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GROUP COMPOSITION, COLLECTIVE CONSUMPTION AND COLLABORATIVE PRODUCTION

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Group Composition, Collective Consumption, and Collaborative Production

By MARTIN C. MCGUIRE*

An enduring question in the analysis of groups is: what determines their composition in terms of their individual constituents? In a universe of diverse people, should homogeneous groups coalesce around like individuals, or should heterogeneous groups of diverse units form (McGuire, 1972, 1974)? Two salient treatments of this problem are the one due to Charles Tiebout (1956) and James M. Buchanan (1965) and that due to Eitan Berglas (1976). Tiebout and Buchanan argue that scale economies in local public good (LPG) consumption should lead to homogeneous groups, provided individual income is unrelated to the group, club, or jurisdiction of residence.[1] Berglas argues that precisely when the place of work and of LPG consumption coincide, mixed "Berglas-groups" should coalesce, and he conjectures—assuming that all groups produce one identical private good—that these jurisdictions should be uniformly heterogeneous across an entire population. Closely related work includes: (i) again Berglas (1976) and John D. Wilson (1987), who show that with two tradable private goods Berglas-groups *must* be nonuniform; (ii) Joseph Stiglitz (1983), who concludes that, even without tradable private goods, dissimilar groups can be Pareto-optimal; (iii) Berglas (1984), who concludes that multiple public goods attenuate the effects of strong taste differences; (iv) Suzanne A. Scotchmer and Myrna H. Wooders (1987), who identify the crucial importance of relative numbers of consumer types in LPG models; and (v) Wooders (1978), who investigates optimal group composition as determined by the equivalence between competitive and core allocations in LPG economies.

Drawing on these studies, this paper builds a concise diagrammatic analysis of the group-composition problem, including conditions under which Tiebout configurations dominate Berglas-groups and conditions under which the conjecture of uniform heterogeneity applies. Throughout, the crucial tension governing group composition remains a conflict between the disadvantage dissimilar people find from cooperating in collective consumption versus the advantage they find from cooperating in production. The analysis is solely normative; questions of equilibria are not treated (see McGuire, 1988).

I. Introductory Efficiency Analysis

A. *Assumptions*

To reduce the analysis to bare essentials, assume a population of two types: 1 and 2. Individuals of each type are identical but different from the other. The two differ both in tastes for collective goods and in productive abilities in private-good supply, with perfect correlation between work abilities and LPG tastes. Private productivities are synergistic, represented by a linear homogeneous production function, $X = F(L_1, L_2)$, with $\partial F/\partial L_i = F_i$. Each individual (types $i = 1,2$) has one unit of L_i and a utility function, $U_i([I_i - c_i], G)$, with L_i representing the number of individuals of type

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[1]Comprehensive summaries may be found in Todd Sandler and John T. Tschirhart (1980).

i. I_i is i's gross income, G is LPG consumption in the group, X is total private good production, c_i is i's cost share for the provision of G, and $Y_i = I_i - c_i$ is i's private consumption after payment of c_i. I assume, that to exploit their complementarity in production, 1's and 2's must "live together" in the same jurisdiction and compromise their differences as to LPG consumption. Scale economies in LPG production are also ruled out to yield C, a cost function for G, with per-person cost per unit of LPG equal to 1:

$$(1) \quad c_1 L_1 + c_2 L_2 = C = G(L_1 + L_2)$$

$$C/NG = 1 \quad N = L_1 + L_2.$$

Thus, I avoid potential integer problems connected with jurisdiction sizing by characterizing G as a publicly provided private good.[2] One might alternatively specify a replicable U-shaped per-person average cost [i.e., $C(G,N)/N$] with minima (where $\partial C/\partial N = C/N$) at $N^*(G)$ and $C(G,N^*(G))/N^*(G) = G$. If N^* is an integer factor of total population, the two formulations are completely equivalent.

B. *The Group Optimization Problem: Dominance of Group Composition over Discriminatory Taxation*

My ultimate concern is the efficient organization of a given national population with given numbers of 1's and 2's. As a preliminary exercise instrumental to analysis of that ultimate goal, I consider the case in which L_1 and L_2 are not given and fixed, but instead are variables of choice. For this problem, I assume a social welfare function $\Sigma \mu_i U_i$ (with welfare weights μ_i) to be maximized over c_i, L_i, I_i, and G; or, equivalently, setting $\mu_1 = 1$ and $\mu_2 = \lambda$ and requiring that all 1's (and all 2's) be treated equally, with U_2^0 the constraint on 2's utility, the problem is

$$(2) \quad \max U_1([I_1 - c_1], G) + \lambda\{U_2([I_2 - c_2], G) - U_2^0\}$$

subject to equation (1) and a provision that pretax private income exhausts private production in each group: $\Sigma I_i L_i = F(\cdot)$. Again, L_1 and L_2 are unrestricted variables of choice in a community maximization problem regarded as an intermediate step toward characterizing efficient allocations. Necessary conditions include (a) $\mu_1 U_y^1 / L_1 = \mu_2 U_y^2 / L_2$ (for optimal distribution, U_y^i being marginal utility of Y_i), (b) $\Sigma L_i \mathrm{MRS}_i = \Sigma L_i$ (the Samuelson condition; where MRS denotes the marginal rate of substitution), and (c) $F_i - I_i + c_i - G = 0$ $(i = 1,2)$ for optimum choice of L_i.

The last condition implies that private income is equal to the marginal product only if costs are equally shared, in which case each resident pays a tax, c_i, equal to the marginal cost he imposes on LPG provision (i.e., G). To sustain this equal-cost-sharing result, the marginal congestion cost each individual imposes on others must be the same for all—the case of anonymous crowding as analyzed by Scotchmer and Wooders (1987). Since marginal costs are identical across residents and the choice of group membership is open, all pay the same taxes. For a more general model, in which marginal costs differ, c_i would differ.

The choice of group membership also is crucial: at the L_i mix selected by the necessary first-order conditions, the relative marginal social values of numeraire private incomes are brought into equality with opportunity costs of income distribution across individuals ($\mu_1 U_y^1 / \mu_2 U_y^2 = L_1 / L_2$) through the choice of (L_1, L_2); therefore, no income transfers are needed to achieve first-best optimality. Thus, the L_i are chosen so that first-best lump-sum income transfers are nil, and therefore each individual pays $c_i =$ marginal cost (MC) $= G$. (Equivalently, if

[2]The so-called "integer problem" first analyzed by Mark V. Pauly (1970) is concisely summarized by David A. Starrett (1988). As pointed out by a referee, my analysis skirts the more general case in which utility is affected not just by the ratio $G/(L_1 + L_2)$, but by both terms individually. The dominance of equal taxation carries through to this more general case, but even with average and marginal per-person costs equal, if these costs per unit of G are nonconstant, then a new source of nonconvexity could arise.

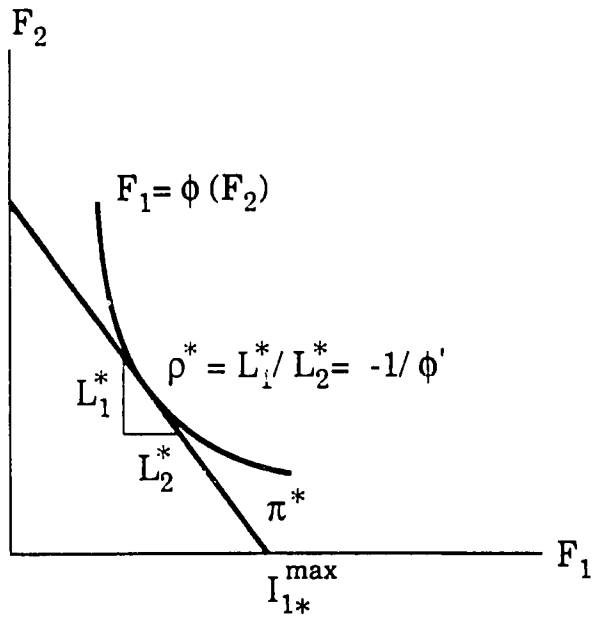

FIGURE 1. THE SUPERIORITY OF MARGINAL PRODUCT CHANGE OVER INCOME REDISTRIBUTION

$I_i \neq F_i$, the excess or deficit must be offset exactly by differential tax shares). For different weights on individual utilities in the social welfare function,[3] the optimal choices of L_i and corresponding marginal utilities U_y^i would also vary. However, at each optimum, taxes still should be equal, and income should be equal to marginal product. Therefore, moving from one optimum to another would never involve income transfers, since these are always dominated by changes in group composition.

