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THE ECONOMICS OF AUTOCRACY AND MAJORITY RULE:
The Invisible Hand and the Use of Force

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Martin C. McGuire and Mancur Olson, " The Economics of Autocracy and Majority Rule"

Develops formal models of tax policy and public good provision under autocracy, democracy, and an idcalited society in which there is a consensus about the distribution of income. These societies are examined in relation to the interaction between how the social order enables the society to obtain the benefits of social cooperation through the provision of public goods and how the gains from such cooperation are shared through the society's arrangements for the distribution of income.

Even an absolute autocrat, because he bears a substantial part of the social loss that occurs because of the incentive-distorting effects of taxation, will limit exactions from the population. Likewise, control of tax receipts also gives the autocrat a significant share of any increase in society's production and, as a result, an incentive to provide public goods. In a democracy with a unified majority and no institutional constraints against exploiting the minority, the majority also has an incentive to limit redistribution to itself and to provide public goods, because it bears much of the deadweight loss from redistributive taxation and receives much of the benefit form the provision of public goods. If a ruling interest is sufficiently encompassing, there is no redistribution whatever; those with no power are treated fully as well as those with total power, and the allocation of resources is the same as in the idealized consensual democracy. Thus non-redistributive regimes incorporate all marginal social benefits and costs into their fiscal decision-making.

A redistributive majority must have a lower tax rate and a larger share of social output than an autocrat. Redistributive regimes ignore a portion of the social benefits of providing public goods and reduce the marginal productivity of the public good, and this reduction is greater the more they redistribute, reducing the amount of public goods that are provided. The more a regime redistrlbures, the less of the public goods is provided.

An autocrat will tax more than will a democratic regime. An unexpected result is that non-redistributive societies may tax so much less than would an autocrat or redistributive majority that they would also provide less of the public good than would these two alternative regimes.

Before this century, it was widely feared that democracy with universal suffrage would inevitably lead to the abolition of private property, since it was considered obvious that a majority would gain from confiscating the property of the wealthy. The present argument suggests that the citizenry with less than median levels of income have an encompassing interest in society: they earn a significant percentage of the national income in wages and, when they control the tax and transfer system of society as well, this gives them a large stake in the productivity of society. If the deadweight losses from the elimination of private property are substantial, the part that would be borne by a low-income majority gives it an incentive to limit confiscation. The present analysis is also consistent with arguments that a "hard" state -- one that does not alter its agenda because of pressures from particular industries and occupations -- is favorable to development.

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by

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INTRODUCTION

Suppose that the leader of a group of roving bandits in an anarchic environment can seize and hold some territory. If the bandit leader becomes a settled ruler with a definite and secure domain. it will pay him to limit the rate of his theft and to provide a peaceful order and other public goods in that domain. In an anarchic environment, there is little incentive for people to invest or produce and. therefore. not much for bandits to steal. But, by choosing a rate of tax-theft that leaves a considerable incentive for his victims to produce and by providing a peaceful order and other public goods, the settled ruler can bring about a great increase in output, and out of this increase obtain more resources for his own purposes than from roving banditry.

This rational monopolization of theft also leaves the bandit's subjects better off: they obtain the increase in income not t&en in taxes. The bandit leader's incentive to avoid confiscatory taxation and tn provide public goods arises because he has an "encompassing interest" in his conquered domain. As the monopoly tax-collector, he bears a substantial part of the social loss that occurs because of the incentive-distorting effects of his taxation, and we prove in this paper that this limits the rate of his tax theft. His control of tax receipts also gives him a significant share of any increase in the society's production and, as we shall here demonstrate, this gives him an incentive to provide public goods. In short, an "invisible hand" gives a roving bandit an incentive to make himself a public-good providing King.[1]

The same invisible hand also influences democratic societies. Suppose the majority in control of a democracy is unified and that no institutional constraints keep it from taking income from the minority for itself. We prove in this paper that the majority in such a situation. even if it has no concern whatever for the minority, necessarily has an incentive to limit any redistribution from the minority to itself and to

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[1] On the other hand, when a rational self-interested autocrat, whether out of fear that he will soon lose power or for any other reason, has a short planning horizon, it pays him to seize all assets whose tax yield over his short time horizon is less than their capital value. That is, his incentive, whenever he has a sufficiently short time horizon, is to *become,* in effect, a roving bandit.

provide public goods for the whole society. This is because it bears much of the deadweight loss from redisuibutive taxation and receives much of the benefit from public goods.

These elemental incentives facing autocrats and majorities have not been seriously addressed -- and certainly not analyzed formally -- in the economics literature. There is, in other words. an unfortunate gap in that literature: most economics takes it for granted that the parties that interact. however much they vary in wealth and in other ways, cannot or will not use coercion to attain their objectives.

But, as Hirshleifer (1994) has pointed out, the same rational self-interest economists usually assume implies that actors with a sufficient advantage in employing violence will use that power to serve their interests: there is also a "dark side to the force. " Economists have not given nearly as much attention to this implication of self-interest as they have to the social consequences of self-interested interaction in peaceful markets. Economists have, of course, analyzed the incentive to use force in conflicts among nations (for example, in Schelling, 1960 & 1966) and with respect to crime and punishment (such as in Becker and Landes, 1974). They have also known that a nation cannot obtain sufficient public goods through voluntary market action and that compulsory taxation power is accordingly a necessary condition for Pareto-efficient provision. They have also observed that those who control or influence the coercive power of government often redistribute income to themselves.

Yet economists have not asked whether those who have coercive power, whether through control of government or by other means, have an incentive to exercise this power in ways partly or wholly consistent with the interests of society and of those subject to this power. Here we shall demonstrate an affirmative answer – that whenever a rational self-interested actor with unquestioned coercive power has an encompassing and stable interest in the domain over which the power is exercised, that actor is led to act in ways that are, to a surprising degree, consistent with the interests of society and of those subject to that power as if the ruling power were guided by a hidden hand no less paradoxical for us than the invisible hand in the market was for people in Adam Smith's time. In fact, when an optimizing entity with coercive power has a sufficiently encompassing interest -- what we define as a super-encompassing interest -

- the invisible hand will lead it, remarkably, to treat those subject to its power as well as it treats itself.

Our models in this paper formalize and extend some of the analyses by Olson (1990) in "Anarchy, Autocracy, and Democracy" later published as "Autocracy, Democracy, and Prosperity" (Olson. 1991), and by McGuire and Olson (1990), together with closely related with analyses of McGuire (1990. 1991), McGuire and Olson (1992), and Olson (1993). We have drawn inspiration from an ethnographic account (Banfield, 1958), an historical example (Sheridan, 1966), and from classics such as those by Hobbes (1651). Ibn Kalduhn (1377), and Schumpeter (1991). Our analysis resonates with earlier analyses of anarchy and the emergence of government (Tullock, 1974). Though our models contradict accounts like those of North (1981), Kiser and Barzel (1991), and Barzel(l993) that feature voluntary exchange and the transactions costs of such exchanges in explaining the origin of government, we underline the value of that line of economic history. The empirical work of DeLong and Shleifer (1993) tends to corroborate the theory we offer. The analysis here emerges partly from the concept of the "encompassing interest" (Olson, 1982), which has also been developed and applied most notably by Cahnfors and Drifiill (1988), Heitger (1987), and Summers, Gruber, and Vergara (1993).

We shall develop formal models of both autocratic and democratic (or, more generally, non-autocratic) government. This will make it possible to compare outcomes of autocracy with various types of democratic and semi-democratic government. In addition to relatively realistic models of autocracy and redistributive democracy, we also develop a purposely idealistic model of a society with consensus about its distribution of income and with each individual paying a Lindahl tax share.

Each of our model societies constitutes a distinct "social order. " One fundamental feature of every social order is that it enables a society to obtain the benefits of social cooperation through the provision of public goods. The other fundamental feature of any social order is that it determines how the gains from social cooperation are shared through its arrangements for the distribution of income. There are also two intimately related but logically distinct interactions between these fundamental features of every social order: that is, two distinct ways in which the redistribution of income and provision of public goods interact. This

paper shows *how* these previously unnoticed interactions affect outcomes in all types of societies.

I

THE PRODUCTIVITY OF PUBLIC GOODS AND DISTORTIONS FROM TAXATION

I.A. The Productivity of Public Goods

In our models, all public goods are in effect public factor inputs or intermediate goods that increase the output of private goods in the society. Accordingly, we can. with the notation set out below, specify an aggregate production function for a society in which total output is a function of the level of provision of public goods. Total output is a flow and so is the provision of the public good. No regime augments its immediate receipts at the expense of the future by confiscating capital goods: this is excluded either by indefinitely long time horizons or, alternatively, by assuming that there are no capital goods.

G = Amount of public factor input (price = 1);
Y = Potential gross private good production;
Y ▪ G = Potential net private good production: and
Y = Y(G).

Y(G) shows the maximum level of national product that can be generated by the labor and other resources in the society in cooperation with G units of the public factor input. Society's entire output is aggregated into the single good Y, which includes all income of everyone. Y is labelled "gross" because the cost of the resources that must be used to produce G has not been subtracted; it is labelled "potential" product because it omits the losses from incentive-distorting taxation, including the taxation necessary to obtain the resources for producing G.

The significance of the definition of "gross potential income" is evident when we make the utopian assumption of lump-sum taxation. Then with no deadweight loss from taxation, potential gross income, Y, is also realized or actual gross income. Since the public good in our models has no direct consumption value, a rational society would not maximize gross product but rather product net of expenditure on the public good. If we assume not only lump-sum taxation but also consensus that the existing income

distribution is right, then this Utopia will have a unique social optimum. such as depicted in Figure 1.

With a price of 1. the total cost. C. of providing G is just the straight line indicated by C(G) = G in Figure 1. The maximum net product available is given by Y(G) - G. At the socially optimal provision of G, the marginal product of G equals its marginal cost, $Y' = 1$. And under lump-sum taxation. society uses the absolute minimum amount of resources necessary to provide G^*, i.e. $C(G^*) = G^*$ and the citizenry enjoys a net income of $Y(G^*) - G^*$.

I.B. Distortions Caused by Taxation

Since no society can rely on lump-sum taxation. the challenge for our analysis is to take account of the deadweight losses from taxation and the productivity of public goods at the same time. We assume that all resources available to government, whether for public good provision or for redistribution, are derived from taxation. Keeping to the simplest possible assumptions, we suppose that taxes are applied at constant average rates on gross income. We use the following notation to capture these ideas:

t	=	constant average "income tax" rate
$r(t)$	=	% of potential Y produced for given t; r(t) is the same for all G; $r' < 0$, $r(0) = 1$.
$1-r(t)$	=	% of Y lost when tax is imposed, i.e. pure efficiency loss. Let us call 1-r(t) the "deadweight loss function."
$u(t)$	=	% of potential Y collected in taxes
$(1-t)r(t)$	=	% of potential Y not taken in taxes
$r(t)Y \equiv I$	=	actual or realized income; if taxation did not distort incentives, Y = I.

An example of these relationships is shown in Figure 2. Though r(t) is depicted as linear, it is more likely, because deadweight losses from taxes presumably rise faster than tax rates, to be convex from above.

Since real-world **regimes,** in contrast with the utopia depicted above, have incentive-distorting taxation (i.e., $r < 1$), the production function must be stated in terms of actual income, I, where $I(G,t)$.

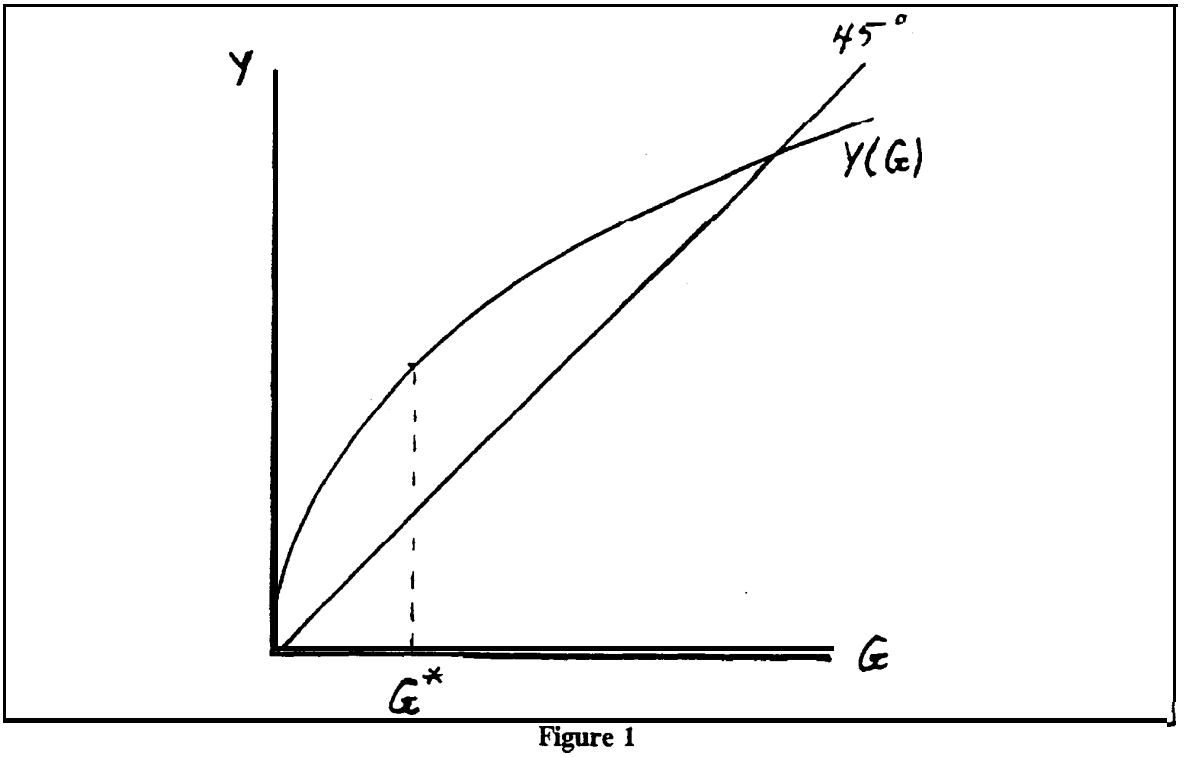

Figure 1

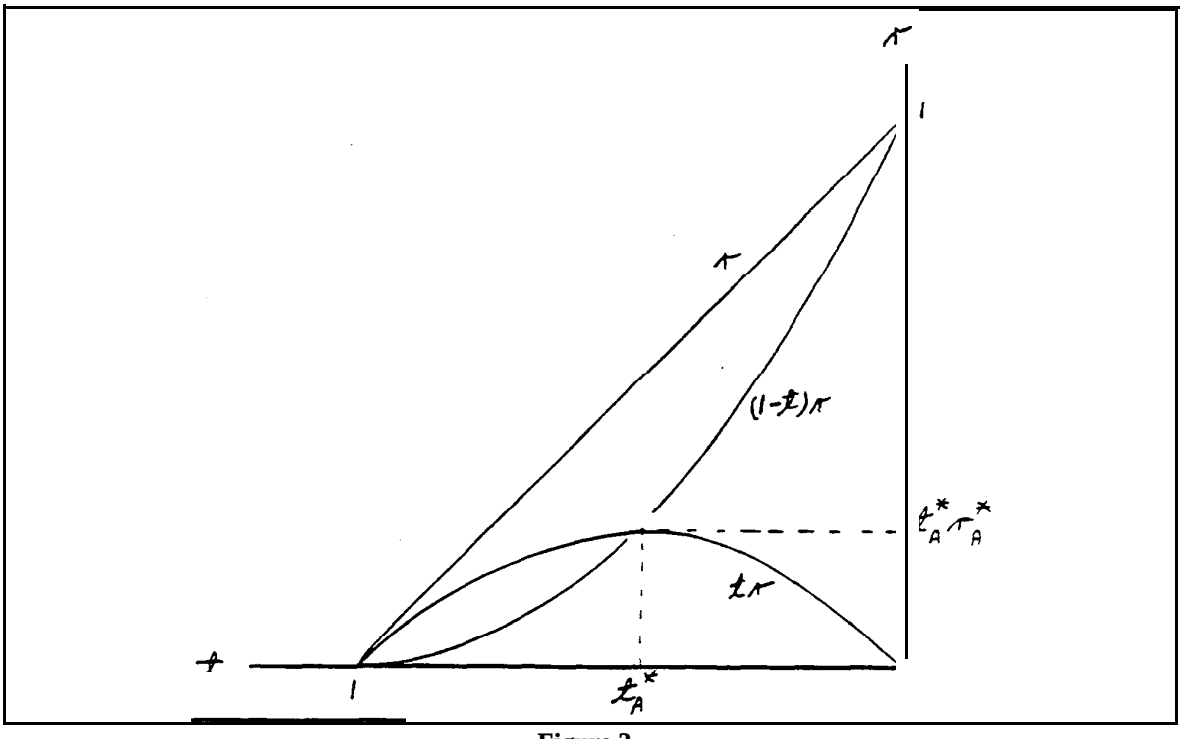

Figure 2

Impartially, we assume that the percent nf potential income lost due to the deadweight losses from taxation. at any given level of taxation, is the same across ail regimes: i.e.. ail face the same deadweight loss (DWL) function. (1 - r(t)). Similarly, all of our regimes are limited by the same production function. Y(G), and (except where we discuss de consequences of relaxing this assumption) all regimes face the same requirement to finance provision of G by proportional taxes at rate t.

