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Fogel,M.M.; Duckstein,Lucien

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PREDICTION OF CONVECTIVE STORM RUNOFF IN SEMIARID REGIONS

Martin M. FOGEL and Lucien DUCKSTEIN
University of Arizona, Tucson, Arizona, USA

SUMMARY

Analysis of 14 years of data taken on the Atterbury Experimental Watershed located near Tucson, Arizona, USA has led to a procedure for predicting semiarid basin runoff from convective storms. The record event for small catchments is usually the result of these localized, high-intensity, short-duration storms.

A general runoff equation was developed based on the mass curves of infiltration and runoff. The relationship included a runoff coefficient defined as the ratio of runoff to effective rainfall (storm total less initial abstractions). Multiple regression techniques indicated that for very small areas (less than two square kilometers), this coefficient remains essentially constant for a given basin. However, for catchments ranging in size up to 20 or 25 square kilometers, additional information can be obtained by characterizing the time distribution of storm rainfall.

Using the storm's maximum 15-minute intensity as the time distribution parameter, it was shown that the runoff coefficient was a function of this parameter. Thus, the probability of equalling or exceeding a given volume of runoff is an event that can be determined by a joint probability. The results from a previously developed model that describes the spatial distribution and frequency of occurrence of convective storm rainfall were combined with the frequency distribution of the storm's intensity to define the joint probability function.

An extreme-value distribution was found to adequately describe the annual maximum series of runoff volumes. Using selected return periods, results from this analysis were compared to runoff predicted from rainfall data. The comparison was favorable, especially for the longer return periods.

Antecedent rainfall and the location of the point of maximum rainfall on the relatively small watersheds had no significant effect, statistically, on runoff.

RÉSUMÉ

PRÉVISION DE L'ÉCOULEMENT DÛ À DES AVERSES CONVECTIVES DANS DES RÉGIONS SEMI-ARIDES

L'analyse de 14 années de données du bassin expérimental d'Atterbury près de Tucson, Arizona a conduit à une méthode pour prévoir l'écoulement d'un bassin semi-aride du fait d'averses convectives. Les événements records pour de petits bassins sont habituellement le résultat de ces averses localisées, de haute intensité et de faible durée. Une équation d'écoulement a été établie, basée sur les courbes cumulées de l'infiltration et de l'écoulement. La relation comprend un coefficient d'écoulement défini comme étant le rapport de l'écoulement à la pluie effective (averse totale diminuée des abstractions initiales). Les techniques de régression multiple montrent que pour de très petits bassins (moins de 2 km^2), ce coefficient reste quasi constant pour un bassin donné. Cependant, pour les bassins de 20 à 25 km^2, des informations additionnelles peuvent être obtenues caractérisant la distribution dans le temps de l'averse.

Utilisant l'intensité maximum de 15 minutes pour l'averse comme paramètre de distribution temporelle, il est montré que le coefficient d'écoulement est fonction de ce paramètre. Il en résulte que la probabilité d'égaliser ou de dépasser un volume donné d'écoulement est un événement qui peut être déterminé par une probabilité combinée. Les résultats d'un modèle primitivement établi qui donne la distribution spatiale et la fréquence de production d'une averse de pluie convective sont combinés avec la distribution des fréquences, de l'intensité d'averse pour définir la fonction de probabilité combinée.

Une distribution de valeurs extrêmes est trouvée pour représenter adéquatement la série des maxima annuels des volumes d'écoulement. En utilisant des périodes de retour choisies, des

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résultats de cette analyse sont comparés aux écoulements déduits des données de pluies. La comparaison est favorable, surtout pour les longues périodes de retour.

Les pluies précédentes et la situation du point de pluie maximum sur des bassins relativement petits n'ont pratiquement aucun effet statistique sur l'écoulement.

INRTODUCTION

The record runoff-producing event that occurs on small watersheds located in the semiarid, southwestern United States will most likely be the result of convective storm rainfall. An individual event occurring on watersheds of less than 100 mile^2 (260 km^2) oftentimes