Figure 1 illustrates this with the factor-price frontier $F_1 = \phi(F_2)$. For constant-returns-to-scale production, ϕ is unique with slope equal to the associated ratio of factor consumers ($-1/\phi' = \rho = L_1/L_2$) where $\phi' = d\phi/dF_2$. For any given ratio, income redistribution possibilities (taken from marginal productivity factor payments as an initial point) are shown by the tangent line π. (At ρ^*, for example, the entire product, if shared equally by L_1^*, gives each L_1 an income of $I_{1*}^{\max}$). Evidently, ϕ dominates all distribution possibilities like π; that is, group composition dominates income transfer for welfare redistribution. The above first-order conditions merely demonstrate that LPG's do not change this conclusion; income transfer remains inferior to group composition as an instrument of welfare change (assuming equality among consumers of the same type). Of course, if population composition were fixed, contrary to the above assumption, then income transfers (or unequal taxation) would be introduced as a potentially desirable instrument. For analysis of the overall national problem in which L_1 and L_2 are fixed (Section III), I will identify cases in which income transfer complements uniformity in individual community compositions and other cases in which income transfer supplants it.

II. The Consumption Set and "Efficient" Group Compositions for a Community

A. *The Opportunity Set*

Figure 2 gives a graphic solution to equation (2). It extends Figure 1 by adding the two consumer quadrants between private and LPG consumption. After-tax consumption of each type depends on its location on the factor-productivity frontier, ϕ, and on how much G is produced and consumed. With marginal product wages and equal LPG costs, Y_i and G are thereby determined for both types. Figure 2 constructs such an LPG-versus-private-consumption opportunity set for type 1 given a fixed utility level for type 2 (curve U_2^0). Combinations (a–b–c) and intermediate points all consistent with U_2^0 form the locus S_1^0 representing 1's corresponding consumption opportunities. For example, consider point c on U_2^0. To pay for the associated G_c at a price of \$1 per unit, type 2 must earn F_2^c.

[3]Using the term "social welfare function" to characterize $\Sigma\mu_i U^i$ when the L_i are to be chosen is stretching terminology somewhat. A conventional (utilitarian) social welfare function might be written $W = \Sigma\beta_i L_i^* U^i$ with population fixed at L_i^*. Here, maximizing W subject to $\Sigma L_i^* I_i = F(L_i^*)$ requires $\beta_1 U_y^1 = B_2 U_y^2$. Suppose the L_i^* are solutions to the first L_i-choice problem; then consistency with the second (conventional social welfare function) requires $\beta_i = \mu_i / L_i^*$, which yields zero transfers (i.e., $I_i = F_i$ in both formulations).

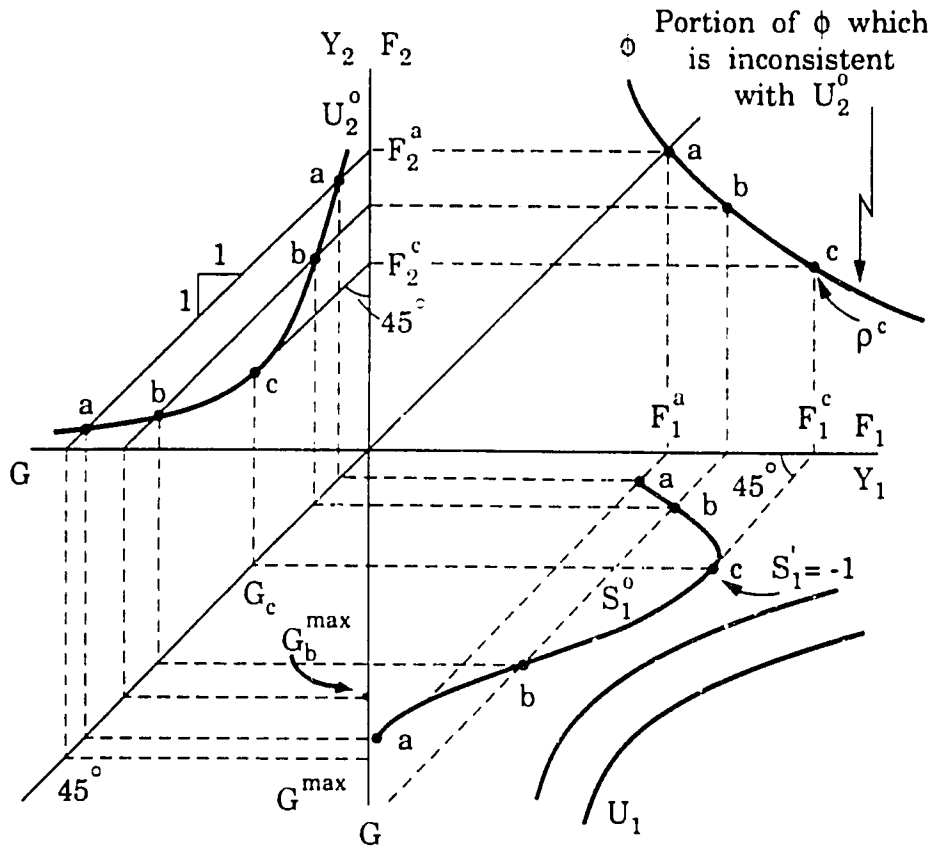

FIGURE 2. DERIVATION OF THE CONSUMPTION OPPORTUNITY SET, S, FROM THE UTILITY CONSTRAINT U_2^0 AND THE FACTOR-PRODUCTIVITY FRONTIER ϕ

Tracing across to ϕ shows that type 1 will earn F_1^c. If the 1's pay \$1 per unit of G_c, then their earnings are reduced along the 45° dashed line moving to c on S_1^0. Other combinations a and b are similarly constructed. If type 2 earns F_2^a, the figure shows two alternative points on U_2^0 labeled "a," two amounts of G, and two associated points on S_1^0, also labeled "a." Since 1's and 2's both pay \$1 per unit of G, the budget lines in the second and fourth quadrants are parallel with slope 45°. With U_2 constrained to a higher level, a smaller opportunity set could be constructed (as in Fig. 3) with a shape similar to S_1^0.

Use of this construct requires a knowledge of its properties, which I will now develop. Each point on any S corresponds to a specific group composition. Clockwise-downward movements along S imply lower L_1/L_2 ratios, that is, more 2's cooperate with 1's in private production up to the smallest value of L_1/L_2 consistent with U_2^0. I denote this limiting ratio of L_1/L_2 as (L_1/L_2)-min at ρ^c (the value of G at this point is G_c). This minimum or limiting ratio of L_1/L_2 shows the lowest L_2 productivity consistent with U_2^0, where the slope of $U_2^0 = -1$ and 2's are supplied precisely the LPG they desire at a price of unity. For values of G on $S(G)$ greater than G_c the implied L_1/L_2 reverses direction, retracing its path and increasing for further movements along S. Thus, for U_2 constrained to U_2^0, only a portion of ϕ is admissible as identified in Figure 2. If L_1/L_2 is too low, F_2 becomes too small for 2's to reach any point on U_2^0 at all. A lower (higher) constraint on U_2 will extend (contract) the allowable range of ρ on ϕ beyond (short of) point c.

B. *Opportunity Maps*

Essential to construction of a consumption-opportunity curve for one type of consumer is specification of a constraining indifference curve for the other. For each

feasible U_2 in the second quadrant, another S_1 curve can be constructed in the fourth. Higher feasible utility for the 2's entails a lesser curve for the 1's. Similarly, opportunity sets for the 2's may be constructed in the second quadrant, with each S_2 curve derived from a specified admissible indifference curve for the 1's, and higher levels of U_1 generating lesser S_2's. When $S_1' = -1$, type-2 consumers just receive the indicated provision of G at a price of unity, receive in private earnings the minimum consistent with U_2^0, and therefore are at their highest proportion (lowest value of L_1/L_2) consistent with that utility. These points of $S_1' = -1$ generate a locus (not shown) in 1's consumption quadrant of amounts of LPG ideally desired by type-2 consumers. In contrast, the LPG ideally desired by type 1 is simply 1's income-expansion path (not shown). Location of the efficient outcome with respect to these loci will show which type receives more than its ideal LPG consumption and which type receives less.