II

THE AUTOCRAT'S TAX AND EXPENDITURE PROBLEM

A dictatorial ruler consumes not only the palaces and pyramids he may build for himself. but also the armies and aggressions that may iift him above the leaders of other governments. He is no more likely to have satiated all his wants than any other consumer. He obtains the resources to satisfy his objectives from the taxes he exacts from his subjects. (We assume he does not seil his labor or any other services in the market.) Because of his rational self-interest, he **extracts the maximum sustainable transfer from the society** -- &at is, he redistributes the maximum possible **absolutc** amount to himself without regard for the welfare of his subjects.

Paradoxically, the same self-interest that leads an autocrat to maximize his extraction from the society also motivates him to be concerned about the productivity of his society. First, his **monopoly**[2] over tax collection induces him to limit those taxes. When the deadweight loss from his taxation reduces the income of society enough at the margin so that his share of social income (i.e., his tax collection) also begins to decrease. he makes no further exactions. Thus a rational autocrat always limits his tax theft: he takes care

[2] This means that competition among autocrats for control over taxpayers (at least if they have no military power of their own) does not curtail the redistribution from taxpayers to autocrats; indeed, it reduces the welfare of taxpayers. Competition reduces the security and shortens the time horizons of rulers and makes them more nearly roving than stationary bandits.

not to increase his rate of taxation above the point where the deadweight losses at the margin are so great that his share of these losses offsets what he gains from taking a higher percentage of income. Second. the Autocrat will provide public goods because doing so increases his tax collections. If, for example, his tax rate is 50%. he will obtain one-half of any increase in national output brought about by provision of public goods. He therefore has an incentive to provide the public good up to the point where his marginal cost of providing it just equals his share of the increase in the national income. In short, as we will demonstrate next, me encompassing mterest or an autocrat. as measured by his chosen tax rate. leads him both to curtail redistribution to himself from the wider society and also to contribute public goods to that society, using in both cases the reciprocal of his tax rate as a governing mechanism for achieving his optimum.

These conclusions follow logically from postulating that the Autocrat solves this maximization problem to find his optimum:

$$\text{(1)} \qquad \max_{t,G} \; tr(t)Y(G) - G \; ; \qquad s.t. \; G \leq tr(t)Y(G)$$

The Autocrat must choose both the level of taxation and the level of public good provision to obtain an optimum. Since the level of provision of the public good, G, affects the level of income, it also affects tax receipts. At the same time, the Autocrat's tax rate determines his share of any increase in income from the provision of more public goods. It would appear. then, that an autocratic ruler cannot choose his optimal level of public good provision without knowing the tax yield this would generate for him and that he cannot know his yield from a tax rate without knowing his optimal public good provision, and that there is no intuitively transparent solution.

II.A. The Autocrat's Optimum

In fact, there is a most simple solution. Though the yield from any tax rate obviously depends on the amount of G, the optimal tax rate for an autocrat does not. The ruler pockets all tax revenues beyond those he spends on G. Thus for any value of G whatsoever he wants to obtain as much product as possible

for his treasury. Just as an autocrat will receive no tax revenue from zero percent or 100 percent tax rates. whatever level of the public good might be available. so we assume that the tax rate that generates maximum revenues does not vary with the level of G. Since the constraint in equation (1) does not bind. the level of G affects the tax yield but not the optimal tax rate. The Autocrat can optimize simply by choosing t so as to maximize tr(t) and then choose G to maximize his rents. Accordingly, he first solves:

(2) $$\underset{t}{\text{Maximize}}\ tr(t)$$

This requires as a necessary condition

(3) $$r + tr' = 0$$

such that at the Autocrat's solution[3]

(4) $$t_A^* = -\frac{r(t_A^*)}{r'(t_A^*)}$$

and therefore the maximum value of the Autocrat's share of potential GNP becomes

(5) $$\text{Maximum Value of } tr(t) = -\frac{(r_A^*)^2}{(r_A^*)'}$$

where the "*" notation means the variable is evaluated at the maximum.

We can now see in a more intuitive way why an autocrat will limit the amount of redistribution to himself. The maximum of tr(t) must come when the effect of the fall in r on the Autocrat's revenues (i.e., tr'dt) just offsets the effect of the increase in t (i.e., rdt). The Autocrat bears t percent of the total deadweight loss that arises from the taxes he imposes to effect the redistribution to himself. Thus he will not gain from further redistribution to himself when the social loss as a proportion of actual income -- i.e., -

[3] The second order condition for t_A^* to give a maximum is that

$$d^2[tr(t)]/dt^2 = d\ [r + tr']/dt < 0$$

when evaluated at t_A^*. The second derivative works out as

$$2r' + tr' < 0$$

To evaluate this expression at the maximum of tr, we incorporate equation (3) above which gives

$$-2(r')^2 + r''r < 0$$

as the second order condition which must obtain at the autocrat's optimum.

$r'(t_A^*)/r(t_A^*)$ -- is the reciprocal of his chosen tax rate: as is clear from equation (4), when the total loss to society, as a proportion of actual income, $-r'(t_A^*)/r(t_A^*)$, reaches $1/t_A^*$, the Autocrat makes himseif worse off by taking more. We shah later see that a simple reciprocal relationship such as this characterizes all redistributive taxation.

Having made this decision on t independently of his decision on G, the Autocrat then goes on to choose the right amount (for him!) of G. To find this he uses his previous choice of t to maximize the absolute amount of his surplus:

$$\underset{G}{\text{Maximize}} \{ [t_A^* r_A^*] Y(G) \} - G \tag{6}$$

This requires

$$Y'(G) = \frac{1}{t_A^* r_A^*} \tag{7}$$

Because of incentive-distorting taxation, this society (the Autocrat and his subjects) does not reaiize its potential income, Y, but instead obtains an actual income of $rY \equiv I$. So, in tcrms of actual income I,

$$[r_A^*] Y'(G) \equiv I'(t_A^*, G) = \frac{1}{t_A^*} \tag{8}$$

This condition states that the Autocrat stops providing public goods when the marginal increase in society's actual realized income from public goods equals the reciprocal of his share of the national income. As we know, the Autocrat curtailed his redistribution to himsclf whcn the proportionate social loss, $-r'(t_A^*)/r(t_A^*)$, was also equal to $1/t_A^*$. Thus the same reciprocal rule applies to both margins because the same linear tax rate determines the extent of the Autocrat's interest, and therefore his share of both the benefits from the public good and the losses from redistributive taxation.

For the sake of a simple example, suppose that the optimal tax rate for an Autocrat is 2/3rds. At this optimum the proportionate social loss from the Autocrat's redistribution to himself, $-r'/r$, is therefore l/t or 3/2's. Then the Autocrat also provides the public good -- with a marginal cost to him of 1 -- up to

the point where its marginal social product ($rY' \equiv I'$) is 3/2's as great as his marginal cost. For the Autocrat (who gets 2/3rds of society's actual product in taxes) his marginal benefit of the last unit of public good is just equal to the marginal cost he must pay; 2/3 times 3/2 = 1.

Returning to equations (7) and (8) and substituting from equation (4), we obtain two more useful relationships:

(9)
$$Y'(G) = -\frac{[r_A^*]'}{[r_A^*]^2} \equiv Q(t_A^*)$$

(10)
$$I'(t_A^*, G) = -\frac{[r_A^*]'}{r_A^*} = P(t_A^*)$$

At $t = t_A^*$, Q necessarily slopes upward,[4] but this does not have to be true of P. Elsewhere $Q(t_A)$ may be increasing or decreasing. But whenever $r'' \leq 0$ both curves will be upward sloping.[5]

We are now in a position to show in a remarkably simple way in one figure how all the optimizing conditions of the Autocrat are simultaneously satisfied and at the same time depict the level of output of the society -- **and also its distribution between the Autocrat's consumption, the subjects' consumption, and the** expenditure on the public good -- plus the extent of deadweight losses. The second quadrant of Figure 3 depicts the choice of optimal t for an autocrat. The product tr(t) is shown as beginning at zero **at the origin,** rising to a maximum and falling off again as t increases. For the reasons explained earlier, a different level of public good provision and income imply a different $tr(t)Y(G)$ product, but the tax rate at which this

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[4] **Differentiating Q gives:** $dQ/dt = [-rr'' + 2(r')^2]/[r]^3$. **Then it follows from the second order condition derived** in the last footnote that in the vicinity of the autocrat's optimum, t_A^*, $dQ/dt > 0$

[5] Marginal deadweight losses from taxation could be U-shaped. The marginal losses could have a high value at low tax rates, then decline as the tax rate increases, and then increase at still higher tax rates. In this case the function **Q(t)** may have a range over which it declines followed by a range over which it increases. The necessary/sufficient condition for Q(t) to decline is $-2(r')^2 + r''r > 0$ or $r''r > 2(r')^2$. The function $r(t) = [c/(c + t)]$ with c any arbitrary constant has the property $r''r = 2(r')^2$. Thus any deadweight loss function with a more positive local second derivative will generate a locally declining Q(t). Since Q(t) must increase in the neighborhood of the Autocrat's optimum, such behavior can only be local.

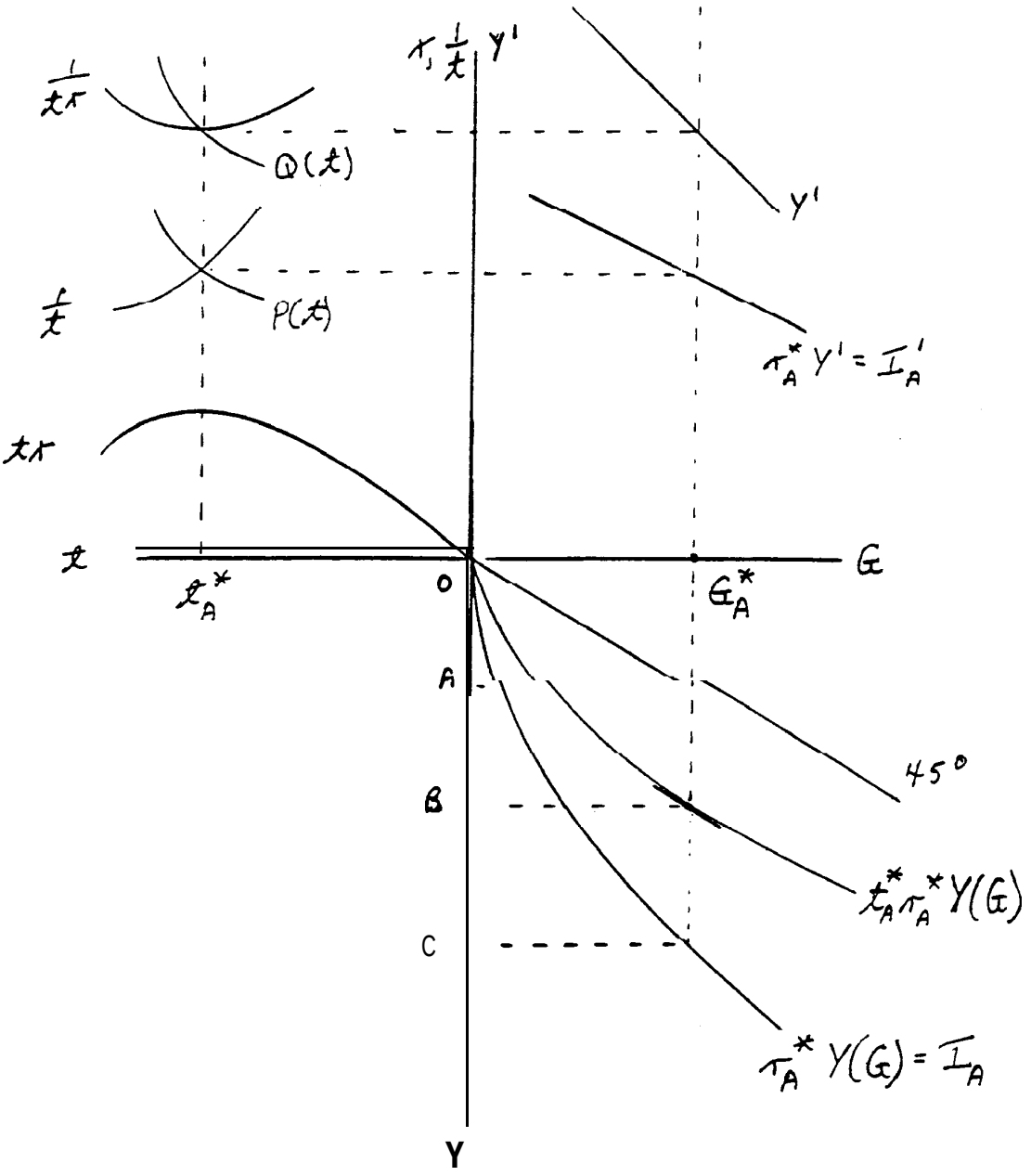
$\frac{1}{t\tau}$
$Q(t)$
$\frac{1}{t}$
$P(t)$
$t\tau$
t
t_A^*
$\tau, \frac{1}{t}$
Y'
Y'
$\tau_A^* Y' = I_A'$
G
G_A^*
O
A
B
C
$45°$
$t_A^* \tau_A^* Y(G)$
$\tau_A^* Y(G) = I_A$
Y

Figure 3

function reaches a maximum is not **affected.** Accordingly. the Autocrat chooses the value oft corresponding to the maximum on u(t), which is also the point where $1/t = -r'^{*}_{A}/r^{*}_{A}$. At the Autocrat's optimal tax rate. t^{*}_{A}, the percentage of potential output realized is r^{*}_{A}, the percentage lost because of efficiency distortions of taxation is $(1-r^{*}_{A})$, and the Autocrat gets his maximum share of income, $t^{*}_{A}r^{*}_{A}$.

Now consider the points directly above the optimal tax rate. From equations (7) and (9), 1/tr and Q(t) at the Autocrat's optimmn must equal Y' and from equations (8) and (10) 1/t and P must equal I'. The first quadrant shows the functions Y' and $r^{*}_{A}Y' \equiv I'$ together with their values at the Autocrat's optimum. We see that an autocrat provides G up to G^{*}_{A} where its marginal product. i.e. $r^{*}_{A}Y'(G)$, equals the reciprocal of his share of the national income. l/t.

Proceeding down, the fourth quadrant shows that the Autocrat equates the marginal cost of G, given by the slope of the 45 degree line and defmed to be 1, with the extra tax revenue he receives out of the increase in national income that additional provision of the public good brings about – shown by the slope of $t^{*}_{A}r^{*}_{A}Y' = tI'$. The Autocrat's tax receipts -- and the income of the society, $rY(G) = I(t^{*}_{A},G)$, -- would have been different had he chosen a different level of taxation, but the choice t^{*}_{A} has already been made: the optimum G depends on the optimum t but not <u>vice versa.</u> We can now see how the national output is used: the total output or income of the society is OC, of which, OA is spent on the public good, AB is the Autocrat's surplus, and BC is consumed by the subjects.

