Achievement of Market-Friendly Initiatives and Results Program (AMIR 2.0 Program)

Funded by U.S. Agency for International Development

Quantitative Analysis

Introduction to Basic Concepts and Comments on Learning Outcomes (CFA III, 2003)

By
Ronald E. Copley
President
Copley Investment Management

Contract No. 278-C-00-02-00210-00

2003

Quantitative Analysis

Introduction to Basic Concepts and Comments on Learning Outcomes (CFA III, 2003)

Note: Candidates will need to refer to the Learning Outcomes (LOs) in your Study Guide for wording of each LO. I have provided you with comments on each LO in each study session, but have not duplicated the LOs in these notes.

by

Ronald E. Copley, Ph.D., C.F.A.
President
Copley Investment Management
5218 Wrightsville Ave.
Wilmington, NC 28403 USA
T: 910-452-7147
F: 910-395-6849
E: RonCopley@AOL.com

Table of Contents

Executive Summary

Linear Regression and Correlation

Correlation analysis is used to portray the relationship (correlation) between 2 or more variables. Originated by Karl Pearson (1900) the coefficient of correlation measures the strength of the relationship between two sets of interval scaled or ratio scaled variables. It is often referred to as Pearson's r and as the Pearson product-moment correlation coefficient. If there is no relationship between the two sets of variables r will be 0. This demonstrates that the relationship is weak. As the coefficients approaches 1 the relationship strengthens. For a coefficient of 1 there is a perfect relationship. Scatter diagrams with divergent points indicate little correlation, while diagrams with data points grouped tightly around the center of the data indicate strong correlation.

The Coefficient of Determination is computed by squaring the correlation coefficient. This is a proportion or percent of the variation in on variable associated with or accounted for by the variation in another variable. A high correlation coefficient or coefficient of determination does not imply causality. The relationship could be spurious.

Regression analysis is a technique used to develop the equation for a straight line. The regression equation is used to estimate y based upon x. It is the mathematical equation used to define the relationship between two variables. The least squares principle determines a regression equation by minimizing the sum of the squares of the vertical distances between the actual y values and the predicted values of $\underset{\cdot}{y}$.The assumptions underlying linear regression are:

1. For each value of x, there is a group of y values and these y values are normally distributed.
2. The means of these normal distributions of y values all lie on the straight line of regression.
3. The standard deviations of these normal distributions are equal.
4. The y values are statistically independent. This means that in the selection of a sample, the y values chosen for a particular x value do not depend on the y values for any other x value.

Confidence intervals report the mean value of y for a given x. Prediction intervals report the range of values for a particular value of x.

William Gossett developed the concept of t in the early 1900's. He observed that z was not correct for small samples. This reflects the fact that smaller samples carry greater error.

Multiple Regression and Correlation

The equation for a multiple regression analysis takes the following form:

$$y = a + b_1x_1 + b_2x_2 + b_kx_k$$

The multiple standard error of the estimate is the measure for estimate errors in multiple regression.

The formula for degrees of freedom in multiple regression is:

$$n - (k +1),$$

where n is the number of observations and k is the number of independent variables. The difference between the estimated and actual value is called the residual.
Assumptions of multiple regression analysis:

1. The independent and dependent variables have a linear, or straight-line relationship.
2. The dependent variable must be continuous and at least interval-scale.
3. The variation in the difference between the actual and the predicted values must be the same for all fitted values of y. That is, (y - y') must be approximately the same for all values of y'. When this is the case the differences exhibit homoscedasticity. Further, the residuals, computed by y - y', should be normally distributed with a mean of 0.
4. Successive observations of the dependent variable must be uncorrelated. Violation of this assumption is called autocorrelation. Autocorrelation often occurs in time series.

The coefficient of multiple determination R^2 is the percentage of variation explained by the regression.