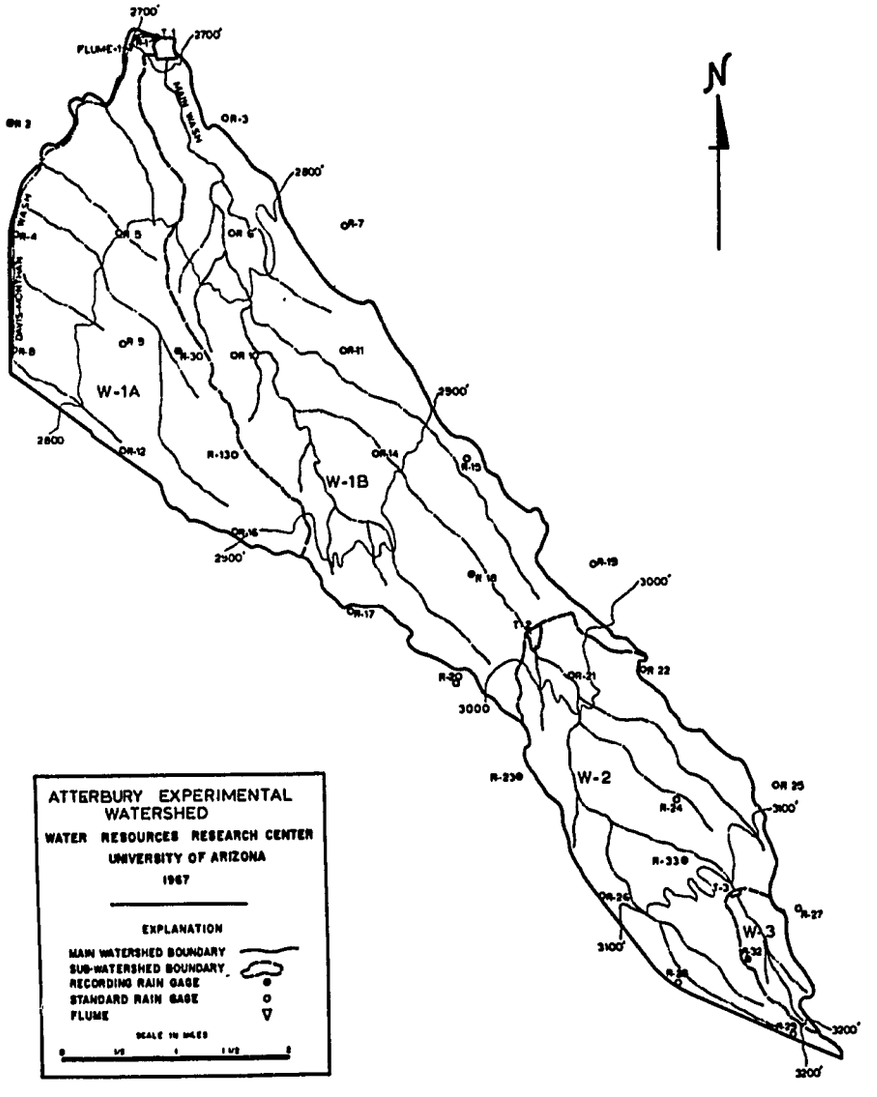

FIGURE 1. *The Atterbury Experimental Watershed*

exceeds the mean annual runoff. While designers of water control structures are cognizant of this wide range of variability, they are faced with a lack of procedures that recognizes the peculiarities of these localized high-intensity, short-duration storms. The problem becomes even more acute in the urban areas of the region where a higher level of water control is generally required than for the rural sections.

Scope of Study

In an effort to fill the above void, 14 years of hydrologic data collected on the Atterbury Experimental Watershed located near Tucson, Arizona, have been analyzed for the purpose of developing useful rainfall-runoff relationships. Specifically, it is the intent of this paper to propose an equation that can be used to estimate runoff from parameters describing convective storm rainfall and the watershed. Inasmuch as rainfall records are usually available for much longer time periods than are runoff records, it seems feasible to develop rainfall-runoff relationships in order to use the frequency distributions of existing precipitation records. The usefulness of such relationships is contingent on the adequacy of frequency distributions of rainfall amounts or intensities and on a means for relating rainfall frequency to runoff frequency. These aspects will be examined in this paper.

Experimental Area

The region in which the experimental catchment is located is characterized by isolated mountain blocks separated by broad alluvial-floored basins whose elevations range up to 4,500 feet (1,372 m) above sea level.

The climate of the region is primarily semiarid with the principal exception being on the summits of the high mountains. Due to a combination of low precipitation, high temperatures and low humidity at altitudes below 5,000 feet (1,524 m), a high evaporation potential exists. Thus, most of the available moisture is returned to the atmosphere or is utilized by vegetation mainly in the areas where rainfall occurred.

In much of the region, there are two distinct periods of precipitation. The catchment area receives approximately 6 inches (152 mm) out of a total of 11 inches (279 mm) during the summer months of July, August, and September. The non-frontal or local thunderstorm accounts for most of the precipitation in this period. Cyclonic storms are prevalent during the period December to March. Prolonged dry spells normally separate the two relatively wet seasons.

