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PREDICTING GROUNDWATER BASIN RESPONSE TO PUMPING AND RECHARGE OPERATIONS USING SURROGATE PARAMETERS

by

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RESEARCH JUSTIFICATION

The general objective of the research contract csd-2162, Agency for International Development, is application of modern water management practices to helping less developed countries become increasingly self sustaining by increasing food production and lessening the need for food importation. In many underdeveloped countries, the groundwater resource is an important focal point, particularly those countries with large arid and semi-arid regions. West Pakistan is a prime example of the need for effective management of groundwater basins.

One of the specific objectives of this contract is "development of surface and subsurface water removal to eliminate the hazards resulting from surface flooding and high water tables, especially on the farm." The "on farm" problem, however, cannot be separated from the larger regional problem, since the groundwater basin is extensive, linking many subunits together. An important foundation for effective groundwater basin management, in conjunction with surface water operations, is the availability of accurate basin models which can be used to predict how the basin will respond to various management schemes. Rational management is severely hampered without the availability of such models. This, then, is the thrust of this particular research effort. The results reported here are generally applicable to basin management problems that arise in both developed and underdeveloped countries. The stress is on development of models which are compatible with available data. The goal is to do the best we can with available data, and point the way to more effective data collection. Unfortunately, in the author's opinion, much current modeling effort has not properly stressed the former point. Models have resulted which, though perhaps theoretically realistic, cannot be properly verified and calibrated with available data.

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BACKGROUND

1. Difficulties in Modeling

Groundwater basins may be classified as distributed-parameter systems. The fundamental nonlinear partial differential equation governing unsteady groundwater flow, based on Darcy's law and the law of conservation of mass, is [5]:

$$\frac{\partial}{\partial x}\left[K(x,y,h) \frac{\partial h}{\partial x} \right] + \frac{\partial}{\partial y}\left[K(x,y,h) \frac{\partial h}{\partial y} \right] = S(x,y,h) \frac{\partial h}{\partial t} \pm Q(x,y,t) \tag{1}$$

subject to appropriate initial and boundary conditions, where

$h(x,y,t)$ = elevation of the water table or pressure head from an arbitrary datum [L].

$Q(x,y,t)$ = pumping or recharge rate per unit area [L/T].

$S(x,y,h)$ = storage coefficient of the porous media [dimensionless].

$K(x,y,h)$ = permeability coefficient of the porous media [L^2/T].

This equation forms the basis for groundwater basin modeling. There is, however, no general analytical solution to Equation 1, so it must be solved numerically via finite-difference approximation. The modeling problem is particularly difficult in that the parameters are not accurately known and tend to vary widely over a large basin. This necessitates solution

of the so-called *parameter identification* or inverse problem. Given well pumpage records $Q_i(t_k)$ and water level fluctuation data $\hat{h}_i(t_k)$ (usually taken at a nearby observation well) for wells $i = 1,\ldots,N$ and discrete time periods $k = 1,\ldots,M$, the problem is to estimate the values of the parameters on the basis of these records, using an arbitrary curve-fitting criterion.

Factors that make accurate solution of the parameter identification problem difficult include:

1. The obvious problem of estimating continuously distributed parameter values from discretely distributed data.
2. Data $\hat{h}_i(t_k)$ at a particular well i are influenced by pumpage at adjacent wells.
3. Boundary conditions are difficult to accurately determine, and may change as water levels at the boundaries fluctuate.
4. There may be considerable interaction between the groundwater basin and surface waters which is difficult to account for in the water level data $\hat{h}_i(t_k)$.
5. There may be instrumentation and measurement error associated with the data.

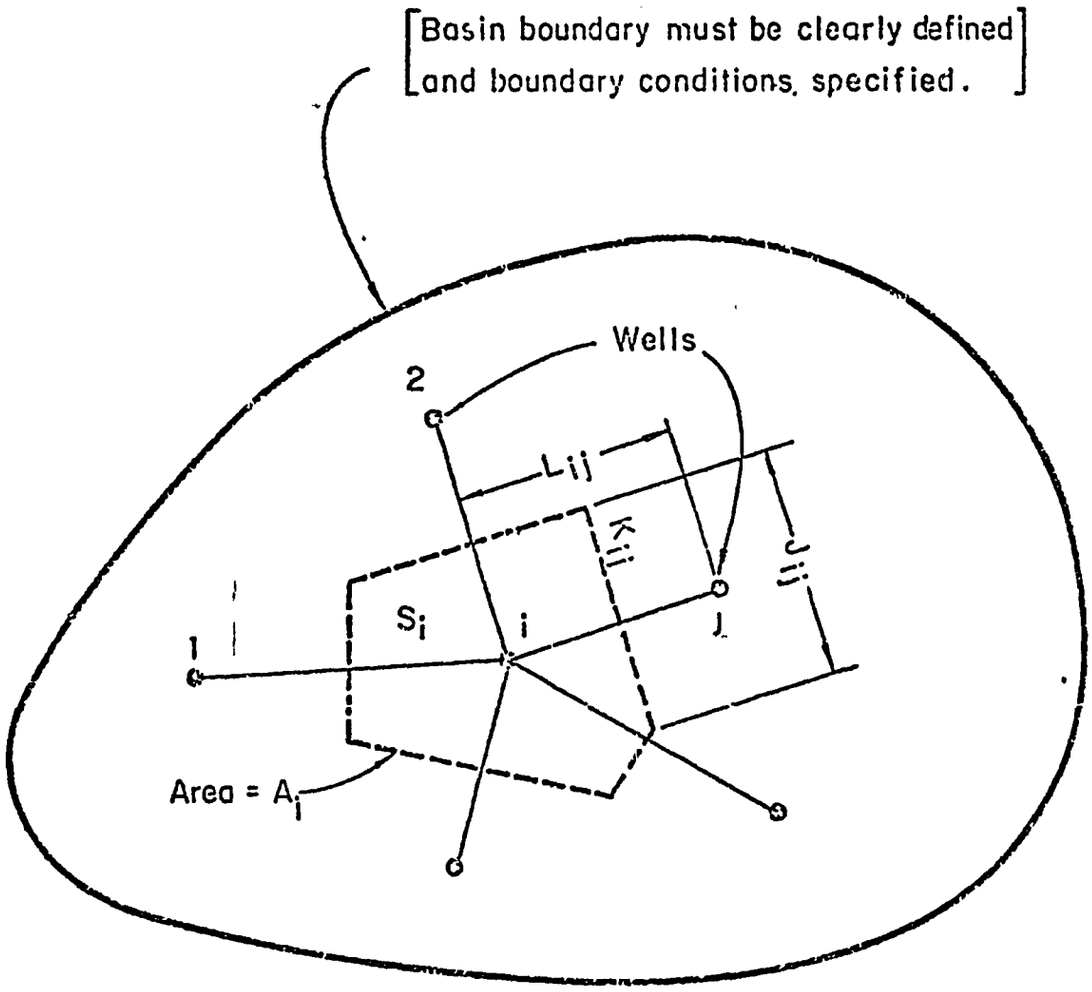
The surrogate-parameter approach described here attempts to deal with 1 to 4. The stochastic element introduced by 5 must eventually be dealt with, but a deterministic approach was taken as a first step.

