{\frac{1}{\epsilon_t^\alpha} W_t^{1-\alpha} \left(r_t^k \right)^\alpha}{\alpha^\alpha (1-\alpha)^{1-\alpha}}$$

This expression comes from the equalization of marginal cost and marginal product of the factors of production:

$$\frac{W_t L_{j,t}}{r_t^k \widetilde{K}_{j,t}} = \frac{1-\alpha}{\alpha}$$

Log-linearizing this expression gives the following expression for desired labor input, $\widehat{L}_t$:

$$\begin{aligned}\widehat{L}_t &= -\widehat{W}_t + (1+\widetilde{\Psi})\hat{r}_t^k + \widehat{K}_{t-1} \\ \widetilde{\Psi} &= \frac{\Psi'(1)}{\Psi''(1)}\end{aligned}$$

where Ψ is the inverse elasticity of the capacity utilization cost function, evaluated at $z_t = \overline{z} = 1$. In order to match the model to data, since there is no measure for labor input in the form of hours worked, Smets and Wouters use an employment measure. They assume that at time given time, only a fraction ξ_e of firms is able to adjust their employment to deired labor input. They use the following auxiliary equation for employment:

$$\widehat{E}_t = \beta E_{t+1} + \frac{(1-\beta\xi_e)(1-\xi_e)}{\xi_e}(\widehat{L}_t - \widehat{E}_t)$$

where $\widehat{E}_t$ represents the number of people employed.

Using the demand for firm j's product as a function of aggregate output and relative prices, the profit function for firm j becomes:

$$\Phi_t^j = \left(p_t^j - MC_t\right)\left(\frac{P_t^j}{P_t}\right)^{-(1+\lambda_{p,t})/\lambda_{p,t}} Y_t$$

Firms which do not re-optimize their price at time t have partial indexing to overall past inflation

$$p_t^j = \left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} p_{t-1}^j$$

The firms which re-set the price at time t follow the same procedure as household do for wage-setting:

$$\frac{\widetilde{p}_t^j}{P_t} = \frac{E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \, y_{t+i}^j (1+\lambda_{p,t+i}) MC_{t+i}}{E_t \sum_{i=0}^{\infty} \beta^i \xi_p^i \left[\frac{(P_t/P_{t-1})^{\gamma_p}}{P_{t+i}/P_{t+i-1}}\right] y_{t+i}^j} \tag{19}$$

The overall price, like the overall wage, is a combination of the optimal and backward-looking price. Using the CES aggregator for the overall wage, we obtain the following expression:

$$(P_t)^{-1/\lambda_{p,t}} = \xi_P \left(\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} P_{t-1}\right)^{-1/\lambda_{p,t}} + (1-\xi_P)\widetilde{p}_t^{-1/\lambda_{p,t}} \tag{20}$$

or simply,

$$P_t = \left[\xi_P \left(\left(\frac{P_{t-1}}{P_{t-2}}\right)^{\gamma_p} P_{t-1}\right)^{-1/\lambda_{p,t}} + (1-\xi_P)\widetilde{P}_t^{-1/\lambda_{p,t}}\right]^{-\lambda_{p,t}}$$

Simplifying this expression, we obtain the hybrid New Keynesian Phillips curve equation in the following log-linear form:

$$\widehat{\Pi}_t = \frac{\beta}{1+\beta\gamma_p}\widehat{\Pi}_{t+1} + \frac{\gamma_p}{1+\beta\gamma_p}\widehat{\Pi}_{t-1} \tag{21}$$

$$\frac{1}{1+\beta\gamma_p}\frac{(1-\beta\xi_p)(1-\xi_p)}{\xi_p}\widehat{MC}_t \tag{22}$$

$$\widehat{MC}_t = \left[\alpha\widehat{r}_t^k + (1-\alpha)\widehat{w}_t - \varepsilon_t^a + \eta_t^p\right] \tag{23}$$

Inflation depends on past and expected future inflation and the current marginal cost, which itself is a function of the rental rate on capital (r_t^k), the real wage (w_t) and the productivity parameter (ε_t^a).

