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9. ABSTRACT

Our objective in this paper is to provide a clear and easily comprehensive illustration of tax and expenditure incidence using a special case of the Harberger model. The special case we analyze is a world in which both production and consumption behavior can be adequately described by Cobb-Douglas functions. Making this particular behavioral assumption has the appealing virtue of allowing us to examine general equilibrium adjustments to taxes and fiscal spending in a sequential rather than simultaneous fashion. We are able, therefore, to manipulate a numerical model of an economy in a way that can be readily followed by those who are unfamiliar with, even frightened by, the differential calculus accompanying the usual presentation of the Harberger model. Since our aim is pedagogical we present no new analytical results but, instead, try to disseminate established ones to a wider readership. In analyzing a tax on capital income in one sector, a tax on all capital income, selective and general commodity taxes, and various expenditure policies, we focus on how a tax or expenditure change affects both the sources and uses sides of income for each of the groups labor, capital and government. Finally, we show that incidence is independent of absolute changes in the price level; only relative price changes have any bearing on the incidence question.

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A Numerical Exposition of the Harberger Model of Tax and Expenditure Incidence

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Winter, 1973

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A Numerical Exposition of the Harberger Model
of Tax and Expenditure Incidence

Arnold Harberger's development of a general equilibrium mathematical model of corporate tax incidence was a major contribution to understanding how this particular tax may affect income distribution.[1] The extension of Harberger's methodology by others to the examination of the incidence of a variety of other taxes and tax and expenditure situations comprises a growing body of literature, by common practice now referred to under the general heading of the Harberger model.[2] While an increasing number of people work with this model, or at least understand it, they would constitute a minority among those interested in budgetary incidence. Due to its mathematical formulation, the model remains largely inaccessible to those who have not had advanced training in economic theory as well as those unaccustomed to using differential calculus. The purpose of this paper is to present some simple algebraic and numerical examples in the effort to make

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[1]The first formal presentation of the model was A.C. Harberger, "The Incidence of the Corporation Income Tax," Journal of Political Economy, Vol. 70 (June, 1962), 215-240.

[2]Among the most important extensions of Harberger's original model are P. Mieszkowski, "On the Theory of Tax Incidence," Journal of Political Economy, Vol. 75 (June, 1967), 250-262; and C.E. McLure, Jr., "Tax Incidence, Macroeconomic Policy and Absolute Prices," Quarterly Journal of Economics, Vol. 82 (May, 1979), 254-267; "The Theory of Tax Incidence with Imperfect Factor Mobility," Finanzarchiv, Vol. 30, No. 1 (1971), 27-48. This literature is surveyed in C.E. McLure, Jr., "General Equilibrium Incidence Analysis: The Harberger Model after Ten Years," Journal of Public Economics, forthcoming.

the important insights of the Harberger model available to a wider readership.

Our goal is to illuminate the inner workings of the Harberger model in the simplest possible fashion. We believe that a set of numerical and algebraic illustrations offers the best expositional means of achieving this aim. Through them we attempt to show clearly the economic factors and principles which combine to determine tax and expenditure incidence in the Harberger model. However, substituting numerical examples for more rigorous mathematical analysis is not costless. We are confined to a numerically tractable special case of the Harberger model which does injustice to the wide range of situations to which the model can be applied. That is, analyzing the incidence of a tax on corporate income, a tax on all income from capital (a property tax), selective and general commodity taxes, and the incidence of government expenditures, we make specific assumptions about the nature of production and consumption for the sake of mathematical convenience. These assumptions permit us to deal sequentially with a general equilibrium model of the economy but limit the generality of the results in some cases. Less restrictive assumptions require solving nonlinear simultaneous equations. The full flavor of the Harberger model's versatility can be appreciated by noting it is possible, though inconvenient, to relax any of our specific assumptions about economic structure. We point to some of the major differences in the analytical outcome if somewhat different assumptions are made. We do this in order to prevent the conclusions in a special case from being accorded undue importance.

There is nothing novel about the conclusions we reach in analyzing a special case. In the case of the corporate income tax these particular

results have been discussed before by Harberger, among others, in his original article on the subject. Since our concern is with enhancing comprehension of the economic processes that generate these results, we make no apology for our reliance on a special case as a device for illustrating the Harberger mechanism of long-run tax shifting. If our presentation of this matter is any improvement over what Harberger has already offered, it is mainly in our explanation of how the real income of different income groups is affected by changes in the cost of purchasing different commodities. We have put more emphasis than Harberger originally gave on the side of the uses or disposition of income in determining tax incidence. We also demonstrate how tax incidence is a function of changes in relative prices rather than changes in the absolute price level. In other words, we are able to show that changes in the distribution of real income, i.e., the incidence of the tax or expenditure, are independent of how a tax may affect an index of the absolute price level. Moreover, we extend Harberger's method of numerical analysis to other taxes and to the expenditure side of the budget.

1. Assumptions of the Analysis

The theoretical incidence of any tax or expenditure is highly sensitive to what is assumed about the competitive nature of the economy and the relevant time period for adjustment to the imposition of taxes. In the Harberger model the economy is assumed to be comprised of two competitive sectors which between them fully employ the fixed amounts of capital and labor. Factor prices are assumed to be sufficiently flexible to insure the full employment of each factor. Competition prevails in both product and factor markets and prevents the emergence of pure profit in either sector and (so long as

both factors are perfectly mobile) the divergence of factor prices between sectors. After the introduction of a government budget, enough time elapses to permit factor markets to achieve the competitive outcome of equality of after-tax returns for all factors irrespective of where they are employed. This period of time defines the long run in economic analysis and is distinguished from a short-run situation in which factor employments are temporarily fixed for a sector. The possibility that the total supply of a factor may change in response to taxation is ignored throughout the analysis. That is, a tax on the earnings of capital is assumed to have no effect on savings rates just as a tax on labor income is assumed to exert no influence on hours or days of work. If these latter assumptions are not satisfied it can be shown that in general in the long run a tax on any factor will reduce the future earnings of both factors.[1]

To facilitate exposition, the following notation is introduced:

a) X and Y denote the levels of output in the economy's two sectors.

b) K and L represent, respectively, the total amounts of capital and labor in the economy. K_X is the amount of capital employed in the production of output X and so forth for K_Y, L_X and L_Y.

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[1]Tax incidence in a growth context has been analyzed by M. Krzyzaniak, for example, in "The Long-run Burden in a General Tax on Profits in a Neoclassical World," Public Finance, Vol. 22, No. 4 (1967), pp. 472-492. It should be noted that the Harberger model is "long-run" in the sense in which economists usually use that term, namely to refer to a time span adequate for factor flows to equalize net factor payments to mobile factors. Because the Harberger approach does not consider effects on capital accumulation, Krzyzaniak has preferred to label its time period the "intermediate-run." The case in which one factor is immobile between sectors is considered briefly in footnote 2, page 20.

c) P_X and P_Y stand for the output prices of X and Y confronting consumers in the market.

d) $P^*_{K_X}$ is the price (cost) of capital to entrepreneurs in industry X when a tax is levied on the income of capital in X. The net or after-tax price of capital received by all capitalists in the long run is P_K, where $P_K = P^*_{K_X}(1-t_{KX})$ and $P_K = P_{K_Y}$, if only a tax t_{KX} is applied to the gross earnings of capital in X. The price of labor is represented by P_L and, in the absence of special taxes on labor income, labor is assumed to be sufficiently mobile to earn the same wage in different employments so that $P_L = P_{L_X} = P_{L_Y}$.

(e) Z denotes the level of net national product in the economy. It is assumed to be fixed in nominal (money) terms throughout the first six sections.

Two other special features of notation merit some discussion. First, since our objective is to analyze changes in income distribution and economic efficiency in response to various types of taxes we use primes to denote after-tax values of the preceding variables. Thus X' and P_X' measure the output level and price of commodity X after the economy has made its competitive adjustments to the imposition of a tax. Second, it is useful and non-restrictive to assume that the units in which factors and output are measured can be defined in such a way that all prices are initially equal to unity. That is, $P_X = P_Y = P_K = P_L = 1$ before taxes are imposed. This harmless convention allows one to use interchangeably physical and value terms for factors or output in the initial no-tax situation.

A particular technology is assumed to link the employment of factors to the level of output in each sector. Each sector is assumed to be character-

ized by a Cobb-Douglas production function. This means that there are constant returns to scale in each activity and that in each sector each factor receives a fixed fraction of the value of the sector's output inclusive of any factor taxes that may be levied.[1] (Factor income shares are allowed to differ between sectors.) Thus production relations are described by the following set of equations:

(1) $K_X P^*_{K_X} = \alpha X P_X$,

(2) $L_X P_{L_X} = (1 - \alpha) X P_X$,

(3) $K_Y P_{K_Y} = \beta Y P_Y$, and

(4) $L_Y P_{L_Y} = (1 - \beta) Y P_Y$, where α and $(1 - \alpha)$, and β and $(1 - \beta)$, are the respective income shares of capital and labor in the two sectors. It perhaps bears emphasizing at the outset that it is mainly this Cobb-Douglas specification of production and a similar assumption about consumption that confine us to dealing with a special case of the Harberger model.

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[1]This Cobb-Douglas model of the economy borrows generously from Harberger's own attempt to simplify the explanation of corporate tax incidence. See A.C. Harberger, op. cit. and "Taxation: Corporation Income Taxes," International Encyclopedia of the Social Sciences, Vol. 15 (1968), pp. 538-45. The two Cobb-Douglas production functions can be written as follows:

$$X = K_X^{\alpha} L_X^{1-\alpha} \quad \text{and} \quad Y = K_Y^{\beta} L_Y^{1-\beta}$$

Competition in both product and factor markets insures that profit-maximizing firms will employ each factor up to the point where the market price of the factor, inclusive of tax, is equal to its marginal contribution to the value of output. This principle defines the demand for factors of production in each sector. Equilibrium capital employment in sector X is given by $P^*_{K_X} = P_X \, \partial X/\partial K$, where $\partial X/\partial K$ is the marginal product of capital in producing output X. Carrying out the indicated differentiation, it is easily established that $K_X P^*_{K_X} = \alpha XP$, as stated in equation (1). Similarly, from the other factor demands we have equations (2)-(4). The constancy of relative factor shares results from what economists call a unitary elasticity of factor substitution.

Another special and limiting assumption is used to describe the behavior of consumers. Analogous to the production side of the economy the utility or preference functions of all participants in the economy, including governments, are assumed to be Cobb-Douglas in nature but not the same for everyone. In concrete terms this means that irrespective of the level of private incomes and the configuration of relative prices for the two commodities, total expenditures on one good in the economy are assumed to comprise the same constant fraction of income.[1] Commodity demand functions, therefore, take the form:

(5) $XP_X = C\ (XP_X + YP_Y)$ and

(6) $YP_Y = (1 - C)(XP_X + YP_Y)$, where C is the aggregate average propensity to spend on good X.

