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THE OPTIMAL ALLOCATION OF INVESTMENT IN EDUCATION

by

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The Optimal Allocation of Investment in Education [1]

I Introduction

In recent years recognition of the fact that education has many of the characteristics of an investment process has led to its formal treatment as a capital good and to attempts to measure its marginal efficiency. The rate of return approach to educational planning that has thus evolved is faced by the same difficulty that confronts the technique when applied to investment problems in general, that it is valid only so long as the investment under consideration is sufficiently marginal that it does not affect the structure of prices. Unless one extends the analysis to take into account the effect of an investment, especially the effect of the increase in output due to the investment, on the relevant markets, its usefulness is confined to making qualitative but not quantitative assessments of desirable investment and it can not be used at all for determining the distribution of such investment over time. For this reason the approach has generally been employed as an indicator of immediate priorities rather than as a planning tool and for the latter purpose it has been displaced by relatively primitive techniques like the manpower requirements approach.

The purpose of this paper is to extend the approach so that it may be used for estimating the return to non-marginal investments in education. As in the classical variation of the approach, wage differentials are used to measure the benefits of education. However instead of taking wage differentials to be exogenous quantities, the effect on them of the growth of the educational system is estimated. This is achieved by calculating the impact of the growth of the educational system on the supplies of different types of labour, and then estimating how changes in the structure of the supply of labour will affect the structure of wage rates. One thus obtains a feedback relationship between the growth of a level of education and its own profitability: the faster the growth of a level of education, the greater will be the supply of graduates from it and therefore the lower

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[1] I am indebted to Marcelo Selowsky for many helpful discussions and suggestions, and to Sam Bowles for numerous comments on earlier drafts.

their wage; meanwhile the smaller will be the supply of persons who enter the labour force immediately instead of receiving the education, and the higher will be their wage; hence the lower will be the wage differential and so the rate of return to that type of education.

Figure 1 illustrates this point. The lines marked 'A' show the course of the rate of return over time to secondary education and higher education in Colombia on the assumption that those levels of education maintain their historical growth rates of 10% and 11% per year respectively. The lines marked 'B' show what happens if the growth rates are changed to 15% and 5% per year respectively. [1]

If one believes that investment in education should be carried to the point where the marginal rate of return is equal to a given social rate of discount, the feedback relationship can be used to estimate the amount of investment required and also to determine its timing. In Figure 1 when higher education grows at 11% its rate of return falls from just over 7% in 1969 to less than 1% by 1985. For a social discount rate of 10% this growth rate would be much too high. Cutting the growth rate back to 5% per year brings the trajectory of the rate of return close to the 10% line, and thus it is shown that the optimal growth rate for that discount rate is near 5%. In principle by making finer, year-to-year, adjustments in the enrolment growth rate one could bring the rate of return trajectory to lie exactly on the 10% line. It should be noted, however, that the growth of one level of education will affect the profitability of another, and so such adjustments should be made simultaneously for all levels.

The key part of the analysis is the relationship between the supplies o different types of labour and wage levels. Studies of the rate of return to education have generally made the somewhat ingenuous assumption that relative wage levels will not change over time. [2] Given that in most countries the educational quality of those joining the labour force for the

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1 In this chart $\sigma = 6$ and $\lambda = 0.04$. The significance of these parameters is explained in Sections IV and V below.

2 Some are downright naive and assume that absolute wage levels will remain unchanged.

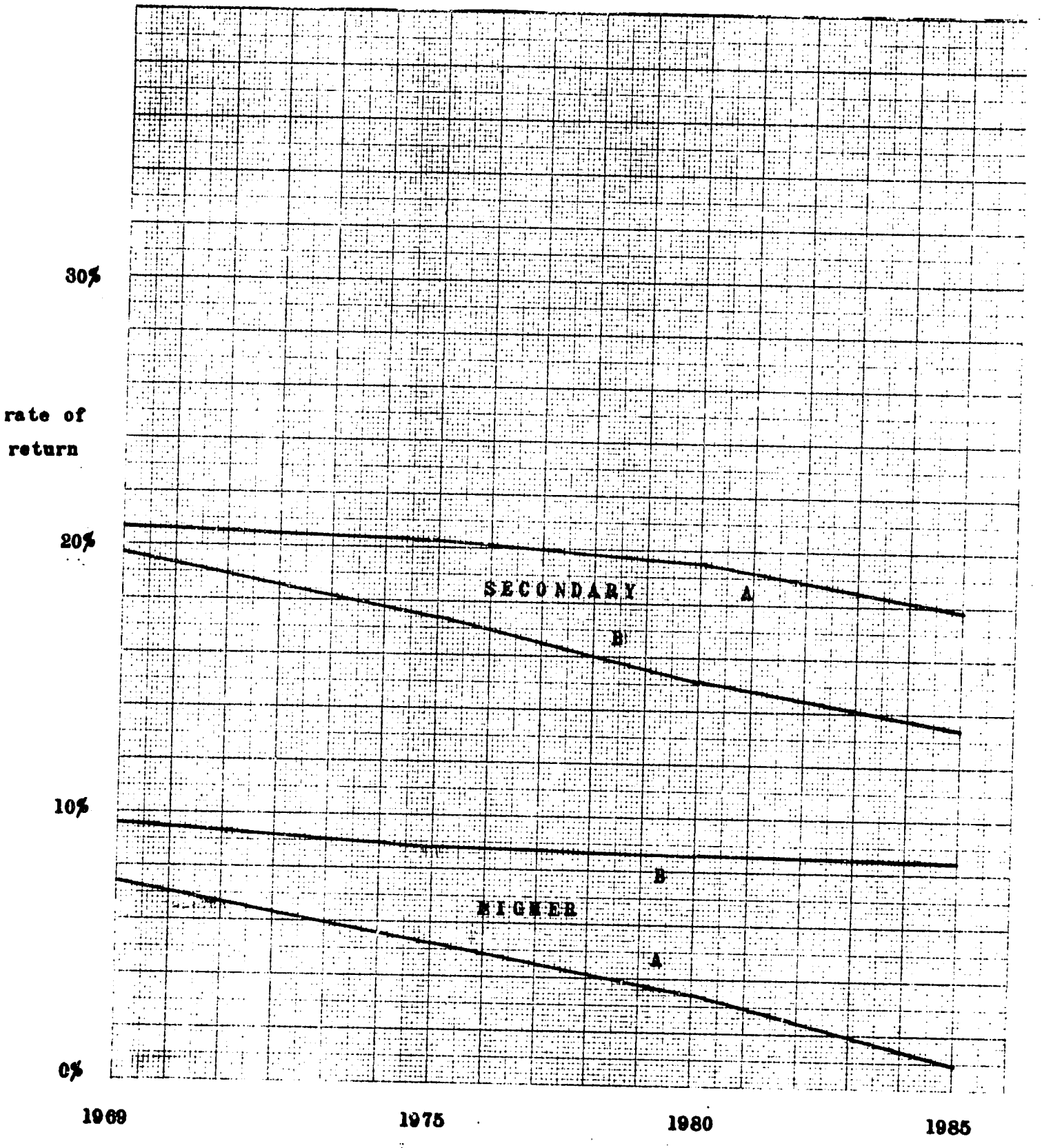
30%
rate of
return
20%
SECONDARY
A
B
10%
B
HIGHER
A
0%
1969
1975
1980
1985

FIGURE 1

first time is appreciably superior to that of the existing stock and that therefore the labour force is being secularly, but in the long run, substantially, upgraded, this assumption implies that the elasticity of substitution between different types of labour is infinite.[1] However casual observation of cross-section data and time series data shows that the greater are the relative supplies of the more highly skilled types of labour, the lower are their relative wages, implying a less-than-infinite elasticity of substitution. This impression has been confirmed by Sam Bowles, who found an elasticity of substitution of between six and eight using two categories of labour and international cross-section data.[2] Appendix 2 describes an experiment using time series data for the United States which yielded an estimate of 3.6 for the elasticity.

The projection of wage rates is important because the estimate of the rate of return to a level of education may be sensitive to yearly earnings far into the future, unless the estimate of the rate of return itself is high.[3] This is true for estimates of the current rate of return to education, and especially true for estimates of the rate of return in future years. Figure 2 shows the sensitivity of estimates of the current rates of return to Colombian education using alternative assumptions about the elasticity of substitution. The lower the elasticity, the faster the relative wage levels are expected to contract. It can be seen that the estimate of the current rates of return to primary and secondary education, both high, are not influenced much by the elasticity, but that of higher education, which is low, is severely affected. Figure 3 shows the effect of the elasticity of substitution on the estimates of the rates of return in 1980. In this case the estimates of both primary and secondary education are significantly affected by the elasticity and the estimate for higher education is very sensitive to it. Table 1 shows base year (1964) wage levels and projected 1980 wage levels corresponding to alternative hypotheses about the elasticity of substitution, in constant 1958 pesos per year.