When the degree of inflation persistence is zero ($\gamma_p = 0$), this equation reverts to the standard purely forward-looking Phillips curve. In other words, the parameter γ_p determines how backward looking the inflation process is. The elasticity of inflation with respect to changes in the marginal cost depends mainly on the degree of price stickiness. When all prices are flexible ($\xi_p = 0$) and the price-mark-up shock, η_t^p , is zero, this equation reduces to the normal condition that in a flexible price economy the real marginal cost should equal one.

2.3 Monetary/Fiscal Policy and Market Clearing

The model is closed by adding a monetary policy reaction function.

In the Taylor rule framework,the Central Bank sets the interest rate as a function of the past interest rate R_{t-1}, expected inflation relative to a target rate of inflation, $\pi_{t+1} - \bar{\pi}_t$), and an output gap measure, $(Y_t - Y_t^p)$:

$$R_t = \rho R_{t-1} + (1-\rho)\{\bar{\pi}_t + r_\pi(\pi_{t+1} - \bar{\pi}_{t+1}) + r_Y(Y_{t+1} - Y_{t+1}^p)\} \tag{24}$$

In this case, the money supply adjusts to the interest rate. This equation in log-linear form becomes

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\{\Pi_t + r_\pi(\widehat{\Pi}_{t+1} - \overline{\Pi}_t) + r_Y(\widehat{Y}_{t+1} - \widehat{Y}_{t+1}^p)\} \tag{25}$$

The central bank will supply the needed quantity of nominal money to support this interest rate according to the money demand function and the current price level:

$$M_t = \left[\frac{1}{1+R_t}\cdot\Lambda_t\right]^{-\frac{1}{\sigma_m}}\left(\frac{1}{\varepsilon_t^M}\right)^{-\frac{1}{\sigma_m}}\cdot P_t$$

or in log-linear form:

$$\widehat{M}_t = \frac{\sigma_c}{(1-h)\sigma_m}\left(\widehat{C}_t^\tau - h\widehat{C}_{t-1}^\tau\right) - \frac{1}{\sigma_m}\widehat{R}_t + \frac{1}{\sigma_m}\left(\widehat{\varepsilon}_t^m - \widehat{\varepsilon}_t^b\right) + \widehat{P}_t$$

The monetary policy-makers gradually respond to deviations of lagged inflation from a time-varying inflation target ($\bar{\pi}_{t+1}$) and to the output gap, defined as the difference between actual and potential output (Y_t^p). Consistent with the model, potential output is defined as the level of output that would prevail under flexible prices and wages in the absence of the three "cost-push" shocks: η_t^Q , η_t^p and η_t^w . The parameter ρ captures the degree of interest rate smoothing.

This equation in log-linear form becomes

$$\widehat{R}_t = \rho\widehat{R}_{t-1} + (1-\rho)\{\Pi_t + r_\pi(\widehat{\Pi}_{t+1} - \overline{\Pi}_t) + r_Y(\widehat{Y}_{t+1} - \widehat{Y}^p_{t+1})\} \tag{26}$$

Government spending following an autoregressive process:

$$\begin{aligned} G_t &= \rho_G G_{t-1} + \varepsilon_t^G \\ & \varepsilon_t^G \sim N(0, \sigma_G^2) \end{aligned}$$

In log-linear first differences, this becomes:

$$\widehat{G}_t = \rho_G \widehat{G}_{t-1} + \widehat{\varepsilon}_t^G$$

The final goods market in in equilibrium if production equals demand by households for production and investment and the government:

$$Y_t = C_t + G_t + I_t + \psi(z_t)K_{t-1}$$

Log-linearizing around the steady-state, we have the following expression for the goods-market clearing:

$$\begin{aligned} \widehat{Y}_t &= (1 - \delta k_y - g_y)\widehat{C}_t + \delta k_y \widehat{I}_t + g_y \widehat{G}_t \\ k_y &= \frac{\overline{K}}{\overline{Y}}, g_y = \frac{\overline{G}}{\overline{Y}} \\ \widehat{Y}_t &= \widehat{\epsilon}_t^\alpha + \alpha \widehat{K}^\alpha_{t-1} + \alpha\psi\widehat{r}_t^k + (1-\alpha)\widehat{L}_t \end{aligned}$$

3 Bayesian Estimation

We followed Smets and Wouters (2003) for the initial calibration of the model. However, we made a number of important changes. The coefficient of capital in the production function was set a value much higher than would be the case for the Euro Area or the USA. The coefficient α was initiall set at .61. We also set the indexation parameters at much lower values, for the sticky wage and price setting behavior. We allowed habit persistence to be higher than in the SWEAR model, with $H = .75$. The coefficient of relative risk aversion in consumption, σ_c was set at 8.00, a value higher than 1.61 used by Smets and Wouters.

