A Solution to the Problem of Externalities When Agents Are Well-Informed

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A Solution to the Problem of Externalities When Agents Are Well-Informed

By Hal R. Varian*

I describe a class of simple two-stage mechanisms that implement efficient allocations as subgame-perfect equilibria for economic environments involving externalities. These mechanisms, known as compensation mechanisms, solve a wide variety of externalities problems including implementation of Lindahl allocations, regulation of monopoly, and efficient solutions to the prisoner's dilemma. (*JEL* D62, D71)

Consider an economic environment in which agents take actions that impose benefits or costs on other agents. The agents involved know the relevant technology and the tastes of all other agents. However the "regulator," who has the responsibility for determining the final allocation, does not have this information. How can the regulator design a mechanism that will implement an efficient allocation?

In this paper I describe a class of simple two-stage games whose subgame-perfect equilibria implement efficient allocations in this sort of environment. In addition to implementing efficient outcomes, the mechanisms also achieve desirable distributional goals. In the case of public goods, the mechanisms implement Lindahl allocations; in the case of a negative externality, the injured parties are compensated. Because payment of "compensation" is an important feature of the mechanisms I describe, I refer to the general class of mechanisms as *compensation mechanisms*. These mechanisms appear to work in a broad variety of economic environments and do not involve substantial restrictions on tastes or technology. They are also quite simple to describe and analyze.

The fact that multistage games and subgame-perfect equilibria may be useful in implementation problems was suggested by Vincent Crawford (1979) and Hervé Moulin (1979, 1981) and was extensively analyzed by John Moore and Rafael Repullo (1988). Moore and Repullo show that in economic environments, almost any choice rule can be implemented by subgame-perfect equilibria. However, as Moore and Repullo (1988 p. 1198) point out, "...the mechanisms we construct ... are far from simple; agents move simultaneously at each stage and their strategy sets are unconvincingly rich. We present such mechanisms to show what is possible, not what is realistic." Moore and Repullo also show that in certain "economic environments" it is possible to use somewhat simpler mechanisms. However, the compensation mechanism appears to be much simpler than the examples Moore and Repullo (1988) examined. For a thorough review of the recent literature on implementation in complete information environments, see John Moore (1992).

It should be emphasized that the solution concept of subgame perfection requires the agents to be informed about the technology and tastes of the other agents. This is, of course, more restrictive than one would like. However, there is a broad set of cases for

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which this assumption may be plausible. For example, consider a group of agents who must design a constitution that describes a mechanism to make group decisions for problems that will arise in the future. At the time the mechanism is chosen, the agents may not know the relevant tastes and technologies, but they will know these things when the mechanism is actually used. In this circumstance, the compensation mechanism may be a useful mechanism. See Moore and Repullo (1988) and Eric Maskin (1985) for further discussion of these issues.

I first describe a very simple example of the compensation mechanism in a two-agent externalities problem and discuss in an intuitive way why the method works. The following sections show how the method can be extended to work in more general environments.

I. A Simple Example of the Compensation Mechanism

Consider the following externality problem involving two agents. For simplicity, think of each agent as a profit-maximizing firm. Firm 1 produces output x so as to maximize profit:

$$\pi_1 = rx - c(x)$$

where r is the competitive price of output and $c(x)$ is a differentiable, positive, increasing, and convex cost function.

Firm 1's choice of output imposes an externality on firm 2; in particular, firm 2's profits are

$$\pi_2 = -e(x)$$

where $e(x)$ is a differentiable, positive, increasing, and convex function of x. All of this information is known to both agents but is not known by the regulator. In general, the level of output chosen by firm 1 will not be efficient, since firm 1 ignores the social cost its choice imposes on firm 2. There are three classic solutions to this problem of externalities.

One class of solutions, associated with Ronald Coase (1960) involves negotiation between the agents. Coase claims that if transactions costs are zero and property rights are well defined, agents should be able to negotiate their way to an efficient outcome. But this is an incomplete solution to the problem of externalities since Coase does not describe a specific mechanism for negotiation. The compensation mechanism described below provides a structure for such negotiations and therefore can be viewed as being complementary to the Coase approach.

A second class of solutions, associated with Kenneth Arrow (1970), involves setting up a market for the externality. If a firm produces pollution that harms another firm, then a competitive market for the right to pollute may allow for an efficient outcome. From the Coasian point of view, a competitive market is a particular institution that allows agents to "negotiate" their way to an efficient outcome. However, as Arrow points out, the market for allocating a particular externality may be very thin—in many cases of interest such markets involve only two participants.

However, a thin market does not necessarily mean a noncompetitive market. There are both theoretical and empirical reasons to believe that certain kinds of market interaction can be competitive even though only a small number of agents are involved. For example, a Bertrand model of oligopoly yields a more or less competitive outcome with only two firms. The real-life implementation of Bertrand competition—competitive bidding—seems to work reasonably well, even with only a small number of bidders. This suggests that markets for externalities with price-setting agents may be a useful model for negotiations among agents. This is a key insight behind the compensation mechanism.

A third class of solutions, associated with A. C. Pigou (1920), involves intervention by a regulator who imposes a Pigovian tax. The difficulty with this solution is that it requires the regulator to be able to compute the correct level of the Pigovian tax; in many cases the regulator may not have access to this information, so the Pigovian solution is also incomplete. The compensation mechanism solves this problem, since it gives the regulator a method to induce the participants to reveal the information necessary to construct the optimal Pigovian tax.

Returning to the example, note that if the regulator had full information, internalizing the externality would be easy. One solution would be for the regulator to impose the costs of the externality on firm 1 by charging it a "tax" of $e(x)$ if it produces x units of output. Firm 1 would then solve the problem

$$\max_x rx - c(x) - e(x).$$

Let x^* be solution to this problem; then x^* satisfies the first-order condition

$$r - c'(x^*) - e'(x^*) = 0.$$

Because of the curvature assumptions on $e(x)$, the regulator could just as well set a "Pigovian tax," $p^* = e'(x^*)$ and let firm 1 solve the problem

$$\max_x rx - c(x) - p^*x.$$

However, I have assumed that the regulator does not know the externality cost function and therefore cannot determine the appropriate value of p^*. The regulator's problem is to design a mechanism that will induce the agents to reveal their information about the magnitude of the externality and achieve an efficient level of production. Here is a version of the compensation mechanism that solves the regulator's problem.

