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Bayesian Decision Theory Applied to Design in Hydrology

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Abstract. The role of Bayesian decision theory in hydrologic design problems is presented both in theory and by example. The theory is applied to an actual flood levee design problem on the Rillito Creek floodplain in Tucson, Arizona. Computer solutions provide a basis for judging the costs of overdesign in the face of uncertainty in the parameters of the flood frequency model (log normal) and for determining the worth of hydrologic data. One conclusion is that decision theoretic analysis looks at the decision situation from the standpoint of the engineer: how can one best decide in the face of limited data and the present knowledge about system behavior?

The purpose of this paper is to demonstrate the case for the use of Bayesian decision theory in hydrologic design problems; the method is illustrated by means of a case study of flood levee design. As we will discuss later, there has been considerable interest recently in studying the risk involved in design decisions and in evaluating the benefits of bringing additional data on design decisions. The problems are obviously intertwined; additional information tends to reduce risk. Current practice separates these problems

Risk is a word that needs defining. Commonly, risk refers to a hazard or peril and the chance of loss due to that hazard or peril. We prefer the definition given by *Klausner* [1969]: '. . . risk is considered to be the consequential effect of possible uncertain outcomes.' Klausner's definition encompasses the common definition and forms a basis for its extension.

Yen [1970] handles risk by a discussion of failure or exceedance probability. The concepts of 'standard project flood' and 'probable maximum precipitation' or the concept of the critical period may be viewed as attempts to eliminate particular uncertain outcomes. By considering risk only in terms of failure, one may overlook the risk of wasting capital by overdesign. Both risks should be evaluated; the theory for doing so should be able to examine the 'consequential effect' (singular) of all 'possible uncertain outcomes' (plural). Bayesian decision theory meets this criterion.

The value of additional data has sometimes been evaluated by examining long historic records [*Mooley and Crutcher*, 1968] to determine when parameter estimates 'settle down.' This evaluation is done by abstracting sequences of various lengths from the historic record. Since this procedure involves sampling from a finite population, it is no surprise that the estimates do settle down (usually within 50–75% of the historic trace used). A more quantitative approach [*Dawdy et al.*, 1970] involves generating synthetic traces based on the historic record and ascertaining the average increase in value of a project designed with an increase in record length. This approach assumes that the parameters used in generating the trace are known with certainty and examines the average results of different sampling periods Bayesian decision theory looks at the problem from the engineer's viewpoint; sample statistics are at hand, possibly from a short record, and the question is: what is the expected value to the project of more data? The two viewpoints are not symmetrical Also, Bayesian decision theory provides an alternate method of handling risk and the value of additional data by incorporating them into design decisions.

Bernier [1967] and *McGilchrist* [1970] improved estimates of streamflow characteristics obtained from historic data by incorporating other types of data into Bayes' theorem. *Con-*

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over [1971] recommends Bayesian methods for parameter estimation but concludes that 'the methods of Bayes' estimation do not lend themselves to numerical methods as performed on a computer.' Although the computer is of little aid in finding the functional form of a Bayes estimator, results presented later in this paper show that a computer is not just useful but indispensable for finding a Bayes solution for a design problem. *Brown* [1970] recommends Bayesian methods for design and gives a simplified example of the use of Bayesian estimation in spillway design *Davis and Dvoranchik* [1971] use Bayesian decision theory to determine the depth of a bridge pier and the worth of additional data, without explicitly estimating the statistical parameters However, a return period is implied for the flood that undermines the piers. Note that Bayesian decision theory focuses on the decision to be made and not on the hydrologic parameters as an end result.

This paper presents in a stepwise manner the procedures involved in making a Bayesian decision analysis The procedures are illustrated by an actual design problem. The example chosen will also show that there are situations that, though yielding a satisfactory Bayes solution to a problem, fail to give an intuitively satisfactory Bayes estimate of the statistical parameters involved; thus the Bayes estimate of the return period of the design flood may be 10^6 years, whereas the Bayesian decision analysis leads to a perfectly acceptable engineering design.