A global test determines whether the dependent variable can be estimated without relying on the independent variables or could R^2 occur by chance.

h_0: $b_1 = b_2 = ...b_k$
h_a: not all b's are equal to zero

The F distribution is used to test the null hypothesis that all multiple regression coefficients are equal to zero. The decision rule is to accept the null hypothesis if the computed value of F is less than the tabular value. If the computed F is greater than the tabular value reject the null hypothesis.

Use the t test to determine if individual regression coefficients are equal to zero. Accept the null hypothesis (b = 0) if the computed t is less than the tabular t.

Qualitative variables may be used in a multiple regression equation. They are called dummy variables.

Stepwise regression enters independent variables into the regression equation in the order in which they explain the most variation (increase R^2) in the independent variable.

Residuals should be normally distributed. Histograms or plots may be used to show that there are no trends or patterns.

Learning Outcomes (please refer to your Study Guide for wording of each LO. I have not duplicated the LOs here. The following are comments I have made to each LO).

Study Session 3, Investment Tools: Quantitative Methods for Portfolio Management, DeFusco, McLeavey, Pinto, and Runkle, "Quantitative Methods for Investment Analysis."

1A) Multiple Regression and Issues in Regression Analysis, Ch. 9

Multiple regression involves two or more independent variables to explain changes in the dependent variable (pg 429)

Assumptions are: (1) linear relationships, (2) independent variables are not random, (3) expected value of error term is zero, (4) variance of error term is constant across all values of the independent variables, (5) error terms are uncorrelated, (6) error term is normally distributed (pg 432).

The standard error of the estimate depends on the sum of the squared residuals (pg 435).

Learning Outcomes
Selected end-of-chapter problems: 1, 7, 15.

a) The relationship between a dependent variable and multiple independent variables is linear. Changes in the independent variables are used to explain changes in the dependent variable (pg 429).

b) Statistical significance for each independent variable is determined by use of the t-statistic since we do not know the true value of the population variance. Do not confuse this test with the F-test that determines the significance of the entire equation (pg 431).

c) The two-tail null hypothesis is that the coefficient (intercept and slopes) = 0. The alternative is that they are not equal to zero. Calculation of the t-statistic: (estimated value of coefficient – hypothesized value of coefficient) / standard deviation of coefficient. If the t value falls outside the acceptance region for a two-tail test, reject the null. If the null is less than or equal to zero (one-tail test), the acceptance region is to the left of the critical value, and vice versa (pg 431).

d) Given an estimated equation with the unknown on the left hand side of the equality sign and coefficients on the right hand side, plug in assumed values of the independent variables to calculate the value of the dependent variable.

e) The confidence intervals for multiple regressions are the same as for simple regressions (pg 391).

f) The standard error of the estimate equals the sum of the squared residuals / (# of observations – 1 + # of independent variables) (pg 435).

d) Given the regression equation, the predicted value of a dependent variable is determined by multiplying each independent variable by its regression coefficient and summing (pg 436).

g) The two types of uncertainty are (1) uncertainty in the regression model itself, as reflected in the standard error of the estimate, and (2) uncertainty about the estimates of the regression model's parameters (pg 436).

h) The F-statistic is used to determine significance of the overall model. The null hypothesis is the all the slope coefficients simultaneously equal zero (pg 437).

i) R2 gives the percentage of variation explained by the regression equation. R2 increases as additional variables are added to the model. Adjusted R2 does not automatically increase when another variable is added to the regression; it is adjusted for df (pg 439).

j) An ANOVA table provides information on the explanatory power of a regression and the inputs for an F-test of the null hypotheses that the regression coefficients all equal zero (pg 437).

k) Dummy variables (independent) take on a value of 1 if a particular condition is true and 0 if that condition is false (pg 439).

l) Heteroskedasticity violates one of the assumptions that the variance of the error term is constant across all observations of the independent variables. In other words, the variance is not constant. Unconditional heteroskedasticity occurs when the heteroskedasticity is not correlated with the independent variables in the model. This is not a problem for statistical inference. Conditional heteroskedasticity is when heteroskedasticity is correlated with the independent variables.