The research area is drained by two ephemeral streams both of which flow into a main reservoir. In addition, there are two smaller reservoirs in the upper reaches of the

TABLE 1. Physical Features of Atterbury Experimental Watershed

Physical feature	Subwatershed			
	W-1A	W-1B	W-2	W-3
Area, square miles	5.17	7.77	4.49	0.47
Average landslope, percent	1.2	2.1	2.6	3.7
Channel slope, percent	0.63	0.82	0.84	1.20
Channel length, thousand feet	32	34	25	10

watershed. As shown in figure 1, the experimental area is divided into four subwatersheds, the physical features of which are presented in table 1. Within the upper watershed W-3, data from two small catchments, each approximately 16 acres (6.5 ha) in size, will also be examined.

Landslopes range up to 5 percent while the long and narrow subwatersheds have relatively flat channel slopes. Vegetative cover is thin, consisting primarily of creosote bush (*Larrea tridentata*), palo verde (*Cercidium microphyllum*), mesquite (*Prosopis juliflora*) and ocotillo (*Fouquieria splendens*). Soils range from sandy or gravelly surfaces on the rounded, gently sloping ridges to soils with loam surfaces in the nearly level water courses that separate the ridges. Much of the area is underlaid with a cemented zone of lime accumulation 6 to 24 inches (15 to 61 cm) below the surface. The main drainageways consist of either broad, grassy swales, or gullies with sandy bottoms that are as deep as 5 feet (1.5 m) and as wide as 20 feet (6 m).

The original basic rain gage network area of nearly 20 square miles (52 km^2) contains 29 gages laid out on approximately a one-mile (1.6-km) grid. Additional gages have been added at various times to study storm patterns as determined from denser networks. Storm runoff volumes for the four subwatersheds are determined from three volumetrically-calibrated reservoirs and by use of a critical depth flume. Runoff from the two small 16-acre watersheds are measured by HL flumes developed and rated by the U.S. Soil Conservation Service.

Convective Rainfall Model

Prior to discussing rainfall-runoff relationships, it is deemed advisable to present some of the characteristics of the previously developed rainfall model (Fogel and Duckstein, 1969).

Areal Distribution

Area-rainfall depth formulas are attempts at describing the spatial distribution of rainfall. Analysis of the patterns of the convective storms that fell on the experimental area resulted in the selection of an exponential relation as best representing this distribution. Assuming circular isohyets, this relation is

$$R = R_0 \exp(-\pi r^2 b) \tag{1}$$

where :

R is the depth of rainfall of the enclosing isohyet located at a distance r from the point of maximum rainfall (storm center);
R_0 is the depth of rainfall at the storm center, and
b is a dispersion parameter found to be dependent on R_0.

A regression analysis with a correlation coefficient of 0.90 produced the equation

$$b = 0.27 \exp(-0.65 R_0) \tag{2}$$

Although the plotted isohyets of the convective storms were found to be elliptical, assuming them to be circular introduces only minor errors.

Depth-Duration Relationship

In examining the rainfall charts, it was noted that many of the storms had a relatively long period of low intensity rainfall either at the beginning or end of the storm which accounted for much of the duration but little of the rainfall. The duration of 90 percent of the total storm depth, therefore, was used in obtaining a depth-duration relationship.

A least-squares analysis with a 75 percent correlation produced the equation

$$R = 2.42 \log D + 1.89 \tag{3}$$

where:

R is the rainfall depth in inches and

D is the duration of 90 percent of the total storm depth measured from the mass center of the storm.

Maximum Point Rainfall

In the previously mentioned paper by Fogel and Duckstein (1969) a procedure for determining the frequency distribution of maximum point rainfall was developed. In general form, the relationship is

$$F_X(a) = \sum_j \{[F_R(a)]^j f_N(j)\} \tag{4}$$

where:

$F_X(a)$ is the cumulative distribution function for a inches of rain (in ½-inch increments);

$F_R(a)$ is the cumulative function for rainfall amount given the occurrence of an event and

$f_N(j)$ is the density function for the number of storms per year.

Analysis of the data strongly indicated that a geometric distribution could be used to describe the distribution of the random variable R, the amount of storm rainfall. A Poisson distribution was used to represent the distribution of the random variable N, the annual number of convective storms. Inserting these distribution functions into equation 4 and using the data to obtain values for the parameters, recurrence intervals for various point rainfall depths were calculated. These probabilities were then compared with frequencies determined by fitting an extreme-value distribution to 75 years of data collected from a nearby U.S. Weather Bureau station. The results shown in table 2

TABLE 2. Comparison of Methods for Calculating Return Periods for Convective Storm Rainfall

Return Period Years	Rainfall, inches	
	Model	Historical Records
100	4.1	3.1
50	3.5	2.7
25	3.0	2.5
10	2.3	2.0
5	1.8	1.7
2	1.0	1.3

indicate that the model exhibits a similar mean but a greater variance and, hence, lower recurrence intervals for the extreme rainfall depths to that obtained from analyzing historical records.