DECISION THEORY

Bayesian decision analysis is a method for choosing and evaluating design alternatives for a project, consideration being given to the effect of uncertain parameters on the performance of the project. Note that three essential ingredients are present: alternatives, performance criteria, and uncertainty in the parameters affecting the project. Decision analysis is not a rigid method but a series of signposts. The following outline of the method is adapted from *Howard* [1966]:

A. Define the goal.
B. Define the decision to be made and identify the alternatives.
C. Analyze the project.
   1. Define the goal function.
      a. Select the state and decision variables.
      b. Set a time preference.
      c. Include a risk aversion.
   2. Make a sensitivity analysis.
   3. Develop the stochastic properties of the knowledge of the values of the state variables as a probability density function.
   4. Calculate the outcomes of the various alternatives and determine the stochastic properties of these outcomes.
   5 Eliminate the dominated alternatives.
D. Make the decision.
   1 Calculate the expected value of the goal function for each alternative.
   2. Choose an alternative to minimize the expected value of the goal function.
E. Evaluate the decision.
   1 Determine the expected opportunity loss due to uncertain parameters in the problem.
   2. Evaluate information-gathering programs.
      a. Determine the expected reduction in the expected opportunity loss with further information.
      b. Determine the full cost of obtaining further information.
      c. Obtain further information if warranted, and repeat the analysis.

The outline shows how Bayesian decision theory through a design decision unifies the treatment of risk as defined earlier and the worth of the data.

Possible uncertain outcomes result from the vagaries of nature. The effect of these outcomes on the performance of the project is expressed by the goal function The uncertainties in our knowledge of nature (or anything else) are expressed as a probability distribution function (pdf) for the uncertain parameters.

The consequential effect of the possible uncertain outcomes is expressed as the expected value of the goal function. The expectation is taken with respect to the pdf of the uncertain parameters. The design decision minimizes the consequential effect if the effect is undesirable. The common view of risk may be incorporated into the procedure by heavily weighting the possibility of loss due to hazards and perils. Step E of the outline evaluates the effect on the decision and the minimum consequential effect of more knowledge about the uncertain parameters.

FLOOD LEVEE DESIGN

The problem studied in this paper is flood control on Rillito Creek in the lightly urbanized reach through the north side of Tucson, Arizona. A broad study of the available alternatives has been made by the *TRW Systems Group* [1969]; however, for the purposes of this paper, the set of alternatives a to be considered are the protection levels corresponding to the maximum flows to be contained by the construction of dikes. The goal function to be minimized is $g(a, \theta)$, where a is the decision variable and θ is the vector of state variables; in this case the goal function is equal to the annual expected costs of flood damage due to floods higher than the protection level, plus the annual costs of the protection system, minus other benefits such as recreation and land enhancement (on an annual basis):

$$g(a, \theta) = E_\theta \text{ [flood damage } (a)] + \text{annual costs of protection } (a) - \text{miscellaneous benefits } (a)$$

The goal function represents the expected annual cost to be incurred by the occupation of the Rillito Creek floodplain. Some values of these costs, which were used by the U.S. Army Corps of Engineers in their analysis [*Davis*, 1971], are shown in Table 1. The components of the state variable vector θ are the parameters of the probability distribution used to obtain the expected costs of flood damage. In this case the pdf of the logarithms x of the peak annual flows was assumed to be log normal, a hypothesis not rejected by a Kolmogorov-Smirnov goodness-of-fit test based on 45 years of data; thus the state variables are the mean μ and variance σ^2 of the log transforms. The annual floods are also independent in time. Estimates of flood damage are based on future values; the time preference is the 4 7/8% interest rate used to discount future values to present values. Risk aversion [*Mayer and Pratt*, 1968], which may be crudely described as the preference of many small losses over one large loss, was assumed to be negligible because the Rillito Creek floodplain occupies a small part of the total area of Tucson.

TABLE 1. Flood Damage and Protection Costs for Rillito Creek

Discharge, cfs	Flood Damages to Future Conditions, $\$10^3$	Annual Costs, $\$10^3$	Other Benefits, $\$10^3$
78,000	49,000	1580	163
37,000	22,000	1195	153
24,000	7,400	815	126
18,000	3,400	...	...
14,000	1,100	...	...

A sensitivity analysis indicated that the two state and the one decision variables strongly affected the goal function; thus all the variables were kept in the analysis.

The uncertainty in the knowledge of the state variables is embodied in the pdf $f(\theta)$. In this case it is a normal gamma pdf $NG(\mu, \sigma^2 | \hat{X}, S^2, n)$ based on the sample mean $\hat{X}$, the sample variance S^2, and the sample size n. This distribution provides the same information about μ and σ^2 as does the normal chi square distribution used to obtain joint confidence intervals for these parameters and is obtained from it by the substitution of nS^2/σ^2 for the chi square variable. The outcomes of the various alternatives are now calculable and their stochastic properties depend on the stochastic properties of the state variables. Two alternatives were eliminated by being found inferior to some other alternative for all values of the state variables. If an alternative of drilling a tunnel under the city to divert floodwaters were under consideration, it presumably would be formally rejected at this stage.

