AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D. C. 20523

BIBLIOGRAPHIC INPUT SHEET

FOR AID USE ONLY

1. SUBJECT CLASSIFICATION

A. PRIMARY: Agriculture — AE10-0000-G302

B. SECONDARY: Agricultural economics--Latin America

2. TITLE AND SUBTITLE

Minimization of unit costs as a basis for making fertilizer recommendations

3. AUTHOR(S)

Cate,R.B.

4. DOCUMENT DATE: I969

5. NUMBER OF PAGES: 32p.

6. ARC NUMBER: ARC 631.8.C357

7. REFERENCE ORGANIZATION NAME AND ADDRESS

N.C.State

8. SUPPLEMENTARY NOTES *(Sponsoring Organization, Publishers, Availability)*

(In Prelim.rpt.no.3)

9. ABSTRACT

10. CONTROL NUMBER: PN-RAA-417

11. PRICE OF DOCUMENT

12. DESCRIPTORS

Unit costs
Fertilizer

13. PROJECT NUMBER

14. CONTRACT NUMBER: CSD-287 Res.

15. TYPE OF DOCUMENT

AID 590-1 (4-74)

7

631.8
C357

Project File

AUGUST 1969

INTERNATIONAL SOIL FERTILITY
EVALUATION & IMPROVEMENT SERIES

Preliminary Report No. 3

MINIMIZATION OF UNIT COSTS AS A BASIS FOR MAKING FERTILIZER RECOMMENDATIONS

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August 18, 1969

Dear Colleague:

Our Preliminary Report No. 1 has provoked a considerable amount of constructive criticism, as we had hoped it would. The present follow-up manuscript is being circulated in the same spirit as the first, and we trust that it will likewise generate helpful discussion.

This paper is divided into five parts: introduction, text, and three appendices. Considerable rearrangement and editing may be needed, but it is felt that the present breakdown will facilitate comments by a wide variety of readers. (Ultimately, two or more articles may be desirable.)

The introduction stresses that yields are affected by many factors and that ideally recommendations should take all of these into account. Unfortunately, much of the pertinent data frequently are lacking.

The text presents the suggested approach in simplified form for those readers who do not wish to enter into the details of the discussion. Essentially, the method is to compare costs per unit of output at varying levels of input (fertilizer). For example: if and only if the cost of producing a ton of potatoes will be reduced (or not be increased) by the application of an additional two-hundred kilos of superphosphate, then these should be applied. In other words, we consider that the goal of the farmer is to minimize his cost of producing a ton of potatoes.

Appendix A contains a more detailed discussion of the economics of cost minimization, including a mathematical proof that overall rate of return is maximized when costs are minimized and that product price does not affect this relationship.

Appendix B is a discussion of the shape of response curves and the interpretation of soil fertility analyses.

Appendix C gives a graphical example of the application of the suggested method for making fertilizer recommendations.

We look forward to your comments.

Sincerely,

Robert B. Cate, Jr.
Research Associate

MINIMIZATION OF UNIT COSTS AS A BASIS FOR MAKING FERTILIZER RECOMMENDATIONS

Robert B. Cate, Jr.[1/]

Introduction

Preliminary Report No. 1 of the International Soil Testing Series (1) was entitled, "Economic Returns from Fertilizer Use - Based on Soil Test Information." Its preface is equally applicable to the present manuscript. Therefore it is quoted, in part, below. "The goal of a successful soil testing program is increased economic yields from the proper use of fertilizers and other soil amendments. The difficult problem is the evaluation of all of the factors involved in crop yields from an economic viewpoint in order to present optimum recommendations.

"Both research and educational activities are involved in the various phases of soil testing programs and include: obtaining representative soil samples, accurate and rapid laboratory analyses, correct interpretation of the data through correlation studies, recommended practices including the use of fertilizer and other soil amendments, and educational follow-up. Of course, the accuracy and value of all of the phases are dependent upon research data.

"Crop yields are influenced by many factors. Fitts (2) has expressed yield as a function of the crop, soil, climate, and management. Each of these general factors is subdivided into several components. Crop involves kind, variety, and population; soil includes fertility, adverse conditions such as acidity, alkalinity, and salinity, water relations, aeration, depth

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of horizons, etc.; climate involves precipitation, temperature, light, wind, etc.; and management includes cultural practices and the control of pests, weeds, insects, diseases, etc.

"In the process of formulating recommendations on the use of fertilizers, all factors in the yield equation should be considered. The greater the amount of research data available for each factor, the more precise can be the recommendations. The ultimate is never reached in any area, and strong research programs are needed everywhere. However, based on existing information in various parts of the world, concepts can be developed which greatly facilitate programs in other regions. Likewise, the concepts can be used to test existing data and to point out areas where more information is needed. We hope that the following will be useful in these respects."

Text

The following proposal for a simplified method of making fertilizer recommendations is based on four assumptions:

1. The economic goal of the farmer should be to to minimize his unit costs; i. e., the cost of producing a unit of output; e. g., a bushel of corn (see Appendix A).
2. Plant response to a particular fertilizer nutrient can be considered, for practical purposes, steeply linear up to about 75-85 percent of maximum yield under given conditions (see Appendix B).

3. Above this level of about 80 percent (on the average), there is increased probability that other factors will be limiting. Therefore even though fields which respond will continue to do so linearly, the average response slope will be less steep (see Appendix B).

4. Soil analyses, properly correlated, can improve the estimate of the probability of a response; and hence the slope of the response line even though they cannot prodict yields.