From equations (1) - (6) we can see just how restrictive is the assumption that both production and utility functions are Cobb-Douglas. Substituting from equations (1) and (2) into equation (5), we obtain the following results:

(1a) $K_X\ P^*_{K_X} = \alpha\ C\ (XP_X + Y\ P_Y)$, and

(2a) $L_X\ P_{L_X} = (1 - \alpha)\ C\ (X\ P_X + Y\ P_Y)$.

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[1] In the jargon of economics, these demand functions imply a unitary price and income elasticity of demand for each commodity. For example, either a 5 percent reduction in the relative price of X or a 5 percent increase in total income will stimulate the overall demand for X by 5 percent. Although these functions constrain the responses of consumers in this way they have enjoyed some success in empirical work. See, for example, R. Stone, "Linear Expenditure Systems and Demand Analysis: an Application to the Pattern of British Demand," Economic Journal, Vol. 64 (1954), pp. 511-27. At a high level of aggregation, such as that used in our numerical model, consumer substitution may occur mainly within rather than between broad expenditure categories. If so, our spending assumption may not be an unreasonable one.

Similar substitutions from equations (3) and (4) into equation (6) produce similar expressions. The crucial point is that, given the three Cobb-Douglas assumptions, the payments that one factor receives in one industry constitute a fixed fraction of total national expenditures on the two products. This outcome restricts the generality of the theoretical results presented here, but it also makes it feasible to derive results for the numerical examples presented below. As noted earlier, we indicate at several points how results are affected by relaxation of the Cobb-Douglas assumptions.

But, before proceeding to the presentation of numerical examples, it might be worthwhile to note that the usefulness of Cobb-Douglas results may not be quite as limited as it at first appears. First, there is substantial evidence that production may be quite nearly Cobb-Douglas. Second, in his original article (p. 234), Harberger shows that for a wide range of realistic values of the various parameters in the more general mathematical formulation of his model, the results may not differ much from those in the Cobb-Douglas case.

In analyzing the distributive impact of a particular tax we assume that there are no taxes and no government in the initial situation. (Otherwise one is implicitly analyzing the interaction of at least two taxes or the presence of a tax package.) On the expenditure side of the newly imposed budget governments are assumed in the first eight sections to spend their tax revenues in a distributionally neutral manner. This means that when private demand declines on account of taxation, public demand for the two goods is assumed to exactly replace this loss of private demand.[6]

[1]The questions of distributionally neutral expenditures and the problems posed by pre-existing taxes are discussed in the survey by McLure mentioned above and in the literature discussed there. Moreover, the incidence of non-neutral expenditures is the subject of section 9 below.

2. The Numerical Structure of the Economy

Certain national income concepts need to be satisfied for a consistent numerical model of our two-sector economy. One requirement is that the value of output in each sector add up to the sum of factor payments and that the sum of sectoral production (or value-added) equal national income. These statements about where and how earnings occur refer to the sources of income. An additional constraint is that the total income received by owners of labor and capital be exhausted in the expenditures made on the two commodities and that the sum of expenditures on either good equal the total value of production of that good. These statements on how income is spent refer to the uses of income.

The equations below and Table 1 express the assumed behavior of producers and consumers on which our examples are based and embody the assumptions set forth in the preceding section:

$$(7) \quad Z = P_X X + P\ Y;$$

$$Z = P^*_{K_X}\ K_X + P_K\ K_Y + P_L\ L, \text{ or}$$

$$Z = P_K\ (K_X + K_Y) + P_L\ (L_X + L_Y) + t_{K_X}\ P^*_K\ K_X.$$

The first expression views national income Z from the side of the uses of income while the second and third focus on the side of the sources of income. National product equals either total outlays on the two goods or the sum of net payments to capital and labor in both sectors plus the tax revenues generated by a tax on capital income in X if that is the only tax in the economy.

In our hypothetical two-sector economy the sources of income are as shown in part 1a of Table 1. National income is $2400, one half of which is

TABLE 1: Sources and Uses of Income

1a. Sectoral source of capital and labor income

Factor	Sector (value of output) X	Y	Total Factor Income
Labor, L:	480	960	1440
Capital, K:	720	240	960
Total Output	1200	1200	2400

1b. Distribution of expenditures by factor group

Consumer Group	Commodity (value) X	Y	Total Expenditure
1. Labor	720	720	1440
2. Capital-before tax	480	480	960
3. Capital-after tax	300	300	600
Government-after tax	180	180	360
Total (1+ 2 or 3)	1200	1200	2400

1c. First alternative expenditure pattern

Consumer Group	Commodity (value) X	Y	Total Expenditure
1. Labor	432	1008	1440
2. Capital-before tax	768	192	960
3. Capital-after tax	480	120	600
Government-after tax	288	72	360
Total (1 + 2 or 3)	1200	1200	2400

1d. Second alternative expenditure pattern

Consumer Group	Commodity (value) X	Y	Total Expenditure
Labor-after tax	388.8	907.2	1296
Capital-after tax	595.2	148.8	744
Government	216	144	360
Total	1200	1200	2400

earned in each sector. Labor earns sixty percent of total income and capital receives the remainder. The production of X, compared to Y, is relatively capital intensive since the share of capital is larger in that sector. (α is .6, but β is only .2.) Alternative descriptions of consumer behavior are presented in parts 1b to 1d of Table 1. In part 1b, each income group (capitalists, workers, and governments) spends half its incomes on each good. In parts 1c and 1d recipients of labor income spend larger proportions of their budget on commodity Y than either government or the recipients of capital income. These two parts portray identical consumption patterns except that government, if its expenditure is to exactly replace private spending, devotes 80 percent of its income to X in part 1c and only 60 percent to X in part 1d. In both parts labor channels 30 percent of its disposable income to expenditures on X while the corresponding allocation on the part of capital is 80 percent. Given the assumed data of Table 1, the stage is set for the analysis of incidence in different tax situations.

3. A Tax on Capital in X

It is instructive to begin with the case of a tax levied on the income of capital earned in only one sector of the economy. This case corresponds to the corporate income tax in which income earned by capital is taxed more heavily in one sector X than in the other (noncorporate) sector. As Harberger has emphasized on a number of occasions, the corporate income tax is a discriminatory tax striking corporate source income more severely than capital income in the noncorporate sector (comprised mainly of housing and

agriculture).[1] This is true even after account is taken of heavier property taxation in the noncorporate sector and the favorable capital gains treatment of retained earnings in the corporate sector. Even within the corporate sector the tax discriminates in favor of firms enjoying unusual tax preferences (especially petroleum) and against capital intensive activities and those firms or industries whose capital structure has a low ratio of debt to equity. The point here is that the corporate income tax is really a tax on corporate equity capital. We do not consider these distinctions as they are not central to our story.

Suppose a tax on capital income earned in sector X is introduced at the rate of 50 percent of the gross price of capital paid by entrepreneurs. The effect of the tax is to reduce the net income received by owners of capital (stockholders) in industry X by $360 and to raise tax revenues by the same amount.[2] With the initial sectoral allocation of capital the net (after-tax) price of capital declines immediately to $.50 in industry X but remains at $1.00 in industry Y. Over time competitive capital market institutions that connect users and owners of capital will react to this divergence in net returns by shifting relatively more of the economy's capital stock into the higher earning sector. Capital will flow from the taxed sector X to the untaxed sector Y until the net earnings of capital are the same in each sector. This is the long run shifting mechanism by which a tax on only one employment of a factor affects the real incomes earned by that factor throughout the economy. Since the net income earned

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[1] See Harberger, op. cit., and "Efficiency Effects of Taxes on Income from Capital," in M. Krzyzaniak, editor, Effects of Corporation Income Tax (Detroit: Wayne State University Press, 1966), 107-17.

[2] Because of the dual Cobb-Douglas assumptions, capital in X receives a gross return of exactly $720, so long as the nominal value of

by all capital falls to \$600 (\$960 - \$360) and the economy is endowed with 960 units of capital, the net return to capital in both sectors is 5/8 of a dollar in the new post-tax equilibrium. A sectoral reallocation of capital occurs until only 576 units of capital remain in industry X (576 = \$360 ÷ \$5/8) and there are 384 units of capital in industry Y (384 = \$240 ÷ \$5/8). Of course, the gross return to capital in industry X rises to 10/8 of a dollar. (The 50 percent tax applied to a gross return of 10/8 gives a net return of 5/8.)[1]

These calculations are sufficient to determine who bears the burden of the tax on the side of sources of income. Since tax receipts are \$360 and the net price of 960 units of capital falls by 3/8, or \$360, the increase in tax receipts is exactly matched by a reduction in the income available

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net national product is unchanged at \$2,400. Relaxation of the assumptions would modify this result once adjustment of the capital stock began. For example, if the elasticity of substitution in industry X were less than unity, capital's share of the (unchanged) value of output of good X would rise in response to tax-induced capital outflows. Relaxation of the assumption that net national product is fixed in nominal terms at \$2,400 is allowed in section 7. It is shown there that in a model such as this incidence depends upon movements in relative prices, which are independent of movements in the nominal level of net national product, and not upon absolute prices, per se.

[1] These results can also be established by substituting the commodity demand expressions for XP_X and YP_Y in equations (6) and (7) and forming the ratio of these two equations. This straightforward manipulation gives:

$$\frac{K_X P^*_{K_X}}{K_Y P_{K_Y}} = \frac{\alpha C}{\beta(1-C)} \text{ or } \frac{K'_X}{K'_Y} \qquad \frac{\alpha C(1-t_{K_X})}{\beta\ (1-C)} .$$

The higher the tax rate t_{K_X} the more capital will be driven out of the taxed sector and the smaller K'_X/K'_Y will be. Initially $K_X/K_Y = 3$. In the new equilibrium position after the tax $K'_X/K'_Y = (6/10 \cdot \frac{1}{2} \cdot \frac{1}{2})(2/10 \cdot \frac{1}{2}) = 1.5$ and $K'_X = 576$, $K'_Y = 384$. In the noncorporate sector $384\ P'_K = 240$ or $P'_K = 5/8$ while in the corporate sector $576 \cdot P^*_{K_X} = 720$ or $P^*_{K_X} = 10/8$. Thus the net price of capital in the two sectors has fallen to \$5/8 while the gross price of capital in sector X has risen to \$10/8.

to recipients of capital income. Thus, on the side of sources of income, capital bears th entire burden of the corporate income tax and labor none of it.[1] On the other hand, the allocation and price of labor are unaffected by the tax. Since labor continues to receive the same fraction of an unchanged national income after the tax is imposed, labor incomes escape unscathed from the tax. But the mobility of capital insures that the corporate income tax is borne on the side of the sources of income by all owners of capital, not just those in the corporate sector.