Returning to the first quadrant, the vertical distance between Y' and I' gives the reduction in the marginal productivity of the public good caused by the Autocrat's incentive-distorting taxation: if all his revenues had been raised by lump-sum taxes, r would have had the value 1 and Y' and I' would have been identical. This reminds us that, if the Autocrat had somehow **been able** to impose lump-sum taxation, the whole situation would have been different; he would have imposed higher taxes and therefore also provided more of the public good. There are also other non-linear tax schemes that could usefully be analyzed, but

we shall not introduce them here, because that would obscure the insights that can come from comparing different forms of government that share the same linear tax system.[6]

Because the Autocrat's taxation causes deadweight losses. it might be argued that there is some Coaseian contract between ruler and subjects that could benefit everyone. Even if we set aside the collective action problem the subjects would have to overcome to bargain for themselves, the Autocrat's interest in preventing his subjects from organizing, and the problem that the Autocrat would control the courts in which any contract could be enforced, this argument is wrong. There is no agreement. given the linear tax, that could make the subjects better off without leaving the Autocrat worse off. Given the constraints. our autocracy is Pareto-efficient.[7]

Though the citizens in our democratic models will enjoy higher post-tax incomes than the Autocrat's subjects, it is nonetheless remarkable how much the encompassing interest of the stable Autocrat leads him to take account of the welfare of his subjects. Our Autocrat has the same motivation as a bandit. Yet, if he has an enduring hold on his domain, an invisible hand leads him to cease redistributing to himself after a point because of the loss in social efficiency his taxation brings about. It also leads him to use some of the resources he collects in taxes to provide public goods that serve the whole society. Moreover, me larger the share of output that the Autocrat takes in taxes, the more encompassing his interest and the closer he

[6] Some recent autocracies have been able to work out complex schemes that, implicitly, came closer to the lump-sum ideal than our flat tax does. In effect me Soviet-type autocracies did this by setting pay for each skill group at a fraction of the market rate with the regime keeping the surplus through control of state enterprises, and combining this with bonus and progressive piece-rate schemes that made marginal income relatively tax free. Though, as is well known, these schemes generated other incentive problems and were not viable in the long run, they did enable the Soviet-type regimes to obtain a larger proportion of the national output for the Autocrat's purposes than any societies have done.

[7] Since we do nnt explicitly take account of the labor-leisure trade-off of the subjects, we cannot say how much their welfare would rise as the tax rate falls. Thus we cannot specify how much they would gain from a reduction in the tax rate in relation to what the autocrat loses. To be sure, if the subjects could organize and impose a lump-sum tax on themselves, they could compensate the autocrat for the loss in revenues from a reduction in the linear tax rate and still be better off. But this merely tells us that lump-sum taxes bring no efficiency loss; lump-sum taxes are no more available to the subjects than they are to the autocrat. Therefore, given the constraint of a linear tax, there is no deal that could make the taxpayers' better off without leaving the autocrat worse off.

comes to taking full account of the gains to society from the public good. This blessing of the invisible hand that we have described has surely been a major source of that substantial part of human progress that has occurred under autocracy.

II.B. The Marginal Private Cost of the Public Good

The Autocrat's choice of G. although dependent on the total deadweight loss of taxation (l-r), is independent of the marginal social deadweight loss. -r´, that his taxes impose. This is because an autocrat, in order to obtain a surplus to redistribute to himself, imposes a higher tax rate than me one that would just pay for his public good provision. Accordingly, in an autocratic social order the public good is financed out of infra-marginal tax receipts. Therefore. the marginal deadweight loss from taxation to finance G does not affect the Autocrat's marginal private cost. We see in the fourth quadrant of Figure 3. at G_A^* the Autocrat equates his private marginal cost of G (given by the slope of the 45 degree line) with the extra tax revenue that he receives out of the increase in national income that additional provision of the public good brings about, given by the slope of $tr(t)Y(G) = t_A^*I$. The marginal social cost -- the aggregate cost to the Autocrat plus his subjects -- of the social order (the public-good-plus-redistribution) that the Autocrat provides -- is dependent on his tax rate, but the marginal private cost of G to him is simply 1.

Just as an autocrat imposes a higher rate of taxation than necessary to finance the public good, so does any government that redistributes income. For any such government, G is financed out of infra-marginal tax receipts. Thus the marginal deadweight loss from the taxation needed to finance G does not affect the marginal private cost of G to any redistributive ruling interest. As we see later, contrary forces are also at work,[8] this factor by itself makes redistributive regimes provide more public

[8] Calculation of marginal social costs and benefits are discussed in more detail, and with reference to alternative regimes including Autocracy in section VI of this paper.

goods than otherwise.

III

A BENCHMARK SOCIETY: THE CONSENSUAL DEMOCRACY

Though it can be prejudicial to autocracy to judge it by the standards of our normatively ideal "Lindahl Consensus" democracy, it is nonetheless important to explore this consensual society next. As it happens. many of the realistic democracies generate allncatinns that fall in hetween the Lindahl Consensus society and the Autocracy. Others, remarkably, under a range of conditions. behave exactly as would the Lindahl Consensus society.

In our consensual democracy as in the autocracy, the public good is the social order. We assume that the society either began with -- or achieved through redistributions in the past -- a distribution of endowments that enjoys social consensus. In addition, each citizen pays a share of the cost of the public good that is exactly proportional to his or her share of the gains (marginal and average) from the social order. Because there is no demand in this society to change the distribution of income, we shall designate it with the subscript "N" for "non-redistributional." Though real societies have neither the naive honesty in preference revelation needed for Lindahl tax shares nor consensus about their income distributions. we abstract from these difficulties to examine public good provision in a Pareto-efficient society with no coercive redistribution of income. [9] As is well known, with perfect Lindahl tax shares, every voter wants the same, socially efficient amount of the collective good. [10] To maintain our assumption of a simple linear tax, we must also assume either that all individuals are identical, or alternauvely that the Lindahl tax shares do not change as more or less of the public good is provided.

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[9] This society is comprised of the same individuals as the autocracy, except that the Autocrat is just another individual. This assumption allows us to make welfare comparisons across regimes.

[10] When public good provision is too low (high), there is unanimous agreement to increase (reduce) it .

III.A. The Optimal Solution for the Lindahl-Consensus Society

The welfare of the Lindahl-consensus society depends on net or post-tax income. e society therefore maximizes:

(11a) $$W = \underset{t}{Max}\,(1-t)r(t)Y(G)$$

Public good expenditures cannot exceed tax revenues. It is feasible for the Lindahl Consensus to collect more taxes than needed to finance public goods and redistribute the surplus to itself. but because this society already has agreement about its income distribution, doing this would cause deadweight losses from incentive-distorting taxation for no purpose. Accordingly, the Lindahl-Consensus society will choose to collect no more in taxes than it spends on the public good. We can then treat the maximization of the Lindahl society as always proceeding with the constraint that tr(t)Y(G) • G = 0. This in effect determines G as a function oft: G = G(t). Because the society's choice oft implies a choice of G, and vice versa, we cannot partition its decision into two phases the way we did with the Autocrat. The consensual society chooses a tax rate such that, when ail tax proceeds are spent on G, the marginal social benefit of the tax as perceived by the consensual government just equals its marginal social cost as perceived by that government. Maximization of (lla), therefore, requires as a necessary condition

(11b) $$\underbrace{[(1-t)r(t)Y'(G)]\frac{dG}{dt}}_{\text{Marginal Post-Tar Benefits of } dt} + \underbrace{Y(G)[(1-t)r'-r]}_{\text{Marginal Post-Tax Costs of } dt} = 0$$

where as shown the first term represents the marginal after tax benefits to the Lindahl society from an incremental change in the tax rate t while the second term indicates the marginal costs due to a change in the tax rate.

An alternative way to characterize the consensual society is to focus on its optimal provision of G. To do this we calculate its income as its gross product reduced by the costs of G. This calls for formulating its social welfare maximization as:

(12a) $$U = \underset{G}{Max}([r(t[G])Y(G)] - G) \; ; \qquad s.t.\; tr(t)Y(G) - G = 0$$

Here the variable of choice is taken as G. with t = t(G) implicit from the constraint. Either of these formulations -- (11) or (12) -- is sufficient to solve the entire problem for the consensual society. But with (12a) marginal resource costs and marginal deadweight losses show up directly and explicitly. Thus the derivative of (12) with respect to G yields:

(12b) $$\underbrace{rY'}_{\substack{\textit{Marginal Pre-Tax}\\ \textit{Benefits of } dG}} + \underbrace{Yr'\frac{dt}{dG}}_{\substack{\textit{Marginal Pre-Tax}\\ \textit{Costs of } dG}} - 1 = 0$$

The marginal cost of G consists of the direct resource cost. given by the term just to the left of the equal sign, and the extra deadweight losses attributable to the additional taxation to finance G, given by the next term to the left. This equation' also shows, as would be expected, that the Lindahl-Consensus society takes account of <u>all</u> of the benefits of the public good (by contrast, the Autocrat's provision of the public good took account only of his share of the benefit, trY'). We shall later show that, whether it has consensus or not, <u>everv regime that abstains from redistribution necessarily takes account of all of benefits and costs of the public good to the society as a whole.</u>

III.B. Marginal Benefit-Cost Comparisons in The Consensual Society

It is clear from (12b) that the marginal costs of financing G themselves depend upon Y'(G). We see this when the constraint $tr(t)Y(G)=G$ is totally differentiated and solved for dG/dt[11] which is then substituted into equation (12b) to obtain the explicit equalization of marginal benefits and cost at the consensual optimum.

For later use let us define the two terms on the right of (13a) as MSC,. MSC, gives the marginal

[11] This gives: $dG/dt = -Y[r + tr']/[trY' - 1]$.

(13a)

Marginal Pre-Tax Benefits of dG	=	Marginal Resource Costs of dG	+	Marginal Pre-Tax Deadweight Losses of dG
rY'	=	1	-	$r'Y\left[\frac{1-tr'}{Y(r+t)}\right]$

social cost of the public good to societies that. like the Lindahl Consensus. are non-redistributional. Collecting like terms yields

(13b)
$$Y'(G) = \frac{r(t) - (1-t)r'(t)}{r^2} \equiv V(r(t),t)$$

as the relation between t and G that must obtain at the optimum. We use t_N^* and r_N^* to denote the solution values of t and r for the Lindahl Consensus and other non-redistributional societies. [12]

Because of incentive-distorting taxation in the Lindahl Consensus, $r < 1$, so potential income Y is not achieved and we observe instead $rY \equiv I$. Accordingly, the actual marginal product of the public good is $r[G_N^*(t_N^*)]Y'(G) = I'(t_N^*,G)$. When we multiply both sides of equation (13b) by r, we obtain

(14)
$$rY'(G) = I'(t_N^*,G) = r(t)V(t) = 1 - (1-t_N^*)\frac{[r_N^*]'}{r_N^*} = MSC_N$$

MSC_N stands for the marginal social cost of one dollar of resources to a non-redistributional society and includes all the effects identified as costs in Equation (13a). On the right side of equation (14), the 1 represents the resource cost of the public good. The expression $\{-[(1-t_N^*)r_N^{*\prime}/r_N^*]\}$ represents the marginal deadweight loss at the society's optimum[13] from the additional taxation needed to obtain the resources to produce another unit of the public good. Since r' is negative, the right side of equation (14) is necessarily greater than 1. As tax rates increase from $t = 0$, V(t) and $r(t)V(t)$ may increase or decrease depending on the specific shape of the deadweight loss function $1 - r(t)$. As taxes increase, however, r must get smaller, and unless there is a sufficiently offsetting reduction in the absolute value of r', MSC_N will rise with the tax

[12] Differentiating V(t) gives: $dV/dt = (1-t)\,[-rr'' + 2(r')^2]/(r)^3$. In the neighborhood of the autocratic maximum, t_A^* by the second order condition $dV/dt > 0$, and V(t) is upward sloping.

[13] The right side of equation (13a) gives a general expression for the marginal social cost for non-redistributional societies. The right side of (14) gives MSC, at the optimum where marginal benefits and marginal costs are equal.

rate.[14] Even if MSC_N does not rise with t, so long as the marginal social product of the public good is declining at a more rapid rate. the second order conditions[15] for a maximum will be fulfilled and the solution to (14) wiil still represent a maximum for the Lindahl society.

These relationships are shown in Figure 4. The second quadrant shows tax shares. tr(t), at each tax rate as before. For illustration V(t) and MSC_N are drawn as increasing throughour and the consensual society's t_N^* is assumed as shown. Above t_N^* we find the marginal social cost of the public good. $1-(1-t)r'/r \equiv MSC_N$, which includes the marginal deadweight losses from the taxation needed to fund the public good. Further up V(t) shows this same marginal cost in terms of potential income. Moving across to the first quadrant shows that actual marginal cost is equated to the actual marginal social product of the public good, I′. The corresponding match of the relevant values of V(t) and Y′ shows marginal costs and benefits in terms of potential income.

Reading down from I′, we observe on the horizontal axis the optimal quantity of the public good G_N^*. The fourth quadrant of Figure 4 then shows actual income $I(t_N^*,G)$, and tax collections $t_N^*I(t_N^*,G)$ as functions of G <u>given</u> that $t = t_N^*$.[16] In contrast with the autocrat, who took account only of his share of the benefit of the public good in deciding how much to provide, the Lindahl Consensus, as we see. equates the entire marginai social cost of the public good – including deadweight losses – to its total marginal social

[14] If the marginal deadweight loss function (ix. $d[1-r(t)]/dt = -r'$) continually increases (i.e. $-r'' > 0$) there is no offsetting decline in the absolute value of r'; but if as t rises the marginal deadweight loss function declines at first and then increases, then MSC, as well as V(t) can decline with increases in t.

[15] For this solution to represent amaximum the second order conditions require

$$d^2[(1-t)r(t)Y(G)]/dt^2 < 0 \; ; \qquad \text{s.t. } tr(t)Y(G) = G$$

Utilization of the expression for dG/dt, its derivative, and of Equation (13) simplifies this condition to

$$-2(r')^2 + rr'' + YY''[(r)^2/(1-t)]^2 < 0$$

Evidently, $r'' < 0$ may be sufficient but not necessary to insure a maximum.

[16] Note that $I(t_N^*,G)$ differs from $r[t(G)]Y(G)$ for $t \neq t_N^*$. Specifically I may be greater than, equal to, or less than r(G)Y depending on whether t is greater than, equal to, or less than t_N^*.

