PN-ABN-615

81494

IRIS Reprint

The Center for Institutional Reform and the Informal Sector (IRIS) has two main purposes: expanding knowledge about institutions in economic development through research, and assisting reform efforts in the third world and in countries undergoing transitions to a market economy. The premise of the IRIS Center is that in unsuccessful economies the existing rules establish poor incentives, often forcing economic activity into the informal economy, and that appropriate reforms improve economic performance. IRIS is especially concerned with the legal and policy framework needed for democratic societies with competitive markets.

IRIS
7100 Baltimore Avenue, Suite 510
College Park, MD 20740
(301) 403-8153

CENTER FOR INSTITUTIONAL REFORM AND THE INFORMAL SECTOR

University of Maryland at College Park

Center Office: IRIS Center, 7100 Baltimore Avenue, Suite 510, College Park, MD 20740
Telephone (301) 403-8153 • Fax (301) 403-8163

THE DEMAND FOR RETAIL PRODUCTS AND THE HOUSEHOLD PRODUCTION MODEL

November, 1989

Roger R. Betancourt and David Gautschi

IRIS Reprint No. 17

This publication was made possible through support provided by the U.S. Agency for International Development, under Cooperative Agreement No. DHR-0015-A-00-0031-00.

Author: Roger R. Betancourt, University of Maryland at College Park and David Gautschi,INSEAD, Cedex, France

Reprinted from *Journal of Economic Behavior and Organization* , 17 (1991) 257-275.

Journal of Economic Behavior and Organization 17 (1992) 257–275. North-Holland

The demand for retail products and the household production model

New views on complementarity and substitutability*

Roger R. Betancourt

University of Maryland, College Park, MD 20742, USA

David Gautschi

INSEAD, 77305 Fontainebleau, Cedex, France

Received June 1989, final version received November 1989

By treating the distribution services provided by retailers as fixed inputs into the household's production activities, we obtain a number of new results with respect to the own and cross-price elasticities of demand for items in a retail assortment as well as with respect to a new concept – the distribution services elasticity of demand for items in a retail assortment. This treatment of the household production model shows that there are pervasive tendencies toward gross complementarity among items in any given retailer's assortment as well as between the items in any given retailer's assortment and the distribution services offered by the retailer. These tendencies are instrumental in understanding the nature of competition among retailers and the creation of retail agglomerations.

1. Introduction

Retail firms provide consumers with a variety of distribution services, such as accessibility of location, product assortment, ambiance, assurance of product delivery at the desired time and in the desired form, and information [cf. Betancourt and Gautschi (1988)]. Higher levels of these services cost the firms more but reduce costs of their customers. The potential shifting of these distribution costs between consumers and retailers is one of the essential characteristics of retail markets and it has important economic consequences.

*The ideas in this paper at various stages of elaboration were presented in seminars at Georgetown University and the University of Maryland. An earlier version of the paper was also presented at the INSEAD Telecommunications Conference (Fontainebleau, June 1987) and at the meetings of the Eastern Economic Association (Boston, March 1988). We thank the participants in these seminars, the discussants of the paper, M. Perry and J. Anderson, as well as Ira Horowitz for their comments without incriminating them for the results. The final version of the paper has benefited from R. Day's editorial suggestions. Both authors also gratefully acknowledge INSEAD's financial support for this research project.

0167-2681/92/$05.00 © 1992—Elsevier Science Publishers B.V. All rights reserved

Best Available Document

For instance, Hotelling's (1929) finding that competition on location is always detrimental to consumers (when prices are fixed), Sharkey (1982 ch. 4), is invalidated if this shifting in distribution costs is allowed to take place [Betancourt and Gautschi (1989)]. More generally, these distribution services are an important source of product differentiation in retail markets. Indeed, the present contribution is a detailed reformulation of demand partially inspired by recent work on product differentiation, e.g., Perry and Groff (1985), and by Chamberlin's (1962) seminal analysis of monopolistic competition.

In this paper we postulate a household production model in which the retailer's distribution services act as fixed inputs into the household's production activities. We then derive the effects of changes in prices and distribution services. The focus of our investigation is on the implications of this formulation for demand analysis. We analyze first the own (uncompensated) price elasticity of demand for retail products. A two-stage formulation of the model leads to a natural decomposition of a price change into a production effect and a consumption effect, which can be analyzed separately and are reminiscent of the Slutsky decomposition of standard consumer theory. Subsequently, by analyzing the (uncompensated) cross-price elasticity of demand for retail items, we are able to explain why the majority of items in the assortment of a retailer will be gross complements from the point of view of the household production model. We then analyze the distribution services elasticity of demand in a manner that generates results perfectly comparable to those obtained with respect to price changes.[1] An important finding is the pervasive tendency for the distribution services of a retailer to be gross complements with the items in his or her assortment. These gross complementarities provide powerful incentives on the demand side for the creation of retail agglomerations including, for example, shopping centers and shopping malls.

