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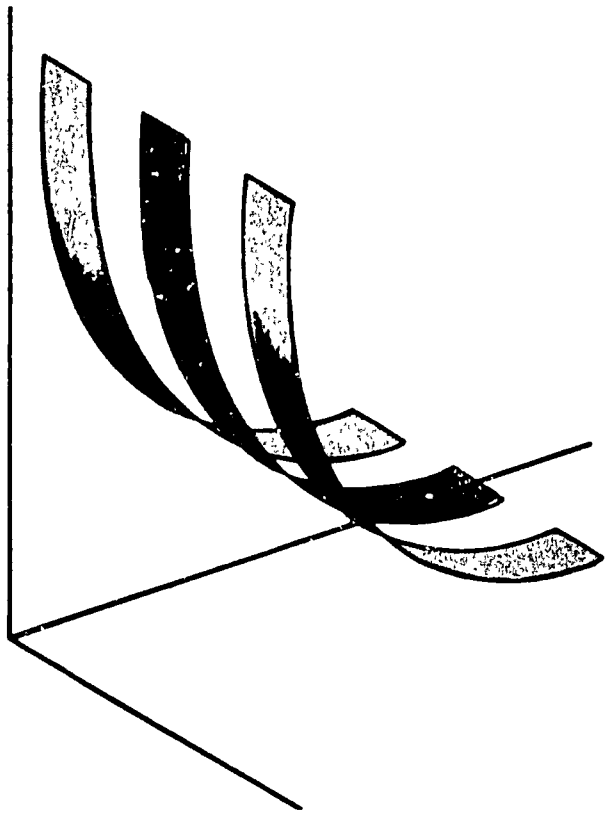
Factor substitution in linear programming:

a methodology and an application to cane alcohol pricing policies in Costa Rica

Rafael Celis



DISSERTATION

Submitted in partial fulfillment of the requirements for the degree of Doctor of Philosophy in Economics

The University of New Mexico
Alburquerque, New Mexico
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FACTOR SUBSTITUTION IN LINEAR PROGRAMMING: A METHODOLOGY AND AN APPLICATION TO CANE ALCOHOL PRICING POLICIES IN COSTA RICA

By

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Licenciado Economía, Universidad de Costa Rica, 1980
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Dedicated to

my Parents, Rafael and Beatriz,

my Wife, Ana,

my Children, Juan Rafael, Gerardo, and Ana Beatriz

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The main methodological contribution of this study is the development of a technique to link activity analysis and neoclassical production functions, in the context of constrained optimization models. The technique has two major components. First, a procedure was developed to incorporate the isoquants into a linear programming model. Second, there is an algorithm to generate numerical isoquants that requires only information on the elasticity of factor substitution, the input ratio and input prices observed at a single point on the hypothetical isoquant. Among the types of applied research that this marriage between activity analysis models and the neoclassical production function make possible are:

- Estimation of neoclassical production function for primary factors, and its inclusion in a model along with activity analysis specifications for intermediate inputs.

- Modelling of conceptual alternatives on factor substitution, such as differentiation of factor substitution possibilities in agriculture between hillside and flat lands, between fragile and fertile soils, between high potential and low potential areas. This possibility offers a powerful tool to evaluate technical options for sustainable agricultural growth.
- The effects of factor substitution, or of technological change for that matter, on distribution of costs and benefits of various policy actions can now be assessed.

To empirically test the technique developed in this study, a model of Costa Rican agriculture was built. This is a price-endogenous, market-simulating model that includes farmers' risk aversion behavior. It also includes downward-sloping demand functions. The basic model was modified to incorporate systematic labor-machinery substitution possibilities along CES isoquants. Then, a cane-alcohol pricing scheme was evaluated using the basic model and the enhanced one. Some of the salient results suggest that:

- there should be an entirely new criterion for designing price support programs if the goal is to increase farmers' incomes.
- there is a need to complement price support programs with technical assistance programs.
- the distribution of benefits of price support programs, as between consumers and producers, depends on labor substitution possibilities.

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CHAPTER I

INTRODUCTION AND SUMMARY

Motivation of the Study

The motivation of this study has been twofold: methodological and empirical. The methodological concern arose from the observation that in the widespread use of linear programming in the industry and in agriculture, the more flexible neoclassical production functions were absent. As a result, activity analysis models could not benefit from the systematic treatment of returns to scale or of elasticities of factor substitution that the neoclassical production function allows for. Conversely, the neoclassical production function analysis could not benefit from the power of activity analysis in using cross-sectional data, and in using limited information to describe real world situations and simulating, for evaluation purposes, policy-relevant scenarios.

A closer examination of the reasons for this apparent divorce between the two traditions indicated, for instance, that because the solution to activity analysis models required handling rectangular (non-square) matrices of coefficients, these models could not be combined with the simultaneous equations models in which neoclassical production functions are normally included. As it is well known, the solution to simultaneous equations systems can occur only in the

context of square matrices of coefficients. Hence one motivation of this study is to develop a methodology to merge the neoclassical and activity analysis traditions in production functions.

On empirical grounds, the interest arose from the need to develop an analytical tool suitable for the evaluation of complex programs such as the substitution of alcohol for gasoline and diesel in motor vehicles. Let us recall that the incentive for governments to implement this kind of fuel substitution has emerged in different circumstances during the last two decades. First, it was in response to soaring oil prices in the early and late seventies. Here are some examples: the United States of America initiated then a massive production of ethanol made out of corn. Brazil and Costa Rica designed programs to produce cane alcohol. The EEC countries have also implemented programs to produce beet alcohol. Second, when oil prices dropped in the early eighties, other circumstances provided the stimulus to implement alcohol fuel programs. In the United States, it was the need to dispose of grain surpluses, particularly during the time of the grain embargo to Russia. In Brazil and Costa Rica, the sharp drop in the prices of sugar in the world market made cane alcohol production relatively more profitable than sugar production.

In all these cases, serious concerns have been raised regarding the social, economic, and environmental impact of massive alcohol production. Furthermore, in the case of Brazil, for example, where the cane alcohol program has been aggressively implemented, the collapse of food crop production and a severe disruption in the labor markets had been predicted as a consequence of that program. However, although a

comprehensive evaluation of the program is not yet available, the doomsday predictions were not correct. It appears then that there is insufficient knowledge about the capability of a country's productive sectors to adjust to major changes in production incentives to be able to make accurate forecasts of sector-wide and economy-wide performance. It is argued here that one major reason for this failure is the lack of sufficiently powerful analytical tools to evaluate, in a multiple commodity framework, the true possibilities for factor substitution at the micro and sectoral levels. The combination of activity analysis and the neoclassical production function in a single framework might be an important step towards filling this vacuum.

The study illustrates this possibility by presenting an extensive evaluation of Costa Rica's sugarcane alcohol program, utilizing the methodological contributions developed herein. Conclusions are developed and supported regarding the policies on sugarcane alcohol pricing, technical assistance to farmers, and other issues. The quantitative results must be taken with caution though, because the values of the elasticities of labor-machinery substitution used in this application were assigned in an arbitrary fashion: therefore, they might not necessarily correspond to the true values. Nevertheless, in this particular case it was observed that large changes in the elasticity substitution were necessary before a noticeable increase in the production of alcohol was observed. This in turn means that the range of elasticity values that give similar results is quite wide; thus, the risk of introducing serious biases in the analysis is reduced.

Methodological Contribution of the Study

The main methodological contribution of this study is the development of a technique to link activity analysis and neoclassical production functions, in the context of constrained optimization models. This is done in the context of a sector-wide or regional model with multiple commodities. The technique has two major components. First, a procedure was developed to incorporate isoquants into a linear programming model (Chapter IV). This allows the use of isoquants estimated by econometric methods. Second, for the situations in which an estimated isoquant is not readily available, there is an algorithm to generate isoquants (Chapter III). A key aspect of this algorithm is that it requires only information on the elasticity of factor substitution, the input ratio and the input prices observed at a single point on the hypothetical isoquant, thereby overcoming the severe constraint of data availability on production functions. While this method is by no means a substitute for the estimation of production function, because the value of the elasticity of substitution still must come from conventional estimates of this parameter, the algorithm offers an interesting analytical tool. For instance, the generation of isoquants can be useful for planning purposes, particularly when one wants to calculate an aggregate elasticity, using estimated values of elasticities for individual crops or for different regions.

Although activity analysis constituted a major breakthrough with respect to input/output models, because it allowed multiple technological choices, the treatment of factor substitution within activity

analysis has been quite informal. Consequently, the marriage between activity analysis model and the neoclassical production function attained in this study adds a great deal of realism and flexibility to activity analysis. Among the types of applied research that will be possible now are:

- Estimation of a neoclassical production function for primary factors, and its inclusion in a model along with activity analysis specifications for intermediate inputs.
- Modelling of conceptual alternatives on factor substitution, such as lower elasticity of substitution in smaller scale enterprises, i.e., more limited technical options. Or differentiation of factor substitution possibilities in agriculture between hillside and flat lands, between fragile and fertile soils, between high potential and low potential areas.
- Production functions estimated in one locality can now be incorporated into a model of similar activities for another locality, provided that the analyst has applied a very rigorous criterion to judge the validity of this "adoption" of data. This choice will be especially useful in modelling agriculture in developing countries.
- The effects of factor substitution, or of technological change for that matter, on the distribution of costs and benefits of various policy actions can now be assessed.

The Empirical Application to Costa Rica

In order to empirically test and illustrate the techrique developed in this study to analyze factor substitution possibilities, a model of the Costa Rican agriculture was built. This is a price-endogenous, market simulating model that includes farmers' risk-aversion behavior. It also includes downward-sloping demand functions. The basic model was modified to incorporate systematic labor-machinery substitution possibilities along constant elasticity of substitution (CES) isoquants. Then, a cane-alcohol pricing scheme was evaluated using both the basic model and the enhanced one.

Some of the salient results suggest that:

- There should be an entirely new criterion for designing price support programs if the goal is to increase farmers' incomes.
- There is a need to complement price support programs with technical assistance programs.
- There is a need to emphasize cost-reducing technologies, and not only yield-enhancing technologies, in agricultural research.
- The distribution of benefits of price support programs, as between consumers and producers, depends on labor substitution possibilities.

The study is organized in seven chapters, including this introduction. The first part of Chapter II contains a review of the theory behind some special aspects of linear programming models that have enabled them to incorporate more realistic economic behavior. In particular, the underlying theory for commodity demands and market

equilibrium, as well as for incorporating producers' risk aversion, in linear programming models is presented. The second part of Chapter II provides the conceptual motivation for the factor substitution specifications developed in this study. Alternative measures of the substitution possibilities and the issues and problems of analysis are discussed also in this chapter.

The development of a new technique to generate isoquants is presented in Chapter III. A review of the existing techniques to generate numerical isoquants precedes the exposition of the new one. Then, in Chapter IV it is shown how to incorporate isoquants into an applied linear programming model. This is done using the case of labor-machinery substitution in Costa Rican agriculture. Chapter V is an illustration of how to implement the newly developed technique using the case of alcohol pricing policies in Costa Rica.

The contrasting results obtained when pricing policies are evaluated with and without factor substitution possibilities are presented in Chapter VI. Finally, in Chapter VII, the policy and methodological implications of the study are presented. A section on recommendations concludes Chapter VII.

CHAPTER II

THEORY BACKGROUND

Special Aspects of Linear Programming Models in Agricultural Sector Analysis

Commodity Demands and Market Equilibrium

Since the early fifties, when linear programming models were first used to analyze agricultural sector behavior, one of the most significant improvements in activity analysis has been the specification of downward-sloping commodity demands. This feature, combined with the upward-sloping supply function for each commodity which is implicitly generated within an activity analysis model, provides the opportunity to simulate competitive as well as non-competitive markets. In other words, prices are endogenously determined as part of the model's solution. Duloy and Norton (1975) first introduced endogenous commodity prices in linear programming models. The linear programming formulation of commodity endogenous prices can be summarized as follows: Using the Samuelsonian objective function of maximizing consumer and producer surpluses, then the objective function for any single commodity j becomes:

$$Z_j = \tfrac{1}{2}(a_j - P_j)Q_j + P_jQ_j - C(Q_j) \qquad (1)$$

where P_j is the price and Q_j is the quantity of commodity j.

The first term of expression (1) represents the consumer surplus and the two remaining terms represent the producer surplus. Notice that the term $C(Q_j)$ is the integral of $C'(Q_j)$ between 0 and Q or the total cost of producing Q units of commodity j (see Figure 1).

If it is assumed that the demand function is linear, the inverse demand function can be written:

$$P_j = a_j - b_j Q_j \tag{2}$$

Substituting (2) into (1) we get the expression:

$$Z_j = (a_j - \tfrac{1}{2} b_j Q_j) Q_j - C(Q_j) \tag{3}$$

To set up the optimization model, it is useful to distinguish between production S_j and sales Q_j and make

$$S_j = Y_j X_j \tag{4}$$

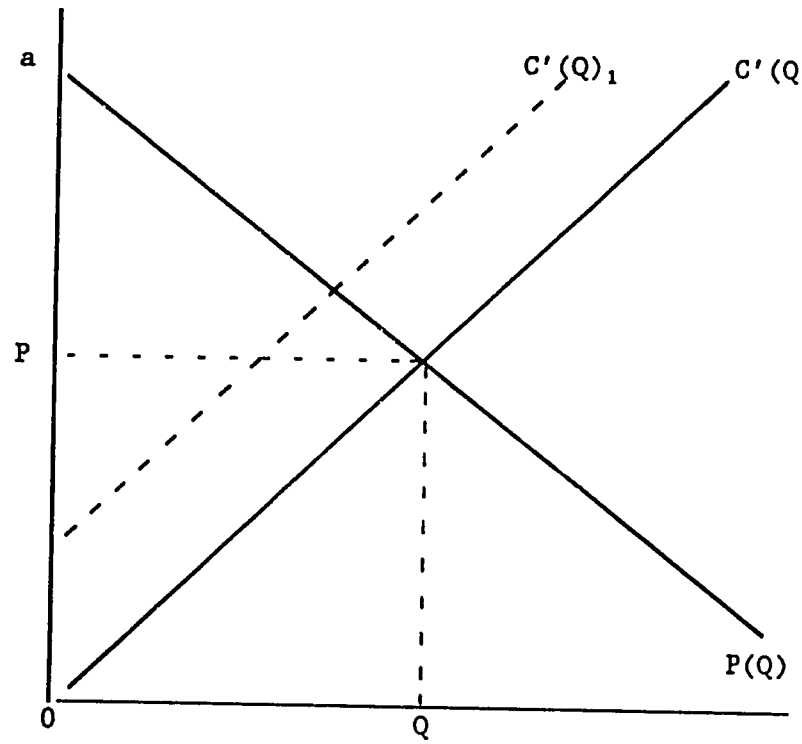
where Y_j is output per hectare and X_j is total number of hectares. The per hectare requirements of fixed resources in production can be denoted by a_{kj}, and resource availability by b_k, for k types of resources. Thus, resource requirements per unit of output would be $d_{kj} = a_{kj}/Y_j$. A sector optimization model with multiple commodities can be specified as:

$$\max Z = \sum_j (a_j - \tfrac{1}{2} b_j Q_j) Q_j - \sum_j C(S_j) \tag{5}$$

such that

$$Q_j - S_j \leq 0, \text{ for all } j \qquad [\phi_j] \tag{6}$$

Figure 1--Commodity market with deterministic and risky supply



$$\sum_j d_{kj} S_j \leq b_k, \text{ for all } k \qquad [\lambda_k] \tag{7}$$

$$Q_j,\ S_j \geq 0, \text{ all } j \tag{8}$$

To show that the solution of this model produces a competitive market equilibrium for the j commodities, let us set up the Lagrangean:

$$L = \sum_j (a_j - \tfrac{1}{2} b_j Q_j) Q_j - \sum_j C(S_j) - \sum_j \phi_j [Q_j - S_j] \tag{9}$$
$$- \sum_k \lambda_k [\sum_j d_{kj} S_j - b_k]$$

Apart from the feasibility requirements, the necessary Kuhn-Tucker conditions are:

$$\frac{\partial L}{\partial Q_j} = a_j - b_j Q_j - \phi_j \leq 0, \text{ for all } j \tag{10}$$

$$\frac{\partial L}{\partial S_j} = - C'(S_j) + \phi_j - \sum_k d_{kj} \lambda_k \leq 0, \text{ for all } j \tag{11}$$

For the cases in which demand and supply are nonzero, these equations imply that

$$\phi_j = a_j - b_j Q_j = P_j \tag{12}$$

that is, at the optimal solution, the model's shadow prices on the commodity balances are equal to the corresponding commodity prices, and

$$P_j = \phi_j = C'(S_j) + \sum_k d_{kj} \lambda_k \tag{13}$$

that is, also at the optimal solution, each commodity price is equal to that commodity's marginal cost of production. Notice that the marginal cost includes both the explicit cost of purchased inputs at the margin, $C'(S_j)$, and the opportunity costs of fixed resources at the margin, $d_{kj}\lambda_k$. Since the dual variable λ_k measures the increment in consumer and producer surplus that would arise from the availability of an additional unit of resource k, and d_{kj} is the amount of resource k required per unit of product j, the second term on the right-hand side of (13) is the resource opportunity cost of an additional unit of product j. Duloy and Norton (1973, 1975) show how to linearize efficiently the quadratic objective function (5) so that the model can be solved as a linear programming model. They also presented a procedure that shows how the linear version leads conveniently into extensions for handling nonzero cross-price effects and nonzero income effects.

Producers' Risk Aversion

Another important advance in the development of linear programming as a tool for analysis of agricultural supply response was the inclusion of risk as part of the model's solution. This followed the broad recognition that farmers are not risk neutral; consequently, as Hazell and Scandizzo (1983) noted:

> "...the omission of risks in programming models is likely to lead to an overestimate of the supply response for farm enterprises with high variance in yields, prices, or both. Furthermore, since there are often high-value enterprises, omission of risk is likely to lead to an overstatement of returns to investment. These biases may be particularly large in models of low income agriculture, in which risk aversion is likely to be greatest."

The only method currently available to introduce price and yield risks plus farmers' risk-aversion behavior in the objective function of agricultural sector models with endogenous prices--of the kind described in the previous section--is the one developed by Hazell and Scandizzo (1974, 1977). A thorough discussion of that method and its empirical applications is presented in Hazell and Norton (1986). The usefulness of Hazell and Scandizzo's contribution to improving linear programming models is conveniently explained here in schematic form. The risk block in a tableau of a properly modified model will look as follows:

$$
\begin{array}{l|cccc|cccccc}
 & X_1 & X_2 & \dots & X_j & R_1 & R_2 & \dots & & R_t & \sigma \\
\hline
\text{Risk } (t = 1) & -50 & -1 & & & -1 & & & & & \\
\vdots & +13 & -24 & & & & -1 & & & & \\
 & -2 & -3 & \dots & \Delta_{tj} & & & \ddots & & & \\
 & -40 & +18 & & & & & & \ddots & & \\
\text{Risk } (t = T) & +75 & +10 & & & & & & & -1 & \\
\hline
\sigma \text{ counter} & & & & & +2 & +2 & \dots & & +2 & -f \\
\text{OBJECTIVE} & & & & & & & & & & -\Phi
\end{array}
$$

In this formulation X_j, as in the previous section, is the total number of hectares planted to crop j; the Δ_{tj} coefficients are revenue deviations per hectare from an average revenue estimated over t periods--these coefficients are also called the mean absolute deviation (MAD). R_t is a risk activity or counter of revenue deviations over t periods; and f is Fisher's (1920) correction factor to convert the

sample MAD to an estimate of the population standard deviation of revenues.

