EdData II

Early Grade Mathematics Assessment (EGMA): A Conceptual Framework Based on Mathematics Skills Development in Children

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Early Grade Mathematics Assessment (EGMA): A Conceptual Framework Based on Mathematics Skills Development in Children

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Abbreviations

AIR	American Institutes for Research
CBM	curriculum-based measurement
EdData	Education Data for Decision Making [project]
EGMA	Early Grade Math Assessment
EGRA	Early Grade Reading Assessment
EMEP	EGMA Mathematics Expert Panel
FRSS	Fast response survey system
FY	fiscal year
ICT	information and communication technology
IEA	International Association for the Evaluation of Educational Achievement
KIE	Kenya Institute of Education
M	mean
NAEP	National Assessment of Educational Progress
NAGB	National Assessment Governing Board
NCDDP	National Clinical Dataset Development Programme
NCES	National Center for Education Statistics
NCTM	National Council of Teachers of Mathematics
NMAP	National Mathematics Advisory Panel
NP	number-to-position [task]
PIRLS	Progress in International Reading Literacy Study
RTI	RTI International [trade name of Research Triangle Institute]
SD	standard deviation
SES	socioeconomic status
T.E.A.C.H.	Teacher Education and Compensation Helps
TEMA	Test of Early Mathematics Ability
TIMSS	Trends in International Mathematics and Science Study
USAID	United States Agency for International Development

Acknowledgments

In January 2009, a panel of university-based experts commented on oral presentations of many of the issues discussed here. This panel consisted of David Chard, Southern Methodist University; Jeff Davis, American Institutes for Research (AIR); Susan Empson, The University of Texas at Austin; Rochel Gelman, Rutgers University; Linda Platas, University of California, Berkeley; and Robert Siegler, Carnegie Mellon University.

We would like to thank the panel for their participation and sharing of information. The January 2009 panel has not reviewed this document, and in any case, all errors or omissions are the responsibility of the authors.

1. Background

All over the world, mathematics skills are essential for adults—employed or not employed—to function successfully in their work, profession, and everyday life. This importance of mathematics skills continues to increase as societies and economies move toward more technologically advanced activities. New learning goals in mathematics are being advocated at the same time that new recommendations for research in this field are emerging (Fuson, 2004; U.S. Department of Education/National Center for Education Statistics [NCES], 2008). As we increase our knowledge through research and the evaluation of programs, we learn what works and what does not. We also establish what children need as a foundation to become successful in learning mathematics in later years.

This background note supplies discussion on the contents of the Early Grade Mathematics Assessment (EGMA), funded by the United States Agency for International Development (USAID). The focus of this tool, and hence of this background paper, is on the early years of mathematics learning; that is, mathematics learning with an emphasis on numbers and operations and on geometry through second grade or, in developing countries, perhaps through third grade. Mathematics here is taken to be broader than, and to include, arithmetic. Although it may seem odd to those unaccustomed to working with these issues, instilling algebraic notions early helps children develop concepts in identification, organization, cohesion, and then representation of information (Clements, 2004b). These are the years in which a young child builds a foundation or base that will be necessary for learning in the years that follow. Without this base, it is possible—but difficult—later, to catch children up to where they need to be (Fuson, 2004).

This note is organized as follows. Section 2 provides a conceptual background derived from the literature on how children develop their earliest conceptual and operational skills related to numbers. This background tends to justify both the use of a tool such as EGMA and its constituent parts. Section 3 provides some background on the universality that seems to exist with regard to the bits of curricular knowledge that are expected in many countries around the world. This knowledge undergirds the choice of tasks for EGMA. Section 4 describes the key tasks within EGMA, and provides some conceptual background on individual tasks. Appendix 1 summarizes the comments from and discussion with a panel of university-based experts in January 2009. Appendix 2 provides technical discussion of the extra measures selected for introduction into EGMA based on the panel of experts' comments.

2. Introduction to Children's Sense of Numbers

Research shows that children develop mathematical skills at different levels before beginning formal schooling. In the United States and in developing countries, it is evident that many students from low-income backgrounds begin school with a more limited skill set than those from middle-income backgrounds (National Council of Teachers of Mathematics [NCTM], 2004b, 2008). Given that children bring different skill levels to school (e.g., from home environment, preschool), the NCTM (2004a) recognizes that some children will need additional support in the early grades to ensure success. The NCTM (n.d.) emphasizes the use of these types of assessments (e.g., curriculum-based assessments; see section 3.3) to provide information for teaching and for potential early interventions. In developed countries, early interventions often take the form of extra support to individual children, or perhaps to small groups of children. In developing countries, early interventions must be geared toward entire systems, as these systems are frequently at the same levels as children who in the developed world are seen as needing special help. In some cases entire developing country systems score at the third or fourth percentile of the distribution of scores in developed-country systems.

Children across cultures seem to bring similar types of skills to school, but do so at different levels (Guberman, 1999). Examples of skills that seem to develop across cultures include counting skills; the use and understanding of number words as numerical signifiers of objects; and the ability to compare small sets of objects (Gelman & Gallistel, 1986; Saxe, Guberman, & Gearhart, 1987). Even before formal instruction, children demonstrate some understanding of addition and subtraction (Guberman, 1999). This suggests that assessing the same kinds of skills in children in developed and developing countries, albeit with adjustment for the level of these skills, makes sense. (In the discussion that follows, curricula of some developed and developing countries are shown to have essentially the same key contents in the early grades.)

One reason children from different social backgrounds may vary in the rate of acquisition of informal mathematics levels is the amount of stimulation available in their environments (Ginsburg & Russell, 1981). Furthermore, the rate of acquisition of mathematical skills can be influenced by the opportunities provided to children in their communities (Guberman, 1999). For example, in observing children in a poor community in Brazil’s northeast coast, Guberman (1996) noted that a majority of parents sent their children to the local stands to purchase goods (e.g., beverages, food) from once to a few times a day. In carrying out these errands, children were participating in an activity that contributed to informal mathematics development. This acquisition also holds true with children’s judgment of “more” or “a lot”: Such judgment develops based on activities children encounter in their environment (Case, 1996; Ginsburg & Russell, 1981). Children make sense of problems and will construct solutions based on such perspectives (Guberman, 1996). Once children begin formal mathematics, they use this previous (informal) knowledge in actively making an effort to complete new tasks (Baroody & Wilkins, 1999; Ginsburg & Russell, 1981).

Children progress in more or less common ways in their construction of number knowledge between ages 3 and 9. In the United States, children begin in kindergarten to integrate some level of mathematics knowledge about quantity and counting. These opportunities allow children to use their existing knowledge in the construction of new knowledge (Griffin, 2004). To allow for these learning opportunities, Griffin and Case (1997) and Griffin (2004) noted the importance of assessing children's current knowledge. Based on the level of knowledge of children's development, systems and teachers can present opportunities that lie within their "zone of proximal development" toward construction of new targeted knowledge (Griffin & Case, 1997).

With formal schooling, children begin to develop new understandings of numbers, the association of numbers with sets of objects, the meaning of symbols such as "=," or the knowledge that 8 is "more" than 5. They begin to develop the use of a mental number line and the association of symbols such as 8 and 5 as places on the number line (Baroody & Wilkins, 1999; Carpenter, Franke, & Levi, 2003; Case, 1996). These are essential precursor skills to further and deeper mathematical knowledge and skills. Children also begin to develop a better understanding of conservation of numbers with the establishment of one-to-one correspondence between two sets of items and their representing numbers, in what Gelman and Gallistel (1986) refer to as the "How-To-Count" principles of counting. These learned principles consist of

- each object or item within a group of objects or array of items being associated with only one number name; and
- the understanding that the final number of objects or items in a grouping is representative of the overall group.

From here, with continued practice, familiarity and confidence with numbers and their values grow. Children progress in their development of counting strategies. This can include advancing to new strategies such as counting from the larger addend (min strategy) when they are shown two numbers representing two groups of objects that are being added together (Siegler & Shrager, 1984). An example of an earlier "sum strategy," or the "counting-all method" (Fuson, 2004), is when a child is asked to solve "5 + 4," and the child counts and shows five fingers on one hand representing the "5," and counts and shows four fingers on the other hand representing the "4," and then counts all: "1, 2, 3, 4, 5, 6, 7, 8, 9." In time, the child may progress to just put his/her fingers up, already knowing that one hand represents "5," and then to count "6, 7, 8, 9" to add the "4" to the "5." That is, as the child progresses with his/her counting skills and is asked to solve a problem such as 5 + 4, he/she may count using the min strategy by counting from the larger addend (5) to get the answer.

With practice, over time, children begin to store information in memory. At first, children may retrieve the answer to a mathematics problem but may not yet have confidence in their answer. They might retrieve the answer and then check it by using a counting strategy (Siegler & Schrager, 1984). With practice, children gain confidence and process information faster in solving mathematics problems. Children may also build confidence in the use of fact retrieval for simpler mathematics problems, such as retrieving knowledge for numbers of equal value such 2 + 2 = 4 (Ashcraft, 1982; Hamann & Ashcraft, 1986; Siegler & Shrager, 1984). But note that there is a level of “automatization” of the knowledge that “2 + 2 = 4” that is preceded by a conceptual stage that requires counting. At the same time, becoming efficient at mathematics does require the automatization of the subsequent stage, rather than a constant recursion to the earlier stages. For more difficult mathematics problems, this extended practice provides the skills and proficiency needed for rapid and accurate processing, freeing up cognitive resources so that children are able to pay attention to more elements of the task at once (Pellegrino & Goldman, 1987). For that reason, children who demonstrate difficulty with single-digit items such as “5 + 6” will find more advanced mathematics more challenging (Gersten, Jordan, & Flojo, 2005). In other words, recursion to more primitive strategies, though it does show understanding of the concept, might impede further conceptual understanding and progression if operational automaticity is not achieved.

As children continue learning and solving addition of single digits, they also learn new strategies such as decomposition of numbers around 5 and 10. An example can be seen in the calculating of 9 + 6, which is equal to 9 + 1 + 5, which is equal to 10 + 5 = 15 (Clements, 2004b; Fuson, 2004). Even with a strategy such as decomposition available for children to use, they use a range of methods in solving problems (Siegler & Jenkins, 1989). It could be that children have available to them more than one algorithm (defined here as a “general multistep procedure”) (Fuson, 2004, p. 120) in solving a problem. These different algorithms can be learned from teachers over several sequential grades, in different schools and classrooms, and from parents. But although many algorithms may be available from teachers and the culture, research has shown that certain algorithms work best both in computation and in the laying of a more solid foundation for more advanced concepts. Students have been very successful with strategies such as using a 10-frame. This has been shown to be a rapid, effortless way to automatically recall the answers to problems requiring addition or subtraction of single-digit numbers (Fuson, 2003, 2004). The unfortunate side is that not all students get introduced to the most efficient methods due to varying teachers, textbooks, and curricular statements of objectives.

In the United States, for example, as opposed to countries with better mathematics results, single-digit subtraction consistently has been taught using the method of counting down. Subtraction in general has been shown to be a much harder task than addition for children to learn. But one method taught in the classroom that has been shown to be an efficient and easier method for subtraction than the counting down method is counting up. An example is a problem such as “9 – 3 = ?” To obtain the answer, children start with 3 and count up to 9 (“4, 5, 6, 7, 8, 9”), while noting the number of digits that have to be counted to obtain the answer: “9 is 6 more than 3” so “9 – 3 = 6” (Fuson, 2004).

As children continue to develop their understanding and become more proficient with skills such as single-digit addition and subtraction, they move to double-digit addition and subtraction problems and also learn place value. They also begin to use more advanced strategies with, for instance, the use of tens and ones. An example is the calculation of 48 + 31, which requires breaking each number down into its specific tens and ones: 40 + 30 = 70 and 8 + 1 = 9. Therefore, the answer is 79 (Clements, 2004b). With continued exposure to and practice of these skills, and the integration of these skills into simple word problems, children are able to work with greater computational and problem solving competence (Fuson, 2003). The understanding of computation and integration of methods, and practice with both, leads to “computational fluency” (Fuson 2003, p. 71). Yet, to get to this point, children must know how to count; they must understand how to simultaneously count and keep track of objects; and then they must continue with this progression, and develop automaticity as the foundation of success with future number operations such as addition, subtraction, multiplication, and division through the following years.

In a joint position statement in 2002, the National Association for the Education of Young Children and the National Council of Teachers of Mathematics emphasized the importance of good early mathematics experiences for children. They noted that without good early instruction, progress to higher-order skills is more difficult (NCTM, 2009b). The NCTM had already signaled its advice on this matter by setting out mathematics standards to be met in the prekindergarten year. These standards are shown in Table 1 (NCTM, n.d.; NCTM, 2009a).

Table 1. Numbers and Operations—Curriculum Focal Points

School Year	Overall Goal	Objectives
Prekindergarten	Developing an understanding of whole numbers, including concepts of correspondence, counting, cardinality, and comparison	Understand whole numbers, recognizing number of objects in small groups; understand that number words refer to quantity; match sets; count objects to 10; understand “how many,” “more than,” and “less than.”
Kindergarten	Representing, comparing, and ordering whole numbers, and joining and separating sets	Use numbers to represent quantities and to solve quantitative problems such as counting objects in a set; create a set with a given number of objects; and compare and order sets or numbers by using both cardinal and ordinal meaning. Children choose, combine, and apply effective strategies for answering quantitative questions; count and produce sets of given sizes; and combine sets and count backward.
First Grade	Developing understandings of addition and subtraction and strategies for basic addition facts and subtraction facts, including whole-number relationships (e.g., tens and ones)	Develop strategies for adding and subtracting whole numbers. With the use of number lines and connection cubes, model “part-whole,” “adding to,” “taking away from,” and “comparing” in solving arithmetic problems. Children also compare and order whole numbers and think of whole numbers between 10 and 100 in terms of groups of tens and ones. They understand sequential order and can represent numbers on a number line.

School Year	Overall Goal	Objectives
Second Grade	Developing understanding of base-ten numeration system and place-value concepts, including fluency with multi-digit addition and subtraction	Develop understanding of base-ten numeration system and place value to at least 1,000. Compare and order numbers, understand multi-digit number place value and properties of addition (commutative and associative). Use efficient and accurate methods to estimate sums and differences or calculate them mentally—depending on context.

Source: National Council of Teachers of Mathematics (2004a). *Overview: Standards for prekindergarten through grade 2.* Retrieved on August 15, 2008, from http://standards.nctm.org/document/chapter4/index.htm; National Council of Teachers of Mathematics (2009a). *Curriculum focal points.* Retrieved on June 2, 2009, from http://www.nctm.org/standards/content.aspx?id=270

3. Commonality of Curricular and Conceptual Goals, Across Countries

3.1 The Influence of NAEP, NCTM, and TIMSS

Over the past decade or so, schools worldwide have increased their focus on mathematics and science (Baker & LeTendre, 2005). There has also been increased discussion of the economic benefits of these competencies (Geary & Hamson, n.d.). International tests such as the Trends in International Mathematics and Science Study (TIMSS)[1] have created awareness among policy makers about their countries' mathematics and science performance relative to that of other countries. The availability of this information tends to spur competition and analysis related to how countries can improve performance. In the United States, for example, a great deal of analysis addresses how curricula, teacher preparedness, and assessment can all be used to improve student performance (Baker & LeTendre, 2005; Mullis & Martin, 2007).

Concern about mathematics performance, over the decades, has led to the creation of national assessments such as the National Assessment of Educational Progress (NAEP). The NAEP is a congressionally mandated assessment developed to test students in grades 4, 8, and 12. The NAEP has been influenced by the TIMSS as well as by input from policy makers, practitioners, and other interested parties such as the National Council of Teachers of Mathematics, and the National Research Council (U.S. Department of Education/National Assessment Governing Board [NAGB], 2006).

The NAEP test structure for mathematics in 2007 provides a useful guide to assessment areas. This structure focuses on five elements: 1) number and operations; 2) measurement; 3) geometry; 4) data analysis and probability; and 5) algebra (U.S. Dept. of Education/NAGB, 2006; U.S. Dept. of Education/NCES, 2008). Forty percent of the test items at the grade 4 level are based on children's knowledge of number and operations;

[1] The TIMSS is administered by the International Association for the Evaluation of Educational Achievement (IEA).

