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DISCONTINUOUS MODELS FOR RAPID CORRELATION, INTERPRETATION, AND UTILIZATION OF SOIL ANALYSIS AND FERTILIZER RESPONSE DATA

INTERNATIONAL SOIL FERTILITY Evaluation & Improvement PROGRAM

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This bulletin combines original research data on soil fertility evaluation with the development of new concepts in handling such data. The illustrations presented to demonstrate the use of these concepts were drawn from research data obtained in Bolivia during the time the senior author served as co-worker in the field trials and Regional Director of the International Soil Fertility Evaluation and Improvement Project. All field studies were cooperative achievements and deep appreciation is expressed for the cooperative spirit, professional dedication, and hard work on the part of the staff of the Department of Soils, Agricultural Experimental Stations, Extension Service, and various national, regional, and local leaders within the Ministry of Agriculture of Bolivia. It was with their help, together with the cooperation of the Rural Development Division of USAID/Bolivia, the Utah State University MIssion and the British Tropical Agriculture Mission that the original research data were obtained. Beyond the original data the concepts and use of the discontinuous models for correlation and interpretation are largely the responsibility of the authors.

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INTRODUCTION

The basic purpose of soil fertility evaluation is to provide information on the nutrient status of the soil and predict the relative response to added nutrients. The nutrient status of cropped soils is variable and continually changing due to the influence of fertilizer addition, nutrient losses by leaching or crop removal, and overall management. Site-specific estimates of the current fertility status of a soil are, therefore, very important to rational fertilizer use. Reliable site-specific information can only be accomplished through an orderly program of soil fertility evaluation in which proper attention is given to the following:

1) Techniques of soil sampling; 2) methods of soil analysis; 3) systems for correlation of soil analysis and crop response; 4) models for the interpretation of fertilizer response in field trials, and 5) procedures for preparing economically sound fertilizer recommendations.

While these points in soil fertility evaluation are basic and clearly important, there is considerable divergence among agronomists as to methods, systems, procedures, and models which are employed to accomplish each phase. The last three phases (correlation, interpretation, and recommendations) are probably the most troublesome, due in large part to difficulty in describing the complex phenomena involved. Complex models often encounter difficulties in practice due to the lack of proper and sufficient input data for deviations of the estimates of the parameters of the system.

There continues to exist a need for rapid and accurate techniques which permit on-the-spot correlation, interpretation, and recommendations. In countries where the International Soil Fertility Evaluation and Improvement project (ISFEI) has been active there has developed a considerable amount of interest in discontinuous models as a means

of implementing a soil fertility evaluation program. In these models, yield response to each nutrient is considered to have a starting point (threshold yield) and an ending point (plateau yield). Relative yield is then defined as threshold yield divided by plateau yield. Correlation of relative yield with soil analysis for a particular nutrient is accomplished either graphically or mathematically, identifying a critical level which separates two distinct and discontinuous soil-crop categories. Field trials pertaining to a specific soil-crop category are then grouped together for interpretation of fertilizer response.

The interpretation of the rate of fertilizer application required to achieve a specific yield response is accomplished with a discontinuous linear response-and-plateau model. In this technique, response to each nutrient is interpreted separately by using the threshold and plateau yield levels. The linear response-and-plateau lines are established and the point of intersection is used to determine the relationship between yield and application rate for each nutrient. The threshold yield for each nutrient and the final plateau yield for the experiment (s) are associated with a specific nutrient or nutrient combination. In effect, this results in a discrete number of recommendation choices, the number being one greater than the number of limiting nutrients. These recommendation choices are easily subjected to step-wise economic evaluation.

The purpose of this bulletin is to outline the theoretical and practical implications in the use of the above-mentioned discontinuous models. The illustrations in the step by step presentation of the use of the various models are based on actual research data obtained in a series of field experiments in Bolivia. Other research data also appear in the text to illustrate the overall concepts which have been developed. It is the hope of the authors that at least some of the new soil fertility research may be directed toward a more complete testing of these concepts against more traditional methods.

I. THE FERTILIZER RESPONSE MODEL IN REVIEW

Most discussions in the literature on fertilizer response are based on field trial interpretation utilizing some version of the Law of Diminishing Returns, i.e., continuous curvilinear functions in which equal additional increments of fertilizer result in steadily smaller responses. The more important curvilinear models in use today are the quadratic, square root, and logarithmic functions. The Mitscherlich model adds constants to the logarithmic function to adjust the interpretation of fertilizer response in accordance with variations in native soil fertility.

With the aid of the computer many additional factors, which are known to affect yield, have been incorporated into these response models in an attempt to further describe the response to applied fertilizers. Each new attempt to characterize the plant growth and fertilizer response model has been thoroughly subjected to economic interpretation in order to provide guidelines for fertilizer use under highly specified conditions. Yet, for all the refinement potentially inherent in these complex models, their use by economists and agronomists is impractical in areas where fertilizer use is relatively new and the science of fertilizer use is in preliminary stages.

There are no established rules for determining which kind of response model is the most appropriate to use in the interpretation of fertilizer response from field trials. If a single experiment is to be evaluated, the researcher may test the data for fit with various linear and curvilinear models and select the function which gives the best R^2 for regression. However, a model which best describes the response data from one experiment may not necessarily be the best for another experiment. Likewise, the more complex models which accurately describe what happened in a given set of response data, for which detailed input information is available, may be of little practical use for predicting future responses when input data is limited or uncertain. This places the user of

such a complex model in a difficult position because the whole purpose of soil fertility evaluation is the accurate prediction of relative response to nutrients under existing conditions.

Difficulties With the Traditional Fertilizer Response Model

In practice a number of problems are encountered in attempting to apply complex fertilizer response models to existing data. Several of these difficulties are summarized below.

Insufficient Information Available to Correctly Apply the Fertilizer Response Model

Yield is a function of many factors which affect the interaction between the genetic potential of the plant and the environmental system in which it is grown (Fitts, 1955). Soil, crop, climate, and management represent four broad categories of factors which are known to affect yield and each of these may be divided into numerous sub-factors. Mathematical models which predict yield must either assign a researched value to each of these input factors or apply some kind of constant in their place when such a value is not available. Most field trials conducted in the past, and even many currently being carried out, simply do not supply sufficient input information to permit the practical application of a detailed, complex response model.

Bias in the Interpretation Model

Most response models have some tendency for bias under certain kinds of conditions. Anderson and Nelson (1971) have described in some detail the limitations and difficulties associated with the quadratic model for describing some single nutrient responses. Of special importance in their study was the observation that the quadratic is particularly biased when there is a marked response to first increment applications followed by little or no response to higher rates. In these cases, the quadratic characteristically

overestimates the measured yield at the zero rate of applied nutrient, then underestimates the yield at the first nutrient rate and overestimates the yield at intermediate rates. The predictions of fertilizer rates required for maximum yield are, therefore, unrealistically high. If a traditional economic marginal analysis is applied to the quadratic under these situations, the predicted application rate which would be most profitable varies considerably with changes in the cost/price ratio, resulting in certain working difficulties.

Variability in Yield Observations Obtained at High Rates of Nutrient Application

Still another difficulty in interpretation revolves around the crop yield or response at high rates of application. Most continuous curvilinear response models are very sensitive to fluctuations in yield at these higher rates. If, for example, high application rates cause a fertilizer burn or create a nutrient disequilibrium there will likely be a yield depression. The quadratic function usually fits this situation quite well. On the other hand, the same high rates of nutrient in question, if applied in a different manner or in a different configuration of nutrient balance, may give yields equal-to or higher-than intermediate rates. The square root or log function usually fits this kind of response better than the quadratic. Unfortunately, factors other than nature of plant growth and nutritional responses are more likely to affect the response at high application rates than at low, such that the model which gives the best fit of the data for one experiment may give the poorest fit for another. The lack of consistency in performance of a fertilizer response model may go unnoticed if one is primarily interested in describing, after the fact, the results of one experiment or the pooled response of a set of experiments. Inconsistency, however, cannot be ignored where predictions of future responses must be made, and where only one model can be used to make that prediction.

Difficulty in Practical Analysis of Complex Response Models If Computers Are Not Available

In many parts of the world field experiments can be conducted on a year-round basis. The time available for analyzing one set of experiments and planning for another is often very limited. Complex curvilinear response models are not very practical for fertilizer response interpretation if a computer is not available.

Alternate Models for Use in Interpretation of Fertilizer Response Data

Swanson (1963) has pointed out an alternative model based on Liebig's Law of the Minimum which should be considered for interpretation of fertilizer response data. This model postulates a linear response to the principal limiting element, halting abruptly when another factor becomes limiting but resuming its linear rise when the limitation is corrected. Eventually, the yield is limited by the genetic capability of the plant when all external limiting factors have been removed. An idealized diagram of these responses and limiting factors is depicted graphically in Figure 1.

Boyd (1970) has summarized a considerable amount of fertilizer response data covering different crops and has shown that most data fit the Liebig-type model very well. In his study most of the fertilizer trials could be characterized by a sloping straight line on the ascending portion of the curve (the yield response) and a horizontal line representing maximum yield (the yield plateau). Bartholomew (1972), summarizing nitrogen response data from large numbers of individual sites and groups of sites, concluded that the response function, while not always strictly linear in the slope segment, can nevertheless best be described with a dual function consisting of a linear response slope and a horizontally linear plateau.

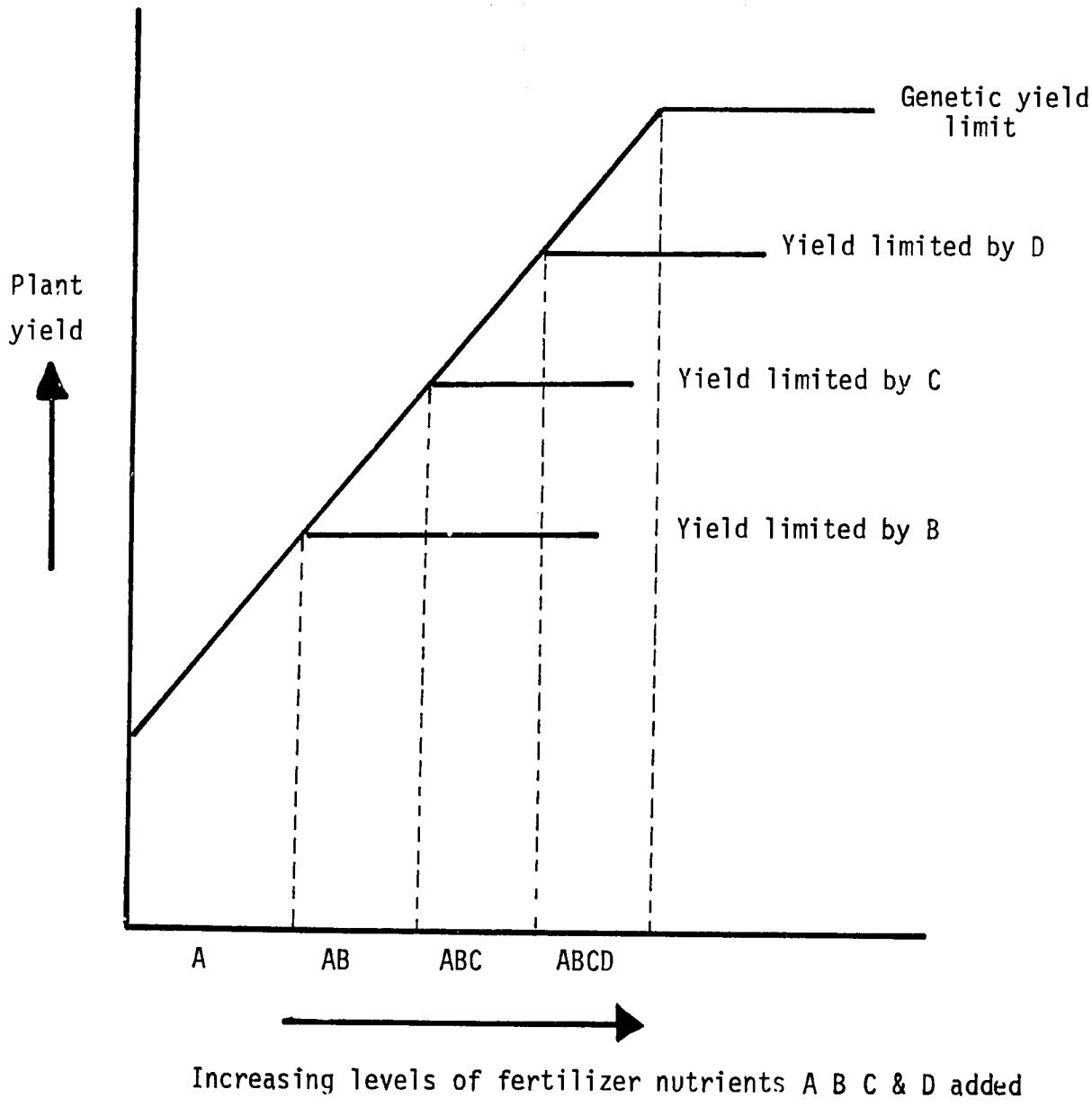

FIGURE 1. The linear response-and-plateau (LRP) function as based on Liebig's Law of the Minimum.

The discontinuous nature of a "linear response-and-plateau" model permits an easy interpretation of fertilizer response data because the point of intersection of the two lines can readily be determined, providing an estimate of nutrient requirement to reach a given yield. Such an estimate would represent a biological optimum (Bartholomew, 1972) being the minimum rate that would be required to reach the plateau yield. A discussion of the economic implications of such a model will be given in Section V.

The Development and Use of a Linear Response-and-Plateau (LRP) Model

The remainder of this bulletin is devoted to a discussion of the theoretical and practical implications of a simple linear response-and-plateau technique to interpret fertilizer response data and determine fertilizer needs. The concepts of yield and yield response to fertilizer as related to soil analysis are discussed next as background to the subject.

II. YIELD, THE LIMITS OF YIELD RESPONSE, AND RELATIVE YIELD

The distinction between yield, yield response, and relative yield is somewhat of a stumbling block to the understanding of the goal and purpose of soil analysis and soil fertility evaluation. Anyone familiar with reading a graph can readily understand the meaning of a plot of yield vs. fertilizer added. The concept of yield increase as a function of fertilizer added seems simple enough, but difficulty often arises when trying to differentiate the limits of response due to a single nutrient from that due to a mixture of nutrients. The meaning of a plot of relative yield vs. soil analysis may be even more troublesome to explain because it involves ratios. The discussion below is provided as a background to the concept and use of these terms.

Yield

Yield is a function of many factors which affect growth. Genetically each species of plant has a certain total potential to transform mineral nutrients, light, carbon dioxide, and water into dry matter. How well any plant can transform these basic ingredients into yield depends on the degree to which all growth requirements are met and balanced. Agronomically, the yield of crops can be related to a wide range of factors as shown in Equation A (Fitts, 1959):

A. Yield = f(crop, soil, climate ....., management)

In soil fertility evaluation work it should be clear from the above equation that a measurement of a given plant nutrient in the soil will not in itself provide enough information to predict what size of yield can be obtained. Clearly, the contribution of all the other factors would have to be quantified mathematically if this were to be done. The goal of soil fertility evaluation is to provide

guidance as to the expected contribution of particular soil nutrients and to predict the likelihood of getting a profitable yield response from the addition of specific fertilizer nutrients and other amendments. Much of the value of the information supplied by soil analysis depends directly on prior experimentation regarding how much a given crop will yield under a known soil fertility condition and under a given set of circumstances. Yield response to added fertilizer must, therefore, be related to crop-specific and site-specific situations.

The Limits of Yield Response to Added Nutrients

Positive yield response to added nutrients can be thought of as resulting from the correction of nutrient deficiency. Each deficiency has a beginning and an end. The farmer's fertilizer dollar should go first to correct the first-order deficiencies because this provides the best return on his money and minimizes his risk. He should then proceed to correct other deficiencies in their order until appropriate economic limits are reached. For each higher yield goal he will likely have a wider range of potential deficiencies to deal with simultaneously. His overall goal must be to add fertilizer nutrients (each according to its degree of limitation and the economics of its correction) so as to eliminate multiple deficiencies simultaneously.

The overall nutrient response which can be obtained will depend heavily on the crop varieties being grown. Varieties developed through selection under infertile soil conditions usually have limited capacities for response to fertilization. On the other hand, modern varieties developed especially for high soil fertility conditions respond markedly to fertilization when placed under favorable growth conditions.

The significance of the gross differences between varieties in their response to fertilization cannot be overemphasized. The plant breeder must define the growth potential of his varieties in terms of soil fertility, and the soil fertility specialist must define soil fertility in terms of the varieties being grown.

This sets the tone of the task for the agronomist who will be called upon to advise the farmer regarding his fertilizer program. The agronomist must evaluate the extent of each of the several deficiencies individually and collectively, ranking and testing the limits of each deficiency so as to determine the amount and ratio of the fertilizer formula most profitable.

The fact that there may be several nutrient deficiencies at the same time, requiring simultaneous but not necessarily equal corrective treatments, means that evaluations of individual nutrient responses must take into account the levels of other nutrients and growth factors which could also affect yield.

Threshold yield

Each soil can be thought of as having a certain level of availability of each nutrient and this marks the lower limit or origin of the response function. According to Liebig's Law of the Minimum the most deficient nutrient will limit the yield to a base or threshold level determined by the level of availability of that nutrient. Except for unusual interactions (see p.43), the yields will not go above this threshold level when fertilizer is applied unless the most deficient nutrient is among those added. The term "threshold yield" [1/] is suggested as a convenient way to describe the yield limited by a single nutrient because it marks the beginning point from which the response to that nutrient should be measured.

Threshold yield refers to the starting point of yield response to a given nutrient under specified conditions. On a graph of yield response to additions of a nutrient it will be the origin of the response function. Experimentally the threshold yield will be established by the treatment in which the nutrient in question is

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[1/] Threshold by definition means the point of beginning, outset, or a place of entering into a physiological effect.

deleted from an otherwise complete or adequate fertilizer application. It follows that the higher the availability of a given nutrient in the soil the higher will be the resultant threshold yield (assuming the other growth factors are not limiting). The direct relationship between the availability of the nutrient in the soil and the threshold yield for a nutrient will be discussed in more detail in reference to relative yield vs. soil analysis correlations.

Plateau Yield

On the upper limit of the response function it is necessary to establish a point where the response to the added nutrient ends. The end point of yield response is established experimentally by the treatment combination(s) where all nutrients are supplied in adequate but not excessive amounts, and where other growth factors are controlled to the extent practical for the situation. Typically, the apparent maximum yield for any given experiment tends to take the form of a plateau or stable area within a certain range of the added test nutrient. Inasmuch as the maximum yield for a given situation is not an absolute maximum for all conditions but only for the specific set of circumstances of the experiment, it is better to use the term "plateau yield" [2/] rather than maximum yield. Plateau yield implies that the yield is at its highest point for the particular conditions and is holding steady over a certain range of nutrient addition. Plateau yield is a meaningful term for describing the yield at which the response to a certain nutrient ceases. Furthermore, plateau yield implies that, while representing the highest obtained, the yield is reflecting the limitations imposed by a given set of growth conditions at a particular site. If these conditions were changed, the plateau yield would also change.

[2/] Plateau means by definition a relatively level area higher than the surrounding area; a uniform or stable area where the progressive advance of a process or condition levels off.

Thus, for a particular nutrient, crop and soil, there is a definite lower limit where the response to fertilization begins (threshold yield) and an upper limit where the response ends (plateau yield). This concept is straightforward when working with positive yield response to fertilization. Yield depression (negative response) to additions of a particular nutrient also has a threshold initial point; the end point, however, is zero. The handling of yield depression data will be discussed in conjunction with relative yield.

The Relative Yield Concept

Relative yield has been more commonly termed percentage yield, expressing the yield with no test nutrient added as a percentage of maximum yield. This measure has been discussed rather thoroughly in the literature as the Mitscherlich-Spillman-Baule-Bray school of thought. Relative yield is a measure of the yield response to a single nutrient when the other nutrients are supplied in adequate but not excessive amounts, and when other site variables, such as climate, are held constant. The main justification for its use is that, being a ratio rather than an absolute amount, it is possible to compare relative yield from different sites.