The above formulation has several practical advantages. First, $[\Delta_{tj}]$ constitute a matrix of *revenue* deviations per hectare; therefore, they include price and yield observed historical variations. Furthermore, $[\Delta_{tj}]$ includes the same information as a variance-covariance matrix. As a result, these coefficients truly capture compensating and reinforcing effects on crop revenues. Additionally, $[\Delta_{tj}]$ shows the type of information that farmers remember: "3 years ago was bad for beans but good for cotton...." In reality, farmers can use information in $[\Delta_{tj}]$ to make their own mental experiments for decision making, such as: "if I had planted a certain cropping mixture last year, I would have lost money, but the year before I would have gained even more money with the same cropping pattern."

The type of farmers' thinking described above is effectively represented in the model because the total outcome (relative to average year) for farmers is the cropping pattern times revenue deviations. Notice that $\sum_j X_j \Delta_{tj}$ in fact gives total revenue performance of a given pattern for each historical year.

Third, $[\Delta_{tj}]$ is the *objective* component of the risk submodel, i.e., how risky each crop or crop combination actually is. Fourth, the *subjective* part (farmers' risk aversion) is Φ. According to various applied studies, the value of Φ has been found to lie between 0.5 and 1.5.

The rest of the submodel, including the Fisher's correction coefficient, links the objective and subjective components of risk and

converts $[\Delta_{tj}]$ into an estimate of the standard deviation. With the inclusion of risk, the objective function (5) becomes:

$$\max Z = \sum_j (a_j - \tfrac{1}{2} b_j Q_j) Q_j - \sum_j C(S_j) - \Phi\sigma \qquad (5)'$$

The effect of the term $- \Phi\sigma$ in (5)' is to shift the implicit model's supply function upwards and to the left, as depicted in Figure 1 by $C'(Q)_1$.

Measuring Factor Substitution Possibilities

The concept of factor substitution appears elsewhere in modern economic literature and is considered of the utmost importance (see Marshall 1920, Hicks 1932, Robinson 1933, Allen 1938). In textbooks of economics, the principles that govern factor substitution are commonly treated in the context of the theory of demand for inputs or what is known as the theory of derived demand for productive services, the marginal-productivity theory of distribution, or simply the distribution theory (see, for instance, Stigler 1966, Layard and Walters 1978, Henderson and Quandt 1980, Gisser 1981).

Marshall (1920) was the first to systematically recognize the importance of factor substitution. In developing the concept of elasticity of derived demand, he postulated what is best known as the second law governing the elasticity of derived demand: "The demand for anything is likely to be more elastic, the more readily substitutes for that thing can be obtained" (Principles, pp. 385-386).

Marshall's second law spurred the interest of economists to measuring the substitution possibilities among factors of production. Two methods were devised: the marginal rate of technical substitution (MRTS) and the elasticity of substitution. The simplest way to measure the easiness with which two factors can be substituted to each other is by calculating the marginal rate of technical substitution. This is attained by finding the ratio of the factors' marginal productivities, output held constant. Alternatively, the slope of an isoquant that relates the two factors of production is also the value of the MRTS. It is important to notice that the MRTS is a physical measure of the additional amount of a factor that is required to produce the same level of output when the amount used of the other factor is reduced:

$$MRTS = \frac{\Delta K}{\Delta L} = - \frac{MPL}{MPK} \tag{14}$$

where K and L are say capital and labor, respectively.

Although the MRTS is a very important concept in the theory of production, its usefulness in derived demand analysis is limited since the MRTS alone does not relate the variations in factor use levels to changes in the factor's respective price. To measure the latter, a more complex coefficient was developed: the elasticity of substitution.

The concept of elasticity of substitution appeared for the first time in Hick's book *The Theory of Wages* (1932). Hicks defined the elasticity of substitution as the proportional change in the capital/labor (or any two inputs) ratio--R--relative to the change in the marginal rate of technical substitution, output held constant:

$$\xi = \frac{\Delta R/R}{\Delta MRTS/MRTS} = \frac{\%\Delta R}{\%\Delta MRTS} \tag{15}$$

In the same book, Hicks reexamined Marshall's third law that governs the price elasticity of derived demand:

> "The demand for anything is likely to be less elastic, the less important is the part played by the cost of that thing in the cost of some other thing, in the production of which it is employed" (Principles, pp. 385-386).

Hicks demonstrated that under a rising supply price of other factors, Marshall's third law holds if and only if the elasticity of commodity demand exceeds the elasticity of substitution. In other words, if and only if consumers can substitute in consumption more readily than producers can substitute in production. Allen (1956) went one step further to demonstrate that if the supply of other factors is perfectly elastic, and the production function obeys constant returns to scale, then the elasticity of the derived demand is:

$$\epsilon_{11} = -(S_1\xi + S_2 |\eta|) \tag{16}$$

where S_1 and S_2 are the shares of the two factors in the total cost, ξ is the elasticity of substitution, and η is the price elasticity of the finished product. The first term on the right-hand side of (16) is the substitution effect and the second term is the expansion effect. Therefore, what Hicks demonstrated for rising price supply functions holds for perfectly elastic supply functions as well. Allen also demonstrated that with flat supply curves of variable inputs and assuming constant returns to scale, the cross-price elasticity of two factors of production is:

$$\epsilon_{12} = S_2 (\xi - |\eta|) \tag{17}$$

The elasticity of substitution is a pair-wise coefficient. Furthermore, in the case in which there are only two factors of production, they must necessarily be substitutes--except, of course, for the fixed-proportion case. In most real world situations though, more than two factors enter in the production process; therefore, when the price of one of the factors changes, it might be expected that any of the other factors change in the same direction than the quantities of the factor whose price changed do. If this is the case, they are said to be complements rather than substitutes. This situation is represented by a negative value of the elasticity of substitution. The complementarity case is discussed by Hicks (1970) and it should be more properly defined as the elasticity of complementarity rather than elasticity of substitution (see Layard and Walters 1978).

It is clear from the above discussion that the elasticity of substitution plays a fundamental role in explaining the response of producers to changes in the prices of the factors of production that they use or to changes in the prices of the finished output.

In production analysis the value of the elasticity of substitution is a convenient parameter to distinguish alternative functional forms of the production function. The fixed proportions or Leontief production function, for instance, has an elasticity of substitution equal to zero. At the other extreme is the production function that obeys infinity elasticity of substitution; this is the case of the linear production function. An intermediate case is represented by the

unitary-constant elasticity of substitution exhibited by the Cobb-Douglas production function. Although these three functions are extensively used in economic analysis, it must be recognized that, from a factor substitution point of view, and particularly in real world situations, they are very constraining. This limitation was partly overcome when Arrow, Chenery, Minhas and Solow (1961) proposed the constant elasticity of substitution (CES) production function. Nevertheless, the constancy of the elasticity of substitution does not seem the most plausible one to explain and evaluate the possibilities of factor substitution, for the elasticity of substitution might well vary with the level of factor use or the level of output. Consequently, the search for a more flexible production function led to the development of the transcendental logarithmic or translog production function. In this functional form the elasticity of substitution varies with output. All the above explicit production functions and the corresponding value ranges of their parameters are presented in Table 1.

Since only in the CES and the translog production functions the value of the elasticity of substitution is unknown, the econometric estimation of the elasticity of substitution has been confined to these two functional forms. On statistical grounds though, the CES production function is easier to handle. Clearly, there exists a tradeoff between mathematical rigidity and difficulty of econometric estimation. Examples of econometric estimations of elasticities of substitution include the works of Griliches (1964) for the agricultural sector,

Table 1--Elasticity of substitution in alternative production functions

Name	Function	Elasticity of Substitution
Leontief	Output = $\min\left(\frac{K}{m}, \frac{L}{n}\right)$, a, b > 0	Zero
Cobb-Douglas	Output = $AK^{\alpha} L^{1-\alpha}$, A > 0 $0 < \alpha < 1$	Unitary
Lineal	Output = $eX_1 + fX_2$, e, f > 0	Infinite
CES	Output = $A\,[\alpha K^{-\rho} + (1 - \alpha)\, L^{-\rho}]^{-h/\rho}$ $-1 < \rho < \infty$ $\rho \neq 0$ $0 < \alpha < 1$ h, A > 0	Constant
Translog	LnOutput = $A + \alpha \ln K + \beta \ln L + \sigma(\ln K)(\ln L) + \delta(\ln K)^2 + \tau(\ln K)^2$	Variable

where the variables are: K = capital, L = labor.

Source: P. R. G. Layard and A. A. Walters, Microeconomic theory (New York: McGraw-Hill Book Company, 1978); Micha Gisser, Intermediate price theory (New York: McGraw-Hill Book Company, 1981).

Ferguson (1965) for the U.S. manufacturing industries, and Atkinson and Halvorsen (1976) for the electric power generation industry.

Factor Substitution in Activity Analysis Models: A Note on the Issues

Activity analysis models were first developed in the 1940s and 1950s to generalize the input-output production function (see Koopmans 1951). The input-output formulation was taken to be valid for a single technique of production, and a more complete description of the production function was obtained by specifying a set of alternative input-output techniques. The production function was then defined as the envelope of the set of vectors differentiated by technique. Possibilities for both factor and input substitution are inherent in the activity analysis approach. The alternative techniques often are specified for the same level of output, and they differ only in their input structure. Moving from one technique to another means increasing the use of some inputs or factors and decreasing others.

These models were also applied to agriculture at a very early stage, the pioneers having been Karl Fox (1953) and Earl Heady (1954). In agricultural applications, it was common practice to standardize the alternative production vectors in terms of a unit of land rather than a unit of output. A higher-yield technique, therefore, would use less land per unit of output, and the ratio of land to other factors and inputs would be different than for other techniques.

Unlike neoclassical production functions, the activity analysis functions are inherently discrete. They are specified for a few dis-

crete values of input-output and input-input ratios. Each technique displays constant returns to scale, but the envelope production function may display non-constant returns to scale. For example, in an agricultural model in which techniques have a spatial dimension--in which they are specified by region and subregion--the most productive techniques for corn may be found in the subregion with the most fertile land. Then expanding corn output may also require use of activities that represent planting the crop in less fertile land, with lower yields. Thus the overall production function may represent decreasing returns to the use of land.

The activity analysis vectors often take into account the use of a large number of inputs. In agricultural models that include monthly land, labor, and irrigation water, plus other inputs, it is not uncommon for the vector to include more than 40 factors and inputs. Since the shadow price of land (its opportunity cost) usually differs by month in agriculture, each month's land is legitimately a different input.

It is clear that this kind of detail of specification is not feasible in estimated neoclassical production functions, especially in developing countries where the available data series are more limited. On the other hand, the activity analysis production functions do suffer a major limitation: their range of variation in factor and input proportions is limited by the combinations actually observed in the base period of the analysis. Bassoco and Rendon (1973) made great efforts to expand the range of input combinations for each agricultural region of Mexico by collecting coefficients on techniques used in other

regions (such as semi-mechanized and mechanized land preparation), and assuming that they also could be applied to the first regions. Effectively, they were able to specify three points on an implicit isoquant in the machinery-labor space, for each crop and each region.

Nevertheless, this set of technical choices still was very limited. In hypothetical experiments to simulate policies that could alter factor price ratios significantly, it could be the case that the indicated new factor proportions simply would not be available in the model's technology set. An estimated neoclassical production function permits us to extrapolate from the observed base-year realities, and thus to better explore the consequences of possible economic changes in the future. To date, little or no progress has been made in combining activity analysis and neoclassical production functions in the same model, in order to exploit the advantages of both.

Some Methodological Problems to be Solved

In order to combine elements of neoclassical and activity analysis production functions, two classes of methodological problems have to be solved. To prepare the way, it must be noted that the fusion of these two kinds of production functions has to occur in the context of mathematical programming models. The reason is that the activity analysis approach leads to a model with more variables than rows, by definition, and therefore the model can be solved only with constrained optimization techniques. In contrast neoclassical production functions are used in simultaneous equation models, and those models cannot accept

activity analysis specifications and still being capable of being solved numerically. Therefore, the challenge is to incorporate neoclassical production functions, in piece-wise form, in a mathematical programming model. Because of its widespread use, especially in agriculture, linear programming has been used here. As indicated previously in this chapter, in the section on "Commodity Demands and Market Equilibrium," nonlinear economic behavior can be incorporated readily in linear programming models.

The other introductory comment is that the methodology developed here incorporates neoclassical behavior in the form of isoquants rather than production functions. It turns out to be more straightforward mathematically this way, and it has the advantage that the only information needed from estimation studies is the elasticity of factor substitution. With this method, it becomes possible to handle substitution among primary factors in the neoclassical way, while handling substitution among the many intermediate inputs in the activity analysis way.

The first of the two methodological problems to be solved is to generate numerical isoquants so that they are consistent with the observed base-year factor proportions and factor prices. It turns out that this problem is not solved by existing procedures, so a new procedure--although fairly straightforward--had to be developed.

The second problem is how to insert the isoquant into an optimization model in a way that is consistent with the rest of the structure of the model. One way would be to replicate each production vector a large number of times, each time varying only (for example) the machin-

ery and labor coefficients, to represent the machinery-labor substitution possibilities along the already-known isoquant. To illustrate this approach, suppose a simplified production vector for corn looked as follows:

Concept	Coefficient	Units
Corn output	2.0	Tons/hectare
Land, May	1.0	Hectares
Land, June	1.0	Hectares
Land, July	1.0	Hectares
Land, August	1.0	Hectares
Land, September	1.0	Hectares
Fertilizer	200	Kilos/hectare
Labor	40	Person-days/hectare
Machinery services	2	Machine hours/hectare

Now suppose 20 person-days of labor were involved in the activities of fertilization and weeding and harvest, and 20 in land preparation. And that the 2 hours of tractor services also were involved in land preparation, and that machinery-labor substitution was possible only in that activity. Knowledge of the shape of the machinery-labor substitution isoquant could permit expansion of the set of production techniques, in the following illustrative way:

Corn output	2.0	2.0	2.0	2.0	2.0
Land, May	1.0	1.0	1.0	1.0	1.0
Land, June	1.0	1.0	1.0	1.0	1.0
Land, July	1.0	1.0	1.0	1.0	1.0
Land, August	1.0	1.0	1.0	1.0	1.0
Land, September	1.0	1.0	1.0	1.0	1.0
Fertilizer	200	200	200	200	200
Labor	62	50	40	30	22
Machinery services	0	1	2	3	4

Here, for illustration, the MRTS is 10 in the first segments adjoining the midpoint, and it changes to 8 and 12 in the extremes. Presumably, these MRTS values would be calculated from the pre-established isoquant (see Chapter III) and then applied to generating the above matrix.

This method is feasible but it has major implications for the computational size of the model. Suppose that originally, without inserting the isoquants, the model has 20 cropping activities for each of 3 farm size classes, for a total of 60 production vectors. (It is not uncommon for agricultural programming models to have hundreds of production vectors, taking into account variations in planting dates, fertilizer dosages, regional characteristics, and so forth.) Then if 10 discrete points were specified on each isoquant, the number of production vectors would expand from 60 to 600.

Likewise, if the model initially had 400 production vectors, inclusion of the isoquants would expand their number to 4,000. To carry the implications further, if each vector contained 30 nonzero coeffi-

cients, then in this last case, the isoquant procedure would be adding 3,600 x 30 = 108,000 coefficients to the model! Clearly, its manageability would be sharply decreased and the possibility of mechanical errors creeping in would be sharply increased.

The method developed here involves specifying the machinery-labor substitution possibilities at a more aggregate level, as for each farm group, or for all grain crops, or for any other designated aggregate. The institutional analogy to the procedure is as follows. Suppose the farmers in a region are accustomed to preparing their land for planting by plowing with oxen: a combination of labor and draft animals. Now many of the farmers have decided to contract the land preparation, because they have found off-farm work or for other reasons. A contractor negotiates with them a price per hectare which essentially is based on their own experiences of how much time it requires to plow a hectare, using their own techniques. Then, before the contractor carries out the plowing, he returns to his shed or office and makes some calculations. He finds that he has some idle tractors that were going to be used elsewhere but now are available. He calculates that it would be cheaper for him to plow with that otherwise idle capacity in machinery, rather than expanding his labor force and renting oxen. Hence labor-machinery substitution occurs at his level--in the aggregate for all the farmers who entered into contracts.

The set of equations used to represent labor-machinery substitution at a more aggregate level is given in Chapter IV (see section "A New Approach to Modelling Labor-Machinery Substitution Possibilities"). They may be summarized briefly here, as follows:

a) A set of equations to add up all demands for machinery and (substitutable) labor, summing over crops, existing technologies, and seasons. (In this model, substitution is allowed to occur separately for each farm group.)

b) An equation for each farm group to translate these demands into demands for traction power.

c) An equation for each farm group to link the total power demand to an exogenous point on the isoquant, that is, to ensure that the isoquant is respected in the choice of factor proportions to meet the demand. In the institutional analogy, this equation transfers the power demands from the farmers to the contractor.

d) A set of equations to spell out the demands for machinery and labor that are implied by the point the contractor has chosen on the isoquant.

e) A set of restrictions on the availability of labor and machinery. In the institutional analogy, these restrictions refer to the contractor's factor endowments; in the applied model for Costa Rica, they refer to region-wide farmers' group endowments.

In addition to these equations, the model's objective function has to be modified so that labor and machinery costs are incurred according to the contractor's decisions on factor use levels, and not according to the levels indicated by the farmers' original technologies of production. This modification is also described in Chapter IV.

In summary, these procedures make it possible to generate and use empirical isoquants, which describe factor substitution possibilities, in activity analysis models. They constitute a marriage of neoclassical and activity analysis production theory in the context of applied models.

Chapter III

DEVELOPING A TECHNIQUE TO GENERATE ISOQUANTS

It is apparent from the previous chapter that substantial flexibility could be added to linear programming models if factor substitution possibilities are allowed for in a more systematic manner. In particular, the scope of these models could be broadened if neoclassical factor substitution surfaces could be built onto the discrete factor proportion possibilities that usually characterize those models. In this chapter a method is developed to generate isoquants that later can be fitted into linear programming models (see Chapter IV). The empirical applications are focused on elasticities of substitution that are different from zero or infinity. The reasons for ruling out the extremes are: i) the first case--the pure input-output case--is of little interest, and ii) in the second case, apart from its theoretical interest, it is fairly unrealistic to think of any production factor that can be perfectly substituted for another one.

