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EFFICIENCY OF HYDROLOGIC DATA COLLECTION SYSTEMS ROLE OF TYPE I AND II ERRORS[1]

Lucien Duckstein and Chester C. Kisiel[2]

ABSTRACT. The efficiency of hydrologic data collection systems is relevant to solution of environmental problems, scientific understanding of hydrologic processes, model-building and management of water resources. Because these goals may be overlapping and non-commensurate, design of data networks is not simple. Identified are four elements of error or risk in such networks. (a) choice of variables and mathematical model for the same process, (b) accuracy of model parameter estimates, (c) acceptance of wrong hypothesis or rejection of correct hypothesis and (d) economic losses associated with error. Of these four, the classical hypothesis testing problem is specifically evaluated in terms of costs of type I and II errors for simple and composite hypotheses, mathematical models for these economic analyses also include costs of sample data and costs of waiting while new data is obtained. An illustrative computational example focuses on the hypothesis that natural recharge might be augmented by a system of pumping wells along an ephemeral channel. The relationship of the hypothesis testing problem to Bayesian decision theory is discussed; it is felt that the latter theory offers a more comprehensive framework for design and use of hydrologic data networks.
(KEY WORDS hydrologic data networks; statistical inference, decision theory; error analysis; groundwater recharge)

INTRODUCTION

The purpose of this paper is to review the overall problem of evaluating the efficiency of hydrologic data collection systems, to present a mathematical basis for evaluating the relationship of risk of Type I and Type II errors to one or more increments of data, and to present examples. The definition of efficiency is taken to include not only the standard statistical concept of relative variance of one estimator in relation to a second estimator but also the broader decision framework in hydrologic science and water management where data collection is accorded intellectual and social merit. The focus is on sample size in space-time, inference and model-building with the data, and the cost of errors associated with the data analysis process. The actual collection, processing and storage of the data is not of concern here.

THE PROBLEM

Any data collection program must be concerned with the tradeoff between errors, costs and benefits. Modeling errors, as illustrated by James, Bower and Matalas [1969] and parameter estimation errors are much more basic considerations in the design of data collection systems than are the truncation and roundoff errors associated, respectively, with approximation and computation. To postulate a model of a process, either deterministic or stochastic,

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and then to estimate model parameters without paying heed to the form or logical structure of the model is a highly questionable activity, but so common. A model must be chosen and data employed to estimate its parameters, but it must not be forgotten that the parameter values are not independent of the model. The available sample size, quality of data, cost of data collection and manipulation, cost of modeling and computing, objectives of the project, and cost of the errors in modeling and parameter estimation determine the choice of model. The quantification of these factors in a decision framework is the challenge as hydrologists and water resource specialists move into the computer age Given the substantial cost of data collection under field conditions, it is a basic tenet of this paper that hydrologic data collection systems, be they ad hoc or permanent, must be linked quantitatively to managerial objectives associated with water problems [Kisiel, 1970].

THE HYPOTHESIS TESTING PROBLEM

The overall process of analysis, synthesis, and problem solution as it relates to data collection systems can be clarified if a distinction is made between at least four aspects of error or risk:

a) choice of mathematical model for the same process, including choice of variable, because quite often a model may be incomplete,
b) accuracy and precision of parameter estimates,
c) acceptance of wrong hypothesis or rejection of correct hypothesis,
d) economic losses associated with error.

Case (a) is concerned with the problem of choosing one from among m model forms being considered as potential descriptors of the phenomenon, a problem attacked by Smallwood [1968]. Case (b) focuses on parameter bias and variance of parameter estimates in relation to the unknown true parameter value, an example of which is given in terms of parameter estimation for a rainfall frequency model by Fogel *et al.* [1971]. Smallwood includes case (b) in his decision model. Case (c) applies to either the testing of scientific hypotheses about hydrologic processes or of economic hypotheses about water resource development, the latter hypothesis testing problem being illustrated later in this paper in terms of costs associated with producer's (Type I) or consumer's (Type II) risk. Case (d) imbeds the parameter estimation problem in a Bayesian theoretic framework wherein economic loss functions (associated with the action and parameter value) are weighted according to the uncertainty associated with that parameter value, an example of which is given by Davis and Dvoranchik [1971] for the problem of designing bridge piers to withstand annual maximum flood flows on the Rillito Creek in Tucson, Arizona.