In the preceding discussion the initial and upper limits of yield response are termed threshold and plateau yields. These two yield parameters can be used to define the relative yield for nutrient X as shown in Equation B:

B. $$\text{Relative yield (X)} = \frac{\text{Threshold yield for X}}{\text{Plateau yield for X}} \times 100$$

The relative yield of a crop at a given site expresses the threshold yield as percentage of the plateau yield. This means that the starting point of the response is being compared to the end point of the response. When threshold yield is small compared to plateau yield there is obviously a large yield response to the addition of the nutrient in question. This results in a low relative yield

because the threshold yield was low compared to the plateau yield. On the other hand, a high relative yield occurs when the threshold yield and plateau yields are similar. Very little response to the nutrient additions results when the threshold yield for the soil is already at or near the plateau yield.

Relative yield will not exceed 100 if these concepts of threshold and plateau yields are followed. If yield depression to added nutrient does occur, the threshold yield (no test nutrient added) should be used as the best measure of plateau yield. This, of course, gives a relative yield of 100. The depressed yields should not be used as divisor because they represent a negative response, with a declining, rather than a stable, basis for comparison. When there is a yield depression to an added nutrient the depression will normally become increasingly severe as further additions are made. If carried to extremes, the end yield would be zero. If any of these depressed yields were to be used as the divisor to calculate relative yield the results would be related primarily to the scale and rate of application of the nutrient and not to the soil analysis for that nutrient. This means that yield depression data are not adequately appraised by relative yield criteria.

III. CORRELATION OF SOIL ANALYSIS WITH FERTILIZER RESPONSE

In order for a soil analysis of any nutrient to be meaningful, it must be related to the availability of that nutrient to plants. Many techniques have been tried for relating soil analyses to plant growth, ranging from the direct attempts to extract from the soil chemically the same amount of nutrient that the plant roots extract biologically, to the indirect or empirical correlations between nutrients extracted and plant growth. The indirect correlation methods have resulted in the most practical means of evaluation despite the difficulty in controlling plant yield factors. Many scientists prefer to initiate correlation work with potted plant studies where yield can be controlled within reasonable limits by having better control over all plant growth factors. Even so, yield is affected by so many factors that studies of the correlation of soil analysis with absolute yield of dry matter - or a related measure, such as the uptake of nutrients- have been less successful than studies involving relative yield. While it is true that nutrient uptake or the amount of nutrient in the plant tissue frequently correlates quite well with soil analysis, there is no convenient way to distinguish between plant requirement and luxury consumption. Therefore, the resultant scale of interpretation for the soil analyses will be arbitrary with no distinct inflection point to indicate when the soil fertility is below or above the yield response range.

Correlation scatter diagrams of relative yield vs. soil analysis are unwieldy to handle as continuous functions due to the fact that the relationship is not linear. In most cases, the function is not even of a higher ordered polynomial form. A rectangular hyperbola or a non-rectangular hyperbola having a 100 percent asymptote would seem to be more appropriate, in which case, a digital computer is a practical necessity for fitting the points to a continuous curve.

Even when a computer is available and some kind of continuous curvilinear model is fitted through the plotted points it is not at all clear what such a correlation means. Frequently the fit is so poor statistically that the researcher concludes no meaningful correlation exists. A further problem with the continuous model is that it is difficult to divide the soil analyses into fertility categories such as low, medium, and high. In altogether too many cases, such division of a continuous curve has little if any mathematical, statistical, economic, or biological foundation in a strictly quantifiable sense.

From a practical standpoint, it would be very helpful to divide soils into fertility categories on the basis of the probability of a profitable yield response, as suggested by Fitts (1955). The United States National Soil Test Work Group (1956) took a step in this direction and made arbitrary class divisions based on the degree of yield response. For the most part, however, no further quantifiable basis for separating categories was proposed until the work of Cate and Nelson (1965).

The Cate-Nelson Critical Level

Graphical Procedure

The work of Cate and Nelson (1965) demonstrated that the peculiar manner in which the points scatter on a plot of relative yield and soil analysis for a particular nutrient is in itself a useful feature. Their observation was that a scatter diagram could be divided into essentially two populations and thus identify a critical level of soil analysis for a given nutrient. The two-population split may be achieved graphically with a technique based essentially on Olmstead and Tukey's nonparametric test of association (1947). The Cate-Nelson technique employs a clear plastic overlay having a pair of perpendicular lines drawn on it to produce four quadrants having

roughly the same relative size as those shown in Figure 2. Then the overlay is positioned on the graph in such a way that the maximum number of points fall in the positive quadrants and the fewest number fall in the negative quadrants. A perfect correlation would put all points in the positive quadrants, thus giving a situation which results in a discontinuous model. In this position, the horizontal line of the overlay splits those relative yields which indicate a large response to the applied nutrient (low relative yield values)from those which indicate small or no response (high relative yields). At the same time, the vertical line identifies the critical level of the soil analysis which splits the two populations into low and high soil fertility values. The end result is that two distinct soil-crop categories are identified on each relative yield plot, each one differing in the kind of response obtained. Below the critical level a large yield response can be expected on that crop with adequate additions of the nutrient in question. Above the critical level, little or no response is expected. Examples of well-developed scatter diagrams are shown in Figure 3 (for P in potatoes, Peru), and Figure 4 (for P in wheat and potatoes, Bolivia).

The critical level obtained depends on the extractant used as well as the crop involved. In practice the Cate-Nelson scatter diagram provides a simple yet highly effective tool for evaluation of soil analytical methods. A good soil extractant for any nutrient should provide an almost perfect split between groups. When correlations are done with potted plant studies, the overall relative yields are usually lower (greater response) than those measured in field trials. Logically, then, the horizontal split on relative yield may not be so much a separation between soils which give large response vs. those which give little or no response, but rather a distinction between degrees of responsiveness. Since potted plant studies are not designed to provide economic interpretation of response, the exact position of the horizontal line should not be counted as having much importance in correlation work.

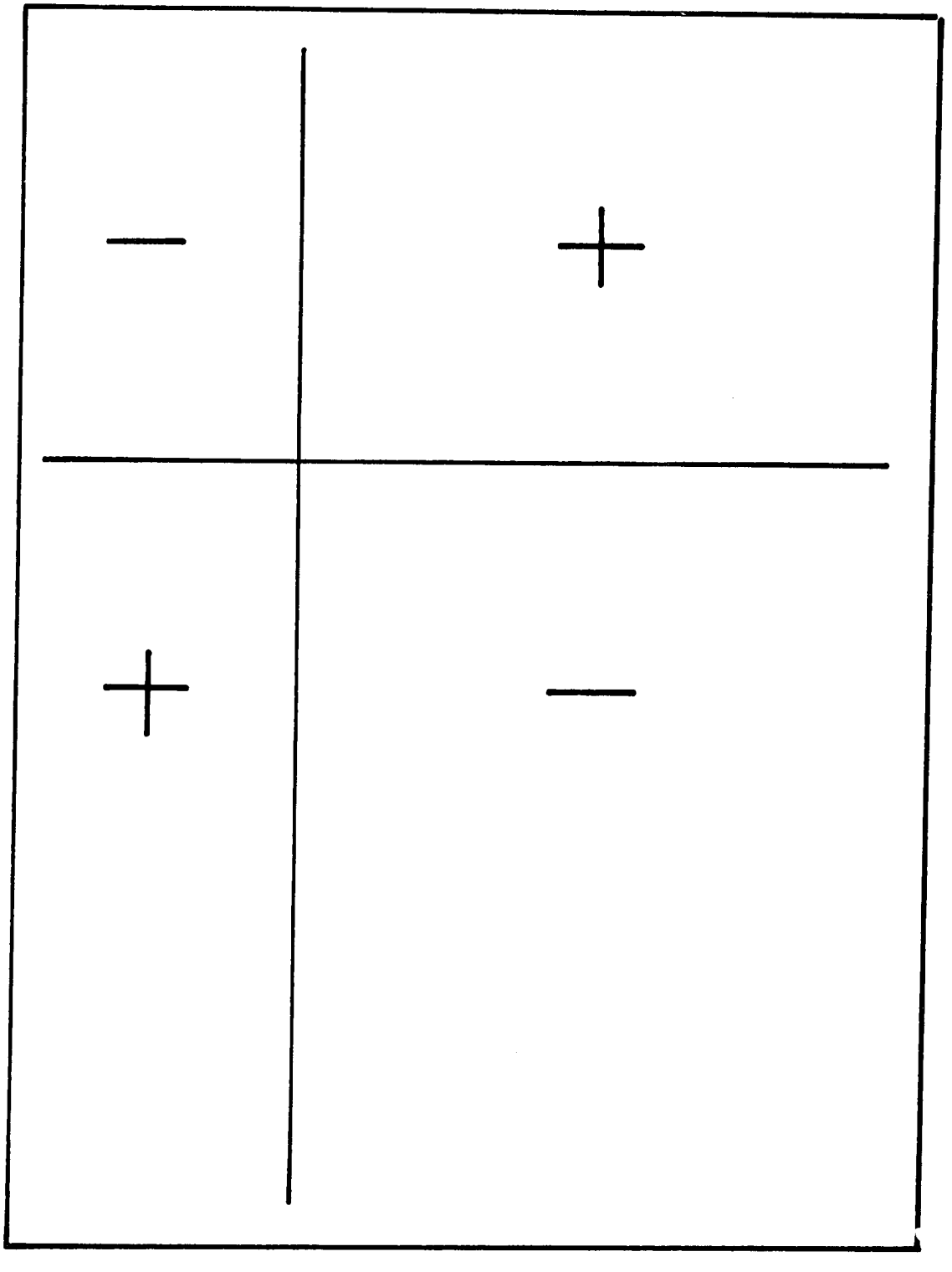


FIGURE 2. Format of clear plastic overlay which is used for finding critical soil test values.

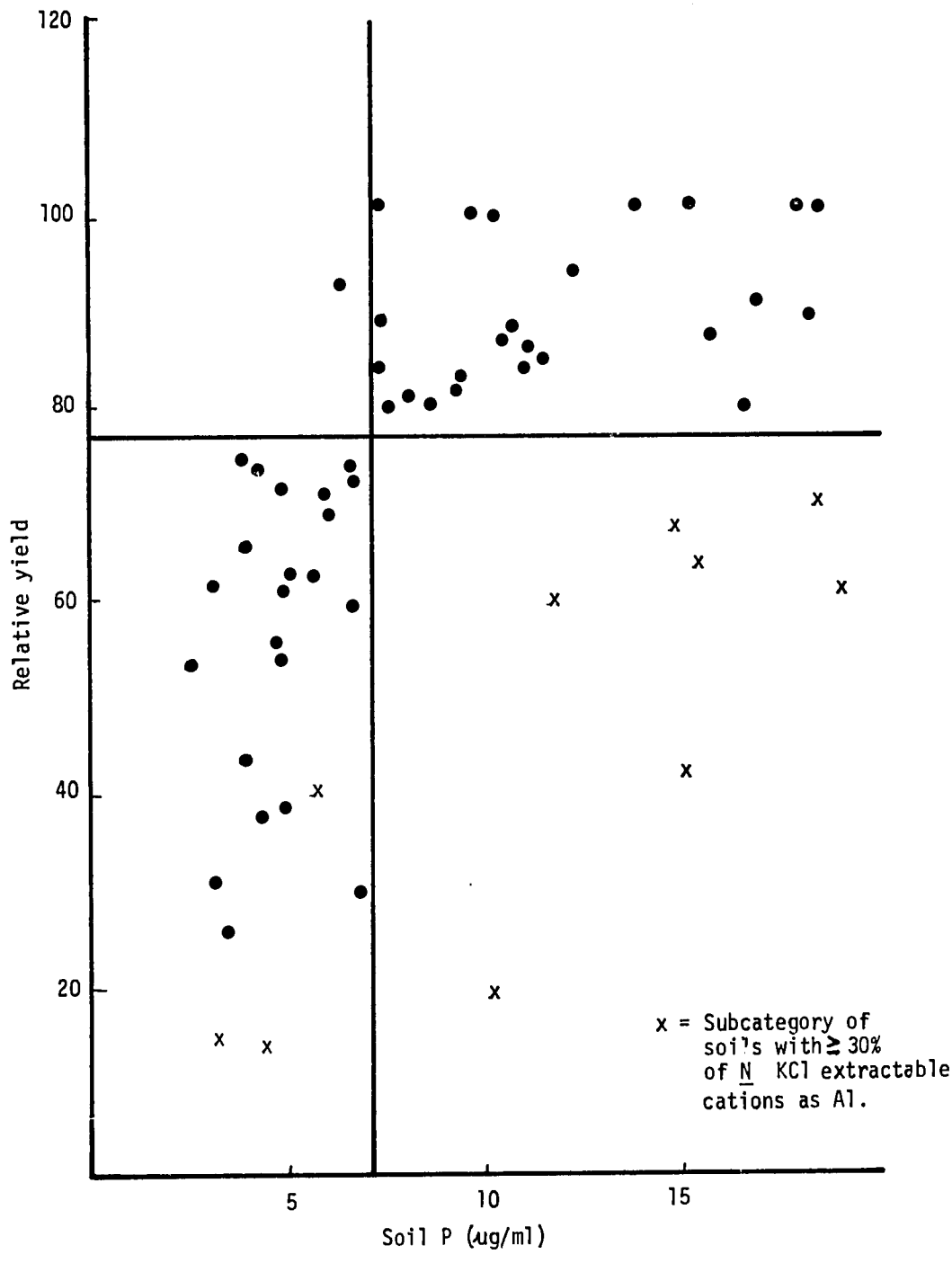


FIGURE 3. Soil P extractable by 0.5 M $NaHCO_3$ (1/10) in relation to relative yield of potatoes in field trials. (From experimental results by the Peruvian National Potato Program in cooperation with International Soil Fertility Evaluation Project, ISFEI Annual Report 1968).

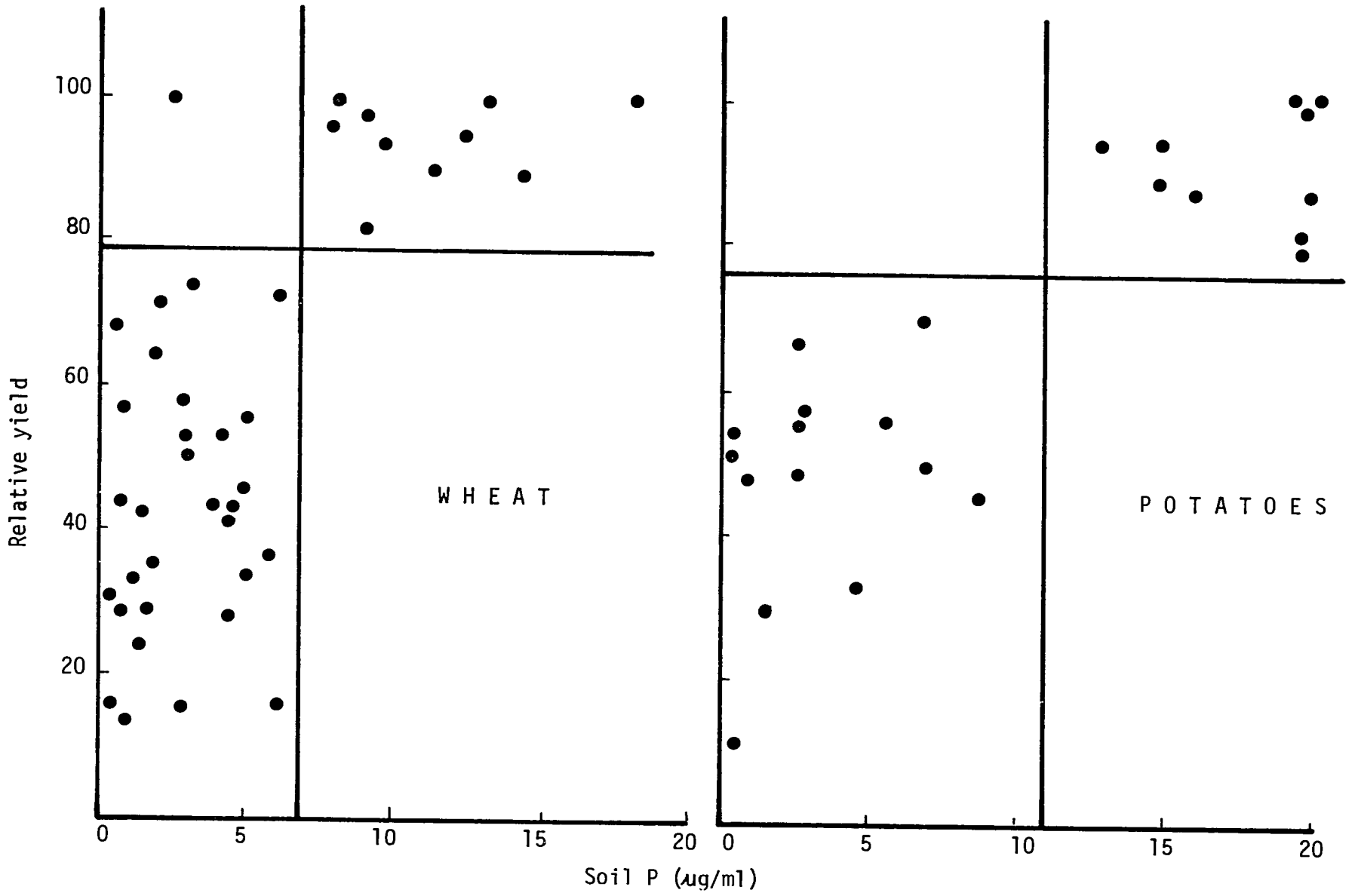


FIGURE 4. Soil P extractable by 0.5 M $NaHCO_3$ (1/10) in relation to relative yield of wheat and potatoes. (From experimental results by Ministry of Agriculture, Bolivia, in cooperation with International Soil Fertility Evaluation Project).

It must be remembered that correlations with field data usually include data from a wide range of conditions, i.e., different locations, soils, climate, and year. The fact that a two-population split can even be recognized at all indicates that the initial soil fertility, with respect to that nutrient, is a more important factor than the uncontrolled yield variables in determining whether or not a response will occur.

A very valuable feature of the Cate-Nelson approach is that outliers among the scatter points can readily be identified and subjected to further study. Careful study of outliers enables the researcher to systematically test different soil extractants or split out subcategories of crop varieties, subsoil fertility, locations, climatic patterns, or any other factors which ultimately result in different soil analysis interpretation or fertilizer recommendation. While most laboratories may begin with a single Cate-Nelson critical level for all crops and all conditions with a given soil analysis, refinements can be readily made as more specific data become available, by splitting out more and more subcategories. Each subcategory will have its own specific critical level. This is done simply by plotting a separate graph for each recognizably distinct set of data and applying the critical level split. Use of numerous two-population splits ultimately provides as much refinement for subcategories as can be justified by available research data. These specific categories and subcategories will be referred to in Section IV as soil-crop categories.

An example of a subcategory soil situation is shown in Figure 3. Phosphorus response in potatoes in unlimed Peruvian soils having more than 30 percent of the N KCl extractable cations as Al is not characterized by the same critical level as are the other acid soils (ISFEI, 1968). Soil pH is not a reliable measure of the Al, but the Al test is easy and effective for sorting out this subcategory (Kamprath, 1967). The Al subcategory soils must also be treated separately in the subsequent interpretation of P response.

Mathematical Procedure

A mathematical model has also been developed for partitioning soil analysis correlation data in two classes (Cate and Nelson, 1971). The procedure amounts to splitting the data into two groups, using successive tentative critical levels to determine the particular critical level which will maximize overall predictability (R^2), the mean of the two classes being the predictor values.

The mathematical model is repeated below as it was originally published (Cate and Nelson, 1971).

"The new technique consists of the following steps:

1) The data are ordered in an array based upon rankings of the X values, i.e., soil test values. The (X, Y) pairs are maintained in this order throughout the analyses. (Table 1).
2) Starting with the X value that will place two or more points to the left of a vertical dividing line, one then calculates the corrected sums of squares of the deviations from the means of the two "populations" that result from moving to each successive X value. The sum of the two corrected sums of squares at each X level is then determined, and this pooled sum of squares is subtracted from the total corrected sum of squares of deviations from the overall mean of all Y observations. The difference between the pooled sum of squares and the total corrected sum of squares (the "between groups" sum of squares) is then expressed as a percentage of the total corrected sum of squares, or as R^2, since this difference represents the additional explanation obtained by fitting two means rather than one. It should be noted that the "between groups" sum of squares could be calculated directly by procedures commonly used in analysis of variance of one-way classification data.
3) By this simple iterative process, one obtains a series of R^2 values for divisions made at various levels of X. One picks the critical level of X as that where R^2 is maximum".