m) Serial correlation occurs when regression errors are correlated across observations. Note that serial correlation can occur in either time series or cross-sectional data. The effect is to misestimate the standard error of the estimates that, in turn, impact hypothesis testing (pg 450).

n) Heteroskedasticity can be observed via a scatter diagram (graph) and corrected in one of two ways: (1) generalized least squares, or (2) computing robust standard errors. Neither method is explained in the text. Serial correlation is tested by use of the Durbin-Watson statistic. It can be corrected in one of two ways: (1) adjust the coefficient standard errors, and (2) modify the regression equation to eliminate the serial correlation (pg 453).

o) The DW statistic approximately equals 2(1-r) for large sample sizes. If DW equals 2.0, error are not serially correlated (pg 452).

p) Multicollinearity occurs when two or more independent variable are highly (but not perfectly) correlated with each other, which causes a problem for interpreting regression ouput (pg 457).

q) Qualitative dependent variables are dummy variables used as dependent variables instead of as independent variables. This technique is used when the outcome is either success or failure (pg 460).

r) The economic meaning of a regression output must have a theoretical basis and not be spurious.

1B) Time Series Analysis, Ch. 10.

Serial correlation in the residuals of a trend model invalidates the model by generating biased estimates (pg 498).

Learning Outcomes
Selected end-of-chapter problems: 1, 3, 9, 15.

a) Predicted trend value involves using a trend in a time series to predict future values of the time series. The trend is estimated with t taking on a value of 1 for the first period, 2 for the second period, and so on. Given the estimated trend model (maybe a simple regression equation), we can predict a value for the dependent variable using the historical trend (pg 491).

b) The factor that determines whether to use a linear trend model or a log-linear trend is the growth rate. If growth is exponential, use of a linear trend model will give nonrandom errors that are serially correlated. In the case of exponential growth, we need to estimate a long-linear model (pg 495).

c) The major limitation of a trend model is serial correlation of the error terms (pg 498).

d) Covariance stationary means that the mean and variance do not change over time (pg 499).

e) An autoregressive model of order p uses t-p lagged independent variables (pg 500).

f) Autocorrelations of the residuals can be used to determine whether an autoregressive time-series model is correctly specified in a three-step process: (1) estimate a particular autoregressive model, (2) compute the autocorrelations of the residuals from the model, and (3) test to see whether the autocorrelations are significantly different from 0 (pg 502).

g) Given the trend model, one-step-ahead forecasts (t+1) are made using a one-period lag value (t) in the model. This process is repeated for t+2 using t+1 value as the independent variable, and so forth.

h) Mean reversion occurs when the value of the variable is above its mean (meaning it will subsequently fall) and when the value is below its mean (meaning it will subsequently rise) (pg 503). For an AR (1) model, this implies x(t) = b0/(1-b1). The AR(1) model predicts that the time series will stay the same if its current value is b0/(1-b1), increase if it current value is below b0/(1-b1), and decrease if its current value is above b0/(1-b1) (pg 504).

i) In-sample forecasts are made within the time period of the data. Out-of sample forecasts are made ahead of the time period of the data (pg 507).

j) Instability of coefficients can come from either the time-series model changing across different sample periods used for estimating the model, or from the choice of model for a particular time series (pg 509).

k) A random walk is a time series in which the value of the series in one period is the value of the series in the previous period plus an unpredictable random error. If a random walk, cannot employ normal regression models discussed in Chapter 9 (pg 512).

l) Unit roots come from lag coefficients that equal 1.0, when the time series is a random walk and is not covariance stationary. Since the time series with unit root is not covariance stationary, modeling it without transforming it to make it covariance stationary is likely to lead to incorrect statistical conclusions, and any decisions made on the basis of these conclusions could be incorrect (pg 516).

m) Unit root transformation involves use of first-difference of the time series data (pg 519).

n) An n-period moving average is calculated by taking the average of the current and past n-1 values of the time series (pg 521).

