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ANALYSIS OF EPHEMERAL FLOW IN ARIDLANDS[a]

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INTRODUCTION

Ephemeral flow in aridlands is characterized by alternating random periods of wetness and dryness. Sustained duration of flow occurs only in those aridland watersheds fed by snowmelt, large aquifers, reclaimed waste waters, or during sustained periods of rainfall. Stochastic analysis of these flows focuses on the properties of the following random variables: (1) Flow duration; (2) temporal distribution of flow within a single wet period and within a long wet-dry sequence; (3) antecedent dry period; and (4) flow volumes.

Such analysis is a prelude to the synthesis of models of ephemeral flow. Models are considered an efficient consolidation of available data, a positive framework for incorporating future data, a means for reducing the inherent physical indeterminacy of hydrologic systems, and a basis for imbedding a priori hypotheses which allow for the possibility of occurrence of events that have not been observed historically. Models may be based primarily on the data, primarily on physical principles, or combinations of both bases. The presumption is that physically-based models in hydrology, either stochastic or deterministic, have greater transfer and regional value than nonphysically based models. However, this generalization must be tempered by the realization that more data are required to implement physically based models and that not all management objectives require such models.

Of concern in the modeling of aridland hydrology is the sparsity of actual rainfall and streamflow events in space and time and paucity of data on these

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events even when they do occur. The high degree of water development in aridlands and the high variability of rainfall and runoff volumes even on a monthly and annual basis are facts that lead to the requirements of much more information than for regions with much more stable streamflow (2). Lustig (24) emphasizes that a distinction must be made between actual length of record and effective length of record; this is a manifestation of redundancy in information (21). Efficient estimates of model parameters are dependent on sample size and naturally lead into questions on the efficient design of hydrologic data collection systems and the worth of additional hydrologic data for water resource planning, design of water supply systems levees, dams, storm drainage systems, and watershed management in aridlands (8,11,15).

The purpose of this paper is to present recent results in the stochastic analysis of ephemeral flows in the metropolitan area of Tucson, Ariz., and to propose a framework for future analyses of such data.

SYSTEM CHARACTERISTICS

Arid versus Temperate.—Relevant to the analysis is a comparison of flows in arid, temperate, and transition climates. In all regions, both area-wide and local storms influence the runoff pattern whether it be in an urban or rural watershed. Analyses in both are conditioned by sparse data but, in general, streamflow records in aridlands are not as long and as dense in space (28). The difference in the precipitation pattern is emphasized by Hershfield (19) who developed empirical rainfall models all over the United States, but found that his approach for determining raingage network density broke down for the Walnut Gulch Watershed near Tombstone, Ariz.

Because of increased mining of aquifers in arid regions, interconnections between the stream channel and aquifer are not as common today except in the upper reaches of a watershed and temporarily during recharge-producing streamflow. As a consequence, base flow is an uncommon component of ephemeral flow, even in the humid islands of mountain watersheds on the periphery of the desert floor. Duration of individual flow events determines the extent of opportunity for natural recharge of aquifers. Flow duration as used in temperate climates is not as meaningful a concept under these circumstances.

Generally speaking, stochastic analysis of ephemeral flows is complicated by the fact that flow is not continuous and that for the same number of years of record fewer events are observed in aridlands (17). This reality mitigates the transfer of properties of point streamflow patterns to other areas. Furthermore, correlation and spectral methods of time series analysis must be applied with care to precipitation and flow sequences with lengthy periods of zero disturbance levels (21). The common interpretations of autocorrelation functions for streamflow in temperate climates must be modified. In the face of the aforementioned realities there is considerable motivation to augment data by appropriate models.

Time Interval.—Flows may be described in terms of their physical and statistical properties according to artificial or natural time sequences. In the latter, nature dictates the time of observation and provides an event series. This is dichotomized against the arbitrary dissection of time and events when annual, monthly, and daily series are formed. Flows occur only during a certain percentage of the time each day, month, or year but the arbitrary series is formed on the premise that flows occur instantaneously at the beginning, middle, or end of the time period as lumps or that flows occur at a constant average rate over that time period. These approaches entail loss of information and are important considerations in the interpretation of such arbitrary series. Furthermore, notice that as far as rainfall duration (DR) is concerned, the usual concept of x-year-y-min rainfall does not seem very useful in a semiarid region; this was noticed for example, by Hershfield (18), who states that "the annual maximum for rainfall durations at the same station come from the same storm. This is particularly true in the desert regions of the southwestern United States . . .". It seems better to consider the concept of rainfall (or runoff) event, defined in the rigorous sense of probability theory.

Managerial Use of Data.—Flood control and water supply problems dominate the managerial use of hydrologic data. Annual and monthly streamflow sequences are studied to define overyear storage to ameliorate future droughts and to forecast within-year storage and supply requirements for irrigation (33). The event series is pertinent to the study of annual flood extremes and their duration. For example, the log-normal distribution has been found to be an appropriate model for annual flood extremes on the Rillito Creek in Tucson; the model in turn is used in a decision theoretic study of the worth of additional hydrologic data in the design of bridge piers against scour and in the design of flood levees (8). Volumes and durations of natural recharge are important in the operation and location of wells along ephemeral channels. However, previous patterns of natural recharge will change, as schemes develop to capture surface water in arid urban areas both for flood control and water supply.