Existing Techniques to Generate Isoquants

There are two existing ways to build isoquants numerically. The first one is to plot a function that relates the two inputs and yields the same level of output for all possible combinations of inputs while

displaying the downward sloping and convexity requirements. Take, for example, the rectangular hyperbola

$$K = \frac{B}{L} \tag{18}$$

where K and L are the two inputs and B is a constant that in this case represents the level of output. This formulation corresponds to the production function $Q = AK^{\alpha} L^{\beta}$, where $A = \alpha = \beta = 1$ and $B = Q$. The marginal rate of technical substitution is given by the slope:

$$MRTS = \frac{\partial K}{\partial L} = -\frac{B}{L^2} \tag{19}$$

Then, for illustration purposes, assuming B = 100, the elasticity of substitution, ξ, is calculated as follows:

K	L	K/L	MRTS	%Δ(K/L)	%ΔMRTS	ξ
1	100	0.01	-100	-	-	-
2	50	0.04	-25	3.00	0.75	4.00
5	20	0.25	-4	5.25	0.84	6.25

As can be seen, ξ takes different values along the isoquant; therefore, the rectangular hyperbola serves to represent non-CES isoquants. Similar behavior is displayed by the following functions suggested by Allen (1964) to represent isoquant systems:

$$\frac{K + h}{h - \sqrt{L + A}} = B, \quad 0 \leq K \leq B\ (h - \sqrt{A}) - h \tag{20}$$

$$(K + h)\ (L + A) = B, \qquad 0 \leq K \leq \frac{B}{A} - h \tag{21}$$

$$K + L + \sqrt{2KL} = B, \qquad 0 \leq K \leq B \tag{22}$$

B in each function represents the production scale, and h and A are positive constants.

A variation of the previous case is attained by taking any CES production function and by fixing the level of output then deriving the equation for the isoquant. For example, let us take the Cobb-Douglas production function $Q = A\ K^{\alpha}\ L^{1-\alpha}$. Then setting $Q = Q_0$ we get

$$K = \left[\frac{Q_0}{A\ L^{1-\alpha}} \right]^{1/\alpha} \tag{23}$$

Expression (18) is a special case of (23). To illustrate this, let us assume $A = 1$ and $\alpha = \frac{1}{2}$ then we get

$$K = \frac{B}{L}, \text{ for } B = Q_0^2 \tag{24}$$

and

$$MRTS = \frac{\partial K}{\partial L} = -\frac{B}{L^2} \tag{25}$$

The difference between (24) and (18) is that in (24) K, L, and B are functionally related in a way that yields a constant elasticity of substitution. Furthermore, in this case the elasticity of substitution

is unitary, which is one of the distinctive features of the Cobb-Douglas production function. The rectangular hyperbola, then, also serves to represent unitary CES isoquants, provided that $B = Q^2$. To illustrate this, let us assume $Q_0 = 4$, or $B = 16$. Thus using (24) and (25) the elasticity of substitution is calculated as follows:

K	L	K/L	MRTS	%Δ(K/L)	%ΔMRTS	ξ
1	16	0.06	-0.06	-	-	-
2	8	0.25	-0.25	3.17	3.17	1.00
4	4	1.00	-1.00	3.00	3.00	1.00

In empirical work though, it would be preferable not to impose a unitary elasticity of substitution. This can be avoided by deriving the function for the isoquant from the more general CES production functions:

$$Q = A\,[\alpha K^{-\rho} + (1 - \alpha) L^{-\rho}]^{-h/\rho}, \qquad \xi = \frac{1}{1 + \rho} \tag{26}$$

$$K = \left[\frac{(Q/A)^{-\rho/h} - (1 - \alpha) L^{-\rho}}{\alpha}\right]^{-1/\rho} \tag{27}$$

$$MRTS = -\frac{1}{\rho}\left[\frac{(Q/A)^{-\rho/h} - (1-\alpha) L^{-\rho}}{\alpha}\right]^{-(1/\rho + 1)} \left[\frac{\rho(1-\alpha) L^{-(\rho + 1)}}{\alpha}\right] \tag{28}$$

If, for the sake of illustration, we assume $h = 1$; that is, constant returns to scale, and make $A = 1$, $\alpha = \frac{1}{2}$, $\rho = -\frac{1}{2}$, and $\xi = 2$, then we get

$$Q = \tfrac{1}{4}\,(\sqrt{K} + \sqrt{L})^2 \tag{29}$$

$$K = 4\,(\sqrt{Q} - \tfrac{1}{2}\sqrt{L})^2 \tag{30}$$

$$MRTS = \frac{\partial K}{\partial L} = -2\,(\sqrt{Q} - \tfrac{1}{2}\sqrt{L})\,(1/2\sqrt{L}) \tag{31}$$

If we make $Q = 25$ then:

K	L	K/L	MRTS	%Δ(K/L)	%ΔMRTS	ξ
39.2	14	2.80	-0.836	-	-	-
37.5	15	2.50	-0.790	0.11	0.055	2.00
36.0	16	2.25	-0.750	0.10	0.050	2.00

A second alternative is that developed by Norton and Bassoco (1983) using linear programming models. It consisted in holding constant the total value of production in a multi-market model and simultaneously changing the price of labor. The result was a set of labor-machinery combinations that served to calculate the sectoral elasticity of substitution. The major limitations of this technique are that one has to hold fixed the level of output while changing the input prices, and that the elasticities of substitution calculated from that procedure are variable and the variation is unpredictable.

A New Technique to Generate Isoquants

Another possibility is proposed here. In this alternative, the departure point is a set of observed values of inputs K and L and their respective prices P_K and P_L, plus a value of a single parameter in the production function, the elasticity of factor substitution.

The major advantage of the technique is that one can generate isoquants using observed or hypothetical information on a single set of values of input levels and input prices without knowing the full production function. It is not necessary to know the level of output either. As indicated in the next chapter, the isoquants generated in this way and incorporated in the linear programming model will shift freely to represent any level of output. This feature represents an advance over previous attempts to estimate input substitution in linear programming models that require fixing the level of output (Norton and Bassoco 1983). The technique produces downward sloping and convex isoquants. Step by step, the procedure is as follows:

Step one. With the observed quantities of the two inputs K_0, L_0 and their prices P_K^0, P_L^0 calculate the initial input ratio (R_0) and the initial marginal rate of technical substitution ($MRTS_0$):

$$R_0 = \frac{K_0}{L_0} \tag{32}$$

$$MRTS_0 = -\frac{P_L^0}{P_K^0} = \frac{\Delta K_0}{\Delta L_0} \tag{33}$$

Notice that for the $MRTS_0$ to be equal to the observed input price ratio at the observed input use level, an important assumption is made: that the input use is optimal in economic terms. That is, that the quantities used of the inputs minimize the cost of production or, equivalently, maximize profits. This might not be the case and a hard evaluation of the fulfillment of the optimality assumption must be made. If not the exact amount of any distortion, at least its direction should be evaluated.

Step two. Choose a value for the Hicksian elasticity of substitution:

$$\xi = \bar{\xi} \tag{34}$$

In empirical work, this parameter can be taken from estimation studies of similar industries in other countries, if appropriate. As explained in Chapter I, the elasticity can be the aggregate of elasticities for individual crops or regions, to approximate the sectoral elasticity. It also can be varied as part of an exercise in which the quantitative consequences of different substitution elasticities--and of policies that affect them--are simulated.

Step three. Choose a new level for one of the inputs. This is an arbitrary decision but it must be as close to the initial level of the input as possible. The reason is that, as proposed in step six, a movement will be made along $MRTS_0$ which in fact takes us away from the isoquant that we try to generate. Although not required, it is preferable to move along the isoquant in constant proportional intervals

because the procedure can then be easily automated. Departing from the initial point, then the quantity of one of the inputs can be increased or decreased in each interval by a given percentage. Call this percentage δ

$$\delta = \bar{\delta} \tag{35}$$

$\delta \gtrless 0$ depending on the direction of movement along the isoquant.

Step four. Calculate a new level for the input chosen in step three. For illustration, let us assume that input K is represented on the vertical axis and that the movement will be upward on the isoquant; then:

$$K_1 = (1 + \bar{\delta})\ K_0 \tag{36}$$

Step five. Obtain the absolute change in K:

$$\Delta K_1 = K_1 - K_0 \tag{37}$$

Step six. Using (37) and (33) obtain the absolute change in L that would keep you on the tangent to the hypothetical isoquant; i.e., on the line representing the MRTS:

$$\Delta L_1 = \frac{\Delta K_1}{MRTS_0} \tag{38}$$

Step seven. Calculate the level of input L associated with K_1:

$$L_1 = L_0 + \Delta L_1 \tag{39}$$

Note that if $\Delta K_1 > 0$, then $\Delta L_1 < 0$ because $MRTS_0 < 0$.

Now that a second point has been constructed for the isoquant, a third point is generated, but in this case making sure both that the preset elasticity of substitution is reflected in the new input combination and that the principle of convexity is respected. This requires several steps.

Step eight. Calculate the new input ratio at the second point on the isoquant:

$$R_1 = \frac{K_1}{L_1} \tag{40}$$

Step nine. Calculate the percentage changes that has occurred in the input ratio, between the initial point and the second point on the isoquant:

$$\%\Delta R_1 = \frac{R_1 - R_0}{R_0} \tag{41}$$

Step ten. By definition of the Hicksian elasticity of substitution, ξ, we know that the MRTS must change in a way that:

$$\%\Delta MRTS_1 = \frac{\%\Delta R_1}{\bar{\xi}} \tag{42}$$

By estimating the percentage change in the MRTS in this way, the value of ξ can be set at a constant level for a CES isoquant or vary along the isoquant for a non-CES isoquant. Additionally, the $\%\Delta R$ is positive because K increased while L decreased, and therefore R went up. Since

ξ also is positive, it follows that the change in the MRTS is positive. Thus the new MRTS is going to be steeper; therefore, the convexity is maintained.

Step eleven. Obtain the value of the new MRTS:

$$MRTS_1 = (1 + \%\Delta MRTS_1)\ MRTS_0 \tag{43}$$

Step twelve. Repeat steps four to eleven to generate as many points on the isoquant as are desired.

This iterative process can be summarized as follows for $i = 1, \ldots, n$:

Initial conditions:

$$K = K_0, \quad L = L_0, \quad R_0 = \frac{K_0}{L_0}, \quad P_K = P_K^0, \quad P_L = P_L^0 \tag{44}$$

$$MRTS_0 = -\frac{P_L^0}{P_K^0} \tag{45}$$

The algorithm to generate n points on the isoquant is:

$$K_i = (1 + \bar{\delta})\ K_{i-1} \tag{46}$$

$$\Delta K_i = K_i - K_{i-1} \tag{47}$$

$$\Delta L_i = \frac{\Delta K_i}{MRTS_{i-1}} \tag{48}$$

$$L_i = L_{i-1} + \Delta L_i \tag{49}$$

$$R_i = \frac{K_i}{L_i} \tag{50}$$

$$\%\Delta R_i = \frac{R_i - R_{i-1}}{R_i} \tag{51}$$

$$\%\Delta MRTS_i = \frac{\%\Delta R_i}{\xi} \tag{52}$$

$$MRTS_i = (1 + \%\Delta MRTS_i)\ MRTS_{i-1} \tag{53}$$

An example of an isoquant generated using this procedure is presented in Table 2 and Figure 2.

In summary, by allowing the MRTS to vary from its initial value, which is given by observed input price ratios, it is possible to generate a family of isoquants, each of which corresponds to a different value of ξ and all of which are consistent with the observed initial factor proportions. The value of this procedure (plus the procedure of Chapter IV below) for empirical work is considerable, for it enables the modelling exercise to make use of estimated substitution elasticities. It also permits a variety of hypothetical and policy-oriented experiments concerning the quantitative effects of variations in elasticities of factor substitution.

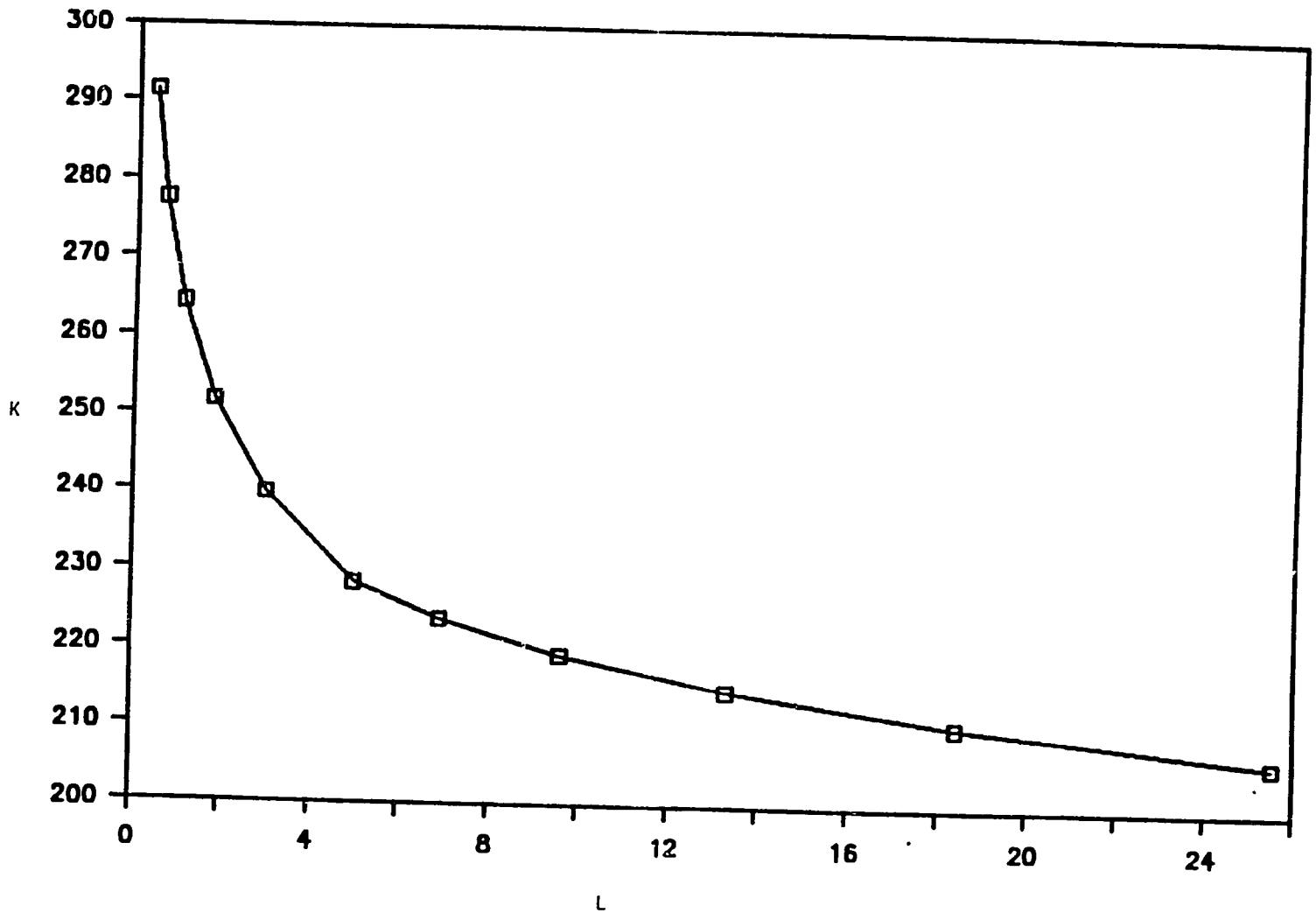
It should be pointed out that this procedure can generate an isoquant with a constant or a non-constant elasticity of substitution. For the latter, the elasticity value can be specified anew at each iteration of the procedure.

Table 2--Coefficients of an isoquant with unitary elasticity of substitution generated using the new technique

Segment	K	L	$MRTS_1$	ΔK	ΔL	R_i	$\%\Delta R_i$	$\%\Delta MRTS_i$
1	291.36	0.40	52.88	13.87	-0.26	726.81	73.73	73.73
2	277.49	0.66	30.44	13.21	-0.43	418.42	73.73	73.73
3	264.27	1.10	17.52	12.58	-0.72	240.85	73.73	73.73
4	251.69	1.82	10.09	11.99	-1.19	138.63	73.73	73.73
5	239.70	3.00	5.81	11.41	-1.97	79.80	73.73	73.86
6	228.29	4.97	3.34	0.00	0.00	45.93	0.00	0.00
7	223.72	6.90	2.36	-4.57	1.93	32.40	-29.46	-29.31
8	219.25	9.58	1.67	-4.47	2.68	22.88	-29.40	-29.25
9	214.86	13.29	1.18	-4.38	3.71	16.16	-29.35	-29.22
10	210.57	18.43	0.84	-4.30	5.13	11.43	-29.30	-29.17
11	206.36	25.53	0.59	-4.21	7.10	8.08	-29.25	-29.12

Note: The information that appears on segment 6, corresponding to the level of inputs K and L, and the relative prices or MRTS, plus the value of the elasticity of substitution is all that is required to generate the isoquant.

Figure 2--Shape of an isoquant generated using the new technique



Chapter IV

A MODEL OF LABOR-MACHINERY SUBSTITUTION IN COSTA RICAN AGRICULTURE

Introduction

This chapter explores the structure of the Costa Rican LP model. As usual in this kind of work, the model is given a name; it is baptized TICO (as nationals of Costa Rica are known everywhere).