Contributions to the marketing literature have often made the point that the role of distribution is more than just providing market goods. For instance, Bucklin (1966) views the 'product' of distribution 'as a mix of market goods in conjunction with an array of services...' More specifically, in the context of retailing, Ingene (1984) notes the possibility of a shifting of distribution costs between consumers and retailers. Nevertheless, until very recently, these ideas were put forth in an informal manner which limited their applicability and further development. Several recent papers have moved the

[1]Our analysis generalizes to any other determinant of consumption behavior that can be viewed as a fixed input into the household's production activities. Incidentally, Barten (1977, pp. 36–37) notes a related feature in the standard approach to the analysis of consumer behavior and Dreze and Hagen (1978, Appendix) provide a similar formalization with respect to the input-output coefficient in Lancaster's linear technology model. Neither one of these contributions, however, exploits the economic implications of the result.

topic closer to formal analysis. For instance, Bliss (1988) postulates a demand side for a retail market in which an indirect utility function depends on the (same) wholesale prices plus a mark-up, which varies according to the store's location, and an income net of transport cost. On the other hand, Lal and Matutes (1989) postulates a demand side for a retail market in which there is a uniform distribution of consumers with respect to various combinations of distribution services offered by stores. Neither paper develops the demand side of the analysis in any detail.

The household production model was introduced in the 1960's by Muth (1966), Becker (1965) and Lancaster (1966). Becker's emphasis on the role of time in consumption activities spurred a large number of applications, especially to non-standard topics such as the economics of the family [Willis (1987)]. Muth and Lancaster were concerned with applications to standard economic topics, for example the use of specific market goods or retail items to produce commodities such as food or nutrition, but the use of restrictive assumptions on the specifications of the household's technology has limited their applicability. The analysis in this paper follows the specifications of the model in the Becker tradition but applies this specification to the analysis of topics in the Muth and Lancaster tradition. Many of the technical properties of the Becker formulation of the household production model were clarified in a contribution by Pollak and Wachter (1975) and in a subsequent exchange (1977) with Barnett (1977). Finally, Deaton and Muellbauer (1980) provide a clear and concise presentation of the model, including the two-stage procedure that provides the point of departure for our own analysis.

The approach presented here should serve as a useful formal characterization of the demand side of retail markets in future theoretical and empirical work. If integrated with the welfare analysis provided in Bockstael and McConnell (1983, section 3), it should also prove useful for policy evaluation.

2. The model

We develop the household production model in the two-stage form used by Deaton and Muellbauer (1980, pp. 245–254). The first stage can be described as follows:

$$\underset{Q}{\text{Min}}\, pQ \quad \text{s.t.} \quad h(Q, D, Z) = 0, \tag{1}$$

where Q is a vector of all the goods and services employed by the household in production, including the goods and services purchased from different retailers as well as the time employed by the household in production activities. p is a corresponding vector of prices, including the opportunity cost of the household's time. D is a vector of distribution services provided

by the retailers which the household patronizes in its purchase activities. Z is the vector of commodities produced by the household, which are the ones that yield satisfaction or utility directly. $h(\cdot)$ is a quasi-convex transformation function.

The result of this optimization procedure is the cost function below

$$C = C(p, D, Z). \tag{2}$$

This function has the following properties: Non-decreasing concave and linear homogeneous in prices, increasing in outputs (the elements of Z) and nonincreasing in distribution services (the elements of D). The last property follows from assuming that the distribution services provided by a retailer act as fixed inputs into the household's production activities. It is in this manner that the shifting of distribution costs between consumers and retailers can be captured formally in the model. It follows from Shephard's Lemma that the conditional or Hicksian demand function for a good purchased from a particular retailer will be given by

$$Q_k = C_k = \partial C/\partial p_k = g_k(p, D, Z), \quad k = 1, \ldots, K. \tag{3}$$

In the second stage the household maximizes utility, by choosing the optimal levels of the commodities that yield satisfaction, subject to the constraint that the household's full income (W) be sufficient to cover the costs of producing the commodities, i.e.,

$$\text{Max } U(Z) \quad \text{s.t.} \quad W \geqq C(p, D, Z),$$

where $U(Z)$ is an increasing, strictly quasi-concave utility function. The first-order conditions for an interior solution are given by

$$U_i(Z) = \lambda C_i(p, D, Z), \quad i = 1, \ldots, I \tag{4}$$

$$W = C(p, D, Z), \tag{5}$$

where $U_i = \partial U/\partial Z_i$, $C_i = \partial C/\partial Z_i$ and λ is the usual Lagrange multiplier. The solution of (4) and (5) yields the demand functions for the commodities, i.e.,

$$Z_i = f_i(p, D, W), \quad i = 1, \ldots, I. \tag{6}$$

Finally, substitution of (6) into (3) yields the Marshallian or uncompensated demand functions for any item purchased from a retailer,

$$Q_k = g_k[p, D, f(p, D, W)], \quad k = 1, \ldots, K. \tag{7}$$

3. The own-p

The two-st
of the effects
standard con
retail item ob

$\varepsilon_{kk} = \varepsilon$

where

$\varepsilon_{kk} = ($

$\varepsilon^*_{kk} = ($

$\omega_{ki} = ($

The first ter
It represents t
or input give
satisfaction. F
cost function.

The second
captures the p
the changes in
costs of produ
in the usage o
Z_i. Througho
production [H
demand of the
item or input.

Just as in th
is useful to d
negative. In co
conditions for
These conditior

Theorem 1. W
in the productio
price of that ite
must be elastic.

[2]The proofs for t
in the Appendix.

4

3. The own-price elasticity of demand for retail items

The two-stage formulation of the model leads to a natural decomposition of the effects of a price change analogous to the Slutsky decomposition of standard consumer theory. That is, the own price elasticity of demand for a retail item obtained from (7) can be written as

$$\varepsilon_{kk} = \varepsilon^*_{kk} + \sum_i \omega_{ki}\eta_{ik}, \tag{8}$$

where

$$\varepsilon_{kk} = (\partial Q_k/\partial p_k)(p_k/Q_k),$$

$$\varepsilon^*_{kk} = (\partial Q_k/\partial p_k \mid Z)(p_k/Q_k),$$

$$\omega_{ki} = (\partial Q_k/\partial Z_i)(Z_i/Q_k) \quad \text{and} \quad \eta_{ik} = (\partial Z_i/\partial p_k)(p_k/Z_i).$$

The first term on the RHS of (8) is the production effect of a price change. It represents the percentage change in the quantity demanded of the kth item or input given the levels of production of the commodities that yield satisfaction. From (3), it must always be nonpositive by the concavity of the cost function.

