AGENCY FOR INTERNATIONAL DEVELOPMENT
WASHINGTON, D. C. 20523

BIBLIOGRAPHIC INPUT SHEET

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0151

1. SUBJECT CLASSIFICATION

A. PRIMARY: Agriculture — AF10-0000-0000

B. SECONDARY: Meteorology and climatology

2. TITLE AND SUBTITLE

Point rainfall frequencies in convective storms

3. AUTHOR(S)

Fogel,M.M.; Duckstein,Lucien

4. DOCUMENT DATE	5. NUMBER OF PAGES	6. ARC NUMBER
1969	9p.	ARC

7. REFERENCE ORGANIZATION NAME AND ADDRESS

Ariz.

8. SUPPLEMENTARY NOTES *(Sponsoring Organization, Publishers, Availability)*

(In Water resources research,v.5,no.6,p.1229-1237)

9. ABSTRACT

10. CONTROL NUMBER

PN-RAA- 095

11. PRICE OF DOCUMENT

12. DESCRIPTORS

Probability theory
Runoff
Thunderstorms

13. PROJECT NUMBER

14. CONTRACT NUMBER

CSD-2457 211(d)

15. TYPE OF DOCUMENT

AID 590-1 (4-74)

10

NOTE: Not supported initially by 211(d), but closely related to subsequent 211(d) activities.

VOL. 5, NO. 6 WATER RESOURCES RESEARCH DECEMBER 1969

Point Rainfall Frequencies in Convective Storms

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Abstract. An exponential relation was developed to describe the spatial distribution of convective storm rainfall in southwestern United States. Given that a storm center has occurred, a geometric distribution was used to describe the frequencies of point rainfall depths. A Poisson distribution was assumed to represent the probability of at least one storm center occurring over a given area a specified number of times during a season. Assuming that the two probability distributions are independent and uncorrelated, maximal and minimal distributions of point rainfall depths were derived. The minimal distribution indicates that with a very high certainty a single rain gage will miss at least one convective storm a year. When compared with frequencies determined from long-term historical records, the maximal distribution exhibits a similar mean, a greater variance, and lower recurrence intervals for the higher rainfall depths.

INTRODUCTION

In the intermountain areas of southwestern United States, convective storms cause almost all of the annual surface runoff from small semiarid watersheds [*Osborn*, 1968]. These storms occur as high intensity, short duration, widely scattered thundershowers principally during the months of July, August, and September. Severe local flooding, which causes heavy damage and may include loss of life, is often the result of such storms. In designing small hydraulic structures to control runoff, it is essential that reliable forecasts be available since detailed hydrologic investigations are usually not warranted in these cases. Current methods for predicting runoff-producing events generally involve estimating rainfall frequencies. *Osborn and Reynolds* [1963] and *Cooper* [1967] have concluded that point frequencies based on rain gage densities of 1 or 2 per 100 square miles are inadequate for obtaining sufficient hydrologic information for use in planning water control works on small watersheds. The procedure presented here is an attempt to develop more accurate predictions of convective rainfall by using a dense network of rain gages.

The analysis of 12 years of rainfall data collected on the Atterbury experimental watershed formed the basis for this study. Located approximately 10 miles southeast from Tucson, Arizona, the long, narrow, and relatively flat drainage basin is representative of valley floors in southern Arizona. About 55% of the mean annual precipitation of 11.0 inches falls during the summer months. The basic rain gage network area of almost 20 square miles contains 29 gages laid out on approximately a 1-mile grid.

SPATIAL DISTRIBUTION OF CONVECTIVE RAINFALL

Area–depth formulas are attempts at describing the spatial distribution of rainfall. *Court* [1961] reviewed the work of previous investigators and compared their area–depth relationships of storm rainfall with one that he suggested. He proposed the use of a bivariate Gaussian distribution of statistics which would give elliptical isohyets. In this model, rainfall depths decrease exponentially away from the point of maximum rainfall (hereafter called a storm center) and follow the familiar bell-shaped curve. Court reasoned that any realistic representation of the distribution of rainfall about the storm center should be smooth at the center. At the other extreme, he felt that an asymptotic approach to zero rainfall with increasing distance appears to be desirable. Formulas in which rainfall depths decrease exponentially with area away from the storm center meet both of these boundary conditions.

