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2003 Review for Level II

Economics
Debt Investments
Derivatives

By
David E. Upton, Ph.D., CFA
Professor of Finance
Virginia Commonwealth University

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David E. Upton, PhD, CFA
Professor of Finance
Virginia Commonwealth University

Tel. 804.828.7169
deupton@vcu.edu
fax: 804.828.3972

2002 Preparation for Level II

Executive Summary

Economics
Debt Investments
Derivative Investments

David E. Upton, PhD, CFA
Professor of Finance
Virginia Commonwealth University

Fax: 804.828.3972
deupton@vcu.edu

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Growth and Accumulation

1. An important concept is the production function. A production function expresses the output of physical goods in an economy in terms of the inputs – e.g., it is of the form:

   $$Y = AF(K,N)$$

   Where Y is output, A is the level of technology, K is the amount of capital employed, N is the amount of labor employed, and F(*) indicates an unspecified relationship.
2. The production function is converted to a "growth accounting equation," in which change (growth) in output is expressed as a function of the changes in the inputs, <u>weighted by the amount of output flowing to the inputs</u> (the input "share"):

   $$\frac{\Delta Y}{Y} = \left[(1-\Theta)\ X\ \frac{\Delta N}{N}\right] + \left[\Theta\ X\ \frac{\Delta K}{K}\right] + \frac{\Delta A}{A}$$

   This simply says that:
   output growth = labor share X labor growth + capital share X capital growth + technical progress
   This relationship can be used to analyze the sources/factor of growth.
3. Growth in total output is $\Delta Y/Y$, but the effect on per capita output is affected by any changes in N. I.e.,

   $$\frac{\Delta Y}{Y} = \frac{\Delta y}{y} + \frac{\Delta N}{N},$$

   where lower case such as y indicates a per capita variable. . The result is a growth accounting equation in per capita terms:

   $$\frac{\Delta y}{y} = \Theta\ X\ \frac{\Delta k}{k} + \frac{\Delta A}{A}$$
4. The neoclassical growth theory (later referred to as the "Solow" growth model) combines two assumptions:
   a) diminishing marginal product of capital – as more capital per capita is added, the gain from each additional unit of capital will decrease,
   b) the investment required to maintain the level of capital per capita varies directly with the level of per capita capital – i.e., no matter what the level of capital per capita, each unit of capital per capita requires the same investment for maintenance/replacement.
5. In the neoclassical model, when the marginal product of capital is above the investment rate, an added unit of capital will contribute more to output than it costs to maintain/replace – more capital will be added. But when the marginal product of capital is below the investment rate, an added unit of capital will cost more to maintain/replace than it will contribute output – capital will be reduced. Because the marginal productivity of capital is declining, the economy will move to a steady state in which marginal product of capital is just equal to investment in maintenance/replacement
6. Because the economy will move to a steady state, and that steady state is determined by the marginal productivity of capital, savings has no effect on the growth rate (although it may affect the level from which growth occurs).

Growth and Policy

1. As a result of the diminishing marginal productivity of capital, the neoclassical growth theory implies that the savings rate does not affect the growth rate – but empirical observation indicates that there is a relationship, with higher savings associated with higher growth.
2. The endogenous growth theory modifies the assumption of diminishing marginal productivity of capital (the text presents a first simplified version in which the marginal productivity of capital is constant). Under this modified assumption, the marginal productivity of capital and the reinvestment rate required for maintenance/replacement no longer intersect and there is no equilibrium – a higher savings rate will lead to higher growth.
3. Unfortunately, the endogenous growth model implies that eventually one firm will emerge – which is also contrary to empirical observation. This problem is solved by assuming that there is a difference between social and private marginal productivity in that private investment is unable to capture all the returns to investment – i.e., there are externalities that benefit society but do not benefit the investing firm.
4. There is some disagreement over the idea that economies will "converge."
    a) Neoclassical growth theory predicts absolute convergence" – i.e., all economies with the same rate of savings, population growth, and technology will converge to the same steady-state income. Economies with different savings rates or different population growth will reach "conditional convergence" – i.e., steady-state incomes will differ, but growth rates will equalize.
    b) Empirical observation indicates that although countries with a high savings rate tend to grow faster, the effect of higher investment on the growth rate is transitory. This implies that endogenous growth rate theory "is not very important for explaining differences in international growth rates," except perhaps in the short run.
5. The basic idea here is that population growth decreases the amount of capital per worker and results in slower growth. There is an extension linking population growth to income:
    a) Population growth is a combination of birth rate and death rate.
    b) Poor countries tend to have high birth rates, but this is offset by high death rates – the result is moderate population growth.
    c) Moderate increases in income tend to cut the death rate while not affecting the birth rate – the result is a high population growth rate.
    d) At high incomes the birth rate tends to decrease and results in a lower rate of population growth.
6. Although limited natural resources would seem to limit economic growth, this argument is offset by two factors:
    a) Technical progress permits us to produce more using fewer resources.
    b) As specific resources come into short supply, their prices rise and producers shift toward substitutes.

The Foreign Exchange Market

1. The reading begins by discussing some basics of foreign trade: spot and forward markets, foreign exchange quotations, spread (which is calculated as a percentage departure from the ask price), cross rates and forward discount or premium. Note that spread is:

a) narrower for widely traded currencies,
b) narrower for les volatile currencies,
c) wider for longer forward contracts.

The spread may also be a function of the position of the particular dealer – i.e., oversold (shortage0 or overbought (excess).

2. A triangular arbitrage opportunity arises when direct conversion between currencies is possible at a different rate than conversion through a third currency.
3. Interest rate parity requires that a difference in return for foreign investments should be offset by changes in the exchange rate, so that the net return is the same in both countries. I.e., interest rate parity requires that:

$$1 + r_h = \frac{(1 + r_f)f_1}{e_0}$$

where r_h is the home country interest rate, r_f is the foreign interest rate, f_1 is the forward exchange rate, and e_0 is the spot rate. If interest rate parity is violated, there is a covered interest rate arbitrage opportunity (or a reverse covered interest rate arbitrage opportunity):
a) borrow in one country,
b) change to another currency and lending in that currency market, and then
c) reconvert to the original currency and pay off the original borrowing (supposedly at a profit).

4. Rewriting the interest rate parity condition we can find the required future rate:

$$f_1 = e_0 \text{ x } \frac{1 + r_h}{1 + r_f}.$$

If this condition does not hold, an arbitrage opportunity exists (in the absence of transaction costs).

5. The presence of transaction costs would result in a range of possible rates for these relationships.

Parity Conditions

1. Purchasing power parity comes in two forms:
   a) absolute purchasing power parity indicates that prices in all countries should be the same after adjustment for exchange rates. This form is not in common use.
   b) relative purchasing power parity indicates that exchange rates will reflect <u>changes</u> in price levels.
      - the price levels are not necessarily equal, but the <u>rate of change</u> in the price levels is equal.
2. The relationship implied by purchasing power parity is:

$$\frac{e_t}{e_0} = \frac{(1 + i_h)^t}{(1 + i_f)^t}$$

or, to find the exchange rate implied by PPP:

$$e_t = e_0 \text{ x } \frac{(1 + i_h)^t}{(1 + i_h)^t}$$

Note that this relationship is in terms of inflation rates i.

3. The real exchange rate e'_t is the nominal exchange rate e_t adjusted for changes in relative purchasing power since time 0:

$$e'_t = e_t \text{ x } \frac{(1 + i_h)^t}{(1 + i_h)^t}$$

4. The (expected) inflation is relatively higher in countries that have higher interest rates or futures rates which indicate devaluation.
5. The discussion assumes that a country's capital market is "integrated" as opposed to segmented – i.e., that rates are determined by global conditions rather than (more than) local conditions.
6. The Fisher effect indicates that the nominal interest rate has two components, the real rate and an inflation premium, and that these two components are related as:

   1 + nominal rate = (1 + real rate)(1 + expected inflation rate)

   a) This is approximated by the formula nominal rate = real rate + expected inflation.

   The generalized version of the Fisher effect also assumes that because of arbitrage, the real rate of return is equal in all countries, so that:

$$\frac{(1+r_h)}{(1+r_f)} = \frac{(1+i_h)}{(1+i_f)}$$

7. Combining the Fisher effect with purchasing power parity implies the International Fisher effect:

$$\frac{e_t}{e_0} = \frac{(1+r_h)^t}{(1+r_f)^t}$$

   where e_t is the expected exchange rate at time t.

   a) This can be written:

$$e_t = e_0 \text{ x } \frac{(1+r_h)^t}{(1+r_h)^t}$$

   to compute the expected exchange rate at time t.

   b) In the absence of biases, this implies that the forward rate should equal the expected exchange rate at time t.
8. These parity relationships provide forecasts based on market conditions. In an efficient market, it would not be possible to better these market-based forecasts. Whether model-based forecasts are better is an open question. Although some tests indicate the possibility that model-based forecasts may outperform market-based forecasts, these tests have been within-sample and so have limited validity.

International Asset Pricing

1. Integrated markets are those with no barriers to capital flow so that new information can be rapidly exploited. Segmented markets have impediments to capital mobility, so that relative mispricing cannot be rapidly exploited. Impediments include:
   a) Psychological barriers.
   b) Legal restrictions.
   c) Transaction costs.
   d) Discriminatory taxation.
   e) Political risks.
   f) Foreign currency risks.
2. Extension of the domestic CAPM to an international context would use the domestic rate for the risk-free rate, and the market capitalization weighted portfolio of all risky assets for the market portfolio. This requires two unreasonable assumptions beyond those of the domestic CAPM:
   a) Investors throughout the world have identical consumption baskets.

b) Purchasing power parity holds exactly, so that real prices of consumption goods are identical in each country.

3. The real exchange rate is defined as the actual exchange rate multiplied by the ratio of the price levels of the consumption baskets in the two countries:

   Real exchange rate = nominal exchange rate X (foreign price level / domestic price level)

   $$X = S\,x(P_F/P_D)$$

4. The foreign currency risk premium is the amount by which the expected appreciation exceeds the appreciation implied by interest rate parity. The expected appreciation can be expressed as a forward premium, and can be approximated as the difference between the two interest rates.
5. The international capital asset pricing model for one foreign currency can be expressed as:

   $$E(R_i) = R_0 + \beta_{iw} x RP_w + \gamma_i x SRP_{FC}$$

   Where R_0 is the risk-free rate, β_{iw} is the world beta, RP_w is the risk premium on the world index, γ_i is the sensitivity of asset i domestic currency returns to the Foreign currency, and SRP_{FC} is the foreign currency risk premium. For multiple currencies, the last term is a series.
6. Currency exposure is essentially the sensitivity of asset returns to currency movements. I.e., if an asset is highly correlated with the movements of a currency, that asset would have a large exposure to that currency.
7. If purchasing power parity held exactly or closely, exchange rate changes would simply reflect differential inflations, and would be unimportant. However, purchasing power parity seems widely violated. It is the movements in the real exchange rate are large relative to purchasing power effects.
8. There are two models of the effect of exchange rates on an economy:
   a) The traditional model: depreciation of the currency tends to increase competitiveness. At the same time the depreciation increases the cost of imports, which increases inflation and reduces real income – resulting in an initial decrease in domestic demand and production. Eventually, however, the increased international competitiveness and increasing exports will reverse the initial decline. The reading warns that if the economy is sluggish to react, the original decline may not be reversed and conditions may continue to worsen.
   b) The money demand model: "In this model, real growth in the domestic economy leads to increased demand for the domestic currency through a traditional money demand equation. This increase in currency demand induces a rise in the relative value of the domestic currency."
9. Since economic activity is one of the major factors affecting equity markets, the effect of interest rate changes on equity markets is in the same direction as the effect on economic activity.
10. The effect of exchange rate movements on bond markets is through the effect on interest rates. Again, there are two theories:

    a) If appreciation of the home currency is associated with an increase in real interest rates, this would have a negative effect on bond prices. The reading notes that an increase in interest rates associated with inflationary expectations, however, could cause a depreciation of the currency – leading to an even greater loss for international investors.

    b) The second theory is associated with policies that contain foreign exchange rate targets. In this case, depreciation of he local currency causes the authorities to increase interest rates to defend the currency, and appreciation would trigger a decrease in interest rates. This situation would result in a positive bond price effect from appreciation of the currency, but a negative effect from depreciation of the currency.

Analyzing the Firm's Environment

1. Industry sales can be estimated using linear regression of the historic relationship with GDP, and a forecast of GDP. Aside from the GDP forecast, this is a straightforward statistical application.
   a) If the regression is in monetary terms (e.g., $ sales), observations should be adjusted for inflation. It the regression is in physical terms (e.g., units sold), observations do not need adjustment for inflation.
2. Firm sales can be estimated from industry sales using a market share estimate.
   a) Market share should be adjusted to take account of economic conditions. E.g., in an economic slowdown, sales of discount stores will be affected differently than sales of full-service or upscale stores.
   b) Short-run forecasts rely heavily on forecasted macroeconomic variables. In the long run, however, macroeconomic variables are assumed to be at an average level, and the emphasis is on firm characteristics.
3. The life cycle (stages of development, growth, maturity, and decline) is useful for forecasts.
   a) It is sometimes difficult to separate the life cycle of a product from that of an industry.
4. Other factors affecting a firm's market share include:
   a) changes in consumer tastes,
   b) entry or exit of competing firms,
   c) relative strengths and weaknesses of industry firms,
   d) changes in general market conditions,
   e) marketing strategies of firms in the industry.

   The effect of marketing strategy is a function of both the effort put forth and the effectiveness of the effort. Market share has been modeled as a function of the effort and effectiveness of the firm as compared to the industry, but the inputs for the model can only be guessed at.

Asset Valuation – Debt Investments

Bond Yields and Prices

1.

Coupon	Maturity	YTM	Price		Δ Price	% Δ
6%	20 yrs	8%	804	A	- 196	- 19.60
6	3	8	949	C	- 51	- 5.10
6	20	6	1000			
6	3	6	1000			
6	20	4	1271	B	+ 271	27.10
6	3	4	1056	D	+ 56	+ 5.60
6	30	20%	320.95			
6	30	18	337.98	E	+ 17.03	+ 5.31
6	20	20	318.26			
6	20	18	357.67	F	+ 39.41	+ 12.38
15	20	8	1687	G	- 345	- 16.98
15	20	6	2032			
15	20	4	2495	H	+ 463	+ 22.79

a) Bond prices move inversely to yield. Compare price changes A to B, C to D.
b) Longer bonds will have larger price changes. Compare price changes A and B to C and D.
c) As maturity increases, volatility (per cent price changes) tends to increase at a decreasing rate. Compare per cent changes A and B to C and D.
   - **NOTE**: this rule is not always true – for deep discount bonds, volatility may actually decrease with maturity. Compare per cent changes E and F. Most texts ignore this, and the usual assumption is that the longer the bond, the greater the volatility.

d) For a given absolute change in the ytm, the price increase due to a decrease in yield is larger than the price decrease due to an increase in yield - i.e., price and volatility is not symmetrical. Compare price and per cent changes A and C to B and D.
e) For a given absolute change in ytm, bonds of higher coupon have less volatility. Compare per cent changes G to A and B, and H to C and D.
f) Note the implications for trading strategies:
   - If yields are expected to increase, price change will be downward and investors should seek the lowest volatility - i.e., short maturities and high coupon.

   b) If yields are expected to decrease, price change will be upward, and investors should seek the highest volatility - i.e., long maturities and low coupons.

2. Yield to maturity is simply the rate that equates price to present value of the cash flows of interest an principal. Problems include:
   a) This assumes a flat yield curve (the discount rate is a constant for all periods).
   b) This implicitly assumes that all cash flows can be reinvested at the yield to maturity.

c) Note that the convention of annualizing using bond equivalent yield (twice semiannual yield) understates annual yield. The correct (but not conventional!) method would be:

$$\text{Annual Yield} = (1 + \text{Semiannual Yield})^2 - 1$$

3. The yield to maturity is sometimes described as the "promised yield" because the actual realized yield, or horizon yield, may be different if reinvestment is not at at the yield to maturity:

EXAMPLE: Assume a 4-year, 8% annual pay bond priced at 94, having a yield to maturity of 9.89%.

Case A: All cash flows reinvested at 9.89%

Time	1	2	3	4
Cash flows	$80(1.0989)^3$ =			106.16
and Value		$80(1.0989)^2$ =		96.60
at Maturity			$80(1.0989)^1$ =	87.91
				1080.00
Total value at Maturity				1370.67

$$\text{Realized Return} = \left(\frac{1370.67}{940}\right)^{\frac{1}{4}} - 1 = 0.0989 \Rightarrow 9.89\%$$

Case B: All cash flows reinvested at 8%

Time	1	2	3	4
Cash flows	$80(1.08)^3$ =			100.78
and Value		$80(1.08)^2$ =		93.31
at Maturity			$80(1.08)^1$ =	86.40
				1080.00
Total value at Maturity				1360.49

$$\text{Realized Return} = \left(\frac{1360.49}{940}\right)^{\frac{1}{4}} - 1 = 0.0968 \Rightarrow 9.68\%$$

4. It is extremely important to realize that bond quotes for semiannual pay bonds are annualized on a bond equivalent yield basis.

a) Bond equivalent yield is twice the semiannual yield:

$$\text{Bond Equivalent Yield} = 2 \text{ X Semiannual Yield}$$

b) This convention understates the true yield, which is calculated as:

$$\text{True Annual Yield} = (1 + \text{Semiannual Yield})^2$$

Duration and Convexity

1. Duration is a measure of the percent change in price for a change in yield. Like so many measures in finance, there are different versions of this measure. The version that is most relevant here is: Duration is the percent change in price for a 100 basis point change in yield.
2. Duration has several uses:

a) It is a measure of volatility or risk – i.e., higher duration means greater volatility. Unlike the four rules given above, duration is a single number reflecting the effects of maturity, coupon, and yield.

b) Given the duration, the percent price change can be quickly computed as:

% change in price = - duration X basis point change in yield / 100

Note the negative – price an yield are inversely related! The placement of the decimal point is easily done by noting that a 1% (100 basis point) change in yield will give a percent change in price equal to duration – i.e., if yield changes by 1%, a bond with a duration of 7 will have a 7% price change, if yield changes by ½ % (50 basis points) a bond with a duration of 7 will have a 3.5% price change, etc.

c) Duration is also useful in portfolio "immunization." This is discussed below.

d) The dollar change in the price of a bond can be computed by using dollar duration:

dollar duration = price X duration

dollar change in price = price X - duration X basis point change in yield /100

= - dollar duration X basis point change in yield /100

Dollar duration is important because it measure absolute changes in price – i.e., two bonds with the same duration, but different prices, will have the same percent price change but different dollar price change.

3. Duration is computed by changing the yield up or down by equal amounts, computing the new prices at these higher and lower yields, and using those prices to compute the price change over a 100 basis point change in yield. I.e., effective duration is computed as:

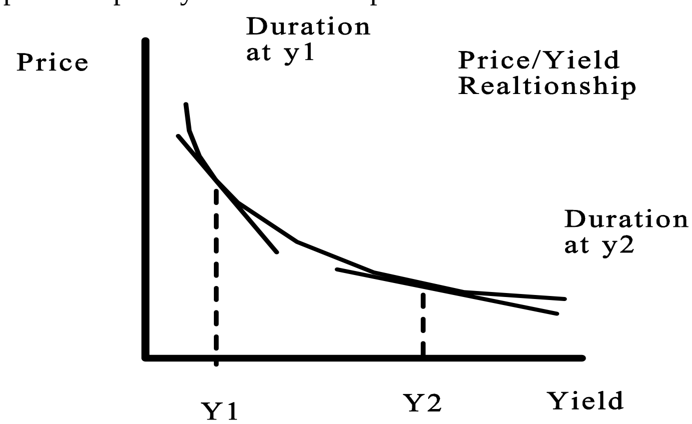
$$\text{duration} = \frac{(P_-) - (P_+)}{2PS}$$

where:

P_- is the price change due to a decrease in yield
P_+ is the price change due to an increase in yield
P is the original price
S is the change in yield expressed as a decimal (i.e., basis point change / 100)

4. Computing the prices to be used in computing duration requires a valuation model. There are different models, and the different models lead to different duration measures:
   - Modified duration – assumes that the cash flows are not affected by the yield change (i.e., that the bond is a "straight" or "option-free" bond.
   - Effective duration – the effects of embedded options are included in the duration calculation.
   - Empirical duration and Implied duration is estimated statistically using historical observations.

5. A different insight is gained from a graphical interpretation of duration. Duration can be shown as the slope of the price/yield relationship:



This diagram demonstrates several important facts:

a) Ceteris Paribus, duration is negatively related to yield. Note that the duration at y1 is greater than the duration at y2.

b) The price change calculated using duration is only an estimate.
   · Since duration is a derivative, it is only correct at the point of estimation, and since it changes with price it will lead to an incorrect estimate for finite price changes.
   · The error of estimation is small for small changes in yield.
   · Duration underestimates price increases and overestimates price decreases.

6. The curvature of the price/yield relationship is measured by "convexity."
   a) Convexity can be used to refine the estimate of the price change by adding the "price change due to convexity" to the "price change due to duration:"

   $$\text{price change due to convexity} = \text{convexity X (change in yield)}^2 \text{ X } 100$$

   where convexity is estimated using the same procedure used to estimate duration:

   $$\text{convexity} = \frac{(P_-) - (P_+) - 2P}{PS^2}$$

   b) Definitions of convexity differ among users and authors, it is necessary to be sur that consistent definitions are used.

7. Another interpretation of duration can be obtained by considering the effects of price risk and reinvestment risk.
   a) Note that price risk is inversely related to yields, while reinvestment rate risk is positively related to yields:

	Yields Increase	Yields Decrease
Effect on Price	Negative	Positive
Effect on Reinvestment	Positive	Negative

Duration can be considered as the point in time at which these two effects cancel- the balance point between price risk and reinvestment rate risk..

EXAMPLE: An 8%, 5-year annual pay bond has a duration of 4.0762 years when the yield is 10%.
➔**Note that the definition of duration has changed from last year. To find the point at which interest rate risk and reinvestment rate risk cancel compute:**

cancellation point = duration X (1 + i/2)

In this case, the cancellation point is at 4.28 years.

i) Consider the value of the position at 4.28 years if yield does not change:
From Reinvestment of interest payments:

Time (year) 1	2	3	4		4.28
$80(1.10)^{3.28}$				=	109.3599
	$80(1.10)^{2.28}$			=	99.4181
		$80(1.10)^{1.28}$		=	90.3801
			$80(1.10)^{0.28}$	=	82.1637
		TOTAL			381.3217

At 4.28 years, the bond will be valued at the present value of the final principal and interest payment:

$$\text{Value of Bond at 4.28 Years} = 1080 / (1.10)^{0.72} = 1008.6943$$

The total value of the position will be 1008.6943 + 381.3217 = 1389.6943.

ii) Consider the value of the position at 4.28 years if yield changes to 10.5%:
From Reinvestment of interest payments:

Time (year) 1	2	3	4	4.28
$80(1.105)^{3.28}$				= 110.9988
	$80(1.105)^{2.28}$			= 100.4514
		$80(1.105)^{1.28}$		= 90.9062
			$80(1.105)^{0.28}$	= 82.2681
		TOTAL		384.6245

At 4.28 years, the bond will be valued at the present value of the final principal and interest payment:

$$\text{Value of Bond at 4.28 Years} = 1080 / (1.105)^{0.72} = 1005.0853$$

The total value of the position will be 1005.0853 + 384.6245 = 1389.7098.
In this simple case of a one-time shift in rates, the increase in return from reinvestment (from 381.3217 to 384.6245 almost exactly balances the decrease in the price of the bond from 1008.6943 to 1005.0853. The difference is partly due to rounding, and partly due to the derivative nature of duration (duration is only exactly valid at one point).

c) Using this approach of setting the duration of the portfolio equal to the point at which a liability will occur, the portfolio can be "immunized" from changes in yields to provide a certain future value.
d) In real application, the process is more difficult - the portfolio will need rebalancing because:
   · There will be multiple shifts in yield, and the portfolio duration changes with the yield.
   · The portfolio duration will not change at the same rate as time.
   · Cash flows will need reinvesting.
   · Individual securities will no longer be suitable due to rating changes or other factors.
e) Another problem is that bonds of the required duration may not be available.

Bonds with Option Characteristics.

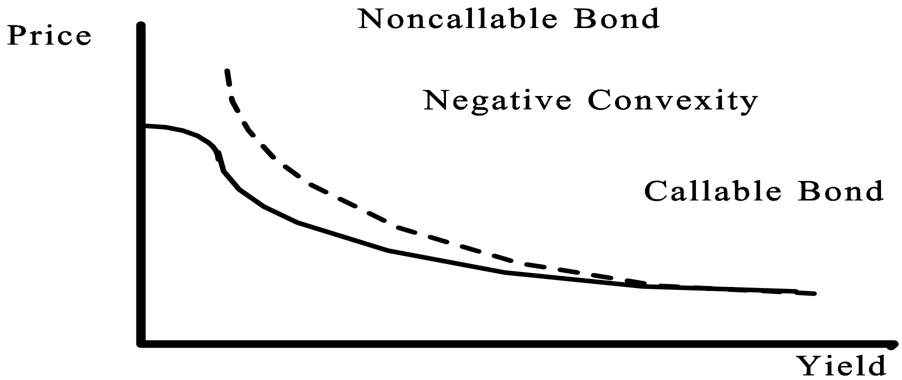
For some bonds, e.g. callable bonds, putable bonds, and mortgage-backed securities, the cash flow may be affected by yield levels.

1. If a bond is callable, the price-yield relationship of the bond will depart from the price-yield relationship for a conventional bond.
   a) The callable bond can be decomposed into a conventional bond and a call option - it is as if a conventional bond were purchased, and then a call on the bond was sold to the firm.
   b) The value of the callable bond would then be the value of the conventional bond , less the value of the call held by the firm:

      Value of Callable Bond = Value of Conventional Bond - Value of Call

   c) The value of the call option would depend on several factors:
      · The level of yields. The firm would not exercise the option at a yield above that implied by the call price. At high yield level, the call would be far out-of-the-money.
      · The volatility of interest rates. Higher volatility would increase the likelihood that the yield would fall below the yield implied by the call price, putting the call in-the-money and making exercise likely.

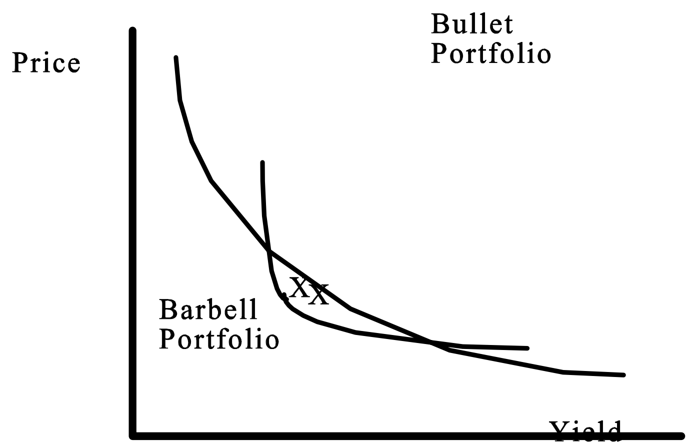
d) The resulting price-yield relationship is shown as:



· At high yield levels, the value of the call is negligible and the relationship is much like that of a conventional bond.
· As the yield level decreases, the value of the call increases as the probability of exercise increases. The value of the callable bond begins to fall measurably below that of a conventional bond.
· At some point, the value of the call will begin to increase at a faster rate than the value of the conventional bond. At this point the convexity of the relationship will change from positive to negative.
· The call price of the bond is an upper limit to the bond price (with some small caveats)

Bullet vs. Barbell

1. A bullet portfolio is a portfolio concentrated at one duration. A barbell portfolio typically is made up of assets concentrated at two different durations. A ladder portfolio is made up of assets spread approximately equally at several duration levels.
   a) The duration of a portfolio is simply the weighted average of the durations of the individual assets:
   $$Duration_{Portfolio} = \sum W_i (Duration_{Asset\ i})$$
   b) For bond portfolios having the same duration, the barbell will have the highest convexity, and the bullet the lowest convexity.
2. Even if the portfolios have the same duration, their performance may differ for two reasons:
   a. Differences arising from different convexity. Given an increasing yield curve, a barbell portfolio with the same duration as a bullet portfolio will have a lower yield. **NB**: the yield of a portfolio is <u>not</u> the weighted average of the yield of the individual assets (although it may be a close approximation)! The barbell will, however, have a higher convexity. This presents a choice between two desirable characteristics:
   · Higher yield is desirable,
   · If the yield changes the capital gain/loss of the higher convexity barbell would be better (i.e., higher gain or lower loss). Graphically:



Note that the durations (slopes) of the price-yield relationship is the same for both portfolios at the present point, but the convexities and yields are different. Which portfolio is preferred depends on the amount of yield change expected.

- · For small changes in yield, the difference in convexity will likely have little effect, and the higher yield of the bullet portfolio is more attractive (In the extreme case of no change in yield, the convexity has no effect and the higher yield of the bullet is clearly more attractive).
- · At the points where the price-yield relationships cross, the price changes and capital gain/loss are equal. The bullet, with equal gain/loss but higher yield, is still preferred.
- · Past the points where the price-yield relationships cross, the capital gain/loss of the barbell is better than the gain/loss of the bullet. At some point, the gain/loss of the barbell will exactly offset the higher yield of the bullet - beyond that point, the barbell is preferred.

In order to make the choice between the bullet and the barbell, the total return (yield plus capital gain) is computed for the expected yield curve change for both portfolios and compared.

- · This can be done using duration and convexity to compute the price change.
- · The yield change may not be a single figure - i.e., there may be a probability distribution.

b) Changes arising from twists (changes in shape) in the yield curve. Since the barbell is at two different maturities, and since the longer maturity is more sensitive, the reaction will differ:

- · If the yield curve is steeply up sweeping and expected to flatten, the rate on the long barbell maturity will exhibit a greater change than the shorter bullet maturity: the barbell outperforms.
- · If the yield curve is flat and short-term rates are expected to fall relative to short-term rates, the bullet maturity will be more affected than the long barbell maturity: he bullet would outperform.

NOTE: The last two points are generalization for large yield curve shape changes!

Term Structure / Yield Curve

1 The term structure is the relationship between the maturity of a bond and the yield of the bond. Properly speaking, "term structure" applies only to spot rates (the theoretical relationship between zero coupon Treasuries and maturity), while the term yield curve applies to the observed relationship between bond maturity and bond yield for coupon-bearing bonds, although the terms are more usually used interchangeably. There are basically three theories of the term structure.

2. Pure (unbiased) expectations theory is based on the same idea as that behind implied forward rates. I.e., If we wish to hold securities for eighteen months, there are many ways to do this. We could hold a series of three six-month securities, a one-year and then a six-month security, an eighteen-month security, or many other combinations. In order for the market to be indifferent, the yield on all of these possible combinations must be the same. I.e., considering the series of three six-month securities and the eighteen-month security, we must have:

$$(1+\text{ytm}_3)^3 = (1+f_1) \times (1+f_2) \times (1+f_3)$$

where ytm_3 is the yield on the eighteen-month security and f_n is the expected yield on a six-month security maturing at the end of six-month period n (referred to as the expected six-month forward rate at period n-1). Solving for ytm_3,

$$\text{ytm}_3 = [(1+f_1) \times (1+f_2) \times (1+f_3)]^{\frac{1}{3}} - 1$$

This equation essentially says that the yield on an n-period security is the Geometric mean of the sequence of expected one-period forward rates.EXAMPLE: Suppose that the anticipated six month rates are:

$f_1 = 6\%$, $f_2 = 7\%$, $f_3 = 8\%$

According to the expectations theory, the (annualized) yield on an eighteen-month bond should be:

$$(([(1+\frac{0.06}{2}) \times (1+\frac{0.07}{2}) \times (1+\frac{0.08}{2})]^{\frac{1}{3}}) - 1) \times 2 = .069984\text{, or } 6.9984\%$$

NOTE - If:

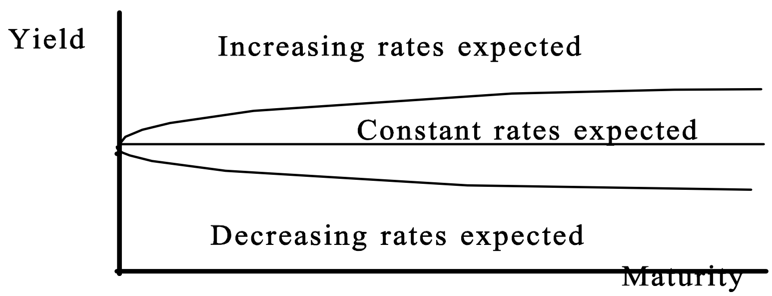
a) the variations in yield are small (<10%), and

b) the time is short (< 20 pds.), the average is a very close approximation of the Geometric mean. I.e.,

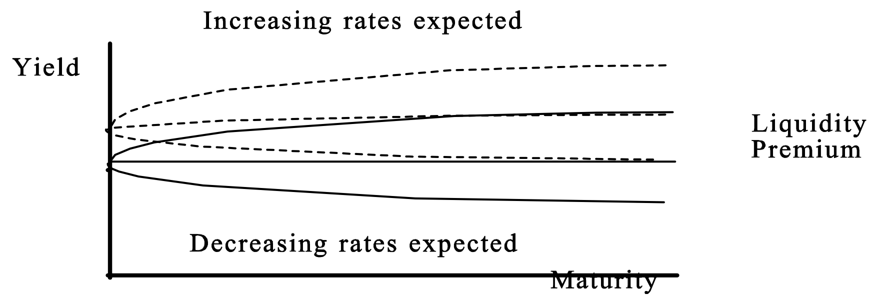
$$([\frac{(1.03)+(1.035)+(1.04)}{3}] - 1) \times 2 = 0.0700\text{, or } 7\%$$

Which is close to our previous answer.

Possible yield curve shapes:



3. The liquidity theory considers the risk involved in holding bonds. It notes that investors in long-term bonds have less liquidity and higher risk, and so will require some risk premium as an inducement. I.e., if the yield curve would otherwise be flat, adding the liquidity premium would result in a gently increasing yield curve. This theory is usually combined with the expectations theory, and the combination results in the possible yield curve shapes shown. The liquidity premium is often suggested as the cause of the "normal," yield curve being upward sweeping or increasing, while a decreasing or downward sweeping yield curve is said to be "inverted."



If liquidity theory were a full explanation, we would not observe down-sloping yield curves - but we do observe them. This does not destroy the liquidity premium, however, since both expectations and liquidity may apply - i.e., expectations may produce a downward-sloping curve, while liquidity reduces but does not eliminate the downward slope:
NOTE: If the liquidity theory holds, the observed yield curve lies above the yield curve implied by the expectations theory, and estimates of expected rates derived from the expectations theory will be upwardly biased.

4. Market segmentation theory suggests that securities markets are not continuous over maturity, but are instead segmented. This segmentation arises because investors can reduce risk by matching the maturity schedule of their assets and liabilities. Thus, investors have a preferred habitat, or position, in the term structure. Under this approach, yields will be determined by supply and demand in each segment. There are many possible yield curve shapes. If there is a strong supply of long-term funds relative to demand, but a shortage of short-term funds relative to demand, the yield curve would be downward-sweeping, if there is a shortage of long-term funds relative to demand, but a strong supply of short-term funds relative to demand, the yield curve would be upward-sweeping.

Spot and Forward Rates

1. The n period spot rate s_n is the discount rate applied to a zero coupon bond maturing in n periods.
2. The forward rate $f_{i,j}$ is the rate over the interval beginning at time i and ending at time j. Normally this will only be one period, although forward rates can also be computed over multiple periods.
3. Forward rates can be implied from the observed yields on zeros (usually government zeroes to avoid credit problems). Unfortunately, pure discount bonds are not available over all maturities. The price and yield of a zero, however, can be sequentially implied through a process referred to as "bootstrapping."
EXAMPLE: A par bond is a bond which is priced at face value (100), so that coupon = yield. Assume the following par bond (semiannual pay) yield curve (expressed on a bond equivalent yield basis):

Maturity (months)	6	12	18	24	32
Yield, %	6	7	7.8	8.4	8.8

a) By definition, the first spot rate is 6%, because the first par bond is a zero. Also by definition, the first one-period forward rate $f_{0,1}$ (i.e., the rate from time zero to the end of the first period) is 6%. (**NB**: the first one-period forward rate is always equal to the first one-period spot rate).
b) In order to compute the 12 month spot rate, we need the price of a one-year zero coupon bond. We can impute this price by subtracting the present value of the first coupon payment from the price of

the one-year coupon par bond. The present value of the first coupon payment is computed using the first spot rate:

$$\text{Imputed Price of 12 month Zero} = 100 - \frac{3.5}{(1.03)} = 96.601942$$

· Note that this is the imputed price of a one-year zero which pays 103.5 - six months coupon plus principal repayment.
· Note that the first coupon payment is 3.5% for 6 months, or half the annual coupon of 7%.
· Note that the discount factor is 1.03, also for 6 months, or half the annualized spot rate of 6%.

The 12 month spot rate is then computed as:

$$\text{Annual Spot Rate} = 2 \text{ X Semiannual Spot Rate} = 2 \text{ X} [(\frac{103.5}{96.601942})^{\frac{1}{2}} - 1] = 0.0701758$$

or 7.02%

· Note that the procedure is to compute the semiannual rate, then multiply by 2 to find the bond equivalent yield.

c) To compute the forward rate over the second period, $f_{1,2}$, we divide the 12 month spot rate by the 6 month spot rate:

$$f_{1,2} = 2 \text{ X} (\frac{1.071407}{1.03} - 1) = 0.080402$$

or 8.04 %.

· Note that this answers the question "What rate over the second six month period will result in the 12 month spot rate?"
· Note that the six month rate is multiplied by 2 to find the bond equivalent annual yield.

d) In order to compute the 18 month spot rate, we need the price of an 18 month zero coupon bond. We can <u>impute</u> this price by subtracting the present value of the first and second coupon payments from the price of the 18 month coupon par bond. The present value of the 6 month coupon payment is computed using the 6 month spot rate, while the present value of the 12 month coupon is computed using the 12 month spot rate:

$$\text{Imputed Price of 18 month Zero} = 100 - \frac{3.9}{(1.03)} - \frac{3.9}{(1.071407)} = 92.573519$$

· Note that this is the imputed price of an 18 month zero which pays 103.9 - six months coupon plus principal repayment.
· Note that the first and second coupon payments are now 3.9 for 6 months, or half the annual coupon of 7.8.
· Note that the first discount factor is 1.03, for 6 months, or half the annualized spot rate of 6%.
· Note that the second discount factor uses the true annual rate:

$$(1 + \frac{\text{bond equivalent yield}}{2})^2 = 1.071407$$

rather than using the quoted but incorrect bond equivalent yield to obtain 1.0701758.

The 18 month spot rate is then computed as:

$$\text{Annual Spot Rate} = 2 \text{ X Semiannual Spot Rate} = 2 \text{ X} [(\frac{103.9}{92.573519})^{\frac{1}{3}} - 1] = 0.0784499$$

or 7.85%.

e) To compute the forward rate over the third period, $f_{2,3}$, we divide the compounded 18 month spot rate by the compounded 12 month spot rate:

$$f_{2,3} = 2 \text{ X } (\frac{1+0.0392249^3}{1+0.03509^2} - 1) = 0.0950977$$

or 9.51%.

· Note that this answers the question "What rate over the third six month period will result in the 18 month spot rate?"

· Note that again the six month rate is multiplied by 2 to find the bond equivalent annual yield.

3. Calculating spot rates requires the iterative procedure shown above, sequentially calculating longer rates. This procedure is called "bootstrapping."
4. Calculation of forward rates is quite a bit simpler - the forward rate over any interval is simply the ratio of the longer spot rate to the shorter spot rate.
5. Possible sources of error to guard against:
   a) Use true yield for calculations, not bond equivalent yield. **NOTE**: This problem disappears if annual pay bonds or annual rates are used - it is necessary to use caution and understand what is required!
   b) Although the spot rate for a given maturity remains constant, so that the discount factors for a given maturity remain the same, the coupon on the par bond will change as the maturity changes (this is, after all, why the bonds are par bonds!) It is necessary to use caution and use the proper coupon.
   c) Specification of the intervals for returns can be confusing - use caution to understand what is required (it is helpful to specify your interpretation).
6. The main influences on the yield curve are:
   a) Expectations of future yields.
   b) The risk premium.
   c) Convexity bias (a downward bias in long rates due to the positive relationship between duration and convexity).