Announcement stage.—Firm 1 and 2 simultaneously announce the magnitude of the appropriate Pigovian tax; denote the announcement of firm 1 by p_1 and the announcement of firm 2 by p_2.

Choice stage.—The regulator makes side-payments to the firms so that the two firms face profit-maximization problems:

$$\Pi_1 = rx - c(x) - p_2 x - \alpha_1(p_1 - p_2)^2$$

$$\Pi_2 = p_1 x - e(x).$$

The parameter $\alpha_1 > 0$ is of arbitrary magnitude.

In this mechanism, firm 1 is forced to pay a tax based on the marginal social cost of the externality as reported by firm 2, and firm 2 receives compensation based on the marginal social cost of the externality as reported by firm 1. Firm 1 must also pay a penalty if it reports a different marginal social cost than firm 2 reports. Any penalty that is minimized when the reports are the same will work, but I have chosen a quadratic penalty for simplicity. Note in particular that the penalty can be arbitrarily small.

II. Analysis of the Compensation Mechanism

There are many Nash equilibria of this game; essentially any triple (p_1, p_2, x) such that $p_1 = p_2$ and x maximizes firm 1's objective function is a Nash equilibrium. However, if the stronger concept of subgame-perfect equilibrium is used, there is a much smaller set of equilibria. In fact, the *unique* subgame-perfect equilibrium of this game has each agent reporting $p_1 = p_2 = p^*$ and firm 1 producing the efficient amount of output.

In order to verify this, one must work backwards through the game. Begin with the choice stage. Firm 1 maximizes its profits, given the Pigovian tax announced in stage 1, which implies that firm 1 will choose x to satisfy the first-order condition

$$r = c'(x) + p_2. \tag{1}$$

This determines the optimal choice, x, as a function of p_2, which I denote by $x(p_2)$. Note that $x'(p_2) < 0$; the higher the tax that firm 2 announces, the less firm 1 will want to produce.

I next examine the price-setting stage of the game. I first examine firm 1's choice problem. If firm 1 believes that firm 2 will announce p_2, then firm 1 will want to announce

$$p_1 = p_2. \tag{2}$$

This is clear since p_1 only influences firm 1's payoff through the penalty term, and the penalty is minimized when $p_1 = p_2$.

Consider now firm 2's pricing decision. Although firm 2's announcement has no *direct* effect on firm 2's profits, it does have an *indirect* effect through the influence of p_2 on firm 1's output choice in stage 2. Differentiating the profit function of firm 2 with respect to p_2, and setting it equal to zero yields

$$(3)\quad \Pi_2'(p_2) = [p_1 - e'(x)]\,x'(p_2) = 0.$$

Since $x'(p_2) < 0$, then it must be that $p_1 = e'(x)$.

Combining (1), (2), and (3) yields

$$r = c'(x) + e'(x)$$

which is the condition for social optimality. Hence, the unique subgame-perfect equilibrium to this game involves firm 1 producing the socially optimal amount of output.[1]

III. Why the Compensation Mechanism Works

The intuition behind the mechanism is not particularly difficult. Firm 2 effectively chooses x by setting the price firm 1 faces. If there is to be an equilibrium, it must be that $p_1 = e'(x)$; otherwise firm 2 would want to change its announcement of p_2 in order to induce firm 1 to change x. Furthermore, firm 1 will always want to set $p_1 = p_2$ so as to minimize its penalty. The only configuration compatible with these conditions is the efficient outcome.

For example, suppose that firm 1 thinks that firm 2 will report a large price for the externality. Then, since firm 1 is penalized if it announces something different from firm 2, firm 1 will also want to announce a large price. If firm 1 announces a large price, firm 2 will be "overcompensated" for the externality—so it will want firm 1 to produce a large amount of output. But the only way firm 2 can give firm 1 an incentive to produce a large amount of output is by reporting a *small* price for the externality. This contradicts the original assumption that firm 1 thinks that firm 2 will report a large price for the externality. The only equilibrium for the mechanisms occurs if firm 2 is just compensated (on the margin) for the cost that firm 1 imposes on it; at this point firm 2 does not want firm 1 to increase or decrease its level of production.

IV. Extensions of the Basic Example

A. *Balance*

The compensation mechanism, in the form presented above, is balanced in equilibrium but not out of equilibrium. However, if there are at least three agents it is easy to choose transfers to balance the mechanism. As Moore and Repullo (1988) point out, one can simply distribute the surplus or deficit generated by each agent's choice among the other agents. Since this lump-sum distribution is independent of agent i's choice, there are no resulting incentive effects.[2]

To see how this works in the simple example considered above, I now suppose that agent 1 imposes an externality on agents 2 and 3. Use the notation p_{ij}^k to represent the price announced by agent k that measures (in equilibrium) the marginal cost that agent j's choice imposes on agent i.

The basic compensation mechanism for this problem has payments of the form

$$\begin{aligned}\Pi_1 &= rx - c(x) - [p_{21}^2 + p_{31}^3]x \\ &\qquad - \|p_{21}^1 - p_{21}^2\| - \|p_{31}^1 - p_{31}^3\| \\ \Pi_2 &= p_{21}^1 x - e_2(x) \\ \Pi_3 &= p_{31}^1 x - e_3(x).\end{aligned}$$

[1] I have not considered the possibility of mixed strategies. However, since the stage-1 game is supermodular and has a unique pure-strategy equilibrium, the results of Paul Milgrom and John Roberts (1990) can be applied to show that there are no mixed-strategy equilibria.

[2] This idea seems to have been first used by Theodore Groves and John Ledyard (1977). Since then it has been used by a number of other authors.

