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Using Moments to Make Risky Choices: Some Alternative Methods

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Mean-variance is shown to be related to approximation of the certainty equivalent. Two different types of formulas which approximate the certainty equivalent using higher moments are derived and are compared for exponential and power utility functions. Using these formulas, the improvement in decisionmaking from adding the third and fourth moments is also considered.

For the case study presented here, use of four moments does provide a better approximation of the certainty equivalent than two or three moments for risk aversion up to the intermediate level for both utility functions. Even for higher levels of risk aversion, when approximation is not very good, the selection of the best choice from a set of risky choices was made using only three moments. In comparison, mean-variance did not lead to good approximations or rankings for risk aversion above the intermediate level.

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With risk, choice among alternatives is usually represented in theoretical literature by expected utility maximization. In optimization problems such as portfolio choice, because expected utility would have to be approximated by numerical integration, and because optimization is more easily done with quadratic programming than with nonlinear programming, mean-variance analysis has usually been used in empirical work.

The relationship between expected utility maximization and mean-variance analysis has been the subject of a large number of articles. One well-known sufficient condition for equivalence of the two methods is that the utility function be quadratic. Alternatively, Freund (1956) showed that mean-variance and expected utility would correspond exactly if utility is of the exponential form and the random variable underlying utility is normally distributed. Often, however, random variables are not normally distributed.

Even when distributions are not normal, studies such as that by Levy and Markowitz (1979) showed that a formula based on mean and variance has a high correlation with expected utility. The correspondence of portfolios which maximize expected utility to the mean-variance frontier has also been considered by Kroll, Levy, and Markowitz (1984); they showed that portfolios along the frontier give expected utility values very close to the actual maximum values.

One explanation for why mean-variance may give results similar to expected utility is based on the approximation of expected utility by a Taylor series. With mean-variance, only the first and second degree terms in the Taylor series are used. When this approximation is good enough, then mean-variance will provide an adequate decision rule.

Tsiang (1972) showed that in many cases, this approximation is quite adequate. He extended this method to consider the effect of third order terms (skewness) on the Taylor series approximation preference. Considering requirements for the Taylor series to converge to the expected utility value, Tsiang concluded that the third and higher moments may be neglected for the exponential utility function if the standard deviation is less than twice the value of the mean. Convergence for the power function was shown to be slower.

Loistl (1976) also considered the convergence of the Taylor series for the power function and the exponential function. In his examples, as many as ten moments were used to approximate expected utility. He concluded that, because of convergence problems, the use of moments more than mean and variance can even make the approximation worse! The power and logarithmic utility functions were shown to exhibit more serious convergence problems than the exponential function. (The approximation formula used by Tsiang and Loistl approximates expected utility directly, rather than approximating its normalization by marginal utility at a base point; thus it differs from the mean-variance formula used by Levy and Markowitz.)

A different approach to relate mean-variance and expected utility ranking was presented by Meyer (1987). Sufficient conditions given require that random variables being compared differ only with respect to their location and scale. This requirement is actually quite restrictive; i.e. if return risk is due to the product of both output risk and price risk, the required condition would not be satisfied.

The topic of this paper is to consider further the question of how information about moments should be used when choices among risky options are to be made consistent with expected utility. Two alternative

approximation formulas are presented. One formula extends the mean-variance formula to higher moments. This formula is shown to be an approximation of the certainty equivalent. The other formula approximates the certainty equivalent directly. We also demonstrate that mean-variance analysis is unsuitable to compare risky choices when risk aversion is large.

Extending Mean-Variance to Higher Moments

Mean-variance analysis uses the following function of mean and variance to compare risky choices:

(1) $$\bar{U} = E\pi - r/2 \text{ Var } \pi;$$

r is the Pratt-Arrow absolute risk aversion coefficient, $E\pi$ denotes the mean of profit and Var π denotes its variance. For example, to choose among alternative stock portfolios, this function is maximized across alternative portfolio combinations. To obtain the mean-variance frontier, the variance of profit is minimized subject to achieving a given mean value. A specific portfolio choice along the frontier then implies a level of risk aversion.