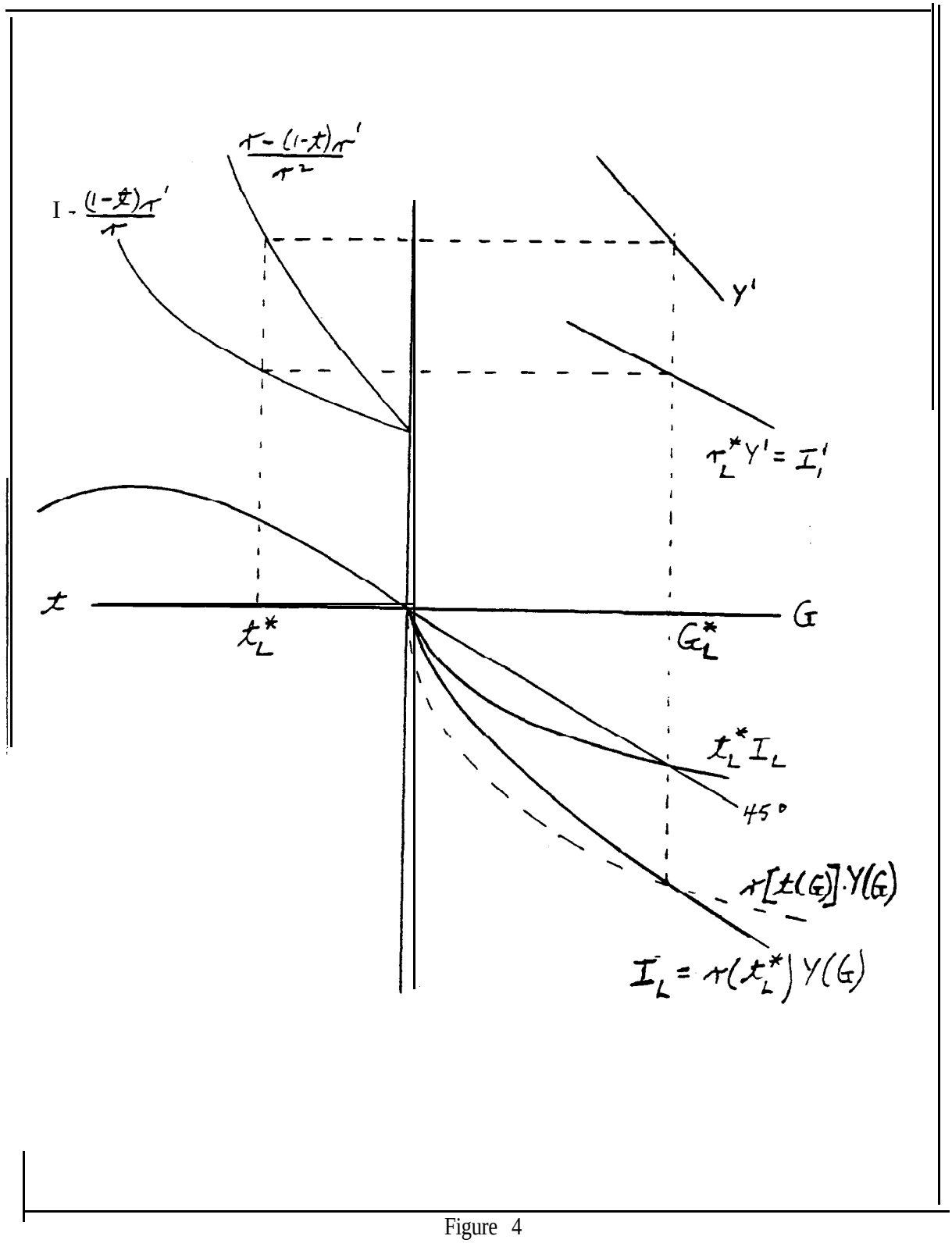
$\frac{\pi - (1-t)\pi'}{\pi^2}$
$1 - \frac{(1-t)\pi'}{\pi}$
Y'
$\pi_L^* Y' = I_1'$
t
G
t_L^*
G_L^*
$t_L^* I_L$
45°
$\pi[t(G)] \cdot Y(G)$
$I_L = \pi(t_L^*) Y(G)$

Figure 4

benefit. Below G_N^* we see that the tax revenues at the optimal tax rate are just sufficient to produce this optimal amount of the public good. The distance from this point on the 45 degree line down to $I(t_N^*,G)$ then shows the amount of actual output left over after taxes as net income for the citizenry. (The I or $r(t_N^*)Y$ curve is not parallel to the 45 degree line at the optimal level of provision of the public good because the resource cost of the public good is only part of its marginal social cost.)

III.C. Redistribution Lowers the Marginal Private Cost of G

We can now see one of the two neglected ways in which the provision of public goods and income redistribution interact in any social order. Though the public good has a price or resource cost of 1, that is not its true marginal cost to the Lindahl democracy. For this society -- and any society that does not redistribute income, and that therefore must raise the tax rate in order to obtain more of the public good -- the marginal deadweight losses from the additional taxation needed to finance more of the public good are part of its marginal cost. This was not the case for the Autocrat: as we saw earlier, he chose the tax rate that would maximize the redistribution to himself and purchased the amount of public goods that was optimal for him out of infra-marginal tax receipts, so the marginal private cost to him of G did not include the deadweight losses from taxation. The total deadweight loss from taxation, by reducing the society's income and the Autocrat's tax receipts, affected the Autocrat's decision about what tax rate was optimal for him. As we shall see when we finish this story in Section VI, it also affects his provision of G. But the marginal deadweight loss of taxation to finance G still is not part of his marginal private cost of G.

As will be evident from the next section, this conclusion for the Autocracy applies to any regime that redistributes. Any ruling interest that uses its control over a government to redistribute income finds that public goods have, other things equal, a lower "private" marginal cost for that ruling interest than they would have had without redistribution. If a government uses the tax system to redistribute as well as to

provide public goods. its decision about redistribution will determine its tax rate. Its provision of public goods will then be financed out of infra-marginal taxation. Therefore, the deadweight loss from additional taxation to finance the public good does not enter into its marginal cost to the ruling interest. This will be evident in the more realistic society -- the redistributive democracy -- to which we now turn.

IV
REDISTRIBUTIVE DEMOCRACIES

Even though the consensual democracy is normatively suggestive and provides some insight into the actual impact of income redistribution on the provision of public goods. it is based on assumptions that do not apply to most democratic or non-autocratic governments. Most elected governments do not enjoy unanimous support, but rather represent some ruling interest, such as a majority, that leaves out part of the society. There is normally a minority of the society (or, in the case of oligarchic democracies with restricted franchises and "minority governments, " even more than a minority) that is not part of the government. Accordingly, we now develop a model of a democratic (or at least non-autocratic) government that does not embody a social consensus, but rather governs the society solely in the interest of a majority or other ruling interest. We shall typically describe the ruling interest as a majority, but the analysis is general and also covers oligarchies and other ruling groups.[17] However, the members of this majority or other ruling interest (unlike the autocrat) earn income as participants in the productive market economy.

All societies that are democratic, even in our broad sense, share three fundamental features. First. they all involve competition for votes to determine who controls the government. Second, they can and often do redistribute income as well as provide public goods. Third, as we shall demonstrate, their behavior

[17] Our original intention was simply to construct a model of majority-rule democracy that paralleled the model of autocracy. We thank Leonid Polishchuk for noting that our model applies to other ruling interests and to nondemocratic settings as well.

depends dramatically on the share of the economy mat parties or office-holders include in their decision calculus -- that is on whether these institutions have broad encompassing constituencies, on the one hand. or narrow ones, on the other. The model that we shall now develop incorporates all three of these features and shows how they affect the allocation of resources and the distribution of income.

When other things are equal, government policies that increase the aggregate income or welfare of the society also make the majority or other ruling interest better off. This introduces a powerful incentive for democratic governments to take account of citizen interests that does not exist in au autocracy.[18]

But the interests of the majority are often served best of all if there is not only a prosperous economy but also a redistribution of income from the minority to the majority. So, in keeping with our motivational assumption about autocrats, we assume no scruples keep democratic political leaders from using the taxpayers' money to obtain the votes of a majority, and we describe this process as if the majority or ruling interest acts as an optimizing monolith. The ruling interests considered in this section of the paper necessarily gain from using their control over the government to redistribute to themselves; we consider majorities that would not redistribute in the next section.

We assume that the majority or other ruling interest is always decisive on decisions about the level of taxation, the provision of public goods, and the redistribution of income through government -- no limitations, apart from those that are needed to maintain political competition, limit what the ruling mterest can do. In practice, government subsidies and transfers cannot be perfectly targeted at a redistributive majority or other ruling interest. Some of the redistribution will not reach its intended targets and thus, from

[18] Extracting the maximum possible revenue from the society for one's personal purposes cannot be a good way to win elections. Voters will obviously never knowingly vote for any candidate who does this. Thus it follows trivially that the first fundamental feature of democracies -- electoral competition to determine who holds office -- tends to prevent the maximtmi-extraction-for-the-leader autocratic outcome. Casual empirical evidence also suggests that most democratic leaders do not accumulate anything approximating the maximum surplus that the societies they govern are capable of producing.

the point of view of the majority, will be lost. Such difficulty in targeting reduces majoritarian redistribution. This difficulty of targeting has no counterpart in the models of autocracy or of the consensual society and thus makes comparisons with these societies less transparent. We shall therefore assume that the ruling majority, like the Autocrat. obtains everything that is redistributed.

We assume that the entire product or income of society (rY) is produced in a market economy and that the majority or other ruling interest earns income in this market economy. Since the society's output is dependent on the provision of the public good, G, some of the product of society has to be spent on G. The remainder. rY - G. is consumed by members of the society. A ruling interest's consumption comes from two sources: (1) the income which its members earn in the market and (2) any redistribution this ruling interest, after defraying the costs of the public good, extracts from the rest of society. We therefore need two additional bits of notation to cover the majoritarian democracy

F = **the fraction** of the total income produced and earned in the market accruing to the redistributive ruling interest: some of the market income in a majoritarian democracy will be earned by the ruling interest and some by the rest of the society, so $0 < F < 1$. This ruling interest consists of the people who produce F. The identity of the ruling interest and its F are exogenously given parameters in our model. If F = 1 everyone would be included in the ruling interest and a consensual model would be appropriate. In an autocracy, where the dictator obtains all of his income through the government and does not sell labor or other factors of production in the market place, $F=0$.

S = the share of the total actual production, rY, of society's income that the ruling interest receives at its optimum from redistribution plus any market earnings. For the ruling interests that actually redistribute. this share consists of both its earnings in the market and what it takes for itself from the "minority" through its control of government. At the redistributive majority's optimum its share of social income is the sum of these two sources as a percentage of the total production of the society: the formula for its share is

$$S = F + (1 - F)t \tag{15}$$

Note that, unlike F. S is not an exogenously given feature of the ruling order. S depends on the value of t which the ruling order chooses, and therefore depends on the entire r(t) function (as well as on F). For

the Autocrat with a constant average tax rate. F = 0 and the share is simply t, the percentage of income the Autocrat takes in taxes. The larger a ruling interest's share, S, the more encompassing its interest. Though we shall normally speak of majority-rule democracy, it is clear that this notation also covers ruling interests other than majorities.

IV.A. Optimization bv Redistributive Ruling Interests

Since in this section we consider only majorities that actually do redistribute income from the minority to themselves, they necessarily collect more in taxes than they spend on the public good ($trY > G$) and give the difference to themselves. Like the autocrats we considered earlier. they first decide what redistributive tax rate best serves their interests and then decide how much to spend on the public good; their tax and public good supply decisions are independent. Because of this independence we can represent the optimization problem of the governing interest as:

$$\text{(16a)} \qquad \underset{t,G}{Max}\ (1-t)r(t)FY(G) + [tr(t)y(G) - G]\ ; \qquad s.t.\ G < tr(t)Y(G)$$

Alternatively, we could let the taxes the majority levies on itself and pays back to itself cancel out and focus only on the transfer from the minority to majority

$$\text{(16b)} \qquad Max\ Fr(t)Y(G) + (1-F)tr(t)Y(G) - G\ ; \qquad s.t.\ G < tr(t)Y(G)$$

Using this formulation would not change the results.

The first term of the objective function in equation (16a) shows the market income of the ruling majority after both deadweighr losses and taxes, and the second term is me **surplus** mat me majority transfers to itself. Given positive redistribution,[19] the first-order conditions[20] for maximization of (16a) are

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[19] We are greatly indebted to Jongseok An for our presentation in this section.

[20] The second order condition with respect to t requires that the derivative of (16a) be negative. This in turn entails $[-2(r')^2 + rr''] < 0$, which implies that the ruling majority's optimum must lie in a region where the curves Q(t) and V(t) are increasing.

(17) $$F[-r+(1-t)r'] + (r+tr') = 0$$

and

(18) $$\{(1-t)rF+tr\}Y'-1 = SrY' - 1 = 0$$

The S and F are as already defined. The optimal tax rate for a majoritarian democracy that redistributes is given by equation (17) and its optimal provision of the public good is given by (18).

IV.B. The Optimal Tax rate for the Maiority that Redistributes

Condition (17) requires that the marginal cost of the tax (of dt) to the majority party -- the negative of the first term in (17) -- be equal to the marginal benefit from redistribution -- the second term. In other words. the majority ceases raising taxes to redistribute to itself when the reduction in its share of market income is exactly as large as what it gains at the margin from redistribution. The majority limits the deadweight losses that it imposes on society because it bears a substantial part of these losses.

In short, the majority is led. as though bv a hidden hand. to limit the extent to which it uses the coercive power of government to redistribute income to itself. Its encompassing stake in the society gives it an interest in moderating the deadweight loss it imposes on society. and thus also the extent of its exactions from the minoritv. We recall that an autocrat (F=O) with coercive power also limited the deadweight losses his taxation imposed upon society because his control over the tax system gave him an encompassing interest in the productivity of the society. As we shall see, a majority's stake ($F > 0$) is necessarily more encompassing than an autocrat's, and this leads to a lower rate of redistributive taxation than an autocrat would impose. Rearranging (17) gives:

(19) $$F = \frac{r + tr'}{r - (1-t)r'} \equiv R(t)$$

Note that, as thc tax rate is increased from t = 0, the function R(t) tends to fall, because as taxes are

increased the deadweight losses at the margin (in the denominator) tend to become relatively greater in relation to the marginal gain from redistribution (in the numerator).[21] The majority increases its tax rate until the function R(t) falls to the point where it equals F, which determines its optimal tax rate. For values oft such that R > F the marginal benefits of further redistribution to the majority exceed the marginal costs and therefore taxes are increased and for R C F the opposite is true.

In short, a redistributing majority stops raising taxes to redistribute to itself when the fraction F of me deadweight loss that it bears is just equal to what it gains at the margin from redistribution. In other words. a redistributive majority ceases redistribution when the resulting loss to the society as a whole -- the drop in the market income of majority plus minority -- reaches 1/F times its gain.

The wide significance of F as a determinant of the degree to which a coercive power limits its redistribution to itself becomes evident when. from (19), we derive the expression for the optimum redistributive tax:

$$t_R^* = -\frac{r}{r'} - \frac{F}{(1-F)} \; ; \quad F \neq 1 \tag{20}$$

Equation (20) quickly and simply makes two important points.[22] First, it confirms the argument of the previous paragraphs that the larger the fraction F of market income that a redistributive majority earns, the lower its optimal tax rate will be. Second, it shows that such a majority levies lower taxes than does an Autocrat. If F = 0, as for the Autocrat, then the equation reduces to equation (4) which gave the optimal tax ratc for an Autocrat. Thus an autocrat will choose a higher tax rate than a majority and

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21 R(t) begins at r(O)/[r(O) - r'(O)]; thus the greater the absolute value of r'(0) the lower is R(0). Depending on the shape of r(t), R(t) may have rising and falling stretches. Differentiating R(t) with respect to t gives,

$$dR/dt = rr'' - 2(r')^2/[r - (1-t)r']^2$$

which is positive when $rr'' - 2(r')^2 < 0$, and negative when the sign is reversed. Note *that* dR/dt must be negative, therefore, in the neighborhood of the Autocrat's optimum, because of the second order conditions on that optimum. Just exactly where R(t) starts the course of its downward slope depends on r(t) and all its derivatives. In the text we generally follow the assumption that deadweight losses from taxes rise more than linearly with the tax rate. and thus assume that R(t) is continuously decreasing in t.

22 We are grateful to Satu Kähkönen for this valuable simplification.

redistribute a larger proportion of the national product.

It may seem natural at this point to ask what would happen when F = 1. but we must note that equation 20 is derived from equation 16. the optimization problem for a majority maximizing the sum of its share of market income plus any rcdistribution to itself from the minority. When F = 1 there can be no minority and we can also see directly that equation (20) has no meaning. And for F= 1. equation 18 similarly loses meaning for societies constrained by distortionary taxation. We shah deal with values of F that equal or approach 1, and with how this analysis relates to no-minority (consensual) societies, later.

IV.C. Public Good Provision bv a Maioritv that Redistributes

How much public good will a majority that redistributes provide? As we know, the marginal cost of the public good for a redistributive majority -- as for any regime that redistributes income -- does not include the deadweight loss of taxes. The marginal benefit of the public good to the redistributive majority must accordingly equal its marginal cost of 1. This is evident from equation (18), where SrY ' represents the Majority's marginal benefit from the public good.

Since a valid solution for the majority that redistributes must satisfy both equations (17) and (18), we combine these two equations to identify the optimal public good provision for that majority on the assumption that the optimal tax rate has also been chosen.

