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Abstract

This paper considers continuous and cyclical farming strategies for the management of soil resources in semi-arid Africa. The results of the Lewis and Scmalensee (1977, 1979) analysis of the optimal use of a renewable resource in the presence of nonconvexities is used to demonstrate that cycles of exploitation (farming) and regeneration (bush fallow) can be the optimal strategy for sustainable agriculture.

CONTINUOUS AND CYCLICAL FARMING STRATEGIES FOR SOIL MANAGEMENT ON SEMI-ARID AFRICA

1. Introduction

The conservation of soil resources and sustainable agriculture is a concern of increasing importance in the semi-arid regions of Africa. Low rainfall is a critical limiting factor for agricultural production. Consequently, farm productivity depends critically upon the soil depth and soil structure, particularly the ability of the soil to capture and retain the limited rainfall. Population growth has led to the clearing of forests to cultivate more land and to more intensive cultivation of the land, particularly through shorter fallow periods. As a result, the rate of soil erosion and the rate of deterioration of the soil quality have increased. Crop productivity has been impaired as the quality of the soil resource has declined from the more intensive cultivation; the viability of sustainable agriculture is threatened. The economic nature of this problem can be characterized as a trade-off between current production and the future productivity of the soil resource.

Optimal control theory provides a useful framework of analysis for intertemporal economic problems; it has been applied to a wide variety of natural resources, including the management of soil resources (Burt, 1981; McConnell, 1983; Bhide, Pope, and Heady, 1982; Segarra and Taylor, 1987). The continuous time framework of optimal control theory is particularly useful for developing general economic insights into how various economic, technological, and policy variables affect the management of natural resource assets. For example, the impacts of the rate of discount, the time horizon, different price scenarios, and uncertainty about land tenure on the conservation of soil resources can be analyzed in the optimal control framework. The discrete-time framework of dynamic

programming is more amenable to empirical applications but some quantitative analysis can be performed with optimal control theory, particularly when simple functional forms can be specified.[1]

Previous treatments of soil conservation in the literature have focused on optimal rates of soil depletion when the net benefit function is strictly concave. For example, McConnell (1983) examines an optimal control model of soil conservation in which current output is a function of the state variable, soil depth, and the control variables, soil loss and an index of other farm inputs. The impact of current farm decisions on the future productivity of the soil is captured in the 'equation of motion', the differential equation which defines the rate of change in soil depth as a function of current decisions. The farmer chooses the time paths of soil loss and other inputs which maximize the discounted net return to the farm over a fixed planning horizon subject to the constraint imposed by the equation of motion.

The first order conditions consist of a static efficiency condition, a dynamic efficiency condition, and a transversality, or boundary, condition which jointly determine the optimal path for the depletion or conservation of the soil as a function of the time paths of input and output prices, the rate of discount, and the length of the planning horizon. In general, it is difficult to obtain a closed-form solution for an optimal control problem but a number of qualitative analyses can be conducted even in the case of nonlinear functional relationships. In some simple cases, this qualitative analysis is relatively straightforward and it provides important economic insights. For example, a higher rate of discount or a shorter planning horizon decreases the present value of future production relative to the present value of current production and results in reduced soil conservation. Since the

security of land tenure affects the time horizon and rate of discount it also is an important factor in determining the conservation of soil resources.

Bhide, Pope, and Heady (1982), examine an empirical dynamic optimization model of soil erosion in Iowa and compare the optimal paths of soil use obtained with different assumptions about the initial soil depth, the rate of discount, technological growth, the farmer's time horizon, and the terminal value of the farm. In the case of a fixed time horizon, the soil loss increases over time with a rapid increase at the end of the time horizon. In contrast, if the planning horizon is infinite, then the optimal level of soil loss declines gradually over time. The optimal rate of soil loss is reduced by a lower rate of discount, a longer planning horizon, or continued technological growth.

Segarra and Taylor (1987) examine the optimal cropping rotation in south-central Virginia over a fifty-year time horizon when the impact of current production on future soil productivity is taken into account. Burt (1981) includes a second state variable, the percent of organic matter in the top 15 centimeters of soil in a model which examines soil erosion and the optimal crop rotation in the Palouse region of Idaho and Washington.

With the assumption of a concave net benefit function, the optimal path is either to asymptotically approach a steady-state equilibrium or to exhaust the soil resource in finite time. Whether or not soil exhaustion is optimal depends upon the rate of return to the soil resource relative to the rate of discount. The rate of return to the soil resource is the sum of the value of the marginal productivity of the soil in the production of crops and the value of its marginal productivity in the regeneration of the soil resource. The optimal steady-state level for the soil resource occurs where the rate of return to the soil

resource is equal to the rate of discount. It is optimal to exhaust the soil resource if its rate of return is less than the rate of discount at all levels of the soil resource.

However, the net benefit function for farming has a nonconvexity if there is a fixed cost to farming (which is not incurred if the land is not farmed) or if the marginal productivity of farming intensity is increasing at low levels of farming intensity. In the case of a net benefit function with nonconvexities, there may be levels of the soil resource for which the maximum net benefits from farming are less than zero. Lewis and Scmalensee (1975, 1977, 1979) use the case of a fishery to demonstrate that if there is a nonconvexity to the net benefit from the harvest of a renewable resource, then the optimal management of the resource can involve continuing cycles of exploitation and regeneration. That is, the fishery is exploited for a period of time, then it is (temporarily) abandoned until the fish stock is sufficient to warrant the resumption of fishing effort.

This paper examines an optimal control model of soil management which allows for nonconvexities in the net benefit function. This can be particularly relevant to the problem of soil management in marginal semi-arid farmlands. The next section of the paper presents a standard one-state variable model in which farm productivity is a function of soil depth and farming intensity and discusses how the rate of discount, the rate of soil regeneration, and the time horizon affect the optimal path for soil erosion. Section 3 of the paper follows Lewis and Schmalensee (1975, 1977, 1979) to demonstrate that in the case of nonconvexities in the net benefit function, it may be that neither a continuous farming strategy nor exhaustion is the optimal soil management strategy. Instead, the land should be used in cycles of exploitation and regeneration. A crop-fallow rotation for the conservation of soil moisture and longer term bush-fallow patterns of cultivation

provide examples of this cyclical farming strategy. A model with two state variables, soil depth and soil quality, and two control variables, soil depletion and soil enhancement practices, is examined in the appendix.

2. Continuous Farming Strategies

Consider a one state-variable model of soil management in which the net benefit from farming the land at time t, denoted by $B(X, s)$, is a function of the state of the soil resource at time t, denoted by $X(t)$, and the control variable, the depletion of the soil resource at time t, denoted by $s(t)$. This general specification of the soil resource can be used to examine the optimal conservation of different soil characteristics. The state of the soil resource could be the moisture content of the soil, the depth of the soil (as in McConnell, 1983), or some index of relevant soil characteristics. Limiting the model to one state variable greatly simplifies the theoretical analysis without changing the basic qualitative nature of the results.[2] An optimal control model of soil management with two state variables—soil depth and soil quality—is presented in the appendix.

The net benefit from farming also depends upon the rate of depletion of the soil resource. The crop yield depends upon the 'farming intensity' which can represent a schedule of various 'activities'—crop selection, tillage, etc.—which affect the yield from the current state of the soil resource and also the depletion of the soil. That is, an alternative formulation of the model could specify farming intensity, say $u(t)$ as the control variable and the depletion of the soil resource would be a function of the level of farming intensity, i.e. $s(t) = f(u(t))$.

In the present formulation, the relationship between farming intensity and soil depletion is embedded within the net benefit function. This formulation simplifies the

exposition of the cyclical farm strategies in the next section. The formulations are the same when there is a monotonic relationship between farming intensity and soil loss.[3] The net benefit function is assumed to be concave with $B_X > 0$, $B_s > 0$, $B_{XX} < 0$, $B_{ss} < 0$, $B_{Xs} \geq 0$. If $B_{Xs} > 0$, then the marginal return to a given level of soil depletion (farming intensity) increases with the state of the soil resource (e.g. soil depth or soil moisture).