20 percent of the distribution is based on measurement; and geometry and algebra each make up 15 percent of the items (U.S. Dept. of Education/NAGB, 2006).

A report prepared by the National Mathematics Advisory Panel (NMAP), comprising 20 expert panelists with backgrounds in education, psychology, technology, and mathematics, is one of the most important sources of current influence on the mathematics curriculum in the United States. Through review of more than 16,000 research publications, policy reports, and testimony from professionals such as algebra teachers and educational researchers, the NMAP emphasized the advantages of a strong start in mathematics (U.S. Department of Education/NCES, 2008); that is, the importance of a strong foundation in the earliest grades. Two key recommendations were that the curriculum for prekindergarten through eighth grade should be more streamlined, and that the goals should be to ensure that students 1) understand key concepts in mathematics but also 2) acquire accurate and automatic execution in solving problems. The design of EGMA reflects these recommendations.

The NMAP recommended reorganization of the components presented for NAEP. The panel believed that some mathematics skills were underrepresented, and it recommended that fractions and decimals be listed as objectives, because proficiency in these skills is an important foundation for later success in algebra. The panel noted that its recommendations are aligned with TIMSS (U.S. Department of Education/NCES, 2008), and thus with international trends (see Table 2).

Table 2. NAEP Recommendations and TIMSS Objectives

Fourth-Grade Objectives	
NAEP	**TIMSS**
Number: Whole Numbers	Number
Number: Fractions and Decimals	Algebra[a]
Geometry and Measurement	Measurement
Algebra	Geometry
Data Display	——

Source: U.S. Department of Education, National Center for Education Statistics (NCES). (2008). *Foundations for success: The final report of the National Mathematics Advisory Panel*. Retrieved August 16, 2008, from http://www.ed.gov/about/bdscomm/list/mathpanel/reports.html

Note: TIMSS mathematics content areas are based on 2003 key features of the TIMSS grade 4 mathematics assessment and the TIMSS items released for grade 4 (retrieved September 5, 2008, from http://nces.ed.gov/timss/educators.asp). For 2007, there has been more consolidation to the major content domains by grade (see Mullis, Martin, & Foy, 2008, Chapter 2: Performance at the TIMSS 2007 international benchmarks mathematics achievement, pp. 65-115. Retrieved September 5, 2008, from the TIMSS & PIRLS Web site, http://timss.bc.edu/TIMSS2007/PDF/T07_M_IR_Chapter2.pdf). For grade 4, the components are Number, Geometric Shapes and Measures, and Data Display. For grade 8, the components are Number, Algebra, Geometry, Data, and Chance.

[a]The TIMMS algebra content for the fourth grade is known as patterns and relationships.

As noted, TIMSS has had an effect on NAEP in the United States, and it has also had an effect on the NCTM in setting the standards or focal points by grade and components recommended for mathematics by grade (Fennell et al., 2008; Phillips, 2007). Table 1 in the previous section shows an example of some of the focal points by grade for the content area "Numbers and Operations."

3.2 A Look at Curricula

In parallel to our reviewing the components and objectives to be met in the United States (as an example of what is done in *one* developed country), we also reviewed the components and objectives that are being set in other countries. Table 3 shows data from only three of the countries we have reviewed: South Africa, Jamaica, and Kenya. Table 2 and Table 3 confirm a great deal of convergence in the curricular objectives of a few countries.

Table 3. Grade 1–3 Objectives Set by South Africa, Jamaica, and Kenya

South Africa	Jamaica	Kenya
Number, Operations, and Relationships	Number and Computation	Numbers and Whole Numbers
Patterns	Pattern and Algebra	Pre-number Activities
Shape and Space	Measurement	Geometry
Measurement	Data Handling	Measurement
Data Handling	Shape and Space	Numbers, Multiplication[a]

Sources:

Department of Education, Republic of South Africa. (2002). *The revised national curriculum statement grades R-9 (schools)*. Retrieved September, 2008, from http://www.education.gov.za/Curriculum/GET/doc/maths.pdf

Ministry of Education and Culture, Jamaica. (1999). *Revised primary curriculum guide, grades 1–3*. Kingston, Jamaica: Author.

Jomo Kenyatta Foundation. (2003a, 2003b, & 2004). *Primary mathematics,* pupils' book and teacher's book for first through third grades. Nairobi, Kenya: Author.

Kenya Institute of Education (KIE). (2002). *Republic of Kenya, Ministry of Education: Primary education syllabus.* Nairobi: Author.

[a]"Data Handling" is not specifically mentioned in the Kenya syllabus. Yet, there is a strong though implicit degree of exposure to these concepts as outlined in the syllabus. It is expected that children will observe and work with picture graphs as they learn about numbers, money, fractions, and multiplication.

The following are examples of some of the benchmarks in TIMSS 2007 (Mullis, Martin, & Foy, 2008) that follow the curriculum and objectives under review in this note. These examples show, once again, a considerable curricular convergence, at least in the basic grades, which can underpin the preparation of an instrument such as EGMA. According to these benchmarks, students should

- demonstrate a level of understanding of whole numbers (e.g., ordering, adding, subtracting).
- demonstrate an understanding of patterns, such as pattern extension with the use of numerical and/or geometric sequence. Here, the goal is for students to respond to what should be next in the sequence.
- recognize both two- and three-dimensional shapes.
- be able to solve multi-step word problems.

It is important to emphasize that students need to be proficient in *computational procedures* and to demonstrate this knowledge (Fennell et al., 2008; U.S. Department of Education/NCES, 2008). This is not just a matter of factual knowledge, but of procedural automaticity, of *skill* and competency. The NMAP also sees the understanding of key concepts and the achievement of automaticity where appropriate to be essential for the progression of mathematics learning in the following years. As indicated by the panel:

> Use should be made of what is clearly known from rigorous research about how children learn, especially by recognizing a) the advantages for children in having a strong start; b) the mutually reinforcing benefits of conceptual understanding, procedural fluency, and automatic (i.e., quick and effortless) recall of facts; and c) that effort, not just inherent talent, counts in mathematical achievement. (p. 13)

This is why some of the EGMA tasks are timed, as discussed below.

Procedural fluency and automaticity are also emphasized internationally. According to the more recent TIMSS Advanced 2008 Assessment Framework, children should by the fourth grade be able to show familiarity with mathematical concepts and able to 1) recall information such as number property and mathematical conventions; 2) recognize different representations of the same function or relation, for example; 3) demonstrate computing information such as solving simple equations; and 4) retrieve information from graphs and other sources (Garden et al., 2006).

3.3 Importance and Use of Research in EGMA Development

Despite this accumulation of findings, the majority of mathematics curricula available today do not incorporate the results of the best and most recent research on how children actually learn, and do not evaluate design and revisions based on student classroom performance (Clements, 2004b). In response, empirical research is available that suggests valid mathematics instruments that can help teachers and systems learn quite specifically where students need support (Gersten, Jordan, & Flojo, 2005).

As seen from the beginning of this paper, developmental theory in children's informal and intuitive mathematics knowledge plays a role as children enter formal schooling and begin to acquire more complex mathematics skills (Baroody, 2004; Griffin, Case, & Capodilupo, 1995).

In the development of EGMA, and of this note as background information, every effort was made to ensure that the measures selected for piloting—which were discussed with a panel of leading experts on mathematics education (see Appendix 1)—drew from the extensive research literature on early mathematics learning and evaluation. We specifically considered research that was provided by some of these individual panel members.

In developing the measures, we closely considered two approaches. The first was to review the curriculum and objectives across a number of states and countries for kindergarten, first-grade, and second-grade goals and objectives to be met. Second, based on Fuch's (2004) approaches for the development of measurement tasks, we used a robust indicators approach. This approach focused on identifying measures that would be representative of each of these grade levels to ensure what we considered a *progression* of skills that lead toward proficiency in mathematics. In addition, we reviewed the objectives that have been set by the NCTM, the findings reported by the National Mathematics Advisory Panel, and finally the influence of the TIMSS on each of them.

In further defining the indicator or measures approach, we also believed it was important to use measures that systematically sample and test skills required during the *early* years, as an indicator of need for intervention (Fuchs, 2004). These kinds of measures are often referred to as curriculum-based measurement (CBM). Clarke, Baker, Smolkowski, and Chard (2008) have characterized CBM as a form of measurement that is quick to administer, can have alternate forms for multiple administrations, and is reliable and valid. A CBM system can monitor and facilitate timely early intervention at the individual or group level. As students begin school with informal and intuitive mathematical knowledge, teachers, schools, and systems can use these sorts of measurements to learn what materials and instruction are needed as students' formal knowledge of mathematics is constructed (Carpenter, Fennema, & Franke, 1996).

Given the age of the children to be assessed with EGMA, we also believed it was important to present each task (e.g., counting objects, quantity discrimination, addition) to each child, and then score the tasks so as to measure—in detail—differences in performance with respect to level of math knowledge in these early years (Hintze, Christ, & Keller, 2002).

In addition, the measurement tools would provide diagnostic feedback that could be made available to teachers and schools (Foegen, Jiban, & Deno, 2007). One of the objectives of EGMA is to choose, and present, the measures in such a way that teachers see how they relate to the curriculum. Waiting until the end of third or fourth grade to see national results only delays the time when these unresolved issues can be identified at the child level and at the country level, making it more difficult to catch students up to the level of mathematics ability they should have reached for their current grade (Fuchs, 2004). Also, the measures should, if possible, even be understandable by community members, to contribute to their and schools' awareness as to where children are in the development of these skills, and where they may need more instruction and development. This can play an important role in increasing parental involvement and in improving accountability.

As discussed earlier, children begin to demonstrate the development of mathematics knowledge well before they begin formal schooling. Although children begin school with some level or form of knowledge about numbers, they are not all at the same level when they begin formal schooling (Gersten & Chard, 1999; Howell & Kemp, 2005). To take this notion a step further, it is known that in the United States, for example, there are large differences in formal schooling across districts, communities, and states (Baker & LeTendre, 2005). We know that in all countries, as shown by Loveless (2007) and Mullis and Martin (2007) using TIMSS data, mathematics achievement varies broadly within and across schools, with more variability between schools being a characteristic of developing countries. These differences are due to reasons such as provincial or state differences in funding, socioeconomic levels, and quality of instruction (Baker & LeTendre, 2005; Fennell et al., 2008). Differences in funding and parental background have consequences for accessibility and quality (Jimerson, 2006; Johnson & Strange, 2005; National Rural Network, 2007). Differences in access to preschool are also large in both developed and developing countries (Rosenthal, Rathbun, & West, 2006). Even within given regions, more specifically within and across districts, preschool education varies in both quality and accessibility (Bryant, Maxwell, & Taylor, 2004; U.S. Dept. of Education/NCES, 2003). Some parents drive (or have their children transported) across districts to obtain high-quality preschool education for their children (National Clinical Dataset Development Programme [NCDDP], personal communication, June 1, 2007; Teacher Education and Compensation Helps [T.E.A.C.H.], personal communication, June 4, 2007). This may also hold true in countries with weaker zoning regulations than in the United States. If children vary in mathematical knowledge among themselves within a given country, it stands to reason that they would also vary between countries, depending on the countries' preschool environments.[2] Furthermore, preschool options are much more limited in developing than in developed countries.

All this implies that EGMA needs to be "reasonable" in the assumptions it makes about likely levels of skills that exist in developing-country classrooms. If the level of difficulty is pitched at a typical developed country level, it will find too many children "bottoming out," or unable to perform at even the lowest established standard (floor effect). Thus, a key design feature in EGMA is to make sure the tool has some tasks that are easy, such as oral counting; and that the tasks in the assessment progress and build on this knowledge. Oral counting fluency and number identification are known as "gateway skills" and are comparable to letter-naming fluency measures in assessing reading ability (Clarke et al., 2008). Quantity discrimination and missing-number identification involve additional knowledge of mathematical relationships and are indicators of mathematical knowledge.

[2] Few societies are environmentally innumerate in the way some societies may be environmentally illiterate, and thus there may be less variation in the mathematics skills with which children come to school as opposed to reading skills.

4. EGMA Measures

This section takes an in-depth look at each of the measures that make up EGMA. Also described is the implementation for each EGMA task.

4.1 Oral Counting, Number Identification, Quantity Discrimination, and Missing Numbers

EGMA uses oral counting, number identification, quantity discrimination, and identification of missing numbers as some of the key measures. The reliability of these sorts of CBM measures is fairly well established (e.g., Fuchs, 2004; Hintze et al., 2002). Clarke and Shinn (2004) look at the validity of some of these CBM measures as “experimental” measures, using others (e.g., Woodcock Johnson Applied Problems, Number Knowledge Test) as criterion measures. The Woodcock Johnson standardized tool requires students to listen to mathematics problems and then perform the needed calculations. It includes counting items, addition, and subtraction items, all of which use visuals; and other items associated with money and time (e.g., “What time does the clock say?”). Items increase in difficulty as students continue through this test. The median reliability of the Woodcock Johnson Applied Problems test is .92 for the age range of 5 to 19 years (Mather & Woodcock, 2001). The methods used for reliability in Clarke and Shinn’s (2004) study were interscorer, alternate form, and test-retest. The interscorer reliability was high, with .99 across the oral counting, number identification, and quantity discrimination tasks. The alternate form method was consistently as high, at .90 for oral counting, number identification, and quantity discrimination. For test-retest of these three tasks, reliability ranged between .78 for oral counting and .86 for quantity discrimination. These levels of reliability seem quite good, if one takes into account the fact that reliability greater than .60 is suitable for lower-stakes decision making about groups of students, and that a reliability standard of .80 is often used as the minimum standard in higher-stakes and more individually oriented assessments (Nunnally, 1978; Nunnally & Bernstein, 1994; Subkoviak, 1988).[3]

Moving beyond reliability, to establish concurrent validity, Clarke and Shinn (2004) established correlations across measures (experimental vs. criterion) for three data collection periods. Table 4 shows some of their results: the range and median for the experimental measures’ correlations with the criterion measures, across the data collection periods. The strongest correlation is in the quantity discrimination measure, and the weakest correlation is in the oral counting measure.

[3] Note that EGMA is mostly oriented at group assessment and policy discussion.

Table 4. Concurrent Validity of EGMA-Type Items with Criterion Tests (Correlations)

Measure	Range	Median
Oral Counting	.49 to .70	.60
Number Identification	.60 to .70	.66
Quantity Discrimination	.71 to .88	.75
Missing Number	.68 to .75	.71

Source: Clarke, B., & Shinn, M. R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*(2), 234–248. Table data are from pp. 242–243.

Note: The intercorrelation among all experimental measures was reported to be high. The concurrent validity correlations among the criterion measures were reported to range from .74 to .79.

Table 5 provides some predictive validity results, with quantity discrimination showing the best median correlation. The oral counting measure demonstrated the weakest median correlation. In general, if we take a median correlation of .50 or higher to be a large effect, then these measures demonstrate a good construct in measuring these specified domains, as discussed in section 4.3 (Nunnally & Bernstein, 1994).

Table 5. Predictive Validity of EGMA-Type Items

Measure	Median Correlations
Oral Sounding	.56
Number Identification	.68
Quantity Discrimination	.76
Missing Number	.72

Source: Clarke, B., & Shinn, M. R. (2004). A preliminary investigation into the identification and development of early mathematics curriculum-based measurement. *School Psychology Review, 33*(2), 234–248. Table data are from pp. 242–243.

These measures were developed for early identification of students who have difficulties with mathematics and to monitor growth of skills over time. Overall, strong relationships have been demonstrated between the measures and the skills being assessed; i.e., the measures demonstrate good validity (Carmines & Zeller, 1979; DeVellis, 2003). EGMA uses these same measures to identify mathematical difficulties across students—more specifically across classrooms and schools. It also uses these measures to assist teachers in monitoring whether students are obtaining the relevant skills. Ensuring that these measures meet criteria for reliability and validity helps teachers, schools, and others to feel confident that they are measuring what they intend to measure (Clarke & Shinn, 2004). Clarke & Shinn (2004) further emphasize the need for caution regarding floor effects when using these measures.