2. Simulation

The simulation approach to modeling groundwater basins starts with division of the basin into asymmetric grids. The number and size of the grids depends upon the number and distribution of wells or other points where data are collected (referred to as nodes). A typical node and grid is shown in Figure 1.

Thus, Equation 1 is approximated by a set of simultaneous ordinary differential equations [2]:

$$\sum_{j \in N_i} (h_j - h_i) \frac{J_{ij}}{L_{ij}} K_{ij} = A_i S_i \frac{dh_i}{dt} + A_i Q_i \ , \quad i = 1,\ldots,N \qquad (2)$$



Parameters associated with homogeneous region surrounding each well and the interface between neighboring sections.

FIGURE 1

ILLUSTRATION OF SIMULATION

where

N = the total number of nodes.

N_i = the set of nodes adjacent to node i.

L_{ij} = distance between nodes i and j.

J_{ij} = length of the perpendicular bisector associated with nodes i,j.

A_i = area of grid surrounding the ith node.

K_{ij} = permeability coefficient between nodes i and j.

S_i = storage coefficient associated with grid i.

Q_i = volumetric flow rate per unit area at node i.

Simultaneous solution of (2) has been carried out on analog [7], digital [5], and hybrid [8] computers. The parameter identification problem is solved by arbitrarily adjusting parameters until a reasonable fit occurs between $h_i(t_k)$, and $\hat{h}_i(t_k)$, $i = 1,\ldots,N$; $k = 1,\ldots,M$, though there is considerable question as to the uniqueness of the final parameter values.

2.1 Weaknesses

The basic weaknesses of simulation can be summarized as follows:

1. There is a danger of high truncation error, implying that the finite-difference scheme is a poor approximation of Equation 2. The accuracy of finite-difference methods depends upon the ability to arbitrarily reduce grid size; an impossibility in this case since nodes are fixed.

2. As previously mentioned, accurate basin boundary conditions are rarely available. Again, accuracy of numerical solution of Equation 1 depends upon the accuracy of given boundary conditions.

3. Arbitrariness of sectioning configurations is clearly evident. There is a large number of possible configurations, each yielding different solutions.

4. Lack of systematic parameter identification methods which assure convergence to the best possible unique global solution.

3. Surrogate-Parameter Approach

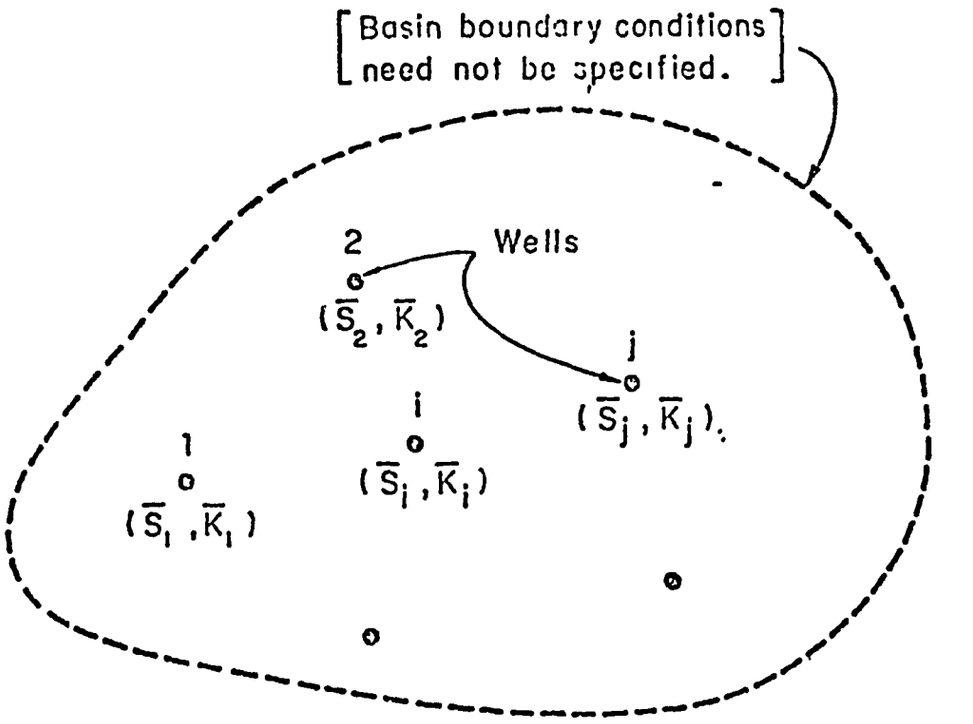
In the surrogate-parameter approach to basin modeling, no attempt is made to delineate homogeneous sections surrounding each well. The complexities of the near-well basin characteristics are lumped into a set of surrogate-parameters $\bar{S}$, $\bar{K}$, which are interpreted differently from S, K. This approach is born from the idea that parameter estimation should be compatible with the amount and location of data. Since groundwater basin data occur at discrete intervals in the basin, it logically follows that parameter estimation should result in values which are also discretely distributed over the basin at the precise points where data are collected. The surrogate-parameters, then, represent a lumping together of the complex porous media properties immediately surrounding the well (Figure 2a).