That the tax is borne fully by capital on the side of sources of income does not mean that it causes no changes in incomes for consumers. This question, the incidence of the tax on the side of the uses of income, requires some knowledge of consumption patterns and how relative product prices are altered by the tax. Since by assumption national income is fixed at its original level in the after-tax situation, if one commodity increases in price the other must decrease in price. Consumers who spend a relatively large proportion of their income on the good which becomes more expensive are made worse off. Similarly, those consumers who purchase

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[1]This particular result illustrates propositions 9 and 10 in the original article on tax incidence by Harberger to the effect that capital carries the full burden of the corporate tax in a Cobb-Douglas world. It will not hold in other situations. Except in the Cobb-Douglas case, capital will always bear a larger burden of the tax on the sources side if the taxed sector is capital intensive relative to the untaxed sector. A Cobb-Douglas production function implies that it is reasonably easy to substitute between factors in production. Fixed factor proportions, on the other hand, mean there is no possibility of changing the composition of capital and labor in production. Harberger's original paper on incidence established the following conclusions. If only the taxed sector has fixed proportions, labor (capital) loses more on the side of the sources of income the more labor (capital) intensive is the taxed sector. However, if only the untaxed sector has fixed factor proportions capital must always be burdened by more than the increase in tax revenues, i.e., there is an income transfer from capital to labor in this case.

relatively large amounts of the good which becomes less expensive benefit from this price reduction. These gains and losses to various consumer groups can be quantified through the use of the following equation for any income group:

$$(7) \quad \Delta R_{\mu} = - (X'\Delta P_X + Y'\Delta P_Y),$$

where R is real income and deltas (Δ)indicate changes in variables. This equation defines a change in a person's real income on the uses side as the sum of the products of the quantity of each commodity purchased in the new after-tax situation and the percentage change in the price of the commodity. It is essentially a Paasche index number formulation of the change in the purchasing power of a dollar of income. It measures the change in the cost of a given consumption bundle, where that bundle consists of the items different groups purchase in the post-tax equilibrium. It can easily be shown that the sum of the income gains and losses experienced by various consumer groups (including governments) is zero.[1]

We have seen above that the cost of capital increases in sector X and diminishes in sector Y and the cost of labor is unchanged. Thus, the cost and price of output will rise in X relative to Y. This shift in relative

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[1]Any index of the "cost of living" that relies on the use of fixed consumption weights in the current rather than the base period is called a Paasche price index. Since each group of income recipients in the economy has a demand function of unitary price elasticity, $\frac{\Delta X}{X'} = \frac{\Delta P_X}{P_X}$. Substituting ΔP_X from this expression into the equation for ΔR_{μ} yields $\Delta R_{\mu} = \Delta XP_X + \Delta YP_Y$. For the economy as a whole increases in the value of production of Y must be counterbalanced by equal value reductions in the value of production of X if national income is unaltered in size. At the aggregate level of the whole economy changes in real income on the uses of income side must necessarily sum to zero. Of course, the excess burden of taxation is ignored throughout this paper, except in section 8.

commodity prices will divert some consumer demand away from X as consumers substitute relatively less expensive Y for X in their budgets. Consumers who strongly prefer corporate sector products will therefore lose and, at the same time, a gain is conferred on those who purchase primarily non-corporate output.

New product prices are found by dividing the constant level of expenditure on each product by the new or after-tax quantity of output of the good. To calculate changes in output, use is made of the following supply relationship:

$$(8) \quad \frac{\Delta X}{X} = \alpha \frac{\Delta K_X}{K_X} + (1-\alpha)\frac{\Delta L_X}{L_X} \quad .$$

This expression says that the percentage change in the output of good X is equal to the weighted average of the percentage change in the two inputs, where the weights α and $(1-\alpha)$ are the income shares of capital and labor in the value of good X.[1] This relationship holds for any production process displaying constant returns to scale, as long as competition prevails in factor and product markets. Inserting the values from the solution we have obtained thus far into the supply equation, we see that $\Delta X/X = .6(-144)/720) = -.12$, so $X' = X(\ .88\ )$ and $X' = 1056$. The analogous result for Y is $\Delta Y/Y = .2(144/240) = .12$ so $Y' = Y(\ 1.12)$ and $Y' = 1344$.

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[1] Any production function, written as $X = f(K,L)$, which possesses the property of constant returns to scale can be expressed in first differences as $\Delta X = f_K \Delta K + f_L \Delta L$ where f_K, f_L are the marginal products of capital and labor respectively. Rewriting this expression gives $\frac{\Delta X}{X} = \frac{f_K K}{X} \cdot \frac{\Delta K}{K} + \frac{f_L L}{X} \cdot \frac{\Delta L}{L}$. If factor markets are competitive, the terms $\frac{f_K K}{X}$ and $\frac{f_L L}{X}$ can be identified as the income shares of capital and labor, which together sum to one.

With this information in hand new product prices are discovered to be $P'_X = 1200/1056 = 25/22$ and $P'_Y = 1200/1344 = 25/28$. Percentage price changes are $\Delta P_X = 3/22$ and $\Delta P_Y = 3/28$. Forming the relative price ratio $P'_X / P'_Y = 28/22$, we see that the price of good X has risen with respect to the price of Y by 27.27 percent.

The average propensity to consume is defined as the ratio of the consumption of a good to a consumer's total income. In the unlikely event that all consumers had identical average propensities to consume good X, the shift in relative product prices would have no redistributive consequences. This result is easily demonstrated by referring to portion 1b of Table 1 which, because of our constant-share expenditure assumptions, exhibits both the initial and post-tax consumption expenditure patterns. For each group physical purchases in the post-tax situation are 22/25 as large as dollar expenditures for good X (because $P'_X = 25/22$) and are 28/25 as large for good Y (because $P'_Y = 25/28$. Using this result and the equation for ΔR_μ the redistribution of real income on the uses side is calculated as:

$$\Delta R_{\mu L} = \$720(22/25)(3/22) - \$720(28/25)(3/28) = 0$$

$$\Delta R_{\mu K} = \$300(22/25)(3/22 - \$300(28/25)(3/28) = 0$$

$$\Delta R_{\mu G} = \$180(22/25)(3/22) - \$180(28/25)(3/28) = 0$$

If each income group and governments have the same average spending pattern there is no redistribution on the side of uses of income.[1]

Alternatively, the impact on income distribution of changes in relative prices occurring in the context of different spending patterns can be assessed by supposing we have the alternative consumption profiles given in section 1c

[1] It should be noted that this result occurs because all consumer groups have the same average propensity to consume good X (in this case, 1/2), and not because that common average propensity happens to be 1/2.

of Table 1. In that case, the following changes in income appear on the uses side:

$$\Delta R_{\mu L} = -\$432(22/25)(3/22) + \$1008(28/25)(3/28) = -\$51.84 + \$120.96 = \$69.12$$

$$\Delta R_{\mu K} = -\$480(22/25)(3/22) + \$120(28/25)(3/28) = -\$57.60 + \$14.40 = -\$43.20$$

$$\Delta R_{\mu G} = -\$288(22/25)(3/22) + \$72(28/25)(3/28) = -\$34.56 + \$8.64 = -\$25.92$$

Both capitalists and the government are harmed by the rise in the relative price of X since they are heavy consumers of this good. Labor, on the other hand, benefits from the fall in the price of Y, the good which labor consumes disproportionately. A reversal in these preference patterns would, of course, reverse these particular results. If we stay with these particular results a table can be constructed to indicate for each factor group and the government the income gains and losses which arise on both the sources and uses side of income, and in total:

Group	Sources	Uses	Total
Capitalists	-$360	-$43.20	-$403.20
Workers	-0-	$69.12	$ 69.12
Government	$360	-$25.92	$334.08

We see that capitalists lose $43.20 in their capacity as consumers, over and above the $360 loss on the side of sources of income. Similarly, labor, which suffers no loss on the sources side actually gains on the uses side. Section 7 shows that the change in real income experienced by any group, though not its division between the sources and uses sides, is insensitive to movements in the absolute price level. Finally, note that the government's loss on the side of the uses of income occurs because its $360 in tax receipts is actually worth $25.92 less than if the revenue had been raised in a way that did not raise the price of good X, the good it prefers, relative to the

price of good Y, the good it buys in smaller proportion than the private sector.

4. A Tax on All Uses of Capital

Consider the outcome if a tax rate of 3/8 or 37.5 percent is imposed on the gross income earned by capital services everywhere in the economy, including the imputed net rental income of homeowners. This tax would generate government receipts of $360 (on a base of $960) and keep tax revenues the same as they were when only a tax on corporate source capital income was levied.[1] In such a case the net return to capital falls immediately to 5/8 or $.625 per unit. Since capitalists earn this return no matter where their capital is employed, this is the equilibrium net price of capital, with the gross price or cost of capital unchanged. Therefore, no competitive pressure emerges for an intersectoral reallocation of factors and output is unchanged in both industries. Moreover, since neither the cost of labor nor the cost of capital is affected by the tax, product prices experience no change. All that happens is that capital bears the entire burden of the tax. A general tax on capital is neutral and quite unlike a selective tax on capital in X, which induces capital to move from the taxed to the untaxed industry, raises the relative price of the taxed good, and causes the amount purchased of the taxed sector's product to fall and that of the untaxed sector to rise. Both taxes have identical effects on the sources side, but the selective tax induces capital flows and provokes a redistribution on the side of the uses of income, whereas the general tax on capital does not.

[1] It should be noted that strictly speaking government revenues are not the same in real terms in the various cases we examine, although revenues are $360 in each case (except in section 7), due to the government's

The neutrality of a tax on capital income earned in all of capital's employments is a robust result which, with few exceptions, would be true no matter what was either assumed or established about the structure of the economy's production and consumption activities. By the same line of reasoning a general tax that is levied on all sources of labor income would be borne entirely by labor. Thus, for example, the nearly comprehensive coverage of the U.S. social security tax makes it highly likely that this tax, including the portion contributed by employers, is paid for entirely by reductions in labor income.

The property tax in the United States has many attributes of a general tax on capital incomes.[1] It strikes the income from all forms of land and capital, and, to the extent it is applied at uniform rates, closely resembles a general tax. In fact, of course, the property tax is not a uniform rate, nationwide tax. It is levied at varying rates by the over 70,000 state and local tax jurisdictions that administer it. However, this diversity can be encompassed by considering property taxes to be composed of two parts.[2] The

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gains and losses on the uses side resulting from tax-induced shifts in relative prices. This minor complication is ignored.