The second term in (8) is the consumption effect of a price change. It captures the percentage change in the demand for the kth item as a result of the changes in the demand for commodities induced by the change in the costs of producing these commodities. ω_{ki} represents the percentage change in the usage of input Q_k as a result of a percentage change in the output of Z_i. Throughout we will assume that there are no regressive inputs in production [Hicks (1946)]; consequently, $\omega_{ki} \geqq 0$. η_{ik} is the elasticity of demand of the ith commodity with respect to a change in the price of the kth item or input.

Just as in the analysis of the income effect of standard consumer theory, it is useful to derive sufficient conditions for the consumption effect to be negative. In contrast to that case, however, there are two sets of sufficient conditions for the consumption effect to be negative which are of interest. These conditions are stated in the following theorems.[2]

Theorem 1. When an item in a retailer's assortment is used by the household in the production of every commodity, the consumption effect of a change in the price of that item will always be negative. Moreover, the demand for this item must be elastic.

[2]The proofs for these two theorems as well as the proofs for all subsequent ones are provided in the Appendix.

A second set of sufficient conditions for the consumption effect to be negative provides a result analogous to the standard case. Namely,

Theorem 2. When an item in a retailer's assortment is used exclusively in the production of a single commodity, the consumption effect will be negative if the commodity is a normal commodity in the sense of having a positive income effect.

These two theorems identify two basic forces leading to a negative consumption effect. First, each product price elasticity of demand for a commodity (η_{ik}) will tend to be negative for the same reason as in the case of standard consumer theory. Secondly, even if some of them are not a weighted average of these elasticities must lead to a negative number by Cournot aggregation. Given that the production effect is always negative, one concludes that the own uncompensated price elasticity of demand for retail items (ε_{kk}) is negative. Less transparently, Theorem 1 suggests that retail items which are used by the household in the production of many different commodities will have more elastic demands than those which are used by the household in the production of a single commiunity, other things equal. An important input that falls into this category is the household's time.

4. The cross-price elasticity of demand for retail items

From the uuncompensated demand function in (7), we obtain the cross-price elasticity of demand for any items in a retail assortment,

$$\varepsilon_{kl}=\varepsilon^*_{kl}+\sum_i \omega_{ki}\eta_{il}, \quad k=1,\ldots,K, \quad l=1,\ldots,K, \tag{9}$$

where

$$\varepsilon_{kl}=(\partial Q_k/\partial p_l)(p_l/Q_k) \quad \text{and} \quad \varepsilon^*_{kl}=(\partial Q_k/\partial p_l \mid Z)(p_l/Q_k).$$

Eq. (9) suggests the following definitions.

Definition 1. Two items in an assortment are *net* substitutes, independent or complements as the production effect (ε^*_{kl}) is positive, zero, or negative, respectively.

Definition 2. Two items in an assortment are *gross* substitutes, independent or complements as the sum (ε_{kl}) of the production effect and the consumption effect is positive, zero, or negative, respectively.

When two items are used in the production of the same commodity, the full range of possibilities encompassed by Definition 1 is available and the classification of items into one category or another must depend on specific circumstances. For example the classifications of coffee and tea or coffee and sugar as substitutes or complements are well defined only when these items are viewed as inputs into the production of the same commodity, and the relevant definition being used is Definition 1. This interpretation is strengthened by the following theorem.

Theorem 3. If there is no joint production of commodities, any two items used exclusively in the production of different commodities will be NET independents.

Turning to the consumption effect, i.e., the second term on the RHS of (9), a number of possibilities must be considered. It is convenient to start by considering two possibilities that correspond to the two theorems of the previous section.

Theorem 1'. If two items in the retail assortment are used by the household in the production of every commodity, the consumption effect will be negative.

Theorem 2'. If two items in the retail assortment are used exclusively in the production of the same commodity, (n), the consumption effect will be negative if the commodity has a positive income effect.

An implication of these two results is that the consumption effect, by being negative, is a force driving all items in the assortment toward gross complementarity. Indeed, a similar force exists in the standard analysis of the consumer under the assumption that goods are normal. This tendency toward gross complementarity, which has not been stressed in the literature, is further illustrated by considering the consumption effect when two items in the assortment are used exclusively in the production of different commodities.

Theorem 4. If two items in a retail assortment are used exclusively by the household in the production of different commodities (n, m), in the absence of joint production, these items will be gross complements even if the commodities are (net) substitutes in consumption, provided that the commodities are normal goods and that the income effect is 'sufficiently strong.'

The main economic implication of the foregoing is that *most items purchased by the household from a retailer will be gross complements.*

To see this, consider items that are used by the household either in the

production of every commodity or exclusively in the production of the same commodity, as discussed in Theorems 1′ and 2′. If these items are net independent or complements in production, they will also be gross complements. Moreover, even if these items are substitutes in production they will still be gross complements if the consumption effect dominates the production effect.