If payments are distributed so as to balance the budget out of equilibrium, the payoffs become

$$(4)\quad \Pi_1 = rx - c(x) - [p_{21}^2 + p_{31}^3]x - \|p_{21}^1 - p_{21}^2\| - \|p_{31}^1 - p_{31}^3\|$$

$$\Pi_2 = p_{21}^1 x - e_2(x) + [p_{31}^3 - p_{31}^1]x + \|p_{31}^1 - p_{31}^3\|$$

$$\Pi_3 = p_{31}^1 x - e_3(x) + [p_{21}^2 - p_{21}^1]x + \|p_{21}^1 - p_{21}^2\|.$$

Straightforward addition shows that this game is balanced. Using the same sort of arguments as before, it is possible to verify that the unique equilibrium of this mechanism is the efficient outcome. In fact, it is not necessary to have penalty terms when there are more than two agents. To see this, set the penalty terms in (4) equal to zero and differentiate the relevant objective functions with respect to the choice variables x, p_{21}^2, and p_{21}^3:

$$r - c'(x) - [p_{21}^2 + p_{31}^3] = 0$$

$$[p_{21}^1 - e_2'(x) + p_{31}^3 - p_{31}^1]x'(p_{21}^2 + p_{21}^3) = 0$$

$$[p_{31}^1 - e_3'(x) + p_{21}^2 - p_{21}^1]x'(p_{21}^2 + p_{31}^3) = 0.$$

Since the derivatives of x must be nonzero due to strict convexity of the cost function, the terms in brackets must be zero. Adding the bracketed expressions in the last two equations together and substituting into the first equation shows that the equilibrium is efficient.

Yet a third way to balance the mechanism is to allow agent 2 to name the cost that agent 1 imposes on agent 3 and vice versa. This is a bit less natural in terms of the information requirements, but it yields a very simple mechanism:

$$\Pi_1 = rx - c(x) - (p_{21}^3 + p_{31}^2)x$$

$$\Pi_2 = p_{21}^3 x - e_2(x)$$

$$\Pi_3 = p_{31}^2 x - e_3(x).$$

Differentiating with respect to each of the choice variables as above shows that the equilibrium of this mechanism is efficient, and it is obviously balanced. Each of these ways of balancing the compensation mechanism works in general as I will demonstrate below.

B. *Adjusting to Equilibrium*

There is a natural adjustment process for the compensation mechanism that will lead naive agents to the subgame-perfect equilibrium. Suppose that two agents play the game repeatedly. In period $t+1$, agent 1 sets p_1 to be whatever price agent 2 announced last period, and agent 2 moves p_2 in a direction that increases its profits if agent 1 sets the same price as it did last period. In the choice stage, agent 1 chooses output to maximize profits, given the current prices. This leads to a simple discrete dynamical system:

$$(5)\quad p_1(t+1) = p_2(t)$$

$$p_2(t+1) = p_2(t) - \gamma[p_1(t) - e'(x(p_2(t)))].$$

Here $\gamma > 0$ is a speed-of-adjustment parameter. The differential-equation analogue of this system is

$$(6)\quad \dot{p}_1 = p_2 - p_1$$

$$\dot{p}_2 = -\gamma[p_1 - e'(x(p_2))].$$

It is easy to show that that (6) is locally stable; the difference-equation version described in (5), will be locally stable if γ is small enough to avoid "overshooting." Note that if the agents use this adjustment procedure neither one needs to know anything about the other agent's technology. All that information is subsumed in the price messages that the agents send back and forth.[3]

C. *Nonlinear Taxes and Compensation Functions*

The basic compensation mechanism described above uses linear pricing. Linear prices are fine in a convex environment, but if the environment is not convex, linear prices will not in general be able to support efficient allocations. However, this difficulty is no problem for a suitable generalization of the compensation mechanism.

Announcement stage.—Firm 1 and 2 each announce the externality cost function for firm 2. Call these announcements $e_1(\cdot)$ and $e_2(\cdot)$.

Choice stage.—Firm 1 chooses x, and each firm receives payoffs given by

$$\Pi_1(x) = rx - c(x) - e_2(x) - \|e_1 - e_2\|$$

$$\Pi_2(x) = e_1(x) - e(x).$$

Here $\|e_1 - e_2\|$ signifies any norm in the appropriate function space. All that is required is that it is minimized when both agents report the same function.

To see that this works, simply note that in equilibrium firm 1 will always want to report the same function as firm 2, so $e_1(x) \equiv e_2(x)$. Maximization of profit by firm 1 in the choice stage implies

$$rx^* - c(x^*) - e_2(x^*) \geq rx - c(x) - e_2(x) \qquad \text{for all } x. \tag{7}$$

However, in the announcement stage, firm 2 can induce any level of x that it wants by appropriate choice of the function e_2. Hence, the equilibrium choice of x must also maximize firm 2's profits:

$$e_1(x^*) - e(x^*) \geq e_1(x) - e(x) \qquad \text{for all } x. \tag{8}$$

Adding (7) and (8) together, and using the fact that $e_1(x) \equiv e_2(x)$ in equilibrium, one obtains

$$rx^* - c(x^*) - e(x^*) \geq rx - c(x) - e(x) \qquad \text{for all } x \tag{9}$$

which shows that x^* is the socially optimal amount.

This argument shows that all equilibria of the mechanism are efficient. However, in general there will be many equilibria of this game. To see this, observe that if e_1 and e_2 are equilibrium announcements, so are $e_1 + F$ and $e_2 + F$ for arbitrary values of F. In order to get uniqueness of equilibrium, it is necessary to restrict the class of allowable messages.[4]

One way to do this is to parameterize the cost function.[5] Suppose that the set of possible externality costs is $e(x,t)$ where t is a real-valued index of type. Suppose that the true type of firm 2 is t_0. In the announcement stage of the game, each firm simply announces the type of firm 2, and firm 1 pays a penalty if its announcement is different from that of firm 2. If t_1 is firm 1's announcement and t_2 is firm 2's announcement, the payoffs will be

$$\Pi_1(x) = rx - c(x) - e(x,t_2) - (t_1 - t_2)^2$$

$$\Pi_2(x) = e(x,t_1) - e(x,t_0)$$

[3] This is, of course, a very special adjustment process. However, Milgrom and Roberts (1991) show that for dominance-solvable games every adjustment process consistent with adaptive or sophisticated learning converges to the dominance-solvable equilibrium. Hence it may be reasonably be expected that a wide class of adjustment mechanisms will work when the second-stage game is dominance-solvable (as it is in this case).