4.2 Oral Counting Fluency, One-to-One Correspondence, Number Identification, Quantity Discrimination, and Missing Number

These items are also known as number-sense items. Okamoto and Case (1996) proposed these measures as a way to identify children's knowledge and skill at using a "mental number line." There has been much discussion as to the definition of "number-sense." However, there is widespread agreement as to the importance of the concept. Perhaps one of the best descriptions or definitions of these sorts of items in the literature refers to fluidity and flexibility with numbers and number concepts, demonstrated through the manipulation of these numbers through quantitative comparisons with limited difficulty (Berch, 2005; Clarke et al., 2008; Floyd, Hojnoski & Key, 2006; Gersten & Chard, 1999). These assessments are known to promote early identification of children at risk (Floyd et al., 2006). Use of these kinds of items to inform instruction can reduce difficulties in mathematics, with particular benefits for students with learning disabilities (Gersten & Chard, 1999). Berch (2005) noted that when children exhibit "number skills" on these kinds of items, they typically possess deeper understanding of the meaning of numbers, have developed strategies for solving a variety of mathematical problems, and can truly use quantitative methods in the interpretation, processing, and communication of information. These number-sense abilities or basic concepts and skills are key in the progression toward the ability to solve more advanced problems and the acquisition of more advanced mathematics skills (Aunola, Leskinene, Lerkkanen, & Nurmi, 2004; Chard, Baker, Clarke, Jungjohann, Davis, & Smolkowski, 2008; Foegen et al., 2007).

The collection of data for these measures by Clarke et al. (2008) and Clarke and Shinn (2004) demonstrates the progression of these skills. Table 6 shows mean score differences for kindergarten children's ability from the beginning of the school year (fall 2005) to the end of the school year (spring 2006). Table 7 shows mean score differences for first-grade children across measures at two different times, in the fall and then in the spring, for a sample of children located in a medium-sized school district in the U.S. Pacific Northwest.

Table 6. Descriptive Statistics for Kindergarten Children in the Fall and Spring

Measure	Fall M (SD)	Spring M (SD)
Oral Counting	22.46 (17.59)	54.38 (25.12)
Number Identification	28.13 (19.84)	49.03 (18.66)
Quantity Discrimination	8.99 (9.41)	22.29 (11.48)
Missing Number	3.85 (4.86)	11.11 (6.89)

Source: Clarke, B., Baker, S., Smolkowski, K., & Chard, D. J. (2008). An analysis of early numeracy curriculum-based measurement: Examining the role of growth in student outcomes. *Remedial and Special Education, 29*(1), 46–57. Data are from table 1.

Note: Data for this study were collected on kindergarten students in the fall (*n* = 230) and the spring (*n* = 222) of the 2005/2006 academic school year. This table presents mean (M) scores and standard deviations (SD) for each time point. Numbers in this task ranged from 1 to 10.

Table 7. Descriptive Statistics for First-Grade Children in the Fall and Spring

Measure	Fall M (SD)	Spring M (SD)
Oral Counting	60.4 (20.5)	74.6 (20.9)
Number Identification	36.0 (15.9)	48.1 (17.8)
Quantity Discrimination	19.2 (10.6)	28.5 (9.9)
Missing Number	11.3 (5.8)	17.4 (17.4)

Source: Clarke, B., Baker, S., Smolkowski, K., & Chard, D.J. (2008). An analysis of early numeracy curriculum-based measurement: Examining the role of growth in student outcomes. *Remedial and Special Education, 29*(1), 46–57. Data are from table 2.

Note: Data for this study were collected on first-grade students at two schools (*n* = 52) at three time points (fall, winter, and spring), with mean score and standard deviations for each time point (*M* = 100, *SD* = 15). Numbers used in this task ranged from 1 to 20.

4.2.1 Individual Discussion of Proposed EGMA Tasks

The following subsections represent each of the tasks that take place in the Early Grade Mathematics Assessment. For each section, a summary of some of the literature and research is provided. The methodologies for the implementation and scoring of each task are also given.

4.2.2 Oral Counting Fluency

The assessment of oral counting fluency targets children's ability to produce numbers fluently. The task usually begins with the number 1, and asks children to continue counting until they reach the highest number they can before making a counting error (Floyd et al., 2006). For EGMA, children are asked to rote count as far as they can. The score is based on the last correct number the child says previous to making an error (see Figure 1) or at the end of a minute (Clarke et al., 2008). (That is, this is a timed task, since the purpose is to elicit a fluency measure.)

Beginning the assessment with oral counting fluency serves as an icebreaker to help children become comfortable with the activities. It also allows us an opportunity to learn students' knowledge of number names (Ginsburg & Russell, 1981). This knowledge includes not only knowing the names of the numbers 1 through 9 (at first), but also understanding the numbers that follow. Baroody & Wilkins (1999) described that children's going on to a new series of 10 names was prompted by the number 9 at the end of one series. For instance, after "9" comes "10," and the numbers from 10 to 19 all begin with 1. After "19," the next numbers begin with a "2" until "29" is reached, and so forth (Baroody & Wilkins, 1999). Gelman and Gallistel (1986) and Baroody and Wilkins (1999) noted that counting experiences contribute to the construction of number concepts. Counting is an important precursor or aid in the development of basic number concepts (Baroody & Wilkins, 1999). Proper counting to higher levels also requires children to understand the rules of generating new series of numbers, which also is a precursor to other important skills (e.g., counting out sets, development of number sense). Thus, even "rote" counting is not as "rote" as it sounds.

Children by the end of first grade should be able to identify and count numbers to 100. (NCTM, 2008). It is important to identify where children are with this knowledge. Figure 1 describes the task as it appears in the EGMA instrument.

Figure 1. Task 1: Oral Counting

TASK 1: ORAL COUNTING

MATERIALS: STOPWATCH

STOP RULE: STOP THE CHILD IF CHILD MAKES AN ERROR WHILE COUNTING OR AT THE END OF A MINUTE

SCORING: RECORD THE LAST SPOKEN CORRECT NUMBER AND THE TIME ON THE STOPWATCH.

DIRECTIONS: START THE STOPWATCH TO COUNT DOWN FROM 60 SECONDS WHEN THE CHILD BEGINS TO COUNT. STOP THE STOPWATCH AND THE CHILD AS SOON AS THE CHILD MAKES AN ERROR OR AT THE END OF A MINUTE. RECORD THE TIME AND THE LAST NUMBER THE CHILD SAYS CORRECTLY, BELOW.

SAY: **I want you to count for me. I am going to use this stopwatch. I will tell you when to begin and when to stop. Count for me from one to as high as you can count. Are you ready? Okay begin, one…**

INTERVIEWER: IF THE CHILD DOES NOT START COUNTING SAY: **Watch me count. One, two, three….ten. Okay, just like me, I want you to count as high as you can. Okay begin, one…**

DID YOU NEED AND USE ANOTHER LANGUAGE OTHER THAN WHAT IS USED FOR THIS TASK?	TIME ON THE STOPWATCH: ___________
IF YOU OR THE CHILD USED ANOTHER LANGUAGE FOR THIS TASK, PLEASE WRITE THE LANGUAGE USED HERE:	LAST NUMBER CHILD SAID CORRECTLY: ____________

4.2.3 One-to-One Correspondence

Gelman and Gallistel (1986) described one-to-one correspondence as "the rhythmic coordination of the partitioning and tagging process" (p. 78). In layperson's terms, it refers to counting objects. Here, children use two processes that need to work together. The first process is recognizing the items they need to count. The second is to recognize, and mentally tag, those items that have *already* been counted. As a child recognizes each item, he or she tags it mentally as needing to be counted. Tagging can be done physically by pointing to the item to keep track of those still needing to be counted, as well as those that have already been counted (Gelman & Gallistel, 1986). Another way to think of this task is an opportunity for the child to represent the collection of objects through the application of number words (Baroody, 2004).

EGMA assesses for enumeration and then cardinality. That is, we assess the number-word counting correspondence, and then, with a prompt, assess whether a child is aware that the last number name signifies the summation of objects that are presented. The goal is to assess the child's understanding that the last number-word counted in a group of objects signifies the value of the group. In other words, the one-to-one correspondence of all objects as a whole is represented by a single, last number (Gelman & Gallistel, 1986).

The materials used in this task are two 8½ × 11-inch sheets of paper, each showing an orderly array of objects (circles). The circles are of the same color and same size so as not to distract children from the counting task. If objects are of different colors or sizes, children may place restrictions on what they count and what they do not count (Bullock & Gelman, 1977; Gelman & Gallistel, 1986).

One of the two sheets of paper has four circles centered on the page. This sheet is used as the practice item for this task. The other sheet has four rows of five circles and is scored.

Circles are no smaller than one inch in diameter. Children have 60 seconds to count all the circles. Assessors instruct the children to point and count the circles. This follows the same methodology used in other studies (e.g., Clarke et al., 2008; Floyd et al., 2006). Furthermore, the last circle counted correctly is scored. Pointing and counting an object twice, or making an error in counting, stops the task. Order of counting objects is irrelevant as long as no object is counted twice. This means that children can start in the middle of a row and begin to count. To assess children's knowledge of cardinality, children are asked "How many circles are there?" when they have successfully counted the circles (see Figure 2).

Figure 2. Task 2: One-to-One Correspondence

TASK 2: COUNTING: ONE-TO-ONE CORRESPONDENCE EXERCISE

MATERIALS: SHEET "A2" AND STOPWATCH **STOP RULE:** STOP THE CHILD IF S/HE DOUBLE COUNTS A CIRCLE, INCORRECTLY COUNTS A CIRCLE, OR IF TIME ON THE STOPWATCH (60 SECONDS) RUNS OUT

SCORING: RECORD 1) THE LAST SPOKEN CORRECT NUMBER; 2) THE RESPONSE THE CHILD GIVES TO YOUR FOLLOW UP QUESTION; AND 3) TIME LEFT ON STOPWATCH

DIRECTIONS: PLACE SHEET "A2" WITH THE 20 CIRCLES IN FRONT OF THE CHILD. START STOPWATCH AT SIXTY SECONDS AS SOON AS THE CHILD STARTS COUNTING.

SWEEP YOUR HAND FROM LEFT TO RIGHT OVER THE CIRCLES AND SAY: **Here are some more circles. I want you to point and count these circles for me. This time, I am going to use this stopwatch and will tell you when to begin and when to stop.**

POINT TO FIRST CIRCLE AND SAY: **Start here and count the circles.**

HOW MANY CIRCLES DID THE CHILD COUNT: _____

IF THE CHILD DOES NOT SAY THE NUMBER AFTER COUNTING THE CIRCLES SAY: **How many circles are there?**

DID YOU NEED AND USE ANOTHER LANGUAGE OTHER THAN WHAT IS USED FOR THIS TASK?	NUMBER OF CIRCLES CHILD SAYS THERE ARE: _____ TIME ON THE STOPWATCH: ______
IF YOU OR THE CHILD USED ANOTHER LANGUAGE FOR THIS TASK, PLEASE WRITE THE LANGUAGE USED HERE:	

4.2.4 Number Identification

The number identification exercise occurs toward the beginning of the EGMA to establish an understanding of children's knowledge and identification of written symbols. Here, students orally identify printed number symbols that are randomly selected and placed in a grid (Clarke & Shinn, 2004).

Number identification or number naming can be expressed differently across countries. In U.S. English, one typically says "forty-three" for a number such as "43," but even in English, the standardization of number identification via words is relatively recent (as in the use, until recently, of constructs such as "four score and seven"). And of course in other languages, such as German, one might say "three-and-forty," or, in Japanese, "four-tens-three," for the same concept (which has the added convenience of making place value explicit in the naming—a feature missing in English). As indicated, Japanese spoken numbers represent the base-ten number system, and explicitly place value within that system. Perhaps as a consequence, Japanese first-grade students are much more efficient in constructing base-ten representations with a better understanding of place value than students in the United States (Miura & Okamoto, 2003).

A review of expectations to be met by a number of states in the United States, a look at curricular standards across a few countries, and a review of the curriculum focal points set by the NCTM (2008) seem to clearly indicate that children should be developing an understanding of and ability to compare and order whole numbers for their grade level. We want to learn whether children can read these numbers. We also want to learn whether children are familiar with the number-word associated with each of the numbers they view. The recognition and understanding that each of the numbers is a constant with one number-word associated with it is crucial in mathematics and is crucial for the following tasks in this assessment.

Based on each of these grade-level expectations, a random sampling of numbers for 1 through 20 for the first 10 items in the exercise and a random sampling of numbers for 21 through 100 for the second 10 items in the exercise is used. Children are stopped from continuing this task if they get four errors one right after the other. Children also have 5 seconds to identify a number. At the end of 5 seconds, the interviewer prompts the child by pointing to the next number and saying, “What number is this?” The number identification task is timed for 60 seconds (see Figure 3).

Figure 3. Task 3: Number Identification

TASK 3: NUMBER IDENTIFICATION ITEM - EXERCISE

MATERIALS: SHEET "B" AND STOPWATCH

STOP RULE: STOP THE CHILD IF S/HE MAKES 4 ERRORS ONE RIGHT AFTER THE OTHER OR IF TIME ON THE STOPWATCH (60 SECONDS) RUNS OUT

SCORING: FOR EACH NON-RESPONSE OR INCORRECT NUMBER WORD, PLACE A "/" THROUGH THE NUMBER.

DIRECTIONS: PLACE SHEET "B1" WITH THE GRID OF NUMBERS IN FRONT OF THE CHILD. START STOPWATCH FOR SIXTY SECONDS AS SOON AS THE CHILD STARTS RESPONDING.

SWEEP YOUR HAND FROM LEFT TO RIGHT OVER THE NUMBERS AND SAY: **Here are some numbers. I want you to point to each number and tell me what the number is. I am going to use this stopwatch and will tell you when to begin and when to stop.**

POINT TO FIRST NUMBER AND SAY: **Start here.**

IF A CHILD STOPS ON A NUMBER FOR 5 SECONDS – TELL THE CHILD WHAT THE NUMBER IS AND POINT TO THE NEXT NUMBER AND SAY: **What number is this?**

				NUMBER CORRECT PER ROW
12	6	9	13	
2	16	19	3	
20	10	32	48	
53	69	25	34	
39	97	46	67	

CHILD SCORE (OVERALL TOTAL CORRECT): ____/20

TIME LEFT ON THE STOPWATCH: ________

4.2.5 Quantity Discrimination

Quantity discrimination in EGMA measures children's ability to make judgments about differences by comparing quantities in object groups. This can be done by using numbers or by using objects such as circles and asking which group has more objects. Quantity discrimination in kindergarten and first grade demonstrates a critical link to an effective and efficient counting strategy for problem solving (Clarke et al., 2008).

As referenced at the beginning of this paper, children begin (or should begin) fairly early to develop new understandings of numbers, such as that the number 8 is "more" than the number 5, and the use of the mental number line and the association of symbols such as where 8 and 5 are located on this number line (Baroody & Wilkins, 1999; Carpenter et al., 2003; Okamoto & Case, 1996). These are essential precursor skills.

Quantity discrimination involves making magnitude comparisons, which can be done with numbers and/or objects. For Clarke et al. (2008), the use of numerals in making comparisons, especially for children in kindergarten and first grade, demonstrates a "critical link to effective and efficient counting strategies to solve problems" (p. 49). For instance, a student who is able to perform a quick magnitude comparison in solving a problem such as 6 + 3 needs to identify that the number 6 is the bigger number or operand. Students who count from the "bigger number" have learned an effective strategy and also make fewer errors in solving these problems. Without this ability, students are more apt to make errors or use less efficient strategies such as counting all or counting from the smaller addend, which obviously takes longer, implies more counting, and is therefore more error-prone.