3.1 Basic Assumptions

There are three assumptions upon which the validity of this approach is based:

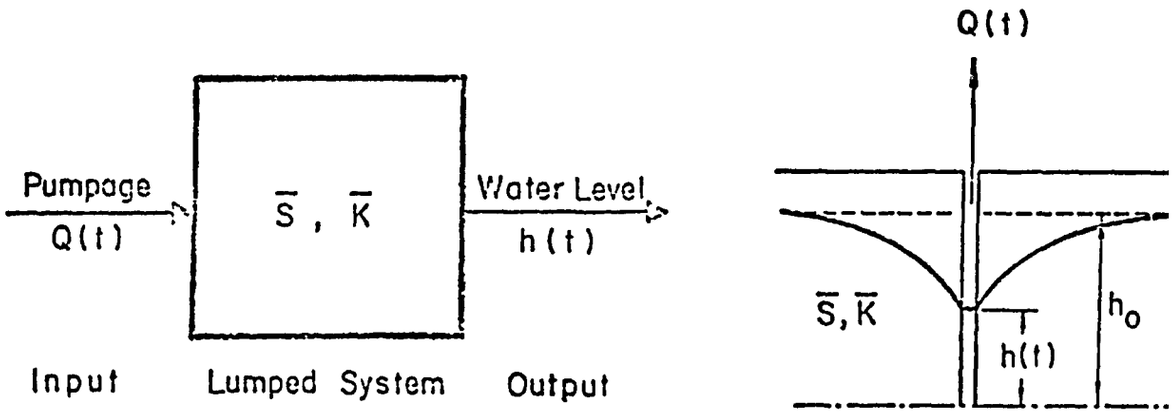
1. The response of a particular well in a nonhomogeneous, nonisotropic aquifer, assuming (i) no interference from adjacent wells, and (ii) no interference from boundary effects, can be predicted with sufficient accuracy (for practical modeling problems) using the same basic flow equation (Equation 1), but replacing the highly variable, continuously distributed parameters $S(x,y)$, $K(x,y)$ with lumped surrogate-parameters $\bar{S}_i$, $\bar{K}_i$ associated with well i and the near-well properties of the porous media.
2. In order to properly consider well interference, we assume that the response at well i due to pumping at well j is *primarily* dependent upon the near-well properties $\bar{S}_j$, $\bar{K}_j$ of well j, rather than those of well i.
3. The total response at well i due to several sources can be determined through linear superposition.

Russell and Prats [6] have concluded that, for practical purposes, a two-layered aquifer can be approximated by a lumped representation.



Surrogate parameters $\bar{S}, \bar{K}$ are associated with the well and the porous media immediately surrounding it — no attempt to define homogeneous sections.

(a)



(b)

FIGURE 2

SURROGATE-PARAMETER APPROACH

We are extending this conclusion to multi-layered, nonhomogeneous aquifers. Assumption 2 is based on the idea that since the great majority of dewatering or pressure reduction due to pumping occurs in the immediate vicinity of the well being pumped, then the degree of response emanating radially outward will be primarily a function of the near-well porous media properties and, of course, the pumping rate.

Application of the finite-element method [3] to various combinations of complex porous media representations (e.g., stratification and spacial variation) appears to have great potential for quantitatively verifying these assumptions.

3.2 Input-Output Analysis

An important advantage of this approach is that complex boundary conditions causing interference with well response can be incorporated into surrogate-parameters $\bar{S}_i$, $\bar{K}_i$, if the conditions are not accurately known. Consider, for example, well i located near an undefined impermeable boundary. The resulting values of $\bar{S}_i$, $\bar{K}_i$ will be lower than the actual characteristics S, K, since no attempt is made to guess these conditions. If the aquifer happens to be sandy then the response $h_i(t)$ will suggest a silty aquifer, due to the influence of the boundary. The emphasis is on prediction, and $\bar{S}_i$, $\bar{K}_i$ mainly act to describe an input-output relationship (Figure 2b). If the boundary conditions are well defined, then the classical *method of images* can be applied, and the resulting $\bar{S}_i$, $\bar{K}_i$ will be comparable to S, K. In the same way natural recharge can also be incorporated into the parameters if quantitative estimation involves a high degree of guesswork. If recharge varies seasonally, then a different set of parameters should be determined for each season. In short, we see that it is unnecessary to make arbitrary guesses with this approach. If accurate information is available, it can be easily incorporated; if not available, the approach produces a model with greater emphasis on accurate prediction and less on physical interpretation of the parameters.

This method should not be referred to as a *black-box* approach, since we are assuming that the basic equations of flow through porous media still apply. It is the parameters that are interpreted differently.