REVIEW OF PREVIOUS RESEARCH

A representative sampling of previous studies of ephemeral flows reveals that most data analyses and modeling efforts have focused on annual and monthly flows and flood peaks.

Use of Descriptive Statistics.—Annual and monthly flows in the Mogollon Rim area of central Arizona were studied in 1960 by McDonald (27). The characteristic climatology of this region, precipitation with low mean values and high coefficients of variation, is typical of arid regions. McDonald finds that coefficients of variation for runoff on annual and monthly bases are distinctly higher than for precipitation and he explains this surprising result in terms of the small difference between precipitation and evapotranspiration losses. This small difference is typically taken as a rough physical definition of aridity. Also reported is the fact that the greater relative variability of winter as compared with summer precipitation is also found in the seasonal differences of streamflow variability. This result has important implications for watershed modification efforts because streamflow augmentation by weather modification and land treatment is projected as an important dimension of future water development in the west. Correlation coefficients between recorded flows and precipitation amounts at adjacent stations are used as a rough measure of spatial homogeneity. For both variables McDonald finds greater spatial homogeneity in winter than summer and he explains this in terms of the widespread cyclonic and frontal storm systems that govern winter precipitation. Isolated orographic and convective thunderstorms account for the greater heterogeneity in the summer.

Floods.—An extensive study of runoff-producing precipitation on semiarid rangeland watersheds is being pursued by the Southwest Watershed Research Center of the Agricultural Research Service in Arizona and New Mexico (10, 31). Their descriptive statistics reiterate the substantial variability of annual precipitation (44 % coefficient of variation for a 10-yr record on Alamogordo Creek watershed). The variability is even greater on a monthly and seasonal basis. Apparently, the short records deter a more comprehensive use of descriptive statistics in the manner of McDonald (27). A measure of the substantial variability of runoff is noted by the fact that during the 10-yr period (1956-1965) about 60 % of the total runoff was recorded in 1960.

Floods on smaller arid streams have been studied more intensively in anticipation of future population growth and urbanization. Historically, most cities in aridlands have no subsurface storm drainage system; streets and highway dips in urban and rural areas are used as drainage channels (20). Flood damages and attendant court suits have been increasing in recent years. Complementing the increasing need for flood control is the increasing economic motivation for capturing surface runoff for urban water supply (23). Stochastic analysis of floods in aridlands is complicated by sparsity of flood data in space and time. Such analysis may focus on annual flood extremes in the manner of Gumbel's theory of extremes for complete duration series or Todorovic's theory of extremes for partial duration series (34), or focus on prediction of the entire flood hydrograph. Most progress has been made in the first case; the problem of flood routing of waves in permeable bed channels in order to predict downstream hydrographs does not have a satisfactory solution as yet. The prediction of flood peaks may be based: (1) On an empirical or envelope relation between flood peak and basin area; (2) on the controversial idea of probable maximum precipitation, including storm transposition (29); or (3) on probability fitting methods with or without a particular probability distribution function in mind. The flood estimate may also be derived from point rainfall estimates obtained from Technical Paper No. 40 of the National Weather Service (formerly U.S. Weather Bureau). For some problems such as spillway design or design of drainage structure in high-value districts, a combination of the aforementioned methods may be used. Recent frequency studies of floods on the Alamogordo Creek Basin in New Mexico as observed over 14 yr, demonstrate that a point rainfall estimate based on Technical Paper No. 40 can be extremely misleading (31). Also noted was the difficulty in fitting a probability distribution to the flood data. Because even 50 yr-60 yr of flood data does not eliminate the hydrologic uncertainty in many flood control problems, recent efforts to apply the decision-theoretic approach merit serious consideration (8).

Modeling of Streamflow Sequences.—The past 10 yr have witnessed substantial progress in the development of deterministic and stochastic models for the synthesis of streamflows. The deterministic watershed models require hypotheses about watershed behavior and actual precipitation series or stochastic models of precipitation as forcing functions. The Stanford watershed model (6) generates long sequences including zeros whereas the U.S. Geological Survey's model generates single-flood hydrographs (9). On the other hand, stochastic models, extensively reviewed by Fiering (14), are noncausal in that there is no explicit consideration of inputs and watershed behavior. Generally speaking, these two modeling strategies are not in opposition in that each serves distinctly different purposes, watershed models being employed to predict floods and the effects of human activity and stochastic models being used for forecasting and simulation of equally likely future input sequences to storage reservoirs and spillways. Where the two models do compete for use on the same problem, there is evidence to support the contention that the two models have a common physical basis (22); after all, even simple rainfall-runoff relations derived by least squares optimization may account for considerable part of the variance (16). The choice between the two should be made on operational grounds—goodness of estimate, data requirements, cost, transfer value to other regions, and sensitivity of management solution to chosen model. There is an important research area involved in the problem of choosing among hydrologic models for both temperate and arid climates. A review of the aforementioned literature leads to the conclusion that both kinds of models have not been critically reviewed in terms of their utility to aridland problems.