(22) $$Y' = \frac{r - (1-t)r'}{r^2} \equiv V(t)$$

(23) $$y' = \frac{1}{rF + (1-F)tr}$$

(24) $$I' = \frac{1}{F + (1-F)t} \equiv \frac{1}{S}$$

Just as the Autocrat chose his optimal tax rate independently of his decision on how much public

good to provide. condition (24) shows that the redistributive democracy does likewise. The public good. G, did not enter into equations (19) and (20). Having chosen the tax rate that gives it the optimal degree of redistribution, this majority then chooses its optimal public good level and finances this out of infra-marginal tax receipts.

An interesting and important feature of equation (22) is that it is the same expression as (13). Condition (13) was, however, derived for the case of a unanimous-consent Lindahl democracy that did not redistribute income. We shall explain why very different regimes meet the same condition (22), and find that this makes it possible for us to offer a new comparison of different social orders. later.

IV.D. A Comprehensive Analysis of Majorities that Redistribute

The majority's total income is given by combining or adding its market income, $FrY(G)$, to the redistribution from the minority to the majority, $(1-F)trY(G)$. If we drop the $rY(G)$ terms we obtain a fraction, $F + (1-F)t$,[23] that indicates the proportion of the society's actual output that the majority receives. Accordingly, in Figure 5, the market income of the ruling interest as a share of potential income is shown by the line Fr, which is simply the fraction of the potential national income it earns in the market times r. The fraction of potential national income collected from the minority is shown by the $(1-F)tr$ curve. After the public good has been financed the remaining tax receipts are available to the majority This majority, therefore, maximizes its share irrespective of the amount of public good it decides to supply. The combined income of this ruling interest as a share of potential income is then $Fr + (1-F)tr \equiv rS \equiv \mathcal{S}$ and the optimal redistribution from the minority to the majority occurs at the tax rate, t_R^*, which corresponds to the maximum of $\mathcal{S}$. Thus, maximization of $\mathcal{S}$ with respect to t (i.e., equation 16b) entails the same first order conditions as (16a), namely those of equation (17). Note that at the majority's optimal level of redistribution, the

[23] The majority's share of social income, after the public good has been financed. can also be given as $t + (1-t)F$.

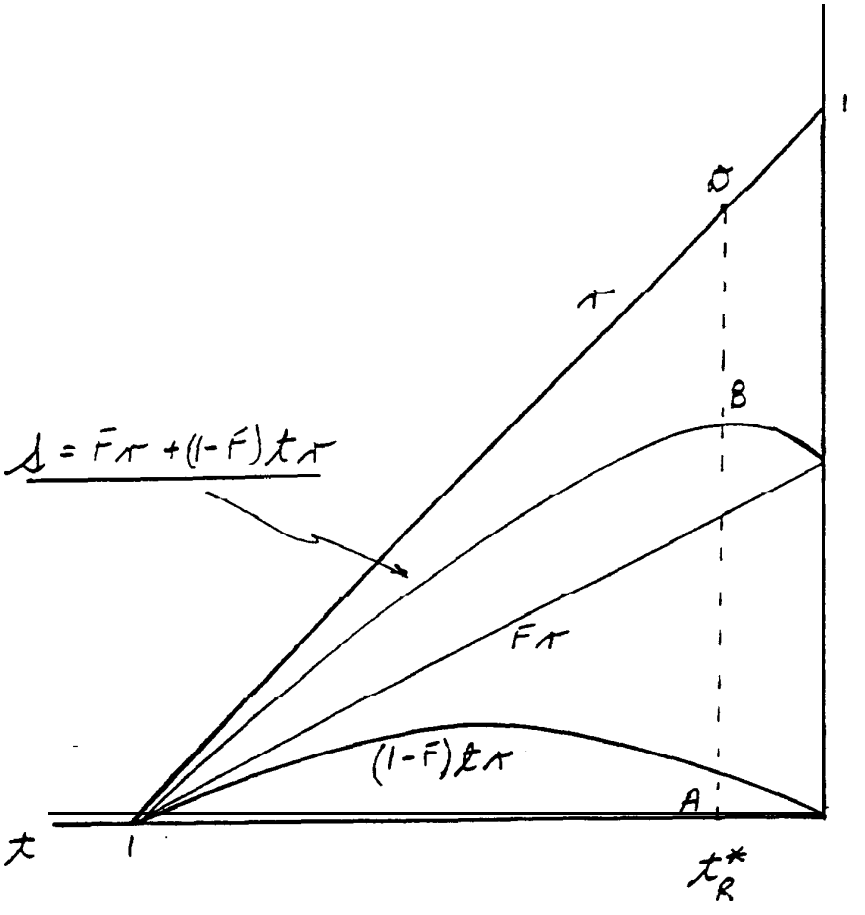

Figure 5

absolute value of the siope of Fr equals the absolute value of the slope of (1-F)tr: at the margin the majority's market income share falls by just as much as the redistribution to it goes up.

This exposition makes it obvious why the majority's optimal redistribution to itself will be higher if it has a smaller F: a smaller value of F makes the decline of Fr. as taxes and deadweight losses increase. less important to the majority, so that the tax rate at which the majority's loss in market income just equals its gain from additional redistribution must be higher. As F approaches zero the majority becomes indistinguishable from an autocracy and the majority's optimal tax rate will be virtually the one that maximizes tax collections.

When the redistributive majority has found the peak of $\mathcal{S}$ and thus its optimal tax rate and redistribution, it then decides how much of the public good to supply. To explore this further we must know what share of the benefits of the public good the majority will receive. At its optimal tax rate, the majority's share of the marginal social output is shown in Figure 5 as AB/AD (which is the same as the fraction S = F + (1-F)t). As Figure 6 illustrates, the deadweight loss from taxation has no effect on the marginal cost of G to the majority. Therefore, the majority equates the marginal resource cost of the public good, 1, to its share, S, of the marginal product of the public good. Therefore, at the optimal value of G, $SI'(t_R^*,G) = 1$. At the optimal tax rate evident from the peak of $\mathcal{S}$ we note that the marginal product of the public good, $I'(t_R^*,G)$ is equal to l/S. The fourth quadrant shows that the majority has purchased the optimal quantity of public goods when the rate of increase in $SI = FI(t_R^*,G) + (1-F)t_R^*I(t_R^*,G)$ just equals the marginal direct resource cost of the public good of unity (slope of the 45 degree line). The national income is then divided as follows: OD is spent on the public good, DE is redistributed to the majority, EF is the market income of the majority, and FG is the post-tax income of the minority.

At the majority's optimum the marginal social product of the public good equals the reciprocal of the ruling interest's share (taking both its market earnings and its redistribution to itself into account) of the

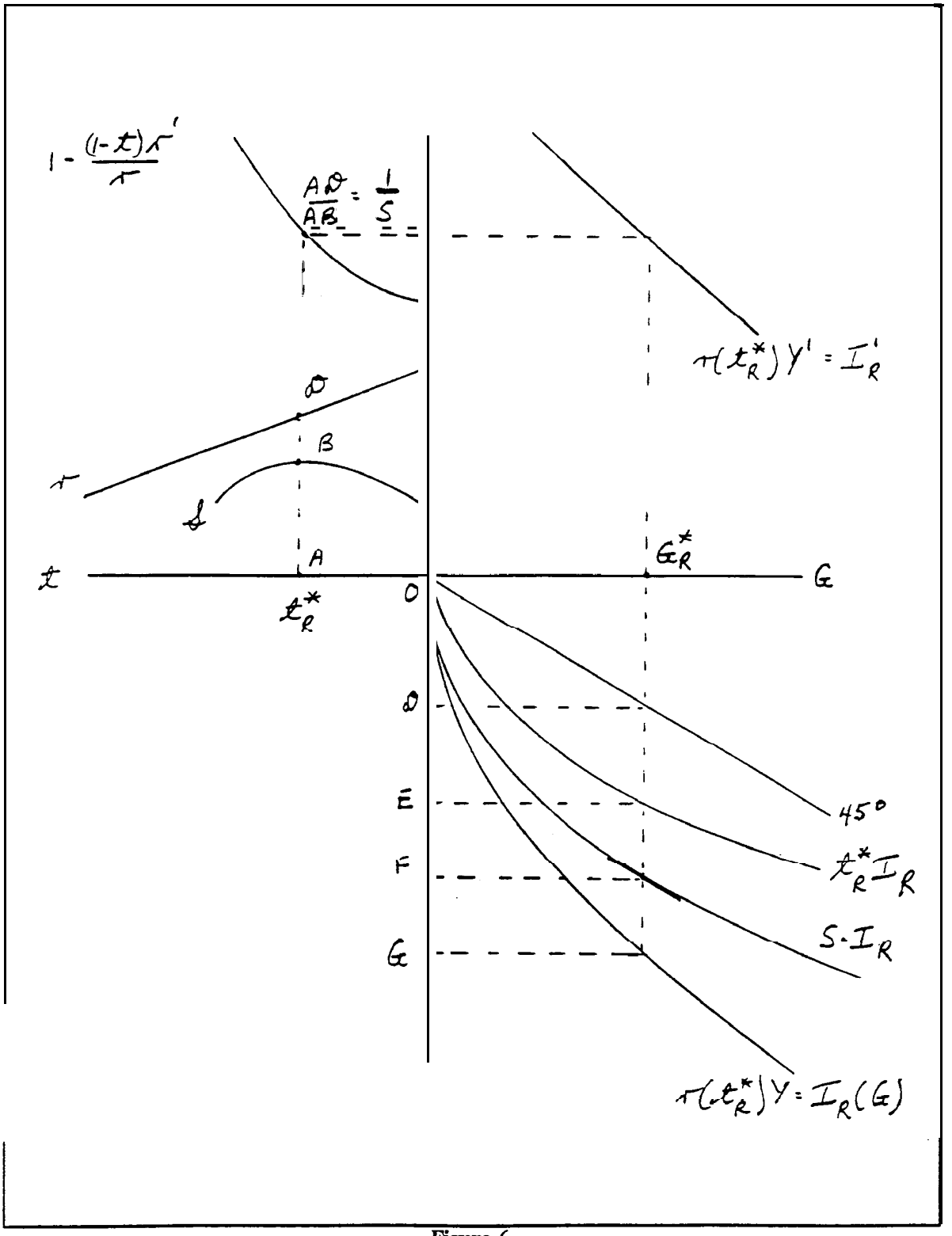
$1 - \frac{(1-t)r'}{r}$
$\frac{AD}{AB} = \frac{1}{S}$
$r(t_R^*)Y' = I_R'$
D
B
r
s
A
t
t_R^*
O
G_R^*
G
D
E
F
G
45°
$t_R^* I_R$
$S \cdot I_R$
$r(t_R^*)Y = I_R(G)$

Figure 6

increase in the income of the society, i.e.. to l/S. This general rule applies to ail redistributive regimes. Recall that the Autocrat's share of social income was simply given by the constant tax rate. and we know from Equation (8) that I ′ is simply equal to the reciprocal of his tax rate.

Any ruling interest that participates in the market economy (i.e.. with F > 0) necessarily has a more encompassing interest than an autocracy. This is evident from examination of equation (16a) and because the majority's share is a linear combination of r(t) and r(t)tY(G). Since the "majority, " like the autocracy, finds mat me public good has a marginal cost of 1, and since it has a more encompassing interest. it always provides more of the public good.

V

NON-REDISTRIBUTIVE MAJORITIES

We now come to the most striking example of the argument that. when coercive power is in the hands of a stable encompassing interest, a hidden hand prevents the disastrous outcomes that might have been expected. As we have seen, self-interested autocrats generate far better outcomes than anticipated. A majority or any ruling interest of participants in the market economy, even though it treats the minority simply as a source of exactions, necessarily generates outcomes that are better than the autocratic outcome for every market participant.[24] We shall see now that the hidden hand that guides encompassing interests can, in circumstances that are by no means rare, make their coercive power totally beneficent. If a ruling interest is sufficientiv encomnassine -- if it is what we call a super-encompassing ruling interest -- there is no redistribution whatever. Those with no Dower are treated fully as well as those with total power and the **allocation of resources is the same as that of our idealized Lindahl-Consensus democracy.**

[24] Redistributive majorities tax less and provide more public goods than autocrats do. Thus everyone except the **autocrat is** better off than under autocracy, although the majority more so. This is rigorously demonstrated in Section VII.

To see why, consider the two driving forces in our whole theory. First. the greater a ruling interest's market fraction. F. the larger its share of any deadweight losses arising from its taxation. and thus the lower the tax rate it desires. Second. the value of S for a ruling interest. the larger its share of the benefits from public good provision and the more it wants to provide. Consider a society in which the ruling interest is replaced by one with a larger F, but in which the r(t) and Y(G) functions remain unchanged. The ruling interest with the higher F has a higher S, i.e. is more encompassing." Thus if a ruling interest becomes more encompassing, ceteris paribus, it wants to tax less and, at the same time, spend more of the taxes it does raise on provision of G.

A point will be reached as F increases -- and with it S also -- where the ruling interest allocates all taxes to public good provision. At this point the ruling interest becomes so encompassing that it ceases redistributing and treats the minority as it treats itself! Such a ruling interest. and any ruling interest that is still more encompassing, will not redistribute to itself. It will, in fact, act the same way the Lindahl Consensus does.

The first of the two driving forces is identified by equation 20

(20 repeated) $$t_R^* = -\frac{r}{r'} - \frac{F}{(1-F)} \; ; \quad F \neq 1$$

This equation shows that t_R^* declines with increases in F. In fact, standing alone this equation implies. for sufficiently large values of F, a tax rate that is zero or even negative. The more encompassing the ruling interest, the larger its share of the deadweight loss from taxes and the sooner it curtails socially-damaging redistribution. eventually up to the point of taxing only to finance productive public goods. Equation (20) was derived from (17), one of the two first order conditions for a redistributive majority. Therefore, the tax rate t_R^* sol&ion from (20) must bc entered in equation (18) -- the first order condition for optimal public

[25] Because $S = F + (1-F)t$, it follows that $dS/dF = [1 - t + (1-F)dt/dF]$. But by the second order conditions for a redistributive majority $dt/dF = [r - (1-t)r']^2/[rr'' - 2(r')^2] < 0$. Substituting F from equation (19) and dt/dF implies $dS/dF > 0$.

good provision. This equation shows that. as F and thus S increases. the solution value of Y ′ declines and therefore G_R^* increases. Once F reaches a high enough value. t_R^* will be so low and G so great that all tax revenue is needed to pay for public goods and there will be no redistribution.

The **second force is** seen in equation **(24)**

(24 repeated) $$I' = \frac{1}{F + (1-F)t} \equiv \frac{1}{S}$$

As F goes up, the ruling interest obtains a larger share of the benefits of the public good. and this makes it want to provide more, thereby requiring that more taxes be **allocated** to provision of G.

The existence nf ruling interests that **leave nut** part of **society, yet act in the interest** of all. are not only a possibility but also (with incentive-distorting taxation and a **sufficiently** large G) a necessity. Assume a society with given r(t) and Y(G) functions. If F = 0 there is an autocrat who levies taxes that obtain a **positive** surplus for him while he provides G_A^* of the public good. By equation (20) there is also a value of $F = F^0 < 1$ that entails that $t_R^* = 0$. At this tax rate, there is no revenue for G. It follows that some value of F, $0 < F < F^0$, will entail a positive tax rate just sufficient to pay for the optimal provision of G. Let us designate the "cross-over" values at this point as $\hat{F}$, $\hat{t}^*$, $\hat{G}^*$. An $\hat{F} \leq F^0$ must exist where the ruling interest is best **served** by a tax rate just sufficient to finance the optimal provision of **public** goods: at $\hat{F}$, $\hat{t}^*\hat{r}^*Y(\hat{G}^*) = \hat{G}^*$. Ruling interests must become "super-encompassing" and thus abstain from redistribution <u>before</u> $F = F^0$ **and** therefore before F = 1. Thus we have proven that. when a majority or **other ruling interest is sufficiently encompassing, it will not redistribute any incomc, and will treat those** subject to its power as well as it treats itself.

V.A. Optimization by a Super-Encompassing Maiority

This same logic is evident in the optimization problem of the highly encompassing ruling interest.