Effects of Embedded Options: Option Adjusted Spread

1. For conventional bonds, the traditional measure of spread has been the spread to an "on-the-run" Treasury security of the same duration/maturity. This traditional measure has two drawbacks:
   a) The term structure of interest rates is ignored,
   b) For callable/putable bonds, the effects of interest rate volatility are ignored.
2. An alternative measure is the "Static Spread."
   a) Static Spread is the constant rate which, when added to the Treasury yield curve, will result in a present value equal to the bond price: I.e., for yield to maturity:

$$\text{Yield to Maturity} : \text{Price} = \sum_{t=1}^{T} (\frac{iF}{(1+y)^t}) + \frac{F}{(1+y)^T}$$

solve for the constant y; for static spread:

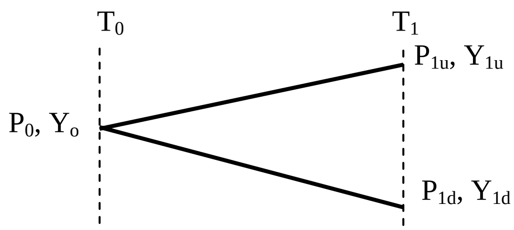
$$\text{Static Spread : Price} = \sum_{t=1}^{T}\left(\frac{iF}{(1+y^*_t+SS)^t}\right)+\frac{F}{(1+y^*_t=SS)^T}$$

where y^*_t is the Treasury spot rate at t, solve for the constant SS.

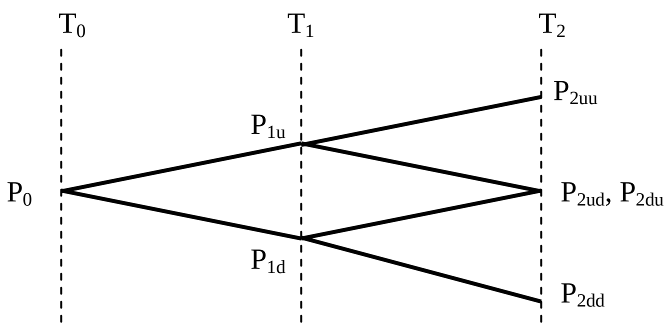
- Static spread takes account of the yield curve, but fails to take account of the embedded call option.

3. Option-Adjusted Spread (OAS) takes account of both the yield curve and the embedded option.
   a) The first step in computing OAS is to generate a (large) number of (equally likely) potential future interest rate paths.
   b) The yield of the bond along each path is computed taking into account the embedded option - i.e., if interest rates along the path would indicate that the bond would be called, the yield to call for the first such point is used.
   c) The yields are averaged across the paths.
4. A typical way of generating the interest rate paths is the binomial model.
   a) In a binomial model, the time until maturity is divided into subperiods.

b) Over each subperiod, the bond price has two equally likely possible outcomes (<u>bi</u>nomial process). This can be visualized as:



   c) Subperiods are combined to provide possible paths over the entire period. I.e., for two subperiods:



5. To find the Option Adjusted Spread:
   a) Choose a possible spread PS.
   b) For each interest rate path, compute the value of the instrument discounted at the relevant interest rate plus PS.
   c) Compute the average of the values AV, and compare it to the market price.

   d) If AV is greater than the market price, return to a) and choose a wider spread. If AV is less than the market price, return to a) and use a smaller spread.
   e) Iterate until AV equals the market price.
6. Note that the OAS is a summary number based on an average. While the summary number is useful as a description, it provides limited information:
   a) It does not tell of the range of outcomes - there might be an outlier which would represent grate risk.
   b) It does not tell the shape of the distribution - the results might be skewed in one direction.
   c) It does not provide information about specific scenarios.
   d) The model used to generate interest rate paths may not adequately reelect the possibilities.

   The remedy to these situations:
   a) Careful choice of models.
   b) Investigation of the entire range of outcomes, perhaps as a probability distribution.
   c) Consideration of outliers or high/low outcomes.
   d) Consideration of specific scenarios thought relevant.

Mortgage Securities

1. Mortgages may be either fixed-rate or adjustable rate.
   a) Fixed-rate mortgages have a constant monthly payment defined by the principal amount and the fixed mortgage rate. The payments have two portions:
      · A principal repayment portion, which reduces the amount of remaining principal.
      · An interest portion, which is the interest on the remaining principal.

      Since the payment is constant, the principal and interest portions will change over time:
      · The principal repayment portion will be small at first, and the first payments will be predominantly interest.
      · As time passes and remaining principal is reduced, the interest portion of the payment will decline, and the principal repayment portion will become larger.
      · The last payments will have a small interest portion and a large principal repayment portion.

      These mortgages can typically be prepaid by the mortgagee (sometimes with a penalty).
   b) Adjustable-rate mortgages are similar, except that the interest rate is periodically reset. This implies that the total amount of the monthly payments will not be fixed.
2. Given the credit risk and the uncertainty of prepayment, individual mortgages are illiquid. Mortgage securities securitize illiquid individual mortgages by combining them into pools. The credit risk and prepayment of the pool is more predictable, and securities may be issued using the pool as collateral.
   a) Mortgage securities are sometimes called pass-throughs because the principal and interest payments arising from the pool are passed through to the individual by the issuer of the securities.
   b) Agency pass-throughs are issued by agencies of the U.S. government, and have the support (directly or indirectly) of the U.S. government. The mortgages included in an agency pool must conform to certain standards.
   c) Nonagency pass-throughs are issued by private concerns, and do not have the backing of the U.S. government., and standards for mortgages included in the pool are somewhat weaker. Because of the weaker credit, they are often credit-enhanced. Nonetheless, yields tend to be higher than on agency pass-throughs.

3. Although more predictable than individual mortgages, the actual rate of payments to a mortgage backed security is uncertain. The ability to prepay can be thought of as a call on the mortgage.
4. There are two major causes of prepayment:
   a) Interest rates make prepayment and refinancing at a lower rate attractive.
      · This is affected by both the level of interest rates and the rate on the existing mortgages
   b) Sale of the property (moving to new home, divorce, etc). This is affected by:
      · General economic conditions
      · Pool characteristics which reflect different social conditions (location, size of mortgages)
      · Seasonal factors (many moves in Spring)
3. Note the multiple effects of interest rates:
   a) High interest rates make prepayment less likely, and extend the duration of the MBS because cash flows occur later. This has a double effect:
      · The price of the bond declines due to the higher discount rate.
      · The price of the bond declines due to the longer duration

   Note that the decrease in prepayment comes at a disappointing time - when interest rates are high investors prefer high prepayment so as to be able to invest at the high rates.
      · These effects of decreased prepayment and increased duration are called "extension risk," - i.e., the duration of the security is extended.
   b) Low interest rates make prepayment more likely, and reduce the duration of the MBS because cash flows occur earlier. This has a double effect:
      · The price of the bond increases due to the lower discount rate.
      · The price of the bond increases due to the shorter duration

      Note that the increase in prepayment comes at a disappointing time - when interest rates are low investors prefer low prepayment so as to retain the higher rate - and this tends to inhibit the bond price increase.
      · These effects of increased prepayment and decreased duration are called "contraction risk," - i.e., the duration of the security contracts (gets shorter).
4. Note, however, that the effects of prepayment for the bondholder will depend on whether the bond is at a discount or at a premium:
   a) If the bond is at a discount, the prepayment provides a capital gain which can be reinvested at a higher rate.
   b) If the bond is at par, prepayment makes no difference (i.e., the rate on the bond is equal to the reinvestment rate).
   c) If the bond is at a premium, the prepayment causes a capital loss and the reinvestment rate is lower.
5. Collateralized Mortgage Obligation (CMOs) and Real Estate Mortgage Investment Conduits (REMICs) are simply securities backed (collateralized) by a pool of mortgages.
   a) The difference between these securities and a simple Mortgage Backed Security lies in the repackaging of the principal, prepayment, and interest payment flows.
      · The various tranches will receive different cash flows, and the repackaging takes many forms.
      · Because of the repackaging, the response of securities of a particular tranche to prepayments may differ widely from the response of securities from a different tranche of the same underlying pool.
   b) The attraction of the tranche securities lies in there different characteristics, which may more closely fit the requirements of various investors.
   c) One type of repackaging divides the securities into "tranches."

· Typically, the first N-1 (out of N) tranches receive interest as accrued. However, the principal payments, including prepayments, are directed to the first tranche. This tranche will be rapidly repaid, and when the first tranche is fully paid, principal payments will be directed to the second tranche. This procedure is repeated until the first N-1 tranches are paid.

· The N^{th} tranche, called the "Z Tranche," will receive no payments until all other tranches have been retired. Until the other tranches are retired, the principal of Z tranche will be increased in lieu of payments. Once the other tranches are retired, all principal and interest payments will be directed to the Z tranche. The timing and amount of payments to the Z tranche are highly uncertain.

d) Another repackaging is the Planned Amortization Class (PAC). This class is often the second tranche, and has a schedule of principal payments which can be sustained over a wide range of conditions (all principal payments are available to the PAC), and are relatively certain.

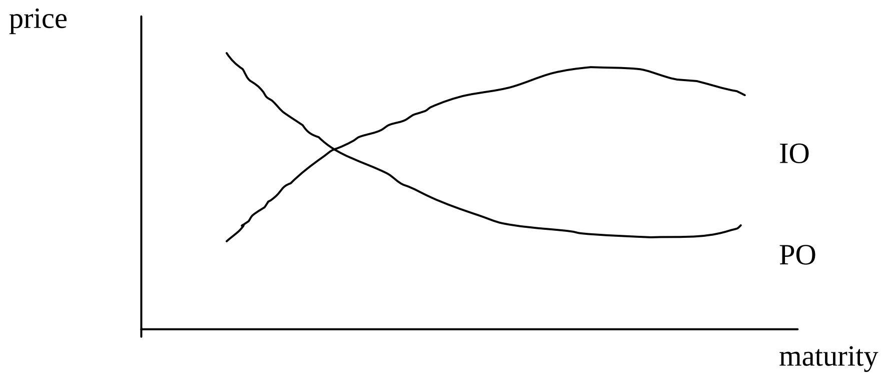
· A variation on the PAC is the Tactical Amortization Class (TAC), which like the PAC has scheduled principal payments. However, the schedule of principal payments cannot be sustained over as wide a set of conditions as for the PAC.

e) The cash flows to the pool may also be divided into Interest Only (IO) and Principal Only (PO) classes, or Stripped Mortgage-Backed Securities.

· IO securities will become increasingly less valuable if mortgage rates go below the rate of the mortgages in the pool. This happens because the prepayment will increase, decreasing the principal and the future interest payments.

· If the mortgage rate increases above the rate of the mortgages in the pool, the rate of prepayment will decrease and the value of the IO will increase because the principal will remain high, and the amount of future interest payments will be greater.

· As the interest rate increases, the effect on prepayment diminishes. If the mortgage rate increases greatly, the impact of higher discount rates will overpower the effect of lower prepayment, and the value of the IO will again decrease:



· The effect will be reversed for the PO. Decreasing interest rates will cause increased prepayment, so that principal is received sooner, while increasing interest rates will cause decreased prepayment and delay receipt of payment. Note the extension risk of both delayed payments and higher discount rate.

· Note that the sum of the price of the IO and the price of the PO should equal the price of the unstripped pass-through.

Structured Securities

1. Adjustable Rate Mortgages have an interest that is reset periodically.
   a) The rate is periodically set to some index plus a "net margin:"

   new rate = index rate + net margin

   b) The interest reset period (interval between resets) varies.
   c) Payments are also reset, but the payment reset schedule may not be the same as the interest reset schedule.
      · If there is a large enough increase in rates, and the payment is not reset to reflect the new rates, the interest due may be larger than the payment. This would cause negative amortization - i.e., the principal would increase.
   d) There may be limits on the adjustments - a cap is an upper limit, a floor is a lower limit..
      · Periodic limits affect the amount of adjustment in a given period, but do not limit the cumulative adjustment.
      · Lifetime limits do not affect the adjustment in a given period, but set a maximum cumulative adjustment.
      · Limits may affect either the interest rate or the payment.
2. The variable rate and the limits make the maturity and duration of the securities uncertain.
   a) Duration: remember that duration is the % change in price for a 1% change in yield.
      · In the extreme case of an interest rate which is continuously reset, the price of the security would always be par. Since interest rate changes would have no effect on price, the duration would be zero.
      · As the reset period becomes longer, the effect of an interest rate change on price becomes larger (although still comparatively small). Duration would increase the longer the reset period.
      · Lower coupon is associated with longer duration - this indicates that low teaser rates (low introductory rates used to attract borrowers) will increase duration.
      · Higher prepayments move payments earlier and result in shorter duration).
      · Higher caps and lower floors are less restrictive of rate and price adjustment and produce lower duration (price sensitivity)
   b) For set rate securities, increased yield is associated with decreased duration. For rate adjustable ARMs, however, a decrease in interest rates lowers duration because of the lower probability that the cap will be reached.
   c) As with other securities with an embedded option, ARMs can exhibit negative convexity.
      · Effective duration, rather than Macauley duration, should be used.
3. Yield on ARMs is sometimes measured by the Net Effective Margin - the yield spread over current index rate:

   Net Effective Margin = yield to maturity - current index rate,

   where the yield to maturity is expressed as a bond equivalent yield (2 X semiannual yield).
   a) The cash flows used in computing the net effective margin take account of adjustments to the existing fully indexed rate (index + net margin).
   b) The net effective margin suffers from the shortcomings of yield to maturity:
      · It assumes a constant index rate over the life of the security.
      · It ignores embedded options (caps/floors and prepayments).
4. Option adjusted spread is a better yield measure for ARM pass-throughs.
   a) This has the advantage of including the effects of caps/floors and prepayments.

b) OAS can be computed with an without any embedded options - the difference will reflect the value of the embedded options.

Asset-Backed Securities

1. Asset-backed Securities (ABS) are simply securities collateralized by a pool of debt assets - car loans, credit card debt, and home equity (second mortgages, as opposed to the first mortgages of MBS)
   a) The characteristics of the underlying debt assets produce somewhat different characteristics for the securities.
   b) In order to gain a higher credit rating, the securities are often "credit enhanced."
      · Credit enhancement is simply some device to raise the probability that scheduled principal and interest payments can be made.
      · Examples of typical enhancements include excess spread, letters of credit, senior/subordinated structure, and reserve funds.
   c) The "zero volatility spread" used to analyze ABS is identical to the "static spread" concept applied to MBS.

Asset Valuation - Derivatives

Cash and Carry Arbitrage.

1. The no-arbitrage condition simply says that it should not be possible to set up a costless position which returns a profit. This condition is widely applied in determining value relationships for derivatives.
2. The name Cash and Carry Arbitrage arises from the arbitrage process - I.e., borrow to pay cash for the underlying security and carry it to the future date, while at the same time selling the futures contract. The payoff to this position will be the difference between the spot purchase price of the asset and the future price received, less the carrying cost (the financing cost of interest on the borrowing), or:

   Payoff = (Futures Price - Spot Price) - Financing Cost

   The no-arbitrage condition implies that this costless position must have zero payoff, so that

   Futures Price - Spot Price = Financing Cost
3. If the cash and carry arbitrage condition is violated by Futures Price - Spot Price > Financing Cost, the futures are (relatively) overpriced. The indicated arbitrage is to sell the (overpriced) futures, borrow the present value of the futures price and buy the stock. The present value of the futures price will be larger than the cost of the stock - an immediate profit (NOTE - there is NO time zero cash flow from the sale of the future). At maturity, the stock is delivered and the futures price received delivered to you against the borrowing - a net of zero. I.e.:

	Action	Cash Flow
At time zero:	Sell Future	Zero
	Borrow PV(Futures Price)	$+ F_T/(1 + r)^T$
	Buy Stock at Spot	$- S_0$
	Net	$F_T/(1 + r)^T - S_0 > 0$
At time T:	Deliver Stock	$+ F_T$
	Pay Borrowing	$- F_T$
	Net	Zero

4. If the cash and carry arbitrage condition is violated by Futures Price - Spot Price < Financing Cost, the futures are (relatively) underpriced. The indicated arbitrage is a reverse cash and carry arbitrage - buy the (underpriced) futures, lend the present value of the futures price and short the stock. The present value of the futures price will be less than the price of the stock - an immediate profit (NOTE - there is NO time zero cash flow from the sale of the future). At maturity, the maturity of the lending pays for the stock delivered against the future, and the stock is delivered against the short - a net of zero.
5. Note that these arbitrage arguments ignore the mark-to-market feature of futures, and actually apply strictly only to forward contracts.
6. Another way of looking at cash and carry arbitrage is to compare the implied financing cost (as an interest rate) to existing interest rates. The implied interest rate is called a "repo" rate. A repurchase agreement is the sale of securities with a simultaneous agreement to repurchase the securities at a later date at a set price, the equivalent to a loan using the securities as collateral. The repo rate is the

interest rate implied by the difference between sale price and repurchase price. In perfect markets where the financing cost is the only carrying charge, the cost-of-carry will equal the implied repo rate - the repo rate implied by the difference between the cash and futures prices. This financing or repo rate is easily
computed as:

$$C = (F_{0,t} / S_0) - 1$$

Note that this assumes a single period for the holding period.
A repo rate above the financing cost indicates a cash and carry arbitrage opportunity, a repo rate below the financing cost indicates a reverse cash and carry opportunity.

7. Cash-and-carry arbitrage in foreign exchange futures is called covered interest arbitrage. Here the arbitrage consists of borrowing domestic currency funds DC, converting the funds to foreign currency FC in the spot market, investing at the foreign interest rate, and initiating a futures contract for reconversion to DC. This is the same as the cash and carry position with an extra exchange rate step. The cash return on this arbitrage is the return on the foreign investment:

$$(\text{DC X Spot Conversion Rate})(1 + \text{Foreign Rate})(\text{Futures Price}) = (\text{DC X } 1/\text{FC})(1 + r_{FC})F_{0.t}$$

less the repayment on the borrowing:

$$DC(1 + r_{DC})$$

so that:

$$\text{Cash Return} = (DC/FC)(1 + r_{FC})F_{0.t} - DC(1 + r_{DC})$$

The no-arbitrage condition is that the return equal zero . This is exactly the same as the interest rate parity condition, except instead of being invested locally, the funds are converted and invested abroad. Setting equal to zero and solving, we obtain the futures price in terms of the spot price of the foreign currency and the relative interest rates in the two countries:

$$F_{0.t} = FC[(1 + r_{DC})/(1 + r_{FC})]$$

8. The effect of interest or dividends is to reduce the financing cost by the future value of the dividends or interest:

$$F_{0,t} = S_0(1 - C) - \sum_{i=1}^{N} D_i(1 + r_i)$$

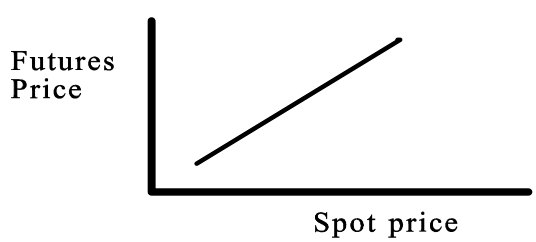
Where D may represent either a dividend or interest payment.

Hedging with Futures

1. Note that the cash and carry arbitrage condition can be rewritten to:

$$\text{Futures Price} = \text{Spot Price} + \text{Financing Cost}$$

This indicates that at any one point in time the difference between the futures price and the spot price is a constant, so that the Futures price will change with the spot price, as indicated in the diagram.



2. Note that this implies that the spot price and the futures price have a correlation = 1.0. Also note that the futures contract is not for one unit but some multiple of units, so that the relationship will not be 1:1 (not 45 degrees).

a) Note this ignores any changes in the basis.

3. The spot price - futures price diagram is for an instant in time. The difference between the futures price and the spot price at a moment in time is called the basis:

$$\text{Basis} = \text{Spot Price} - \text{Futures Price} = S - F_{o,t}.$$

   a) The basis will change with time, and at maturity of the future the spot price will equal the future price, so that the basis will be equal to zero.
   b) If the basis changed over time exactly along the path indicated by the futures rate, the rate would be locked-in for all periods. The basis will, however, also vary with interest rates or other factors, so that:
      - over finite time periods the relationship will not hold exactly,
      - if the hedge is lifted before maturity, the return may not equal the futures rate
      - hedges lifted before maturity are subject to basis risk.
4. Although the uncertainty about the basis indicates that the perfectly certain linear relationship of the diagram does not occur in reality, the relationship is nonetheless a reasonably close approximation. Because of this relationship, it is possible to hedge - to reduce the risk of a position by taking an offsetting position in futures. Note that the risky position may be actual (you are actually long a risky asset) or anticipated (you anticipate that you will be long the risky asset).
5. Hedges may be either short or long, with the appropriate hedging position being the position such that the gain or loss moves oppositely to the gain or loss on the hedged position. Given that the relationship between spot and futures prices is positive, a short hedge, or sale of the futures contract, is appropriate when the exposure is due to an actual or anticipated long position in the asset. A long hedge, or purchase of the futures contract, is appropriate when the exposure is due to an actual or anticipated short position (or need for) the asset.
6. In some cases, futures based on the asset to be hedged do not exist. In that case, it is possible to create a hedge if there is a futures contract available on an asset which moves similarly to the asset to be hedged - i.e., an asset which is highly correlated with the target asset. This is called a cross hedge.
7. Since the futures price and the asset price will not move in perfect unison, and since the futures contract is for a set number of units of the underlying asset, the number of contracts to be used for the hedge is not exactly defined. The number of contracts to be used is often expressed as the hedging ratio - the value of contracts per unit of the underlying asset. I.e., a 1:1 ratio would be one contract per unit of the underlying asset, or contract size equal to the amount of the asset being hedged. Alternately, the value of the hedge position is:

$$V_t = S_t + HRf_t$$

   where HR is the Hedge ratio, S_t is the Stock Value, and f_t is the future value, so that:

$$\Delta V_t = \Delta S_t + HR\Delta f_t$$

   The purpose of the hedge is to reduce ΔV_t to zero, so that:

$$\Delta S_t + HR\Delta f_t = 0$$

   or:

$$HR = - \Delta S_t / \Delta f_t$$

   Note that the hedge ratio is negative - i.e., to hedge you take the opposite of the position you wish to hedge:

   To hedge a real or anticipated long position, take an offsetting short position (sell futures).
   To hedge a real or anticipated short position, take an offsetting long position. (buy futures).

8. If the underlying asset and the asset being hedged moved perfectly together, a naive 1:1 hedge would eliminate risk. Given the uncertainty in the spot/futures price relationship, however, it is unlikely that this naive approach would be optimal.
9. An alternative approach is to choose the hedge ratio which will result in the least (statistical) variance in the net outcome. This can be achieved by the same Markowitz diversification approach used in portfolio theory. The variance of the net position, σ^2_P, is:

$$\sigma^2_P = \sigma^2_S + (HR)^2\, \sigma^2_F + 2HR\rho_{SF}\, \sigma_S\, \sigma_F = \sigma^2_S + (HR)^2\, \sigma^2_F + 2HRCov_{SF}$$

   This is minimized when:

$$HR = \rho_{SF}\, \sigma_S\, \sigma_F / \sigma^2_F = Cov_{SF} / \sigma^2_F$$

   Another way of saying this is that this approach attempts to minimize "tracking error."
10. This hedge ratio can then be estimated by estimating the covariance of the spot and futures prices and the variance of the futures price. Alternately, this ratio is often estimated as the β in the regression:

$$S_t = \alpha + \beta F_t + \varepsilon_t$$

   with the R^2 of the regression providing a measure of the effectiveness of the hedge. Unfortunately, there are econometric problems in this approach.
11. With interest rate futures, it is important to realize that you do not "lock in" the existing rate, but rather you lock in the existing forward rate implicit in the futures price.
12. When using Index futures to set up a hedge against a portfolio, the idea is:

$$\text{Change in Portfolio Value} = - \text{Change in Index Value} = N\, \Delta V_F$$

$$V_P\, \beta_P = - V_I\, \beta_I = N\, V\, \beta_F$$

   Since the beta of the Index is 1.0, the change in value for the portfolio and the change in value of the Index will be the same when:

$$(V_P/V_F)\, \beta_P = \text{number of contracts}$$

   This formula is critically dependent on two assumptions: 1)that β_P accurately measures the relative volatility of the portfolio over the period (i.e., the portfolio is well-diversified), and 2) that the Index future moves exactly in tandem with the spot index.
13. If C_U is the cost of the transaction in the hedged currency as predicted from the futures exchange rates:

$$C_U = \text{Amount of foreign currency X futures exchange rate at time 0}$$

   and C_A is the actual cost in the hedged currency:

$$C_A = \text{Amount of foreign currency X actual exchange rate at time t}$$

   the gain or loss (measured from the expected cost)on an unhedged position is $C_U - C_A$. The intent of a foreign exchange hedge is to avoid this uncertainty and provide a certain future exchange rate and so a known cost. The gain or loss on the hedged position includes the hedging gain or loss, H:

$$\text{gain or loss on hedged position} = C_U - C_A + H$$

   If the hedge is perfect there is no gain or loss:

$$C_U - C_A + H = 0,$$

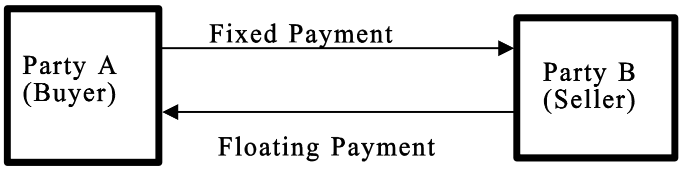
   or:

$$H = -(C_U - C_A)$$

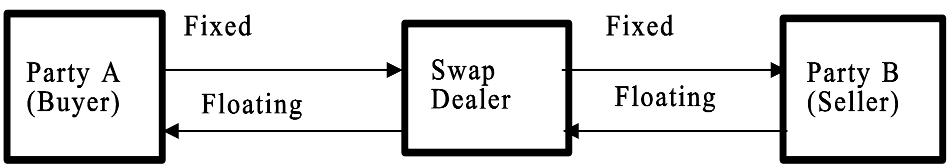
   Two reasons why the hedge may not be perfect (i.e., why $H \neq -(C_U - C_A)$): 1) the maturity of the futures contract may not be equal to the time of the contract (maturity mismatch), and 2) the amount needed may not be a whole multiple of contract size (principal mismatch). To compute the success of the hedge, the result of hedge, H, is compared to the result of the unhedged position, $C_U - C_A$. Note that a "successful" or perfect hedge should produce neither losses nor gains.

Swaps

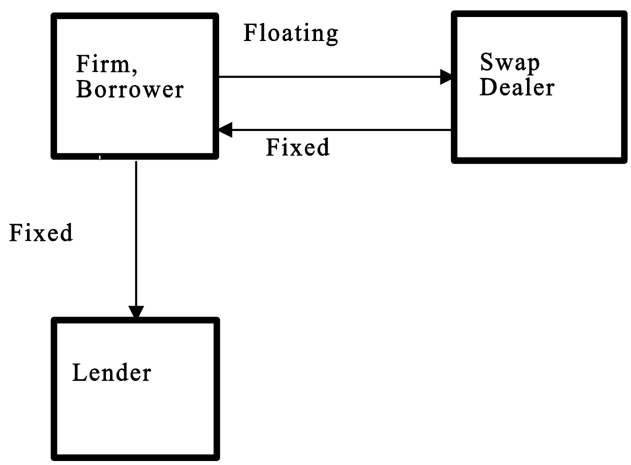
1. Swap diagrams are very useful for visualization and explanation of swaps:
   "Plain Vanilla" interest rate swap:



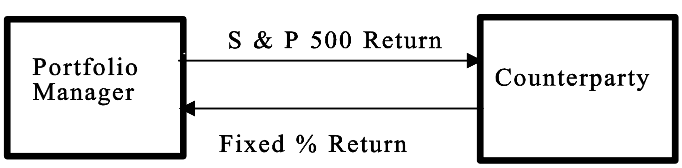
"Plain Vanilla" interest rate swap with intermediaries:



Converting Fixed Rate Debt to Floating Rate Debt:



Equity Swap:

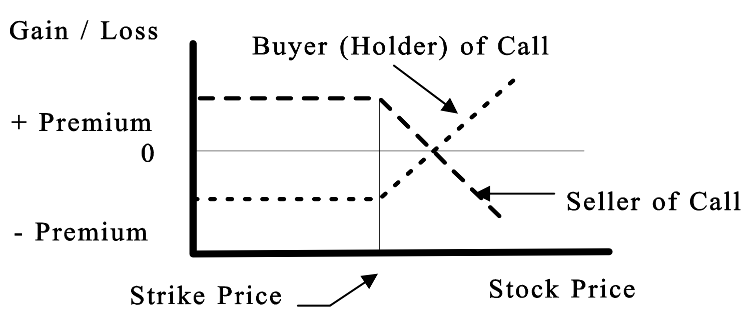


It is often easiest to create a situation by drawing the diagram of the result you want, and then proceed to the specification.

2. In a swaption, remember that the buyer of the swap is the pay fixed party - the party who will pay the fixed rate. Thus:
   a) The buyer of a call swaption will buy the swap (pay fixed, receive floating) upon exercise.
   b) The seller of the call swaption will sell the swap (receive fixed, pay floating) upon exercise.
   c) The buyer of a put swaption will sell the swap (receive fixed, pay floating) upon exercise.
   d) The seller of a put swaption will buy the swap (pay fixed, receive floating) upon exercise.
3. An important point: in an interest rate swap the principal is notional and is not exchanged. In a currency swap, the principal is actually exchanged at the beginning and the end of the swap with both exchanges at the original exchange rate. In interest rate swaps, the payments are netted - i.e., only the difference is exchanged.
4. Comparison of swaps and futures:
   a) Futures contracts are highly standardized with specific contract terms which cannot be altered, while the terms of swaps are very flexible.
   b) Futures are exchange traded, leading to some loss of privacy, while swaps are private agreements.
   c) Futures and options trading is subject to considerable government regulation, while swaps have virtually no government regulation.
   d) Futures exchanges provide a central market, while finding swap counterparty can be difficult.
   e) Futures positions can be modified or terminated by offsetting trades, while swaps are an agreement between two parties and may be difficult to alter or terminate.
   f) The exchange guarantees the performance of futures contracts, while swaps have no such guarantee and require greater attention to creditworthiness.
5. A swap can be visualized as a series of exchanges. Each of these exchanges can be thought of as a forward contract on an interest rate instrument. The gains or losses on the forward contract will result in the same net cash flows as the floating rate of the swap. Thus, a swap could be replicated by a series of forward contracts.

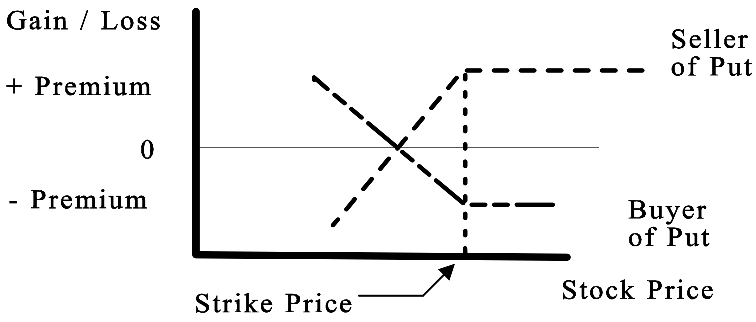
Option Payoff Diagrams and Strategies

1. The payoff diagram to any option strategy can be constructed from the payoff diagrams for the underlying instruments:
   a) The payoff pattern to a call:



Note that:

· The premium is the largest gain for the seller, the largest loss for the buyer.
· For both buyer and seller, the gain / loss line changes slope at the Strike Price.
· For both buyer and seller, the gain / loss is equal to zero when the stock price exceeds the strike price by the per share premium.



b) The payoff pattern for a put:

Note that the rules are the same as for calls:

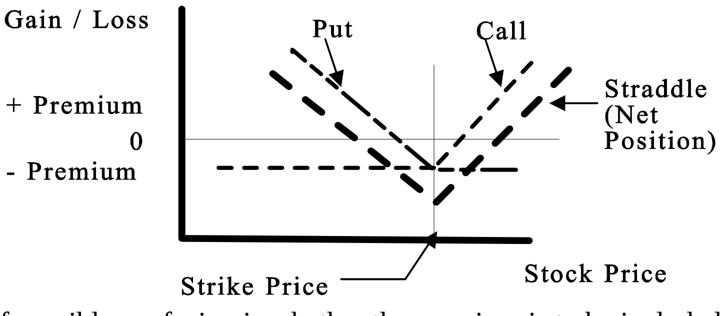
· The premium is the largest gain for the seller, the largest loss for the buyer.
· For both buyer and seller, the gain / loss line changes slope at the Strike Price.
· For both buyer and seller, the gain / loss is equal to zero when the stock price is under the strike price by the per share premium.

Only the shape of the relationship has changed!

2. The payoffs to any strategy can be found by simply summing the payoffs - e.g., for a straddle position made up of a long call with a per share premium of $2 and a strike price of $40 plus a long put with a per share premium of 1.50 and a strike price of $40.

Note that, when the premium is included to find the profit or loss:

· When the stock is at the strike price, both premiums are lost for a loss of $4 per share.
· When the stock price is above or below the strike by the amount of the per share premium, one option has zero gain/loss, but the premium is lost on the other option.
· The position will have a gain if the stock price is above or below the strike by twice the amount of the per share premium.



3. A point of possible confusion is whether the premium is to be included in the gain or loss, or whether the diagram is for the payoff at expiration without the premium. Caution is advised, accompanied by an explicit statement concerning the premium.

4. The motivation behind various option strategies is easily read from the payoff diagram. E.g., the straddle investor expects a price change, but is uncertain of the direction of the change. An example would be a firm in a substantial legal suit of uncertain outcome.

Synthetic Assets and Put-Call Parity

1. Consider the payoffs to a position consisting of a long position in the asset, a long position in a put on the asset, and a short position in a call on the asset, both options at strike price X:

Payoff:	Asset	+	Long Put	-	Short Call	=	Net
$S < X$	S	+	(X - S)	-	0	=	X
$S > X$	S	+	0	-	(S - X)	=	X

No matter what the final stock price, the payoff is X!

2. An alternative investment having the same payoff is to lend the present value of X:
$$PV(X) = Xe^{-r(T-t)}$$
(Note that this formulation uses continuous compounding at rate r for T-t periods)
3. One consequence of the equivalence of these two positions is put-call parity.
a) The two alternatives have the same outcome - if they had different costs, arbitrage would be possible.
b) Under the no-arbitrage condition, then, if S is the asset price, P the put premium, and c the call premium:
$$S + P - C = Xe^{-r(T-t)}$$
or:
$$C - P = S - Xe^{-r(T-t)}$$
This is the put-call parity relationship - the relationship between the price of a call and the price of a put.
Note again that this formulation uses continuous compounding. It would perhaps be more general to express the relationship using the present value operator: C - P = S - PV(X). In application, the present value (or simply the amount which must be invested today the obtain X at expiration of the option) may be computed various ways.
4. Another consequence of the equivalence of these two alternatives is that synthetic assets can be created.
   a) We can interpret the put-call parity condition $S + P - C = Xe^{-r(T-t)}$ as: "The combination of a long asset plus a long put plus a short call is equivalent to lending the present value of X."
   b) Looked at this way, we see that the combination of a long asset plus a long put plus a short call can be used to create a synthetic lending position.
   c) By algebraic manipulation, we can obtain:
   $$S = - P + C + Xe^{-r(T-t)}$$
   I.e., we can create a synthetic asset by shorting a put (negative sign), buying a call (positive sign), and lending (positive sign) the present value of X.
   d) Similarly,
   $$P = C + Xe^{-r(T-t)} - S$$
   I.e., we can synthetically create a put by buying a call, lending the present value of the strike price, and shorting the asset.

e) Finally,

$$C = - Xe^{-r(T-t)} + P + S$$

I.e., we can create a synthetic call by buying a put, borrowing the present value of the strike price, and buying the asset.

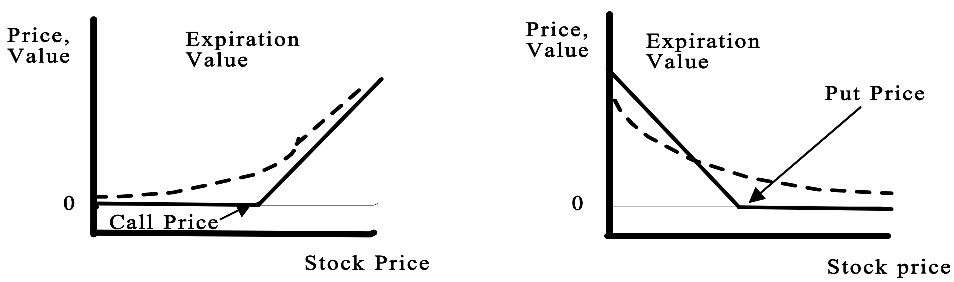
f) Note that these relationships hold strictly only for European options, which cannot be exercised before expiration.

Option Pricing and Hedging

1. There are a number of factors that affect option premiums:

Factor	Put	Call
Price of the underlying asset	negative	positive
Strike Price	positive	negative
Volatility of the Underlying Asset	positive	positive
Time to Expiration	positive	positive
Dividend Rate of Asset	positive	negative
Short Term Interest Rate	negative	positive

2. The relationship between the price of the underlying asset and the premium is diagramed as:



Deep out-of-the-money options will not be sensitive to the price of the underlying asset, deep in-the-money options will move in value almost perfectly with the price of the underlying asset. The change in the relationship is greatest around the strike price.

3. The relationship between option prices and the underlying asset indicates that options can be used to hedge the underlying asset.
   a) The "delta" (δ) of an option is the first derivative of the value of the option with respect to the value of the underlying asset:

$$\delta = \partial c/\partial S.$$

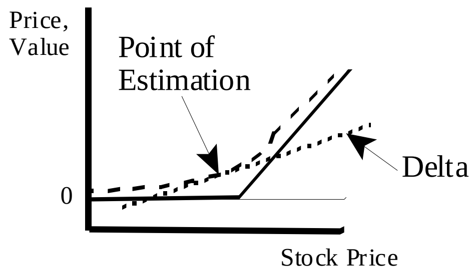
   b) Expressed in finite or approximate terms, this is:

$$\delta = \Delta c/\Delta S,$$

   where Δ indicates a (small) change. I.e., delta is the change in the value of the option for a small change in the value of the underlying asset.

   c) Rearranging, we obtain $\Delta c = \delta\ \Delta S$, or $\Delta S = (1/\delta)\ \Delta c$. Since the arbitrage is meant to eliminate any change in the position, shorting $(1/\delta)$ options is indicated - or alternately, a short call for every δ units of the underlying asset.

4. Delta can be graphically interpreted as the slope of the relationship between option value (premium) and stock price:

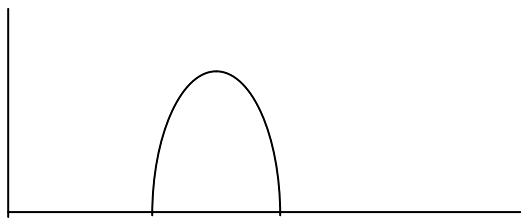


Delta, as a derivative, is only a valid estimate at a single stock price. As can be seen from the diagram, as the stock price moves from the point of estimation, delta becomes inaccurate. As a result of this inaccuracy, as the stock price changes the hedge must be revised - a dynamic hedge.

