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COLLECTIVE UTILITY: A SYSTEMS APPROACH TO WATER PRICING POLICY

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ABSTRACT: Collective utility of a region based on Lesourne's theory is used to compare water pricing policies in Tucson (Arizona) over a single time period. The present method of marginal pricing (regressive block rate) is compared to a hypothetical conservation pricing (progressive block rate). Well-defined hypotheses are made about the behavior of consumers served by the water distribution company. The value of the change in collective utility determines which price structure should be adopted once model parameters are known and once a value has been imputed to conservation. A progressive rate structure decreases the total consumption. When peak consumption is lowered, lower capital expenditures occur. Extensions of the model to multi-period policies and uncertainty of future conditions are presented. The analysis is appropriate wherever a central water agency exists. Collective utility as an approach to analysis of water resource systems is contrasted with the techniques of cost-effectiveness and benefit-cost analyses.

RESUME: Utilité Collective d'un Système de Tarification d'eau

Deux systèmes de tarification d'eau à Tucson (Arizona) sont comparés durant une periode simple, du point de vue de l'utilité collective de Lesourne. La méthode presente de tarification par valeur marginale (taux degressif) est comparée à une tarification hypothetique de conservation (taux progressif). Des hypothèses simples et bien définies décrivent le comportement des consommateurs d'eau. La valeur du changement d'utilité collective permet de déterminer le système de tarification à adopter en fonction des paramètres du modèle et de la valeur imputée a la conservation. Une tarification progressive réduit la consommation totale, ainsi que la consommation de pointe, ce qui permet de faire des économies d'investissement. Les cas multi-périodiques et d'avenir incertain sont ensuite présentés. Le concept d'utilité collective est applicable chaque fois qu'une compagnie regionale de distribution d'eau existe. Cette méthode d'aborder les problèmes de ressources en eau est comparée aux techniques d'analyse côut-efficacité et côut-bénéfice.

1.0 Introduction

In regions where water is a scarce resource, such as in the semi-arid southwestern United States, the distribution of water in metropolitan areas is often controlled by a City Public Utility Company. Such an agency controls consumption by two means: 1) metering of every individual user, and 2) setting a price structure. The purpose of this paper is to illustrate the influence of this price

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structure on the welfare of the community. The measure chosen for the welfare is a collective utility based on the theory of Lesourne (1964).

In order to simplify the presentation, only the residential category of users is considered; according to Whitford (1970), this category corresponds on the average to 41 percent of the water consumed in metropolitan areas. The contention is that residential users are more likely to react to a rate structure change than industrial, commercial or other users, which consume respectively 24 percent, 18 percent and 17 percent of the water. Furthermore, the methodology proposed here can easily be extended to study how changes of rate structure, within each category of user, are reflected in the change of collective utility.

The model is essentially developed for one period (say five years), with no uncertainty; extensions in those two directions are then briefly discussed.

2.0 Price Structures

There are basically three price structures for assessing water rates. First, the flat rate system assesses the consumer a fixed price per billing period which is dependent upon many factors, e.g., type of dwelling (apartment, farmhouse), business establishment, or type of plumbing fixtures. Second, the step rate system has a unit rate that is dependent on the total quantity consumed, for example:

Consumption	Rate
0- 700 ft^3	\$0.45/100 ft^3
800-3,700 ft^3	0.41/100 ft^3
3800- ft^3	0.40/100 ft^3

Here, an individual who consumes 4100 ft^3 of water pays 41 x .40 = \$16.40. Finally, the block rate system prescribes a new unit price system for each block of the total quantity consumed, for example,

Consumption	Rate
0- 700 ft^3	\$3.15 (flat)
800-3,700 ft^3	.41/ft^3
3800- ft^3	.40/ft^3

Here, an individual who consumes 4100 ft^3 of water pays: \$3.15 + 30 x \$0.41 + 4 x .40 = \$17.05. Each of these structures may be regressive or progressive. In a regressive pricing structure, rates decrease as quantity increases; this corresponds to a policy of non-conservation. In a progressive pricing structure, rates increase as consumption increases, which corresponds to a policy of conservation. The City of Tucson presently uses a regressive block rate pricing structure.

Pricing structure can be changed either permanently, in which case one has to consider the steady state situation established several months after the change has taken place; or seasonally, in which case the transient behavior of consumers ("shock" effect) is the dominant factor in the analysis. The structure change considered here is from a regressive to a progressive block rate.