[1]Although the tax is levied on the asset value of a piece of property it is possible to convert this tax into an equivalent tax on the income from the property through the use of simple discount formulas. For example, if property worth $100 earns $10 per year perpetually, a tax of one percent on the value of the property is the same as a 10 percent tax on the income from the property.

[2]This conceptualization of the property tax corresponds to the treatment by P. Mieszkowski, "The Property Tax: An Excise Tax or a Profits Tax?", Journal of Public Economics, Vol. 1, No. 1 (April, 1972), 73-96. We must note that the verbal discussion in the text is not fully consistent with the Cobb-Douglas assumptions underlying the examples in this paper. In particular, if production and utility functions are Cobb-Douglas, none of the differential property taxes can burden or benefit land or labor. They can, of course, affect relative product prices and incidence on the uses side in a way analogous to the discriminatory corporation income tax. But if demand has an elasticity greater than unity, as is likely in a

first is an average rate of property taxation that applies uniformly to all of the various jurisdictions while the second is the positive or negative deviation of each taxing authority from the national average. The average rate of property taxation would be a general tax on all capital incomes and would be borne by all recipients of income from capital in the form of lower net returns.

The portion of the property tax that is a deviation from the national average burdens some uses of capital more heavily than others. The incidence of this part of the tax is more difficult to assess, since there is a larger number of possible outcomes. A certainty is that communities with above-average rates of property taxation will pay more for the services of capital than communities charging below average rates if capital in each jurisdiction is to earn the same net return everywhere. The reallocation of capital in response to property tax differentials will closely resemble the capital

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regional context, earnings of land and labor can be reduced (increased) substantially by geographic differentials in property taxation, even if production is carried on under Cobb-Douglas conditions.

More attention is not devoted to taxation levied under conditions of less than perfect factor mobility in the present paper because of the similarity of results in the mobility and immobility cases in the Cobb-Douglas world. Briefly, a general income or consumption tax is borne in proportion to income or consumption, as in the complete mobility case (see below). A tax on an immobile factor will be borne entirely by owners of that factor, with no repercussions on factor allocation, relative product prices, or the earnings of other factor groups. In the Cobb-Douglas case a tax on a mobile factor will have exactly the effects described for the complete mobility case in the previous section. Finally, an excise on one commodity is equivalent to factor taxes on the two factors used to produce it and can be analyzed from the last two results just described. Further discussion of the effects of taxes levied in the context of incomplete factor mobility is contained in McLure, "The Theory of Tax Incidence with Imperfect Factor Mobility," op. cit., "The Inter-regional Incidence of General Regional Taxes," Public Finance, Vol. 24 (1969), 457-83, and "Taxation, Substitution, and Industrial Location," Journal of Political Economy, Vol.78, (January-February, 1970), 112-32. Those not versed in mathematics may find it useful to read McLure, "A Diagrammatic Analysis of the Harberger Model with One Immobile Factor," Journal of Political Economy (January-February, 1974), pp.

market adjustments that ensue whenever capital incomes are taxed at higher rates in the corporate sector. In high tax areas the higher cost of capital will be passed on in higher consumer prices if demand conditions permit. If they do not, because output prices are fixed by outside competition, owners of immobile labor and land services will bear the tax.[1] However, the losses sustained by consumers, workers or landowners in high tax areas will be matched by gains to these groups in low tax jurisdictions.

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[1]The general part of the property tax would not affect property values, because both income from capital and the net rate of return used to capitalize this income would decline by equal proportionate amounts. (If the average rate of property tax reduces the net return of capital from 10 to 9 percent, a $100 piece of property will now earn only $9 and be discounted at 9 percent.) On the other hand, property tax differentials will tend to be capitalized into asset values, especially land values if labor is mobile and can escape the tax. If, however, local tax differentials correspond to benefit differentials provided by varying levels of public expenditure, the capitalization of these benefits will oppose any depression of asset values due to capitalization of the tax.

While on the subject of capitalization it is interesting to examine how the corporate income tax will influence asset values. Will higher rates of corporate income taxation reduce the value of corporate stockownership? The answer is yes if only the short run is considered, but no if the nature of the long- un equilibrium adjustment is analyzed. In the short run an increase in corporate income taxes will reduce corporate earnings by the amount of the tax and the capitalization of this smaller earnings stream at the net rate of return on capital in other parts of the economy will diminish the value of ownership claims to this income. In the long run the net return in other sectors will be driven downward to equality with the net return in the corporate sector and the value of corporate stocks will be untouched by higher corporate income taxes. The numerical example of Section 3 illustrates this proposition. After the imposition of a 50 percent corporate income tax the gross price of capital in the corporate sector rises to 10/8 and the net return to capital is 5/8 in both sectors. Thus a unit of capital would earn about $1.25 before taxes and $.625 after taxes in the corporate sector. If the price of capital was 10 percent before the tax, a unit of capital earning $1 before the tax was worth $10. After taxes the same unit of capital invested in the corporate sector earns only $.625 but capital everywhere yields only 6.25 percent, so the original asset value of $10 would still prevail after taxes.

5. A Tax on Good X

An alternative to levying a tax on the receipt of income is to impose a tax on the expenditure of income. This may be done in either a selective or general manner. Suppose that an excise tax of thirty percent is applied against the gross price of good X, that is a sales tax of 42.86 percent applies to the net price of good X. With this tax rate government receipts will be the same as before, if net national product is unchanged. This tax is most easily treated as an equal rate tax on both capital and labor used in producing good X because payments to these factors completely exhaust the value of the product. This implies that the analysis of Section 3, the effects of a tax on only one use of a factor, will apply in the case at hand, but to each factor.[1] As the tax reduces the net return of both factors involved in producing X by .3, capital and labor will flow out of sector X until their net returns are the same in both sectors. The operation of this competitive principle will produce the pattern of net factor earning exhibited below:

Factor group	X	Y	Total Income	Net factor price
Labor	336(=480 · 7/10)	960	1440 · 9/10=1296	9/10
Capital	504(=720 · 7/10)	240	960 · 31/40=744	31/40
Government	360(=1200 · 3/10)		360	
Total Value	1200	1200	2400	

[1]This does not, however, imply that the tax is, in some sense, borne solely by "producers," instead of "consumers," as conventional analysis suggests. As always, we must consider both the sources and uses side. And as explained in detail in section 7, whether or not the value of net national product rises in response to the tax is immaterial so far as incidence analysis is concerned.

The calculation of net factor prices is illustrated for labor. Since 480 . 7/10 is the after tax income of labor in the taxed sector X, that sum plus labor payments in Y must add to total labor income times the net price of labor P'_L in all sectors: $480(7/10) + 960 = 1440 \cdot P'_L$. Thus the wage rate P'_L equals $(480 \cdot 7/10 + 960)/ 1440 = 9/10$. Note that the net price of labor (9/10 = .9) does not fall by as much as the net price of capital (31/40 = .775). Capital's return declines by 22.5 percent compared to a decline of ten percent in the net price of labor. Capital bears a relatively greater burden of the tax simply because sector X is relatively capital intensive compared to sector Y. If the factor intensities were reversed and sector X was relatively labor intensive, labor would bear proportionately more of the tax. If there were no difference in factor intensities both factor prices would diminish by the same percentage amount and the tax would be borne in proportion to initial shares of national income on the sources side.

The final allocation of factors between sectors is determined by dividing the net payments received by any factor in a given sector by that factor's net price. Thus:

	Sector X	Sector Y	Total
Labor in:	480 . 7/10 ÷ 9/10 = 373 1/3 ;	960 ÷ 9/10 = 1066 2/3	1440
Capital in:	720 . 7/10 ÷ 31/40 = 650 1/3;	240 ÷ 31/40 = 309 2/3	960

Given these factor reallocations, changes in industry output may be estimated using equation (8) above:

$$\Delta X/X = .6(69\ 2/3 \div 720) + .4(106\ 2/3 \div 480) = -14.695\% = 1023\ 2/3$$

$$\Delta Y/Y = .2(69\ 2/3 \div 240) + .8(106\ 2/3 \div 960) = 14.695\% = 1376\ 2/3$$

These supply changes provide the information needed to evaluate the after-tax prices of the two goods:

$P'_X = 1200/\ 1023\ 2/3 = 1.1723;\quad P'_Y = 1200/1376\ 2/3 = .8717;$

$P'_X\ /\ P'_Y = 1376\ 1/3\ /\ 1023\ 2/3 = 1.345.$

In this example, the price of X rises relative to the price of Y by less than the 42.86 percent tax on the net price. From the side of the uses of income the tax appears to be almost completely shifted forward to consumers as is conventionally assumed for a selective ad valorem tax.[1] If the expenditure pattern depicted in section 1d of Table 1 prevailed,[2] changes in real income due to the altered relative product prices would be:

$$R_{\mu L} = -\$388.8\left\{\frac{1023\ 2/3}{1200}\right\}\left\{\frac{176\ 1/3}{1023\ 2/3}\right\} + \$907.2\left\{\frac{1376\ 1/3}{1200}\right\}\left\{\frac{176\ 2/3}{1376\ 2/3}\right\} =$$

$$= \$518.4(14.695\%) = \$76.18$$

$$R_{\mu K} = -\$446.4(14.695\%) = -\$65.60;\quad R_{\mu G} = -\$72(14.695\%) = -\$10.58$$

Combining real income changes on both the sources and uses sides of income for the different factor owners and government gives the pattern of income redistribution shown below:

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[1] If factor intensities were the same in the two sectors, the price of X would rise by exactly 42.86 percent. Since in that case the tax would be neutral on the side of sources of income, we could say that the tax was borne exclusively by consumers of good X. Equal factor intensities produce constant opportunity costs, that is, a completely elastic supply of good X at initial prices.

[2] Total expenditures by capitalists and workers in part 1d of Table 1 are consistent with the reduced factor incomes described in the previous two paragraphs of the text. In this part of Table 1 as well as in part 1c average propensities to consume good X, both before and after tax, are .3 for labor and .8 for capital. Government's average propensity for the same good has been changed from .8 to .6 in order to maintain the neutrality of government expenditure. The problem of non-neutral government expenditure patterns is examined in Section 9.