Since farming depletes the soil resource, it affects the state of the soil resource in future periods. The natural regeneration of the soil is denoted by the function $g(X)$. In most of the literature on soil erosion, it has been assumed that the soil regenerates at the constant rate k (for example, McConnel, 1983). However, it is possible that the regeneration of the soil could depend upon its current state. For example the regeneration of the soil can depend upon the deposition of residue organic matter in previous years which, in turn, depends upon the state of the soil resource. In addition, there can be an upper limit to the state of the soil resource. Consequently, the natural rate of regeneration of the soil can be low when the state of the resource is low and then increase, reach a peak, and then decline as the state of the soil increases. Similarly, in the case of soil moisture, the carry-over of seasonal rainfall from a year of fallow can depend upon the soil moisture content of the soil and there can be an upper limit to soil moisture.

For this general specification of soil regeneration, then, it is assumed that there exists a maximum state of the soil resource $\tilde{X}$ such that $g(\tilde{X}) = 0$. It also is possible that there is a state of the soil resource below which the soil does not regenerate. This state is denoted by $\underline{X}$ so $g(\underline{X}) = 0$. It also is assumed that $g(X)$ is concave for $X \geq 0$ (there can be a discontinuity at $X = 0$). Then the equation of motion is given by: $\dot{X} = g(X) - s$. Both the more general specification and the special case $g(X) = k$ are examined here and

they are depicted in Figures 1a and 1b. It also is assumed that $X(0) > \underline{X}$ and that there exists an $s(0)$ such that $B(X(0), s(0)) > 0$ (this implies that farming is desirable at some time). Finally, it is assumed that $B_s(X, 0) > 0$.

The net benefit function and the equation of motion capture the notion that farming the soil entails a trade-off between current returns and the future productivity of the soil. That is, more intensive farming increases current profit but depletes the soil and this depletion reduces the future returns to farming.[4]

The farmer selects the time path for soil depletion which maximizes the present value of net returns from the land over a given time horizon. Formally, the problem is to choose the time path for soil depletion, $s(t)$, which maximizes:

$$\int_0^T e^{-\delta t} B(X(t), s(t))\, dt + e^{-\delta T} V(X(T)) \tag{1}$$

subject to:

$$\dot{X}(t) = g(X) - s \tag{2}$$

$$s(t), X(t) \geq 0 \; \forall t, \tag{3}$$

where T denotes the length of the planning horizon (which could be endogenous or exogenous, finite or infinite), $V(X(T))$ denotes the value of the state of the soil resource at the terminal time, and δ denotes the rate of discount.

The current-value Hamiltonian is given by:

$$H(X, s, \lambda) = B(X(t), s(t)) + \lambda(g(X) - s(t)) + \alpha_1 s + \alpha_2 X \tag{4}$$

where λ is the costate variable for the equation of motion governing soil depth and can be interpreted as the current value shadow price, or marginal value of the soil resource,

and the α_i's are the Lagrange multipliers associated with the non-negativity constraints. The first order conditions for an interior solution are:

$$\partial H/\partial s = B_s(X,s) - \lambda = 0 \tag{5}$$

$$\dot{\lambda} = \delta\lambda - \partial H/\partial X = \delta\lambda - B_X(X,s) - \lambda g'(X) \tag{6}$$

Equation (5) is the static efficiency condition that the marginal net benefit from soil depletion is equal to the marginal cost of soil depletion (where the latter is the value of soil resource times the marginal soil loss). Equation (6) is the dynamic efficiency condition which insures that the rate of return to the soil asset is equal to the rate of discount and, therefore, to the rate of return to other assets in the economy. The rate of return to the soil asset is the capital gain on its value, $\dot{\lambda}/\lambda$, plus the marginal productivity of the soil in terms of the numeraire, B_X/λ, and the marginal productivity of the soil in the regeneration of soil, $g'(X)$.[5] If natural rate of soil regeneration is constant, then $g(X) = k$, and the last term of the return to the soil asset is zero.

The transversality, or boundary, conditions are:

$$X(T) \geq 0, \quad \lambda(T) \geq dV/dX, \quad \text{and} \quad X(T)[\lambda(T) - dV/dX] = 0 \tag{7}$$

if $X(T)$ is free but must be non-negative, and:

$$B(X(T), s(T)) + \lambda[g(X(T)) - s(T)] + V(X(T)) = 0 \tag{8}$$

if the time horizon is endogenous. Equation (7) requires that if the terminal state of the soil resource is positive, then the terminal shadow price of the soil resource is equal to the marginal value of the terminal state of the soil. If the terminal value of the soil is zero,

then this implies that either the resource is exhausted at the end of the time horizon or the shadow price of the soil resource is zero.[6]

Equation (5) implicitly defines the optimal soil depletion $s(t)$, as a function of the state of the soil resource, $X(t)$, and its shadow price, $\lambda(t)$. Consequently, at any point in time, the entire system is described by the state of the soil and its shadow price at that time. Indeed, the optimal trajectories can be characterized by the differential equations:

$$\dot{X} = M(X, \lambda) = g(X) - s(X, \lambda) \tag{9}$$

$$\dot{\lambda} = N(X, \lambda) = \lambda[\delta - g'(X)] - B_X(X, s(X, \lambda)) \tag{10}$$

Then the optimal path $s^*(t)$ satisfies equations (9) and (10), the initial condition $X(0) = X_0$, the non-negativity conditions, and the transversality condition(s). Thus, the optimal solution $s^*(t)$ is a function of the initial soil depth, the time horizon, the rate of discount, and the parameters and functional forms of the net benefit and soil loss equations.

If $g(X) = k$, then the dynamic efficiency condition, equation (6), can be integrated with respect to time to yield:

$$\lambda(t) = \lambda(T)e^{-\delta(T-t)} + \int_t^T e^{-\delta(\tau-t)} B_X(X(\tau), s(\tau))\, d\tau. \tag{11}$$

The current value shadow price for soil depth can be separated into two parts. The first term on the left hand side is the current value, at time t, of the scarcity rent associated with soil depth. This term is marginal terminal value of the soil resource and is equal to zero if there is no terminal value to the soil and the soil is not exhausted over the time horizon (equation (7)).

The second term can be viewed as a 'degradation charge' which represents the discounted value of the cost imposed on the future because the current soil loss increases

the marginal cost (in terms of reduced productivity) of any future soil losses. Both the scarcity rent and the degradation charge must be covered by the marginal net benefit from farming intensity.

The value of the degradation charge depends upon the entire future path of the system. For example, if soil depletion is positive ($g(X) < s$) for all τ from t to T, and if $B_{Xs} = 0$, then the degradation charge must be greater than the present value of the current marginal productivity loss over the remaining time horizon. In this case, the soil loss today increases the marginal cost of future soil losses because of the concave relationship between the soil resource and productivity.

With the more general specification of the soil regeneration function, the additional term $\int_t^T \lambda(s)g'(X)e^{-\delta(\tau-t)}$ is added to the right hand side of equation (11) and the evolution of $\lambda(t)$ cannot be determined independently of the rest of the system. This additional term captures the value of the soil's marginal productivity in soil regeneration.

Differentiating the static efficiency condition, equation (5), with respect to time, and using the dynamic efficiency condition, equation (6), yields the time derivative of the control path for soil depletion:

$$\dot{s}(t) = \frac{(\delta - g'(X))\lambda - B_X - B_{Xs}(g(X) - s)}{B_{ss}} \tag{12}$$

The denominator of equation (12) is negative. The numerator is positive when λ is increasing and the soil resource is decreasing. In this case, then, soil depletion is decreasing over time.[7]

The qualitative nature of the optimal time path for soil depletion, and therefore soil loss, depends critically upon the time horizon. If the time horizon is finite, then the time path for the exploitation of the soil depends upon the length of the time horizon and the

terminal value of the soil resource. This is apparent from equation (11) where the first term in the expression for the current value of the soil is the present value of the terminal shadow price of the soil.