In the hypothesis testing problem the following elements are always encountered (see Table 1);

- the states of natures $\theta_1, \theta_2, \ldots$,
- the actions a_1 and a_2 which consist in accepting or rejecting the null hypothesis H_0, respectively,
- the sample or experimental data (x_i) which serves as a basis for taking the action a_1 or a_2.

Regarding the action, two cases can be distinguished:

1) a_2 is the rejection of H_0 (alternative hypothesis is composite)
2) a_2 is the acceptance of a specific or simple alternate hypothesis H_1.

Hydrologic examples of case (1) include:

a) H_0 states that the mean values of a split discharge record exhibit no trend, that is, their

differences are not significantly different from zero.

b) H_0 is that natural or human activities have no influence on watershed responses. Moments of hydrologic time series or hydrograph properties (time to peak, peak discharge, total runoff volume) do not change over time as judged by split samples.

c) H_0 is that the annual maximum flood is a log-normally distributed variate.

d) H_0 is that regression between precipitation and elevation in a certain mountainous area is not significantly different from zero.

e) H_0 is that channelization of an ephemeral stream will not adversely affect natural recharge patterns.

TABLE 1. Error Structure of Simple Hypothesis Testing Problem (Case 1)

State of nature	Action concerning H_0	
	a_1: Accept H_0	a_2: Reject H_0
$\theta_1 = H_0$ is true	No error	Type I error = α error
$\theta_2 = H_0$ is false	Type II error = β error	No error

Hydrologic examples of case (2) include:

a) H_0 is that no additional natural recharge is induced beneath an ephemeral stream by installation of a system of wells along the stream; H_1 is that 50 percent additional natural recharge is obtained by that method. (This hypothesis is to be subjected to a cost analysis later in the paper.)

b) H_0 is that human action on a watershed does not significantly change the rainfall-runoff relationship; H_1 is that the water yield after manipulation is 20 percent higher than before.

c) H_0 is a mean value of runoff such that the frequency and amount of rainfall over a constant area of land treated to capture runoff efficiently for use on a ranch or farm will not justify the investment in land treatment; H_1 is a specific runoff value such that the frequency and amount of rainfall justifies the investment on the average over the long run.

d) H_0 is that a transpiration inhibitor in a forested watershed has no significant effect on water yield; H_1 is that it reduces transpiration losses by 40 percent.

e) H_0 is that annual floods are not serially correlated at a certain design site for a dam; H_1 is that annual floods are serially correlated ($r_1 = 0.3$) because of a strong base flow component [Carrigan and Huzzen, 1967].

Note the strategy used to define the null hypothesis H_0. because acceptance of H_0 really means non-rejection, H_0 should be so stated such that no action follows if H_0 is accepted; whereas the rejection of H_0, which has a much stronger level of certainty, should be made to imply a positive action (take more data, in particular) To illustrate, in case (1a) the acceptance of H_0, that is, no trend in mean value of a discharge time series, signifies only that a trend is not detectable with the existing data, whereas rejection of H_0 is a much stronger statement. Even a larger sample size may not ascertain that there is not a trend. A more striking example is given by case (1c) wherein a chi-squared test may show that either the Gumbel or log-Pearson distributions for annual maximum flows may not be rejected, that is, the possibility for using either one has been established. If it is vital to distinguish between the two distributions before action can be taken, then three directions are possible: to take more

data, to formulate an extreme value theory based on physical reasoning about atmospheric and terrestrial processes, or to pursue both concurrently. On the other hand, if the Gumbel distribution, for example, is rejected, then we have established strong ground for rejection and need not reconsider this density function further

In the above examples simple and composite hypotheses have been formed in both cases involving the decision a_2 When testing a simple hypothesis H_0 against a simple alternative H_1, there are exactly two points (μ_1, μ_2) in the parameter space μ and two points (a_1, a_2) in the decision or action space A (see Figure 1). In simple hypothesis testing the parameter space is divided by a critical point μ_c into the sets Ω_1 (containing the value of the hypothesized null parameter μ_0) and Ω_2 (containing the value of the hypothesized alternative parameter μ_1). In the composite hypothesis case, to test H_0 versus H_1, either one or both may be composite hypotheses in which case the parameter space Ω_j, $(j = 1, 2)$, has more than one point [de Groot, 1970].