[4] One could also refine the solution concept. In this example it may be reasonable for agent 1 to assume that agent 2 will announce the largest possible value of F consistent with agent 1's participation.

[5] In the convex case one can think of the efficiency prices as being a particularly convenient parameterization for the type space.

Differentiating with respect to x, t_1, and t_2, one has

$$r - c'(x) - \frac{\partial e(x,t_2)}{\partial x} = 0$$

$$t_1 - t_2 = 0$$

$$\left[\frac{\partial e(x,t_1)}{\partial x} - \frac{\partial e(x,t_0)}{\partial x}\right] x'(t_2) = 0.$$

Assuming that $x'(t_2) \neq 0$, it is easy to see that these equations imply

$$r = c'(x) + \frac{\partial e(x,t_0)}{\partial x}$$

which is the condition for social efficiency.

Of course, this argument requires sufficient regularity so that the various derivatives exist. If the environment is not suitably convex, an argument can be constructed along the lines given above in inequalities (7)–(9). Note that in a nonconvex environment one needs to assume that firm 2 can induce firm 1 to choose any desired level of x by choosing an appropriate value of t_2; this is simply a "global" version of the assumption that $x'(t_2) \neq 0$.

V. A General Externalities Problem

The externalities problem examined up until now is rather special. Only one agent makes a choice, and both agents have quasi-linear objective functions so there are no income effects. In this section I consider a more general externality problem. For simplicity, I continue to examine a two-agent problem, but the argument is easily generalized to n agents.

In the general model there are two choices, x_1 and x_2, and one transferable good, y. Agent i makes choice x_i, and has a quasi-concave utility function $u_i(x_1, x_2, y_i)$. Initially, agent i has w_i units of the transferable good, which can be thought of as money.

A. *Efficient Choices*

In the absence of any transfers between the agents, agent i will choose x_i to maximize his own utility. The first-order condition characterizing these choices can be written as

$$\frac{\partial u_1/\partial x_1}{\partial u_1/\partial y_1} = 0$$

$$\frac{\partial u_2/\partial x_2}{\partial u_2/\partial y_2} = 0.$$

By contrast, an efficient allocation of choices must satisfy the first-order conditions

$$\frac{\partial u_1/\partial x_1}{\partial u_1/\partial y_1} + \frac{\partial u_2/\partial x_1}{\partial u_2/\partial y_2} = 0 \tag{10}$$

$$\frac{\partial u_2/\partial x_2}{\partial u_2/\partial y_2} + \frac{\partial u_1/\partial x_2}{\partial u_1/\partial y_1} = 0.$$

These conditions simply require that the sum of the marginal willingnesses to pay for activity i should be zero.

Define

$$p_{ij} = \frac{\partial u_i/\partial x_j}{\partial u_i/\partial y_i} \qquad \text{for } i \neq j.$$

Then one can write the efficiency conditions (10) as

$$\frac{\partial u_1/\partial x_1}{\partial u_1/\partial y_1} + p_{21} = 0$$

$$\frac{\partial u_2/\partial x_2}{\partial u_2/\partial y_2} + p_{12} = 0.$$

This form suggests that the efficient allocation can be achieved if each agent faces the correct "price" for his choice. The problem is how to determine the correct price. Here is a description of the general compensation mechanism that solves this problem.

Announcement stage.—Agent 1 announces p_{12}^1 and p_{21}^1, and agent 2 announces p_{12}^2 and p_{21}^2.

Choice stage.—Each agent chooses x_i and y_i so as to maximize utility subject to a budget constraint:

$$\max_{x_1, y_1} u_1(x_1, x_2, y_1)$$

such that

$$p_{21}^2 x_1 + y_1 = w_1 + p_{12}^2 x_2 - \|p_{21}^1 - p_{21}^2\|$$

and

$$\max_{x_2, y_2} u_2(x_1, x_2, y_2)$$

such that

$$p_{12}^1 x_2 + y_2 = w_2 + p_{21}^1 x_1 - \|p_{12}^2 - p_{12}^1\|.$$

I show below that the subgame-perfect equilibria of this game are precisely the efficient allocations that satisfy the budget constraints. However, before providing that proof, I will make a few observations.

First, each agent i is facing a price, p_{ji}^j, for his own choice x_i. He is also receiving compensation $p_{ij}^j x_j$ for the choice that the other agent makes. Both prices p_{ji}^j and p_{ij}^j are set by the *other* agent. Each agent i also pays a penalty based on how different his announced price, p_{ji}^i, is from the price that the other agent j announced for i's choice, p_{ji}^j.

As one might suspect, in equilibrium it must be that $p_{ji}^i = p_{ji}^j$. This means that no penalties will be paid and that the payment made by agent i for his action will just be equal to the compensation paid to agent j. Hence, in equilibrium, the aggregate budget constraint will balance.

I will now show that the equilibrium of this game must be efficient. I provide two proofs. The first proof simply involves writing down the first-order conditions for the utility-maximization problems. There are three choice variables for each of the two agents, x_i, p_{ij}^i, and p_{ji}^i, so there are six first-order conditions. Choosing the quadratic norm for computational simplicity, the first-order conditions are:

$$\frac{\partial u_1}{\partial x_1} - \frac{\partial u_1}{\partial y_1} p_{21}^2 = 0 \tag{11}$$

$$\left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_1}{\partial y_1} p_{21}^2\right) \frac{\partial x_1}{\partial p_{12}^1} + \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_1}{\partial y_1} p_{12}^2\right) \frac{\partial x_2}{\partial p_{12}^1} = 0 \tag{12}$$