With a finite time horizon, the terminal value is exogenous but implicitly assumes some optimal use of the soil resource in the periods after the terminal date (or it assumes that the soil resource is not valuable after that date). That is, there must be, at least implicitly, a valuation for the terminal level of soil depth, $V(X(T))$. Since this terminal valuation gives the present value of the net benefit of the soil resource at the end of the planning horizon, it essentially represents a summary statement of all the economic and technological conditions which are expected to prevail after the terminal time. Thus, any statement of the problem which imposes a fixed time horizon does not avoid the difficult task of projecting the costs and benefits of different actions out to an indefinite future.[8]

If the soil resource has little or no value at the end of the time horizon, then complete exhaustion by the end of the time horizon can be optimal. Early in the planning period, the farmer conserves the soil for its future productivity but depletes the soil rapidly as the terminal time approaches. Thus, the assumption of a finite time horizon and low terminal value can force the optimal level for farming intensity to increase over time whereas an infinite planning horizon would yield a monotonically decreasing time path for farming intensity and soil loss (McConnell, 1983; Bhide, Pope, and Heady, 1982).

With an infinite planning horizon, the value of the resource at any point in time is determined endogenously. Any infinite time horizon problem can be represented as a series of fixed time horizon problems since the optimal path for the infinite time horizon

problem implies a value for the soil resource for any chosen point in time. This is depicted graphically in Figure 2.

In Figure 2, the steady-state value for the state of the soil resource is denoted by $\bar{X}$. In this case, the initial value for the state of the soil resource, denoted X_0, is greater than the steady-state soil depth. The state of the soil resource decreases monotonically along the optimal path and asymptotically approaches its steady-state level. At time $\tilde{T}$, there is a maximum value to the soil resource, $V^*(X(\tilde{T}))$. The shadow price of the soil resource along the optimal path, $\lambda^*(t)$, gives the marginal value of the soil resource at that time. If $\tilde{T}$ is chosen as a fixed time horizon and $V^*(X)$ is chosen as the terminal valuation for soil depth, then the optimal solution to this finite time horizon problem requires $\lambda(T) = dV^*/dX(\tilde{T})$. Consequently, the optimal path for the soil resource in the solution of the fixed time horizon problem is the same as the optimal path for the infinite time horizon problem over the interval $[0, \tilde{T}]$.

The conditions describing the steady-state of an infinite planning horizon can be determined in a straightforward manner. A steady-state equilibrium occurs when $\dot{\lambda} = \dot{X} = 0$ (or $\dot{s} = \dot{X} = 0$). Thus, the soil resource is is in a steady-state if and only if $\bar{s} = g(\bar{X})$, where $\bar{X}$ and $\bar{s}$ denote the steady state of the soil resource and the steady state soil depletion from farming. The shadow price for soil is constant if and only if $\delta = g'(X) + B_X(X, s)/\lambda$. Static efficiency requires $\lambda = B_s(X, s)$. Therefore, the steady state of the soil resource, $\bar{X}$, is defined by the condition:

$$\frac{B_X(\bar{X}, g(\bar{X}))}{B_s(\bar{X}, g(\bar{X}))} + g'(\bar{X}) = \delta \tag{13}$$

A sufficient condition for the existence of a steady-state equilibrium is $g'(\bar{0}) > \delta$.[9]

The phase diagram in the λ, X space is depicted in Figure 3, both for the general case of a concave soil regeneration function and the special case of constant soil regeneration ($g(X) = k$). Let $M(X, \lambda) = g(X) - s(X, \lambda)$ denote the combinations of X and λ for which the state of the soil resource is constant. Then

$$\left.\frac{d\lambda}{dX}\right|_{\dot{X}=0} = -\frac{M_X}{M_\lambda} = \frac{g'(X) + (B_{sX}/B_{ss})}{B_{ss}}. \tag{14}$$

In the general case, $g'(X)$ is positive for low values of X and negative for large values of X; $g'(X) = 0$ in the case of a constant soil regeneration. The second term in the numerator is negative and the denominator also is negative. Thus the slope of the curve $\dot{X} = 0$ is negative for small values of X and positive for large values of X; this slope is strictly positive in the case of constant soil regeneration.

The $\dot{X} = 0$ curve represents combinations of the state of the soil resource and its price where the intensity of farming is just equal to the natural regeneration of the soil. As such, it can be interpreted as the steady state supply function for the soil resource (see Smith, 1977). The general specification for the natural regeneration function makes it possible for this 'supply curve' to have a downward sloping segment. The value of $M(X, \lambda)$ is positive (negative) for combinations of X and λ which lie above the $\dot{X} = 0$ curve. If the shadow price for the soil resource is above (below) its equilibrium supply price, then the supply of the soil resource increases (decreases) toward its equilibrium.

Let $N(X, \lambda) = \lambda[\delta - g'(X)] - B_X(X, s(X, \lambda)) = 0$ denote the combinations of X and λ for which the shadow price of the soil resource is constant. Then

$$\left.\frac{d\lambda}{dX}\right|_{\dot{\lambda}=0} = -\frac{N_X}{N_\lambda} = \frac{\lambda g''(X) + [(B_{XX}B_{ss} - B_{Xs}^2)/B_{ss}]}{\delta - g'(X) - (B_{Xs}/B_{ss})} < 0. \tag{15}$$

The curve $\dot{\lambda} = 0$ represents combinations of the state of the soil resource and its shadow price for which the rate of return to the soil asset is just equal to the rate of discount in the absence of capital gains. As such, it can be interpreted as the steady state demand function for the soil resource (Smith, 1977). The value of $N(X, \lambda)$ is positive (negative) for combinations of X and λ which lie to the right (left) of the $\dot{\lambda} = 0$ curve. If the shadow price of the soil resource is above (below) its equilibrium demand price, then the demand for the soil resource decreases (increases) toward its equilibrium.

The introduction of a concave soil regeneration function and the productivity of the soil resource make it possible for multiple steady-state equilibria to occur. If a steady-state equilibrium is unique, then it is a saddle point equilibrium and the equilibrium is stable in that the optimal path moves the system toward the steady state. If there are multiple steady-state equilibria, then they will alternate between stable and unstable equilibria. The initial state of the soil resource then determines the optimal steady-state. It is possible that exhaustion is optimal for some initial values of the state of the soil resource while movement toward a steady-state with a positive soil resource is optimal from other initial values for the state of the soil resource.

Total differentiation of equation (11) and using $\bar{s} = g'(X)d\bar{X}$ from equation (2) yields:

$$\frac{\partial \bar{X}}{\partial \delta} = \frac{-B_s}{(\delta - g')[B_{ss}g' - B_{Xs}(1 + g')] - g'B_{Xs} - B_{XX}} < 0 \tag{16}$$

if $\bar{X}$ is a stable equilibrium. In the special case $g(X) = k$, this reduces to:

$$\frac{\partial \bar{X}}{\partial \delta} = \frac{B_s}{B_{XX} - \delta B_{Xs}} < 0 \tag{17}$$

In the special case, the value of k also affects the steady state of the soil resource:

$$\frac{\partial \bar{X}}{\partial k} = \frac{B_{Xs} - \delta B_{ss}}{(\delta B_{Xs} - B_{XX})} > 0 \tag{18}$$

As the discount rate increases (decreases), the $\dot{\lambda} = 0$ curve shifts downward (upward). That is, the steady state demand for the soil resource decreases (increases) as alternative investments become more (less) attractive. Consequently, the steady state of the soil resource decreases (increases), the equilibrium shadow price decreases (increases). The latter results in an increase (decrease) in the rate of soil depletion at each state of the soil resource.

As the soil regeneration parameter k increases (decreases), the $\dot{X} = 0$ curve shifts downward (upward). That is, the steady state supply curve shifts out (back), the steady state shadow price for soil decreases (increases) and the steady state of the soil resource increases (decreases). The rate of soil depletion is greater (smaller) at each state of the soil resource.