Next consider items used by the household exclusively in the production of different commodities, as discussed in Theorem 4. If these items are used in the production of commodities that are net independent or complements in consumption, the items will be gross complements. Even if these items are used in the production of commodities that are net substitutes in consumption, the items will still be gross complements if the income effect dominates the substitution effect. While situations in which two items purchased from a retailer are gross substitutes can in principle be constructed, the analysis in this section suggests that such situations are far less common than situations where gross complementarity prevails.[3]

5. The services elasticity of demand for retail items

In this section we turn to consider the most novel and important feature of the model for the analysis of retail markets, namely, the services elasticity of demand. Retail firms offer consumers a set of explicit products or services for purchase *together* with various levels of distribution services such as accessibility of location, information, etc. In the present model the levels of these distribution services appear as fixed inputs in the household's production activities, i.e., as elements of the D vector. Hence, if a retailer increases the levels of these services the costs to the household of producing a given level of commodities must not increase and will usually decrease.

Distribution services will be classified as *common* or *specific* in the subsequent discussion. A common distribution service, such as accessibility of location, is one that is available to all the explicit products offered by a retailer. If two stores instead of a single store are located in a given market area, the increased accessibility of location is provided for all items that the household may purchase from each store. By contrast a specific distribution service, for example information on the price of an item, is one that is available to one particular item that the household may purchase.

From the uncompensated demand function in (7), we can also obtain the

[3]Parenthetically, the identification of items that are gross complements and have high (absolute) values of the cross-price elasticity of demand underlies the rationale for market basket pricing and for what the retailing literature calls 'one way cross-price elasticities of demand' [Albion (1983)]. In a related paper addressed to the marketing literature, Betancourt and Gautschi (1990), we derive the implications of our analysis for this issue and show that, regardless of the type of usage of an item in household production, the items most useful as traffic builders are those that constitute a large share of the consumer's budget.

services elasticity of demand for any item in a particular retailer's assortment with respect to the *jth* distribution service. Namely

$$\varepsilon_{kj}=\varepsilon^*_{kj}+\sum_i \omega_{ki}\eta_{ij}, \quad k=1,\ldots,K,\ j=1,\ldots,J, \tag{10}$$

where

$$\varepsilon_{kj}=(\partial Q_k/\partial D_j)(D_j/Q_k), \qquad \varepsilon^*_{kj}=(\partial Q_k/\partial D_j \mid Z)D_j/Q_k,$$

$$\omega_{ki}=(\partial Q_k/\partial Z_i)(Z_i/Q_k) \quad \text{and} \quad \eta_{ij}=(\partial Z_i/D_j)D_j/Z_i.$$

Eq. (10) suggests the following definition.

Definition 3. A distribution service of a retailer and an item in this retailer's assortment that may be purchased by the household, i.e., $k=1,\ldots,K$, are *net* complements, independent or substitutes as the production effect (ε^*_{kj}) is positive, zero or negative.[4]

While one would expect the distribution services of a retailer to be *net* independent or complements with the items in the retailer's assortment that the household may purchase, it is difficult to rule out in general, except by assumption, special sets of circumstances in which a relation of *net* substitutability may arise.

One important relation of net substitutability arises with respect to the input of the household's time in the production of commodities. Several distribution services, by their very nature, are net substitutes in production with the household's own time in purchase activities. We are referring in particular to accessibility of location, extent of product assortment, degree of assurance of product delivery and level of information services. Higher levels of these distribution services diminish the need for the household to employ its own time in travel and search activities at any given level of Z_i's.

Eq. (10) also suggests the following definition.

Definition 4. A distribution service of a retailer and any item in this retailer's assortment that may be purchased by the household are *gross*

[4]Incidentally, this definition is perfectly consistent with Definition 1 of section 4. The reason for the difference in sign is that the elasticity is being defined with respect to a change in a quantity rather than a price. For instance, in a conceptual experiment where the distribution services were offered by a retailer at an explicit price (p_j), we would have: an increase in p_j decreasing D_j by the concavity of the cost function), which in turn would imply a decrease in Q_k if Q_k and D_j are complements. Since the behavior of the restricted cost function must be consistent with the behavior of the unrestricted one, Definition 3 is consistent with this conceptual experiment.

complements, independent or substitutes as the sum (ε_{kj}) of the production effect and the consumption effect is positive, zero or negative.

Given this definition, it is of interest to explore the circumstances generating positive consumption effects. These circumstances are summarized in the following theorems:

Theorem 5. If an item in an assortment is used in the production of every commodity by the household, the consumption effect of a change in distribution services will always be positive.

Theorem 6. If an item is used exclusively in the production of a single commodity (n), the consumption effect of a change in a specific distribution service will always be positive if the commodity has a positive income effect.

Theorem 6'. If an item is used exclusively in the production of a single commodity (n), the consumption effect of a change in a common distribution service, which affects all marginal costs in the same proportion, will always be positive if the commodity has a positive income effect.

The most direct economic implication of these results is that *the distribution services provided by a retailer will tend to be gross complements with every item in this retailer's assortment that may be purchased by the household.* It is useful to examine some of the conditions under which the results will be strongest, in the sense of leading to the largest (positive) values of the distribution services elasticity of demand. First, the percentage reduction in the costs to the household of an increase in a distribution service (S_j) by the retailer is likely to be much higher for distribution services that are common inputs and affect many of the household activities than for those that are specific inputs and only affect the costs of purchasing a particular item. Second, the same common input, for example accessibility of location, may be a greater source of gross complementarity in an area where the opportunity cost of time is high than in an area where it is low, because an increase in this service generates a larger reduction in costs for a household with a high opportunity cost of time than for one with a low valuation of time. Finally, specialty retailers who provide items that are a small share of the consumer budget may be able to increase demand for these items considerably by providing a distribution service that is a net complement with these items and that contributes to a substantial reduction in costs.