Table 1: Bayesian Posterior Estimation

Parameters

	prior mean	posterior mean	confidence interval		prior distribution
α	0.610	0.756	0.756	0.756	beta
H	0.750	0.948	0.946	0.949	beta
ρ_a	0.822	0.969	0.965	0.972	beta
ρ_b	0.882	0.989	0.989	0.990	beta
ρ_g	0.900	0.710	0.738	0.770	beta
σ_c	8.000	7.907	7.915	7.928	norm
σ_l	1.000	1.021	1.014	1.016	norm

St.Dev shocks

	prior mean	posterior mean	confidence interval		prior distribution
η_a	0.010	0.351	0.372	0.399	inverse gamma
η_b	0.324	0.668	0.628	0.711	inverse gamma
η_g	0.331	0.045	0.039	0.053	inverse gamma

Table 1 shows a number of important results. The persistence parameters for productivity and private spending, given by ρ_a and ρ_b, are higher than the persistence parameter for government spending, given by ρ_g. The relative risk aversion coefficient and the disutility of labor, given by σ_c and σ_l, do not depart far from the prior means. Finally we see that the standard deviations of the shocks to productivity and private sending, given by η_a and η_b, are quite large relative to the size of the standard deviation to goverment spending shocks, given by η_g. This results indicate that fiscal policy fundamentals, given by spending volatility, is not a major source of overall volatility in the economy, in comparison with overall demand or productivity volatility.

We note in Table 1 that for a few parameters, the mean posterior falls outside of the 90% confidence region. This shows the relevance of plotting the confidence regions rather than the simple means of the parameters. When the means fall outside of the confidence regions, this means that there are some extreme values in the distributions which are pulling the means outside of the 90% confidence region of the distribution.

Figure 1 pictures a multivariate diagnostic test of convergence for the montecarlo simulations. Specifically, the two lines represent specific measures of the parameter values both within and between Markov chain simulations. For the results to be sensible, they should be relatively close, and should converge, for all three moments. The multivariate diagnostic is based not on one specific parameter but on an aggregate principal component measure of all of the parameters.

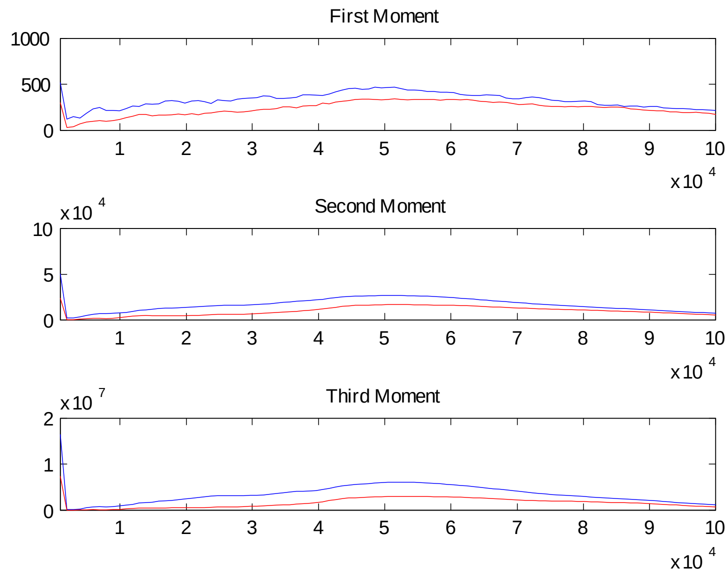

Figure 1: Multivariate Diagnostic Test of Convergence

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