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1 Providing that there do not exist relative complementarities between some types of labour and other factors of production, e.g. between highly skilled workers and capital. If such relative complementarities do exist, it would be possible for relative wage levels to remain constant with less-than-infinite elasticities of substitution.

2 The Aggregation of Labor Inputs in the Study of Growth and Planning Project for Quantitative Research in Economic Development, Harvard University, Economic Development Report No. 122.

3. For a discussion of this point see Section VI below.

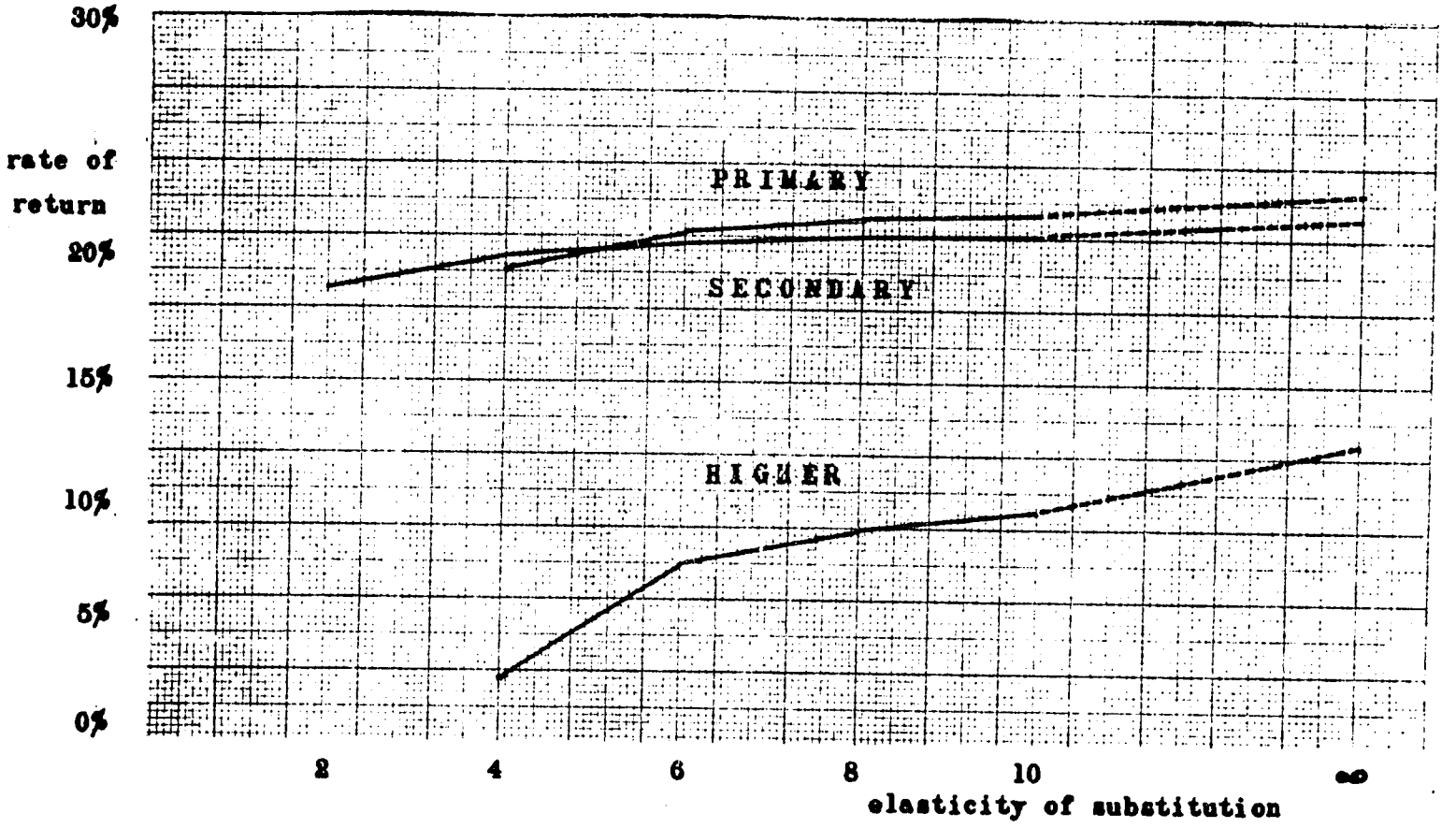

FIGURE 2 : 1969

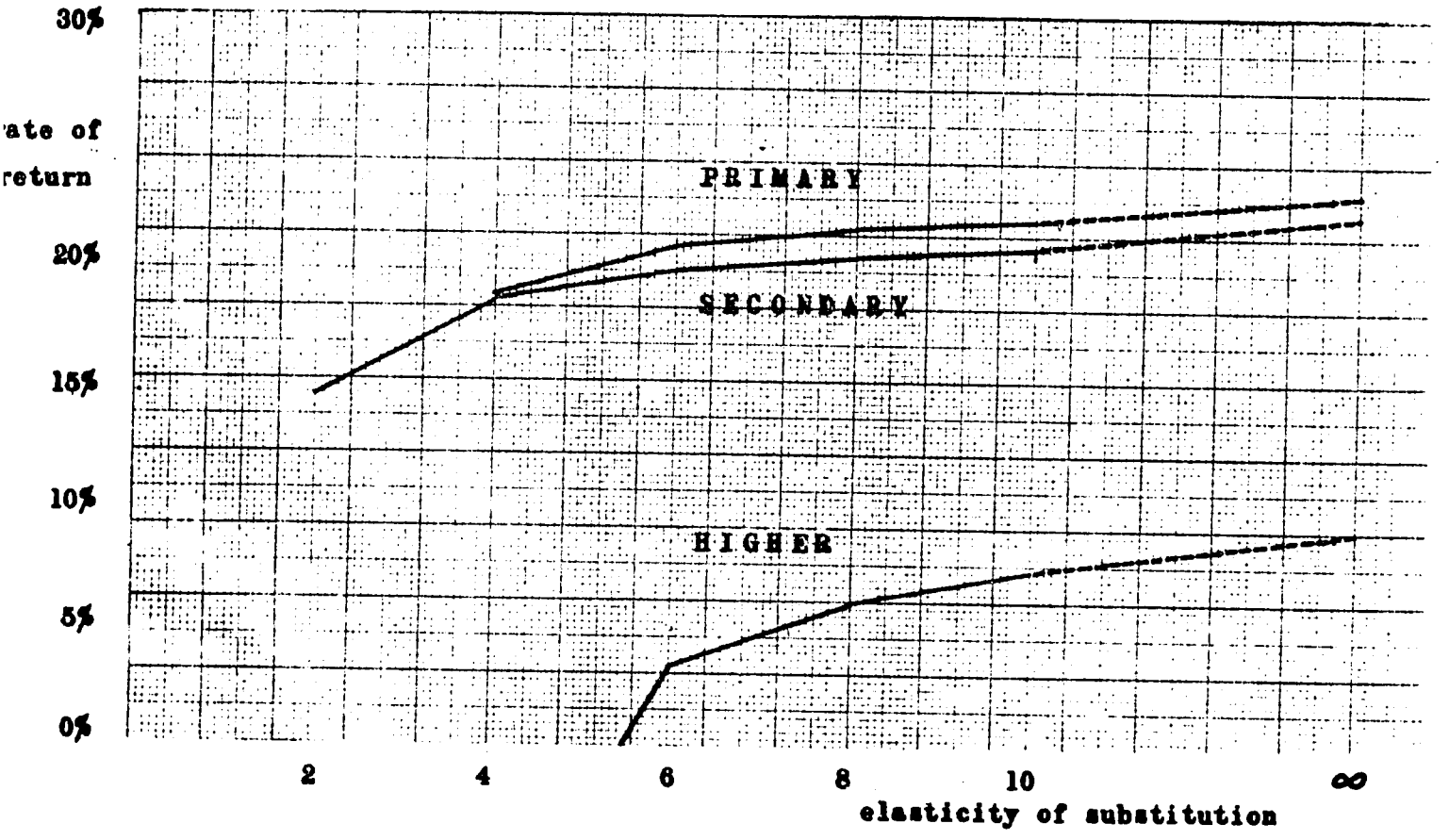

FIGURE 3 : 1980

TABLE 1

Projected wage levels in Colombia [1]
(thousand 1958 pesos per year)

		Illiterates	Primary 1-3	Primary 4-5	Secondary 1-4	Secondary 5-6	Higher 1-3	Higher 4-6
1964 (base year)		1.50	2.26	3.27	5.61	12.65	17.07	21.61
1980	$\sigma = 4$	2.45	2.98	4.56	6.15	18.83	17.32	23.04
	6	2.27	2.96	4.46	6.51	17.99	18.81	24.61
	8	2.19	2.95	4.40	6.69	17.57	19.58	25.41
	10	2.14	2.95	4.37	6.80	17.32	20.06	25.90
	00	1.94	2.92	4.22	7.24	16.33	22.03	27.89