An example of a convective storm that was centered over the watershed is shown in Figure 1. Reviewing the pattern of nearly 200 con-

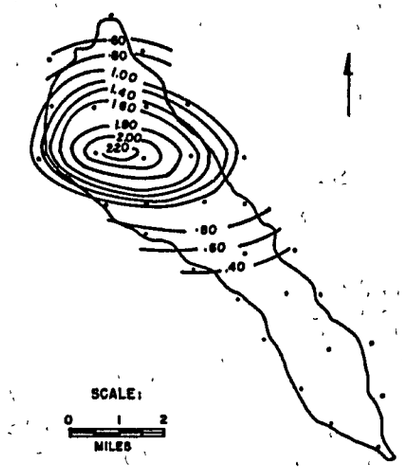

Fig. 1. Isohyetal map of storm, September 9, 1960, over Atterbury experimental watershed.

vective storms that fell on the experimental area, the enclosed isohyets exhibited a marked tendency to be elliptical. On the average, however, the major axis was about one and one half times the length of the minor axis. When the radius of the circle is computed as the average of the axes, the difference between areas enclosed by a circle and by an ellipse is less than 10%. Thus a circular pattern which simplifies computations appears to be a reasonable approximation.

Radar studies show that a thunderstorm cell moves very little during its life span [*Ackerman*, 1959]. The fact that a storm does move is usually assumed to be the reason for the plotted isohyets to depart from circularity. This horizontal movement of a cell tends to support the idea of a rather flat distribution of rainfall around the storm center. An exponential model was therefore selected to represent the spatial distribution of convective rainfall. The area-rainfall depth formula in which circular patterns are assumed is then of the form

$$R = R_0 e^{-\pi r^2 b} \tag{1}$$

where R is the depth of rainfall along an isohyet which is at a distance r from the storm center, R_0 is the storm center depth, and b is a dispersion parameter to be determined.

When storms were selected for analysis, only those of less than two hours duration were chosen. Thunderstorms often consist of a group of three or more cells adjacent to each other. The longer durations provide greater opportunities for the neighboring cells to move over the rain gage network and affect the pattern of the cell in question.

The selected storms were grouped according to their center depth to determine the value of the dispersion parameter for each group of storms. A least squares regression analysis was performed on each group, the results of which are presented in Table 1. The calculated regressions accounted for at least 80% of the variance of the observations in every instance.

In an effort to extend the applicability of equation 1, available data from three record storm events on Agricultural Research Service (ARS) experimental watersheds were also analyzed. As reported by *Osborn and Reynolds* [1963], two of these exceptional storm events were recorded on the Alamogordo Creek watershed located in New Mexico and the third occurred on the Walnut Creek watershed located near Tombstone, Arizona. The respective storm center depths were 4.09, 3.55, and 2.53 inches; their durations were approximately one hour. It is of interest to note that *Keppel* [1963] reported that the rainfall intensities for the 4.09-

TABLE 1. Relationship of Parameter b to Storm Center Depth R_0 as Determined from Regression Analyses

Storm Center Depth, inches		No. of Storms	b	Standard Error of the Estimate
Range	Mean			
2.00–2.70	2.31	9	0.069	0.005
1.50–1.95	1.67	7	0.078	0.004
1.25–1.42	1.31	8	0.092	0.007
1.00–1.18	1.08	8	0.129	0.008
0.75–0.99	0.86	10	0.191	0.016

inch storm exceeded the maximums on record for first order U. S. Weather Bureau stations up to a period of 30 minutes.

The analysis indicated that the proposed exponential formula could satisfactorily depict the area-depth relationship for the ARS reported storms. Values for the parameter b were calculated for each of these storms and plotted with Atterbury data (Figure 2). An equation of best fit was found to be the exponential relationship

$$b = 0.27e^{-0.67R_0} \qquad (2)$$

Thus the model as given by (1) and (2) depicts the spatial distribution of rainfall for a given storm center depth (Figure 3).

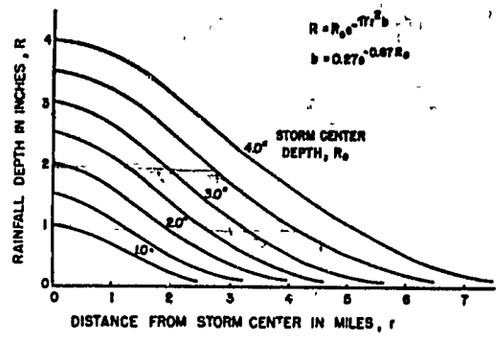

Fig. 3. Spatial distribution model of convective storm rainfall.