The selection of numbers used in the EGMA quantity discrimination task included careful attention to the research. The discussion in this paragraph also shows, via examples, the sorts of knowledge being assessed. Nuerk, Kaufmann, Zoppoth, and Willmes (2004) demonstrated that numbers placed farther apart on a visible number line were easier to discriminate. They also noted slower reaction time when children were asked to identify numbers with larger unit digits. For instance, comparing groups such as "59" and "65" may take more time than comparing groups such as "51" and "65." In this last case, the reason for the longer processing time is the larger unit digit ("9") in "59." Furthermore, "compatible" comparisons are easier. A comparison between groupings such as "52 and 67" is known as a compatible comparison because both the "decade digits" (e.g., 5< 6) and the "unit digits" (e.g., 2<7) lead to the same decision (Nuerk et al. 2004, p. 1200), both being smaller in the first than in the second grouping. In a study by Nuerk et al. (2004) with second- through fifth-grade students, strong main effects were reported: Compatible trials, large decade distance trials, and large unit distance trials demonstrated the least number of errors. Processing capacity also improved with student's age.

The EGMA instructions for assessing children are similar to those used by Clarke and Shinn (2004). Each item presented to children consists of two numbers. The children are asked to identify the larger number (e.g., "Which one is bigger?"). All items to which the children identify the smaller number, respond that they do not know, or do not respond are counted as incorrect. Children have 3 seconds to respond before the interviewer moves on with the prompt "Let's try the next one."

Children should also be developing the ability to compare and order whole numbers. Again as seen with the number identification task, a random sampling of numbers 1 through 20 is completed for the first five items in this exercise, and then a random sampling of numbers from 1 to 100 is completed for the second five items in this exercise. The stop rule for this task is four consecutive errors in a row. Children who get four consecutive errors are stopped from continuing the task and are moved on to the next task. Figure 4 shows the EGMA quantity discrimination task.

Figure 4. Task 4: Quantity Discrimination Measure

TASK 4: QUANTITY DISCRIMINATION MEASURE – EXERCISE

MATERIALS: SHEET "C", COUNTERS, AND COVERSHEET

STOP RULE: STOP THE CHILD IF S/HE GETS 4 ERRORS ONE RIGHT AFTER THE OTHER.

SCORING: FOR EACH ITEM, CIRCLE THE NUMBER THE CHILD SAYS. IF THE CHILD SAYS AN INCORRECT NUMBER FOR AN ITEM (A NUMBER NOT LISTED FOR THE ITEM), RECORD THE NUMBER ON THE LINE FOR THE ITEM.

DIRECTIONS: PLACE SHEET "C1" WITH THE COVERSHEET COVERING THE ITEMS IN FRONT OF THE CHILD. WHEN YOU ARE READY TO BEGIN, BRING COVERSHEET DOWN TO REVEAL THE FIRST NUMBERS TO THE CHILD.

P1. TAKE OUT THE COUNTERS AND FORM TWO STACKS. THE LEFT STACK SHOULD HAVE 10 COUNTERS AND THE RIGHT STACK SHOULD HAVE 4 COUNTERS. ONCE YOU HAVE FINISHED MAKING THE PILES, POINT TO THE STACKS AND SAY: **Look at these two stacks. Tell me which one is bigger? Point to it.**

CHILD POINTS TO STACK WITH (CIRCLE THE CHILD'S RESPONSE): 10 CHIPS | 4 CHIPS

P2. TAKE THE COUNTERS AND FORM TWO STACKS. THE LEFT STACK SHOULD HAVE 12 COUNTERS AND THE RIGHT STACK SHOULD HAVE 8 COUNTERS. POINT TO THE STACKS AND SAY: **Look at these two stacks. Tell me which one is bigger? Point to it.**

CHILD POINTS TO STACK WITH (CIRCLE THE CHILD'S RESPONSE): 12 CHIPS | 8 CHIPS

PUT THE COUNTERS AWAY AND GET SHEET C1 OUT.

SAY: **Now, let's try some with numbers. Look at these numbers. Tell me which one is bigger? Tell me the number name.**

IF THE CHILD DOES NOT RESPOND TO AN ITEM, REPEAT THE QUESTION ONCE, WAIT THREE SECONDS, MARK WITH A SLASH "/" IF THERE IS STILL NO ANSWER, MOVE ON TO THE NEXT ITEM.

REPEAT THE QUESTION FOR EACH OF THE ITEMS. CIRCLE THE CHILD'S RESPONSE FOR EACH ITEM.

1.	5	16	
2.	8	7	
3.	15	20	
4.	14	17	
5.	19	18	
6.	40	96	

CIRCLE THE ANSWER GIVEN BY THE CHILD. FOR NON-ANSWERS, PLACE A SLASH "/"ON THE SPACE AT THE END OF THE ROW.

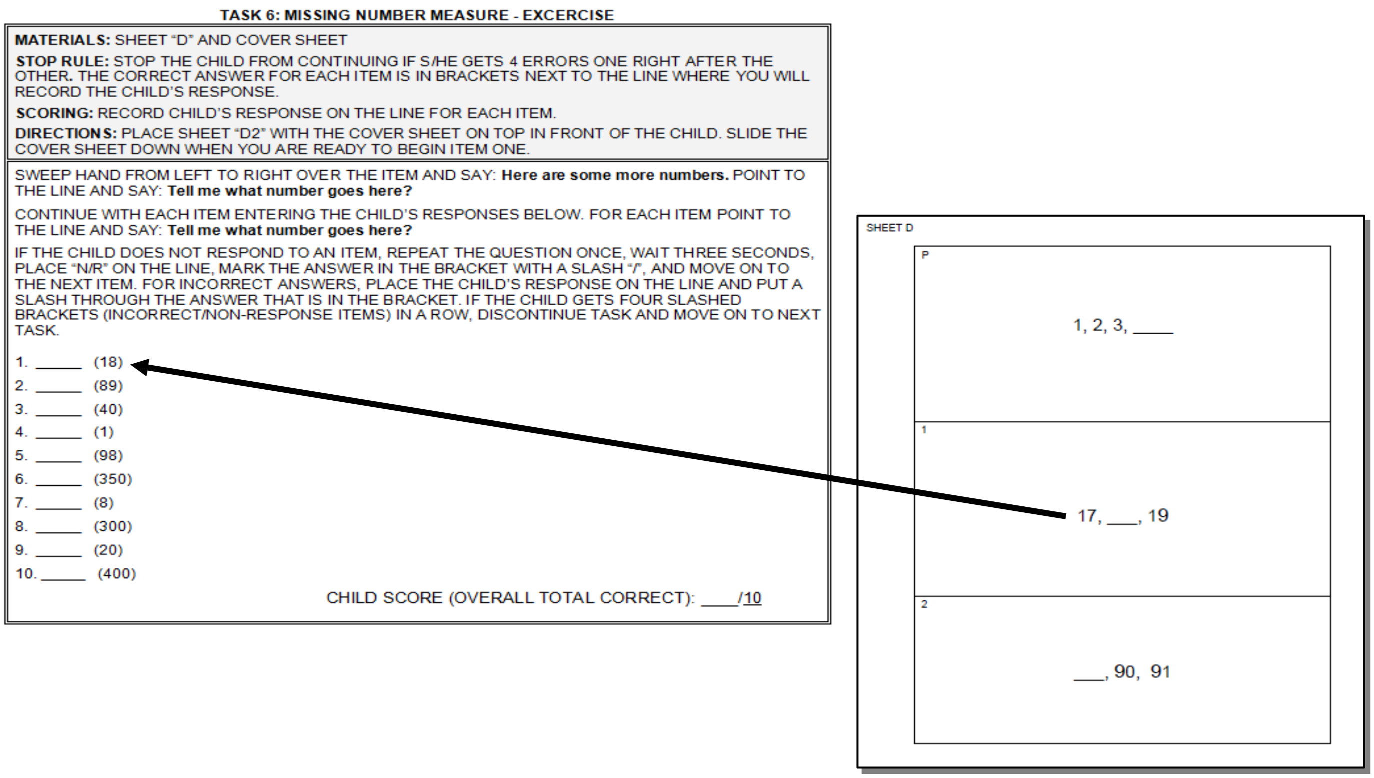
PLACE THE TOTAL RESPONSES CORRECT ON THE LINE FOR CHILD SCORE.

4.2.6 Missing Number

In this task, children are asked during EGMA to name a missing number in a set or sequence of numbers. Based on the objectives set by NCTM (2008) and national and international assessments (e.g., NAEP, TIMSS), children need to be familiar with numbers and able to identify missing numbers. In the early grades, children should be counting by ones, twos, fives, and tens (NCTM, 2008). Children also should be able to count backward. In general, children should be able to identify missing numbers and strategically demonstrate their knowledge of these numbers (Clarke & Shinn, 2004). Also, using our example of "6 + 3" from the discussion of quantity discrimination above, good performance on a missing number task demonstrates the depth of the child's understanding that he or she needs to count 3 more numbers from 6, and those numbers are "7, 8, 9" (Clarke et al., 2008).

For EGMA, similar to Clarke and Shinn's (2004) description of the missing number task, children are presented with a string of three numbers with the first, middle, or last number in the string missing. Children are instructed to tell the assessor what number is missing. Children have 3 seconds to correctly identify each of these numbers. At the end of 3 seconds, the assessor prompts the child and moves on to the next item. We also assess for counting backward, and note whether children have any difficulty in transitioning to a new series of numbers (e.g., 29 signals the end of the "twenties" and the start of a new series of numbers, the "thirties"). Figure 5 shows the EGMA missing number task.

Figure 5. Task 6[4]: Missing Number, from Booklet and Stimulus Sheets (Math Sheets)


TASK 6: MISSING NUMBER MEASURE - EXCERCISE

MATERIALS: SHEET "D" AND COVER SHEET

STOP RULE: STOP THE CHILD FROM CONTINUING IF S/HE GETS 4 ERRORS ONE RIGHT AFTER THE OTHER. THE CORRECT ANSWER FOR EACH ITEM IS IN BRACKETS NEXT TO THE LINE WHERE YOU WILL RECORD THE CHILD'S RESPONSE.

SCORING: RECORD CHILD'S RESPONSE ON THE LINE FOR EACH ITEM.

DIRECTIONS: PLACE SHEET "D2" WITH THE COVER SHEET ON TOP IN FRONT OF THE CHILD. SLIDE THE COVER SHEET DOWN WHEN YOU ARE READY TO BEGIN ITEM ONE.

SWEEP HAND FROM LEFT TO RIGHT OVER THE ITEM AND SAY: **Here are some more numbers.** POINT TO THE LINE AND SAY: **Tell me what number goes here?**

CONTINUE WITH EACH ITEM ENTERING THE CHILD'S RESPONSES BELOW. FOR EACH ITEM POINT TO THE LINE AND SAY: **Tell me what number goes here?**

IF THE CHILD DOES NOT RESPOND TO AN ITEM, REPEAT THE QUESTION ONCE, WAIT THREE SECONDS, PLACE "N/R" ON THE LINE, MARK THE ANSWER IN THE BRACKET WITH A SLASH "/", AND MOVE ON TO THE NEXT ITEM. FOR INCORRECT ANSWERS, PLACE THE CHILD'S RESPONSE ON THE LINE AND PUT A SLASH THROUGH THE ANSWER THAT IS IN THE BRACKET. IF THE CHILD GETS FOUR SLASHED BRACKETS (INCORRECT/NON-RESPONSE ITEMS) IN A ROW, DISCONTINUE TASK AND MOVE ON TO NEXT TASK.

1. _____ (18)
2. _____ (89)
3. _____ (40)
4. _____ (1)
5. _____ (98)
6. _____ (350)
7. _____ (8)
8. _____ (300)
9. _____ (20)
10. _____ (400)

CHILD SCORE (OVERALL TOTAL CORRECT): ____/10

4.2.7 Addition and Subtraction Word Problems

Three types of oral word problems are discussed here based on research by Carpenter, Hiebert, and Moser (1981). Table 8 shows these word problems. Each of these oral word problems has been used in studies by Carpenter et al. (1981), Carpenter and Moser (1984), Okamoto and Case (1996), and Riley and Greeno (1988). Using concepts similar to those in Table 8, for instance, Riley and Greeno (1988) defined three categories of word problems, similar to those originally presented by Carpenter et al. (1981). One example of Riley and Greeno's "combine" task is similar to the joining of two quantities and figuring out their combination or sum. An example of Riley and Greeno's "change" task with an unknown result is similar to the combine subtraction example in Table 8. The third type of item, "compare," is very similar to Carpenter and Moser's (1984) "compare" task, in which the aim is to determine the difference between two numbers.

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[4] Task 5 was added to EGMA after the meeting with the EGMA Mathematics Expert Panel (EMEP) in January 2009. An explanation of Task 5, number line estimation, can be found in Appendix 2.

Carpenter et al. (1981) used word problems to analyze children's informal concepts of addition and subtraction by following the strategies children used to solve certain items presented to them. Carpenter and Moser (1984), in a 3-year longitudinal study with a sample of children from grades one through three, found that, even with formal instruction, children still used informal knowledge and strategies as they continued to learn number facts outside of school. For Carpenter et al. (1981), children's exposure to oral word problems in the mathematics curriculum enhanced their ability to apply mathematics concepts they had already learned to analyzing problems.

Table 8. Types of Verbal Problems with Examples

Types/Classes of Verbal Problems	Definition	Example
Joining/Separating	Initial quantity with some direct or implied action that causes a change in the quantity.	Addition: Johnny had 3 fish. His father gave him 8 more fish. How many fish did Johnny have altogether? Subtraction: John had 8 pieces of candy. He gave 3 pieces to his friend. How many pieces of candy did he have left?
Combine (Part-Part-Whole)	Relationship involves two distinct quantities that are parts of a whole.	Addition: Some children were fishing. Three were girls and 8 were boys. How many children were fishing altogether? Subtraction: There are 11 children at the school. Three children are boys and the rest are girls. How many girls are at the school?
Comparison	1) Difference between two quantities, or 2) difference between one quantity, and the solution with the second quantity as the unknown.	Addition: Johnny has 3 pieces of candy. Sam has 8 more pieces than Johnny. How many pieces of candy does Sam have? Subtraction: Johnny caught 3 fish at the lake. His sister Jane caught 8 fish at the lake. How many more fish did Jane catch than Johnny?

Source: Based on the breakdown of classes of verbal word problems presented to first-grade children, from Carpenter, T. P., Hiebert, J., & Moser, J. M. (1981). Problem structure and first-grade children's initial solution processes for simple addition and subtraction problems. *Journal for Research in Mathematics Education, 12*(1), 27–39.

Carpenter et al. (1981) and Carpenter and Moser (1984) developed these word problems based on problems included in mathematics textbooks and elementary school mathematics, and on younger children's ability to solve them. In addition, the construction of these problems takes into account syntax, vocabulary, sentence length, and familiarity of the situations provided in the problems (Carpenter et al., 1981).

The strategies children use in solving these problems are very similar to those described in the next task (below) for addition and subtraction with numerically stated problems such as "8 + 7" or "12 + 4." Table 9 demonstrates some of the strategies used by children in solving these addition/subtraction problems, as well as the skill level implied by the strategy chosen by the children. For example, progression between levels might imply progressing from "counting all" to solve an addition problem (level 1) to fact retrieval (level 3) (Carpenter & Moser, 1984). Carpenter and Moser (1984) found that students in first grade typically use manipulatives to solve word problems; whereas second- and third-grade students use more counting and/or recall of number facts.

Riley and Greeno (1988) collected data from children in kindergarten through third grade. Reliable differences were observed in the children's success on word problems, depending on their grade and the type of problem presented to them. They also demonstrated the importance of well-defined sets and relations between sets within a word problem as differences were notable across first- and second-grade students.