TICO is a model of the agricultural sector. Although it is spatially defined for the Northwestern region of Costa Rica, known as *Pacifico Seco*[1] (see Figure 3), it is linked to the rest of the agricultural sector in the country and to the international markets via commodity trade activities. On the production side, TICO includes a disaggregation into three groups of farmers. Each group's use of land, labor, and machinery is specified on a monthly basis, and the physical amount of two kinds of fertilizers and the monetary value of all other inputs such as pesticides and management, are also accounted for by group. There are cropping and livestock production activities with alternative technologies for each crop. Uncertainty regarding farmers' revenues from crop activities and risk-averse behavior by farmers is

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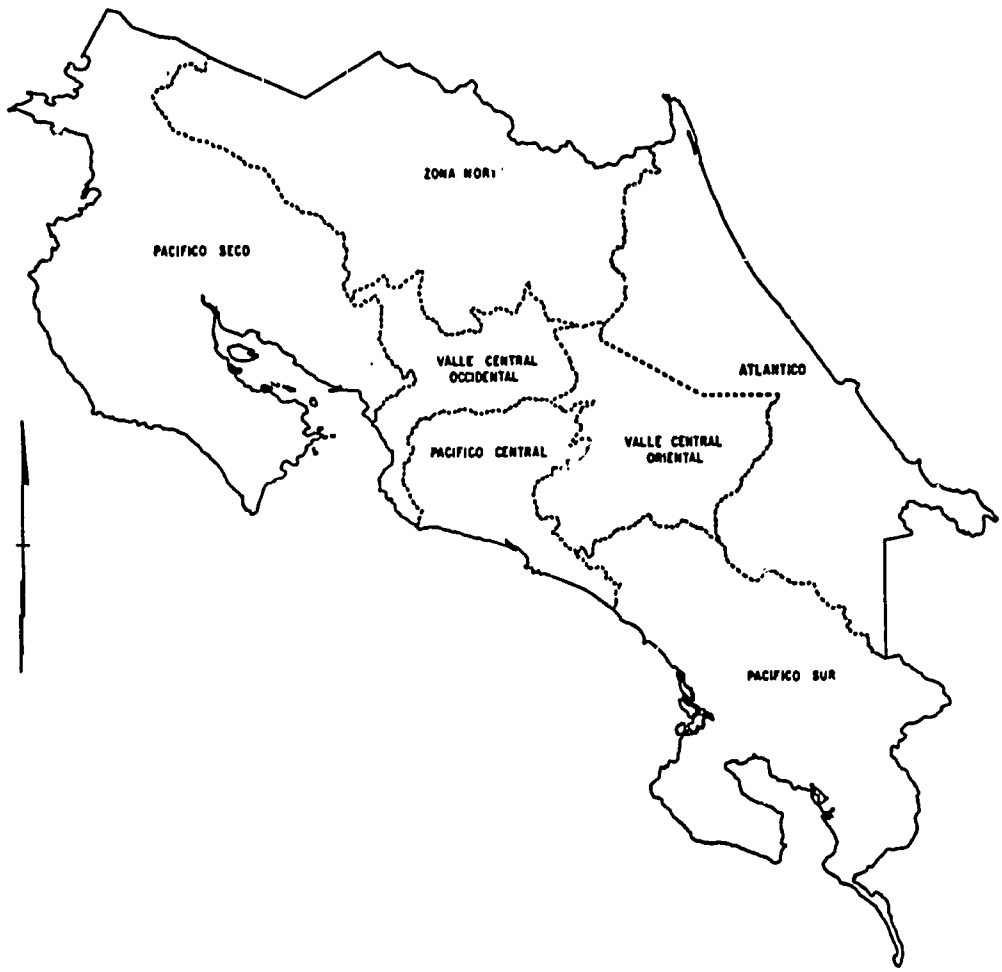
[1] The name of the region and its main characteristics used here correspond to those prevailing in the agricultural census of 1973. More recently the Northwestern region changed name and boundaries; the salient features relevant for this study remain the same.

Figure 3--The *Pacífico Seco* region in Costa Rica



Source: Dirección General de Estadística y Censos, Censos nacionales de 1973 agropecuario regiones agrícolas (San José, Costa Rica: DGEC, Sección de Publicaciones, 1975).

accounted for in the model using the (E,σ) criterion described in Chapter II.

On the input market side of the model, TICO contains activities that reflect the hiring of labor and machinery on a monthly basis and the purchase of fertilizers and other inputs on a "cropping season" basis. All input prices are exogenous to the model. The input availability constraints are defined for each month in the case of land, labor, and machinery. Fertilizers and other inputs are not explicitly constrained in availability; however, they are indirectly constrained through the maximum amount of credit available to each group of farmers. The labor market in TICO reflects not only the use of landless labor by each one of the groups of farmers, but also intergroup labor migration.

The structure of the commodity markets is formulated in several steps. First, requirements for home retentions at the farm level are specified exogenously for each group of farmers; then, the endogenous transfers of unprocessed products to the market by each group are pulled together and subjected to processing, transport, and marketing activities to define the regional marketed supply for each commodity. This endogenous regional supply is added to the national initial inventory, the endogenous domestic supply from other regions and the endogenous imports; the result is the total supply for each commodity. This supply is then equated to total demand, which also is endogenous as a function of prices, and which is the sum of domestic demand, exports, and final national inventory. The equilibrium levels of supply and demand determine prices. Demand functions are explicit in the model

and supply functions are implicit, so the equilibrium quantities of both will be influenced by prices.

The procedure used for the purpose of modelling market equilibrium is that developed by Duloy and Norton (1983, 1975) which utilizes the grid linearization techniques of Miller (1963). In TICO, commodity demands are assumed to be independent, i.e., cross-price elasticities are set equal to zero.

Algebraic Formulation of the Basic Model

The set of equations and variables that comprise TICO are presented and explained in detail in this section; summary tables are also included to support the exposition.

Input Balances

$$\sum_i \sum_j a_{ktijg} X_{ijg} - S_{ktg} \leq 0, \text{ all } k, t, g \qquad (54)$$

Input demand derived from production activities	-	Input supply or purchase of inputs	≤ 0

where the variables are defined as:

X_{ijg} = number of hectares of crop i planted using technology j by group of farmers g. TICO also includes livestock activities in which case this variable is defined as number of animal units in livestock activity i produced under technology j by group of farmers g.

S_{ktg} = quantity of input k purchased in period t by group of farmers g,

and the <u>parameter</u> is defined as:

a_{ktijg} = quantity of input k required in period t to produce one hectare of crop or one unit of livestock activity i using technology j by group of farmers g,

and the range values the <u>subscripts</u> may take on are:

$g = 1,\ldots,3$ corresponding to small, medium, and large-scale group of farmers,

$i = 1,\ldots,14$ corresponding to rice, maize, beans, sorghum, cotton, sugarcane, plantain, oranges, pastures, cattle breeding, cattle fattening, cattle breeding and fattening, and dairy cattle,

$j = 1,\ldots,32$ corresponding to 5 technologies for rice, 5 for maize, 10 for beans, 3 for sorghum, 1 for cotton, 1 for sugarcane, 1 for plantain, 1 for orange, 1 for pasture, 1 for cattle breeding, 1 for cattle fattening, 1 for cattle breeding and fattening, and 1 for dairy cattle,

$t = 1,\ldots,12$ corresponding to 12 months,

$k = 1,\ldots,11$ corresponding to land, labor, machinery, nitrogen fertilizer, compound fertilizer, other inputs, credit, and cows, bulls, calves, and oxen for each of the livestock activities, respectively.

Therefore, the maximum number of equations (rows) in this section is g x t x k, i.e., 396. Actually, TICO has only 132 in this section, since only land, labor, and machinery balances are specified by month.

<u>Uncertainty on Revenues and Risk Aversion</u> (E,σ approach)

$$\sum_i \sum_j \gamma_{ijlg} X_{ijg} \quad - \quad R_{lg} \leq 0, \text{ all } l, g \tag{55}$$

Positive and negative revenue deviations for all production activities - Positive deviation counters in dollars ≤ 0

$$2\sum_l R_{lg} \quad - \quad f\sigma_g \quad = \quad 0, \text{ all } g \tag{56}$$

Sum of mean absolute deviations - Fischer coefficient = 0

where the new <u>variables</u> are defined as:

R_{lg} = risk activity or counter of revenue deviations for group of farms from sample mean obtained over l periods of historical information (l becomes just the number of observations when the revenue data come from a cross-sectional sample),

σ_g = model's endogenous estimate of standard deviation of revenues associated with the optimal production plan of group of farmers g,

and the new parameters are defined as:

γ_{ijlg} = observed per hectare revenue deviation in crop i planted using technology j by group of farmers g, from sample mean obtained over l periods or observations,

f = coefficient of Fischer,

$$f = \frac{T}{\sqrt{F}} \text{ and } F = \frac{T\pi}{2(T-1)}$$

T is the maximum number of l periods or observations and π is the mathematical constant,

and the new subscript values are:

l = 1, ..., 10.

This section contains (l x g) + g = 33 rows.

Regional Commodity Balances

$$\sum_j y_{ijg}X_{ijg} - T_{ig} \geq A_{ig}, \text{ all } i, g \qquad (57)$$

Total farm production	-	Production transferred to the market	$\geq$	Home retentions at farm level

where the new variable is defined as:

T_{ig} = marketing and/or transformation activity of crop i by group of farmers g. It accounts for the quantities of each raw product send to the market, in tons,

and the new parameters are:

y_{ijg} - yield per hectare of crop i planted by group of farmers g, using technology j,

A_{ig} - minimum quantity in tons to be produced to satisfy self-requirements of crop i by group of farmers g.

This section contains a maximum of i x g = 42 rows.

National Commodity Balances

$$-\sum_g t_{sg}T_{ig} + \sum_h q_{sh}D_{sh} - M_s + E_s$$

- Commodity supply from Pacífico Seco + Commodity domestic demand - Commodity imports + Commodity exports

$$- L_s + F_s - K_s \leq \sum_g t_{sg} A_{ig}, \text{ all } s \qquad (58)$$

- Commodity initial inventory + Commodity final inventory - Commodity supply from other regions ≤ Home retention

$$\sum_h D_{sh} \leq 1, \text{ all } s \qquad (59)$$

Convex combination constraints

where the new variables are defined as:

D_{sh} - activity level at point h on the demand function for processed commodity s. This variable is an interpolation weight, so its value ranges from zero to unity,

M_s - imports of commodity s, in tons,

E_s - exports of commodity s, in tons,

L_s - initial inventory of commodity s, in tons,

F_s - final inventory of commodity s, in tons,

K_s - supply of commodity s from the rest of the country in tons,

and the new <u>parameters</u> are defined as:

t_{sg} - yield of commodity s from marketing and/or processing crop i transferred to the market by group of farmers g. This coefficient accounts for wastes of raw products and its value is the percentage of commodity s obtained from raw product or crop i,

q_{sh} - quantity demanded for domestic consumption at point h of the demand function for final product s,

and the values of the new <u>subscripts</u> are:

$S = 1,\ldots,15$ corresponding to rice, maize, beans, sorghum, vegetable oil, white sugar, raw sugar (panela), plantain, oranges, meat, and milk. TICO also includes fixed price demands for cotton, cotton cake, alcohol (ethanol), and molasses. In the case of these fixed price demands, equation (59) is not required and in equation (58), the term $\sum_h q_{sh}D_{sh}$ is replaced by a simple quantity variable D_s.

$h = 1,\ldots,15$ corresponding to 15 points on each linearized demand function.

The RHS of equation (58) adds to the supply of each commodity the amounts consumed at the farm level. This computation is necessary in TICO because the domestic demand functions were calculated from demand quantities for the entire population, including all families. This section has a maximum of 2s = 30 equations; however, because there are only 12 linearized demand functions in TICO, the actual number of equations is 27.

Resource Constraints

$$\underset{\text{Resource purchases}}{S_{ktg}} \leq \underset{\text{Resource endowment}}{b_{ktg}}, \text{ all } k, t, g \qquad (60)$$

where the new parameter is defined as:

b_{ktg} = availability of resource k in period t for use by group of farmers g.

This section has a maximum of k x t x g = 684 equations. As mentioned before, not all the resources are specified in TICO on a monthly basis and some of them are not directly constrained; furthermore, purchasing activities are not specified for all the resources. On the other hand, TICO allows for three different kinds of labor constraints: family labor in the group of small farms, family labor in the group of medium-sized farms, and landless labor. Thus, this section in TICO has only 75 equations: 36 for labor, 36 for machinery, and 3 for credit.

Commodity Trade and Stock Constraints

$$M_s \leq M_s^0, \text{ all } s \tag{61}$$

$$E_s \leq E_s^0, \text{ all } s \tag{62}$$

$$L_s \leq L_s^0, \text{ all } s \tag{63}$$

$$F_s \leq F_s^0, \text{ all } s \tag{64}$$

$$K_s \leq K_s^0, \text{ all } s \tag{65}$$

This section has a maximum of 5s = 75 equations.

Objective Function

$$\text{Max } Z = -\sum_k \sum_t \sum_g P_{ktg} S_{ktg} - \sum_g \Phi_g \sigma_g - \sum_i \sum_g \tau_{ig} T_{ig} + \sum_s \sum_h W_{sh} D_{sh}$$

$$+ \sum_s (-m_s M_s + e_s E_s - k_s K_s) - \sum_s \mu_s (F_s + L_s)/2 \tag{66}$$

where the new parameters are defined as:

P_{ktg} = price of purchased input k in period t, paid by group of farmers g,

Φ_g = aggregate risk-aversion coefficient for group of farmers g,

τ_{ig} = marketing and/or processing cost of crop i transferred to the market by group of farmers g,

W_{sh} = area under demand curve at point h of the demand function for commodity s,

m_s = import price per ton of commodity s,

e_s = export price per ton of commodity s,

μ_s - storage cost per ton of commodity s,

k_s - other regions' supply price per ton of commodity s.

Table 3 shows the schematic representation of TICO; Table 4 amplifies the scheme to show the disaggregation by group of farmers. Table 5 shows the inter-group relationships in the labor market. In the three tables, the I's represent identity matrices; the demand for credit is calculated in the credit constraint rows by introducing the prices of the inputs below the variables that account for the physical quantities purchased. For the sake of illustration, two conventions are introduced in Table 5; first, the subscript k has taken the value of 1, representing the labor resource. Second, the purchasing activities have now an additional subscript indicating the source of hired labor. For instance:

S_{1t11} - labor "hired" in period t by group 1 (small-sized farms) from group 1. This is the family labor use,

S_{1t14} - labor hired in period t by group 1 from group 4 (landless workers),

S_{1t21} - labor hired in period t by group 2 (medium-sized farms) from group 1.

In the objective function, P^*_{1t1} and P^*_{1t2} are the reservation wages paid by small- and medium-scale group of farmers when they use their own so-called family labor. Notice also that all farmers pay the same market wage for hired labor, i.e., $P_{.t1} = P_{1t2} = P_{1t3} = \omega$. It is also assumed that group 3 farmers do not work in the fields themselves, but rather

Table 3--Aggregate basic model

	X_{ijg}	S_{ktg}	R_{lg}	σ_g	T_{ig}	D_{sh}	M_s	E_s	L_s	F_s	K_s	RHS
INPUT BALANCES	a_{kijtg}	-1										≤ 0
Risk	γ_{ijlg}		-1									≤ 0
σ_g			$2 \ldots 2$	$-f$								$= 0$
Regional commodity balances	γ_{ijg}				-1							$\geq A_{ig}$
National commodity balances					$-t_{sg}$	q_{sh}	-1	-1	-1	-1	-1	$\leq \Sigma t_{sg} A_{ig}$
Convexity constraints						$1 \ldots 1$						≤ 1
						$\vdots$						$\vdots$
						$\ldots 1$						≤ 1
Input constraints		1										$\leq b_{ktg}$
Credit constraints		P_{ktg}										$\leq C_g^o$
Other constraints												
Imports							1					$\leq M_s^o$
Exports								1				$\leq E_s^o$
Initial inventory									1			$= L_s^o$
Final inventory										1		$\geq F_s^o$
Supply from other regions											1	$= K_s^o$
OBJECTIVE		$-P_{ktg}$		Φ_g	$-\tau_{ig}$	W_{sh}	$-m_s$	e_s	$-\mu_s/2$	$-\mu_s/2$	$-k_s$	MAX

Table 4--Basic model with farmers group disaggregation

	X_{ij1}	X_{ij2}	X_{ij3}	S_{kt1}	S_{kt2}	S_{kt3}	R_{L1}	$\hat{\sigma}_1$	R_{L2}	$\hat{\sigma}_2$	R_{L3}	$\hat{\sigma}_3$	T_{i1}	T_{i2}	T_{i3}	D_{sh}	M_s	E_s	L_s	F_s	K_s	RHS
Input balances group 1	a_{kijt1}			-1																		≤ 0
Credit constr. G1				P_{kt1}																		$\leq C_1^0$
Input balances group 2		a_{kijt2}			-1																	≤ 0
Credit constr. G2					P_{kt2}																	$\leq C_2^0$
Input balances group 3			a_{kijt3}			-1																≤ 0
Credit constr. G3						P_{kt3}																$\leq C_3^0$
Risk group 1	$Y_{ij\ell1}$						-1															≤ 0
$\hat{\sigma}_1$							2...2	-f														$= 0$
Risk group 2		$Y_{ij\ell2}$							-1													≤ 0
$\hat{\sigma}_2$									2...2	-f												$= 0$
Risk group 3			$Y_{ij\ell3}$								-1											≤ 0
$\hat{\sigma}_3$											2...2	-f										$= 0$
Comm. balances G1	Y_{ij1}												-1									$\geq A_{i1}$
Comm. balances G2		Y_{ij2}												-1								$\geq A_{i2}$
Comm. balances G3			Y_{ij3}												-1							$\geq A_{i3}$
National comm. balances													$-t_{s1}$	$-t_{s2}$	$-t_{s3}$	q_{sh}	-1	1	-1	1	-1	$\leq \sum_g t_{sg} A_{ig}$
Convexity constraints																1...1						≤ 1
																⋮						⋮
																1...1						≤ 1
Input constraints G1				1																		$\leq b_{kt1}$
Input constraints G2					1																	$\leq b_{kt2}$
Input constraints G3						1																$\leq b_{kt3}$
Import constraints																	1					$\leq M_s^0$
Export constraints																		1				$\leq E_s^0$
Initial inv. constraints																			1			$= L_s^0$
Final inv. constraints																				1		$\geq F_s^0$
Supply constr. other reg.																					1	$= K_s^0$
OBJECTIVE				$-P_{kt1}$	$-P_{kt2}$	$-P_{kt3}$		$-\Phi_1$		$-\Phi_2$		$-\Phi_3$	$-\tau_{i1}$	$-\tau_{i2}$	$-\tau_{i3}$	W_{sh}	$-m_s$	e_s	$-\mu_s/2$	$-\mu_s/2$	$-k_s$	MAX

Table 5--Labor market in the basic model

	X_{ij1}	X_{ij2}	X_{ij3}	S_{1t11}	S_{1t14}	S_{1t21}	S_{1t22}	S_{1t24}	S_{1t31}	S_{1t32}	S_{1t34}	RHS
Labor balance G1	a_{1ijt1}			-1	-1							≤ 0
Labor balance G2		a_{1ijt2}				-1	-1	-1				≤ 0
Labor balance G3			a_{1ijt3}						-1	-1	-1	≤ 0
Family labor G1				1		1			1			$\leq b_{1t1}$
Family labor G2							1			1		$\leq b_{1t2}$
Landless labor					1			1			1	$\leq b_{1t4}$
OBJECTIVE				$-P^*_{1t1}$	$-\omega$	$-\omega$	$-P^*_{1t2}$	$-\omega$	$-\omega$	$-\omega$	$-\omega$	MAX

Note: The landless labor is sometimes referred to as regional labor in the text. Its wage rate is the same as any hired labor, ω. The reservation wage for family labor working on its own farm is P^*_{1tg}.

they supervise other labor. Including several accounting equations, not shown in the preceding formulation, TICO has 382 rows and 551 columns or variables.

A New Approach to Modelling Labor-Machinery Substitution Possibilities

The input substitution possibilities developed through the algorithm in Chapter III are introduced here in TICO for the case of labor-machinery substitution. In this section, the subscript k takes the value of 1 for labor and 2 for machinery. The equations here implement the concepts explained in the last section of Chapter II. To guide the reader, each set of equations is introduced by reference to the analogy of the farmers and the contractor that was presented in the last section of Chapter II.