FLOOD LEVEE DECISION

The preliminaries are over, and the decision may be made. The consequential effect (the expected value of the goal function) may be calculated for each alternative: $R(a) = \int g(a, \theta) f(\theta)\, d\theta$. The term $R(a)$ is appropriately called the Bayes risk. The decision is made by choosing the alternative a^* (the Bayes solution) that minimizes the Bayes risk:

$$R(a^*) = \min_a \int g(a, \theta) f(\theta)\, d\theta$$

Next, according to step E of the outline, the decision is evaluated. If the true values θ_T of the state variables were known, the alternative chosen a_T would be the one that minimized the goal function for θ_T:

$$g(a_T, \theta_T) = \min_a g(a, \theta_T)$$

The decision has been made to use alternative a^*; it may be a nonoptimal choice. In making a

nonoptimal choice we suffer an opportunity loss (OL):

$$OL(a^*, \theta_T) = g(a^*, \theta_T) - g(a_T, \theta_T)$$

We do not know the true value θ_T of the state variable vector, but we have a pdf $f(\theta)$ that enables us to calculate an expected opportunity loss (XOL):

$$XOL(f) = \int [g(a^*, \theta) - \min_a g(a, \theta)] f(\theta)\, d\theta$$

The XOL represents the expected value of perfect information and may be used to judge the effect of uncertainty as embodied in $f(\theta)$ on the performance of the project.

Should more information be gathered? Should the decision for the Rillito Creek flood control problem be delayed 1 or more years to obtain additional peak flow data? Another sequence of observation x would give revised knowledge of the state variable by means of Bayes' theorem:

$$f(\theta \mid x) = \frac{f(\theta) \cdot l(x \mid \theta)}{\int f(\theta) \cdot l(x \mid \theta)\, d\theta}$$

where $l(x \mid \theta)$ is the likelihood of the observation sequence x, $f(\theta)$ is the prior distribution, and $f(\theta \mid x)$ is the posterior distribution. In the case studied here, $l(x \mid \theta)$ is normal $N(x \mid \mu, \sigma^2)$, and the prior and posterior distributions are normal gamma. For the observation x, a new XOL may be calculated. $XOL[(f(\theta \mid x)]$. However, the observation x is uncertain. Its pdf, which is the denominator of the previous equation, may be calculated: $l(x) = \int l(x \mid \theta) f(\theta)\, d\theta$. Term $l(x)$ is the predictive distribution of x; for Rillito Creek it is a form of the t distribution [*Raiffa and Schlaifer*, 1961, chap. 7]. By means of the predictive distribution, the expected expected opportunity loss $XXOL$ may be calculated: $XXOL\{x\} = \int XOL[f(\theta \mid x)] l(x)\, dx$. The expected decrease in the expected opportunity loss is called the expected value of sample information ($XVSI$): $XVSI\ x\} = XOL[f(\theta)] - XXOL\{x\}$.

There is a cost of sampling that usually consists of the actual costs of making the observation and, if the project is delayed, the loss of benefits for the period of delay. These costs may be stochastic, as is certainly the case in delaying the construction of a flood levee. These costs constitute the expected costs of sampling $XCS\{x\}$. For a flood control project, the actual cost of sampling a peak flow is relatively low compared with the expected cost of delay. The expected cost of delay for a flood control project is the Bayes risk for the alternative 'no project' minus the Bayes risk for the optimal alternative.

The expected net gain of sampling $XNGS$ is: $XNGS\{x\} = XVSI\{x\} - XCS\{x\}$. If $XNGS$ is positive, further sampling is indicated. If $XNGS$ is negative, cessation of sampling is not categorically indicated; a larger sampling program could still be in order. If more than one source of information about the state parameters is available, the values of the information they give can be compared by these concepts.

