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Hurricane Return Period Estimation

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1. Introduction

Return periods capture the essence of uncertainty in extreme meteorological phenomena (storm surge, wave, and wind) associated with hurricanes. Unfortunately, a detailed historical record of hurricane activity in the Atlantic Basin is only slightly more than one hundred years, making return period estimation of "storm-of-the-century" effects problematic. The purpose of this paper is to describe statistical methodology to address the problem of return period estimation. Initially conceived in a project for the Organization of American States (Montego Bay, Jamaica), the methodology was subsequently refined and expanded in the context of various consulting projects and a large contract with the Division of Community Affairs for the state of Florida involving hazard mitigation. Output from our work includes both maximum likelihood estimates of extremes and their associated uncertainties. Hence, our approach can provide rational assessments of the uncertainty that is more technically sophisticated than the commonplace but extremely conservative method of inflating the Saffir-Simpson category.

In addition to outlining the methodology, the presentation will demonstrate applications to the full North Atlantic basin with GIS-based output products and extensive validation/verification tests. An additional benefit of the approach is that it affords us the opportunity to assess directly the impacts of El Nino and la Nina by partitioning the historical hurricane record appropriately. Other phenomena outside the Atlantic Basin presumed to impact hurricane frequency and intensity are characterized according to their impacts on the return period estimates and corresponding uncertainty. Overall, the methodology offers a sounder statistical basis than the conventional scan radius method.

2. Return Period Estimation

The statistical methodology for return period estimation and attendant confidence and prediction limits has been detailed by Johnson and Watson (1998). We summarize the basic steps in this section. Following Simiu and Scanlan (1996), return period is defined as the reciprocal of the probability of observing a specific hurricane effect (or more extreme effect) in a single year. The method used at an individual site for a single phenomena (wind, wave or storm surge) is applied at every site in a study region. Hence, it suffices to explain the methodology for the generic situation in which we have 112 years (1886-1997) of annual maxima at a site, say $x_1, x_2, \ldots, x_{112}$. The steps are given, as follows:

1. Compute the maximum likelihood estimates (MLEs) of the shape parameter α and the scale parameter β, where the Weibull density function f(x) is proportional to $x\alpha^{-1} \exp[-(x/\beta)\alpha]$. To do so, the following equation is solved for α :

$$n/\beta - n \ln \beta \, \Sigma \ln x_i - \Sigma (x_i/\beta)\alpha \ln (x_i/\beta) = 0, \text{ where}$$

$$\beta = [(\Sigma x_{i\alpha}) / n]^{1/\alpha}.$$

2. Point estimates of return period values are computed by evaluating the inverse distribution function at the MLEs. For an n-year value, take p = 1 – (1/n) and evaluate

$$x_{1-p} = [-\beta \ln(1-p)]^{1/\alpha},$$

where the parameters represent the computed MLEs.

3. Compute the asymptotic variances and covariances of the MLEs using the observed Fisher information matrix. This involves evaluating the following three quantities in terms of the MLEs:

$$\partial^2 \ln L / \partial \alpha \partial \alpha = - n/\alpha^2 - \Sigma (x_i/\beta)\alpha [\ln (x_i/\beta)]^2$$

$$\partial^2 \ln L / \partial \alpha \partial \beta = - n/\beta + (\alpha/\beta) \Sigma (x_i/\beta)\alpha \ln (x_i/\beta) + (1/\beta) \Sigma (x_i/\beta)\alpha$$

$$\partial^2 \ln L / \partial \beta \partial \beta = (n\alpha/\beta^2) \{ 1 - [(\alpha+1)/(n\beta\alpha)] \Sigma x_{i\alpha} \}$$

where ln L is the logarithm of the likelihood function and all summations proceed from i equal one to n (n equals 112 if the full HURDAT data set is being used).

4. Compute the observed Fisher information matrix D as the inverse of the symmetric 2x2 matrix, as follows:

$$D = \begin{bmatrix} \partial^2 \ln L / \partial\alpha\partial\alpha & \partial^2 \ln L / \partial\alpha\partial\beta \\ \partial^2 \ln L / \partial\alpha\partial\beta & \partial^2 \ln L / \partial\beta\partial\beta \end{bmatrix}^{-1}$$

5. For the simulation step given shortly, we need as preliminary calculations:

$$\sigma_1 = \sqrt{D_{11}}$$

$$\sigma_2 = \sqrt{D_{22}}$$

$$\rho = D_{12} / (\sqrt{D_{11}} \sqrt{D_{22}})$$

where D_{ij} is the $(i, j)^{th}$ element of the matrix D.