C. *Nonconvexities in the S Curve*

The shape of the S curve will be crucial to show when an overall national configuration of groups requires uniform heterogeneity for efficiency and when it requires nonuniform heterogeneity. Its equation, $Y_1 = S(G)$, is obtained from $Y_1 = F_1 - c(G)$, where $F_1 = \phi(F_2)$, $F_2 = Y_2 + c(G)$, $Y_2 = V_2(G)$, $c(G)$ is per-person cost of G [i.e., $c = C(N,G)/N$], V_2 is the utility constraint for type-2 individuals, and F_i and Y_i are as defined already. Substituting successively gives

$$(3)\quad Y_1 = \phi[V_2(G) + c(G)] - c(G) \equiv S(G)$$

$$(4)\quad dY_1/dG = (V_2' + c')\phi' - c' \equiv S'.$$

Here, $c' = dc(G)/dG$ is the marginal per-person cost of G, and V_2' is the slope of U_2^0. Assuming from equation (1) that G is perfectly congestible (a publicly provided private good) gives $c' = c/G = 1$, and (4) simplifies to

$$(4')\quad dY_1/dG = (V_2' + 1)\phi' - 1 \qquad V_2' < 0 \quad \phi' < 0.$$

Differentiating (4) and substituting ($c' = 1$, $c'' = 0$) gives

$$(5)\quad d^2Y_1/dG^2 = \underset{(-)}{\phi''}\underset{(-)}{\left(V_2' - 1\right)^2} + \underset{(-)(+)}{\phi' V_2''} \equiv S''$$

with components as signed. This confirms that S is concave from below ($S'' < 0$) near $S' = -1$ and may have two or more points of inflection (and $S'' > 0$ downward along the G-axis in the fourth quadrant). If V'' and ϕ'' were constant, there would be just two inflection points, one on either side of $V_2' = -1$ or $S' = -1$. Exact locations of nonconvex regions of S will depend on the rates of change in ϕ'' and V'', making precise a priori specification of S impossible (nonconstant values of ϕ'' and V'' might give many inflection points and regions of nonconvexity).[4] Still, these nonconvexities are pointedly relevant to uniform versus nonuniform jurisdiction structures and will be used presently to demonstrate the logical possibility that nonuniformity is superior.

D. *Effect of Freezing Community Composition*

If community compositions were frozen at a particular L_1/L_2, a construction similar to Figure 2 would generate a different U_2-constrained opportunity set, say D, for each value of ρ. Figure 3 shows U_2^0, ϕ, and S_1^0 from Figure 2. With group composition frozen at ρ^b, income-transfer possibilities

[4]This illustrates the prevalence of nonconvexities in LPG models (e.g., Stiglitz, 1983; Wilson, 1987). I assume that production/utility sets are convex and eliminate integer considerations but still come up with nonconvexity. A referee has pointed out how $S(G)$ can also be derived from the following simple maximization program: Given G, choose Y_2, L_1, and L_2 to maximize

$$[F(\cdot) - L_2Y_2 - (L_1 + L_2)G]/L_1$$

subject to $U_2(Y_2, G) = U_2^0$. The quantity maximized is the surplus generated per type 1. $S(G)$ is the maximized value of this surplus. Equations (4) and (5) then follow using the envelope theorem. First-order conditions require that differences between marginal product and private consumption equal G for each type. Thus, each type is added to the community until its surplus, the difference between its marginal product and consumption of resources ($Y + G$), is zero at the margin.

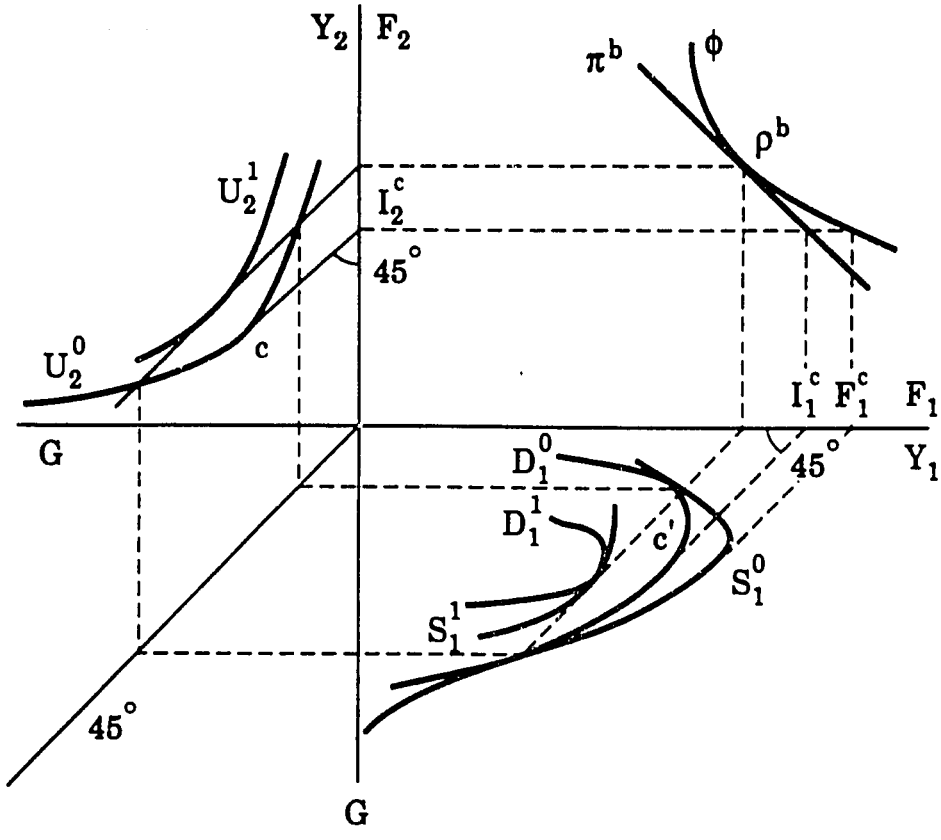

FIGURE 3. IF FACTOR ENDOWMENTS ARE FROZEN AT ρ^b, CONSUMPTION OPPORTUNITIES ARE RESTRICTED TO D_1^0, IN CONTRAST TO WHEN GROUP COMPOSITION IS ALLOWED TO CHANGE ALONG ϕ

are given by π^b. Now consider the effect this new constraint has on the 1's consumption opportunities. For example, to sustain point c on U_2^0, income I_2^c is necessary for type-2 individuals, and therefore, only I_1^c is achievable for the 1's. F_1^c is not permitted in this case because movement along ϕ has been eliminated by freezing L_1/L_2 at ρ^b. Instead, movement in the first quadrant is restricted to π^b, the locus of income-transfer options. Consequently, from point c in the second quadrant, c' can be reached in the fourth quadrant. Curve D_1^0, therefore, rather than S_1^0, shows the set of feasible consumption points restricted by π^b and U_2^0. A similar construction for U_2 limited to $U_2^1 > U_2^0$ gives S_1^1 and D_1^1. Thus each D curve (incorporating income transfers as the only redistribution mechanism available when group composition is frozen) is inferior to the corresponding S curve, except where the income-transfer option is not exercised. S may be regarded as an envelope of such D curves, one for each L_1/L_2. Therefore, if the population is fixed, a map of opportunity curves like $D_1^0, D_1^1, \ldots$ becomes relevant in addition to S^0.

E. "*Pareto-Efficient*" *Community Compositions and LPG Resource Allocations*

When $\lambda > 0$, efficiency[5] requires tangency between U_i and S_i, and from (4') Samuelson's condition obtains again:

$$(6)\quad \mathrm{MRS}_i = dY_i/dG = (\mathrm{MRS}_j + \mathrm{MC})(-L_j/L_i) - \mathrm{MC}$$

[5]Here, by "efficiency" I mean points on the utility-possibility frontier between representative types when the number or proportion of such types itself varies. Since the set of agents is ordinarily fixed in definitions of efficiency and optimality, I have used quotation marks in the heading above.