In addition to possibly more accurate predictions of convective rainfall amounts, this procedure has the added advantage in that results can be used to evaluate the effectiveness of hydrologic processes employed to conserve water or to increase water yield. For example, in some of the more arid climates, research is being conducted on methods for increasing the runoff volume through the manipulation of soils and/or vegetation. The

economics of these processes requires knowledge of the probabilities of both the number as well as the magnitude of runoff-producing events. The more conventional procedures concern themselves with the forecasting only of rainfall amounts.

Runoff from Convective Storms

A rainfall-runoff relationship specifically for the short-duration, varying intensity convective storm can be derived in a manner similar to the runoff equation developed by the U.S. Soil Conservation Service (1964). In this latter procedure, the storms are considered to be more of the frontal type in which storm durations are considerably longer than for convective storms and the rainfall intensity is assumed to be constant. Most convective storms in the southwestern United States are over within one hour and practically all within two hours and their mass curves are distinctly curvilinear as indicated by equation 3 and shown in the schematic diagram, fig. 2.

General Equation

If it is assumed that initial abstractions I_a (interception and depression storage) occur during the first 10 or 15 minutes of a storm (a similar assumption is made in the Soil Conservation Service development), the remaining portion of the mass rainfall curve can be made to fit a power function of the form

$$R_e = KD^a \tag{5}$$

where R_e can be considered as effective rainfall (total rainfall less initial abstractions), D is the duration of effective rainfall and K and a are constants. Reviewing the mass curves of maximum point rainfall indicated that the exponent a in equation 5 ranged mostly from 0.4 to 0.6. An exponential value of slightly higher than 0.5 was obtained when

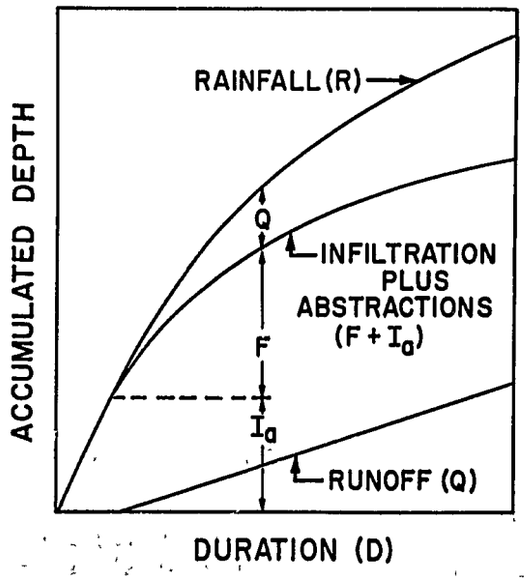

Figure 2. *Schematic diagram of accumulated values for rainfall, runoff and infiltration plus initial abstractions*

equation 3 was analyzed in a similar manner. Thus, equation 5 can be approximated by

$$R_e = KD^{1/2} \tag{6}$$

It is of interest to note that in the rainfall depth-duration relationship of the world's greatest rainfalls as developed by Jennings (1950), the exponent turned out to be 0.486.

Insofar as the mass infiltration curve is concerned, Philip (1957) by an approximation to a rigorously developed mathematical formula derived the infiltration equation

$$F = St^{1/2} + At \tag{7}$$

where t is the time and the first term including the constant S describes the contribution to infiltration due to capillarity, while the second term containing the constant A essentially represents the contribution due to gravity.

Commenting on the work of Philip, Rose (1966) stated that for "small" times after the surface application of water to initially dry soil, the potential gradients due to gravity are negligible compared to those due to capillarity. As such, the infiltration rate at the surface decreases as $t^{-1/2}$ from very high initial values. Thus, neglecting the gravitational component to infiltration for the short duration convective storm, equation 7 can be reduced to

$$F = SD^{1/2} \tag{8}$$

Using the relationship

$$Q = R_e - F \tag{9}$$

and noting that the exponents in equations 6 and 8 are the same, it can readily be shown that

$$Q = C_r(R - I_a) \tag{10}$$

in which C_r is a constant.

Effect of Storm's Temporal and Spatial Distribution

In an effort to determine the relative importance of the storm's distribution in time and space, multiple regression techniques were used. Assuming the linear form of the regression equation, the model for making this determination can be written as

$$Q = B_0 + B_1 R + B_2 T + B_3 S + e \tag{11}$$

where Q is storm runoff, R is total storm rainfall, T is a time distribution factor, S is a space factor which indicates where the heaviest rainfall was located on the watershed, B_i (i = 0, 1, 2, 3) are coefficients to be estimated and e is the error of estimation.