3.0 Collective Utility

3.1 Conceptual framework. The concept of collective utility developed by Lesourne (1964) postulates the existence of an Economic Decision Maker (EDM) who can construct a real functional U[S(1), ..., S(k), ..., S(K)] of the satisfaction functions S(k) of individual k. Since the functional forms of S(k) and U are very difficult, if not impossible, to determine, the theory deals with marginal economic transformations, that is, compares neighboring economic states; in the case considered here, an economic state corresponds to a price structure and the variables describing this state are the quantities of water consumed in the various price categories. The EDM can be visualized as the manager of the City Utility who has the power to determine the rate and price structure of water.

The following indices and symbols are used in the basic model:

i: goods, principally the various categories of water, and services, such as labor: i=1, 2, ..., I

k: individual consumers: k=1, 2, ..., K

s: economic states: s=1, 2.

Q(i,k): quantity of good or service of type i consumed by individual k.

Note: Q(i,o) is the initial quantity of good i; the total quantity consumed by all persons is: $Q(i) = \sum_k Q(i,k)$

p(i): price of good i for individual k (the same for all individuals)

r(k): revenue or income obtained by k

r: income of water utility company

$n(i) = \frac{\Delta Q(i)}{Q(i)} \cdot \frac{p(i)}{\Delta p(i)}$: elasticity of good i

S(k) = S(k; Q(1), ..., Q(I)): individual satisfaction function

U = U[S(1), ..., S(K)]: collective utility

Additional notation will be defined later for the extensions of the basic model.

The working hypotheses are:

H.1. Each individual k has a satisfaction function (S(k)) which is a non-decreasing function of the quantity of goods consumed and which he maximizes subject to his revenue constraint; thus, for each set of revenues r(k):{k=1, ..., K}, an equilibrium state of prices and quantities becomes established.

H.2. Every individual pays the same price for a given good i.

H.3. The price of good i depends on the quantity Q(i) only.

H.4. The change in consumption of good i by individual k when the price of i changes is determined by the elasticity of good i (same for all k).

H.5. The EDM considers that the flow of goods in $ units determines the collective utility of an economic state; he assigns the same value to a dollar of good regardless of the individual who produced or consumed that good.

H.6. Interaction between individuals, substitution of other goods for water, and nonlinearities are second-order effects.

H.7. Second-order effects are negligible.

Under these hypotheses, the change in collective utility between two price structures can be written as:

$$dU = \sum_{i,k} [p(i)dQi,k) + Q(i,k)dp(i)] \qquad [1]$$

This expression for dU implies a knowledge of p(i), Q(i,k), dp(i), and dQ(i,k). The variable dQ(i,k) is the most difficult to obtain and the most controversial to define accurately; demand schedules should be used to obtain it.

Another method is used here. The Tucson Department of Water and Sewers is currently in the process of developing computer programs designed to estimate the revenue gained from water each month. Thus, there are some reliable data available but at this time these data are not suited to our mathematical model, and are far from complete. Without having to go through each user's bill, it is possible to use the data from the computer print-out with some assumptions. So the following application of the model is not intended to be accurate but will demonstrate the applicability of the model.

Tucson is presently metered in four areas under a regressive block rate structure. There are three rates; the first is a flat rate while the succeeding two are incremented. To see how the collective utility formulas apply, let us just take one area and one service type, whose present characteristics are:

Consumption	Rate	# Users (583 total)	
0- 700 ft^3	\$7.50	128	
800-3700 ft^3	\$ .40/100 ft^3	425	[2]
3800- ft^3	\$ .40/100 ft^3	30	

Let the above rate structure define state 1.

Let us hypothesize another rate structure as state 2:

0- 700 ft^3	\$7.50	
800-3700 ft^3	.41/100 ft^3	[3]
3800- ft^3	.43/100 ft^3	

Note: If individual (user) k consumes more than 700 ft^3, he must pay the second rate for all quantity greater than 700. Every k pays the flat rate; every k in the

third rate system pays 7.5+.41(30)+.43(Q(k)-37). So there are three types of goods or i=1, 2, 3. The basic formula for dU will be used in finite difference form:

$$U = \sum_{i=1}^{3} [Q(i,k)\Delta p(i)+\Delta Q(i,k)p(i)] \qquad [4]$$

We have hypothesized a $\Delta p(i)$; then $\Delta Q(i,k)$ should be computed by using a marginal rate of substitution of one level of water use for another one. To find the rate of substitution, however, is a problem. If a city water agency estimated revenue based strictly on elasticities of demand, substitution of i=2 for i=3 can force consumer k to drop his consumption enough to fall into the next lower rate. But the above method does not take this into account. The problem of substitution can be solved if care is taken to sum over the correct set of individuals for a given good i. Let K(i,s) be the number of users of water of type i in state s; i=1, 2, 3 and s=1, 2. The complete set K of users, in state 1, is written as;

$$K = K(1,2)+K(2,1)+K(3,1)$$

Let K(i,j) denote the number of users that went from type i in state 1 to type j in state 2. In state 2, the set K is written as:

$$K = K(1,2)+K(2,2)+K(3,2)$$
$$= [K(1,1)+K(2,1)] + \Delta[K(2,1)+\Delta K(3,2)-\Delta K(2,1)] + [K(3,1)-\Delta K(3,2)].$$