STORM CENTER CHARACTERISTICS

Storm center locations. MacDonald [1959] has stated that the locations of air mass thunderstorms in fairly level country may be regarded as almost randomly located. *Osborn and Reynolds* [1963] have concluded that convective storms are randomly distributed. The Kolmogorov-Smirnov test was used to test the assumption that a storm center can occur anywhere in the rain gage network area with equal likelihood.

The test is applied by deriving the cumulative frequency distributions for both a hypothetical or expected distribution and an observed distribution. The maximum discrepancy between the step functions of the two distributions becomes the test statisic D which is defined as

$$D = \max |F_n(x) - F(x)| \qquad (3)$$

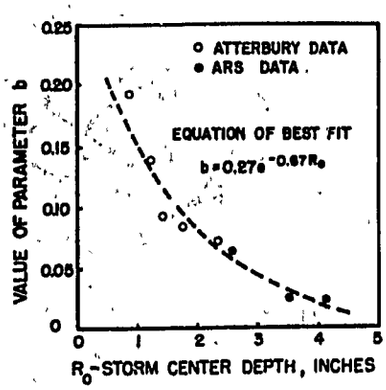

Fig. 2. Relationship of parameter b in area-rainfall depth formula to storm center depth.

where $F_n(x)$ is the cumulative frequency distribution for the hypothetical data and $F(x)$ is that for the observed data. Any wide deviation between the two distributions would suggest a rejection of the null hypothesis, which states that the two sets of data are drawn from the same population. The probability of rejecting the null hypothesis when it is true is the significance level of the test which for this study is taken to be 0.10.

The network area was first divided into polygons by the Thiessen method of determining the average precipitation over an area. Each of the storm centers was located with respect to one of the polygon-shaped areas surrounding a rain gage. The observed distribution values were simply the ratio of the number of storm centers that occurred within each polygon to the total number of such events. Assuming a uniform distribution, the expected distribution values were based on the ratio of the polygon area to the total area. The Kolmogorov–Smirnov statistic was calculated and compared to the tabulated value. Since the calculated value was less than the tabulated value at the chosen significance level, the null hypothesis was not rejected.

Storm centers per event. During the 12-year period of record, 79 storm centers occurred in 64 separate events over the 20-square-mile rain gage network area. Only 1 center occurred in 52 events, whereas 2 centers were located in 9 events and 3 centers in 3 events. In this instance an event is defined as the occurrence of at least one storm center over the rain gage network area. While a frequency distribution may be used to describe the number of storm centers

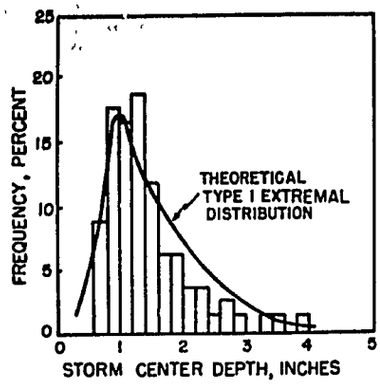

Fig. 4. Frequency distribution of storm center depths.

per event, insufficient data suggested the use of a simple average of 79/64 or 1.23 centers per event.

Storm center magnitudes. The type 1 extremal distribution first proposed by *Gumbel* [1941] for estimating flood frequencies was assumed to represent the distribution of storm center magnitudes (Figure 4). Performing a chi square test resulted in the null hypothesis' not being rejected at the 70% level of significance indicating a relatively good fit between the observed and the type 1 extremal distributions. The data were also fitted to log normal and log Pearson type 3 distributions with no significant improvement in goodness of fit.

POINT RAINFALL FREQUENCY

The probability of an occurrence in which there is at least one storm center within a given area with a storm center depth equal to a specified amount can be considered as a joint probability of the simultaneous occurrence of two events [*Woolhiser and Schwalen*, 1959]. That is, given a storm center occurrence, there is a probability associated with a center over a given area combined with the probability that the center will have a given magnitude. It is assumed for this study that these two probabilities are independent and uncorrelated.