In comparison to (1), approximation of expected utility in terms of a Taylor series about moments of profit gives the formula

(2) $$Eu \approx u(E\pi) + u''(E\pi)/2 \text{ Var } \pi + u'''(E\pi)/6\, M_3(\pi) + u^{(4)}(E\pi)/24\, M_4(\pi)\cdots$$

where $M_i(\pi)$ denote higher order moments. Applying this formula to the exponential function case, in which

(3) $$u(\pi) = -e^{-r\pi},$$

leads to the Taylor series approximation with higher moments

(4) $$Eu \approx -e^{-rE\pi}[1 + r^2/2 \text{ Var } \pi - r^3/6\, M_3(\pi) + r^4/24\, M_4(\pi)\cdots].$$

For the power function case,

(5) $$u(\pi) = (k + \pi)^{1-s}/(1-s),$$

the expected utility approximation with higher moments is

$$(6)\quad Eu \approx (k + E\pi)^{1-s}/(1-s)\ [1 - (1-s)\{\frac{s}{2(k+E\pi)^2}\ \mathrm{Var}\ \pi - \frac{s(s+1)}{6(k+E\pi)^3}\ M_3(\pi) + \frac{s(s+1)(s+2)}{24(k+E\pi)^4}\ M_4(\pi)\cdots\}].$$

These formulas, corresponding to those presented by Tsiang and Loistl, are in terms of moments. Clearly, even considering only the first two moments, these formulas are different from the mean-variance form (1)!

A formula more similar to the mean-variance form can be derived from (2). First, use the Mean Value Theorem to obtain $u(E\pi)$ in terms of $E\pi$:

$$(7)\qquad u(E\pi) = E\pi\ u'(\xi),\ \text{where } 0 < \xi < E\pi.$$

Also, expanding $u'(\xi)$ about $E\pi$,

$$(8)\quad u'(\xi) = u'(E\pi) + u''(E\pi)\ (\xi - E\pi) + u'''(E\pi)/2\ (\xi - E\pi)^2 + \cdots.$$

Substituting (7) and (8) in the expression (2) for Eu and dividing both sides by $u'(E\pi)$,

$$(9)\quad \frac{Eu}{u'(E\pi)} \approx E\pi + \frac{1}{2}\frac{u''(E\pi)}{u'(E\pi)}\ \mathrm{Var}\ \pi + \frac{1}{6}\frac{u'''(E\pi)}{u'(E\pi)}\ M_3(\pi) + \frac{1}{24}\frac{u^{(4)}(E\pi)}{u'(E\pi)}\ M_4(\pi)\cdots + R(\xi - E\pi)$$

where $R(\xi-E\pi)$ denotes a remainder term in terms of powers of the difference $(\xi-E\pi)$. This remainder term will be positive for risk averse preferences assuming that the Taylor series converges. Denote the right hand side of (9), omitting the remainder term, by $\bar{U}$:

$$(10)\quad \bar{U} = E\pi + \frac{1}{2}\frac{u''(E\pi)}{u'(E\pi)}\ \mathrm{Var}\ \pi + \frac{1}{6}\frac{u'''(E\pi)}{u'(E\pi)}\ M_3(\pi) + \frac{1}{24}\frac{u^{(4)}(E\pi)}{u'(E\pi)}\ M_4(\pi)\cdots\ .$$

$\bar{U}$ will be an underestimate of $Eu/u'(E\pi)$ because of the remainder term in (9). $\bar{U}$ is clearly an extension of the mean-variance formula to higher order moments. The coefficient of $M_3(\pi)$ is the skewness preference coefficient

defined by Tsiang (1972). It shows whether the Pratt-Arrow coefficient is constant, decreasing, or increasing as $E\pi$ increases.

The $\bar{U}$ expressions for exponential and power functions differ in terms of the coefficients of the moments. If the utility is of the exponential form, then the coefficients of the moments for the exponential function are constants which depend on the risk aversion coefficient r:

$$\bar{U} = E\pi - \frac{1}{2} r \operatorname{Var} \pi + \frac{1}{6} r^2 M_3(\pi) - \frac{1}{24} r^3 M_4(\pi) \cdots . \tag{11}$$

For the power function form, the corresponding form is

$$\bar{U} = E\pi - \frac{1}{2} \frac{s}{(k+E\pi)} \operatorname{Var} \pi + \frac{1}{6} \frac{s(s+1)}{(k+E\pi)^2} M_3(\pi) - \frac{1}{24} \frac{s(s+1)(s+2)}{(k+E\pi)^3} M_4(\pi) \cdots . \tag{12}$$

For the power function, the coefficients of the moments are not constant; they vary inversely to the mean profit of a risky choice. In particular, the power function exhibits a decreasing Pratt-Arrow risk aversion coefficient with respect to $E\pi$.