Table 9. Strategies Used by Children in Carpenter and Moser's (1984) Study

Strategy	Description	Level
Addition		
Counting all	Both sets are represented with manipulatives (e.g., counters, blocks) or fingers, and then combined and counted from 1, to get at the total.	Level 1
Counting from smaller number	Counting is done mentally, with use of fingers or manipulatives, starting with the smaller number. Example, "3 + 4 = ?" with child starting with first number "3" and counting up from "4, 5, 6, 7" to come up to 7.	Level 2
Counting from larger number	Counting is done mentally, with use of fingers or manipulatives, starting with the larger number. Example, "3 + 4 = ?" with child recognizing the larger number and counting up from "4, 5, 6, 7" to come up with 7.	Level 2
Number fact	Answer based on known addition facts.	Level 3
Heuristic	Based on facts such as "4 + 4 = 8" so "4 + 6" is 2 more, which equals "10."	Level 3
Subtraction		
Separating	Using manipulatives, the smaller quantity is removed from the larger quantity. A backward counting sequence can be used, with the last word spoken being the answer.	Level 1
Counting down from	Manipulatives are counted out for the larger set, and the child removes one at a time until the remainder is equal to the second given number. Counting the number of manipulatives (e.g., cubes) removed gives the answer. Backward counting can also be used, with the last spoken word being the answer. Example: "8 – 5 = ?" so "8, 7, 6, 5, 4…the answer is 3."	Level 1

Strategy	Description	Level
Adding on	Starting with the smaller quantity and then adding on to this set until it is equal to the larger numbers. With manipulatives or fingers where possible, those added are counted and the answer is arrived at.	Level 2
Matching	Child puts out two sets of manipulatives, each standing for the numbers given. Then, the sets are matched one-to-one until one set is exhausted. The remaining manipulatives are counted for the answer.	Level 2
Uncodable	Interviewer unable to determine strategy.	—

Source: Strategies used by children in a study of addition and subtraction concepts in the early grades (first through third grade). Carpenter, T, P., & Moser, J. M. (1984). The acquisition of addition and subtraction concepts in grades one through three. *Journal for Research in Mathematics Education, 15*(3), 179–202.

Carpenter et al.’s (1981) study demonstrated that children in first grade often succeed in solving oral problems. Both the modeling of children’s actions in solving the problems and an understanding of the operations were observed. Children in first grade demonstrated some variation in strategy use, but this may have been due to limited formal instruction. The authors reported that the use of manipulatives may influence the strategy that a child uses in answering the questions (e.g., counting all vs. counting on).

According to Carpenter and Moser (1984) there has been “over 50 years” of consistency in the findings on children’s strategies with word problems, with a key finding being that care needs to be taken with the semantic structure of the word problems (Carpenter et al., 1981). This was further emphasized by Riley and Greeno (1988) in their review of the construction of the semantic structure of a word problem presented to a child, and how this can influence the strategies used by children. Furthermore, Riley and Greeno (1981) defined their word problems by level with a progression of difficulty. Carpenter and Moser’s (1984) problems reflected this progression as well. In Riley and Greeno’s (1988) study, a significant number of kindergarten and first-grade children demonstrated level 2 knowledge for combine (joining) and change (separating) items, whereas only a few kindergarten and first-grade children demonstrated level 1 ability for compare items. Okamoto and Case (1996) showed very similar findings: Kindergarten through second-grade students performed much better on the combine and change items. Carpenter et al. (1981) saw the same trend. Riley and Greeno (1988) reported the use of manipulatives (e.g., counters, blocks) by first-grade children, whereas Carpenter and Moser (1984) observed more recall of number facts with these types of tasks by second and third graders.

Okamoto and Case (1996), using some of Riley and Greeno’s (1988) word problems, demonstrated a correlation between children’s level of number knowledge and level of word problems successfully solved.

In EGMA, administration of word problems reflects the semantic format provided by Carpenter and Moser (1984) and Riley and Greeno (1988). Based on Carpenter et al.’s (1981) administration of the items, two of each type (joining/separating, part-part-whole, compare) are administered. In Carpenter and Moser’s (1984) study, if a child incorrectly answered three out of the first four items or only used the “count all” strategy for the word problems, he/she would not continue with the comparison items. With EGMA, if a child incorrectly answers the first two items or only uses the “count all” strategy for these word problems, he/she does not continue with the following two items. Based on the level of difficulty seen with the comparison items, only joining, separating, and combining are assessed.

Also based on Carpenter and Moser’s (1984) study, the smaller addend always appears first in the EGMA addition problems. This was done to observe whether a child uses the counting-on method from the first (smaller) or the larger addend. As for the subtraction items, the same format with the larger number first is present for all of these items.

EGMA’s instruction/administration of these items (see Figure 6) is based on the instruction used by Carpenter and Moser (1984). Here, the interviewer reads the entire word problem to a child before he/she can begin the task. If the child needs a word problem reread, the interviewer rereads it in its entirety. It can be reread as often as the child needs, as it may help the child continue with the identification of the numbers while solving the problems. The interviewer also tells the child that he/she has some counters that can be used in solving the problems.

Figure 6. Task 7: Word Problems

TASK 7: WORD PROBLEMS

MATERIALS: COUNTERS

STOP RULE: NOT INCLUDING THE PRACTICE ITEM, STOP THE CHILD IF S/HE GETS 2 ERRORS ONE RIGHT AFTER THE OTHER. IF AN ITEM IS TAKING A CHILD MORE THAN 30 SECONDS TO COMPLETE, PLACE A CHECK MARK DIRECTLY AFTER WHERE IT READS "TOOK MORE THAN 30 SECONDS" AND MOVE ON TO THE NEXT ITEM. ITEMS THAT TAKE MORE THAN 30 SECONDS ARE COUNTED AS ERRORS.

SCORING: ENTER CHILD'S ANSWER ON THE LINE WHERE INDICATED FOR EACH ITEM. FOR EACH OF THE QUESTIONS: IF THE CHILD DOES NOT RESPOND, CIRCLE THE NR. IF THE CHILD REFUSES TO ANSWER, CIRCLE THE RF. IF THE CHILD SAYS THAT HE OR SHE DOES NOT KNOW, CIRCLE THE DK.

DIRECTIONS: PLACE COUNTERS TO THE SIDE OF THE CHILD.

SAY: **I have some problems that I am going to ask you to solve for me. Here are some things to help you count. You can use them if you want to, but you don't have to. Listen very carefully. If you need me to, I will repeat the question for you. Okay, let's get started.**

PRACTICE ITEM: SAY: **Tommy had 2 candies. Sarah had 3 candies. How many candies did they have altogether?**

IF THE CHILD CORRECTLY ANSWERS THAT THEY HAD 5 CANDIES ALTOGETHER, SAY: **That's right. They had five candies altogether. Let's do some more.**

IF THE CHILD DOES NOT CORRECTLY ANSWERS, SAY: **They had 5 candies altogether. Watch me.** USE THE COUNTERS AND READ THE PROBLEM DEMONSTRATING THREE COUNTERS FOR TOMMY AND TWO COUNTERS FOR SARAH. IF THE CHILD STILL DOES NOT ANSWER THE PROBLEM, SAY: **There are 5 altogether. Let's do some more.**

FOR THE FOLLOWING QUESTIONS, IF THE CHILD DOES NOT RESPOND TO AN ITEM, REPEAT THE QUESTION ONCE, WAIT THREE SECONDS AND MOVE ON TO THE NEXT ITEM. RECORD THE SCORES BELOW.

SAY: **For these next questions, I will read the questions to you. Let me know if you would like for me to repeat the question. Remember some of these questions are hard for even older children, so don't feel bad if you do not know the answer. Remember, you can use these** POINT TO THE COUNTERS **to help you answer the questions. Okay, let's get started.**

QUESTION 1: SAY: **Jane had 4 pieces of candy. Her father gave her 5 more pieces of candy. How many pieces of candy does Jane have altogether?**

CORRECT ANSWER: 9	NR RF DK
CHILD'S ANSWER: ___	

4.2.8 Addition and Subtraction Problems

Children already have some very basic addition and subtraction concepts before entering formal schooling. For example, children realize that the size of a group of objects grows when more objects are added (e.g., Johnny gets another piece of candy, so Johnny has more candy). This knowledge is seen in children as young as 3 to 5 years (Cooper, Starkey, Blevins, Goth, & Leitner, 1978; Starkey & Cooper, 1980).

Studies over more than 50 years have investigated children's addition knowledge, and have focused on children's ability to solve addition problems. Some of the abilities assessed include the time it took to solve a problem, the size of the problem that was solved, and the strategy used in solving the problem (e.g., Groen & Parkman, 1972; Groen & Resnick, 1977). These studies confirm that children use a variety of methods to solve problems (e.g., counting from one on fingers, counting from the larger addend) (Siegler & Robinson, 1982). The studies also show ability prior to any schooling. For example, preschool children in the United States demonstrated the knowledge that the number that answers an addition problem is greater than the largest addend in the problem (Siegler & Shrager, 1984).

Children also use many methods to solve subtraction problems, starting with some of the simplest. For example, a "finger-based" method commonly used by children starts with the representation of the larger: Children count up to the larger number using their fingers or by just putting up the larger number of fingers. Once children have the representation of the larger number on their fingers, they lower fingers to represent the smaller number, and then count the remainder of fingers still raised (Siegler & Shrager, 1984).

Table 10 shows some of the methods or strategies children use for addition and subtraction at ages four, five, and six. The table shows that children use overt strategies such as counting their fingers for problems they think are difficult. Young children who use overt strategies to solve problems that they perceive as difficult tend to make fewer errors (Siegler & Robinson, 1982; Siegler & Shrager, 1984). But overt methods tend to be slower, and are thus less efficient. "Finger" strategies will always be the slowest (Siegler & Shrager, 1984). Children may also use multiple strategies. They may start by first retrieving their current knowledge as a way to solve the problem and then feel a need to use an overt strategy to ensure confidence in their answer, such as fingers or some countable objects. The observation of strategy use (e.g., use of fingers, use of counters) for solving the addition and subtraction problems is included for this EGMA task. This should result in a better understanding of children's ability (Siegler & Shrager, 1984).

Table 10. Examples of Addition and Subtraction Strategies

Strategy	Description of Strategy
Sum	Child puts fingers up or uses counters to represent both addends. An example of this is "3 + 2." Here the child puts up three fingers on one hand and two fingers on his/her other hand. Then, the child starts counting from one, counting each finger "1, 2, 3, 4, 5."
Finger Recognition	Child puts up fingers and then says "5" without counting. An example of this is "3 + 2." Here the child puts up three fingers on one hand and two fingers on the other hand. Child looks at fingers and says "5."
Min	Child may use fingers or counters in solving from the larger addend. An example of this is "3 + 2." Here the child may start counting from three, put up two fingers or counters to represent the two, counting only these two fingers or counters "4, 5."
Retrieval	Child says an answer and explains that he/she just knew it.
Guessing	Child says an answer and it can only be explained as guessing.

Source: The strategies in this table are based on some of the strategies of Siegler and Jenkins (1989), table 3.2, p. 59. Siegler, R. S., & Jenkins, E. (1989). *How children discover new strategies.* Hillsdale, New Jersey: Lawrence Erlbaum Associates.

Through age and experience, children add to their existing knowledge. Their perceptions as to degree of difficulty of addition and subtraction problems change, and then so do the strategies they use in solving problems (Siegler & Shrager, 1984). Experience with numbers contributes to a decrease in errors over time (Ashcraft, 1982).

The format represented in EGMA is based on that used by Jordan, Hanich, and Kaplan (2003). Children are shown a visual representation of the mathematics problem, and also have the problem read aloud to them. Children also have counters available to them. They can use any method in solving the problem. The addition and subtraction items in this task (Figure 7) were based on the development of mathematics problems used by Siegler and Shrager (1984). Per feedback from Robert Siegler (personal communication, January 16, 2009) on the numbers used for addition, harder numbers were generated to be used in this task. For instance, the first two addition items have addends equal to or less than nine with a sum less than or equal to 10. The last addition problems have addends greater than 11 with sums up to 25. The subtraction problems are the inverse of the addition problems.

Based on Siegler and Shrager (1984) and Siegler and Jenkins (1989), we originally proposed that the interviewers record the method used by children for *each* item, but this is not possible in developing countries. The burden on the interviewer has been judged too great, particularly in a non-experimental context. We want to ensure that the interviewers in these countries are paying attention and collecting the answers that the children provide. Currently the interviewers record the method if a child used his/her fingers, the counters, or a combination of methods on *any* item in the task, but they record only at the *end* of the task. Similarly, the original intention was to record times for each item with a stopwatch. Unfortunately, experience has taught that this is nearly impossible and that an overall time for the items is needed. We currently have a rule that if a child has not responded or attempted to solve a problem after 10 seconds, the interviewer prompts the child once, waits 5 seconds, and if the child still does not respond, continues to the next problem.

Figure 7. Task 8: Addition and Subtraction

TASK 8: ADDITION/SUBTRACTION PROBLEMS – EXERCISE

MATERIALS: SHEET "E1", COUNTERS, AND STOPWATCH

STOP RULE: FOR THE ADDITION PROBLEMS (ITEMS 1-5), STOP THE CHILD FROM CONTINUING IF S/HE GETS 3 ERRORS ONE RIGHT AFTER THE OTHER AND MOVE ON TO ITEM 6. IF THE CHILD GETS THREE ERRORS FOR ITEMS 6-10 ONE RIGHT AFTER THE OTHER, STOP. IF AN ITEM IS TAKING A CHILD MORE THAN 30 SECONDS TO COMPLETE, PLACE A CHECK MARK DIRECTLY AFTER WHERE IT READS "TOOK MORE THAN 30 SECONDS" AND MOVE ON TO THE NEXT ITEM. ITEMS THAT TAKE MORE THAN 30 SECONDS ARE COUNTED AS ERRORS.

SCORING: ENTER CHILD'S ANSWER ON THE LINE FOR EACH ITEM. IF THE CHILD DOES NOT RESPOND, CIRCLE THE NR. IF THE CHILD REFUSES TO ANSWER, CIRCLE THE RF. IF THE CHILD SAYS THAT HE OR SHE DOES NOT KNOW, CIRCLE THE DK.

DIRECTIONS: PLACE THE COUNTERS TO THE SIDE OF THE CHILD. PLACE SHEET "E1" IN FRONT OF THE CHILD. POINT TO EACH ADDITION / SUBTRACTION PROBLEM WHILE YOU READ THEM TO THE CHILD. IF THE CHILD DOES NOT RESPOND TO AN ITEM, REPEAT THE QUESTION ONCE, WAIT FIVE SECONDS AND MOVE ON TO THE NEXT ITEM. RECORD THE SCORES BELOW.

ADDITION SECTION:

POINT TO THE FIRST ADDITION PROBLEM ON MATH SHEET E1 AND SAY:

1. **How much is 4 plus 5?**

(IF CHILD DOES NOT UNDERSTAND SAY: **How much is 4 and 5 altogether?**)

CORRECT ANSWER: 9 CHILD'S ANSWER: ___	NR RF DK

2. **How much is 8 plus 2?**

(IF CHILD DOES NOT UNDERSTAND SAY: **How much is 8 and 2 altogether?**)

CORRECT ANSWER: 10 CHILD'S ANSWER: ___	NR RF DK

3. **How much is 20 plus 4?**

(IF CHILD DOES NOT UNDERSTAND SAY: **How much is 20 and 4 altogether?**)

CORRECT ANSWER: 24 CHILD'S ANSWER: ___	NR RF DK

4.2.9 Geometry

As with arithmetic, children learn both intuitive and explicit knowledge about shapes and patterns through their everyday experiences even before beginning formal schooling (Clements, 2004a). In the United States, the NCTM (2004a, 2008, 2009a) stresses the development of spatial reasoning beginning in prekindergarten through hands-on exploration of shapes, and the ability to communicate information as to the location of shapes in the children's environment (see Table 11). Children at age four can begin to compare, sort, and classify shapes according to similarities and differences (Clements, 2004a; Kersh, Casey, & Young, 2007).

The EGMA measures for shape recognition and pattern extension assess whether students are developing these geometry concepts early, as it has been shown that early learning of shapes and spatial reasoning leads to both later success in mathematics problem solving ability through a more developed understanding of how mathematics and geometry are linked, and more choices in strategy use (Battista, 1990; Casey, 2004; Kersh et al., 2007).