Deterministic Watershed Models of Multiple Event Series.—As suggested herein, there are management problems where modeling of the multiple event series is important, namely, (1) Routing of floods through complex drainage systems and (2) operation of surface storage reservoirs and well pumping systems contiguous to recharge channels. The Stanford watershed model and the University of Texas version in Fortran IV (5) tend to generate many zero flows in response to the many quiescent periods in the precipitation input and the regimen of the arid watershed. Aside from the questions generated in the fitting of such daily flow series, would it not be more meaningful to look at the statistical properties of wet and dry periods in terms of a stochastic renewal process and then to use deterministic hypotheses (e.g., the aforementioned watershed models) to reconstruct and predict the amplitude of flows during the wet period? Furthermore, this line of reasoning suggests a probabilistic structure for finding the combination of rainfall events in space and time that could have given rise to the observed flow event.

Stochastic Models for Multiple Flow Event Series.—Stochastic models of daily, monthly, or annual streamflow are generally in the Markovian idiom. Almost without exception these models are assumed to apply to sequences with long periods of zero flow. Fiering (14), however, in a notable exception, explicitly states: "The linear autoregressive schemes proposed in this book certainly are not the only acceptable models for flow synthesis. In some arid regions of the world, precipitation patterns are characterized by bimodal distributions; there is either no rain or an intense cloudburst. The cyclic, smoothly varying seasonal precipitation characteristic of humid regions—like that on which our recursion relations are based—does not occur." One instance where the zero flows were differentiated from flow events is the effort by Allen (1) who considered the diverse flow regimes in California; a Markov lag-one monthly streamflow generator was derived along with methods: (1) To increase the sample size by combining unrelated streamflows and (2) to match streams with similar watersheds in order to provide data for streams without records. Allen reports considerable variation in the mean, variance, and skew properties of flows from month to month. The number of months with no flow reduced the number of flow months such that the sample size for estimating higher-order moments is meager. (The writers present similar results later in connection with flows on Rillito Creek in Tucson.) Variance of estimates is severe to the extent that their subsequent use to generate synthetic traces is fraught with uncertainty. No wonder that some hydrologists prefer to stay with the classical critical period estimate for required reservoir storage, as determined from the historical record alone.

To circumvent the problem of zeros the frequent prescription is to augment all flows by a small positive number or some other simple augmentation formula. This is a prelude to subsequent logarithmic transformation to achieve normality (3). Even if the streamflow modeling is done in the original untransformed space, negative flows arise in the simulation and are then arbitrarily set at zero. Negative values can arise also in sum-of-harmonics process models (32).

In all of this process of modeling and computer generation, the writers believe that important questions need to be answered.

1. What effect does addition of positive numbers and logarithmic transformation have on the preservation of the original statistics including those features of importance to design and operation, namely, extreme droughts and floods?

2. On the assumption that the problem in 1 does not exist, what effect does historical or small sample bias in the estimates of the mean, variance, skew, and serial correlation coefficients have on the subsequent use of synthetic traces in simulation and mathematical programming studies of water resource systems?

3. Is there a more natural statistical structure for modeling of streamflows?

Without having undertaken an exhaustive theoretical and simulation study of question 1, the writers' experience suggests that the simulated extremes are generally depressed or lower than their historical counterparts, even though in principle Monte Carlo simulation should lead to convergence to the historical (but biased) parameters.

The writers believe that decision theory (8) offers one of the best avenues for reconciling the issues raised in question 2. Question 3 suggests that more basic hydrologic knowledge must somehow be coupled to the stochastic modeling effort.

Distributed (Space-Time) Models of Streamflow.—Two approaches are possible for generating streamflows at multiple points in a river basin.

1. Use of existing streamflow records in order to find a spatial correlation structure that reflects a measure of the regional hydrometeorology.

2. Use of hydrodynamic or watershed models in order to predict response to observed or model precipitation patterns over the basin.

The mathematical problems involved in approach 1 have been reviewed elsewhere (3,7,25,26). Inconsistent matrices arise because the streamflow records at multiple sites over the basin are not of the same length. The method basically relies on the available record and it does not invoke prior knowledge about the drainage network, meteorology, and watershed dynamics. Continuity of flows at gaged and ungaged sites are not necessarily preserved in currently developed models.

To move in the direction of approach 2 would require another class of hydrologic data. This strategy leads to a more intuitive feel for the spatial correlation structure mentioned herein. An excellent example of approach 2 is the work of Eagleson (12) who studies the one-dimensional cyclonic and convective storms. While there has been considerable idealization of the natural process, the work (12) does represent an important conceptual step in space-time models of hydrologic phenomena. More refined or distributed models of the hydrologic processes may be the only route to take to get at the problem of network density and the prediction of response statistics of regulated systems.

MODEL OF STREAMFLOW AND OPPORTUNITY FOR NATURAL RECHARGE

In this section, a methodology is set forth to model and analyze rainfall and runoff events from the standpoint of contribution of runoff to natural recharge. This is intended to demonstrate the utility of basic concepts of probability theory to the general modeling problem. As said earlier, the random variable of prime importance for recharge of aquifers is the duration, DF, of the flow; the occurrence of a precipitation and of a flow constitute events. Recharge is taken to be the downward percolation of flood waters to an aquifer not in permanent connection with the river bed. In actuality, the transformation of channel losses to true natural recharge is a complex problem that remains to be modeled satisfactorily. In this section, of prime concern is the opportunity for natural recharge as a consequence of enduring flows in the channel prior to complete dissipation of the wave through channel losses.