Because $\bar{U}$ approximates expected utility divided by $u'(E\pi)$ and omits the remainder term, assuming its convergence as a series, $\bar{U}$ could be regarded as ranking alternatives equivalently to expected utility only under certain conditions :

(i) all alternatives have nearly the same mean; or $u'(\pi)$ is nearly constant (because u is nearly linear);

(ii) the remainder term is small relative to $\bar{U}$ (because $E\pi$ is small or because risk preference coefficients are small).

However, as in the Levy and Markowitz results, $\bar{U}$ could represent preferences adequately in other cases. Conditions for this to occur are explored below, both theoretically and empirically.

Approximating the Certainty Equivalent

Below, we show that $\bar{U}$ in (11) and (12) is actually an approximation of the certainty equivalent, respectively for the exponential power functions. When this approximation is close, since the certainty equivalent ranks outcomes equivalently to expected utility, then $\bar{U}$ would provide a ranking of risky alternatives similar to the expected utility ranking, thus explaining why the mean-variance formula (1) may often result in rankings similar to expected utility. To show the relation between the certainty equivalent and $\bar{U}$, an expression approximating the certainty equivalent directly in terms of moments is first described and then its relation to $\bar{U}$ is developed.

In the exponential function case, substituting the Taylor series approximation (4) for Eu in the definition of the certainty equivalent (CE),

(13) $$-e^{-rCE} = Eu$$

$$\approx -e^{-rE\pi}[1 + \frac{r^2}{2}\,\mathrm{Var}\,\pi - \frac{r^3}{6}M_3(\pi) + \frac{r^4}{24}M_4(\pi)\cdots].$$

Denote the term in brackets by [1+K]. Note that K is defined by

$$K = r\,(E\pi - \bar{U}).$$

Solving for the certainty equivalent above, the certainty equivalent is approximated by:

(14) $$CE \approx E\pi - \frac{1}{r}\,\ell n(1+K).$$

To examine further the relationship between $\bar{U}$ and (14), expand the Taylor series of $\ell n(1+K)$ about $K = 0$:

(15) $$\ell n(1+K) = K - K^2 + K^3 - \cdots .$$

Ignoring K^2 and higher order terms,

(16) $$CE \approx E\pi - \frac{1}{r}\,[\frac{r^2}{2}\,\mathrm{Var}\,\pi - \frac{r^3}{6}M_3(\pi) + \frac{r^4}{24}M_4(\pi)\cdots]$$

$$= E\pi - \frac{r}{2} \text{Var } \pi + \frac{r^2}{6} M_3(\pi) - \frac{r^3}{24} M_4(\pi) + \cdots.$$

$$= \bar{U}.$$

That is, if K is sufficiently small, then $\bar{U}$ approximates the certainty equivalent.

Similarly for the power function, substituting (6) for Eu in the definition of the certainty equivalent:

$$(17) \qquad \frac{(k+CE)^{1-s}}{1-s} = Eu$$

$$\approx \frac{(k+E\pi)^{1-s}}{1-s} [1 - (1-s)\{\frac{1}{2} \frac{s}{(k+E\pi)^2} \text{Var } \pi - \frac{1}{6} \frac{s(s+1)}{(s+E\pi)^3} M_3(\pi) + \frac{1}{24} \frac{s(s+1)(s+2)}{(k+E\pi)^4} M_4(\pi) \ldots\}].$$

Denote the term in brackets by [1-G]. Note that

$$G = (E\pi - \bar{U})\ (1-s)/\ (k + E\pi).$$

Then solving for the certainty equivalent, the approximate CE is:

$$(18) \qquad CE \approx [1-G]^{1/(1-s)}\ (k+E\pi) - k.$$

To relate the approximate CE and $\bar{U}$, expand (18) about G = 0:

$$(19) \qquad [1-G]^{1/(1-s)} \approx 1 - \frac{1}{1-s} G + \frac{s}{(1-s)^2} G^2 \ldots .$$

Substituting (19) in (18),

$$(20) \qquad CE \approx (k+E\pi)[1 - \frac{1}{1-s} G + \frac{s}{(1-s)^2} G^2 \cdots] - k.$$