The appropriate Lagrange function is:[26]

(25) $$L = (1-t)r(t)FY(G) + tr(t)Y(G) - G + \lambda\{tr(t)Y(G) - G\}$$

The Kuhn-Tucker condition is $\lambda[tr(t)Y(G)-G]=0$, $\lambda \geq 0$, and $[tr(t)Y(G)-G] \geq 0$.

First assume that $trY = G$. Then $\lambda > 0$ and the first order conditions with respect to t yield

(26) $$\frac{F}{1+\lambda} = \frac{r + tr'}{r - (1-t)r'} \equiv R(r)$$

or

(27) $$F = (1 \quad \lambda)\, R(r)$$

From differentiating with respect to G. we obtain

(28) $$\frac{F}{1+\lambda} = \frac{1 - trY'}{(1-t)rY'}$$

Equation (27) gives the condition for optimal distribution when the majority just supplies the public good out of tax collections with nothing left over for cash redistribution. When zero cash redistribution is imposed as a constraint and $\lambda > 0$ it follows that the marginal costs of redistribution exceed the marginal benefits. If it were possible to reduce taxes toward equality of marginal costs and benefits, condition (26) says that the ruling interest would do so; however to reduce taxes would provide insufficient revenue to finance the desired public good. The second equation (28) indicates that at the constrained optimum of G, the marginal benefits of G exceed marginal costs.[27] Moreover, both (26) and (28) indicate that every ruling majority with an F so high that it rejects redistribution behaves just like a majority with $F = \hat{F}$. All ruling interests that are forced by the constraint $trY = G$ not to redistribute behave as if their $F = F$ and they had chosen $trY = G$. That is, for all $F > \hat{F}$, $F/[1+\lambda] = \hat{F}$.

When equations (26) and (28) are combined, we find that the same general condition for the optimal level of G holds when redistribution is constrained to be nil as held when there was positive redistribution.

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26 We thank Jongseok An for suggesting this set up.

27 More detail to demonstrate this can be found in McGuire and Olson (1994).

namely equation 22. This is also the same condition that held for the Llndahl Consensus. This means that every non-redistributive ruling interest. whatever its F. will make the same decisions about public good provision it would have made had its F been $\hat{F}$ and will have the same tax rate $\hat{t}^*$. This is evident in two ways. If we think of the ruling interest in the redistributional model and thus as having a marginal private cost of the public good of 1, then it has the $\hat{S}$ corresponding to $\hat{t}^*$, and $\hat{S}$ is the effective share of me every super-encompassing ruling interest, so $\hat{S}r(\hat{t}^*)Y$ '(G) = 1. and MSC = $1/\hat{S}$. Alternatively, we can think of societies with F ≥ F as explicitly recognizing that the marginal cost of G includes the deadweight COSTS of taxation. Then for all super-encompassing and Lindahl Consensus societies we must take S as S^u, i.e., always equal to unity, and then $S^u r(\hat{t}^*)Y'(G)$ = MSC. Both accounts give the same answer and every ruling interest with F ≥ $\hat{F}$ makes exactly the same choices as the Lindahl Consensus.

The absence of redistribution implies that both the maiority and the minority each pay their fair or Lindahl share of the tax burden. The majority receives F percent of the benefits of the public good and pays F percent of the tax. It therefore chooses exactly the same level of public good provision as me Lindahl Consensus. Thus the society ruled by a super-encompassing majority is twice blessed: the ruling interest not onlv abstains from redistributive taxation. but it also chooses an ideal level of public good orovision that reflects the minoritv's interests as its own.

Since all super-encompassing (F ≥ $\hat{F}$) interests generate the same outcomes as the Lindahl Consensus, we use the same notation for both. Thus t_N^*, where the subscript "N" means "non-redistributing," refers both to the idealized Lindahl Consensus and the super-encompassing majority. It remains true that $t_A^* > t_R^* > t_N^*$ and $r_A^* < r_R < r_N^*$.

V.B. The Prevalence of Super-encompassing Majorities

Ruling interests so encompassing that they abstain from redistribution are by no means oddities.

Consider those super-majorities required for major decisions in political systems with numerous checks and limits on the use of power. such as Switzerland and the United States, or even simple majorities composed mainly of those with above-the-median incomes (Niskanen. 1992). It is easily possible for such majorities to represent. say, three-fourths of the income-earning capacity of a country, in which case they would cease any redistribution to themselves when the last dollar redistributed brings a marginal deadweight loss of one-third of a dollar. Suppose that at the same time the Y(G) function is such that it pays the majority to spend a fourth of the national product on public goods. In such circumstances, it does not require any remarkable deadweight loss function. l-r, for tax rates of .25 to make the deadweight loss from the last dollar raised in taxes a third or more of a dollar, and in this case the majority will not redistribute. Thus coalitions so encompassing that they abstain from redistribution are a feature of reality.[28]

VI

EFFECTS OF TAXATION ON THE PRODUCTIVITY AND COST OF PUBLIC GOODS

We now come to the second way in which the two fundamental features of a social order -- the public good provision that makes social cooperation possible, and the distribution of the gains from this cooperation -- interact. The second interaction[29] arises because taxation affects both the marginal social cost and the productivity of public goods.[30]

We begin with how taxation affects the marginal social costs in each regime. The marginal cost to society of the public-good-with-redistribution is the good's resource cost plus the deadweight losses of the taxes

[28] Tom Bozzo has demonstrated this by computer simulations over a broad range of F and r(t) values.

[29] The first interaction was the effect of redistribution on the marginal cost of public goods to a ruling interest.

[30] From the point of view of society as a whole -- the citizens plus any redistributive ruling interest -- net income is r[t(G)]Y(G) - G, whence the realized marginal product of G can be written rY´, and the marginal social cost of providing more G can be written l-r ´Ydt/dG. See equation 12b.

imposed. Though a redistributive ruling interest is only indirectly concerned about social costs. they are of direct interest to the economist-observer. not to mention those who are taxed. Thus we need to identify these social costs and to see how they vary at the margin for each regime. In addition, we want to identify the marginal costs and marginal benefits that determine each regime's decision about how much public good to supply.

We begin with the non-reciistributive Lindahl and super-encompassing societies in which all social costs and benefits are included in the decision calculus. The marginal social costs of these societies were already shown in equation 12a and Figure 4 and their equilibrium or realized vaiues are set out in equation (14).

(14 repeated) $$rY'(G) = I'(t^*,G) = r(t)V(t) = 1 - (1 - f\frac{[r^*]'}{r^*}) = MSC$$

Equation (14) has an immediate intuitive meaning: the left hand side shows the actual marginal social product of public goods; the realized marginal cost on the right is simply the direct unit cost of the resources needed to produce the public good, 1, plus the marginal loss of net output arising from the marginal deadweight cost of taxation. This expression gives the equilibrium marginal social cost of the public good in terms of actual income $rY \equiv I$ in any regime, whether it is an autocracy, a redistributive majority, or a non-redistributive society. To see this, divide equation 14 by r to obtain equation 13b, which states the same equilibrium condition in terms of potential income, Y.

(13b repeated) $$Y'(G) = \frac{r(t) - (1-t)r'(t)}{r^2} \equiv V(t)$$

We found in the last section that this equation applies not only to the equilibria of non-redistributive societies but also to majorities that redistribute: equation 22 was identical to 13b. This may seem surprising: super-encompassing and Lindahl-consensus governments set their tax rates only for the allocative purpose of providing G, whereas redistributive majorities base tax rates solely on redistributive objectives. Nonetheless. as equations (13b) or (22) show, the functional form of the expression for the marginal social costs at the optimum provision of G is the same in these different cases, even though levels of public goods provision differ. Moreover, an autocrat, the equivalent of a redistributive majority with an F = 0, chooses r + tr' = 0 (equation 3) and when this is true equation (13b=22) reduces to the autocrat's optimal $Y' = -r'/r^2$ (equation 9). Some regimes choose higher tax rates than others and thus bring about lower values of r, but the marginal social cost of the public good

at each optimum can still be expressed in the same functional form.

This symmetry is instructive. The marginal social cost of the public good depends only on the regime's tax rate. and each regime bears the same share, S, of the marginal social cost of the public good as it receives of its benefits. Thus in equilibrium even redistributive regimes that have no direct interest in social costs or benefits find that, when they have equated marginal private costs of the public goods to marginal private benefits. they also equate marginal social costs to marginal social benefits. We can capture this symmetry in the formula

$$\textbf{(29)} \qquad \textit{Marginal Social Costs of } G = \frac{1}{S} = \textit{Marginal Social Benefits of } G$$

Though different regimes have both different tax rates and public good provision levels. and thus generate different levels of marginal social costs and benefits, within each regime these marginai benefits and costs are equalized. This entails that, with all regimes in equilibrium on all margins. their different marginal social costs/benefits of the public good can all be stated in terms of the same general expression -- they are simply at different points the same marginal social cost curve.

The foregoing is illustrated in Figure 7. The different tax rates that each regime imposes, and the varying shares, S, of the output that they encompass, are shown in the second quadrant. Think of Figure 7 as depicting a single society with unchanged r(t) and Y(G) functions, but under various alternative regimes. If the society were under an autocrat, he would find his revenue-maximizing tax rate, t_A^*, where his share ot the social loss from taxation falls by as much as he gains. If the society were under a redistributive democracy with a given F, the tax t_R^* would be given by the peak of the $\mathcal{S}$ curve. Super-encompassing and Lindahl-consensus governments would necessarily have a tax rate lower than t_A^* and t_R^*.

If we start at the autocratic optimum and think of alternative redistributive ruling interests with progressively larger values of F (and thus lower tax rates), we move to the right. When $F = \bar{F}$, we reach the non-redistributive or Lindahl tax rate. Each of this infinite set of alternative regimes would have an equilibrium marginal social cost of the public good in terms of actual income, given by equation 14, and an equilibrium marginal social cost in terms of potential income. given by equation 13b or 22.

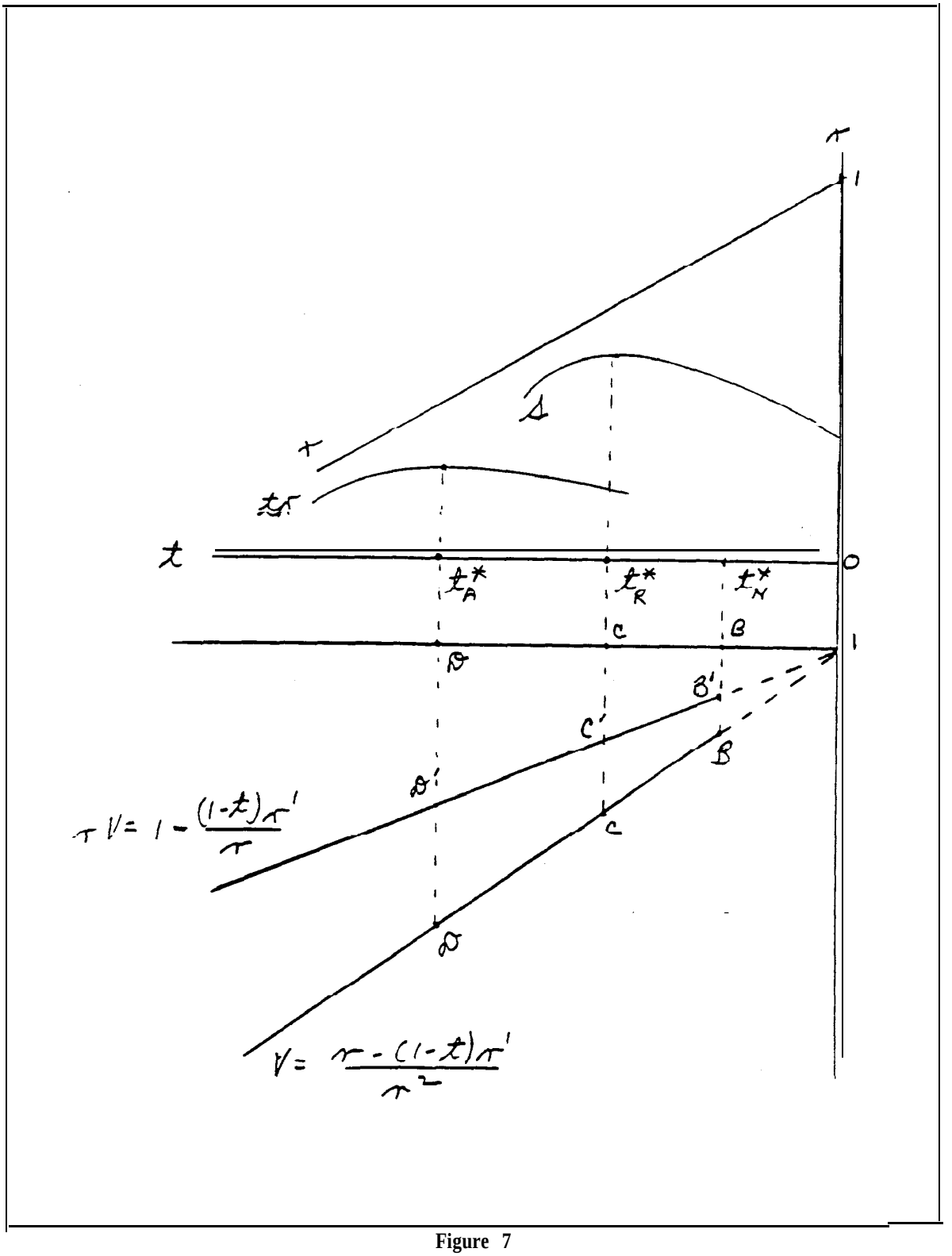

Figure 7

Curve V(t) in the third quadrant shows the equilibrium values of equation 13h at each tax rate. and rV(t) shows the corresponding equilibrium value for equation 14: that is. the marginal social cost of G in terms of potential income. Y. and in terms of actual income. I, respectively. Since the values of V(t) and rV(t) to the right of the non-redisuibutive optimum do not correspond ιω achieved equiiibria. we show these portions of the curves as dotted lines. At t = 0. r(0) = 1 and thus V = rV. (As it approaches zero the values V = rV ≥ 1.)

The marginal social cost of resources in terms of potential income. Y. is the direct unit cost. 1. plus all marginal deadweight losses at each regime's equilibrium. These marginai deadweight losses are given by the vertical distance between 1 and V(t) at each regime's equilibrium tax rate. For the non-redistributive regimes this distance is BB. for the illustrated redistributive democracy it is CC, and for the Autocracy, DD.[31]

Now we turn to the marginal social benefit from each social brdes. o c i a l p r o d u c t o f G is simply Y ′ in terms of potential income and rY′ ≡ I ′ in terms of actual income. Remember that Y ′ is the marginal social product of the public good had there been no dcadwcight loss from taxes. Thus the different regimes, though they choose different tax rates and bring about different values of r, all face the same Y ′ schedule. The I ′ functions (I ′ = r(t)Y ′), by contrast, shift with the tax rate. so each regime has a different I' function. All of these I' functions are expressed in terms of the same variables: t, r. and G,[32] but each regime chooses different values of these variables. We will now use the I ′ functions to show how taxes reduce the marginal productivity of G.

We use Figure 8 to illustrate. The vertical axis measures Y ′ and I ′ as well as V(t) and rV(t). Let

[31] The direct resource cost of the public good serves as a numeraire, so we have the deadweight losses and marginal social costs in terms of both actual and potential income measured in absolute units.

[32] Because the Lindahl Consensus chooses t and G together, for it I' = r[t(G)]Y′; that is, r and Y′ both decline with G. For redistributive regimes which choose t and G independently, r(t*) is a constant in I' = r(t*)Y′.