If a steady-state equilibrium exists, then the state of the soil resource is monotonically decreasing (increasing) and the shadow price of soil is monotonically increasing (decreasing) along the optimal path when the initial soil depth is greater (less) than the steady-state soil depth. If the net benefit function and equation of motion are linear in the rate of soil depletion, then the approach to the steady-state is a 'most rapid approach path' (Kamien and Schwartz, 1981). In this case, if the soil resource is below its steady-state level, then it is optimal to not farm the land until the soil depth has reached the steady-state level. If the state of the soil resource is better than its steady-state level, then it is optimal to farm the land at an exogenously given maximum level of intensity until the steady-state of the soil resource is reached.[10]

This section of the paper assumed that the net benefit function was concave in the control variable, soil depletion. If there are non-convexities in the net benefit function, then it can be optimal to farm the land in periodic cycles of exploitation and regeneration (Lewis and Schmalensee, 1975, 1977, 1979). This could occur, for example, if there are fixed costs which do not vary with the level of exploitation of the resource but are not incurred in the absence of exploitation. (The opportunity cost of fuelwood and forage which can be provided if the land is in bush fallow can be viewed as a fixed cost of farming the land.) Crop-fallow rotations for managing soil moisture in semi-arid lands and the longer cycle bush-fallow pattern of cultivation can represent this type of farming strategy.

3. Nonconvexities and Cyclical Farming Strategies

If there is a nonconvexity in the net benefit function, then a continuous farming strategy leading to a steady-state equilibrium may be only a local maximum. That is, there can be a non-continuous farming strategy which gives a higher present value to the soil resource.

A nonconvexity is introduced into the net benefit function if there are fixed costs to farming which are not incurred if the land is not farmed. In this case, there is a discontinuity in the net benefit function at $s = 0$. If the productivity of the soil declines sufficiently as the state of the soil resource declines, then net benefits may be negative at all levels of farming intensity (soil depletion)—see Figure 4. Then the farmer is clearly better off by avoiding the fixed cost and not planting a crop and letting the productivity of the soil regenerate before planting a crop in a later year. Of course, the farmer's land tenure must be secure both during and after a fallow cycle.

Nonconvexities in the net benefit function can also occur in other ways. For example, if there is a return to the land in the absence of farming which is not received when the land is farmed, then the opportunity cost of farming includes the forgone return to unfarmed land. This, then, is a type of fixed cost to farming. Another source of nonconvexity is present if there is a discontinuity in the soil loss function at a farming intensity of zero. That is, if $s(t) \geq \sigma > 0$ when the land is farmed at any level of cultivation but $s(t) = 0$ if farming intensity is zero, then the net benefit function can have a nonconvexity. Finally, a nonconvexity occurs if there is a nonconvexity in crop yields at low levels of farming intensity (and therefore soil depletion).

In the presence of nonconvexities, the optimal soil management strategy can be to farm the land in cycles of exploitation and regeneration. Indeed, there are conditions which are sufficient to insure that a cyclical farming strategy is optimal. The most straightforward case is when there is a fixed cost to farming. The discussion here follows that of Lewis and Schmalensee (1975, 1977, 1979) which examine optimal cyclical fishing strategies in the presence of fixed costs to fishing effort.

Let F denote the fixed cost to farming. It is clear that there is a fixed cost which is large enough for it to be optimal not to plant a crop. For a given fixed cost, let $\hat{X}$ denote the largest X for which $B(s(X,0),X) \leq F$. That is, $\hat{X}$ is the greatest lower bound for the state of the soil resource for which it is possible to have non-negative net benefits (including the fixed cost). Clearly, if the state of the soil resource falls below $\hat{X}$, then it is better not to plant a crop. If this occurs, then the option is to abandon the land permanently or to return to farm the land at a later date when the soil resource has regenerated sufficiently to overcome the fixed cost of farming.[11] Whether or not

permanent abandonment is optimal depends upon the cost of returning to farm the land, denoted by R.

If returning to the land to farm is not possible ($R = \infty$), then abandonment must be permanent. A unique abandonment strategy exists if the net benefit (including the fixed cost) from farming at the steady-state of the continuous farming strategy is negative ($B(g(X^*), X^*) \leq F$). A unique abandonment strategy also exists if the initial state of the soil resource is less than its optimal steady-state under continuous farming and net benefits are negative without further depletion of the soil resource ($X_0 < X^*$ and $B(g(X_0), X_0) \leq F$) (Lewis and Scmalensee, 1977, Proposition 6) or if the intial state of the soil resource is low (Lewis and Scmalensee, 1975, Proposition 3.4). In both cases, the productivity of the soil is insufficient to overcome the fixed cost of farming unless the rate of soil depletion is greater than the rate of depletion which allows the maintenance of the soil resource.[12]

The most interesting case is when the cost of returning to farm the land is positive but finite ($0 < R < \infty$).[13] In this case, sufficient conditions for it to be optimal to farm the land in cycles of exploitation and regeneration are given by: (a) the net benefit (including the fixed cost) from farming at the steady-state of the continuous farming strategy is negative ($B(g(\bar{X}), \bar{X}) \leq F \; \forall \; \bar{X}$); (b) there is a level of the soil resource greater than the minimum viable level for which the maximum net benefit from farming (including fixed costs) is positive ($\hat{X} > \underline{X}$); and (c) there is a level for the soil resource where the present value of returning to farm the land is greater than the cost of returning to farm the land (there exists an $X < \bar{X}$ such that $V_a(X) > R$ where V_a denotes the discounted net benefit from an optimal abandonment strategy) (Lewis and Scmalensee, 1977, Proposition 11).

The economic intuition behind these sufficient conditions is straightforward. The fixed cost is such that the net return to farming is negative in the steady-state. Thus, a continuous farming strategy cannot be optimal since eventually the net return to farming is negative (condition (a)) and it is better to not farm the land. However, the (temporary) abandonment of the land occurs at a viable level of the soil resource (condition (b)). Consequently, the soil resource regenerates while it is not farmed. However, the land is not permanently abandoned since eventually the soil resource reaches a level at which the present value of farming the land (even with the intent of abandonment) is greater than the cost of returning to farm the land. But since a continuous farming strategy is not optimal, eventually it again becomes better to not farm the land and the cycle repeats itself. Security of land tenure is essential for the individual's optimal management of the soil to be consistent with socially efficient management of the soil. Without secure property rights in the regenerated productivity of the soil, the farmer's incentives to manage the land as an investment in soil is greatly diminished.

A crop-fallow rotation for the conservation of soil moisture provides an example of this phenomenon of cycles of exploitation and regeneration. The planting of a crop is profitable only if the soil moisture at planting time and the seasonal rainfall are sufficient to produce a profitable yield. In many semi-arid regions, the seasonal rainfall can be insufficient for a profitable crop and so a crop is planted only after the previous season's rainfall has been conserved in the soil by a year of fallow. The optimal number of consecutive seasons to plant a crop or the optimal number of consecutive fallow seasons depends upon the effect of soil moisture, X, on crop yields (and therefore net benefits), the carry-over of rainfall

from a year of fallow,[14] and the cost of fallow, which can be viewed as a re-entry cost (for example, see Young, 1986).

Clearly these conditions are stronger than necessary. Even if the net benefit (including fixed costs) is positive in the steady-state, this may only be a local maximum. It would be better to farm the land in cycles of exploitation and regeneration if the present value of the increased productivity from higher states of the soil resource is greater than the present value of the net benefits foregone while the soil resource is regenerating.[15] In this case, the globally optimal strategy would have to be determined by a comparison of the present values for each farming strategy in the particular case.[16]

As long as the soil regeneration and net benefit functions, the fixed cost to farming, the cost of returning to farm the land, and the rate of discount are independent of time, the optimal strategy is stationary. That is, the optimal soil depletion is the same whenever the same state of the soil resource is encountered. A farming cycle, then, is defined by the state of the soil resource at which farming is abandoned, denoted by S^A, and the state of the soil resource at which farming resumes, denoted by S^R. The cycle interval is then defined to be $[S^A, S^R]$.

Increases in fixed costs reduce the optimal time spent farming the land (Lewis and Scmalensee, 1979, Proposition 3). This provides some insight into the effect of changes in price or cost on the optimal cycle. To some degree, an increase (decrease) in price is similar to a decrease (increase) in fixed cost and so the land is exploited for longer (shorter) periods when the price is higher (lower). However, the shift in the net benefit function is parallel for a change in fixed costs but not necessarily for a price change and

so the analogy is not exact. Similarly, a decrease (increase) in operating costs would shift the net benefit curve upward (downward) as would a decrease (increase) in fixed costs.