Then, for G sufficiently small, ignoring terms of higher order than G,

$$(21) \qquad CE \approx E\pi - \frac{1}{2} \frac{s}{(k+E\pi)} \text{Var } \pi + \frac{1}{6} \frac{s(s+1)}{(k+E\pi)^2} M_3(\pi) - \frac{1}{24} \frac{s(s+1)(s+2)}{(k+E\pi)^3} M_4(\pi) + \cdots$$

$$= \bar{U}.$$

Thus, in both exponential and power function cases, $\bar{U}$ of the appropriate form ((11) or (12)) is actually shown to be an approximation of the certainty equivalent. For this approximation to be a good one, the appropriate series of moments for $\bar{U}$ must converge and K or G must be sufficiently small. Since the certainty equivalent ranks equivalently to expected utility, these conditions (less restrictive than those presented in the previous section) then describe when $\bar{U}$ will rank risky choices similarly to expected utility.

Alternatively, the certainty equivalent can be approximated directly from (14) or (18). In this case, the only requirement for a good approximation of the certainty equivalent is that K or G must simply be well-defined (not necessarily small).

Comparison of Approximation Methods

Above, two different methods of approximating the certainty equivalent were presented for both the exponential and power utility functions. Satisfaction of the requirement for one method, that K (or G) be "sufficiently small", is relative to the particular application. Here, we use examples to compare both methods for both exponential and power functions.

To compare methods of alternative types, approximated certainty equivalent values are tested against actual certainty equivalent values. Approximation with three and four moments is compared with mean-variance. In addition to the degree of approximation, we also consider whether correct choices (in terms of picking the best choice from a set) would be made using the alternative methods.

Rather than evaluating portfolios, a simpler problem is studied. Seven choices concerning corn yield risk are to be ranked. The seven choices are six insecticide treatments and "no treatment"; each choice produces a different distribution of yield and profit risk. The treatment with the greatest certainty equivalent for profit per acre (revenues minus cost of treatment and other variable costs) is to be selected. Although yield distributions are negativegly skewed (represented by Beta distributions), profit distributions are differently skewed because of the presence of price risk (more details are given in Loehman, Boyd, Turpin, and Tongo, 1989).

Table 1 shows the moments of profit corresponding to the alternative treatments; profit distributions with both positive and negative skewness are included. Tables 2-4 and 5-7 compare the actual certainty equivalent values with approximations for the exponential function and the power function. Treatments do not satisfy Tsiang's convergence requirement that the standard deviation be less than twice the mean. Regardless, approximations are close to actual certainty equivalent values for lower levels of risk aversion.

Three different approximations are compared in Tables 2-7:

(i) mean-variance formula (1);

(ii) approximate certainty equivalent ((15) or (18) as appropriate) with four moments;

(iii) $\bar{U}$ (formula (11) or (12) as appropriate) with four moments.

Appendix Tables A1-A6 similarly compare actual certainty equivalents with approximate CE and $\bar{U}$ using three moments in each case.

Table 1. Moments of Profit per acre ($) for Pesticide Choices

treatment a	pesticide	mean	variance	skewnesss	kurtosis
0	Untreated	143.7	6842.6	0.126	2.62
1	Counter	152.1	7588.2	0.214	2.64
2	Dyfonate	149.4	7638.8	-0.014	2.57
3	Furadan	149.2	6905.6	0.121	2.65
4	Lorsban	158.2	8233.7	-0.063	2.58
5	Mocap	140.5	6619.4	0.069	2.67
6	Thimet	150.6	7770.4	0.092	2.59

Table 2. Exponential utility function, r = .00565

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	124.93	124.37	124.86	123.82
1	131.58	130.66	131.51	130.27
2	127.97	127.82	127.99	126.64
3	130.21	129.69	130.17	129.11
4	134.89[b]	134.93[b]	134.96[b]	133.37[b]
5	122.09[w]	121.80[w]	122.10[w]	121.11[w]
6	129.18	128.64	129.16	127.80

Table 3. Exponential utility function, r = .01130

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	107.44	105.03	107.62	99.16
1	113.04[b]	109.22	113.10	103.09
2	107.41	106.23	108.24	97.00
3	112.47	110.17	112.72	104.05[b]
4	112.31	111.67[b]	113.56[b]	100.13
5	104.66[w]	103.09[w]	105.02[w]	96.85[w]
6	109.31	106.69	109.71	98.63