Assuming that accurate boundary conditions are not available, we will alter Equation 1 by (i) introducing surrogate-parameters $\bar{S}_j$, $\bar{K}_j$, (ii) changing to radial coordinates, and (iii) removing the spacial boundary conditions. Therefore, the radial response of well j ($h_j(r,t)$) due to pumping rate $Q_j(t)$ is governed by:

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,\overline{K}_j h_j \frac{\partial h_j}{\partial r}\right] = \overline{S}_j \frac{\partial h_j}{\partial t} + Q_j(t) \tag{3}$$

subject to

$$h_j(r,0) = \lim_{r \to \infty} h_j(r,t) = h_0 \tag{3a}$$

Define

$$d_{ijk}(\overline{S}_j,\overline{K}_j) = h_0 - h_j(r_{ij},t_k) \tag{4}$$

as the drawdown at well i, at discrete time t_k, produced by pumping at well j; r_{ij} is the radial distance from well i to well j (r_{ii} is the radial distance from well i to its observation well).

The total drawdown at well i, at discrete time t_k, due to its own pumpage, plus pumpage at adjacent wells, is given by...

$$\sum_{j \in M_i} d_{ijk}(\overline{S}_j,\overline{K}_j) \tag{5}$$

$(i = 1,\ldots,N;\ k = 1,\ldots,M)$

where

M_i = the finite set (of dimension m_i) representing well i, plus all adjacent wells causing measurable interference with well i [Note: all wells in the basin are given distinct real numbers].

Let

$$\hat{d}_{ik} = h_0 - \hat{h}_i(t_k), \qquad \text{for all } i,k \tag{6}$$

Notice that (3) can be accurately solved by finite-difference methods since radial grids emanating outward can be arbitrarily reduced for desired accuracy (assuming that there are no stability difficulties).

3.3 Advantages

We can now summarize the advantages of the surrogate-parameter approach in conjunction with the basic difficulties encountered in modeling groundwater basins and the weaknesses of current methods:

1. Groundwater basins which are highly nonhomogeneous and nonisotropic can be effectively modeled. The highly variable parameters are represented by lumped surrogate-parameters which are discretely distributed over the basin.

2. Complex boundary conditions need not be guessed. This represents a considerable reduction in arbitrariness.

3. Natural recharge to the groundwater basin, which is difficult to accurately measure, can be incorporated into the surrogate-parameters, so that further reduction in nonuniqueness results.

4. There is no need to arbitrarily divide the basin into homogeneous sections, as in simulation-type methods.

5. The parameter identification problem can be formulated as a mathematical programming problem.

Solution of the parameter identification problem constitutes the major difficulty associated with the surrogate-parameter approach. A great deal of emphasis has been placed on the uniqueness of this approach, so that is is extremely important that convergence to global solutions to the mathematical programming problem be assured. In general, however, this problem is large-scale (a large groundwater basin can have over 200 interfering wells) and nonconvex. Standard gradient-type methods will converge to various local solutions, depending on the initial guess. This source of arbitrariness is dealt with by decomposing the large-scale problem into smaller-dimensional subproblems for which global solutions can be attained with a greater degree of assurance than for the original large-scale problem. The choice of error criterion is an important factor in properly carrying this out. Two error criteria have been developed (called Problems A and B), and have been proven to be equivalent, under certain mild conditions [4]. It has been shown that judicious combination of these criteria results in subproblems involving no more than three decision variables.

<u>Problem A</u> is

$$\min_{\bar{S}\in S,\,\bar{K}\in K} f(\bar{S},\bar{K}) \tag{7}$$

where

$$f(\bar{S},\bar{K}) = \sum_{i=1}^{N} \sum_{k=1}^{M} \gamma_{ik} \left\{ \sum_{j\in M_i} d_{ijk}(\bar{S}_j,\bar{K}_j) - \hat{d}_{ik} \right\}^2 \tag{8}$$

$$\overline{S} = (\overline{S}_1,\ldots,\overline{S}_N),\ \overline{K} = (\overline{K}_1,\ldots,\overline{K}_N)$$

$$S = S_1 \times\cdots\times S_N,\ K = K_1 \times\cdots\times K_N$$

$$\gamma\varepsilon(E^{MN})^+ = \text{vector of weighting factors}$$

and <u>Problem B</u> is

$$\min_{\overline{S}\varepsilon S,\overline{K}\varepsilon K,w\varepsilon E^p} \overline{f}(\overline{S},\overline{K},w) \tag{9}$$

subject to

$$\sum_{j\varepsilon M_i} w_{ijk} = 1 \qquad \begin{cases} i = 1,\ldots,N, \\ k = 1,\ldots,M \end{cases} \tag{10}$$

where

$$\overline{f}(\overline{S},\overline{K},w) = \sum_{i=1}^{N}\sum_{k=1}^{M} \rho_{ik} \sum_{j\varepsilon M_i} \{d_{ijk}(\overline{S}_j,\overline{K}_j) - w_{ijk}\hat{d}_{ik}\}^2 \tag{11}$$

$$w = (w_{ijk},\ \text{for all } j\varepsilon M_i,\ i = 1,\ldots,N;\ k = 1,\ldots,M)$$

$$p = M\sum_{i=1}^{N} m_i, \qquad \rho\varepsilon(E^{MN})^+$$

COMPUTATIONAL EXPERIENCE

1. Example Problem

Consider a system of four wells tapping a confined aquifer, as shown in Figure 3. The circle surrounding well i depicts the maximum measureable extent of the uninfluenced cone of depression for well i. These radii can be based on conservative, homogeneous estimates of aquifer parameters and maximum expected pumping rates.