Table 1. STEP 1, Correlation.
Compilation of soil P values and yields: Potato data, Bolivia.[1/]

Soil analysis for P	Variety	Yield of potatoes		
		Threshold yield [2/]	Plateau yield [3/]	Relative yield [4/]
(ug/ml)		(T/Ha)	(T/Ha)	(%)
1	Radosa	8.6	15.3	55
1	S. Imilla	11.4	21.6	53
3	Gineke	9.6	16.3	59
3	S. Imilla	21.2	30.3	70
3	S. Imilla	10.8	16.6	65
3	Alpha	10.6	19.6	54
7	Gineke	6.2	13.8	44
7	S. Imilla	10.8	16.7	64
15	S. Imilla	11.8	13.5	87
15	Gineke	16.0	18.7	85
20	S. Imilla	15.0	17.3	86
20	Alpha	14.1	16.4	85
35	Radosa	14.0	14.0	100
35	S. Imilla	21.6	21.6	100

[1/] Data after Saravia and Waugh.

[2/] Threshold yield = treatment NK (no P) (see discussion page 11)

[3/] Plateau yields determined graphically as shown in Section IV. (see discussion page 12).

[4/] Relative yield $= \left(\frac{\text{Threshold yield}}{\text{Plateau yield}} \times 100\right)$

World wide compilation of correlation data by the International Soil Fertility and Improvement Project (Cate, 1971) has shown that well-conducted field trials commonly give two-population coefficients of correlation of 0.80 or higher, even when dealing with a rather wide range of soil and crops. Correlation coefficients obtained with continuous curvilinear models are usually no better and often not as good as the two-population or discontinuous model, as shown in Appendix Table A (Cate, 1971).

Illustrations of STEPS 1, 2, and 3 involved in making the correlation scatter diagrams are given in Tables 1 and 2, and Figure 5, based on the Bolivian potato data.

Good correlation between soil analysis and crop response requires that (1) effective methods of analysis are available for the types of soil being analyzed and (2) pot or field trials are properly conducted to provide correct relative yield information. Currently most laboratories cooperating with the ISFEI project have established at least preliminary critical levels for phosphorus and potassium. Many others have correlations for secondary and some micronutrients as well. As the need for better information regarding secondary and micronutrients intensifies, the critical level concept should be an especially valuable tool in soil fertility work.

The data from Bolivia used here include all the experiments carried out in one given area using paired varieties at each site. Absolute yields varied greatly between varieties and between sites and there was only limited relationship to soil P levels (Figure 6). Relative yields, on the other hand, correlated very well with soil analysis even though they include different varieties whose absolute yields varied widely (Figure 5). The correlation of relative yield with soil P gives a clear-cut picture of the sites which did respond (soil-crop category I) and those which did not respond to P (soil-crop category II). The statistical coefficients of determination (R^2) for the two-mean discontinuous model (Table 2) show that the

Table 2. STEP 2, Correlation.
Calculation of coefficient of determination (R^2) and critical level for best two-population split using the 2-mean discontinuos model: Potato data, Bolivia.

Last value of soil P included in Population 1	Population 1		Population 2			
	Mean relative yield	Corrected sum of squares of deviations from mean (CSS^1)	Mean relative yield	Corrected sum of squares of deviations from mean (CSS^2)	Postulated critical level between values	R-square for postulated critical level 1/
1	54	2	75	3,619	1 and 3	.17
3	59	233	81	2,470	3 and 7	.38
7	58	476	90	273	7 and 15	.83 2/
15	64	1,730	93	211	15 and 20	.56
20	67	2,530	100	0	20 and 35	.42

1/ $R^2 = \frac{TCSS - (CSS^1 + CSS^2)}{TCSS}$; (Total corrected sum of squares, $TCSS_{,} = 4370$)

2/ Critical level lies between 7 and 15.

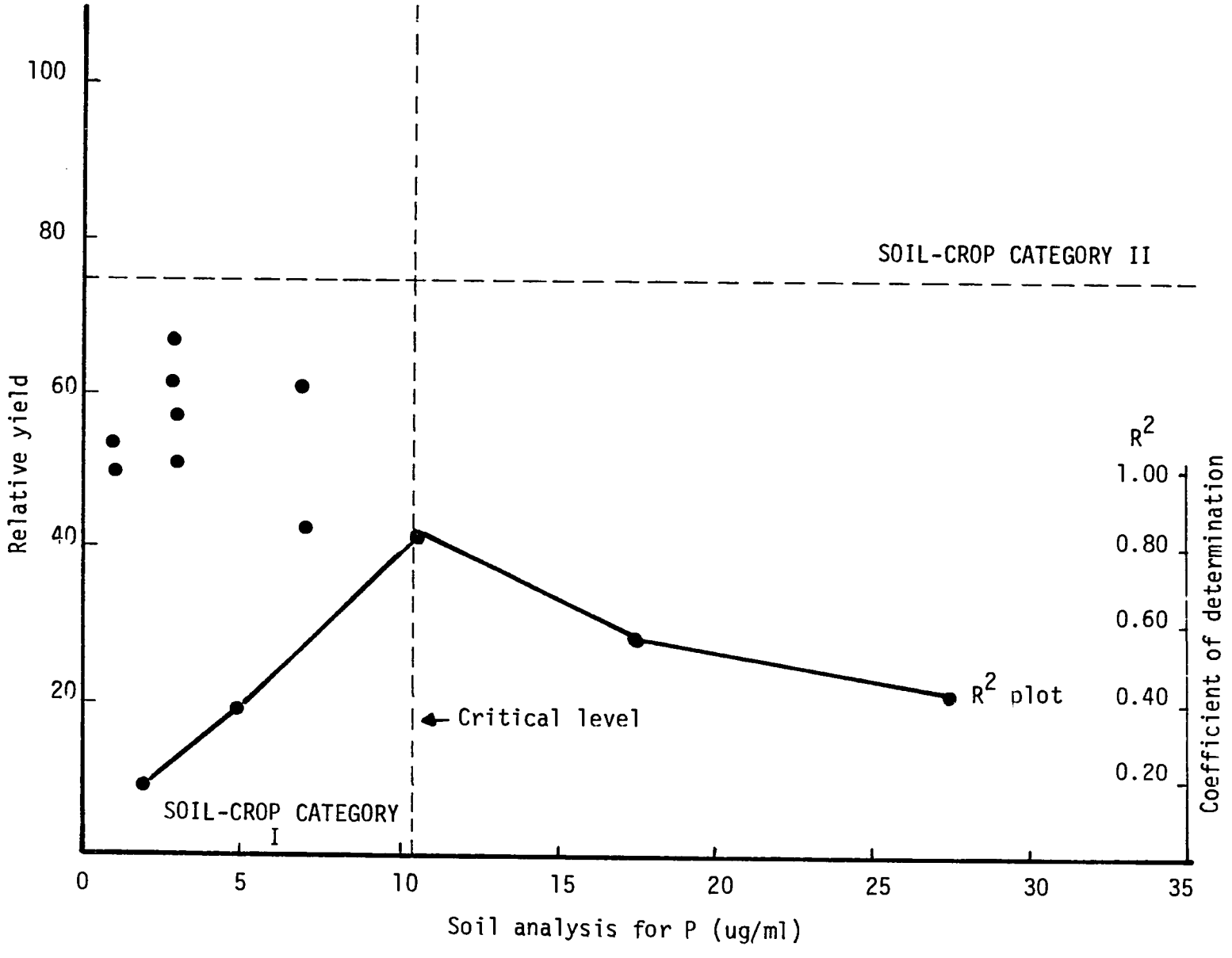


FIGURE 5. STEP 3.
Correlation and determination of soil-crop categories by preparation of scatter diagram (relative yield vs. soil P) and plot of coefficient of variation (R^2) for two-mear. discontinuous model: Potato data, Bolivia.

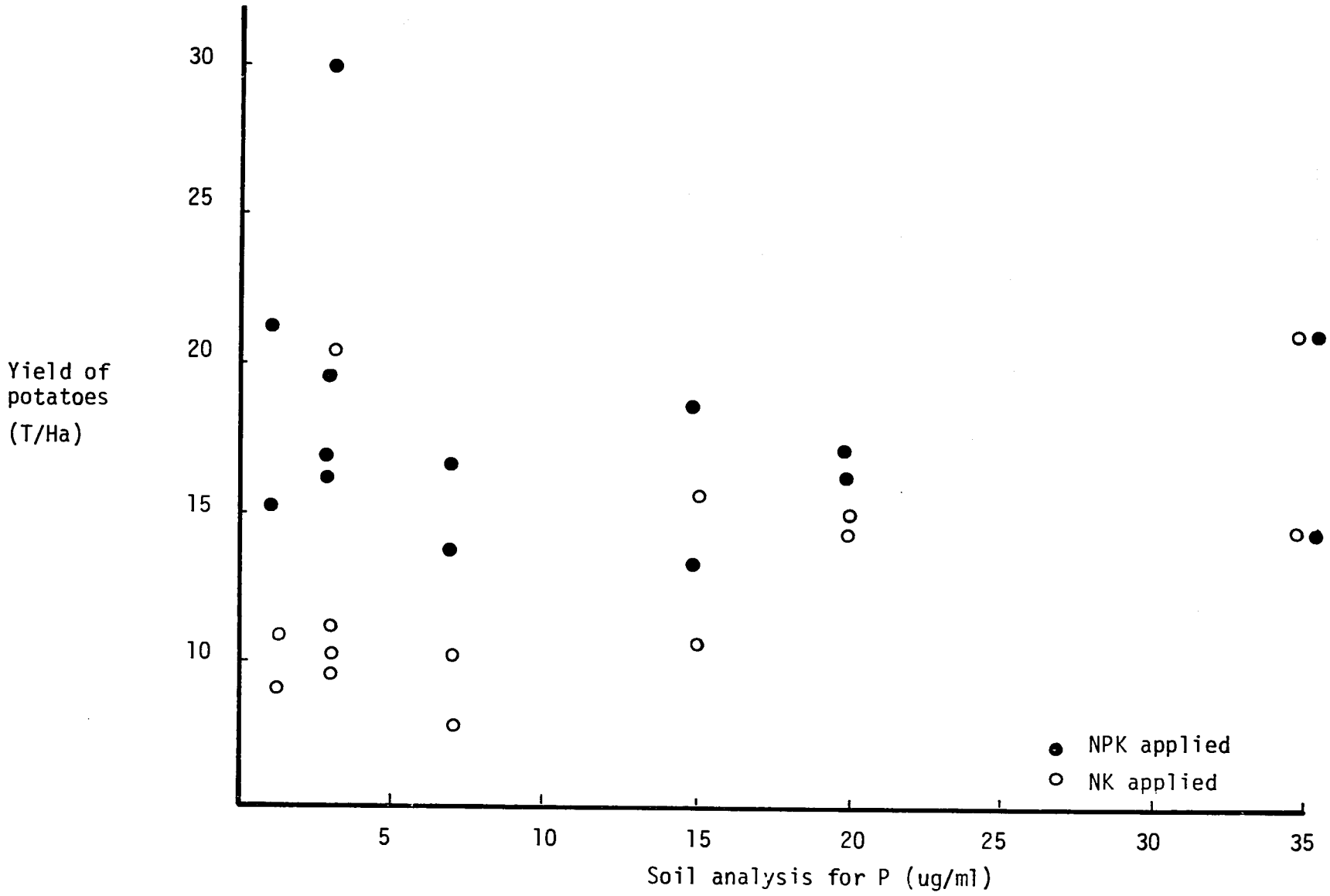


FIGURE 6. Relationship between absolute yield and soil P. Potato data, Bolivia. (Note limited correlation between absolute yield and soil analysis for P).

the critical level lies between 7 and 15 ppm. Normally a fully developed scatter diagram over various years and locations will provide more points in the immediate area of the critical level making the determination more precise (see Figures 3 and 4). The relationship between the scatter diagram and the coefficients of determination (R^2) for the two-mean discontinuous model is shown in Figure 5. STEP 3 of the correlation may be done with either the scatter diagram or the statistical determination of R^2 by the two-mean discontinuous model; both approaches should give the same critical level.

A final point about correlations between soil analysis and relative yield is that they are useful only in determining the critical level for soil-crop categories and subcategories and predicting when responses are likely to occur. Such a correlation does not establish how much fertilizer to apply to reach a given yield. In other words, soil analysis does not predict yield, but rather it predicts relative yield. In order to establish the relationship between yield response and the amount of fertilizer application associated with that response it is necessary to work with another kind of relationship, i.e., the fertilizer response model. The interpretation of fertilizer response as a function of fertilizer applied will be covered in Section IV.

IV. INTERPRETATION OF FERTILIZER RESPONSE DATA

The Purpose of Fertilizer Response Interpretation

In order to adequately select the appropriate fertilizer response model the purpose of interpretation must be defined. In this bulletin the following definition is used:

The main purpose of the interpretation of fertilizer response data is to establish how much of each nutrient must be applied to bring about a given yield response within a predictable soil-crop category.

The key to the above definition lies in the term predictable soil-crop category. Any farmer or agronomist knows that all crops cannot be lumped together when one is trying to determine how much fertilizer to apply. Likewise, it is just as important when making fertilizer recommendations not to mix unlike categories of soil fertility i.e., those that are responsive and non-responsive to a given nutrient. Ideally, when the nutrient response is interpreted within a specific soil-crop category the resultant interpretation is both situation-specific and site-specific.

The predictability of a soil-crop category depends on how well a soil analysis can separate response and no response with respect to a particular nutrient. The correlation scatter diagram (see Section III) is a convenient tool for this separation. When the scatter diagram of relative yield and soil analysis gives a well-defined critical level it is clear that two distinct soil fertility categories exist for that crop (under the range of conditions studied). Subsequent interpretation of fertilizer response should then be made with each soil-crop category separately.

Studies of Interpretation Using the Bolivian Data

This bulletin summarizes at this point the results obtained and conclusions drawn in a series of studies involving the interpretation of fertilizer response data. The purpose of this summarization study was to determine if the linear response-and-plateau (LRP) model results in sufficiently reasonable estimates of nutrient requirements to be used as a rapid method for interpretation of fertilizer response data. The data analyzed were obtained from experiments in Bolivia carried out during the period 1969-1970. The same central composite rotatable design had been used in all of the trials, with only the center-point treatment replicated. The center-point treatment for those experiments was coded as 2-2-2 for N-P_2O_5-K_2O applications, respectively, and the range of code levels for each nutrient was 0 to 4. Application rates for P_2O_5 and K_2O were based on soil tests and previous knowledge of crop response. At the time of data summary and analysis the sites were grouped into soil-crop categories derived by the critical level technique described in Section III. The treatments and yields for each group are summarized in Appendix Table B.

Attention is drawn to the fact that the data in Appendix Table B represent unreplicated observations except for the center point treatment 222. Experiments in soil-crop categories I and II each employed 8 replications of treatment 222, while III employed 3 replications, and IV and V employed 5 replications. Table 3 summarizes the treatments included in the weighted means for single nutrient yield data for each group.

Table 3. Summary of treatments included in single-nutrient yield means for each soil-crop category at five levels (0-4) of N, P, K application 1/

Nutrient	Level	Treatments included	Yield means for each soil-crop category				
			I	II	III	IV	V
			Potato trials			Wheat trials	
			T/Ha				
N	0	022	8.5	8.6	7.3	1.12	1.00
	1	111, 113, 131, 133	17.0	14.8	13.9	1.75	1.63
	2	222, 242, 224	18.8	16.0	16.0	1.82	2.16
	3	311, 313, 331, 333	19.8	16.2	14.0	1.96	1.86
	4	422	19.7	15.6	12.5	1.94	1.98
P	0	202	11.1	15.4	4.9	0.89	0.79
	1	111, 113, 311, 313	17.6	14.8	12.3	1.69	1.69
	2	222, 422, 224	18.8	15.9	15.0	1.93	2.20
	3	131, 133, 331, 333	19.2	16.2	15.7	2.02	1.80
	4	242	19.5	16.9	19.8	2.22	2.28
K	0	220	17.7	11.7	10.9	2.07	1.63
	1	111, 311, 331, 131	18.0	16.0	14.1	1.93	1.73
	2	222, 422, 242	18.8	16.3	15.8	2.03	2.19
	3	113, 133, 313, 333	18.7	15.0	18.5	1.78	1.77
	4	224	19.9	12.7	14.3	1.88	2.33

1/ From Appendix Table B.

Fertilizer Response Interpretation at Individual Locations

Preliminary interpretation of the data was done first by regression analysis of individual locations using a conventional quadratic model including terms representing the interaction among the nutrient elements. The results of these analyses are rather lengthy and cannot be presented here. The results were useful, however, in showing that all three nutrients contributed to the fertilizer response in at least certain experiments, and also that the interaction among the nutrients was significant for a number of the experiments. For example, in soils responsive to both N and P the response to N could not be studied until sufficient P was supplied so as not to be limiting. Therefore, in order to facilitate subsequent single nutrient comparisons between models in the absence of interaction terms, the zero treatments of accompanying nutrients had to be deleted from the analysis. Specifically, the 202 (zero P) and 220 treatments (zero K) were deleted when the nitrogen response was being processed. (The 1-level of the accompanying nutrients was assumed to be sufficiently high to permit a realistic picture of response to the variable nutrient being studied). Also, the 022 and 220 treatments were deleted in determining the P curve, whereas the 022 and 202 treatments were deleted from the K curve. The 000 treatment was not used in any of the response analyses since it does not meet the requirements of the threshold yield (See Section II).

Regression analyses were then carried out with the computer on the data deck for each of the 28 experiments, using quadratic, square root, and logarithmic models. A further regression analysis was made on the same data deck with two linear response-and-plateau (LRP) models, in one case assuming that treatment levels 0 and 1 were on the response slope while levels 2, 3, and 4 were on the plateau, and in the other case assuming that levels 0, 1, and 2 were on the response slope while levels 3 and 4 were on the plateau. The R^2 for fit was determined as follows:

$$R^2 = \frac{\text{S.S. for regression (2 d.f.)}}{\text{S.S. for treatments (12 d.f.)}}$$

The results of the R^2 of regression analysis of the data for each model are given in Appendix Table C (only those functions which gave significant yield response are shown).

These results, obtained by fitting the response data of individual experiments with the various models, point out the fact that each function fits some data much better than others. (In some cases the R^2 for all models is very low due to the fact that there was very little nutrient response). The function which best describes the response in one field trial is sometimes the poorest for describing the response in another. A logical conclusion to this might be that, for predictive purposes, any of the algebraic forms would serve equally well to describe the response functions. Nevertheless, the linear response-and-plateau (LRP) model did result in the best R^2 fit of the data in 27 out of the 37 significant response functions studied in comparison with 3, 6, and 1 for quadratic, square root, and logarithmic models, respectively (Appendix Table C). The linear response-and-plateau (LRP) model should be recognized as an appropriate working model for describing and interpreting fertilizer response from individual experiments.

Fertilizer Response Interpretation for Pooled Locations

Comparisons were also made between the different models on data for pooled experiments rather than on individual experiments. These results are given in Appendix Table D and are plotted graphically in Appendix Figures A through J for all pooled locations. Whereas the linear response -and-plateau (LRP) model was clearly better than curvilinear models for the majority of the individual locations, there was less difference between models when working with pooled locations. This is probably due to the fact that the averaging together of data from individual sites, whose slope of response and plateau yield are

different, will mathematically result in curvilinear average response, even when the individual responses are essentially characterized by the two straight lines of the linear response-and-plateau (LRP) model (Boyd, 1970; Bartholomew, 1972).

Plots of the response data for each group are given in the Appendix. These plots of the response data use the means of all experiments in the pool of locations, thereby making it easier to see the range of observations and the weighting effect of unequal numbers of observations at each level (Appendix Figures A through J). The predicted response line is drawn through the points for each response model studied. A discussion of the interpretation of the response is presented later in this Section.