5. Vega (v) is the first derivative of the value of the option with respect to the volatility of the underlying asset:

$$v = \partial c / \partial \sigma_S.$$

and measures the size of the volatility effect (sensitivity of the option value to volatility). Although option value is positively related to volatility, vega changes with option price:



Interest-Rate Caps and Floors

1. An interest-rate cap is used to limit the buyer's exposure to increases in yield beyond a certain level.
   a) The seller of the cap is obligated to pay interest to the buyer of the cap if the yield on a reference index rises above a stated strike rate. The interest payment will be based on the excess of yield over the strike rate, and a nominal principal amount. I.e.,
   $$\text{Interest Payment} = \text{MAX}[0, (\text{Index Yield} - \text{Strike Rate}) \text{ X Nominal Amount}]$$
   Note that the nominal amount is reference only, and is not exchanged.
   b) A floating rate borrower wishing to limit exposure to increases in yield would purchase a cap. If the floating rate rose above the strike rate, the cash flows from the cap would offset the higher payments on the borrowing.
2. An interest rate floor is used by the buyer to limit exposure to decreases in yield below a certain point.
   a) The seller of the floor is obligated to pay interest to the buyer of the floor if the yield on a reference index falls below a stated strike rate. The interest payment will be based on the excess of the strike rate over the yield, and a nominal principal amount. I.e.,
   $$\text{Interest Payment} = \text{MIN}[0, (\text{Strike Rate} - \text{IndexYield}) \text{ X Nominal Amount}]$$
   Note that the nominal amount is reference only, and is not exchanged.
   b) A a floating rate lender wishing to limit exposure to decreases in yield would purchase a cap. If the floating rate fell below the strike rate, the cash flows from the cap would offset the lower payments from the lending.
3. The premium paid for the cap/floor can be expressed as basis points (on the nominal amount)t, or as a stated amount.
   a) The effect of the premium is to increase the effective cost of the borrowing or to decrease the effective yield on the lending. The effective borrowing/lending rates can be found by annualizing

the premium. This is done by assuming that the premium is borrowed. The required payments on the borrowing can be expressed in terms of basis points on the original amount borrowed or lent.

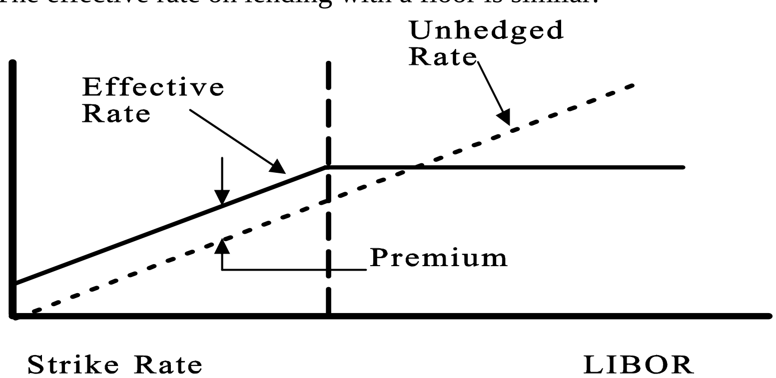
Example: A borrower a floating rate obligation at LIBOR, quarterly settlement. In order to protect against extreme increases in LIBOR, she buys a four-year cap on a nominal amount of $25,000,000, referenced to the three-month LIBOR with a premium of 170 basis points and a strike rate of 9%, quarterly settlement on an actual/360 day basis.
If the borrower can finance the premium at 7% fixed, what is the annual premium equivalent to be added to find the effective interest rate on the underlying obligation?

ANS. The amount of the premium is (0.0170)X($25,000,000) = $425,000. The quarterly payments if this amount was borrowed would be the quarterly annuity with a present value equal to the borrowing at the financing rate, or $30,684.82. This is an annual payment of $122,739.28 or ($122739.28)/(25,000,000) = 49.1 basis points of the principal borrowed.

If the same premium had been paid on a similar floor, the annual premium equivalent would be subtracted to find the effective yield on the underlying obligation.
What is the effective interest rate as a function of LIBOR?

ANS. If LIBOR is below the strike rate of 9%, there is no cash flow from the cap. However, the annual equivalent of the premium must be added to the interest rate on the original borrowing, so that if LIBOR is less than 9% the effective interest rate is (LIBOR + annual equivalent)
If LIBOR is above the strike rate, the cap will provide offsetting interest payments. The effective interest rate is then LIBOR plus the annual equivalent of the premium.

The effective rate on lending with a floor is similar.



Convertibles

1. There are two points to the dilutive effect of convertibles:
   a) more shares are created,
   b) the new shares are sold below market price (otherwise the convertible would not be exercised).
2. The payback period of a convertible is calculated as:

$$\text{Payback} = \frac{\text{Bond Price} - \text{Conversion Value}}{\text{Bond Income} - \text{Income} \quad \text{Equal Investment in Stock}}$$

This assumes that the interest payment on the bond is greater than the dividends that would be received if the common stock were purchased instead of the bond, and asks "How long would it take the difference in payments to make up the conversion premium?" A potential point of confusion is that the "Equal Investment in Stock" is not the number of shares that would result from conversion - it is the number of shares that would result from investing the value of the bond in stock at the market price.

a) E.g., consider a bond, priced at 90, convertible into 40 shares of common, when the price of common is $20 per share. The Equal Investment in Common Stock" would be $900/$20 = 45 shares - not the 40 shares resulting from conversion.

3. A convertible bond is likely to also be callable. Thus, the observed price of the bond has three components:

Observed Price = Value of Straight Bond + Value of Conversion - Value of Call

I.e., it is as if the firm sold you the bond and a call on the stock, and purchased a call on the bond from you.

a) This indicates that the difference between the observed price and the value of a comparable straight bond has two components:

Observed Price - Value of Straight Bond = Value of Conversion - Value of Call

b) In order to then find the value of the call, it is necessary to find the value of the conversion feature.

c) Since the conversion feature can be considered as a call on the number of shares resulting form conversion, the value of the conversion feature is the per share value of a call times the number of shares resulting from conversion.

d) The value of a comparable straight bond is the present value of the payments to be received from the bond (assuming no conversion) at the yield on similar straight bonds.

e) The per share value of a call can be estimated using option pricing models such as the Black-Scholes or binomial models.

2002 Preparation for Level II

ABSTRACTS

Economics
Debt Investments
Derivative Investments

David E. Upton, PhD, CFA
Professor of Finance
Virginia Commonwealth University

Fax: 804.828.3972
deupton@vcu.edu

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TABLE OF CONTENTS

STUDY SESSION 4
INVESTMENT TOOLS
ECONOMICS FOR VALUATION

1. *Macroeconomics*, 8th edition, Rudiger Dornbusch, Stanley Fischer, and Richard Startz (McGraw-Hill/Irwin, 2001).

A. Growth and Accumulation (Ch. 3)

SS 4. 1. A. a) Analysis using production functions. *(pp37 – 38) A production function expresses the output of physical goods in an economy in terms of the inputs – e.g., it is of the form:*

$$Y = AF(K,N)$$

*Where Y is output, A is the level of technology, K is the amount of capital employed, N is the amount of labor employed, and F(*) indicates an unspecified relationship. Note that the relationship between output Y and the inputs K and N is modified by the level of technology – i.e., a higher level of technology permits higher output from the same inputs.*

In analysis, the production function is converted to a "growth accounting equation," in which change (growth) in output is expressed as a function of the changes in the inputs, <u>weighted by the amount of output flowing to the inputs</u> (the input "share"):

$$\frac{\Delta Y}{Y} = \left[(1-\Theta)\,X\,\frac{\Delta N}{N}\right] + \left[\Theta\,X\,\frac{\Delta K}{K}\right] + \frac{\Delta A}{A}$$

This simply says that:

output growth = labor share X labor growth + capital share X capital growth + technical progress

Note that the shares to labor and to capital must sum to one, so that if capital share is Θ, labor share is (1– Θ). Another way of putting this is that the effect (on output) of increasing an input depends not only on the size of the increase in the input, but also depends on it's "share", and the "share" is not the same for all inputs. The definition of "shares" is somewhat tortured in the reading. It can be thought of as the increase in output if the input is increased by 1%.

SS. 4. 1. A. b) Growth in input vs. growth in total productivity. *(p. 38) In the above equation, there are two ways that output can grow. One way is to increase the amount of inputs – input growth. The second way is to get more out of the <u>same amount</u> of inputs – total factor productivity growth.*

SS 4. 1. A. c) Importance of factor shares. *(p. 38) As mentioned, the definition of "shares" is somewhat tortured in the reading. It can be thought of as the increase in output if the input is increased by 1%. The "share" of an input is clearly of great importance, since increasing the input with the largest "share" will have the greatest effect on output!*

SS 4. 1. A. d) Calculating growth in output. *(p. 38) This is simply application of the "growth accounting equation:"*

output growth = labor share X labor growth + capital share X capital growth + technical progress

The only example in the reading does not include any technical progress.

SS 4. 1. A. e) Growth in total output vs. growth in per capita output. *(p.39) Growth in total output is ΔY/Y, but the effect on per capita output is affected by any changes in N. I.e.,*

$$\frac{\Delta Y}{Y} = \frac{\Delta y}{y} + \frac{\Delta N}{N},$$

where lower case such as y indicates a per capita variable. Note that the math here is a little murky. The result is a growth accounting equation in per capita terms:

$$\frac{\Delta y}{y} = \Theta \text{ X } \frac{\Delta k}{k} + \frac{\Delta A}{A}$$

SS 4. 1. A. f) Convergence and the growth accounting framework. *(pp. 39-40) "Convergence" is "the process of one economy's catching up with another economy." This is analyzed using the per capita growth accounting equation. The portion of growth not explained by increase in per capita capital (Θ X Δk/k) would, in this model, be attributed to technical progress (ΔA/A).*

SS. A 4. 1. A. g) Influence of natural resources and human capital) *(pp. 41-44) Natural resources can be very important to growth, but no specific linkage is described. Human capital is not simply the supply of labor, but the supply of labor conditioned by the skills an talents of workers. The factor share of human capital is large in industrialized countries.*

SS 4. 1. A. h) Neoclassical theory and economic growth. *(pp. 44-47) Neoclassical growth theory (later referred to as the "Solow" growth model) combines two assumptions:*

- *a) diminishing marginal product of capital – as more capital per capita is added, the gain from each additional unit of capital will decrease,*
- *b) the investment required to maintain the level of capital per capita varies directly with the level of per capita capital – i.e., no matter what the level of capital per capita, each unit of capital per capita requires the same investment for maintenance/replacement.*

If the marginal product of capital is above the investment rate, an added unit of capital will contribute more to output than it costs to maintain/replace – more capital will be added. If the marginal product of capital is below the investment rate, an added unit of capital will cost more to maintain/replace than it will contribute output – capital will be reduced. The economy will move to a steady state in which marginal product of capital is just equal to investment in maintenance/replacement.

The text goes on to discuss the implication that the steady-state is not affected by the savings rate, and further to discuss the effect of population growth and technological innovation, but these do not seem to be part of the LOS.

SelfTest Question: What is the "share" of a factor in the neoclassical growth model?

Suggested Answer: Although the text describes the "share" as "the fraction of total output that goes to compensation" for the factor, this seems misleading. A clearer explanation is that the "share" is the per cant increase in output if the factor is increased by 1%.

B. Growth and Policy (Ch. 4)

SS. 4. 1. B. a) Endogenous growth theory vs. neoclassical growth theory. *(pp. 64-68) The neoclassical growth theory assumes diminishing marginal productivity of capital and, as a result of this assumption, indicates that an economy will achieve a long-run equilibrium state in which the growth rate is not affected by the savings rate. This is at odds with empirical observations. The endogenous growth theory modifies the assumption of diminishing marginal productivity of capital (the text presents a first simplified version in which the marginal productivity of capital is constant). Under this modified assumption, the marginal productivity of capital and the reinvestment rate required for maintenance/replacement no longer intersect and there is no equilibrium – a higher savings rate will lead to higher growth.*

SS 4. 1. B. b) Private return vs. social returns. *(pp. 67-68) An objection to the simplified endogenous growth model is that constant marginal capital productivity implies increasing returns to scale – which in turn implies the eventual domination of a single firm. This problem is patched over by considering the difference between private returns to a firm and public returns. It is assumed that when an individual firm increases capital, it is unable to capture all of the benefits, some of which leak out to other parts of society. Thus, even if one firm has constant returns to all factors, there will be no tendency to monopolization.*

NB: This point is not well explained in the text – the "explanation" is more of a stipulation.

SS 4. 1. B. c) Absolute vs. conditional convergence. *(pp.69-70) Neoclassical growth theory predicts absolute convergence" – i.e., all economies with the same rate of savings, population growth, and technology will converge to the same steady-state income. Economies with different savings rates or different population growth will reach "conditional convergence" – i.e., steady-state incomes will differ, but growth rates will equalize. Empirical observation indicates that although countries with a high savings rate tend to grow faster, the effect of higher investment on the growth rate is transitory. This implies that endogenous growth rate theory "is not very important for explaining differences in international growth rates, although it may be quite important in explaining growth in countries on the leading edge of technology" (the last point concerning "countries on the leading edge of technology" is not explained.*

SS 4. 1. B. d) Population growth and economic growth. *(pp72-73) The basic idea here is that population growth decreases the amount of capital per worker and results in slower growth. There is an extension linking population growth to income. Population growth is a combination of birth rate and death rate. Poor countries tend to have high birth rates, but this is offset by high death rates – the result is moderate population growth. Moderate increases in income tend to cut the death rate while not affecting the birth rate – the result is a high population growth rate. At high incomes, however, the birth rate tends to decrease and results in a lower rate of population growth.*

SS 4. 1. B. e) Analysis of growth. *(pp. 73-79) This LOS is quite similar to LOS 4. 1. A. a. Note also that although this LOS specifically mentions the "Asian Tigers," the reading also discusses Eastern Europe and "Truly Poor" countries. The basic idea is to apply the "growth accounting equation" of Chapter 3 to find out which of the input factors have contributed the most to growth. While the analysis here uses the factors in a somewhat different way, the basic idea remains. In Chapter 3, the "growth accounting equation" used labor growth and capital growth (and later included human capital growth), with any residual assigned to technical progress/total factor productivity. In Chapter 4, Table 4-1, presents data on total factor productivity TFP (technical progress), Δ % labor force participation (assumedly a measure of labor growth), and Δ % secondary education or higher (assumedly a measure of human capital). The text indicates that TFP growth has not been high, but that labor growth and human capital growth have been high. The authors conclude that the rapid growth of the "Asian Tigers" is "old-fashioned hard work and sacrifice."*

SS 4. 1. B. f) Key factors in reforming formerly centralized economies. *(pp. 75-76) The list of features developed by the IMF and the World bank was:*

- *a) Restore macroeconomic stability by bringing the budget close to balance and pursuing tight monetary and credit policies.*
- *b) Liberalize prices by removing price controls and allowing markets to begin operating.*
- *c) Privatize government-owned firms by selling them, or even giving them away to the citizens.*
- *d) Liberalize foreign trade, allowing domestic firms and consumers to have access to world markets.*
- *e) Establish a social safety network so that people who become unemployed do not become destitute.*

SS 4. 1. B. g) Natural resources and limits on economic growth. *(p. 77) Although limited natural resources would seem to limit economic growth, this argument is offset by two factors:*

a) *Technical progress permits us to produce more using fewer resources.*
b) *As specific resources come into short supply, their prices rise and producers shift toward substitutes.*

SS 4. 1. B. h) Factors likely to promote economic growth. *(p77) It is unclear exactly what this LOS refers to, since there is no direct response in the reading. However, the first point in the Summary on p. 77 indicates that:*

a) *Technological growth is important (in all models).*
b) *In the endogenous growth models, technological progress depends on saving, especially when invested in human capital.*

While this answer seems unsatisfactory, it does seem to be consistent with the tone of most of the analysis presented.

SelfTest Question: How can the “causes” of past economic growth be analyzed?
Suggested Answer: The basis of analysis if the “growth accounting equation.” I.e., the total growth can be subdivided into the relative contributions to growth from the factors of labor, capital, and human capital .

2. *Foundations of Multinational Financial Management,* 3rd ed., Alan C. Shapiro (Prentice Hall, 1998)

A. The Foreign Exchange Market (Ch. 5)

SS 4. 2. A. a) Calculating spread. *(pp.36-37) Spread is calculated as a percentage departure from the ask price – i.e., the ask price is the denominator. E.g., if the bid is 1.4090$US/$JD, and the ask is 1.4105$US/$JD, the spread is (1.4105 – 1.4090)/1.4105 = 0.11%*

SS 4. 2. A. b) Factors affecting spread. *(p. 37) Spread is:*

a) *narrower for widely traded currencies,*
b) *narrower for less volatile currencies,*
c) *wider for longer forward contracts.*

The spread may also be a function of the position of the particular dealer – i.e., oversold (shortage) or overbought (excess).

SS 4. 2. A. c) Cross rates. *(p37) If two currencies are quoted in terms of a third currency – e.g., DM/$US and $JD/$US - the cross rate for the two currencies can be calculated. E.g., if DM/$US = 1.7130 and $JD/$US = 1.4095, $JD/DM = ($JD/$US)/(DM/$US). One possible complication is the inclusion of spreads. If bid and ask quotations are given, the cross rate is calculated assuming you buy at the ask, and sell at the bid – i.e., the worst possible combination.*

SS 4. 2. A. d) Triangular arbitrage. *(pp. 39-40) Triangular arbitrage exploits mispricing/unbalanced exchange rates among currencies. Consider two paths for converting $ into £:*

a) *$ directly into £,*
b) *$ into DM, DM into £ .*

If these two paths result in a different number of £, triangular arbitrage is possible. The arbitrage would be to buy £ low (using the path that would give the most £ per $), and sell £ high (reversing the that would give the least £ per $). E.G., using the example in the reading:

a) $ directly into £: $1 => 0.504821£.

b) $ into DM, DM into £: $1 => 1.599744DM => 0.505448£.

The arbitrager would buy £ cheaply using the $ into DM, DM into £ path, and sell the £ at a higher rate by reversing the $ directly into £ path (i.e., £ directly into $). The first step would result in 0.505448£, which would be exchanged directly for 0.505448/0.504821 = 1.001242$ (it is more likely that the arbitrager would do this for a large amount).

The key to detecting the arbitrage is to look for the imbalance. The profit is easily calculated.

SS 4. 2. A. e) Effect of spreads on arbitrage. *(p. 40) The effect of spreads or other transaction costs would be to create a range of exchange rates within which arbitrage would not be feasible, since the arbitrage profit would be smaller than the transactions costs.*

SS 4. 2. A. f) Forward discount and premium. *(p. 43-45) "A foreign currency is at a forward discount if the forward rate expressed in dollars is below the spot rate, whereas a forward premium exists if the forward rate in dollars if above the spot rate. The forward discount or premium is calculated as a percent, based on the spot rate. It is annualized on a simple interest (no compounding) basis:*

$$\text{forward premium or discount} = \frac{\text{forward rate} - \text{spot rate}}{\text{spot rate}} \text{ x } \frac{360}{\text{days to maturity}}$$

SS 4. 2. A. g) Interest rate parity. *(pp. 46-51) Essentially, interest rate parity indicates that there should be no opportunities to arbitrage interest rates. I.e., any potential gain or loss of converting to another currency and lending should be offset by a change in the exchange rates. This implies that a country with a higher interest rate then country A should experience devaluation of its currency relative to country A. E.g., if interest rates in Britain are higher than the U.S., the pound would devalue vs. the dollar. Any higher gain from converting to pounds and lending would be wiped out by the decrease in the dollar value of a pound.*

Note that interest rate parity requires that:

$$1 + r_h = \frac{(1 + r_f)f_1}{e_0}$$

where r_h is the home country interest rate, r_f is the foreign interest rate, f_1 is the forward exchange rate, and e_0 is the spot rate. Alternately, this can be written as:

$$\frac{1 + r_h}{1 + r_f} = \frac{f_1}{e_0}.$$

This makes it quite easy to find the interest rate required by interest rate parity.

SS 4. 2. A. h) Covered interest arbitrage. *(pp. 46-51). Covered interest arbitrage is*

a) borrowing in one country,

b) changing to another currency and lending in that currency market, and then

c) reconverting to the original currency and paying off the original borrowing (supposedly at a profit).

E.g., from the example in the text (p. 47):

Borrow \$1,000,000 @ 7% → *Repay \$1,070,000*
Convert to £571,428.57 → *Convert to \$1,075,200* (*Net \$5,200*)
Lend £ at 12% → *Receive £640,000*

This should not be possible (for very long). The possibility of covered interest arbitrage is the motivation behind interest rate parity.
Note that interest rate parity requires that:

$$1 + r_h = \frac{(1 + r_f)f_1}{e_0}$$

where r_h is the home country interest rate, r_f is the foreign interest rate, f_1 is the forward exchange rate, and e_0 is the spot rate. Alternately, this can be written as:

$$\frac{1 + r_h}{1 + r_f} = \frac{f_1}{e_0}.$$

This makes it quite easy to find the interest rate required by interest rate parity. A close approximation is sometimes used:

$$r_h - r_f = \frac{f_1 - e_0}{e_o}$$

Perhaps the easiest way of testing for an arbitrage opportunity is to rewrite the interest rate parity condition to find the required future rate:

$$f_1 = e_0 \text{ x } \frac{1 + r_h}{1 + r_f}.$$

If this condition does not hold, an arbitrage opportunity exists (in the absence of transaction costs). The arbitrage is easily verified.

SS 4. 2. A. i) Forward rate under interest rate parity. *(p. 49) see SS 4. 2. A. h.*

SS 4. 2. A. j) Covered interest differential with transaction costs. *(p. 50) The is the difference in interest rates earned by domestic investment vs. exchanging to and investing in another currency, and converting back to the original currency, after transaction costs. Note again that you would pay the ask and receive the bid. The calculation is straightforward. The example in the reading leaves this as a \$ gain, it would be better expressed as a difference in the rate earned.*

B. Parity Conditions in International Finance and Currency Forecasting. (Ch. 7)

SS 4. 2. B. a) Purchasing power parity. *(pp. 83-88) Purchasing power parity comes in two forms. The absolute form indicates that prices in all countries should be the same after adjustment for exchange rates. This form is not in common use. The relative form indicates that exchange rates will adjust to reflect changes in price levels. Note that in the relative form, the price levels are not necessarily equal, but the rate of change in the price levels is equal:*

$$\frac{e_t}{e_0} = \frac{(1+i_h)^t}{(1+i_f)^t}$$

or, to find the exchange rate implied by PPP:

$$e_t = e_0 \text{ x } \frac{(1+i_h)^t}{(1+i_h)^t}$$

where e_t is the exchange rate at time t and i_x is the rate of inflation for country x.

SS 4. 2. B. b) Real exchange rate. *(p86-87) The real exchange rate is the nominal exchange rate adjusted for changes in relative purchasing power since time 0:* $e'_t = e_t \text{ x } \frac{(1+i_h)^t}{(1+i_h)^t}$

where e'_t is the real exchange rate. Note that this is quite similar to the preceding formula for the PPP exchange rate. This implies that if the relative changes in purchasing power exactly offset the change in the nominal exchange rate, the real exchange rate will remain unchanged.

SS 4. 2. B. c) Relative levels of inflation. *(pp.83-87) As can be seen from the preceding two topics, we can infer that (expected) inflation is relatively higher in countries that have higher interest rates or futures rates which indicate devaluation.*

SS 4. 2. B. d) Purchasing power parity and exchange rate movements. *(pp. Relative purchasing power parity indicates that the exchange rate between two currencies will adjust to reflect changes in the price levels of the two countries:*

$$\frac{e_t}{e} = \frac{(1+i_h)^t}{(1+i_f)^t},$$

where e_t is the exchange rate at time t, e_0 is the exchange rate at time 0, i_h is the home country inflation rate, and i_f is the exchange rate in the foreign country. This can be rewritten as:

$$e_t = e_0 \text{ x } \frac{(1+i_h)^t}{(1+i_f)^t}.$$

"Currencies with high rates of inflation should devalue relative to currencies with lower rates of inflation."

SS 4. 2. B. e) Capital market integration / segmentation. *(pp. 91-95) A capital market is said to be integrated if real interest rates are determined by the global supply and demand for funds. A capital market is said to be segmented when real interest rates are determined by local credit conditions.*

SS 4. 2. B. f) Fisher effect. *(pp. 88-91) The Fisher effect indicates that the nominal interest rate has two components, the real rate and an inflation premium, and that these two components are related as:*

1 + nominal rate = (1 + real rate)(1 + expected inflation rate)

This is approximated by the formula nominal rate = real rate + expected inflation.

The generalized version of the Fisher effect also assumes that because of arbitrage, the real rate of return is equal in all countries, so that:

$$\frac{(1+r_h)}{(1+r_f)} = \frac{(1+i_h)}{(1+i_f)}$$

Computing the components (given the others) is an algebraic exercise.

SS 4. 2. B. g) International Fisher effect. *(pp. 96-99) Purchasing power parity indicated that:*

$$\frac{e_t}{e_0} = \frac{(1+i_h)^t}{(1+i_f)^t}$$

while the Fisher effect indicated that:

$$\frac{(1+r_h)}{(1+r_f)} = \frac{(1+i_h)}{(1+i_f)}.$$

The result of extending the Fisher effect with purchasing power parity is the International Fisher effect:

$$\frac{e_t}{e_0} = \frac{(1+r_h)^t}{(1+r_f)^t}$$

Where e_t is the expected exchange rate at time t. This can be written:

$$e_t = e_0 \text{ x } \frac{(1+r_h)^t}{(1+r_h)^t}$$

to compute the expected exchange rate at time t.

SS 4. 2. B. g) Expected future spot rates. *(99-101) Given no biases, the forward rate should be exactly equal to the expected spot rate at maturity.*

3. "International Asset Pricing," Ch. 5, *International Investments*, 5th edition, Bruno Solnik and Dennis McLeavey (Addison Wesley Longman, forthcoming).

SS 4. 3. a) International market integration and international market segmentation. *(pp.3-4) While there is no direct definition of international integration of markets, "An integrated world financial market would achieve international efficiency, in the sense that capital flows across markets would instantaneously take advantage of any new information throughout the world." This would imply that integrated markets are those with no barriers to capital flow so that new information can be rapidly exploited. A segmented market, on the other hand, would have impediments to capital mobility so that relative mispricing could not be rapidly exploited.*

SS 4. 3. b) Impediments to international capital mobility. *(p. 4) Possible impediments are:*

- *a) Psychological barriers.*
- *b) Legal restrictions.*
- *c) Transaction costs.*
- *d) Discriminatory taxation.*
- *e) Political risks.*
- *f) Foreign currency risks.*

SS 4.3. c) Factors favoring international integration. *(p. 4-5) Although the impediments to integration are explicitly spelled out, the factors favoring integration are implied. The argument is that total capital mobility is not necessary for rapid exploitation of mispricing/new information. In many countries both private and institutional investors are extensively invested abroad, and all major corporations have truly multinational operations.*

SS 4. 3. d) Extending the domestic CAPM to the international CAPM. *(pp. 8-10) An extension of the domestic CAPM to an international context would use the domestic rate for the risk-free rate, and the market capitalization weighted portfolio of all risky assets for the market portfolio. This requires two unreasonable assumptions beyond those of the domestic CAPM:*

a) Investors throughout the world have identical consumption baskets.
b) Purchasing power parity holds exactly, so that real prices of consumption goods are identical in each country.

The first is simply the domestic assumption applied internationally, the second is necessary to make the first work.

SS 4. 3. e) Changes in the real exchange rate. *(pp 8-9) The real exchange rate is defined as the actual exchange rate multiplied by the ratio of the price levels of the consumption baskets in the two countries:*

Real exchange rate = nominal exchange rate X (foreign price level / domestic price level)

EXAMPLE: Suppose we start out with the price of the consumption basket \$1 or 3$\$_F$, and an exchange rate of 3$\$_F$/\$. The real exchange rate is 1 to 1: 3 X (1/3) = 1. If the price of the consumption basket goes to 4$\$_F$ but stays constant at 1\$, the exchange rate necessary to maintain a real rate of 1/1 is 4$\$_F$/\$. If it was anything but 4$\$_f$/\$, the real rate would have changed.

SS 4. 3. f) Expected exchange rates and expected holding returns on foreign bonds. *(pp.9-10) The real exchange rate is computed as:*

Real exchange rate = nominal exchange rate X (foreign price level / domestic price level)

Or:

$$X = S\ x(P_F/P_D)$$

The expected exchange rate under the assumption of a constant real rate of exchange would be:

$$S' = X\ x\ (P'_D/P'_F)$$

Where primes are end-of-period values.

EXAMPLE: Suppose that the initial spot rate is 2D/1F, and the consumption basket is priced at 2F or 1D. The real exchange rate is then 1D/2F X 2F/1D = 1 (1 to 1). If domestic inflation is 3%, and foreign inflation is 1%, and the real exchange rate is constant, we would have at the end of the year:

$$S' = 1\ x\ (2.06/1.01) = 2.039604D/1F$$

Or approximately 2.04D/1F = (0.04/2) x 100 = 2%

Note: it is necessary to keep the numerator and denominator straight – it helps to notice that in computing the real exchange rate the numerators and denominators cancel, leaving a pure ratio!

Not explicitly mentioned in the text is the use of an approximation for currency appreciation:

currency appreciation = domestic inflation rate – foreign inflation rate

The text uses a domestic inflation of 3% and a foreign inflation of 1%, arriving at a currency appreciation of 2%. The expected rate of return is then:

(1 + foreign interest rate) x (1 + currency appreciation)

(1.03) x 1.02) = 1.0506 => 5.06%

This may also be approximated by simply summing the two rates 2% + 3% = 5%.

Alternately, the expected domestic currency holding period return is simply the return using the expected end-of-period exchange rate. I.e., assume a foreign security originally was priced at F100, for a domestic price of D200. With a 3% interest rate, at the end of the period the foreign security will be priced at F103. The expected domestic value of the security is then 2.04 X 103 = 210.12, and the expected return is (210.12/200) - 1 = 0.0506 => 5.6%

SS 4. 3. g) Actual exchange rates and return. *(p. 9-10) This is a twist on the preceding LOS. In this case you are given S', X, and the inflation rates. The new real exchange rate is:*

$$X' = S'\ x\ [P_F(1 + i_F)/P_D(1 + i_D)]$$

The example in the text uses an S' of 1.8,so that

$$X' = 1.8\ x\ [1(1.01)/2(1.03) = 0.88$$

The realized return is once again (1 + foreign interest rate) x (1 + currency appreciation). This time the currency appreciation is (1.8/2.0) –1 = -0.1 = -10%. I.e., instead of getting 2D for each F, we now get 1.8F for each F – which in this case is a depreciation. The rate of return is then (1.03) x (0.9) = 0.972 =>-7.3%.

SS 4. 3. h) Explanation of foreign currency risk premium. *(p.12) The foreign currency risk premium is the amount by which the expected appreciation exceeds the appreciation implied by interest rate parity. The expected appreciation can be expressed as a forward premium, and can be approximated as the difference between the two interest rates.*

SS 4. 3. i) Calculating a foreign currency risk premium. *(p. 11) If interest rate parity implies a 2% rate of appreciation, but the expected rate of appreciation is 3%, 1% of the expected appreciation is attributed to the risk premium. The appreciation (or forward premium) implied by interest rate parity can be approximated as the difference between the two interest rates. E.g., if the domestic interest rate is 5%, and the foreign interest rate is 3%, interest rate parity implies a 2% appreciation.*

SS 4. 3. j) Expected returns using the international capital asset pricing model. *(p. 15-16) The international capital asset pricing modelfor one foreign currency can be expressed as:*

$$E(R_i) = R_0 + \beta_{iw} x RP_w + \gamma_i x SRP_{FC}$$

Where R_0 is the risk-free rate, β_{iw} is the world beta, RP_w is the risk premium on the world index, γ_i is the sensitivity of asset i domestic currency returns to the Foreign currency, and SRP_{FC} is the foreign currency risk premium. For multiple currencies, the last term is a series.

SS 4. 3. k) Currency exposure and correlation. *(pp. 13-15, 20-21) Currency exposure is essentially the sensitivity of asset returns to currency movements. I.e., if an asset is highly correlated with the movements of a currency, that asset would have a large exposure to that currency.*

SS 4. 3. l) Market segmentation and the International CAPM. *(pp.16-17) The ICAPM applies to all securities only in an integrated market. Markets are segmented if securities that have the same risk have different values in different markets. The result is that a much more complicated model is required.*

SS 4. 3. m) Estimating exchange rate exposure and the impact of real and nominal exchange rate changes. *(p. 22-23) If purchasing power parity held exactly or closely, exchange rate changes would simply reflect differential inflations, and would be unimportant. However, purchasing power parity seems widely violated. It is the movements in the real exchange rate are large relative to purchasing power effects. The exchange rate exposure of a firm depends on the currency structure of it's exports, imports, and financing. It may also be exposed through the actions of it's competitors. The exposure could be estimated by regressing stock returns on market and currency returns. "This estimate can be refined by a detailed analysis of the activities and financings of the company; the geographical distribution of it's sales and investments, the currency origin of it's profits and costs, the currency structure of it's debt, etc." There are examples on p. 23 of the reading.*

SS. 4. 3. n) Currency exposures of national economies, equity markets, and bond markets. *(pp. 23-25) There are two models of the effect of exchange rate movements on economic activity – these two models are discussed in the next LOS. The effect of exchange rate movements on equity markets is a result of the impact of the changes on the economy – i.e., "…economic activity is a major determinant of stock market returns, so the influence of exchange rate movements on domestic activity may explain the relation between exchange rate movements and stock returns." In the traditional competitiveness model, the short-run effects are negative but the long-run effects are positive and "The stock market… may be positively or negatively affected, depending on whether the short-term or long-term effect dominates." In the money demand model, appreciation of the domestic currency is associated with increased real growth and so with an increase in real stock returns.*

The effect of exchange rate movements on bond markets is through the effect on interest rates. Again, there are two theories:

a) *If appreciation of the home currency is associated with an increase in real interest rates, this would have a negative effect on bond prices. The reading notes that an increase in interest rates associated with inflationary expectations, however, could cause a depreciation of the currency – leading to an even greater loss for international investors.*
b) *The second theory is associated with policies that contain foreign exchange rate targets. In this case, depreciation of he local currency causes the authorities to increase interest rates to defend the currency, and appreciation would trigger a decrease in interest rates. This situation would result in a positive bond price effect from appreciation of the currency, but a negative effect from depreciation of the currency.*

SS. 4. B. o) Models of exchange rates and economic activity. *(p. 23-25) The tow models are:*

a) *The traditional model: depreciation of the currency tends to increase competitiveness. At the same time the depreciation increases the cost of imports, which increases inflation and reduces real income – resulting in an initial decrease in domestic demand and production. Eventually, however, the increased international competitiveness and increasing exports will reverse the initial decline. The reading warns that if the economy is sluggish to react, the original decline may not be reversed and conditions may continue to worsen.*
b) *The money demand model: "In this model, real growth in the domestic economy leads to increased demand for the domestic currency through a traditional money demand equation. This increase in currency demand induces a rise in the relative value of the domestic currency."*

4. "Analyzing the Firm's Environment," Ch. 5, *Corporate Finance: A Valuation Approach*, Simon Benninga and Oded Sarig (McGraw-Hill, 1997)

SS 4. 4. a) Estimating industry sales using historical gross domestic product. *(pp. 23 – 27) This is a straightforward application of linear regression:*

$$\text{Annual industry sales} = \alpha + \beta(\text{GDP growth})$$

Note that the regression is in terms of GDP growth, rather than GDP value. Also note that where dollar figures are used the regression is based on constant dollars, where the base year could be any year but the reading suggests using the present year as the base.

SS 4. 4. b) Forecasting industry sales using the estimated relationship. *(pp. 24-25) This is again straightforward application of linear regression, but again note that GDP growth is the independent variable.*

SS 4. 4. c) Interpreting regression analysis. *(p.24) Interpretations are:*

a) *The intercept (α) is industry sales if GDP does not change.*
b) *The slope (β) is the increase in the number of cars sold for a 1% increase in GDP.*
c) *The industry sales is the sum of a) and b).*

SS 4. 4. d) Effects of inflation. *(pp. 143-144) If a variable is measured in dollar terms, adjustment for inflation is called for (because the value of a dollar changes). If a variable is in terms of physical goods (e.g., units of production, number of whatevers), adjustment for inflation is not necessary because the price of the item will change.*

SS 4. 4. e) Industry company performance over economic cycles. *(pp. 23-25) This LOS seems to have been covered in the preceding LOSs. However, on p. 23 we have: "… to project the industry's (short-run!) sales based on macrogrowth rates… you:*

a) Estimate the relation between the sales of the industry and growth rates by using historical values
b) Forecast industry sales by applying the estimated relation to the forecasted growth rate
The reading presents a regression analysis, it is likely that this would be given to you.

SS 4. 4. f) Market share and macroeconomic conditions. *(pp. 28-29) The idea here is that the performance of companies in the same industry will be different, depending on the fundamentals of the company and it's marketing strategy.*

a) *"The different positioning of firms in the same industry implies that, depending on economic conditions, consumers switch from one brand to another to match the characteristics of their consumption to the desired consumption basket." (Economics is the art of stating the obvious in incomprehensible terms.) What this rather tortured statement means is that if you make less you don't go out to fancy restaurants as much as you used to (and to McDonalds more).*
b) *There are many ways to consider this – pricing policy, efficiency, etc. The text suggests an historical comparison of the ratio of sales for different firms to the level of GDP.*

SS 4. 4. g) Life cycle and forecasting. *((pp. 31-34) Generally, life cycle analysis divides the life of a product into stages such as:*

a) *development: low sales, low competition. Profits are indeterminate: perhaps high due to high margins, perhaps low if price is held down to generate interest and future sales.*
b) *expansion: rapidly increasing sales, high profits, increasing competition.*
c) *maturity: sales slowly increasing, increasing competition, decreasing profits.*
d) *decline: decreasing (or steady) sales, normal profits and competition.*

Examination of the life cycle of similar products in the industry my help in forecasting potential life cycle.

SS 4. 4. h) Life cycle of product vs. life cycle of industry. *(pp. 33-34) The life cycle of the product and the life cycle of the industry are two different things. I.e., products may grow and decline as the industry develops new or improved products and carries on. The product life cycle is simply a starting point for examining the industry life cycle.*

SS 4. 4. i) Regression toward the mean: *(p. 34) Simply put, regression to the mean implies that in the long run the industry's profits tend to return to normal levels. I.e., if profits are high, competition will be attracted and profits will drop. If profits are low, some competitors will drop out of the market and profits will rise. Duh.*

SS 4. 4. j) Factors affecting market share. *(pp. 35-36) Factors that affect a firm's market share include:*

a) *changes in consumer tastes,*
b) *entry or exit of competing firms,*
c) *relative strengths and weaknesses of industry firms,*
d) *changes in general market conditions,*
e) *marketing strategies of firms in the industry.*

SS 4. 4. k) Marketing and market share. *(pp. 35-36) The reading discusses a model in which market share is a function of the relative amount and effectiveness of firm marketing effort compared to the total amount and effectiveness of industry marketing effort. The implication of the model is that the change in a firm's market share is proportional to the percentage change in its marketing efforts.*

SS 4. 4. l) Growth rate: *(p.36) Note first that if the market share of the company does not change, the company sales growth rate will be the same as the industry growth rate. If the company's market share changes, however:*

(1 + growth of firm sales) = (1 + growth of industry sales) x (1 + fractional change of market share)

STUDY SESSION 13
Asset Valuation
Debt Securities: Credit Analysis

Study Session 13: 2. "General Principals of Credit Analysis," Ch. 9, *Fixed income Analysis for the Chartered Financial Analyst Program: Level I and II Readings*, Frank J. Fabozzi (Frank J. Fabozzi Associates, 2000).

This reading deals with credit analysis – the probability that the borrower will be able to meet its Obligations in timely fashion. This is not simply a 0/1 or on-off situation – the probability is variable, and the implications of changing credit quality themselves create risk, even if the borrower does not eventually default. Further, there are degrees of default, ranging from late payments or interruption of payments through bankruptcy and dissolution of the firm – and even then there may be appreciable recovery.

1. Three types of credit risk are identified – default risk, credit spread risk, and downgrade risk. Note that only one of these deals with outright default.
2. The analysis of risk is an appraisal of four characteristics of the buyer (the four Cs):
   a) Character – the quality of management.
   b) Capacity – the ability to generate sufficient cash flow to meet obligations.
   c) Collateral – the support if cash flows are insufficient.
   d) Covenants – requirements to perform, or to refrain from, certain actions.
3. Analysis of capacity involves traditional analysis of accounting ratios (profitability, debt levels and coverage) and of cash flow.
4. There are many types of covenants, which may be "affirmative" (require an action) or "negative" (require that an action not be taken).
5. High yield bonds require special considerations: analysis of the debt structure of the firm, analysis of the corporate structure, and a refined examination of the covenants with an eye toward insights as to corporate strategy. For high yield bonds, there is some merit to taking an equity analyst's perspective.
6. Asset backed securities and non-agency mortgage-backed securities (MBS) (agency MBS, remember have the real or implied backing of the government) also call for additional analysis in such areas as:
   a) Collateral quality,
   b) Quality of the seller or servicer (this is similar to quality of an issuer),
   c) Analysis of cash flows (including prepayment and other possible effects),
   d) Legal structure.