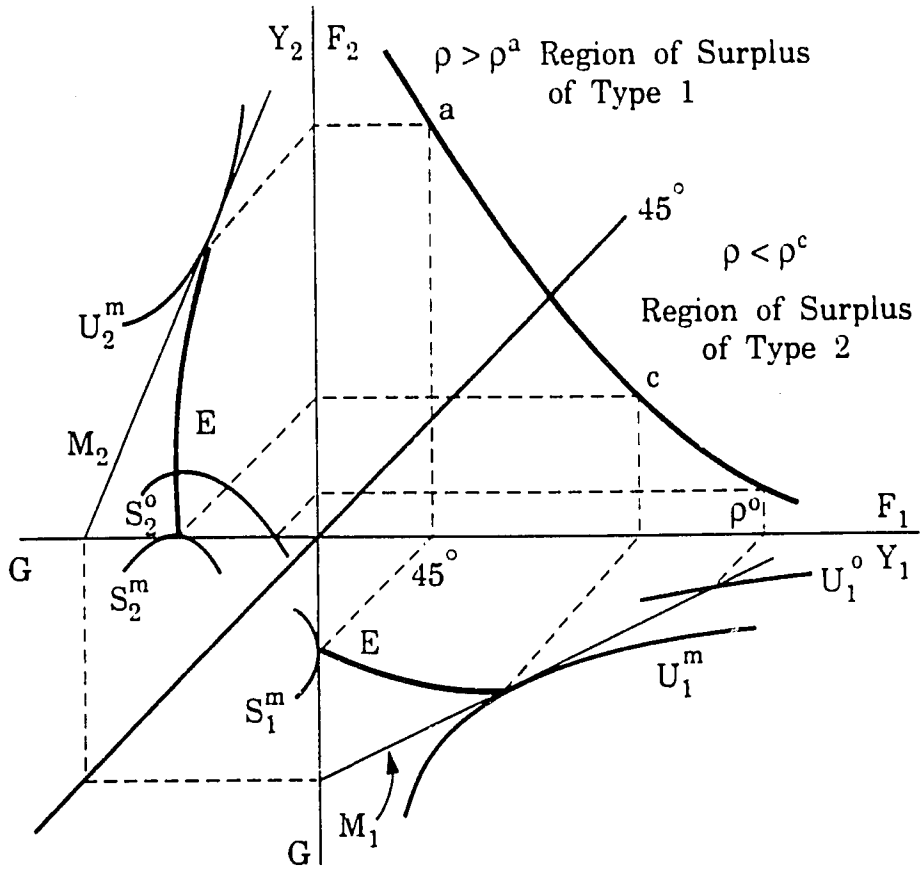

FIGURE 4. E LOCUS OF "EFFICIENT" GROUP COMPOSITIONS: AS ENDOWMENT RATIO L_1/L_2 CHANGES, UNIFORM GROUPS ARE EFFICIENT, BUT FOR $\rho > \rho^a$ AND $\rho < \rho^c$, BOTH U_1 AND U_2 DECLINE

or, with $i = 1$ and $j = 2$,

$$(7) \qquad L_1 \mathrm{MRS}_1 + L_2 \mathrm{MRS}_2 = -(L_1 + L_2)\mathrm{MC}.$$

Figure 4 shows E, the efficiency locus of such tangencies, in the 1's consumption quadrant (and symmetrically in the 2's quadrant) for the "regular" case in which S is convex throughout. Every tangency of U_i and S_i implies a value of ρ, and by continuity every admissible ρ entails a particular welfare distribution. E terminates at the tangency of U_i with M_i. As developed in Appendix A, this occurs when $\lambda = 0$, and the Samuelson condition becomes $L_i \mathrm{MRS}_i = \Sigma L_j$. Both cases demonstrate the consistency between the diagrammatic and mathematical analyses.

III. Efficiency Analysis and the Structure of Group Composition

I have developed the foregoing tools assuming the membership composition of the group to be unconstrained and assuming income transfers to be an inferior distribution instrument. Now I can apply these tools to the situation in which national population size and composition are fixed and, therefore, in which income transfers may be required to achieve a socially desirable distribution of utility. At a general level, the extension of Figure 4 to include the case with fixed population is straightforward. E was constructed on the assumption that L_1/L_2 is a variable of choice. If instead, L_1/L_2 is frozen at a given value, then following the logic of Figure 3, "constrained" E-curves could be constructed im-

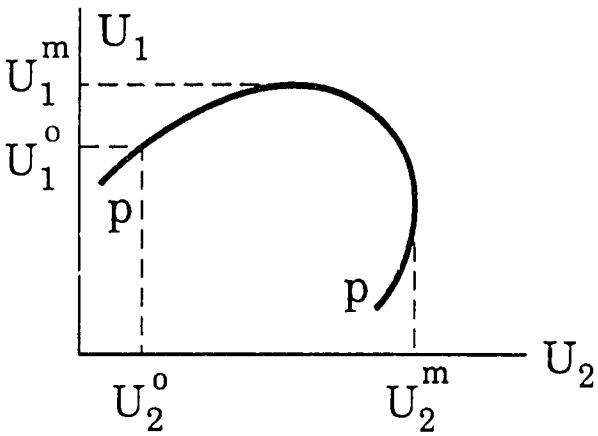

FIGURE 5. UTILITY-POSSIBILITY FRONTIER BETWEEN REPRESENTATIVE TYPES CORRESPONDING TO FIGURE 4

plying in turn a constrained utility-possibility curve wholly inside curve pp of Figure 5 (except at a point of tangency). It could happen that the endowment of factor consumers ρ^0 just happens to match the optimal value of ρ for the welfare distribution picked out by the social welfare function of equation (2). However, if the given L_1/L_2 is not the same as would be optimally chosen, then income transfers (or unequal taxation[6]) along a line π^0 tangent to ϕ at ρ^0 (and along a corresponding D^0 curve) are indicated.

At a more specific level, the constructs developed above will also be useful for (i) discerning when homogeneous Tiebout groups dominate heterogeneous Berglas groups, (ii) appraisal of the conjecture that Berglas-groups should be uniformly heterogeneous, and (iii) more specific identification of efficient group configurations.

A. *Tiebout versus Berglas*

Consider first comparisons between pure uniform Berglas and Tiebout groups; the diagrams indicate which is superior. Figure 6 shows ϕ frontiers B_1 and B_2 with low and high elasticity of substitution (σ), respectively, and, at point b, the Berglas configuration and utility distribution (assuming convexity of S, uniform heterogeneity, and zero income transfers to be optimal). The endowed proportion is ρ^b, the slope of B_1 or B_2 at b. Now consider a noninteractive technology (infinite σ) with productivity of L_1 and L_2 fixed at MP_1^t and MP_2^t. As shown, point t can make 1's and 2's just as well off in Tiebout configurations as they are at b in Berglas clubs. The lower productivity of each at t is just offset by the benefits of organizing for LPG consumption in homogeneous groups. Moreover, any other noncooperative technology along line T (also with slope ρ^b) is potentially equivalent to t since from any point on T, t can be reached via income transfer. It follows that every noncooperative technology above T dominates b. Moreover it can be efficient to mix a noncooperative or Tiebout configuration (shown as a point below T) with Berglas groups along ϕ (not at point b), since this exploits the superiority of population composition for altering the utility distribution. However, if t is below line T, it could never be efficient to form Berglas clubs together with homogeneous Tiebout groups for *both* types.

Next, consider forming segregated Tiebout clubs with the same B technology. Could these ever be superior? Figure 6 provides the answer. It depends on the shape of B. If the factor-price frontier is tightly curved as in the the case of B_2 (indicating high σ), then little is lost by forming Tiebout groups, and combination b_2 dominates t and therefore b. But if σ is low as with B_1, then segregation implies productivities at b_1 inferior to t and therefore to b. In the diagram, both type 1 and type 2 gain or lose in a switch from Tiebout to Berglas groups. However, if B_1 or B_2 were not symmetric about b, the change could harm one factor and aid the other; but in this case *potential* Pareto superiority can be determined by examination of the income-transfer possibilities along a line with slope $-L_1/L_2$ through b_1 or b_2. Consumer preferences also obviously play a crucial role in comparison of Tiebout versus Berglas clubs. Greater taste differences between consumer types

[6]Although transfers and tax-price discrimination usually differ, for public good provision they are known to be equivalent (McGuire and Henry J. Aaron, 1969).