6. Implications

The above analysis suggests that the consumption effect is a powerful force

driving the demand for the items in a retailer's assortment toward gross complementarity. With respect to price changes, gross substitutability is indeed possible, but it requires the net substitution effect in production to be stronger than the consumption effect for items used in the production of the same commodity or the net consumption effect to dominate the income effect for items used in the production of different commodities that are net substitutes in consumption. In the case of distribution services, since the net production effect is almost always likely to be positive, gross substitutability is far less likely.

These results raise the question of how competition takes place in the context of our model. Consider a change in the price of an item in the assortment of retailer B. It will have an effect on the demand for an item in the assortment of retailer A described by eq. (9) in Section 4. A rise in B's price of brand X of coffee is likely to generate a larger positive net production effect on brand X of coffee in A's assortment than on brand Y of coffee in B's assortment. Therefore, it is more likely to dominate the consumption effect and thus lead to gross substitutability.

Indeed, it is entirely possible for retailer B to drive an item out of the bundle that the household purchases from A's assortment through price decreases, because the size of the consumption effect is directly related to the value of the usage term for A, ω_{ki}, which will be driven to zero as a result of a series of price decreases when there is gross substitutability between the item in the two assortments.

With respect to changes in distribution services, one can similarly analyze the effect of a change in a distribution service by retailer B on the demand for an item in the assortment of retailer A through the use of eq. (10). A change in a distribution service in the assortment of retailer B will usually have two types of net production effects on the items in B's assortment, $\varepsilon^*_{kj}(B,B) \gtreqless 0$, as the items are net complements or independents with the jth distribution service, respectively.[5] This means that in the case of net independence, there will be no net production effect on the item in A's assortment, i.e. $\varepsilon^*_{kj}(A,B)=0$. Because the consumption effect will be positive, however, the demand for items in A's assortment will increase as a result of the increase in B's jth distribution service.

Those items in B's assortment that are net complements with the jth distribution service will generate a more complex pattern of changes as a result of an increase in B's jth distribution services. The net production effect on the item in A's assortment, $\varepsilon^*_{kj}(A,B)$, will be positive, zero or negative as these items are net production substitutes, independent or complements with the items in B's assortment, $\varepsilon^*_{ki}(A,B) \gtreqless 0$, that are net complements with the

[5]Our notation is such that the order of identification of the retailer inside the parentheses is always coupled with the order of the subscripts. Thus, $\varepsilon^*_{kj}(A,B)$ would mean the direct production effect of B's jth distribution service on the kth item in A's assortment.

jth distribution service, $\varepsilon_{ij}(B, B) > 0$. Moreover, the demand for those items in A's assortment will tend to increase, since the consumption effect is positive, unless the net production effect for those items that are substitutes, $\varepsilon^*_{ij}(A, B) < 0$, is sufficiently strong to dominate the consumption effect. Hence, allowing for the role of different retailers generates opportunities for a distribution service of one retailer to exhibit gross substitutability with items in another retailer's assortment, if these items are net substitutes in production with items in the assortment of the retailer that alters the level of the distribution service.

7. Conclusions

Our characterization of the demand for retail products suggests three main general insights into the interactions between retailers in a market. First, distribution services provide retailers with tools for nonprice competition but these tools have limits on their usefulness. They must work through the same channels as price competition, because net substitutability in production between items in different assortments is a necessary condition for the nonprice competition to take place. Furthermore, common distribution services are not precise instruments because if gross complementarity prevails they may affect the demand for many different items in the different assortments in the wrong direction. Second, the effects of both price and nonprice competition between retailers are asymmetric, especially when the assortment and the mix of distribution services offered by any two retailers are very different in the initial situation. Third, the tendency toward gross complementarity between retailers through the provision of distribution services is one of the most important forces leading to the creation of retail agglomerations.

The model also helps explain why two department stores with very broad but similar assortments might agree to locate in the same mall but two specialty stores with very narrow and similar assortments might be reluctant to do so. This is because net substitution in production between all the items in the narrow assortment of the specialty stores is far more prevalent than between all the items in the broad assortments of the department stores. Therefore, gross substitutability and intense price and nonprice competition are more likely to arise for the specialty stores than for the department stores.

Since most distribution services are net substitutes in production with the household's use of its own time in consumption activities, a significant part of the 'competitive effect' of increases in most distribution services by any one retailer is likely to be borne by the household through its time allocation. If prices are fixed such changes always enhance welfare, because the household can maintain the same pattern of consumption as before the

increase in distribution services. Alternatively, increases in the opportunity cost of time, through rising wage rates, provide an incentive for retail institutions to offer higher levels of distribution services that economize on this household resource directly or indirectly, i.e., by allowing a relaxation on the constraints that may limit the timing of its employment.

Appendix

The proofs of Theorem 1, 1′ and 5 are all based on a condition analogous to Cournot aggregation in the standard model and we take them up first.

Proof of theorem 1. Differentiating (5) with respect to the price of the kth product and manipulation of the results leads to the following condition:[6]

$$\sum_i \theta_i \eta_{ik} = -S_k, \tag{A.1}$$

where $\theta_i = C_i Z_i / C$ and is the marginal budget share of the ith commodity. $S_k = p_k Q_k / C$ is the budget share of the kth input or item in an assortment.