Within a given type of rate, the frequency density of users is given, so that the K(i,j)'s can be calculated. Until such a density can be calculated from empirical data, a uniform distribution will be assumed.

The formula (4) for dU can now be set up with the proper summation sets; see Motlor and Duckstein (1971) for the details of the formulation and computations.

To illustrate the above, initially the price rate will be changed on a permanent basis in the hope of conserving water.

3.2 Permanent Rate Change. The behavior of users in the month of November will be considered representative of the average behavior all year around. A complete study would consider dU for each month and each service area separately, then add up over a year.

To make the computations, one finds from the literature that the average elasticity n = n(i) of water is given as -0.4 by Howe and Linaweaver (1967); -0.35 by Clausen (1970), who states that Tucson's is much higher; and as 0.5 by Whitford (1970). A value of n = -0.5 is chosen since it seems reasonable to illustrate the method. For the situation depicted by [2] and [3], one finds

1. $\Delta Q(2) = -29\ ft^3$, $\Delta Q(3) = -154\ ft^3$;
2. $\Delta k(2,1) = 4$, $\Delta k(3,2) = 5$; and [5]
3. $dU = \$(19.64 - 13.35) = \$6.29$ (for one month.

Both the change in total water consumption and change in collective utility are small as compared to the total consumption and monetary flow in the service area. However, if the above figures are representative of other areas and of monthly averages, they must be multiplied by about 1,200 for Tucson (50,000 users instead of 500, 12 months instead of one). In that case, since water can be conserved with a positive change in collective utility, it would be advantageous to go from a regressive price structure to a progressive one. Note that it would be a matter of straight computation to obtain dU for every month and every service,area, if the corresponding data were available in proper form.

3.3 Seasonal Rate Change. In an effort to reduce peak consumption during the summer months, the EDM may wish to examine if the present regressive rate structure from October 1 to March 31 combined with a progressive rate for the remaining 6-month period represents an improvement in collective utility. Let the month of September be representative of the summer period; price structures 1 and 2 are as shown earlier in [2] and [3]; however, the number of users in state 1 is 106, 436, 41 instead of 128, 425 and 30 for the same value of K = 583.

The absolute value of the elasticity for the summer is higher than for the winter, because the water bill is higher, so that people are more sensitive to water prices. The value n = -0.7 suggested by Howe and Linaweaver (1967) will be chosen here. The results are as follows:

1. $\Delta Q(2) = -38.5\ ft^3$, $\Delta Q(3) = -251\ ft^3$;
2. $\Delta k(2,1) = 5$, $\Delta k(3,2) = 9$; and [6]
3. $dU = \$-38.1$ (for one month)

The figures are noticeably larger than for a change based on the month of November given in [5], which is caused by higher consumptions and a higher elasticity (in absolute value).

Let the EDM assign a value \$C to conservation: if C > 38.1, he will prefer economic state 2 to state 1. Another possibility is to assign a value c/ft^3 to a reduction in consumption because this also corresponds to a reduction in peak consumption, hence a savings on capital investment costs; then, if (38.5 + 251)c > 38.1, again state 2 is preferred to state 1. Finally, both considerations can be combined, to yield the policy:

1. If C + (38.5 + 251)c ≤ 38.1, stay in state 1(digressive rate), and [7]
2. If C + (38.5 + 251)c > 38.1, change to state 2 (progressive rate).

Conversely, if the EDM decides to stay in state 1, the value he imputes to conservation is less than 38.1 - (38.5 + 251)c, under the assumptions of this model.

4.0 Discussion

4.1 Summary of Hypotheses and Results. In addition to H.1 to H.7, which could be postulated in any case, the following hypotheses had to be made in order to obtain numerical results: 1) One service area was chosen out of four and one type of service out of five; 2) November is a typical month of the year and September a typical summer month; 3) users are uniformly distributed with respect

to quantity consumed within each price category; an exponential distribution in the highest category would be more likely, but it would strengthen our result; 4) the elasticity is the same for all individuals (it could vary with income of users); the yearly average is $n = -0.5$ and the summer value is $n = -0.7$. Under these conditions, the following results are found for one year:

For a permanent change: $\Delta Q = -2{,}030\ ft^3$; $dU = \$75$; $\Delta r = \$-300$. [8]

where Δr is the change of income of the water utility company.