The calculations required by the analysis are executed in six stages. Exogenously given projections of the number of students entering the first year of each educational level are used to estimate the additions to the different categories of workers and to calculate the demand for teachers and for investment in fixed capital in schools. This information is used to project the number of workers in each of the different categories by which the labour force is classified. A constant elasticity of substitution function is then used to aggregate these projections into a single measure of the labour force which takes into account not only the increase in the total number of workers but also the improvement in its quality. This index is used together with an estimate of growth due to factors of production other than labour in order to estimate the rate of growth of GDP for the country. This enables the marginal product of each different category of worker to be calculated, and hence one obtains projections of the wage rate for each category. The wage rate projections can then be used to evaluate the present discounted values of the benefits and the costs of each level of education, and one arrives at an estimate of their rates of return. If it is desired that the rate of return to each level of education should be equal to some predetermined social discount rate, these results can be used to modify the enrolment projection. Those branches of the educational system with rates of return greater than the social discount rate should be

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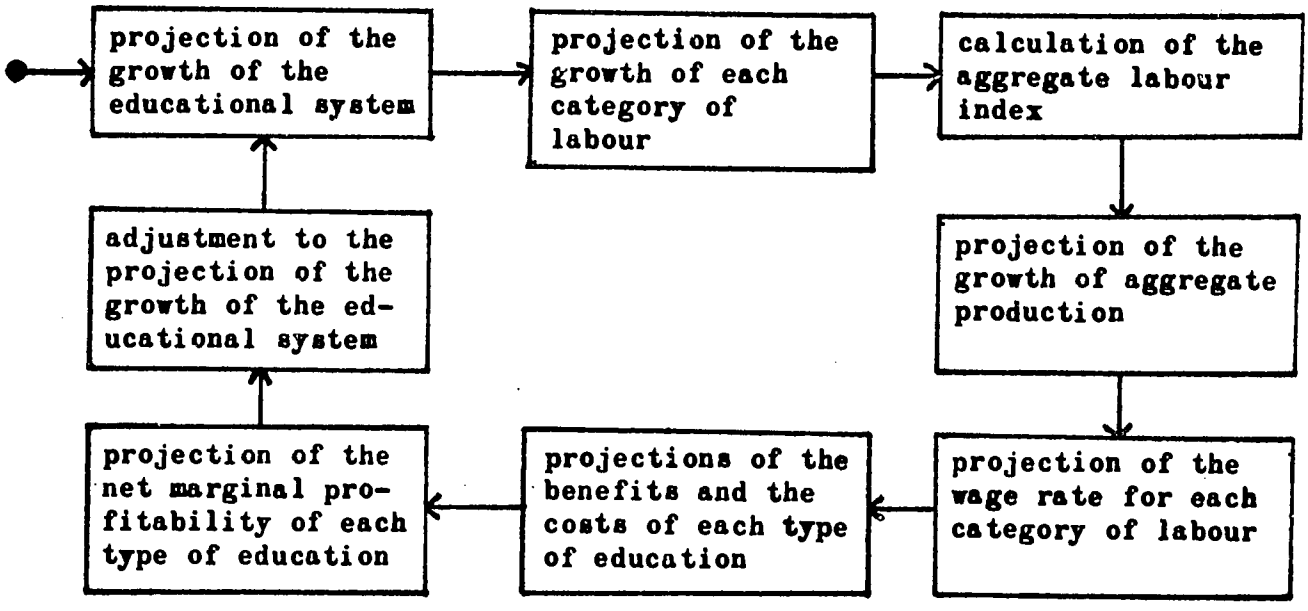
[1] All the parameters except σ were set at their reference values. See Section VII below.

increased, and those with rates of return less than it should be reduced. The initial projections can thus be adjusted iteratively so that their rates of return are equal to the discount rate over the whole of the planning horizon. When this point is reached the supply and the social demand for each type of labour are in balance and the educational system is attaining its optimal growth path. The diagram below represents the process schematically.



The remaining sections of this paper describe each of the stages in more detail with reference to an application of the methodology to education in Colombia.

II The Educational System

In Colombia primary education lasts five years, secondary education six, and higher education up to five or six.[1] Two kinds of information are required for the projection of the growth of each level of education:

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[1] That is, if a student does not repeat any year. In fact repeating rates are high, so that, for example, the average time taken to complete primary education is approximately six years.

(1) the basic parameters, such as drop-out rates and student-teacher ratios, and (2) the projection of the enrolment in the first grade[1] of each educational level for each year of the planning period.

The first task is to transform, for each educational level, the basic parameters into what will be referred to as an "impact table", a table which shows all the repercussions of increasing the enrolment in the first grade of that educational level by 100 students in year n. In order to shorten the exposition the construction of the impact table will be described here in detail for Colombian secondary education only. The procedures for primary and higher education are similar and are described in Appendix 3.

The basic parameters needed for the analysis are (i) the student-teacher ratio; (ii) the composition of the teaching staff according to educational level; (iii) the capital cost of providing a place for one student; and (iv) the drop-out and repeating rates for each grade.

With these basic data the effects of enrolling 100 students in the first grade of secondary education in year n are calculated. The number of teachers required by the students is estimated using the student-teacher ratio and this is divided between teachers with higher education and those with secondary education only. The capital stock associated with the students is also calculated.

The next step is to calculate the position at the beginning of year n+1. By this time most of the students will be entering second grade. Some will be repeating first grade. The remainder will have dropped out. The total enrolment will be smaller than 100 by the number that have dropped out, and the number of teachers and the amount of capital required will have been reduced proportionately. The investment in the second year will be equal to the depreciation on the capital stock during the first year less the reduction in the capital stock due to the drop-out of a fraction of the students, and hence is likely to be negative. The fraction of the students that dropped out during the previous year represents a potential

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1 In order to avoid ambiguous use of the word "year" this will be replaced by the word "grade" when referring to instruction within any educational level. For example the first year of secondary education will be referred to as the first grade of secondary education.

addition to category 4 of the labour force.[1]

The process is repeated for each succeeding year. In year n+6 the first graduates appear - those students that have neither dropped out nor repeated. In the next few years more graduates appear, these being students who have repeated one or more grades. By eight years after the initial enrolment most of the students have dropped out or graduated and will have been assigned either to category 4 or to category 5 of the labour force.

The results of these calculations are gathered together in the impact table (Table 3 overleaf). Having constructed this table it is possible to calculate the effect on the secondary school system and on the labour force of a projection of the number of students entering the first grade of secondary education from 1969 onwards.

As an illustration take the enrolment projection shown in the second column of Table 2, which is based on the assumption that the present 10% annual growth rate is maintained.

TABLE 2

Calculation of the number of university trained teachers required by secondary education in 1978

Year	Number of students entering first grade of secondary school (thousands)	Requirements in 1978 of university trained teachers (thousands)
1969	164	0.0
1970	180	0.0
1971	198	0.1
1972	218	0.4
1973	240	1.8
1974	264	2.7
1975	290	3.9
1976	319	5.6
1977	351	7.7
1978	386	11.7
1979	424	-
1980	467	-
TOTAL		33.9

1 For the purposes of this paper the Colombian labour force is classified into seven categories according to educational level. For the definitions of these categories see Section III.

TABLE 3

Impact table for secondary education: the consequences of enrolling 100 students in the first grade of secondary education in year n

Year	n	n+1	n+2	n+3	n+4	n+5	n+6	n+7	n+8
Total enrolment	100.0	71.7	57.6	44.7	33.4	24.8	6.3	1.0	0.1
Enrolment in 1st grade	100.0	5.0	0.3	0.0	-	-	-	-	-
Enrolment in 2nd grade	-	66.7	6.7	0.5	0.0	-	-	-	-
Enrolment in 3rd grade	-	-	50.6	7.6	0.8	0.1	-	-	-
Enrolment in 4th grade	-	-	-	36.6	7.3	0.9	0.1	-	-
Enrolment in 5th grade	-	-	-	-	25.3	6.3	1.0	0.1	-
Enrolment in 6th grade	-	-	-	-	-	17.5	5.3	0.9	0.1
Secondary school trained teachers	4.57	3.27	2.63	2.04	1.53	1.13	0.29	0.05	0.01
University trained teachers	3.04	2.18	1.75	1.36	1.02	0.76	0.19	0.03	0.00
Capital requirements (thousand 1988 pesos)	235.3	168.7	135.4	105.1	78.7	58.4	14.8	2.4	0.3
Depreciation on capital in previous year	-	7.1	5.1	4.1	3.2	2.4	1.8	0.4	0.1
Gross investment	235.3	-59.5	-28.2	-26.3	-23.3	-17.9	-41.9	-11.9	-2.0
Drop-outs from grades 1 - 4 in previous year	-	28.3	14.2	12.9	11.2	2.1	0.3	-	-
Cumulative drop-outs from grades 1 - 4	-	28.3	42.5	55.3	66.6	68.6	68.9	68.9	68.9
Drop-outs from grades 5 - 6 in previous year	-	-	-	-	-	6.5	1.6	0.3	0.0
Cumulative drop-outs from grades 5 - 6	-	-	-	-	-	6.5	8.2	8.4	8.4
Graduates at end of previous year	-	-	-	-	-	-	16.7	5.0	0.9
Cumulative graduates	-	-	-	-	-	-	16.7	21.7	22.5