   Agency ratings of asset-backed securities are on a different basis and procedure than the ratings of corporate bonds.
7. Municipal bond analysis depends on the type of bond.
   a) General obligations bonds depend on the taxing power of the municipality, and analysis focuses on the tax base and burden, the economic situation and forecast, and the fiscal soundness of administration.
   b) Revenue bonds are supported by revenues from a specific project, and the analysis focuses on the viability and cash-generating power of the project.
8. Analysis of sovereign bonds is in some ways similar to municipal bond analysis, but with greater emphasis on economic forecasts and political ability and the will to meet obligations.

SS 13. 2. a) Default risk, credit spread risk, and downgrade risk. *They are:*

a) *Default risk: the risk that the borrower will fail to satisfy the terms of the obligation with respect to the timely payment of interest and repayment of the amount borrowed.*
b) *Credit spread risk: the risk that the value of a bond will decline due to an increase in the credit spread (the premium over similar Treasury issues).*
c) *Downgrade risk: the risk that the value of a bond will decrease due to a downgrade in the credit rating of the issue.*

SS 13. 2. b) Borrower's character, borrower's capacity to pay, underlying collateral, and covenants in credit analysis. *(pp. 665). These are the "four C's" of credit:*

a) *Character – Includes the ethical reputation, business qualifications and operating record of directors, management, and executives responsible for use and repayment of the borrowed funds,*
b) *Capacity – the ability to repay obligations,*
c) *Collateral – the assets pledged to secure the debt* <u>*and*</u> *the quality of unpledged assets controlled by the issuer.*
d) *Covenants – the terms and conditions of the lending agreement, which lay down restrictions to protect the lender.*

SS 13. 2. c) Factors in assessing the quality of management, and their importance. *(pp. 665-666). Fabozzi quotes Moody's Investor Service: "Although difficult to quantify, management quality is one of the most important factors supporting an issuer's credit strength. When the unexpected occurs, it is a management's ability to react appropriately that will sustain the company's performance." Factors include:*

a) *strategic direction,*
b) *financial philosophy,*
c) *conservatism,*
d) *track record,*
e) *succession planning,*
f) *control systems.*

SS 13. 2. d) Sources of liquidity and their importance. *(pp. 666-667). Sources of liquidity include:*

a) *securitization of assets,*
b) *third-party guarantees,*
c) *backup credit facilities (lines of credit, etc.)*

These sources of liquidity are very important in adverse conditions such that traditional financing sources are not available.

SS 13. 2. e) Short-term solvency ratios, capitalization ratios, and coverage ratios. *(pp. 668-670). As follows:*

a) *Short-term solvency ratios are used to judge the adequacy of short-term assets for meeting short-term obligations. These ratios include:*
   - *current ratio = current assets / current liabilities; measures the coverage of current liabilities by current assets*
   - *acid-test ratio = (current assets – inventories – accruals – prepaid items) / current liabilities; takes into account the composition of current assets – i.e., that some current assets may not be liquid.*
b) *Capitalization ratios are used to determine the extent to which the firm is trading on it's equity (using leverage). These ratios include:*

- *Long-term debt to capitalization:*

$$\frac{\text{long - term debt}}{\text{long - term debt + shareholders equity including minority interest}}$$

- *Total debt to capitalization:*

$$\frac{\text{current liabilities + long - term debt}}{\text{long - term debt + shareholdrs equity including minority interest}}$$

c) *Coverage ratios are used to test the adequacy of cash flows generated through earnings to meet debt and lease obligations:*

- *EBIT interest coverage ratio:*

$$\frac{\text{earnings before interest and taxes}}{\text{annual interest expense including capitalized interest}}$$

- *EBITDA interest coverage ratio:*

$$\frac{\text{earnings before interest, taxes, depreciation and amortization}}{\text{annual interest expense including capitalized interest}}$$

- *Funds from operations / total debt and free operating cash flow / total debt both indicate the amount of funds from operations relative to the amount of total debt. Free cash flow is the cash flow from operations minus fixed charges.*

SS 13. 2. f) Calculating and applying key ratios used in credit analysis. *(pp. 668-670). The calculation is clear from d). The text gives little guidance as to "standards" for the ratios, but in general higher solvency ratios, lower capitalization ratios, and higher coverage tests would be considered better. Typically, a candidate would be given some sort of averages or typical levels for different ratings, and the candidate would be expected to evaluate the rating in terms of the typical figures.*

SS 13. 2. g) Limitations of traditional ratios. *(pp. 670-672). The traditional ratios discussed above have not performed well as indicators of financial distress. Fabozzi relates this to the lack of consideration of cash flows, which provide better indicators of financial distress: "Yet none of the traditional ratios discussed above take into account the cash flows from operations." Note however, that funds from operations and free operating cash flows, which do consider cash flows, were discussed. Other authors have suggested that limitations include the reliance on accounting data, which is both backward looking and often difficult to compare.*

SS 13. 2. h) Cash flows, the ability of an issuer to service debt, and financial flexibility. *(pp. 670-672). Examination of the level and trend of cash flows has provided signals of financial distress in situations in which the traditional ratio analysis has failed. The analysis proceeds as follows:*

a) *Compute the "basic cash flow as the sum of net earnings, depreciation, and deferred income taxes less items in net income that do not provide cash.*
b) *Deduct nondiscretionary items (e.g., maintenance).*
c) *Deduct increases in working capital (excluding cash and payables).*
d) *The result is "operating cash flow."*
e) *Deducting capital expenditures from "operating cash flow" gives the discretionary cash flow.*

A useful ratio is "cash flow from operations to capital expenditures." A higher ratio is generally considered better, but the trend and reasons for the trend must be examined.

SS 13. 2. i) Bond covenants and their importance. *(pp.673-674). Covenants may be classified as "affirmative" (promises to do certain things) or "negative" (promises not to do certain things). There are many types of covenants, but in general:*

a) Affirmative covenants may require:
- *payment of interest, principals, and premiums on a timely basis.*
- *pay taxes and other claims on a timely basis (unless contested).*
- *Maintain all properties in good condition.*
- *Submit periodic certificates of compliance.*

b) Negative covenants may include:
- *limitations on the ability to incur debt.*
- *"maintenance test" – a minimum required ratio of earnings available for fixed charges.*
- *"debt incurrence test," - a minimum required ratio of earnings available for fixed charges for new debt to be incurred.*
- *cash flow tests*
- *working capital maintenance*

Some indentures may put limitation on subsidiaries. In computing covenants, some subsidiaries (called restricted subsidiaries) may be required to be consolidated, while consolidation may not be required of other subsidiaries (called unrestricted subsidiaries).

SS 13. 2. j) Debt structure of a high-yield issuer and it's effect on risk. *(pp. 674-676). The typical elements are:*

a) bank debt,
b) broker's loans or "bridge loans,"
c) reset notes,
d) senior debt,
e) senior subordinated debt,
f) subordinated debt (payment in kind).

Interrelationships include the following considerations:

a) Bank debt has priority over other forms of debt, is short-term, and because it is short-term it is strongly affected by short-term interest rates, requiring an added dimension of analysis.
b) Repayment of the short-term bank debt can come from operating cash flow, making projections of those flows more critical.
c) If operating cash flows are inadequate refinancing or sale of assets must be utilized. Sale of assets requires analysis of the assets available fro sale and the effect of the asset sales on future cash flows. Refinancing typical involves broker loans (bridge loans) or reset loans. In this case the forecast of future rates becomes more important. High interest rates / reset levels may cause the borrower to turn to sale of assets.
d) In the presence of higher-priority bank debt, the term "senior securities" can be misleading.
e) Deferred coupon bonds, which allow postponement of interest or principal to a later date, may adversely affect subordinated securities as the amount of deferred payments increases over time.

SS 13. 2. k) Importance of holding company corporate structure of a high-yield issuer. *(pp. 676-677). In a holding company structure, there may be multiple subsidiaries. It is necessary to understand the corporate structure to assess how cash will flow will occur between the subsidiaries and the parent, and between the subsidiaries. The interrelationships may provide cash, but may also use cash.*

SS 13. 2. l) Equity perspective in analyzing the credit worthiness of high-yield users. *(pp. 677-678). Some investors advocate using an equity perspective when analyzing high-yield bonds because:*

a) *The risk (standard deviation of returns) of high-yield bonds is greater than that of high-grade bonds (although less than that of common stock),*
b) *High-yield bond returns are more highly correlated with equity returns than with investment-graded bond returns.*

SS 13. 2. m) Collateral credit quality, seller / servicer quality, cash flow stress and payment structure, and legal structure in ratings analysis. *(pp. 678-680). As follows:*

a) *Credit quality of the collateral considers the borrower's ability to pay and the borrower's equity in the asset. The latter is important as to whether the borrowing will be defaulted or paid of by sale of the assets. Further, the larger the number of borrowers in the underlying pool, the greater the diversification and the less the risk of default.*
b) *Quality of the seller / servicer refers to the ability of the seller / servicer to collect payments, follow up on delinquent accounts, and in general process payments in a timely manner. Important factors include servicing history, experience, underwriting standard, servicing capabilities, human resources, financial condition, and competitive situation.*
c) *Cash low stress and payment structure: The payment structure may be pass-though, in which payments are paid to the investors proportionately as received, or pay-through, in which payments are divided among participants according to a specified payment structure. The cash flow stress refers to the likelihood that the cash flows from the payments will be sufficient to meet the promised payments. Cash flow stress is more critical for pay through structures.*
d) *Legal structure: If a firm directly issues securities backed by receivables, in bankruptcy the firm still has an interest in the receivables and they can be seized to satisfy the obligations to other security holders. To avoid this possibility, the firm may create a Special Purpose vehicle – a subsidiary which purchases the receivables from the parent and issues the receivables-backed securities. These securities, having been sold by the parent, are no longer subject to the claims of the parent's creditors and will receive a higher credit rating (pay a lower yield).*

SS 13. 2. n) Analyzing the creditworthiness of tax-backed and revenue municipal bonds. *(pp.682-684). The analysis of tax-backed and revenue bonds is quite different. The analysis of tax-backed bonds centers on the ability and will of the municipality to impose and collect taxes sufficient to meet the obligations. Factors include:*

a) *the overall debt burden and its structure.*
b) *the issuers ability and political discipline to maintain sound budgetary policy.*
c) *the local taxes and intergovernmental revenues available to meet the obligation, and the ability to collect those taxes.*
d) *assessment of the overall socioeconomic environment.*

The analysis of revenue-backed municipal securities centers on the ability of the project to generate sufficient revenues to meet the obligations, and is similar to the analysis of corporate bonds. Note the importance of covenants in the areas of:

a) *limit of the basic security,*
b) *flow of funds structure,*
c) *rate or user charge conditions,*
d) *priority-of-revenue claims,*
e) *additional-bonds test,*
f) *miscellaneous requirements (e.g., insurance, reporting, outside inspections).*

SS 13. 2. o) Economic and political risks in rating sovereign debt. *(pp. 685-686). The key factors considered by Standard and Poor's for local currency debt are:*

a) the stability of political institutions and degree of popular participation in the political process,
b) income and economic structure,
c) fiscal and budgetary flexibility,
d) monetary policy and inflationary pressures,
e) public debt burden and debt service track record.

When assessing the quality of foreign currency debt, Standard and Poor's considers:

a) the interaction of domestic and foreign government policies,
b) the country's balance of payments,
c) The structure of the country's external balance sheet.

It is interesting to note the similarity between the analysis of sovereigns and the analysis of municipal bonds.

SS 13. 2. p) Factors considered in local and foreign currency ratings. *(pp 685-686). One rating is in terms of local currency, the other rating is in terms of foreign currency. The two ratings are given because the ability of the government to generate sufficient local currency (by raising taxes or other means) may differ from its ability to generate foreign currency.*

SS 13. 2. q) Credit analyses required for corporate bonds vs. asset-backed securities vs. sovereign debt. *(pp. 665-687). Fabozzi does not present a clear-cut answer to this LOS, especially in the comparison of corporate and sovereign debt. Analysis of corporate debt focuses on the ability of the firm as a whole to generate the required funds and on the legal safeguards. For corporate debt, there is a higher legal authority. While the analysis of foreign debt also focuses on the ability of the country to generate sufficient funds, the ere is no higher legal authority and the analyst must focus on willingness to pay and the soundness of political systems. It is similar to the question "Is the glass half empty, or half full?" Standard and Poor's, with a dual rating system, seems to say that the glass is half empty; Fridson seems to say that the glass is half full. Thus, there are similarities between corporate and sovereign analysis, but sovereign analysis also has complicating factors. The analysis of asset-backed securities centers on the soundness of the underlying pool of assets, and secondarily on the ability of the issuer to process the payments. This is quite different from the analysis of the firm required by corporate bonds.*

SS 13. 2. r) Traditional ratios and downgrading. *(pp667-672). The traditional ratios may be classified as profitability ratios or debt and coverage ratios. The emphasis in Fabozzi is not on the ratios themselves (although candidates should know the ratios), but on their use and ineffectiveness. While the use of the ratios is straightforward, Fabozzi points out that these traditional ratios do not take account of cash flows. He suggests that analysts should also consider the discretionary cash flow.*

SelfTest Question: Define credit quality, and relate credit quality to credit risk.

Suggested Answer: Credit quality is the probability that the issuer of a bond will meet all obligations in a timely fashion – the higher the probability, the higher the credit quality. This is the dimension desribed by rating agencies such as Standard & Poor's. Although credit quality is the probability, credit risk is also concerned with changes in the level of quality. An issuer may not default and meet all of its obligations, but if the issue is downgraded the required rate of return on the issue will increase and the bondholders will suffer a capital loss. Also included in credit risk is spread risk, i.e. the risk of changes in the spread for a risk / rating class, since increased spreads may cause a capital loss.

STUDY SESSION 14
Asset Valuation
Debt Securities: Credit Analysis

Study Session 14: 1. , *Fixed Income Analysis for the Chartered Financial Analyst Program*, Frank J. Fabozzi (Frank J. Fabozzi Associates, 2001)

A. Introduction to the Measurement of Interest Rate Risk (Level 1, Ch. 7) This reading covers the price-yield relationship and the concept of duration.
➔ **NB: The candidate should be aware that the terms used in this reading to describe various aspects of duration may be inconsistent with the terms used in previous Fabozzi articles and in other texts.**

SS 14. A. a) Full valuation vs. duration/convexity. *(pp. 244-247) In evaluating interest rate risk, the full valuation approach re-values each security in a portfolio individually, and then sums to get the change in value of the portfolio. The duration/convexity approach uses the portfolio duration and convexity to estimate the change in portfolio value. Both approaches have advantages. The full valuation approach is more accurate (assuming that the valuation model is accurate), particularly if the portfolio contains a high proportion of assets (putable, callable, or prepayable instruments) for which duration and convexity are inaccurate. Also, all bonds may not undergo the same change in yield under a particular scenario. The full valuation approach is time-consuming for largerportfolios. The duration/convexity approach offers a quicker and easier, although less accurate, alternative.*
SS 14. A. b) Stress testing . *(p. 247) Stress testing refers to the use of extreme scenarios to assess exposure to interest rate exposures.*
SS 14. A. c) Computing the duration of a bond. *The duration of a bond is defined as:*

$$\text{duration} = \frac{V_- - V_+}{2(V_0)(\Delta y)}$$

Note:

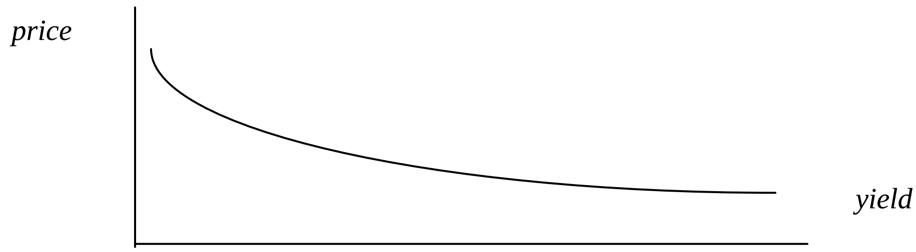
a) *This is an increase in price (V_-) associated with a decrease in yield and a decrease in price (V+) associated with an increase in yield. The + and – refer to the direction of the change in interest rate, not to the change in price.*
b) *This can also be interpreted as the <u>fractional</u> change in the price divided by the change in yield – i.e., not the dollar change.*

SS 14. A. d) Percent change in price using duration and yield change. *This is simply the previous problem slightly changed. Solving for the change in price, we obtain:*

approximate percent change in price = - duration x Δy x 100.

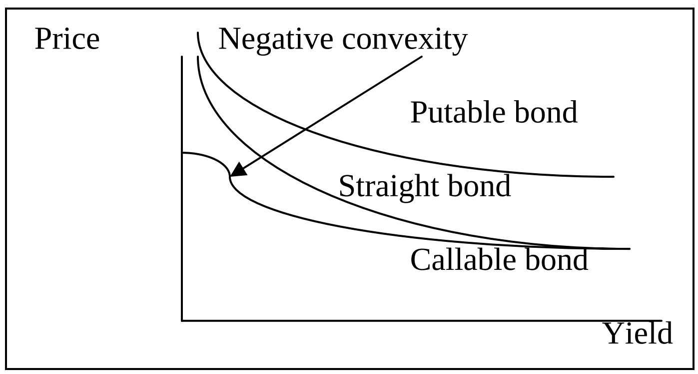
Note the negative to reflect the inverse relationship between price and yield, and the multiplication by 100 to change from a fraction to a percent.

SS 14. A. e) Price – yield relationship of an option-free bond and approximation of price change.
The price-yield relationship for an option-free bond is graphed as:



Note that the relationship is curved in a convex manner. Duration is a linear approximation, so that over small yield changes the curvature will have a small effect on the calculation, but this effect will become larger as the yield change increases.

SS 14. A. f and SS 14. A. g) Price-yield relationships for putable and a callable securities.



a) *The callable bond will not go above the call price. At high yields, the right to call the bond will have little value and the bond will act like a straight bond. As yield drop, the right to call the bond will become increasingly more valuable, and the price increase of the bond will slow and change from positive convexity (increasing at an increasing rate) to negative convexity (increasing at a decreasing rate). Note that positive convexity implies that the price increase from a decrease in interest rates will be larger than the decrease in price from a same size increase in interest rates.*
b) *The putable bond price will not go below the put price. A low yields, however, the right to put the bond will have little value and the putable bond will act like a straight bond*

SS 14. A. h) Effect of interest rate change on the calculation of duration. *There are three considerations:*

a) *The formula assumes that the rate changes do not affect the cash flows from the bond. If the bond has option characteristics, however, this assumption may be in error. In that case, duration computed on small changes may fail to reflect the optionality effect.*
b) *The price-yield relationship is convex – i.e., the price changes are asymmetric. Larger rate changes may encounter this asymmetry.*

c) *The model used to predict the price changes resulting from interest rate changes is crucial to accurate estimation of duration. This is particularly difficult when small rate changes are used, since inaccuracy in the model will be magnified.*

SS 14. A. i) Modified duration vs. effective duration. *"Modified" duration is computed (for a 100 basis point change) assuming that the cash flows are unaffected by the interest rate changes. As noted, this is incorrect for bonds with option characteristics. If the price changes are computed taking account of the effect of the rate changes, the resulting duration is "effective duration."*

SS 14. A. j) Effective duration and embedded options. *The calculation of effective duration takes into account any effect of the rate changes on the cash flow to the bond, whereas modified duration ignores any such effects. For bonds with embedded options, effective duration is more accurate.*

SS 14. A. k) Portfolio duration and it's limitations. *The duration of a portfolio is simply the weighted average of the assets in the portfolio:*

$$\text{portfolio duration} = \sum_i \text{w}_{\text{i}} D_i$$

Alternately, the price change for every individual asset in the portfolio, for a 100 basis point change, can be used. The problem with a portfolio duration is that it assumes that very asset in the portfolio has the same yield change, but in actuality various assets in the portfolio will likely have different yield changes.

SS 14. A. l) Computing convexity. *Convexity can be calculated using the formula:*

$$\text{convexity} = \frac{V_+ - V_- - 2V_0}{2V_0(\Delta y)^2}$$

Again, note that this is an increase in price(V_-) associated with a decrease in yield, and a decrease in price(V+) associated with an increase in yield from an original price V_0. The + and – refer to the direction of the change in interest rate, not to the change in price.

SS 14. A. m) Convexity and estimated price change. *Duration assumes that the price change due to an interest rate change moves along a straight line. The price-yield relationship, however, is curved (convex). Convexity provides an adjustment to reflect the curvature:*

$$\text{convexity adjustment} = \text{C x } (\Delta y)^2 \text{ x } 100$$

Again, note that this formula may not be the same as the formula found in previous Fabozzi readings, or in other texts, since several definitions of convexity are in use.

SS 14. A. n) Estimated price change using both the duration and convexity. *This is simply the sum of the duration0based estimate and the convexity adjustment:*

$$\textit{Estimated price change} = -\textit{ duration x } \Delta y \textit{ x } 100 + C \textit{ x } (\Delta y)^2 \textit{ x } 100$$

SS 14. A. o) Convexity variation among vendors. *Simply because they use different scaling, depending on whether they use %, decimal, or basis points or whether they use annual or semiannual duration. The real point is that the convexity measure must be consistent with the duration measure.*

SS 14. A. p) Modified convexity vs. effective convexity. *As with modified and effective duration, modified convexity assumes that yield changes have no effect on cash flows (i.e., no optionality), while effective duration includes the potential effect of yield changes on the cash flows (i.e., reflect optionality).*

SS 14. A. q) Dollar value of a basis point. *The price (dollar) value of a basis point is the change in the price of a bond when the yield changes by one basis point. It can be found by using duration to find the % price change for a 1 basis point change in yield, and then applying the % price change to the price to find the dollar price change:*

$$\text{PVBP} = \frac{\text{\% price change}}{100} \text{ x price}$$

Fabozzi presents this somewhat differently:

$$PVBP = duration \times \Delta y \times 100 \times price$$

Where Δy is measured in decimal form (i.e., 1 basis point = 0.0001), rather than in basis points.. You can avoid confusion over the decimal point by focusing on what you are doing, rather than memorizing the formulas. Just remember that you are computing % change in price and applying that to the price to find the dollar change in price, and put the decimals accordingly.

SS 14. A. r) Duration and the dollar value of a basis point. *As presented in Fabozzi,*

$$PVBP = duration \times \Delta y \times 100 \times price$$

Again, note that in this form Δy is measured in decimal form (i.e., 1 basis point = 0.0001), rather than in basis points.

SS 14. A. s) Importance of yield volatility in interest rate risk. *Note that the effect of an interest rate change on the value of a bond has two components. The first component is the duration – the change in price for a given change in yield. The second component is the change in the yield itself. If bond A and bond B have the same duration, but bond B is subject to grater yield changes (is exposed to greater yield volatility), bond B has greater interest rate risk. This difference in yield volatility could come about because bonds are in different segments of the bond market, or because the duration arises from a different set of yield, coupon, and maturity characteristics.*

B. The Term Structure and the Volatility of Interest Rates (Level II, Ch. 1)

This reading discusses the history, measurement, implications, and theories describing yields and changes in yields.

1. The most common relationship between yield an maturity has been an upward-sweeping curve, although flat or downward curves do occur. The steepness of the curve is measured by the difference between short and long rates.
2. Although parallel shifts in the yield curve are sometimes assumed for ease of computation, assumed, various non-parallel shifts in the yield curve are usually observed.
3. Changes in the term structure can be separated into three factors:
   a) changes in the level of yields (largest effect),
   b) changes in the slope of the yield curve (small effect)
   c) changes in the curvature of the yield curve (relatively little effect).
3. The spot rate curve is generated through bootstrapping of on-the-run treasuries, but this leaves gaps in the maturities of the observed bonds. The bootstrapping results are sometimes supplemented by bootstrapping of off-the-run issues or treasury strips.
4. Two major theories of the term structure of interest rates have evolved:
   a) Pure expectations, which essentially holds that any series of bonds representing investment over a given horizon should have the same return. The drawback to this theory is that it ignores the greater risk of longer-term bonds. There are several conflicting interpretations of the theory.
   b) Two variants of the expectations theory are described.
      - Liquidity theory, which adds a "liquidity premium" that increases with the maturity of the bond.
      - Preferred habitat theory, which asserts that investors will want to reduce risk by matching the maturities of assets with the maturities of assets. The result is a preference for a particular maturity (a "preferred habitat"). The term structure will then reflect both expectations and a premium required to move investors from their preferred habitat to balance supply and demand.

c) The other major theory is the "market segmentation" theory. It is an extreme form of the preferred habitat argument in which investors will not leave their preferred habitat, and the yield curve is defined by the supply and demand for securities within each maturity sector.

5. Duration assumes a parallel change in the yield curve. An alternative approach is to consider the effect of a yield change at a given maturity, with all other yield held constant. A duration calculated this way is called a "rate duration." By computing this rate duration for various key maturities, the effect of changes in the yield curve can be more directly considered.
6. Historical yield variability can be measured by the standard deviation of yields over time, and used for statistical statement concerning the future volatility of rates. The usual statistical problems of sample size versus stability of parameters arise.

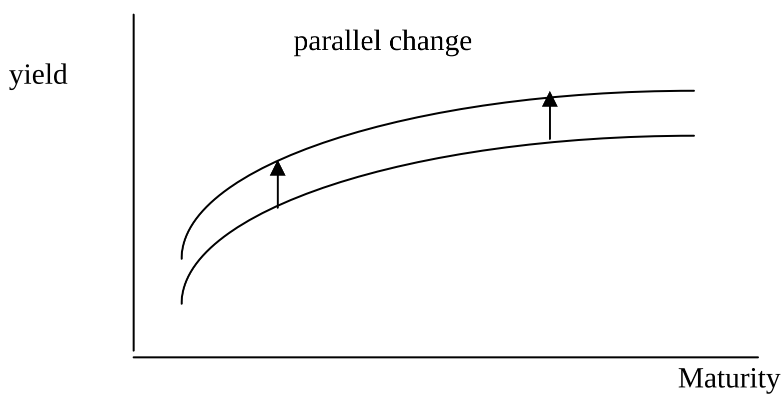
SS 14. B. a) Treasury yield curve shapes and interpretation. *(p292-294). There are three general shapes that have been observed:*

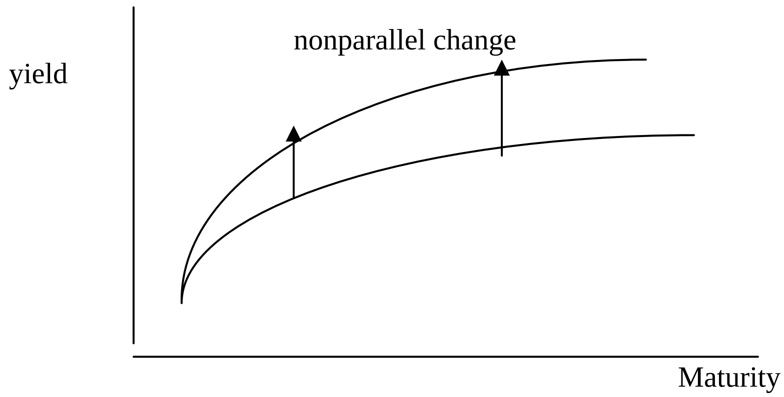
a) positively sloped: longer maturities have higher yields, often referred to as "normal,"
b) flat: little difference in yields between short and long maturities,
c) negatively sloped: longer maturities have lower yields, often referred to as "inverted."

The interpretation of the yield curve depends on the theory referenced. For the Expectations theory, the shapes indicate that short-term rates are expected to increase (positive slope), remain constant (flat), or decrease (negative slope). For the market segmentation theory, the slope is the product of the relative levels of supply and demand for loanable funds at various maturities (NB: a high yield indicates that supply is low relative to demand, while a low yield indicates that supply is large relative to demand). For the preferred habitat theory, both expectations and supply/demand have an effect. Under the liquidity premium theory, the yield curve has an upward bias. See also SS 14. g).

SS 14. B. b) Slope of the yield curve. *(p294). The slope of the yield curve is the difference between long-term treasury yields and short-term treasury yields. Note that there is no universally-accepted definition of "short" and "long."*

SS 14. B. c) Parallel and nonparallel yield curve shifts. *(pp. 294-295). A parallel change in the yield curve is an identical change in yield for all maturities, a nonparallel change is a change which is different for different maturities. Graphically:*





SS 14. B. d) Yield curve twists and curvature changes. *(p. 295). A yield curve twist is a change in the yield curve that is unequal, i.e., in which the slope of the yield curve changes. A twist in which the slope steepens is shown above to illustrate a nonparallel shift. Note that here the change was an increasing function of maturity. A "butterfly shift" is a shift in which the yield change at intermediary maturities differs from the change at short or long maturities.*

SS 14. B. e) Factors driving Treasury returns. *(pp. 297-298). Fabozzi describes three factors contributing the Treasury returns. In order of importance:*

a) changes in the level of yields, greatest explanatory power,
b) changes in the slope of the yield curve, about one-tenth as significant as changes in the level of rates.
c) changes in the curvature of the yield curve, relatively little explanatory power.

SS 14. B. f) Advantages and disadvantages of types of treasury securities used to construct the theoretical spot rate. *(p298 – 301). The universe consists of:*

a) On-the-run treasury issues. There is an observed yield for each of the on-the-run issues, but in order to avoid the complication of tax treatments the "par" yield must be estimated. Requires bootstrapping calculations. There are wide gaps between maturities
b) On-the-run treasury issues and selected off-the-run treasury issues. Used to avoid the wide gaps between maturities. Par value must be estimated, and bootstrapping is required.
c) All Treasury Coupon Securities and Bills. Both of the above fail to recognize information embedded in other treasury security prices. Some screening is desirable to avoid market effects. Requires more complex methodologies.
d) Treasury Strips. The observed yield is already a spot rate. However, strips are less liquid, have a different domestic tax treatment than coupon securities, and may have a different tax treatment for foreign investors.

SS 14. B. g) Pure expectations theory, liquidity theory, and preferred habitat theory and their implications for the shape of the yield curve. *(pp. 302-309). As follows:*

a) Pure expectations theory asserts that the entire term structure at a given time reflects the market's expectations of future short-term rates. Under this theory, a rising term structure will indicate that short-term rates will increase, while a flat yield curve indicates unchanging rates, and a decreasing term structure indicates decreasing future short-term rates. The problem with this theory is that it ignores the two sources of risk / uncertainty in holding bonds: the price at the end of the investment horizon, and the return from reinvestment of coupon payments.

b) *Liquidity theory accepts the role of future short-term rates, but adds the consideration of risk, indicating that investors will hold longer, more risky maturities only if given a premium over pure expectations. The liquidity premium thus results in a liquidity premium that increases with maturity, and a term structure which lies above the pure expectations term structure. This does not mean that the term structure must be positively sloped – since the term structure is the result of both expectations and the liquidity premium, it is possible for a positive premium to be overcome by expectations of decreasing short-term rates. The resulting term structure can be positively sloped, negatively sloped, or flat, depending on the expectations and size of liquidity premium.*

c) *Preferred habitat theory suggests that borrowers and lenders will reduce risk by matching the maturity schedule of their assets and their liabilities, and so have a preferred maturity (preferred habitat in the term structure). Borrowers and lenders will leave their preferred habitat to take advantage of temporary market conditions, but only if such risk taking is rewarded by a premium. The primary determinant of the term structure is a combination of supply and demand and the premiums required to induce investors to leave their preferred habitat. Again, this can lead to almost any term structure shape, depending on the supply and demand situations.*

SS 14. B. h) Forward rate definitions. *(pp.307-308). There are three interpretations of forward rates given in the readings. Two of these are based on arbitrage arguments, and are discussed immediately below. The third is inherent in expectations theory. In the context of pure expectations, the forward rate is the "market consensus" expected rate over the future period.*

SS 14. B. i) Arbitrage based interpretations of forward rates. *(p307-308). The two arbitrage-based interpretations of forward rates are:*

a) *It is the "break-even rate," the rate over some future period that will just make an investor indifferent between assets of different maturities. I.e., if the forward rate from T_1 to T_2 is 6%, then the investor will be indifferent between i) a security with maturity T_2, or ii) a security with a maturity of T_1, rolled over into a security of maturity $T_2 - T_1$.*

b) *It is the rate which can be "locked in" over some forward period – i.e., by buying a security maturing at the end of the forward period, you will get the forward rate over the forward period regardless of what happens to interest rates.*

SS 14. B. j) Key rate duration. *(pp. 309-312). The duration of a security is dependent in part on the yield at which the duration is computed (higher yield produces lower duration). Except for a flat spot rate curve, the duration of payments to be received at different times will be different – I.e., the duration of a security or a portfolio is actually a vector of durations for payments at different times. In order to make the vector manageable, the durations for only 11 points on the spot rate curve (called the "key rates") are considered. There are two important points:*

a) *If the duration vectors for two securities are different, the securities will in general have different value changes for non-parallel shifts in the yield curve.*

b) *The effective duration of the portfolio or security is computed as the weighted average of the "key rate" durations. Securities with the same effective duration will have the same change in value for parallel shifts of the yield curve. It is possible, however, to come up with the same effective duration from different key rate duration vectors, so that securities or portfolios having the same effective duration may react differently to changes in the shape of the yield curve.*

SS 14. B. k) Historical yield and yield volatility. *(pp312-316). There are two important points here. The first is the definition of "yield". Fabozzi uses "continuously compounded" change in yield, rather than periodic or "simple compounded" change in yield.*

I.e., periodic or "simple compounded" change in yield is:

$$\text{simple compounded yield} = \frac{\text{yield on day 2 - yield on day 1}}{\text{yield on day 1}},$$

while the "continuously compounded" change in yield is:

$$\text{continuously compounded yield} = \ln\left(\frac{\text{yield on day 2}}{\text{yield on day 1}}\right).$$

The second important point is that Fabozzi defines volatility as the standard deviation of the continuously compounded change in yield. This is computed as:

$$\text{volatility} = \sigma(\text{historical changes}) = \left[\frac{\sum_{t=1}^{T} (X - \bar{X})^2}{T - 1}\right]^{1/2}$$

where X is the continuously compounded change in yield.

➔ **It is important to realize that volatility as defined in Fabozzi is not the volatility of yield itself, but is rather the volatility of the continuously compounded rate of change in yield.**

The interpretation of volatility must keep in mind that it is not the standard deviation of the yield itself. Since the volatility is defined on the change in yield, rather than on the yield itself, the standard deviation of yield itself is computed by multiplying the observed yield by the standard deviation of the change in yield:

standard deviation of yield = yield X standard deviation of the change in yield.

E.g., if the yield is 7%, and the standard deviation of the change in yield is 15%, the standard deviation in yield is 0.15 X 7% = 1.05% or 105 basis points.

SS 14. B. l) Issues in calculating yield volatility. *(pp. 317-319). The number of days to be used in computing the volatility depends on the situation – in particular, on the length of the investment horizon. The longer the horizon, the larger the number of days used. Note that the measured volatility may be quite sensitive to the observation period chosen. The daily standard deviation is annualized by multiplying by the number of days in a year. There are three choices for the number of days in a year:*

a) 365 calendar days,
b) 260 trading days (52 weeks X 5 days per week),
c) 250 actual trading days after subtracting non-trading holidays.

There is no specific guidance given.

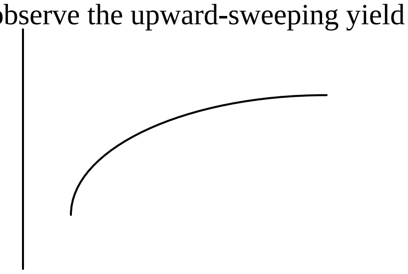
SS 14. B. m) Historical yield volatility vs. implied yield volatility. *(pp. 319-320). Historical yield volatility is based on observed changes in yield. Implied yield volatility is based on volatility as implied from the observed prices of options and caps.*

SS 14. B. n) Forecasting yield volatility. *(pp. 320-322). There are essentially four techniques described in Fabozzi:*

a) Use of the most recently observed moving average for the expected value of the change in yield.
b) Use of zero for the expected value of the change in yield.

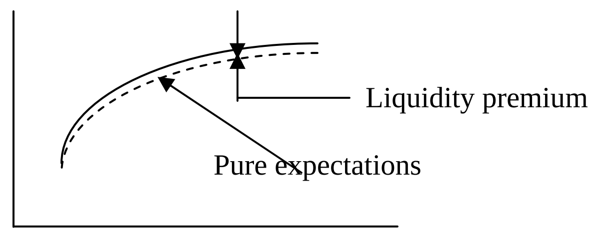
c) Use of a weighted average approach. In this approach, the more recent observations are weighted more heavily in computing volatility. This approach also requires a choice of the most appropriate expected value – the "weighted volatility" formula given by Fabozzi uses zero as the expected value.
d) Models based on recent prior volatility, which are not further described by Fabozzi.

Selftest Question: You observe the upward-sweeping yield curve shown.



a) Assuming that the observed curve reflects the liquidity premium, indicate by drawing on the diagram where the pure expectations curve would lie.
b) Interpret what the observed curve implies according to the expectations and the market segmentation theories.

Suggested Answer: a)



b) Under the pure expectations theory, the observed curve indicates that short-term interest rates are expected to rise in the future. Under the preferred habitat theory, the observed curve indicates that the demand for longer-term securities is larger, relative to supply, than the demand for short-term securities,

C. Valuing Bonds with Embedded Options (Level II, Ch. 2)

This reading discusses the added problems in bond valuation when there are embedded options. Since the exercise of the options is dependent on the level of interest rates, considering the possible future path of interest rates is essential to valuation.

1. Exercise of embedded options, which affects the cash flows to a bond, depends on the level of interest rates. In order to compute the cash flows, it is necessary to decide on the time path of interest rates. The exact time path of rates is not known, but it is possible to estimate probability distributions for changes in rates. Several models are available to predict the future rates using probability trees, usually binomial. Although the basic approach is common, it is necessary to remember that the result

is model-specific, and different models, or different assumptions in the same model, will result in different conclusions.
2. Binomial probability trees can be constructed based on assumptions about the future volatility of interest rates. The rates in the tree are actually "prediction" of forward rates.
3. The value of a bond is taken as the average of the present values of the possible period ending values. In order to be arbitrage-free, the model's value of a treasury bond at a node must be equal to the observed market value of the on-the-run Treasury corresponding to that node. This requires iterating to the required rate, using a backward induction technique.
4. Applying the binomial model to bonds with embedded options must recognize the effect of those options. This requires further assumptions or modeling about the conditions that will trigger exercise of the options.
5. Nominal spread and zero volatility spread do not consider he effect of options. The "Option Adjusted Spread" (OAS) is computed as the (constant) spread that will equate price and present value over all of the interest rate paths considered in an option-adjusted valuation model.
6. The option-adjusted valuation model can also be used to compute "effective" duration (duration which takes into account the effect of options on bond price). The valuation model is used to produce bond value after positive and negative interest rate shocks, and duration is computed based on these values.
7. Convertible bonds can be exchanged for common stock at a stated exchange rate. A convertible bond is valued as the higher of either the value of the common stock obtained from conversion, or as a bond. The bond can be considered as a "straight" bond plus an option on common stock.