The actual method is presented graphically in Appendix C (with additiona discussion) and arithmetically as follows:

A. Estimate yield per hectare without a given input.

B. Estimate costs per hectare without a given input.

C. Divide B by A.

D. Estimate yield per hectare with a given input.

E. Estimate costs per hectare with a given input.

F. Divide E by D.

G. If F is equal to or less than C, apply the input.

Perhaps this is clearer if stated as follows:

$$\frac{B}{A} = \frac{\text{costs/ha without input}}{\text{yield/ha without input}} = C$$

$$\frac{E}{D} = \frac{\text{costs/ha with input}}{\text{yield/ha with input}} = F .$$

If $F \leq C$, apply input; and

if $F > C$, do not.

This simply states that an input should be applied if, and only if, it will not increase unit costs. The desirability of adding additional amounts of the same input can be determined in the same manner. (See Appendix A for the reasoning underlying this rule.) The principal problems which arise are: (1) the estimate of the size of the expected yield increase, and (2) the decision as to which costs to include. The yield increase estimate must be based on experimental data and other knowledge. Yield increase estimates are discussed further in Appendix B and Appendix C. Costs should include all cash expenses attributable to the particular field; such as:

Machine cost (prior to harvesting)
Seed
Pesticides
Rent (if actually paid)
Hired labor (prior to harvesting)
Irrigation .

These represent money <u>currently</u> being invested in the field, which could have been invested elsewhere. The farmer's problem is to decide how much fertilizer cost to add to these other <u>current</u> costs in order to maximize his <u>current</u> rate of return on this field.

Several other types of cost might be included; e. g.:

Overhead
Depreciation
Interest
Farmer's own labor
Returns to management
Returns to capital

Returns to land
Property taxes
Income taxes
Harvesting costs
Marketing costs
Crop shares .

Tentatively, the author suggests that these should not be included since they are not part of the current capital investment in the field. Also, most of these costs are very difficult to calculate, particularly before the crop is fertilized. For more discussion on these points, the reader is referred to the literature on capital rationing. Dean (3) provides a good introduction. However, it might be mentioned that the completely empirical method presented in our Preliminary Report No. 1 (4) seems to work because of its close relationship to the current operating ratio of agriculture; i. e., $\frac{\text{current expense}}{\text{current income}}$ (about .50 in recent decades in the U. S.).

It should be stressed that the proposed method is primarily useful for the case of the individual field after the choice of crop has been made. The actual crop choice is made on the basis of some estimate of the probable rate of return, as well as other considerations. However, as pointed out in Appendix A, the minimization of unit costs automatically maximizes the rate of return--regardless of price. If price estimates are also available, the method will help in making the most rational choice of crop since it permits the point of maximum rate of return to be determined for each crop (or other operation).

Although somewhat unrelated to the main theme of this paper, the author would like to suggest that some of the material presented here (especially in Appendix B) may have relevance to the possible relationship of excess fertilization to water pollution (eutrophication) and plant disease susceptibility because minimum unit costs will usually be attained at the point where plant nutrient needs are satisfied but not exceeded.

APPENDICES

APPENDIX A

Some Considerations on Cost Minimization

Most of what has been written for laymen on the economics of fertilizer use has emphasized the general rule: Apply a given nutrient until the cost of the last additional unit is equal to the value of the increase in yield which results. In practice, this rule is difficult to follow because of problems in predicting both yields and crop prices. Also, unless it is modified to include other investment opportunities, the rule is only applicable to cases where unlimited capital is available. In the more common situation of limited capital, total profit for the farm is maximized when the average rate of return is maximized. This occurs when average total costs are minimized for each field utilized, which means that costs per unit of output are minimized. Obviously, this is also the best competitive position for the farmer because he is better able to undersell other farmers or meet price cuts. In addition, the point of minimum unit cost (and maximum rate of return) has the great advantage for the decision-maker that it is independent of the price of the product. The following graph may aid in visualizing this concept. (An investment level of A for NPK is optimum at all prices.)