The consequences of this projection for, say, the year 1978 can be calculated using the impact table. As an example, the total demand of the secondary school system for university trained teachers will be estimated. The number of university trained teachers needed by the cohort of students entering secondary education in 1978 will be 386/100 x 3.04 = 11.7 thousand. The number needed in 1978 by the cohort which entered in 1977 will be 351/100 x 2.18 = 7.7 thousand. Generalizing, the number needed by the cohort entering in the year (1978 - n) will be equal to the amount of the enrolment in that year multiplied by 1/100 of the coefficient for year n+1 in the impact table. It can be seen from Table 2 that the closer is a cohort to 1978 the larger is its requirement of teachers. This is due to the fact that the later is its date of entry the larger is its initial size and the less it will have been reduced by desertion and graduation by 1978. The aggregate requirement of university trained teachers by secondary education in 1978 is shown as the total in Table 2. In the same way one may calculate the requirement of teachers with secondary education, the gross investment in fixed capital needed, the drop-outs to labour category 4, the drop-outs to labour category 5, and the number of graduates in 1978. Likewise all these quantities may be calculated for the secondary school system for every year from 1969 until the terminal year of the planning period. And using the same methodology the consequences of enrolment projections for primary and higher education may be calculated.[1] The next stage is to compute the combined effects of these enrolment projections on the supply of each category of labour.

III The Labour Force

In the application to Colombia the labour force is divided into seven categories according to the following definitions:

C 1 : illiterates

C 2 : those with some primary education but not more than three complete years of it

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1 For details see Appendix 3.

C 3 : those with more than three years of primary education but with no secondary education

C 4 : those with some secondary education but no more than four complete years of it.

C 5 : those with more than four complete years of secondary education but no higher education

C 6 : those with some higher education but not more than three complete years of it

C 7 : those with more than three years of higher education

The projection for C 5 is described in detail here. Since the projections for the remaining categories are made in similar fashion their descriptions have been relegated to Appendix 4.

C5 contains five separate subcomponents:

(i) those workers remaining from the initial stock of the category in 1969, the year the calculations for Colombia begin. This subcomponent declines slowly as its members retire or die.

(ii) primary school teachers. Using the projected enrolment in primary education and the student-teacher ratio for that level one can calculate the requirement of teaching staff. This figure forms a negative contribution to the labour force available for the rest of the economy.

(iii) secondary school teachers. In the same way one can calculate the number of secondary school graduates who will be withdrawn from the potential labour force in order to teach secondary school.

(iv) drop-outs from secondary school. Using the projections of enrolment in secondary education and Table 3 in Section II one can calculate the number of students who drop out from the last two grades of secondary education and who are therefore assigned to C5.

(v) graduates from secondary school who do not enter higher education. Using the projection of secondary school students and Table 3 above one can estimate the number of graduates from secondary school for each year. From the projection of graduates one subtracts the projection of the enrolment in the first year of higher education and the remainder are assigned to C5.

Subcomponents (iv) and (v) represent potential additions to C5 in the sense that they are estimates of students who leave the school system

qualified to enter that labour category. However a significant proportion, mostly women, for some reason or other will not actually enter the labour force. The figures for these two subcomponents are therefore multiplied by an adjustment factor calculated from data obtained in the 1964 census. Similar adjustment factors are calculated for the other six labour categories.

The table below shows a typical projection made for C5. Although the projection has to be calculated for each year it is sufficient to show here the results at five year intervals. It should be emphasized that each of the subcomponents (ii) - (v) is dependent upon the particular projections of the enrolments in the different levels of the educational system.

TABLE 4

Example projection of the growth of labour category 5 (thousands)

Year	Residue from initial stock	Primary school teachers	Secondary school teachers	Cumulative drop-outs from secondary school	Cumulative graduates from secondary school not continuing to higher education	TOTAL
1970	272.0	-53.7	-28.0	20.4	7.2	232.4
1975	266.0	- 92.4	- 37.7	82.5	17.6	236.1
1980	257.0	- 114.9	- 61.5	160.9	17.6	259.0
1985	242.0	- 142.9	- 99.1	289.5	17.6	307.1

The projections for the other six categories of labour are made in the same way. These are now combined using a constant elasticity of substitution function in order to arrive at an aggregate measure of the labour force which takes into account the educational level of the workers as well as their number.

IV The Aggregate Labour Index

International cross-section data indicate that most countries are experiencing two major long term changes in their economies: the structure of the labour force is changing as the proportion of the more highly educated categories increases; and the structure of relative wages is contracting. These two processes work together in complementary fashion in order to secure the absorption of those students leaving the educational system into the labour force. If techniques of production change readily in response to small changes in relative wages, then one can expect relative wages to change slowly. If on the other hand techniques are rigid then the structure of wages may have to change considerably in order for the supply of new workers to be absorbed. If the rigidity is extreme the increase in the relative supply of some type of worker may not be entirely absorbed at any acceptable wage and the result would be unemployment.

A useful concept for the analysis of this interaction is the elasticity of substitution between factors of production. In the case of a process using only two factors, the elasticity of substitution is defined as the rate at which the relative use of two factors changes divided by the rate at which the ratio of their marginal products changes, measured along a production isoquant. The concept has been generalized to cover more than two factors of production in several different ways, of which the most widely used is due to Allen.[1] The higher is the elasticity of substitution the smaller is the fall in the relative price of a factor that is required in order to induce the absorption of a given increase in the relative supply of that factor into production. When the elasticity is infinite, any quantity of the factor can be absorbed without requiring a fall in its relative price. When the elasticity is unity, the relative price of the factor must fall at the same rate as the relative supply is increased. When the elasticity is zero the production process is of the Leontief fixed coefficients form and in this case it is impossible to eliminate a surplus of the factor however much the price changes.

It is important to notice that the elasticity of substitution is a time-dependent concept. In the short run it is difficult to adapt techniques

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1 R.G.D. Allen Mathematical Analysis for Economists MacMillan 1938 pp. 503 - 509 .

of production and hence an increase in the relative supply of a factor is likely to cause a temporary surplus and a significant fall in its price. The long run price elasticity of demand, however, may well be appreciably greater than the short run elasticity. The initial fall in the relative price will induce a modification of the production process towards greater relative use of the factor. The increase in the relative demand for the factor may then restore the relative price part of the way towards its former level. Thus the factor could possess both a low short run elasticity and a high long run elasticity of substitution.

The same comment applies to the case in which the relative supply of a factor is being continually increased. The result will be a continuous decline in the relative price of the factor. The relationship between the rate of decline of the relative price and the rate of increase of the relative supply can be expected to be sensitively dependent upon the magnitude of the latter. A small rate of increase in the relative supply might cause virtually no change in the relative price. A slightly larger rate of increase might cause an equal rate of decline in the relative price. A substantial rate of increase might not be capable of being absorbed at all and might lead to a drastic rate of decline in the relative price.

In view of these comments one might expect the elasticity of substitution between different types of labour to be high, since the ratio of the annual increase of each type of labour to its existing stock is quite small and the rates of relative increase even smaller. It is therefore not surprising that some preliminary results of empirical measurements by Sam Bowles and Marcelo Selowsky[1] indicate that the elasticities of substitution between different types of labour are large, sometimes not statistically different from infinity, even when the number of types of labour distinguished is small.

The aggregate labour index is calculated using the straightforward constant elasticity of substitution function

$$(1) \qquad L = (a_1 C_1^{\theta} + \dots + a_7 C_7^{\theta})^{\frac{1}{\theta}}$$

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1. Bowles: op. cit. Selowsky: unpublished experiments with U.S. manufacturing data.

where $\theta = \frac{\sigma - 1}{\sigma}$

σ = elasticity of substitution

C_i = number of workers in labour category i

and the a_i are constants.

For Colombia the a_i are calculated using 1964 census data and information on relative wage rates.[1] There is at present no reliable estimate for σ, and so the calculation of the labour index is made for a range of reasonable values of this parameter. There exist more sophisticated forms of the CES function, notably the two level version introduced by Sato[2] which permits a partial relaxation of the assumption of a single elasticity of substitution between any two types of labour.[3] However, in the Colombian case, given the data limitations it has appeared advisable to keep the form of the index as simple as possible.[4]

Table 5 shows some projections of the categories of the labour force for some of the years of the planning period (based on underlying projections of the enrolment in the different branches of the educational system), and the aggregate labour index calculated on the assumption that the elasticity of substitution is equal to six.