For this example

$$M_1 = \{1\} \qquad m_1 = 1$$
$$M_2 = \{1,2\} \qquad m_2 = 2$$
$$M_3 = \{2,3\} \qquad m_3 = 2$$
$$M_4 = \{2,4\} \qquad m_4 = 2$$

$$R_1 = \{1,2\} \qquad r_1 = 2$$
$$R_2 = \{2,3,4\} \qquad r_2 = 3$$
$$R_3 = \{3\} \qquad r_3 = 1$$
$$R_4 = \{4\} \qquad r_4 = 1$$

For confined aquifers, as discussed Labadie [4], the coefficient $K(x,y,h)$ can be replaced by $T(x,y)$ in Equation 1. Therefore, we can replace the term $\overline{K}_j h_j$ in Equation 3 with a surrogate transmissibility coefficient $\overline{T}_j$. Assuming constant discharge Q_1 over time, the new governing equation is

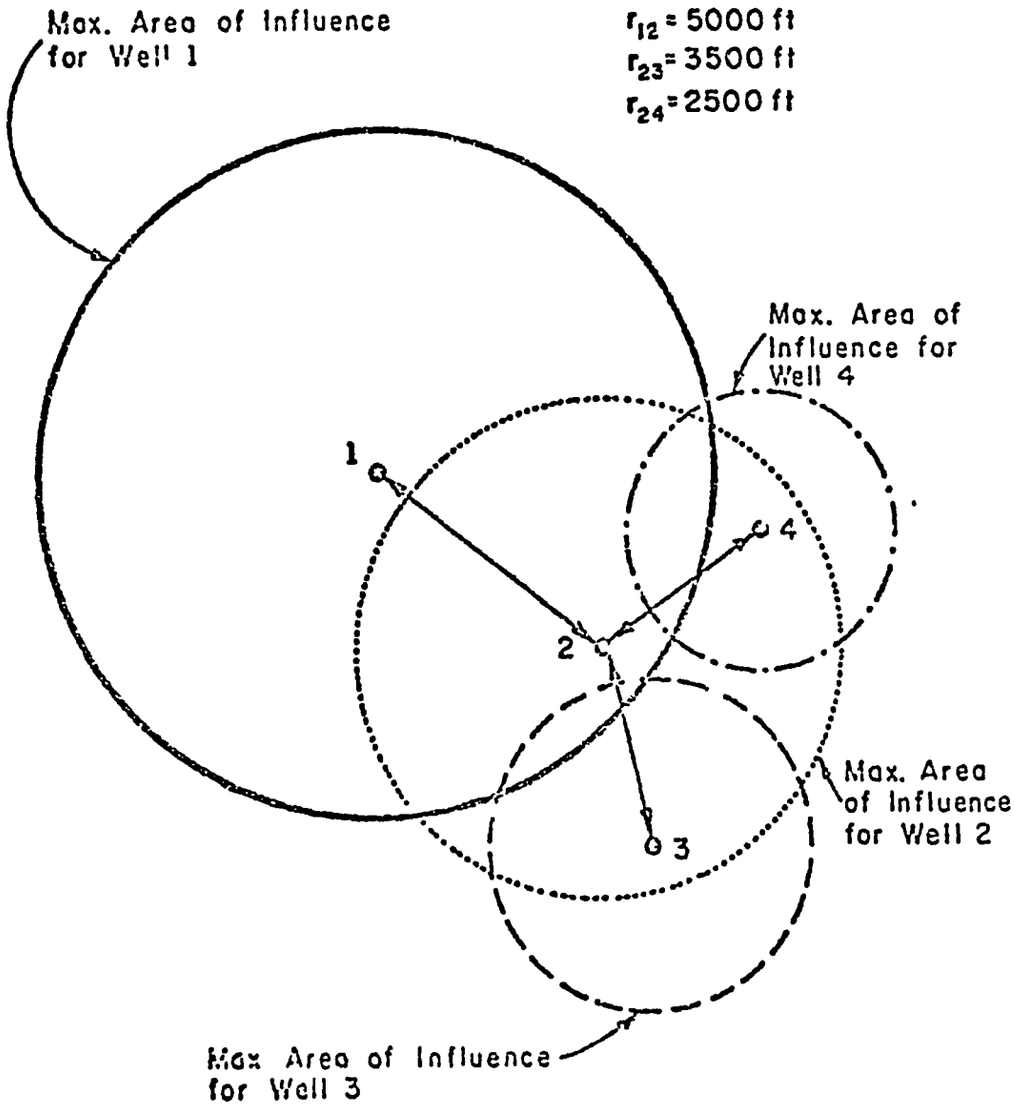


FIGURE 3

EXAMPLE WELL CONFIGURATION

$$\frac{1}{r}\frac{\partial}{\partial r}\left[r\,\overline{T}_j\,\frac{\partial h_j}{\partial r}\right] = \overline{S}_j\,\frac{\partial h_j}{\partial t}\quad , \tag{12}$$

subject to

$$h_j(r,0) = \lim_{r\to\infty} h(r,t) = h_0$$

$$\lim_{r\to 0} r\left(\frac{\partial h_j}{\partial r}\right) = \frac{Q_j}{2\pi\overline{T}_j}$$

Equation 12 has a closed-form solution (due to Theis [5])

$$d_{ijk}(\overline{S}_j,\overline{T}_j) = \frac{Q_j}{4\pi\overline{T}_j}\int_{\frac{\overline{S}_j r_{ij}^2}{4\overline{T}_j t_k}}^{\infty}\frac{e^{-u}}{u}\,du \tag{13}$$

[Note: Variable discharge $Q_j(t_k)$ can be considered through superposition of Equation 13 for various constant discharges (or recharge), where pumping (or recharge) is initiated at appropriate times.]