When using a multiple regression analyses, it is assumed that the variables R, T, and S are all independent. It is readily apparent that storm rainfall and storm center location on the watershed are independent variables. The relationship between total depth of rainfall and the time distribution of rainfall is not as definitely independent. However, it was noted that the correlation between the total rainfall and the maximum depth or intensity for a given time period within the storm decreased as the time period became shorter. With knowledge of the limits of accuracy in measurement and in processing the data, the maximum intensity for a 15-minute period was selected as being representative of a short burst of runoff-producing rainfall that is sufficiently uncorrelated with storm total to justify consideration as being an independent variable.

Several parameters were considered to represent the storm's space or location factor S. These included the distance from the storm center or point of maximum rainfall to the

watershed outlet and the percentage of the watershed that received at least a specified amount of rainfall. After numerous trials on the subwatersheds, it was concluded that none of the selected parameters significantly improved the prediction of runoff. This may have been due to the small watershed areas so that in using the mean value for the total storm rainfall the effect of S was masked.

It should be mentioned at this time that the effect of antecedent rainfall on runoff was investigated and also produced negative results. This affirms the work of Keppel (1965) and Schreiber and Kincaid (1967) whose studies were also located in the semiarid southwestern United States. This does not necessarily mean that antecedent soil moisture has no effect on runoff but rather that the likelihood of two or more storms occurring on the same area within a short time span is remote.

In determining the relative effects of various convective rainfall characteristics on runoff, the final regression equation used was

$$Q = b_0 + b_1 R + b_2 i_{15} \tag{12}$$

where:
Q is as before the runoff volume;
R is the mean depth of storm rainfall determined by the isohyetal method;
i_{15} is the maximum 15-minute intensity and
b_0, b_1, and b_2 are regression coefficients.

Experimental Results

Table 3 summarizes the results of the regression analysis for the 16-acre catchments, W-3HL, and for three of the four larger subwatersheds. The fourth subwatershed, W-1A, was not used since some of the hydrologic characteristics of the area were altered during the period of record.

TABLE 3. Prediction Equations for Runoff Volumes from Convective Storms

Prediction Equation	Watershed			
	W-3HL (16 acres)	W-3 (302 acres)	W-2 (2875 acres)	W-1B (4974 acres)
$\hat{Q} = b_0 + b_1 R$				
b_0	−0.1531	−0.0886	−0.0491	−0.1101
b_1	0.4843	0.2150	0.1487	0.2936
r^2	0.87	0.83	0.64	0.62
S_e	0.07	0.05	0.04	0.06
$\hat{Q} = b_0 + b_1 R + b_2 i_{15}$				
b_0	−0.2006	−0.1302	−0.1541	−0.2300
b_1	0.4054	0.1858	0.1384	0.2671
b_2	0.0746	0.0298	0.0403	0.0468
r^2	0.91	0.87	0.82	0.78
S_e	0.06	0.05	0.03	0.05

In the first set of prediction equations in which the depth of storm rainfall was the only independent variable used, the coefficient of determination, r^2, ranged from 62 to 87 percent. The smaller the watershed, the greater the correlation between rainfall and runoff.

Introducing the time distribution variable into the regression equation, significantly reduced the unexplained variance, especially for the larger watersheds. Addition of this variable was significant at the one percent level for the two largest watersheds, W-1B and W-2, at the five percent level for the smaller 300-acre (121.4 ha) W-3 watershed and at the 10 percent level for the very small 16-acre (6.5 ha) catchments.

Analyzing the data from the small 16-acre watersheds tended to substantiate the general runoff equation for convective storms developed earlier. In this equation, runoff is related linearly with storm rainfall. Results of the regressison analysis indicated that the depth of storm rainfall accounted for nearly 90 percent of the variation in runoff. Similar results were obtained by Osborn and Lane (1969) on very small watersheds located in an area that receives about 25 percent more annual precipitation than does the Atterbury Experimental Watershed. They reported that the total storm rainfall accounted for 76 to 89 percent of the variance on four watersheds ranging in size from 1/2 to 11 acres (0.2 to 4.5 ha).

From table 3, the equation for predicting runoff from the 16-acre watersheds in which there is only one independent variable, storm rainfall, is

$$\hat{Q} = -0.1531 + 0.4843R \tag{13}$$

Rearranging and using the above equation in a deterministic manner results in

$$Q = 0.48(R - 0.32) \tag{14}$$

The similarity between equation 14 and the general runoff equation, equation 10, is readily apparent. The term within the parenthesis in the above equation can be considered as effective rainfall in which the initial abstractions (I_a) are equal to 0.32 inches (8.1 mm). Similarly, the value for the runoff coefficient (C_r), or ratio of runoff volume to effective rainfall volume, is 0.48. Thus, for very small watersheds, a simple linear relationship appears to be adequate for estimating runoff volume from convective storm rainfall.