SS 14. 4. C. a) Interest rate models. *(pp. 336-338). An interest rate model is a probabilistic description of how interest rates can change over the life of a bond.*
SS 14. 4. C. b) Interest-rate trees. *(pp. 339-342). More properly, this should refer to a "binomial" interest rate tree – a tree in which at each point there are two possible changes. I.e., starting at the present, the one-period interest rate is set. At the end of the first period, there are two possible new one-year rates. These two possible new one-year rates are the result of a higher (H) change and the lower (L) change (note that we do not know here the absolute level of either rate, we only know that one is higher than the other). At the end of year two, interest rates change again. However, now there are two possible rates that can change, with two possible changes for each (the higher change and the lower change). This would mean four possibilities. In the model, however, the result of high change and then low change is forced to give the same result as low change high change (this is called a recombinant model). Since the sequence HL must equal the sequence LH, we are left with only three possible outcomes after two changes: HH, HL=LH, and LL. Similarly, after four periods we have four possible outcomes: HHH, HHL=HLH=LHH, LLH = LHL = HLL, and LLL. I.e., any rate resulting from an equal number of H and L changes will be the same, regardless of the order of the changes. The rates over the future periods are forward rates describing the path of interest rates over time, and the tree indicates the possible time-paths of interest rates.*
SS 14. 4. C. c) Backward induction. *(pp.342-343). Fabozzi does not go into details. Essentially, the "backward induction methodology" means starting at the final possible values of the bond and working backward to the present value of the bond. I.e., at maturity the bond is worth face value plus the coupon payment. Its value at the "node" one period before is the average of two possible present values: coupon plus the present value at the higher rate, and coupon plus the present value at the lower rate. This value is then one of the values for the previous node, and so on.*

This would be easy except that the result for the first node must be equal to the observed price of the security – otherwise, arbitrage would be possible. This requires repetitive calculations.

SS 14. 4. C. d) Valuing an option-free bond using an interest rate tree. *(pp. 345-346). Consider the following one-period (one-node) problem (from Fabozzi, Exhibit 6, p. 346, year 2 Node N_{HH} from year 3 Nodes N_{HHH} and N_{HHL}): the two possible ending values from the node are 97.529 present value plus 6.5 coupon payment = 104.029 at the high rate, and present value of 99.041 plus 6.5 coupon payment = 105.541 at the low rate. We must find the average present value of the two ending values at a (provided) discount rate "r_*," where $r_* = 7.0053\%$:*

$$\text{Value at Node} = \frac{1}{2}\left[\frac{V_H + C}{1 + r_*} + \frac{V_L + C}{1 + r_*}\right]$$

$$= \frac{1}{2}\left[\frac{104.029}{1.070053} + \frac{105.541}{1.070053}\right] = 97.295$$

In order to compute the value of the bond, it is necessary to start at the final nodes and work back through the tree one year at a time.

SS 14. 4. C. e) Finding the value of a callable bond from an interest rate tree. *(pp. 347-349). This is exactly the same procedure as computing the value of a non-callable bond <u>except</u> that if the bond "violates" the call rule, it is considered called and the call price is substituted for the computed value. Note that the tree does not end if the bond is considered called, since the cash flow from the funds is considered reinvested.*

SS 14. 4. C. f) Valuing an embedded call option. *(p. 349). The value of the embedded option is the difference between the value of the bond computed without recognizing the possible effects of the embedded option, and the value of the bond with the possible effects of the embedded option included.*

SS 14. 4. C. g) Values of a callable (putable) bond, the corresponding option-free bond, and the embedded option. *(p.349) As follows:*

value of embedded option = value of option-free bond – value of bond with option

SS 14. 4. C. h) Volatility and the arbitrage-free value of an option. *(p.350) The value of an option increases with increasing volatility. Note that this indicates that the arbitrage-free value of a callable bond will decrease with increasing volatility, but the value of a putable bond will increase with increasing volatility.*

SS 14. C. i) Using the binomial model to calculate the option-adjusted spread. *(pp. 350-351). Assume a theoretical value, as computed from an interest rate tree that includes the affect of embedded options, and a market price which is different (lower than) from the theoretical value. The "option-adjusted spread" is the constant amount that, if added to the interest rate at each node, will produce a theoretical value equal to the market price.*

SS 14. C. j) Option-adjusted spread, nominal spread, and benchmark interest rates. *(pp. 350-353). As described by Fabozzi, the spread of a security relative to Treasury issues can be thought of as having three components:*

a) spread due to credit risk (a function of the credit risk of the security),
b) spread due to liquidity (a function of the liquidity of the security)
c) spread due to embedded options in the security.

Nominal spread reflects all three components. OAS relative to a Treasury-based benchmark reflects only the first two components – i.e., it removes the spread due to the embedded options. If the spread of a security is measured relative to an issuer-based benchmark, it will reflect only the spread components *associated with liquidity and embedded options. Finally, if the OAS spread is measured relative to an issuer-based benchmark, it reflects only the spread due to the liquidity of the particular issue.*

SS 14. C. k) Computing effective duration and effective convexity using the binomial model. *(pp. 353-356). Remember that "effective" duration and convexity take into account the effect of embedded options. In order to compute "effective" duration and convexity, it is necessary to find the change in value of a bond under small changes in yield including the option effects. First, compute V_+ using the following steps:*

a) *Compute the OAS based on the market price.*
b) *Construct a new yield curve by shifting the on-the-run yield curve up by a few basis points, and compute a new value based on this new yield curve.*
c) *Construct a new binomial tree for an option-free bond, based on the new value.*
d) *Add the OAS to each period rate in the new binomial tree to produce an "option-adjusted tree."*
e) *Compute V_+ from the "option adjusted tree."*

I.e., the observed / computed OAS is added to the option-free binomial tree implied by a shifted yield curve and used to produce a shifted value for a bond with an embedded option. The procedure to compute V_- is exactly the same except that the yield curve is shifted down instead of up. At this point, the effective duration and convexity can be computed.

SS 14. C. l) Valuing of a putable bond using an interest rate tree. *(pp.356-358). The procedure is exactly the same as finding the value of a callable bond: if the bond "violates" the put rule, it is considered put and the put price is substituted for the computed value.*

SS 14. C. m) Binomial model with multiple embedded options. *(p. 358). The procedure is to simply reflect the effect of the options – i.e., at each node, if the bond "violates" the rule for a particular option, the price resulting from exercise of option is inserted.*

SS 14. C. n) Binomial model applied to a step-up callable note. *(pp. 358-363). A step-up note is a callable instrument whose coupon rate is increased ("stepped up") at specified times. This type of instrument is valued by replacing the value at each node by the value indicated by the call rule, and reflecting the changing coupon of the security.*

SS 14. C. o) What is the appropriate "nodal decision" within the backward induction methodology for: putable bond, callable bond, putable/callable bond, multiple step-up bond, and a floater with a cap. *(pp. 343-363). Rephrased, at each node, what is the proper value and coupon for the security? Simply reflect the call / put exercise rule (which must be supplied or assumed) and any change in coupon.*

SS 14. C. p) Basic features of a convertible bond. *(pp. 364). A convertible bond can be thought of as the combination of a bond and an option on common stock. In the case of a convertible bond, the common stock underlying the option is the common stock of the issuer of the bond, while in an exchangeable bond the underlying common stock is that of a firm other than the buyer. The number of shares of common stock to be received on conversion is called the "conversion ratio." The conversion ratio can also be stated as a conversion price, i.e., the face value of the convertible divided by the number of shares of common stock to be received at conversion. Where the convertible is also putable, the put may be "hard" (paid in cash only) or "soft" (payable in cash, common stock, subordinated notes, or some combination of the three at the discretion of the issuer).*

SS 14. C. q) Conversion value, straight value, market conversion price, market conversion premium per share, market premium conversion ratio, premium payback period, and premium over straight value. *(pp. 364-368). As follows:*

a) *Conversion value = market price of common X conversion ratio. It is the value of the stock to be received if the bond is converted immediately.*

b) Straight value is the value of the bond if the conversion feature were not attached – i.e., the value of a "straight" bond. It is computed from a valuation model.

c) Market conversion price is the market price of the convertible divided by the conversion ratio. It reflects the effective price per share paid for the common stock if acquired through conversion.

d) Market conversion premium per share = market conversion price – current market price. It is the difference between the cost of a share acquired through purchase in the market, and the cost of a share acquired through conversion, i.e., the premium or discount on shares acquired through conversion.

e) Market premium conversion ratio is the market conversion premium per share divided by the market price of common stock. It is the ratio of the cost of a share acquired through conversion to the cost of a share acquired through market purchase – i.e., it is the premium or discount on shares acquired through conversion expressed as a ratio.

f) Premium payback period is the market conversion premium per share divided by the favorable income differential per share. I.e., it is the per period difference between the interest payment on the bond, divided by the conversion ratio, and the dividends to be received on a share of common stock acquired through conversion. The favorable income differential per share is the difference between the interest payment on a "per share" basis and the dividends which would be received on a share after conversion. I.e., on a per share basis, how much will the cash flow increase if the bond is converted? Assuming a positive market conversion premium per share, premium payback period is how long it takes for this higher cash flow to pay back the premium paid for a share acquired by conversion.

g) Premium over straight value is:

$$\text{premium over straight value} = \frac{\text{market price of convertible}}{\text{straight value}} - 1$$

It is the premium paid to acquire the conversion feature, expressed as a decimal or fraction.

SS 14. C. r) Components of a convertible bond's value in an option-based valuation approach. *(p.368-369). The full range of potential components would include:*

a) the straight value,
b) the value of the call option implied in the convertible feature (a positive),
c) the value of the call held by the issuer (a negative),
d) the value of a put held by the bondholders.

SS 14. C. s) Risk-return characteristics of a convertible bond vs. the risk-return characteristics of the underlying common stock. *(pp. 369-370). Generally, a convertible has less downside risk, but less upside potential, than an equivalent stock position:*

a) If the stock price declines, the security will be primarily valued as a bond. Note that this is sometimes held to be a "floor" value to the security, but that this "floor" may itself decrease if interest rates increase. Nonetheless, the bond downside risk will be less than that of the equivalent common stock position.

b) If the stock price remains constant, generally the interest return on the bond will be greater than the dividend return on the equivalent common stock position.

c) If the stock price increases, the return on the bond will be less than the return on the equivalent stock, since the bond included the option premium and the common stock was "purchased" at a premium.

SelfTest Question: The binomial model provides an "arbitrage free" value. What does this mean, and why is it important?
Suggested Answer: "Arbitrage free" means that the value generated by an interest-rate tree for a security is equal to the market price of the security. Remember that arbitrage is the generation of a riskless, costless profit. If the model produced a value different from the market price, it would indicate that an arbitrage opportunity exists for the security. But generation of a riskless, costless profit should not be possible, and it is more likely that the model is incorrect than that an arbitrage opportunity actually exists.

D. Valuation of Interest Rate Derivative Instruments, (Level II, Ch. 8)

Derivative instruments derive their value from the underlying assets (hence, "derivative"). This reading discusses the approaches to determining this derived value. Central to derivative valuation is eh "no arbitrage" condition – i.e., it should not be possible to make a riskless, costless profit.

1. Interest rate futures contracts are agreements to exchange predetermined assets at some predetermined future time. It is important to remember that there is *no* exchange at the creation of the future.
2. An alternative to buying a futures contract is to borrow, purchase the asset, and hold it until the future date. Both the borrow-and-buy strategy and the futures strategy have the same result: ownership of the asset. In one case the borrowing must be repaid, in the other the futures price must be paid. In order for there to be no arbitrage, the two alternatives must have the same cost. If the future is valued higher, a "cash and carry" arbitrage would be possible; if the future is valued lower a "reverse cash and carry" arbitrage would be possible. This indicates that the futures price must be equal to the future value of the value of the assets today.
2. The "cash and carry" arbitrage model is simpler than the real world. Problems include:
   a) Actual futures require margin and a "marked to market" at the end of each trading day.
   b) The model assume that borrowing and lending rates are equal.
   c) In treasury futures, the deliverable bond is not known exactly – due to trading specifications, there will be one "cheapest to deliver" bond, but that bond is not known in advance with certainty.
   d) Also because of trading arrangements, the exact day of delivery on treasury futures is not known in advance with certainty.
3. Interest rate swaps are agreements to exchange a series of cash flows (actually, they are netted), based on an interest rate times a notional principal. The principal is not exchanged. The interest rates can be any combination of fixed and floating rates, but typically one rate is fixed and the other is floating.
4. As with futures, there is no exchange at the creation of the interest rate swap (there is an exception called an "off-market swap"), because the rates of the swaps are set by equating the present value of the expected cash flows. I.e., a swap is an exchange of items of equal value.
5. After the swap has been created, conditions will almost certainly differ from expectations. The present values of the two cash flows will no longer be equal, and the value of the swap will be the difference between the present values of the cash flows.
6. Options give the purchaser the right, but not the obligation, to buy (call) or sell (put) defined assets at a defined price. The intrinsic value of an option is the payoff if exercised immediately, while the time value is any value exceeding the intrinsic value. The value of the option is affected by a number of factors.
7. Options may be written on actual securities, or they may be written on futures contracts on the securities. The factors affecting value are similar.

8. The Black-Scholes model is the most popular model for pricing equity options, but three underlying assumptions of the Black-Scholes model are less applicable to fixed-income securities:
   a) the probability distribution allows negative values,
   b) the short–term interest rate is constant,
   c) the variance of prices is constant over the life of the option.
9. The binomial model can be applied to interest rate options on securities in a manner similar to its application to securities. A model developed by Black is commonly used for options on futures.
10. Similar to duration, there are measure of the sensitivity of option price to factors, including changes in the value of he underlying security, changes in the time to expiration and changes in volatility.
11. Caps and floors are upper and lower limits on interest rates. The cap is achieved by having the cap (floor) have a payoff if interest rates exceed (are lower than) the cap (floor). Since the payments are periodic, caps and floors can be treated as a series of options.

SS 14. D. a) Theoretical price of a futures contract. *(pp. 602-606). The theoretical price of a futures contract is determined by finding the futures price at which no arbitrage profit is possible. I.e., if you sell the futures contract, and simultaneously borrow to buy the bond, at maturity you will have the bond to deliver at the futures price, and you will owe the future value of the amount borrowed. Note that the profit or loss is determined at time zero, since the futures price and the interest rate are set at that time, and there is no net investment. If the futures value of the amount borrowed is not equal to the amount received for the bond (the futures price), you will have a profit or loss. It should not be possible to make a riskless profit with no investment, so there should be no profit. There should be no loss because if there was a loss, it would be possible to reverse the position (i.e., buy the future, short the bond, and invest at the proceeds) and make a profit. Some other ways of saying the same thing:*

a) The futures price must equal the future value of the amount needed to buy the bond.
b) The present value of the futures price must equal the amount required to buy the bond.

Note that these two statements are equivalent, they merely state values at different times.

➔ **Note that the form of the pricing equation use here is based on simple interest, ignoring compounding.**

SS 14. D. b) Theoretical price of a Treasury futures contract. *(pp. 605-606). The theoretical price of a Treasury futures contract can be computed as:*

$$F = P + Pt(r-c)$$

where:

a) F = futures price
b) P= cash market price of bond
c) r = financing rate
d) c = current yield,
e) t = time to futures delivery date.

Some explanation is required. The term Pt(r-c) is the future value of the amount borrowed – i.e., the amount which must be paid back at maturity. It looks strange because:

a) the effective borrowing rate is the interest rate r minus the rate of return c.
b) the future value is computed using simple interest, which ignores compounding. I.e., we can write the right-hand side as P[1+ t(r-c)]. The term t(r-c) is time periods (or fraction thereof) X effective interest rate, or increase in value. E.g., for thirty days at an annual rate of 6%, the simple interest future value of $1.00 is $1.00 X 30/360(0.06) = $1.005.

Over short periods, the difference between compounding and simple interest is small- e.g., the compounded future value would be:

$$\text{compounded future value} = \$1.00 \text{ X } (1.06)^{30/360} = 1.00486755$$

SS 14. D. c) Effects of delivery options on the theoretical price of a treasury bond futures contract. *(pp. 607-608). There are two delivery option problems with the simple model presented above. The first is that the option seller can deliver a range of options (the quality option), and since this option is at the hands of the seller, the futures price will be reduced. Secondly, the delivery date is not exactly specified, but is variable at the discretion of the seller. Again, the futures price is reduced. The net effect of the delivery options, then is to reduce the futures price.*

SS 14. D. d) Extending the standard arbitrage pricing model to the valuation of treasury bond and note futures contracts. *(pp. 606-607). There are two assumptions that ignore reality and reduce the validity of the arbitrage pricing model.*

a) *Futures mark-to-market daily through the margin account. This causes interim cash flows (cash flows during the life of the futures contract). Further, these cash flows are stochastic, so that a deterministic model is not achievable. There may also be coupon interest payments, but these can be more easily handled.*
b) *The arbitrage model assumes equal lending and borrowing rates. In actuality, the borrowing rate is higher than the lending rate. The result is a band within which the futures price may vary without arbitrage being possible.*

SS 14. D. e) Floating-rate payments for a swap. *(pp.610-611). The floating rate payments for the swap discussed by Fabozzi are computed on an actual/360 basis – i.e., the fraction of a year is taken as the actual number of days in the period divided by 360. The interest is computed on a simple interest basis, with the interest rate the rate in force at the beginning of the period. I.e., if the number of days in the period is N, and the interest rate in force at the start of the period is r, then the floating rate payment for the period is:*

$$\text{Floating rate payment} = \text{principal X } r \text{ X } N/360$$

E.g., for a 90-day period, with the interest rate at 6% at the beginning of the period, the floating rate for the period for a $1,000,000 swap is:

$$\text{Floating interest payment} = \$1{,}000{,}000 \text{ X } 0.06 \text{ X } 90/360 = \$15{,}000$$

Note that the future floating-rate payments can be locked in using Eurodollar CD futures. These "locked in" future floating-rate payments are computed in the same way as above, using the rates for Eurodollar CDs over the future periods. These "locked in" payments are very important, because the present value of these payments is the value of the swap to the parties.

Candidates should be ready to compute payments using assumptions other than those used by Fabozzi.

LOS 14 D. f) Determination of swap rate and swap spread. *(pp. 616-618). The key here is to remember that a swap is an exchange of items of equal value. In this case the value is the present value of the future payments. To the fixed-rate receiver, it is the present value of the fixed payments, but to the floating-rate receiver it is the present value of the floating rates. These two present values will be equated at some discount rate, and so the swap rate will be the discount rate that equates the present value of the fixed and floating-rate payments. Problem – we do not know what the future floating rate payments will be! Not a problem – the proper floating rates to use are those that can be locked in using futures.*

The swap spread is computed as the spread from the on-the-run Treasury rate for a maturity equal to the maturity of the swap.

SS 14. D. g) Valuing a swap. *(pp. 618-620). Once the swap has been established, the value of the swap is the difference between the present value of the floating payments that could be locked in using futures, and the present value of the fixed payments.*

SS 14. D. h) Swap rate, swap spread, and value of a swap. *(pp. 616-620). This is simply the application of the points already discussed:*

a) the swap rate is the rate which will, at the time the swap is established, equate the present values of the fixed and floating payments (i.e., produce a value of zero),
b) the swap spread is the spread from an on-the-run treasury of the same maturity as the swap,
c) after the swap has been established, the value of the swap is the difference between the present value of the fixed payments and the present value of the floating payments.

SS 14. D. i) Factors affecting option price components. *(pp. 621-622). The two components of the option price are:*

a) The intrinsic value, which is the economic value of the option if it is exercised immediately. Note this is the immediate payoff, not the gain or loss – i.e., the premium is not considered. If immediate exercise would produce a profit, the option is said to be "in the money;" if immediate exercise would produce a zero payoff, the option is said to be "at the money;" if immediate exercise would produce a loss, the option is said to be "out of the money."
b) The time value, which is the amount by which the option price exceeds the intrinsic value of the option. This premium arises from the possibility of future increases in the intrinsic value of the option.

The factors that affect the value of an option, and the direction of the relationship for a call (put) are:

a) the price of the underlying security, positive (negative),
b) the strike price, negative (positive),
c) the time to expiration, positive (positive),
d) the expected interest rate volatility over the life of the option, positive (positive),
e) the short-term risk-free rate over the life of the option, positive (negative),
f) coupon interest payments over the life of the option, negative (positive)

SS 14. D. j) Limitations of the Black-Scholes stock-option pricing model for interest rate options. *(pp. 624-626). The limitations are based on assumptions that may be violated:*

a) The B-S model assumes a probability distribution that is unlimited in price – i.e., the price can go infinitely high. Actually, the price of a bond is limited because the interest rate does not go below zero.
b) The B-S model assumes a constant short-term rate over the life of the option, and this is an unlikely occurrence.
c) The B-S model assumes that the volatility of the bond is constant over the life of the option, but this is an unlikely occurrence.

SS 14. D. k) Price options using the arbitrage-free binomial model. *(pp. 626-629). An option is valued using the relevant portion of the interest rate tree for the underlying bond, using option value rather than bond value. For the node corresponding to expiration of the option, the value of the option, based on the bond value, is inserted. I.e., if the option is in-the-money, the intrinsic value of the option is inserted; if the bond is at-the money or out-of-the-money, zero is inserted. The backward induction procedure is then applied, using the option prices and the discount rate from the bond interest rate tree.*

SS 14. D. l) Using the Black model to value options on futures. *(pp.630-632). From Fabozzi:*

a) the Black model is the most popular model for valuing short-term options on Treasury futures,
b) the Black model does not overcome the problems of the Black-Scholes model,
c) the Black model applies to European options, while Treasury options are American options.

SS 14. D. m) Measuring the sensitivity of an option's value to the changes in factors. *(pp. 630 – 632). The following are the factors and how sensitivity to the factor is measured:*

a) change in the price of the underlying bond: measured by delta:

$$\text{delta} = \frac{\text{change in option price}}{\text{change in price of underlying bond}}$$

Note that, like duration, delta is not a constant. There is a second measure gamma, with a role similar to convexity, which measures the change in delta:

$$\text{gamma} = \frac{\text{change in delta}}{\text{change in price of unerlying bond}}$$

b) time to expiration; measured by theta:

$$\text{theta} = \frac{\text{change in price of option}}{\text{decrease in time to expiration}}$$

c) interest rate volatility; measured by kappa:

$$\text{kappa} = \frac{\text{change in option price}}{\text{1\% change in expected interest rate volatility}}$$

Note that the change is not the change in interest rate; it is the change in the <u>volatility</u> of interest rates.

LOS 14 D. n) Valuing a cap or a floor using the arbitrage-free binomial model. *(pp. 632-636). The cap or floor can be considered as a series of options (or 'caplets"). The procedure for a cap is to value each "caplet," and then sum the values to find the value of the cap. For each caplet, the procedure is the same as for an option: the relevant portion of the interest rate tree for the underlying bond is used for backward induction, with the value of the caplet being inserted at the node corresponding to expiration of the caplet. Note that this is done separately for each caplet. The procedure for a floor is analogous.*

SelfTest Question: The swap rate and the value of a swap depend on the present values of the payments to be exchanged. If one of the payments is based on floating rates, how can we overcome the uncertainty of the future floating rate payments?

Suggested Answer: It is correct that the future path of the floating rate, and of the payments based on the floating rate, are not known with certainty. The payments can, however, be treated as a series of futures contracts, and the rate on those future contracts can be locked in by hedging with futures contracts. These locked-in rates can then be used to compute the payments and present values.

STUDY SESSION 15
Asset Valuation
Debt Securities: Structured Securities

Study Session 15: 1. "Fixed Income Analysis for the Chartered Financial Analyst Program: Level I and Level II Readings," Frank J. Fabozzi (Frank J. Fabozzi and Associates, 2000).

A. Mortgage-Backed Securities (Ch. 3, Level II)

Mortgage backed securities, whether pass-through or Collateralized Mortgage Obligations (CMOs) have very complex cash flows. In the pass-throughs this is due to the uncertainty about prepayments. In the CMOs there is an added layer of designed complexity in the various "tranches" into which the payments are divided.

1. There are a variety of mortgage types, differing in interest rates (fixed or adjustable-rate), amortization (fully amortizing , balloon, growing equity) and other factors. The most widely used mortgage is the fixed-rate, fully amortizing mortgage with level payments.
2. A complicating factor is that the borrower typically has the right to pay in full or in part – referred to as the "prepayment" option. Uncertainty about this prepayment means that the cash flows from the mortgages are uncertain in amount.
3. As individual instruments, mortgages are illiquid due to the significant counterparty risk and cash flow uncertainties. Mortgages may be pooled by institutions, however, and the counterparty risk and cash flow uncertainty reduced. Bond issues using a pool of mortgages are liquid instruments.
4. The simpler form of mortgage-backed security (MBS) is the pass-through, in which the payments (interest, scheduled principal payment, and prepayment) are simply passed through to the investors pro rata.
5. The actual prepayment is not known, and assumptions must be made (this is not unusual – remember the assumptions made for other bonds!) in order to compute rates of return or to make comparisons. These assumptions are often in the form of a multiple of a basic cash flow pattern assumption referred to as the Public Securities Association prepayment benchmark.
6. The prepayment rate is a function of the characteristics of the underlying mortgages and of general economic conditions, but the most important factor is the level of mortgage rate. This is because prepayment is often associated with refinancing the mortgage at a lower rate.
7. The prepayment rate affects the effective life of the MBS. Increasing prepayment shortens the effective life of the MBS, while decreasing prepayment increases the effective life. Given this uncertainty, the "average life" of the mortgages in the pool is often used as a measure of maturity.
8. Unlike passthroughs, Collateralized Mortgage Obligations (CMOs) redirect the cash flows from the mortgages, so that different classes of bonds (called tranches) receive different streams of cash. The motivation is to provide investors with cash flows that more closely match their desired characteristics.
9. There are many tranches, with different characteristics. Some tranches are designed to have little uncertainty about cash flows, while other tranches may have increased uncertainty. The principal payments may be directed first to some tranches while others receive only interest. Other ranches may have increasing principal, etc.
10. "Stripped" MBS are divided into Interest Only (IO) and Principal Only (PO) classes.

11.MBS may be issued by government agencies or by private institutions. The major difference is in the counterparty risk or credit quality. As a result, private ("non-agency") MBS are often credit enhanced, i.e., carry some added form of quality improvement.

SS 15. 1. A. a) Mortgage loans. *(pp. 390-391). A mortgage is a loan secured by some specific real estate property. There are many types of mortgage designs. Typical mortgages are fully amortizing, and may have fixed or floating interest rates. Fixed rate mortgages are normally level payment.*

SS 15. 1. A. b) Cash flows of a fixed-rate, level payment, fully amortized mortgage loan. *(pp.391-392). The characteristics of fixed-rate, level payment, fully amortized mortgage loans are:*

a) Fixed-rate indicates that the interest rate will not change over the life of the mortgage.
b) Level-payment indicates that all payments on the mortgage are of the same amount.
c) Fully amortized means that there is no principal payment due after the last scheduled payment, the principal amount having been fully paid in the scheduled payments.

Typically, payments are on a monthly basis. Each payment has two components: the interest due for the month, and a repayment of principal amount. Since each month's payment reduces the principal amount, the interest due each month decreases over the life of the mortgage and, since the payments are equal, the amount of repayment of principal increases each month.

SS 15. 1. A. c) Prepayments and prepayment risk. *(p. 393). A mortgage loan is often prepayable – i.e., the borrower has the right to accelerate or prepay principal amounts, often without penalty. The prepayment may be partial or full, so that the mortgage holder does not know the timing and amount of the (interest and principal) cash flows from the mortgage.*

SS 15. 1. A. d) Investment characteristics of mortgage passthrough securities. *(pp. 393-394). The cash flow to a passthrough is the cash flow received from the underlying mortgages, less a servicing fee. Because of the servicing fee, the passthrough coupon rate is less than the mortgage rate for the underlying pool. There is also some difference in the timing due to the delay in processing the cash flows. The mortgages in some pools may have differing maturities and/or rates, so that the pool may be described by a weighted average coupon rate or a weighted average maturity. Note that passthroughs originated by the Government National Mortgage Association is guaranteed by the "full faith and credit" of the U.S. government, while passthroughs from other sources do not carry the full faith and credit guarantee.*

SS 15. 1. A. e) Importance of prepayments in the estimation of the cash flow of a mortgage-backed security. *(p. 393). Prepayments shift cash flows to the earlier life of the passthrough. I.e., when principal is prepaid, cash flows are higher. However, this either removes mortgages from the pool (for full prepayment), or shortens the life of some mortgages (for partial prepayment or curtailment). Since the timing and amount of prepayments are uncertain and can only be approximated, the estimation of the cash flows to the security are in turn uncertain and can only be approximately estimated.*

SS 15. 1. A. f) Weighted average coupon and weighted average maturity of a mortgage pool. *(p. 393-394). The weighted average coupon (WAC) is:*

$$\text{Weighted Average Coupon} = \Sigma \text{w}_\text{i} C_i$$

and the Weighted Average Maturity (WAM) is:

$$\text{Weighted Average Coupon} = \Sigma \text{w}_\text{i} M_i$$

where W_i is:

$$\text{W}_\text{i} = \frac{\text{principal of mortgage i}}{\Sigma(\text{principal of all mortgages in the pool})}$$

SS 15. 1. A. g) Conditional prepayment rate and the single monthly mortality rate. *(pp. 396-397). The conditional prepayment rate (CPR) is the estimated fraction (expressed as a decimal) of remaining mortgages that will prepay over a year. When this annual rate is converted to a monthly rate, it is called*

the "single-monthly mortality rate" (SMM). The rule for conversion is:

$$SMM = 1 - (1 - CPR)^{\frac{1}{12}}$$

Since CPR is the fraction of mortgages prepaid or exiting the pool, 1 – CPR is the fraction of mortgages remaining in the pool. The twelfth root would then give the fraction of mortgages remaining in the pool after one month. Subtracting this from 1 then gives the fraction of mortgages exiting the pool in one month. Note we cannot simply divide the annual rate by twelve.

SS 15. 1. A. h) Public Securities Association (PSA) prepayment benchmark and conditional prepayment rate. *(pp397-399). The PSA prepayment benchmark is a market convention regarding the prepayment pattern (Fabozzi strongly asserts that it is NOT a prepayment model). The PSA assumes that the conditional prepayment rate (CPR) on mortgages depends only on the age of the mortgage. Over the first thirty months of mortgage age, the CPR begins at zero and increases by 0.2% each month, to reach a CPR of 6% at the thirtieth month. After the thirtieth month, the CPR is assumed to remain constant at 6%. Note that PSA is based on the months since origination of the mortgage, not on the months since origination of the passthrough.*

SS 15. 1. A. i) Prepayment amount for a month. *(p.397). Computation of the amount of prepayment in any month is based on the principal at the <u>end</u> of the month, so that the scheduled principal payment for the month must be subtracted before multiplying by the SMM:*

prepayment amount = SMM X (beginning principal – scheduled principal payments)

SS 15. 1. A. j) Factors affecting prepayments. *(p. 403-405). Fabozzi identifies four factors that affect prepayment:*

- *a) prevailing mortgage rate,*
- *b) characteristics of the underlying mortgage pool,*
- *c) seasonal factors,*
- *d) general economic activity.*

SS 15. 1. A. k) Contraction and expansion risks. *(pp. 404-405). Contraction risk is associated with decreases in mortgage rates. As interest rates decline, the value of bonds increases. However, reasons for prepayment include refinancing the mortgage at a lower rate and movement to another home. As interest rates drop, both refinancing and moving to a new home become more attractive, and the prepayment rate will increase. This has two effects:*

- *a) The prepayments have the effect of shortening the maturity of the mortgage, so that the sensitivity to interest rates becomes smaller and the increase in price will be less.*
- *b) The prepayments increase at the worst time – as interest rates, and therefore reinvestment rates, are declining.*

Extension risk is associated with increases in mortgage rates. As interest rates decline, the value of bonds increases. However, as interest rates increase the rate of prepayment will decrease. Again, there are two effects:

- *a) The maturity of the mortgages increase, so that the drop in price is larger.*
- *b) The prepayments decrease at the time that reinvestment becomes more attractive.*

SS 15. 1. A. l) Average life of a mortgage-backed security vs. maturity. *(pp. 405-406). The stated maturity is not a useful measure because it simply specifies the time of the last payment, ignoring what is happening to all other payments. I.e., prepayments shift cash flows to the earlier payments, making the* *early payments larger and the later payments smaller. This has no effect on the maturity of the security. However, the average reflects the effect on all of the payments, and the effect of prepayment is reflected in the (weighted) average life:*

$$\text{(weighted) average life} = \sum_{t=1}^{T} \frac{\text{t X (estimated) principal received at time t}}{\text{12 X total principal}}$$

SS 15. 1.A. m) Collateralized Mortgage Obligations (CMO) and types of structures. *(pp.406-424). A CMO is created by redistributing the cash flows from the mortgage payments among different bond classes. The idea is to structure bond classes with risk-return characteristics more closely matching those of different classes of investors. The various bond classes are called "tranches" (French for "slice," i.e., the cash flows are sliced into various classes). Some structures are:*

a) *Sequential-pay tranches: the various tranches are retired sequentially. I.e., all tranches receive interest payments (not that the coupon rate for different tranches need no be the same), but tranche A receives all of the principal payments. This will continue until tranche A has been retired, at which time the principal payments will flow to tranche B, etc.*
b) *Accrual tranche: the accrual tranche (sometimes called the Z tranche) receives neither principal nor interest until the other tranches have been retired. In place of interest, the principal of the Z tranche is increased. This shortens the life of the other tranches, and the Z tranche has no reinvestment risk until those tranches are paid off.*
c) *Floater tranches: the interest payments for a tranche can be split into floating-rate and inverse-floating-rate instruments. The difference between these securities and corporate floater (and inverse-floaters) is that the principal amount declines over time as principal payments are made.*
d) *Planned Amortization class (PAC) bonds: these tranches have a prespecified principal repayment schedule, which can be satisfied over a large range of prepayment rates. The greater certainty of cash flow comes because the PACs have priority over all other tranches in the CMO structure. The tranches that must absorb the prepayment risk for the PACs are called "support tranches" or "companion tranches." Finally, a "support tranche with a schedule" is a support tranche which itself has scheduled principal repayment, but is subordinate to the PAC if the PAC schedule cannot be otherwise met.*

SS 15. 1. A. n) CMO tranches and matching of assets and liabilities. *(pp.406-424). It is important to note first that CMO's cannot change the amount of risk – they can only redistribute it among the various tranches. In sequential-pay tranches the different tranches have shorter and longer average lives than the collateral. The shorter life tranches are better protected against extension risk, while the longer life tranches are better protected against reinvestment risk. The Z tranche absorbs much of the prepayment risk, shortening the average lives of the other tranches, and is attractive to some investors because, with no interest payments, it is protected against reinvestment risk until the other tranches are paid off. Similarly, the PAC tranches have much greater predictability of payments, but at the expense of the other tranches who absorb the risk. Clearly, the creation of floaters and inverse floaters from the same tranche allows a different risk distribution for investors with different risk profiles.*

SS 15. 1. A. o) Risk characteristics and performance of CMO tranches. *(pp. 406-428) This has actually been covered in the previous two LOSs. Although this is a new LOS, it is difficult to see how it provides anything not already covered. About the only thing not covered so far would be IO (Interest Only) and PO (Principal Only) strips. These are created by "stripping" the interest payments from mortgage backed securities, and selling the interest payments and the principal payments as separate securities. These are covered below.*

SS 15. 1. A. p) Initial PAC collar and effective collar. *(pp.416-421). The PAC has a specified repayment schedule that is supported by subordinating the claims of the support tranches to principal repayments, and the schedule can be met over a large range of prepayment rates. The initial collar is the range of prepayment rates within which the PAC repayment schedule can be met. If there are multiple PAC tranches, the repayment schedule for the shorter tranches can be met over a wider range of prepayment rates, so that the "effective collar" for these shorter tranches is wider.*

SS 15. 1. A. q) Support tranches and prepayment risk. *(p. 424). The support tranches have the greatest prepayment risk because they have not only their own prepayment risk, but also absorb the prepayment risk of the PAC tranches.*

SS 15. 1. A. r) Stripped mortgage-backed securities, principal mortgage strips (principal-only securities), and interest mortgage strips (interest-only securities). *(p.426). A stripped mortgage-backed security is created by redirecting the principal and interest cash flows. The most common arrangement is to direct all principal payments to one set of securities (the principal-only mortgage strips), and all of the interest payments to another set of securities (the interest-only mortgage strips).*

SS 15. 1. A. s) Investment characteristics of principal-only and interest-only mortgage strips. *(pp. 426-428). The principal-only mortgage strips are inversely related to interest rates. This is partly due to the present value effect of lower interest rates, but the principal-only bonds are also affected by the increasing prepayment rates at lower interest rates. This has the effect of shifting principal repayments to earlier dates, increasing the present value even more. The result is a greater sensitivity to interest rates than regular bonds. At low rates, the effect is somewhat moderated by the low reinvestment rates, and negative convexity results.*

The interest-only mortgage strips are positively related to interest rates. The effect of prepayment is to reduce the amount of interest to be received – i.e., if a mortgage is repaid, there will be no more interest paid on that mortgage. Note that there is both a present value and a prepayment effect. Decreasing interest rates will increase the present value of payments, but the higher prepayment rates reduce the amount of payments. The net result is a reduction in the value of the interest-only security as rates decline. Increasing interest rates have an inverse effect. Note that the effect of higher interest rates is limited – i.e., it cannot increase the interest payments above the no-prepayment amounts. At higher interest rates the present value effect may cause the value of the interest-only mortgage strip to decline.

SS 15. 1. A. t) Agency vs. nonagency mortgage-backed securities. *(p429). The major difference between agency and nonagency mortgage-backed securities is in the guarantees. The agency securities carry the explicit (GNMA) or implicit (FNMA, FHLMC) guarantee of the US government, while nonagency securities do not. Because of this lack of a government guarantee, nonagency securities are credit enhanced, i.e., they have additional support against default.*

SelfTest Question: Why is contraction risk especially important to interest-only tranches?
Suggested Answer: Interest-only (IO) tranches receive only the interest portion of payments. Contraction risk is the shortening of the life of the loan due to higher prepayment in times of lower interest rates. But prepayment results in fewer / smaller interest payments, and so directly affects the amount the interest-only tranche will receive. Note that extension risk is viewed favorably by the IO tranche, since interest will be received longer / in greater amounts.

Study Session 15: 1. B. Asset-Backed Securities (Ch. 4, Level II)
The same approach used to convert mortgages into liquid securities has been applied to other assets. There basic structure is similar to MBS and CMOs, but with somewhat different arrangements to fit the differing characteristics of the underlying assets. Since these asset-backed securities are issued by private concerns they, like non-agency MBS, often contain various credit enhancements.

1. Some asset-backed securities, such as auto loans, are based on fully-amortizing loans. Some asset-backed securities, however, are based on nonamortizing loans in which the required minimum payments may not reduce, and may actually increase, principal.
2. Lacking federal credit guarantees of agency MBS, asset-backed securities usually are credit-enhanced. This enhancement may be by guarantees from external parties, or they may be built-in or internal to the structure of the issue. The latter include reserve funds, overcollateralization, and senior / subordinated structures.
3. Home equity loans, manufactured housing loans, and auto loans have a similar structure, with slightly different characteristics. E.g., prepayment rate differ. Securities based on student loans, Small Business Administration loans, and credit card receivables have somewhat different characteristics.
4. A Collateralized Bond Obligation is similar to an MBS.

SS 15. 1. B. a) Amortizing assets vs. non-amortizing assets. *(p. 450). An amortizing asset is a loan that has a prespecified schedule for payments to reduce the principal balance. Payments on principal beyond the scheduled amount are prepayments. A non-amortizing asset is a loan that does not have a prespecified schedule for reduction of the principal balance, although it may have other required payments. While it is possible in a non-amortizing loan to make a payment to reduce the principal balance, this is not considered a "prepayment" since no payment was scheduled.*
SS 15. 1. B. b) External vs. internal credit enhancement. *(p. 452-453). External credit enhancement is provided by a third party external to the entity issuing the bond. Internal enhancement is provided by the entity issuing the bond through modification of the conditions of the bond*
SS 15. 1. B. c). Types of external credit enhancement and associated. *(p. 452). The types of external guarantees are:*

a) Corporate guarantees. This is a guarantee made by the entity that sponsored (i.e., arranged and sold) the issue. The guarantee is typically for a fraction of the principal amount.
b) Bank letter of credit. This is a standing bank account that can be drawn on to make up any losses.
c) Bond Insurance. This is an "insurance" or guarantee provided by a third party (often an insurance company), typically for a fraction of the principal amount.