Group	Sources	Uses	Total
Capitalists	- $216	- $65.60	- $281.60
Workers	- $144	$76.18	- $ 67.82
Government	$360	- $10.58	$349.42

With an original income of $960 capital owners lose $281.60 or about 29 percent of their initial income while workers suffer a reduction of $67.82 from their initial income of $1440 or about 4.7 percent. Capitalists are made relatively worse off by the tax both because it is imposed on the capital intensive commodity and because the taxed commodity has a relatively large weight in capitalists' budgets. This redistributive pattern may be compared with the one which arises from invoking the common rule of thumb that tax on a single commodity is shifted forward and borne in proportion to the consumption of the good. Allocating the burden of $360 worth of tax receipts according to the share of each factor group in the total consumption of X results in a burden of $142.2 for capitalists and $217.8 for workers. In the present case, this mechanical procedure attributes too much of the burden to workers, too little to capitalists, and none to government.[1]

6. A Tax on Both Goods or Income

In the Harberger model with no net saving, imposition of a uniform rate tax on both commodities is the same as introducing a value-added tax of the income type. Referring back to the national income equation (1), it is readily seen that a general tax on expenditures has the same incidence as a proportional income tax levied on all sources of income.

[1] Of course in the constant cost (equal factor proportions) case the traditional result holds, provided that part of the burden is allocated to the government as a purchaser of good X. See also fn. 1, p. 25.

In our example a tax of 3/20 or 15 percent on income or the gross value of all purchases (or 3/17 or 17.647 percent on the net value) would yield the same revenue, $360, as in the other tax situations. In the post-tax equilibrium the returns to all factors would decline by the same percentage as the tax rate. After taxes the prices of both labor and capital would be $.85 instead of the original price of unity. There would be no change in relative output prices, no factor reallocations and thus no modification of output levels. A broad-based income or expenditure tax would be neutral with respect to resource allocation decisions and would be borne in proportion to initial shares in national income. It should be noted that this result does not depend upon the particular specifications of production and demand relations used here.

7. Relative versus Absolute Price Changes

All of the conclusions that have been reached regarding the incidence of particular taxes are independent of how taxes may affect, or coincide with changes in, the absolute price level. Up to now we have assumed for the sake of convenience that aggregate demand policies keep the level of total monetary income at its original nominal amount ($2400). In this section additional proof is provided for the critical link between tax incidence and changes in relative product and factor prices by demonstrating the insensitivity of some previous results to alterations in absolute price levels.

In the previous example of a uniform tax on all goods one might more realistically assume that factor prices are rigid downwards and monetary policy would be sufficiently expansive to permit the tax to be shifted

forward in higher product prices. If both product prices rose by the extent of the tax, national income would increase from \$2400 to \$2823.53. Factor prices would remain unchanged while the price of both goods rose by 17.647 percent. Total tax receipts would be \$423.53, which in real terms could purchase the same outputs as \$360 could when national income is \$2400 (\$423.53 ÷ 1.17647 = \$360). One could say that the tax is borne in proportion to total consumption if both product prices rise by the same amount instead of being borne in proportion to initial income shares when product prices are unchanged. This is an example in which all of the incidence is on the sources side if product prices, and therefore the absolute price level, are held constant or is all on the uses side if product prices increase by the amount of the tax. In general, more of the incidence of a tax will be borne on the side of the uses of income if the increase in taxation is associated with higher absolute price levels. Nonetheless, no matter how incidence is divided between the sources and uses sides as absolute price levels vary, the total change in the real income of any group will always be the same.[1]

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[1]The total change in real income experienced by capitalists, for example, can be written as $\Delta R = \Delta R_S - \Delta R_\mu$, where $\Delta R_S = K\Delta P_K$, is the change on the sources side and $\Delta R_\mu = X_K\Delta P_X + Y_K\Delta P_Y$ is the change on the uses side. X_K and Y_K are the expenditures on the two commodities X and Y by capitalists. Define Δ_1 as the change (in price) when there is no tax-induced inflation (the value of net national income is fixed) and let Δ_2 be the change (in price) attributable solely to inflation of the average price level. The real income gain or loss for capitalists in the inflation case can be written as: $\Delta R = K(\Delta_1 P_K + \Delta_2 P_K) - X_K(\Delta_1 P_X + \Delta_2 P_X) - Y_K(\Delta_1 P_Y + \Delta_2 P_Y)$. Since inflation does not affect relative prices and all prices are initially unity it follows that $\Delta_2 P_K = \Delta_2 P_X = \Delta_2 P_Y = \Delta_2 P_L = \Delta P_i$ where ΔP_i represents the common rate of price inflation. Thus ΔR can be rewritten as

$$\Delta R = K\Delta_1 P_K + X_K\Delta_1 P_X + Y_K\Delta_1 P_Y + \Delta P_i(K - X_K - Y_K).$$

Recalling that initial prices are unity, the expression $(K - X_K - Y_K)$ can be recognized as expressing the budget constraint of capitalists and is therefore equal to zero. The last term for ΔR above then disappears, taking with

Nothing happens to disturb this conclusion in the slightly more complicated situation where only one of the commodities is taxed. In this case, assume that upon introduction of the 30 percent tax on X national income increases to $2,823.53 from $2400. (This choice is discussed further below.) As before, this higher income level will be divided evenly in consumption between goods X and Y. Consumers will spend $1,411.765 on good X and a like amount on good Y. In industry Y the total of $1,411.765 is available for factor payments, but in industry X only 7/10 of $1,411.765 is so available, the remaining $423.53 being paid into government coffers as a result of the excise tax on X. Of the total amount of income earned in industry Y, two tenths, or $282.353, will be spent on capital services and eight tenths, or $1,129.412, on labor services. From the net income earned in the other industry X, capital will receive six tenths, or $592.941, and labor four tenths, or $395.294. Comparing these after-tax payments to labor and capital with those in the absence of both the tax and the change in national income valuation provides the measurements for the after-tax prices of the factors, at the higher level of national income. For labor the price after tax is $\frac{\$1,524.706}{\$1,440}$ = 1.0588 while for capital the net return is $\frac{\$875.294}{\$960}$ = .91176. The ratio of these after-tax rewards is the same as when the tax is applied to X and the national income is held constant (.9/.775). Moreover, the expenditure of 1,411.765 dollars on good Y will buy only $1,376\frac{1}{3}$ units of good Y so the price of Y rises to 1,411.765/1376 1/3 or 1.0255 in the after-tax equilibrium. Since relative factor prices are unaffected

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it the apparent impact of inflation on real income changes. Similarly, inflation does not disturb the real income position of other income recipients. However, it should be clear that inflation will affect the division of any real income change between the sources and uses sides with more of the burden being carried on the uses side (but being offset on the sources side), the higher the rate of inflation.

by the increase in only the value of national income, the allocation of factors between the two sectors, and hence the absolute amounts of output, must have also experienced no change. The production of X will therefore remain unchanged at 1023 2/3 units. In addition to all factor prices net of tax and the price of good Y having a value close to unity, the price of good X is 1,411.765/1023 2/3 = 1.3791 under this particular set of assumptions about what happens to the total dollar amount of national income. Relative product prices, $P'_X / P'_Y = \frac{1411.765}{1376\ 2/3} \div \frac{1,411.765}{1023\ 1/3} = \frac{1376\ 1/3}{1023\ 2/3} = 1.345$, are the same as before. Since both factors and good Y have almost the same prices as in the pre-tax situation, it is possible to assert, if one wishes, that consumers of good X pay nearly the full burden of the tax.

More importantly, it is easily seen that relative prices are what matter for tax incidence, not absolute prices. Relative to the previous case, total expenditure was assumed to rise by 17.647 percent from $2400 to $2,823.53 and all prices went up with this increment to spending by the same percentage amount. The price of X rose from 1.1723 to 1.3791; the price of Y rose from .8717 to 1.0255; the net price of labor rose from .9 to 1.0588 and the net price of capital went from .775 to .91176. However, no real magnitudes are affected by this 17.647 percent escalation of all factor and product price levels. Relative product and factor prices display the same relationships in the post-tax equilibrium, regardless of the assumption made about the level of national output in dollar terms. Since factor allocations and output levels depend exclusively on these relative prices, it follows that the quantities of factor employment and sectoral production have also undergone no changes. All that has happened is that in the post-tax equilibrium every factor and output has experienced an equal percentage

increase in price. Thus no one can be better or worse off than before the price increase and consequently the incidence of the tax is divorced from this change in absolute prices.

This logical deduction is supported by further numerical example. Compared to the original no-tax situation capitalists lose \$84.706(=\$960x.08824) on the sources side because the price of capital falls from unity to .91176. By the same token labor gains \$84.706 (\$1440 x .0588) on the sources side. Thus after-tax incomes are \$1524.706 and \$875.294 for labor and capital respectively. From section 1d of Table 1 the average propensity to consume X is .3 for labor and .8 for capital. The same propensity for government is .6. Therefore, expenditure on good X after taxes amounts to \$457.412 for labor and \$700.235 for capital. Government also spends \$254.118 of its receipts on X. The price of X has risen to 1.3791 so the percentage change in P_X is 37.91% and physical purchases are only 1/1.3791 = .7251 as large as dollar purchases. P_Y increases to 1.0255, a percentage change of 2.55%, while physical purchases are only 1/1.0255 = .9751 as large as before. On the uses side of income, labor, capital and government sustain the following losses:

$$\Delta R_{\mu L} = -\$457.412(.7251)(37.91\%) - \$1067.294(.9751)(2.55\%) =$$
$$= -\$125.736 - \$26.538 = -\$152.28$$

$$\Delta R_{\mu K} = -\$700.235(.725)(37.91\%) - \$175.059(.975)(2.55\%) =$$
$$= -\$192.485 - \$4.353 = -\$196.84$$

$$\Delta R_{\mu G} = -\$254.118(.725)(37.91\%) - \$169.412(.975)(2.55\%) =$$
$$= \$69.8533 - \$4.2124 = -\$74.07$$

Pulling together both sides of the income statements generates the incidence pattern below:

Group	Sources	Uses	Total
Capitalists	-$84.706	-$196.84	-$281.55
Workers	$84.706	-$152.28	-$ 67.57
Government	$423.53	-$ 74.07	+$349.46

Except for a slight discrepancy due to rounding error, these real income losses of capitalists and workers are identical with those shown earlier for the same tax. They also sum to $349.12 which matches the real income transfer to the government sector of $349.46.

No inconsistency has been inserted into this numerical example by comparing $360 in tax revenue in one instance with $423.53 in revenue in the other. In real terms $423.53 will buy exactly what $360 would buy under the assumption that national income is pegged at $2400. If all prices go up by 17.647 percent an index of prices would be 1.1765 in the new equilibrium as opposed to 100 in the original situation. Deflating the $423.53 in revenue by the index value of 1.1765 gives $360 of revenue in real terms. This change in the total value of tax receipts is related to the general price level and is distinct from the previous problem of the composition of government spending between the two goods.

In order to clarify the point of this section, we might come at the issue from yet another angle. Among the many alternative assumptions that could be made about the level of national income in this illustration, two have special appeal. One is that national income rises by enough that the price of good Y is unchanged from the pre-tax situation. This approach has the advantage that the price effect on the side of uses of income is given directly by the change in the price of good X and looks very much like the traditional partial equilibrium result. In the case under examination, it

would appear that the tax would be borne in large part by consumers of good X and in a minor degree by factor owners.