It is apparent, however, that there must be areal limits for which an equation that uses only the depth of storm rainfall as an independent variable is applicable for estimating runoff. As shown in table 3, the simple correlation between rainfall and runoff decreases as the catchment size increases. This is to be expected as the convective rainfall variability in time and space increases with area, to say nothing of the added complications due to channel storage and transmission losses.

Staying within the arbitrary limits of watershed area of 10 square miles (26 km^2), it is still possible to use the general runoff equation as it does explain at least 60 percent of the variance. The unexplained variance can be significantly reduced with the inclusion of a variable such as the maximum 15-minute intensity. In simple regression analyses, the variance of the slope of the regression can be used to explain the deviations about the regression line. Assuming that these deviations are the result of short bursts of runoff-producing rainfall, it can be similarly assumed that the runoff coefficient, C_r, in the general runoff equation is affected by the storm's maximum 15-minute intensity. Thus, for a given drainage basin, the runoff coefficient is a function of both the characteristics of the watershed and the rainfall intensity. This conclusion is consistent with procedures that employ a coefficient of runoff in estimating the peak rate of runoff. An example of such a method is the use of the well-known Rational Method as recommended by the Water Research Foundation of Australia (1965). This procedure is applied to the estimation of the peak rate of flood runoff on catchment areas of 10 square miles or less.

Runoff Prediction

To predict runoff from ungaged catchments requires a method for evaluating watershed characteristics. In order to use the maximum 15-minute intensity with the general runoff

equation as suggested by the multiple regression analysis, a relationship between this time distribution parameter and the runoff coefficient is needed. These are subjects that will be discussed in the following section in connection with developing a procedure for estimating runoff from convective storms.

Evaluating Watershed Characteristics

It seems advisable that wherever possible, readily available procedures should be used to describe the hydrologic reaction of a variety of watersheds. For example, in the U.S. Soil Conservation Service method for estimating runoff, a runoff curve number, sometimes called a watershed index, is assigned to a particular hydrologic soil-cover complex with a given antecedent moisture condition. From this index, an empirical relationship is used to determine a potential infiltration term. Initial abstractions are estimated to be a fraction of this term. The infiltration term is then used with a runoff equation to relate rainfall to runoff.

The coefficient in the rational method for estimating peak runoff rates is another example of characterizing the runoff-producing capabilities of watersheds. It is a measure of the losses occurring on a watershed during a design storm. A value for a given watershed is selected to reflect the characteristics of the basin such as soil type, nature and extent of cover and land slope. Values range from 0.10 for flat lands with relatively high infiltration capacities to 0.95 for impervious surfaces. In some adaptations of the rational method, the coefficient is also dependent on the intensity of the design rainfall insofar as it governs infiltration rates.

The proposed equation for estimating convective-storm runoff from ungaged watersheds requires an estimate of two terms, namely, a runoff coefficient and initial abstractions. The latter term can be estimated by the U.S. Soil Conservation procedure, which estimates it to be 20 per cent of the potential infiltration term for most situations. In semiarid areas, this percentage is probably too high due to the relative scarcity of the vegetative cover. The more important term, however, in predicting runoff by this procedure is the runoff coefficient. An error in estimating the initial abstractions results in a constant error for all storms, while an error in the coefficient gives runoff estimates that are progressively poorer with increasing storm rainfall.

As at least a first approximation, it appears that the runoff coefficient in the rational method can be used as an estimate of the runoff coefficient in the proposed procedure. Both of these coefficients depend on infiltration losses to about the same degree. In the simple regression analysis shown in table 3 the regression coefficient b_1 is equivalent to the proposed runoff coefficient. These values are comparable to those that would be estimated for the rational method runoff coefficient for watersheds similar to the experimental area. It goes without saying that additional ressearch on other areas will be needed to substantiate this hypothesis.

Relating Runoff Coefficient to Time Distribution Parameter

The multiple regression analysis indicated that the maximum 15-minute intensity contributes significantly to predicting runoff. Also, it was previously mentioned that certain procedures use rainfall intensity as an added variable to watershed characteristics in evaluating the capabilities of a catchment for producing runoff. The following is a procedure which is used to relate these two parameters.

Assume that the runoff coefficient, C_r, and a time distribution parameter, T, are linearly related such that

$$C_r = \bar{C}_r + m(T - \bar{T}) \tag{15}$$

where $\bar{C}_r$ is the known tabulated runoff coefficient for a given watershed, $\bar{T}$ is a mean

value obtained from rainfall records and m is a constant to be determined. Inserting equation 15 into the general runoff equation 10 gives

$$Q = -(\bar{C}_r - m\bar{T})I_a + (\bar{C}_r - m\bar{T})R - (mI_a)T + mRT \tag{16}$$

Equation 16 suggests that R and T are not independent as initially assumed and that the multiple regression equation should be in the form

$$Q = b_0 + b_1 R + b_2 T + b_3 RT \tag{17}$$

The regression analysis, therefore, was rerun for the larger subwatersheds, W-1B and W-2. Results indicated that the regression coefficients differed from that shown in table 3 even though the contribution of the RT term in explaining the variance was not significant. Thus, as there is no significant interaction between the amount of rainfall and the maximum 15-minute intensity, these variables are still considered to be relatively independent.