The external enhancements are normally the "first line" of defense, and are used along with internal enhancement. The problem is the reliance on a third party. If the third party credit rating is downgraded, "weakest link" reasoning makes it likely that the enhanced assets will also be downgraded.
SS 15. 1. B. d) Internal credit enhancement reserve accounts and senior-subordinated structures. *(pp.452-453). There are actually three types of internal guarantees:*

a) Reserve funds. Cash reserve funds are funds obtained from the issue and set aside. Excess servicing spread accounts are created when the payments are greater than needed to make payments and cover servicing fees. The excess spread is set aside in a reserve account.
b) Overcollateralization. The amount of bonds issued is less than the principal amount of the underlying assets.
d) Senior / subordinated structures. This is the most popular form of credit enhancement. Subordinated or "non-senior" tranches are created, and any losses will be absorbed by these tranches, in order.

SS 15. 1. B. e) Shifting interest mechanism in senior-subordinated structures. *(p. 454). In a mortgage-backed senior / subordinated structure, the subordinated tranches are a percentage of the total mortgage balance. Prepayments will alter the percentage, i.e., "shifting" the subordinate interest. A shifting interest mechanism will allocate prepayments so as to maintain a specified level of protection for the senior securities.*

SS 15. 1. B. f) Passthrough structure vs. pay through structure. *(p. 455). For asset-backed securities, a "passthrough" structure is one in which the payments are simply passed through to security holders on a "pro rata" (i.e., weighted share) basis, without modification. In a "pay through" structure, the payments to the senior tranches may be reallocated, much like a CMO. The subordinated tranches are usually not reallocated, but remain on a "pro rata" basis.*

SS 15. 1. B. g) Optional clean-up call provisions. *(p. 456). This right is granted to the trustee. The right to call at par may arise:*

- *a) after the remaining collateral falls below a percentage (often 10%) of the original collateral,* ***OR***
- *b) after the amount of bonds outstanding falls below a percentage of the original amount of bonds outstanding,* ***OR***
- *c) after a specified date,* ***OR****,*
- *d) at the first occurrence of a) or c).*

SS 15. 1. B. h) Cash flows of securities backed by: closed-end home equity loans, open-end home equity loans, manufactured housing loans, student loans, and Small Business Administration loans. *(pp. 457-466). The cash flows are:*

- *a) Closed-end home equity loan: the cash flow consists of interest, regularly scheduled fully amortizing principal payments, and prepayments.*
- *b) Open-end home equity loans: during the revolving period, the borrower can draw against a line of credit. Interest is usually floating. At the end of the revolving period there may be either a balloon payment or an amortization schedule.*
- *c) Manufactured housing loans: these may be either mortgage loans or consumer loans. Cash flow consists of interest, regularly scheduled, fully amortizing principal payments, and prepayments.*
- *d) Student Loans: Student loans need not be paid during the "deferment period," the period that the student is in school. After the student leaves school, there is also a "grace" period during which no payments are due. After that point, payments are made which consist of interest and amortization of principal. Interest is floating.*
- *e) Small Business Administration loans: The monthly cash flow consists of interest, principal payment, and prepayment. These interest and principal payment are both variable – the size of the payment is recomputed at the start of each month to find the payment which would be fully amortizing on a level payment basis.*

SS 15. 1. B. 1) Prospectus payment curve for home equity loan-backed securities. *(p.457). The prospectus payment curve is the benchmark prepayment speed for the issue, an assumed rate or base case prepayment assumption.*

SS 15. 1. B. j) Available funds cap and adjustable-rate home equity loans. *(pp. 458-459). Typically, a home equity loan pool serving as collateral will contain both fixed-rate and variable-rate securities. The variable-rate securities will have both lifetime and periodic caps, and the interest payments will depend on these caps and on the servicing fees. As a result, the cap on a home equity loan floater is not fixed, but depends on the amount of funds generated by the net coupon on the principal, less fees.*

SS 15. 1. B. k) Non-accelerating senior tranches and planned amortization class tranches in a home equity loan-backed structure. *(pp. 459-461). A non-accelerating senior tranche receives a prespecified percentage of its pro rata principal payment. This prespecified percentage is smaller at first (protecting against contraction risk) and higher later (protecting against extension risk). As a result, the average life of the senior non-accelerating tranche is stable over a wide range of prepayment speeds. A planned prepayment class for a home equity loan-backed security is similar to the planned amortization class for a CMO, although there may be somewhat different collars for the fixed-rate and the floating-rate loans.*

SS 16 1. B. l) Prepayments resulting from refinancing for manufactured housing-backed securities and automobile loan-backed securities. *(p. 462). Manufactured housing is generally not refinanced for the following reasons:*

a) loan balances are typically small, with less savings possible through refinancing,
b) depreciation is rapid, and the value may actually be less than or close to principal.
c) typical manufactured home borrowers are of low credit quality and find refinancing difficult.

Automobile loans are generally not refinanced because of the smaller size of the loans and because of the already attractive interest rates on the loans (the low interest rates are a sales tool).

SS 15. 1. B. m) Absolute prepayment rate vs. conditional prepayment rate. *(p. 462). The difference between the "absolute" prepayment rate and the "conditional" payment rate is the base on which the rate is computed. The "absolute" prepayment rate is computed as a percentage of the original collateral amount. The "conditional" prepayment rate is computed as a percentage of the prior month's remaining balance.*

SS 15. 1. B. n) Structure of a credit card receivable-backed security. *(p466-467). Credit-card receivable-backed securities are structured as a master trust – i.e., several series can be sold from the same trust. The cash flow consists of finance charges collected, fees (e.g., late payment fees), and principal. Interest is paid to security holders periodically, but at first there is a "lockout period" in which principal repayments are retained and reinvested to maintain the size of the pool. After this lockout period, principal payments are paid to investors in a "principal-amortization period." This amortization period can have three structures:*

a) passthrough – i.e., passed through to investors pro rata as received (with processing delay.
b) controlled-amortization - i.e., a scheduled amortization, set long enough to be supported over a wide range of repayment.
c) bullet

SS 15. 1. B. o) Early amortization triggers for a credit card-backed receivable. *(p. 468). Early amortization may be triggered by certain provisions. A typical provision would be failure of the pool to provide enough cash flow to cover the obligation for three months.*

SS 15. 1. B. p) Basic structure of a collateralized bond obligation. *(pp. 468-469). A collateralized bond obligation (CBO) is similar to a CMO, except that it is collateralized by a pool of bonds. The bonds in the pool may include:*

a) high-yield corporates,
b) emerging market bonds,
c) bank loans to corporations.

Typical tranche structures include:

a) senior tranche (floating rate, generally the largest tranche),
b) subordinated or junior tranches (fixed rate),
c) an equity tranche, which receives any interest not paid to the senior and junior tranches.

SS 15. 1. B. q) Collateralized bond obligations and interest rate derivatives. *(p. 469). Note that the largest, senior tranche receives a floating rate, while the high-yield corporates and the emerging market bonds (and some bank loans) are fixed rate. This is a floating rate obligation financed by fixed rate assets. In order to correct this imbalance, the CBO manager will convert the fixed-rate payments into floating rate payments using interest rate swaps.*

SS 15. 1. B. r) Start-up phase, reinvestment phase, and pay down phase of a collateralized bond obligation. *(p. 471). As follows:*

a) *In the start-up phase, the manager uses the funds from the issue to establish the portfolio.*
b) *In the reinvestment phase, the manager maintains the portfolio, reinvesting principal payments. This phase typically lasts three to five years.*
c) *In the paydown phase over the remaining life of the issue, principal payments are made to the senior and junior tranches.*

SelfTest Question: Discuss the most important risks common to asset-backed securities, and how these risks are reduced.

Suggested Answer: The most important risks common to asset-backed securities are credit risk and prepayment risk. Credit risk is reduced by credit enhancement in the form of external support or by modifications in the structure of the issue. There is no direct solution to prepayment risk, but the prepayment risk of an issue can be repackaged in ways more suitable to the needs of different investors.

Study Session 15: 1. C. Valuing Mortgage-Backed and Asset-Backed Securities (Ch. 5, Level II)

Prediction of the cash flows from an asset is a basic requirement in the valuation of the asset. In the case of Mortgage-Backed and Asset-Backed securities, the prepayment option makes such prediction very difficult. Probabilistic models and simulation have been used. Also important in valuation is risk, as measured by duration. Duration, however, also depends on the prediction of cash flows, and several duration concepts have been suggested. Given the optionality of Mortgage-Backed securities, an option adjusted duration is proper.

1. The zero-volatility spread is the spread that will make the present value of the cash flows, when discounted at the treasury spot rates, equal to the observed price. Unfortunately, the prepayment option causes uncertainty in the cash flows. Monte Carlo simulation of mortgage rates over arbitrage-free paths, along with a model of prepayments based on mortgages rates is applied.
2. Given a set of interest rate / prepayment / cash flow paths generated by the Monte Carlo simulation, a zero-volatility spread can be computed for each path. The Option Adjusted Spread (OAS) is the spread which, when used as the zero volatility spread on each of eth simulated paths, sets the average present value equal to the price. Note that this is similar to the OAS for bonds, but is based on a large number of possible mortgage rate paths produced by the Monte Carlo simulation.
3. Duration and convexity can also be applied to MBS and ABS. There are several methods of calculation.

SS 15. 1. C. a) Cash flow yield of a mortgage-backed or asset-backed security . *(p.492). This is computed in exactly the same way as a yield to maturity or an internal rate of return: it is the discount rate which will set the present value of the expected cash flows equal to the market price (including accrued interest – i.e., the full or dirty price).*

SS 15. 1. C. b) Bond-equivalent yield of a cash flow yield. *(p. 493). The bond-equivalent yield is twice the semiannual yield. Since the cash flow yield is given as a monthly rate, it is necessary to convert this to a semiannual rate by compounding, and then multiply by two to find the bond equivalent yield annual rate:*

$$\text{bond-equivalent yield} = [(1 + i_M)^6 - 1] \times 2$$

SS 15. 1. C. c) Limitations of the cash flow yield measure. *(p. 493). Similar to yield to maturity, two assumptions underlie the calculation of cash flow yield:*

a) the expected cash flow are reinvested at the cash flow yield,

b) the security is held to final payout (based on some payout pattern assumption).

In practice, there is a risk that cash flows will be reinvested at some rate less than the cash flow yield, and the actual payout may differ due to defaults, prepayments, etc.

SS 15. 1. C. d) Limitations of the nominal spread and the zero-volatility spread for a mortgage-backed security. *(pp. 493-494). The nominal spread is computed as the spread to Treasuries at one point on the yield curve, and so ignores the yield curve. It also ignores any embedded options. The Z-spread takes the yield curve into account, but ignores any embedded options. In this case the embedded option is the prepayment option held by the borrowers.*

SS 15. 1. C. e) Valuing a mortgage-backed security using Monte Carlo simulation. *(p494-499). The Monte Carlo method generates a large number of possible interest rate paths, which are "adjusted" to be arbitrage free. Cash flows from those interest rate paths are determined by prepayment, default, and recovery models. Given the cash flows and the relevant spot rates, the value of the security can be computed for the specific interest rate path as the present value of the cash flows. Averaging the values over the interest rate paths then produces a generalized value.*

SS 15. 1.C. f) Backward induction method and mortgage-backed securities. *(pp. 494-495). The binomial model, and backward induction in general, assumes that the decision to exercise an option is not path-dependent – i.e., does not depend on how interest rates evolved over time. However, for mortgages, the decision to prepay is a function of the interest-rate history of the pool. E.g., if mortgage rates dip below the rate for the underlying pool, making refinancing attractive, some but not all mortgages will be prepaid and refinanced. If interest rates increase and then go back to the same level that made refinancing attractive the first time, more mortgages will be prepaid and refinanced – but a smaller percentage than the first time. This is the "burnout" referred to by Fabozzi.*

SS 15. 1. C. g) Critical assumptions of the Monte Carlo model. *(pp.495-497). Critical assumptions of the Monte Carlo model are:*

a) The interest rate volatility assumption. This assumption is common to all interest rate models.

b) The relationship between Treasury rates and the refinancing rates.

c) A prepayment model.

d) A model of defaults and recovery.

SS 15. 1. C. h) Monte Carlo simulation and arbitrage-free value. *(p. 496). There is little to explain here – the Monte Carlo model relies on interest rate that are generated randomly, but have no restrictions that would make the interest rate paths arbitrage-free. It is necessary to "adjust" the paths to ensure that they are arbitrage-free. Fabozzi gives no details as to the process of adjustment.*

SS 15. 1. C. i) Computation and interpretation of option-adjusted spread using the Monte Carlo model. *(pp.499-500). The option-adjusted spread is the (constant) spread that, when added to all the spot rate on all the interest rate paths used in the model, will result in a value equal to the market price. It is the spread which links value to price, a measure of the yield of the security relative to treasury spot rates.*

SS 15. 1. C. j) Identifying rich/cheap mortgage-backed securities using the option-adjusted spread analysis. *(pp. 500-506). Note that analysis can be across issues or within an issue. As a first approximation, the tranche with the highest OAS would be the "cheap" (i.e., highest return for the purchase price) tranche. There are other factors to consider, however. The option cost, computed as:*

option cost = zero volatility spread – option-adjusted spread,

measures the prepayment risk embedded in the security, and the duration of the security is also a factor. A combination of the highest OAS with the lowest option cost would be the "cheapest" security, the lowest OAS with the highest option cost would be the most "rich" (expensive, lowest return for the purchase price) security.

SS 15. 1. C. k) Using Monte Carlo simulation model to compute duration. *(pp. 507-508). The procedure is similar to the procedure previously discussed: the yield curve is "shocked" up or down a small amount (i.e., a small amount is added to or subtracted from the yield curve), and the Monte Carlo simulation model is applied to the new yield curves to find the increased and decreased values to be used in the effective duration calculation.*

SS 15. 1. C. l) Major assumptions behind differences in reported effective duration. *(pp. 507-508). There are four differences discussed in Fabozzi:*

a) Differences in the amount of the shock used. Since duration assumes a linear relationship, while the actual relationship has variable convexity, larger "shocks" can reflect the effect of convexity.
b) Differences in prepayment models. Differences in prepayment models produce different values and hence different duration estimates.
c) Differences in OAS. Different OAS will produce different values and hence different duration estimates. Note that this brings along all of the assumptions built into the Monte Carlo model used to determine the OAS.
d) Differences in the relationship between short-term interest rates and refinancing rates. This will in turn produce different cash flows, different values, and different duration estimates.

SS 15. 1. C. m) Cash flow duration, coupon curve duration, and empirical duration and their limitations. *(pp. 508-512) There are three alternative duration measures discussed by Fabozzi:*

a) Cash flow duration. In cash flow duration a base prepayment assumption is used to compute cash flows and cash flow yield, based on the market price. The cash flow yield is then increased and decreased by some Δy. Based on some prepayment model new prepayment rates, cash flows, and values are computed. The effective duration model is then applied. I.e., in cash flow duration the values in the duration formula are based on a valuation model. The major problem here is that there is incomplete recognition of the effect of interest rate changes on prepayment rates.
b) Coupon curve duration. In coupon curve duration, a schedule of price and coupon is employed. Effective duration is computed based on the prices of higher and lower coupon bonds. I.e., in coupon curve duration, the observed market prices of slightly different coupons are used to compute duration. The major problem is that the price-coupon relationship is developed for generic mortgage-backed securities and does not fit mortgage derivatives such as CMOs.

c) Empirical duration is based on historical price-yield relationships, with the effect of changes in yield estimated by regression. The problems here are that sufficient observations may not be available, the effect of embedded options may not be fully reflected, and the volatility of the spread to treasuries may distort the result.

SS 15. 1. C. n) Computing cash flow duration for a mortgage-backed security computed, and comparison to Monte Carlo-based modified duration and effective duration. *(pp. 510-511). In cash flow duration a base prepayment assumption is used to compute cash flows and cash flow yield, based on the market price. The cash flow yield is then increased and decreased by some Δy. Based on some prepayment model new prepayment rates, cash flows, and values are computed.*
The difference between modified duration and effective duration is that modified duration does not consider the possibility of changes in the cash flow when yield changes – i.e., it ignores the effect of embedded options. The cash flow duration does consider the possibility of changes in the cash flow, and so is an effective duration. Fabozzi, forcefully states that cash flow duration "is based on a naïve assumption about how prepayments may change."

SS 15. 1. C. o) Zero-volatility spread approach vs. Monte Carlo option-adjusted approach in the valuation of asset-backed securities. *(p. 513). If an asset-backed security has a prepayment option, and that option is likely to be exercised when interest rates change, the OAS approach should be used, since it will include the effects of prepayment. If, however, the security has no prepayment option OR it is unlikely that the prepayment option will be exercised due to interest rate changes, the simpler zero-volatility approach can be used. I.e., the question hinges on the effect of interest rate changes on the cash flows.*

SS 15. 1. C. p) Nominal spread vs. zero-volatility spread vs. option-adjusted spread in valuing a specific fixed income security. *(p. 513-514). The nominal spread, which is compute at only one point on the yield curve, should not be used to assess the value of a security. The zero-volatility spread is applicable to securities that are option-free. The OAS (estimated from the binomial model) should be used for securities with embedded options that are not path dependent. The OAS (estimated from the Monte Carlo simulation model) should be used for securities with embedded option that are path dependent.*

SelfTest Question: Why is Monte Carlo analysis applied to MBS and to ABS, but not to other assets with embedded options such as callable bonds.

Suggested Answer: The reason for the difference in treatment is that exercise of the call option is total and final, while in MBS and ABS the exercise of the option is partial and path dependent. I.e., in a callable bond, the bond is either called or not called – a 0 / 1 situation. In MBS and ABS, exercise of the option – prepayment – is not 0 / 1. Not all of the borrowers will prepay as mortgage rates change. Consider, for example, prepayment to refinance the mortgage at a lower rate. Contrary to the expectations of economists, not all borrowers will take advantage of the lower rates at the same time or even at all. Further, if interest rates decline, increase, and then decline again, the increase in the rate of prepayment in the second decline will be smaller than in the first decline – i.e., the relationship between prepayment rate and mortgage rate is path dependent.

Study Session 16: 1. D. A Framework for Assessing Trading Strategies (Ch. 6, Level II)

"Look before you leap" is a well-worn adage. This reading considers where to look before leaping into a trade or trading strategy by considering the potential risk and return.

1. Leverage is a two-edged sword, affecting both return and risk. Repurchase agreements ("repos") are often used to finance security holdings, and the characteristics of repos must be considered.
2. Total return is the return from all sources over the life of an investment, and is composed of cash flows (interest), interest on reinvested cash flows (interest-on-interest), and capital gains. Where cash flow is uncertain it must be modeled.
3. Risk analysis includes not only the risk of the individual trade, but of the effect of the trade in a portfolio context.

SS 15. 1. D. a) Advantages and disadvantages of leverage. *(pp. 530-532). Leverage is the borrowing of funds in the hope of earning a return in excess of the cost of funds. The advantage of borrowing is that return may be magnified. The disadvantage of borrowing is that losses may be magnified.*
SS 15. 1. D. b) Effects of leverage on a trading strategy. *(p530-532). There are two keys. The first is that return is based on the investor's equity, which, given leverage, is smaller. The second is that the absolute amount of the return is reduced by the cost of the leverage.*
SS 15. 1. D. c) Repurchase agreements. *(p. 533). A repurchase agreement is the sale of a security with a commitment to buy back (repurchase) the security at a specified time for a specified amount. This is really a loan in which the collateral is the "sold" security.*
SS 15. 1. D. d) Dollar interest of a repurchase agreement. *(pp. 533-534). The dollar interest is computed as:*

Dollar interest = amount borrowed X repo rate X repo term/360

Note that the repo term is given in days, and that the interest is simple interest (compounding is ignored) based on actual days in the period and a 360 day year.
SS 15. 1. D. e) Credit risks associated with a repurchase agreement. *(pp. 536). The credit risk of the repo depends in large part on the delivery arrangement. The simplest concept is to deliver the underlying security to the lender (the purchaser of the security) and receive it back at the end of the repo. This arrangement is referred to as "delivering out," and credit risk is minimal. "Delivering out" is, however, expensive for short-term repos. An alternative is to deliver the security to a third party custodian – usually the borrower's clearing bank. This has some risk because of the reliance on the third party. Another alternative is for the borrower (the seller of the security) to hold the security in a segregated account. This is called "hold-in-custody". This has higher credit risk, since the credit is in the hands of the borrower and may be used fraudulently.*
SS 15. 1. D. f) Factors affecting the repo rate. *(p. 536-537). Fabozzi lists six factors that affect the repo rate:*

a) Quality of collateral – higher quality indicates a lower repo rate.
b) Term of the repo – depends on the "very short end of the yield curve."
c) Delivery requirement – "delivering out" would have the lowest rate, third party custody and hold-in-custody would have progressively higher rates.
d) Availability of the collateral – the more scarcer the particular type of collateral, the lower would be the repo rate.
e) Prevailing federal funds rate – this sets the general level of the repo rate, while the other factors set the rate for the specific repo.
f) seasonal factors.

SS 15. 1. D. g) Total return and expected total return over a given investment horizon. *(pp. 537-540). The total return is the return over the investment horizon, expressed on an annualized basis. I.e., the return over the entire period is given by:*

return over total period = total future dollars / initial investment

This is then converted to a <u>*semiannual*</u> *return by taking the* h^{th} *root:*

$$\text{semiannual return} = (\text{return over total period})^{1/h},$$

where h is the number of semiannual periods. The semiannual return is then annualized, either as a bond equivalent yield or as an effective rate.

The total future dollars will be the future value of any payments received over the investment horizon (i.e., interest plus interest-on-interest), plus the final value of the investment.

SS 15. 1. D. h) Using option adjusted spread (OAS) to compute a horizon price. *(p. 540). This is done in the same way as for a bond, but with predicted variables. The value of the security at the horizon can be computed using the predicted OAS. The OAS is commonly assumed to remain at the same level as at the date of purchase.*

SS 15. 1. D. i) "Scenario analysis". *(p. 541). A "scenario" is a set of assumptions, here the assumptions are the values of the variables at the investment horizon. Scenario analysis is the use of differing chosen scenarios to evaluate the result of a strategy. I.e., scenario analysis is the evaluation of the outcome of an investment under various sets of assumptions thought to be relevant.*

SS 15. 1. D. j) Controlling interest rate risk in a trade. *(pp. 543-544) Interest rate risk is controlled in a trade by equating the change in dollar value when interest rates change. Since "dollar duration" measures the change in the value of an investment as interest rates change, the change in value of two investments will be equated if the dollar durations of the positions are equal.*

SS 15. 1. D. k) Maintaining the dollar duration of a portfolio. *(p. 543-544). Remember that dollar duration is the change in value for a 100 basis point change in yield, and is given by:*

$$\text{dollar duration} = D_{\$} = \text{value } X \text{ (duration / 100)}$$

In order to control interest rate risk, the dollar change in value of both positions must be the same, i.e.,

$$D_{\$Y} = D_{\$Z}$$

or:

$$(\text{Value of } Y) \text{ X } (D_{\$Y} / 100) = D_{\$Z}$$

The market value of Y required to control interest rate risk (maintain equal dollar duration) is:

$$(\text{Value of } Y) = D_{\$Z} / (D_{\$Y} / 100).$$

SS 15. 1. D. l) Using total return analysis and scenario before executing the trade. *(p. 544). I am tempted to simply point out that you should not lock the barn after the horse has been stolen. It is likely, however, that this topic is attempting to get at the point that total return analysis and scenario analysis can "identify the range of possible outcomes and therefore provide the manager with a feel for the risk associated in a trade."*

SS 15. 1. D. m) Using scenario analysis and other analytical tools to analyze trades. *(pp. 544-546). This would require the computation of total return under various assumptions, using the techniques already discussed. Analysis would be mainly straightforward calculation, but it would also be important to evaluate the strategy based on the results of the analysis.*

SelfTest Question: Discuss the input to and process of computing horizon return.
Suggested Answer: Horizon return is the return over the entire life of an investment (the horizon), expressed on an annual return basis. The inputs are the predicted cash flows during the life of the investment, the rate of return on the reinvested cash flows, and the terminal value of the investment. The procedure is to:

a) compound forward the cash flows to find the future value at the horizon,
b) add this future value to the terminal value to final the total value at the horizon,
c) compute the Geometric Mean periodic rate of return over the horizon.

Study Session 15: 2. "Swap Contracts, Convertible Securities, and Other Embedded Derivatives," Ch. 25, pp. 1071-1082, *Investment Analysis and Portfolio Management*, 6th edition, Frank K. Reilly and Keith C. Brown (Dryden, 2000)

1. Structured notes are debt issues that have their principal or coupon payments linked to some other underlying variable. Distinguishing features include:
   a) Structure notes are designed for and targeted to a specific investor with a very particular need.
   b) After structuring the financing to meet the investor's needs, the issuer typically will hedge that unique exposure with swaps or exchange-traded derivatives.
2. Traditionally, the bond market was oriented toward the needs of the borrower, with the choice of maturity or coupon structure driven by the economic situation faced by the borrower. The structured note market, however, is oriented toward and driven by the economic situation of the investor.
3. Dual currency bonds can be viewed as a combination of two simpler instruments:
   a) a single-currency fixed-coupon bond, with cash flows consisting of the original outflow of purchase price, subsequent inflows of coupon payments, and final payment of the principal.
   b) a forward contract to exchange the bond's principal into a predetermined amount of a foreign currency, with cash flows at the time of maturity.
      - Note that if the final exchange is "off market," i.e., not at the prevailing forward rate, there will also be a cash flow at origination of the contract.

   This combination results in a cash flows consisting of:
   a) An original outflow consisting of bond purchase price plus any premium for the "off market" exchange of currency at maturity of the forward contract.
   b) Periodic coupon payments.
   c) Final receipt of principal (including final coupon payment) and exchange for foreign currency.
4. Equity-index linked notes can be viewed as a combination of a long bond plus a call option on the index.

   The cash flows are:
   a) The cash flows to the bond (which may be a zero).
   b) A "supplemental redemption amount" determined by the level of the index above a reference point.
   c) This cash flow allows a bond return plus participation in possible returns from equity appreciation.
5. A commodity-linked Bull and Bear bond can also be viewed as a combination of a bond and an option, but is divided into two "tranches:"
   a) The bull tranche can be viewed as a long bond plus a long call on the commodity.

- The cash flows to the bull tranche as the regular cash flows and principal repayment to the bond, plus the payoff to the call if the commodity price increases.
- The payoff to the call is in the amount of the bond principal times the fraction by which the commodity price exceeds a reference point.
- The bull tranche thus has regular bond payments but a principal repayment which may contain a premium if the commodity price increases.

b) The bear tranche can be viewed as a long bond plus a put on the commodity.
- The payoffs are similar to the bull tranche, but the principal repayment will contain a premium if the price of the commodity decreases.

c. Note that the bull tranche may have a cap, while the bear tranche may have a floor.

6. Swap-linked notes are a combination of variable -rate borrowing plus entering a receive-fixed swap to convert the rate paid back to a fixed rate.
   a) The cash flows are the variable-rate coupon paid on the borrowing, and the cash flow from / to the swap.
   b) If the borrowing is constructed in such a manner that the payments could have a minimum (such as a reverse floater with leverage), the arrangement might also include a cap to offset higher payments on the swap.

SS 15. 2. a) Structured notes vs. regular fixed-income securities. *Distinguishing features include:*
a) Structure notes are designed for and targeted to a specific investor with a very particular need.
b) After structuring the financing to meet the investor's needs, the issuer typically will hedge that unique exposure with swaps or exchange-traded derivatives.
The traditional bond market was oriented toward the needs of the borrower, with the choice of maturity or coupon structure driven by the economic situation faced by the borrower. The structured note market, however, is oriented toward and driven by the economic situation of the investor
LOS 15. 2. b) Cash-flow characteristics of dual-currency bonds, equity-index linked notes, commodity-linked bull and bear bonds, and swap-linked notes. *Dual currency bonds can be viewed as a combination of two simpler instruments: a single-currency fixed-coupon bond, with cash flows consisting of the original outflow of purchase price, subsequent inflows of coupon payments, and final payment of the principal, and a forward contract to exchange the bond's principal into a predetermined amount of a foreign currency, with cash flows at the time of maturity. Note that if the final exchange is "off market," i.e., not at the prevailing forward rate, there will also be a cash flow at origination of the contract. This combination results in a cash flows consisting of:*
a) An original outflow consisting of bond purchase price plus any premium for the "off market" exchange of currency at maturity of the forward contract.
b) Periodic coupon payments.
c) Final receipt of principal (including final coupon payment) and exchange for foreign currency.
Equity-index linked notes can be viewed as a combination of a long bond plus a call option on the index. The cash flows are:
   a) The cash flows to the bond (which may be a zero).
   b) A "supplemental redemption amount" determined by the level of the index above a reference point.
   c) This cash flow allows a bond return plus participation in possible returns from equity appreciation.
A commodity-linked Bull and Bear bond can also be viewed as a combination of a bond and an option, but is divided into two "tranches." The bull tranche can be viewed as a long bond plus a long call on the commodity. The cash flows to the bull tranche as the regular cash flows and principal repayment to the

bond, plus the payoff to the call if the commodity price increases. The payoff to the call is in the amount of the bond principal times the fraction by which the commodity price exceeds a reference point. The bull tranche thus has regular bond payments but a principal repayment which may contain a premium if the commodity price increases.
The bear tranche can be viewed as a long bond plus a put on the commodity. The payoffs are similar to the bull tranche, but the principal repayment will contain a premium if the price of the commodity decreases.
Note that the bull tranche may have a cap, while the bear tranche may have a floor.
Swap-linked notes are a combination of variable -rate borrowing plus entering a receive-fixed swap to convert the rate paid back to a fixed rate. The cash flows are the variable-rate coupon paid on the borrowing, and the cash flow from / to the swap. If the borrowing is constructed in such a manner that the payments could have a minimum (such as a reverse floater with leverage), the arrangement might also include a cap to offset higher payments on the swap.

SelfTest Question: Given the complexity of structured notes, why are they attractive?

Suggested Answer: The complexity of structured notes lies at the heart of their attraction. The complexity arises from designing the notes to have features that are desirable to investors. While the complexity itself is annoying, it is the result of making the notes more attractive to individual investors.

STUDY SESSION 16
Asset Valuation
Derivative Investments: Futures and Swaps

FUTURES PRICING

Study Session 16: Futures, Options & Swaps, 3rd. Edition, Robert W. Kolb (Blackwell, 1999)

1 - A. Futures Prices (Ch 3, pp. 43-76)

1. Futures price quotes are given in tabular form.
   a) The first column is the month of maturity, arranged by increasing term.
      - The contract that matures next (shortest time to maturity) is called the nearby contract.
      - Contracts which mature later in time are called distant or deferred contracts.
   b) The second, third and fourth columns are the opening, high and low prices for the day.
   c) The fourth column is the settlement price, which is the price at which contracts are settled (marked-to-market) at the end of the day.
      - Although the settlement price is often the last trading price, infrequent trading may result in the settlement price being set different than the last trade price.
   d) The fifth, sixth, and seventh columns are the change in settlement price from the previous day, the lifetime high price for the contract, and the lifetime low price for the contract.
   e) The final column is the open interest for the contract, or the number of contracts for which delivery is currently obligated.
      - Open interest is essentially the number of contracts which have not been offset by traders through reversing trades, i.e., by canceling long/short positions.
   f) Additional information given below the table applies contracts for a given commodity <u>for all maturities</u>.
      - The information includes the estimated volume for the day, the actual volume on the preceding day, the open interest for all maturities, and the change in open interest from the preceding day.
2. The price for immediate delivery is called the spot price (or sometimes the cash price or the current price).
   a) Basis is the difference between the current (cash, spot) price and the futures price:
      $$\text{Basis} = \text{Spot Price} - \text{Futures Price} = S - F_{o,t}$$
      - The spot price and the basis may differ geographically due to transportation costs.
      - Basis is usually refers to the nearby futures contract, but could also apply to other maturities.
      - If the basis is negative (spot price < futures price), the market is said to be normal. If the basis is positive (spot price > futures price) the market is said to be inverted.
   b) The basis will exhibit convergence – i.e., it will approach zero as the future approaches maturity.
      - At times before maturity, the futures price can depart substantially form the spot price - i.e., the basis may be large.
      - At maturity, however, the spot price and the futures price will differ only by transaction costs - i.e., the basis will be close to zero.
      - Outside of the convergence property, basis tends to be relatively stable.
   c) The difference between the prices of futures of different maturity is called the spread.

3. Cash-and-carry arbitrage is the creation a zero cost position which results in a profit.
   a) The position consists of a long position in the asset, borrowing equal to the cost of the asset, and a short future position.
      - There is no initial cost to the arbitrageur – the amount borrowed equals the amount paid out.
   b) The arbitrageur will hold (carry) the asset until the future matures, and deliver it against the contract.
      - The arbitrageur will pay costs of carrying the asset such as storage, insurance, and interest on the amount borrowed. These costs are called carrying costs.
   b) The payoff to this position will be the difference between the spot purchase price of the asset and the future price received, less the carrying costs:

$$\text{Payoff} = (\text{Futures Price - Spot Price}) – \text{Carrying Costs}$$
$$\text{Payoff} = (F_{0,t} - S_0) - CS_0$$

   where $F_{o,t}$ is the futures price, S_0 is the spot price, and C is the cost-of carry (expressed as a fraction of the spot price.
4. The cost-of-carry or carrying charge is the total cost to carry a good forward in time (hold the good over the time period), usually expressed as a per cent (of the value of the good) rate over time.
   a) Carrying cost has several components, such as storage, insurance, and transportation, but these are overwhelmed by the cost of financing.
   b) The financing cost will be close to the repo rate implied from a repurchase agreement.
      - A repurchase agreement is the sale of securities with a simultaneous agreement to repurchase the securities at a later date at a set price. This is equivalent to a loan using the securities as collateral.
      - The repo rate is the interest rate implied by the difference between sale price and repurchase price.
   c) In perfect markets where the financing cost is the only carrying charge, the cost-of-carry will equal the implied repo rate - the repo rate implied by the difference between the cash and futures prices:

$$C = (F_{0,t} / S_0) – 1$$

5. The possibility of cash-and-carry arbitrage implies that the future price must (in perfect markets) be related to the spot price and the cost-of-carry.
   a) Arbitrage possibilities should not exist in perfect markets:
      - If $F_{0,t} > S_0 (1 + C)$, the futures price is greater than the spot price plus the cost-of-carry, and arbitrageurs could enter into a cash-and-carry arbitrage (i.e., they could long the underlying asset using borrowed funds, sell the future, and make a profit with no investment)
      - If $F_{0,t} < S_0 (1 + C)$, the futures price is less than the spot price plus the cost-of-carry arbitrageurs could enter into a reverse cash-and-carry arbitrage (i.e., short the underlying asset, lend the proceeds, buy a future, and make a profit with no investment).
   c) Since arbitrage possibilities should not exist, in a perfect market, the futures price should not be greater than or less than the spot price plus the cost-of-carry, so that it must be equal to the cost-of-carry: $F_{0,t} = S_0 (1 + C)$.
      - This relationship determines the fair price of a future.
   d) This same reasoning can be applied to the difference between two futures contracts of different maturity to arrive at the conclusion that the difference in price between two futures contracts must be less than the cost-of-carry between the contracts.
      - The difference between the prices of two contracts is called the spread:

$$\text{Spread} = F_{0,d} - F_{0,n} = F_{0,n}(1 + C), \text{ or}$$
$$F_{0,d} = F_{0,n}(1 + C)$$

6. The cost-of-carry will not hold exactly in imperfect markets:
   a) Traders face a variety of transaction costs, including:
      - Commissions and exchange-imposed fees.
      - Bid-ask spreads.

   Traders cannot arbitrage unless the profit is greater than the added cost.
      - The result is a "corridor," or "no-arbitrage" bounds, within which arbitrage is not possible:

   $$S_{0,t}(1 - T)(1 + C) <= F_{o,t} <= S_o(1 + T)(1 + C)$$

   Where T is the transactions costs.

   b) Borrowing and lending rates are not equal:
      - As a result of unequal lending and borrowing rates, the "no-arbitrage" bounds are not equally spaced:

   $$S_{0,t}(1 - T)(1 + C_L) <= F_{o,t} <= S_o)1 + T)(1 + C_B),$$

   where L and B indicate the lending and borrowing rates.

   c) There are restrictions on short selling – the short seller may not have full use of the proceeds of the short sale.
      - Since this implies that the short seller can only invest a portion of the short-sale funds, the result is that the lower boundary on the futures price is lower:

   $$S_{0,t}(1 - T)(1 + fC)_L <= F_{o,t} <= S_o)1 + T)(1 + C_B),$$

   where $f < 1.0$ is the fraction of the short-sale proceeds available to the short-seller.

   d) There are limits to the storability of some commodities. To the extent that the storability of the commodity is limited, the above cash-and-carry relationships will be inapplicable.

SS 16. 1. A. a) Spot (cash) and futures prices covergence of basis to zero at expiration.
The spot price (or sometimes the cash price or the current price) is the price today for immediate delivery of the underlying asset, while the futures price is the price paid at a future date for delivery at that time. As the future date approaches, the future maturity approaches zero, and at maturity the future becomes immediate delivery – equivalent to the spot contract. Since both spot and future are equivalent at maturity, the futures price will converge to the spot price since an asset will only have one price in perfect markets.

SS 16. 1. A. b) Arbitrage profit opportunities, execution, and profit. *In perfect markets, an arbitrage profit exists any time the cash-and-carry arbitrage condition:*

$$F_{0,t} = S_0 (1 + C),$$

is violated. In imperfect markets, the more complex condition:

$$S_{0,t}(1 - T)(1 + fC)_L <= F_{o,t} <= S_o)1 + T)(1 + C_B),$$

is used, to reflect other costs beyond financing costs. The equations simply point out that if the amount to be received from the futures contract at maturity is more (less) than the cost of buying and holding, you can set up a profitable arbitrage (reverse arbitrage).

EXAMPLE: Suppose that the spot price of gold is \$400/oz., the future price of gold for delivery in one year is \$450/0z., and the interest rate is 10%. Then we have:

$$450 > 400(1.10).$$

This indicates an arbitrage opportunity.

The appropriate arbitrage depends on the direction in which the no-arbitrage condition is violated. If $F_{0,t} > S_0 (1 + C)$, the future is relatively more expensive than buying and holding the asset: sell (short) the future and buy and hold the commodity (buy low, sell high!). If $F_{0,t} < S_0 (1 + C)$, the future is relatively cheaper than buying and holding the asset: sell (short) the asset and buy the future (sell high, buy low!).

The payoff to the arbitrage will be the difference between the spot purchase price of the asset and the future price received, less the carrying cost, or Payoff = (Future Price - Spot Price) – Cost-of-Carry EXAMPLE: Suppose that the spot price of gold is $400/oz., the future price of gold for delivery in one year is $450/0z., and the interest rate is 10%. Then we have 450 > 400(1.10). This indicates that the future is overpriced relative to the spot price. The indicated arbitrage is to sell the overpriced future, borrowing to buy gold to hold and deliver.

The profit from this action can be taken at either the end of the year, or at the inception of the position:

- *If profit is taken at the maturity of the contract, the arbitrageur would borrow $400 and buy gold at $400/oz.. At the end of the year, he/she would deliver the gold, receive the $450, and pay $400(1.10) = $440, a net profit of $10.*
- *Profit can be taken immediately. In this case the arbitrageur would borrow the present value of the future price, or $450/1.10 = $409.09. He/she would then purchase gold at $400/oz., and have an immediate profit of $9.09. At maturity, he/she would simply deliver the gold, collect the $450, and pay the $450 owed on the borrowing.*

Note that the two answers are equivalent, since the $9.09 invested at 10% would produce $10 at the end of the year (within rounding). Either answer is correct, as long as the timing is specified. However, Kolb tends to compute profit at maturity.