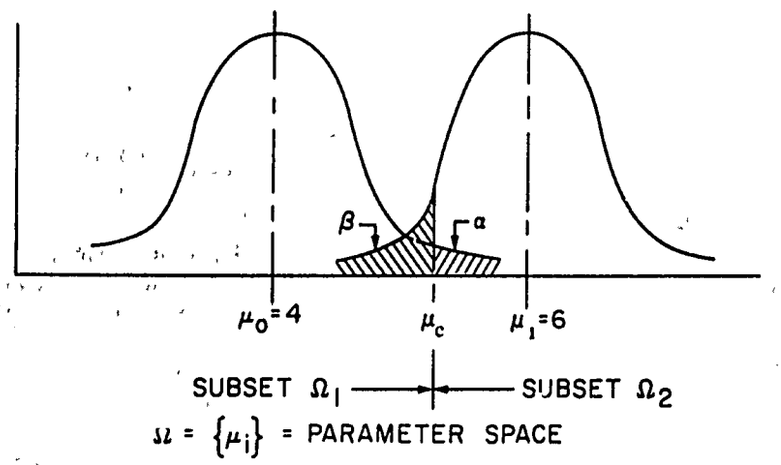

Figure 1. Parametric spaces in the hypothesis testing problem.

Both scientific and managerial questions abound in the above hypotheses [Skellam, 1969]. Quite often in the environmental sciences, the same data must be used to answer both sets of questions. An important requirement is that H_0 be chosen in a serious vein as the best available description of nature. Typically, it is the generally accepted explanation of nature and not just a "straw man" to be knocked down by artificial analysis. In many instances rejection of H_0 would disrupt the established order and engender needless waste in the consideration of alternative hypotheses. Even so, continued data collection in many cases is intended to re-evaluate its veracity. In hypothesis testing, it is common practice to make the probability of rejecting H_0 a small one. Bayesian decision theorists criticize this approach for its informality and for excessive reliance on unaided intuition and judgment, they also argue that no economic account is taken of the risks and errors [Hamburg, 1970, p. 741]. The latter point is also made strongly by Wilson [1952, p. 59] in the context of scientific research. In defense of the scientist who does not use decision theory to assess costs of uncertainty or errors, Hooke [1963, p. vi] argues that the scientist is not making a decision His role is to draw conclusions. Hooke states that "the scientist announces results on the basis of which many other people may make decisions, each with a different set of costs attached to the various errors." Furthermore, "the scientist does not know what these costs are, and, because of their different

amounts in different situations, probably could not use them if he did." Hence, he contents himself with construction of confidence intervals and does not count their costs. Another reason for the rare use of decision theory by the academic researcher in drawing his conclusions is that the nature of the prior probabilities and posterior consequences is likely to be unclear, that scientists have been able to ignore pertinent information without being questioned, and that scientific or research problems are too new to allow for the formulation of any meaningful prior beliefs as required by decision theory [Morgan, 1968, p. 110]. Rather strikingly Tukey [1960] distinguishes between conclusions and decisions by emphasizing that practical decisions "let us decide to act *for the present as if*" alternative A was better (or worse) than alternative B. The *decision* must be made *now* and the decision maker must act as if the scientific generalization or hypothesis is acceptable now or decide not to act at all if the hypothesis is weak. Whereas, the scientist must not be rushed in making *conclusions* whose long-run effects must be judged by their "truth" with respect to the whole body of scientific knowledge, not by specific consequences of specific actions.

Notwithstanding these assertions, there is a growing conviction that scientists must face up to the decision aspects of their conclusions in the face of diminishing margins for errors in proposed "solutions" of environmental problems. To design optimal data collection systems to sample the environment is at best a difficult task in view of the seemingly non-commensurate goals of scientists and decision makers. The former seeks "perfect" knowledge and understanding whereas the latter cannot wait but must make decisions here and now. Perhaps, joint decision analysis in the spirit of decision theory by both groups may serve to facilitate communication and better designed data networks.