Point rainfall probability per event. The quantity of rain that is measured at a point, given the occurrence of an event in which there is an average of 1.23 centers per event over 20 square miles, is a variate whose frequency distribution can be determined at this time. First, however, it must be assumed that all summer rainfall is associated with thunderstorms where the spatial distribution can be idealized by using equations 1 and 2.

Referring to Figure 5, the probability that a point in space (x, y) receives a given amount of rain R can now be calculated. To obtain R inches of rainfall at x, y will require that a storm center $R_0(r_{k-1})$ occur at a distance r_{k-1} such that the variate R equals at least R. Similarly, another storm with a center at r_k will require a slightly larger storm center depth to produce R inches at x, y. By doing so in a discrete manner, it is possible to sum up in space the joint probabilities of having a storm center occur anywhere within the incremental areas between r_k and r_{k-1} with the required storm center magnitudes sufficient to give R inches of rain at x, y. Expressed mathematically, this becomes

$$\text{prob}\,\{|\mathrm{R} - R| > 0 \text{ at } x, y\} = \sum_k [\text{prob}\,\{C_k\}][\text{prob}\,\{D_k\}] \qquad (4)$$

where C_k represents the occurrence of a center at r_k and D_k is the depth of the center $R_0(r_k)$ such that the variate R equals R. For a uniform distribution, the probability of a center at r_k is

$$\text{prob}\,\{\text{center at } r_k\} = \gamma\pi(r_k^2 - r_{k-1}^2) \qquad (5)$$

in which γ is the number of storm centers per

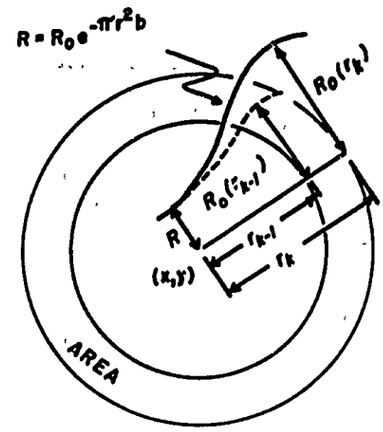

Fig. 5. Schematic diagram for illustrating development of point rainfall frequency formula.

event (1.23) per square mile (20), or 0.062 in this case.

Point rainfall frequencies per event were calculated from (4) and (5) for half-inch increments of rain. Equation 5 was used to determine the probability of a center's occurring in the incremental area bounded by r_k and r_{k-1}, a distance of 0.2 miles. Then assuming the occurrence of a storm center at the midpoint between the two radii, its magnitude required to produce the predetermined amount of rain at x, y was obtained graphically from Figure 3. The probability of getting this amount of rainfall was determined from the theoretical extremal distribution shown in Figure 4.

Given that a storm center has occurred, consider rainfall at a point and assume that its amount is an integral number of half-inch increments of rain to coincide with the calculated point rainfall frequencies per event. In this case the physical system has two possible states, rain or no rain. Treating the amount of rainfall per event as a random variable **R**, it can be shown that the calculated rainfall frequencies closely fit a geometric distribution with p, the conditional probability of rain having a value of 0.48 (Figure 6). As no known rain gage in the Tucson area has ever recorded more than 4.5 inches per short duration storm, it was decided to truncate the distribution of **R** after five inches. Thus, the truncated distribution can be written as

$$f_R(n) = \frac{(1-p)p^n}{1-p^{11}} \qquad n = 0, 1, \cdots, 10 \quad (6)$$

Neglecting the truncation constant $1 - p^{11}$ as it is very nearly 1, the cumulative function of **R** is

$$F_R(a) = \sum_{n=0}^{a} (1-p)p^n = 1 - p^{a+1}$$

$$a = -1, 0, 1, \cdots, 10 \qquad (7)$$

Number of yearly occurrences of events. During the 12 years of record there were 64 events in which at least one storm center occurred over the 20-square-mile network area. The range was from 2 to 10 with an average of 5.33 per year. Although the number of years of record are limited, there appears to be a trend to indicate that a Poisson variate **N** can adequately describe the distribution for the number of events per year. Since there are only a limited number of