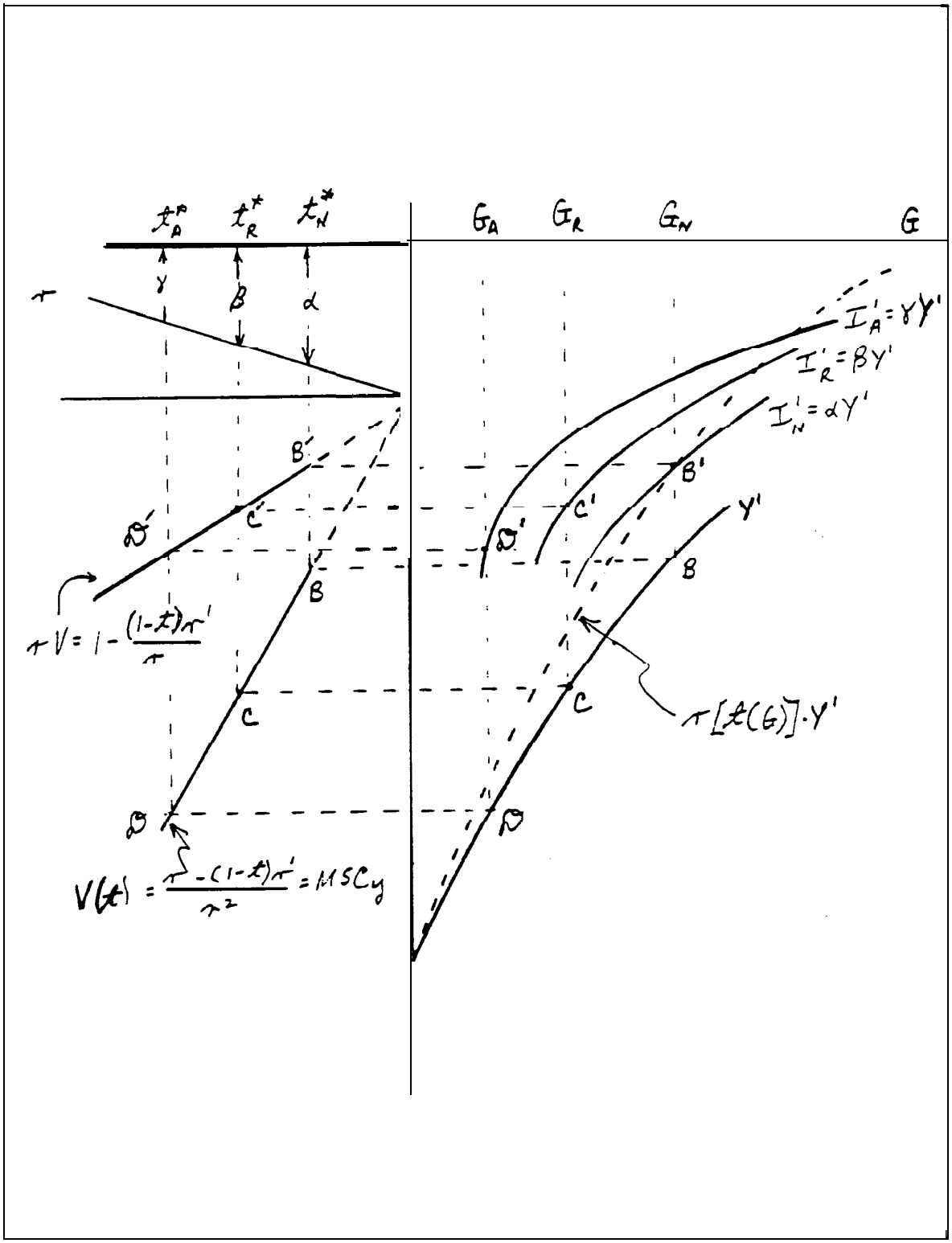
t_A^*
t_R^*
t_N^*
γ
β
α
r
G_A
G_R
G_N
G
$I_A' = \gamma Y'$
$I_R' = \beta Y'$
$I_N' = \alpha Y'$
Y'
B'
C'
D'
B
C
D
$rV = 1 - \frac{(1-t)r'}{r}$
$V(t) = \frac{r - (1-t)r'}{r^2} = MSC_y$
$r[t(G)] \cdot Y'$

Figure 8

the vertical axis serve additional duty as an axis for r down to r = 1 so that the r(t) curve may be depicted in the thrid quadrant. The proportions of potential output achieved for each illustrative regime are given as $I_N' = \alpha Y'$, $I_R' = \beta Y'$, $I_A' = \gamma Y'$ and the proportions (l-r) are as drawn. In quadrant 4 we show with Y' the gross marginal product of G before any erosion due to taxation. We also show. with the separate I ′ curve for each regime. the realized marginal productivity of G schedule. The distance between the Y′ curve and any given regime's I' curve, which is simply (l-i), must give that regime's proportionate loss in marginal productivity of G due to taxation. Thus for the each regime depicted. its I ′ schedule must at its optimum G be as far away from the Y' curve as rV is from V at the optimum tax rate.[33] If the r for a regime is. say, .67, then its I ′ is two-thirds of the distance between the horizontal axis and Y ′ . with I ′ being below Y' by one-third. For each regime, its optimum on the rV = MSC, curve is in line with its. optimum on its respective I ′ curve.

We showed earlier that for non-redistributional regimes, the deadweight loss from the taxation needed to provide G enters directly into their decisions about how much to supply. But for redistributive regimes, which finance public goods out of ma-marginal tax receipts, public goods have a marginal private cost of only 1. That was the first interaction between the two fundamental features of a social order.

We have now seen the second interaction between the fundamental features of a social order: <u>all taxation lowers the actual marginal product of the public good,</u> and <u>redistributive</u> taxation lowers that productivity more than is necessary to provide public goods.[34] More precisely, redistributive taxation not

[33] The economic meaning of the additional social loss given by the distance between 1 and rV is explained in McGuire and Olson (1994), which also offers a delineation of the marginal social losses that arise from limited provision of G as well as from the productivity eroding effects of taxation.

[34] The intuitive and elemental character of this point is evident the moment one thinks of the relationship between the productivity of a producer's public good and the incentives facing the producers. If, for example, the public good is a flood control system that protects farmers' fields. the productivity of a flood control system will depend upon the productivity of the farmers and, therefore, also on the extent to which they are confronted with incentive-distorting taxation.

only reduces the post-tax incomes of the victims and makes a social order more costly for the society as a whole; it also reduces the marginal product schedule (I t´)n of the public good. a redistributive regime's chosen tax rate. the higher $(1-t)r'/r$ becomes and therefore the greater the marginai social deadweight loss.[35] Although redistribution makes the marginal private cost of the public good to the ruling interest fall to 1, it also shrinks the entire productivity schedule of the public good both for the society and for that ruling interest, as shown by the inward shift of I ´. More than this. the higher t^* chosen by a more redistributive regime helps determine its less encompassing share. S^*,[36] and it is to this share of I' (i.e., SI') that a redistributive ruling interest equates its marginal private cost of the public good. 1. Thus. with more redistribution. and associated smaller S, the drop in I ´ is greater. thus giving a still lower SI' schedule that intersects me marginal private cost curve (equal to 1) at necessarily lower values of G. This guarantees that the less encompassing the ruling interest in a redistributive regime the less public good it supplies.[37]

VII

A UNIFIED PRESENTATION OF THE CONTINUUM OF REGIMES

Figure 8 did not show how the proportion of the marginal benefits of the public good in each regime, together with the marginal private costs it must pay, determine its decision about how much to provide. To explain this decision, we must understand each regime's calculated balance between marginal benefits and costs. We therefore remrn to the share, S. We know that the Autocrat's share S is given by

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[35] The rate of change of the realized optimal marginal deadweight loss is

$$d[(1-t)r'/r]/dt = \frac{(1-t)[r''r-(r')^2]-rr'}{r^2}$$

[36] Recall that for greater values of F, t^* declines and S increases.

[37] The general conclusion that as F increases G_R^* must increase for positively redistributing regimes ($0 \leq F < \hat{F}$) follows from consideration of equations (17), (18) and (19). From footnotes 15 and 16 second order necessary conditions require $dt/dF < 0$ and $dV(t)/dt > 0$. Therefore, when F increases and t declines V(t) = Y'(G) must decline. Because $Y' > 0$ and $Y'' < 0$ this entails that G increase.

his tax rate t Similarly, any majority that gains from redistribution -- i.e.. any ruling interest within the range $0 < F < \hat{F}$ -- will receive a share $S = F + (1-F)t$ of the society's income. Both consider only their share S of the benefits from the public good, equating this -- i.e. SrY' -- to 1. Thus when the actual marginal product of the public good is $1/S$, i.e., when $rY' \equiv I' = 1/S$, each of the foregoing redistributive regimes is at its private optimum (see Equations 18 or 23). That is. redistributive regimes incorporate only part of marginal social benefits and part of marginal social costs in their decision-making. In these regimes, the marginal social costs of resources given by the margin of taxation exceed the marginal private costs -- equal to 1. Similarly, only part of the marginal social benefit. SrY'. is taken into account. and in equilibrium these marginal private benefits are set equal to 1.

Everyone in the Lindahl consensus pays a share of the costs of the public good equal to his or her share of the benefits (i.e., his marginal evaluation of the social order). Therefore, each voter best serves his or her interest by voting for the amount of public good that balances all of its marginai henefits and costs to the entire society -- a choice that maximizes the net product of the entire society (see equation 12b). As we showed earlier, a society with a super-encompassing ruling interest ($F \geq \hat{F}$) is paradoxically twice blessed by the invisible hand: first, though there is a minority to exploit and the majority has no scruples about exploiting it, self-interest leads the majority to abstain from redistribution: second, society is blessed agam because absence of redistribution implies that the costs of the public good are shared in proportion to the benefits. So, with marvelous synecdoche,[38] the majority acts as though it were the whole and chooses the same amount of public goods as an idealized Linda&consensus society. Thus all non-redistributive regimes incorporate all marginal social benefits and costs in their decision calculus.

Figure 9 depicts the foregoing logic. In panel a, depicting the Lindahl Consensus. the V(t) and rV(t) curves of Figure 8 appear in the second quadrant: the first quadrant shows marginal social costs and benefits as functions of G. The marginal social benefit schedule for the Lindahl regime. $r[t(G)]Y'$. is steeper than

[38] This two-dollar word comes from the Greek and means having "the part" of something stand for, or take account of, the "the whole" of it.

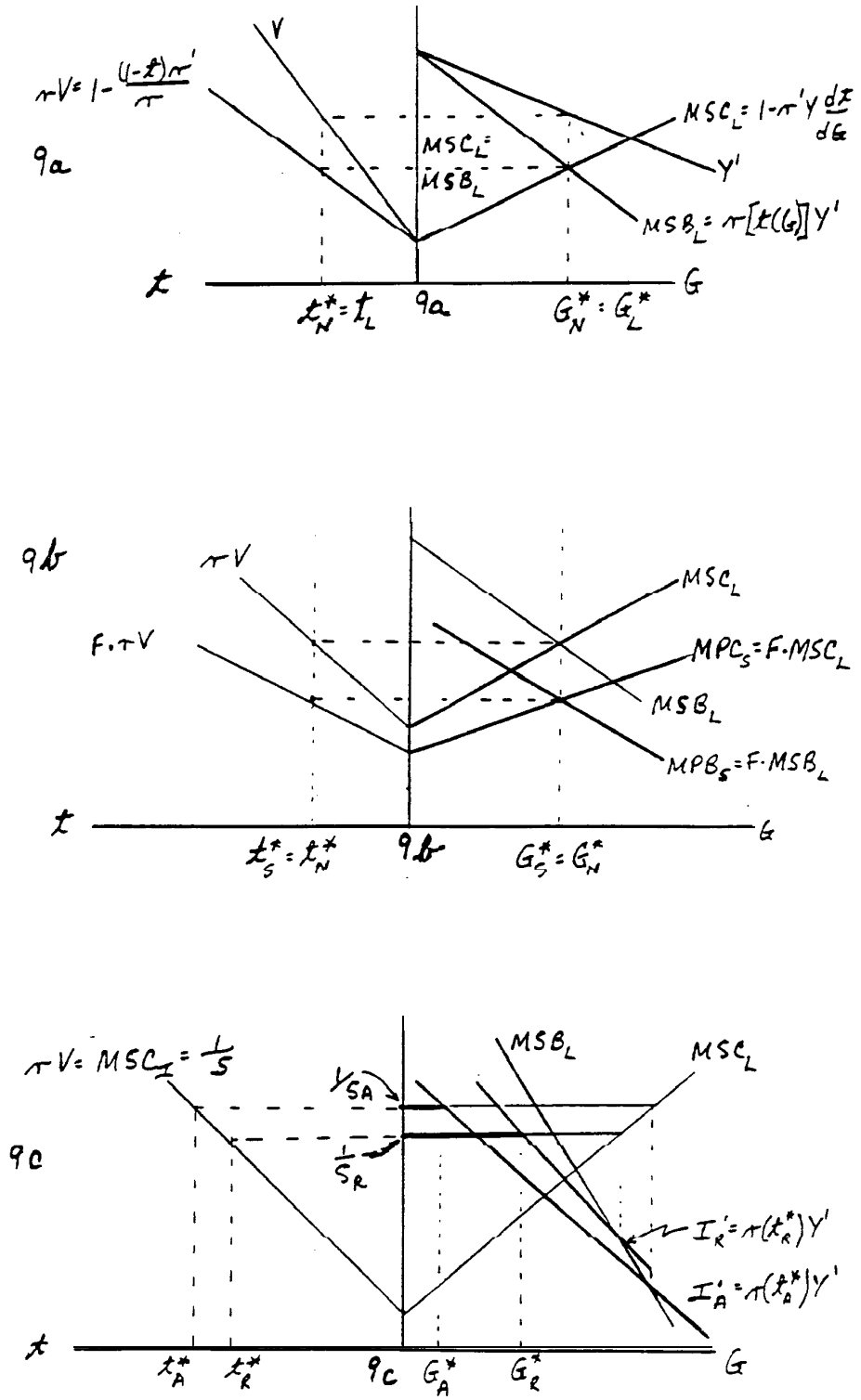
9a
V
$\tau V = 1 - \frac{(1-t)\tau'}{\tau}$
$MSC_L = 1 - \tau' Y \frac{dt}{dG}$
$MSC_L = MSB_L$
Y'
$MSB_L = \tau[t(G)]Y'$
t
G
$t_N^* = t_L$
9a
$G_N^* = G_L^*$
9b
τV
MSC_L
F·τV
$MPC_S = F \cdot MSC_L$
MSB_L
$MPB_S = F \cdot MSB_L$
t
G
$t_S^* = t_N^*$
9b
$G_S^* = G_N^*$
$\tau V = MSC_I = \frac{1}{S}$
MSB_L
MSC_L
$1/S_A$
9c
$\frac{1}{S_R}$
$I_R' = \tau(t_R^*)Y'$
$I_A' = \tau(t_A^*)Y'$
t
G
t_A^*
t_R^*
9c
G_A^*
G_R^*

Figure 9

any single $r(t)Y' = I'$ curve. This is because the Lindahl regime must increase the tax rate **to spend more on** the public good. so it lowers r as it obtains more G and thus has a different I' curve at each level of public good expenditure. [39] Going up from t_L^* in the second quadrant to the $rV \equiv 1 - [(1-t)r']/r$ curve giving the actual marginal social cost **of the** public good. and across to the MSB, curve. we find the optimum (private and social) for the Lindahl Consensus. Since all tax proceeds are spent on G and since its marginal social cost is the direct resource cost of G plus the deadweight loss of taxation, we can derive those marginal costs as a function of G. Start from any tax rate. read up to rV = MSC, then go directly across to me first quadrant to the amount of G which the constraint (G=trY) allows. This generates marginal social cost in terms of units of G. i.e.. $1 + r'Ydt/dG$ (as in equation (12b)). Thus MSC, in the first quadrant expresses the same value as its counterpart in the second.

Panel b in the panel below depicts the same MSB_L and MSC_L curves and the same outcome. But, to illuminate decision-making for super-encompassing ruling interests and to distinguish private and social costs for such interests, it depicts a majority with a given $F \geq \hat{F}$. Because $F \geq \hat{F}$ there will be no redistribution, either via explicit transfers or through a disproportionate sharing of the costs of G. The ruling interest receives F% of the marginal social benefits, **so its marginal private** benefit is $F(MSB_L) = MPB_S$, where the subscript "S" stands for super-encompassing. It also pays F% of the direct resource costs of the public good and bears F% of the society's deadweight losses. so its marginal private cost is $F(MSC_L) = MPC_S$. Just as in the Lindahl Consensus. where each voter's share of the marginal costs of the public good equals that voter's share of the marginal benefits, the super-encompassing ruling interest's share of marginal social costs is also equal to its share of benefits. It therefore chooses the same outcome as the Lindahl Consensus, in effect treating the minority the same way it treats itself. Since the outcomes are the same, we can return to using the subscript "N" for non-redistribution to describe both the Lindahl Consensus and the super-encompassing ruling interest.