For a seasonal change: $\Delta Q = -1{,}620\ ft^3$; $dU = \$-228$; $\Delta r = \$130$. [9]

The reason dU and Δr do not have comparable values comes from H.6 and H.7: if individuals spend more money for water, the effect on their consumption of other goods is negligible.

The considerations about conservation and lowered peak consumption presented in 3.3 can clearly be repeated with values [8] and [9].

Extensions of the model will now be briefly considered.

4.2 Multi-period Policies. Let y be a control variable to be changed every year: an example is the water rate in the highest category of consumption. If the EDM wishes to control water consumption in a dynamic fashion over a period of 20 years in order to insure, for example, that the groundwater supply will not be exhausted before other sources become available, a sequential decision-making scheme can be put forth (Masse, 1956; Bellman, 1957) in the following manner:

$$dU(Q,n) = \max_{y \in Y} \{r[Q(y,n)] + dU[Q-Q(n-1), n-1]\} \qquad [10]$$

with $Q(n-1) = F[Q(y,n)]$ and in which

Q is the total quantity of water available over the 20 years;

n is the number of years of decision left;

$r[Q(y,n)]$ is the revenue of the utility company, with a rate y and n years of decision left

$dU(Q,n)$ is the maximum change of collective utility that can be obtained with Q units of water and n years of decision left.

F is a function describing how water consumption is affected by rate and previous consumption.

Before formulation [10] can be applied, enough data must be gathered to determine the functions r and F.

Another extension of the model, which can be combined with this sequential approach, introduces uncertainty in future states of the world.

4.3 Uncertainty. Additional notation is needed in this section, namely:

u = state of the world (inflation, technological change, low pressure during peak hours)

P(u) = fixed probability of such a state

p(i,u) = price of water of type i given u

R(j,k) = quantity of uncertain water of type j that individual k intends to consume (to fill a pool at peak consumption time, for example).

$\alpha(i,j,u)$ = quantity of water i that can be obtained from one unit of j given u.

To illustrate further the concept of the state of the world, consider the situation where uncertainty of the future consists:

1. in consumptions, which can change by ± 10 percent or stay constant, all else being fixed;
2. in elasticities, which can change by ± 25 percent, or stay constant, all else being fixed.

We thus have nine possible states, with a self-evident notation:

u	1	2	3	4	5	6	7	8	9
Δ consumption	0	0	0	+	+	+	-	-	-
Δ elasticity	0	+	-	0	+	-	0	+	-

Under fairly general conditions (Lesourne, 1964), the change in collective utility can be written as

$$dU = \sum_{u,i,k} P(u) \{p(i,u)dQ(i,k) + p(i,u)\,\alpha(i,j,u)dR(j,k)\} \qquad [11]$$

Due to space limitations, no further elaboration of the uncertainty case will be done here, so that in the next section a succinct comparison between collective utility and other approaches can be made.

4.4 Comparison with Cost-Effectiveness and Cost-Benefit Analyses. As pointed out by English (1968, Chapter 7) cost-effectiveness techniques are not resource allocation methods and imply the existence of alternative systems to reach a goal; criteria of effectiveness must be developed to compare alternate systems. The collective utility approach has the advantage that two pricing structures (or systems) can be compared without any specific goal in mind; the unique criterion of effectiveness is the value of the monetary change of goods flow. In this sense, the interesting study presented by deNeufville (1970), where no precise goal is discernible, could have been approached by the method proposed here. On the other hand, cost-effectiveness techniques would be almost worthless for sequential allocation or pricing, as presented in 4.2.

Cost-benefit analyses imply detailed design of a fixed channel, structure or a water distribution system and do not take individual satisfaction into account. Many implicit hypotheses are made to arrive at a benefit figure, so that inconsistencies may arise from one project to another one.

5.0 Conclusions

Although a progressive price structure goes against the established principle of "marginal pricing" (the more water is produced, the cheaper it is sold), preliminary calculations show that a permanent change to such a structure increases the collective utility; a seasonal change may be advantageous if the value of conservation and of lowering peak consumption is taken into account.

The collective utility approach can be used for sequential decision-making on price structures; uncertainty of future states of the world can be taken into account. Only neighboring states should be compared; finally, the method postulates the existence of an EDM (Economic Decision-Maker).

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