The residuals of the regression analysis are perhaps even more worthy of mention than the fit of the model to the response function. Ideally, the residuals should be random (i.e., distributed equally on the negative and positive sides of the reference line). However, the curvilinear models showed some consistent residual trends for the individual experiments in this study, indicating a tendency for bias. When the linear response-and-plateau (LRP) model was used, on the other hand, these residuals appeared to be appropriately random. The work of Boyd (1970) and Anderson and Nelson (1971) indicated a similar bias with the quadratic model. An example of the pooled residual plots for six locations in Group 2, nitrogen,is given in Appendix Figure K. In this example, the quadratic model shows negative residuals at the 0-and 3-levels and positive residuals at the 1-and 4-levels. The residual trends of the square root and log models are not shown but they appeared to be intermediate between the quadratic and the linear response-and-plateau (LRP) models in nearly all cases.

Graphical Interpretation of Response Data

Many agronomists have very little time to analyze results from one set of experiments before they begin to plan for a subsequent experiment. A rapid graphical method of interpretation of response data would, therefore, be invaluable for evaluating the yield response and the yield plateau portions of the response function. Similarly, a rapid technique would be useful to those in charge of making fertilizer recommendations based on soil analysis by aiding them in taking new data and readily determining a suitable level of fertilization. The following graphical procedure ultimately produces approximately the same result as the interpretation obtained with the computerized version of the linear response-and-plateau (LRP) model. (STEPS 4 and 5 are continuations of the STEPS 1-3 illustrated in Section III).

Organize the Single-Nutrient Response Data

Compute the mean of yields which correspond to the various application rates, i.e., level 0, level 1, level 2, etc. following the model in Tables 3 and 4. If the design contains more or less than five levels a similar procedure may be followed, being careful to delete incomplete or inadequate nutrient combinations (those with 0-levels of the accompanying nutrients) in the means of the complete treatments being evaluated. This step is necessary to meet requirements of the threshold yield concept (Section II). Prepare a single nutrient plot of yield vs. nutrient application for each nutrient using the means of yield for each level. The best procedure to follow for a plot of this type is to arrange the plots for each nutrient side by side as shown in Section V, p. 48. If the single-nutrient response plots are placed side by side there will be a plateau yield common to all nutrients, providing more points through which the plateau yield line can more easily be plotted by sight alone.

Table 4. STEP 4, Interpretation.
Computation of mean yield at each nutrient level for each soil-crop category [1/]

Soil-crop category I			Soil-crop category II		
P level	Yield	No. of observations included	P level	Yield	No. of observations included
	T/Ha			T/Ha	
0	11.1	8	0	15.4	6
1	17.6	32	1	14.8	24
2	18.6	80	2	15.9	60
3	19.2	32	3	16.2	24
4	19.5	8	4	16.9	6

[1/] Summarized from Table 3.

Establish Statistical Significance in Yield Among Treatment Levels

Compute the Least Significant Differences (LSD's) among single-nutrient means. These least significant differences may be useful in determining which of the yield points are the same and, therefore, should be considered to be on the plateau. Techniques for calculation of the LSD's together with an example of the calculations are reported in Appendix Table E.

Establish The Yield Plateau

Draw a best-fit horizontal line through all points which differ by no more than experimental error (Figure 7). If this plateau line includes the 0-level then obviously there is no response for that nutrient. If the plateau line includes levels 1 through 4 (excluding only the 0) then the rate of nutrient application necessary to reach that yield is assumed to be the 1-level or less. If the plateau yield line includes only points at the 2-level or greater, a response (yield increase) slope must be established.

Establish the Response Slope

Connect the threshold and plateau yields, drawing a best-fit straight line through all points not already included in the yield plateau (Figure 7). If the 2-level application rate is already on the plateau line, the response line is established by connecting the yields at the 0-and 1-level upward with the same slope until it intersects with the yield plateau line (soil-crop category I, Figure 7). If the 2-level or 3-level is not on the plateau, the response line would be fitted through those points as well as the 0-and 1-levels (Figure 8).

Establish the Yield Inflection Point

Drop a perpendicular line from the point of intersection of the two lines to the X axis to obtain the estimate of nutrient requirement.

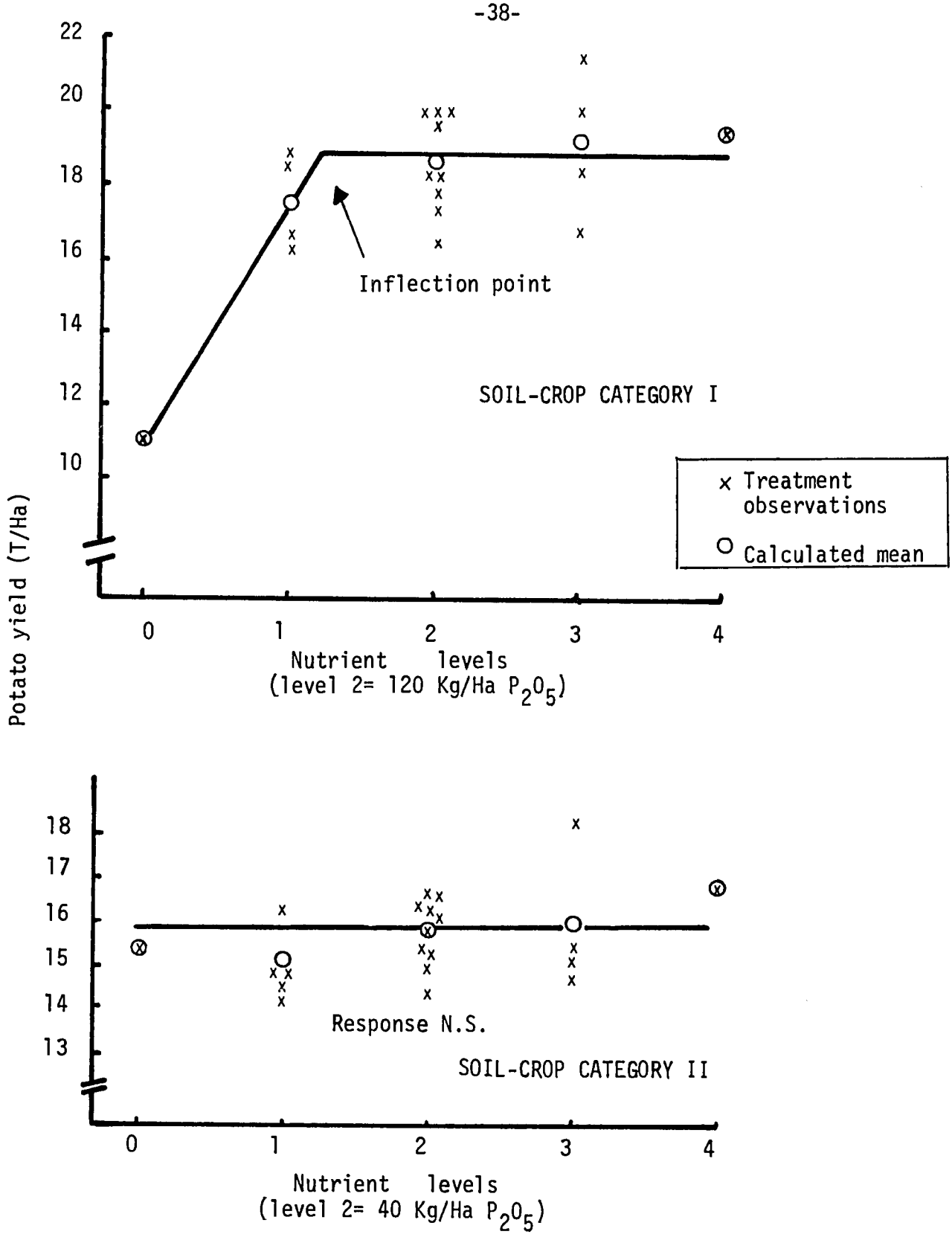


FIGURE 7. STEP 5, Interpretation.
Determination of linear response-and-plateau (LRP) lines on response function (LSD's p. 65).

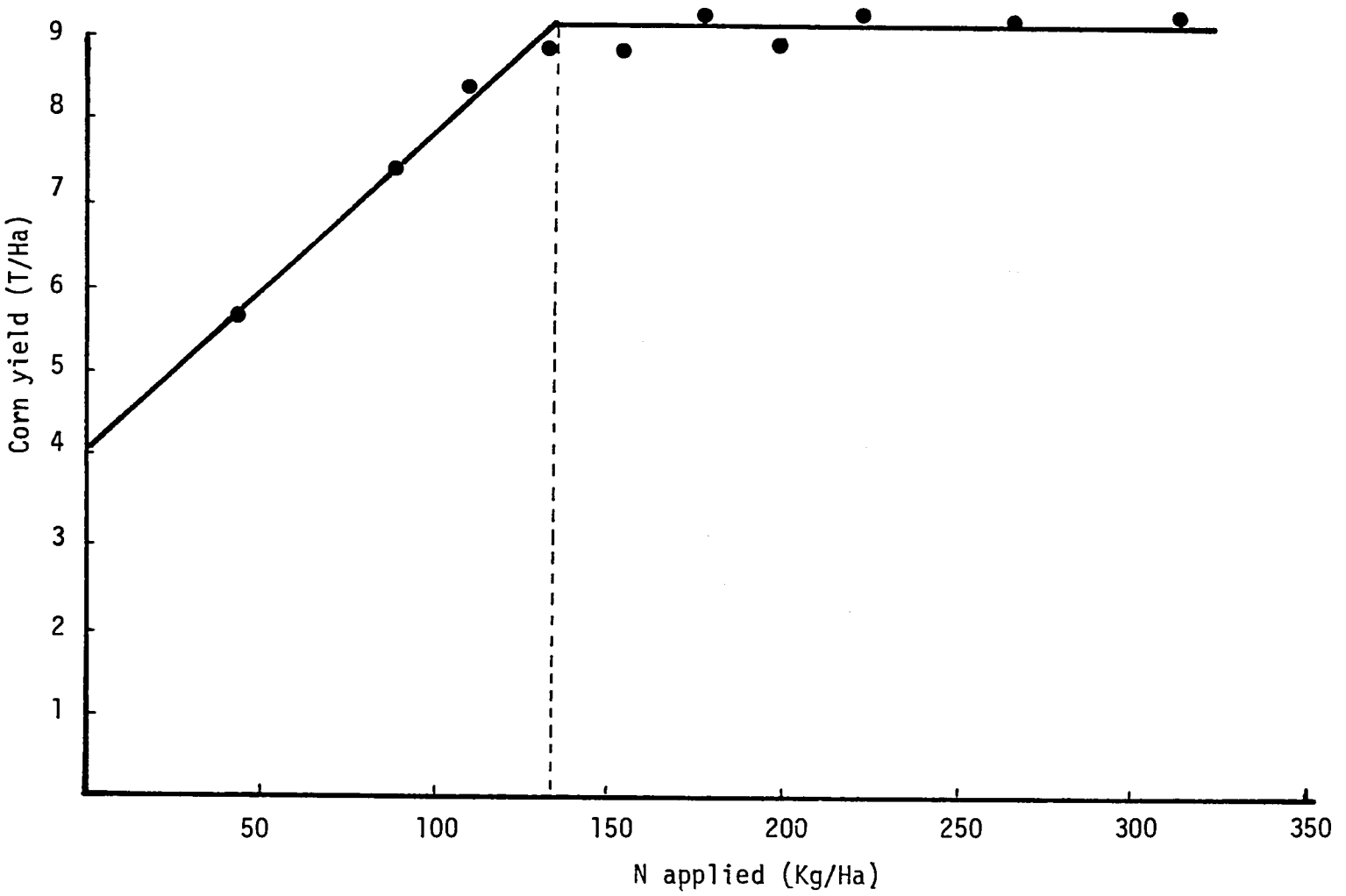


FIGURE 8. Example use of the linear response-and-plateau model (LRP) for interpretation of trials having many levels of nutrient applied. (After Hunter and Youngen, 1955).

The response line (slope) established by the graphical procedure must not pass to the right of the first yield point included on the plateau. For example, notice in soil-crop category I that a response line drawn through the 0- and 1-levels of K would intersect the plateau to the right of the 2-level (p.48). When this is the case, draw the response line connecting the 0-and 2-level application rates. Normally a low yield point at a low level of application indicates that some factor has prevented the crop from growing normally for that rate of fertilization. This represents the so-called sigmodial effect. Such a condition could also develop, for example, if P fixation was very strong and the rates of application were relatively low. In that case, it would be more appropriate to work with the yield at the 2-level of application since the efficiency of fertilizer use at that level is better than at the 1-level.

The yield points in the example data (Figure 7) are weighted means based on unequal numbers of observations at each level. With uniformly replicated experiments this would not be the case. The number of observations included in each mean is shown in STEP 4, Table 4. Notice that the most precise point (highest number of observations) is the replicated 2-level. The least precise yield points are the 0-and 4-levels. This in part is why the 4-level yield for soil-crop category I falls above the line.

Experimental Designs and Fertilizer Response Interpretation

The data employed to illustrate the step by step procedure in correlation and interpretation are taken from field trials carried out with the central composite rotatable design. This design has been employed in a number of countries by researchers anxious to develop yield contours from which can be predicted the combination of nutrients required to reach a given yield. The quadratic response model is traditionally employed in connection with use of the central composite design, resulting in a response function similar

to those shown for the quadratic in the Appendix figures. In the study of the Bolivian data, the R^2 for fit was consistently lower with the quadratic than with the other models. Furthermore, the quadratic model predictions of nutrient requirement to reach maximum yield were high and varied widely between locations within a given soil-crop category. A striking example of the gross difference between the quadratic and the linear response-and-plateau (LRP) models is depicted graphically in soil-crop category I, P, Appendix Figure B. Where all models are likely to exhibit bias either on the high or low of an ideal interpretation, the larger bias (always on the high side) appears to be associated with the quadratic model.

The use of the linear response-and-plateau (LRP) concept permits a tentative estimation of nutrient requirement even for response data based on only three nutrient levels. Figure 9 shows the three possible outcomes of such an interpretation of nutrient requirement: the 1-level or less, between levels 1 and 2, the 2-level or greater. In view of the enormous number of field trials that have been carried out with only three levels of nutrients applied (usually 3x3x3 factorials) some kind of response model (other than the traditional quadratic) is greatly needed. Data summaries conducted by the International Soil Fertility Evaluation and Improvement project indicate that LRP interpretation of nutrient response in 3x3x3 factorials does give logical estimates of nutrient requirement in most cases. Furthermore, the estimates are surprisingly consistent when interpretations are made within properly formed soil-crop categories. On the other hand, curvilinear interpretation of such data commonly gives predictions of maximum yields which are higher than any measured observations, with resulting predictions of nutrient requirement which are unreasonable.

The interpretation of fertilizer response data is, of course, much easier and more precise when there are larger numbers of nutrient levels being studied. Most of the field trials summarized by the rectilinear model of Boyd (1970) had seven or more levels,

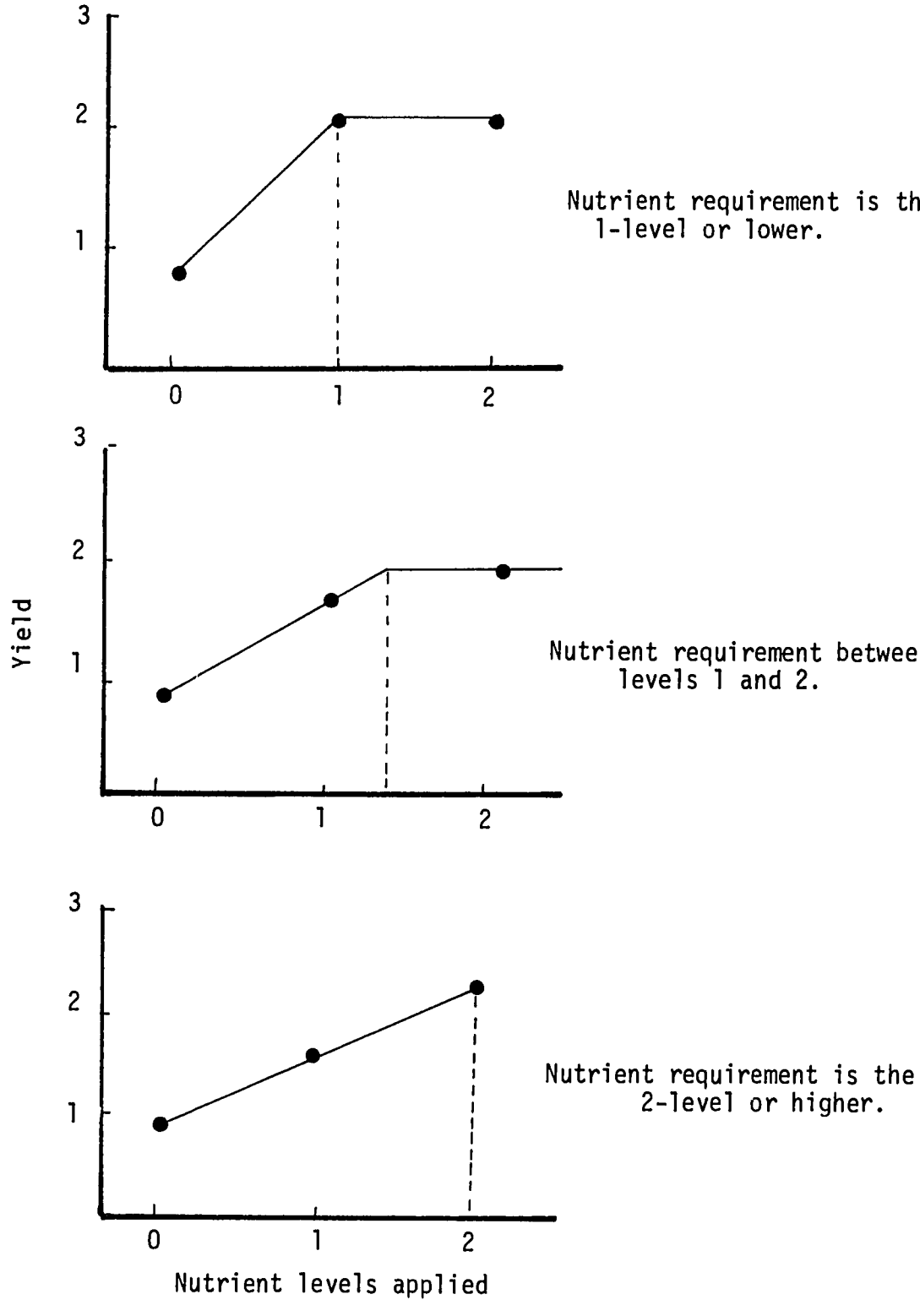


FIGURE 9. Estimation of nutrient requirement with linear response-and-plateau (LRP) model in experiments with only three nutrient levels applied.

providing a clear picture of the linearity of both the response and plateau portions of the response function. Bartholomew (1972), in his summary of nitrogen response, also showed that dual linear functions plotted through a larger number of points provided for easy determination of the inflection point and realistic estimation of the amount of nitrogen that should be applied for the plateau yield. An example of corn response to 10 levels of N application was shown in Figure 8, (Hunter and Youngen, 1955). It would seem, then, that single nutrient trials with at least seven levels of nutrients would be highly recommended for future studies designed for determining optimal fertilizer levels.

Interaction Between Nutrients and Single-Nutrient Response

A principle objective of experimental designs in the past has been that of measuring nutrient interaction. Experimental results will exhibit interaction in a number of ways, the most common occurring when more than one nutrient simultaneously contribute to a given response. In such cases, any one limiting nutrient will affect response to any other nutrient (See Figure 1). For example, N and P commonly give this kind of interaction in soils low in both nutrients.

A different type of interaction may occur if application of one nutrient produces an associated response which is partly due to a second factor. An example of an associated response was mentioned with P response in non-limed Peruvian soils high in Al. In these soils, increasing rates of P as simple superphosphate also reduced Al toxicity and supplied additional Ca. The end result was that the response could be attributed to correction of P deficiency, reduction of Al toxicity (liming effect) or both (p. 21). Another example of an associated response is that caused by release of soil organic N through microbiological action simulated by addition of P. In both of the above examples, all of the yield response which occurs

cannot be attributed to a P effect alone. These associated response interactions may not be detected with conventional fertilizer treatments and statistical analyses. Their detection is readily made, however, by examining outlier observations in Cate-Nelson correlation diagrams (p. 19) or by noting unusual response characteristics in LRP response models.