The second attractive assumption, and the one actually made, is that national income increases by enough that the nominal value of factor payments is unchanged. Thus the result must be that in the aggregate there are no losses on the side of sources of income, so far as the two factor groups are concerned. (Of course, the government gains the amount of the tax revenues on the sources side). Moreover, it turns out in this case that consumers of both goods lose on the uses side since the price of both goods rise. But the crucial fact is that in total the real burdens and benefits experienced by each factor group and the government are invariant to the assumption made about the nominal level of national income. Workers gain relative to capitalists on the sources side and lose less in relative terms on the uses side. On balance, workers lose $68 and capitalists lose $282, both in real terms, regardless of the level of national income. And the taxes collected by the government are worth only $349, valued at pre-tax prices.

8. Taxation and Economic Efficiency

So far the discussion has dwelled entirely on the equity question of how tax changes are related to income distribution. The Harberger model also lends itself to an explicit determination of how tax changes may impair or improve the efficiency with which the economy's resources are allocated. Losses in efficiency due to taxes, or the excess burden of taxation, will be briefly considered here.

If resources are efficiently allocated in the pretax equilibrium it follows by definition that taxes which alter this original allocation will impose an efficiency cost on the economy. If the original allocation of

factors is inefficient the introduction of taxes could conceivably improve the allocation and confer an efficiency gain on the economy. We ignore this possibility since nothing of a general nature can be said about it. In contrast with the incidence analysis where changes in net factor earnings were crucial, the analysis of efficiency turns on how taxation provokes differences in gross factor earnings.

Taxes carry an efficiency cost along with them if, in the post-tax equilibrium, there is any sectoral divergence in gross factor prices. If a divergence occurs the value of total output could always be increased if resources were transferred to activities where their gross payments would be higher. This potential gain in the value of total output is the welfare loss occasioned by taxation. Neutral taxes, those with a zero efficiency cost, are those which avoid creating a discrepancy in gross factor prices among sectors. Looking back over the set of numerical examples that have been employed, it can be seen that this criterion of neutrality was satisfied in only two instances. These were a tax on all uses of a factor and a tax on all expenditures (or an income tax, since the two are equivalent in the Harberger model).

Unless the original situation is distorted, one can feel comfortable with the conventional wisdom that selective or discriminatory taxes are more inefficient (less neutral) than general or nondiscriminatory taxes.[1] We use the results of Section 4 to illustrate excess burden of a particular discriminatory tax: the corporate income tax. In the post-tax equilibrium of

[1]Recently Ballentine and Eris have modified the original Harberger model to consider how tax incidence analysis is affected by the existence of initial factor and product taxes. See J. Gregory Ballentine and Ibrahim Eris, "On the General Equilibrium Analysis of Tax Incidence," Discussion Paper no. 38, Program of Development Studies, Rice University, 1973. Their paper shows how incidence and efficiency aspects of taxation can be dealt with simultaneously instead of in the traditional serial fashion which we have followed.

our numerical example, 144 units of capital have fled from the corporate sector seeking higher net returns in the non-corporate sector. The net earnings of capital are \$5/8 in both sectors. This is also the gross return to capital in the non-corporate sector. In the corporate sector the gross earnings of capital are the higher value of \$10/8. Thus, if the last of the 144 units of misallocated capital were transferred to the corporate sector total income in the economy would increase by \$5/8. Successive transfers back to the corporate sector would reap smaller income gains as the gross earnings of capital fell in the corporate sector and rose in the non-corporate sector. When all but one of the 144 units of capital had been returned to the corporate sector there would be a negligible income advantage of transferring back the last unit of capital. The sum of the income gains resulting from such a retransfer process constitutes the income loss associated with the corporate income tax. If the demand curves (schedules of marginal revenue products) for capital are linear in both sectors this income loss can be measured by the formula $1/2\ \Delta K_X(P^*_{K_X} - P'_K)$.[1] Applying this formula to the numerical example of Section 4 results in a welfare cost of \$1/2 . 144 (5/8) = \$45. This reduction in total income amounts to

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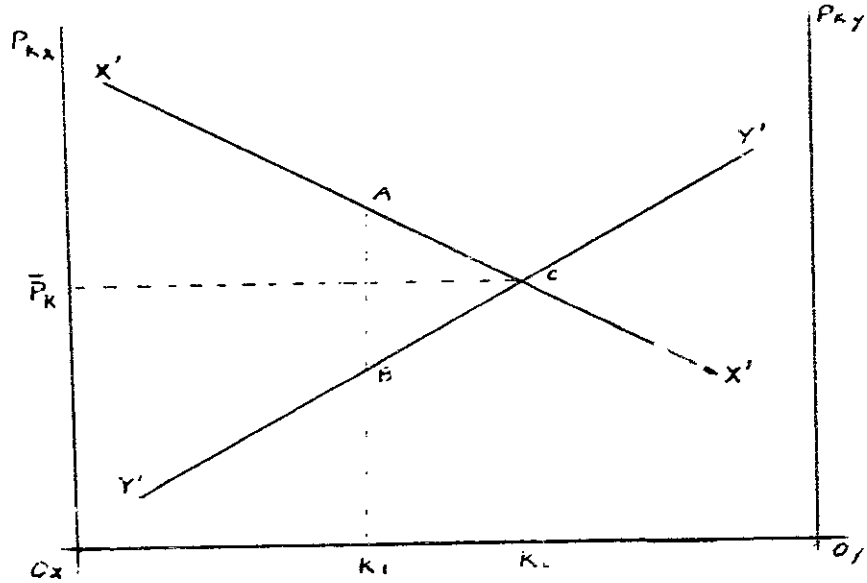
[1] In the Cobb-Douglas world, the demand curves for capital are not linear. Thus the expression given in the text is only an approximation to the actual welfare loss. It should be noted that the welfare loss calculation presented here differs in one important respect from that presented in A.C. Harberger, "Efficiency Effects of Taxes on Income from Capital," in M. Krzyzaniak, ed., Effects of Corporation Income Tax (Detroit: Wayne State University Press, 1966), pp. 107-17. In a sense the present analysis is a calculation of the welfare cost of financing certain budgetary expenditures through a corporation income tax, rather than in a neutral way (such as through a general sales on income tax or a tax on all capital). The Harberger analysis measures, for given government spending, only the welfare costs of taxing capital in a non-neutral way in that it considers differences between gross rates of return in the corporate and non-corporate sectors. In other words, Harberger compares the difference between the gross rate of return in the corporate sector and the average rate of return to all capital, whereas by assumption there is no tax on non-corporate capital in our analysis.

1.88 percent of total income and 12.5 percent of the total tax yield of $360.

Figure 1



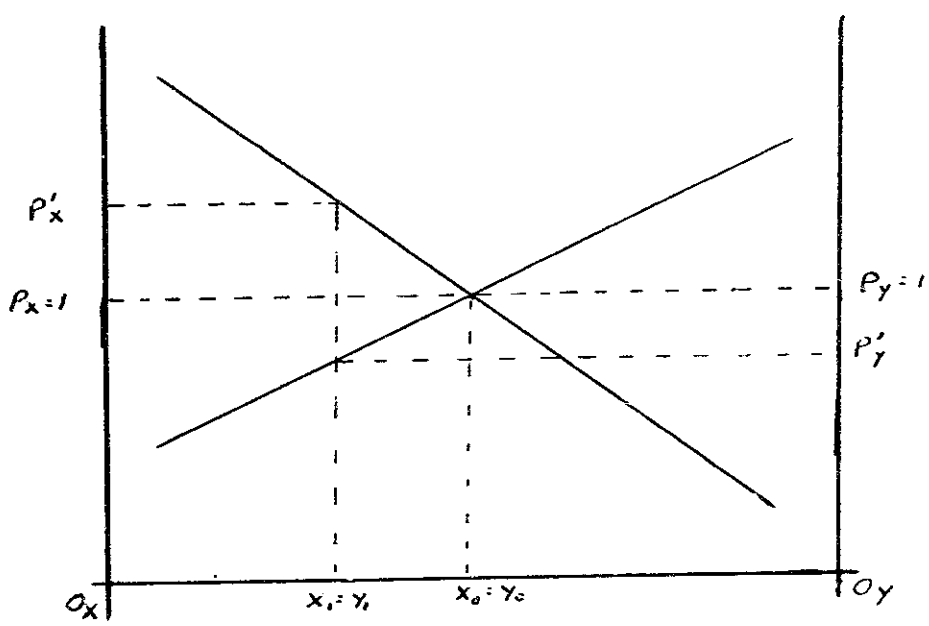
A diagram of factor market equilibrium shows more explicitly the basis for the formula used to calculate the tax induced efficiency loss. Figure 1 depicts the sectoral allocation of capital at various rates of return to (prices of) capital. X' - X' describes the demand for capital in sector X while Y' - Y' does the same for sector Y. Between them these two sectors employ the total capital stock of O_X - O_Y. In the absence of taxes levied on capital income earned only in sector X competition will tend to establish the common rate of return $\bar{P}_K$ in both sectors at which O_X - K_o amount of capital will be used in sector X and O_Y - K_o in sector Y. In the post-tax equilibrium K_o - K_1 units of capital have left sector X for sector Y until the same net rate of return is achieved everywhere. As noted above, the welfare change in transferring a unit of capital from sector X to sector Y is simply the difference in the gross rate of return the capital can earn in the two sectors. Since this difference reflects the distance AB

between the demand (or marginal revenue product) curves for capital in the two industries, the welfare loss resulting from the tax-induced distortion of capital use is simply the area of the triangle ABC, or $1/2\ \Delta K_X(P^*_{K_X} - P_X)$.

The same economic considerations are involved in determining the efficiency loss attributable to selective commodity taxation. In section 5 we saw that an excise tax imposed only on the value of production in sector X caused a contraction in the output level of X by 176 1/3 units and an increase in sector Y's output level of the same amount. In the post-tax equilibrium gross prices of both capital and labor were higher in sector X than in Y indicating that resources are inefficiently allocated in this after-tax situation. What has happened of course is that consumers have reacted to the relatively higher price of output X by buying more of the now cheaper product Y and less of the more expensive product X.