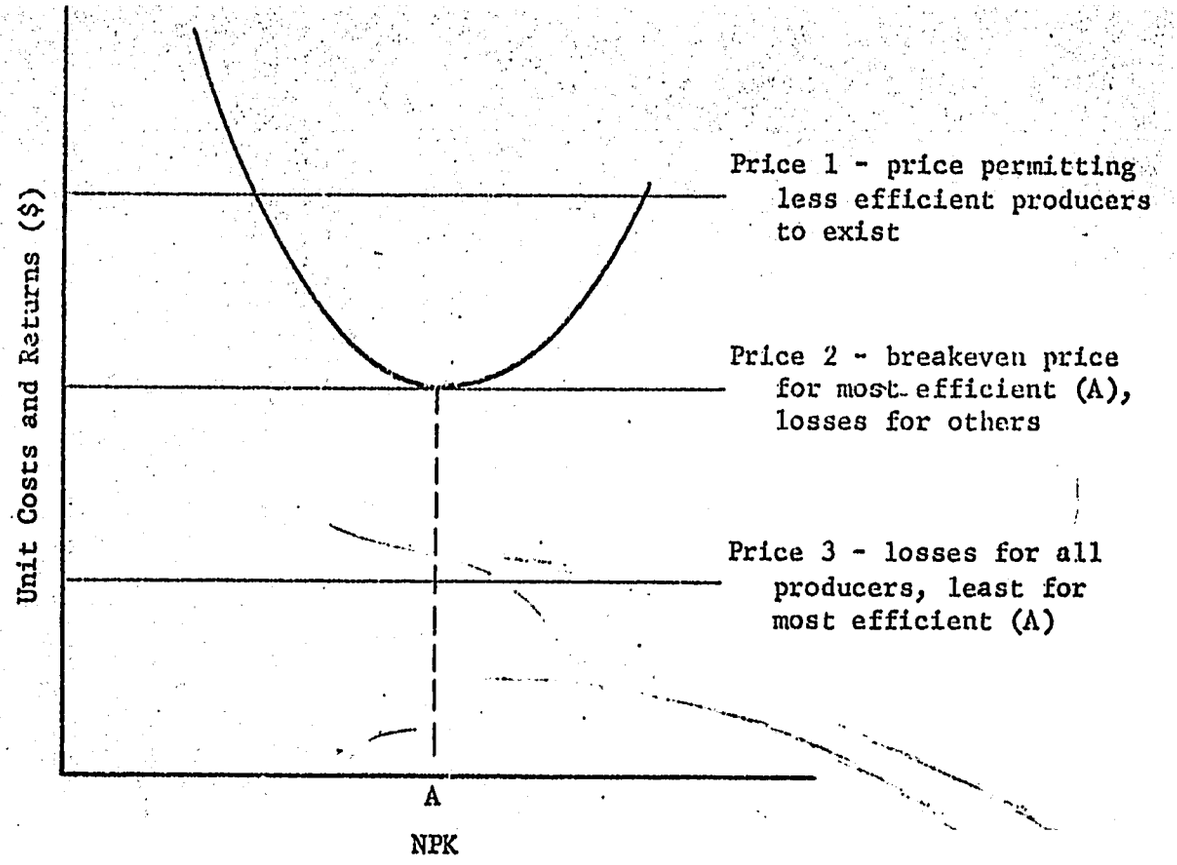

FIGURE 1. Unit costs (and returns at various product price levels) with varying investment in NPK

There follows a simple proof that rate of return is maximized when unit costs are minimized, and that product price levels do not affect the determination of the optimum input.

General formula for differentiation of a ratio function:

$$\frac{d}{dx}\frac{(u)}{(v)} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$$

Definitions:

$$\text{Rate of return} = \frac{TR - TC}{TC} = \frac{PyfX - (FC + PxX}{(FC + PxX)} = r$$

$$\text{Unit cost} = ATC = \frac{TC}{Y} = \frac{FC + PxX}{fX}$$

TC = Total Cost = Fixed cost + (price of variable input times variable input) = (FC + PxX)

TR = Total Revenue = Price of product times yield (= fX) = PyfX

Px = price of X. Py = price of Y fX = function of X

Minimum unit cost is at the level of input X where:

$$\frac{fXPx - (FC + PxX)f'X}{fX^2} = 0 = \underline{\underline{(FC + PxX)f'X - fXPx}} \quad \text{(see below)}$$

Maximum rate of return is at level of input X where:

$$\frac{(FD + PxX)\ (Pyf'X - Px) - Px\ [PyfX - (FC + PxX)]}{(FC + PxX)^2} = 0$$

Eliminating the denominator and multiplying out the term in brackets gives:

$$(FC + PxX)\ \ (Pyf'X - Px) - (PxPyfX - PxFC - PxPxX) = 0$$

Removing the third set of parentheses gives:

$$(FC + PxX)\ \ (Pyf'X - Px) - PxPyfX + PxFC + PxPxX = 0$$

Multiplying out the remaining terms in parentheses gives:

$$FCPyf'X + PxXPyf'X - PxFC - PxPxX - PxPyfX + PxFC + PxPxX = 0$$

Cancelling out plus and minus terms and dividing by Py gives:

$$FCf'X + PxXf'X - PxfX = \underline{\underline{(FC + PxX)f'X}} - fXPx = 0 \quad \text{(see above)}$$

It is thus shown that the formula for determining the point of minimum unit cost is the same as that for maximizing the rate of return, and that the price of the product (Py) is not needed.

The above differs from the conclusions reached by Carlson (5) because he considered capital to be fixed rather than equal to TC as has been done above. Therefore, he maximized rate of return as is shown on the next page. This is mentioned because Carlson has given one of the most complete "classical" discussions of the rate of return, and the reader will note that the last formula given in Carlson's presentation is the equivalent of the rule given in the first sentence of this appendix; i. e., marginal cost should equal marginal revenue. Thus, this paper differs from Carlson because of a difference in treatment of rate of return.

The comparison with Carlson leaves out the question of interest. However, the rational farmer would be expected to invest if his average rate of return is greater than the prevailing rate of interest. It seems evident that this is not the same as Carlson's rm = im, except in the unusual case where only one input is involved. A final point is that equating marginal cost with marginal revenue will only produce optimal results at the breakeven point; i. e., when Rm - Cm = 0. In all other cases the increased revenue from one input is more than offset by reduced

Carlson's mathematical treatment is as follows (his symbols have been altered to conform with the ones used in this paper. Other symbols are defined as follows: i = interest; L = Loan; m = marginal; and K = fixed capital).

$$\text{Rate of return } (r) = \frac{\text{Total revenue} - (\text{Total Cost} + \text{Loan interest})}{\text{Fixed Capital}}$$

$$r = \frac{TC - (TC + iL)}{K}$$

Maximum rate of return is at level of input X where:

$$\frac{dr}{dx} = \frac{1}{K}\left[\frac{dTR}{dx} \quad \frac{dTC}{dx} \quad \frac{d(iL)}{dL}\right]\frac{dL}{dx} = 0$$

which, eliminating K, gives: $Rm - Cm - Cmim = 0$

$$\text{or: } \frac{Rm - Cm}{Cm} = im$$

$$\text{or: } rm = im$$

$$\text{or: } Rm = Cm\,(1 + im)$$

It can be seen that Carlson considers that r is maximum when $rm = \frac{Rm - Cm}{Cm}$, while it is shown by our proof that r is actually maximum when $(FC + PxX)(f'X) - fXpx = 0$. The discrepancy results because Carlson does not differentiate r as a ratio function, and this leads him to elimination of K.