In field experiments the total number of plots which can be conveniently handled is such that the researcher must limit the total number of plots. Thus, it usually becomes a choice of (1) restricting the number of application rates in order to study main effects and interactions with several nutrients, or (2) controlling,ignoring or by-passing the interaction and studying response to a larger number of application rates of a single nutrient. The latter approach will supply more reliable response information for a given nutrient provided the levels of accompanying nutrients are supplied in adequate but not excessive amounts. In most cases a good researcher can establish adequate levels of accompanying nutrients by drawing from preliminary laboratory studies and/or results from previous field experiments. In cases where inappropriate rates are applied the LRP interpretation of response will readily show what adjustments would be necessary for future studies.

V. PREPARATION OF ECONOMICALLY SOUND FERTILIZER RECOMMENDATIONS

The discussions in the preceding sections dealt first with techniques for correlation of soil analysis and plant response to fertilizer as a tool for establishing predictable soil-crop categories. The next discussions concerned the interpretation of fertilizer response data from individual sites or pooled sites belonging to a particular soil-crop category. Taken together these discontinuous models for correlation and interpretation supply the basic information regarding the amount of fertilizer required to bring about a given yield response. The final step, before a recommendation can be made, is to apply some kind of economic criteria to determine if the venture will be profitable for the farmer.

The preceding sections have emphasized that yields themselves are very hard to predict because of the influence of numerous uncontrollable factors affecting them. This situation in practice can be shown graphically using as an example the soil-crop category I data (Figure 10). Of the eight separate experiments shown in Figure 10, all were conducted by the same agronomist the same year, in the same general locality, using improved varieties. Yet the absolute yields vary by nearly 150 percent. On the other hand, relative yields vary only about 40 percent, resulting in a reasonably predictable response. Stated another way, it is possible to predict with greater accuracy that a certain fertilizer application will increase yield by a certain relative amount than to predict what the absolute increase in yield will be. The economic evaluation of fertilizer recommendation options should take the limits of prediction into account.

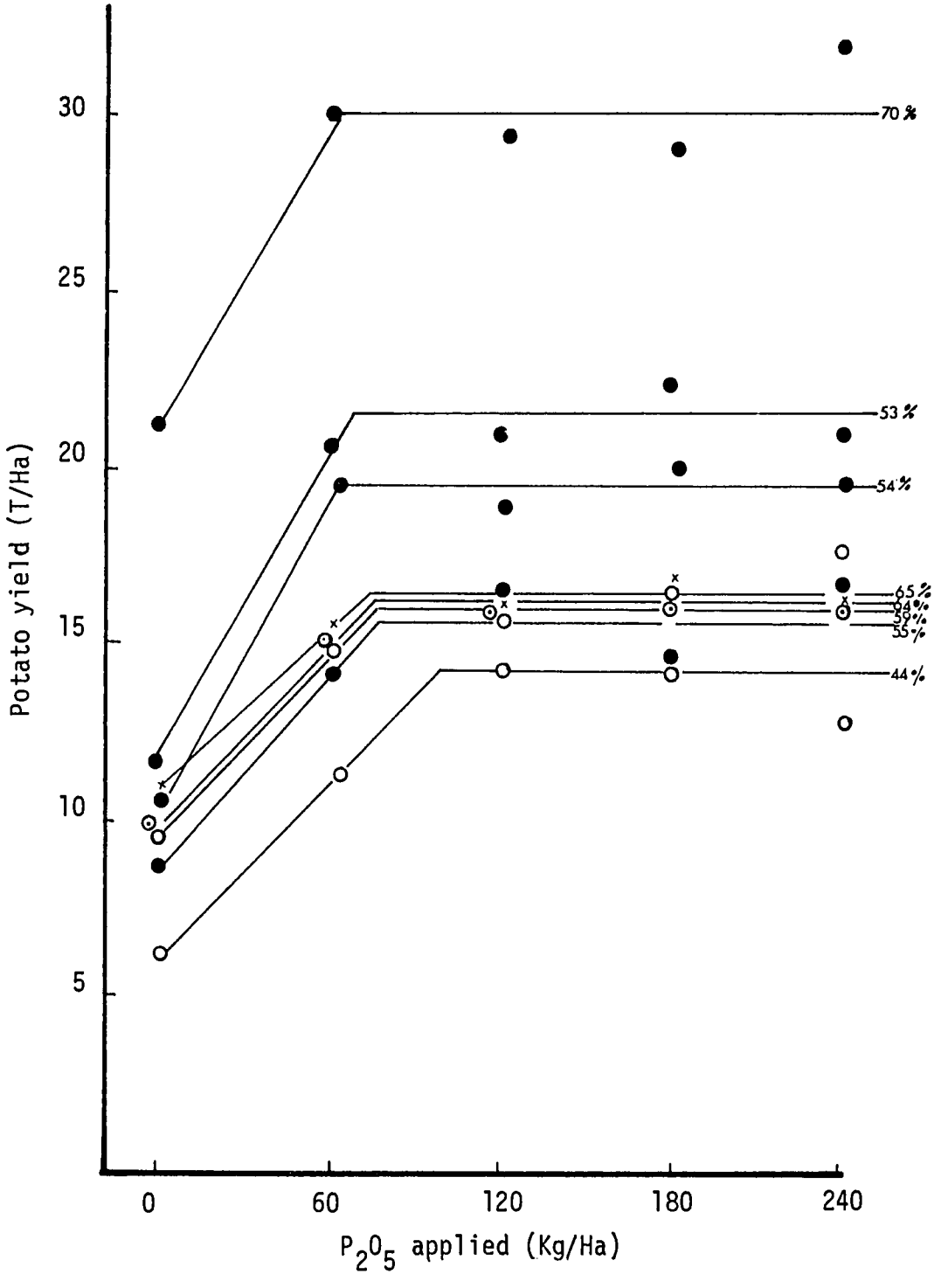


FIGURE 10. Yield variation among individual sites belonging to soil-crop category I, Bolivian data. 1/

1/ Numbers in parentheses are relative yield = $\frac{\text{Threshold yield}}{\text{Plateau yield}} \times 100$

Single- and Multi-nutrient Choices in Making Recommendations

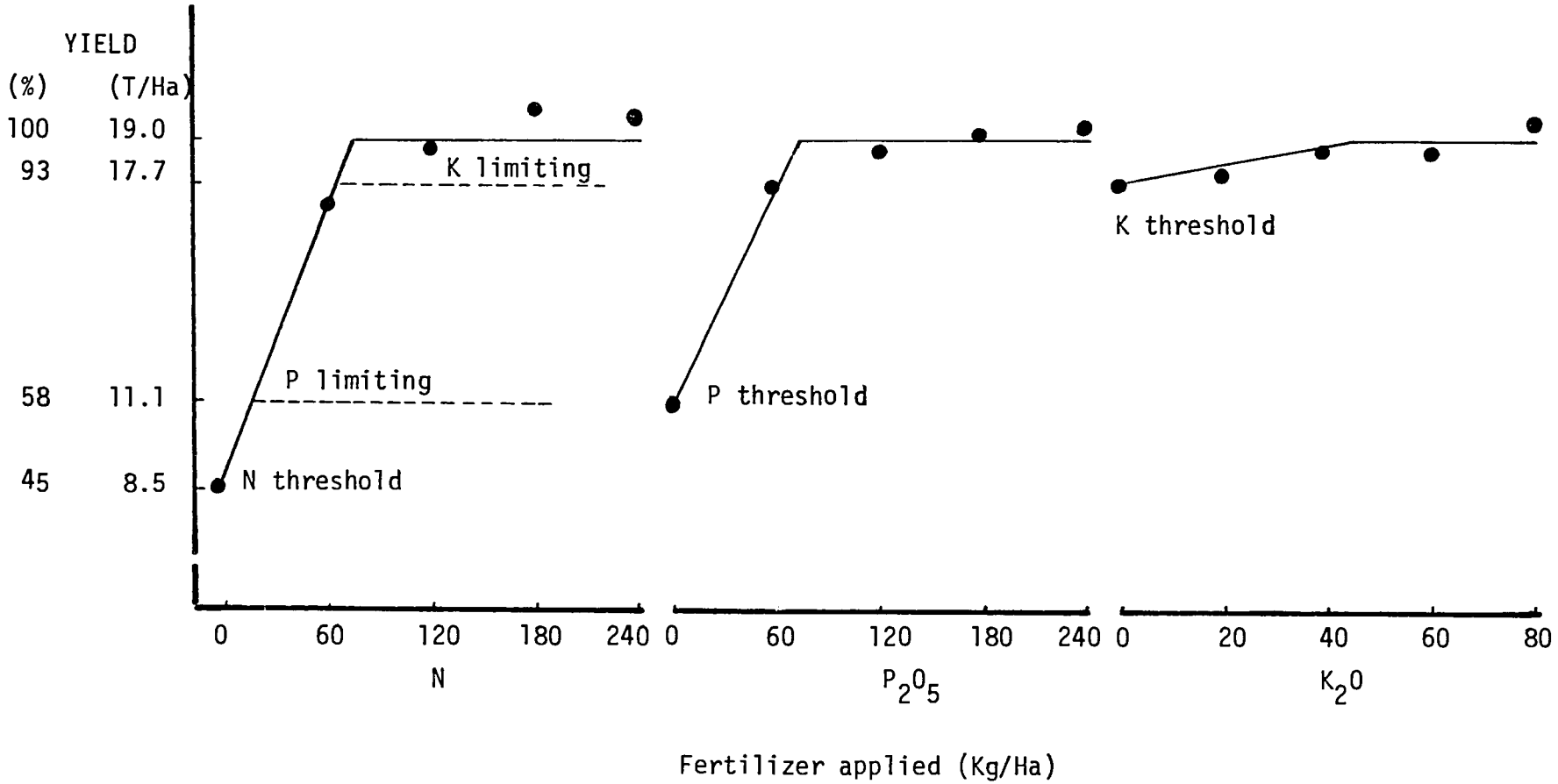
STEP 5, which illustrates the interpretation of fertilizer response data (Figure 7, Section IV) deals with single-nutrient functions. When a single-nutrient response is being evaluated one must include only those treatments where the other nutrients were supplied in adequate amounts.

It must be recalled that the concept followed here for the practical use of single-nutrient responses is essentially that of Liebig (an idealized picture of this concept was shown in Figure 1). If the limiting factors B, C, and D in Figure 1 are taken to be other fertilizer inputs, then a threshold yield can be established for each nutrient. The threshold yield is the yield at which the response to a particular nutrient begins. When several nutrients respond simultaneously it is, of course, necessary to interpret those responses together.

The multinutrient interpretation of N, P, and K response for soil-crop category I is illustrated in Figure 11. The plateau yield requires all three nutrients. Nitrogen is the most limiting nutrient in this soil-crop category, such that some improvement appears possible with N application alone for this particular group. (Other areas, however, usually show approximately equal limitation to both N and P when soil P is low. When this is known to be likely it is unwise to place much confidence in the possible effect of a low level application of a single nutrient. The remainder of the interpretation does, in fact, show that N and P are needed about equally).

Soil-crop category II shows a large response to N, a moderate to K, and no response to P (Figure 12). Recall that this category represents soils which are above the critical level where little or no response is expected.

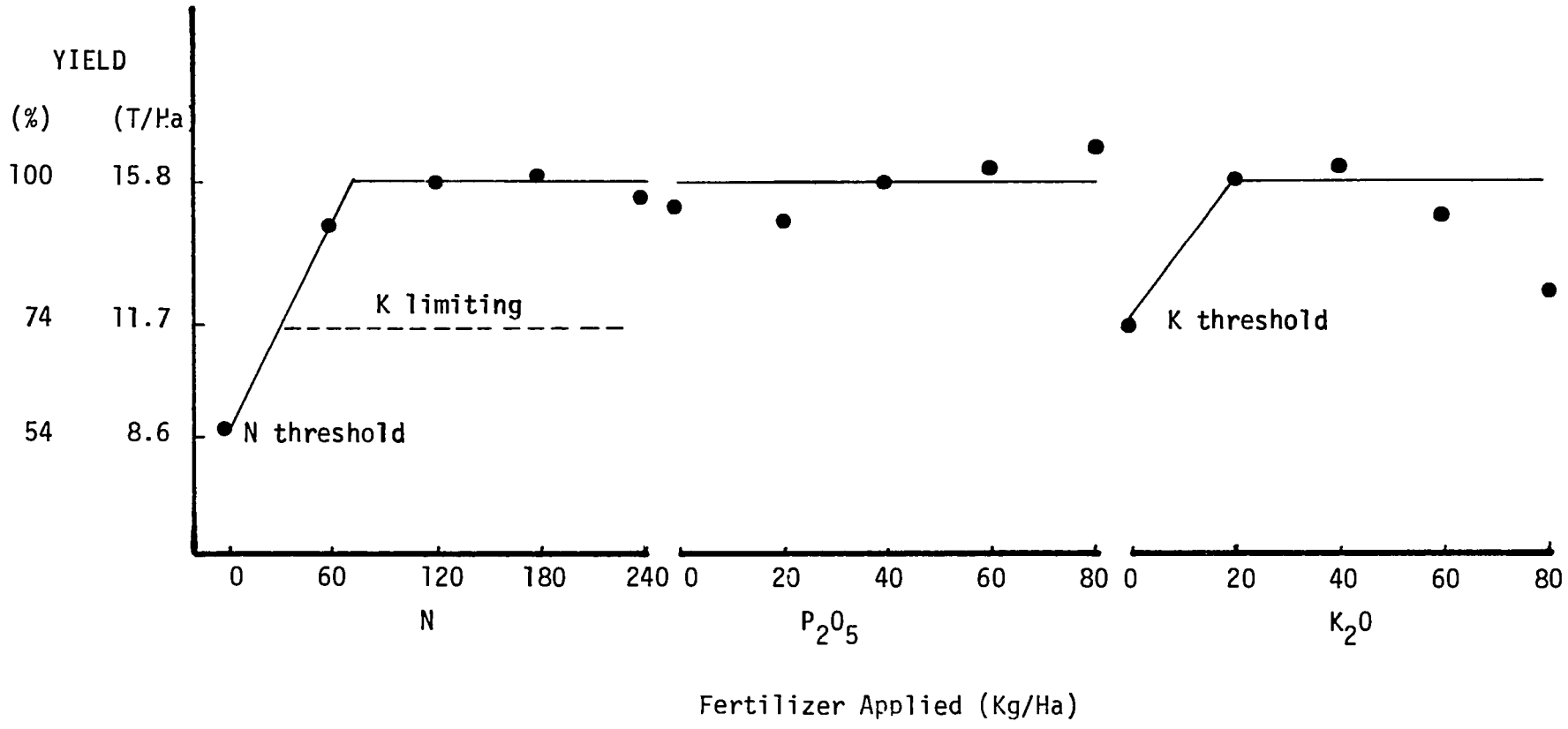
The number of choices for recommendation is limited to the fertilizer treatments necessary to reach threshold yield levels for each nutrient, and to reach the plateau when all responding nutrients



Recommendation choices					
1.	To yield plateau	75-70-20	3.	To P threshold	25-0-0
2.	To K threshold	65-60-0	4.	To N threshold	None

See LSD's in Appendix Table E.

FIGURE 11. STEP 5-a.
Multinutrient response interpretation and development of fertilizer recommendations using combined single-nutrient LRP models: soil-crop category I. (Potatoes).



Recommendation choices			
1. To yield plateau	70-0-20	3. To N threshold	None
2. To K threshold	30-0-0		

See LSD's in Appendix Table E.

FIGURE 12. STEP 5-b.
Multinutrient response interpretation and development of fertilizer recommendations using combined single-nutrient LRP models: soil-crop category II. (Potatoes).

are present. The number of choices will always be one greater than the number of nutrients affecting yield, i.e., the extra choice will be the do-nothing option. Thus, for N-P-K responsive situations, there will be four recommendation choices as shown in Figure 11, each of which is determined directly from the graph. In Figure 12, there are only three choices, since the response was due only to N and K.

An additional illustrative example for multinutrient interpretation is given for soil-crop category V (wheat) in Appendix Figure L. Several points of interest should be brought out. The most obvious feature of this set of data is that there is considerable scatter among yield observations beyond the 2-level. This scatter makes an eye-ball graphing technique more difficult in locating the yield plateau. However, since all nutrients ultimately reach the same yield plateau the collective points from all three curves can be used to sight in the yield plateau across all nutrients. (More correctly the actual yield plateau, 2.15 in this case, should be the calculated mean of all levels on the plateau over all nutrients). Note also in Appendix Figure L that P is more limiting that N, and that the 1-level treatment of the K function cannot be used in establishing the slope (see Section IV, p. 40).

Economic Interpretation of Recommendation Choices

The linear response-and-plateau model (LRP) is a simple tool not only for establishing the yield inflection points for each nutrient response, but also for completing an economic interpretation. The linear response portion, if sufficiently steep, indicates that the most profitable rate to apply will be synonymous with the inflection point rate of application (at the intercept with the yield plateau). In economic terms, this will minimize unit cost since the response is considered to be linear up to the yield plateau. This minimization of unit cost, of course, would be true if only one nutrient were involved. The situation with more than one nutrient

is that each successive nutrient deficiency which is corrected adds the cost of a new nutrient to the previous nutrient cost. To increase yield to each successive threshold yield adds progressively to the cost because greater quantities and more nutrients are involved. Thus, when three nutrients are involved, with a resultant four recommendation choices (Figure 11), it is necessary to evaluate the step-wise economics of each option. This can be readily done making a simple comparison between the additional crop value and additional fertilizer cost from each recommendation choice to the next one higher. Another technique, which may be particularly valuable when crop prices fluctuate greatly or are generally unknown, is that of comparing changes in relative cost and relative yield between each of the recommendation choices. With this latter technique the rule is: "apply the nutrient or fertilizer combination if the resultant percentage increase in cost does not exceed the predicted percentage increase in yield" (Cate, et al., 1971).

The Percentage-Increase-in-Cost-and-Yield Approach

Cate et al. (1971) point out that it is useful to differentiate between relative yield and percentage yield increase. Mathematically these two quantities are inversely related because they employ opposite bases of reference. Relative yield employs the plateau yield as the divisor, whereas percentage increase in yield uses the threshold yield as the divisor. By way of illustration a 50 percent relative yield means that the threshold yield was only half as much as the plateau yield, i.e., the application of a particular fertilizer nutrient doubled yield or produced a percentage increase of 100 percent.

The procedure for calculating percentage increases in yield and cost due to fertilizer use is illustrated in Table 5 using the Bolivian data. In soil-crop category I each additional level of fertilization reduces the relative unit cost, and the percentage

Table 5. STEP 6, Economic evaluation of fertilizer recommendation.
Calculation of percentage increases in yield and cost and the change in relative unit cost at each choice of fertilizer recommendation 1/

Fertilizer recommendation choices	Yield data			Cost data				
	Relative yield at each nutrient threshold	Change from next lower recommendation choice		Fertilizer cost	Total production cost	Change from next lower recommendation choice		Relative unit cost 4/
		Increase in relative yield	Percentage increase in yield 2/			Increase in cost	Percentage increase in cost 3/	
			(%)	(U.S. $)	(U.S. $)	(U.S. $)	(%)	
Soil-crop category I. (From Figure 11)								
0-0-0	45	--	--	0	340	--	--	7.6
25-0-0	58	13	29	10	350	10	3	6.0
60-55-0	93	35	60	40	380	30	9	4.1
70-65-40	100	7	8	60	400	20	5	4.0
Soil-crop category II. (From Figure 12)								
0-0-0	54	--	--	0	340	--	--	6.3
30-0-0	74	20	37	10	350	10	3	4.7
70-0-20	100	26	35	32	372	22	6	3.7
Soil-crop category V. (From Appendix Figure L)								
0-0-0	37	--	--	0	66	--	--	1.8
0-15-0	46	9	24	5	71	5	8	1.5
60-45-0	76	30	65	37	103	32	45	1.4
100-70-35	100	24	32	72	138	35	34	1.4

1/ Based on potato and wheat production data with Bolivian cost estimates for seed, land, labor prior to harvest, pesticides, interest on borrowed money, all nutrients @ U.S.$0.35/Kg.

2/ Increase in relative yield ÷ the relative yield at each nutrient threshold, i.e., previous relative yield.

3/ Increase in cost ÷ total production cost for previous recommendation choice

4/ Total production cost ÷ relative yield at each nutrient threshold.

increase in yield is well above the percentage increase in cost. The correct recommendation, for a farmer with no constraints on fertilizer purchases, would be the complete formula, 70-65-40.

In the case of soil-crop category II the last step addition (70-0-20) would also be recommended under the yield vs. cost rule.