METHOD OF COMPUTATION

The calculations indicated in the preceding section were done for the Rillito Creek case study with the aid of a CDC 6400 computer. The necessary integrations, including the double integration over the two state variables to obtain the Bayes risk, were done by Simpson's rule. The number of points used in each integration was specified by the accuracy level required. The alternative giving the minimum Bayes risk or opportunity loss was found by using a modified Golden Search procedure. A notable feature of this method is the use of a continuous pdf and an infinite set of alternatives throughout the analysis. Discretization occurs only at the computational level, where the use of Simpson's rule and the Golden Search enables us to carry out the integration over the pdf and the minimization to an acceptable level of accuracy.

The decision was made in less than 30 seconds of computer time; evaluation of the decision (step E of the method) took another 6 minutes. If procedures tailor-made for this study had been used instead of a fairly general numerical analysis method, these times could have been cut considerably.

The cost and damage functions were interpolated by using polynomial and rational functions. Extrapolations were performed linearly. Details on the preceding procedures can be found in *Davis* [1971].

RESULTS

The results presented in Table 2 were calculated by means of varying lengths of Rillito

TABLE 2. Decision Analysis of a Flood Protection Project

Years from 1915	$\hat{X}$	S^2	Protection Level, cfs	Bayes Risk, $10³ Minimum	Bayes Risk, $10³ No Protection	*XOL*, $10³	*XVSI*, $10³	*XNGS*, $10³
5	3.96	0.027	72,000	2300	2480	885	323	148
8	3.93	0.049	77,000	1670	1894	505	73	−149
9	3.89	0.054	none	1579	1579	479	100	100
10	3.83	0.080	79,000	1737	1830	507	68	−25
11	3.80	0.080	none	1478	1478	413	81	81
14	3.72	0.095	none	1053	1053	191	14	14
15	3.76	0.116	79,000	1693	1722	404	41	12
16	3.75	0.109	none	1473	1473	348	59	59
20	3.75	0.093	none	971	971	115	2.5	2.5
44	3.71	0.101	none	720	720	12	<0.100	<0.100

$\hat{X}$ and S^2 are based on the logarithm of the annual peak flow in cubic feet per second.

Creek data starting in 1915. This procedure implicitly assumes no prior knowledge about floods before 1915. The worth of the decision analysis procedure can be evaluated by noting how the results change with additional data As discussed earlier, a positive *XNGS* in Table 2 corresponds to the desirability of waiting for more data, whereas a negative *XNGS* requires judgment as to the worth of waiting. We will discuss this point in detail later The *XXOL* and *XCS* are not included in Table 2; they are implicitly there, however. The *XVSI* is obtained by subtraction of the *XXOL* from the *XOL*. The *XCS* may be obtained by subtracting the minimum Bayes risk from the no protection Bayes risk.

The sample mean decreased, and the sample variance increased over the period analyzed The Bayes risk, *XOL*, *XVSI*, and *XNGS* exhibited a downward trend during this time. This trend may be attributed to he reduction of uncertainty due to an increase in the available data. With 16 or more years of data, the alternative chosen was no protection. Prior to that time (1930) the decision fluctuated between no protection and protection around the 75,000-cfs (cubic feet per second) level Note that this figure originates from a combination of historic flow data and cost function values (Table 1): an action can be taken without worrying about the physical realizability of a 75,000-cfs flood. Does this fluctuation indicate an instability and/or an unreliability of Bayesian decision analysis? With 14 years of data, the decision is to provide no protection. Then the largest flood on record (24,000 cfs) appears during the fifteenth year (1929) and is reflected in the decision to protect at the 79,000-cfs level. Moreover, in 1930 the decision is again to provide no protection Note that with 15 years of data the expected net gain of sampling *XNGS* is positive, so that the decision in 1929 should have been to wait for more data. With 8 and 10 years of data, however, the *XNGS* is negative, and protection around 75,000 cfs is indicated The negative *XNGS* indicates that delaying the decision just 1 year would not be justified; a delay of more than 1 year was not evaluated, as the numerical procedures become horrendous However, for 8 and 10 years the *XOL* is $500,000, which is 30% of the optimal Bayes risk, and the Bayes risk of the nonoptimal decision of no protection is less than 15% higher than the Bayes risk for the optimal decision On the basis of this information, a judgment to delay the decision may have to be seriously considered If the cost of delay had been based on present values of the floodplain, which are smaller than the future (discounted) values used to compute the Bayes risk, the *XCS* would have been small enough to make the *XNGS* positive, thus a delay in the decision would have been called for The expected cost of sampling and the expected value of sample information were expressed on an annual basis. The *XCS*, the cost of a year's wait, is certainly an annual quantity On the other hand, the *XVSI* is a reduction in annual cost that may be expected to last for the life of the project. If our ultraconservative treatment of the com-

ponents of the expected net gain of sampling were corrected (i.e., if the *XCS* were lowered and the *XVSI* raised), the *XNGS* would have been positive for all lengths of record considered. Then the decision analysis would consistently call for delaying the project to obtain more data.