rates of return on other inputs. On the other hand, the technique presented in this paper will maximize total farm revenue because it makes the marginal rate of return on each individual input equal to zero, assuming that all costs are estimated and allocated properly. The following graph may help to clarify some of the points discussed in this appendix. In this graph, NM is maximum net revenue; and optimum output (with this as the criterion) will vary with price. (At Price 1, it is greater than Price 2; at Price 2, NM = PQ = 0; i. e., this is the breakeven point.) However, PQ divided by QR is the rate of return; and optimum output on this basis will <u>not</u> vary with price, being always maximum at output R.

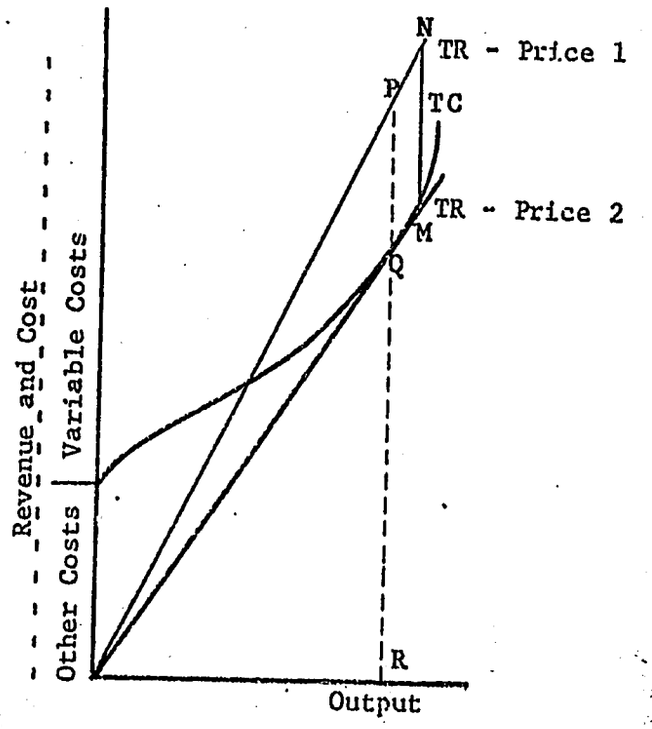

FIGURE 2. Outputs for optimum rate of return versus (price-dependent) optimum return

In conclusion, despite the divergencies from Carlson, the equality shown on page ten (maximum rate of return = minimum unit cost) appears to be a corollary of the linear programming - dual problem discussion on pages 80-82 of Henderson and Quandt (6) in which maximum R = minimum Z, where R = total revenue and Z = the imputed value of the entrepreneur's endowment; i. e., the costs of the farmer's inputs. Much of the reasoning presented is also similar to that of Dean (3) regarding the measurement of capital productivity. The problem of what to include in "other costs" appears to be common to all approaches.

APPENDIX B

A Discussion of Response Curves and Production Functions

Response curves (functions) are based on scatter diagrams which usually have an appearance similar to Figure 3.

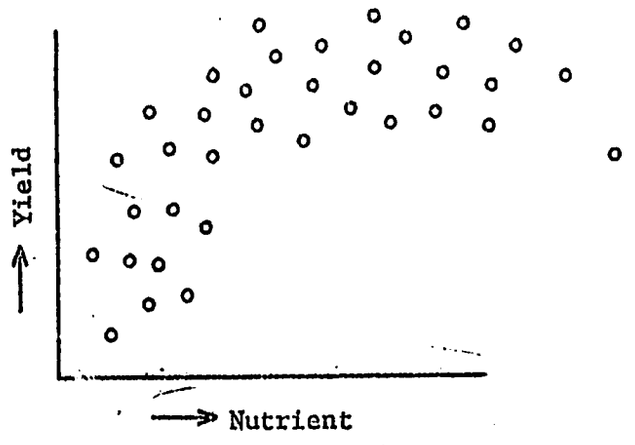

FIGURE 3.

The best known generalization of such data is probably the Mitscherlich-Spillman (7) function, $Y = A\ (1 - e^{-cx})$, in which each additional increment gives 50 percent of the yield increase produced by the preceding increment as is shown in Figure 4.

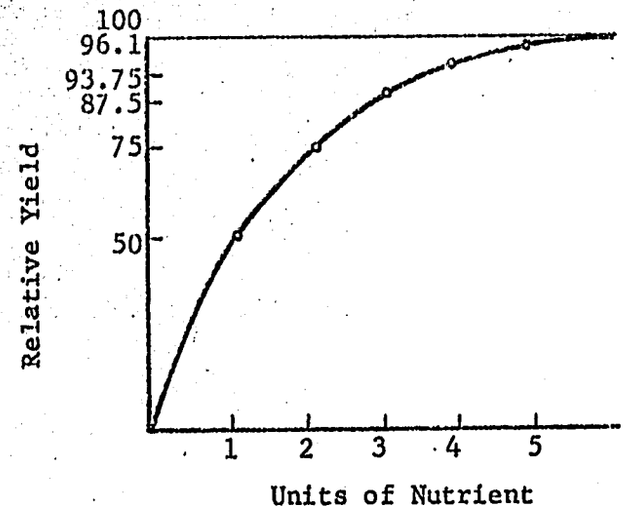

FIGURE 4.