Furthermore, an increase (decrease) in fixed cost increases (decreases) the state of the soil resource at which farming is resumed (Lewis and Scmalensee, 1979, Proposition 5. Thus an increase in the price of farm output can induce the land to be farmed again at a lower state of the soil resource. Determinate conditions on the effect of fixed cost on the state of the renewable resource for which it is optimal to temporarily stop farming could not be derived (Lewis and Schmalensee, 1979, p. 688).

An increase (decrease) in the cost of returning to farm the land increases (decreases) the state of the soil resource at which return occurs and decreases (incresases) the state of the soil resource at which the land is abandoned. This increases (shortens) the cycle interval and increases (decreases) the time period between successive returns (Lewis and Schmalensee, 1975, Propositions 5.5 and 5.6).

Whether a continuous or cyclical farming strategy is optimal depends upon the available agricultural technology, the geological and biological conditions governing soil regeneration, and the economic conditions (prices of crops and inputs) in a specific location. The introduction of a new technology, such as tied ridges—which increase yields and decrease soil depletion, could result in a shift from a cyclical to a continuous farming strategy.

4. Summary and Conclusions

The optimal management of a soil resource depends upon a variety of factors including the marginal productivity of the soil resource, the value of the agriculture production, the cost of agricultural inputs, the rate at which the soil resource regenerates, the rate of

discount, the planning horizon, and the value of the soil resource at the end of the planning horizon. Of particular interest from a broad social perspective is the management of the soil resource so that agricultural production can be sustained over a long-term planning horizon.[17]

In principle, it can possible to sustatin agricultural production through either a steady-state continuous farming strategy or through a farming strategy with cycles of exploitation and regeneration. Given the prevalent pattern of bush-fallow farming in many semi-arid regions in Africa and the decreasing length of the fallow periods (Asante, 1987), a careful analysis of the optimality of cycles of exploitation and regeneration is warranted.

Of particular interest is whether or not the natural regeneration of the soil is great enough so that the there exists a steady state equilibrium where the present value of the net return to to continuous farming is positive and greater than the present value of the net return to a cyclical farming strategy. The key empirical relationships which must be determined include the natural regeneration of the soil at different states of the soil resource (including the state of the soil resource at which regeneration effectively becomes zero) and the net benefits from farming and soil conservation practices as the state of the soil resource declines.

The uncertainty about the functional relationship between farming activities and net benefits and the stochastic nature of rainfall and therefore output indicate the importance of risk analysis as a topic for further research. The risk analysis should include the risk associated with the possibility of exploiting the land beyond the point at which regeneration is possible. Population growth has increased the length of the period of

exploitation and shortened the length of the fallow period to where the potential for permanent damage to the soil resource is an immediate concern. The soil can be exploited to such an extent that it would not regenerate for many years, if ever.

The models presented here have implicitly assumed that the net benefit function is stationary. In general, the net benefit function can be changing over time due to changes in input and output prices and technological changes in the agricultural production function. The models can be extended to examine how the conservation of the soil resources is affected by changing price patterns and technological progress.

Appendix–Two State Variable Control Model

The separate characteristics of the soil resource can be modeled as different state variables. For example, the amount of moisture in the soil can depend upon the depth of the soil and the ability of rainfall to penetrate through the soil. The latter can depend upon the quality of the soil (which is affected by past farming practices). This is particularly true in the semi-arid regions of Africa where the water infiltration capacity of the soil is critical.

Day and Aillery (1987) report that poor soil quality in Mali exacerbates the problem of low rainfall. Crusting and sealing of the soil results in low absorptive capacity in a region where rainfall is infrequent but intense and water is the limiting factor of production. Farmers' disposable income could increase two to four times with an increase of rainfall infiltration from 40 to 60 percent. Improved soil quality and enhanced water infiltration also can reduce the level of soil erosion from a given level of farming intensity.

An alternative formulation of the model which distinguishes between different soil charactistics requires two state variables. In addition to the state variable, $X(t)$, which will represent soil depth, there is a second state variable, the quality of the soil at time t, denoted by $Q(t)$.

There are farming practices, such as crop residue management, which increase soil quality over time and thereby enhance the yield from a given soil depth and farming intensity. Let $s(t)$ and $z(t)$ denote the control variables soil loss and soil quality enhancement at time t. Then the farmer chooses the time paths $s(t)$ and $z(t)$ which maximize:

$$\int_0^T e^{-\delta t} B(X, Q, s, z)\, dt + e^{-\delta T} V(X(T), Q(T)) \qquad (A1)$$

subject to:

$$\dot{X} = k - s(t) \tag{A2}$$

$$\dot{Q} = z(t) \tag{A3}$$

$$X, Q, s, z \text{ nonnegative for all } t$$

where the variables are as previously defined. For simplicity, the case of constant soil regenration ($g(X) = k$) is discussed here. Equation (3) is the equation of motion governing soil quality. It should be noted that the complexity of the relationship between farming practices and soil loss and soil quality enhancement is embedded in the net benefit function.[18]

The current-value Hamiltonian is:

$$H(X, Q, s, z, \lambda, \gamma) = B(X, Q, s, z) + \lambda(k - s(t)) + \gamma z(t) + \alpha_s s + \alpha_X X + \alpha_z z + .alpha_Q Q \tag{A4}$$

where γ denotes the shadow price of soil quality and, as before, λ denotes the shadow price of soil depth and the α's are Lagrange multipliers associated with non-negativity constraints.

The first-order necessary conditions for an interior solution are:

$$H_s = B_s - \lambda = 0 \tag{A5}$$

$$H_z = B_z + \gamma = 0 \tag{A6}$$

$$\dot{\lambda} = \delta\lambda - B_X \tag{A7}$$

$$\dot{\gamma} = \delta\gamma - B_Q \tag{A8}$$

The interpretation of these conditions is the same as before in that the marginal benefit of soil loss is equal to the marginal cost of soil loss, the marginal value of soil

quality enhancement is equal to its marginal cost (where B_z is the cost of enhancing soil quality), and the rates of return to soil depth and soil quality are equal to the rate of discount and therefore to each other.

Differentiating equations (A5) and (A6) with respect to time and using equations (A5), (A6), (A7), and (A8) yields the time derivatives of the optimal paths for the control variables soil loss and soil quality enhancement.

$$B_{Xs}\dot{X} + B_{Qs}\dot{Q} + B_{ss}\dot{s} + B_{sz}\dot{z} = \dot{\lambda} \tag{A9}$$

$$B_{Xz}\dot{X} + B_{Qz}\dot{Q} + B_{sz}\dot{s} + B_{zz}\dot{z} = -\dot{\gamma} \tag{A10}$$

Writing the two equations in matrix notation yields:

$$\begin{bmatrix} B_{ss} & B_{sz} \\ B_{sz} & B_{zz} \end{bmatrix} \begin{bmatrix} \dot{s} \\ \dot{z} \end{bmatrix} = \begin{bmatrix} \dot{\lambda} - B_{Xs}\dot{X} - B_{sQ}\dot{Q} \\ -\dot{\gamma} - B_{Xz}\dot{X} - B_{zQ}\dot{Q} \end{bmatrix} \tag{A11}$$

Solving for $\dot{s}$ and $\dot{z}$ gives:

$$\dot{s} = D^{-1}[(\dot{\lambda} - B_{Xs}\dot{X} - B_{Qs}\dot{Q})B_{zz} + (\dot{\gamma} + B_{Xz}\dot{X} + B_{Qz}\dot{Q})B_{sz}] \tag{A12}$$

$$\dot{z} = D^{-1}[-(\dot{\gamma} + B_{Xz}\dot{X} + B_{Qz}\dot{Q})B_{ss} - (\dot{\lambda} - B_{Xs}\dot{X} - B_{Qs}\dot{Q})B_{sz}] \tag{A13}$$

where $D = B_{ss}B_{zz} - B_{sz}^2 > 0$ if B is concave.

Further qualitative analysis depends on the signs and magnitudes of the second-order partial derivatives. In general, it is not possible to place unambiguous signs on the time derivative of the control variables. Even with restrictive assumptions about the cross partial derivatives, there can still be some ambiguity about the movement of the level of farming intensity and soil conservation practices depending upon whether soil depth and soil quality are increasing or decreasing. In addition, placing signs on the direction of the

movement of the system is complicated by the interaction of the variables within the net benefit function.