o) Unlike a simple moving average that applies equal weights to the current and past n-1 values of a time series, MA(q) places different weights on the q values (pg 524, and Ex 10-15).

p) For a moving average of order q, MA(q), the first q autocorrelations will be significantly different from 0, and all autocorrelations beyond that will be equal to 0. This result is critical for choosing the right value of q for an MA model (pg 524, and Ex 10-15).

q) Autoregressive and moving average (see Ex 10-15, pg 524).

r) A seasonal lag occurs one year before the current period. To test for a seasonal lag, the seasonal lag is included as an extra term in an autoregressive model. If significant, include the seasonal lag in the autoregressive model (pg 528).

s) If the seasonal lag is 12 months, a forecast for growth (t+1) would include growth (t-1) and growth (t-12) values (Ex 10-17, pg 528-530).

t) The major limitations of ARMA models is that they can be very unstable, depending on the data sample used and the particular ARMA model estimated (pg 532).

u) A test for conditional heteroskedasity involves statistical analysis of whether the variance of the error in a particular time-series model in one period depends on the variance of the error in previous periods. This test is called an ARCH(1) model (pg 532).

v) Using a model with ARCH errors will cause the variance of the error in the next period to be even larger (pg 532).

1C) Portfolio Concepts, Ch. 11

Learning Outcomes
Selected end-of-chapter problems: 1-4, 10.

a) Mean-variance analysis provides the theoretical foundation for the trade-off between risk and return, Its assumptions are: (1) investors are risk averse, (2) known expected returns, (3) known variances and covariances, (4) normal distribution of returns, (5) no transaction costs (pg 560).

b) two-asset expected returns equals weighted average of expected returns on each asset
three-asset expected returns equal weighted average of expected returns on each asset
No matter how many assets in portfolio, expected portfolio return does not depend on correlation. This is a key point of portfolio management theory (pg 561)

$$\sigma(\text{portfolio}) = \left[\sum_{i=1}^{n} w_i^2 \sigma_i^2 + 2 \sum_{i=1}^{n} \sum_{\substack{j=1 \\ i \neq j}}^{n} w_i \sigma_i \sigma_i \rho_{ij}\right]^{1/2}$$

2-Asset case:

$$\sigma_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + w_2 w_1 \sigma_2 \sigma_1 \rho_{21}$$
$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2\, w_1 w_2 \sigma_1 \sigma_2 \rho_{12}$$

3-Asset case:

$$\sigma_p = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma^2_3 + w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + w_1 w_3 \sigma_1 \sigma_3 \rho_{13} + w_2 w_3 \sigma_2 \sigma_3 \rho_{23} +$$
$$w_2 w_1 \sigma_2 \sigma_1 \rho_{21} + w_3 w_1 \sigma_3 \sigma_1 \rho_{31} + w_3 w_2 \sigma_3 \sigma_2 \rho_{32} +$$
$$= w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + w_3^2 \sigma^2_3 + 2 w_1 w_2 \sigma_1 \sigma_2 \rho_{12} + 2 w_1 w_3 \sigma_1 \sigma_3 \rho_{13} + 2 w_2 w_3 \sigma_2 \sigma_3 \rho_{23}$$

c) see above for 3-asset case. Be sure to understand importance of correlation (pg 562).

d) The plot is the combinations depend on the correlation of returns (see Fig 11-4, pg 568).

e) The min variance frontier shows the minimum variance of all portfolios for given levels of expected return. The global minimum variance is the one portfolio farthest to the left on the efficient frontier (pg 564).

f) usefulness of efficient frontier allows investors making portfolio choices in terms of mean return and variance of return can restrict their selection to portfolios lying on the efficient frontier, which simplifies their task (pg 565).

g) diversification benefits occur when the pairwise correlations are each less than +1.0.

h) As pairwise correlations decline toward –1, holding other values constant, the potential benefits to diversification increase.

i) To determine the optimal portfolio we must solve for the portfolio weights that will result in the lowest risk for that level of expected return, which is called optimizing (pg 571).