If the weights in the consumption effect, ω_{ki}, are proportional to the marginal budget shares of the ith commodity, the consumption effect will be negative by (A.1). This is indeed the case. These weights will either be zero, if the kth product is not used in the production of the ith commodity, or proportional to the marginal budget shares. For

$$\begin{aligned}\omega_{ki} &= (\partial Q_k/\partial Z_i) Z_i / Q_k = (\partial Q_k/\partial Z_i)(p_k/C_i) C_i Z_i / p_k Q_k, \\ &= [(\partial Q_k/\partial Z_i)(p_k/C_i)] \theta_i / S_k.\end{aligned} \tag{A.2}$$

The term in square brackets will either be zero or unity, because $(\partial Q_k/\partial Z_i) p_k$ is the marginal cost of input k in the production of the ith commodity and it must, therefore, equal C_i, or the marginal cost of producing the ith commodity, if it is used in the production of the ith commodity or zero if it is not used.

Substitution of (A.2) in (8) and use of (A.1) yields

$$\varepsilon_{kk} = \varepsilon^*_{kk} - 1 \tag{A.3}$$

Proof of theorem 2′. Under the hypothesis of the theorem, $\omega_{ki} = \theta_i / S_k$ for all i; hence, the second term in (9) becomes

[6]Incidentally, eq. (A.1) is simply the Cournot aggregation or adding up condition in the household production model.

$$\sum_i \theta_i \eta_{il}/S_k = -S_l/S_k < 0, \qquad \text{(A.4}$$

where the equality follows from Cournot aggregation, (A.1). □

Proof of theorem 5. Under the hypothesis of the theorem, the second term in (10) becomes

$$\sum_i (\theta_i/S_k)\eta_{ij} = (1/S_k)\sum_i \theta_i \eta_{ij} = -S_j/S_k > 0, \qquad \text{(A.5}$$

where $S_j = (\partial C/\partial D_j)D_j/C$. S_j represents the percentage reduction in costs as a result of percentage increases in the levels of the jth distribution service hence it will be negative; indeed, $(\partial C/\partial D_j)$ may be viewed as the negative of the shadow price of the jth distribution service. The second equality in (A.5 follows from differentiating (5) with respect to the jth distribution service and some manipulation. It is exactly analogous to the Cournot aggregation condition that arises in the household production model as a result of a price change. Since S_j is negative, the consumption effect will be positive. □

The proof of Theorem 3 is unrelated to the previous ones as well as to the subsequent ones and is taken up next.

Proof of theorem 3. In the absence of joint production, the total costs of producing two commodities, say, Z_1, Z_2, can be specified as

$$C(p_k, p_l, \bar{p}, Z_1, Z_2, D) = C^1(p_k, \bar{p}, Z_1, D) + C^2(p_l, \bar{p}, Z_2, D),$$

where $\bar{p}$ is a vector of prices other than p_k and p_l. The conditional demand for item k will be given by $C_k = Q_k = \partial C^1/\partial p_k$, since $\partial C^2/\partial p_k = 0$ because there is no joint production.[7] Hence,

$$(\partial Q_k/\partial p_l \mid Z) = 0 = (\partial Q_l/\partial p_k \mid Z),$$

and

$$\varepsilon^*_{kl} = 0 = \varepsilon^*_{lk}(S_l/S_k). \qquad \square$$

Finally, the proofs of theorems 2, 2′, 4, 6, and 6′ are based on a comparative statics analysis of the second stage of the optimization problem of section 2 and we will take them up in the course of presenting this analysis. For simplicity of exposition, let us consider the case in which there are two commodities (Z_1, Z_2), but we allow $s = 1, \ldots, K$ items used by the

[7]Actually, all we need for the result in the theorem is that both inputs k and l be net independents with the inputs that are the source of jointness in production.

14

household and $j=1,\ldots,J$ distribution services provided by a particular retailer.

Total differentiation of the first-order conditions, (4) and (5) in the text, yields after rearrangement

$$(U_{11}-\lambda C_{11})\,\mathrm{d}Z_1+(U_{12}-\lambda C_{12})\,\mathrm{d}Z_2-C_1\,\mathrm{d}\lambda$$
$$=\lambda\sum_s C_{1s}\,\mathrm{d}p_s+\lambda\sum_j C_{1j}\,\mathrm{d}D_j, \tag{A.6}$$

$$(U_{21}-\lambda C_{21})\,\mathrm{d}Z_1+(U_{22}+\lambda C_{22})\,\mathrm{d}Z_2-C_2\,\mathrm{d}\lambda$$
$$=\lambda\sum_s C_{2s}\,\mathrm{d}p_s+\lambda\sum_j C_{2j}\,\mathrm{d}D_j, \tag{A.7}$$

$$-C_1\,\mathrm{d}Z_1-C_2\,\mathrm{d}Z_2+0\cdot\mathrm{d}\lambda=\sum_s Q_s\,\mathrm{d}p_s+\sum_j C_j\,\mathrm{d}D_j-\mathrm{d}W, \tag{A.8}$$

This system can be written more compactly as

$$H\,\mathrm{d}Z=\mathrm{d}x, \tag{A.9}$$

and its solution will be given by

$$\mathrm{d}Z=H^{-1}\,\mathrm{d}x. \tag{A.10}$$

If only the price of the kth product is allowed to change, (A.10) yields

$$\partial Z_1/\partial p_k=h^{11}\lambda C_{1k}+h^{12}\lambda C_{2k}+h^{13}Q_k \tag{A.11}$$