Even though the hypothesis testing problem can be imbedded in a Bayesian decision framework [Larson, 1969, p. 315; DeGroot, 1970, p. 237], it is not clear how to compute the probability of Type I and II errors. Nonetheless, the Bayesian approach appears to offer more to the hydrologist in his efforts to efficiently collect data because prior degrees of belief and subjective knowledge explicitly enter the decision analysis. There is much room for further research in these areas. It seems a truism that complete rationalization of design of data networks is not possible.

ECONOMIC ANALYSIS OF TYPE I AND II ERRORS

In case (1) as posed above, two kinds of decision errors arise (see Table 1): rejecting H_0 when in fact it is true, or accepting H_0 when it is not true. α specifies probability of the first kind of error (Type I) whereas β is the probability of the second kind of error (Type II). In performing the test in a traditional sense, specify H_0 and α; then select the critical value C, such that $(1 - \beta)$ is as large as possible. Ackoff [1962, p. 299], Vance [1966], Aigner [1968], Myers and Melcher [1969] and Hamburg [1970]. point out that the statistical and applications literature generally gives no guidelines for choosing α (or β) except to say that α is chosen at either the five or one percent levels and that α and β errors can be reduced by increasing n, the sample size, or improving the accuracy of parameter estimates, that is, by reducing the variance $\sigma^2(x)$. While sample size and accuracy are important, the payoff is in terms of the economic loss functions, $L(a_2/\theta_1)$ and $L(a_1/\theta_2)$, associated with α and β errors (see Table 2).

The problem is then one of obtaining reliable and realistic estimates of these functions, which, in example (2a) can be interpreted as follows: $L(a_1, \theta_1)$ is the loss incurred by installing wells along the river—where it is impractical—for a zero gain in natural recharge. $L(a_1, \theta_2)$ is an opportunity loss incurred by not installing wells and failing to increase natural recharge.

TABLE 2. Payoff Table for Natural Recharge Problem (Case 2)

State of Nature	Actions	
	a_1	a_2
θ_1: recharge = 4×10^4 AF/yr	0	$L(a_2/\theta_1)$
θ_2: recharge = 6×10^4 AF/yr	$L(a_1/\theta_2)$	0

More specifically, assume that we want to test the simple null hypothesis H_o versus the simple alternate hypothesis H_1 wherein:

H_o: the mean recharge μ remains unchanged when wells are drilled, near the river bed, i.e., $\mu_o = 4$ in terms of 10,000 acre-feet (AF)/year,

H_1: the mean annual recharge μ_1 is increased by 50 percent if wells are installed, that is, $\mu_1 = 6 \times 10^4$ AF/year. (Henceforth, the 10^4 AF/year will be dropped and understood to apply to both mean μ and variance σ^2 of the example.) By fixing $\mu_1 = 6$ we imply that benefit due to natural recharge offsets the cost of well system.

From previous studies we know that the recharge has a variance of $\sigma^2 = 4$. Also, assume that we deal with normal or normalized data for the sake of simplicity. The numbers used in this example have some resemblance to a real world problem but the results of the analysis are presented only to demonstrate the economic aspects of α and β errors.

Let Q_c be the critical value of the mean recharge, such that when a sample of recharge measurements q_i of size n $(q_1, q_2, ..., q_n)$ is taken at each ith site in space over one time period with $Q = \frac{\Sigma q_i}{n}$ (or some more complex measure of recharge volume based on the dynamics of groundwater motion [Moench and Kisiel, 1970]) one will take the following actions:

$$a_1 \text{ if } Q < Q_c \tag{1}$$

$$a_2 \text{ if } Q > Q_c$$

If x_α is the abscissa along the standard normal curve for the random variable $\underset{\sim}{X}$ so that

$$P\left\{\underset{\sim}{X} > x_\alpha\right\} = \alpha = 1 - \Phi\left(\frac{Q_c - \mu_o}{\sigma/\sqrt{n}}\right) \tag{2}$$

and

$$P\left\{\underset{\sim}{X} < x_\beta\right\} = \beta = \Phi\left(\frac{Q_c - \mu_1}{\sigma/\sqrt{n}}\right) \tag{3}$$

(in which Φ is the cumulative standard normal probability), then the relationships between Q_c, n, α and β can be written as:

$$\Phi_1(Q_c, \alpha, n) = Q_c - \mu_o - x_\alpha \frac{\sigma}{\sqrt{n}} = 0 \tag{4}$$

$$\Phi_2(Q_c, \beta, n) = Q_c - \mu_1 - x_\beta \frac{\sigma}{\sqrt{n}} = 0 \tag{5}$$

We thus have two equations between at least four variables of interest: Q_c, n, α and β. We can fix any two variables and determine the two other ones.