SS 16. 1. A. c) Computing the implied repo rate. *The repo rate is $F_{o,t}/S_0 - 1$. This is easily computed, but it is necessary to carefully specify the period – i.e., the repo rate should specifically be stated as "daily," "weekly," or whatever time period. The repo rate is often converted to an annual rate by multiplying the rate for the period by the number of periods in a year.*

SS 16. 1. A. d) Transaction costs, unequal borrowing and lending rates, restrictions on short selling, and upper and lower no-arbitrage bounds on futures prices. *(pp. 60-67). Cost-of-carry arguments indicate that:*

$$S_0(1 + C) \leq F_{0,t} \leq S_0(1 + C)$$

The presence of imperfections changes this relationship by changing the cost-of-carry for arbitrage and reverse arbitrage. The effect of transactions costs T is to change the cost of carry both ways:

$$S_0(1 + C)(1 - T) \leq F_{0,t} \leq S_0(1 + C)(1 + T)$$

Adding unequal borrowing and lending rates C_L and C_B:

$$S_0(1 + C_L)(1 - T) \leq F_{0,t} \leq S_0(1 + C_B)(1 + T)$$

Finally, restrictions on short selling reduce the proceeds from short selling by a factor f:

$$S_0(1 + fC_L)(1 - T) \leq F_{0,t} \leq S_0(1 + C_B)(1 + T)$$

SS 16. 1. A. m) Conditions causing differing futures price. *(pp. 72-76). The argument is based on two parts. The first is that speculators will not enter a market unless there is an expected profit. The second is that the market may have and imbalance between hedgers desiring short positions and hedgers desiring long positions, which must be made up by speculators. If there is an excess of short hedgers, futures prices must be above nearby prices in order to provide a profit that would induce speculators to take the balancing long positions – a condition called " normal backwardation." If there is an excess of long hedgers, futures prices must be below nearby prices in order to provide a profit that would induce speculators to take the balancing short positions – A condition called "cotango." Finally, the "net hedging hypothesis" argues that the excess begins with short hedgers, but later shifts to long hedgers. Net hedging results in futures prices above cash prices at first, but futures prices below cash prices as the contract moves toward maturity.*

Self Test Question: Define "cash-and-carry arbitrage" and explain it's implications.
Suggested Answer: Cash-and-carry arbitrage is a zero cost position that results in a profit. The position consists of a long position in the asset, borrowing equal to the cost of the asset, and a short future position which has no initial cost to the arbitrageur. The payoff to this position will be the difference between the spot purchase price of the asset and the future price received, less the carrying cost (which includes financing cost), or Payoff = (Futures Price - Spot Price) - Financing Cost = ($F_{0,t}$ - S_0) - CS_0. The possibility of cash-and-carry arbitrage implies that the future price must (in perfect markets) be related to the spot price and the cost-of-carry. I.e., since arbitrage possibilities should not exist, so that Payoff ≤ 0, or $F_{0,t} \geq S_0 (1 + C)$. Further, if the futures price is greater than the spot price plus the cost-of-carry arbitrageurs could enter into a reverse cash-and-carry arbitrage (i.e., short the underlying asset, lend the proceeds, and buy a future), so that $F_{0,t} \leq S_0 (1 + C)$. As a result, in a perfect market $F_{0,t} = S_0 (1 + C)$, determining the fair price of a future.

B. Interest Rate Futures: Introduction (Ch. 5, pp. 112-117 and 125-130)

1. Treasury bill futures traded on the International Monetary Market of the Chicago Mercantile Exchange call for delivery of $1 million face value of 90-day bills.
   a) Prices are quoted as a discount from the IMM Index value:
   $$\text{IMM Index} = 100.00 - \text{Discount Yield.}$$
   Or:
   $$\text{Discount Yield} = 100 - \text{IMM Index.}$$
   NOTE: despite being called a "Yield," this is a discount from face value, and should not be confused with the actual yield on the bill.
   b) The Discount Yield is given in annualized (yearly) terms, using simple (uncompounded) interest and a 360 day year. To find the discount for an N-day T-bill, it is necessary to multiply by the fraction of a year before maturity:
   $$\text{N day Discount Yield} = \text{Annualized Discount Yield X } \frac{N}{360}$$
   For a 90 day T-bill, this would be 90/360 = 0.25
   c) The price which must be paid at delivery for 90-day T-bills is the face value of the bill minus the discount:
   $$\text{Bill Price} = \$1{,}000{,}000 - \frac{\text{Annualized 90-day Discount Yield X } \$1{,}000{,}000 \text{ X Days to Maturity}}{360}$$
   c) The contract also allows delivery of 91 and 92 day T-bills. The Invoice Amount is computed exactly as for the 90-day T-bills, except that the yield is the yield for the bills delivered (all delivered bills must be of the same maturity):
   $$\text{Invoice Amount} = \$1{,}000{,}000 - \frac{\text{T-bill Yield X } \$1{,}000{,}000 \text{ X Days to Maturity}}{360}$$
   where "T-bill Yield" is the annualized discount yield on T-bills of the given maturity, and "invoice Amount is the price of the T-bill to be delivered.
2. Eurodollar futures contracts are often based on the LIBOR (London Interbank Offer Rate).
   a) This contract is settled in cash, rather than by delivery of the underlying asset.

b) The yield quote on Eurodollar contracts is on an add-on basis, and is computed as:

$$\text{Add-On Yield} = \left(\frac{\text{Discount}}{\text{Price}}\right)\left(\frac{360}{\text{Days To Maturity}}\right)$$

Note that the add-on yield is greater than the corresponding discount yield.

3. Applying cash-and-carry arbitrage to interest rate futures proceeds in the manner previously indicated, but requires recognition of the peculiarities of the underlying Treasury securities.
   a) It is necessary to recognize the difference between the Treasury security purchased, and the Treasury security which underlies the futures contract.
      - The security purchased must be longer than the underlying security by the term to maturity of the contract – i.e., if you wish to arbitrage an N-day contract on 90-day T-bills, you must use 90 + N day T-bills so that the T-bills will have 90 (or 91 or 92) days until maturity *at the time the futures contract matures*.

EXAMPLE: If I wish to arbitrage a 35-day futures contract which calls for delivery of 90-day T-bills, I must use T-bills which will mature in 90 + 35 = 225 days (or 91+35= 226 days or 92 + 35 = 227 days)!

NOTE: Pricing conventions of T-bills (discount basis) and Eurodollars (add-on yield) differ.

   b) The no-arbitrage condition is that the yield on the underlying security must equal the yield obtained by buying the future and holding the delivered security until its maturity.
      - For example, the yield from purchasing a (90 + N)-day T-bill must equal the yield from holding an N-day T-bill contract and then holding the 90-day T-bill obtained from exercising the future:
        Yield on (90 + N) day T-bill = (Yield on N-day bill)(Yield on [futures] 90 day T-bill).
      - Again, this implies that the futures price must equal the spot price plus the cost-of-carry.
      - Note that the difference in price of two futures of differing maturity must equal the cost-of-carry for the time between maturities.
   c) Alternately, the no-arbitrage condition requires that the cost-of-carry equals the implied repo rate, where the implied repo rate is the rate which would be obtained by selling the future and buying the underlying asset. The implied repo rate is found by dividing the futures price by the spot price of the underlying asset and subtracting 1:

      Implied Repo Rate = (Futures Price /Spot Price) – 1 = $(F_o/S_0) - 1$

   NOTE this gives the rate over the period of the futures contract, and must be annualized by multiplying by the number of periods in a year before comparison to the (annualized) cost-of-carry.
   d) Two rules for arbitrage:
      - If the implied repo rate exceeds the financing cost, use a cash-and-carry arbitrage : Borrow funds, buy the underlying security, sell futures; hold the bond and deliver against the futures.
      - If the implied repo rate is less than the financing cost, exploit a <u>reverse</u> cash-and-carry arbitrage: Buy futures, sell the underlying security, and lend the proceeds; take delivery to repay the short position.

2. From the no-arbitrage (zero-arbitrage) condition, the futures yield is given by:

   (Yield on [futures] 90 day T-bill) = (Yield on (90 + N) day T-bill) / (Yield on N-day bill).
   a) This is the <u>forward rate</u> over the life of the underlying asset (the asset received at exercise of the future).

3. The treatment of instruments that pay interest must take the payments into account.
   a) The cost-of-carry is reduced if interest payments are received on the asset held.
   b) At delivery of a bond, the seller will receive accrued interest. The futures price must be adjusted to include this accrued interest amount.

SS 16. 1. B a) Discount yield and price quotes on U. S. Treasury bill (T-bill) futures contracts. T*he price which must be paid at delivery for N-day T-bills is the face value of the bill minus the discount:*

$$\text{Bill Price} = \$1{,}000{,}000 - \frac{\text{N Day Discount Yield X } \$1{,}000{,}000 \text{ X Days to Maturity}}{360}$$

The yield quote on Eurodollar contracts is on an add-on basis, and is computed as:

$$\text{Add - On Yield} = \left(\frac{\text{Discount}}{\text{Price}}\right)\left(\frac{360}{\text{Days To Maturity}}\right)$$

where the "Add-On Yield is the LIBOR yield. Solving this equation for the price of the Eurodollar contract:

$$\text{Price} = \left(\frac{\text{Discount}}{\text{Add - On Yield}}\right)\left(\frac{360}{\text{Days to maturity}}\right)$$

SS 16. 1. B. b) Cash-and-carry arbitrage and reverse cash-and-carry arbitrage, appropriate trades, and arbitrage profits. *The appropriate arbitrage depends on the direction in which the no-arbitrage condition is violated:*

If $F_{0,t} > S_0 (1 + C)$, the commodity is relatively more expensive than the future – use an arbitrage: sell (short) the commodity and borrow to buy the future (buy low, sell high!).

If $F_{0,t} < S_0 (1 + C)$, the commodity is relatively cheaper than the future – use a reverse arbitrage: sell (short) the future and borrow to buy the commodity (sell high, buy low!).

The payoff to the arbitrage will be the difference between the spot purchase price of the asset and the future price received, less the carrying cost (for Interest rate futures this is the financing cost), or Payoff = (Future Price - Spot Price) - Financing Cost. .

SS 16. 1. B. h) Implied repo rate and annualized repo rate.

The implied repo rate is again found by dividing the futures price by the spot price of the underlying asset and subtracting 1:

Implied Repo Rate = (Futures Price – Spot Price) – 1 = (F_o/S_0) -1

Note that this gives the rate over the period of the futures contract, and must be annualized by multiplying by the number of periods in a year:

Annualized Repo Rate = Implied Repo Rate X (Days to Maturity)

Self Test Question: Define the "implied repo rate" and explain how it could be computed for futures.
Suggested Answer: The repo rate is the interest rate implied by the difference between sale price and repurchase price. It is the ratio of the futures price divided by the spot price of the underlying asset minus 1:

Implied Repo Rate = C = (Futures Price / Spot Price) - 1= $(F_{0,t} / S_0)$ - 1

C. Stock Index Futures: Introduction (Ch 7, pp. 202-212)

1. Application of the arbitrage conditions to stock index futures must include the effects of dividends.
   a) Dividends have two effects:
      - The dividends themselves are a cash inflow.
      - The dividends may be reinvested and the proceeds received further reduce the cost-of-carry.
   b) For stocks, the cost-of-carry is the financing cost, less the dividends received and interest on the dividends received.

$$\text{Cost-of-Carry} = S_o C - \sum_{i=1}^{N} D_i (1 + r_i)$$

      - Note that the interest term $D_I(1+r_I)$ must be adjusted for each individual stock I and summed.
2. The no-arbitrage condition for index futures is that the futures price must equal the cost of acquiring and holding the stocks in the index less the future value (at the time of maturity) of the dividends

$$F_{0,t} = S_0(1 + C) - \sum_{i=1}^{N} D_i(1 + r_i)$$

   received:
3. The "fair value" of an index futures contract is the price the satisfies the no-arbitrage condition. This can be calculated by the following steps:
   a) Compute the cost of buying the stocks in the index (cost of replicating the index).
   b) Compute and add the cost of interest on the cost of the borrowing required to purchase the stocks, over the period of the futures contract.
   c) Estimate the dividends on the stock, and subtract the sum of dividends and return from reinvesting the dividends over the period of the futures contract.
   d) Convert the resulting number to an index value (for a price weighted / Dow Jones type index, this is done by dividing by the Index Divisor).

   This index value is the "fair value" of the index futures contract.
4. If the price of the index futures contract is not equal to the "fair value" of the contract, arbitrage is possible.
   a) Index arbitrage requires simultaneous sale of the futures contract. This is a difficult task.
   b) The arbitrage is actually implemented by using a computer program to communicate the required orders. This is called "program trading."
   c) Aside from the problems of simultaneous buying and selling, there are a number of other difficulties in index arbitrage:
      - Dividends, and the return on reinvested dividends, must be estimated. For large indexes, this can be difficult.
      - Other market imperfections such as direct transaction costs, unequal lending and borrowing rates, margins and restrictions on short selling, and storage costs complicate arbitrage.

SS 16. 1. C. a) Index arbitrage and program trading. T*he no-arbitrage condition for index futures is that the futures price must equal the cost of acquiring and holding the stocks in the index less the future value (at the time of maturity) of the dividends received. Index arbitrage requires simultaneous sale of the futures contract. Since this is a difficult task, the arbitrage is actually implemented by using a computer program to communicate the required orders. This is called "program trading."*

SS 16. 1. C. b) Dividends and the cost-of-carry model. *Dividends have two effects. The dividends themselves are a cash inflow, which must be deducted from the cost-of-carry. Also, the dividends may be reinvested and the proceeds received further reduce the cost-of-carry. The dividends and returns from reinvestment of dividends can be thought of a negative cost-of-carry.*
SS 16. 1. C. c) Fair value of a stock index futures contract. *The "fair value" of an index futures contract is the price the satisfies the no-arbitrage condition. This can be calculated by the following steps:*

a) *Compute the cost of buying the stocks in the index (cost of replicating the index).*
b) *Compute and add the cost of interest on the cost of the borrowing required to purchase the stocks, over the period of the futures contract.*
c) *Estimate the dividends on the stock, and subtract the sum of dividends and return from reinvesting the dividends over the period of the futures contract.*
d) *Convert the resulting number to an index value (for a price weighted / Dow Jones type index, this is done by dividing by the Index Divisor).*

This index value is the "fair value" of the index futures contract.
SS 16. 1. C. d) Difficulties in implementing an index arbitrage strategy. *Aside from the problems of simultaneous buying and selling, there are a number of other difficulties in index arbitrage. Dividends, and the return on reinvested dividends, must be estimated. For large indexes, this can be difficult. Other market imperfections such as direct transaction costs (commissions, exchange fees, and bid-ask spread), unequal lending and borrowing rates, margins and restrictions on short selling, and storage costs complicate arbitrage. These imperfections create a "band" within which arbitrage is unprofitable, because the yield will be less than the cost.*

Self Test Question: Describe the no-arbitrage condition for index futures.
Suggested Answer: The no-arbitrage condition for index futures is that the futures price must equal the cost of acquiring and holding the stocks in the index less the future value (at the time of maturity) of the dividends received:

$$F_{0,t} = S_0(1 - C) - \sum_{i=1}^{N} D_i(1 + r_i)$$

The no-arbitrage condition is applied to the monetary value of the underlying stock to find the monetary value of the futures contract. However, the futures price is expressed in index values, so the monetary value must be converted to an index value.

D. Foreign Exchange Futures (Ch. 9, pp. 261-266)

1. Interest rate parity asserts that exchange rates will adjust in such a way that a trader will receive the same return regardless of choice of currency - i.e., the return from investing in the domestic currency DC is the same as the rate of return from converting the domestic currency into a foreign currency FC at the spot rate, investing in FC, and initiating a futures contract to convert the proceeds back to DC:

DC Return = (DC X Conversion Rate) X (FC Return) X (Futures Price)

DC(1 + Domestic Rate) = (DC X Conversion Rate)(1 + Foreign Rate)(Futures Price)

$$DC(1 + r_{DC}) = (DC \text{ X } FC/DC)(1 + r_{FC})F_{0.t},$$

where $F_{0.t}$ is expressed as an exchange rate $FC_t/DC_{t.}$ at time t.

2. Cash-and-carry arbitrage in foreign exchange futures is called covered interest arbitrage.
   a) Covered interest arbitrage consists of borrowing DC funds, converting the funds to FC in the spot market, investing at the foreign interest rate, and initiating a futures contract for reconversion to DC.
   b) The return on this arbitrage is the return on the foreign investment less the repayment on the borrowing. The no-arbitrage conditions is that this return is equal to zero:

Return = (DC X Conversion Rate) X (FC Return) X (Futures Price) – (DC Return) = 0
Return = (DC X Conversion Rate)(1 + Foreign Rate)(Futures Price) -DC(1 + Domestic Rate) = 0

$$(DC \text{ X } FC/DC)(1 + r_{FC})F_{0.t} - DC(1 + r_{DC}) = 0$$

   c) The no-arbitrage condition is that the return equal zero . This is exactly the same as the interest rate parity condition.

SS 16. 1. D. a) Theoretical futures price and arbitrage opportunities. *The theoretical futures price is the price that satisfies the no-arbitrage condition – i.e., the price that makes the return to arbitrage zero. From the no-arbitrage condition,*

Return = (DC X Conversion Rate) X (FC Return) X (Futures Price) – (DC Return) = 0

So that:

$$\text{Theoretical Futures Price} = \frac{\text{DC Return}}{\text{(DC X Conversion rate)(FC Return)}} = \frac{DC(1 + r_{DC})}{(DC \setminus FC)(1 + r_{FC})} = FC(\frac{(1 + r_{DC})}{(1 + r_{FC})})$$

More simply, the theoretical futures price is simply the spot price of the foreign currency times the ratio of the domestic return to the foreign return, where both returns are expressed as Holding Period Returns (1 + r).

EXAMPLE: If the local rate of return is 6%, the foreign rate of return is 8%, and the cash price of the foreign currency is 5 of the domestic currency, and the futures price for a 90 day contract on 1000 units of the foreign currency is 4800 units of the domestic currency, is there an arbitrage opportunity?

ANS. The theoretical 90 day futures price (for one unit of the foreign currency) is:

Cash Price of foreign currency = 5

R_{DC} *= 0.06 X (360/90) = 0.015*

r_{FC} *= 0.08 X (360/90) = 0.02*

$$\text{Theoretical Futures Price} = 5 \text{ X } \frac{1 + 0.015}{1 + 0.02} = 4.9506$$

Since the theoretical futures price of 4.8/unit is different from the spot price of 4.9506/unit, there is an arbitrage opportunity.

SS 16. 1. D. b) Strategies to exploit arbitrage opportunities. *If the market futures price is above the theoretical price, the future is relatively expensive (overpriced) and should be sold. If the market price is below the theoretical price, the future is relatively cheap (underpriced) and should be purchased.*

NOTE: *Buy low, sell high!*

Once the futures position has been decided, the other positions follow. If the future is purchased, it will result in a long position in the underlying asset at maturity, and the position in the underlying asset must result in a short position in the underlying asset.

In the previous example, the theoretical price of the future is above the existing price, indicating that the future is underpriced. The underpricing indicates that the arbitrageur would buy (buy low, sell high!) the relatively cheap future:

TIME 0: - Buy the contract at 4800.
- Borrow, in the foreign country, the present value of the amount of F to be received in 90 days: PV(F_{90}) = 1000/1.02 = 980.3922
- Convert the PV(F_{90}) D: D = F X 5 = 4901.9608
- Invest the amount of D received domestically at 6%

90 DAYS: - Collect on the D investment: D_{90} = 4901.9608 X (1.015) = 4975.4902
- Under the contract, exchange 4800D for 1000F
- Deliver the 1000F to pay the foreign borrowing

Note that the net position in F at the end of the arbitrage must be zero. In this case purchase of the future results in receiving 1000F in 90 days. Borrowing the present value of 1000F in 90 days (at an annual rate of 8%) will require payment of F in 90 days, resulting in a net zero position. This amount borrowed is converted to D and invested locally to provide the D required by the contract. If the F had been invested in the foreign country at the 8% annual rate, the outcome would be uncertain, since we do not know what the exchange rate will be in 90 days.

Self Test Question: Describe cash-and-carry arbitrage in foreign exchange markets.
Suggested Answer: Cash-and-carry arbitrage in foreign exchange futures is called "covered interest arbitrage". It consists of borrowing domestically DC funds, converting the funds to foreign funds FC in the spot market, invested at the foreign interest rate, and initiating a futures contract for reconversion to DC. The return on this arbitrage is the return on the foreign investment less the repayment on the borrowing:

Return = (DC X Spot Conversion Rate)(1 + Foreign Rate)(Futures Price) -DC(1 + Domestic Rate)

$$= (DC/FC)(1 + r_{FC})F_{0,t} - DC(1 + r_{DC})$$

The no-arbitrage condition is that the return equal zero.

2. RISK MANAGEMENT WITH FUTURES

2 - A. "Using Futures Markets," Ch. 4, pp. 98-105.

1. A hedger enters the futures market to reduce risk, either over some hedging horizon or on a continuous basis.
   a) The relationship between futures prices and spot prices allows the use of futures to hedge exposure to risk / uncertainty arising from positions in asset.
      - The hedge may offset an actual present position, or offset an anticipated position at some future time (an anticipatory hedge).
2. Hedges may be either short or long, with the appropriate hedging position being the position such that the gain or loss moves oppositely to the gain or loss on the hedged position.
   a) A short hedge, or sale of the futures contract, is appropriate when the exposure is due to an actual or anticipated long position in the asset.
   b) A long hedge, or purchase of the futures contract, is appropriate when the exposure is due to an actual or anticipated short position (or need for) the asset.

3. Although hedging is often described as insurance, with the argument that speculators are needed to assume risk, the argument is limited.
   a) Long and short hedgers take positions with each other, so that speculators are only needed if there is an imbalance between long and short hedgers.
4. In actual markets, there may be a difference between the position the trader wishes to hedge (the <u>hedged</u> position) and the position actually being hedged (the <u>hedging</u> position), resulting in cross-hedging.
   a) One difference between the hedged and hedging positions may be the time span covered - i.e., the futures contract may not match the hedging horizon.
   b) Another difference may be that the quantity of the asset being hedged may not correspond to the quantity available through futures contracts.
   c) Another difference may be that the asset underlying the futures contract may not be the same as the asset being hedged.
5. The futures price and the spot price of the asset being hedged do not move in perfect unison, resulting in risk or uncertainty about the effectiveness of a continuous hedge - i.e., the gains and losses on the hedged position and the hedging position may not cancel.
   a) The hedging position is described in terms of the hedge ratio HR:

   Hedge Ratio = HR = Futures Position / Cash Market Position

   b) One approach is a 1:1 hedge - a futures position in the same amount of the asset as the position being hedged.
      - A 1:1 hedge will eliminate risk only if the hedged and hedging positions move in perfect unison.
   c) An alternative approach is to choose the hedge ratio which will result in the least (statistical) variance in the net outcome.
      - The variance of the net position, Φ^2_P, is:

      $$\Phi^2_P = \Phi^2_S + (HR)^2\, \Phi^2_F + 2HR\Delta_{SF}\, \Phi_S\, \Phi_F = \Phi^2_S + (HR)^2\, \Phi^2_F + 2HRCov_{SF}$$

      - This is minimized when:

      $$HR = \Delta_{SF}\, \Phi_S\, \Phi_F / \Phi^2_F = Cov_{SF} / \Phi^2_F$$

   d) In practice, this ratio is often estimated as the ∃ in the regression:

   $$S_t = \alpha + \beta F_t + \varepsilon_t$$

      - The R^2 of the regression provides a measure of the effectiveness of the hedge.

SS 16. 2. A. a) Short hedge vs. long hedge for a given risk exposure. *The position in the futures contract must be the opposite of the exposure, or anticipated exposure, to the underlying asset. If the exposure to the underlying asset is a long position, a short position in the future is used. The holding of the underlying asset (the long position) is offset by the delivery of the underlying asset required by the futures contract. If the exposure to the underlying asset is a short position, a long position in the future is used. Here the requirement to deliver the underlying asset (the short position) is offset by the receipt of the underlying asset from the futures position.*

SS 16. 2. A. b) Cross-hedging. *A cross-hedge occurs when the hedged and hedging positions are not the same – i.e., the underlying asset of the futures contract differs from the asset being hedged.. Reasons for mismatching of hedged and hedging positions include:*

a) The futures contract may not match the hedging horizon – a difference in time span..
b) The asset being hedged may not correspond to the quantity available through futures contracts.
c) The asset underlying the futures contract may not be the same as the asset being hedged.

SS 16. 2. A. c) Number of futures contracts needed to create a risk-minimizing hedge . *Risk minimization hedging is the choice of the hedge ratio which will result in the least (statistical) variance in the net outcome. The variance of the net outcome is minimized when:*

$$HR = \Delta_{SF}\ \Phi_S\ \Phi_F / \Phi^2_F = Cov_{SF} / \Phi^2_F$$

In practice, this ratio is often estimated as the β in the regression:

$$S_t = \alpha + \beta F_t + ,_t$$

with the R^2 of the regression providing a measure of the effectiveness of the hedge.

Self Test Question: Explain when a long or a short hedge is appropriate
Suggested Answer: The appropriate hedging position is the position on which the gain or loss moves oppositely to the gain or loss on the hedged position. A short hedge, or sale of the futures contract, is appropriate when the exposure is due to an actual or anticipated long position in the asset. A long hedge, or purchase of the futures contract, is appropriate when the exposure is due to an actual or anticipated short position (or need for) the asset.

Study Session 17: Futures, Options & Swaps, 3rd. Edition, Robert W. Kolb (Blackwell, 1997)

2 - B. "Interest Rate Futures: Introduction," Ch. 5, pp. 138-145.

1. The interest rate hedger seeks to reduce risk by taking a position that will produce a gain or a loss that will cancel the movement of the market.
   a) It is important to realize that the purpose is to reduce uncertainty about the future, and may produce a loss that offsets a market gain.
2. The asset underlying the futures contract is an interest-paying security, such as a T-bill, and the uncertainty being hedged is the purchase or sale price of the asset.
   a) Since price and yield, or interest rate, are linked the hedge will also offset any changes in the yield of the asset - leading to the name "interest rate futures."
   b) As with commodities it is not necessary to hold the contract until maturity.
      - The position can be canceled using an offsetting trade, with the gain or loss on the futures contract used to offset the change in the price of the underlying asset.
3. It is important to realize that the interest rate which is being hedged (via the price hedge) is not the spot rate, but the forward rate at the time the hedge is established.
   a) The futures yield is equal to the forward rate at the time the futures contract is established.
4. The basis (spot yield - futures yield) will change over time.
   a) Expected or anticipated change in the basis does not change the effectiveness of the hedge, so long as the difference between the futures yield and the forward rate remains constant.
   b) Unexpected changes in the basis will change the effectiveness of the hedge.
      - If a cross-hedge is used, the spot price at maturity for the hedged asset may be different from the spot price at maturity for the asset being used for the hedge (the asset underlying the futures contract).
      - If the spot price of the hedging asset does not equal the spot price of the hedged asset, the gain or loss on the hedge will not equal the loss or gain on the hedged asset, and any changes in yield will not be exactly offset.

SS 16. 2. B. a) Long hedge vs. short hedge. *The rule is that the futures position must result in a position opposite to the position in the underlying asset. Thus, if the hedger wished to acquire the underlying asset, he is short the underlying asset and should take a long position in (buy) the future. If the hedger will acquire the underlying asset, he is long the underlying asset and should take a short position in (sell) the future.*

SS 16. 2. B. b) Total cash flows of a hedged position vs. cash flows of an unhedged position. *It is crucial to remember that there are no cash flows at the original purchase or sale of the future. Cash flows occur only if the contract is reversed or matures. Rather than being held for delivery, most futures are reversed, and the gain or loss offsets the changed spot price.*

Example: A money manager expects to purchase $1,000,000 face value of 90 day T-bills 43 days from now. He is quite satisfied with the existing forward rate of 9%, but is afraid that rates will change before he receives the money. In order to "lock in" the forward rate (hedge any changes in the rate), he purchases 90 – day futures contracts. The futures price at the time of purchase is $978,685, but there is no cash flow at this time. If held to maturity, these contracts would result in delivery of $1,000,000 face value of T-bills paying the 9% forward rate at the time of purchase. When the contract is at maturity, the rate has indeed dropped to 8%, so that $1,000,000 of 90 day T-bills would cost $980,944**. Rather than holding for delivery, however, the manager reverses the position by selling an offsetting contract for $980,944. This reverses (cancels) his original contract, and gives him a profit on the contract of $2,259. He then purchases the $1,000,000 face value of bonds for $980,944, but because of his profit on the contract his net outlay is the original $978,685:*

Time	*Action*	*Cash Flow*
0	*Purchase futures contract, price = $978,685*	*None*
43 days	*Sell Futures contract, price = $980,944*	*Net profit = $ 2,259 inflow*
	Purchase $1,000,000 face value T-bills	*Purchase = $980,944 outflow*
	NET Cash Flow	*Net flow = $978,685*

The manager has purchased the T-bills at a cost which implies a 9% annualized rate of return.

* *The price of 90-day T-bills with a true annual return of 9% would be* $\$1{,}000{,}000/(1.09)^{1/4}$.

** *The price of 90-day T-bills with a true annual return of 9% would be* $\$1{,}000{,}000/(1.08)^{1/4}$.

SS 16. 2. B. c) Effect of cross-hedging on total cash flows. *The treatment on pp.141-145 of Kolb is confusing because it begins with an example in which the hedger has incorrect expectations. This is apparently an attempt to point out that it is the forward rate that is "locked in," rather than the existing spot rate. The question faced here is simpler - it would be the same as the situation in SS 17 2.B. c), except that the instrument being hedged and the instrument being used to create the hedge are different. Since the rates on the hedged and the hedging instruments may move differently, the hedge would not exactly offset any changes in rates. The example on p.144 of Kolb makes this point.*

Self Test Question: Comment on the following statement: "I am afraid that interest rates will decrease over the next month, so I will lock in today's rate using futures."

Suggested Answer: It is important to realize that the interest rate that is being hedged is not today's rate (the spot rate), but the forward rate at the time the hedge is established.

2 - C. Stock Index Futures: Introduction (Ch. 7, pp. 214-217)

1. A short hedge using Index futures would be appropriate when a downturn in the market is expected.
   a) Hedging positions must take account of the relative volatility of the hedged and hedging assets.
   b) For Index futures, the beta of the portfolio is a measure of relative volatility.
   c) The change in value for the portfolio and the change in value of the Index will be the same when
   $$(V_P/V_F)\ B_P = \text{number of contracts}$$
   d) This formula is critically dependent on two assumptions:
      - That B_P accurately measures the relative volatility of the portfolio over the period.
      - That the Index future moves exactly in tandem with the spot index.
2. A long hedge using Index Futures can be used to protect against a rise in the market when funds are expected at some future time.
   a) In this case the appropriate hedge ratio is 1:1 - the manager should buy futures contracts equal in value to the investment amount anticipated.

SS 16. 2. C. a) Hedge ratio Long vs. short hedge and the number of contracts required. *Hedging positions must take account of the relative volatility of the hedged and hedging assets. For Index futures, the beta of the portfolio is a measure of relative volatility, and the change in value for the portfolio and the change in value of the Index will be the same when the number of contracts = $(V_P/V_F)\ \beta_P$. The appropriate hedging position is one that will produce a result opposite the result of the position (or anticipated position) of the hedger. I.e., if the hedger has or anticipates a short position (needs to buy), the appropriate hedge is a long position (buy the future); if the hedger has or anticipates a long position (holds the portfolio), the appropriate hedge is a short position (sell the future).*
For the long hedge, the hedger should purchase futures contracts equal in cost to the amount of the planned investment – i.e., number of contracts = amount to be invested / futures price (this may require rounding to a whole number).
The appropriate number of contracts for a short hedge is the hedge ratio, or $(V_P/V_F)\ \exists_P$. The first term, V_P/V_F, adjusts for the size of the portfolio (note that V_F is a multiple of the index value). The second term $\exists_P$, adjusts for the change in portfolio value relative to the change in the index value.
SS 16. 2. C. b) Total cash flows of a hedged vs. an unhedged position.

The cash flows are:	*Hedged*	*Unhedged*
Change in value of futures position	$F_T - F_0$	NA
Less change in value of portfolio	$P_T - P_0$	$P_T - P_0$

Self Test Question: Explain why a hedging position on a stock portfolio must consider the beta of the portfolio.
Suggested Answer: The beta gives the volatility, or change in return for the portfolio, relative to the change in the market. In order to match changes in the portfolio and the market, the ratio of portfolio value to futures principal should equal to beta of the portfolio.

2 - D. Foreign Exchange Futures (Ch. 9, pp. 273-277)

1. Exchange Risk may be classified into two types:
   a) Transaction exposure occurs when the trader faces actual exchange of one currency for another.
   b) Translation exposure occurs when the currencies will not actually be exchanged, but it will be necessary to restate one currency in terms of the other.
2. Profit or loss on a transaction is calculated as the difference from the unhedged cost as predicted from the futures exchange rates.
   a) Thus, if C_U is the cost of the transaction in the hedged currency as predicted from the futures exchange rates:
   $$C_U = \text{Amount of foreign currency X futures exchange rate at time 0}$$
   and C_A is the actual cost in the hedged currency:
   $$C_A = \text{Amount of foreign currency X actual exchange rate at time t}$$
   the gain or loss on the unhedged position is $C_U - C_A$.
   b) The gain or loss on the hedged position includes the hedging gain or loss, H:
   $$\text{gain or loss on hedged position} = C_U - C_A + H$$
3. There are two reasons why the hedge may not be successful (i.e., why $H \neq -(C_U - C_A)$):
   a) The maturity of the futures contract may not be equal to the time of the contract (maturity mismatch).
   b) The amount needed may not be a whole multiple of contract size (principal mismatch).
   This assumes that the hedge is not a cross-hedge.

SS 16. 2. D. a) Long or short hedge and the number of foreign exchange futures required. *The appropriate hedging position is one that will produce a result opposite the result of the position (or anticipated position) of the hedger. I.e., if the hedger has or anticipates a short position (needs to buy), the appropriate hedge is a long position (buy the future); if the hedger has or anticipates a long position (holds the portfolio), the appropriate hedge is a short position (sell the future). The appropriate number of contracts is determined by the number of contracts which will most closely match the position (or anticipated position) being hedged.*
Example: You will need ¥39 million to pay an obligation due in four months. The exchange rate of 100¥ to $1embedded in the futures price is acceptable, and you decide to hedge. Since the ¥ contract is for ¥12.5 million, you would like to purchase 39/12.5 = 3.12 contracts. Since you must purchase a whole number of contracts, however, you would purchase 3 contracts. Note that this imperfect size match would leave 1.5 million of your anticipated position unhedged.

SS 16. 2. D. b) Total cash flows of a hedged position vs. an unhedged position.
Total cash flows of hedged position = cash flow of unhedged position + gain or loss on futures.
Example: In the above example, you purchased three futures with an implied exchange rate of 110¥ per $1 (no cash is exchanged at the purchase of the futures) to hedge an expected short position of ¥39 million. At maturity, the actual exchange rate is 105¥ per $1. The cash flows to the hedged position are:

Sell futures, receive 3 X [(12.5 million/105) – (12.5 million/110)]	*+$ 16,200*
Buy ¥39 million, pay 39 million / 105	*- $371,400*
Net Cash Flow of Hedged Position	*- $355,200*

The cash flows to the unhedged position are:

Buy ¥39 million, pay 39 million / 105	*- $371,400*
Difference in cash flows between hedged and unhedged positions:	*$ 16,200*

** The futures are a claim to 3 X •12.5 = •37.5 million at a cost of 37.5 million / 110 = $340,900. But at the exchange rate prevailing at the time the futures are sold, the •37.5 million are worth 37.5 million / 105 = $357,100. The profit on the 3 futures contracts is $357,000 - $340,900.*

Note that the hedge is not perfect. In a perfect hedge, the ¥ could have been obtained for 39 million / 110 = $354,500. A perfect hedge of 3.12 contracts would have had a profit of $371,400 – $354,500 = $16,900.

Self Test Question: Differentiate "transaction exposure" from "translation exposure."
Suggested Answer: Transaction exposure occurs when the trader faces actual exchange of one currency for another, while translation exposure occurs when the currencies will not actually be exchanged, but it will be necessary to restate one currency in terms of the other.

3. SWAPS

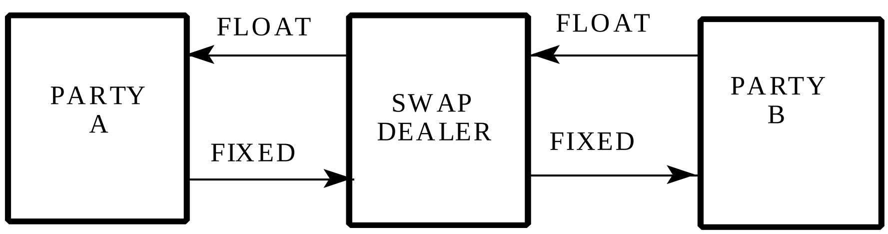
Study Session 16: Futures, Options & Swaps, 3rd. Edition, Robert W. Kolb (Blackwell, 1999)

3 - A. The Swaps Market: Introduction (Ch. 20, pp. 608-625 and 632-643)

1. While both futures and swaps are agreements to exchange assets at some future date, there are some distinct differences between the two agreements.
   a) Futures contracts are highly standardized with specific contract terms which cannot be altered, while the terms of swaps are very flexible.
   b) Futures are exchange traded, leading to some loss of privacy, while swaps are private agreements.
   c) Futures and options trading is subject to considerable government regulation, while swaps have virtually no government regulation.
   d) Futures exchanges provide a central market, while finding a counterparty for a swap can be difficult.
   e) Futures positions can be modified or terminated by offsetting trades, while swaps are an agreement between two parties and may be difficult to alter or terminate.
   f) The exchange guarantees the performance of futures contracts, while swaps have no such guarantee and require greater attention to creditworthiness.
2. There are two basic motivations for swaps:
   a) Commercial needs - the need to alter the characteristics of cash flow requirements arising in the course of normal business. I.e., transformation of a floating rate asset or liability into a fixed rate asset or liability (or vice versa).
   b) Comparative advantage - firms with comparative advantages in different markets can exploit the advantage by borrowing in the market in which they have the advantage and swapping cash flows.
3. Swaps may either with or without intermediaries:
   a) A "plain vanilla" swap without intermediaries can be diagramed as:

- In this case the cash flow exchange is between counterparty A (pay fixed) and counterparty B (receive fixed).
- Note that while there may be third parties present, they are not intermediaries in the cash flow.

b) A "plain vanilla" swap <u>with</u> intermediaries can be diagramed as:



- In this case the cash flow exchange is between counterparty A (pay fixed) and the intermediary, and between counterparty B (receive fixed) and the intermediary.

4. The cash flows in a swap are determined by the rates and the notional principal, and are netted.
   a) For an interest rate swap, the fixed payment is simply the fixed rate times the notional principal, while the floating payment is the floating rate times the notional principal; the actual cash flow is the net difference between the two:

      Net Cash Flow = Floating Payment - Fixed Payment

      Net Cash Flow = (Fixed Rate X Notional Principal) - (Floating Rate X Notional Principal)

      Net Cash Flow = (Fixed Rate - Floating Rate) X Notional Principal

      - The floating rate is usually expressed as a premium over a reference rate such as LIBOR.

   b) For a currency swap there are two sets of cash flows:
      - At the start of the swap, the notional principal is exchanged at the then-prevailing exchange rate, while at the end of the swap the original amounts are re-exchanged. These flows <u>are not netted</u>!

        NOTE: The amounts exchanged at the start and end of a currency swap are the same, and are not affected by changes in the exchange rate.
      - The interim payments are determined by the applicable rates times the notional amount; the cash flows <u>are netted</u> at the exchange rate in effect at the moment:

        Net Cash Flow = (Rate for Currency A - Rate for Currency B) X Exchange Rate
      - The currency swap cash flows are complicated by the multiple possible rates - i.e., the rate for either currency may be fixed or floating.

5. Swap dealers face a number of risks, all of which will affect the pricing of the swap.
   a) Creditworthiness of the counterparties, the possibility of default by one counterparty.

b) Interest rate risk, the possibility that interest rates may move against the dealers position.
c) Basis risk, the possibility that the normal relationship between the two swap rates may change.

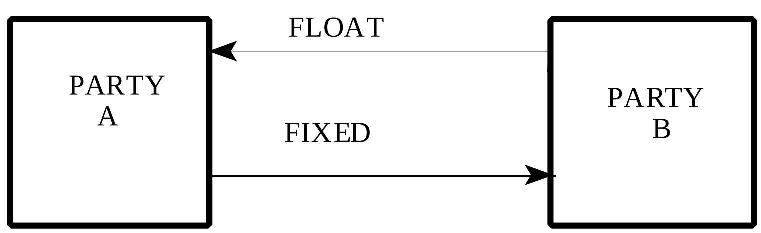
6. In order to reduce risk, the dealer will take a portfolio viewpoint, and arrange swaps so that the dealer is only an intermediary rather than a participant. Under this approach, risk may be classified as:
   a) Mis-match risk, the risk that the exact needed offsetting position cannot be found. The mis-match may involve differences in size or in currency or in reference rate.
      - This is also described as the availability of additional counterparties.