If soil depth and soil quality are decreasing over time, then the shadow prices of the two assets, λ and γ, are increasing over time. If B_{X_s} and B_{Q_s} are both positive, then this implies that soil depletion is decreasing over time. Essentially, the soil exploited early in the time horizon which reduces the optimal level of soil depletion in the future. On the other hand, if soil depth and soil quality are increasing over time, then the time derivative of soil depletion is positive. Essentially, it is optimal to allow the two soil assets to accumulate in order to allow more intensive cultivation in the future.

A similar argument can be made for soil conservation practices. If it is optimal for soil depth and soil quality to decrease (increase) over time, then the current level of soil conservation practices is lower (higher) than in the future.

The movement of the system along the optimal path depends upon the initial values of the two assets, the time horizon, and the terminal valuation of the two assets. It is possible for it to be optimal to have one asset increasing while the other asset is decreasing. This complicates the direction of the movement of the two control variables. It is even possible to have cyclical movements in the variables (although this does not necessarily imply cycles of farming and not farming).

If the time horizon is infinite, then an important determinant of the movement of the system is whether the initial values of soil depth and soil quality are above or below their steady-state equilibrium values. A steady-state equilibrium is defined by the condition $\dot{X} = \dot{Q} = \dot{\lambda} = \dot{\gamma} = 0$ which requires:

$$k - s = 0 \tag{A14}$$

$$z = 0 \tag{A15}$$

$$\lambda\delta - B_X = 0 \tag{A16}$$

$$\gamma\delta - B_Q = 0 \tag{A17}$$

Total differentiation of these four equations and equations (A5) and (A6) yields the following matrix equation which can be solved for the impact of the rate of discount and the rate of soil regeneration on the steady state values of X and Q:

$$\begin{bmatrix} 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 \\ B_{XX} & B_{XQ} & B_{Xs} & B_{Xz} & -\delta & 0 \\ B_{XQ} & B_{QQ} & B_{Qs} & B_{Qz} & 0 & -\delta \\ B_{Xs} & B_{Qs} & B_{ss} & B_{sz} & -1 & 0 \\ B_{Xz} & B_{Qz} & B_{sz} & B_{zz} & 0 & 1 \end{bmatrix} \begin{bmatrix} dX \\ dQ \\ ds \\ dz \\ d\lambda \\ d\gamma \end{bmatrix} = \begin{bmatrix} dk \\ 0 \\ \lambda d\delta \\ \gamma d\delta \\ 0 \\ 0 \end{bmatrix} \tag{A18}$$

The impact of the rate of discount on the steady state is complicated by the interaction between the two state variables. That is, the direct effect of a lower rate of discount is to induce greater accumulation of both assets. However, greater accumulation of one soil asset can indirecty affect the incentive to accumulate the other soil asset. The relative magnitude of the direct and indirect effects depends upon the productive relationship between the two assets.[19]

Notes

[1]The dynamic programming framework requires the specification of a time horizon and terminal value function. This specification can greatly affect the optimal path for soil conservation. This point is discussed more fully in Section 2 below.

[2]Of course, the construction of such an index is not trivial.

[3]McConnell (1983) also uses soil loss as the control variable. In McConnell's model, the relationship between farming intensity and soil loss is subsumed within the production function. Krautkraemer (1987) develops the current model with farming intensity as the control variable.

[4]This implicitly assumes that the schedule of farming activities is monotonic. That is, if $s = f(u)$, then $f'(u) > 0$ so that $\partial B/\partial u > 0$.

[5]The corner solution which occurs if $B_s(X, 0) < 0$ has been ruled out by the assumption that there is an $s(0)$ such that $B_s(X(0), s(0)) > 0$. If this was not the case, then it is optimal to never farm the land. This case has obvious differences with the case of abandonment discussed in Section 3 below. If $\dot{\lambda}/\lambda + B_X(0, s) + g'(X) < \delta$, then the rate of return to soil asset is less than the rate of return to other assets and it is optimal to exhaust the soil resource in finite time.

[6]It can be shown that this condition also must hold in the limit if the time horizon is infinite.

[7]If the time horizon is infinite, then the movement of the system depends upon its starting point relative to the steady-state equilibrium. The discussion below establishes that the state of the soil resource is monotonically increasing (decreasing) and the shadow price is monotonically decreasing (increasing) along the optimal path if the initial state of the soil resource is below (above) its steady state.

[8]Arrow and Kurz (1970, p. 30) note, "If we choose to stop our analysis at any fixed date, it will be necessary,...,to include ... some scrap value for the stock of capital at the end of the period. But the only logically consistent way of doing so is to determine the maximum [benefit] attainable in the future beyond our given date stating with any given stock of capital."

[9]Since capital gains are zero if the value of the soil depth is constant, this insures that the marginal productivity of the soil depth covers the interest charges on holding soil depth as an asset.

[10]For example, if soil loss is proportional to farming intensity (i.e. $s = \beta u$ where u denotes farming intensity), then $B(X, s) = pF(X)u - cu = pF(X)[s/\beta] - c[s/\beta]$, and $0 \leq s \leq \bar{s}$, where p is the price of output, c is the constant unit cost of farming intensity, $F(X)u$ denotes output, and $\bar{s}$ is a maximum level of soil depletion determined by the maximum level of farming intensity, then the optimal path is a most rapid approach path.

[11]If the land is abandoned, then the terminal state of the soil resource, $X(T)$, should be $\hat{X}$. If $X > \hat{X}$, then net benefits from farming are positive and it is not optimal to abandon; if $X < \hat{X}$, then it would have been better to abandon earlier. If $\hat{X} < \underline{X}$, then abandonment implies exhaustion.

[12]Note that this includes the case in which the cost of soil conservation is too great relative to the returns to farming.

[13]If returning to farm the land is costless ($R = 0$), then the nonconvexity essentially disappears. This is because abandonment and return can take place infinitely often in a finite period of time. This is the case of 'pulse fishing' (Lewis and Schamalensee, 1975).

[14]The equation of motion for soil moisture can depend upon the level of soil moisture even when rainfall is a constant k if the carry-over to the next planting season depends upon the soil moisture level.

[15]Of course, the alternative uses of the land many also provide net benefits.

[16]Lewis and Schmalensee (1975) refers to Lewis (1975) as a source for a computational algorithm.

[17]This may be a difficult task since the subsistence nature of cultivation in the region results in farmers having a short planning horizon and/or a high rate of time preference (Mansius, 1987). This generally implies a rapid rate of depletion of the soil resource.

[18]That is, soil loss, s can be a function of soil depth, X, soil quality, Q, farming intensity, u, soil enhancement activities, denoted by $w(t)$. That is, $s(t) = f^1(X, Q, u, w)$. Similarly, $z(t) = f^2(X, Q, u, w)$. Then $B(X, Q, s, z) = B(X, Q, f^1(X, Q, u, w), f^2(X, Q, u, w))$.

[19]For an example in a different context, Krautkraemer (1988) demonstrates that a lower rate of discount can actually reduce the optimal level of permanently preserved natural environments.