As before, this welfare loss can be shown diagrammatically. (See Figure 2)

Figure 2



In the pre-tax case, the price of both goods is one and outputs are $O_X X_o$ and $O_Y Y_o$. When a tax is imposed on good X, its output falls to $O_X X_1$ and the output of industry Y rises to $O_Y Y_1$, due to the tax-induced transfer of resources between sectors. The problem, then, is to value the loss of real income as welfare associated with the fall in the production of good X and the increase in welfare resulting from the increased output of industry Y and compare the two. Consumers behave as though the production of X has become more costly when in fact it has not from a real resource point of view. Consequently, consumers are induced to forego more value in the reduced production and consumption of X than they receive in the form of an expanded value of production and consumption of Y. The loss in value experienced by consumers (and the economy) is the efficiency cost of the tax.

Subject to certain restrictions, it is usually accepted that the prices paid for goods are a reasonable measure of the value consumers place on them. Thus, the value of a given unit of consumption of good X or good Y is given by the height of the relevant demand curve. For the units of consumption labelled X_o and Y_o in the non-distorted situation, the price--and value--of both goods is one, by assumption. On the other hand, the units labelled X_1 and Y_1 have values of P'_X and P'_Y, respectively, and intermediate units have intermediate values. Thus, the decrease in the amount of good X consumed can be seen to have a total value equal to the area under the demand curve for X from X_1 to X_o, and analogously, for Y the increased consumption is equal to the area under the demand curve from Y_o to Y. In total then, the loss in welfare due to tax-induced distortion is equal to the difference in these two areas, which is approximately $1/2(P_X-1)(X_o-X_1)-1/2(1-P_Y)(Y_1-Y_o)$.[1]

[1] If supply and demand curves were linear (which they are not in a Cobb-Douglas world) no approximation would be involved. Note that if an

Because of the assumption that both goods have initial prices of unity, units of the two goods are interchangeable, and this expression can be written as $1/2\ (\Delta P_X - \Delta P_Y)(X_o - X_1)$, or $-\ 1/2\ (\Delta P_X - \Delta P_Y)\ \Delta X$. From the example of section 5 ΔP_X is (1200/1023 2/3 - 1) and ΔP_Y is (1200/1376 1/3 - 1) or \$.1723 and -\$.1283 respectively. Since ΔX is 176 1/3, the cost of the selective tax on commodity X is given by 1/2 (176 1/3)(\$.3006)= \$26.50.

These two examples illustrate, but do not exhaust, the many ways in which taxation may distort the allocation of resources in an economy. It is worth noting that the loss induced by the excise tax turns out to be significantly smaller than that which arises from the imposition of an equal yield tax on the income earned by capital in the same sector even though in real terms the tax is larger in this case. This is easily explained. Both taxes distort consumer choices (relative product prices)--in one case by 27.3 percent (corporation income tax) and in the other by 34.5 percent (excise tax). But the corporation income tax also distorts producers' choice of production technique and factor employments--whereas the excise tax is neutral so far as relative factor usage is concerned. This added distortion on the production side far outweighs the difference in distortion on the consumption side.

9. Expenditure Incidence

Up to now we have assumed that government expenditure policy is neutral, that is, that it merely replaces the private spending that does not occur because of the tax-induced reductions in private incomes. This assumption

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excise tax on product Y existed initially the imposition of an equal rate excise tax on X would give rise to an efficiency gain instead of a loss.

has been convenient in that it allows us to focus directly upon the incidence of taxation, without the issue being obscured by changes in real income that result from shifts in relative product and factor prices induced by a change in the composition of overall demand. But in the real world, government demand does not typically offset tax-induced reductions in private demand. Thus, it will be useful to investigate briefly expenditure incidence or the effects of non-neutral expenditure policies that do not merely offset the fall in private expenditures.[1] By analogy with the way in which we analyzed tax incidence while abstracting from expenditure incidence, we shall assume that the non-neutral expenditures are financed in a neutral way, in particular, through a tax on all income from capital.[2] A further extension of this paper would be to consider in detail the balanced-budget incidence of non-neutral financing of non-neutral expenditures. But that analysis is straightforward once the separate analyses of both tax and expenditure incidence are mastered and is rendered impossible here by space constraints in any case.

The approach to be employed can be called differential expenditure analysis. That is, we assume that $360 is raised from the general tax on capital and that it is spent in various alternative ways, instead of in a

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[1]The terminology is from R. Musgrave, The Theory of Public Finance (New York: McGraw-Hill, 1959), pp. 213-215. A more detailed treatment of this subject is found in C. McLure, "The Theory of Expenditure Incidence," Finanzarchiv (1972), pp. 432-53. It should be noted that expenditure incidence is concerned with the question of how non-neutral government spending affects relative prices and real incomes, and not with the question of who gets the benefits of publicly provided services, which the Harberger model is not useful in answering.

[2]It may be objected that this tax is not distributionally neutral in that it reduces the real incomes of only capitalists, and not those of labor. The point is that it is allocationally neutral, unlike the tax on capital in X or the excise tax on X itself. Neutral finance via a tax on all income or value added could be considered, but the choice actually made simplifies the analysis and exposition somewhat.

neutral way. Then we compare the real incomes of each factor group under the alternative expenditure assumptions with those assuming neutral expenditure in order to determine the differential effects of shifting from neutral expenditures to the various non-neutral spending patterns. The two non-neutral expenditure policies that we examine are (a) a shift of $96 of government purchases from good X to good Y and (b) spending of the entire $360 of tax revenue to hire one factor labor.[1] In addition, in subpart (c) we compare the effects of using tax revenue to hire labor with those of conscription.

Unfortunately, this analysis does not lend itself to the sequential approach followed in previous sections dealing with tax incidence. The problem is that consumption by the two groups of factor owners depend upon their incomes, which depend in turn upon consumption patterns. But some simple algebra will suffice to allow us to surmount the difficulties posed by simultaneity, and then we can proceed as before to solve for prices, outputs, factor allocations, and changes in real incomes originating on the sources and uses sides.

a) Shift of demand to product Y

Suppose that instead of following the expenditure pattern indicated in part 1c of Table 1, the budget provided for a shift of $96 of demand from good X to good Y, so that the government spent only $192 on good X. In that case total expenditure on good X would be as described in the following equation:

$$XP_X = .3L\ P_L + .8K\ P_K + 192. \qquad (9)$$

[1] Because total capital income is not constant in this case, as it is in the previous cases, we cannot simply assume that a tax rate of 37.5 percent (360/960) is used to raise the $360 of revenue. It is necessary to solve for the tax rate needed to raise the $360. In the first case the rate is 39.5 percent (360 ÷ 912 x 100) and in the second it is 49.6 percent (360 ÷ 726 x 100).

Since everything not spent on good X must be spend on good Y, we also have the following relationship:

$$Y\ P_Y = 2400 - X\ P_X. \tag{10}$$

In order to utilize equations (9) and (10) we must substitute into equations (11) and (12) for the total wage bill and the net income of capitalists. Total labor income is described as follows:

$$L_X P_L = .4\ X\ P_X\ ;\ L_Y P_L = .8\ Y\ P_Y$$

$$L\ P_L = .4\ X\ P_X + .8\ Y\ P_Y \tag{11}$$

Similarly, remembering the \$360 tax imposed on capital, we have:

$$K_X\ P_K^* = .6\ X\ P_X\ ;\ \ K_Y\ P_K = .2\ Y\ P_Y$$

$$K\ P_K = .6\ X\ P_X + .2\ Y\ P_Y - 360. \tag{12}$$

Substituting, we have

$$X\ P_X = .3(.4\ X\ P_X + .8\ Y\ P_Y\ ) + .8(.6\ X\ P_X + .2\ Y\ P_Y - 360) + 192 =$$

$$= (.2 + .48)X\ P_X + (.24 + .16)\ Y\ P_Y - 288 + 192 =$$

$$= .6\ X\ P_X + .4(2400 - X\ P_X) - 96$$

$$.8\ X\ P_X = 960 - 96$$

$$X\ P_X = 864 \div 0.8 = 1080$$

$$Y\ P_Y = 2400 - 1080 = 1320$$

These new equilibrium values due to the expenditure change can be summarized in the following table analogous to parts 1a and 1c of Table 1.

Changes on the side of sources of income can be read immediately from the top part of this table. Because government purchases have shifted toward the relatively labor intensive product, the total wage bill has risen by \$48, from \$1440 to \$1488. By the same token, both the gross and net returns to capital have fallen by \$48. Stated in the language of cost-benefit analysis,

Production

Factor	Sector X	Sector Y	Total Factor Income
Labor	432	1056	1488
Capital	648	264	912
Total	1080	1320	2400

Consumption

Consumer Group	Commodity X	Commodity Y	Total Expenditure
Labor	446.4	1041.6	1488
Capital	441.6	110.4	552
Government	192	168	360
Total	1080	1320	2400

the shift in public spending involves a pecuniary external economy for labor of $48 and an equal sized pecuniary external diseconomy for capitalists.

In addition to these effects on relative factor rewards, we can calculate the effects on relative product prices and incidence on the side of uses of income. To do this we proceed as before:

$$P_L = L\ P_L \div L = 1488 \div 1440 = 1.0333$$

$$P_K^* = K\ P_K^* \div K = 912 \div 960 = 0.95$$

$$P_K = K\ P_K \div K = 552 \div 960 = 0.575$$

These calculations confirm that the shift has raised the price of labor and reduced both the gross and net returns to capital.

$$K_X' = K_X\ P_K^* \div P_K^* = 648 \div .95 = 682.11$$

$$\Delta K_X'/K_X = -37.89 \div 720 = -\ .0526$$

$$K_Y' = K_Y\ P_K^* \div P_K^* = 264 \div .95 = 277.89$$

$$\Delta K\ /K = 37.89 \div 240 = .1578$$

$$L'_X = L_X P_L \div P_L = 432 \div 1.0333 = 418.06$$

$$\Delta L_X/L_X = -61.94 \div 480 = -.1290$$

$$L'_Y = L_Y P_L \div P_L = 1056 \div 1.0333 = 1021.94$$

$$\Delta L_Y/L_Y = 61.94 \div 960 = .0645$$

$$\Delta X/X = -.6(.0526) - .4(.1290) = .0831$$

$$\Delta X = -99.72 \qquad X' = 1100.28$$

$$P'_X = X P_X \div X' = 1080 \div 1100.28 = .9815$$

$$\Delta Y/Y = .2(.1578) + .8(.0645) = .0831$$

$$\Delta Y = 99.72 \qquad Y' = 1299.72$$

$$P'_Y = Y P_Y \div Y' = 1320 \div 1299.72 = 1.0156$$

As one would suspect, the shift in public procurement toward good Y bids up the relative price of that good. As a result, consumers who spend relatively large fractions of their budget on Y, the good toward which government demand has shifted, lose on the side of uses of income, and others gain. These gains and losses can be quantified as before:

$$\Delta R_{\mu K} = - X_K \Delta P_X - Y_K \Delta P_Y = \frac{(441.6)(.0185)}{.9815} - \frac{110.4}{1.0156}(.0156)$$

$$= 8.32 - 1.70 = 6.62$$

$$\Delta R_{\mu L} = \frac{(446.4)(.0185)}{.9815} - \frac{1041.6(.0156)}{1.0156} = 8.41 - 16.00 = -7.59$$

$$\Delta R_{\mu G} = \frac{(192)(.0185)}{.9815} - \frac{(168)(.0156)}{1.0156} = 3.62 - 2.58 = 1.04$$

As in earlier sections, we see that except for a small rounding error the losses to labor, the relatively heavy consumers of good Y, are exactly counterbalanced by the gains to capitalists and the government, both of which are relatively heavy consumers of good X, the good whose relative price has

fallen.[1] Finally, we can summarize the pattern of the gains and losses on the sides of sources and uses in the following table:

	Sources	Uses	Total
Capital	- 48.00	+ 6.62	- 41.38
Labor	48.00	- 7.59	40.41
Government	--	+ 1.04	1.04
Total	0	0	0

b) Use of tax proceeds to hire labor

Suppose now that instead of spending its $360 of tax proceeds in a neutral way the government simply used them to hire labor, as in the case, for example, of a volunteer army. In this situation equation (9) above would be replaced with the following:

$$X P_X = .3L P_L + .8K P_K . \tag{9a}$$

By the same token, equation (10) would be altered to the following:

$$Y P_Y = 2040 - X P_X \tag{10a}$$

Finally, equation (11) would become

$$L P_L = .4 X P_X + .8 Y P_Y + 360 \tag{11a}$$

Equation (12) would be as before.