$$\left(\frac{\partial u_1}{\partial x_1} - \frac{\partial u_1}{\partial y_1} p_{21}^2\right) \frac{\partial x_1}{\partial p_{21}^1} + \left(\frac{\partial u_1}{\partial x_2} + \frac{\partial u_1}{\partial y_1} p_{12}^2\right) \frac{\partial x_2}{\partial p_{21}^1} - 2\frac{\partial u_1}{\partial y}(p_{21}^1 - p_{21}^2) = 0 \tag{13}$$

$$\frac{\partial u_2}{\partial x_2} - \frac{\partial u_2}{\partial y_2} p_{12}^1 = 0 \tag{14}$$

$$\left(\frac{\partial u_2}{\partial x_2} - \frac{\partial u_2}{\partial y_2} p_{12}^1\right) \frac{\partial x_2}{\partial p_{21}^2} + \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_2}{\partial y_2} p_{21}^1\right) \frac{\partial x_1}{\partial p_{21}^2} = 0 \tag{15}$$

$$\left(\frac{\partial u_2}{\partial x_2} - \frac{\partial u_2}{\partial y_2} p_{12}^1\right) \frac{\partial x_2}{\partial p_{12}^2} + \left(\frac{\partial u_2}{\partial x_1} + \frac{\partial u_2}{\partial y_2} p_{21}^1\right) \frac{\partial x_1}{\partial p_{12}^2} - 2\frac{\partial u_2}{\partial y_2}(p_{12}^2 - p_{12}^1) = 0. \tag{16}$$

Here I have assumed that the equilibrium choices in the second stage are differentiable functions of the price announcements made in the first stage. As I show in the next section, this is not necessary for the argument, but it does help to see why the method works. Note that when agent 1 chooses p_{12}^1, for example, he recognizes that both his own choice, x_1, and the other agent's choice, x_2, may respond to changes in p_{12}^1.

One must assume that $\partial x_2/\partial p_{12}^1$ and $\partial x_1/\partial p_{21}^2$ are not zero. Now simply observe that this assumption and (11)–(13) together imply that $p_{21}^1 = p_{21}^2$. Similarly, (14)–(16) imply that $p_{12}^1 = p_{12}^2$. Finally, combine (11) and (15) to get

$$\frac{\partial u_1/\partial x_1}{\partial u_1/\partial y_1} + \frac{\partial u_2/\partial x_1}{\partial u_2/\partial y_2} = 0$$

and combine (12) and (14) to get

$$\frac{\partial u_2/\partial x_2}{\partial u_2/\partial x_2} + \frac{\partial u_1/\partial x_2}{\partial u_1/\partial y_1} = 0.$$

These are precisely the first-order conditions given in (10). Therefore, the subgame-perfect equilibrium is efficient.

Note that the equilibrium is a *particular* efficient allocation, namely, one that satisfies the "natural" budget constraint involving the efficiency prices. In general, such allocations will be a small subset of all efficient allocations. By analogy with the public-goods literature, I call these allocations *generalized Lindahl allocations*. I show below that when the externalities problem is a public-goods problem, the prices in the compensation mechanism are Lindahl prices.

B. *A More General Proof*

The above proof shows clearly why an equilibrium of the compensation mechanism must be an efficient allocation. However, being a calculus proof, it does not deal very well with corner solutions, additional constraints, nondifferentiabilities, and the like. Here is another argument that handles these difficulties easily.

I need one assumption for the proof, an invertibility assumption that says that each agent can set a price for the other agent that will induce the other agent to make any desired choice. That is, if agent 1 would like agent 2 to make some choice, there is some price that agent 1 can set that will induce agent 2 to make this choice. This is analogous to the assumption that $\partial x_2/\partial p_{12}^1 \neq 0$ in the previous proof. As in the differentiable case, the demand functions only need to be locally invertible if the environment is suitably convex.

ASSUMPTION 1 (Local Invertibility): *Let $x = (x_1, x_2)$ be the outcome of some set of price announcements. Let $\hat{x}_i$ be a choice close to x_i that agent j prefers to x_i. Then there is some $\hat{p}_{ji}^j$ that agent j can announce that will make $\hat{x}_i$ an optimal choice for agent i.*

Local invertibility says that agent i can manipulate agent j's choices through agent i's price announcements. This is a very weak assumption. If the agents' demands are differentiable functions of price with nonzero derivatives than the inverse-function theorem implies local invertibility.

THEOREM 1: *Let preferences be convex and continuous. Then every subgame-perfect equilibrium of the compensation mechanism is Pareto efficient.*

PROOF:

Let (x, y, p) be a subgame-perfect equilibrium of the compensation mechanism. First I show that in equilibrium $p_{21}^1 = p_{21}^2$. To see this, consider the agents' budget constraints:

$$p_{21}^2 x_1 + y_1 = w_1 + p_{12}^2 x_2 - \| p_{21}^1 - p_{21}^2 \|$$

$$p_{12}^1 x_2 + y_2 = w_2 + p_{21}^1 x_1 - \| p_{12}^2 - p_{12}^1 \|.$$

Note that agent 1 can influence agent 2's choice of x_2 through both the "income term," $p_{21}^1 x_1$, and the "price term," p_{12}^1. However, by Assumption 1, any choice of x_2 that can be achieved through the income term can also be achieved by an appropriate choice of the price term, p_{12}^1.

Suppose that there were an equilibrium in which $p_{21}^1 \neq p_{21}^2$. Let agent 1 set $p_{21}^1 = p_{21}^2$ and adjust p_{12}^1 so as to induce the original equilibrium value of x_2. This must reduce agent 1's penalty and thereby increase agent 1's utility. This contradicts the assumption that there is an equilibrium. It follows that an equilibrium must exhibit zero penalty terms for all agents.