First, climatic considerations lead to a distinction between summer and winter phenomena. Inspection of data taken between 1945 and 1967 at the Tucson Arroyo gaging station, displayed in Figs. 1, 2, and 3, confirms that summer and winter flows tend to belong to different populations. In fact, summer precipitation is mainly caused by convective thunderstorms, while winter rain is of a frontal nature; sometimes, in September, both types of rain can occur simultaneously in the form of hurricane activity from the Pacific Ocean. As a first approximation, this mixed regime will be included in the summer precipitation study. In a semiarid land, the chain of events can be studied as follows.

Summer Rainfall Frequency.—In a previous study (15) it was found that a Poisson distribution to describe the frequency of point rainfall occurrences was generally accepted in the scientific community; experimental results found in the southwest did not modify this hypothesis. Thus it can be inferred that summer storms occur in an independent manner; flows caused by these storms should occur the same way.

As no rainfall measurements have been taken in the watershed of the Tucson Arroyo under study herein, it will be assumed that climatic conditions are similar to those above the Atterbury Watershed, where a mean number, m, of at least five summer precipitation events per year have been observed over a period of 15 yr. In fact, more than five events per year have occurred; however, several events were not analyzed although they were runoff-producing events because the center of the storm cell was located outside of the gaged area.

For the Tucson Arroyo, a small watershed in the same area range as Atterbury (18 sq miles), it will thus be assumed that the probability function of the number, N, of rainfall events per summer season is Poisson:

$$P(N = j) = f_N(j) = \frac{e^{-m} m^j}{j\,!} \quad j = 0, 1, 2, \ldots \quad m > 5 \quad \ldots\ldots\ldots (1)$$

This model assumes independence of rainfall events within the season. The

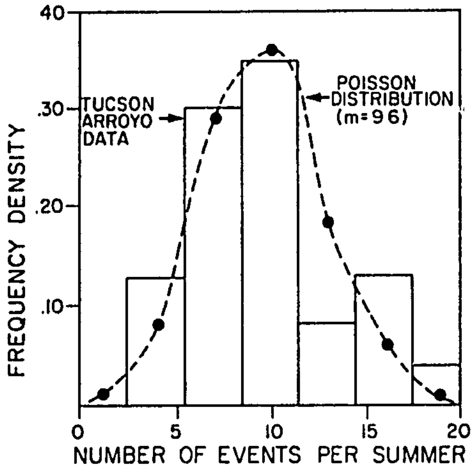

FIG. 1.—NUMBER OF EVENTS PER SUMMER, FREQUENCY

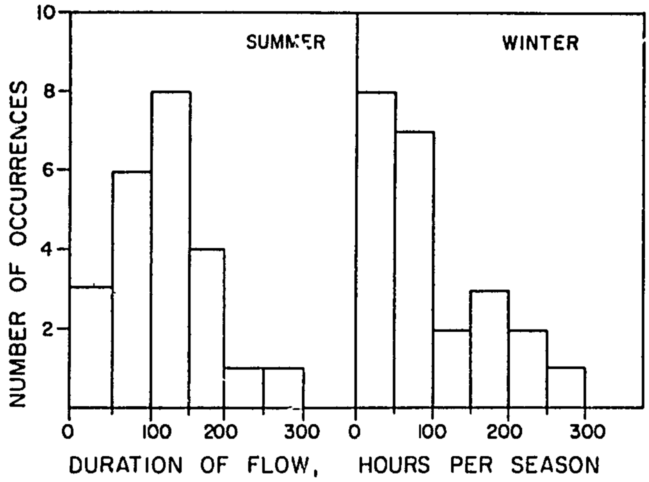

FIG. 2.—DURATION OF FLOW, HOURS PER SEASON

generating function of f_N is

$$F_N(s) = e^{-m+ms} \quad \ldots\ldots\ldots (2)$$

The concept of generating functions $F(s)$ is introduced to obtain the summation of a known or random number of random variables.

$$F(s) = \sum_x f(x)\, s^x \quad x = 0, 1, \ldots \qquad (3)$$

This permits computation of the mean and variance by evaluating the following equations at $s = 1$:

$$\text{Mean} = F'(s)\Big|_{s=1} \qquad (4)$$

$$\text{Variance} = \{F''(s) + F'(s) - [F'(s)]^2\}\Big|_{s=1} \qquad (5)$$

Summer Rainfall Event—Amount, Duration, and Shape Factor.—The amount, R, of point rainfall, given that a rainfall event occurs over the area considered, was found to be geometric (15):