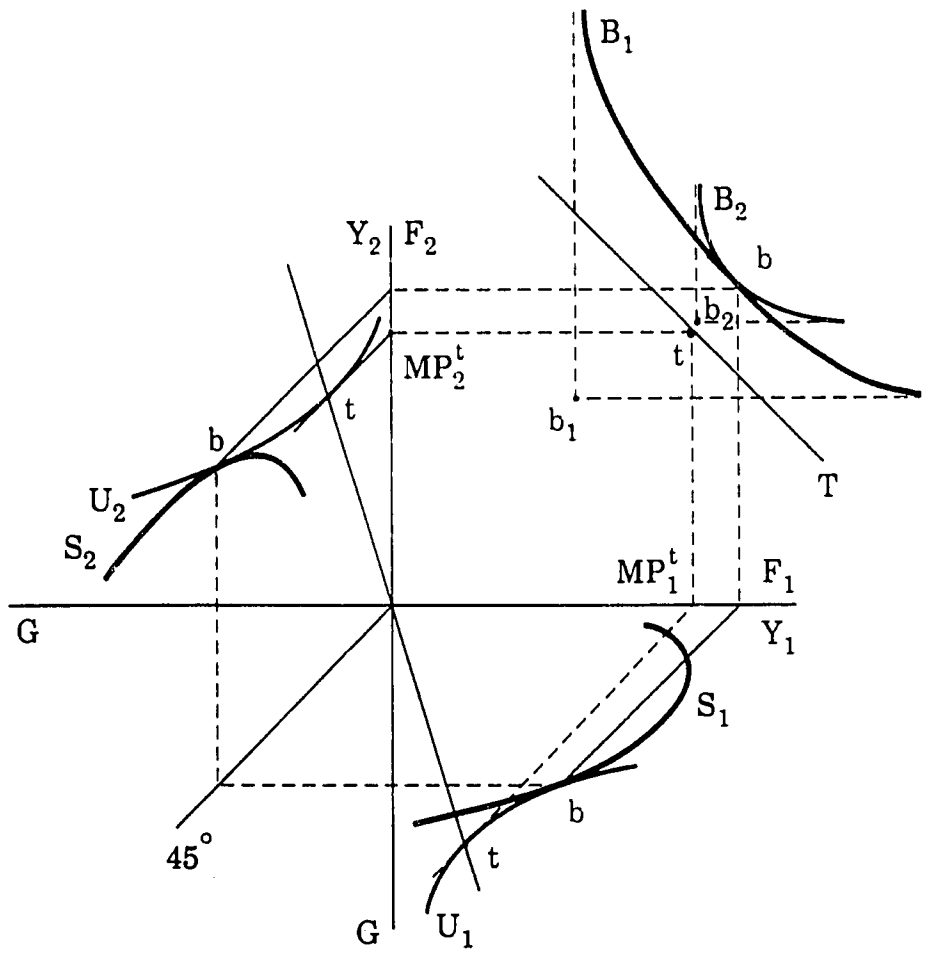

FIGURE 6. COMPARISONS BETWEEN BERGLAS AND TIEBOUT GROUPS

and *lower* elasticities of substitution between public and private good consumption favor Tiebout configurations.[7]

B. *Uniformity of Berglas Jurisdictions with S Convex*

Next I ask whether within this model it could be efficient to organize a fixed national (L_1/L_2) endowment into dissimilar groups. If so, it could not be due to integer problems, for I have allowed no scale economies in public or private production. The issue of uniform versus nonuniform heterogeneity depends entirely on convexity of consumption-opportunity S sets.

With S convex, diversity of groups is unambiguously inferior; it is never optimal to form diverse groups and then equalize utilities within types by transfers. For instance, suppose ρ^0 in Figure 7 is a fixed national endowment. Rather than organize the entire population at that ratio, groups of two compositions might be formed with $\rho^1 > \rho^0$ and $\rho^2 < \rho^0$, each sized just to absorb the endowed (L_1, L_2). Then, to equalize utility among the 1's and 2's, transfers are made along D^1 (π^1) and D^2 (π^2) until U_1 is the same in both groups, a result shown as U_1^*, which is necessarily Pareto-inferior to ρ^0 and U_1^0. With S convex, no combination of ρ^1 and ρ^2 can surpass ρ^0. This result continues to hold (where S is convex) even if income transfers are required by the social

[7]Berglas (1984), however, shows how multiple public goods cause taste differences actually to favor Berglas-groups.

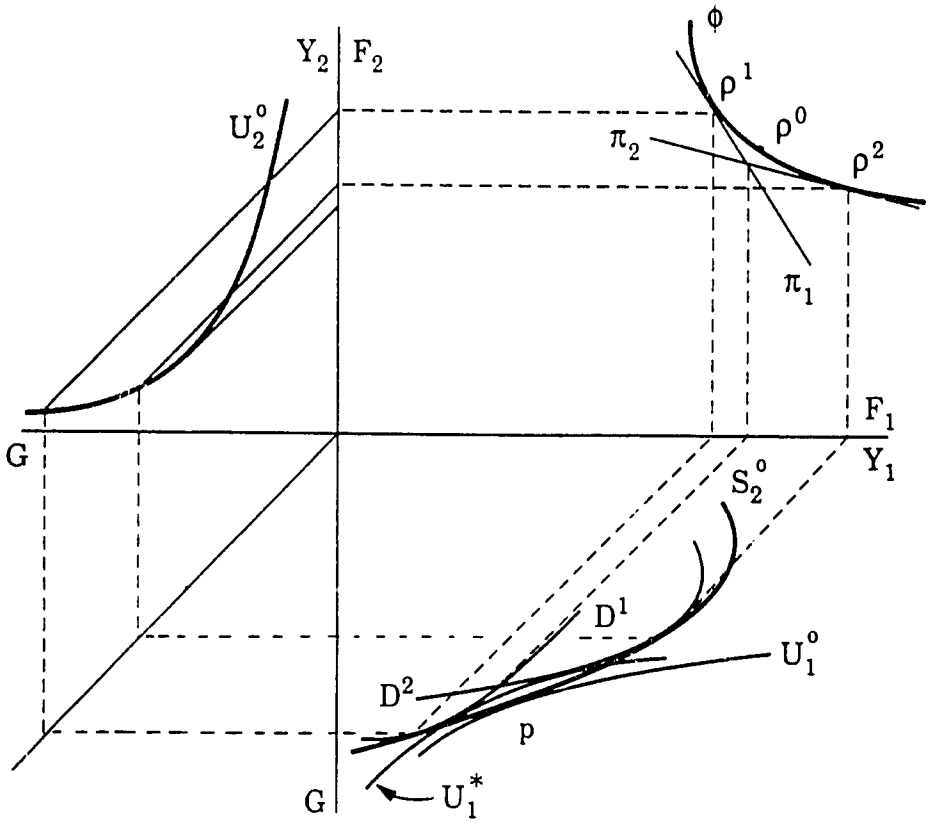

FIGURE 7. THE SUPERIORITY OF UNIFORMLY MIXED GROUPS WHEN S IS CONVEX: ENDOWMENT RATIO ρ^0 ALLOWS U_1^0 AND U_2^0 AT P WHEN ALL GROUPS USE ρ^0; NONUNIFORM GROUPS AS AT $\rho^1-\rho^2$ ALLOW ONLY $U_1^*-U_2^0$, WHICH IS INFERIOR

welfare function. For example, with population fixed at ρ^0 in Figure 7, income transfers to increase the utility of type 1 require southeast movement along a line tangent to ϕ at ρ^0 (not shown) and along D^0 (a corresponding curve tangent to S_1^0 at p). Again, no combination such as $\rho^1 > \rho^0$ and $\rho^2 < \rho^0$ can surpass ρ^0.

C. *Efficient Groups: Consumption Opportunities Not Convex*

If S is not convex (as in Fig. 8), then organizing some groups with composition ρ^1 and others with ρ^2 achieves a distribution (U_1^*, U_2^0) that dominates uniform organization at ρ^0 (for all $\rho^2 < \rho^0 < \rho^1$) and the associated inferior welfare distribution (U_1^0, U_2^0) at n.[8] Appendix B produces a numerical example to demonstrate more concretely the logical possibility that nonuniform groups may be superior. Moreover, this potential for nonuniform groups to Pareto-dominate uniform groups may interact with utility distribution and income transfer in a particularly sensitive fashion.[9] This is illustrated in Figure 9.

Assume that the endowed national population ratio is ρ^0, that the social welfare

[8]Similar closely related results are obtained by Stiglitz (1983 p. 64 and figs. 2b, 11), Wilson (1987), and Berglas (1976) when two tradable private goods are postulated. Stiglitz's result derives from diminishing marginal product of land, whereas the present model follows solely from substitution elasticities and taste differences. Berglas and Wilson show that the envelope ϕ frontier for two goods must have a kink (as in Fig. 8), thereby generating dissimilar groups. Figure 8 is a specific case to show that uniform groups may be inferior to a mix of diverse groups; the general case would be a serpentine-shaped S_1 curve tangent to one U_1 curve at several points.

[9]This conclusion is also demonstrated in Scotchmer and Wooders (1987).