j) Instability of the efficient frontier occurs because in practice, there is uncertainty about the expected returns, variances, and covariances used in tracing out the frontier (pg 576).

k) When using historical data, the inputs are only point estimates that contain variation. Additionally, the efficient frontier is not necessarily stable in the future (pg 577).

l) Given an equally-weighted portfolio, the variance equals [(1/n)x(average variance) plus (n-1)x(average covariance)]. You can see from this equation that as n increases, the importance of variance diminishes, which means a greater emphasis on covariance. This is the key to MPT (pg 580).

m) Using the equation in (l) above and given the necessary inputs, you can rather easily calculate the ratio for a small n relative to a large n (pg 581).

n) Given a risk-free asset, the standard deviation of a two-asset portfolio = (weight of risky asset x variance of risk asset)^(1/5). Note that the variance of the riskless is zero and that the covariance between the risky and riskless assets is zero (pg 585).

o) The capital allocation line represents combinations of the risk-free asset and a broad combination of risky assets. It is the line from the risk-free rate of return that is tangent to the efficient frontier of risky assets (pg 587).

p) The slope of CAL measures the expected return to risk tradeoff (pg 587).

q) The CML is the CAL with the market portfolio as the tangency portfolio.

r) The CAL = CML when the tangency portfolios both equal the market portfolio of risky assets (pg 592).

s) The market price of risk is the slope of the CML [E(Rm) – Rf]/standard deviation of M. It indicates the market risk premium for each unit of risk (pg 592).

t) CAPM: $E(R_i) = R_f + B_i\,[E(R_m)]- R_f$ comes from the SML (be sure you fully understand the difference between CML, SML and market model. They all look similar. The CAPM describes the expected return on any asset (or portfolio) as a linear function of its beta.

u) The CAPM: $E(R_i) = R_f + B_i\,[E(R_m)]- R_f$ depends on efficiency of the unobservable market portfolio in order to make the CML a straight line. In reality, we do not know if the market portfolio is efficient because it is unobservable (pg 593).

v) Financial market equilibrium says that the CAPM identifies mispriced securities. That is, if the CAPM is correct and financial markets are in equilibrium, all securities will be priced such that a security's price is perfectly aligned with its risk. If a security is mispriced, market forces will bring the price back into equilibrium.

w) Beta is a measure of a security's return sensitivity to movements in the market.

x) The market model assumes that the return to each asset are correlated with the returns to the market. The slope of the line is beta (pg 594).

y) Market model predicts that the expected return for an asset depends on the expected return to the market, the variance of the return depends on the variance of the return to the market, and the covariance of the returns between assets i and j depend on the variance of the market and the effect of market return on the return of asset i and j (pg 595).

z) Correlation between i and j = Betai x Bj x variance M (pg 596).

aa) An adjusted beta is based on the belief that the historical beta is not a random walk, but follows an autoregressive model. Unadjusted betas assume that the historical beta is the best predictor of the future beta (pg 597).

ab) An adjusted beta (t+1) depends on beta (t) plus an error term. This is a first-order autoregressive model (pg 597).

ac) A multifactor model is one in which expected returns are a function of more than one factor. The three categories of multifactor models are (1) macroeconomic factor models, (2) fundamental factor models, and (3) statistical factor models (pg 598).

ad) Macroeconomic factor models are driven by surprises in macroeconomic variables that significantly explain equity returns. The raw descriptors would include: 1) interest rates, 2) inflation, 3) real business activity, 4) long-term bond yield changes, 5) short-term T Bill yield changes (pg 599).

ae) Priced risk is risk for which investors require a return. Systematic factors should thus help explain returns (pg 598).

af) Systematic factors affect the average return of a large number of different assets (pg 598).

ag) Factor sensitivities (factor betas) measures the impact of a surprise in a macroeconomic variable on the stock's return (pg 600 & 603 fn).

ah) The predicted stock return is the return expected without surprises. The actual return will differ from what was expected depending on any surprises that occur (pg 601).