TABLE 5

Example projections of the seven categories of the labour force and of the aggregate labour index, sample years[1]

Year	C 1	C 2	C 3	C 4	C 5	C 6	C 7	L
1970	1481.0	2808.4	1321.8	504.7	232.4	30.6	43.7	380.2
1975	1378.0	3852.9	1442.9	877.9	236.1	68.3	96.6	499.1
1980	1258.0	4291.5	1968.2	1477.9	259.0	119.8	200.0	663.6
1985	1118.0	4424.3	2281.2	2463.4	307.1	195.3	316.6	849.2

1 See Section V.

2 K.Sato, A Two-Level Constant-Elasticity-of-Substitution Production Function, Review of Economic Studies April 1967

3 Bowles, op. cit., has estimated such a function using three types of labour and subaggregating two of them.

4 Some further problems in the use of this aggregate labour index are discussed in Appendix 1. Appendix 2 describes the results of fitting it to data for the United States.

V The Projection of Wages

The last section described the estimation of an aggregate index for labour over the planning period. The next step is to use this index in order to project the marginal productivity of each category of labour using the equation

$$(2) \qquad w_i = \frac{dY}{dC_i} = \frac{\partial Y}{\partial L} \cdot \frac{dL}{dC_i} \quad [1]$$

where w_i = marginal product of labour category i

Y = aggregate output

The term $\frac{dL}{dC_i}$ can be calculated by differentiating equation (1). The projection of the value over time of the term $\frac{\partial Y}{\partial L}$ is less straightforward. One may expect it to be a decreasing function of L and an increasing function of the supplies of other factors of production, for example capital. One may also expect it to be an increasing function of time, ceteris paribus, as a result of the effects of what may loosely be termed technical progress.

In order to project the value of the term $\frac{\partial Y}{\partial L}$ it is therefore essential to construct some sort of aggregate production function. An attempt was made to estimate for Colombia the coefficients of a Cobb-Douglas function using regression analysis. This was abandoned when it was found that an exponential time trend by itself would explain 99.6% of the variance in production over an observation period of thirteen years and that it was highly collinear with the other independent variables used.

With some reluctance the function

$$(3) \qquad Y = A\, L^{\beta}\, e^{\lambda t} \quad [2]$$

has been used instead. The term $e^{\lambda t}$ is intended to cover all sources of growth of output other than labour, λ being the total rate of growth due to

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1 Some of the implications of this equation are discussed in Appendix 1.

2 The average share of labour in GDP in recent years, 0.55, has been used to estimate β.

these sources. It is not possible to predict with any confidence the value of λ for Colombia, and hence all the calculations have been repeated using a range of values for this parameter. The results obtained in each case are therefore conditional on the particular value of λ associated with them. The higher the value of λ, the faster is the projected growth of GDP and thus of the demand for labour, and hence the greater is the need for improving the educational level of the labour force and the greater is the rate of return to education.

The marginal product of each category of labour can now be calculated using equations (1), (2) and (3).

$$(4) \qquad w_i = a_i \beta Y C_i^{\theta-1} L^{-\theta} \qquad (i=1,\ldots,7)$$

where all the symbols have been defined above.

The values of the parameters a_i were estimated using equation (4) and the values of the w_i and the C_i in 1964. Using equation (1) one has

$$w_i = \frac{a_i \beta Y C_i^{\theta-1}}{\sum_j a_j C_j^{\theta}} \qquad (i=1,\ldots,7)$$

$$w_i \sum_j a_j C_j^{\theta} = a_i \beta Y C_i^{\theta-1} \qquad (i=1,\ldots,7)$$

These equations are homogeneous in the a_i, and therefore possess an infinite number of solutions if consistent, and no solution at all if not consistent. Consistency is proved by multiplying equation i by C_i and summing:

$$\sum_i \left(w_i C_i \sum_j a_j C_j^{\theta} \right) = \sum_i a_i \beta Y C_i^{\theta}$$

since $\quad L \sum_i w_i C_i = L \beta Y$

Table 6 shows the projection of the wage rates for the different categories using the projection of their numbers shown in Table 5 and values of six for σ and 0.04 for λ.

TABLE 6

Example projection of the wage rates for the seven categories of labour, sample years (thousand 1958 pesos per year)

Year	C 1	C 2	C 3	C 4	C 5	C 6	C 7
1970	1.74	2.50	3.74	6.11	14.38	18.54	24.88
1975	1.99	2.68	4.17	6.30	16.22	18.33	24.66
1980	2.27	2.96	4.46	6.51	17.99	18.81	24.61
1985	2.64	3.36	4.95	6.81	19.92	19.75	25.96

In Table 5 labour category 6 is projected to grow sixfold over the period 1970 - 1985, and as a result by the latter date its shadow wage would fall below that of C 5. In reality one would expect the declining profitability of three years of higher education to encourage potential students either to opt for professional studies leading to C 7 instead of technical studies, or not to enter higher education at all but to go straight into the labour force on graduating from secondary school.

VI Calculating the Rate of Return to Education

The value of education has many components, some of them "economic", like the increase in the productivity and the occupational mobility of an individual, some of them "non-economic", like his cultural enrichment and the satisfaction he derives from the improvement in his social status. Of these only the increase in the marginal product of an individual is readily

susceptible to measurement,[1] and for this reason the analysis below is confined to this subcomponent.[2]

In Section II it was described how one can calculate the impact of a cohort of 100 students entering the first grade of each level of the educational system. The calculations were described in detail for secondary education. The impact was estimated in real terms: the number of teachers required, the number of graduates produced, the number of drop-outs generated, the stock of capital required. Using these figures alone it is impossible to evaluate the worth of the educational system. They must be converted to social value terms by multiplying them by the relevant shadow prices calculated in Section V.

This yields a series of benefits and costs for each year from the date of entry of the cohort into secondary school until the retirement of the last of its members from the labour force. The rate of return is then that rate of discount for which the net present value of the series is zero.[3]

There are four sources of cost:

(i) The opportunity cost to society of withdrawing 100 primary graduates from the labour force and putting them into secondary education. The actual deduction from labour category 3 (the category to which primary graduates would be assigned if they entered the labour force instead of enrolling in secondary education) is less than 100 because a large proportion would not have entered the labour force in any case. The estimated participation rate for labour category 3 is 56.4% and this figure is used in the estimation

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1 And this only at the expense of assuming that the marginal product of an individual is equal to his wage. For lack of a better alternative, this assumption is being used here.

2 The omission of the other components therefore constitutes a downward bias in the calculation of the value of education in this paper. A bias in the other direction should also be noted. In most countries there exists a positive correlation between the innate ability of an individual and the education he receives. Thus to some extent salary differentials ascribed to education are in fact due to differences in ability. However it appears that in Colombia this correlation is relatively weak, and no attempt is made here to adjust for it.

3 In principle such a discount rate may not exist, and if it does exist it may not be unique. The latter problem was not encountered in the calculations for Colombia, and when the former occurred the economic interpretation was clear.

of the cost. Hence the opportunity cost in year u of enrolling 100 primary graduates in secondary school in year t ($u \geqslant t$) is 56.4 x the wage of a C 3 worker in year u.

(ii) The cost of secondary school educated teachers. In Section II it was shown that the number of teachers with this level of training required by a cohort of 100 students entering secondary school in year t would be 4.57 in year t, 3.27 in year t+1, and 2.63, 2.04, 1.53, 1.13, 0.29, 0.05, 0.01 in years t+2 through t+8. The decline in the number of teachers required during the first six years is due to the reduction in the size of the cohort caused by desertion. In the seventh year the number is reduced still further with the graduation of those students who have gone through secondary school without dropping out or repeating a grade. By the ninth year all but a fraction of hardy triple-repeaters have left secondary school and the number of teachers required is negligible.

The social cost of these teachers in year u is equal to the number required in that year multiplied by the wage for that year for the labour category 5, to which these teachers belong.

(iii) The cost of university educated teachers. The calculation of their cost is made in the same manner as that of the secondary school educated teachers. In this case the shadow price used to value them is the marginal product of the labour category 7, the category to which workers with this level of education belong.

(iv) Overheads. In addition to teachers' salaries there is a certain amount of expenditure on administration, maintenance and materials. It is assumed that this will continue to bear the historically observed relationship to expenditure on salaries.

(v) Capital costs. The cost of fixed capital is calculated on an implicit cash flow basis. The stock of capital required by a cohort of 100 students entering secondary school in year t is calculated and registered as an expenditure in year t. By year t+1 part of the cohort has dropped out and the amount of capital required has diminished. The reduction in capital is then registered as a receipt for year t+1, after deducting from it a charge for depreciation. In the succeeding years the rest of the capital will be returned as the cohort is reduced by further desertion and then, in years t+6 onwards, graduation. The cost of capital is thus a

series consisting of one large expenditure followed by a number of small receipts whose total is less than the expenditure by the total of depreciation charges. The net discounted cost is thus an increasing function of the discount rate used.