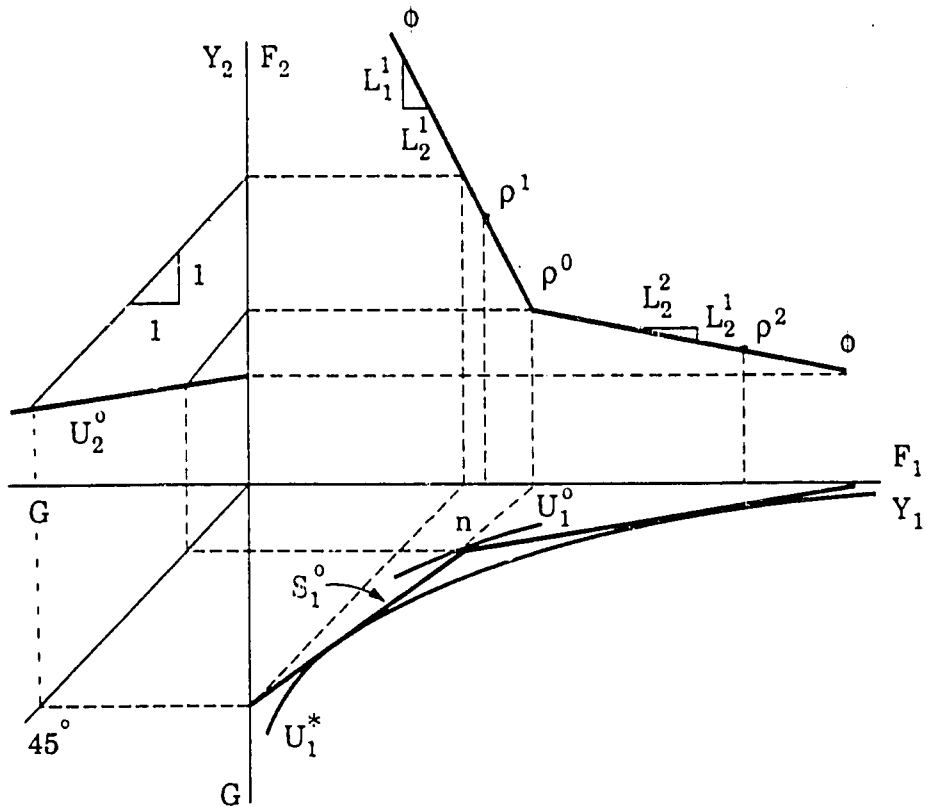

FIGURE 8. EXAMPLE OF A NONCONVEX CONSUMPTION OPPORTUNITY SET, S_1^0, AND NONUNIFORM GROUPS ρ^1 AND ρ^2 SUPERIOR TO UNIFORM GROUPS AT ρ^0

function picks U_2^0 as the constraint, and that maximizing U_1 without income transfers and with equal cost shares allows U_1^0 to be reached. All communities are identical. Now suppose the central government decides to transfer lump-sum income from type-2 to type-1 individuals. This redistribution policy can be represented as a translation of the factor productivity curve from ϕ to $\hat{\phi}$.[10] The new $\hat{\phi}$ shows productivity cum

[10]If the efficient group configuration is uniform, transfers are all intracommunity, but if the efficient configuration is nonuniform (as in Fig. 8), transfers both within and across jurisdictions are necessary to equalize utilities of similar consumers. This conclusion is representative of the interdependencies that hold among variables at an optimum: in particular, whether at efficient solutions (i) population composition is optimal, (ii) taxes within communities are equal, (iii) transfers across individuals and across communities are nonzero, and (iv) L_1/L_2 ratios diverge across groups (or all are uniform). The relevant overall maximization problem can be written as follows. Let $j = \mathrm{a,b}$ index community types, and let L_i denote population endowments. Choose L_i^j, I_i^j, c_j^j, and G^j to maximize

$$\begin{aligned}
&\mu_1 U_1^a[(I_1^a - c_1^a), G^a] + \mu_2 U_2^a[(I_2^a - c_2^a), G^a] \\
&\quad + \theta_1(U_1^a - U_1^b) + \theta_2(U_2^a - U_2^b) \\
&\quad + \theta_3[F^a(\cdot) + F^b(\cdot) - L_1^a I_1^a - L_2^a I_2^a - L_1^b I_1^b - L_2^b I_2^b] \\
&\quad + \theta_4(L_1 - L_1^a - L_1^b) + \theta_5(L_2 - L_2^a - L_2^b) \\
&\quad + \theta_6[L_1^a c_1^a + L_2^a c_2^a - (L_1^a + L_2^a)G^a] \\
&\quad + \theta_7[L_1^b c_1^b + L_2^b c_2^b - (L_1^b + L_2^b)G^b].
\end{aligned}$$

First-order conditions are interpreted as follows. First suppose that $\theta_4 = \theta_5 = 0$; that is, the population constraints vanish either because the L_i are choice variables or because the given L_i are optimal. In this case, (a) taxes must be equal for all within each group, and (b) zero intergroup and intertype transfers are optimal, but (c) communities may nevertheless diverge because of nonconvexities in $S(\cdot)$. Next let $\theta_4 \neq \theta_5 \neq 0$, meaning that population $L_1 - L_2$ is given at nonoptimal values. Then, at an optimum it follows that (a) $\theta_4/\theta_5 = -L_1/L_2$, and therefore θ_4 and θ_5 must have opposite

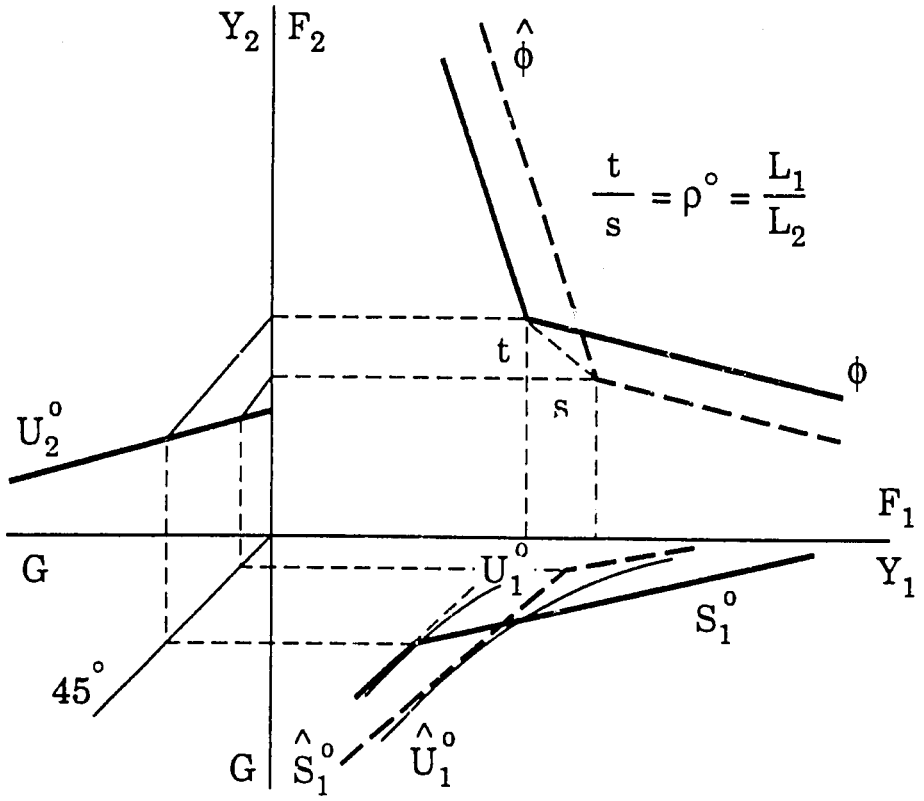

FIGURE 9. EFFECTS OF LUMP-SUM INCOME TRANSFERS ON EFFICIENT GROUP STRUCTURE WHEN THE OPPORTUNITY SET IS NONCONVEX: BEFORE LUMP-SUM TRANSFERS, UNIFORM JURISDICTIONS ($L_1/L_2 = \rho^0$) GIVE U_1^0 AND U_2^0; PROPER CHOICE OF TAX, t, AND SUBSIDY, s, GIVES U_2^0 AND $\hat{U}_1^0 > U_1^0$ FOR NONUNIFORM GROUPS

transfers. Every point on ϕ is lowered by t, the amount of the tax on type-2 individuals, and moved to the right by s, the amount of the subsidy to 1's. Thus, the ratio t/s must equal the overall population ratio L_1/L_2. This construction also produces a new after-transfer opportunity map for type-1 individuals. Figure 9 shows one of these new $\hat{S}$ curves: in particular, the one corresponding to U_2^0. Note that the new $\hat{S}$ curve makes a nonuniform jurisdiction structure both feasible and superior to the original uniform structure (i.e., $\hat{U}_1^0 > U_1^0$). Obviously this example is only one possibility; the opposite phenomenon could be easily constructed.

D. *Nonconvexity, the Efficiency Locus, and the Utility-Possibility Frontier*

S curves may be nonconvex over a range of utility distributions. This has important implications for the interdependence among social welfare optima, income distribution, and community organization. To capture the fundamentals of that interdependence Figure 10 shows nonconvexity over an entire range of factor consumer ratios.[11] Several S

signs; (b) taxes of L_1 and L_2 in each group cannot be equal (i.e., transfers across types are mandatory); (c) if intergroup transfers are allowed but chosen to be zero (with each community therefore self-supporting), then L_1/L_2 is the same in all groups; and (d) if intergroup transfers are positive, then L_1/L_2 differs among groups, which can only be the case if $S(G)$ is nonconvex.

[11]A much sharper-cornered ϕ curve is required to generate the S set shown. The curve is drawn to show sets α, β, γ, and δ more distinctly.