Suppose now that (x', y') is a feasible allocation that Pareto-dominates the equilibrium allocation. I will show that the existence of such an allocation leads to a contradiction. By convexity and continuity of preferences, one can assume that (x', y') is arbitrarily close to the equilibrium allocation. According to Assumption 1, agent 1 can induce agent 2 to choose x'_2 simply by choosing an appropriate level of p^1_{12}; furthermore, agent 1 can directly choose (x'_1, y'_1). If agent 1 decides *not* to choose this preferred allocation, it must be because it lies outside his budget set. The same argument applies to agent 2, and this gives the inequalities

$$p^2_{21}x'_1 + y'_1 > w_1 + p^2_{12}x'_2$$

$$p^1_{12}x'_2 + y'_2 > w_2 + p^1_{21}x'_1.$$

Summing these inequalities and using the fact that $p^i_{ji} = p^j_{ji}$, one obtains

$$y'_1 + y'_2 > w_1 + w_2$$

which shows that the Pareto-dominating allocation must be infeasible.

Note that the logic of this proof is quite general. In particular, the taxation and compensation functions do not need to be linear functions. All that is necessary is that each agent can manipulate the other agent's choice without incurring any cost himself. If the economic environment is convex, one only needs local invertibility; if the economic environment is nonconvex, global invertibility may be necessary.

It can also be shown that, if the environment is convex, any Pareto-efficient allocation is an equilibrium of this game for a suitable choice of initial endowments. The proof is a simple variation on the second welfare theorem and is omitted for the sake of brevity.

VI. Balancing the Mechanism

In the simple example discussed above, the compensation mechanism can be balanced by distributing the budget surplus generated by each agent among the other agents. The same procedure works in general; here I examine the simple case of quasi-linear utility.

The appropriate payoff to agent i is

$$u_i(x) + \sum_{j=1}^{n} B_{ij} - T_i$$

where

$$B_{ij} = p^j_{ij}x_j - p^j_{ji}x_i - \| p^i_{ji} - p^j_{ji} \|$$

$$T_i = \frac{1}{n-2} \sum_{k \neq i} \sum_{j \neq i} B_{kj}$$

$$= \frac{1}{n-2} \left[\sum_{k=1}^{n} \sum_{j=1}^{n} B_{kj} - \sum_{j=1}^{n} B_{ij} - \sum_{j=1}^{n} B_{ji} \right].$$

It is obvious that $B_{ii} = 0$, and it is not hard to show that

$$\sum_{i=1}^{n} \left[\sum_{j=1}^{n} B_{ij} - T_i \right] = 0. \tag{17}$$

Note that B_{ij} depends on the vector of prices and the vector of choices. It is important to observe that the only price term that agent i determines is the price in the penalty term, p^i_{ji}; all other prices are independent of i's choices. To emphasize the fact that the payment depends on x, I write $B_{ij}(x)$ in the following paragraphs.

By local invertibility, each agent can induce any desired allocation in the choice stage by choosing the appropriate prices in the announcement stage. Therefore, a subgame-perfect equilibrium allocation must satisfy

$$u_i(x^*) + \sum_{j=1}^{n} B_{ij}(x^*) - T_i(x^*)$$

$$\geq u_i(x) + \sum_{j=1}^{n} B_{ij}(x) - T_i(x)$$

for all x. Summing over the agents and using equation (17) yields

$$\sum_{i=1}^{n} u_i(x^*) \geq \sum_{i=1}^{n} u_i(x)$$

for all x, which shows that the subgame-perfect equilibrium is Pareto efficient.

Note that this argument does not use the linear structure of the $B_{ij}(x)$ terms; indeed, the only feature used is that agent j can report a $B_{ij}(x)$ term that will induce agent i to make the choice x_i that agent j wants him to make. In a convex environment linear prices will generally have this property, but in other environments other sorts of pricing functions may be necessary.

Note further that this argument for efficiency does not use the penalty terms; if all the penalty terms are set equal to zero, the proof of efficiency still goes through. However, the penalty terms will in general be necessary if the equilibrium allocation is to be a Lindahl allocation. Why? In order to be a generalized Lindahl allocation each agent must satisfy his budget constraint when each choice is priced at its supporting efficiency price. For this to be the case, the $T_i(x)$ term must be zero in equilibrium. If the penalty terms are present, each agent will have an incentive to set $p_{ji}^i = p_{ji}^j$, which will ensure that this will occur.

VII. A Different Information Structure

The compensation mechanism described above is appropriate for a "bilateral" information structure: if agent i imposes costs on agent j, both i and j know the magnitude of these costs. Another structure that one might imagine is that there is some third party, k, who knows the magnitude of these costs. In this case, one can use a slightly different type of compensation mechanism to achieve efficient outcomes.

Consider the following example with three agents. Agent i chooses x_i, holds "money" y_i, and has a quasi-linear utility function $u_i(x_1, x_2, x_3) + y_i$. The prices that support an efficient allocation will have the form $p_{ij} = \partial u_i(x)/\partial x_j$. Let p_{ij}^k denote the report of person k about the appropriate magnitude of the price p_{ij}, and let $\mathbf{x} = (x_1, x_2, x_3)$ be the vector of choices.

In this variant of the compensation mechanism, the payoffs to the agents will be:

$$(18)\quad \begin{aligned} &u_1(x) - (p_{31}^2 + p_{21}^3)x_1 + p_{12}^3 x_2 + p_{13}^2 x_3 \\ &u_2(x) - (p_{32}^1 + p_{12}^3)x_2 + p_{21}^3 x_1 + p_{23}^1 x_3 \\ &u_3(x) - (p_{23}^1 + p_{13}^2)x_3 + p_{31}^2 x_1 + p_{32}^1 x_2. \end{aligned}$$

Note that the payoffs are balanced, even out of equilibrium. No sidepayments or penalties are necessary in this case.

One way to prove that the subgame-perfect equilibrium is efficient is to differentiate the payoffs with respect to each of the choice variables. However, one can also apply the logic of the previous section. Simply replace the definitions used there with

$$B_{ij}^k(x) = p_{ij}^k x_j - p_{ji}^k x_i$$

$$T_i(x) = 0$$

where k takes on all possible values $1, \ldots, n$, but $k \neq i, j$. Note that when $n = 3$ these are the payoffs given in (18). These definitions imply that

$$\sum_{i=1}^{n} \sum_{j=1}^{n} B_{ij}^k(x) = 0$$

and this is all that is required for the proof given in the previous section to work. The resulting allocation is automatically Lindahl.