The calculations for potatoes in Table 5 are typical for an intensive crop, where production costs are rather high even without fertilizer. Thus, the percentage increase in yield is typically much higher than percentage increase in cost due to fertilization. Another way of interpreting this situation is that the producer already has considerable input investment to protect and, therefore, the relatively small added cost to fertilize to maximum yields is easily justified.

The wheat data (Table 5) are presented to show the case for a less intensive crop, where base costs of production without fertilizer are much lower. Soil-crop category V data result in a situation where all three nutrients are required for the last step increase to plateau yield and the percentage increase in cost for that step is slightly higher than the expected percentage increase in yield. According to the yield vs. cost rule therefore, the most economical recommendation would be 60-45-0 in this example. A similar case might be seen in non-mechanized agriculture where cheap labor and available land can be substituted for fertilizer and other inputs. Typically the percentage increase in cost under these conditions runs high when fertilizer is used, approaching or surpassing the predicted percentage increase in yield.

The Additional-Crop-Value-and-Fertilizer-Cost Approach

The farmer is usually concerned about knowing how much his fertilizer will cost and how much added revenue the investment will produce. In its simple form such an economic interpretation can also be based on a step-wise evaluation, as shown in Table 6. The linear response-and-plateau (LRP) model for the three nutrient responses (Figures 11 and 12) conveniently provide the basic information required for this evaluation. The final economical recommendation of soil-crop categories I and II would be 70-65-40 and 70-0-20, respectively. The final step of fertilizer application would be profitable and return at least 2:1 on fertilizer investment in both cases. On the other hand, the recommendation for soil-crop category V could be either 60-45-0 or 100-70-35. Given the cost of fertilizer as high and the value of wheat on the local market as low as the example provided, a conservative farmer may not want to risk only a 1.5:1 return even though it is slightly profitable. Of course, given either a cheaper fertilizer or a better price for the crop the ratio of added crop value to added fertilizer cost would improve and eventually reach at least a 2:1 margin.

The margin of safety which should be allowed will depend on many factors, among them being the risks associated with the overall production program. In situations like this the agronomist must draw on other pertinent facts about the overall production program, such as type of credit available, type of crop which follows in the rotation, and possibilities of considering part of the P or K application as a capital investment. The agronomist who is aware of any fundamental changes or trends regarding crop value is in a good position to help decide borderline cases.

A complete discussion of economic evaluation by either of the two economic approaches is beyond the scope of this bulletin. The two approaches will often give the same conclusion. More complete economic evaluation than those outlined here would require input data beyond that available to most soil fertility evaluation programs.

Table 6. STEP 6a.
Economic evaluation of fertilizer recommendations by comparing added cost and added crop value[1/]

Fertilizer recommendation choices	Average expected yield	Crop value	Increase in crop value from next lower recommendation choice	Fertilizer cost	Increase in fertilizer cost from next lower recommendation choice	Returns to fertilizer: Ratio of added crop value to added fertilizer cost	Returns to fertilizer: Net return to last fertilizer increments
	T/Ha	$	$	$	$		$
Soil-crop category I. (Potatoes)		(From Figure 11)					
0-0-0	8.5	255	--	0	--	--	--
25-0-0	11.1	333	78	10	10	7.8	68
60-55-0	17.7	531	198	40	30	6.6	168
70-65-40	19.0	570	39	60	20	2.0	19
Soil-crop category II. (Potatoes)		(From Figure 12)					
0-0-0	8.6	258	--	0	--	--	--
30-0-0	11.7	351	93	10	10	9.3	83
70-0-20	15.8	474	123	32	22	5.6	101
Soil-crop category V. (Wheat)		(From Appendix Figure L)					
0-0-0	0.79	79	--	0	--	--	--
0-15-0	1.00	100	21	5	5	4.2	16
60-45-0	1.63	163	63	37	32	2.0	31
100-70-35	2.15	215	52	72	35	1.5	17

1/ All nutrients figured at estimated Bolivian prices of fertilizer nutrients @ U.S.$0.35 Kg, potatoes @ $30/T, and wheat @ $100/T.

2/ Increase in crop value from next lower recommendation choice minus increase in fertilizer cost from next lower recommendation choice.

In addition, the much emphasized difficulty in predicting yield is a key limitation which is common to all except the most highly controlled cropping systems. The important feature of these simple economic approaches is that they maximize use of the kinds of data which are already available to good soil fertility programs, and as such, they offer a reasonable interpretation.

A final point about these recommendation results is that all the data employed here represent specific varieties, situations, and sites. The final outcome of the recommendations, therefore, apply only to the specific conditions under which these data were obtained. The conclusions, while illustrative, do not represent generalized recommendations for these crops.

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A P P E N D I X

Appendix Table A. Coefficients of determination of various models fitted to correlation data from various countries 1/

Country and data set code	Continuous models			Discontinuous two-mean model
	Mitscherlich	Reciprocal	Logarithmic	
00001	.38	.67	.59	.73
00013	.18	.06	.14	.49
00019	.16	.33	.34	.51
00035	.18	.38	.32	.41
00054	.31	.34	.41	.62
00074	.61	.38	.65	.72
00115	.57	.80	.84	.74
00122	.64	.87	.82	.88
00135	.76	.48	.83	.67
00144	.47	.14	.30	.42

1/ Data after Cate, 1971.

Appendix Table B. Summary of treatments and average yields for each soil-crop category in Bolivia trials 1/

Treatment code	Soil-crop categories									
	Potato trials						Wheat trials			
	I Low P - High K soils 2/		II High P - High K soils 2/		III Low P - Low K soils 3/		IV Low P - High K soils 4/		V Low P - Low K soils 4/	
	Treatment	Yield	Treatment	Yield	Treatment	Yield	Treatment	Yield	Treatment	Yield
	Kg/Ha	T/Ha	Kg/Ha	T/Ha	Kg/Ha	T/Ha	Kg/Ha	T/Ha	Kg/Ha	T/Ha
000	0- 0- 0	6.8	0- 0- 0	6.0	0- 0- 0	4.8	----	----	-----	-----
222*	120-120-40	18.6	120-40-40	16.3	120-120-120	15.6	120-100-40	1.94	120-100-40	2.06
022	0-120-40	8.5	0-40-40	8.6	0-120-120	7.3	0-100-40	1.12	0-100-40	1.00
202	120- 0-40	11.1	120- 0-40	15.4	120- 0-120	4.8	120- 0-40	0.90	120- 0-40	0.79
220	120-120- 0	17.7	120-40- 0	11.7	120-120- 0	10.9	120-100- 0	2.07	120-100- 0	1.63
422	240-120-40	19.7	240-40-40	15.6	240-120-120	12.5	240-100-40	1.94	240-100-40	1.98
242	120-240-40	19.5	120-80-40	16.9	120-240-120	19.4	120-200-40	2.33	120-200-40	2.28
224	120-120-80	19.9	120-40- 80	12.7	120-120-240	14.4	120-100-80	1.88	120-100-80	2.33
111	60- 60-20	16.7	60-60-20	14.8	60- 60- 60	13.1	60- 50-20	1.69	60- 50-20	1.68
113	60- 60-60	16.2	60-20-60	13.9	60- 60-180	14.8	60- 50-60	1.78	60- 50-60	1.68
131	60-180-20	16.8	60-60-20	15.2	60-180- 60	15.3	60-150-20	1.74	60-150-20	1.56
311	180- 60-20	18.7	180-20-20	14.3	180- 60- 60	11.5	180- 50-20	1.72	180- 50-20	1.72
133	60-180-60	18.3	60-60-60	15.4	60-180-180	12.9	60-150-60	1.77	60-150-60	1.60
331	180-180-20	20.0	180-60-20	19.5	180-180- 60	16.6	180-150-20	2.56	180-150-20	1.95
313	180- 60-60	18.9	180-20-60	16.4	180- 60-180	9.7	180- 50-60	1.56	180- 50-60	1.70
333	180-180-60	21.5	180-60-60	14.6	180-180-180	18.3	180-150-60	2.01	180-150-60	2.08

1/ Experiments conducted under direction of ISFEI project and Bolivian Ministry of Agriculture. (Categories I and II had 8 reps. of treatment 222, while III had 5 reps., and IV and V had 3 reps.).

2/ Original data after Saravia et al., Chinoli Exp. Sta. (14 trials).

3/ Original data after Claure et al., Toralapa Exp. Sta. (3 trials).

4/ Original data after Manzano, Carrera, Iriarte, Lujan, Zuleta, Hinojosa et al., Cochabamba and Tarija (11 trials).

Appendix Table C. Regression analysis R^2 obtained by fitting four response models to potato and wheat field data: by location

Soil-crop category	Location	Response model: Linear response-and-plateau	Quadratic	Square root	Log
		N Data			
I	7	0.294*	0.307*	0.308*	0.309*
	8	0.909**	0.834**	0.938**	0.911**
	9	0.947**	0.520*	0.720**	0.651**
	10	0.869**	0.855**	0.896**	0.891**
II	4	0.826**	0.616**	0.815**	0.749**
	5	0.754**	0.611**	0.764**	0.712**
III	13	0.775**	0.735**	0.672**	0.712**
	14	0.389*	0.580*	0.523*	0.576*
	15	0.467*	0.351	0.455*	0.424*
IV	25	0.909*	0.888**	0.868**	0.894**
	26	0.340*	0.132	0.107	0.106
	27	0.273*	0.207	0.151	0.180
	28	0.562*	0.264	0.544*	0.440*
V	29	0.370*	0.307	0.347	0.345
	30	0.423*	0.381*	0.390*	0.396*
	32	0.474*	0.454*	0.476*	0.512*
	33	0.625**	0.426*	0.370*	0.413*
	34	0.556*	0.785**	0.454*	0.617*
		P Data			
I	7	0.345*	0.421*	0.312*	0.363*
	8	0.718**	0.605**	0.716**	0.690**
	9	0.534*	0.522*	0.620*	0.567*
	10	0.552*	0.408*	0.540*	0.488*
III	13	0.348*	0.296	0.292	0.301
	14	0.421*	0.462*	0.503*	0.487*
	15	0.718**	0.690**	0.650**	0.704**
IV	24	0.361*	0.235	0.346	0.305
	25	0.592*	0.487*	0.571*	0.561*
	26	0.376*	0.256	0.311	0.260
	27	0.569*	0.545*	0.518*	0.535*
V	29	0.463*	0.399*	0.346*	0.381*
	30	0.371*	0.258	0.194	0.238
	31	0.300*	0.250	0.190	0.227
	32	0.433*	0.144	0.368*	0.286
	33	0.584*	0.432*	0.524*	0.514*
		K Data			
III	15	0.401*	0.270	0.212	0.243
V	29	0.342*	0.206	0.306	0.293
	33	0.467*	0.323	0.310	0.321
Total number of experiments in which model gave best R^2		27	3	6	1

Appendix Table D. Regression analysis R^2 obtained by fitting four NPK response models to potato and wheat field trial data pooled into soil-crop categories 1/

Nutrient response studied	Soil-crop category	Response model			
		Linear response-and-plateau	Quadratic	Square root	Log
Nitrogen	I	0 .872	0.813	0.906	0.881
	II	0.737	0.664	0.738	0.730
	III	0.507	0.503	0.468	0.524
	IV	0.642	0.574	0.616	0.630
	V	0.745	0.592	0.625	0.636
Phosphorus	I	0.706	0.617	0.720	0.690
	III	0.698	0.677	0.719	0.715
	IV	0.818	0.744	0.838	0.816
	V	0.745	0.552	0.614	0.630
Potassium	V	0.444	0.157	0.150	0.167

1/ R^2 is based on treatment sum of squares = $\frac{\text{S.S. regression (2 d.f.)}}{\text{Total S.S. treatments (12 d.f.)}}$

Appendix Table E. Least Significant Differences for comparing single-nutrient mean pairs and their calculation

The Least Significant Difference (LSD) provides a guide as to how large a difference must be before it is considered to be statistically significant. This statistic is very useful in comparing single nutrient mean yields to ascertain whether there is a response to addition of a fertilizer nutrient and also to provide a guide as to where the plateau begins.

The LSD calculation requires an estimate of experimental error, s^2, which is obtained from experimental error mean square in the analysis of variance. Using the degrees of freedom from which error mean square was estimated, a tabular "t" value for the chosen level of probability (.05 or .01) is obtained from a table of values of Student's "t". The LSD is then calculated using the formula:

$$LSD_{\alpha} = t_{\alpha}(\text{error d.f.})\sqrt{\frac{s^2}{r_1} + \frac{s^2}{r_2}}$$

where α = the probability level chosen

s^2 = estimate of experimental error mean square as discussed above

r_1 = number of observations used in determining the first mean

r_2 = number of observations used in determining the second mean

If the r_i values differ for the various sets of pairs of means being compared as they did in this study, it is necessary to calculate a separate LSD for comparing distinct pairs of means. All values in the formula remain the same from one pair to the next except r_1 and r_2.

Example of calculation of LSD for level 0 vs. level 1 for one of the nutrients (N, P, or K). Soil-crop category II.

$$LSD_{.05} = t_{\substack{.05 \\ 42\ d.f.}} \sqrt{\frac{s^2}{6} + \frac{s^2}{24}}$$

$$= (2.02) \sqrt{\frac{5.5718}{6} + \frac{5.5718}{24}}$$

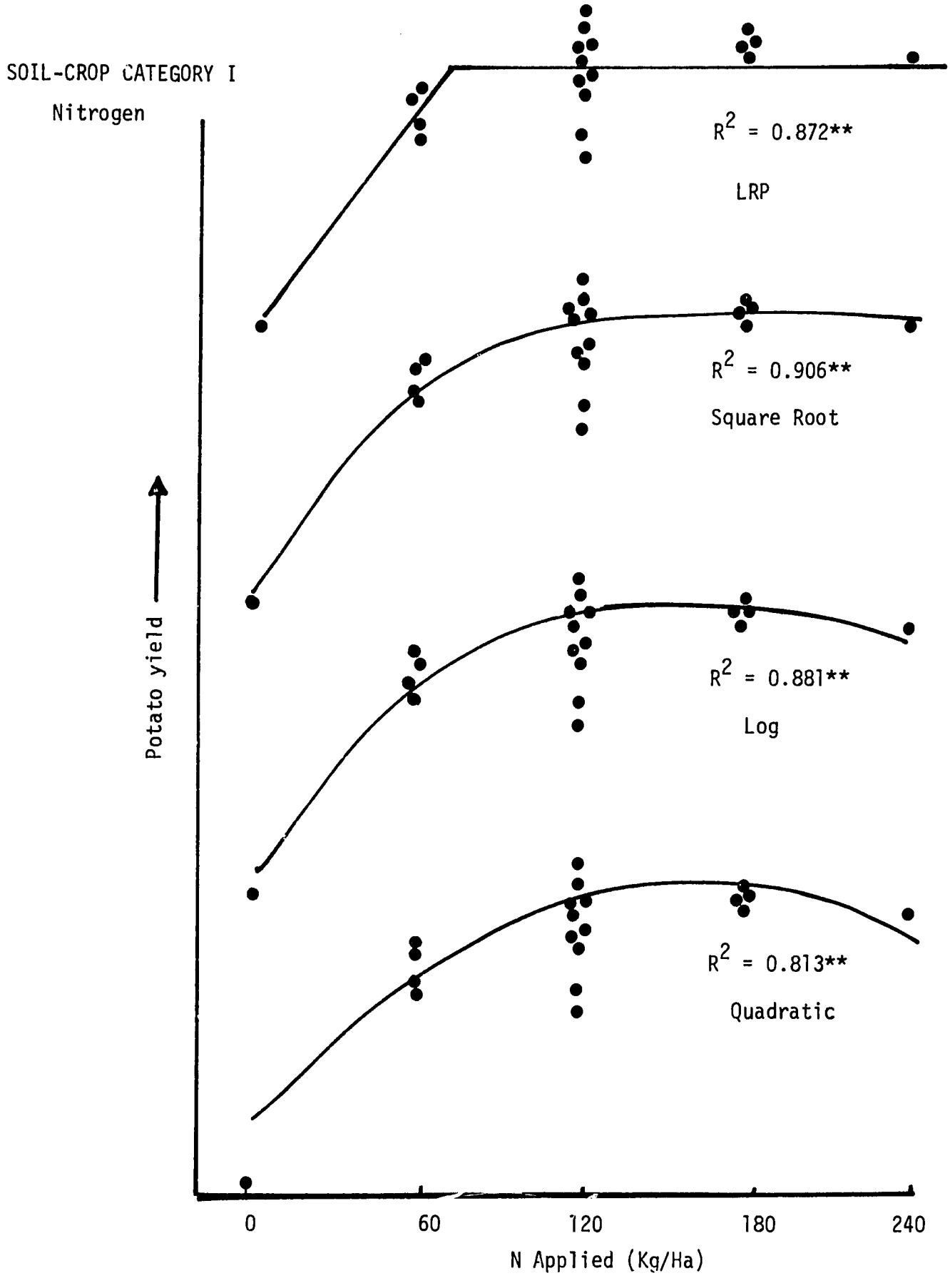
$$= (2.02)\ (1.08)$$

$$= 2.18$$

The LSD's reported below were calculated using the technique described above, substituting in different values of r_i depending upon which pair of means is being compared.

	Soil-crop category I		Soil-crop category II	
	Probability level .05	.01	Probability level .05	.01
Level 0 vs. levels 1, 2, 3, 4	2.26	3.02	2.00	2.69
Level 0 vs. 2, 3, 4	2.28	3.04	2.01	2.69
Level 0 vs. 1 or 3 vs. 4	2.11	2.81	2.18	2.91
Level 0 vs. 2 or 2 vs. 4	1.98	2.63	2.04	2.73
Level 0 vs. 4	2.67	3.55	2.75	3.69
Level 1 vs. 2 or 2 vs. 3	1.12	1.49	1.15	1.54
Level 1 vs. 3	1.34	1.78	1.38	1.84

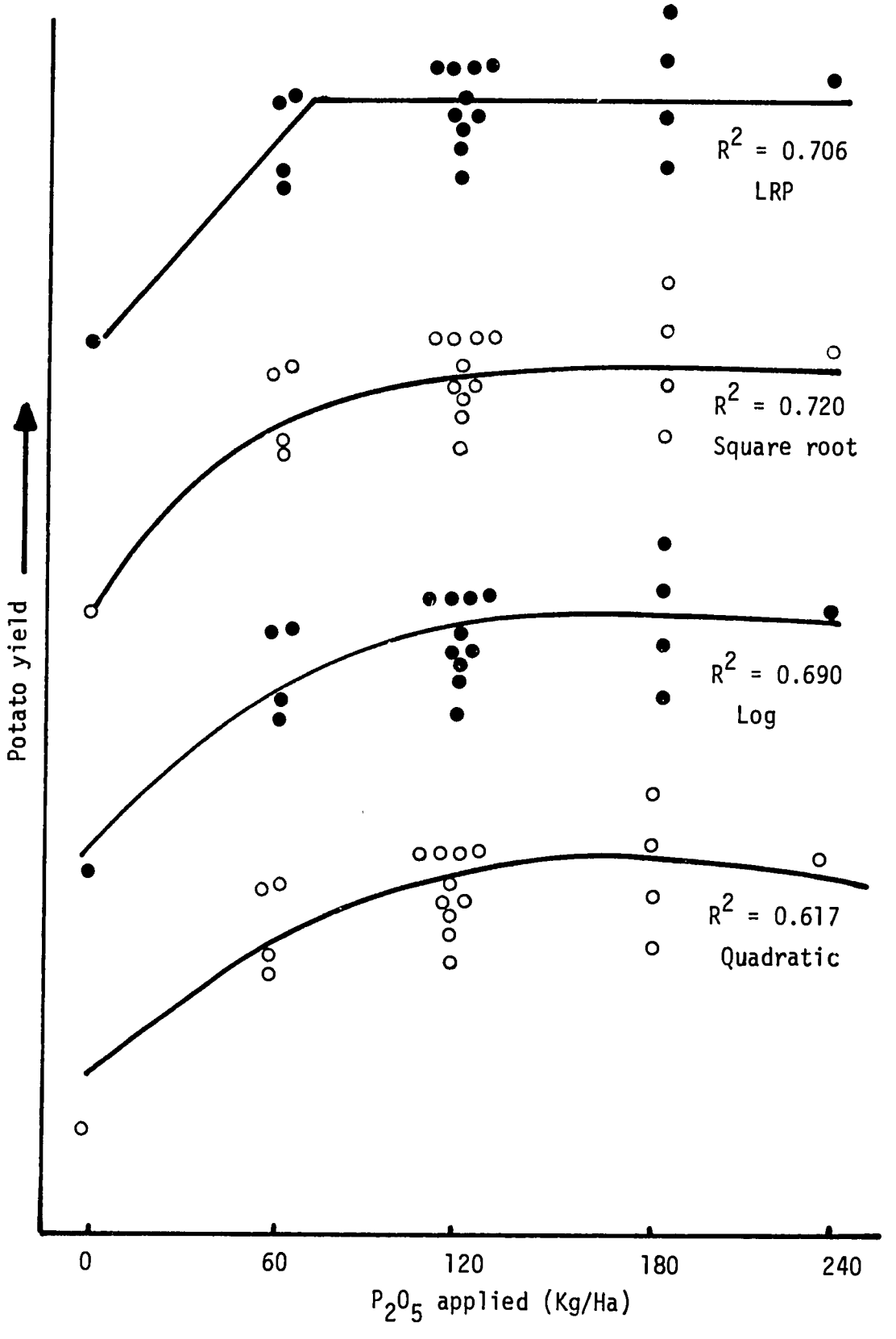
Perhaps one of the most useful comparisons is 0 vs. 1, 2, 3, and 4. This compares the first level mean with the mean of the four other treatment levels. This is used in cases where response, if any, is to the first increment of fertilizer only and the test is for purposes of establishing that there was in fact a significant response to the first increment. If results of such a test fail to establish a significant response to the first increment, further analyses of the data including economic interpretation are usually unwarranted.