We will assume for this problem that $\overline{T}_j^*$, $j = 1,\ldots,4$, are accurately known. Problem A is therefore (letting $\gamma_{ik} = 1$, for all i,k)

$$\min_{\overline{S}_1,\ldots,\overline{S}_4}\ \sum_{i=1}^{4}\sum_{k=1}^{7}\left[\sum_{j\in M_i} d_{ijk}(\overline{S}_j,\overline{T}_j) - \hat{d}_{ik}\right]^2$$

and Problem A' is

$$\min_{\overline{S}_{ij},\ \text{for all } j\in M_i}\ \sum_{i=1}^{4}\sum_{k=1}^{7}\left[\sum_{j\in M_i} d_{ijk}(\overline{S}_{ij},\overline{T}_{ij}) - \hat{d}_{ik}\right]^2$$

$i=1,\ldots,4$

subject to

$$\bar{S}_{21} - \bar{S}_{11} = 0$$
$$\bar{S}_{32} - \bar{S}_{22} = 0$$
$$\bar{S}_{42} - \bar{S}_{22} = 0$$

The Lagrangian is

$$L = \sum_{i=1}^{4} L_i$$

where

$$L_1(\bar{S}_{11},\lambda_{21}) = \sum_{k=1}^{7} \left[d_{11k}\ (\bar{S}_{11},\bar{T}_{11}) - \hat{d}_{1k} \right]^2 - \lambda_{21}\bar{S}_{11}$$

$$L_2(\bar{S}_{21},\bar{S}_{22},\lambda_{21},\lambda_{32},\lambda_{42})$$
$$= \sum_{k=1}^{7} \left[d_{21k}(\bar{S}_{21},\bar{T}_{21}) + \hat{d}_{22k}(\bar{S}_{22},\bar{T}_{22}) - \hat{d}_{2k} \right]^2$$
$$+ \lambda_{21}\bar{S}_{21} - (\lambda_{32} + \lambda_{42})\bar{S}_{22}$$

$$L_3(\bar{S}_{32},\bar{S}_{33},\lambda_{32})$$
$$= \sum_{k=1}^{7} \left[d_{32k}(\bar{S}_{32},\bar{T}_{32}) + d_{33k}(\bar{S}_{33},\bar{T}_{33}) - \hat{d}_{3k} \right]^2$$
$$+ \lambda_{32}\bar{S}_{32}$$

$$L_4(\bar{S}_{42},\bar{S}_{44},\lambda_{42})$$
$$= \sum_{k=1}^{7} \left[d_{42k}(\bar{S}_{42},\bar{T}_{42}) + d_{44k}(\bar{S}_{44},\bar{T}_{44}) - \hat{d}_{4k} \right]^2$$
$$+ \lambda_{42}\bar{S}_{42}$$

and the dual function

$$h(\lambda) = \sum_{i=1}^{4} \min_{\bar{S}_{ij},\text{for all } j\in M_i} L_i$$

where

$$\lambda = (\lambda_{21}, \lambda_{32}, \lambda_{42}) \in E^3$$

The original 4-dimensional nonconvex problem has been decomposed into three 2-dimensional problems, and one 1-dimensional problem, plus a 3-dimensional, unconstrained concave dual problem.

2. Numerical Results

A simple gradient approach was utilized for solving the dual problem.

$$\lambda_{ij}^{(n+1)} = \lambda_{ij}^{(n)} + \theta \, [\bar{S}^*_{ij}(\lambda^{(n)} - \bar{S}^*_{jj}(\lambda^{(n)})]$$

$$(\text{for all } j \in M_i,\ i=1,\ldots,N,\ i \neq j)$$

where $\theta > 0$, and the subproblem minimizations were carried out by a 2nd-order gradient search procedure.†

Hypothetical field data for the system of wells of Figure 3 are given in Tables 1 and 2. Initial guesses for parameters $\bar{S}_i^{(0)}$ are given, as well as the known values for parameters $\bar{T}_i^*$.

Computational results are illustrated in Figures 4 and 5. Figure 4 depicts the maximization of the dual problem. Convergence is not monotonic until after the 3rd iteration, due to rather large step sizes initially.

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† Subroutine MO CAL NMIN, Computer Center Library, University of California, Berkeley.

TABLE 1

GROUNDWATER BASIN DATA

	WELL			
	i = 1	i = 2	i = 3	i = 4
h_0(ft.)	1000	1000	1000	1000
r_{ii}(ft.)	1	1	1	1
Q_i(gal/min)	1000	600	450	350
$S_{ii}^{(0)}$	.00042	.00048	.0072	.0072
$T^*_{ii}\left[\frac{gal/day}{ft}\right]$	9250	6500	4850	4025

TABLE 2

DRAWDOWN OBSERVATIONS $\hat{d}_{ik}$ (ft.)

time(t_k)		WELL			
k	days	i = 1	i = 2	i = 3	i = 4
1	30	236	194	145	150
2	45	241	203	152	157
3	60	245	210	157	162
4	90	250	219	164	170
5	180	258	234	177	183
6	360	267	250	191	197
7	720	275	266	206	211

Figure 5 indicates that a saddle point was found since the total absolute error

$$e = |\bar{S}^*_{21} - \bar{S}^*_{11}| + |\bar{S}^*_{32} - \bar{S}^*_{22}| + |\bar{S}^*_{42} - \bar{S}^*_{22}|$$

converged to zero.

The initial set of multipliers were set equal to zero ($\lambda^{(0)}_{ij} = 0$, for all i,j). The final optimal values were

$$\lambda^*_{21} = -7.368 \times 10^4$$
$$\lambda^*_{32} = -1.0028 \times 10^3$$
$$\lambda^*_{42} = -1.0028 \times 10^3$$

Table 3 lists the optimal parameter solutions $\bar{S}^*_{1i}$, $i = 1,\ldots,4$; and final drawdown values. The final squared-error between computed and observed drawdown was

$$E^* = 8.5106 \text{ ft}^2$$

3. Discussion

The following observations were noted during computation:

1. After the first iteration of the dual problem, subproblem solutions converge more rapidly. Solutions from previous iterations are utilized as accurate initial approximations for subproblem solutions in succeeding iterations of the dual problem.
2. Since a gradient method was utilized for solving the nonconvex subproblems, there was never absolute assurance the global solutions were attained at intermediate iterations