VIII. Examples of the Compensation Mechanism

I have described the general form of the compensation mechanism; here I illustrate how it works in some specific cases.

A. *Pure Public Goods*

The special case of a pure public good is of some interest, since it is a well-known and much studied example of a particular type of externality. Let x_1 and x_2 be two agents' monetary contributions to a public good. Let y_i be agent i's private consumption. In the absence of any transfer mechanism, agent 1's maximization problem in a public-goods contribution game takes the form[6]

$$\max_{x_1, y_1} u_1(x_1 + x_2, y_1)$$

such that

$$x_1 + y_1 = w_1 \qquad x_1 \geq 0.$$

Since there is now a positive externality between the agents, it is natural to think of the agents as subsidizing each other rather than taxing each other. Applying the subsidy payments appropriate for the compensation mechanism, the budget constraint facing agent 1 becomes

$$(1 - p_{21}^2)x_1 + y_1 = w_1 - p_{12}^2 x_2 - \| p_{21}^1 - p_{21}^2 \|.$$

Here agent 1's contributions are subsidized at a rate p_{21}^2 which is chosen by agent 2; this subsidy is recovered by a tax on agent 2. Agent 1 also sets the rate at which agent 2's contributions should be subsidized, and in equilibrium he ends up paying $p_{12}^2 x_2 = p_{12}^1 x_2$ to cover this subsidy. In the compensation mechanism the taxes and subsidies that each agent faces are chosen by the other agent(s). See Varian (1994) and Leif Danziger and Arne Schnytzer (1991) for similar mechanism in which the agents set some of the subsidy rates for themselves. Joel Guttman (1978) describes a related mechanism in which agents choose the rate at which they will match other agents' contributions to a public good. Guttman's mechanism is of some interest since matching contributions are a commonly used method to encourage contributions to a public good.

Since public goods are just a special kind of externality, the proof of efficiency given earlier still applies. Note that the noncalculus proof is the appropriate version here, due to the presence of the nonnegativity condition. However, given the special form of the public-goods externality, one can say a bit more about the equilibrium prices. Suppose that there is an interior solution to the public-goods game so that x_1 and x_2 are both positive. Since x_1 and x_2 are perfect substitutes in consumption, they must have the same price in equilibrium.

By inspection of the budget constraint, it follows that $1 - p_{21}^2 = p_{12}^2$. Therefore, the budget constraint facing agent 1 can, in equilibrium, be written as

$$p_{12}^2[x_1 + x_2] + y_1 = w_1.$$

It follows that an equilibrium value of p_{12}^2 is simply the Lindahl price of the public good for agent 1, and the equilibrium allocation is simply a Lindahl allocation. Hence, the compensation mechanism gives a way to decentralize Lindahl allocations by giving each agent the incentive to reveal the appropriate Lindahl prices.

B. *Pure Private Goods*

Agent 1 is a consumer who consumes an x-good and a y-good and has a quasi-linear utility function $u(x) + y_1$. Agent 2 is a monopolist that can produce the x good at cost $c(x)$; its objective function is $y_2 - c(x)$. How can the monopolist be induced to produce the socially optimal output?

If one is only interested in efficiency, this is not terribly difficult: simply have one of the agents dictate a production level and a transfer. In this full-information environment there will be an efficient amount of x regardless of which agent chooses it; only the transfer will be different. However, if one wants to get a *particular* efficient allocation—say, the competitive outcome—it is not so obvious how to proceed. However, the compensation mechanism solves the problem quite readily.

[6]The nonnegativity constraint is natural in a model of voluntary contributions: one may choose to contribute a positive amount to a public good, but one is typically not able to make a *negative* contribution. The equilibrium of this contribution game has been studied extensively by Theodore Bergstrom et al. (1986).

Announcement stage.—The consumer announces how much he values the good, p_1, and the producer announces how much the consumer values the good, p_2.

Choice stage.—The producer chooses x, and the payoffs are

$$\Pi_1 = u(x) - p_2 x$$

$$\Pi_2 = p_1 x - c(x) - \| p_2 - p_1 \|.$$

Note that this problem is very similar to the simple externalities problem used to motivate the compensation mechanism, illustrating the Coasian point that externalities are just a special case of private goods.[7] Applying the standard argument shows that in equilibrium

$$p_1 = p_2 = u'(x) = c'(x)$$

which are the conditions that characterize the competitive allocation.

C. *Regulation of Duopoly*

There are now three agents: the consumer (indexed by 0) and two firms. Firm 1 produces x_1 at cost $c_1(x_1)$, firm 2 produces x_2 at cost $c_2(x_2)$, and the consumer has utility function $u(x_1, x_2) + y_0$. The standard compensation mechanism involves payoffs of the form

$$\Pi_0 = u(x_1, x_2) - p_{01}^1 x_1 - p_{02}^2 x_2$$

$$\Pi_1 = p_{01}^0 x_1 - c_1(x_1) - \| p_{01}^1 - p_{01}^0 \|$$

$$\Pi_2 = p_{02}^0 x_2 - c_2(x_2) - \| p_{02}^2 - p_{02}^0 \|.$$

Here the consumer is setting the prices that the firms face, and the firms are setting the prices that the consumer faces.

However, in the case of duopoly it is natural to think that the firms may know more about each other's technology than the consumer knows. Hence it makes sense for each firm to report the price that the *other* firm should face. This yields payoffs of the form

$$\Pi_0 = u(x_1, x_2) - p_{01}^2 x_1 - p_{02}^1 x_2$$

$$\Pi_1 = p_{01}^2 x_1 - c_1(x_1)$$

$$\Pi_2 = p_{02}^1 x_2 - c_2(x_2).$$

Note that the consumer chooses both x_1 and x_2 and that each firm sets the price for the *other* firms' product.

The arguments given earlier show that the competitive allocation is the unique equilibrium of this game. But it is useful to think about how it works here. Consider the classic Bertrand cases in which the two goods are perfect substitutes. Suppose that each firm has announced the competitive price. Why would firm 1 not want to raise the price of firm 2's product, creating more demand for its own output? If firm 1 raised the price facing firm 2, then the consumer would demand more output from firm 1, which it would be *forced* to supply. But since the price that firm 1 faces equals its marginal cost, this would reduce firm 1's profit.