$$f_R(x) = (1 - p)\, p^x \quad x = 0, 1, \ldots \qquad (6)$$

The parameter, p, is the probability that a point receives a runoff-producing

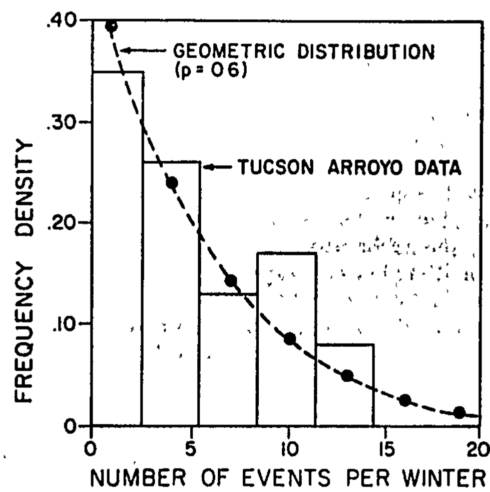

FIG. 3.—NUMBER OF EVENTS PER WINTER

amount of rain. The generating function corresponding to Eq. 3 is

$$F_R(s) = \frac{1 - p}{1 - ps} \qquad (7)$$

Clearly, summer runoff cannot be described using rainfall at one point only (although this is the quantity measured physically). For a small watershed, the sum of two or three point measurements may be sufficient to describe the contribution of rainfall amount to runoff. The distribution of the sum of k_0 independent and identically distributed geometric random variables is negative binomial with generating function (13):

$$G_{k_0}(s) = \left(\frac{1 - p}{1 - ps}\right)^{k_0} \qquad (8)$$

It will be assumed that the areal rainfall amount distribution is given by Eq. 8. If the point measurements are dependent, k_0 is smaller than the number of measurement points in space. This naturally suggests a pathway for the design of rainfall networks and the evaluation of the worth of rainfall data. In order to get at the prediction of opportunity for natural recharge, it seems natural to consider the flow duration.

In addition to depending upon the amount, R, of rainfall and the parameter, k_0, the flow duration, DF, depends also, among other variables, upon rainfall duration DR and a shape factor of the hyetograph TM. The joint probability density of DR and TM should thus be known before the probability density of DF can be determined; however, data that would permit such analysis are either not available or not yet retrieved, so that this step is left for further research. In the meantime, rough hypotheses will allow the analysis to proceed.

Summer Streamflow Event—Frequency.—In a semiarid country, assume that the number of events occurring per year is such that the streamflow frequency density is the same as the rainfall frequency density and is given by a Poisson distribution (Eq. 1). As shown in Fig. 1, a Poisson distribution with mean m = 9.6 may be acceptable; the use of a Kolmogorov-Smirnov test, shown in Fig. 4, illustrates that this hypothesis cannot be rejected at the 0.10 level of significance. The variance, s^2, is found to be 16, which indicates that the distribution may be flatter than a Poisson distribution because by definition the mean and variance should be identical. This may be explained by the fact that some flows are caused by more than one rainfall event. For a larger watershed, the flow frequency will be obtained by adding a random number of Poisson rainfalls. If the number of rainfall cells over a watershed is geometrically distributed, the probability density of streamflow occurrences will be a compound Poisson distribution. Experimental confirmation of this hypothesis is another area for further study.

Summer Streamflow Event—Duration.—A priori considerations on a minimum set of factors probably affecting the duration of flow DF lead to the following relation

$$\mathrm{DF} = f(R, \mathrm{DR}, \mathrm{TM}, \mathrm{WT}) \quad (9)$$

in which R = areal rainfall amount; DR = duration of rainfall; TM = time to mass center, a shape factor of the hyetograph; and WT = water table depth below streambed. Other factors such as storm direction and antecedent watershed conditions might be included.

As none of the quantities, DR, TM and WT are known, it is hoped that the relation, DF = $f(R)$, will explain a substantial part of the variability in DF.

As a first approximation, assume that DF has the same density function as R, namely, negative binomial with parameters p and k_0 and generating function given by Eq. 8. Experimental results are shown in Fig. 5; the hypothesis that the duration of flow per event DF (with a 4-hr grouping) follows a negative binomial distribution with parameters p = 0.56 and $k_0 \approx 2$ cannot be rejected using a Kolmogorov-Smirnov test at the 0.10 level. The parameters were found by the method of moments on the grouped data. Note that the mean of the actual and assumed distribution are the same:

$$\frac{k_0 p}{1 - p} = \frac{2 \times 0.56}{0.44} = 2.5 \quad (10a)$$

Summer Streamflow—Number of Hours of Flow per Year.—The total number of hours of flow X per summer season is obtained by summing a random number, N, of independent identically distributed random variables DF. The generating function of X is then (13):

$$X(s) = F_N\left[G_{k_0}(s)\right] \qquad (10b)$$

in which $G_{k_0}(s)$ = the generating function corresponding to DF. Using Eqs. 2 and 8:

$$X(s) = \exp\left[-m + m\left(\frac{1-p}{1-ps}\right)^{k_0}\right] \qquad (11)$$

is obtained.