Mitscherlich justified this curve by its similarity to others found in nature; e. g., declines in radioactivity and disappearances of recessive genes. However, there does not appear to be any particular biological reason why this "law" should apply to nutrient response by plants.

Another criticism of the Mitscherlich-Spillman function is that it does not include the possibility of yield depression caused by excessive amounts of the nutrient studied. (Note that on a relative yield graph such depressions, or negative responses, will show as points with values greater than 100.) Mitscherlich later modified his equation in an attempt to answer this criticism.

Another modification has been suggested by several investigators, especially Steenbjerg (8) who observed that on very poor soils the

response curve is sigmoid; i. e., it demonstrates increasing returns during its first portion. This can be explained by fixation, nutrient imbalance (Steenbjerg called it "blocking"), or simply uptake inefficiency at the very low levels of application. Inasmuch as these effects are noted primarily on soils which are extremely low in a particular nutrient, they are not characteristic of response curves on the soils of relatively productive agricultural regions. In other words, response curves normally intercept the Y axis somewhat above the origin. This means that the most general shape is similar to that shown in Figure 5, but the possibility of a sigmoid shape to the initial portion should not be overlooked.

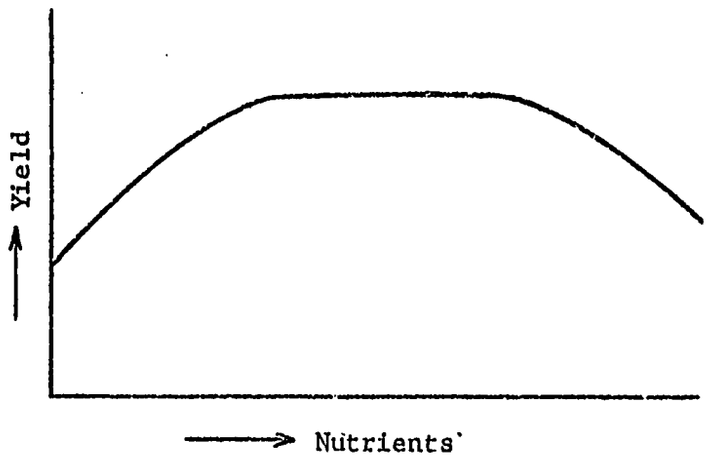

FIGURE 5.

Bateman (8) has proposed a rather intriguing hypothesis to explain this shape. He postulated that many growth factors inherently possess both positive and negative affects, and that observed responses reflect the dominance of one or the other (or an average of the two). Bateman spoke specifically of "crossed parabolas," but his data would also tend to fit a combination of an exponential (e. g., $Y = X^c$) function and a Mitscherlich equation (i. e., postive and negative exponentials).

Another theoretical explanation has been advanced by Specht (9) who analyzed the individual coefficients of variation of the foliar analyses of ten replicates at each of up to fifteen levels of added nutrient. Typical results are shown in Figure 6.

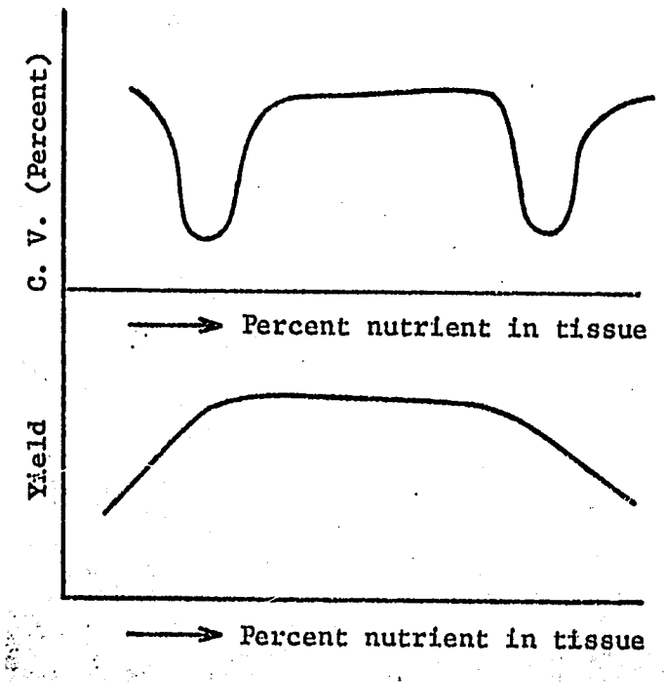

FIGURE 6.

Specht concluded that plants have a self-regulating mechanism which distributes a particular element evenly among the cells when the concentration of the element is either critically deficient or excessive. Similar work was done later by Friis-Nielsen (11) who obtained the same kind of results and elaborated Specht's theory into a general guide for the interpretation of foliar analysis.

Specht's discovery fits well with the hypothesis elaborated by Blackman (12) who considered the response curve to be essentially linear until some other factor becomes limiting. This was later disputed by Harder (13) who presented data indicating that the slopes of the response curves differed. The contrasting concepts are illustrated in Figure 7.