The apparent instability in the decisions regarding protection level, when the first 16 years of data are considered, can be explained by noting the small difference between the Bayes risks for no protection and for protection at the 75,000-cfs level (Figure 1) This effect results from the configuration of each particular Bayes risk curve, which may have two local minimums The dip in Figure 1 from a peak at 20–21 × 10^3 cfs for the 15- and 16-year curves indicates the point at which flood protection is beginning to pay off. The dips contain a local minimum: in the 15-year curve the right-hand minimum corresponding to protection at 75,000 cfs is global; in the 16-year curve, however, the left-hand minimum corresponding to no protection is global On the other hand, both the 14-year and the 44-year curves have only one local minimum and thus indicate that the Bayes risk is minimum when no protection is provided The top three curves start to rise again at about 84,000 cfs because the cost of overdesign becomes higher than the benefits of protection. Note that the Rillito Creek starts to flood at 10,000 cfs.

If the values of the Bayes risk at the local minimums do not differ much, seemingly different decisions will produce very similar results. This phenomenon looks counterintuitive at first, but if we realize that the Bayes risk is an expected value, we can sense that the statement 'protection at 75,000 cfs and no protection correspond to the same Bayes Risk,' really means 'if many Rillito Creeks are to be protected against floods, then on the average protecting all the creeks at 75,000 cfs or none of them would cost the same' We could also say that if funds were available to protect at 75,000 cfs we have nothing to lose to go ahead, on the other hand, if funds were limited it is better not to protect than to protect at, say, 20 000 cfs. Furthermore, if we want to allocate flood protection funds between several projects along the same river or along identical streams with different cost functions, we should first fund

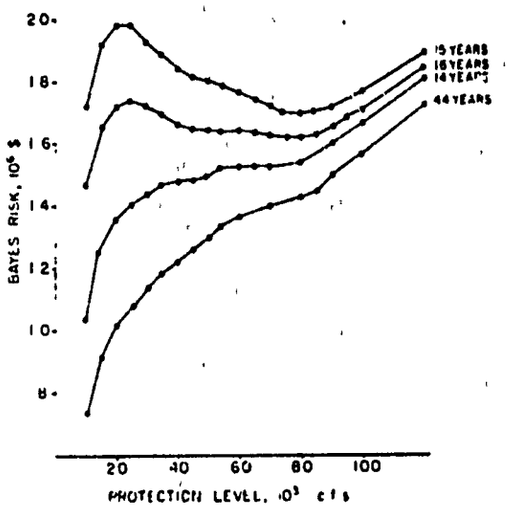

Fig. 1. Bayes risk versus protection level data starting in 1915.

the project that corresponds to the largest reduction in Bayes risk from the no protection level

The curves in Figure 1 also illustrate the difficulty of obtaining a meaningful estimate of the state variables, which in the present case are the parameters of either the log normal pdf of yearly floods or the return period implied by the solution of the problem by the Bayes estimation procedures The difficulty stems from the fact that the estimates are discontinuous with respect to the sample statistics. The discontinuity of the decision is mitigated by the continuity of the Bayes risk; no such amelioration is available for the Bayes estimate or a 'Bayes return period.' The return period corresponding to the 75,000-cfs protection level is orders of magnitude higher than the return period corresponding to no protection Yet as we have seen, these differences result from a small change in sample statistics A further problem in Bayesian estimation is the lack of unique values for μ and σ^2 corresponding to the chosen decision. However, such unique values are important only for scientific or 'understanding' reasons. Practically, the results indicate that the design should be chosen from the available alternatives and not from parameter estimates.

The 44-year curve in Figure 1 may be used to indicate the cost of a nonoptimal decision. A decision to protect at the 75,000-cfs level will result in an annual excess cost of about $700,000, i.e., the nonoptimal Bayes risk (for protection at 75,000 cfs) minus the Bayes risk of the optimal solution.