D. *Prisoner's Dilemma*

Consider the following asymmetric prisoner's dilemma:

	Column	
Row	Cooperate	Defect
Cooperate	5,5	2,6
Defect	7,1	3,3

How can one induce the Pareto-efficient outcome? Let $x_i = 1$ if agent i cooperates and $x_i = 0$ if agent i defects, and let $u_i(x_1, x_2)$ be the payoff to agent i taken from the above game matrix.

Announcement stage.—Agent 1 names p_{12}^1, how much agent 1 should be paid if he cooperates, and p_{21}^1, how much agent 2

[7] Or is it the other way around?

should be paid if he cooperates. Similarly agent 2 names p_{21}^2 and p_{12}^2.

Choice stage.—Each agent chooses whether to cooperate or defect. The agents receive payoffs

$$\Pi_1 = u_1(x_1, x_2) + p_{21}^2 x_1 - p_{12}^2 x_2 - \|p_{21}^1 - p_{21}^2\|$$

$$\Pi_2 = u_2(x_1, x_2) + p_{12}^1 x_2 - p_{21}^1 x_1 - \|p_{12}^2 - p_{12}^1\|.$$

Note the sign change: since there is now a positive externality between the two agents, it is natural to subsidize good behavior rather than to penalize bad behavior.[8] Using the by now standard argument, it can be shown that it is a subgame-perfect equilibrium for both players to cooperate. The supporting prices satisfy the conditions

$$4 \geq p_{21}^1 = p_{21}^2 \geq 2$$

$$3 \geq p_{12}^1 = p_{12}^2 \geq 1.$$

(If the inequalities are strict, the cooperative equilibrium will be unique.) As usual, the compensation mechanism produces an efficient outcome in this game—or in any game, for that matter. However, the prisoner's dilemma has a special structure, and it turns out that a related, but simpler, mechanism is available.

Announcement stage.—Agent 1 names p_2^1, how much he is willing to pay agent 2 to cooperate, and agent 2 names p_1^2, how much agent 2 is willing to pay agent 1 to cooperate.

Choice stage.—Each agent chooses whether to cooperate or defect. The agents receive payoffs

$$\Pi_1 = u_1(x_1, x_2) - p_2^1 x_2 + p_1^2 x_1$$

$$\Pi_2 = u_2(x_1, x_2) - p_1^2 x_1 + p_2^1 x_2.$$

In this game, each agent sets the rate at which he will subsidize the other agent's cooperation. As in the other games examined, the subgame-perfect equilibrium yields an efficient outcome. It has long been known that the ability to make binding preplay commitments allows for a solution to the prisoner's dilemma. What is interesting about this example is how simple the first-stage commitments can be and still support efficient outcomes.

IX. Related Literature

There is a vast literature on mechanism design that is concerned with how to implement various social-choice functions. Much of this literature is concerned with whether a particular social-choice function can be implemented by a decentralized game. My concern is not so much with the *existence* of a mechanism, but rather finding a suitably simple mechanism. Most of the attempts to find "simple" solutions to externalities problems have been concerned with the case of public goods, so I provide a very brief review of that literature insofar as it relates to the work described here. Moore (1992) provides a thorough review of the recent literature.

The well-known demand-revealing mechanism of Edward Clarke (1971) and Theodore Groves (1976) implements the efficient amount of a public good via a dominant strategy equilibrium. However, this mechanism only works with quasi-linear utility, and it is not balanced, even in equilibrium. Furthermore, it does not in general yield a Pareto-efficient outcome.

Groves and John Ledyard (1977) describe a quadratic mechanism that yields efficient Nash equilibria for the public-goods problem, but the equilibrium allocations are not Lindahl allocations. Leo Hurwicz (1979) and Mark Walker (1981) also describe mecha-

[8]One could also formulate this problem so as to have each agent announce how the other agent should be fined if he defects.

nisms that implement Lindahl allocations. In the Hurwicz mechanism, each agent proposes an amount of the public good and a Lindahl price; agents pay a quadratic penalty if they announce different levels of the public good. Walker's mechanism avoids such penalty terms. Groves (1979) and Groves and Ledyard (1987) provide a nice survey of these results.

Turning to the more recent literature on simple mechanisms, Mark Bagnoli and Bart Lipman (1994) and Matthew Jackson and Hervé Moulin (1992) examine the special case of a discrete public good with quasi-linear utility. The Bagnoli-Lipman mechanism is very simple: each agent offers a voluntary contribution. If the sum of the contributions covers the cost of the public good, it is produced; otherwise the contributions are returned. This mechanism implements the core of the public-goods game in undominated perfect equilibria.

The Jackson-Moulin mechanism implements an efficient allocation using undominated Nash equilibria. They also describe a variation using subgame-perfect equilibrium. Their mechanism is reasonably simple and works with a broad family of cost-sharing rules. However, it appears that both the Bagnoli-Lipman and Jackson-Moulin mechanisms work only in the special case of indivisible public goods and quasi-linear utility.

Varian (1994) describes some mechanisms for the public-goods problem that are closely related to the compensation mechanism. In the case of two agents with quasi-linear utility, there is a very simple mechanism that achieves a Lindahl allocation: in the first stage each agent offers to subsidize the contributions of the other agent; in the second stage, each agent makes a voluntary contribution and collects the promised subsidies from the other agent.[9] That paper also describes some other variations on the compensation mechanism for public-goods problems involving many agents and general utility functions.

[9]This mechanism is related to the mechanism of Guttman (1978), which involves offering to match contributions. See also Danziger and Schnytzer (1991) for a related model.

X. Summary

The compensation mechanism provides a simple mechanism for internalizing externalities in economic environments. Transfer payments can be chosen so that the compensation mechanism is balanced, and penalty payments, when they are used, can be chosen to be arbitrarily small. The main problem with the mechanism is that it requires complete information by the agents. In many cases, a simple dynamic adjustment model will converge to the subgame-perfect equilibrium.

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