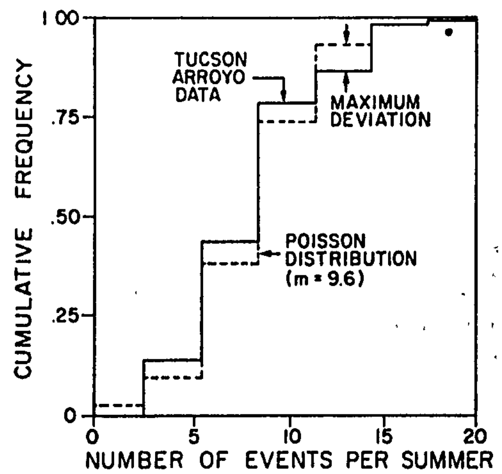

FIG. 4.—NUMBER OF EVENTS PER SUMMER, CUMULATIVE

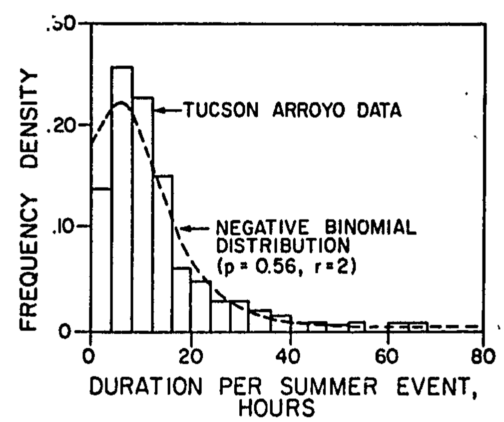

FIG. 5.—DURATION PER SUMMER EVENT, IN HOURS

The distribution corresponding to Eq. 11 with $m = 9.6$, $p = 0.56$, and $k_0 = 2$, has a mean and variance of 110 hr and 113 hr^2, respectively; whereas, the data shown in Fig. 2 have a mean and variance of 118 hr and 167 hr^2, respectively. The discrepancy may be explained by the fact that the number of events per year follows a distribution with a larger variance than the Poisson. This is still another point left for further study. Also, distributions should, strictly speaking, be truncated, but this effect is not significant as compared to small sample error. The actual distribution can be found from Eq. 11, by taking successive derivatives which are evaluated at $s = 0$. There is a need to computerize this process. A less detailed analysis will be performed for winter events, so that total hours of flow on a yearly basis can be computed.

Winter Streamflow Event—Frequency.—Here the data on rainfall frequency, the amount, duration, and shape factor of rainfall events are not available; obtaining or retrieving and analyzing the data, or both, is left for further investigations. This study will be restricted to an examination of the available data in order to set forth a methodology.

Data shown in Fig. 3 indicate that winter flow events may follow a geometric distribution with parameter $p_0 = 0.6$ and generating function given by Eq. 7:

$$F_M(s) = \frac{1 - p_0}{1 - p_0 s} \qquad (12)$$

A reason for departure from a Poisson distribution is due to meteorological factors such as persistence, which often prevents the occurrence of more than one yearly flow. As pointed out earlier, winter and summer flow events thus seem to belong to different populations.

Winter Streamflow Event—Duration.—In the absence of any rainfall data, it can only be said that a negative binomial distribution for the duration of flow DF, with parameters $p_1 = 0.65$ and $k_1 = 2$, cannot be rejected, using a Smirnov test, at the 0.10 level (Fig. 6). Therefore, the generating function of DF is given by Eq. 8:

$$G_{k_1}(s) = \left(\frac{1 - p_1}{1 - p_1 s}\right)^{k_1} \qquad (13)$$

Winter Streamflow—Number of Hours of Flow Per Year.—The generating function, $Y(s)$, of the total number of hours of flow per winter season Y is obtained by the formula derived in a similar fashion as for summer flows:

$$Y(s) = F_M\left[G_{k_1}(s)\right] \qquad (14a)$$

Using Eqs. 12 and 13:

$$Y(s) = (1 - p_0)\left[1 - p_0\left(\frac{1 - p_1}{1 - p_1 s}\right)^{k_1}\right]^{-1} \qquad (14b)$$

is obtained in which $p_0 = 0.6$; $p_1 = 0.65$; and $k_1 = 2$. The mean and variance computed from Eq. 14*b* are, respectively, 85 hr and 450 hr^2, as compared to the actual values 93 hr and 294 hr^2. The actual distribution is shown in Fig. 2. As for the number of hours of summer flow it would be desirable to plot the theoretical density represented by Eq. 14*b*.

Summer and Winter.—The total number of hours of flow per year Z can be obtained using the convolution $Z = X + Y$, because winter and summer flows are, by construction, independent. The generating function of Z is then

$$Z(s) = X(s)\,Y(s) \quad \ldots\ldots \quad (15)$$

in which $X(s)$ and $Y(s)$ are given, respectively, by Eqs. 11 and 14b. The mean and variance computed from Eq. 15 are, respectively, 194 hr and 563 hr^2 as compared to the actual values of 211 hr and 461 hr^2.