Since by the assumptions of Theorem 2 and 2′, the kth product is used exclusively in the production of the first commodity, we have, given the continuity of the cost function, $C_{2k}=C_{k2}=\partial Q_k/\partial Z_2=0$. Thus, (A.11) reduces to an expression analogous to the income and substitution effects of standard consumer theory, i.e.,

$$\partial Z_1/\partial p_k=(\partial Z_1/\partial p_k\,|\,U)-Q_k(\partial Z_1/\partial W), \tag{A.12}$$

which expressed in elasticity terms yields

$$\eta_{1k}=\eta^*_{1k}-S_k\eta_1, \tag{A.13}$$

where η_1 is the income elasticity of demand of commodity 1. The substitution effect, η^*_{1k}, can also be expressed as

$$\eta^*_{1k} = p^*_{11}\pi_{1k}, \tag{A.14}$$

where $p^*_{11} = (\partial Z_1/\partial C_1 \mid U)C_1/Z_1$ and $\pi_{1k} = C_{1k}P_k/C_1$. In terms of interpretation p^*_{11} is simply the compensated own (shadow) price elasticity of demand of commodity 1 and π_{1k} is the percentage change in the shadow price of commodity 1 (C_1) as a result of a percentage change in the price of product k. Under the assumption that commodity 1 is a normal good ($\eta_1 > 0$), (A.13) must be negative.

Proof of theorem 2. Under the hypothesis of the theorem, the consumption effect can be written in general as

$$\sum_i \omega_{ki}\eta_{ik} = \omega_{kn}\eta_{nk}, \tag{A.15}$$

because $\omega_{ki} = 0$ for $i \neq n$. By the same argument as with respect to (A.13), however, it follows that a positive income effect ($\eta_n > 0$) insures

$$\eta_{nk} = \eta^*_{nk} - S_k\eta_n < 0, \tag{A.16}$$

where $\eta^*_{nk} = (\partial Z_n/\partial p_k \mid U)(p_k/Z_n)$ and $\eta_n = (\partial Z_n/\partial W)W/Z_n$. Since $\omega_{kn} > 0$, the consumption effect will be negative. □

Proof of theorem 2'. Since $\omega_{ki} = 0$ for $i \neq n$, the second term in (9) becomes

$$\sum_i \omega_{ki}\eta_{il} = \omega_{kn}\eta_{nl}. \tag{A.17}$$

But, by the same argument as in Theorem 2, η_{nl} must be negative if the income effect (η_n) is positive. □

If only the price of the lth product, which is used exclusively in the production of, let us say, the second commodity (Z_2), changes we have from (A.11)

$$\partial Z_1/\partial p_l = h^{12}\lambda C_{2l} + h^{13}Q_l,$$

or

$$\partial Z_1/\partial p_l = \partial Z_1/\partial p_l \mid U - Q_l\,\partial Z_1/\partial W. \tag{A.18}$$

46

of commodity 1. The substitu-

(A.14)

$_kP_k/C_1$. In terms of interpre-
ı (shadow) price elasticity of
centage change in the shadow
rcentage change in the price of
ımodity 1 is a normal good

the theorem, the consumption

(A.15)

ıent as with respect to (A.13),
.t ($\eta_n > 0$) insures

(A.16)

$/\partial W)W/Z_n$. Since $\omega_{kn} > 0$, the

: second term in (9) becomes

(A.17)

2, η_{nl} must be negative if the

ch is used exclusively in the
ty (Z_2), changes we have from

(A.18)

This expression can be rewritten in elasticity terms as

$$\eta_{1l} = \eta^*_{1l} - S_l\eta_1 = p^*_{12}\pi_{2l} - S_l\eta_1, \tag{A.19}$$

where p^*_{12} is the compensated cross price elasticity of demand of commodity 1 with respect to a change in the shadow price of commodity 2. Since we only have two commodities, they must be net substitutes in consumption ($p^*_{12} > 0$) although in the general case they can also be net independent or complements. In any event, even though the commodities are net substitutes in consumption, the consumption effect can still be negative if the commodities are normal goods ($\eta_1 > 0$) and the income effect is 'sufficiently strong,' i.e., the magnitude of the second term in (A.19) exceeds in absolute value the magnitude of the first term.

Proof of theorem 4. Under the hypothesis of the theorem, the two items are net independents (by Theorem 3). Hence, by Definition 2, gross complementarity will be determined by the consumption effect which is given by

$$\sum_i \omega_{ki}\eta_{il} = \omega_{kn}\eta_{nl}. \tag{A.20}$$

The previous argument with respect to (A.19) implies that

$$\omega_{kn}\eta_{nl} = \omega_{kn}(p^*_{nm}\pi_{ml} - S_l\eta_n) < 0, \tag{A.21}$$

where p^*_{nm} is the compensated cross price elasticity of demand of commodity n with respect to a change in the shadow price of commodity m and π_{ml} is the percentage change in the shadow price of commodity m as a result of a percentage change in the price of item l in the assortment. Incidentally, if the two commodities are net or Hicksian independent or complements in consumption ($p^*_{nm} \geqq 0$), the sign of (A.21) follows solely from the commodities being normal goods. □

If there is only a change in a distribution service, the effect on the first commodity, let us say, is given from (A.10) as

$$\partial Z_1/\partial D_j = h^{11}\lambda C_{1j} + h^{12}\lambda C_{2j} + h^{13}C_j. \tag{A.22}$$

If the distribution service is a specific one in that it affects only the kth item used exclusively in the production of the first commodity, we then have $C_{2j} = 0$ and (A.22) becomes

$$\partial Z_1/\partial D_j = \partial Z_1/\partial D_j \,|\, U - (\partial Z_1/\partial W)C_j. \tag{A.23}$$

This result can be expressed in elasticity terms as

$$\eta_{1j} = \eta^*_{1j} - S_j\eta_1 = p^*_{11}\pi_{1j} - S_j\eta_1. \tag{A.24}$$

Since $S_j < 0$, $\pi_{1j} < 0$ and $p^*_{11} < 0$, (A.24) must be positive if $\eta_1 > 0$.