ai) For a two-stock portfolio, the return equals the expected return plus two surprise component returns that come from each factor (pg 602, Example 11-13).

aj) APT: $E(R_i) = R_f + B_{i,m}[E(R_m - R_f] + B_{i,F1}[E(R_{f1} - R_f] + ... + B_{i,FK}[E(R_{FK}) - R_f]$. If the only factor in the APM is market risk, the APM reduces to the CAPM. APT describes the expected return on aan asset (or portfolio) as a linear function of the risk of the asset (or portfolio) with respect to a set of factors (pg 602).

ak) Three assumptions of APT are: (1) a factor model describes asset returns, (2) investors can form well-diversified portfolios that eliminate asset-specific risk, and (3) there are no arbitrage opportunities (pg 602).

al) APT places a restriction on the intercept term in the multifactor model in the sense that the APT model tells us what its value should be.

am) Given factor sensitivities, the expected return on a portfolio equals Rf + the sum of risk premiums for each factor proportional to the factor beta (pg 603).

an) An arbitrage portfolio presents a riskless opportunity to generate a positive return with no investment (pg 602).

ao) An arbitrage opportunity has no risk but provides a positive net cash flow. An arbitrage opportunity exists when the investment requires no initial cash outflow but generates a subsequent positive cash flow (pg 602 and Ex 11-15, pg 605).

ap) A factor portfolio is where there is only one factor driving the return to the portfolio (pg 604).

aq) Weights of a factor portfolio are solved mathematically (Ex 11-17, pg 608).

ar) A tracking portfolio is a portfolio that has a desired or target configuration of factor sensitivities (pg 610).

as) Weights of a tracking portfolio are selected in such a way as to create a tracking portfolio from his individual stock picks. This way, the manager hopes to manage the tracking risk of his portfolio while continuing to pick stock individually (Ex 11-18, pg 611).

at) A portfolio manager uses factor and tacking portfolios to control the risk of his portfolio. By selecting factors that differ from the benchmark portfolio, the manager can evaluate the risk of a particular strategy (pg 610).

au) The CAPM says that an investor should invest in some combination of Rf and M, depending on his/her risk preferences. The investor may want to tilt away from an index fund after considering dimensions of risk ignored by the CAPM that are not ignored by multifactor models and, in the process, gain a premium unrelated to market movements (pg 618-619).

av) The CAPM says to invest in only one portfolio of risky assets, that of M. The CAPM is, thus, a single factor model where M represents the single factor of risk. Multifactor models consider many factors of risk and allow the investor to select the risk to which he/she wants exposure (pg 618-619).

A few additional comments related but not directly included in the required reading.

The Active versus passive debate rages on. Theoretically the optimum portfolio to own is the market portfolio combined with some investment in the risk free asset. This combination will lie on the efficient frontier. That is the portfolio will generate the maximum rate of return for any given level of risk. It will dominate all possible portfolios in risk-return space

Historically active managers have not been able to outperform the market index. The author contends that the failure of a positive relationship to exist between the CAPM beta and realized stock returns means either (1) the market is inefficient in the sense that expected returns differ significantly from realized returns or (2) The market is efficient but investors do not base their decisions on CAPM or true betas are not equal to historical betas used in the studies. A number of studies have examined anomalies such as he January effect, the turn of the month effect, the size effect and the low P-E effect.

Behavioral arguments are important:

- Investors are not rational
- Investors are overconfident.

Overconfidence can explain the anomalies. Over confident investors will over- and under-react if they have limited information. They will over-react to new information which supports their position and they will under react to negative information as well. This behavior explains the momentum effect.

Investors do not behave as if the Market is efficient. If the market is efficient investors must behave as if it is. If investors quit gathering information then prices will no longer reflect all publicly available information. The important question is how do investors form expectations? Also how do those expectations impact market prices? Theorists have some ideas, but nobody is sure.

Active managers fail to outperform the market for three reasons: (1) poor investment decisions, (2) poor risk control, A (3) high fees and expenses.