The benefit derived from entering a cohort of 100 students in secondary school is the value of their earnings after they have left that level of education. 68.9 members of the cohort will drop out during the first four grades and will therefore be assigned to labour category 4. 8.4 will drop out during the last two grades and will be assigned to C 5. The remaining 22.7 will graduate and their value will again be the marginal product of C 5. (Some of the students who graduate will proceed to higher education and have their marginal products increased further; but this fact is not relevant to the estimation of the benefits of the secondary school system.)

Table 7 below shows the present discounted values of each of the series of costs and benefits of entering 100 students in the first grade of secondary education in 1969, using the example wage projections in Table 6 and two different discount rates. With a discount rate of 5% the net discounted value per student is 52,782 (constant 1958 pesos); with a discount rate of 20% the net value falls to 342 pesos. It can therefore be seen that in this example calculation the rate of return to entering an additional student in the first grade of secondary education is just over 20%.

TABLE 7

Present discounted value of the costs and benefits of entering 100 students in the first grade of secondary education in 1969 (example calculation)[1] (thousand constant 1958 pesos)

		Discount Rate 5%	Discount Rate 20%
COSTS			
(i)	Lifetime earnings of C 3 workers	5038.5	1375.7
(ii)	Secondary school educated teachers	208.1	168.4
(iii)	University educated teachers	235.7	192.4
(iv)	Overheads	259.2	210.7
(v)	Capital cost	55.9	114.6
Total		5591.0	1855.4
BENEFITS			
(i)	Earnings of the 68.9 drop-outs to C4	3675.4	879.0
(ii)	Earnings of the 8.4 drop-outs to C5	2025.5	309.9
(iii)	Earnings of the 22.7 graduates	5168.3	700.7
Total		10869.2	1889.6
NET VALUE		5278.2	34.2

These calculations can be repeated for students enrolling in the first grade of secondary school in 1970 and in subsequent years. The rate of return to secondary education will change from year to year reflecting principally the changes in the shadow wages for categories 3, 4 and 5 of the labour force, these being the most important shadow prices used in estimating the benefits and the costs associated with this educational level.

Similarly these calculations may be made for the other branches of the educational system. Table 8 below shows typical estimates of the rate of return to enrolling an additional student in primary, secondary and higher education for 1969, 1975, 1980 and 1985.

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1 Assumptions: $\sigma = 6$, $\lambda = 0.04$, enrolments grow at historical rates.

TABLE 8

The rate of return to the additional enrolment of one student in primary, secondary and higher education, selected years (example calculation) [1]

Year	Primary	Secondary	Higher
1969	21.1	20.7	7.4
1975	20.7	20.3	5.2
1980	20.5	19.5	3.4
1985	19.8	17.8	0.9

From Table 7 it may be observed that the direct cost of secondary education, i.e. the salaries of teachers, overheads, and the cost of fixed capital, is considerably smaller than the discounted values of the earnings of the workers withdrawn from C 3 and of the weighted earnings of those who graduate or drop out from secondary school. It should therefore be evident that the estimate of the rate of return to secondary education is at least as sensitive to the assumptions used for projecting the earnings streams as to those used for calculating the direct cost. In the case of higher education the discounted values of the earnings are very much greater than the direct cost and the rate of return is not much affected by substantial changes in the latter. In the case of primary education the direct cost is more important.

VII Optimizing the Growth of the Educational System

In the example calculation in Section VI the rate of return to entering an additional student into the first grade of secondary school in 1969 is just over 20%. This fact implies that if the social discount rate were

1 Assumptions: $\sigma = 6$, $\lambda = 0.04$, enrolments grow at historical rates.

20% or less it would profit society to increase the entry for that year above the number shown in the original projection. This would have the effect of increasing the supply of C 4 and C 5 labour in the future, and hence of reducing the shadow wages of those categories. At the same time the number of primary school graduates entering labour category 3 in that year would be decreased, causing a slight rise in the C 3 shadow wage. Both these effects would tend to reduce the difference between the C 4 and C 5 shadow wages and the C 3 shadow wage, with the result that the marginal rate of return to secondary education would decline. So long as the rate of return to adding an additional student to the enrolment in the first grade is greater than the social discount rate, the enrolment ought to be increased. If it should happen that the rate of return is less than the social discount rate, this would imply that society is making a loss on educating the marginal students and the enrolment should be cut back.

Ideally the enrolment should be adjusted so that the rate of return to secondary education is equal to the social discount rate. This observation applies to every year in the projection of secondary school enrolment. The enrolment for each year of the planning period should be adjusted so that the rate of return to enrolling an additional student in the first grade becomes equal to the social discount rate for each year.

Likewise the projection of enrolment in primary and higher education should be adjusted until the marginal rates of return to those levels of education become equal to the social discount rate for each year. Since the rates of return to the three levels of education are dependent on the projected enrolment of each (through their common dependence on the structure of wage rates) the adjustment of the three enrolments must be done simultaneously or by using an iterative process.

When the adjustments are completed and the rate of return to each educational level is equal to the discount rate for each year, the educational system has achieved its optimal growth path. It should be emphasized that the optimal growth path thus calculated is conditional on the assumptions made about the key parameters, in particular the social discount rate and the elasticity of substitution.

It is possible that the optimal projections may show unrealistic rates of growth for some of the branches, at least for the first few years. When

this is the case it is necessary to examine the factors limiting growth and to estimate the maximum rate. The revised optimal projections then should show for each branch for each year either a rate of return equal to the social discount rate or constrained maximum growth in enrolment over the previous year. As will be seen in the next section, this proved to be the case for secondary education in Colombia; an upper bound of 15% per year was imposed upon its growth rate, and this bound should be attained for at least the next ten years unles the elasticity of substitution is very low or the social discount rate very high.

VIII Results for Colombia

The first part of this section reports results based on the assumption that secondary and higher education continue to maintain their recent growth rates of 10% and 11% per year respectively. The projection for primary education is made in a special way and is described in Appendix 3. The second part of the section describes the results of following the procedure outlined in Section VII for optimizing the projection of enrolments in secondary and higher education.

A. Results based on present trends

The estimates of the rates of return to the different levels of education in Colombia are dependent on the assumptions concerning a number of parameters whose values are open to debate. In view of this fact a reference analysis was executed using plausible values for these parameters, and then a series of sensitivity analyses were made varying each of the parameters in turn.

In the reference analysis σ, the elasticity of substitution, was set at six; λ, the rate of growth of GDP due to factors of production other than labour, was set at 4% per year; and β, the share of labour in GDP, was set at 0.55. The illustrations of the stages in the analysis given in Sections II through VI above all assume that the parameters take their reference values. Table 4 shows the construction of the projection of labour category 5 using the hypothesis that present enrolment trends in secondary

and higher education are maintained. Table 5 shows the projections for the other six categories using the same assumption and the projection of the labour index using the reference value of six for the elasticity of substitution. Table 6 gives the projection of wages corresponding to the projections of the labour categories in Table 5 on the assumption that takes its reference value of 4% per year. Table 7 shows the calculation of the rate of return to secondary education, and Table 8 shows the rates of return to each of the three levels of education for selected years. In Figure 4 overleaf these rates of return are plotted over time together with the results of three sensitivity analysis experiments described below.

The parameter which is the most obvious candidate for sensitivity analysis is the elasticity of substitution. Table 9 shows the rates of return for each of the levels for the years 1969, 1975, 1980 and 1985 for different values of this parameter. The results for the years 1969 and 1980 have already been plotted in Figures 2 and 3 above.

TABLE 9 [1]

The rates of return as function of σ

	primary				secondary				higher			
σ	1969	1975	1980	1985	1969	1975	1980	1985	1969	1975	1980	1985
2					18.8	17.3	14.4	8.8	-	-	-	-
4	19.6	19.0	18.5	17.1	20.2	19.5	18.3	15.9	2.7	-	-	-
6	21.1	20.7	20.5	19.8	20.7	20.3	19.5	17.8	7.4	5.2	3.4	0.9
8	21.7	21.4	21.2	20.7	21.0	20.7	20.1	18.7	8.9	7.3	5.9	4.0
10	22.0	21.7	21.6	21.1	21.1	20.9	20.5	19.3	9.7	8.4	7.3	5.6
∞	23.0	22.9	22.7	22.3	21.8	21.8	21.9	21.5	12.5	12.2	11.9	10.9

Other important parameters are λ and β, both of which appear in the production function, equation (3). Sensitivity analysis showed that for

1 Notation: "-" marks a negative rate of return. No rate of return existed for primary education when $\sigma = 2$. The present discounted value of the series of benefits and costs was negative for all values of the discount rate.