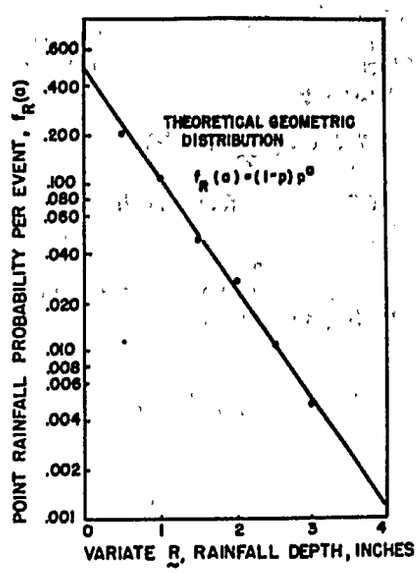

Fig. 6. Comparison of calculated point rainfall probability per event with theoretical geometric distribution.

days in a season when convective storms occur, the distribution of **N** should also be truncated. In this case, where there was an annual maximum of 10 events, the distribution was truncated after 12. The frequency distribution for the number of events per year is therefore

$$f_N(j) = 1.0035 \frac{e^{-\lambda}\lambda^j}{j!} \qquad j = 0, 1, \cdots, 12 \quad (8)$$

where the parameter λ is the mean number of events per year.

Development of the maximal distribution. If there is one event per year, the frequency distribution of maximum point rainfall is that of R and is given by (4). If there are one or two events per year, the cumulative distribution of maximal rainfall Q is $F_Q(a)$, which is obtained as follows (derivation suggested by S. Yakowitz, University of Arizona, Tucson):

$$F_Q(a) = \text{prob}\,(Q \leq a)$$

$$= \text{prob}\,(R \leq a)\,|\text{prob}\,(N = 1)$$

$$+ \text{prob}\,(R_1 \leq a, R_2 \leq a)|\,\text{prob}\,(N = 2)$$

or

$$F_Q(a) = F_R(a)f_N(1) + [F_R(a)]^2 f_N(2)$$

Similarly, if there is a random number of **N**

events per year with a frequency distribution $f_n(j)$, then the maximal distribution function can be written as

$$F_Q(a) = \sum_j \{[F_P(a)]^j f_N(j)\} \tag{9}$$

Equation 9 assumes that the distributions for **R** and **N** are independent. Note also that $F_Q(-1) = 0$ and $F_Q(\infty) = 1$ so that equation 9 is a true distribution function. Substituting (7) and (8) into (9) results in

$$F_Q(a) = 1.0035e^{-\lambda} \sum_j (1 - p^{a+1})^j \frac{\lambda^j}{j!} \qquad j = 0, 1, \cdots, 12 \tag{10}$$

A closed form of equation 10 can be obtained if the summation covers the interval from 0 to ∞, since in that case

$$e^{-\lambda} \sum_j (1 - p^{a+1})^j \frac{\lambda^j}{j!} = e^{-\lambda+\lambda'}$$

where

$$\lambda' = \lambda(1 - p)^{a+1}$$

This closed form can then be used if the tail of the distribution from 13 to ∞ is considered. Thus

$$F_Q(a) = 1.0035[e^{-\lambda+\lambda'} - r_Q(a)] \tag{11}$$

in which $r_Q(a)$ is the tail and is given by

$$r_Q(a) = 1.0035e^{-\lambda} \sum_{j=13}^{\infty} \frac{\lambda'^j}{j!}$$

or

$$r_Q(a) = 1.0035e^{\lambda'-\lambda} \sum_{j=13}^{\infty} \frac{e^{-\lambda'}\lambda'^j}{j!} \tag{12}$$

Substituting equation 12 into equation 11 yields

$$F_Q(a) = 1.0035e^{\lambda'-\lambda}\left(1 - \sum_{j=13}^{\infty} \frac{e^{-\lambda'}\lambda'^j}{j!}\right) \tag{13}$$

in which the summation can be evaluated using a table of the Poisson distribution. In summary, (13) gives the probability that the maximum point rainfall in any year be less than or equal to a inches. The mean or expected recurrence interval T_r for rainfall equal to or greater than a inches is simply

$$T_r = \frac{1}{1 - F_Q(a)} \tag{14}$$

Development of the minimal distribution. In certain instances involving water yield from surface runoff it may be desirable to determine the distribution of the minimum yearly point rainfall. This can readily be done by using a procedure similar to the one in which the maximal distribution was obtained.