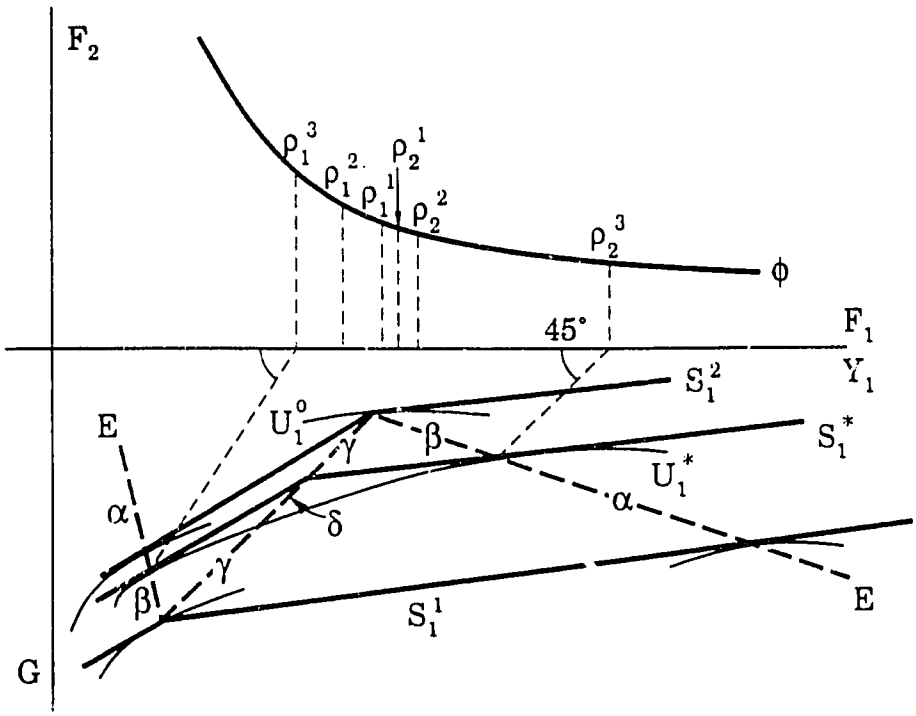

FIGURE 10. UTILITY DISTRIBUTION BETWEEN L_1 AND L_2 ORGANIZED IN UNIFORM MIXED GROUPS AS ENDOWMENT L_1/L_2 CHANGES AND OPPORTUNITY SETS ARE NONCONVEX.
FOR SOME FACTOR ENDOWMENTS (IN α), UNIFORM GROUP ORGANIZATION IS EFFICIENT; FOR SOME (IN δ), UNIFORM GROUPS ARE ALWAYS INEFFICIENT; FOR OTHER ENDOWMENTS (IN β AND γ), DIVERSITY OF GROUPS IS SUPERIOR ONLY IF INCOME CAN BE TRANSFERRED

curves are shown as S_1^k corresponding to U_2^k, with $U_2^1 < U_2^2 \ldots$ generating $S_1^1, S_1^2 \ldots$. The efficiency locus E is then constructed in Figure 10 assuming that 1's have homothetic utility. Each point on E designates a particular community composition, LPG provision, and utility distribution. E now consists of several subsets of tangencies between the S_1^k and U_1 maps; first, α, consisting of two branches connected by the indifference curve U_1^*; second, β, consisting of two branches connected through a "trough"; third, γ, consisting of allocations in the nonconvex trough where $U_1 > U_1^*$ or $U_2 > U_2^*$; and fourth, the set δ, consisting of allocations with $U_1 < U_1^*$ *and* $U_2 < U_2^*$. These subsets have corresponding segments on ϕ and the utility frontier. When S sets are convex throughout, E collapses to a single curve as in Figure 4, and the kinks in Figure 10 disappear. Very much more complex patterns than these could easily be constructed from less regular preference maps for 1's or 2's.

E. *Implications of Nonconvexity for Community Organization*

Although Figure 10 necessarily pictures only a specific case, it is indicative nevertheless of the more general implications of nonconvexities. These implications fall into two categories: those for a single community able to choose its composition and those for the nation as a whole with a fixed $L_1 - L_2$ endowment. For the first problem, a constraining utility level for one type is fixed, and free movement along ϕ is permitted. For the second problem, an endowed point on ϕ is given so that average factor proportions cannot be changed. Still, the endowment may be divided into different mixes on either side of the average, and the total may be absorbed by appropriate numbers.

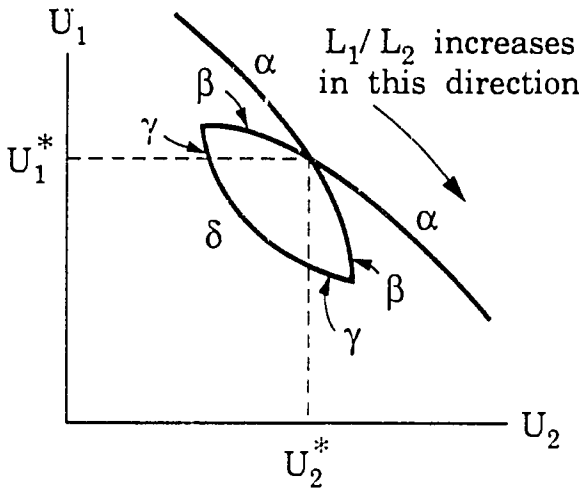

FIGURE 11. UTILITY-POSSIBILITY FRONTIER WHEN CONSUMPTION-OPPORTUNITY SETS, S, ARE NONCONVEX

For the single-community case the diagrams show the following. (a) Absent nonconvexities, every welfare distribution implies one community-composition ratio, and within the range of "admissible" or "viable" factor-consumer ratios vice versa. However, as discussed below, some ratios may be inadmissible. (b) With population a variable of choice and nonconvexities (Fig. 10), some subsets such as $\beta-\gamma-\delta$ ($\rho_1^3 > \rho > \rho_2^3$) will be forbidden: these are dominated by groups in set α ($\rho \leq \rho_2^3$ or $\rho_1^3 \leq \rho$). For every ρ in $\beta-\gamma-\delta$, one or more values in α can increase both U_1 and U_2 (see Fig. 11). (c) Diverse group compositions may yield the same welfare distribution. This occurs at (U_1^*, U_2^*) in Figure 11 [i.e., at (ρ_1^3, ρ_2^3) in Fig. 10].

For the problem of how to organize a given national endowment, the results may be summarized as follows. (a) For endowed ratios in the set α, uniform jurisdictions are efficient, with all groups having the same L_1/L_2 as the nation as a whole. Absent nonconvexities, every first-best feasible welfare distribution is achieved by lump-sum transfers within these identical communities. (b) Assuming nonconvexities (i.e., segments such as $\beta-\gamma-\delta$ along E), national endowment ratios within set δ (between ρ_1^1 and ρ_2^1) always require nonuniform groups at ρ_1^3 and ρ_2^3 which increase both utilities to U_1^* and U_2^*. If transfers are required to optimize social welfare, the specific mixtures may be influenced by the amount and direction of the transfer, but uniform communities within δ are inferior whether transfers are allowed or not. (c) If the overall endowed population is in set β or γ ($\rho_2^3 \leq \rho \leq \rho_2^1$ or $\rho_1^1 \leq \rho \leq \rho_1^3$) then, absent income transfers, uniform community structure cannot be bettered. However, if income can be lump-sum redistributed (as in Fig. 9), then a mix of diverse groups Pareto-superior to uniform organization in γ or β will be available. This is most readily seen for an endowed ρ in γ, for then a nonuniform mix of groups is readily found which keeps one type at its uniform-mix utility level and increases utility of the other type in both kinds of groups. The welfare of the two types who gain can then obviously be equalized by lump-sum transfers between them (and therefore between kinds of groups). (d) Endowed ratios over certain ranges such as $\rho < \rho^c$ and $\rho > \rho^a$ in Figure 4 are of special note and are discussed in Appendix A.

V. Conclusions

This paper has examined club configurations when groups serve as sites both for private production and LPG consumption. Special attention has been given to efficiency of uniform versus diverse heterogeneous groups. A diagrammatic technique has been introduced with the following results. (a) Absent nonconvexities, when group composition is open to choice, changing that composition is never inferior as an instrument of distribution to lump-sum income transfers. (b) Efficient organization of Berglas clubs is achieved by creating uniformly heterogeneous groups only under specified conditions of convexity, and even with convexity, some endowments may simply contain a surplus of entire consumer types (Appendix A). For some configurations of taste and productivity (themselves individually convex), diverse mixes of heterogeneous clubs can be optimal due to nonconvexities that result from the interplay of the components. When diverse mixes are efficient, it

means that all consumers of one type consume more of the LPG than they demand and all of the other type consume less than ideally desired (at unit price). In groups with high LPG provision, the ratio of the first type to the second is lower than in groups with low amounts of LPG provision. This difference in factor proportions is just adequate to yield the right wage difference to compensate each individual for belonging to the (from his perspective) less desirable group. (c) Comparisons of Tiebout and Berglas club configurations can be resolved by reference to differences in taste for public versus private goods among consumers, and differences in private factor productivities. Knowledge of substitution elasticities in production and consumption is also required to compare mixed interior and segregated corner solutions. (d) None of these conclusions depends on economies of scale or integer problems of any sort, whether in private or LPG production.