Appendix Figure A. Nitrogen response for potatoes fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.

SOIL-CROP CATEGORY I

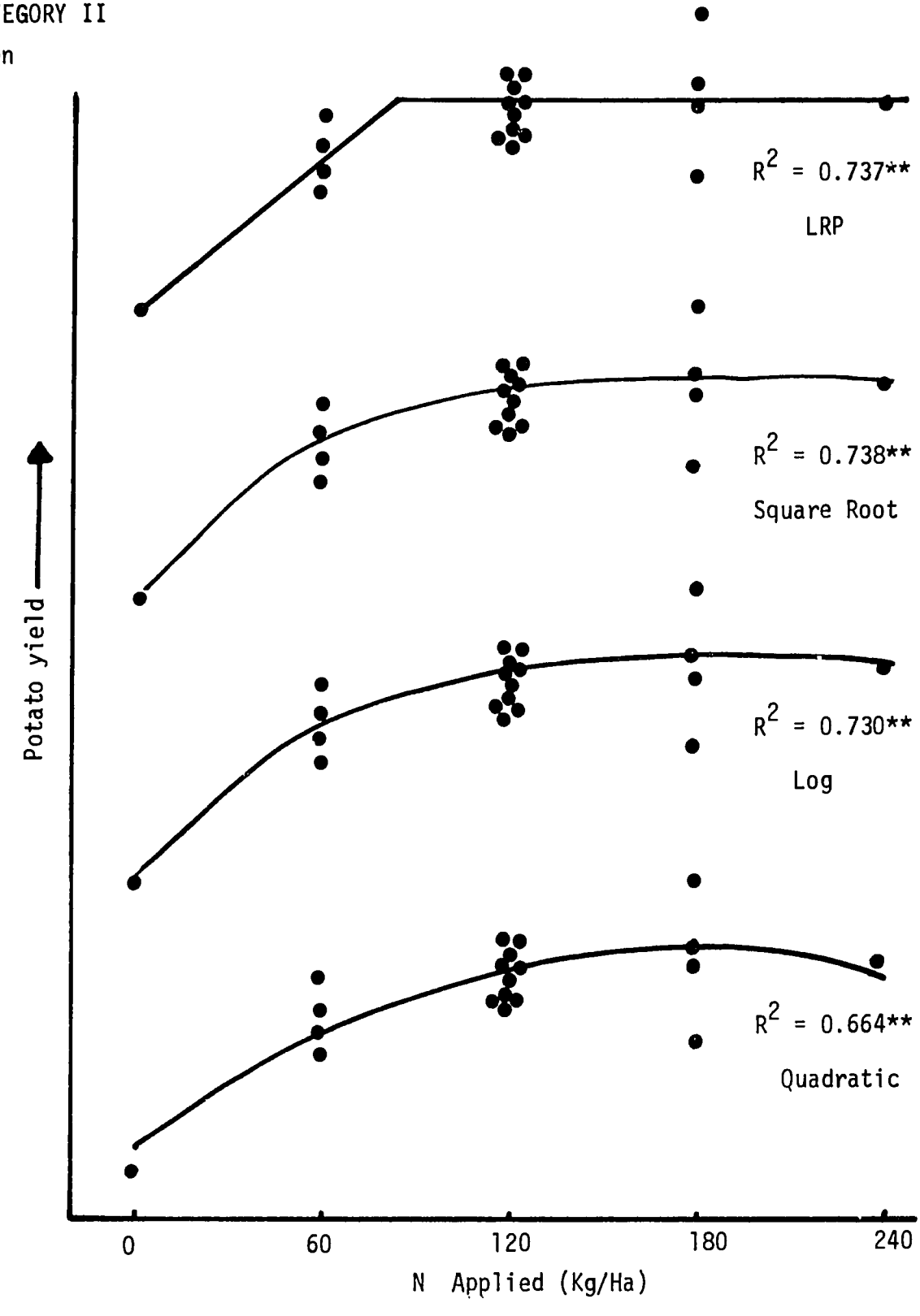
Phosphorus



Appendix Figure B. Phosphorus response for potatoes fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.

SOIL-CROP CATEGORY II

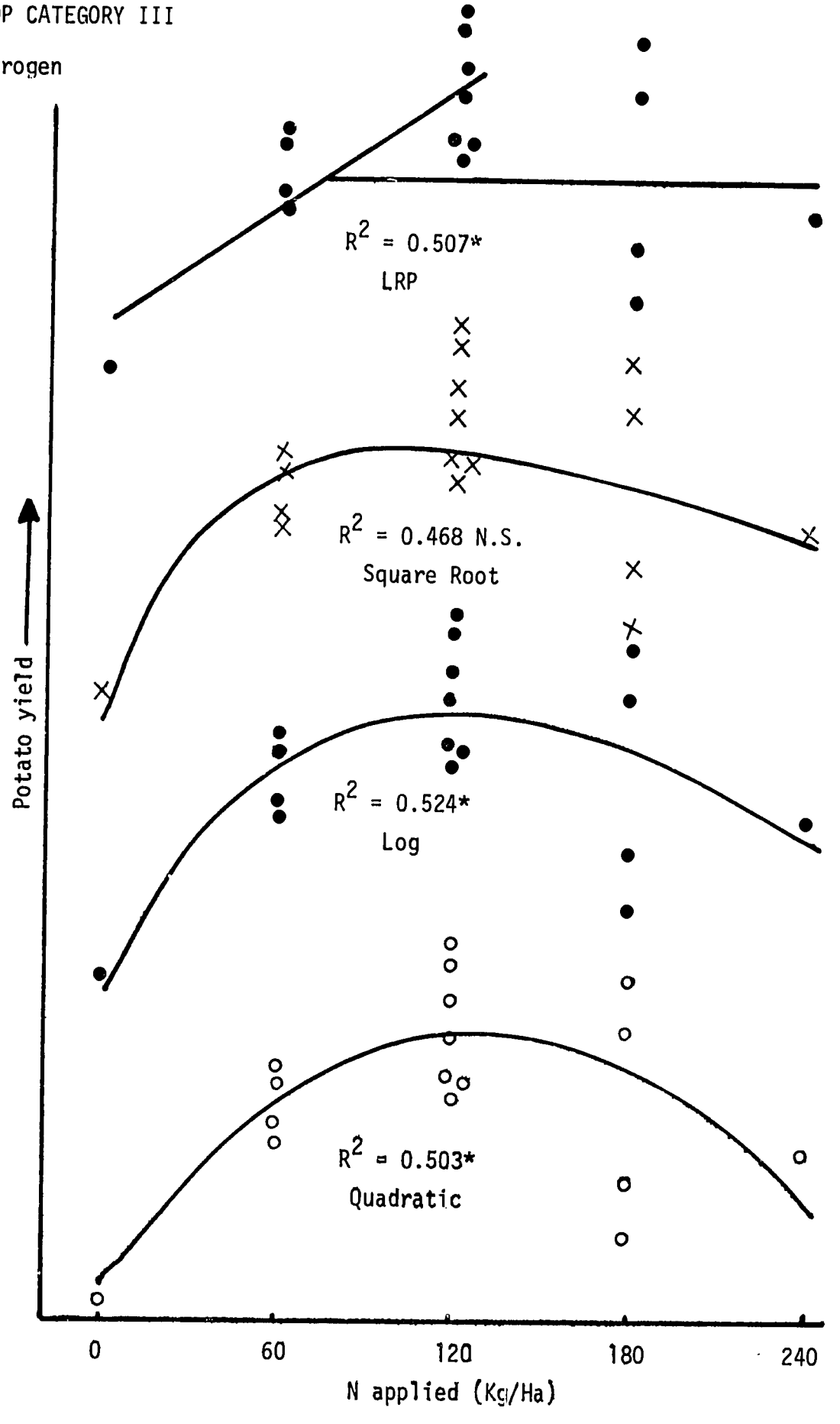
Nitrogen



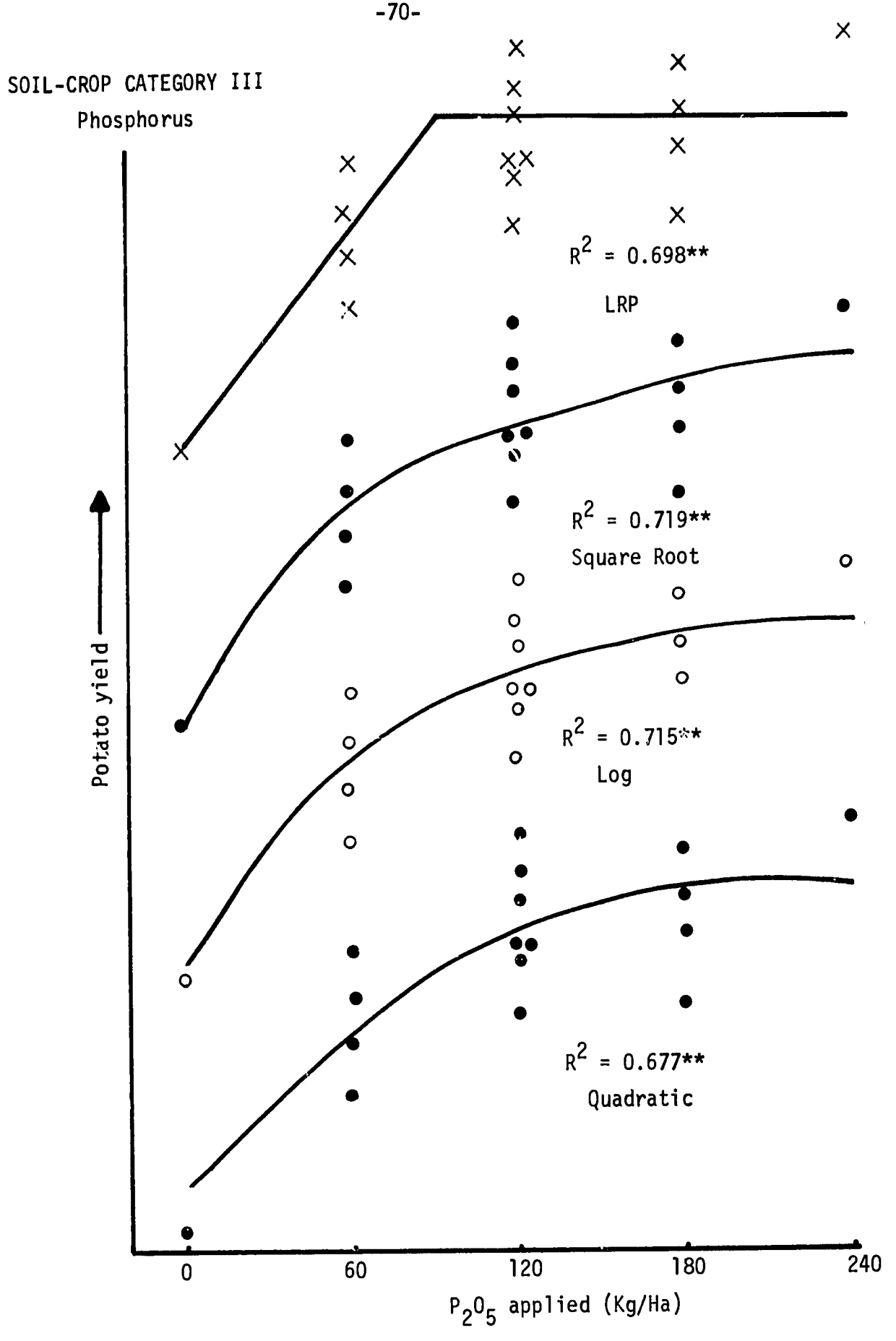
APPENDIX FIGURE C. Nitrogen response for potatoes fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.

SOIL-CROP CATEGORY III

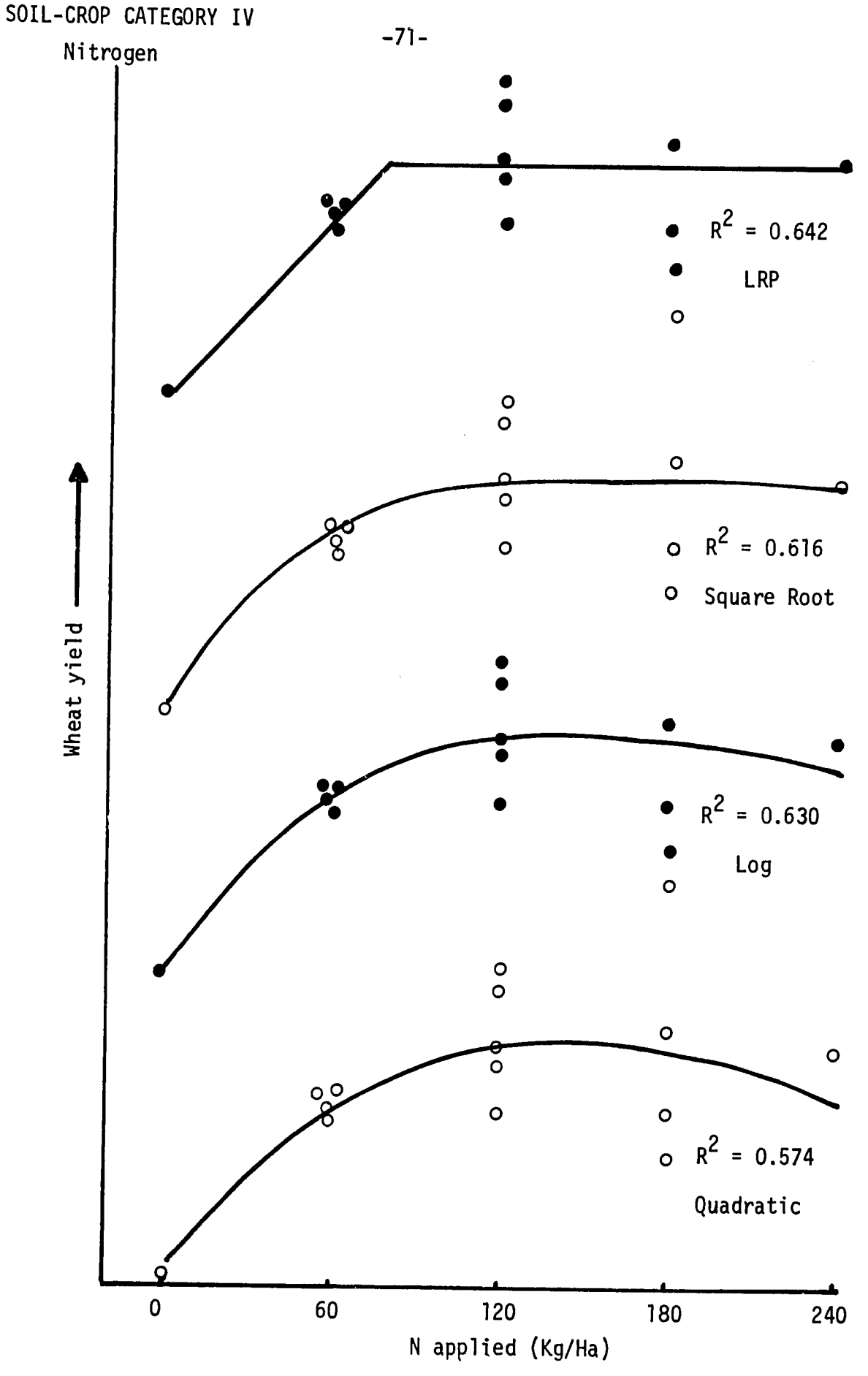
Nitrogen



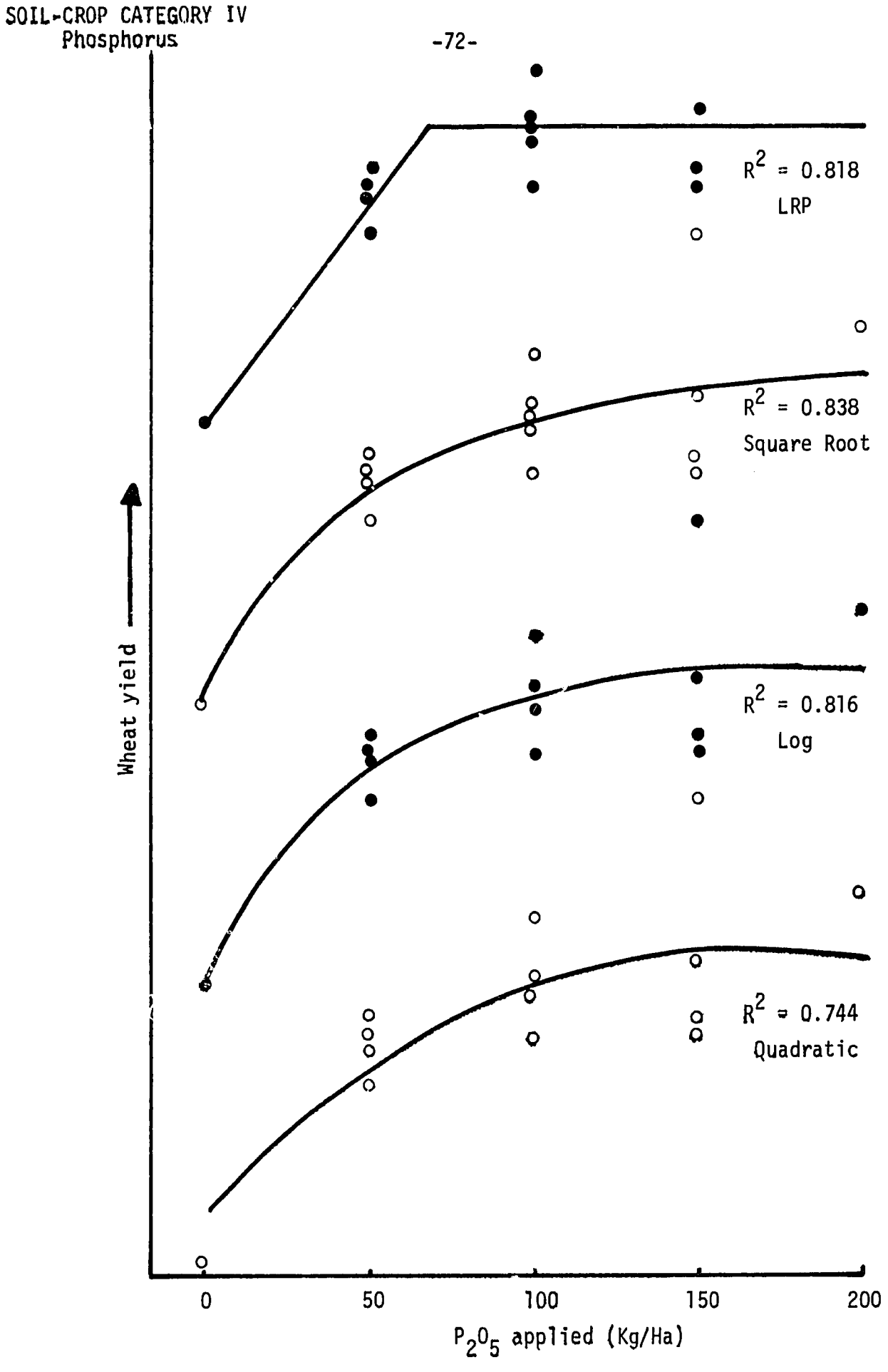
APPENDIX FIGURE D. Nitrogen response for potatoes fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