By definition

$$F_U(a) = \text{prob}\,(\mathbf{U} \leq a)$$

But

$$\text{prob}\,(\mathbf{U} > a) = \sum_j \text{prob}(\mathbf{R}_1 > a, \mathbf{R}_2 > a, \cdots, \mathbf{R}_j > a) f_N(j)$$

and

$$\text{prob}\,(\mathbf{U} > a) = \sum_j \{[1 - F_R(a)]^j f_N(j)\} \qquad j = 0, 1, \cdots, 12 \quad a = 0, 1, \cdots, 10 \tag{15}$$

Inserting (7) and (8) into (15) and again neglecting the truncation constant of the geometric distribution, we can obtain

$$\text{prob}\,(\mathbf{U} > a) = 1.0035 \sum_{j=0}^{12} e^{-\lambda} \frac{(\lambda p^{a+1})^j}{j!} \tag{16}$$

$$\text{prob}\,(\mathbf{U} > a) = 1.0035e^{-\lambda'} - r_U(a)$$

with

$$r_U(a) = 1.0035 \sum_{j=\infty}^{13} e^{-\lambda} \frac{(\lambda p^{a+1})^j}{j!}$$

Letting

$$\lambda'' = \lambda p^{a+1} = \lambda - \lambda'$$

we have

$$r_U(a) = 1.0035e^{-\lambda'} \sum_{j=13}^{\infty} e^{-\lambda''} \frac{(\lambda'')^j}{j!} \tag{17}$$

Thus the cumulative minimal distribution is given by

$$F_U(a) = 1 - 1.0035e^{-\lambda'} + r_U(a) \tag{18}$$

or

$$F_U(a) = 1 - 1.0035e^{-\lambda'} \left[1 - \sum_{j=13}^{\infty} e^{-\lambda''} \frac{(\lambda'')^j}{j!}\right] \tag{19}$$

TABLE 2. Cumulative Probabilities for Maximal and Minimal Distributions

Depth of Rainfall, R inches	Maximal Distribution, prob $[Q \geq R]$	Minimal Distribution, prob $[U \leq R]$
0	0.9224	0.9342
0.5	0.7061	0.9835
1.0	0.4446	0.9913
1.5	0.2459	0.9936
2.0	0.1265	0.9945
2.5	0.0628	0.9948
3.0	0.0306	0.9950
3.5	0.0148	0.9951
4.0	0.0071	0.9951
4.5	0.0035	0.9952
5.0	0.0016	0.9952

RESULTS AND DISCUSSION

The cumulative probabilities for the maximal and the minimal distributions were calculated using (13) and (19), respectively. The results are shown in Table 2. To compare the maximum point rainfall probabilities determined by the model, historical records from a nearby U. S. Weather Bureau station were used. This station, known as Tucson (University of Arizona), has been in continuous operation for over 70 years.

For the months of July, August, and September the annual maximum 24-hour precipitation was assumed to be the result of the short duration, high intensity thunderstorm. According to *Sellers* [1964], 'summer precipitation is definitely of a showery nature, starting abruptly, continuing intermittently for a brief period, usually less than one-half an hour, then slowly tapering off until ceasing altogether.' In determining their frequencies, these maximum 24-hour amounts were grouped into one-half-inch increments as was done in the model. The frequency densities determined from both the model and the historical records are compared in Figure 7. For illustrative clarity the model frequencies are shown as a continuous function although they were calculated on a discrete basis as indicated by the points shown.

The grouped historical records had a mean value for the summer maximum 24-hour rainfall of 1.27 inches with a standard deviation of 0.54 inches. Corresponding values for the model are 1.28 and 0.86 inches.

To determine the maximum 24-hour rainfall for a given recurrence interval, the ungrouped historical records were fitted to a type 1 extremal distribution as shown in Figure 8. For purposes of comparison, the cumulative probabilities of the model shown in Table 2 were also plotted. Since the model was based on an extremal distribution of storm center depths, these values tended to plot as a straight line on the extremal probability paper.