SENSITIVITY ANALYSIS

As demonstrated by *James et al.* [1969], uncertainty in the economic parameters of a project may have a large effect on the decision reached. To examine the sensitivity of our results to changes in the goal function, protection costs were reduced by 25–50%; protection costs may be reduced by new technology and errors in earlier evaluation. The results of the calculations are shown in Table 3, where the data base is 1940–1949; consequently, these results are not directly comparable with those of Table 2. This data base was chosen because it provided a suitable illustration. The alternative chosen with reduced protection costs was a high level of protection in both cases, whereas use of the original costs led to the decision to provide no protection. Closer examination shows that a 25% reduction in protection costs reduced the minimum Bayes risk by only 3%. A reduction in protection costs does not change the Bayes risk of the no protection alternative. A change of decision will come about only when the right-hand local minimum becomes a global minimum. This change barely happens at a 25% reduction in costs, but a significant difference does arise for a 50% reduction.

Next, damage costs were changed by assuming the decision maker to be risk averse. These costs were raised to the 1.04 power, which doubles flood damage costs for a 75,000-cfs flood (three times the historic maximum). The effect of this operation is to weight large losses more heavily in the decision-making process. The alternative chosen was to protect at the 107,000-cfs level, whose physical realizability, as stated earlier, is of no concern for our assumptions. This decision, which has a 20% lower Bayes risk than the no protection decision, is not optimal in the case in which there is no risk aversion; it costs about $500,000 more per year. This cost may be considered the insurance premium to be paid for protection from a large loss. In this sense it is an actuarial measure of risk.

LIMITATIONS OF THE APPROACH

Bayesian decision analysis evaluates only the information put into it. All results must be viewed with this limitation in mind. In the case detailed here, the decision and the worth of more data are for the specific problem of Rillito Creek flood control and the specific statistical model of the log normal pdf of floods. The uncertainties were hydrologic in nature; uncertainties in structural design, in models of channel hydraulics during the presence of levees, and in economic assessment were not considered. The solution would be different if a log Pearson type 3 distribution were assumed or if a maximum peak flow (uncertain) were postulated. To obtain a meaningful decision, all factors affecting the problem must be included in the analysis. The results of the analysis should state the information and models used. In particular, it should be noted that the worth of additional data is not measured on an absolute scale and is valid only within the framework of

TABLE 3. Change in Goal Function for Flood Protection Project for Decision Based on 1940–1949

	Protection Costs, %			
	100	75	50	100
Damage cost exponent	1	1	1	1.04
Sample mean $\bar{X}$	3.60	3.60	3.60	3.60
Sample variance S^2	0.140	0.140	0.140	0.140
Minimum Bayes risk, $\$10^3$	1,653	1,603	1,157	2,716
No protection Bayes risk, $\$10^3$	1,653	1,653	1,653	3,338
Optimal protection level, cfs	none	106,000	136,000	107,000
XOL, $\$10^3$	426	583	405	916

the specific project confronting the hydrologist, engineer, or decision maker.

Thus decision theoretic analysis essentially gives specific results conditioned on specific information and model assumptions. These limitations may be advantageous since the analysis has integrated the economic, engineering, statistical, and scientific aspects of the problem and has provided a point of focus for communication between the various groups working on a problem. Decision analysis is obviously a form of systems analysis.

The method can be extended to cover uncertainties in the choice of a model and in the economic parameters; the computer time required would be the only limitation.

Although the results are for a particular case, the method is very general in application. The computer program used [*Davis*, 1971] is easily adaptable to other goal functions, provided that the uncertain parameters originate from a (log) normal distribution Other distributions would require a more extensive adaptation of the program.

Interestingly enough, the questions asked in the particular framework of the Rillito Creek flood control project may be important queries for research of a more scientific and general nature.

CONCLUSIONS

Decision theoretic analysis provides a unified and rational method for making decisions, handling risk, and evaluating the worth of additional data in the face of uncertainty. The concepts of expected opportunity loss and expected expected opportunity loss enable the decision maker to see how uncertainty is affecting the problem by evaluating the decision made and by deducing the worth of an information-gathering program. The concepts of risk and risk aversion enable the decision maker to evaluate the hazards of the project and the costs of heavily weighting some hazards. Sensitivity analyses are readily provided by the method to determine the effects of changes in the problem statement.

Decision theoretic analysis examines a design problem from the viewpoint of the design engineer; decisions are made for the specific problem of concern by using the data on hand.