The first set of equations sums up the demands for labor and machinery that are implicit in the farmers' original technologies of production, by farm group and by month:

Monthly Labor and Machinery Balances

$$\sum_i \sum_j a_{1tijg} X_{ijg} - S_{1tg} \leq 0, \text{ all } t, g \tag{67}$$

$$\sum_i \sum_j a_{2tijg} X_{ijg} - S_{2tg} \leq 0, \text{ all } t, g \tag{68}$$

The next set of equations sum over months the previous total demands for machinery and labor according to the original technologies:

Accounting of Total Annual Labor and Machinery Demands

$$\sum_t S_{1tg} - TL_g = 0, \text{ all } g \tag{69}$$

$$\sum_t S_{2tg} - TM_g = 0, \text{ all } g \tag{70}$$

where the new variables are defined as:

TL_g = total annual labor demanded by group of farmers g,

TM_g = total annual machinery demanded by group of farmers g.

Now the original total (annual) demands for machinery and labor are converted into demands for traction power. Subsequently, it is this demand for total traction power that must be met by the contractor.

Accounting of Total Annual Draft Power Demand

$$\alpha_1 TL_g + \alpha_2 TM_g - TP_g = 0, \text{ all } g \tag{71}$$

where the new variable is defined as:

TP_g = total annual draft power demanded by group of farmers g. In the case of labor, sometimes the draft power is supplied by the animals that work with the labor, but it is equivalent to represent it as a function of labor only.

and the new parameters are defined as:

α_1 = units of power obtained from each unit of labor,

α_2 = units of power obtained from each unit of machinery.

In TICO, the following conventions have been assumed regarding the values of these two parameters:

(i) $\alpha_1 = 1$,

(ii) $\alpha_2 = 8\alpha_1 = 8$,

(iii) α_1 and α_2 are the same for all groups of farmers.

Now come the equations that show how the total power demand is met by the contractor. In equation (72) below, the first term represents power <u>demand</u>, and the second term represents power <u>supply</u> (by the contractor). Equation (72) requires that this power supply be derived from a linear combination of points Q on the isoquant. The choice variable (or interpolation variable) Z_{Qg} gives a weight to each isoquant point. If the isoquants are not shifted to represent higher or lower levels of output, then the variables Z_{Qg} will take on values between 0 and 1, with $\sum_Q Z_{Qg} = 1$. It is important to emphasize that the role of equation (72) is to force the model to observe the isoquant in supplying the total power needed. The isoquant represents the contractor's array of choices of labor-machinery combinations that he may use to meet the total power demand.

<u>Power, Labor, and Machinery Balances Along the Labor-Machinery Isoquants</u>

$$TP_g - \sum_Q \psi_{Qg} Z_{Qg} = 0, \text{ all } g \qquad (72)$$

Equations (73) and (74) reverse the role of the isoquant (the contractor). In equation (72), its role (the contractor's role) was to

supply the farmers' demand for power. Now the contractor has signed his contracts with the farmers, and he becomes a demander of inputs in order to fulfill his contracts. Thus, the first term in equation (73) is the total labor the contractor demands to meet his obligation to group of farmers. Recall that he may fulfill his contract by using either more or less labor than the farmers originally thought was necessary. He can move along the isoquant. That demand by the contractor (for inputs to his operation) can be met in two ways: by hiring farmers from other groups (the variables V_{1gg*}) or by convincing farmers to remain on their own farms and hiring them on the spot (or deducting the value of their labor input from the agreed contract amount); this second option is represented by the variables V_{1g}.

Balances for Meeting the Labor Demands Implied by the Choice of Points on the Isoquant

$$\sum_{Q} \theta_Q Z_{Qg} - \sum_{g*} V_{1gg*} - V_{1g} = 0, \text{ all } g \tag{73}$$

Similarly, equation (74) spells out the contractor's demand for machinery to fulfill his contractual obligations, as a function of the factor proportions he has selected through his choice of Z_{Qg}.

Balances for Meeting the Machinery Demands Implied by the Choice of Points on the Isoquant

$$\sum_{Q} \nu_{Qg} Z_{Qg} - \Sigma V_{2g} = 0, \text{ all } g \tag{74}$$

where the new variables are defined as:

Z_{Qg} - choice variable for selecting the point on the labor-machinery isoquant of farmers of group g, ($0 \leq \sum_{Q} Z_{Qg} \leq 1$ if the isoquant is not shifted outward),[2]

V_{1gg*} - quantity of labor hired by farmers of group g from group g*,

V_{1g*g} - quantity of labor transferred to group g* by farmers of group g,

V_{1g} - total quantity of labor in group that remains to work on the farms of group g,

V_{2g} - quantity of machinery used on farms in group g.

and the new parameters are defined as:

ψ_{Qg} - amount of power obtained from the combination of labor and machinery at point Q on the labor-machinery isoquant of farmers of group g,

θ_{Qg} - quantity of labor at point Q on the labor-machinery isoquant of farmers of group g,

ν_{Qg} - quantity of machinery at point Q on the labor-machinery isoquant of farmers of group g,

---

[2] Note that the isoquant's position is endogenous to the model; $\sum_{Q} Z_{Qg} < 1$ means it is shifted inward from its original position; $\sum_{Q} Z_{Qg} > 1$ means it is shifted outward.

and the new subscript takes the following values:

$g^* = 1,\ldots,4$ corresponding to the three groups of farmers and to the regional market. For the case of labor, for instance, group of farmers 1 may hire their own labor and labor from the regional market (landless). In this case the variable V_{kgg*} will take the values V_{k11} and V_{k14}, for $k = 1$. The group of farmers 1 may also transfer labor to group of farmers 2 and group of farmers 3; therefore, the variable V_{kg*g} will take the values V_{k21} and V_{k31}, for $k = 1$.

Now that the contractor's demands for labor and machinery have been defined, they can be subjected to the constraints on the total regional availabilities of labor and machinery. There will be an interaction between the constraints and the contractor's decision. If his initial instinct is to hire, say, machinery in excess of the amount available for the cropping cycle, then he will modify his decision by selecting a different point (points) on the isoquant. The endowment of labor in group g (L_g) provides an upper bound on the sum of labor working on its own farm (V_{1g}) and labor working in other farm groups (V_{1g*g}).

Annual Labor and Machinery Constraints

$$V_{1g} + \sum_{g*} V_{1g*g} \leq L_g, \text{ all } g \tag{75}$$

$$V_{2g} \leq M_g, \text{ all } g \tag{76}$$

where the new parameters are defined as:

L_g - annual availability of labor from group or source g,

M_g - annual availability of machinery from group or source g.

Finally, the model's original objective function has to be modified so that it is not charged for the amounts of labor and machinery that the farmers originally estimated to be necessary, but rather for the amounts actually deployed by the contractor.

Objective Function

The labor and machinery components of the first term from equation (66) are replaced by the following expression:

$$- \sum_{Q} \sum_{g} \zeta_{Qg} Z_{Qg} + \sum_{g=1}^{3} \Lambda_g V_{1g} \tag{77}$$

where the new parameters are defined as:

ζ_{Qg} - cost of total labor and machinery at point Q of labor-machinery isoquant of group of farmers g,

Λ_g - difference between reservation wage and market wage paid for labor hired by group of farmers g. (When the contractor hires labor on their own farms, he has to pay them less than if he were inducing them to migrate to other areas, so the total labor cost in the first term is adjusted for this differential.)

The first term in equation (77) accounts for the cost of the labor-machinery bundle at each point on each isoquant. The input

prices used are the base year market prices. Then, the second term makes an adjustment by crediting for the difference between the market wage and the reservation wage accruing from the family labor used.

The introduction of linearized isoquants required several changes in the basic structure of TICO, which was described by equations (54) to (66). These changes are shown in schematic form in Table 6. First, recall from Chapter III that the input substitution could be formulated at different levels: by crop, by combination of crops, by group of farmers, at regional level, at national level, and in different time settings (monthly, quarterly, or yearly). On computational grounds, the labor-machinery substitution possibilities are formulated in TICO by group of farmers and on a yearly basis. This implies that although it is still possible to specify the labor and machinery demand under the pre-substitution situation by group on a monthly basis, their total supply and availability from different sources is now accounted for on an annual basis, equations (69) to (76). Similarly, in the objective function two major changes are introduced. First, the cost of labor and machinery appears combined in a single figure, in the parameters ζ_{Qg}. Second, the monthly disaggregation of purchases of these two inputs is replaced by an annual figure, the same parameters ζ_{Qg}.

Table 6--Formulation of labor-machinery substitution: Illustration for group 1

	x_{ij1}	S_{1t1}	S_{2t1}	TL_1	TM_1	TP_1	$Z_{11}\ldots Z_{Q1}$	V_{11}	V_{114}	V_{121}	V_{131}	V_{21}	RHS
Monthly labor balances	a_{1ijt1}	-1											≤ 0
Monthly machinery balances	a_{2ijt1}		-1										≤ 0
Total annual labor demand		1...1		-1									$= 0$
Total annual machinery demand			1...1		-1								$= 0$
Total annual draft power demand				$+\alpha_1$	$+\alpha_2$	-1							$= 0$
BALANCES ALONG ISOQUANT													
Power						+1	$-\psi_{11}\ldots-\psi_{Q1}$						$= 0$
Labor							$+\Theta_{11}\ldots+\Theta_{Q1}$	-1	-1	+1	+1		$= 0$
Machinery							$+\nu_{11}\ldots+\nu_{Q1}$					-1	$= 0$
ANNUAL CONSTRAINTS													
Family labor								+1		+1	+1		$\leq L_1$
Landless labor									+1				$\leq L_4$
Machinery												+1	$\leq M_1$
OBJECTIVE							$-\zeta_{11}\ldots-\zeta_{Q1}$	$+\Lambda_1$					MAX

Chapter V

IMPLEMENTING THE MODEL: EVALUATION OF A CANE ALCOHOL PROGRAM IN COSTA RICA

The Cane Alcohol Program: Key Policy Issues

The cane alcohol program in Costa Rica originated in the improvement of technologies to substitute ethanol for gasoline and diesel. This technological breakthrough occurred in Brazil, where an aggressive alcohol fuel program was launched at the end of the seventies. Additionally, two major economic events provided the incentive to consider an alcohol fuel program in Costa Rica. First, the sharp increase in oil prices during the seventies. Second, when oil prices receded, the drop in the world prices of sugar and the reduction in the U.S. sugar import quotas during the eighties. Both events make the production of cane alcohol a more profitable option for farmers and millers.

The substitution of ethanol for gasoline can be done in two different ways: mixing ethanol and gasoline in a proportion of up to 20 percent of anhydrous ethanol and 80 percent of gasoline--this mixture is known as gasohol, or by modifying the cars' engines to run entirely with hydrate ethanol. The difference between anhydrous and hydrate ethanol lies in the alcohol content--98 percent in the first and up to 95 percent in the second. The fact that Costa Rica has a heavy

rainfall season that lasts for over six months translates into high humidity levels in the atmosphere. This in turn makes gasohol a very unstable mixture; therefore, the people who conceived the alcohol program considered that the use of engines run solely with hydrate alcohol is a more viable option. Since diesel and alcohol cannot be mixed, this would also be the only option to substitute for diesel.

Ethanol can be produced by fermentation and subsequent distillation of almost any material that contains sugar. This is what makes sugarcane particularly suitable for ethanol production. Furthermore, cane alcohol production can take two different processes: using molasses that are obtained as a byproduct of sugar production or using the totality of sugarcane juices. Traditionally, alcohol in Costa Rica has been produced using molasses. These molasses are delivered from the sugar mills to the national distillery. Under this process, since the bulk of the sugar content in the sugarcane is used to produce sugar, a massive production of alcohol would require enormous amounts of land. It is estimated that a total of 1.7 hectares would be required to produce a thousand liters of alcohol. This contrasts with the second alternative, that is, to use all cane juices to produce alcohol. Under this technology, a total of 0.2 hectare would be required to produce a thousand liters of alcohol.[3] Consequently, the second option is considered the most appropriate if a large quantity of alcohol is to be produced.

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[3] This estimate is based on a yield of sugarcane of 70 metric tons per hectare, 29.7 kilograms of molasses per ton of sugarcane, and 0.28 liter of alcohol per kilogram of molasses. Or alternatively, 64.4 liters of alcohol per ton of sugarcane when cane juices are used.

Nevertheless, any program aimed at substantially substituting for gasoline and diesel will require large areas of land and enormous amounts of labor. The land requirement ranges from 8,000 hectares to substitute for 20 percent of gasoline (the gasohol option) to 179,000 hectares to substitute for total current consumption of gasoline and diesel (the hydrate alcohol option).[4] Under current available technology, this implies a requirement of approximately 713,000 labor days for 20 percent substitution of gasoline and 18 million labor days for total substitution of gasoline and diesel. More importantly, the labor requirement is concentrated during the harvesting period--40 percent of labor is used during the period December through February.

Precisely, it has been the huge land and labor requirements and its seasonality that has prompted the need to consider the expansion of cane in areas that can be mechanized. Of course, the land constraint could only be resolved by incorporating new land into cane production or by increasing cane yields per hectare. Since Costa Rica has practically reached the frontier of arable land, area expansion to cane can only occur if other crops are displaced. Increasing cane yields, on the other hand, is at its best a long-term option. To be realistic, no major breakthroughs should be expected, since Costa Rica has one of the highest productivities in sugarcane production in the tropical world--average yield in Costa Rica is 70 tons per hectare as compared to 50 tons in Brazil, for instance.

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[4] These estimates are based on a cane yield of 70 metric tons per hectare and an output of 57 liters of anhydrous alcohol per metric ton of cane or 64.4 liters of hydrate alcohol per ton of cane.

As a result, a cane alcohol program to substitute for gasoline and diesel poses a series of policy questions that need to be resolved. Following is a comprehensive list of them:

1. What would be an appropriate price of alcohol, relative to the prices of other agricultural commodities, to encourage its production?

2. Given the enormous land requirements and the fact that Costa Rica appears to have hit the land bound, what crops would most likely be displaced?

3. What would be the impact of a cane alcohol program on agricultural relative prices? In particular, how would it affect supply and prices of food crops?

4. What would be the impact of the cane alcohol program on total labor demand and agricultural wages?

5. How would seasonal labor demand change as sugarcane area planted expands?

6. What would be the role of mechanization in resolving the anticipated labor shortage? More importantly, what would be the relative prices of labor and machinery that would encourage substitution of labor for machinery?

7. What would be the impact of changes in relative prices of labor and machinery?

8. What would be the impact of a cane alcohol program on demand and prices of intermediate inputs?

9. What would be the net impact of a cane alcohol program on the balance of payments?

10. How would demand for credit and interest rates be affected?

11. What would be the environmental impact of massive production and use of alcohol?

12. What, finally, would be the distributional effects?

The focus of this study is to document how the answers to policy questions like those outlined above can substantially vary when labor-machinery substitution possibilities are allowed for in the analysis. The remaining part of this chapter is devoted to explaining the technical coefficients used in the implementation of TICO, the model whose economic and mathematical properties were presented in the preceding chapters.

The *Pacífico Seco* was chosen as the region to build the model described in the previous section for three main reasons: first, because the relative importance of the region as part of the agricultural sector is such that a disaggregate model at this level provides a very good representation of the impact of an Alcohol Fuel Program (AFP) on key variables such as food production, food prices and trade, income distribution among different groups of farmers, employment, etc. Second, the government considers this region as the most promising one for the implementation of an AFP; actually, the only distillery that has been built in Costa Rica for this purpose is located in the *Pacífico Seco*. Third, when compared to other agricultural regions of Costa Rica, the *Pacífico Seco* offers the greatest potential for mechanization; this fact gives the opportunity to simulate the labor-machinery substitution within a realistic framework.

Even though the AFP was conceived and initiated at the end of the seventies, 1973 was chosen as the model's base period on the basis of

data availability. Census data are available for that year and other relevant information such as technology set, existing empirical estimates of price demand elasticities, etc. could be properly combined with census information.

Overview of the *Pacífico Seco* Region in Costa Rica

The *Pacífico Seco* Region is located in the Northwestern part of Costa Rica. It shares boundaries with Nicaragua in the North, with other Costa Rican agricultural regions in the East and the South, and with the Pacific Ocean in the West (see Figure 3).

Its total area is approximately 1.1 million hectares, which represent 35.2 percent of the agricultural land of Costa Rica. From a physiographical point of view, the *Pacífico Seco* is very diverse. First, it has a continental zone formed by a ridge of mountains known as Guanacaste and Tilarán sierras with an average altitude of 1,000 meters above sea level. Then, extending towards the ocean is the Tempisque Valley, a depression averaging 30 meters of altitude. Third, lying on the shore there are two peninsular areas known as the Santa Elena and the Nicoya Peninsulas with an average altitude of 300 meters above sea level. The fertility of the soils varies widely within the region; nonetheless, a study by the Instituto de Fomento y Asesoria Municipal (IFAM 1976) estimated that half of the total area could be used for agricultural purposes, but that only 10 percent have a high potential to increase productivity via irrigation and other modern technologies. Most of these soils with high potential are still

exploited under extensive agricultural systems. As shown in Table 7, more than two-thirds of the land are grasslands and only 9 percent is under cultivation; the remaining area is occupied by forests and others.

Pacífico Seco is the driest region in Costa Rica. The average annual rainfall varies from 1,500 millimeters to 4,000 millimeters; however, approximately 88 percent of the rains concentrate during the months of May through October. The Tempisque Valley normally experiences a drastic drop in rainfall during July-August, making cultivation particularly risky in that area. On the other hand, temperatures vary from 16°C in the higher zones to 27°C in areas below 150 meters of altitude. The prevailing ecosystem is typical of a dry tropical zone.

The region's infrastructure is in general very well developed. Access to the region is possible using the Pan-American Highway, a paved road that runs from south to north. A secondary system of gravel and paved roads that connect with this highway facilitates the access to all agricultural areas within the region, and from these to the market centers. Of particular relevance is the fact that producers have access to three big ports on the Pacific Ocean: Punta Morales, Puntarenas, and Caldera. The first one is a specialized port for sugar and ethanol exports. Agro-processing infrastructure includes rice millers, grain storages, sugarcane millers, fruits and vegetable packing, etc. One of the sugarcane millers, CATSA, has an annex distillery with a total production capacity of 36 million liters of ethanol over a period of 150 harvesting days. Other infrastructure available includes 50 airports for small planes and one airport for medium to large-sized

planes. It is also worth mentioning that in the Tempisque Valley, there is an irrigation project under way. It will use water resources from the Arenal Hydroelectric Project, and it is expected to irrigate a total of 58,000 hectares.

Marketing of agricultural products is performed in its majority by existing public and private institutions at national level. The Consejo Nacional de Producción (CNP), which is the price stabilization institution, intervenes mainly in the basic grains market and to a lesser extent in the beef market. CNP sets the guarantee prices for rice, maize, sorghum, and beans and it has a network of depots in the region to collect harvested products. The totality of the marketing operation of sugarcane is performed by the Liga Agrícola e Industrial de la Caña (LAICA). The other two major products, beef and milk, are marketed by the Cooperativa Nacional de Montecillos and the Cooperativa de Leche Dos Pinos, respectively. For marketing of other crops such as fruits and vegetables, there exists several permanent and periodical markets scattered all over the region.