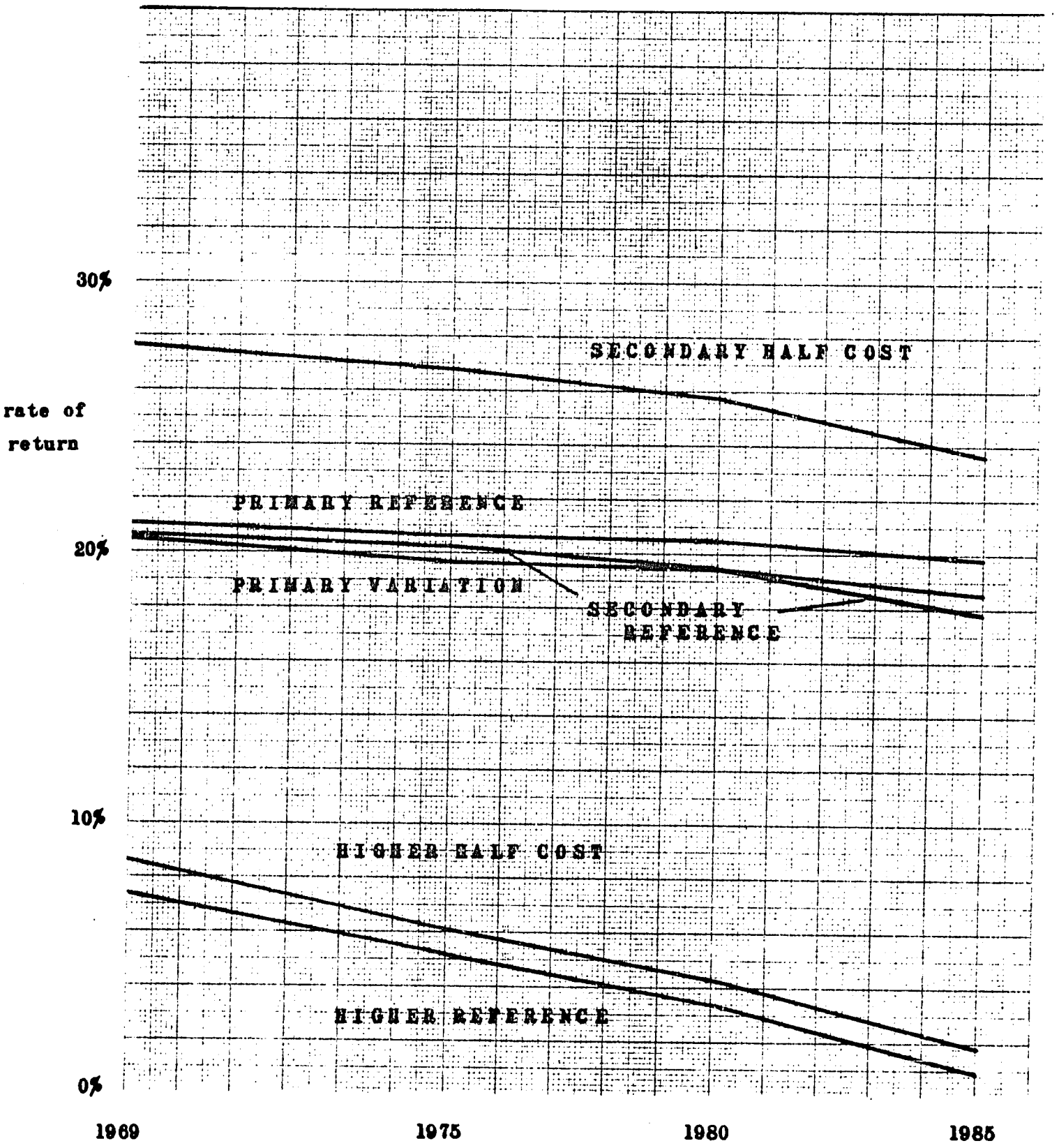
30%
rate of
return
20%
10%
0%
SECONDARY HALF COST
PRIMARY REFERENCE
PRIMARY VARIATION
SECONDARY
REFERENCE
HIGHER HALF COST
HIGHER REFERENCE
1969
1975
1980
1985

FIGURE 4

each one per cent increase in λ the rates of return to each level of education would increase by between 1.4% and 1.6%. It was not easy to estimate β from the official Colombian statistics. Its value was calculated to be 0.55, but it is possible that the true value may be lower. Reducing the estimate to 0.45 had the effect of decreasing the rates of return to primary and secondary education by 1% and of decreasing the rate of return to higher education by 0.7%.

The results of three other experiments are shown together in Figure 4. The curve marked "primary variation" shows the effect over time on the rate of return to primary education of increasing substantially the projection of the enrolment in the first grade. As is explained in Appendix 3, the projection assumes that all seven-year-olds are enrolled in first grade and that their number grows exponentially at a rate of 3.2% per year from a base of 700,000 in 1969. An adjustment to this is made for the backlog of students who in the past failed to enter school at age seven. It is assumed that the additional enrolment in first grade due to this source is 300,000 in 1969 and declines over the next ten years at a rate of 30,000 per year. In the variation on the enrolment projection the figure of 700,000 was raised to 750,000 , its growth rate remained the same, and the additional enrolment due to the backlog was assumed to be 400,000 in 1969, decreasing at a rate of 40,000 per year. The effect on the rate of return is to reduce it, but not significantly.

The other two variations involved assuming that the costs of, first secondary, and then higher, education were halved. In both of these levels the typical unit is too small. In secondary education the average number of students per school is just over one hundred, which inevitably means that, given the high drop-out rates, the number of students per grade, and therefore the student-teacher ratio, in grades 4 through 6 is unreasonably small. In higher education the failure of the universities to organize a division of labour among themselves has led to excessive duplication of undersized departments, again leading to unnecessarily high costs and probably to a loss of quality as well.

The effect of halving the costs is much more pronounced for secondary education, whose rate of return rises from 20.7% to 27.7% for 1969, than for higher education, whose rate of return rises from 7.4% to 8.6% for the same year. This asymmetry reflects the fact that the direct costs of education are smaller, relative to the discounted values of the relevant earnings

streams, for higher than for secondary education. It is also partly due to the fact that the rate of return itself is much higher for the latter.

The results of the experiments show that, assuming that secondary and higher education maintain their present growth rates of 10% and 11% respectively, and that enrolment in primary education accelerates in the way described in Appendix 3, the rates of return to primary and secondary education are high and that to higher education is low. Under any plausible set of assumptions the rates of return to primary and secondary education are both around 20% and will remain near that level for the next fifteen years. By contrast, unless one assumes a very high elasticity of substitution, the rate of return to higher education is well below 10% and can be expected to fall substantially over time. If one assumes as low a value for the elasticity as four, the rate of return is less than 3% in 1969 and rapidly becomes negative.

These conclusions suggest that secondary education should grow faster than its present rate and higher education more slowly.

B. Optimizing the enrolment growth rates

For the calculations discussed below the projection of the enrolment in primary education, described in Appendix 3, was kept fixed. Colombia would have difficulty increasing the enrolment at a rate faster than that foreseen by the programme, and since the rate of return to primary education remains above the highest social discount rate considered here, should not allow it to grow more slowly.

In principle, as outlined in Section VII, the optimization process of the enrolment growth rates for secondary and higher education should have been executed on a year-to-year basis over the forty year planning period. This would have required a search for the optimal values of eighty variables and, given the complexity of the analysis, was not practicable even with the aid of a large computer. In order to simplify the problem the planning period was divided into three parts: 1969-1978, 1979-1988, 1989-2008, and it was assumed that within each part the growth rate of each level of education would be constant.[1] Further, an upper bound of 15% per year was put on the

1 The third subperiod was made twice as long as the other two on the ground that the results for those years would be of less interest.

growth rate of both levels in the belief that this would be the maximum administratively feasible.

With this drastic reduction in the complexity of the task it was possible to search for the solutions on a trial-and-error basis. The solutions thus obtained are sub-optimal but have the advantage of suppressing the jerkiness that year-to-year optimization might have yielded.

The solution using the reference values for the key parameters and a discount rate of 10% is shown below. Secondary education grew at the maximum rate until 1982, when the number of students entering secondary school became equal to the number of primary school graduates the previous year. After 1982 the growth of secondary education was limited to the growth of the supply of primary school graduates.

TABLE 10

Optimal solution, social discount rate = 10%
(parameters at reference values)

growth rates

	1969-1978	1979-1988	1989-2008
secondary	15%	max[1]	max[1]
higher	2%	5%	5%

rates of return

year	primary	secondary	higher
1969	22.0	19.4	10.3
1975	22.7	17.1	9.9
1980	23.4	14.8	9.9
1985	24.3	13.0	9.9

1 max: until 1982 this was 15% per year. After 1982 this was the growth rate of primary school graduates which, according to the primary school projection outlined in Appendix 3, is 5.1% in 1982 and falls to the population growth rate of 3.2% per year by 1990.

The descent in the rate of return to secondary education is caused by the rapid increase in the supply of C 5 labour in the labour force, more due to the reduction in the absorption of secondary school graduates by higher education than to the increase in the rate of their formation.

It was assumed that higher education should have a non-negative growth rate. If the social discount rate is raised to 15%, the lower bound of zero growth is attained. The rate of return is 11.6% in 1969 and by 1985 it still has not reached 15%. Secondary education grows at its maximum rate for the first period and then should show slightly slower growth[1] for the few years until it reaches the primary graduate supply constraint.