Table 4. Exponential utility function, r = 0.02260

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	77.64	66.36	82.96	13.31
1	82.16[b]	66.34	87.38	5.30
2	71.67[w]	63.06	81.85	-10.02
3	82.02	71.15[b]	87.81[b]	16.24[b]
4	72.87	65.14	86.13	-23.15[w]
5	74.01	65.68	80.38[w]	12.55
6	75.91	62.78[w]	83.44	-7.07

*four moments used
[b]best choice
[w]worst choice

Table 5. Power function, s = 1.5

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	122.03	124.37	122.74	121.41
1	129.42	130.66	129.90	128.45
2	123.56	127.82	125.74	124.07
3	127.84	129.69	128.47	127.20
4	130.22[b]	134.93[b]	133.21[b]	131.40[b]
5	118.64[w]	121.80[w]	119.68[w]	118.36[w]
6	125.88	128.64	127.13	125.50

Table 6. Power function, s = 3

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	95.25	105.03	102.61	90.63
1	101.43[b]	109.22	108.79	95.82
2	83.60	106.23	103.23	88.04
3	100.66	110.17	108.44	96.97[b]
4	82.22[w]	111.67[b]	109.44[b]	92.92
5	88.90	103.09[w]	99.56[w]	87.50[w]
6	93.54	106.69	104.90	90.16

Table 7. Power function, s = 6

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	38.29	66.36	78.87	-18.53
1	38.61[b]	66.34	84.10	-21.36
2	-25.61	63.06	78.78	-41.51
3	38.03	71.15[b]	84.31[b]	- 9.50[b]
4	-49.72[w]	65.14	84.15	-46.01[w]
5	7.71	65.68	75.99[w]	-22.28
6	12.46	62.78[w]	80.29	-37.38

*four moments used
[b]best choice
[w]worst choice

A simulation program (SLAM) was used to compute certainty equivalents and moments given specified probability distributions. With the availability of simulation packages and other computer methods, there is little difference in computational difficulty for approximations using two, three, or four moments, or using different utility functions, or using different types of approximations.

The well-known advantage of the power function model is that preferences are independent of scale; that is, if treatment is to be considered for farms of different sizes, then multiplying profit in (5) by a constant A (acres planted) would not change the ranking of preferences whereas (3) would be affected by such scale changes. Similarly, changes in monetary units (eg. due to inflation) would not affect ranking.

To make results comparable for the two different utility functions, risk preference parameters were defined so that the Pratt-Arrow risk aversion coefficients for exponential and power functions are the same for the base (untreated) case with mean value $E\pi^0$. The following association between the coefficients in the exponential and power function models is made for this purpose:

$$r = \frac{s}{(k+E\pi^0)}.$$

Risk aversion levels for s values of 1.5, 3, and 6 were used in the analysis. s values of about 1.5 are considered to represent intermediate levels of risk aversion whereas levels of three or higher represent severe risk aversion (Binswanger, 1980).

For an intermediate level of risk aversion and exponential utility, ($r = .00565$), the approximate CE was closer to the actual certainty equivalent than $\bar{U}$. For corresponding risk aversion ($s = 1.5$) for the power

function, $\bar{U}$ generally gave a better approximation of the actual CE value than the approximate CE. All methods correctly selected the best and worst outcomes. We may infer that these conclusions would also hold for lower levels of risk aversion.

For more severe risk aversion levels ($r = .0113$ and $r = .0226$) and exponential utility, the approximate CE again gave a better fit than $\bar{U}$ to the actual certainty equivalent. For the most risk averse case ($r = .0226$), neither approximation was very good. For corresponding risk aversion ($s = 3$) for the power function, a better fit to the actual CE values was obtained with $\bar{U}$ than with the approximate CE. For the most risk averse case for the power function ($s = 6$), both the CE approximation and $\bar{U}$ were very different from the actual CE.

Generally for both the exponential and power functions and all levels of risk aversion, neither method of approximation selected the best outcomes correctly except in the lowest risk aversion cases, but the selected choices were close in value to the actual best choices.

As expected, use of three moments gave a worse approximation of the certainty equivalent than four moments (too large because of the alternating signs in (11) and (12)). However, surprisingly, the selection of the best outcome in each risk aversion case was made correctly with three moments and $\bar{U}$ for both the exponential and power functions. The worst outcome was not always correctly identified.

Conclusions

Two different methods of approximating the certainty equivalent were compared for exponential and power utility functions. $\bar{U}$ is a direct extension of mean-variance to higher moments. The approximate CE is a different type of formula than mean-variance which is also expressed in terms of moments.