With the assumption that in equation 17 runoff is predicted with no error, inspection of equations 16 and 17 indicates that these equations should produce equivalent results, as both equations contain similar terms. This observation allows estimates to be made for the constant m in equation 15. Equations 16 and 17 will lead to similar results only if the coefficients or constants for like terms are equal. With an additional restriction that I_a is a constant for a given watershed, inspection of the T term indicates that m must be equal to $-b_2/I_a$. Similarly, from the RT term, m and b_3 are equal. If equation 16 is exact and its values can be measured without error and if equation 17 explained all the variance, m would have equal values regardless of which relationship was used. Since none of these assumptions are entirely correct, a mean value for m was used.

The values for C_r and I_a for subwatershed W-1B as obtained from the simple regression results shown in table 3 are 0.29 and 0.37, respectively. Based on rainfall records of convective storms that were located over the experimental area, the mean value for the maximum 15-minute intensity was 2.83 inches per hour (72 mm/hr). The grouped data which had a standard deviation of 0.80 inches per hour (20 mm/hr) was fitted to a log

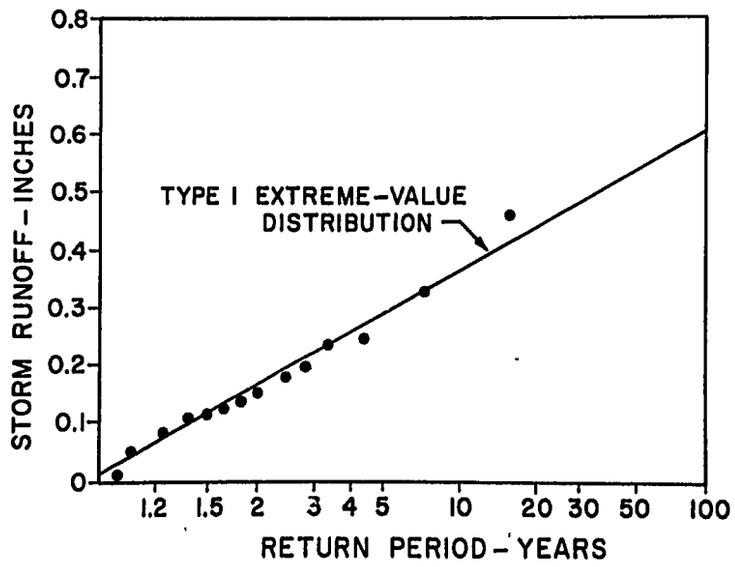

FIGURE 3. *Frequency distribution for runoff volume from Watershed W-1B*

normal distribution with good results. Thus, a probability can now be associated with any value of the variable T, the maximum 15-minute intensity. For selected values of T, table 4 presents cumulative probabilities of its occurrence and corresponding values for C_r as calculated from equation 15.

Comparing Rainfall and Runoff Frequencies

As a check on the rainfall-runoff relationship, runoff frequencies were determined by two separate procedures. First, an annual series of maximum convective strom runoff volumes was prepared for the 14 years of record. The data were fitted to several frequency distributions using techniques proposed by the United States Geological Survey (1968). As shown in figure 3, the type 1 extreme-value distribution appears to adequately describe the annual maximum series for subwatershed W-1B. Similar results were obtained with the other two subwatersheds. Figure 3 was then used to obtain anticipated runoff volumes for various return periods.

The other procedure in which the frequency distribution of convective storm rainfall was used, required that a design storm with a given storm center depth be superimposed over the watershed. Point rainfall depths for selected return periods are shown in table 2. Assuming each of these amounts to be the maximum depth of rainfall of a convective storm, the mean storm rainfall was determined by the isohyetal method. This is the value that is used in the general runoff equation.

Since it has been established that the runoff coefficient depends on the maximum 15-minute rainfall intensity, it becomes evident that the probability of equalling or exceeding a given volume of runoff is an event that can be defined by a joint probability. This joint probability can be stated as the probability of receiving a given amount of rainfall with at least a specified maximum 15-minute intensity. Continuing with the assumption that these variables are independent, the joint probability is simply the product of the two individual probabilities.