Other services such as schools, hospitals, agricultural research stations, banks, telephone and telegraph communications, etc. are extensively available all over the region.

As mentioned before, the *Pacifico Seco* region is predominantly agricultural. This is also reflected in the fact that 71 percent of its population was classified as rural in the 1973 census. The region had about 260,368 inhabitants, one-third of which was at working wage. Two-thirds of these workers were wage earners and only one-third were independent workers. Even though the employment rate in 1973 was high,

intercensal comparisons show a negative migration balance. This phenomenon has been explained by IFAM (1976) as the result of the rapid development of low labor-intensive ranching activities. It is also known that during the harvesting season, farmers employ migrant workers who come from other regions of Costa Rica and even from Nicaragua. No statistics exist to account for seasonal migration, therefore, it was not possible to adjust labor availability accordingly.

Groups of Farmers in the Model

TICO allows for disaggregation at farm group level. Three groups of farmers are distinguished and their main characteristics are described in this section. The main feature that differentiates the group of farmers is the farm size: small, less than 10 hectares; medium, 10 to less than 200 hectares; and large, greater than 200 hectares. To a large extent, this classification was arbitrary, but some attempts were made to reflect well-known characteristics typical of each group. Small farmers devote most of the land (46.5 percent) to cropping activities, while medium-scale farmers and large-scale farmers devote only 12.7 percent and 6.2 percent, respectively, for that purpose. Those farmers above 10 hectares assign the higher proportion of their holdings to grassland (see Table 7). On the other hand, the small farmers group has a very low participation in the total area planted to annual and perennial crops in the region, approximately 9 percent in each case (see Table 8).

Table 7--*Pacífico Seco*: Land use in chosen farm size categories

Farm size (hectares)	Area in thousand hectares					
	Annual crops	Perennial crops	Grassland	Forests	Others	Total
< 10	7.9 (41.2)	1.0 (5.3)	8.2 (43.4)	0.2 (1.1)	1.7 (9.0)	19.0
10 to < 200	41.6 (11.6)	4.0 (1.1)	253.9 (70.7)	24.0 (6.7)	35.5 (9.9)	359.0
≥ 200	38.1 (5.3)	6.4 (0.9)	474.1 (65.7)	111.4 (15.4)	91.3 (12.7)	721.3
Regional total	87.6 (8.0)	11.4 (1.0)	736.2 (67.0)	135.6 (12.3)	128.5 (11.7)	1,099.3

Source: Dirección General de Estadística y Censos, Censos nacionales de 1973 agropecuario regiones agrícolas (San José, Costa Rica: DGEC, Sección de Publicaciones, 1975).

Note: Figures in parentheses are percentages with respect to total land in each farm size category.

Table 8--*Pacífico Seco*: Land use distribution among chosen farm size categories

Farm size (hectares)	Annual crops	Perennial crops	Grassland	Forests	Others	Total
			(percent)			
< 10	9.0	8.8	1.1	0.1	1.3	1.7
10 to < 200	47.5	35.1	34.5	17.7	27.6	32.7
≥ 200	43.5	56.1	64.4	82.2	71.1	65.6
Regional total	100.0	100.0	100.0	100.0	100.0	100.0

Source: Table 7.

The prevailing land tenure system across groups is that of ownership. It must be noticed, however, that rental of land is a common practice in the region, particularly among small farmers (see Table 9). Rental is observed more frequently for livestock activities. Accordingly, TICO includes activities that account for "transfers" of grassland among groups.

As Table 10 shows, the three groups have different patterns of cultivation. The main difference among them is the degree of mechanization. The group of small farmers utilize a total of 13 non-mechanized technologies and only 5 mechanized technologies. The medium-scale farmers utilize about the same number of mechanized and non-mechanized technologies--13 and 14, respectively--whereas the large-scale farmers utilize mostly mechanized technologies. Notice also that the small group is not allowed for cultivation of sorghum, cotton, and sugarcane. In brief, the classification adopted distinguishes two extreme cases: low mechanization among small farmers and high degree of mechanization among large-scale farmers, and one intermediate case--between these two extremes, the medium-scale farmers.

Additional distinctions between groups are captured in the model via access to labor markets, risk-aversion behavior, and of course by different resource endowments. The interrelations between groups in the labor market were illustrated in Table 5. The basic assumption to differentiate the perception towards risk in production shown by each group of farmers is that farmers are risk averse and that the degree of aversion diminishes with the size of the farm. The coefficients Φ_g in the objective function, equation (66), carry this assumption. Several

Table 9--*Pacífico Seco*: Land tenure systems in chosen farm size categories

Farm size (hectares)	Area in thousand hectares			
	Owned	Rented	Other tenure	Total
< 10	12.9 (67.9)	2.0 (10.5)	4.1 (21.6)	19.0
10 to < 200	314.8 (87.7)	5.7 (1.6)	38.5 (10.7)	359.0
≥ 200	654.2 (90.7)	10.2 (1.4)	56.9 (7.9)	721.3
Regional total	981.9 (89.3)	17.9 (1.6)	99.5 (9.1)	1,099.3

Source: Dirección General de Estadística y Censos, op cit.

Note: Figures in parentheses are percentages with respect to total land in each farm size category.

Table 10--*Pacífico Seco*: Number of technologies adopted by each group of farmers

Crop and technology	Farm size (hectares) < 10	10 to < 200	≥ 200
Rice			
Non-mechanized	1	1	0
Mechanized	0	4	4
Maize			
Non-mechanized	2	2	0
Mechanized	3	3	3
Sorghum			
Non-mechanized	0	0	0
Mechanized	0	3	3
Beans			
Non-mechanized	8	8	0
Mechanized	2	2	2
Cotton			
Non-mechanized	0	0	0
Mechanized	0	1	1
Sugarcane			
Non-mechanized	0	0	0
Mechanized	0	1	1
Plantain			
Non-mechanized	1	1	1
Mechanized	0	0	0
Oranges			
Non-mechanized	1	1	1
Mechanized	0	0	0
Total			
Non-mechanized	13	13	2
Mechanized	5	14	14

Source: Table 13.

solutions of TICO were obtained changing the values of Φ_g to choose those that yield a solution closest to the actual situation observed in the base year. The final values obtained were $\Phi_1 = 1.5$, $\Phi_2 = 1.0$, and $\Phi_3 = 0.5$. Table 11 shows that the retentions in the two smaller groups are considerably higher as compared to the largest farmers. As one should expect, however, retentions of basic grains are higher than those of other products. Finally, the resource endowments of each group are shown in Table 12. Several remarks must be done from that table. First, the input prices shown in the first column are averages and they represent the explicit cost. The model assigns the same explicit cost to the three groups of farmers, however, the implicit cost which takes into account the restrictions faced by each group as well as the production technologies adopted by each one, will vary substantially among groups. Second, the number of animals per hectare goes from 4.76 for the small farmers to 1.38 for the medium-scale farmers and to 0.88 for the large-scale farmers. This is an indication of intensity in the use of land and, more importantly, it already suggests that expansion of croplands is more likely to occur first in the larger farms. The actual results will be discussed in the next chapter.

The Technology Set

Reliable information on costs of production is very scarce in Costa Rica. In order to obtain the technical coefficients to build a

Table 11--*Pacifico Seco*: Crop production retained at farm level for consumption

Crop	Farm size (hectares) < 10	10 to < 200	≥ 200
	(metric tons)		
Rice	575.4 (24)	992.1 (10)	189.5 (7)
Maize	2,019.0 (60)	3,342.7 (61)	260.5 (9)
Beans	332.3 (54)	885.8 (41)	53.3 (36)
Sorghum	0.0 (0)	32.9 (8)	245.9 (6)
Sugarcane	242.0 (5)	888.0 (2)	49.0 (.01)
Plantain	154.0 (7)	293.5 (9)	47.9 (4)
Oranges	134.0 (33)	258.0 (25)	25.7 (12)

Source: Dirección General de Estadística y Censos, op cit.

Note: Figures in parentheses indicate the proportion relative to total production in each farm size category.

Table 12--Average prices and availability of resources

Resource	Unit	Price	Availability by farm size: < 10	10 to < 200	≥ 200
		($)			
Crop land	Hectare	*	8,684	45,561	44,571
Grass land	Hectare	*	8,194	253,875	473,862
Family labor	Man-day	0.87	264,572	311,542	0
Landless labor**	Man-day	1.74	...	...	779,216
Machinery	Tractor-hour	5.81	50,240	88,480	114,720
Fertilizer (nitrogen)	Kilogram	0.10	§	§	§
Fertilizer (NPK)	Kilogram	0.10	§	§	§
Credit	Million $	8% and 10%	1.08	11.8	23.9
Animals for beef					
Cows	Number	*	16,925	132,689	143,426
Female calves	Number	*	11,263	95,233	85,658
Male calves	Number	*	5,623	69,851	92,940
Oxen and bulls	Number	*	2,678	35,487	90,465
Animals for dairy					
Cows	Number	*	291	1,996	83
Female calves	Number	*	246	1,340	65
Male calves	Number	*	107	1,022	60
Oxen and bulls	Number	*	51	519	58
Animals for beef and dairy					
Cows	Number	*	753	4,534	871
Female calves	Number	*	646	3,993	793
Male calves	Number	*	280	2,612	675
Oxen and bulls	Number	*	133	1,327	657

Sources: Dirección General de Estadística y Censos, op. cit.; Ministry of Agriculture, Statistical bulletins (San José, Costa Rica, 1973-1980); Oficina de Planificación Sectorial Agropecuaria, Studies of agricultural credit (San José, Costa Rica, 1978).

* No explicit price is assigned in the model.

** This amount of landless labor is available to all three groups of farmers.

§ Not directly constrained in the model.

set of production vectors, partial studies for the various crops were used and then adjusted on the basis of discussions with agronomists in the agricultural research stations located in *Pacifico Seco* and further complemented with discussions with farmers all over the region. The result is the set of technological alternatives included in Table 13. These production options incorporate different times of planting and harvesting, different levels and combinations of input use, and different yields per hectare. An effort was made to put in the model the actual data available, and no attempt was made to systematize the input-output relations due to lack of empirical evidence to do so. Therefore, the chosen technologies represent discrete production possibilities that may or may not lie on the same continuous production functions. As it is well known, however, these vectors put within the context of a linear programming model which simultaneously incorporates other relevant economic variables, generates supply functions that represent a continuous and non-linear aggregate production function.

The variation between some of the technologies is strikingly high, but it was retained after confirmation of its validity on the basis of the responses from both the agronomists and the farmers. The variation is better illustrated in the total use of labor, which in turn is disaggregated on a monthly basis in Table 14. When these production vectors are assigned to the three groups of farmers, a total of 61 alternatives result (see Table 10).

An integral part of the set of production possibilities in TICO is the revenue deviations associated with each alternative technology. These deviations, as explained in Chapter II, are the Mean Absolute

Table 13--Pacífico Seco: Resource requirements and yields per hectare for alternative crop technologies

Crop and technology	Season	Labor	Machinery	Nitrogen	NPK	Others	Yield
		(man-days)	(hours)	(kg)	(kg)	($)	(kg/ha)
Rice							
I	May-Dec	23	...	46	46	23.72	1,150
II	Jun-Dec	5	6	92	184	133.37	2,300
III	Jul-Dec	10	4	388	...	76.29	1,966
IV	May-Oct	5	7.5	295	...	97.24	1,757
V	Jul-Oct	25	4	230	...	49.07	1,647
Maize							
I	Apr-Aug	29	...	...	92	25.58	1,300
II	Jul-Dec	27	...	...	92	25.58	1,200
III	Mar-Sep	18	7	92	92	22.44	2,100
IV	Jul-Dec	18	7	92	92	22.44	1,800
V	Apr-Sep	34	6	111	111	20.83	2,434
Sorghum							
I	Sep-Dec	10	3	92	92	34.30	1,840
II	Oct-Jan	5	5	75	75	51.70	2,295
III	Aug-Dec	7	7.5	75	125	35.00	2,145
Beans							
I	May-Sep	20	...	...	...	34.30	575
II	Sep-Jan	20	...	...	...	34.30	520
III	Aug-Nov	31	...	...	...	20.30	424
IV	Sep-Dec	19	...	...	...	34.91	340
V	Apr-Sep	24	10	300	...	50.81	920
VI	Aug-Dec	30	10	300	...	50.81	880
VII	May-Sep	31	...	300	...	45.35	690
VIII	Sep-Dec	53	...	92	123	68.05	800
IX	Sep-Dec	31	...	300	...	45.35	700
X	Sep-Dec	59	...	91	130	63.86	854
Cotton							
I	Jul-Feb	40.5	5	200	140	56.23	1,000
Sugarcane							
I	Jan-Dec	101	8	750	...	31.63	70,000
Plantain							
I	Jan-Dec	90	...	140	230	23.25	15,600
Oranges							
I	Jan-Dec	98	...	630	645	55.93	31,450

Sources: Ministry of Agriculture, Technical reports (San José, Costa Rica, 1973-1980); Banco Central de Costa Rica, Credit provisions (San José, Costa Rica, 1973-1988); Banco de Crédito Agrícola de Cartago, Costs handbook 1973 (San José, Costa Rica, 1973); Consejo Nacional de Producción, Statistical bulletins (San José, Costa Rica, 1973-1988).

Table 14--Labor use seasonality by technology

Crop and technology	Number of man-days per hectare per month												
	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec	Total
Rice													
I					6.0	2.0	4.0	3.0	1.0	3.5	3.5		23.0
II						5.0							5.0
III							3.0	5.0	2.0				10.0
IV					2.0		3.0						5.0
V								14.0	11.0				25.0
Maize													
I				4.0	10.0	3.0		12.0					29.0
II							4.0	10.0	3.0		6.0	4.0	27.0
III					6.0	1.0	1.0	6.0	4.0				18.0
IV								6.0	1.0	1.0	6.0	4.0	18.0
V				5.0	3.0	3.0	3.0	8.0	10.0				32.0
Sorghum													
I									2.0	3.0	3.0	2.0	10.0
II										3.0	2.0		5.0
III								3.0	2.5	1.5			7.0
Beans													
I	5.0				2.0	5.0		8.0	5.0				20.0
II									2.0	5.0	8.0		20.0
III								7.0	6.0	5.0	13.0		31.0
IV									1.0			18.0	19.0
V							8.0	12.0	4.0				24.0
VI										8.0	18.0	4.0	30.0
VII					5.0	8.[illegible]	2.0	10.0	6.0				31.0
VIII									20.0	11.0	9.0	13.0	53.0
IX									5.0	8.0	6.0	12.0	31.0
X									22.0	9.0	13.0	15.0	59.0
Cotton													
I	12.0	8.0					6.0	3.0	4.0	2.0	3.5	2.0	40.5
Sugarcane													
I	14.0	18.0	14.0	15.0	23.0		5.0		2.0		10.0		101.0
Plantain													
I	15.0	3.0	3.0	6.0	3.0	3.0	9.0	20.0	6.0	6.0	9.0	7.0	90.0
Oranges													
I	12.0	12.0	12.0	12.0	10.0	6.0	6.0	5.0	5.0	5.0	5.0	8.0	98.0

Sources: Own estimates based on field work and the agricultural calendar published by the magazine Agroindustrias (San José, Costa Rica, 1973-1978).

Deviations (MAD) used to get estimates of the standard deviation of farm incomes. The calculation of deviations was performed as follows: the basic information was the 10-year series of average yields and prices presented in Table 15. To obtain a similar series for each technology, the yields were adjusted applying the proportional relationship between the yield shown in Table 15 for 1973 and the yield associated to each technology; then, multiplying the adjusted yields by the corresponding prices, a series of revenues per hectare was obtained, and further, the deviations from the sample means were calculated to obtain the MAD.

Livestock production has four major possibilities. Breeding, which consists of the production of male and female calves that are sold at age of six months. Fattening, performed by ranchers who buy six-month-old male calves from breeders and raised them up to the age of two years when the animals are sold for beef production. Breeding and fattening, which is a combination of the above two possibilities, performed within the same ranch. In this case, only a small proportion of calves is sold in the market and most of them are retained for fattening. Dairy, performed with the main purpose of producing milk. Ranchers involved in this activity frequently buy female calves from breeders.

The above given description of livestock activities is necessarily very simplistic. Cattle production can take not only numerous modalities, but also requires a complex process of capital investment that could be better captured in the context of a dynamic or multi-period optimization model. However, given its economic importance in the

Table 15--Producer prices per ton and average yields per hectare used to estimate MAD

Crop		1964	1965	1966	1967	1968	1969	1970	1971	1972	1973
		(kilograms and dollars)									
Rice	Yield	1,356	1,408	1,461	1,477	1,500	1,501	1,130	1,440	1,130	1,630
	Price	134	91	79	83	82	75	86	74	85	97
Maize	Yield	1,072	1,052	1,053	1,052	1,053	1,052	1,410	1,470	1,520	1,430
	Price	52	57	57	58	60	55	70	70	75	137
Beans	Yield	298	271	250	230	304	300	250	450	690	660
	Price	108	122	105	134	149	139	175	173	154	412
Sorghum	Yield	1,632	1,646	1,633	1,655	1,643	1,643	2,630	2,630	1,970	1,762
	Price	45	48	51	48	47	41	50	50	46	101
Cotton	Yield	1,720	1,960	1,468	1,212	1,188	1,176	500	500	1,000	500
	Price	419	381	379	430	379	428	414	414	588	903
Sugarcane	Yield*	46.04	34.06	47.85	48.45	48.95	49.38	49.50	44.92	47.55	58.04
	Price	5.58	5.36	5.20	5.32	5.81	5.80	5.75	5.59	6.14	6.53
Plantain	Yield	6,000	6,439	7,035	7,483	8,983	9,914	10,018	11,090	12,050	12,940
	Price	79	64	63	65	66	66	67	68	77	88

Sources: Banco Central de Costa Rica, National accounts (San José, Costa Rica, 1964-1973); Ministry of Agriculture, Statistical bulletins (San José, Costa Rica, 1973-1980); Consejo Nacional de Producción, Statistical bulletins (San José, Costa Rica, 1973-1988).

* in metric tons.

region, omitting livestock production in TICO could lead to erroneous interpretations of the actual process of resource allocation and, therefore, to miscalculation of their true opportunity cost. To be consistent with this reality, a static approximation to the above mentioned possibilities was included in TICO. One technology for each possibility was chosen and the relevant technical coefficients are presented in Table 16.

Notice in that table that by convention, the cow was chosen as the production unit to model breeding, breeding and fattening, and dairy activities, while the male calf was used for the fattening activity. This implies, for instance, that for each cow in the typical breeding herd, there are 0.67 female calves, 0.33 male calves, and 0.15 oxen an bulls. The requirements of grassland, labor, and other inputs are those necessary to sustain these animals altogether, and yields represent the average physical output obtained from the operation per year.