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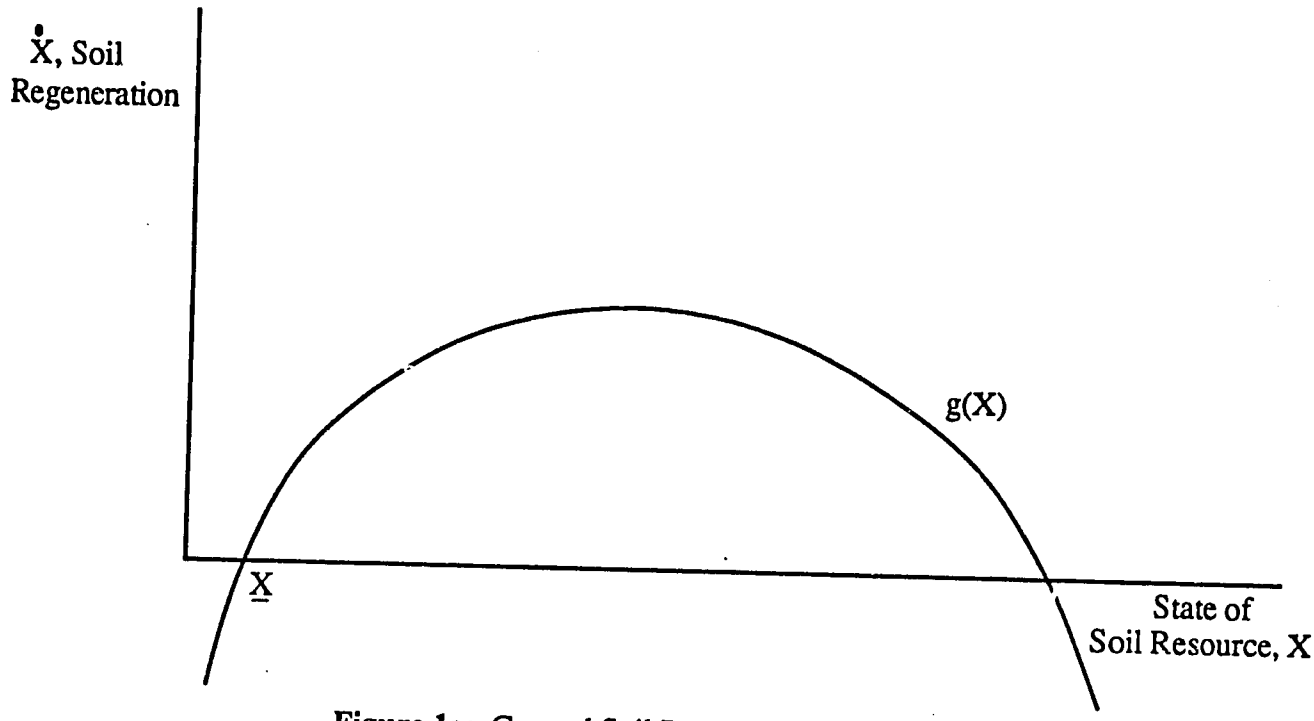

Figure 1a: General Soil Regeneration Function

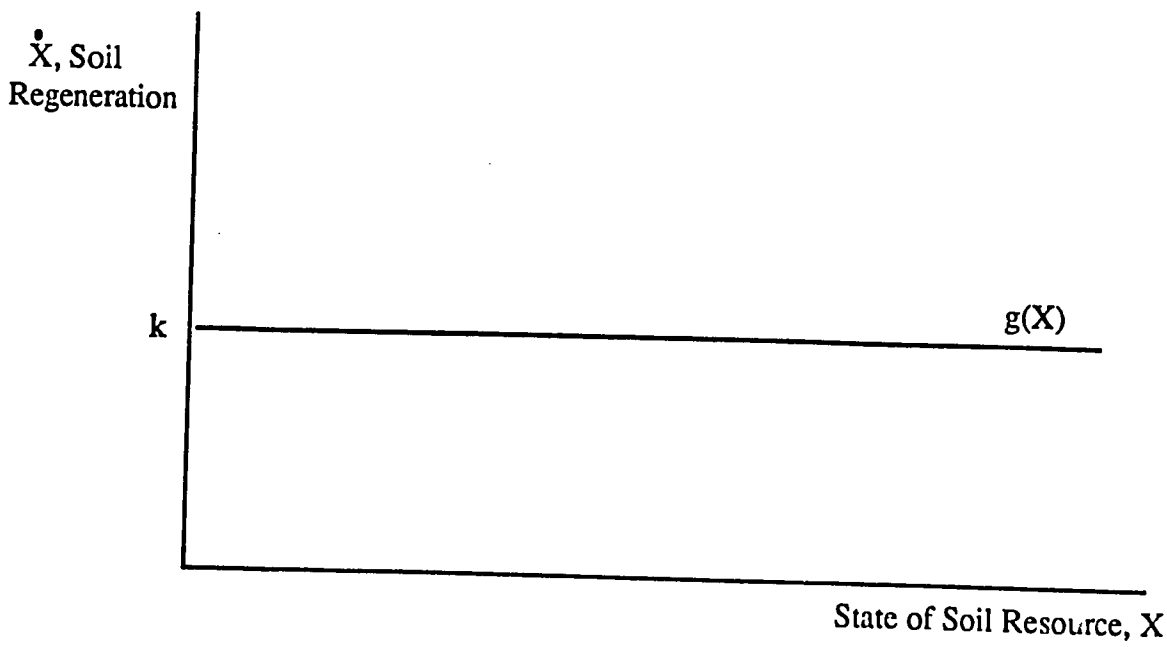

Figure 1b: Constant Soil Regeneration Function

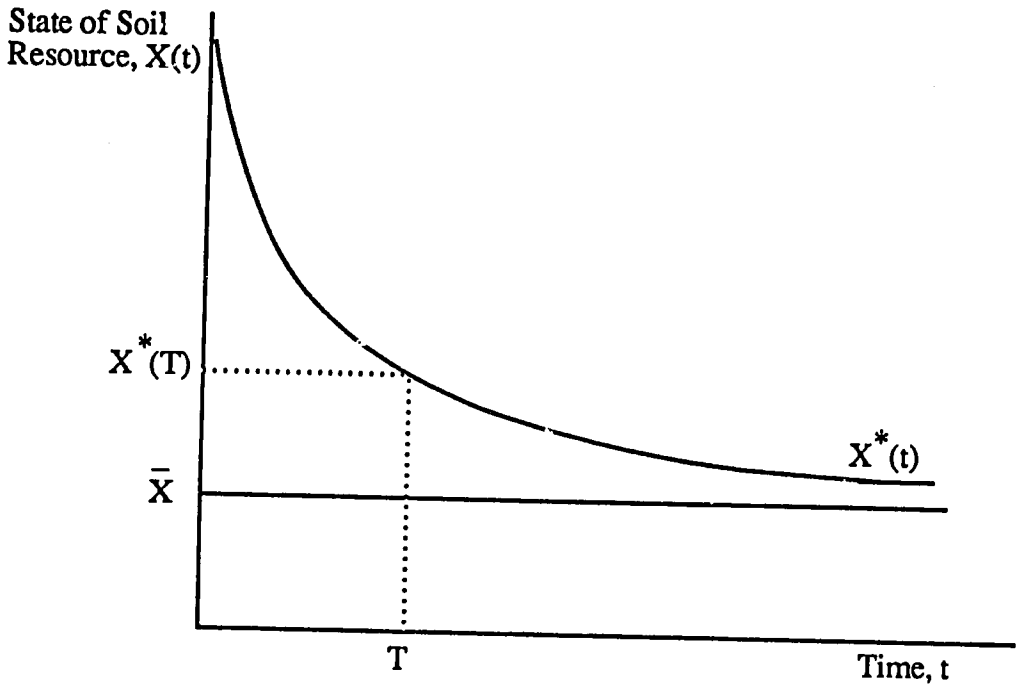

Figure 2

$X^*(t)$ denotes the optimal path for the infinite time horizon problem. If T is the terminal time for a finite horizon problem, then $X^*(T)$ is the optimal terminal state for the soil resource if the terminal value function gives the value of the soil resource over the remaining time horizon.