Less encompassing ruling interests with $F < \hat{F}$ select, as shown in Figure 9c, t_R^* (t_R^* is found by

[39] **The MSB_L curve, in effect. consists of a series of points on I' curves**

equating R(t) with F as shown in equation (19) or by locating the peak of $\mathcal{S}$ in Figure 6). Their selection of t_R^*, necessarily higher than that of the non-redistributing societies. determines I_R' and $1/S_R$. Thus me marginal social cost of the public good they provide. $1/S_R$, is necessarily higher -- and so its realized marginal social product I_R' is necessarily also higher -- than for non-redistributive regimes.[40] The redistributive ruling interest ceases providing G when 1/S equals $r(t_R^*)Y' = I_R'$. Since $1/S_R$ for any redistributive regime is equal to me marginal social cost generated by its tax rate. its equilibrium is found where MSC = $1/S_R$ and I_R' intersect. Finally, an autocrat with F = 0 chooses t_A^* and $S_A = 1/t_A^*$. His independent choice of t determines I_A' -- shown as shifted down even further. And the Autocrat's optimum G is shown by reading horizontally across from the appropriate MSC = $1/S_A$ to the intersection with I;.

Figure 10 allows us to summarize by depicting ail regimes together. The ordinate below the origin provides the scale for l/S. Each redistributive regime equates its 1/S in the third quadrant to its respective marginal social product or I ′ curve shown in the fourth quadrant. At this point a redistributive regime's share of the marginal social product of the public 'good is just equal to the regime's marginal private cost of 1. But. we recall, just as a redistributive regime obtains S percent of the marginal benefits of the public good. so it also bears S percent of the marginal social costs of G: the 1/S for redistributive regimes is necessarily on the MSC curve (i.e.. the $rV = 1 - (1-t)r'/r$ identified by equation 14). If the optimal tax rate for an autocrat, for example, was .5, he would not only obtain half of the marginal benefits of the public good (equation 8), but also bear half of the deadweight loss caused by his taxes (remember that his share of the deadweight loss is what kept him from taking all income): since the marginal private cost at the Autocrat's optimum is 1, me marginal social cost must be 1/S or. in this example, 2. A redistributive majority must have a lower tax rate. a higher marginal social product of the public curve (I ′), and a larger S than an autocrat, but it also bears S percent of the marginal social costs and obtains S percent of the benefits, of the public good. It follows that all redisuibutivc regimes effectively incorporate all marginal

[40] I_R' is constructed as follows. For any t_R^* read up to $rV = 1/S_r$, thence read horizontally across to MSC,. This gives the G available if all taxes are spent on the public good. Directly below at this value of G, the $I_R'(t_R^*)$ curve intersects MSB,. I_R' is necessarily less steep than MSB_L.

social costs and benefits m their decision calculus. but that they do so indirectly. mediated by the tactor l/S (see equation (29). Redistributive taxation raises the marginal sociai cost of G above the necessary minimum. But. reflecting the symmetry mentioned earlier. redistributive regimes restrict provision of G so that its marginal social benefit equals this -- higher than necessary - marginal social cost.

In non-redistributive regimes, all decision-makers face private incentives that make them choose the quantity of G that equates MSC_N with $r_N^* Y' = I' = MSB_N$, i.e. that make sure that the society internalizes all of the benefits and costs of the public good. Whereas an autocrat with a privately optimal tax rate of .5 would take account of 50% of both marginal social costs and benefits. the non-redistributive societies wouid take account of 100% of both: they are at the point marked "S^u" in the third quadrant of Figure 10.[41] Unless something akin to a lump-sum tax could be found. the socially optimal quantity of the public good could not be obtained at a lower social cost. All choose t_N^* and therefore choose an optimum on their marginal social product curve, I_N^* at its Intersection with MSB_L.

Since ail redistributive governments finance the public good out of infra-marginal tax receipts, they equate their share of its marginal benefits to its direct resource cost of 1, so in the first quadrant marginal private benefits from G are parallel to the 45 degree line. Because the Lindahl-consensus and the super-encompassing interest take the deadweight losses directly into account in deciding how much public good to provide, they equate the (total) marginal sociai benefits of the public good to the total marginal sociai cost: the marginal private deadweight loss is not zero for them. as it was for the redistributive regimes.

Figure 10 reveals that redistributive regimes ignore a portion of the social benefits of G and reduce its marginai productivity schedule. and also that this reduction is greater the more they redistribute. Both the neglect of a portion of the social benefits of the public good and the erosion of its marginai productivity through redistribution reduce a regime's provision of G. Both of the foregoing effects reduce the amount of G redistributive regimes provide. even though the marginal private cost of the public good to any regime

[41] That is, S^u and $1/S^u = 1$. There is an alternative way of looking at the super-encompassing ruling interest: it can be considered to have a marginal private cost of 1 and an S= S as indicated in footnote 30. This gives the same answer.

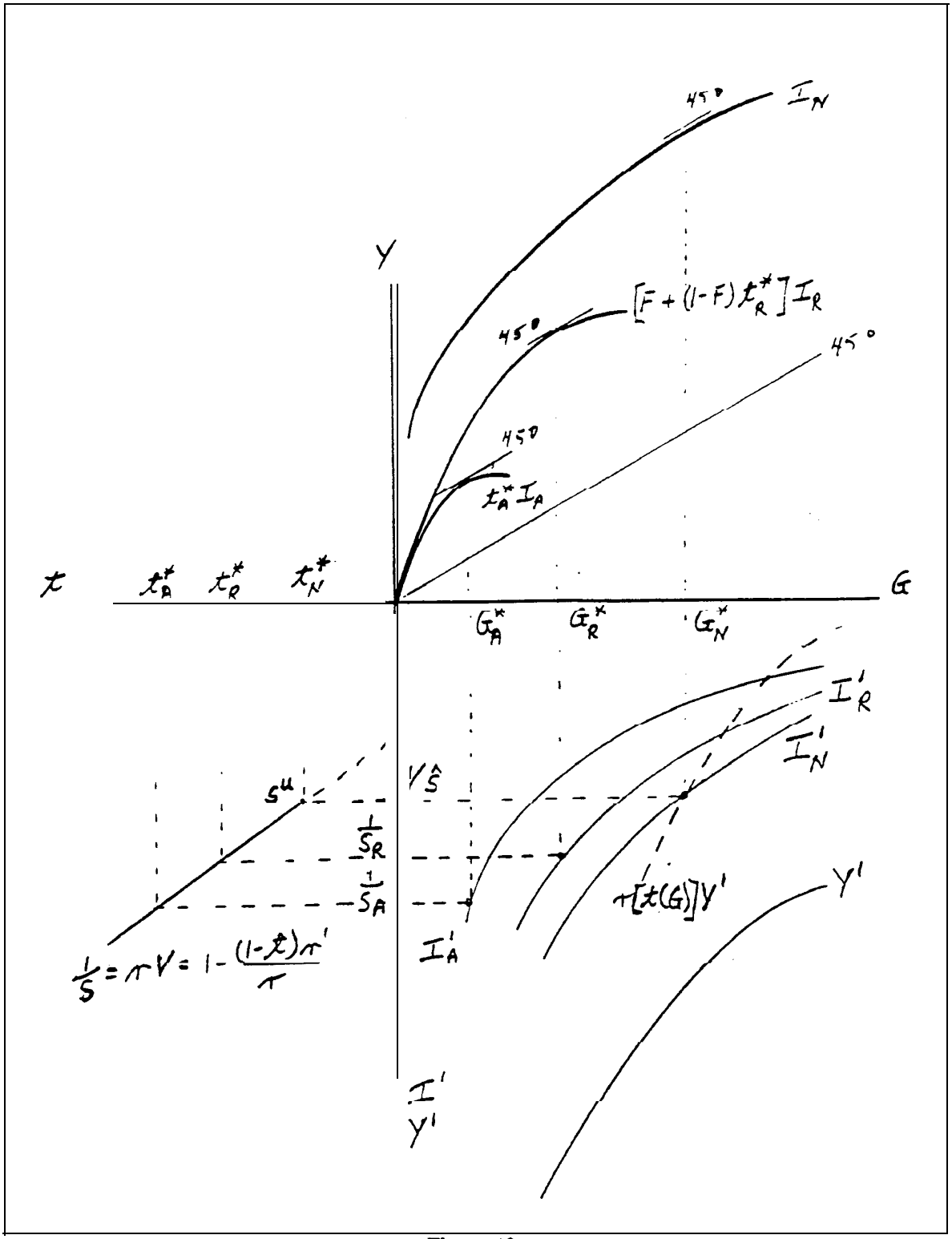
45°
I_N
Y
$[F + (1-F)t_R^*]I_R$
45°
45°
45°
$t_A^* I_A$
t
t_A^*
t_R^*
t_N^*
G
G_A^*
G_R^*
G_N^*
I'_R
I'_N
s^u
$1/\hat{s}$
$\frac{1}{s_R}$
$\frac{1}{s_A}$
$r[t(G)]Y'$
Y'
$\frac{1}{s} = rV = 1 - \frac{(1-t)r'}{r}$
I'_A
I'
Y'

Figure 10

that redistributes. however much or little it redistributes. is less than for non-redistributive regimes. The more a regime redistributes. moreover. the less G it supplies and the higher the marginal social value of its optimal provision of G.

The analysis of the optimal choices of alternative regimes isolates the crucial importance of two functions in the interplay between costs and benefits under alternative regimes: 1) the productivity of public good function Y(G), and 2) the deadweight loss function i- r(t). Evidently the more productive the public good and the less responsive the citizenry to taxation. the more we should expect a society to spend or "invest" in public good provision for any given form of governance.

More remarkably. however. the foregoing analysis implies an unexpected paradox that derives from the interaction between these two functions. The paradox is that the non redistributivc **socicties may tax so** much less than would an Autocrat or a redistributive democracy that they provide less public good than would either of the redistributive regimes. This can only happen if the function. V(t), which was first introduced by Equation 13b. is U-shaped. so that marginal deadweight losses from taxation are very high at low tax rates, and lower at some higher rates.

(13b repeated) $$Y'(G) = \frac{r(t) - (1-t)r'(t)}{r^2} \equiv V(t)$$

On the basis of this condition, comparison of the tax and public good supply outcomes under autocracy versus majority governance is straightforward and unambiguous. Equation (20) established that the Autocrat will tax more than the Majority; $t_A^* \geq t_R^*$. Second order conditions require that V(t) slope upward at each of these values of t; thus assuming V(t) to be increasing throughout this range -- i.e. barring great irregularities in V(t) – we know that the redistributivc majority must face a marginal **social product curve** that is further to the right or more productive that of the autocrat. and also take account of a larger proportion of the marginal social benefits of G. But this result does not necessarily extend to comparisons between the Autocrat and majoritarian regimes, on the one hand. and the non-redistributive regimes, on the other. The reason is that V(t) could be U-shaped. In this case V(t) may be larger at some low vaiues of

t than at some high values. making the marginal cost of me public good to the non-redistributive regimes extremely high, thereby making them choose less of me public good than even an autocratic regime. which has a marginal private cost of G of 1; in this case $G_N^* < G_A^*$. Discovery of further anomalies may await a more complete comparative static analysis of this system than presented here.[42]

VIII.

QUALIFICATIONS AND IMPLICATIONS FOR FURTHER RESEARCH

In the interest of unity and manageable length, the foregoing analysis has abstracted. from some most important aspects of the matter at hand. Most notabiy, it has abstracted from the great problems that arise when coercive power is dispersed among many individuals or groups, each with only a narrow or minuscule interest in society, and it has only mentioned in passing the problems that arise from short time horizons.

With respect to narrow interests. we have not analyzed. for example, the problems that arise when individuals have only a tiny stake in the success of society at large, yet may in me aggregate exact significant tribute from society. Criminal behavior is an example: the typical criminal in this country obviously does not have any incentive to moderate his depredations because of his stake in the society. Thus the invisible hand does not, of course, prevent crime.

Similarly, me foregoing models do not explain me social losses from special-interest groups, each of which constitutes only a minute part of the economy and thus has only a narrow interest in society. Thus these groups have virtually no incentive to limit the deadweight losses they impose upon society as they use their political influence or collusive power in their own interest. These narrow special interests face incentives far more perverse for society than those that confront a secure stationary bandit. To the extent

[42] McGuire and Olson (1994) present an example of this paradoxical result.

that such interests prevail in a democracy, the democracy will perform very much worse than the majoritarian redistributive democracy or the super-encompassing democracy depicted in this paper. The neglect of this aspect of the matter has tended to bias our analysis in favor of democracy and against strong autocrats.

By giving only passing attention to short time horizons. we have. on the other hand. tended to bias the analysis in favor of autocracy. An autocracy is by definition a society where one person is above the law. When that person has a short time horizon he will gain from confiscating all capital goods whose tax-yields over the horizon are less than their capital value: he wiii. in effect. revert to roving banditry. Under a democratic rule of law, there is no individual who can use the power of the state to seize assets for himself. Thus our analysis here has ignored the inherent connection between democratic-(or at least non-autocratic) governance and individual rights, especially with respect to private property and contract enforcement.

Thus this paper is very far indeed from being sufficient to fill in the gap in the economics literature with which we began. Nonetheless. it does offer. with the simple r(t)Y(G) analytical machinery, a tool of thought that can help in generating the needed literature. This analytical framework may, we think, also prove adaptable to other types problems in which it is necessary to deal simultaneously with an output-increasing force and a related output-depressing factor.

This paper has also demonstrated rigorously that there is a hidden hand that leads encompassing and stable interests with unquestioned coercive power to act, to a significant and surprising degree, in the interests of the society and of those who are subject to their force. The outcome from stationary banditry is not nearly as bad as might initially have been supposed. and thus the analysis helps explain the puzzling amount of human progress that has occurred under self-interested and extravagant autocrats.

The clearly superior results that must emerge from an optimizing redistributive majority with a stake

in the market economy also have great practical significance. It was once generally believed that democracy with anything approaching universal adult suffrage would inevitably lead to the abolition of private property: a low-income majority would. it was thought, obviously gain from confiscating all the property of those with wealth and redistributing to themselves. In fact. there is not a single democracy that has eliminated private property. The present argument suggests that the citizenry with less than median levels of income have an encompassing interest in the society: they earn a significant percentage of the national income in wages and. when they control the tax and transfer system of the society as well. this gives them a large stake in the society. If. as is plausible. the deadweight losses from the elimination of private property would be substantial. it is easy to see why even that part of the social loss from the abolition of property that would be borne by a low-mcome majority would give that majority an incentive to avoid confiscating all wealth.

Some observers of economic development, especially in East Asia, argue that a "hard" state -- one that does not alter its agenda because of pressures from particular industries or occupations -- is favorable to economic development. To the extent that this argument has a theortical basis. it is the theory offered here.

The argument here also helps to explain why, in me United States. Presidents. irrespective of party, seem to have a iesser propensity to favor pork barrel projects and special-interest measures than do members of Congress. again irrespective of party. No President cm be re-elected without pleasing a nationally encompassing constituency, but that is not true of the individual member of Congress. nor (given the weakness of political parties in this country) of any large optimizing majority in the Congress. The argument here also suggests that there is much to be said for a two-party system with disciplined political parties. since large and disciplined parties may approximate optimizing entities with encompassing interests. but small or weak political parties do not.

Finally, there can be no doubt that the hidden hand does lead to the benign -- even the beneficent -

use of force when there is a super-encompassing interest. and that super-encompassing interests can readily arise. A super-encompassing majority, even when it thinks only of itself and has no concern for the losses of the minority, abstains from redistribution and treats the majority as well as it treats itself. Economic research should not ignore this remarkable phenomenon or the other ways in which encompassing interests bring society the blessings of the invisible hand.

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