Given the density function of the total number of hours of flow per year, the next step is to relate duration of flow to flow magnitude and to recharge rate into the aquifer. Not only would this require well data on water level fluctuation proximate to the channel but also the postulation of some deterministic hypothesis in the transformation of streamflow to natural recharge. To arrive at estimates of the stochastic properties of natural recharge over the entire recharge zone associated with the river channel would require ex-

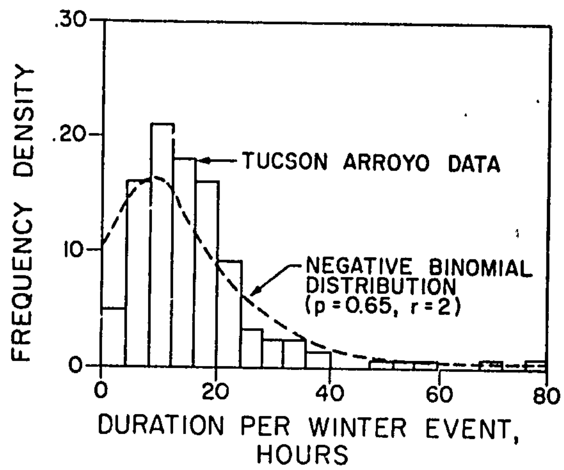

FIG. 6.—DURATION PER WINTER EVENT, IN HOURS

trapolation of observed streamflow fluctuations at the gaging station to other locations. This may be done by appropriate flood routing models for permeable bed channels. The ground-water hydrologist tends to circumvent the aforementioned questions by water balance studies (on an annual basis) of well water levels in the vicinity of the river. Little is known about the statistical reliability of such estimates in a formal sense.

TIME SERIES ANALYSIS OF MONTHLY RILLITO CREEK STREAMFLOW

The utility of the methods of time series analyses of streamflow data has been amply demonstrated for channels with perennial flows (14). However, when the time series contains on the order of 53 % zero flow, as is the case for Rillito Creek (see Table 1), serious questions arise about the interpretation of correlograms and variance spectra for such sequences. Serial correlations found between streamflows of adjacent months (as shown in Table 2) do not necessarily reflect the storage capabilities of the basin because there is no base flow. The only explanation for such persistence in flows would be

TABLE 1.—STATISTICS OF TOTAL MONTHLY STREAMFLOW[a]

Month (1)	Mean[b] (2)	Variance (3)	Standard deviation (4)	Coefficient of variation (5)	Skewness (6)	Number of years with zero flows (7)	Zero flows, as a percentage (8)
October	2.54	7.32	2.71	1.07	7.54	38	66
November	8.20	188.3	13.72	1.67	7.53	41	71
December	115.80	63,800.	252.60	2.18	7.83	40	69
January	54.32	17,080.	130.70	2.41	7.54	31	53
February	54.26	9,080.	98.03	1.81	7.67	31	53
March	31.62	1,910.	43.71	1.38	7.54	25	43
April	4.46	36.91	6.08	1.36	7.52	44	76
May	4.16	65.63	8.1[illegible]	1.95	7.51	54	93
June	3.11	12.46	3.53	1.14	7.51	47	81
July	36.80	8,072.	89.84	2.44	7.54	6	10
August	45.25	2,573.	50.73	1.12	7.53	4	07
September	22.29	2,519.	50.19	2.25	7.56	12	21

[a] Excluding zero flows.
[b] In acre-feet (AF).
Note: There were 373 months with zero flows over a total of 696 data points. The percentage of zero flows over this record is 53.

TABLE 2.—MONTHLY FLOWS ON RILLITO CREEK

Month (1)	October (2)	November (3)	December (4)	January (5)	February (6)
October	1.00	0.25	0.39	0.27	0.33
November	0.25	1.00	0.31	0.20	0.56
December	0.39	0.31	1.00	0.40	0.88
January	0.27	0.20	0.40	1.00	0.45
February	0.33	0.56	0.88	0.45	1.00
March	0.28	0.30	0.55	0.40	0.53
April	0.37	0.47	0.62	0.37	0.69
May	-0.06	-0.04	-0.03	-0.04	-0.05
June	-0.13	-0.09	-0.08	-0.08	-0.10
July	-0.11	-0.05	-0.08	-0.08	-0.09
August	-0.2[illegible]	-0.17	-0.14	-0.10	-0.10
September	-0.14	-0.11	-0.04	-0.05	-0.01

[a] Confidence limits on the cross correlation coefficients were based on a two tailed - 0.28; upper limit of confidence band is approximately + 0.28.

in the persistence of regional meteorological conditions. Even though this factor dominates flows in more temperate climates, it seems more natural to model aridland streamflows in terms of alternating sequences of wet and dry periods and to include in the model a provision for meteorological transitions.

It would seem that this approach is important to future use of stochastic models of streamflow in subsequent operations research studies of water resource systems in aridlands.

Tables 1 and 2 summarize the results of a statistical study of total monthly streamflow on Rillito Creek for the period 1909 to 1966. A bimodal population of flow and noflow periods is clearly indicated in the results. But the 53 % of the time in which zero-flow occurred is deceptive because in many of the months flows were recorded only a few days within a month. This condition repeats itself in many other streamflow records and provides a strong argument for stochastic models that are distinctly different from the class of models described by the lag-one or first-order Markov process model.

The coefficients of variation noted in Table 1 are quite typical of aridland conditions. The skewness reflects the wide spread in observed flows in each month. Table 2 gives the cross-correlation matrix for each month with respect to the remaining 11 months. Of significance in this table are the substantial positive cross-correlations between the set of months from October to May. These strong seasonal linkages and the extreme variability of monthly flow totals are important features to be preserved in any proposed stochastic model.