Comparing the degree of approximation for alternative methods with a case study, the approximate CE with four moments gave the best estimate of the actual certainty equivalent for intermediate and higher levels of risk aversion for the exponential utility function. For intermediate risk aversion and the power function, $\bar{U}$ with four moments (of the appropriate form) gave the best approximation. Approximation of the certainty equivalent for severe levels of risk aversion was generally not very good with any method. Similar to findings in other studies, approximation was generally better for the exponential utility function than for the power function because of convergence problems.

In real applications, correct selection of the best choice from a set of risky choices is the main goal of analysis and approximation is just a means to this end. Therefore here the ranking of risky choices was of concern in addition to the accuracy of alternative methods in approximating the certainty equivalent. For our application, the actual best choices were correctly selected for all risk aversion levels and both preference models using $\bar{U}$ of the appropriate form with only three moments. Apparently, correct selection of the best choice can be made even when approximations are not very close to the actual certainty equivalent values!

For the case study presented here, mean-variance analysis both approximated certainty equivalents and ranked choices adequately for low to intermediate levels of risk aversion. However, mean-variance analysis did not rank alternatives appropriately for higher levels of risk aversion.

Future work should test the generality of these conclusions with other case studies. If these results generalize to other applications, then the implication for empirical work is that mean-variance analysis should be extended at least to include skewness when distributions are not normal, particularly for higher levels of risk aversion. The appropriate formulas to use for this purpose, depending on the form of the utility function, were presented here.

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Appendix: Three Moment Evaluations

Table A1. Exponential utility function, r = .00565

Treatment	Actual CE	Mean-variance	Approx CE*	$\bar{U}$*
0	124.93	124.37	125.69	124.75
1	131.58	130.66	132.53	131.41
2	127.97	127.82	128.99	127.77
3	130.21	129.69	131.02	130.06
4	134.89[b]	134.93[b]	136.12[b]	134.68[b]
5	122.09[w]	121.80[w]	122.90[w]	121.99[w]
6	129.18	128.64	130.20	128.98

Table A2. Exponential utility function, r = .01130

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}$*
0	107.44	105.03	112.68	106.55
1	113.04[b]	109.22	119.19	112.23[b]
2	107.41	106.23	114.11	106.03
3	112.47	110.17	117.90	111.65
4	112.31	111.67[b]	120.16[b]	110.67
5	104.66[w]	103.09[w]	109.86[w]	103.88[w]
6	109.31	106.69	115.84	108.03

Table A3. Exponential utility function, r = 0.2260

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}$*
0	77.64	66.36	101.24	72.44
1	82.16[b]	66.34	108.71[b]	78.38[b]
2	71.67	63.06	101.25	62.26
3	82.02	71.15[b]	106.41	77.06
4	72.87[w]	65.14	106.82	61.13[w]
5	74.01	65.68	97.90[w]	68.85
6	75.91	62.78[w]	104.05	68.14

*three moments used
[b]best choice
[w]worst choice

Table A4. Power function, s = 1.5

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	122.03	124.37	125.94	125.00
1	129.42	130.66	133.50	132.50
2	123.56	127.82	129.37	128.19
3	127.84	129.69	131.58	130.67
4	130.22[b]	134.93[b]	137.04[b]	135.76[b]
5	118.64[w]	121.80[w]	122.85[w]	121.91[w]
6	125.88	128.64	130.87	129.73

Table A5. Power function, s = 3

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	95.25	105.03	113.24	107.06
1	101.43[b]	109.22	120.68	114.30[b]
2	83.60	106.23	114.83	106.88
3	100.66	110.17	118.84	112.86
4	82.22[w]	111.67[b]	121.57[b]	112.88
5	88.90	103.09[w]	116.95	103.72[w]
6	93.54	106.69	104.90[w]	109.50

Table A6. Power Function, s = 6

Treatment	Actual CE	Mean-variance	Approx* CE	$\bar{U}^*$
0	38.29	66.36	102.52	73.45
1	38.61[b]	66.34	110.55[b]	82.17[b]
2	-25.61	63.06	102.67	63.99[w]
3	38.03	71.15[b]	107.85	79.35
4	-49.72[w]	65.14	108.60	65.75
5	7.71	65.68	98.90[w]	68.56
6	12.46	62.78[w]	105.68	70.94

*three moments used
[b]best choice
[w]worst choice

COVER SHEET

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