6. To obtain 90 percent prediction limits for an n-year return period, a simulation scheme is used:

   - Generate independent normal variates X and Y and a uniform variate U.
   - Compute

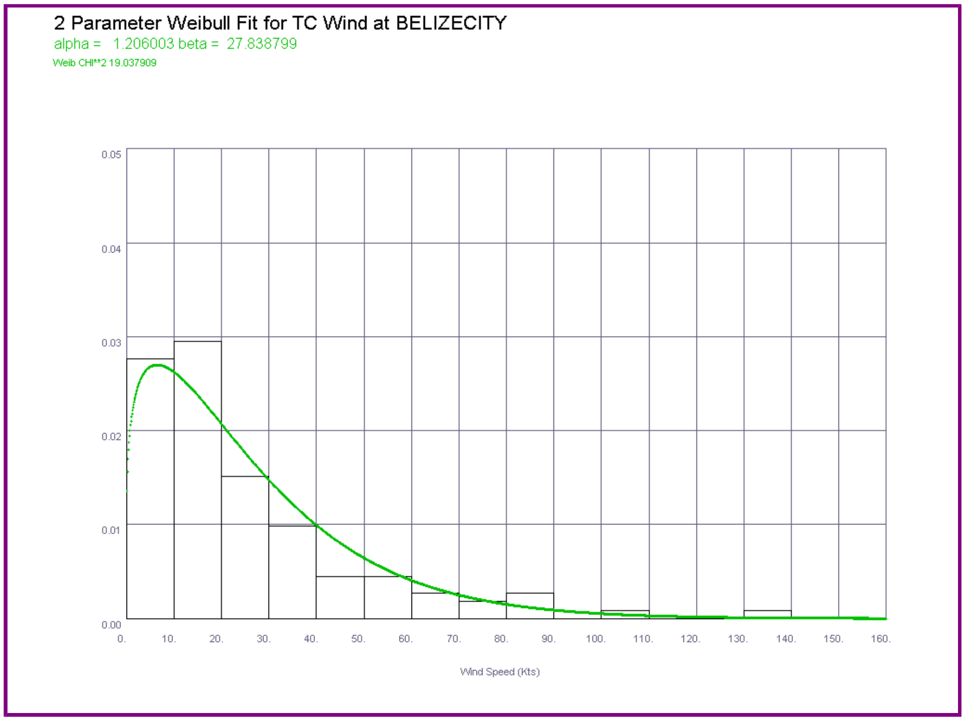
$$V = \sigma_1 X + \alpha$$
$$W = \sigma_1 [\rho X + \sqrt{(1-\rho^2)} Y] + \beta$$
$$R = W [- \ln (1 - U^{1/n})]^{1/V},$$

   where α and β are the MLEs. Repeat this simulation process a large number, say N times (1000 should suffice for up to 99% prediction limits), storing the generated R values. Sort them in ascending order and the $(0.90 \times N)^{th}$ item is the simulated 90% prediction limit.

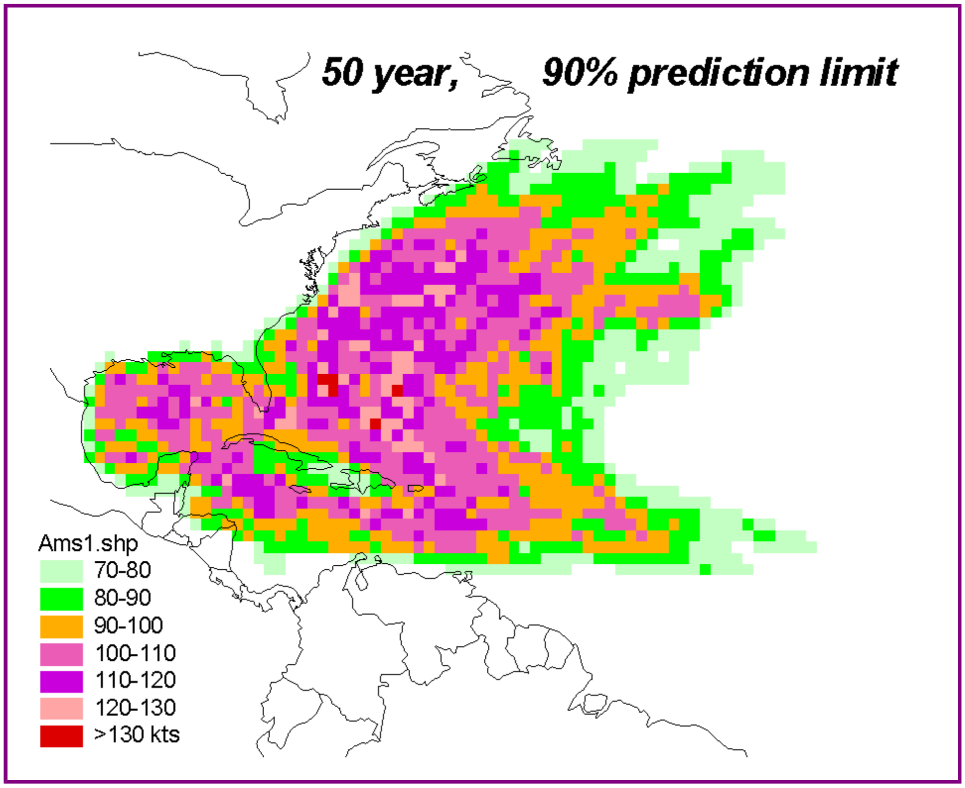
Further details on implementation of the above steps can be found in Johnson and Watson (1998).

3. Sample Applications

To illustrate the methodology of Section 1, consider the location Belize City, Belize. Figure 1 provides a plot of the fitted Weibull distribution overlayed with a histogram of the annual maximum winds.



Of greater interest may be basin wide results. Figure 2 gives upper 90% prediction limits for 50 year return period winds. Additional applications of the return period methodology can be found in Johnson and Watson (1998).



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