Output Processing Activities

After output is collected at the farm level, it still requires additional transformation before it is ready for consumption. This transformation process varies widely among products: in one extreme, we have the case of beans, plantains, and oranges that involves only transportation to the marketplace; in the other extreme, we have rice, sugarcane, and cotton that, in addition to transportation, require a series of industrial and marketing activities. In both cases, the amount actually delivered to the market differs from the amount finally

Table 16--*Pacífico Seco*: Technologies for livestock production

Resources	Beef			Dairy
	Breeding	Fattening	Breeding and fattening	
Animals				
Cows	1.00	...	1.00	1.00
Female calves	0.67	...	0.86	0.67
Male calves	0.33	1.00	0.37	0.51
Oxen and bulls	0.15	...	0.18	0.26
Grassland (hectares)	1.94	0.68	1.91	1.62
Labor	5.52	2.16	7.08	4.56
Other (dollars)	32.59	25.25	46.66	115.20
Yield (kilograms)	270.10	220.88	322.35	1,427.00

Sources: Alain Vigne, Costos de producción de ganado de carne en Costa Rica (San José, Costa Rica: MAG, 1974); Ministry of Agriculture, Technical reports (San José, Costa Rica, 1973-1980).

available for consumption. The major sources of discrepancy are the waste that normally occurs due to loss of weight and product damages, on the one hand, and the losses resulting from the industrial processes, on the other hand.

Table 17 shows both the waste rate and the milling extraction rate for the crops included in TICO.[5] Taking rice as an example, these two rates combined means that for every 100 kilograms of paddy rice produced at the farm level, 97 kilograms reach the mill and at the end of the industrial process, 60.63 kilograms of husked rice are obtained. Notice that cotton gives rise simultaneously to three different final products: lint, oil, and cake. Notice also that sugarcane can be submitted to three different processes: in the first one, sugar and molasses are the main outputs; in the second case, only brown sugar is produced; and in the third case, only alcohol is produced.[6] The "net coefficient" column contains the values of parameters t_{sg} of equation (4.5).

Domestic and International Output Markets

There are three components that complete the output market representation in TICO: the supply from other regions, the domestic demand, and the international trade. The output supply from other regions is

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[5] Beef and milk are not included in the table because yields at the farm level are already expressed in final product equivalents.

[6] In the three processes, other byproducts. such as bagasse, are obtained; however, they are not included in the table because they do not have any commercial value.

Table 17--Coefficients of transformation of raw products into final products

Raw product	Final product	Waste rate	Milling extraction	Net coefficient
			(percent)	
Paddy rice	Husked rice	3.0	62.5	60.63
Corn	Corn	5.0	100.0	95.00
Sorghum	Sorghum	5.0	100.0	95.00
Beans	Beans	4.0	100.0	96.00
Raw cotton	Lint	n.a.	35.0	35.00
	Oil	...	8.1	8.10
	Cake	...	25.9	25.90
Sugarcane - 1	Sugar	1.0	9.3	9.20
	Molasses	...	3.0	2.97
Sugarcane - 2	Brown sugar	...	11.5	11.39
Sugarcane - 3	Hydrate alcohol	1.0	6.5	6.44
Plantain	Plantain	15.0	100.0	85.00
Oranges	Oranges	15.09	100.0	85.00

Sources: Consejo Nacional de Producción, Statistical bulletins (San José, Costa Rica, 1973-1988); GAFICA, Perspectivas para el desarrollo y la integración de la agricultura Centroamericana (Guatemala, 1974).

entered at a fixed level. The quantities for each product are shown in Table 18, and they correspond to those observed in 1973. Imports and exports are entered as upper bounds, these being the levels observed in 1973. Table 19 shows the quantities and prices of imports and exports. As explained before, the domestic demands are entered in the form of linearized functions. The procedure applied to obtain the corresponding technical coefficients is explained in the following paragraphs.

Two major assumptions underlie the endogenization of prices in TICO: first, that product demands are independent, hence cross-price elasticities are equal to zero; second, that the demand functions are linear and consequently, the inverse demand function can be written:

$$P_s = a_s - b_s Q_s \tag{78}$$

The starting parameters needed for each demand function are the own-price elasticity of demand (η_s), the initial price (P_{so}), and the initial quantity (Q_{so}). This basic information is presented in Table 20.

Using the formula for the elasticity, we can obtain b_s:

$$b_s = \frac{dP_s}{dQ_s} = - \frac{P_{so}}{\eta_s Q_{so}} > 0 \tag{79}$$

Substituting (79) into (78), we get:

$$a_s = P_{so} + b_s Q_{so} > 0 \tag{80}$$

The relevant range of the demand function was chosen to be $(P_s^l, P_s^u) = (0, 2P_{so})$ which in turn is translated to the quantity axis as:

Table 18--Product supplies from other regions in 1973

Product	Quantity
	(metric tons)
Husked rice	39,394
Corn	38,674
Sorghum	3,076
Beans	7,765
Cotton	0
Vegetable oil	0
Cotton cake	0
Sugar	140,414*
Molasses	45,329
Brown sugar	25,976*
Plantain	64,823
Oranges	6,453
Beef	64,025
Milk	246,861

Sources: Dirección General de Estadística y Censos, op cit.; Banco Central de Costa Rica, National accounts (San José, Costa Rica, 1973).

* According to information from Liga Agricola Industrial de la Caña (LAICA), in 1973, 8 percent of sugarcane production was transformed into sugar and 13 percent into brown sugar.

Table 19--Imports and exports of products in 1973

Product	Imports		Exports	
	Quantity	Price	Quantity	Price
	(metric tons)	(dollars)	(metric tons)	(dollars)
Rice	349	269.8	149	251.2
Corn	43,164	75.6	1,536	96.5
Sorghum	504	305.8	...	...
Beans	5,936	215.1	106	101.2
Lint	1,263	538.0	123	476.3
Cotton oil	6,683	359.3	150	291.9
Palm oil	262	794.2	418	280.2
Corn oil	4	595.4	...	...
Cotton cake	28,136	122.1	...	...
Sugar	...	...	113,169	146.5
Molasses	0.6	511.6	...	...
Beef	...	...	n.a.	1,191.9

Source: Dirección General de Estadística y Censos, Anuario de comercio exterior (San José, Costa Rica, 1973).

Table 20--Price elasticities, per capita consumption, and consumer prices of selected commodities

Commodity	Elasticity	Per capita consumption	Price
		(kilogram)	($/ton)
Rice	-0.38	51.2	226.7
Corn	-0.17	55.0	160.0
Beans	-0.30	22.0	360.5
Sorghum	-0.38	37,450.0*	98.1
Lint	n.a.	n.a.	904.4
Vegetable oil	-0.37	27.1	824.4
Cotton cake	n.a.	n.a.	134.2
Sugar	-0.30	52.8	126.3
Brown sugar	-0.10	4.1	175.4
Molasses	n.a.	n.a.	19.8
Plantain	-0.24	50.9	85.7
Oranges	-0.58	63.8	36.4
Beef	-0.20	27.7	860.0
Milk	-0.45	104.3	154.7

Sources: Dirección General de Estadística y Censos, Consumer price index (San José, Costa Rica, 1973); LAICA, Statistical bulletins (San José, Costa Rica, 1973); Miguel Gómez and Carlos Quintana, Estimaciones del consumo de granos básicos (San José, Costa Rica, 1978), mimeo.

* Total consumed by feedstock industries.

$$Q_s^l = \frac{a_s - P_s^u}{b_s} \tag{81}$$

$$Q_s^u = \frac{a_s}{b_s} \tag{82}$$

Fifteen was chosen as the number of segments for each demand function. The length of each segment was then obtained as:

$$K_s = \frac{Q_s^u - Q_s^l}{h - 1} \tag{83}$$

where h is the number of segments.

The quantities at each point on the function are:

$$\begin{aligned} q_{s0} &= Q_s^l \\ q_{s1} &= Q_s^l + K_s \\ q_{s2} &= Q_s^l + 2K_s \\ &\vdots \\ q_{sh} &= Q_s^l + hK_s = Q_s^u \end{aligned} \tag{84}$$

Finally, the values of W_{sh}, calculated on the basis of the foregoing information, are:

$$W_{sh} = a_s q_{sh} - \tfrac{1}{2} b_s q_{sh}^2 \tag{85}$$

Although not shown in Tables 3 and 4, TICO also includes equations to account for gross revenues. Those are obtained as:

$$R_{sh} = a_s q_{sh} - b_s q_{sh}^2 \tag{86}$$

Labor-Machinery Isoquant Coefficients

To generate the coefficients that account for labor-machinery substitution, the algorithm developed in Chapter III was applied.[7] The values of the elasticity of substitution were set first at very low levels and then increased to observe the impact of substitutability on alcohol production. The values of elasticity reported correspond to those at which the increase in production of alcohol was noticeable. This procedure is arbitrary and in consequence the quantitative results must be taken with caution. Nevertheless, the direction of the relationships that are discussed in Chapter VI can be regarded as valid. Additionally, in this particular case, large increases in the value of the elasticity had to occur before a noticeable reaction of alcohol output took place. This might be taken as indicative of good stability of the simulation. Numerical values of isoquants so generated are shown in Tables 21 and 22 and then plotted in Figure 4.

The power equivalent of the labor-machinery bundle is obtained by multiplying the labor quantity by unity and the machinery quantity by 8, i.e., the coefficients α_1 and α_2 introduced in equation (71). These values are arbitrary and, of course, one can experiment with different values to see the sensitivity of the model solution to different labor to power and machinery to power relationships. Ideally, one would like to have empirical evidence on the substitutability between labor and

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[7] Constant elasticity of substitution (CES) isoquants are generated here because there is no empirical or theoretical basis to indicate how the elasticity of substitution would change along the isoquant.

Table 21--Labor-machinery substitution in small-sized farms

(elasticity of substitution = 0.2)

Labor	Machinery	Power	Total Cost
291.4	2.6	311.9	656.3
277.5	2.7	299.2	640.3
264.3	2.9	287.6	628.9
251.7	3.2	277.5	625.6
239.7	3.8	270.1	637.9
228.3*	5.0*	268.1	686.0
227.4	5.5	271.2	713.5
226.9	6.0	275.0	744.0

Source: Own computations using the new technique to generate isoquants, developed in Chapter III.

Note: Labor is reported in thousands of man-days, machinery in thousands of machinery-hours, power in thousands of units, and cost in thousands of dollars. Total cost is calculated using input prices reported in Table 12.

* This is the observed labor-machinery bundle used as the initial point to generate the isoquant.

Table 22--Labor-machinery substitution in small-sized farms*

(elasticity of substitution = 0.7)

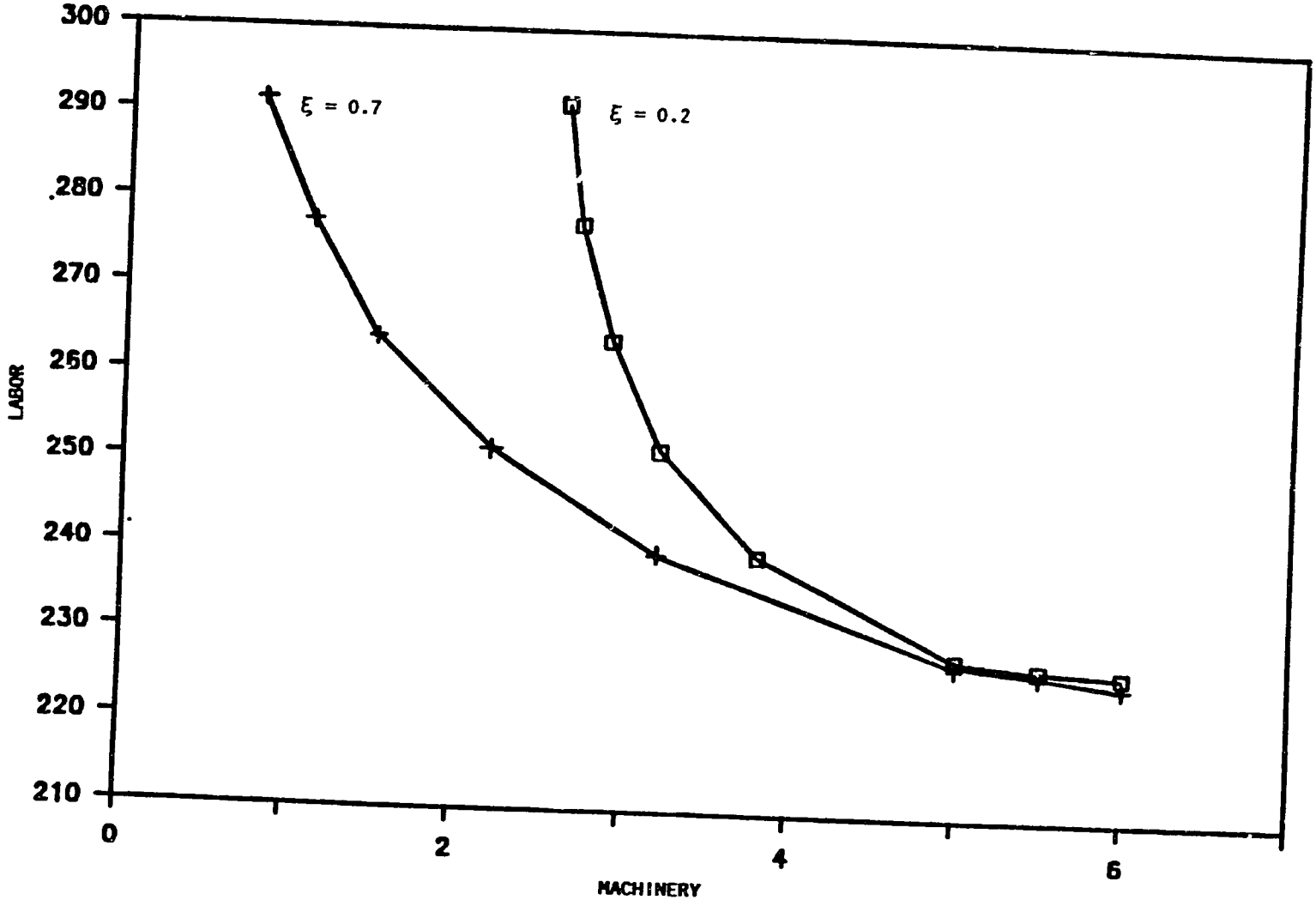
Labor	Machinery	Power	Total Cost
291.4	0.8	297.9	554.6
277.5	1.1	286.3	546.7
264.3	1.5	276.4	547.6
251.7	2.2	268.9	562.9
239.7	3.2	265.2	602.4
228.3*	5.0*	268.1	686.0
226.9	5.5	270.6	712.5
225.5	6.0	273.6	741.6

Source: Own computations using the new technique to generate isoquants, developed in Chapter III.

Note: Labor is reported in thousands of man-days, machinery in thousands of machinery-hours, power in thousands of units, and cost in thousands of dollars. Total cost is calculated using input prices reported in Table 12.

* This is the observed labor-machinery bundle used as the initial point to generate the isoquant.

Figure 4--Labor-machinery substitution in small-sized farms



machinery both for different types of labor and machinery and for different tasks to be performed. Furthermore, this evidence should go one step more ahead in providing information on the variation of substitutability under different soil, weather, and other conditions. This, however, is beyond the scope of this study. In the Book of CHAC (Norton and Solis 1983, p. 191), it was estimated that each tractor of approximately 60HP displaced 10 to 12 jobs.

One interesting feature to notice in Tables 21 and 22 is that when one moves from one point to another on the isoquant, the total power may increase or decrease. This is so because we have assumed that α_1 and α_2 remain constant. Therefore, the change in power is given by:

$$dP = \alpha_1 \Delta L_h + \alpha_2 \Delta M_h \tag{87}$$

and since ΔL_h and ΔM_h are opposite in sign, we have that:

$$dP \gtrless 0 \quad \text{iff} \quad \left| \frac{\Delta L}{\Delta M} \right| \gtrless \left| \frac{\alpha_2}{\alpha_1} \right| \tag{88}$$

A similar situation is observed in the total cost of each labor-machinery combination. In this case, it must be recalled that perfect competition was assumed in the input markets; therefore, the input prices (explicit cost) remain unchanged.

Chapter VI

ANALYSIS OF RESULTS

Introduction to the Numerical Results

This chapter presents the results of using the Costa Rican LP model to evaluate the output response to an scheme of administered prices of alcohol made out of sugarcane, under different scenarios of labor-machinery substitution possibilities for all crops in the cane-growing region. The experiments performed consist of subsequently increasing the price of alcohol using three different versions of the model. The first version is one in which labor-machinery substitution is not allowed for (first column of Table 23). The second version corresponds to a modified model in which labor-machinery substitution is endogenized following the technique presented in Chapter III. In this case, constant elasticities of substitution are assumed for each group of farmers: 0.2 for smallholders, 0.5 for medium-scale farmers, and 1.0 for large-scale farmers. These elasticities represent what could be considered as low levels of substitution possibilities (second column of Table 23). In the third version of the model (third column of Table 23), the values of elasticities of substitution are increased. This implies a higher degree of factor substitutability than that of the second version. As previously indicated, the quantitative results are subject to the caveats originating in the arbitrary choice of

Table 23--Alcohol cane production response to increases in alcohol prices (level of production in millions of metric tons)

Alcohol price	Without factor substitution possibilities	With modest factor substitution possibilities*	With larger factor substitution possibilities**
($/liter)	(1)	(2)	(3)
3.3	0	0	2.3213
3.4	0	0	2.7948
3.5	0	0	3.1014
3.6	0	2.8102	3.1023
4.0	0.6815	3.7176	3.8485
5.0	1.8632	5.0938	5.5281

Source: Simulation results of the model TICO.

* Elasticities of factor substitution = 0.2, 0.5, and 1.0 for the small, medium and large farms, respectively.

** Elasticities of factor substitution = 0.7, 1.5, and 2.0 for the small, medium and large farms, respectively.

elasticities of substitution and the optimality assumption regarding the use of labor and machinery in the generation of isoquants. However, the direction and approximate order of magnitudes of the results can be taken as valid.

It is important to emphasize that the alcohol price increases must be regarded as administered prices set by the government to induce alcohol production. This is a realistic case of modern price interventions and in fact corresponds to the Brazilian and Costa Rican alcohol program experiences. Notice also that the prices are increased until the supply response becomes nil. This is due to the limiting resource endowment specified in the model and the fact that the model is static; therefore, it does not include investment possibilities to enhance the resource base.

In the next section, the more aggregate results and their implications are reviewed, and then the more disaggregated results are presented.

Analysis of the Aggregate Results

Table 23 presents the response of alcohol cane output to increases in government-administered price of alcohol. The first thing to notice is that the minimum required price to induce cane alcohol production is $3.3 per liter, which corresponds to the situation under larger factor substitution possibilities. This price is very high as compared to the price of gasoline of $0.18 per liter, or to the price

of diesel of $0.06 per liter.[8] This is simply a reflection of the opportunity cost of the factors of production used in agriculture, which in turn is determined by the market of agricultural commodities. Furthermore, the high price of alcohol required to induce its production implies that if the government is willing to encourage the substitution of alcohol for gasoline or diesel, it should be prepared to give a large subsidy to consumers. After correcting for the differences in the efficiency of motor vehicle engines when run with each one of the four alternative fuels (gasoline, diesel, gasohol, or hydrate alcohol), it is estimated that the necessary subsidy to induce alcohol consumption needs to be at least $3.05 per liter of anhydrous alcohol, $3.75 per liter of hydrate alcohol in gasoline vehicles, and $4.86 per liter of hydrate alcohol in diesel vehicles.[9] And this is without including the cost of modifying the engines. Thus, the alcohol fuel program appears to be a very costly enterprise for Costa Rica to pursue.