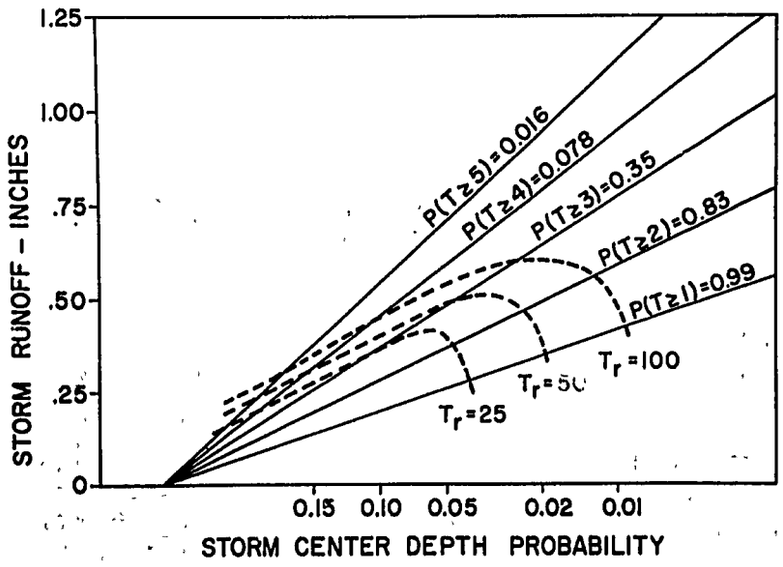

FIGURE 4. *Storm runoff return periods (T_r) as a function of maximum depth of rainfall and maximum 15-minute intensity (T)*

TABLE 4. Relationship between Runoff Coefficient and Time Distribution Parameter

T, in/hr	Prob ($i_{15} \geq T$)	Runoff Coefficient W-1B	Runoff Coefficient W-2
1	0.99	0.16	0.08
2	0.83	0.23	0.14
3	0.35	0.31	0.21
4	0.078	0.38	0.27
5	0.016	0.45	0.34

Figure 4 represents a graphical solution to the problem of obtaining the return period for a given volume of runoff.

Initially, the runoff coefficients shown in table 4 are used to obtain a family of lines relating rainfall to runoff based on the simple linear relationship

$$Q = C_r(R - I_a)$$

Since each coefficient or slope is dependent on a given T, the maximum 15-minute rainfall intensity, it is associated with a pre-determined cumulative probability. From a log normal distribution of T, the expected values of C_r and T are equivalent inasmuch as they have been assumed to be linearly related.

Along the horizontal axis, plotted are the probability densities of watershed W-1B receiving a convective storm with a point of maximum rainfall sufficient to produce a specified mean rainfall over the entire watershed. Thus, the probability of equalling or exceeding a given runoff volume is the product of the probability of receiving a given volume of rainfall and the cumulative probability of the storm having at least a specified rainfall intensity. The inverse of this joint probability is the return period, T_r.

TABLE 5. Comparison of Methods for Estimating Runoff Volumes for Selected Return Periods

Subwatershed	Period, Years	Storm Center Depth, Inches	Mean Rainfall, Inches	Runoff Inches by annual series	Runoff Inches by rainfall-runoff eq.
W-1B	100	4.05	2.82	0.60	0.60
	50	3.54	2.42	0.52	0.51
	25	2.96	2.00	0.45	0.42
	10	2.30	1.55	0.35	0.31
W-2	100	4.05	3.43	0.48	0.47
	50	3.54	2.92	0.42	0.40
	25	2.96	2.47	0.35	0.32
	10	2.30	1.93	0.27	0.24

A comparison of the two methods for estimating the frequency of runoff is shown in table 5. Figure 4 illustrates that there are many combinations of storm amounts and maximum 15-minute intensities that can occur for a particular return period. The maximum runoff volume for a selected return period was used since a comparison is being made with the annual maximum series. This check on the rainfall-runoff relationship indicates good agreement for the longer return periods but with increasingly greater

divergence for the shorter time periods. The results do, however, tend to substantiate the use of the point rainfall frequencies calculated from the procedure developed by Fogel and Duckstein (1969). Using the rainfall frequencies determined from relatively long-term historical records (table 2) would result in runoff estimates that are considerably lower for all return periods than would be obtained from the annual maximum series.

Thus, this study indicates that in semiarid regions, a linear rainfall-runoff relationship can be used to predict convective-storm runoff from small watersheds ranging in size up to at least 300 acres. For the larger watersheds, those up to about 5000 to 6000 acres (20 to 25 km^2), the use of the storm's maximum 15-minute intensity significantly increased the accuracy of prediction. The procedure is contingent on being able to accurately predict the amount of convective rainfall, a difficult task in itself. It remains to be seen whether the method has more universal applicability than just for the region within which the experimental area is located.

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