The literature for geometry and spatial reasoning is not as well developed as the literature on early child numeracy, even though it is known that geometry is a foundation for mathematics skills and for other subject learning. Nonetheless, children's geometric knowledge and skills have been studied for many years. Fairly important early research included Williams's (1934) work on "perception of symmetry" which analyzed children's familiarity with and ability to manipulate two-dimensional and then three-dimensional shapes. Williams's study was conducted with children 5 to 8 years of age and demonstrated an age-based progression in children's ability to manipulate objects.[5] Case (1996) saw most children as having the ability to represent three-dimensional shapes/objects on a two-dimensional surface (e.g., a tree's branches and leaves as a circle and its trunk as a line extending vertically from the ground up into the circle). This representation of objects could be thought of as the beginning of an ability to represent concrete three-dimensional objects as geometrical abstractions in a two-dimensional surface. Children also continue to build on this ability as they learn objects' types and shapes, location in space, and attributes (e.g., number of sides, corners, edges), and as they continue exploration of objects in their environment (Case, 1996; Clements, Swaminathan, Hannibal, & Sarama,1999; Greenes, 1999). Children's continued drawing of shapes demonstrates their representation and understanding of the world, and their ability to render the concrete as abstract. As children learn more about their surroundings, they also learn how to communicate their position in relation to these surroundings (Case, 1996). Table 11 shows some examples of curriculum focal points, by grade, that were developed by the NCTM (2004a; 2009a).

Table 11. Geometry—Curriculum Focal Points

School Year	Overall Goals	Definitions
Prekindergarten	Identifying shapes and describing spatial relationships	Developing spatial reasoning by examining shapes of objects and inspecting their relevant positions. Finding shapes in their environment, being able to describe them, combining two- and three-dimensional shapes, and understanding vocabulary such as "above," "below," and "next to."
Kindergarten	Describing shapes and space	Interpreting the physical world with geometric ideas and using corresponding vocabulary. Ability to identify, name and describe a variety of shapes, including three-dimensional shapes (e.g., spheres, cubes). Modeling objects in the environment using basic shapes and spatial reasoning.
First Grade	Composing and decomposing geometric shapes	Demonstrating an understanding of part-whole relationships as well as the properties of the original and composite shapes (composing and

[5] Williams (1934) had children insert shapes, which varied from very simple and symmetrical to more complex, into the correct insets. More shapes than insets were provided for each of the tasks to ensure that the relationship between the shapes and insets was not too obvious.

School Year	Overall Goals	Definitions
		decomposing plane and solid figures). Recognizing figures from different perspectives and orientations and describing their geometric attributes and properties. Beginning to develop measurement concepts.
Second Grade	______	Estimating, measuring, and computing lengths in solving problems involving data and space. Using geometric knowledge and spatial reasoning to develop foundations for understanding area, fractions, and proportions.

Source: National Council of Teachers of Mathematics (NCTM). (2009a). *Curriculum focal points.* Retrieved on September 14, 2009, from http://www.nctm.org/standards/ content.aspx?id=270

Proficiency and familiarity with both two- and three- dimensional shapes through continued awareness and experiences within and outside the classroom contribute to the familiarity needed for later tasks (Clements, 1999). As shown in Table 11, among the mathematics skills children first develop is the ability to communicate information about shapes in the environment. By second grade, children are, or should be, applying their knowledge to tasks in measurement as well as the integration of counting skills (e.g., reading graphs, solving fraction problems) (NCTM, 2009a).

As with number skills, children also bring to school a level of informal geometry skills such as perceptions of shape and space. Many studies have demonstrated these informal understandings in infancy (e.g., Craton, 1996; Van de Walle & Spelke, 1996) and in many human societies that are otherwise not very numerate. When a child begins formal schooling he/she should be provided opportunities to build on existing knowledge through the use of materials and curricula that teach differences and names for shapes (Greenes, 1999). In addition, Clements (1999) suggests caution in the use of pictures used to represent shapes in assessing children's knowledge of shape names. The fact that pictures and diagrams of shapes tend to be presented very conventionally in textbooks and other materials can undermine children's shape naming and recognition ability. For example, in many early textbooks, triangles are presented with the base on the horizontal (relative to the bottom edge of the page). This may undermine children's recognition and understanding of a triangle as a three-sided shape. This was demonstrated by research by Clements et al. (1999) on children's perceptions of shapes. Table 12 shows each of the shapes presented to the children, and the outcomes from the presentation of each of these shapes. The outcomes affirm Clements's (1999) concern that materials available to children may be too "rigid" in teaching about shapes. This also stresses the importance of hands-on activities in working with shapes in the environment (NCTM, 2008) and opportunities for children to identify shapes that are rotated or under various transformations (Clements, 1999). We discuss these issues not as an academic digression, but because they affect the way shape knowledge is assessed in EGMA.

Table 12. Shapes Presented to Children and Outcomes

Shapes Presented to Children	Usual Visual Prototype of Shapes	Outcomes/Observations
Circles	——	Identified accurately by children. Differences—only a few younger children chose other shapes (e.g., ellipse)
Squares	With horizontal base	Identified "fairly" well by children Difference—younger children chose non-square rhombi.
Triangles	Equilateral or isosceles with horizontal base	Identified less accurately by children. Orientation did not seem to have much of an effect. Lack of symmetry had an effect, with children rejecting a triangle if the point at the top was not in the middle.
Rectangles	Horizontal, elongated, and twice as long as they are wide	Identified less accurately by children. Difference—many children accepted long parallelograms or right trapezoids. Children seem to make selection based on the ratio of height to base.

Source: Clements, D. H., Swaminathan, S., Hannibal, M., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192–212.

Note: Clements, Swaminathan, Hannibal, & Sarama (1999) conducted a study with children 3 years 6 months through 6 years 9 months of age. An interviewer asked a child to mark a specific shape on an 8½ x 11-inch sheet of paper. Additional data were collected for this study according to inquiries of the children, based on their shape selections.

Note that, per the report presented by the Task Group on Conceptual Knowledge and Skills (Fennell et al., 2008), familiarity with shapes in the early grades was found to be an essential and a critical foundation for later algebra skills. This familiarity and experience with shapes lays the grounding needed for children in the United States to solve problems such as those involving perimeter and area of triangles by the end of fifth grade, and to go on to further learning of concepts such as slope in algebra and the concepts of parallelism and perpendicularity (NCTM, 2008).

4.2.10 Geometry—Shape Recognition Task

Our work on shape recognition is informed by work such as that of Clements et al. (1999) in which the authors conducted interviews with children in a one-on-one setting. Interviewers asked children to identify and select specific shapes when presented with an 8½ × 11-inch piece of paper containing shapes (e.g., "put a mark on each of the shapes that is a circle"). The children were expected to respond by identifying and marking all the shapes that corresponded to the specific task/shape requested by the interviewers. The shapes used in this task were circles, squares, triangles, and rectangles. For EGMA, circles, squares, triangles, and rectangles are presented to the children. Table 13 shows the mean correct numbers of answers and standard deviations, by age, in the study conducted by Clements et al. (1999). Here, 6-year-old children were shown to be performing significantly better than younger children ($F = 5.54$, $p < .005$). Clements et al. (1999) also noted that progress in shape recognition and in understanding of their properties is determined by instruction more than by age. Therefore, one can take these sorts of results only as a very rough initial precursor of benchmarks appropriate to developing countries, where instruction and the environment likely imply results that are poorer than or different from those observed by Clements et al. (1999) in a U.S. setting.[6]

Table 13. Mean Scores by Age in Shape Selection Task

Shapes Presented	Possible Scores	Age Groups		
		4 years (*n*=25)	5 years (*n*=30)	6 years (*n*=42)
Circles	15	13.76 (2.0)	14.33 (1.4)	14.86 (0.4)
Squares	13	10.64 (2.7)	11.17 (2.7)	11.79 (1.7)
Triangles	14	7.92 (2.7)	8.17 (2.6)	8.48 (2.2)
Rectangles	15	7.68 (3.9)	7.7 (2.9)	8.79 (2.9)

Source: Clements, D. H., Swaminathan, S., Hannibal, M., & Sarama, J. (1999). Young children's concepts of shape. *Journal for Research in Mathematics Education, 30*(2), 192–212.

Note: This table represents the data from table 2 in Clements et al. (1999). Included are 4-year-old children from the study, as this represents student scores based not only on age, but also on the beginning of formal instruction. For students in developing countries in the first grade, this may be their first exposure to formal learning of shapes.

For EGMA, an interviewer asks a child to identify and point to all representations of one shape on an 8½ × 11-inch sheet of paper. As the child is pointing to the specific shapes on the paper, the interviewer documents the shapes identified by the child in the EGMA booklet (Figure 8). At the end of the assessment, the interviewer bases the score on the number of correct shapes and incorrect shapes that were marked. The interviewer uses four sheets to ask the child to identify squares, circles, triangles, and rectangles.

[6] Interestingly, even in developing-country environments extremely different from those found in the United States, basic skills are often present, reinforcing the notion that some of these skills are inherent in most human societies and are measurable in nonschooled children or in unschooled situations. Dehaene, Izard, Pica, & Spelke (2006), through research with the Mundurukú, an Amazonian indigene group that lacks formal schooling and has few words for mathematics or geometric concepts beyond the basic, demonstrated that people nonetheless have knowledge of the concepts of geometry (e.g., lines, points) and geometrical figures (e.g., circles, squares).

Figure 8. Task 9: Shape Recognition

TASK 9: SHAPE RECOGNITION

MATERIALS: SHEETS "F1" THROUGH "F4"

STOP RULE: N/A

SCORING: THE CORRECT SHAPES ARE MARKED WITH A PATTERN OF DOTS ON THEM. FOR EACH CORRECT SHAPE THE CHILD IDENTIFIES, HE OR SHE GETS A POINT. ENTER THE SCORE ON THE LINE THAT FOLLOWS EACH FIGURE.

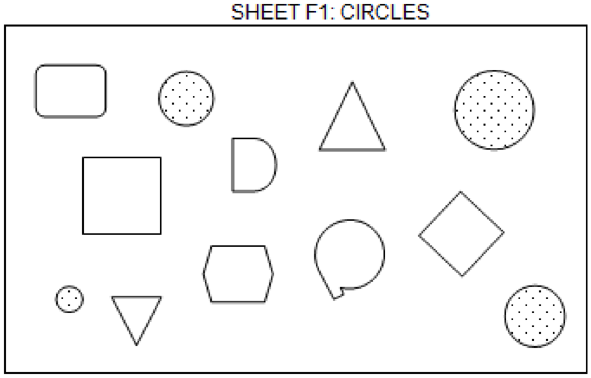
DIRECTIONS: FOR EACH OF THE SHAPES BELOW, ONCE THE CHILD FINISHES PLACING THE COUNTERS ON TOP OF THE SHAPE SHEET, MARK THE SHAPES WITH A "√" ON YOUR SHEET. THEN, RECORD THE NUMBER OF SHAPES THAT WERE COUNTED CORRECTLY. MAKE SURE TO FOLLOW THESE INSTRUCTIONS FOR EACH SHAPE SHEET PRESENTED TO THE CHILD.

SAY: **Now we are going to play another game with shapes.**

PLACE THE COUNTERS TO THE SIDE OF THE CHILD.

POINT TO THE COUNTERS AND SAY: **I want you to place the counters on all of the circles you find on this sheet.** PLACE THE SHEET IN FRONT OF THE CHILD. **Let me know when you are done.**

SHEET F1: CIRCLES



CIRCLE FIGURES IDENTIFIED CORRECTLY: ___/ 4
CIRCLE FIGURES IDENTIFIED INCORRECTLY: ____

4.2.11 Patterns

Patterning is one of a number of content areas (e.g., part-whole relations, shape attributes, mapping) that enhance the development of spatial thinking (Casey, 2004), and hence pattern recognition is an important predictor or proxy for ability in spatial thinking. Pattern recognition requires children to identify similarities and differences among the objects that make up a pattern. Children review and identify the number of objects making up the pattern and the groups and replication of the objects making up the patterns, and based on this information, make predictions on how the pattern continues (Greenes, 1999). Clements (2004a) noted the order, cohesion, and predictability used in pattern identification as the beginnings of algebraic thinking (e.g., the ability to deduce beyond available data/information).

Solving pattern problems through extension of patterns is only one task of many that, as Greenes (1999) indicates, teach children to "reason inductively and prepare them for later work with functions and concepts of probability" (p. 43). Most development in pattern recognition ability takes place with formal instruction that builds on knowledge of shapes. In research conducted by Klein & Starkey (2004) through The Berkeley Mathematics Readiness Project, it was noted that younger children of preschool age were able to duplicate patterns, but were not developmentally ready to extend patterns. The understanding of patterns gradually develops during these early years. This was further affirmed by the geometry curricula presented by Grande and Morrow (1993) for NCTM for kindergarten and first grade. Prekindergarten is a time for children to get familiar with objects; the kindergarten years are a time for children to explore, compare, classify, and arrange objects (e.g., relationships of size, position of object). By first grade, children are, or should be, working with shapes in constructing linear patterns.

Children can work with different types of pattern tasks in the early grades. Working with movement patterns, for instance, involves physical activity such as having children predict the next move or moves that follow two hops and two claps; or to interpret auditory and/or visual stimuli (e.g., musical notes with different tones, dots, and lines demonstrating pattern of tone) (Greenes, 1999). These tasks, as well as those previously noted for shape, work with number concepts in children's further learning of measurement and data tasks.

Pattern extension is a way for children to begin recognizing and testing elements in the continuation of patterns. To do this, children need to retain the attributes of the shapes in memory while recognizing and testing what shape(s) come next (Grande & Morrow, 1993; NCTM, 2008). In EGMA, the goal of the proposed task of pattern extension is to learn of children's spatial ability in their recognition of the different shapes and the embedded pattern to be added to.

The presentation of items for this task is modeled after the items used by Klein & Starkey (2004) and Clements (2004a). Children in the early grades should be able to identify and predict units (e.g., AB), repetitive units (e.g., ABAB), and grow units (ABAABAAAB) (Clements, 2004a). For pattern recognition, children are presented with a pattern and asked to select a response option for the object necessary to complete the pattern (e.g., ABABA? = A or B; see Figure 9). Unlike Klein and Starkey (2004), EGMA does not use small colored blocks for this task. Instead, EGMA's pattern extension task is similar to Clements, Sarama, & Liu's (2008) representation of pattern extension. For EGMA, the pattern is presented on an 8½ × 11-inch sheet of paper with response options from which the child may choose. The interviewer introduces each pattern extension task to the child and points to the blank(s), asking the child which of the response options will complete the pattern. Children's solutions to this task are scored as correct/incorrect.

Figure 9. Task 11[7]: Pattern Extension

TASK 11: PATTERN EXTENSION

MATERIALS: SHEETS "H1 THROUGH H5" **STOP RULE:** STOP THE CHILD FROM CONTINUING IF S/HE GETS 3 ERRORS ONE RIGHT AFTER THE OTHER.

SCORING: RECORD ONE POINT FOR EACH CORRECT RESPONSE.