Appendix A

The Social Constraint on LPG Supply and "Surplus" Factor Endowments

This appendix examines further the notion of "efficient" community compositions, when community membership is a variable of choice or when a separate group can be carved out of a larger set by excluding some people. The topic bears further analysis because the expression maximized in equation (2) contains an implicit constraint in addition to those stated in the text, and this additional constraint can lead to the possibility of an "absolute surplus" of some consumers. From equation (2), each type's utility is limited by the utility of the other type and by the technology (ϕ curve). In addition, however, each type's utility is also limited by the necessity to provide equal LPG to all in the group. This means that the maximum LPG feasible and consistent with (2) is limited to the earnings of the less productive type. For example, in Figure 2 point G_b^{max} shows the maximum G consistent with $\rho^b = (L_1/L_2)^b$; at that ratio, 1's could not obtain greater LPG unless 2's spent more than their total earnings on G, or unless 1's allocated more than $c = G$ of earnings to LPG costs or both. The absolute maximum of LPG supply cannot exceed the maximum of the smaller marginal productivity (which occurs at a factor ratio where these productivities are equal: $F_1 = F_2 = G^{max}$). More generally, as L_1/L_2 declines, and consequently as F_1 increases and F_2 declines, the maximum G (absent cross subsidy between 1's and 2's) will decline along with F_2. Connecting these maximum LPG points identifies the social limit on LPG supply (M_i in Fig. 4, $i = 1, 2$). Equations of these frontiers in parametric form are $I_i - c_i \equiv Y_i = F_i(\rho) - F_j(\rho)$ and $G = F_j(\rho)$ for M_i ($i = 1, 2$; $j = 2, 1$). For each point on M_1 (M_2), there is a corresponding point on the G-axis for type 2 (type 1), where all 2's (1's) earnings are allocated to G. The M frontier thereby places an outer limit on the consumption-opportunity curves S^i. The absolute maximum utility for either consumer occurs at the tangency in Figure 4 between U_i and M_i (shown also in Fig. 5 at the extremes of the utility-possibility frontier). This tangency corresponds to a special case in the original formulation of equation (2), specifically, when only one type's utility is valued in equation (2). For example, let $\lambda = 0$; then, 2's utility has value only as enhancing the utility of the 1's. From equation (1), however, society still must provide the LPG to all. Now necessary conditions include $F_1 - I_1 + c_1 - G = 0$ and $F_2 - G = 0$, showing all 2's product is spent on the LPG exactly as above; the Samuelson condition changes to $L_1 \mathrm{MRS}_1 = \Sigma L_i$.

The primary implication of this analysis is that there can be "too many" factor consumers of a particular type. This occurs when the endowment is beyond the cutoff points on ϕ at which the complementary factor consumer's utility reaches an absolute maximum. Returning to Figure 4, one sees some factor proportions for which the more numerous consumer type is in what might be termed "absolute surplus"; the relative number of persons of a particular type can become so great that their productivity is less than their contribution to LPG congestion. Thus, at composition ρ^a in Fig-

ure 4. 1's factor earnings are so low and group demands for LPG so high that efficient provision of G requires the 1's to spend their entire incomes on G, which places the 1's at point a, giving the best possible outcome for type-2 individuals (i.e., U_2^m tangent to frontier M_2). From this point, the 2's could transfer income to 1's; 2's would lose and 1's would gain, though not as much as if the population mix were changed to fewer 1's. However, any ρ higher than ρ^a will lower utility of both 1's and 2's. In this region of corner solutions, if 2's now transfer income to 1's, both 2's and 1's may gain, and both will necessarily gain if (for small amounts) all the transfer is allocated to LPG provision. Similarly, for $\rho < \rho^c$, there are "too many" type-2 individuals; for at ρ^c all 2's earnings are absorbed by LPG costs. Thus, if the nation must include all consumers when endowed proportions are ρ^0 in Figure 4, then both 1's and 2's have lower utility (U_1^0, U_2^0) than the included 1's and 2's would have if (excluding some 2's altogether) they organized at ρ^c. The "efficient" region of the utility frontier between included types therefore terminates at the welfare extremes $U_1^m - U_2^m$ with factor consumer ratios at a and c (for further development, see McGuire [1986]).

Appendix B

Example in which Nonuniform Groups Are Superior

This appendix gives an example of the general case illustrated by Figures 8 and 9 in which the existence of two different types of communities (with different mixes of L_1 and L_2 and with different efficient supplies of LPG) is Pareto-superior to uniform communities, all with the same $L_1 - L_2$ mix, and efficient LPG supply.

Assumptions:

(i) Population: total $L_1 = 195$ and total $L_2 = 600$.

(ii) Utility functions: $U_2 = Y_2 + 0.5G$ and $U_1 = Y_1^a G^{1-a} = Y_1^{0.5963} G^{0.4037}$.

(iii) Private good production: two fixed-proportion "Leontief" technologies, "a" and "b," with $x_a = \min[(99/8)L_1, (99/64)L_2]$; $x_b = \min[3L_1, 6L_2]$. The two can be mixed in any linear combination giving marginal products of $\mathrm{MPL}_1 = 2.375$ and $\mathrm{MPL}_2 = 1.25$. Therefore, the combined production function becomes

$$x = x_a + x_b$$
$$= \min[3L_1, 2.375L_1 + 1.25L_2, (99/64)L_2].$$

Constraints:

(i) Consumer 2's utility/indifference curve: $U_2 = Y_2 + 0.5G = 1$.

(ii) Factor-productivity curve: the production function gives a ϕ curve kinked at point ($\mathrm{MPL}_1 = 2.375$, $\mathrm{MPL}_2 = 1.25$), as at ρ^0 in Figures 8 and 9. Its equations are $2F_1 + F_2 = 6$ (upper-left segment) and $F_1 + 8F_2 = 12.375$ (lower-right segment).

(iii) Consumer 1's opportunity set: $U_2 = 1$ and ϕ determine the S curve, kinked (as in Figs. 8 and 9) at point ($Y_1 = 1.875$, $G = 0.5$). Equations of S are $Y + 5G = 4.375$ (the upper-right segment) and $0.8Y + G = 2$ (the lower-left segment).

Composition of Alternative Uniform and Diverse Communities:

	Two nonuniform jurisdictions		One uniform jurisdiction
	a	b	Mixed a & b
L_1:	$\frac{67}{536} = \frac{1}{8}$	$\frac{128}{64} = \frac{2}{1}$	$\frac{1}{8} < \frac{195}{600} < \frac{2}{1}$

Solution Showing That Nonuniform Groups Give Higher Utility

L_1's consumption and utility ($U_1 = Y_1^{0.5963} G^{0.4037}$):

Y_1:	2.6088		1.4905		1.8750
G:	0.3532		0.8072		0.5000
U_1:	1.1637	=	1.1636	>	1.099

L_2's consumption and utility ($U_2 = Y_2 + 0.5G$):

Y_2:	0.8234	0.5974	0.7500
G:	0.3532	0.8072	0.5000
U_2:	1.000	1.000	1.000.

Conclusion.—Organizing communities all with the same L_1/L_2 ratio (195/600) and

an efficient amount of LPG can be inferior to creating type-a communities with $L_1/L_2 = 1/8$ and type-b communities with $L_1/L_2 = 2/1$ (in proportions 67:64) with an efficient amount of LPG in each. Note that these solution values of Y_1, Y_2, and G are located on the proper segment of ϕ and S, that MRS_1 equals the slope of the appropriate segment of S, and that the $\Sigma MRS = MC$ condition obtains in each group For the one uniform jurisdiction, the MRS_1 at the kink in S is calculated as 2.5388, and the $\Sigma MRS = MC$ condition obtains there too.

Example of a Bergson / Samuelson Social Welfare Function that Selects this Pareto-Optimal Configuration.—The weighted sum of utilities of representative types in community a,

$$W = 0.4675U_1^a([F_1^a - c^a + T_1^a], G^a) + 0.5325U_2^a([F_2^a - c^a + T_2^a], G^a)$$

when maximized subject to the constraints $U_1^a = U_1^b$ (with Lagrangian multiplier $L = 0.2440$), $U_2^a = U_2^b$ (with $L = 0.0568$), and $\Sigma T = 0$ (with $L = 0.8874/1.000$) can be solved for $T^i = 0$ ($i = a, b$; $j = 1, 2$) for the above values of Y and G and for $U_2 = 1$ and $U_1 = 1.164$, as above.

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