ANALYSIS

Insight into the results shown in Table 1 can be obtained by considering the results of the event-based analysis of the Tucson Arroyo. The noticeable features of the flow are:

(1909-1966), CROSS CORRELATION OF MONTHS[a]

March (7)	April (8)	May (9)	June (10)	July (11)	August (12)	September (13)
0.28	0.37	-0.06	-0.13	-0.11	-0.24	-0.14
0.30	0.47	-0.04	-0.09	-0.05	-0.17	-0.11
0.55	0.62	-0.03	-0.08	-0.08	-0.14	0.04
0.40	0.37	-0.04	-0.08	-0.08	0.0	-0.05
0.53	0.69	-0.05	-0.10	-0.09	-0.10	-0.01
1.00	0.46	0.40	0.16	-0.08	-0.12	-0.10
0.46	1.00	-0.04	-0.10	0.21	-0.12	-0.09
0.40	-0.04	1.00	0.52	-0.05	-0.11	-0.05
0.16	-0.10	0.52	1.00	0.01	-0.02	-0.05
-0.08	0.21	-0.05	0.01	1.00	0.50	-0.00
-0.12	-0.12	-0.11	-0.02	0.50	1.00	0.26
-0.10	-0.19	-0.05	-0.05	-0.00	0.26	1.00

test at the 5 % significance level: lower limit of confidence band is approximately

1. The flow duration per event is similar in summer and in winter (Figs. 5 and 6).

2. The number of events per season looks radically different in summer than in winter (Figs. 1 and 3). The number of summer events is approximate-

ly Poisson, so that mean and variance are equal; the number of winter events is geometric, so that the variance is $1/(1 - p_0) = 2.5$ times larger than the mean.

3. Thus, winter flows would differ from summer flows principally because the number of events in winter is much more variable than in summer, as confirmed by Table 1.

4. Notice that by studying only the yearly duration of flow, the reason for different distributions in summer and winter disappears.

5. By studying monthly flows, the seasonal differences appear; however, there is no reason other than the Gregorian calendar to cut up flows (if any) the first of each month.

6. The advantages of an event-based analysis have thus been spelled out regarding differentiation between seasons, and correlation with known meteorological factors (independence in summer, persistence in winter).

7. Parameters m, p, k_0, p_0, k_1, and p_1 may have a value for analysis of regional properties of streamflow in aridlands and possibly in more temperate climates.

SUMMARY AND CONCLUSIONS

A framework has been presented for the analysis and modeling of streamflows in aridlands. The basic event series of streamflows as observed in nature is found to be more informative than monthly and annual series. The frequency of occurrence of summer and winter events at the Tucson Arroyo gaging station are described, respectively, by a negative binomial distribution and a geometric distribution. Meteorologically this result is explained in terms of the temporal and spatial independence of summer thunderstorms and the persistence of winter cyclonic storms. The negative binomial distribution also describes the duration of flow per event in both winter and summer.

Meriting further exploration in the study of ephemeral flows but not considered herein are the following: (1) Zero-crossing theory (30); (2) theory of forecasting (4); (3) extreme value theory for a random number of random variables (34); (4) multivariate time series models to allow for inclusion of other causal variables that influence flow prediction; (5) a decision-theoretic approach to estimation of parameters in deterministic and stochastic models (8); and (6) the study of time series of ground-water level fluctuations in wells adjacent to aridland channels periodically recharging the unconfined aquifer.

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APPENDIX II.—NOTATION

The following symbols are used in this paper:

DF = duration of flow;
DR = duration of rainfall;
$F(s)$ = generating function;
$F_M(s)$ = generating function of M;
$F_N(s)$ = generating function of N;
$F_R(s)$ = generating function of R;
$f_N(j)$ = discrete distribution of N;
G_{k_0} = generating function of areal rainfall amount in summer;
$G_{k_1}(s)$ = generating function of DF in winter;
j = index;
k_0 = parameter for DF in summer;

k_1 = parameter for DF in winter;
M = number of rainfall events per winter season;
m = mean of Poisson distribution;
N = number of rainfall events per summer season;
p = probability that point receives runoff-producing rain;
p_0 = parameter for number of events per winter season;
p_1 = parameter for DF in winter;
R = amount of point rainfall;
s = variable in generating functions;
s^2 = variance;
WT = water table depth below streambed;
X = total number of hours of flow per summer season;
$X(s)$ = generating function of X;
Y = total number of hours of flow per winter season;
$Y(s)$ = generating function of Y;
Z = total number of hours of flow per year; and
$Z(s)$ = generating function of Z.

8455 ANALYSIS OF EPHEMERAL FLOW IN ARIDLANDS

KEY WORDS: arid lands; channel flow; hydrographs; hydrology; monthly; probability theory; runoff; stochastic models; stream flow; water loss

ABSTRACT: An important parameter is found to be the flow duration, which describes the opportunity for natural recharge. Arid-land hydrology is, in effect, characterized by the absence of base flow, special precipitation patterns, and scarcity of data. A stochastic model of flow duration is developed for a stream near Tucson, Ariz. using a regional model of rainfall. The number of flow events per summer seems to follow a Poisson distribution and that in winter, a geometric distribution, whereas the flow duration per event seems to follow a negative binomial distribution in both cases. Time series estimates of monthly streamflow of the Rillito Creek, Ariz. are given to illustrate the correlation between various months and the difficulty of interpreting such time-lumped data.

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