The EMH and the CAPM may fail as descriptive theories but they are good proscriptive theories. They are good models for how investors should behave. Most investors do not measure risk properly. They do not avoid active management if they do not have a competitive advantage. They do not alter risk exposure through borrowing and lending. They do not build portfolios using optimizers.

Markowitz developed the idea that risk could be quantified using the variance or standard deviation of a stocks return. This concept is referred to as total risk.

The total risk of a portfolio is not the sum of total risk of each stock in the portfolio but rather the covariance or correlation between each pair of stocks in the portfolio.

Markowitz defined an efficient portfolio as one, which minimized the total risk of a portfolio for any level of expected return. Subsequent analysis of risk developed two principles:

- Systematic risk is the only risk investors are rewarded for assuming in efficient capital markets. Systematic risk cannot be diversified away by investors.
- Unsystematic risk may be diversified away and earns no return in efficient markets.

Asset pricing models show the relationship between the expected return for a stock and risk factors. The two dominant asset pricing models are: the Capital Asset Pricing Model (CAPM) and the Arbitrage Pricing Model (APM). The CAPM holds that there is only one form of systematic risk. This risk is movement of the market in general.

In theory the market is defined as all assets. Practically it is the rate of return on a broad-based market index such as the S&P 500. The CAPM holds that the expected return on an individual asset is a positive linear function of its index of systematic risk as measured by Beta. Only an assets beta determines its expected return.

There are two approaches to estimating Beta. First, using monthly or weekly returns and running a linear regression against some market index may estimate it. The Beta statistic generated using this approach is referred to as the historical beta. The regression equation is called the market model. The problem with the historical beta is that it is based upon historical data while the fundamental attributes of a company will change in the future. The second approach to estimating beta attempts to resolve this problem. First calculate the historical beta for all firms using the market model. Next identify the fundamental factors that affect the betas for firms. Estimate the following regression equation:

$$B_i = a_0 + a_1 x_{1,i} + a_2 x_{2,i} + ... + a_k x_{k,i} + e_i$$

Given the estimates for the parameters, a company's beta is estimated by substituting into the previous equation the company's fundamental attributes. The resulting beta is called a fundamental beta. The weakness of this approach is that it is assumed all firms are equally impacted by a fundamental factor.

Stephen Ross developed the Arbitrage Pricing Model in 1976. The APM holds that there may be more than one systematic risk. APM does not specify the fundamental factors. The APM states that investors want to be compensated for all factors that systematically affect the return of a stock. The compensation for assuming risk is the sum of the products of each factor's systematic risk. The model is:

$$E(R_i) = R_f + B_{i,m}[E(R_m - R_f] + B_{i,F1}[E(R_{f1} - R_f] + ... + B_{i,FK}[E(R_{FK}) - R_f]$$

If the only factor in the APM is market risk the APM reduces to the CAPM.

CAPM and APM are equilibrium models that tell us the relationship between risk and expected return. Factor models are empirically derived models that seek to identify the risk factors that explain stock returns. There are three types of factor models: (1) statistical factor models use historical and cross-sectional data on stock returns to derive factors which best “explain” stock returns; (2) macroeconomic factor models use historical stock returns and macroeconomic data to determine the economic variables that best “explain” stock returns, and (3) fundamental factor models use company in industry attributes as well as market data to “explain” stock returns.

Fundamental Factor Models (FFM) use company and industry attributes and market data as basic inputs to the model. Important factors are called raw descriptors. The output of a FFM is the expected return for a stock after adjusting for all of the risk factors. This return is called the expected excess return. From the expected excess return for each stock, a weighted average expected excess return for a portfolio comprised of stocks can be computed. Similarly the sensitivity of a portfolio to a given risk factor is a weight average of the factor sensitivity of the stocks in the portfolio. The set of factor sensitivities is the portfolio’s risk exposure profile.

Portfolio Managers can compute expected excess returns and develop risk exposure profiles for a market index in order to measure performance.