DIRECTIONS: DO NOT PLACE THE SHEET IN FRONT OF THE CHILD UNTIL YOU SAY: **I am going to show you a pattern.**

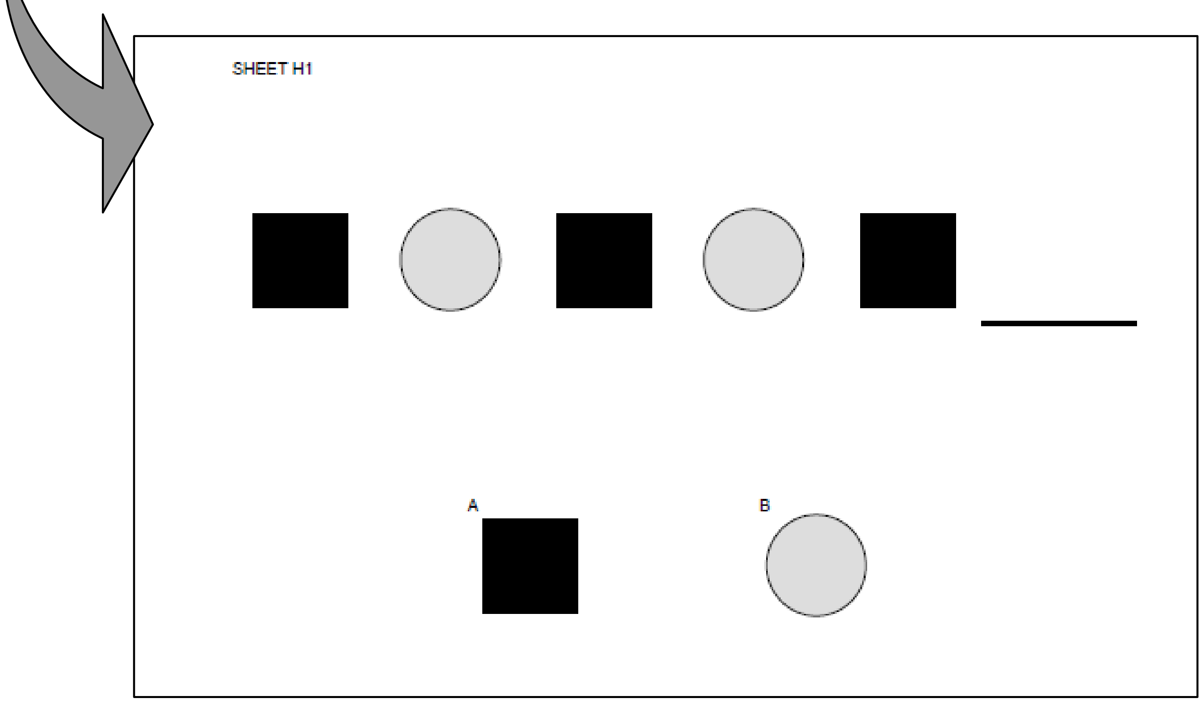
PLACE SHEET 11 IN FRONT OF THE CHILD. POINT TO THE PATTERN MOVING HAND FROM LEFT TO RIGHT OVER PATTERN AND SAY: **I want you to finish this pattern for me.**

THEN MOVE HAND ACROSS RESPONSE OPTIONS AT BOTTOM OF PAGE AND SAY: **Which one of these goes here?** POINT TO THE BLANK AT THE END OF THE PATTERN.

REPEAT THE INSTRUCTIONS ABOVE FOR EACH OF THE ITEMS. CIRCLE THE CHILD'S RESPONSES BELOW. FOR ANY ITEMS THE CHILD DOES NOT ANSWER, PLACE A "/" ON THE LINE FOR THAT ITEM.

1.	A	B		
2.	A	B	C	
3.	A	B	C	
4.	A	B	C	
5.	A	B	C	

CHILD SCORE (OVERALL TOTAL CORRECT): ____/5



[7] Task 10 was added to EGMA after the meeting with the EMEP in January 2009. An explanation of Task 10, shape attributes, can be found in Appendix 2.

4.3 Additional Information for Measures

4.3.1 Floor and Ceiling Effects

For a number of the EGMA tasks, a practice item introduces and provides feedback to the children. This assists in avoiding any floor effects due to their not understanding what is required of them for a task. Moreover, stimulus materials are used for some of the tasks. Overall, these items measure students' understanding of numbers and geometric and quantitative concepts by second grade. To avoid ceiling effects (in which students routinely score at the top of the expected scale), the items gradually get harder as the child progresses.

4.3.2 Timing

Studies with school-age children have demonstrated the importance of using a timing method on mathematics tasks as a way to reveal differences in the processing of numerical information. Furthermore, this method provides information in addition to accuracy scores (Berch, 2005). The following are some studies and outcomes that demonstrate the role that timing plays.

Passolunghi & Siegel (2004) compared two groups of children, one with difficulties in mathematics and normal reading ability and the other with normal mathematics and reading ability. Both were tested for accuracy and speed based on the time it took each child to complete each task (timed from when the child started the task to when the child finished the task). Included in the mathematics tasks were two tasks used in EGMA, namely the oral counting task and the one-to-one counting tasks. The group of children who were known to have mathematics difficulties but normal reading skills performed more slowly and with less accuracy on mathematics processing tasks (e.g., number comparisons, identification of correct arithmetic operation in a simple word problem) than the children in the group known ahead of time to have normal mathematics and reading ability. There were also significant differences in a counting span task (similar to one-to-one counting).

In an additional task on listening span completion, children with mathematics difficulties recalled fewer items than children in the group with normal mathematics ability. Note that children with mathematics difficulties showed deficits in some of the tasks, but not all. Passolunghi and Siegel's (2004) findings were consistent with those of Case, Kurland, and Engle (1982), in that there may be a correlation between time taken to retrieve numerical information from long-term memory and processing speed in some memory tasks. Berch (2005) and Nuerk et al. (2004) have demonstrated time effects in adults tested at tasks involving distance comparisons between two-digit numbers (e.g., "51 and 56," "59 and 65"). Results in Nuerk et al. (2004) demonstrate that comparing numbers that are further apart takes less time in processing. Based on their research conducted with children, Nuerk et al. (2004) demonstrated that numbers further apart took less time to process with less error than those closer together. Differences were also seen across grades, with faster processing as age advanced. These studies make us cognizant of both the numbers used in the EGMA tasks (e.g., quantity discrimination task) and the timing of EGMA tasks (e.g., one-to-one correspondence task).

Appendix 1: Summary January 2009 Meeting with Expert Panel

As a way to guide the development of EGMA, RTI appointed an expert panel of mathematics experts to guide, critique, and provide peer review of work done on the Early Grade Mathematics Assessment (EGMA). A meeting with the experts was held on January 15–16, 2009. These notes informally summarizing that discussion were prepared in February 2009.

Members of the Expert Panel

David Chard, Southern Methodist University
Jeff Davis, American Institutes for Research (AIR)
Susan Empson, The University of Texas at Austin
Rochel Gelman, Rutgers University
Linda Platas, University of California, Berkeley
Robert Siegler, Carnegie Mellon University

Roundtable Discussions

The proposed EGMA tasks were presented to the members of the EGMA Mathematics Expert Panel (EMEP). The following is a list of the tasks that were presented:

- Counting fluency (60 seconds)
- Counting one-to-one (fluency, 60 seconds)
- Number identification (fluency, 60 seconds)
- Quantity discrimination (not timed)
- Missing number (not timed)
- Word problems (not timed)
- Addition/subtraction (not timed)
- Shape recognition (not timed)
- Pattern extension (not timed)

The following list is a brief summary of some of the recommendations that were made after the presentation. Following this list is a more in-depth description of recommendations for each of the tasks in EGMA.

General or Summary Recommendations

1. Consider more items to truly test fluency (if and when this is what we are looking for), especially with the quantity discrimination task.
2. For quantity discrimination, children should be instructed to say the number, not just point to the correct number. This is important to thoroughly understand children's concept of numbers.
3. Note that counting and number identification generally have low predictive power, but these tasks are important nonetheless and may be more predictive or better screens in low-skills environments.

4. It is important to add a number line task (analog/cardinal number line, not just order; high correlation with other skills); easy to test with lots of items, and establish linearity.

Task 1: Counting

Rochel Gelman pointed out the importance of using a counting task to assess children's knowledge of the generative rule, and noted that having children count only to 30–40 would be considered insufficient for this task. The missing number task could be used for this; for example by asking children to find the missing number in 98, 99, 100, ? , or in 998, 999, 1,000, ? . Alternatively, requiring counting to a higher number would help, but would be very time consuming. (Also note that this is affected by whether the language in question has a fairly transparent generative rule.)

Note: As of February 2009, to make this task as easy as possible to administer in the field, we will keep the current instructions and time for 60 seconds. For scoring purposes, we will note that counting to 30–40 is sufficient for this task.

Task 2: Counting One-on-One: Correspondence

The only comment for this task was to check the size of the circles that the children count. As of January 2009's draft version, the circles looked too large. There was some inquiry about the format of the task. The task was formatted into four rows of five circles after a study done by Floyd, Hojnoski, & Key (2006) with 3- to 6-year-old children. The circle size was approximately one inch in diameter. The task was timed for 30 seconds. We will time the task for 60 seconds and will instruct the child to go as fast as he/she can "but to be right."

Task 3: Number Identification Task

As of the January 2009 draft shared with EMEP, Exercise One had 12 items and Exercise Two had 20. We will add items to Exercise One so both exercises have a total of 20 items. Exercise One will have numbers in random order from 1 to 20. Exercise Two will have numbers in random order from 21 to 100.

Task 4: Quantity Discrimination Task

Robert Siegler and David Chard affirmed this task to be important in understanding children's knowledge of magnitude comparisons.

Children will have 60 seconds to complete this task.

In the original draft shared with the EMEP, children were instructed to point to the bigger number. It was recommended that children *say* the bigger number (e.g., "I want you to point to and tell me the name of the bigger number"). This will allow a better understanding as to where children are in the development of a mental number line.

It was recommended that more items (both single-digit and double-digit) be added for children to complete within the minute.

There was also emphasis on ensuring item balance between digits (e.g., compatible comparison, single-digit differentiation) presented to the children (Nuerk, Kaufmann, Zoppoth, & Willmes, 2004).

Task 5: Missing Number Task

It is important that we learn the degree to which children understand counting and numbers over 100. We agreed to add a couple of items with numbers over 100 to learn whether children do have this understanding. Numbers over 100 are more predictable.

It was recommended that the missing number task be moved to the end of the pattern extension task. We agreed we would rename the pattern extension task, and have a part one with geometrical shapes and a part two with numbers, *if possible*.

Note: In February 2009, this task—based on the feedback from the EMEP—was considered, but the current placement of the task plays a role in understanding what knowledge children have before getting to the word problems and subtraction/addition task (e.g., counting by five, tens, counting backward, "___, 90, 91"). For this reason, we decided to leave the missing number task where it is currently located.

Task 6: Word Problems

One panel member recommended the need to include a change/unknown problem. An example of a problem to add to the word problem tasks was also introduced to the group, "I have $7. How many more dollars do I need to buy an $11 toy?" We agreed to consider this.

Task 7: Addition/Subtraction

One question asked of the panel was whether the addition and subtraction problems should be mixed together. For example, the first item could be an addition problem and the next item a subtraction problem. The response was to keep them as they are now. Addition and subtraction problems should be kept separate.

The following recommendations were also made:

- Change the language in the questions that are asked of the child in the following sense. Currently the question reads, "How much is 2 and 3 altogether?" and it was suggested by the EMEP that this be changed to read, "How much is 2 plus 3?" If a child does not understand the question, the enumerator should follow up with the original question ("How much is 2 and 3 altogether?").

- Have children identify the numbers in the problem they are presented when asked to solve, for example, "5 + 2 = __". (Due to the time that will be involved, we will test the instrument locally in both formats.)

- We will update the addition and subtraction problems by adding more two-digit numbers. This will also help increase the diversity of problems.

Task 8: Shape Recognition

One of the panel members recommended removal of the circle and square recognition tasks. Not all of the panel members agreed. We suggest leaving them in this task. One reason for leaving squares in this task is to understand children's knowledge of shape orientation.

A panel member asked if there is a way to assess spatial reasoning that is not related to shape symbols. Another suggested more of an understanding as to what children know about shapes. The first-referenced panel member suggested the reading by Dehaene, S., Izard, V., Pica, P., & Spelke, E. (2006): Core knowledge of geometry in an Amazonian indigene group, *Science, 311*(5759), 381–384.[1]

A recommendation was made to see what types of tasks are available to learn of children's knowledge of properties of shapes. Further discussion on properties of shapes and shape identification was tabled. Based on concern as to children's knowledge, in February–March 2009, RTI researched a proposed task to add to EGMA. The task could have children identify objects based on characteristics (e.g., "Which shape has four sides that are equal in length?"). This task has been developed, but due to the growing length of the instrument it may not be used. The shape recognition and pattern extension tasks may capture both the identification of shapes (shape recognition) and the recognition of differences in shapes, although it will not assess explicit (definitional) knowledge of shape characteristics.

Task 9: Pattern Extension

It was suggested that the pattern extension task be changed to "pattern and number extension." The first part of this task would be with geometrical shapes. The second part of this task would be with numbers. The missing number task would be moved here as part of Task 9.

Note: Based on a further review of the measures in February 2009, Pattern Extension will stay in its current location separate from the missing number task.

Vote on Existing and Proposed Measures

A list of all items currently in EGMA and proposed items was presented to the EMEP members. The panel was asked whether any of the current measures in EGMA should be removed. There was unanimous agreement that there were no tasks to be removed.

The EMEP and others participating at the meeting voted on a list of measures that had been proposed for addition to the instrument. These measures are described below:

- Number line estimation received a total of seven votes to be added to EGMA. To obtain a strong and complementary prediction to children's knowledge of number representation and magnitude comparison, it was recommended to add a number line estimation task to EGMA. The numerical estimation task will follow the quantity discrimination task.

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[1] See abstract at http://www.sciencemag.org/cgi/content/abstract/311/5759/381. For more detail, see http://www news harvard.edu/gazette/daily/2006/01/19-amazon.html.

- Properties of shapes received a total of five votes to be added to the instrument. This task is to look closer at a geometric description of the shapes (e.g., rotation, symmetry). Further discussion noted that for any of the geometric tasks, the predictive validity is in general not understood. Yet, it is important that children are familiar with these concepts, so it will be included.

- Counting by hundreds to fairly large numbers received four votes and will be added to an existing task (missing number).

- Knowledge/use of formal prepositions and rank order of numbers each received two votes. It was concluded that the objective of quantity discrimination would already get at rank order of numbers.

Based on the outcome from the votes, it was decided that number line estimation and properties of shapes would be added to the instrument (see Appendix 2). Moreover, it was agreed that once the instrument was updated, it would be sent to the EMEP members for review. Once reviewed, RTI would retest the instrument. Based on the new times for children to complete the updated instrument, RTI would work on cutting back on some of the tasks, such as shape recognition (e.g., circles) and pattern extension.

One item that did not receive any votes was children understanding statements such as "show me more" and "show me less." RTI will ask children to "show me which one is bigger," "show me which one is smaller," "show me which one has more" with the use of counters. Based on the conversation, it is important that we learn whether a child understands these terms or whether the child is having difficulty with the task that uses these terms. In other words, it is important that we know whether the child is having difficulty with the *language* or the *mathematics skill* that is the focus of the quantity discrimination task. RTI will add these items to the beginning of EGMA, directly before quantity discrimination.

Updated Early Grade Mathematics Assessment

The following list represents the changes made to the EGMA tasks based on the recommendations made by the EMEP in January 2009:

1. Counting fluency (not timed)
2. Counting one-to-one Correspondence (not timed)
3. Number identification (fluency, 60 seconds)
4. Quantity discrimination (60 seconds)
5. Number line estimation
6. Word problems (not timed; children will be guided through the items to keep a good pace)
7. Addition/subtraction (not timed; children will be guided through the items to keep a good pace)
8. Shape recognition (not timed; children will be guided through the items to keep a good pace)
9. Shape attributes (not timed; children will be guided through the items to keep a good pace)
10. Pattern and number extension (not timed; children will be guided through the items to keep a good pace)

Possible Critiques of the Approach

Legitimacy and balance in backing of EGMA tasks. One discussant suggested not being too reliant on the National Council of Teachers of Mathematics (NCTM) and the Trends in International Mathematics and Science Study (TIMSS). In response, it was noted that the initial curriculum comparison *in each country* is a critical part of the process, and EGMA will need to be validated for use in each country. In response, the EMEP noted that the use of the TIMSS and the National Assessment of Educational Progress (NAEP), as well as the review of the literature and curriculum to create the items, was good. The instrument needs to have legitimacy and backing in the creation of the items it contains. It was also noted that although some tasks may not be, *a priori*, familiar in a given society, or included in the curriculum, one could argue for including these tasks in the instrument because research shows their importance. Understanding of the number line is one such issue. Research shows this to be a critical skill, and thus one may want to interest countries in making sure that an understanding of the number line becomes a curricular objective.