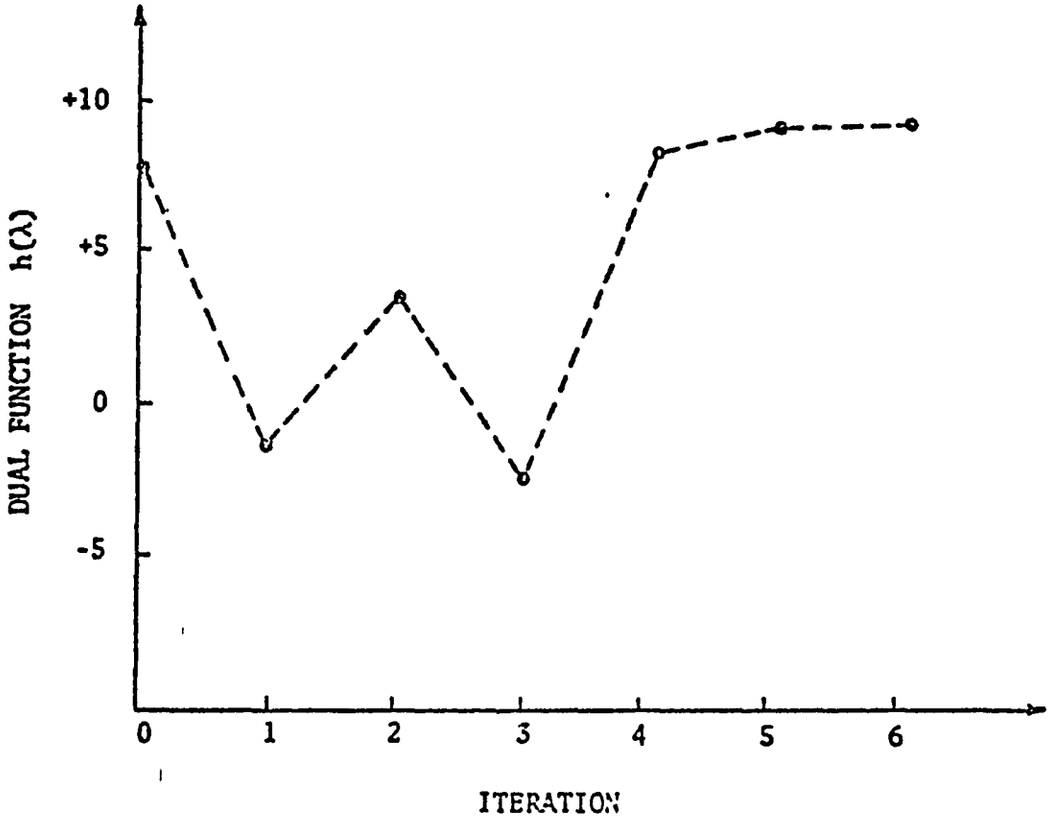

FIGURE 4

MAXIMIZATION OF THE DUAL FUNCTION

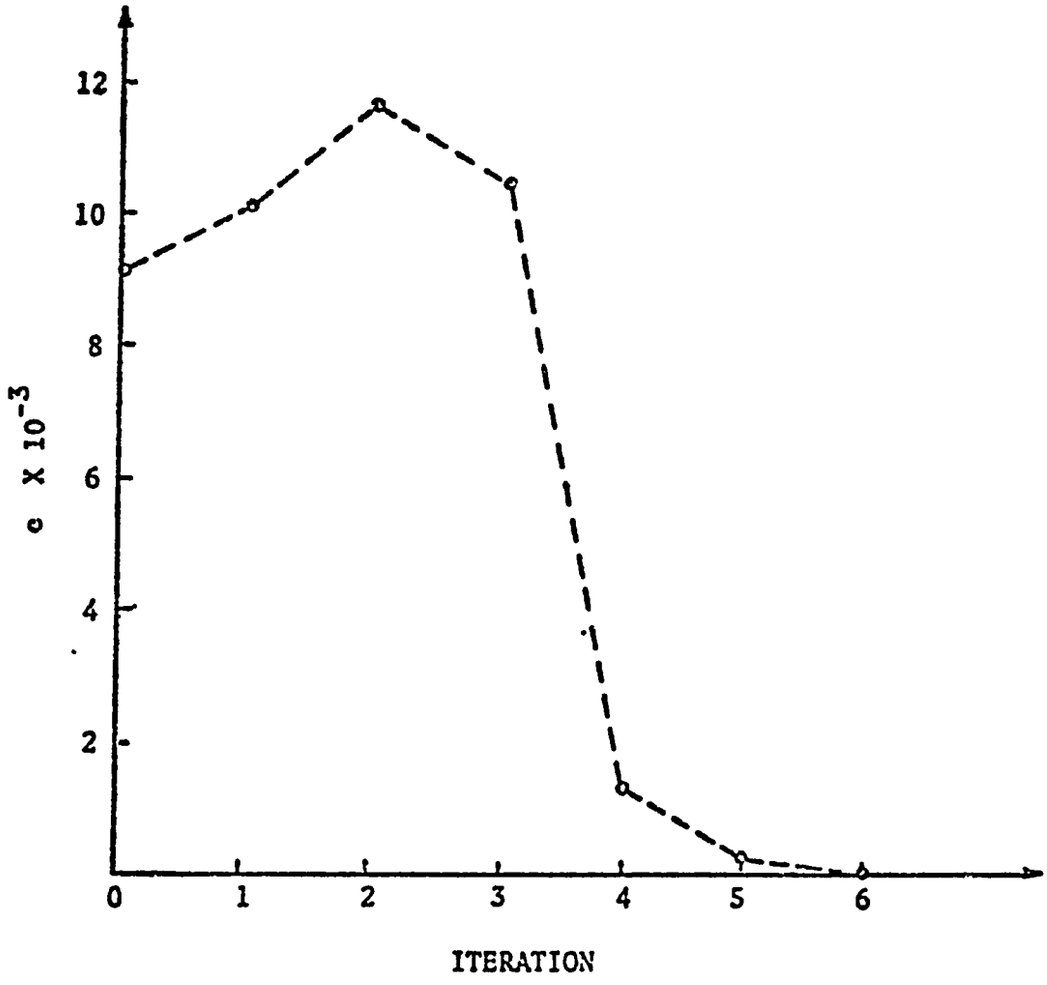

FIGURE 5

MINIMIZATION OF ERROR

TABLE 3

OPTIMAL SOLUTIONS

Time t (days)	WELL			
	i = 1	i = 2	i = 3	i = 4
30	235.9	194.1	145.9	150.5
45	240.9	203.1	152.3	157.6
60	244.4	209.5	157.3	162.8
90	249.5	218.7	164.6	170.4
180	258.1	234.5	178.0	184.0
360	266.6	250.3	192.0	197.8
720	275.2	266.2	206.3	211.9
S^*_{ii}	.000499	.00718	.064	.020

of the dual problem. The final subproblem solutions, at the last convergent iteration of the dual problem, are global solutions since these values were known *a priori* and used to generate the hypothetical data. Even without this assurance of intermediate global solutions, it can be seen from Figures 4 and 5 that convergence was quite rapid.

3. It seems unnecessary that subproblem solutions converge to highly accurate values. Total computation time can be significantly lessened by allowing subproblem solutions to fall within some ϵ-neighborhood of the true optimum. Then, as the dual problem begins to converge, let $\epsilon \to 0$. This idea raises some interesting questions of stability and trade-off between decreased subproblem computation time and increased number of iterations needed to solve the dual problem.

4. It is interesting to note that multipliers λ^*_{32} and λ^*_{42} turned out to be equal. This is an extremely significant result in that it shows that the constraints

$$\bar{S}_{32} - \bar{S}_{22} = 0$$

and

$$\bar{S}_{42} - \bar{S}_{22} = 0$$

could have been added together to form

$$\bar{S}_{32} + \bar{S}_{42} - 2\bar{S}_{22} = 0$$

and a new Lagrangian written, with the resulting dual problem involving one less dual variable. This reduced problem would then have solved the original problem. In general, this

suggests the possibility of adding the constraints

$$\overline{S}_{ij} - \overline{S}_{jj} = 0, \quad \text{for all} \quad i \varepsilon R_j, \quad i \neq j$$

and

$$\overline{T}_{ij} - \overline{T}_{jj} = 0, \quad \text{for all} \quad i \varepsilon R_j, \quad i \neq j$$

thus reducing the number of dual variables by $2 \sum_{i=1}^{N} (r_i - 1)$, and seeing if this reduced formulation will solve the original problem.

SUMMARY AND CONCLUSIONS

A new surrogate-parameter approach to modeling geohydrologic systems or groundwater basins has been presented which has several advantages over current simulation-type approaches, including:

1. conduciveness to modeling nonhomogeneous and nonisotropic basins.
2. less arbitrariness, since complex boundary conditions and natural recharge to the basin need not be guessed.
3. not requiring arbitrary division of a basin into homogeneous sections.
4. formalization of the parameter identification problem as a mathematical programming problem.