Usually α and β are fixed, and then Q_c and the sample size n, necessary to perform the test, are calculated. If the number of tests, n, which can be performed are limited by physical or financial considerations, then only α or β may be preset, but not both, on the basis of economics or prior knowledge. It is important to note that in either case the focus is on a single pertinent state of nature for which there exists some single true value of the parameter μ to be estimated in the parameter space Ω. The action space $A = \{a_1, a_2\}$ contains only two decisions. These are in contrast to Bayesian theory in which the uncertainty over a range of true parameter values is explicitly considered and the action space contains two or more decisions [DeGroot, 1970, p. 238].

The losses and costs of sampling and testing attributable to the actions A may be formed as an optimization problem, that is,

$$\underset{\alpha, \beta, n, Q_c}{\text{minimize}}\ Z(\alpha, \beta, Q_c, n)$$

subject to two constraints represented by Equations 4 and 5 and a budgetary constraint:

$$\phi_3\ (n) = c_s n - B \leqslant 0 \tag{6}$$

Particular classes of the optimization problem are based on the nature of the null and alternative hypotheses.

1) Simple H_o: $Q = \mu_o$, simple H_1 : $Q = \mu_1$.
2) Simple H_o: $Q = \mu_o$, composite $H_1 = Q > \mu_o$.
3) Composite H_o, composite H_1.

and on the number of variables in the set (α, β, n, Q_c) that are fixed:

a) Fix one of the four, vary one of the remaining variables and determine the remaining two; 12 permutations result, for example

- Fix β; vary α, Q_c or n.

b) Vary two of the four variables and determine the remaining two; six permutations result. The final form of the optimization problem depends on the degree of prior knowledge possessed by the decision maker. As more of the four variables (α, β, n, Q_c) are prespecified, fewer degrees of freedom exist in choosing among alternatives.

Consideration of simple H_o - simple H_1. Consider first the simple H_o - simple H_1 situation wherein the total cost function is

$$Z = \alpha C_1(\alpha) + \beta C_2(\beta) + (c_w + c_s)\, n + C_3 \tag{7}$$

in which $C_1(\alpha)$ = cost of Type I error (or cost of installed wells in the case of example (2a)).
$C_2(\beta)$ = cost of Type II error (or lost water revenue for example (2a)).
c_w = unit cost of waiting for more data to sharpen estimate of Q (or opportunity costs associated with lost water revenues for example (2a)).
c_s = unit cost of sampling (either in space, time or both).
C_3 = cost of conducting decision analysis (may be a function of n).

The three constraints are represented by Equations 4, 5, and 6. Combining Equations 4 and 5,

one obtains

$$\mu_1 - \mu_0 = (x_\alpha - x_\beta)\frac{\sigma}{\sqrt{n}} \tag{8}$$

The problem as stated is one of minimizing a linear objective function Z with nonlinear constraints.

Given the above general problem formulation for the simple H_0 - simple H_1 case, the following conditions identify objective functions Z_k (k=1, 2, ..., 18) according to the decision maker's prior knowledge and degrees of freedom in setting α, β, Q_c and n. In each condition, the probability density function $f(\underset{\sim}{Q} = Q)$ is assumed to be normal with unknown mean μ and known variance $\sigma^2(Q)$ so as to facilitate demonstration by example. The first 12 objective functions are obtained by fixing one variable, for instance:

Fix n and vary α to find the minimum value of Z_1. Hence, minimize $Z_1(\alpha, \beta, Q_c)$. But $Q_c = \phi_1^{-1}(\alpha)$ when n = constant and $\beta = \phi_2^{-1}[\phi_1^{-1}(\alpha)]$ so that

$$Z_1 = Z_1\ [\alpha, \phi_1^{-1}[\phi_1^{-1}(\alpha)], \phi_1^{-1}(\alpha)] = Z_1(\alpha) \tag{9}$$

wherein the inverse function ϕ_1^{-1} is simply a solution of Equations 4 or 5.