Rainfall amounts for recurrence intervals of five to 10 years differ very little between those given by the model or determined from the historical records. For recurrence intervals be-

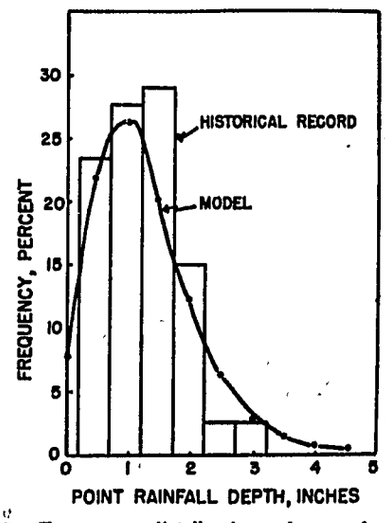

Fig. 7. Frequency distribution of annual maximum convective rainfall for Tucson, Arizona.

low five years, corresponding amounts of rainfall given by the model are lower. This may be partially due to the fact that the model assumes no accumulation of rainfall from more than one storm cell. Associated with the higher recurrence intervals, the model exhibits increasingly larger rainfall quantities. A clue may be obtained by observing the cumulative probabilities of the minimal distribution shown in Table 2, which indicates that there is a very high probability (about 93%) that a single rain gage will miss at least one storm during the season. Some of these missed storms may well be the maximum 24-hour rainfall. This also accounts for the model having a greater variance than the distribution of the historical records.

Although the historical records and the model agree as far as the mean annual maximum 24-hour rainfall is concerned, the sensitivity of the model to the assumptions made in the development of the model warrants consideration. Given that a storm center has occurred, a geometric distribution with a parameter equal to 0 48 was used to describe point rainfall frequencies. Involved in this determination were the assumptions of circular isohyets, independence of events, uniform distribution of storm center locations, extremal distribution of storm center magnitudes, the exponential spatial distribution of rainfall, and the number of storm centers per square mile per event. While these all appear to be reasonable assumptions, Figure 9 indicates that point rainfall recurrence intervals are sensitive to values of p, particularly for the higher rainfall amounts. Additional evidence is needed to verify this important parameter.

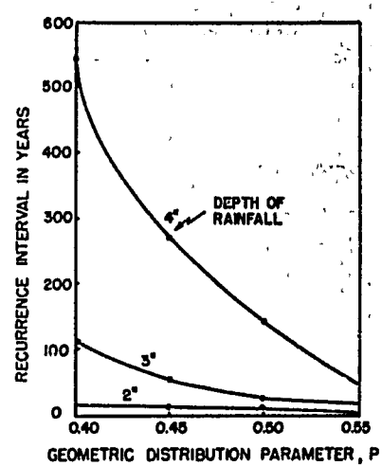

Fig. 9. Relationship of parameter p to recurrence intervals for convective rainfall depths.

The model is also sensitive to the parameter λ, the expected number of annual events in which at least one storm occurred over the rain gage network area. This distribution was assumed to be a Poisson one with λ equal to 5.33. Twelve years of records are hardly sufficient to determine the adequacy of the distribution of an annual event. Fortunately, however, the model is not as sensitive to λ as it is to p. Truncations of the distributions of **R** and **N** are relatively unimportant.

Other than more realistic point rainfall probabilities primarily for the greater magnitudes, the model has an advantage over historical records in that statistical properties of the distribution can be kept, taking into account that we have only one random sample out of many possibilities. We will also be able to draw statistical inferences in ungaged areas if the distributions are known. We can now begin to determine the required density of a rain gage network to ascertain within prescribed limits the volume of convective storm rainfall that falls on a watershed.

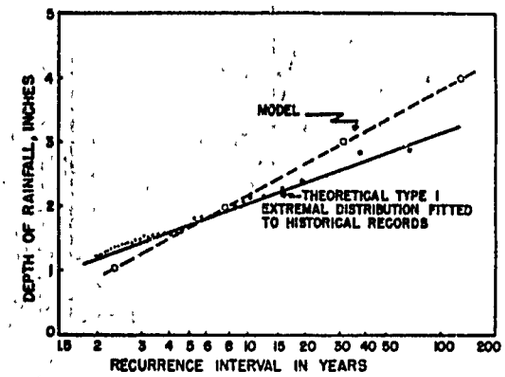

Fig. 8. Recurrence intervals for depths of convective rainfall for Tucson, Arizona.

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(Manuscript received May 22, 1969;
revised August 21, 1969.)