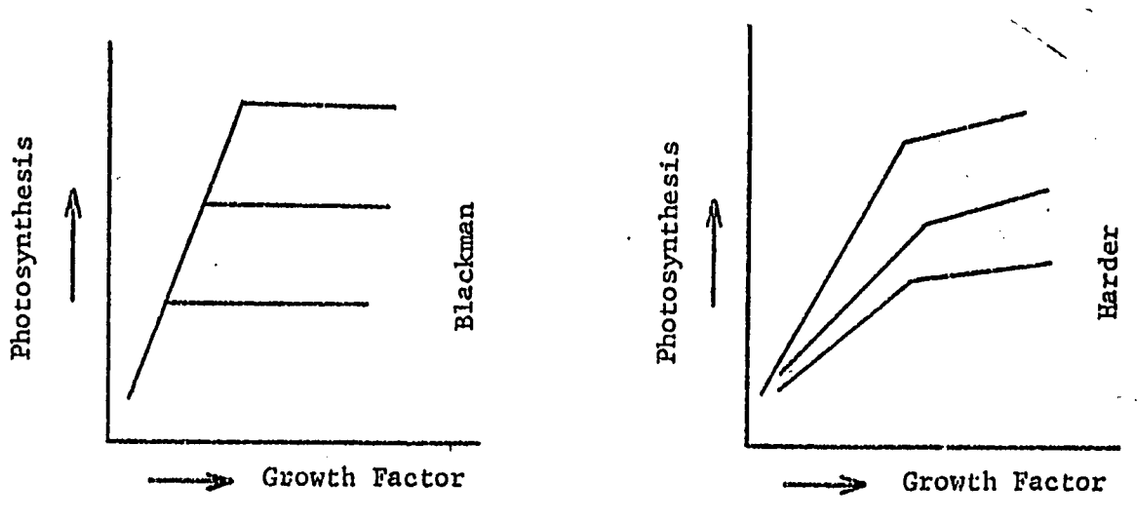

FIGURE 7.

However, even though this controversy has raged for years, primarily in the literature on photosynthesis, Figure 8 raises doubts as to its significance since the two contrasting sets of curves in this graph simply represent the same data plotted to different scales!

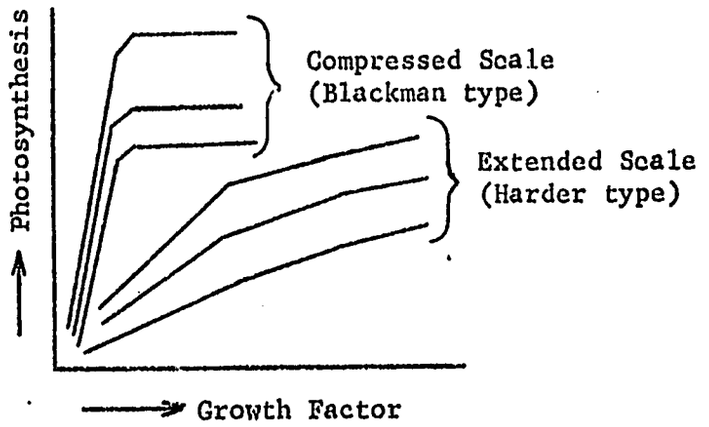

FIGURE 8.

In any event, much of the above discussion indicates that there may be some basis for considering response curves as reflections of Liebig's Law of the Minimum, with relatively abrupt inflection points when some factor becomes limiting. In this connection, it is interesting to note in Figure 9 the distortion often created when one arbitrarily fits a curvilinear function to data which (in fact) are discontinuous or have a sharp inflection point. In effect, the curvilinearity is an artifact which would disappear if the data were split into two populations. This

is a potential pitfall in empirical regression formulae of the type presented by Shrader et al. (14). Additional problems associated with the empirical approach have been discussed by Swanson (15), including the difficulty of fitting and of testing the "goodness of fit."

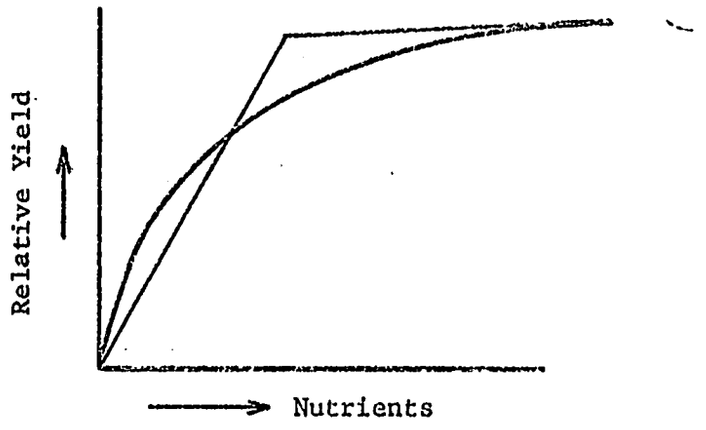

FIGURE 9.

From the above, it can be seen that the literature on plant responses might be considered as split between two positions:

1. Responses are best represented by a continuous, concave downward curve equivalent to the "Law of Diminishing Returns."
2. Responses are linear so long as a particular nutrient is limiting, with a subsequent level component when some other factor becomes limiting, equivalent to the "Law of the Minimum."

A possible compromise is implied in a paper by Cate and Nelson (1) suggesting a simple method for dividing the points on a soil test-yield

scatter diagram into two populations, those likely to give a large response and those unlikely to do so. This is illustrated in Figure 10 (The dashed lines are drawn to maximize the number of points in the positive quadrants.)