The last 6 objective functions are obtained by letting 2 of the 4 variables determine the optimum Z.

By fixing any two of the four variables the decision maker manifests a strength of conviction and substantial prior knowledge. Consider the following examples based on the hypothetical natural recharge problem.

Example 1

Given: Q_c fixed at 5.5×10^4 AF/yr by decision maker,

n fixed at 16, and

σ^2 assumed at 4.

Problem: What are the implied α and β errors?

Solution: $H_0 : \mu_0 = 4$ and $H_1 : \mu_1 = 6$.

$\alpha = P(Q > Q_c \mid \mu_0 = 4 = 1 - \Phi(3.0) = 0.0013$

$\beta = P(Q > Q_c \mid \mu_1 = 6) = \Phi(-1.0) = 0.1587$

Discussion: α and β errors are completely determined by prior results and hypotheses especially H_1. The α error is too conservative. No effort is made to compute the economic losses associated with risks in this case. Q_c was preset either by prior economic analyses or simply guessed.

Example 2

Given: $\sigma = 2 \times 10^4$ AF/yr and

$Q = 5.5$

Total cost function: $Z = \alpha C_1(\alpha) + \beta C_2(\beta) + (c_w + c_s)\, n + C_3$

in which $C_1(\alpha) = \$1{,}000$

$_2(\beta) = \$1{,}000$

$_w = \$10$

$_s = \$1$

$C_3 = \$50$

Problem: Find n to minimize total cost.

Solution: Table 3 is based on computation of Z for each setting of n for fixed Q_c (which in turn determines the α and β error). The starred value of $Z^* = \$386.60$ per decision identifies the minimum cost as occurring at n = 10 at which point $\alpha = 0.0019$ and $\beta = 0.1587$.

Discussion: The above solution is conservative in that equal likelihood of the alternative states of nature is assumed. In contrast to Example 1, explicit specification of the loss functions, $L(a_2/\theta_1)$ and $L(a_1/\theta_2)$, as $C_1(\alpha)$ and $C_2(\beta)$, respectively, along with other costs have led to an optimum tradeoff between the two kinds of risk. Only one managerial use of the data is considered. For multiple uses of the same data set, the problem set up would require a much more involved objective function. It is possible that each use would not require the same sample size. No budgetary constraints have been imposed on the total cost functions and the cost of decision analysis is irrelevant, since it is a constant.

TABLE 3. Tabular Solution of Optimum α and β Error for Example 2

n	α	β	$\alpha c_1(\alpha)$	$\beta c_2(\beta)$	$c_w n$	$c_s n$	c_3	z
1	.2266	.4013	\$226.60	\$407.30	\$ 10	\$ 1	\$50	\$688.90
4	.0668	.3085	66.80	308.50	40	4	50	469.30
9	.0122	.2266	12.20	226.60	90	9	50	387.80
10	.0019	.1587	1.90	152.70	160	16	50	386.60*
25	.0001	.1056	.10	105.60	250	25	50	430.70
36	0	.0668	0.00	6.68	360	36	50	452.68
49	0	.0322	0.00	3.22	490	49	50	591.22

*Minimum cost at n = 10, $\alpha = .0019$, $\beta = .1587$.

Example 3

Given: $C_1(\alpha), C_2(\beta), c_s, c_w,$

Problem: Find Q_c and n that minimizes Z.

Solution: $\underset{Q_c,\, n}{\text{Min}}\ Z = \alpha C_1(\alpha) + \beta C_2(\beta) + (c_s + c_w)\, n$

subject to Equation 8. But because the alternatives H_0 and H_1 are not equally likely, we parameterize this objective function by introducing Γ as a weighting parameter. Vance [1966] calls it the unconditional probability that H_0 is true. Rewriting Z,

$$\underset{Q_c,\, n}{\text{Min}}\ Z = \Gamma\alpha C_1(\alpha) + (1-\Gamma)\beta C_2(\beta) + (c_s + c_w)\, n \tag{10}$$

in which $\Pr[\mu = \mu_0] = \Gamma$

$\Pr[\mu = \mu_1] = 1-\Gamma$

$\Gamma\alpha$ is the "true" frequency of Type I error and $(1-\Gamma)\beta$ that of Type II error. Differentiating Equation 10 with respect to Q_c and n, respectively, and setting each equal to zero, we get the critical value Q_c^* and optimum sample size n^*:

$$\frac{\partial Z}{\partial Q_c} = 0 \rightarrow \Gamma C_1 \text{ erf } x_0 = (1-\Gamma) C_2 \text{ erf } x_1$$

$$\frac{\partial Z}{\partial n} = 0 \rightarrow \Gamma x_0 \text{ erf } x_0 = (1-\Gamma) x_1 \text{ erf } x_1 + n(c_s + c_w)$$

where

$$x_0 = \frac{Q_c - \mu_0}{\sigma/\sqrt{n}}, \quad x_1 = \frac{Q_c - \mu_1}{\sigma/\sqrt{n}}, \quad \text{erf } x = \frac{1}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right).$$

Iterative solution is necessary. An example is given by Vance [1966].

Discussion: The solutions are highly sensitive to Q_c, n, and Γ. Hence, one must be fairly certain as to the costs and prior probabilities assigned to the costs as pointed out by Vance [1966]. By choosing Γ at large levels, smaller optimal sample sizes are required and the minimum cost Z^* or risk is smaller. The arbitrariness of Γ suggests formulation of the same problem in a Bayes framework.

Consideration of simple H_0 - composite H_1. In the case when we have a simple null hypothesis H_0 $(\mu = \mu_0)$ and a composite alternate hypothesis $(\underset{\sim}{\mu} > \mu_0)$, the value of β becomes undetermined, since β is a function of μ_1 and μ_1 is not known. The cost minimization problem becomes

$$\text{Min } Z = \alpha C_1(\alpha) + \underset{\sim}{\beta} C_2(\underset{\sim}{\beta}) + (c_w + c_s) n \tag{11}$$

subject to

$$Q_c - \mu_0 = \frac{x_\alpha \sigma}{n} \quad \text{or} \quad \alpha = 1 - \Phi\left(\frac{Q_c - \mu_0}{\sigma/\sqrt{n}}\right) \tag{12}$$

The quantity β is written as a variate because it depends on the variate $\underset{\sim}{\mu}$:

$$\underset{\sim}{\beta} = \beta(\underset{\sim}{\mu}_1) = 1 - \Phi\left(\frac{Q_c - \underset{\sim}{\mu}_1}{\sigma/\sqrt{n}}\right) \tag{13}$$

If relation 13 is substituted into the objective function 11, the cost minimization problem is then

$$\text{Min } \underset{\sim}{Z} = Z(\alpha, n, Q_c, \underset{\sim}{\mu}_1) \tag{14}$$

subject to

$$Q_c - \mu_0 = \frac{x_\alpha \sigma}{\sqrt{n}}$$

The only way this problem can be solved is by taking a Bayesian type of approach, which necessitates an a priori knowledge of the distribution of $\underset{\sim}{\mu}_1$.

For example, if we are testing

H_0: the mean value of recharge is $\mu = 10^4$ AF/year

versus

$$H_1 : \mu > 10^4 \text{ AF/year},$$

then we have to answer the question: what is a likely probability density function pdf for μ_1 between 10^4 AF/year and infinity? Reasonable guesses could be an exponential, a log gamma with mean 1.2×10^4, etc. Assume this pdf $f_{\mu_1}(\mu)$ is known. Now the expected value of Z in Equation 14 can be taken and we can proceed with the problem

$$\text{Min } E(\underset{\sim}{Z}) = \int_{\mu_0}^{\infty} Z(\alpha, n, Q_c, \mu) f_{\mu_1}(\mu) d\mu$$

or

$$\text{Min } E(\underset{\sim}{Z}) = \alpha C_1(\alpha) + \int_{\mu_0}^{\infty} \beta(\mu) C_2(\beta(\mu)) f_{\underset{\sim}{\mu_1}}(\mu) d\mu + (c_w + c_s) n + C_3 = G(n, \alpha, Q_c) \quad (15)$$

subject to

$$\alpha = 1 - \Phi\left(\frac{Q_c - \mu_0}{\sigma/\sqrt{n}}\right) \quad (16)$$

In this problem we can either fix one of the three variables n, α, Q_c and look for the minimum with respect to another one (six conditions), or search for the minimum with respect to two variables (three conditions). In either case, constraint 16 determines the value of the third variable If we fix two variables, then the problem is completely determined and no optimization is possible (see Lindgren [1968, p. 237], where n and Q_c are fixed). Another way to express this is to remark that we have two degrees of freedom in the cost optimization problem for simple H_0 - composite H_1. We shall next examine the case where both H_0 and H_1 are composite.