Proof of Theorem 6. Under the hypothesis of the theorem, the second term in (10) becomes

$$(\theta_n/S_k)\eta_{ij}=(\theta_n/S_k)(p^*_{nn}\pi_{nj}-S_j\eta_n)>0, \tag{A.25}$$

where π_{nj} is the proportionate change in the marginal costs of the nth commodity as a result of a proportionate change in the level of the jth distribution service. The equality in (A.25) follows from an argument analogous to the one leading to (A.24). Note that $p^*_{nn}<0$, $S_j<0$, and $\pi_{nj}<0$; hence, if the nth commodity has a positive income effect, the consumption effect will be positive. □

If the distribution service is a common one and it affects the marginal costs of producing the second commodity, then (A.22) implies

$$\partial Z_1/\partial D_j=(\partial Z_1/\partial C_1\,|\,U)C_{1j}+(\partial Z_1/\partial C_2\,|\,U)C_{2j}-(\partial Z_1/\partial W)C_j \tag{A.26}$$

or in elasticity terms

$$\eta_{1j}=p^*_{11}\pi_{1j}+p^*_{12}\pi_{2j}-S_j\eta_1. \tag{A.27}$$

Proof of Theorem 6'. Under the hypothesis of the theorem, the second term in (10) becomes

$$(\theta_n/S_k)\eta_{ij}=(\theta_n/S_k)\left(\sum_s p^*_{ns}\pi_{sj}-S_j\eta_n\right), \tag{A.28}$$

where π_{sj} is the proportionate decrease in the marginal costs of commodity s as a result of a proportionate increase in the level of common distribution service j. The equality follows from the analysis leading to (A.27). If for simplicity, we assume that the common distribution service affects the marginal costs of producing all commodities and in the same proportion we have $\sum_s p^*_{ns}\pi_{sj}=\pi\sum_s p^*_{ns}=\pi\cdot 0=0$.[8] Hence (A.28) becomes

$$-\theta_n S_j\eta_n/S_k>0, \tag{A.29}$$

where the inequality follows from assuming a positive income effect for the commodity. □

[8]Even if all marginal costs do not change in the same proportion, there will be a tendency for this term to be close to zero since if the marginal costs were to change in the same proportion the condition would have to be satisfied. Therefore, this term is likely to be outweighed by the income effect in almost every instance.

References

Albion, Mark S., 1983, Advertising's hidden effects: Manufacturer's advertising and retail pricing (Auburn House, Boston, MA).

Barnett, William A., 1977, Pollak and Wachter on the household production approach, Journal of Political Economy 85, 1073–1082.

Barten, Anton P., 1977, The systems of consumer demand functions approach, Econometrica 45, 23–52.

Becker, Gary S., 1965, A theory of the allocation of time, Economic Journal 75, 493–517.

Betancourt, Roger R. and David Gautschi, 1988, The economics of retail firms, Managerial and Decision Economics 9, 133–144.

Betancourt, Roger R. and David Gautschi, 1990, Demand complementarities, household production and retail assortments, Marketing Science 9, 146–161.

Betancourt, Roger R. and David Gautschi, 1989, Two essential characteristics of retail markets and their economic consequences, Maryland Working Paper no. 89–6.

Bliss, Christopher, 1988, A theory of retail pricing, Journal of Industrial Economics 36, 372–391.

Bockstael, Nancy E. and Kenneth E. McConnell, 1983, Welfare measurement in the household production framework, American Economic Review 73, 806–814.

Bucklin, Louis P., 1966, A theory of distribution channel structure (Institute of Business and Economic Research, University of California, Berkeley, CA).

Chamberlin, Edward H., 1962, The theory of monopolistic competition, (Harvard University Press, Cambridge, MA), 8th ed.

Deaton, Angus, and John Muellbauer, 1980, Economics and consumer behavior (Cambridge University Press, Cambridge).

Dreze, Jacques H. and Kare P. Hagen, 1978, Choice of product quality: Equilibrium and efficiency, Econometrica 46, 493–513.

Hicks, John R., 1946, Value and capital (Oxford University Press, Glasgow), 2nd ed.

Ingene, Charles, 1984, Productivity and functional shifting in spatial retailing: Private and social perspectives, Journal of Retailing 60, 15–36.

Lal, Rajiv and Carmen Matutes, 1989, Price competition in multimarket duopolies, Rand Journal of Economics 20, 516–537.

Lancaster, Kelvin J., 1966, A new approach to consumer theory, Journal of Political Economy 74, 132–157.

Muth, Richard F., 1966, Household production and consumer demand functions, Econometrica 34, 699–708.

Perry, Martin K. and Richard H. Groff, 1985, Resale price maintenance and forward integration into a monopolistically competitive industry, Quarterly Journal of Economics 100, 1293–1312.

Pollak, Robert A. and Michael L. Wachter, 1975, The relevance of household production and its implications for the allocation of time, Journal of Political Economy 83, 255–277.

Pollak, Robert A. and Michael L. Wachter, 1977, Reply: Pollak and Wachter on the household production approach, Journal of Political Economy 85, 1083–1086.

Willis, Robert J., 1987, What have we learned from the economics of the family?, American Economic Review: Papers and Proceedings 77, 68–81.

19