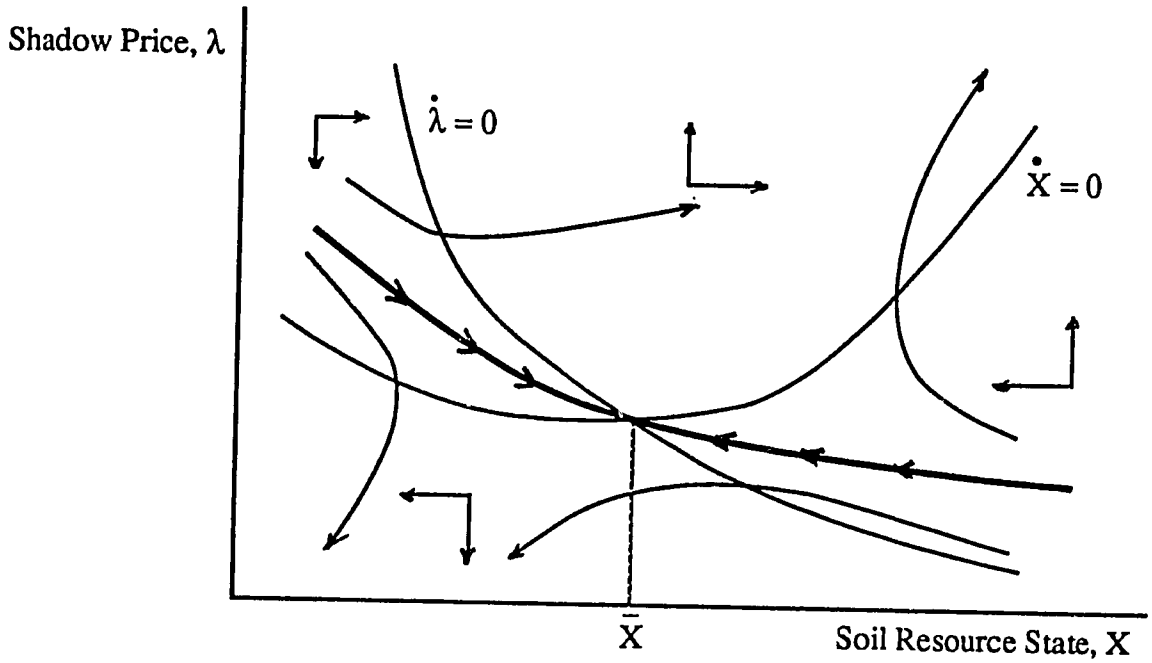

Figure 3a: λ , X Phase Diagram for the General Regeneration Function

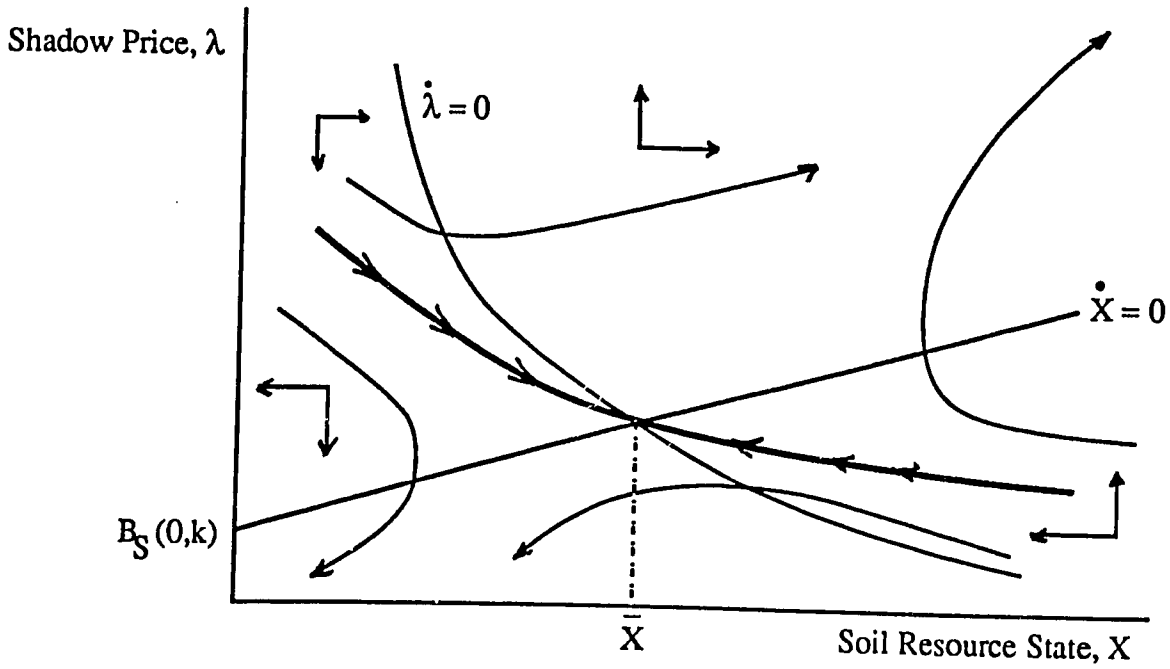

Figure 3b: λ, X Phase Diagram for the Special Case g(X) = k

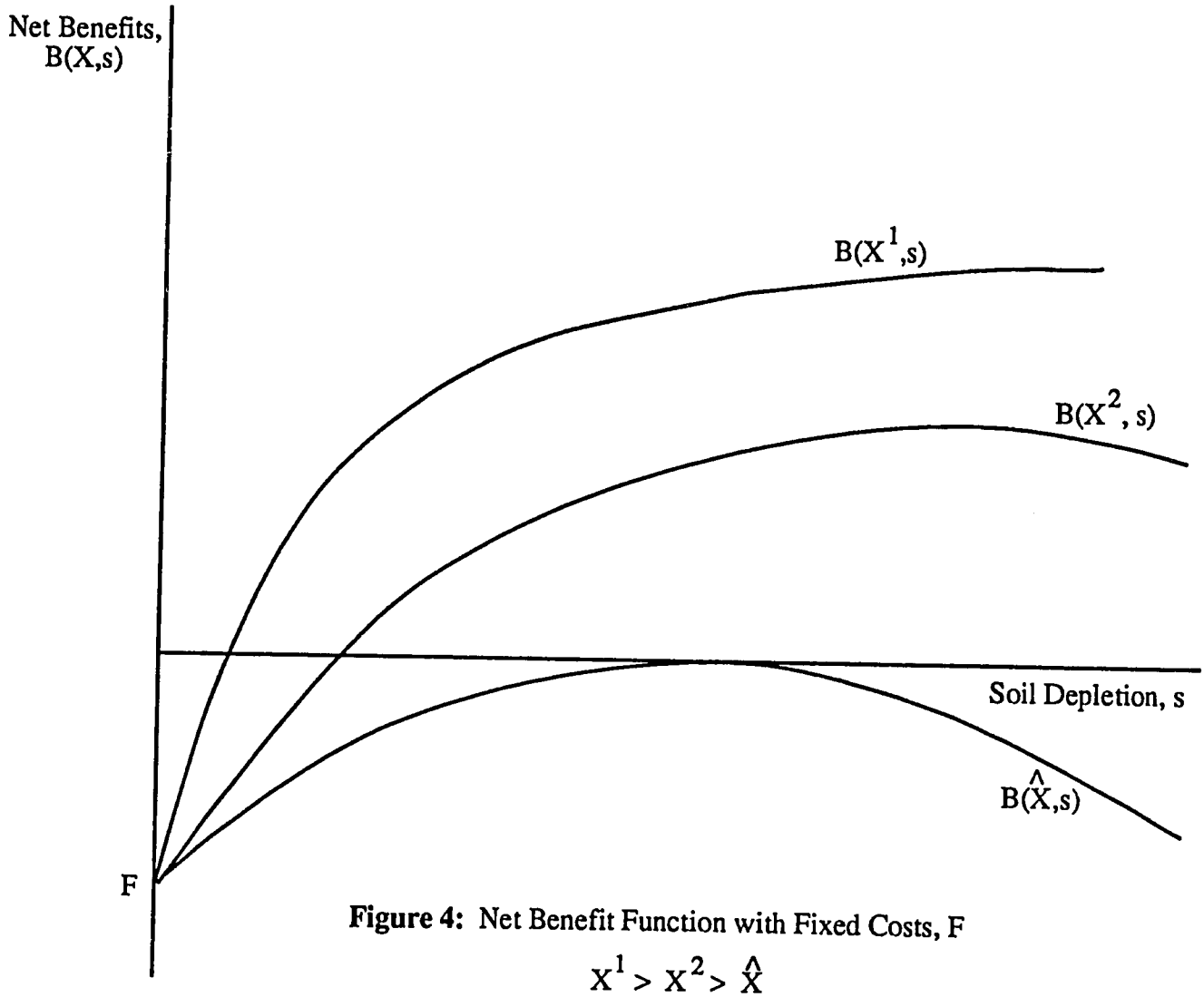

Figure 4: Net Benefit Function with Fixed Costs, F

$X^1 > X^2 > \hat{X}$