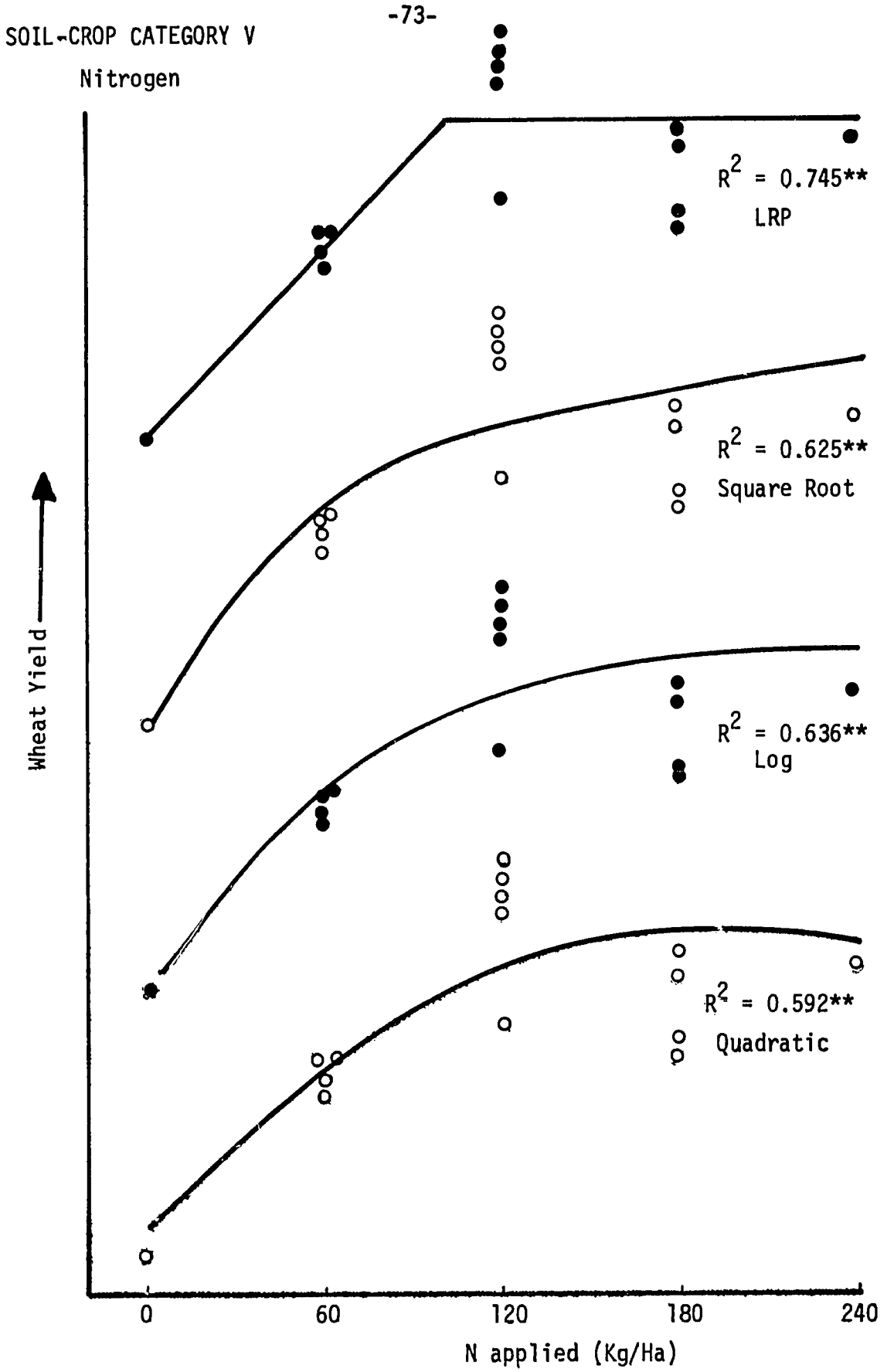
APPENDIX FIGURE E. Phosphorus response for potatoes fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



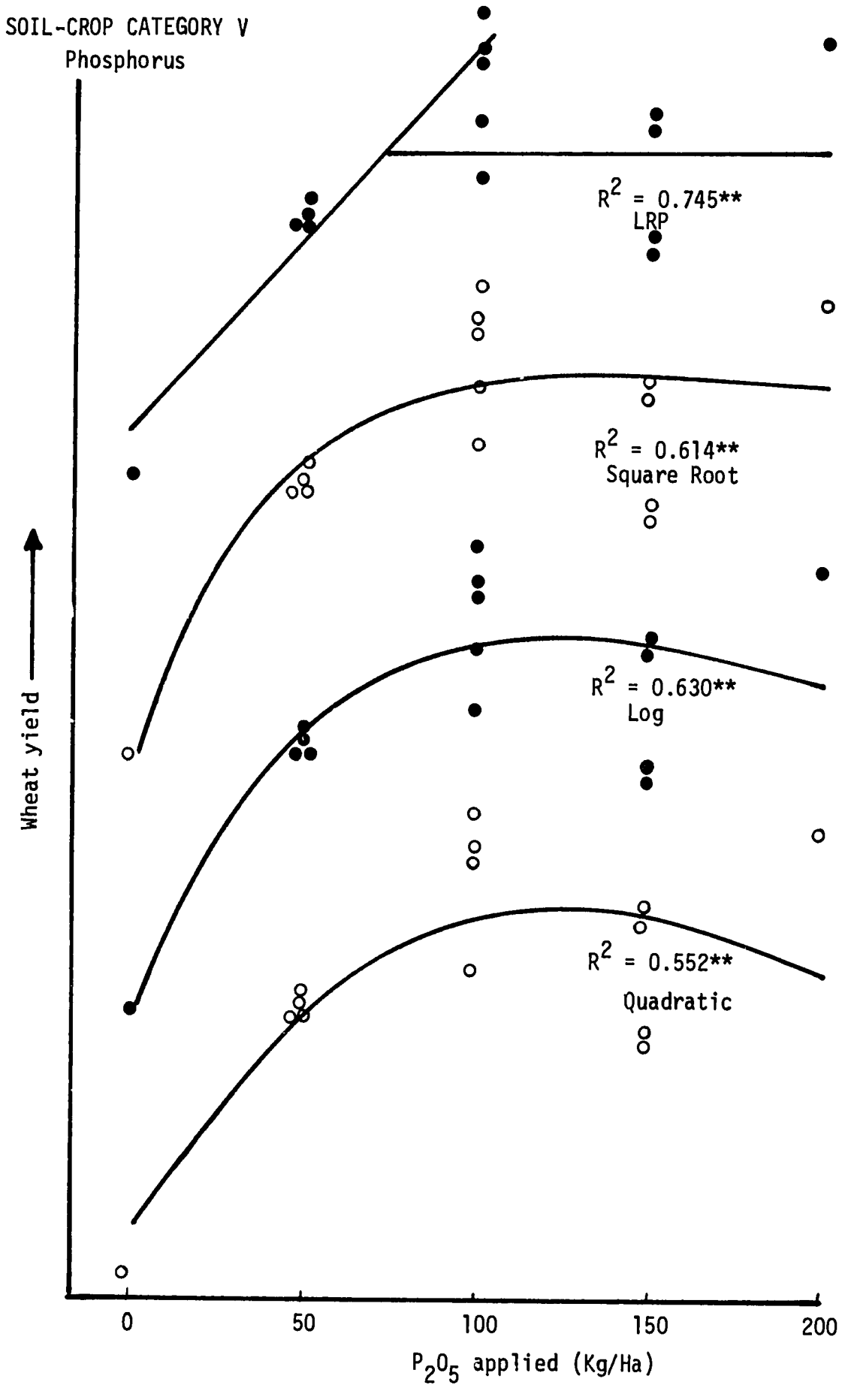
APPENDIX FIGURE F. Nitrogen response for wheat fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



APPENDIX FIGURE G. Phosphorus response for wheat fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



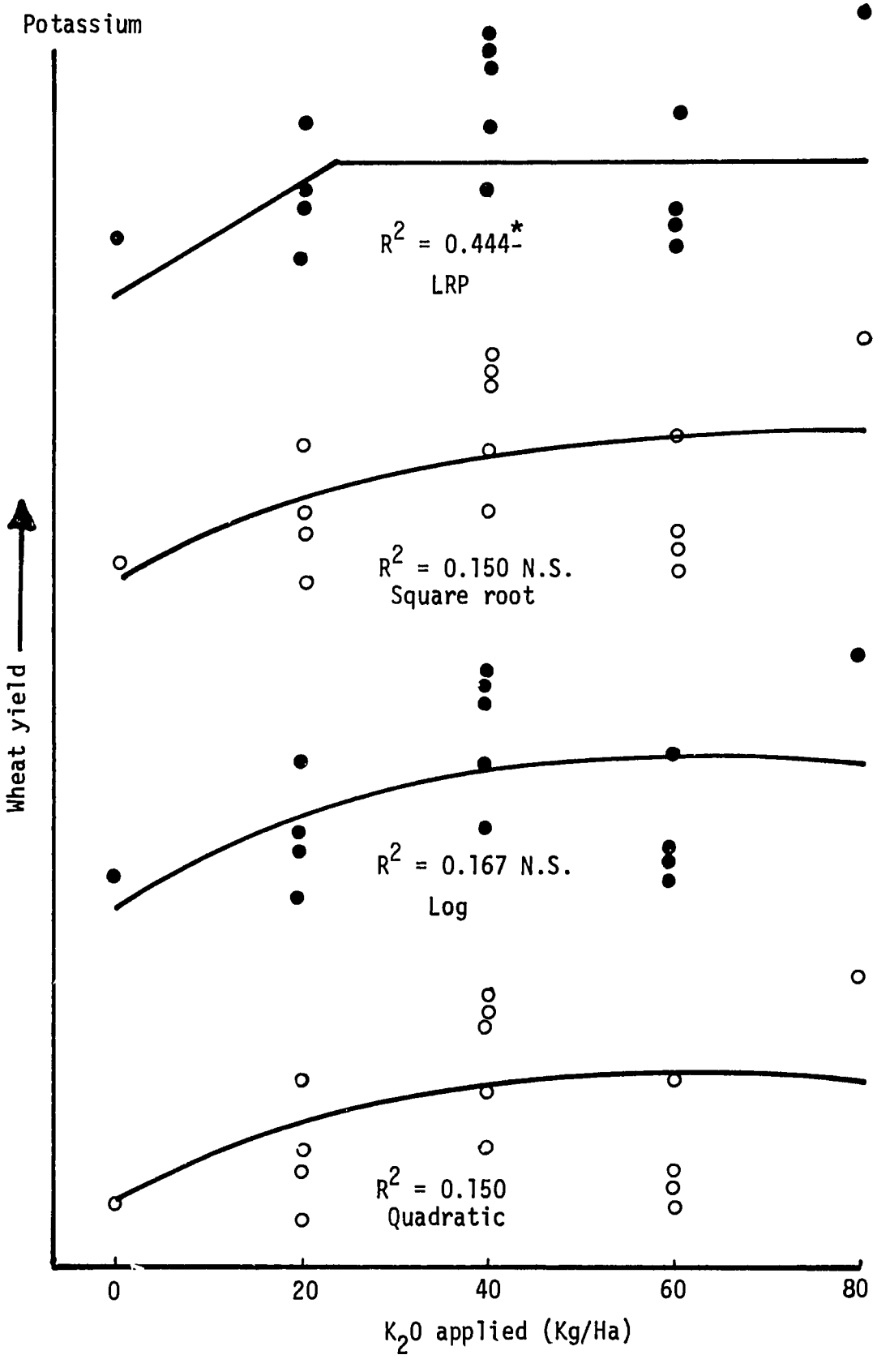
APPENDIX FIGURE H. Nitrogen response for wheat fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



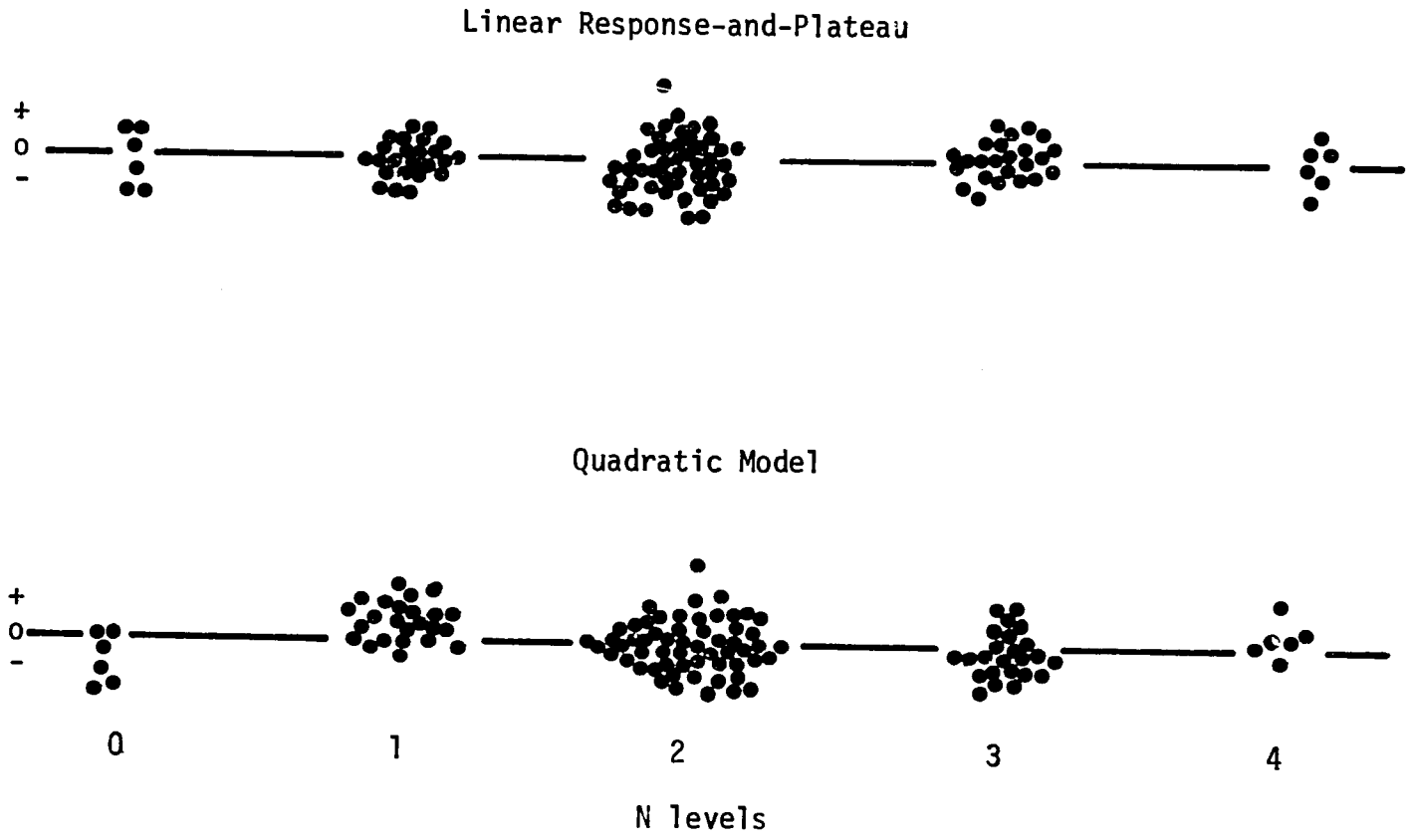
APPENDIX FIGURE I. Phosphorus response data for wheat fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.

SOIL-CROP CATEGORY V

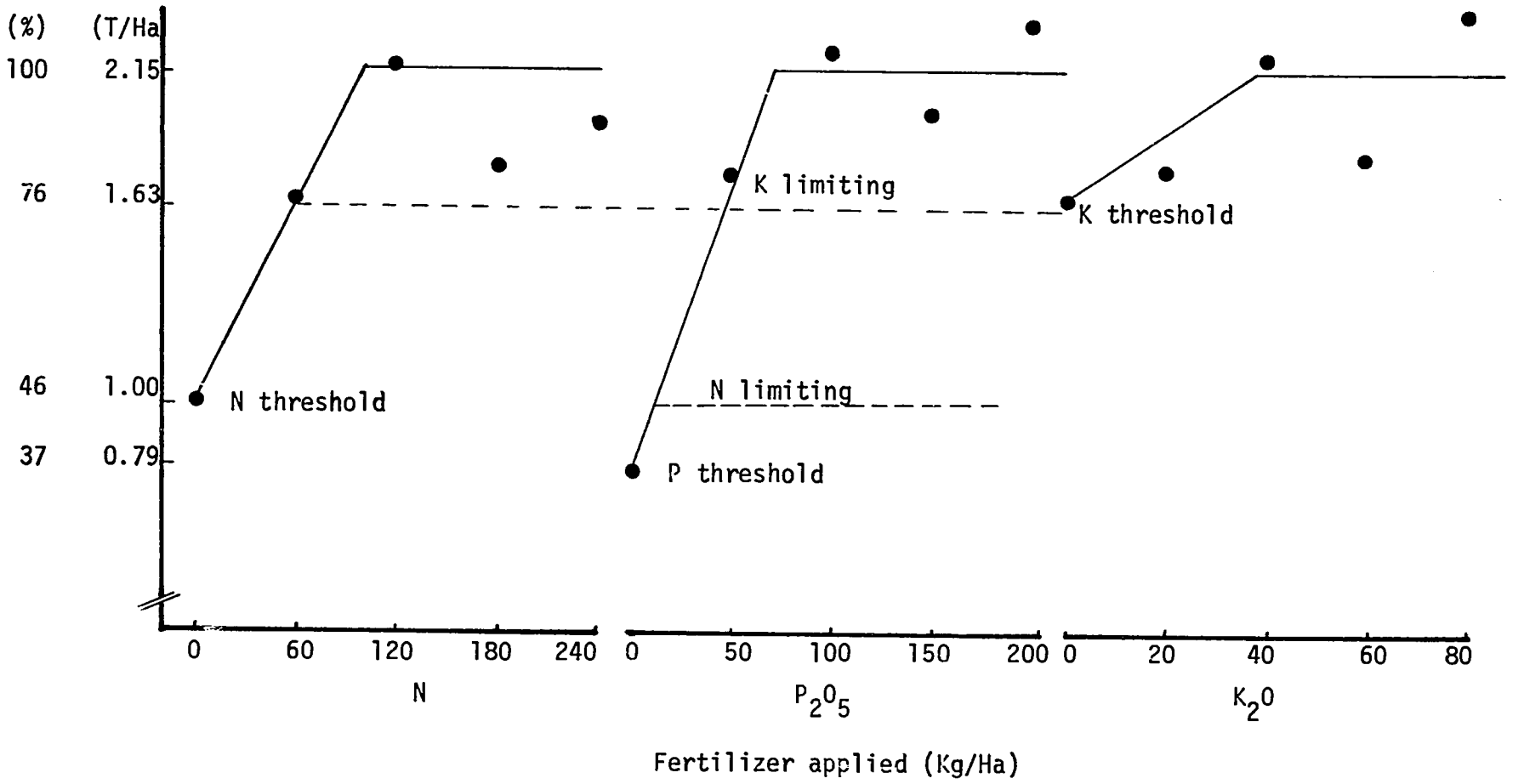
Potassium



APPENDIX FIGURE J. Potassium response for wheat fitted to linear response-and-plateau (LRP), square root, log, and quadratic models.



APPENDIX FIGURE K. Plots of regression residuals for the pool of six individual nitrogen experiments in soil-crop category II.



Recommendation choices			
1. To yield plateau	100-70-35	3. To N threshold	0-15-0
2. To K threshold	60-45-0	4. To P threshold	None

APPENDIX FIGURE L. Multinutrient response interpretation and development of fertilizer recommendations using combined single-nutrient LRP models: soil-crop category V (wheat).