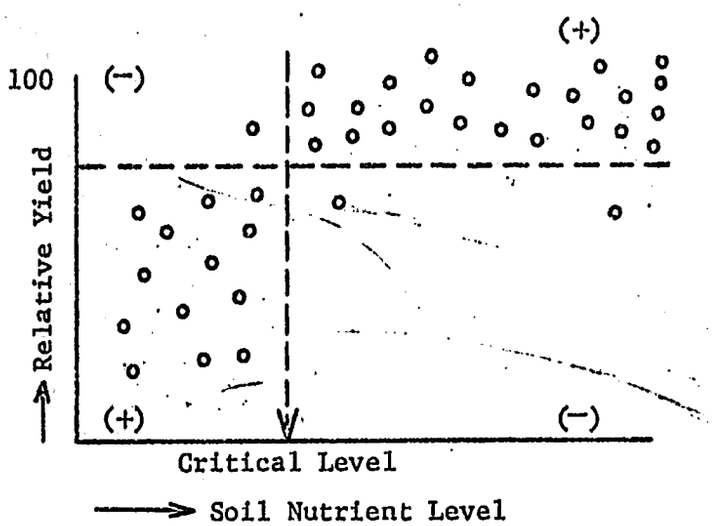

FIGURE 10.

This sort of graph could be interpreted to mean that the <u>average</u> response would be much steeper on the low side of the Cate-Nelson critical level than on the high side. In Figure 10 each point represents a separate field trial. Similar reasoning could be applied to the means of replicates at different nutrient levels in one field trial. The end result would be the type of curve shown in Figure 11.

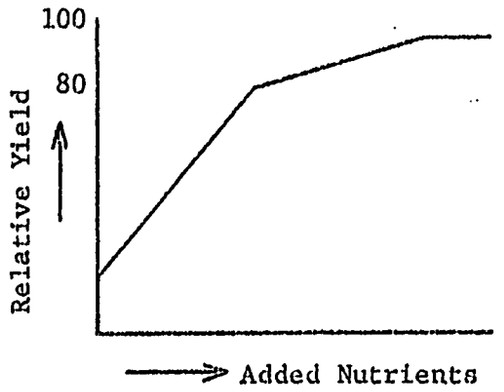

FIGURE 11.

This is essentially Swanson's (15) Modified Liebig Model. As Swanson has pointed out, economic analysis of responses is dependent on the biostatistical interpretation of the data. Unfortunately, Cox and Cochran's (16) discussions of variability suggest that Blackman's (12) comments in 1905 are still essentially valid.

> "The way of those who set out to evaluate exactly the effects of changes in a single factor upon a multi-conditioned metabolic process is hard, and especially so when the process is being pushed toward the upper limits of its activity . . . at present our science entirely lacks data that will stand critical analysis."

Nevertheless, from a practical point-of-view, it seems reasonable to conclude from the material presented in this appendix that responses are essentially linear (and steeply so) up to about 75-85 percent of the maximum yield possible under given conditions. The remaining 15-25 percent

probably cannot be accurately predicted using present data and techniques, except through some use of probability estimates. Incidentally, Bray (17) also concluded that field experiments could only be used to determine the low-medium split; i. e., the Cate-Nelson critical level.

In conclusion, it might be noted that the above discussion may have application to a number of similar problems in both the biological and the social sciences. For example, the "Phillips curve" used in macroeconomics to relate price and employment may actually be a "two-population" situation with a "critical level" for full employment without inflation. (See Samuelson and Solow (18).)

APPENDIX C

Graphic Solution of the Proposed Method

1. Draw a graph in which the cost of the fertilizer nutrient under consideration is measured on the X axis, to the right of the origin, while all other costs are measured on the X axis to the left of the origin; and yield is measured on the Y axis. Then draw a line which is tangential to the yield "curve," from the appropriate "other costs" point on the X axis.

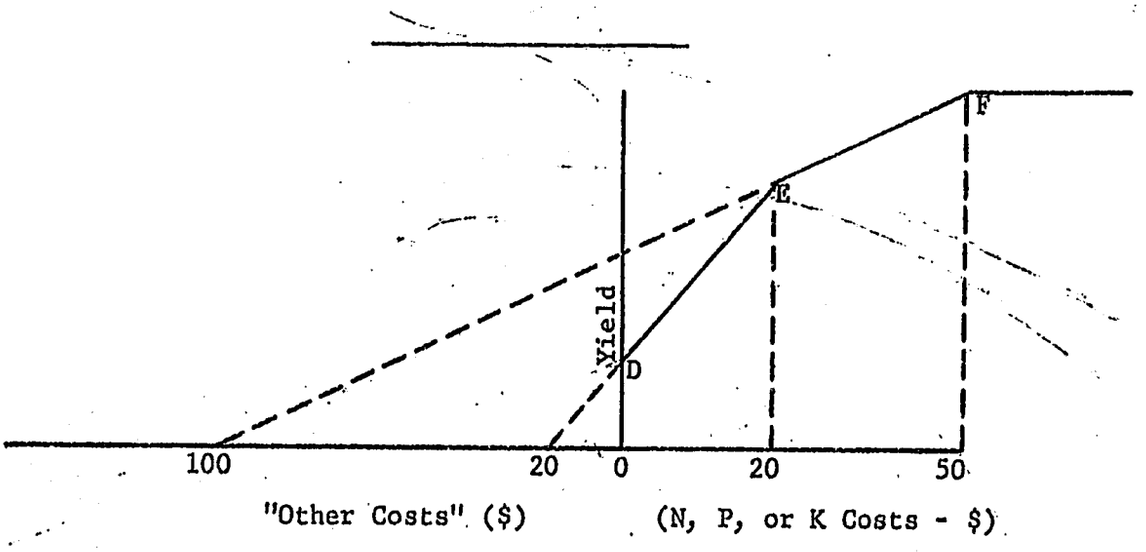

FIGURE 13. Graphical solution for estimating optimum fertilizer expenditures when capital is limited. (This graph is similar to that presented by Richard C. Lindberg, USDA, in a manuscript dated about 1960.)