SS 16. 3. A. a) Swap contracts vs. futures contracts. *While both futures and swaps are agreements to exchange assets at some future date, there are some distinct differences between the two. Futures contracts are highly standardized with specific contract terms that cannot be altered, while the terms of swaps are very flexible. Futures are exchange traded, leading to some loss of privacy, while swaps are private agreements. Futures and options trading is subject to considerable government regulation, while swaps have virtually no government regulation. Futures exchanges provide a central market, while finding a counterparty for a swap can be difficult. Futures positions can be modified or terminated by offsetting trades, while swaps are an agreement between two parties and may be difficult to alter or terminate. The exchange guarantees the performance of futures contracts, while swaps have no guarantee and require greater attention to creditworthiness.*

The reference to payment date versus expiration date is unclear. It may refer to the fact that the swap is a series of payments on set dates, while the future expires on a single date. Note that this implies that a swap is similar to a series, or strip, of futures. Alternately, it may refer to the fact that the maturity of exchange-traded futures is shorter than the tenor of swaps.

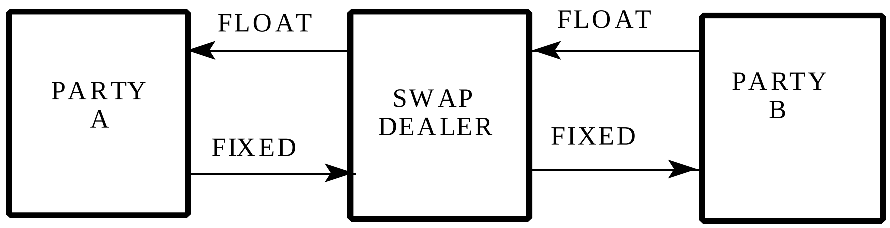
There are two basic motivations for swaps. One motivation is commercial needs - the need to alter the characteristics of cash flow requirements arising in the course of normal business, such as the need to transform a floating rate asset or liability into a fixed rate asset or liability (or vice versa). The other motivation is comparative advantage - firms with comparative advantages in different markets can exploit the advantage by borrowing in the market in which they have the advantage and swapping cash flows.

SS 16. 3. A. b) Diagrams for a plain vanilla swap contract (including intermediaries). *A "plain vanilla" swap <u>without</u> intermediaries can be diagramed as:*



In this case the cash flow exchange is between counterparty A (pay fixed) and counterparty B (receive fixed). Note that while there may be third parties present, they are not intermediaries in the cash flow.

A "plain vanilla" swap <u>with</u> intermediaries can be diagramed as:



In this case the cash flow exchange is between counterparty A (pay fixed) and the intermediary, and between counterparty B (receive fixed) and the intermediary. Note that again there may be non-intermediary third parties.

SS 16. 3. A. c) Cash flows of a swap. *The cash flows in an interest rate swap are determined by the rates and the notional principal, and are netted. For an interest rate swap, the fixed payment is simply the fixed rate times the notional principal, while the floating payment is the floating rate times the notional principal; the actual cash flow is the net difference between the two:*

Net Cash Flow = Floating Payment - Fixed Payment

Net Cash Flow = (Fixed Rate X Notional Principal) - (Floating Rate X Notional Principal)

Net Cash Flow = (Fixed Rate - Floating Rate) X Notional Principal

Note that the floating rate is usually expressed as a premium over a reference rate such as LIBOR.

For a currency swap there are two sets of cash flows:

At the start of the swap, the notional principal is exchanged at the then-prevailing exchange rate, while at the end of the swap the original amounts are re-exchanged. These start and end flows <u>are not netted</u>!

NOTE: *The amounts exchanged at the start and end of a currency swap are the same, and are not affected by changes in the exchange rate.*

The interim payments are determined by the applicable rates times the principal amount; these cash flows <u>are not netted</u> at the exchange rate in effect at the moment time of the flow:

Net Cash Flow = (Rate for Currency A - Rate for Currency B) X Exchange Rate

The currency swap cash flows are complicated by the multiple possible rates - i.e., the rate for either currency may be fixed or floating.

In an interest rate swap, all cash flows are netted (note that the principal is <u>not</u> exchanged). In a currency swap, no cash flows are netted, and the principal is exchanged at the start and the end of the swap.

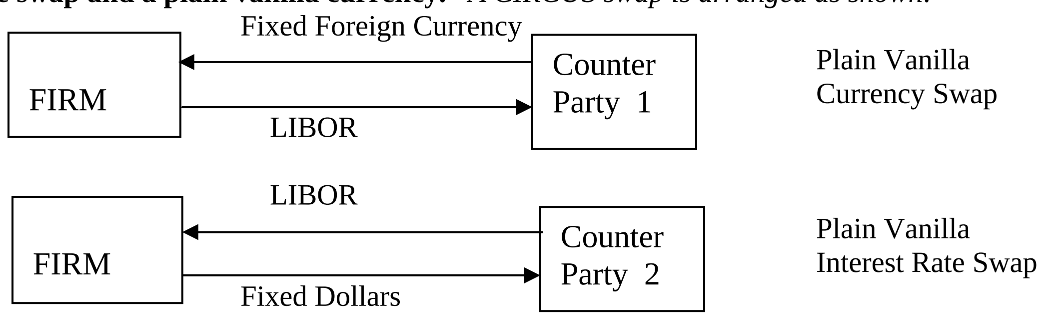
SS. 16. 3. A. d) Net borrowing/lending rates for swap counterparties. *The diagrams have been presented above, the actual net rates would be computed as the difference between the rates paid and received. Note that this may not be a constant, since some of the rates may be floating.*

SS 16. 3. A. e) Plain vanilla interest rate swap, an amortizing swap, an accreting swap, a seasonal swap, a roller coaster swap, an off market swap, a forward swap, a yield curve swap, a constant maturity swap, a basis swap, and a diff swap. *The types are defined as:*

- *a) Plain vanilla interest rate: an interest rate fixed-floating swap with constant notional principal.*
- *b) Amortizing swap: the notional principal reduces over time.*
- *c) Accreting swap: the notional principal increases over time.*
- *d) Seasonal swap: the notional principal varies according to a fixed plan.*
- *e) Roller Coaster swap: The notional principal first increases, then decreases to zero*
- *f) Off market swap: the fixed-pay rate is set above market (requires a payment by the floating-rate party.*

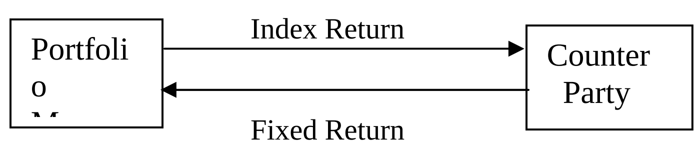
g) Forward swap: the swap will begin at some future time.
h) Yield Curve swap: both parties pay a floating rate, but the two rates are pegged to different points on the yield curve.
i) Constant maturity swap: a form of yield-curve swap in which the rates are based on a constant maturity index.
j) Basis swap: both parties pay a floating rate, but the two rates are based on different indexes.
k) Diff (rate-differential) swap: the two rates are based on indexes in two different countries, but the payments are in a single currency.

SS 16. 3. A. f) Form a combined interest rate and currency swap (CIRCUS) from a plain vanilla interest rate swap and a plain vanilla currency. *A CIRCUS swap is arranged as shown:*



The net result is a fixed receipt of the foreign currency, and a fixed payment of dollars.

SS 16. 3. A. g) Net periodic swap cash flow for an equity swap. *The diagram is:*



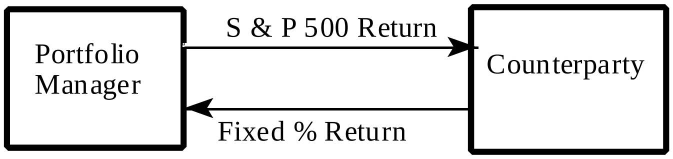
The cash flows would be:
a) From Counterparty to Portfolio Manager: index return X notional principal.
b) From Portfolio Manager to Counterparty: fixed rate X notional principal.
Note that a negative return on the index would result in a cash flow to the portfolio manager.

Self Test Question: Illustrate two basic motivations for swaps
Suggested Answer: There are two basic reasons for swaps. The first is commercial needs - such as the desire to convert a floating rate obligation to a fixed obligation to reduce uncertainty, or to convert a fixed inflow to a floating inflow to match a floating obligation. The second is comparative advantage - a borrower having an advantage in a particular market might borrow in that market and convert to another market through a swap.

3 -B. The Swaps Market: Refinements (Ch. 21, pp. 648 – 671)

1. In an equity swap, the "floating rate" is the return on a specified stock portfolio, or on an index such as the S&P 500. The fixed cash flows are then simply the fixed rate times the notional principal, while the variable cash flows are the return on the portfolio times the notional principal.

This can be diagramed as:



a) Note that if the return on the portfolio is negative, the cash flow is reversed. I.e., if the underlying portfolio has a negative return, the cash flow will all be from the "pay fixed" counterparty to the "receive fixed" counterparty.

2. Swaptions are simply an option on a swap.
   a) The option can be either a "receiver option" or a "payer option.".
      - The buyer / owner of a receiver swaption has the right (but not the obligation) to enter the specified swap as the fixed-rate receiver.
      - The buyer / owner of a payer swaption has the right (but not the obligation) to enter the specified swap as the fixed-rate receiver.
3. A swap may be extendable or cancelable. I.e., the tenor of an extendable swap may be increased at the discretion of the owner, while a cancelable swap may be ended at the discretion of the owner.
   a) An extendable or cancelable swap can be thought of as a plain vanilla swap with an embedded swaption.
      - With the exception of the tenor, the embedded swaption for an extendable swap would be the same as the original swap.
      - The embedded swaption for a cancelable swap would differ in both tenor (running to the end of the original swap) and also in direction, since the embedded swaption would be an offsetting swap to the original swap. I.e., for a cancelable receive-fixed swap, the embedded swaption would be a pay-fixed swap at the same fixed rate and notional principal as the original swap, but with a tenor equal to the remaining life of the original swap.
4. A swap agreement can be viewed as a pair of bond transactions:
      - The receive-fixed party can be thought of as buying a bond (which provides fixed payments) and at the same time issuing a floating rate note (which requires variable payments)
      - The pay-fixed party can be thought of as issuing a bond (which requires fixed payments) and buying a floating rate note (which provides variable payments).
5. A swap is directly comparable to a series of forward rate agreements.
   a) Consider a single interim cash flow of a swap: this is a fixed payment netted against a floating rate payment.
      - For the pay-fixed counterparty, this will be an inflow (gain) when the floating rate is above the fixed rate, a loss when the floating rate is below the fixed rate.
   b) This is the same result obtained by sale of a futures contract on an interest rate instrument t:
      - When the rate at maturity is above the yield implied by the forward contract, the position results in a profit; when the rate at maturity is below the yield implied by the forward contract, the position results in a loss.
      - In essence, the forward contract is the exchange of a fixed payment for a floating rate payment - the same as the single swap payment.
   c) This indicates that it is possible to replicate a swap by entering a series of forward contracts.

SS 16. 3. B. a) Swap agreement as a combination of capital market instruments or as a portfolio of forward rate agreements.

a) *A swap agreement can be viewed as a pair of bond transactions. The receive-fixed party can be thought of as buying a bond (which provides fixed payments) and at the same time issuing a floating rate note (which requires variable payments). The pay-fixed party can be thought of as issuing a bond (which requires fixed payments) and buying a floating rate note (which provides variable payments).*

b) *Consider a single interim cash flow of a swap: this is a fixed payment netted against a floating rate payment. For the pay-fixed counterparty, this will be an inflow (gain) when the floating rate is above the fixed rate, a loss when the floating rate is below the fixed rate. This is the same result obtained by sale of a forward contract on an interest rate instrument: When the rate at maturity is above the yield implied by the forward contract, the position results in a profit; when the rate at maturity is below the yield implied by the forward contract, the position results in a loss. In essence, the forward contract is the exchange of a fixed payment for a floating rate payment - the same as the single swap payment. Since any single cash flow in the swap can be replicated by a forward contract, it is possible to replicate the entire swap by entering a series of forward contracts.*

SS 16. 3. B. b) Interest rate swap agreement as a strip of Eurodollar futures contracts and a pair of interest rate options. *As follows:*

a) *A strip of Eurodollar futures contracts is simply a sequence of Eurodollar futures contracts with successive expirations. The situation would be exactly the same as the series of forward rate agreements discussed above.*

b) *A "cap" is an instrument that pays the difference between a variable rate payment on a notional principal and a fixed rate payment on the same notional principal <u>if</u> the variable rate is <u>above</u> the fixed rate (it can be considered as a series of long interest rate call options). A "floor" is an instrument that pays the difference between a variable rate payment on a notional principal and a fixed rate payment on the same notional principal <u>if</u> the variable rate is <u>below</u> the fixed rate (it can be considered as a series of interest rate put options).*

A combination of a long call at strike rate Y, and a short put at lower strike rate (Y – X) (with other features being the same), is called a "collar." Since the long cap pays a gain equal to the added interest if the variable rate is above Y, while the short floor results in a loss equal to the interest savings if the variable rate is below (Y – X), the collar can be used to hedge the effective rate paid on a variable rate borrowing to fall between Y and (Y – X). If the strike price of the call and the strike price of the put are equal, so that Y = (Y – X) (i.e., X = 0), the payoff pattern of the two options is exactly that of an interest rate swap. The two options can be used to hedge away all variation in the variable rate borrowing, resulting in a fixed rate.

Although the combination of the two options has the same effect as an interest rate swap, the combination of the two options may differ from an interest rate swap by having a non-zero cost. If the premium on the long call (a cost) equals the premium on the short put (a gain), however, the collar is a zero-cost collar and is equivalent to an interest rate swap

SS 16. 3. B. c) Forward Rate Agreements. *An FRA (forward rate agreement) is an agreement to exchange a variable future rate (typically LIBOR) on a notional principal and a set rate on the notional principal. Note that the set rate is decided at the initiation of the FRA, while the future rate will be the rate prevailing at the maturity of the FRA. The gain or loss on the FRA will be the difference between the variable rate and the future rate, times the notional principal:*

Gain / loss on FRA = (variable rate – fixed rate) X notional principal.

If the variable rate is below the fixed rate, the variable-rate receiver will have a loss, and if the variable rate is above the fixed rate, the variable rate receiver will have a gain.

If a call on the underlying rate at the same notional principal as the FRA is purchased, the gain / loss to the call will:

Equal the gain to the holder of the FRA if the variable rate is above the fixed rate
Equal zero if the variable rate is below the fixed rate

If a put on the underlying rate at the same notional principal as the FRA is sold, the gain / loss on the put will:

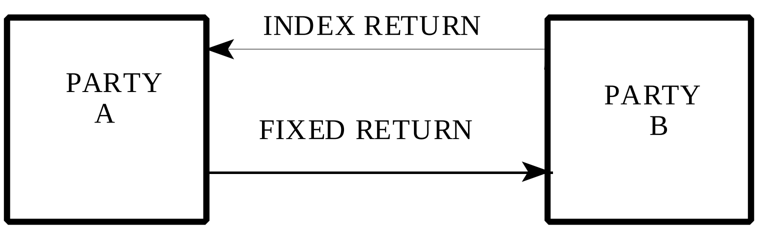
Equal zero if the variable rate is above the fixed rate
Equal the loss to the holder of the FRA if the variable rate is below the fixed rate

The combination of a put and a call will have a gain / loss equal to the gain / loss on the FRA.

SS 16. 3. A. f) Swap agreement as a portfolio of caps and floors. *A combination of a long call at strike rate Y, and a short put at lower strike rate (Y – X) (with other features being the same), is called a "collar." Since the long cap pays a gain equal to the added interest if the variable rate is above Y, while the short floor results in a loss equal to the interest savings if the variable rate is below (Y – X), the collar can be used to hedge the effective rate paid on a variable rate borrowing to fall between Y and (Y – X). If the strike price of the call and the strike price of the put are equal, so that Y = (Y – X) (i.e., X = 0), the payoff pattern of the two options is exactly that of an interest rate swap. The two options can be used to hedge away all variation in the variable rate borrowing, resulting in a fixed rate.*

Self Test Question: Diagram and describe the cash flows for an equity swap.

Suggested Answer: In an equity swap, the notional principal is expressed in terms of a stock portfolio and the "floating rate" is the return on the portfolio. The cash flows are then simply the fixed and the return on the portfolio times the portfolio amount. This can be diagramed for a positive portfolio return as:



Note that if the return on the portfolio is reversed, the cash flow is reversed. I.e., if the underlying portfolio has a negative return, the cash flow will all be from the "pay fixed" counterparty to the "receive fixed" counterparty.

PRACTICE EXAM #1
LEVEL II

Question #1

The concept of "zero volatility") spread used in the analysis of Asset Backed Securities and the concept of "static spread" used in the analysis of bonds are identical.

A. **Describe**:
- a) The problem this concept is meant to overcome.
- b) The measure and its method of computation.

(**10 minutes**)

Due to the shortcomings of the zero volatility concept, "Option-Adjusted Spread" has been developed.

B. **Describe**:
- a) The problem the "Option Adjusted Spread" is meant to overcome.
- b) The measure and its method computation.

(**10 minutes**)

Question #2

Interest rates can be expressed in several ways. The analyst must understand each of these ways.

A. **Describe** the following interest rate concepts:
- i. yield curve
- ii. par yield curve
- iii. spot rate
- iv. forward rate

(10 minutes)

You observe the following rates:

Maturity(yrs)	1	2	3	4	5
Par Rate, %	6	7	8	8.5	9
Spot Rate, %	6	7.03535	X	8.6585	9.23779
Forward Rate, %	6	8.0808	10.295	10.3177	X

B. **Compute** the missing entries (show all computations).

(10 minutes)

The expected return on a bond is sometimes expressed as the "rolling yield." You observe the following yields on zero coupon bonds:

Maturity, yrs	1	2	3	4	5
Yield,%	6	7	8	8.5	9

C. Assuming that the yield curve does not shift, **compute** the one-year yield for a 4-yr bond (show all calculations).

(10 minutes)

Question #3

A client has noticed that gold, which is presently priced at $450 per ounce, has a futures price of $472.50 per ounce. In an attempt to understand what is happening, she asks you to explain the difference between "repo rate," and "cost-of-carry," and how they relate to the difference between spot and futures prices of gold.

A. **Calculate** the implied "repo rate," and **illustrate** the meaning of the two terms, making reference to the spot and future prices of gold.

(**15 minutes**)

Question #4:

Table 1 presents data on Treasury Securities.

A. Using the data in Table 1, **compute** the theoretical spot rates for the period given. Assume annual pay bonds.

Table 1

Maturity	Coupon	Yield to maturity	Price
1 year	0.000	0.0800	96.15
2 years	0.100	0.0824	102.70
3 years	0.145	0.0846	114.16
4 years	0.085	0.0515	97.40

(10 minutes)

Term structure can also be described in terms of forward rates.

B. Using the data in Table 1, **compute** the implied *forward rate* on a one-year bond purchased at the end of year three. **Explain** the difference between a forward rate and a spot rate.

(10 minutes)

Table 2 presents data on two annual pay bonds.

C. Using the spot rates developed in part A., and the information in Table 2, **recommend** *either* bond X or bond Y for purchase. **<u>Justify</u>** your choice.

Table 2

	Bond X	Bond Y
Maturity	3 years	4 years
Coupon	12%.	7%
YTM	9.04%	9.17%
Price	107.50	93

(10 minutes)

Question #5:

As part of the analysis of a bond, you use a popular computer software routine to compute the modified duration and convexity of the bond as 6.7 years and 55.5 yrs.

A. **Compute** the estimated % price change of the bond if the market rate of interest goes from 10% to 10.5%, using both the computed duration and convexity.

(3 minutes)

B. **Explain** why convexity is used to provide a better estimate of price change.

(5 minutes)

C. **Explain** why higher convexity is desirable.

(3 minutes)

You now note that the bond is callable at 110. Reviewing the recent price of the bond as interest rates changed, you find the following:

YIELD-TO-MATURITY	PRICE
8.7%	108.5
8.6%	109.1
8.5%	109.6
8.4%	110.0
8.3%	110.3

D. **Explain** why modified duration and convexity are not appropriate for this situation.

(5 minutes)

E .Using the observed price behavior, **calculate** the "effective" duration for the bond.

(4 minutes)

Question #6:

Pricing relationships for derivative instruments are often based on an arbitrage argument.

A. **Define** the concept of arbitrage.

(**5 minutes**)

B. **State** and **explain** the "cost-of-carry" arbitrage condition for futures.

(**10 minutes**)

C. **Describe** the appropriate actions if the "cost-of-carry" arbitrage condition is violated for violations in *both* directions.

(**10 minutes**)

D. The "cost-of-carry" arbitrage condition can also be described in terms of the "implied repo rate.'"

(i) **Explain** the term "implied repo rate."

(ii) **State** the repo rate in terms of a rearranged "cost-of-carry" arbitrage condition.

(iii) **Describe** the appropriate actions to take advantage of violation of the implied repo rate condition for violations in *both* directions.

(**15 minutes**)

Question #7

A client of your firm thought that Mortgage Backed Securities were a wonderful investment, but has become uncertain since being reminded that prepayment of mortgages is common. She has heard of both “extension risk” and “contraction risk,” and has come to you with several questions.

A. What are the effects of prepayment on MBS when:
   i) yields increase
   ii) yields decrease
Be sure to indicate the nature of both extension risk and contraction risk.

B. Do prepayments always have a negative effect on investment performance?

SUGGESTED ANSWERS
PRACTICE EXAM #1
LEVEL II

Question #1

PART A

a) Use of traditional spread analysis, which considers the difference between the yield of a security and the yield of an on-the-run Treasury bond of the same maturity, suffers from two shortcomings:

i) It does not consider the term structure of interest rates, in effect assuming a flat yield curve at the treasury yield.

ii) It does not consider any changes in the cash flows which may arise from embedded options

b) The concept of "Static Spread"/"Zero Volatility Spread" is meant to correct the first shortcoming. It is the spread that will make the present value of the cash flows, when discounted at the treasury spot rate plus the spread, equal to the security price. Essentially, it is computed using a trial-and-error process using different spreads (or, using a form of interpolation).

PART B

a)The "Option-Adjusted Spread" (OAS) is meant to correct for the second problem mentioned above - the possibility of changes in the cash flows due to embedded options. It is usually used in the context of callable bonds. The return is computed over various possible "paths" of future cash flows, based on a probability distribution of outcomes and associated cash flows. The average of these calculated returns is the "option-adjusted spread." In practice, it is often infeasible to compute all possible paths, so that large-sample Monte Carlo simulation is used.

Question #2

PART A

i) The yield curve is a graph of bond yields versus their maturities (or sometimes against their durations). All of the bonds in the graph would be similar except for their maturities (or durations).

ii) The par yield curve is the yield curve for theoretical bonds whose prices equal par.

iii) The spot rate is the discount rate which would be applied to a single future cash flow - e.g., to a zero coupon bond.

iv) The forward rate is the rate of return on a bond between any two dates in the future, contracted today.

PART B

1) Since the spot rate is the discount factor applied to a single future cash flow, it is necessary to isolate the present value of the final principal and interest payment on the par bond. This is done by subtracting the present values (at the spot rates) of the other interest payments from the par price. For the third year, the present value of the first two interest rates is:

$$\text{PV of Interest Payments} = \frac{80}{1.06} + \frac{80}{(1.073535)^2} = 145.30$$

The present value of the final interest and principal payment is then $1000.00 - $145.30 = $854.70. The discount which would produce this present value is:

$$1 + S_3 = \left(\frac{1080}{854.70}\right)^{1/3} = 1.0811102 \Rightarrow 8.11\%$$

2) The forward rate is simply the rate of return over the forward period. This is computed by finding the rate which, given the fourth spot rate, will produce the fifth spot rate:

$$(1 + S_4)^4) \text{ X } (1 + f_{4,5})^5 = (1 + S_5)$$

$$(1 + f_{4,5}) = \frac{(1 + 0.0923779)^5}{(1 + 0.086585)^4}$$

$$\text{or } f_{4,5} = 11.5860.$$

PART C

The return on the four-year zero will be the forward rate between year between years three and four. This forward rate will have two components:

a) The four-year spot rate. Since the above bonds are all zero's, the spot rate will be 8.5%

b) A capital gain/loss due to the change in the yield on the bond. This component is called the "rolling yield." It can be approximated by noting that the duration is equal to maturity for a zero, and then using the year-end duration to estimate the price change:

$$\%\Delta P = - 3\ (0.005) = 1.5\%$$

Question #3.

PART A

The spot and futures prices must differ by the cost of buying and storing the asset over the maturity of a forward contract - the "cost of carry.". If the spot and forward prices differed by more than this cost of carry, you could sell the future, buy the asset, and make a profit. In the case of gold, if the cost of storing gold were less than $472.50 - $450.00 = $22.5, say $20, you could sell the future, borrow to buy the gold at $450.00, hold it for one year, and at the end of the year deliver the gold, pay $20.00 in costs, and have a $2.50 profit. This type of arbitrage opportunity cannot exist more than momentarily, so that we would expect that the cost of carry is actually $22.5 per ounce.

A different way of looking at the transaction is to consider selling the gold at $450.00, and entering a forward contract to buy gold at $472.50. This is equivalent to a "repurchase contract," in which an item is sold and at the same time the seller enters an agreement to repurchase it at a given time for a given price. I.e., it is as if the seller borrowed the $450.00 using the gold as collateral, and agreed to pay principal and interest at maturity. The "repo rate" is the implied interest rate on this repurchase/borrowing arrangement. In this case the implied repo rate is the interest rate paid, or:

effective interest rate = "repo rate" = ([$472.50 - 2]/$450.00) - 1 = 0.046 => 5%

If the interest rate is below the "repo rate," the interest payment is too high or, equivalently, the future is priced too high.

Question # 4

Part A

The spot rates are found by the process of *bootstrapping*:

The spot rate on the one-year security is it's yield to maturity, which is given as 8%.

The spot rate on the two-year maturity is found by solving for X in the equation:

$$\frac{10}{1.08} + \frac{110}{(X)^2} = 102.70$$

resulting in the value X = 1.085, or 8.5%.

The spot rate for the three-year security is found by solving for X in the equation:

$$\frac{14.5}{1.08} + \frac{14.5}{(1.085)^2} + \frac{114.5}{(X)^3} = 114.16$$

resulting in the value X = 1.09, or 9%.

The spot rate on the four-year security is found by solving for X in the equation:

$$\frac{8.5}{1.08} + \frac{8.5}{(1.085)^2} + \frac{8.5}{(1.09)^3} + \frac{108.5}{(X)^4} = 97.40$$

resulting in the value X = 1.094, or 9.4%.

Part B

The comparison proceeds by treating each asset as a series of zero-coupon instruments, and finding the value of the series to imply the "no-arbitrage" price. In the case of Bond X, the "non-arbitrage" price is:

$$\frac{12}{1.08} + \frac{12}{(1.085)^2} + \frac{112}{(1.09)^3} = 107.79$$

For Bond Y, the "no=arbitrage" price is:

$$\frac{7}{1.08} + \frac{7}{(1.08)^2} + \frac{7}{(1.09)^3} + \frac{107}{(1.094)^4} = 92.53$$

Since Bond X is priced slightly *below* the "no-arbitrage" price, while Bond Y is priced *above* the "no-arbitrage" price, Bond X is preferred.

Question #5

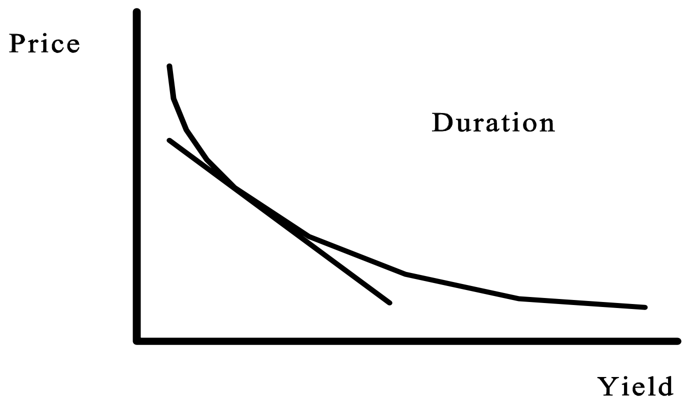
Part A

Per cent and dollar price changes:

a) % change from duration = - (6.7/1.05) X (0.005) X 100 = - 3.19%
b) % change from convexity = (0.5) X (55.5) X (0.005)2 X 100 = 0.07%
c) Total % change = 3.12%

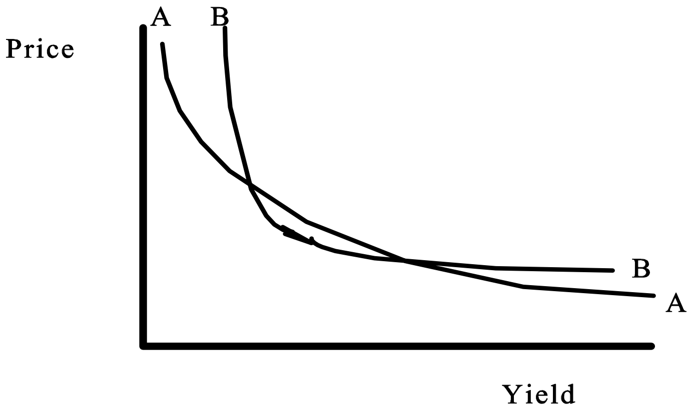
PART B

As shown in the figure, the price - yield relationship is convex, whereas duration assumes change along a straight line tangent to the curve at the prevailing yield. This will overestimate price decreases and underestimate price increases.



PART C

The figure shows bonds of different convexity.



For a given change in yield, bond B will exhibit a smaller price drop (or a larger price gain) than will bond A.

PART D

A Callable bond can be decomposed into a bond and an option. As the yield decreases to the point that the bond price begins to approach the call price, the possibility of call will impose an upper bound on the bond price. The price - yield relationship for a callable bond will depart from the price - yield relationship for a noncallable bond, as shown in the figure.

PART E

The effective duration can be computed as:

effective duration = % price change / % yield change

effective duration = [(110.0 - 109.1)/(109.6]/[(0.084 - 0.086)/(1.0425)

effective duration = 4.28 years

Question #6

PART A

The concept of arbitrage is the idea that if equivalent assets are priced differently, it is possible to make a riskless profit by simultaneously selling the expensive asset and purchasing the equivalent but cheaper asset. Such situations should not last long, which is the basis to the “Law of One Price.”

PART B

The concept of arbitrage and the law of one price leads to the "cost-of-carry" or "spot-futures parity" relationship:

Futures Price = Spot Price + Cost-of-Carry - Income

Buying a future and buying the underlying asset both result in holding the asset at the expiration of the future, so that buy and hold is equivalent to the purchase of a future. Equivalent assets must have equivalent prices, since arbitrage is possible if they are different. The "price" or "cost" of buying and holding a security is the immediate purchase price or spot price, plus the costs (e.g., storage and insurance) of holding the asset, less any income received.

PART C

Violation of the "cost-of-carry" condition arise if:

a) Futures Price > Spot Price + Cost-of-Carry - Income, or
b) Futures Price < Spot Price + Cost-of-Carry - Income

The general rule in any arbitrage is to sell the relatively overpriced asset while simultaneously buying the underpriced asset for delivery. In condition a), this would indicate selling the future and buying the asset. In condition b), this would indicate buying the future and selling (shorting) the asset.

PART D

In the case of securities, the "cost-of-carry" is the interest income foregone by buying and holding the asset. This gives the relationship:

$$\text{Futures Price} = \text{Spot Price}(1 + rt/360)$$

which can be solved for r, the implied repo rate:

$$r = [(\text{Future Price} / \text{Spot Price}) - 1](360/t)$$

If the risk-free rate is below r, you would like to lend at r, and you would do this by buying the security and selling the future.
If the risk free rate is above, r, you would like to borrow at r, and you would do this by selling(shorting) the security and buying the future.

Question #7

PART A

Since the prepayment option is similar to a call option, when interest rates decline prepayment can be expected to increase. This will have two consequences:

i) the price of the MBS will have less upside potential
ii) funds received from prepayments will be reinvested at a lower rate.

These consequences are referred to as “contraction risk.”

In a period of increasing yields, the prepayment rate will decline. Now the price of the bond will decrease, and the decrease will be made larger because the decrease in cash flow from prepayment effectively makes the maturity longer. These effects are referred to as "extension risk."

Part B

The effect on investment performance will depend on whether to bond was purchased at a premium or at a discount:

i) Discount: The prepayment is beneficial because there is a capital gain and because the funds will be reinvested at a higher rate than was being received on the MBS (remember, it was selling at a discount, indicating a lower than market coupon rate)

ii) Premium: for a bond purchased at a premium, both effects will be reversed.

PRACTICE EXAMINATION #2

LEVEL II

Question #1

Given the diagram, discuss the four quadrants:

	High Growth	Low Growth
High Leverage	**A**	**B**
Low Leverage	**C**	**D**

(10 minutes)

Question #2

Define the following indenture provisions:

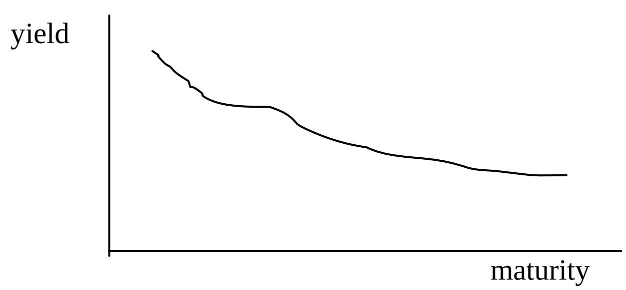
a) Negative pledge clause
b) Limitations on sale and leaseback
c) Dividend test

(10 minutes)

Question #3

A. Interpret the implications of the the yield curve at the right in terms of:

a) Pure expectations
b) Liquidity premium
c) Preferred habitat
d) Segmented markets



(10 minutes)

B. Interpret forward rates in terms of:
a) Pure expectations
b) Liquidity premium
c) Preferred habitat
d) Segmented markets

(10 minutes)

Questions 4 - 20 are multiple choice. 2 MINUTES EACH

4. Traditional credit analysis ratios:
   a) are primarily based on cash flows.
   b) have proven to be of limited use.
   c) are most useful in analysis of sovereign bonds.
   d) none of the above.

5. Municipal bonds:
   a) are tax-free.
   b) may be subject to state taxes.
   c) will typically have higher yield.
   d) are always credit-enhanced.

6. A restrictive indenture covenant:
   a) requires the firm to undertake certain actions.
   b) sets ROE goals for management
   c) requires bondholders to submit disagreements to binding arbitration.
   d) none of the above.

7. Corporate bond ratings:
   a) are meant as a guide to the attractiveness of investing in a bond issue.
   b) are based primarily on the volatility of rates in the particular sector.
   c) are based on the probability a firm will declare bankruptcy.
   d) indicate the likelihood of timely payment of obligations.

8. Which of the following is *not* one of the main influences on the shape of the yield curve mentioned by Ilmanen?
   a) bond risk premia.
   b) duration bias.
   c) market expectations.
   d) convexity bias.

9. Which of the following is *incorrect*?
   a) Convexity is more important when rates are low.
   b) Convexity bias leads to an increase in long rates
   c) Convexity is generally greater in longer bonds.
   d) Duration decreases with higher yields.

10. A bond with a duration of 7 years has a yield of 12.35%. If the yield changes to 12.72, the percent change in price estimated from duration will be:
   a) 2.44%
   b) 2.59%
   c) 3.7%
   d) 5.18%

11) You regress industry sales in thousands of units on GDP growth in %, and obtain the following output:

Constant	5.3594267
Std Err of Y Est	0.6384572
R Squared	0.6938571
No. of Observations	35
Degrees of Freedom	33
X coefficien(s)	0.298765
Std Err of Coef.	0.005473

   a) If GDP growth in the next period is estimated to be 3.5%, industry sales will be 6,405 units.
   b) The equation to estimates sales is: Sales = 0.298765 + 5.3594267(estimated growth x 100).
   c) The equation to estimates sales is: Sales = 5.3594267 + 0.298765(estimated growth x 100).
   d) None of the above.

12) Which of the following is correct?
   a) In the above regression, sales observations should have been adjusted for inflation.
   b) The above regression should have been based on GDP itself, rather than growth in GDP.
   c) The regression should be applied to every firm in the industry without adjustment.
   d) None of the above.

13) Spread is:
   a) wider for widely traded currencies,
   b) wider for less volatile currencies,
   c) narrower for longer forward contracts.
   d) none of the above.

The following exchange rate data applies to questions 14 – 16:

Currency	to Currency	Rate
A	B	2/1 (A/B)
B	C	1/3 (B/C)
B	D	1/2 (B/D)
A	C	2/3 (A/C)
A	D	5/4 (A/D)

14) The cross rate for A to C is:
   a) 1/1
   b) 2/3
   c) 3/2
   d) none of the above.

15) The triangular arbitrage opportunity is:
   a) A to B to C to A
   b) A to B to D to A
   c) C to B to A to C
   d) none of the above

16) The arbitrage profit is:
   a) 33%
   b) 133%
   c) 25%
   d) none of the above.

17) The exchange rate between A and B is 3A/1B. If the expected annual inflation rate in A is 3%, while the expected annual inflation rate in B is 10%, what is the futures rate implied by purchasing power parity?
   a) 0.9634
   b) 2.8091
   c) 3.2039
   d) none of the above.

18) We observe that the existing exchange rate is 1.27853 X/Y, while the futures exchange rate is 1.47923 X/Y. Which of the following is (ceteris paribus) *inconsistent* with this observation?
   a) Interest rates in Y are higher than interest rates in X.
   b) Inflation is expected to be higher in Y than in X.
   c) Y is expected to have a trade surplus with X.
   d) none of the above.

19) Which of the following is true?
   a) Economics is the art of stating the obvious in incomprehensible terms.
   b) Economists are good with numbers but lack the personality to become accountants.
   c) If all of the economists in the world were laid end to end, they would still point in different directions.
   d) All of the above.

SUGGESTED ANSWERS
PRACTICE EXAMINATION #2

Question #1

Quadrant A: This is where high growth firms sometimes wind up, but the problem is that this will limit their financial flexibility. The borrowing rate will also be high. They would be better advised to be in or move toward quadrant C, to the extent possible.
Quadrant B: This is the likely location of low-growth firms. These firms are likely to have higher and less volatile cash flow to support high leverage, and the higher leverage will help ROE.
Quadrant C: This is the preferred location for high growth firms, but they are often cash-starved and may move toward quadrant A.
Quadrant D: This is not the optimal location for low growth firms, unless there are other factors which limit the advisability or availability of financing.

Question #2

a) A negative pledge clause limits the use of a particular asset for collateral.
b) Sale and leaseback is a form of financing, in which an asset is sold and immediately leased back by the firm. A limitation on sale and leaseback prevents this.
c) A dividends test limits the amount of dividends a firm can pay, dependent on stated ratios.

Question #3

A. a) Based on pure expectations, short-term yields are expected to decline in the future.
b) Based on liquidity premium, we would expect that short-term yields will decrease in the future, and the decrease will be larger than that indicated by pure expectations alone.
c) Based on preferred habitat, the supply of longer-term loanable funds is larger, relative to the demand for long-term funds, than is the supply of short-term funds, relative to the demand for short-term funds.
d) Based on segmented markets, the interpretation is the same as the interpretation for preferred habitat. The difference is that under preferred habitat, some borrowers will move to shorter maturities, while some lenders will move to longer maturities.

B. a) Based on pure expectations, the forward rate is the expected rate over the forward period.
b) based on the liquidity premium, the forward rate is the expected rate over the period *plus* a premium (i.e., it is an overestimate of the expected rate over the period).
c) Based on preferred habitat, the forward rate reflects the supply and demand for loanable funds with maturities at the forward period.
d) Bases on segmented markets, , the forward rate reflects the supply and demand for loanable funds with maturities at the forward period.