The computer can satisfactorily handle the computations involved in decision theoretic analysis. Decisions can be made with more than one uncertain parameter. Realistic goal functions of a complicated functional form may be handled Continuous probability density functions need not be discretized before the computational step

Bayes estimation may not be a satisfactory parameter estimation method when the Bayes risk has more than one local minimum, owing to problems of continuity and uniqueness. These problems are circumvented by deciding among alternatives

Bayesian decision analysis gives specific results for specific information and assumptions. It is a form of systems analysis and promotes communication.

Although specific results come from decision analysis, its informational needs may require that much generalized scientific knowledge be brought to bear on a problem. As a focus of communication it will point out research needs.

NOTATION

- a, design alternative;
- a^*, optimal design decision,
- $E\{\cdot\}$, expected value;
- $f(\theta)$, prior pdf on uncertain parameters $\theta = \{\mu, \sigma^2\}$;
- $f(\theta/x)$, posterior distribution of θ, given x;
- $g(\cdot)$, goal function;
- $l(x)$, predictive distribution of x;
- $l(x/\theta)$, likelihood of the observation sequence x, given θ;
- OL, opportunity loss,
- pdf, probability density function;
- $R(\cdot)$, Bayes risk;
- S^2, estimate of σ^2;
- T, subscript identifying true value of μ and σ^2;
- x, logarithm of the annual peak how (called observation or data);
- X, estimate of μ;
- XCS, expected cost of sampling;
- $XNGS$, expected net gain in sampling information;
- XOL, expected opportunity loss;
- $XVSI$, expected value of sample information;
- $XXOL$, expected expected opportunity loss;
- θ, parameter vector;
- μ, mean of the logarithms of annual peak flow;
- σ^2, variance of the logarithms of annual peak flow.

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REFERENCES

Bernier, J., Les méthodes Bayesiennes en hydrologie statistique, *Proceedings of the International Hydrology Symposium*, pp. 459–470, Colorado State University, Fort Collins, 1967.

Brown, B W., Probabilistic selection of project design floods, paper presented at the National Water Resources Engineering Meeting, Amer. Soc. of Civil Eng, Memphis, Tenn., January 1970.

Conover, W. J., Introduction to Bayesian methods using the Thomas-Fiering model, *Water Resour. Res.*, *7*(2), 406–409, 1971.

Davis, D. R., Decision making under uncertainty in systems hydrology, *Tech. Rep. 2*, pp. 1–179, Dep. of Hydrol. and Water Resour., Univ. of Ariz, Tucson, 1971.

Davis, D. R., and W. Dvoranchik, Evaluation of the worth of additional data, *Water Resour. Bull.*, *7*(4), 700–707, 1971.

Dawdy, D. R., H. E Kubik, and E. R Close, Value of streamflow data for project design—A pilot study, *Water Resour. Res.*, *6*(6), 1045–1050, 1970.

Howard, R, Decision analysis: Applied decision theory, in *Proceedings of the Fourth International Conference of Operations Research*, pp. 55–71, John Wiley, New York, 1966.

James, I. C., B. T. Bower, and N. C. Matalas, Relative importance of variables in water resources planning, *Water Resour. Res.*, *5*(6), 1165–1173, 1969.

Klausner, R. F., Evaluation of risk in marine capital investment, *Eng. Econ.*, *14*(4), 183–214, 1969.

McGilchrist, C. A., T. G. Chapman, and K. D. Woodyer, Recurrence intervals between exceedances of selected river levels, 3, Estimation and use of a prior distribution, *Water Resour. Res.*, *6*(2), 499–504, 1970.

Meyer, R., and J. Pratt, The consistent assessment and fairing of preference functions, *IEEE Trans. Syst. Sci. Cybern.*, *SSC-4*(3), 270–278, 1968.

Mooley, D. A, and H C Crutcher, An application of the gamma distribution to Indian rainfall, *Tech Rep 5*, pp 29–37, Environ. Sci. Serv. Admin., Silver Spring, Md, 1968.

Raiffa, H., and R. Schlaifer, *Applied Decision Theory*, pp. 211–241, Harvard University Press, Cambridge, Mass, 1961.

TRW Systems Group, A methodology for flood plain development and management, report submitted to U.S Army Eng. Inst for Water Resour., pp 114–173, Alexandria, Va., 1969.

Yen, B. C, Risks in hydrologic design of engineering projects, *J. Hydraul. Div., Amer. Soc. Civil Eng.*, *96*(HY4), 959–967, 1970.

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