2. Using Figure 13 as an illustration with hypothetical costs, if the point of tangency is D ("other costs" less than 20), do not apply the nutrient. If the point of tangency is E ("other costs" equal

to or more than 20, but less than 100), apply 20 monetary units of this nutrient. (Note that the local cost of the nutrient has been taken into account in the construction of the graph.) If the point of tangency is F ("other costs" equal to or greater than 100), apply 50 monetary units of this nutrient.

The reader will note that the breakpoints on the "other costs" axis, 20 and 100, represent the points where tangential lines will have the same slope as the yield "curve."

Another point to note is that the function of soil analyses is to facilitate the drawing (prediction) of the yield "curve." As is indicated in Appendix B, if the soil or plant analysis is above the Cate-Nelson critical level, the yield "curve" will be truncated as is illustrated in Figure 14.

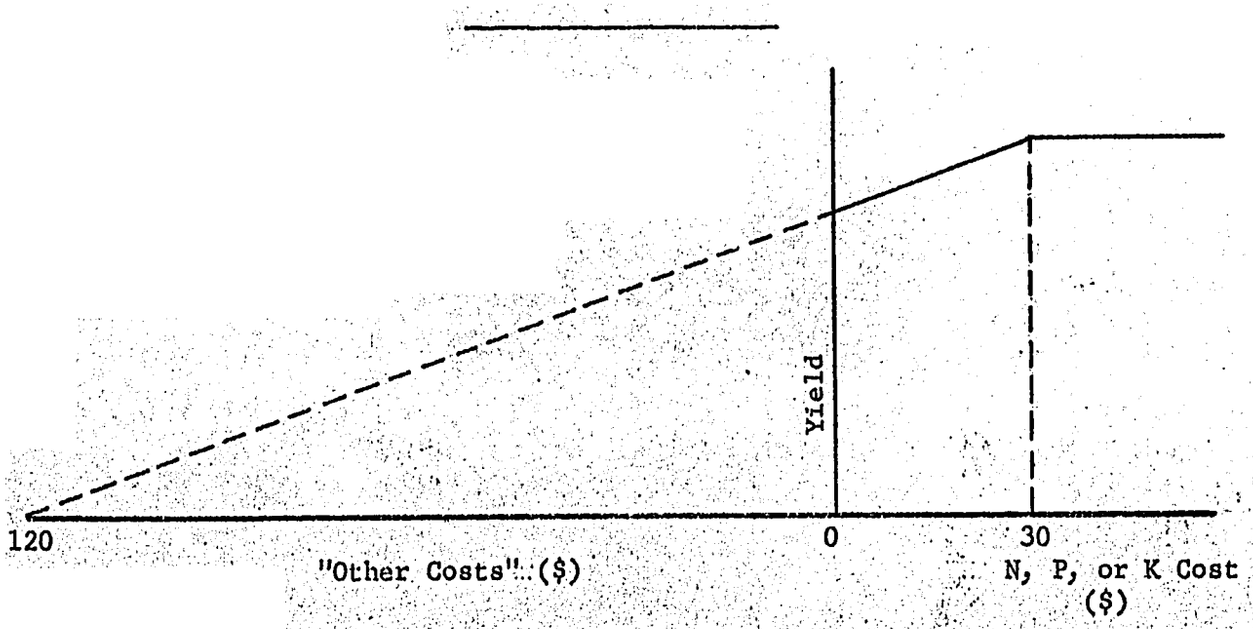

FIGURE 14. Optimum expenditure point with soil test above critical level

This will cause the intercept, or breakpoint, on the "other costs" axis to be 120 rather than 100. In other words, an analysis which falls above the critical level should (under these conditions) only receive the particular nutrient if "other costs" are equal to or greater than 120. The amount to apply (if any) would be 30 monetary units, which would produce the maximum yield. (The figure 30 is derived by subtracting the "soil contribution" of 20 from the 50 units hypothesized for maximum yield in Figure 13.)

Data on nutrient needs of different crops at various levels of production can also be helpful in drawing the yield "curve." The case of nitrogen is discussed in detail in two of the technical bulletins of this project to be published soon. Essentially, the technique is to use the estimated yield without nitrogen (in conjunction with plant uptake data) as an indication of the amount that the soil can supply. In other words, yields without nitrogen (assuming other nutrients adequate) are used in lieu of soil tests.

Another possible approach is to perform laboratory fixation tests to determine the amount of fertilizer needed to raise the soil above the critical level. This technique is especially applicable to phosphorus and lime needs. In these cases the economic value of residual effects should also be taken into account. However, the high discount rates which must often be applied (especially in agriculture) will tend to diminish the economic significance of these residual effects. Incidentally, it is interesting to note that the actual rate of return on investment

in fertilizer (even at the fixed point of minimum unit cost) will depend not only on the price of fertilizer and the value of the crop; but also on the number of nutrients which are limiting (and how limiting they are) since these factors determine the extent to which NPK costs must be spread.

Clearly, the scale on the righthand side of the X axis (and the yield "curve" itself) will have to be derived from experimental data, plus knowledge of local fertilizer costs. The intercept on the lefthand portion, the "other costs," must be determined by economic studies of prevailing costs under various crop-management situations--or by the farmer himself. The "other costs" should include all expenses that are directly attributable to the field, <u>including the costs of fertilizer nutrients other than the one being studied</u>. The question of what other costs to include has already been raised in the text section of this paper and need not be repeated here. Nevertheless, it requires considerable attention and thought!

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