Consideration of composite H_0 - composite H_1. Let, for example, the hypothesis be, respectively,

$$H_0 : \mu \leqslant a$$

$$H_1 : \mu > a$$

Both $\underset{\sim}{\alpha}$ and $\underset{\sim}{\beta}$ are variates:

$\underset{\sim}{\alpha}$ depends on a variate $\underset{\sim}{\mu_0}$ defined in $(-\infty, a)$

$\underset{\sim}{\beta}$ depends on a variate $\underset{\sim}{\mu_1}$ defined in $(a, +\infty)$

Let the unknown mean $\underset{\sim}{\mu}$ have an a priori pdf $f(\mu)d\mu$. Then the cost minimization problem can be written as

$$\text{Min } E(\underset{\sim}{Z}) = \int_{-\infty}^{a} C_1(\alpha(\mu))\alpha(\mu) f(\mu) d\mu + \int_{a}^{\infty} C_2(\beta(\mu))\beta(\mu) f(\mu) d\mu + (c_w + c_s) n + C_3 \quad (17)$$

subject to

$$\alpha(\underset{\sim}{\mu_0}) = 1 - \Phi\left(\frac{Q_c - \underset{\sim}{\mu_0}}{\sigma/\sqrt{n}}\right) \quad (18)$$

$$\beta(\underset{\sim}{\mu_1}) = \Phi\left(\frac{Q_c - \underset{\sim}{\mu_1}}{\sigma/\sqrt{n}}\right) \tag{19}$$

When Equations 18 and 19 are used in the objective function 17, one obtains the unconstrained optimization

$$\text{Min } E(\underset{\sim}{Z}) = H(Q_c, n) \tag{20}$$

One can still find Q_c and n that minimize the total cost; however, there is no choice of α and β, which are given by the distributions 18 and 19.

Note that the expected loss due to α error (for example) has the characteristics of a convolution of the cumulative density function (c d.f), $\Phi(x)$, with the pdf $f(\mu)$ around Q_c; (take $C_1(\alpha(\mu)) = C_1$ to simplify):

$$\int_{-\infty}^{a} C_1\,\alpha(\mu) f(\mu) d\mu = C_1 \int_{-\infty}^{a} \Phi\left(\frac{Q_c - \mu}{\sigma/\sqrt{n}}\right) f(\mu) d\mu \tag{21}$$

This type integral is best evaluated if $f(\mu)$ is taken as the conjugate distribution of Φ; this point is illustrated by Davis and Dvoranchik [1971] who perform the numerical convolution of a normal with a chi-square to compute an expected opportunity loss (or cost).

SUMMARY AND CONCLUSIONS

In this paper, we have set forth a systematic classification of the various aspects of the following problem: minimize an *economic* cost function subject to constraints describing the relationships between α, β, Q_c and n. Specific examples of cost functions and constraints have been given. Three cases distinguishing between simple and composite hypotheses for H_0 and H_1 have been discerned. For simple H_0 - simple H_1, any two out of the four variables α, β, Q_c and n can be chosen as independent variables For simple H_0 and composite H_1, β becomes a variate so that any two of the three variables α, Q_c and n can be chosen as independent variables provided a Bayesian approach is taken (prior probability density function known). For both composite H_0 and composite H_1, only Q_c and n are left as independent variables, since both α and β are variates, here again the problem can be solved only if a prior probability density function is known.

The analysis has focused primarily on an economic framework for the classical hypothesis problem as applied to hydrologic data collection systems It is important (and has rarely been done) to put α and β errors in such a framework. Equations have been put forward for determining optimum sample size for levels of risk in making Type I and II errors These results emphasize the importance of cost of α errors when the hydrologist is checking a theory and cost of β errors (social costs) when people problems are being attacked. β errors are important to a young evolving science whereas concern over α errors tends to dominate an older science. β errors are important in poorly understood hydrologic regimes; a strategy of multiple hypotheses is consistent with this viewpoint

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