Substituting as before, we have

$$X P_X = .3(.4X P_X + .8Y P_Y + 360) + .8(.6X P_X + .2Y P_Y - 360)$$

$$= .6X P_X + .4(2040 - X P_X) + 108 - 288$$

$$.8X P_X = 636$$

$$X P_X = 636 \div .8 = 795$$

$$Y P_Y = 2040 - 795 = 1245$$

[1]Note that although government demand shifts toward good Y, the government remains a relatively heavy consumer of good X.

Summarizing in tabular form, we have this picture of the economy after adjustment to the expenditure switch:

Production

	X	Y	G	Total
Capital	477	249	-	726
Labor	318	996	360	1674
Total	795	1245	-	2400

Consumption

	X	Y	L	Total
Capital	292.8	73.2	-	366
Labor	502.2	1171.8	-	1674
Government	-	-	360	360
Total	795	1245	360	2400

As we would expect, the policy of hiring labor raises the income of workers on the side of the sources of income and lowers that of capitalists relative to the case of neutral expenditure, in this case by $234. The effects on the unit prices of factors are as follows:

$P_L = 1674 / 1440 = 1.1625$

$P_K^* = 726/960 = 0.7562$

$P_K = 366/960 = 0.3812.$

Changes in factor utilization and output are as follows:

$K_X' = 477/0.7562 = 630.74$

$\Delta K_X/K_X = -89.26/720 = -.1239$

$K_Y' = 249/0.7562 = 329.26$

$\Delta K_Y/K_Y = 89.26/240 = .3719$

$L_X' = 318/1.1625 = 273.55$

$\Delta L_X/L_X = -206.45/480 = -.4301$

$L'_Y = 996/1.1625 = 856.77$

$\Delta L_Y/L_Y = -103.23/960 = -.1075$

$L'_G = 360/1.1625 \qquad 309.68$

$\Delta X/X = -.6(.1239) - .4(.4301) = -.2463$

$X' = (.7537)(1200) = 904.44$

$\Delta Y/Y = .2(.3719) - .8(.1075) = -.0117$

$Y' = (.9883)(1200) = 1185.96$

Several aspects of these results deserve discussion. The first is that the government hiring of workers results in hardly any drop in the production of good Y, once demand adjustments are allowed, but a substantial drop in production of good X. This occurs because of the large increase in the real incomes of workers on the sources side, together with the high propensity of workers to spend the additional income on good Y. This tendency is offset in part, however, by consumer substitution away from good Y since good Y is relatively labor intensive, and therefore more adversely affected by the relative shortage of labor and induced increase in the relative wage rate than is good X. The second thing worth noting is that capital is reallocated from industry X to industry Y in order that the induced demand for good Y can be met. But in relative terms labor is also allocated relatively more toward industry Y, in the sense that the percentage outflow of labor from industry X is far smaller than that from industry Y.

As before, we can utilize the information gained up to now to solve for the tax-induced changes in product prices and real incomes on the uses side. The new product prices are:

$P'_X = 795 \div 904.44 = .8790$ and

$P'_Y = 1245 \div 1185.96 = 104.97.$

The shift in demand toward labor raises the price of good Y for two distinct reasons. First, good Y is relatively more labor intensive than good X, so that it would be affected more by the induced rise in the wage rate. Second, as noted above, workers are relatively heavy consumers of good Y, so that when wage income rises the price of good Y is bid up. Of course, in general these two forces need not be reinforcing.

Changes in real income distribution on the side of uses of income are as follows:

$$\Delta R_{\mu K} = \frac{292.8}{.8790}(.1210) - \frac{73.2}{1.0497}(.0497) = 36.83$$

$$\Delta R_{\mu L} = \frac{502.2}{8790}(.1210) - \frac{1171.8}{1.0497}(.0497) = 13.68$$

$$\Delta R_{\mu G} = -\ 309.68\ (.1625) = -\ 50.32$$

These calculations differ from those presented previously in that government is a heavy loser on the uses side because it bids up the price of the one thing it uses all its budget to purchase, labor.[1] Thus we see that in terms of redistribution on the uses side the 16 percent rise in the price of labor, which neither workers nor capitalists consume directly, is more important than the 5 percent rise in the price of good Y, especially when the 12 percent drop in the price of good X is taken into account.

Finally changes in income on the uses and sources side can be summarized for this analysis as follows:

	Sources	Uses	Total
Capital	- 234	36.83	197.17
Labor	+ 234	13.68	247.68
Government	--	- 50.32	- 50.32
Total	0	0	0

[1]Note that if the price of labor had not risen, the government could have purchased 360 units of labor, instead of merely 309.68.

c) Conscription

As a final exercise in the use of the Harberger model, we consider the case of conscription of labor by the government. By so doing we demonstrate the enormous versatility of this relatively simple general equilibrium economic model. Moreover, the results for this case can usefully be compared with those for the case just examined, in which the government hires labor and pays it the market wage rate. In a sense this is a crude comparison of the income distribution effects of the draft with those of the voluntary army.

Suppose that the government drafts 360 workers, paying nothing for their services, and that its macroeconomic policies are consistent with a total national product, excluding the output of these workers, of $2,040.[1] The equations describing the adjustments in the private portion of the economy are as follows:

$$XP_X = .3\ L\ P_L + .8K\ P_K \tag{13}$$

$$LP_L = .4\ X\ P_X + .8\ Y\ P_Y \tag{14}$$

$$KP_K = .6X\ P_X + .2Y\ P_Y \tag{15}$$

$$YP_Y = 2040 - X\ P_X \tag{16}$$

These can be solved to yield the following results:

$X\ P_X = \$1,020$ $\qquad$ $Y\ P_Y = \$1,020$

$L\ P_L = \$1,224$ $\qquad$ $K\ P_K = 816$

[1] It might well be objected that the government must pay labor at least a subsistence wage. This complication is ignored in the present analysis for the sake of convenience. But it can easily be included by adding to total labor income whatever the government pays to its conscripts and subtracting from the proper point in the circular flow of income the taxes needed to pay labor, as in the extreme case in which labor working for the government receives the market wage. The choice of macroeconomic assumption is irrelevant to the real outcome of the analysis, as demonstrated in

$P'_L = 1.1333$	$P'_K = .850$
$L'_X = 360$	$K'_X = 720$
$L'_Y = 720$	$K'_Y = 240$
$X' = 1{,}080$	$Y' = 960$
$P'_X = .9444$	$P'_Y = 1.0625$

We see that expenditures on the two goods both fall by 15 percent, the fall in nominal national income. This means that payments to the two factors must also fall by 15 percent, given the Cobb-Douglas production functions. (The equiproportionate fall in factor payments explains the equiproportionate fall in expenditures on the two goods.) This does not, however, imply that there is no redistribution of income. The unit price of capital falls by the same fraction as total payments to capital, since the capital stock is unchanged. On the other hand, the wage rate rises by 13 1/3 percent, since the labor supply available to the private sector falls by 25 percent and total wage payments fall by only 15 percent. Stated in yet another way, the 1,080 units of labor that are not drafted earned $1,224 in wages, whereas they earned only $1,080 before the initiation of the draft. Thus on the side of sources of income there has been a transfer of $144 in real income from capitalists to workers, over and above the transfer of $360 from conscripted workers to the government.

The capital stock is not reallocated between sectors, since payments to capital fall by 15 percent in both sectors. Similarly, the number of workers employed in each sector falls by 25 percent in each sector, since total wages

section 7. The one actually made is consistent with national income falling by the $360 no longer paid to conscripted labor. Note that the comparison between a conscript and a volunteer army involves different sized armies. Under conscription 360 men belong to the army while only about 310 have joined our volunteer army. One could, if he prefers, make alternative comparisons which hold constant the size of the army.

fall by the same fraction in both sectors. But because Y is the relatively labor intensive good, its output falls by a greater fraction (.20) than does the output of X (.10).[1] Because total expenditures on each of the two goods falls by 15 percent, the price of good Y rises by 1/16, while the price of good X falls by 1/18. Seen from yet another angle, the rise in the price of labor and fall in the price of capital result in a higher price of good Y, the labor intensive good, and a lower price for X. Of course, labor loses on the side of uses of income, since it is the relatively heavy purchaser of good Y, and capitalists gain. The amount of this redistribution is $28.80.

The gains experienced on the sources side by capitalists, the government, and the two classes of workers and on the uses side by capitalists and unconscripted labor are as shown in the following table:

Factor	Sources	Uses	Total
Drafted labor	- 360	--	- 360
Market labor	+ 144	- 28.8	+ 115.2
Total labor	- 216	- 28.8	+ 244.8
Capital	- 144	+ 28.8	- 115.2
Government	+ 360	--	+ 360
Total	0	0	

This example completes our treatment of expenditure incidence but does not nearly exhaust the almost limitless opportunities for real world application of the concept.

[1]The reason output of Y falls a great deal under a volunteer but not a conscripted army is that in the former case income increases for the group (labor) which strongly prefers Y while income falls for the same group in the latter case.

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31. "A Diagrammatic Exposition of General Equilibrium Tax and Expenditure Incidence Analysis with One Immobile Factor" (1972), 36 pp..... .....Charles E. McLure, Jr.

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