The results presented in Table 23 also indicate that output response to price increases is larger, the larger the possibilities for labor-machinery substitution. They also indicate that when substitution possibilities are limited, the price increments needed to induce a

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[8] These are the average prevailing prices in 1973, which is the base year of the model TICO.

[9] The following technical relationships were used to make this estimate: 1.0 liter of anhydrous alcohol substitutes for 1.4 liters of gasoline (the gasohol option); 1.19 liters of hydrate alcohol substitute for 1.0 liter of gasoline (alcohol-run engine); 1.49 liters of hydrate alcohol substitute for 1.0 liter of diesel (alcohol-run engine). These technical coefficients are reported in the study of the Instituto de Investigaciones en Ciencias Económicas (1981).

given level of output are substantially higher. In other words, the price increase required to induce a given level of alcohol production is in inverse relation to ξ, the elasticity of factor substitution in agriculture in general. The results in Table 23 are also depicted in Figure 5. There, it can be observed that the increased substitutability is reflected in a shift of the response curve downwards and to the right. This result is similar to the introduction of improved technologies, and although *strictu sensu*, a higher value of the elasticity of substitution can be taken as representing a technological change, the above findings have other implications, as will be discussed later in this chapter.

One of the major criticisms of alcohol production programs is that they have enormous social costs in the form of food price increases caused by shortages of food resulting from the displacement of food crop areas by the expanding alcohol-cane. The role of labor-machinery substitution in ameliorating the negative impact of alcohol production is illustrated in Table 24.[10] Evidently, increasing the production of alcohol increases food prices. The social cost of a price increase has two components that work like two sides of the same coin: higher food prices mean simultaneously a loss in consumer surplus and a gain in producer surplus--although the latter is not always the case. The net effect on social cost can be measured through changes in consumer and

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[10] Recall from Table 23 that at an alcohol price of $3.6 per liter, no production is induced when labor-machinery substitution is not allowed for, whereas alcohol cane production increases to 2.8 million metric tons and 3.1 metric tons under modest and larger factor substitution, respectively.

Figure 5--Response of alcohol cane production to increases in alcohol prices

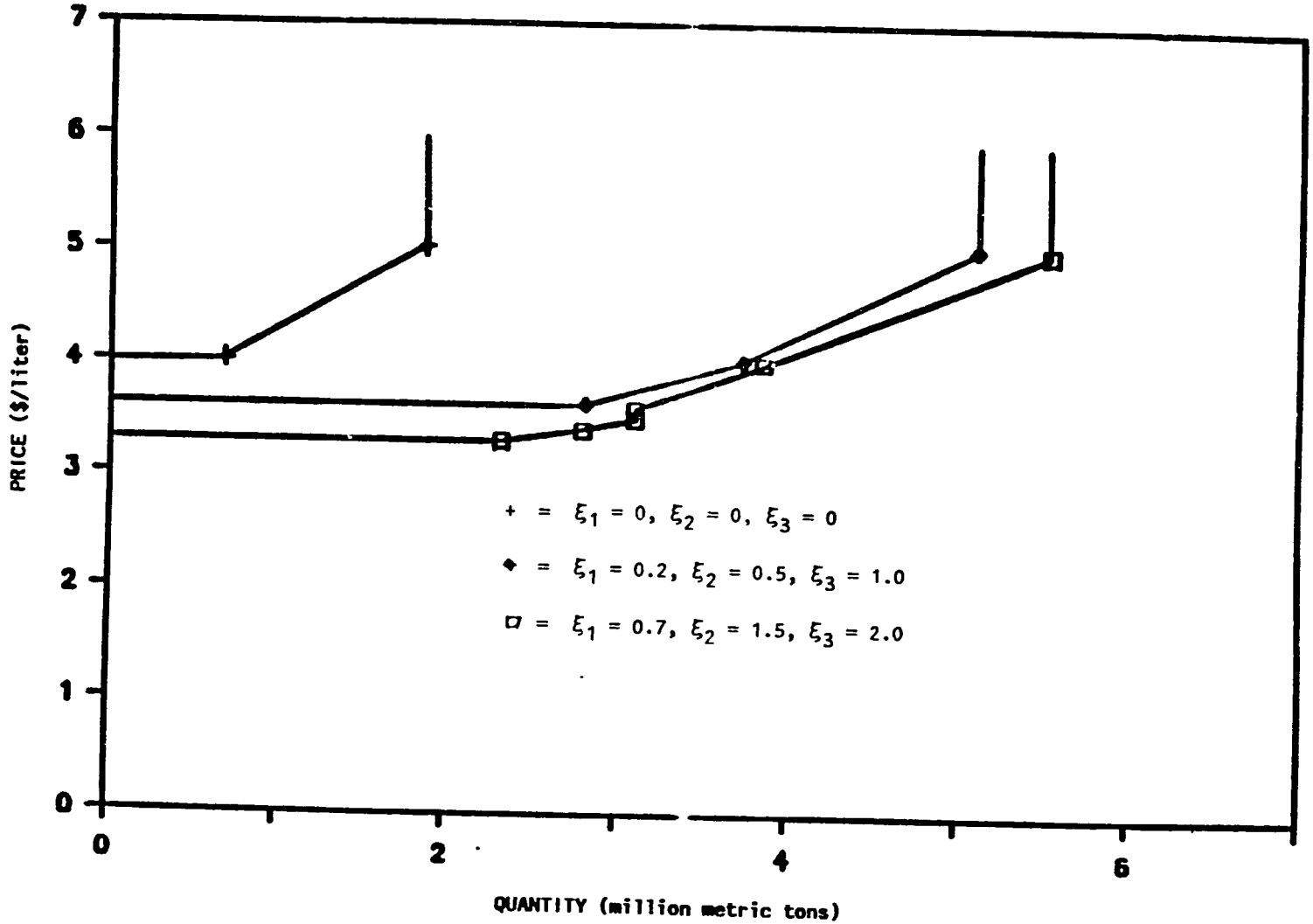

Table 24--Social cost of alcohol cane production under alternative factor substitution possibilities

Item	With no substitution possibilities	With modest factor substitution possibilities*	With larger factor substitution possibilities**
	(1)	(2)	(3)
Alcohol price	3.6	3.6	3.6
Consumer food prices ($/ton)			
Rice	241.6	394.5	333.4
Maize	144.5	160.0	132.8
Beans	386.2	386.2	386.2
Sugar	126.3	146.5	146.5
Plantain	79.6	101.9	91.8
Beef	1,191.9	1,191.9	1,191.9
Consumer surplus (million $)	309.9	291.9	300.2
Producer surplus (million $)	101.3	152.0	175.0
Total surplus (million $)	411.2	443.9	475.2
Total employment (1,000 man-days)	6,223.1	7,574.5	7,574.5

Source: Simulation results of the model TICO.

* Elasticities of factor substitution = 0.2, 0.5, and 1.0 for the small, medium, and large farms, respectively.

** Elasticities of factor substitution = 0.7, 1.5, and 2.0 for the small, medium, and large farms, respectively.

producer surpluses, measured over all agricultural product markets, as shown in Table 24. It must be noted that the values reported in that table are nevertheless partial because they do not include the market for gasoline.

Here again, it is interesting to note that at higher values of elasticity of substitution, the negative effects of an increase in the alcohol price on consumer surpluses are lower, whereas the positive effects on producer surpluses are higher. The positive impact of alcohol production on employment is also enhanced at higher levels of labor-machinery substitution.

Some Implications of the Aggregate Results

Two general conclusions can be drawn from the preceding analysis: first, output response to price changes is larger, the larger the value of elasticity of factor substitution. Second, the social cost of policies of price administration is lower when applied to commodities where the elasticity of factor substitution is higher. From these conclusions, a series of policy implications can be derived:

1. If the policymaker's goal is to increase output of a given product via price management, the effectiveness of a price incentive scheme and its social efficiency are enhanced if a technology package aimed at facilitating factor substitution is simultaneously promoted. This may be, for example, programs to facilitate labor migration, oxen cultivation or other means of mechanization, building roads or other forms of transport facilities, training farmers or their families, etc.

2. If the policymaker's goal is to increase farmers' incomes, price incentives should be aimed at regions, commodities, or particular groups of farmers where the factor substitution possibilities are larger. This means that a single commodity price scheme might not be the most attractive one. Instead, it could be efficiently substituted by a scheme where commodities, regions, and farmers are chosen according to their highest potential to attain the income-generating goal. The additional administrative costs associated with a multiple crop scheme must, of course, be taken into account.

3. In a world in which technological change is strongly oriented towards the introduction of high-yielding varieties, new reconsideration should be given to ways of reaching more efficiently the current levels of production per hectare, because the release of factors relatively scarce could in turn be used to increase total output and to improve the overall socioeconomic situation in the sector.

A methodological corollary of this is that new attention should be given to estimation of elasticities of substitution, or other parameters to measure the potential of factor substitutability.

The Disaggregated Results

Table 25 shows, for selected crops, the response of area planted to higher alcohol prices, under different labor-machinery substitution possibilities. Notice that with a larger value of elasticities of labor-machinery substitution, there tends to be more sugarcane area planted at any price of alcohol. Although this is not a surprising

Table 25--Response of planted areas to higher alcohol prices under different factor substitution possibilities, selected crops

	Alcohol price ($ per liter)			
	3.3	3.6	4.0	5.0
	(1)	(2)	(3)	(4)
No factor substitution				
Rice	25.7	25.7	25.7	25.7
Maize	11.0	11.0	14.9	14.9
Sorghum	0.1	0.1	0.6	0.1
Beans	1.7	1.7	1.7	2.5
Sugarcane	8.2	8.2	9.7	26.6
Plantain	0.4	0.4	0.4	0.4
Pastures	729.2	729.2	729.2	0.0
Modest factor substitution*				
Rice	25.9	5.8	1.0	1.0
Maize	18.5	4.1	4.1	4.1
Sorghum	0.2	0.2	0.1	0.1
Beans	1.7	1.7	2.0	2.0
Sugarcane	2.9	40.2	53.1	72.8
Plantain	0.5	0.2	0.0	0.0
Pastures	729.2	727.4	678.0	0.0
Large factor substitution**				
Rice	20.3	12.2	1.0	1.0
Maize	20.5	20.5	4.1	4.1
Sorghum	0.1	0.1	0.1	0.1
Beans	2.5	2.5	2.0	2.0
Sugarcane	36.1	44.4	55.0	79.0
Plantain	0.5	0.4	0.0	0.0
Pastures	729.2	678.0	663.1	0.0

Source: Simulation results of the model TICO.

* Elasticities of factor substitution = 0.2, 0.5, and 1.0 for the small, medium, and large farms, respectively.

** Elasticities of factor substitution = 0.7, 1.5, and 2.0 for the small, medium, and large farms, respectively.

result, the response of sugarcane area shows two distinctive features: it is more continuous and is larger in acreage. Changes in area planted to other crops, such as rice, plantain, and pastures, are also smoother. And this occurs despite the fact that labor is fully occupied at its annual maximum of 7,574.6 thousands man-days (see Tables 24 and 26). An important implication of these effects is that if labor migration were allowed for in the model, the response of alcohol production and the response of sugarcane area planted would probably be even much smoother. This is an interesting consequence in that it shows that a higher degree of mechanization in agriculture in general could lead to higher labor occupation, also that the adjustment process is by no means as abrupt as one would have expected.

Another interesting result from Table 25 is that including factor substitution possibilities can lead to a reversal of the predictions about crop complementarity and crop substitution on the supply side. Notice that when factor substitution is not allowed for, area expansion of sugarcane is accompanied by an expansion of the areas of maize and beans due mostly to a switch to non-mechanized technologies in these two crops. In contrast, when labor-machinery substitution possibilities are enhanced, the area planted to maize and beans actually declines. Nevertheless, these effects underline the perverse effects that an agricultural program, such as the cane alcohol program, sometimes can have on production technology. Specifically, under the situation of limited labor-machinery substitution possibilities--which could be the case on hillside lands--as the cane alcohol price increases, production of maize and beans shifts to a lower level of

technology to free up machinery and labor for the cane fields. This lower technology also has lower yields, so to meet the demand for maize and beans the area planted in those crops actually increases as the cane area increases.

By far, the largest substitution effect in supply occurs with pastures, but it is very important to notice that this effect is indirect, through the labor market, rather than direct through the land market. This is so because as it may be recalled, TICO does not allow for the investments that would be required to expand crop cultivation in the pasture lands. The displacement of pastures has no effect on beef domestic prices because the change in beef supply is subtracted from exports, and exports of beef face a horizontal demand curve.

Finally, Table 26 shows that mechanized land increases smoothly and by a large acreage when labor-machinery substitution is allowed for. It also shows that employment increases directly with higher substitution possibilities. Moreover, inclusion of labor-machinery isoquants leads to full employment of the work force. However, important changes take place in the composition of employment. The most notorious one is that as labor-machinery substitution possibilities increase, farmers have higher incentive to work in other farms--those planting sugarcane. This is possible because by introducing mechanization, farmers can free up labor from their own farms, while taking advantage of higher returns to their time in the sugarcane plantations.

Table 26--Mechanization, employment, and exports under different alcohol prices and factor substitution possibilities

Factor	Alcohol price ($ per liter)			
	3.3	3.6	4.0	5.0
No factor substitution				
Total mechanized area	45.1	45.1	51.0	67.4
Total non-mechanized area	731.9	731.9	731.8	3.0
Total farm employment	6,222.9	6,223.1	6,272.4	3,320.3
Total work on own farms	2,091.5	2,091.5	2,028.3	1,080.7
Total work on other farms	611.8	715.2	674.0	597.3
Total landless labor	3,519.6	3,416.4	3,570.1	1,642.3
Total machinery use	236.0	236.0	236.0	288.7
Labor-machinery ratio	26.4	26.4	26.6	11.5
Average labor per hectare	8.0	8.0	8.0	47.2
Average machinery per hectare	0.3	0.3	0.3	4.1
Sugar exports	113.2	113.2	97.0	97.0
Beef exports	128.2	128.2	128.2	28.5
Modest factor substitution*				
Total mechanized area	17.1	40.7	53.6	73.3
Total non-mechanized area	762.2	739.3	685.3	7.3
Total farm employment	7,574.5	7,574.5	7,574.5	7,574.5
Total work on own farms	3,161.9	2,091.8	2,062.4	1,936.8
Total work on other farms	294.8	1,364.9	1,395.2	1,519.9
Total landless labor	4,117.6	4,117.8	4,117.8	4,117.8
Total machinery use	36.7	508.0	720.0	583.8
Labor-machinery ratio	206.7	14.9	10.5	13.0
Average labor per hectare	9.7	9.7	10.3	94.0
Average machinery per hectare	0.0	0.7	1.0	7.2
Sugar exports	113.2	97.0	97.0	97.0
Beef exports	128.2	127.9	121.1	28.5
Large factor substitution**				
Total mechanized area	44.7	54.0	55.5	78.5
Total non-mechanized area	787.1	706.5	670.4	7.3
Total farm employment	7,574.5	7,574.5	7,574.5	7,574.5
Total work on own farms	2,740.4	2,378.1	2,024.3	1,937.9
Total work on other farms	708.3	1,078.6	1,432.4	1,518.8
Total landless labor	4,117.8	4,117.8	4,117.8	4,117.8
Total machinery use	487.9	600.1	743.4	711.6
Labor-machinery ratio	15.5	12.6	10.2	10.6
Average labor per hectare	9.3	10.0	10.4	87.3
Average machinery per hectare	0.6	0.8	1.0	8.2
Sugar exports	113.2	97.2	97.0	97.0
Beef exports	128.2	121.1	118.2	28.5

Source: Simulation results of the model TICO.

Note : Areas are given in thousand hectares, exports in thousand tons, employment in thousand man-days, and machinery in thousand machine-hours.

* Elasticities of factor substitution = 0.2, 0.5, and 1.0 for the small, medium, and large farms, respectively.

** Elasticities of factor substitution = 0.7, 1.5, and 2.0 for the small, medium, and large farms, respectively.

Chapter VII

CONCLUSIONS: THE ROLE OF FACTOR SUBSTITUTION IN EVALUATING ECONOMIC ALTERNATIVES

This study has addressed and resolved two important issues in applied economic analysis. One methodological: how to bring together the neoclassical and the activity analysis production function traditions. One empirical: how to overcome the sometimes severe scarcity of data in production function analysis. What difference does it make to improve the tool of analysis for evaluating economic alternatives?

From the results discussed in Chapter VI, it is clear that the systematic treatment of factor substitution possibilities--labor and machinery in this case--within the context of programming models, could effectively change the decisions that policy makers would make concerning the implementation of large programs such as the production of alcohol to substitute for gasoline and diesel in motor vehicles.

On what price would be required in order to encourage production, the knowledge of factor substitution possibilities can lead to set prices at a much lower level than the lack of knowledge on those possibilities would actually lead to. *Prices of finished products are inversely related to the elasticity of substitution among the factors that go into the production of such finished products.* This principle, that emerges from the present study, was somehow imbedded in Marshall's (1920) third law as well as in the subsequent reexaminations of the

same law by Hicks (1932) and Allen (1938). If the own-price elasticity of demand for a factor of production was related to the elasticity of substitution of that factor and to the price elasticity of demand for the finished product, one would expect that some functional relationship should exist between the elasticity of factor substitution and the price elasticity of demand for the final product. By implication, one would further imagine that a functional relationship should exist between factor substitution possibilities and the price of the finished commodity. The methodological contribution of this study and its empirical application help to clarify these relationships.

On what are the complementarity and substitution effects that take place among commodities on the supply side as a result of changes in their relative prices, the knowledge of factor substitution possibilities can be used to make predictions that might turn out to be *opposite* to the predictions that would be made ignoring factor substitution possibilities.

On what would be the social efficiency of economic policies, the knowledge of factor substitution possibilities can help in making a more accurate ranking of those policies. This study has demonstrated, for instance, that *the distribution of benefits of price support programs, as between consumers and producers, depends on labor substitution possibilities*.

On the effectiveness of price policies to increase farmers' incomes, the knowledge of factor substitution possibilities would call for *multi-commodity and spatially differentiated pricing policies*, as opposed to single commodity and panterritorial pricing policies. More

importantly, this finding suggests that the effectiveness of pricing policies could be substantially increased if they are accompanied by the implementation of technology packages aimed at improving substitution possibilities among factors of production.

In sum, the knowledge of factor substitution, i.e., the estimation of elasticities of factor substitution, and its use in the context of programming models, could lead to a better assessment of economic alternatives or policies, and in some instance to the reversal of the assessment that would otherwise be made.

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