An unconstrained and a constrained error criterion were utilized in formulating two mathematical programming problems (Problems A and B, respectively). The equivalence of these problems was established, under certain mild conditions, through use of generalized duality theory. Both were subsequently decomposed via duality theory and the so-called *multi-level* approach. It was shown that exclusive use of Problem A could result in subproblems of unwieldy dimension. On the other hand, utilization of only Problem B resulted in a dual problem involving a massive quantity of dual variables. It was decided that the identification problem should be formulated in terms of Problem A, with Problem B replacing those subproblems of high dimension that result from decomposition of A. Decisions as to when Problem B should be applied must be based on the particular characteristics of the identification problem being considered.

Computational experience with Problem B is incomplete, but a hypothetical problem involving four interacting wells was solved by decomposing Problem A. Transmissibility coefficients were assumed to be known, and storage coefficients were determined through solution of the resulting dual problem. A second-order gradient method was utilized, so that globality of subproblem solutions during intermediate iterations of the dual problem could not be guaranteed. In spite of this, convergence to a saddle-point was rapid.

In conclusion, the surrogate-parameter approach appears to be a valid alternative to the current heavy emphasis on simulation approaches to groundwater basin modeling. Considerable future investigation, however, of both a theoretical and computational nature, is required. Some suggested goals are:

1. Attempt to formally (or informally, through numerical results) prove the existence of a saddle-point for some special cases (e.g., when the Theis solution is applicable) and investigate the possible uniqueness (or g-uniqueness) of subproblem solutions.
2. Obtain a great deal of computational experience with Problem B, in order to establish guidelines as to when it should be utilized in place of Problem A.
3. Incorporate additional complexities into the surrogate-parameter model, including artificial recharge, leaky confined aquifers, highly stratified aquifers, rising water, etc.
4. Develop a generalized computer program designed to deal with a wide variety of groundwater basin parameter identification problems, with particular emphasis on efficiency and accuracy.

5. Application of the surrogate-parameter concept to development of management schemes which seek to meet the demand for groundwater, while minimizing pumping costs and protecting the basin from irreversible damage.
6. Possible conjunctive use of surrogate-parameter and simulation approaches. For example, it appears that artificial recharge operations can be more easily incorporated into simulation models than they can in the surrogate-parameter approach. Recharge through injection wells can be easily incorporated into the latter. But it is difficult to apply an input-output structure to large spreading operations, where we are trying to explicitly define the response of the basin water level to these operations.
7. Application of the surrogate-parameter identification and management models to a wide variety of groundwater basins.

APPENDIX

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