One of the panel members added that there are two separate issues here. A first issue relates to cultural awareness and factors that might detract from validity: If some tasks are too foreign and not related to the curriculum, they might be seen as invalid or pointless. A second issue is that this is not a high-stakes instrument: It can be used to generate discussion about items that perhaps should be in the curriculum or in learning objectives in the country.

Street mathematics. The issue of "street mathematics" came up. There was some discussion of the fact that in some countries children appear to have good arithmetic skills in the marketplace (making change, etc.), and that these same children often test badly in mathematics assessments. Countries' policy makers then sometimes wonder whether the assessments are assessing poorly. The panel noted that in these cases the arithmetic skill children display in "street mathematics" are very concrete and limited to their specific needs, and don't often transfer to other situations. They may not be based on knowledge of principle. Thus, while acknowledging that "street mathematics" is useful, the panel noted that one still needs to assess in a formal way.

Timing. Some of the panel members noted that timing can be an issue; there may be criticism of the idea of timing children in completing a task. However, it was further noted by the EMEP that timing seldom bothers children; it does seem to bother adults. Timing of some of the tasks is vital to establish fluency, to reduce the time taken to complete the assessment, and to relieve the stress children might feel in trying to perform a task unsuccessfully for an indefinite period of time. Children sometimes see the fact that the assessment is timed as making it more of a game. In any case it will be important to build rapport with the child at the beginning of the assessment. It was also noted that

- In the instructions for timed tasks, the instruction to be read by the assessors to the children must let the children know that they are being timed; and
- The instruction should also emphasize getting the items in the task correct, but using the best strategy possible.

Counters. The panel did not see possible criticism due to the use of counters. Counters will allow a more efficient method of solving problems, and children may have difficulty with other methods. Counters seem to be a universal method to solving problems.

Data entry and quality data collection. The group discussed using a data entry strategy that would enable all data to be entered. This would create an opportunity for assessing differences in children's abilities within tasks. This information may be too detailed for teachers, but it would allow us to look across and within tasks for reporting purposes.

In any case, the intent has not been to produce a single summary score. One panel member noted that, importantly, if this kind of tool is put into teachers' hands, it is even more important not to have just a single summary score. (Having a single summary score is simply not practical, unless a great deal more research is done.)

To ensure quality data collection in a standardized fashion across all locations, a strategy of spending at least 5 days on training and explaining was recommended and agreed upon. These 5 days would include introducing the project and the assessment, practicing each task, and having trainers observe enumerators doing the assessment and recording responses to questions, to ensure standardization and accuracy. Enumerators will be certified once they demonstrate the ability to successfully implement the assessment.

Language. It will be important to take care with language, customs, country-specific myths, and unfounded opinions. Furthermore, it will be important that we understand any differences between the language being used in the current version of EGMA versus the language being used in a culture. For example, apparently in Ethiopia, at least in one key language, there is no specific traditional distinction between a rectangle and any four-sided shape. One may thus want to learn about and use the formal names in the language/culture where the assessment will be used or the types of terminology typically used in teaching and learning (e.g., "X plus Y equals what?" versus "what is X and Y altogether?"). A World Bank commentator noted that we may need to make sure that a very clear language policy is followed; and it may make sense that it be the same as in a written test for which concurrent or predictive validity is used to calibrate the EGMA tool.

Another panel member argued for a policy of flexibility in the instruction giving; namely, to explain things in a specific language, but to use appropriate code switching to maximize the child's chance to understand. However, this may be a problem if one wants to have some sort of standardization and understanding. For now, some flexibility will be allowed, but with guidance as to alternative ways to instruct or prompt the children in order to achieve uniformity.

Legitimate concerns versus misunderstandings. The panel acknowledged the difference between possible legitimate concerns regarding the tool, and concerns based on misunderstandings. It was recommended by a World Bank commentator that we must ensure the test is not seen as an accountability tool, but more for system diagnostics and then for use by teachers (with adaptation).

In addition, there was legitimate concern that children may not be used to being tested in this way. This is another reason it is important to present this as a research or diagnosis tool, not as an accountability or high-stakes tool. We have to emphasize low individual consequences for anyone. In any case, we will not be gathering or attaching the children's names to the assessments. We will continue to emphasize this in the consent form that we read to the children. It was also suggested by one of the panel members that in the medium term, it may be wise to involve teachers as research partners.

Computation versus conceptual skills in the assessment. The EMEP congratulated RTI for not overemphasizing computation. The panel reported that EGMA is well-balanced and has a good conceptual emphasis. It was also noted that some people might not understand some of the assessment as conceptual, as some of the aspects appear purely computational. Tasks such as quantity discrimination, word problems, and the number line are all conceptual, even if in some cases they might be tied to the notion of fluency.

The group believed that there are enough pure fact-retrieval items, and that in any case, fact retrieval is demonstrated in such tasks as the word problems and addition/subtraction problems. In fact, for some tasks, fluency is emphasized, and fluency generally increases when children use a fact-retrieval or automatized strategy. Note that in each task (not in each item), at least for some key tasks, assessors will be asked to judge whether children are using less-efficient strategies.

Informing Teachers and Classroom Practice

The panel noted that teachers almost universally fear mathematics. They often don't believe that they can teach mathematics. EGMA can help dilute some fear by making the goals more explicit. Teachers often fall back on memorization and definition of facts because they don't have a way to grasp the conceptual tasks themselves in a simple way; EGMA helps them do that. In addition, many teachers don't know what to set as standards, other than computation. This is why teachers sometimes fall back only on computation.

Many of the panel members emphasized that the beauty of EGMA is that it is actually a test one *wants* teachers to teach "to" and teach "from." This is a positive feature, because the balance between the computational and the conceptual is good, and because the computational aspects are building blocks to the conceptual.

Two levels of the "instructional response" to an assessment such as EGMA were identified: 1) the system level, and 2) the teacher level. The system level needs answers to questions such as how children are doing in mathematics, and can use these answers to feed deliberations regarding curricular improvement and teacher training improvement. Teachers can use the tasks contained in EGMA to improve their technique, track individual children, and determine whether changes in technique are having an effect.

It was also noted by a couple of the panel members that another instructional implication is the connection between some of these tasks and clear instructional strategies such as board games (e.g., in the U.S., Chutes and Ladders). As children play Chutes and Ladders, they are exposed to number-words they have said and heard. Children are also using their fingers and pennies to count results. Their sense of the number line is enhanced. Knowledge and practice with these skills make kids better at arithmetic problems. Using well-structured games as part of instructional strategy may be something to experiment with.

The panel was asked to review the list of recommended tasks for EGMA, and to give an opinion as to whether they all had instructional implications. There was general agreement that the tasks do have instructional implications. One panel member pointed out the Test of Early Mathematics Ability (TEMA), with instructional implications for each testlet, and suggested touching base with Herbert Ginsburg, the developer of TEMA.

It was emphasized to the group that the main aim of EGMA is as a sort of system diagnostic, and it does not aim to produce child-level data, in general, unless and until teachers use it in the classroom.

It was also emphasized that the items in the current version of the EGMA instrument were chosen because of their predictive power, and because there is evidence that even very young children, if they have had proper instruction, can carry out these tasks quite easily. Also, the items are predictive and they interlock in producing a good overall picture of capabilities. For instance, good skill at quantity discrimination underpins one of the more efficient early methods for addition (counting from the largest addend).

A question was raised as to whether one could gather background data such as socioeconomic status (SES) and perform correlations, and that this could be used to target resources (e.g., by area of the country, SES). RTI responded that such background data are gathered in the Early Grade Reading Assessment (EGRA), but not very much of it has been used thus far. Some data will continue to be gathered. On the other hand, the tool has not generally been used as a targeting tool when applied on a sample basis. It can be used in schools and by teachers to improve, though, if they feel they are below where they need to be in terms of either curricular standards or normative results (which are not available yet).

<u>Norms and standardization</u>. Some discussion addressed growth norms. A proposal was made to apply the assessment in good schools, or schools that do well on national assessments, or school-leaving exams, as a way to begin to set norms. Another proposal focused on taking normed achievement tests such as the Stanford, which has a large experiential record and large sample sizes, and comparing some children's concurrent results to EGMA as a way to set some norms or standards regarding growth paths. This could be very helpful with instruction. It was also noted that it could be useful to look at the standards by country, and to look at any national-level mathematics tests currently being implemented.

As for reliability and validity, it was noted that the quantity discrimination task is reliable and sensitive to instruction. Obtaining reliability and validity (concurrent and predictive) of the instrument will be important.

One of the panel members noted that because mathematics is hierarchical, it might be acceptable to tell teachers how children score, without standardization.

Caution was also noted in regard to the interpretation of results due to variables that can vary by country, such as gender differences.

Are There Interesting Technological Applications to Either Assessment or Remediation?

The group discussed the application and implementation of EGMA through information and communication technology (ICT). One option presented was the use of a stylus and electronic pads. Some panel members voiced concern with this method due to screen sensitivity. Further conversation noted that children have a definite learning curve, and it can take children many hours to master the technology, which makes it difficult to use the technology in assessment. There was also concern about children's fine motor skills, especially when they are being asked to make a mark—e.g., in the number line task.

Additional conversation took place about the expense of having engineers develop the software to be used for tasks or for programs for children's use in learning; e.g., programs that could be developed and used on an Apple iPhone. A couple of the EMEP members believed that testing the tool in an ICT-based platform would be labor-intensive and expensive.

Regarding the use of ICT for remediation (discussion thus far had focused on assessment), one person mentioned Douglas Clements' Building Blocks[2] as an approach worth exploring.

It was suggested that to explore the role of ICT in these areas, the following individuals would be good contacts: 1) John Sealey Brown[3] with the Xerox Corporation; 2) Roy Zimmerman[4] of AIR, who did some ICT work on early childhood for a pilot project in Nicaragua; and 3) John Bransford.[5]

In all of this discussion, it was noted that the weakest link, and thus the most important thing to consider, is teacher training, and that ICT can play only a modest role if teachers are not well trained.

[2] Building Blocks: http://www.gse.buffalo.edu/org/buildingblocks/index_2.htm
[3] John Sealey Brown: http://www2.parc.com/ops/members/brown/cv.html
[4] Roy Zimmerman: http://blog.ompt.org/2009/01/19/week-of-conversations/
[5] John Bransford: http://faculty.washington.edu/bransj/; and the following: http://net.educause.edu/NLII051/Program/1804?PRODUCT_CODE=NLII051/GS01

Appendix 2: Discussion of New Measures Based on Expert Panel Recommendations

Development of Number Line Skills

A numerical estimation task was proposed at the January 2009 meeting by Robert Siegler (see Appendix 1 for more detail), who has conducted studies using this form of estimation with children from kindergarten through second grade. Estimations on a number line do not require knowledge of measurement units (Siegler & Booth, 2004).

Children's number sense can be estimated with tasks such as quantity discrimination and number identification (Dehaene, 1997). But one can go further in the assessment of number sense and assess children's awareness and accuracy of use of the number line. Dehaene (1997) claims that the use of the number line can demonstrate the association between written number notation and space, or what he defined as "the mental representation of numerical quantities" (p. 81). Siegler and Booth (2004) and Siegler and Opfer (2003) have documented that children's representation of numbers on a number line, in relation to where the numbers actually are, follows a logarithmic pattern in the early grades (e.g., kindergarten in the United States, but maybe later in poorer countries), and then a linear pattern in later grades.

Also, young children (e.g., grade 2, but maybe later in poorer countries) may be able to place numbers linearly on the number line if the number line represents only, for instance, 0 to 100, but may place them nonlinearly if the number line represents 0 to 1,000, and this may have to do with practice. As children become more experienced with the formal number system (e.g., counting, number identification, number value), they begin to develop a linear representation of numbers even on number lines ranging from 0 to 1,000 (Siegler & Booth, 2004), and they place numbers on the number line where they should be. With age and experience, choice of strategy and representation in estimation change (Siegler & Opfer, 2003).[1] Siegler and Booth (2004) have also demonstrated a positive correlation between the linearity of children's representation of numbers on the number line and the actual position of the numbers, and math achievement scores such as those obtained on the Stanford Achievement Test (see Siegler & Booth, 2004). That is, linearity of representation is an important precursor skill.

In EGMA, the number-to-position (NP) task asking children to identify the position of a specific number on a number line will be used. The number line estimation will be based on the studies conducted by Siegler and Opfer (2003) and Siegler and Booth (2004). The number line for each problem will be 23 centimeters in length with a "0" labeled at the left end and a "100" labeled at the right end of the number line. The number that children will be asked to place will be located above the center of the number line. There will be a total of 10 number line items, with two per page. The interviewer will give the child a pencil and ask the child to place a line through the number line for the number that is displayed. Two practice items will assist in teaching the child how to respond to the problems for this task. These two practice items will be modeled after those used by Booth and Siegler (2008). Children will have 5 seconds for each item. There will

[1] Siegler & Opfer (2003) observed that children's accuracy in estimation in second grade followed a logarithmic pattern more than a linear pattern. Linear patterns were observed with sixth graders.

be a total of 10 items. Children will work on the 10 randomly ordered numbers and then go through them again in the same random order. The two estimates of the same number will be averaged for a reliable estimate of the child's intent. The numbers that will be used for this task will be based on the numbers used by Booth and Siegler (2008).

Shape Attributes

All children have some level of knowledge about shapes. The ideal period for children to learn about shapes is between the ages of 3 and 6 years (Clements, 2004a). As children learn about shapes, they also learn more in-depth concepts about shapes, such as the fact that triangles have three angles and three sides; and may also learn that triangles are not always isosceles. Children will learn that although triangles always have three sides and three angles, they can have different combinations of angles and different sizes and still be categorized as triangles (Clements, 2004a; Clements, Swaminathan, Hannibal, & Sarama, 1999). Through learning and primarily observing and drawing attributes, children have an opportunity to develop language and form ideas about shape properties (e.g., a square has four equal sides). The descriptive level of geometry builds off knowledge and familiarity with shapes, and lays a foundation for future geometric problem solving (Clements et al., 1999).

There is also evidence that in geometry, as in arithmetic (and as opposed to reading), some knowledge may be relatively "built in" to human cognition. In research conducted by Dehaene, Izard, Pica, & Spelke (2006) on basic concepts of geometry, an array of six objects was presented to participants from the Mundrurukú tribe in the Amazon, a tribe known to have a less developed sense of mathematics than many other pre-state peoples. (Work with such a tribe would allow one to test hypotheses about "base" levels of mathematical sense in human culture.) Five of the images represented a specific shape, but with different sizes and orientations (e.g., squares, but differently sized and oriented). The sixth object violated the attributes of the other five objects (e.g., it might be a rectangle). The Mundrurukú were asked to point to the one object that was "weird" or "ugly." They performed this part of the study very well. The mean performance of American children and Mundrurukú children was highly similar on this task (r = 62%), which suggests a shared "base" knowledge of geometry, and also a similar "base" level of ability to deal with problems of similar difficulty (Dehaene et al., 2006). The skill to be able to judge shapes to be similar, even though they are differently sized and oriented, however, can also be worked on. Clements (2004a) stressed the importance of children experiencing many different examples of a type of shape, including nonexamples similar to the attributes of the type of shape being observed, but different in one of its attributes. This allows children experiences and opportunities for further discussion and learning of shape characteristics.

To ensure that children are familiar with and recognize the attributes of shapes, EGMA will present a page with six shapes to children. All but one shape will share similar attributes but they will all have different sizes and orientations. Children will be asked, based on a simple description of the shapes (e.g., shapes with three sides and three angles), to point to the shape that does not fit the description. This will test the tendency of some children to be confused by size and particularly by orientation. As Clements (2004a, p. 152) notes, young children's perceptions of shapes can be based on "imprecise visual qualities and irrelevant attributes, such as orientation, in describing the shapes while omitting relevant attributes." This task will consist of four questions, based on a triangle, rectangle, circle, and square. Children will have to compare the properties of shapes based on the description. Through the comparison, they will need to identify the shape that does not belong.

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