If the elasticity of substitution is lowered to four and the social discount rate set at 10%, the growth rate of higher education should be approximately 2% in each period. Secondary education should be at its maximum growth rate until it becomes equal to the supply of primary graduates.

Finally the optimal rate of growth was calculated for higher education assuming that direct costs could be cut to half their present levels. With the elasticity of substitution equal to six and a social discount rate of 10%, the optimal growth rate would be roughly 5% for each period. This is hardly different from the solution shown in Table 10, and indicates again that direct costs are relatively unimportant in the evaluation of higher education.

Conclusions

The extension to the rate of return approach described in this paper has made it possible to calculate in quantitative terms optimal enrolment projections for Colombia. Since the methodology thus provides an alternative to the manpower requirements approach to educational planning it may be worthwhile to point out the essential differences between the two.

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1 The exact rate of growth was not calculated.

The manpower requirements approach, in its usual fixed coefficients form, implies that there is a zero elasticity of substitution between different categories of labour. The rate of growth of the demand for each type of labour can thus be calculated given the rate of growth of output of each industry and the rate of increase of productivity within it. Given the projection of aggregate demand for each kind of labour one can derive the need for education. No attempt is made to value this need or to place costs on the manpower bottlenecks or surpluses that would arise if the actual growth of the educational system is different from the required growth. There is therefore no way in which one may evaluate the return to the investment of real resources in education.[1]

The present technique assumes that there is sufficient substitutability between different types of labour for their supplies to be absorbed by the labour market. The projection of the educational system determines the supplies, and the demands are equated to them by movements of relative wage rates. The concept of "manpower requirements" thus loses most of its significance. This of course does not imply that any projection of the growth of the educational system is as good as another. The technique takes into account explicitly the diminishing returns effect of the growth of the educational system on its own profitability and this feedback can be used to calculate the optimal enrolment projections and the derived optimal supply projections for the different kinds of labour.

The results for Colombia show that, for the next ten years at least, the country should accelerate the rates of growth of primary and secondary education and, unless the social discount rate used is very low, should cut back the growth of higher education. Even if the forced march towards universal primary education described in Appendix 3 were implemented, the rate of return to primary school would remain around 20% for the next fifteen years. Likewise even if secondary education were to grow at 15% per year for the next ten years, and even if one makes the pessimistic assumptions that the present inefficient student-teacher ratio and the high drop-out rates are not improved, the rate of return would remain around 20% for

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1 Unless the analysis were imbedded in an optimizing model, for example a linear programming model. But to my knowledge this has never been done. I am indebted to Robert Repetto for this point.

the next fifteen years. If it were possible for secondary education to grow at a faster rate without reducing the quality of instruction, this would certainly be justified. Eventually of course the growth of secondary education would be limited by the supply of primary school graduates.

On the other hand the rate of return to higher education is low. If the present growth rate of 11% per year were maintained the rate of return would be less than 10% now and would fall over time, probably approaching zero by 1985. If the present mix of courses remains unchanged, higher education should grow at no more than 5% per year for the next ten years if one assumes a social discount rate of 10%, and it should not grow at all if one assumes a social discount rate of 15%. If one were to assume a social discount rate as low as 5% the present growth rate would be justified until 1975 but should be reduced substantially after that date.

It should be remembered that the calculations for higher education were made on the assumption that the present mix of courses will not change. Doubtless many of the courses yield much higher rates of return than that for higher education as a whole. If these courses were identified and future growth restricted to them, then the university system would be justified in growing faster than suggested in the preceding paragraph.

Finally it should be emphasized that in this paper the analysis has been confined to the narrowly economic aspects of education. The policymaker should modify the recommendations above taking into account the social and political aspects of education, particularly its role in the redistribution of income and in increasing social mobility. Nonetheless it is unlikely that the basic findings of this paper, that primary and secondary education should be expanded as fast as possible for at least the next ten years, would be changed by such considerations.

Appendix 1

Some Problems with the Use of the Aggregate Labour Index

In the analysis two strong implicit assumptions have been made:

(i) separability between labour and all other factors of production in the aggregate production function

(ii) separability between the different categories of labour within the aggregate labour index

The first assumption is implied by the form of the aggregate production function and the second by the use of a simple constant elasticity of substitution function in order to calculate the labour index.

Neither of these assumptions is realistic. The first implies that each category of labour has the same elasticity of substitution with all the factors of production responsible jointly for the term $e^{\lambda t}$ in the production function. In particular it implies that the amount of capital per worker influences the demand for labour as a whole but does not otherwise have any special effect on the demand for an individual category. There is however reason to believe that there exists a correlation between the capital intensity and the demand for more educated types of labour. The greater the capital intensity, it may be argued, the greater is the sophistication of the technology being used and hence the greater is the demand for specialists. The reason for this relationship is the fact that those countries with the greatest stocks of physical capital per capita have been the industrial pioneers and have been responsible for the application of new scientific discoveries to industrial processes. These countries have also had relatively highly educated labour forces, largely as a result of social pressures, and in consequence the new industrial processes have tended to be designed to make use of the skills available. Hence one may expect a supply-induced joint use of capital and skilled labour in advanced countries to imply a complementary demand for them in less developed countries.[1]

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1 The existence of such complementarity together with relatively fast growth of the more educated categories of labour would lead to an upward bias in the estimates of the elasticity of substitution between labour categories. This may partially account for the very high elasticities found by Sam Bowles and Marcelo Selowsky. In fact it would not be surprising if "greater than infinity" estimates of the elasticity were encountered.

The assumption of separability between different categories of labour within the CES function implies that the demand for any given category of labour is a function of the supplies of the other categories but not of the structure of those supplies. However it is unlikely that the elasticity of substitution is independent of the quantities used. It is even more unlikely that the elasticity is the same for each pair of categories. One would expect it to be an increasing function of the closeness of their educational levels. The use of a two-level CES function would partially overcome this problem, but data for deciding upon groupings of categories and for estimating the elasticity of substitution within each group are lacking in Colombia.

The use of equations (1) and (3) in the analysis therefore reflects a present inability to find superior functions rather than an unawareness of the influences of complementarities on the future structure of wage rates. Empirical research in this area is still in its infancy, and is likely to progress slowly because time series on wage rates by level of education, essential for such research, are scarce.

Appendix 2

Fitting the CES Function to U.S. Data

In Appendix F of his study "Why Growth Rates Differ" (Brookings, 1967) Denison gives the following data on the structure of the labour force and wage levels, according to years of schooling, for the United States in 1949 and 1959. For both years wages are expressed as a percentage of wages earned in that year by those workers with eight years of education. The labour structure is given in percentage terms.

Years of Schooling	WAGES			EMPLOYMENT		
	1949	1959	1959/1949	1949	1959	1959/1949
0	50	51	1.020	2.0	1.2	0.600
1 - 4	65	61	0.938	6.6	4.9	0.742
5 - 7	80	83	1.083	14.5	11.1	0.766
8	100	100	1.000	20.8	15.9	0.764
9 - 11	115	109	0.948	20.2	20.2	1.000
12	140	124	0.886	22.6	27.2	1.204
13 - 15	165	139	0.842	6.8	9.1	1.338
16+	235	181	0.770	6.4	10.5	1.641

The assumption of a constant elasticity of substitution function would imply the relationships

$$w_i = a_i C_i^{\theta-1} L^{1-\theta} \frac{\partial Y}{\partial L} \qquad (i=1,\ldots,8) \quad \text{for 1949}$$

$$w_i' = a_i C_i'^{\theta-1} L'^{1-\theta} \frac{\partial Y'}{\partial L'} \qquad (i=1,\ldots,8) \quad \text{for 1959}$$

where

w_i , w_i' = wage of labour category i in 1949, 1959

C_i , C_i' = size of labour category i in 1949, 1959

L , L' = aggregate labour index in 1949, 1959

Y , Y' = aggregate output in 1949, 1959

Since both the wage data and the category size data in the table have been scaled,

$$w_i = \lambda p_i \qquad w_i' = \lambda' p_i'$$

$$C_i = \mu q_i \qquad C_i' = \mu' q_i'$$

where p_i and p_i' are the relative wages of category i , q_i and q_i' are the percentages of category i in the total labour force, and λ , λ', μ and μ' are some unknown scaling factors, in 1949 and 1959 respectively.

Hence one obtains the theoretical relationships

$$\lambda p_i = a_i (\mu q_i)^{\theta-1} L^{1-\theta} \frac{\partial Y}{\partial L} \qquad (i=1,\ldots,8) \quad \text{for 1949}$$

$$\lambda' p_i' = a_i (\mu' q_i')^{\theta-1} L'^{1-\theta} \frac{\partial Y'}{\partial L'} \qquad (i=1,\ldots,8) \quad \text{for 1959}$$

$$\therefore \quad p_i'/p_i = (q_i'/q_i)^{\theta-1} (L'/L)^{1-\theta} \left(\frac{\partial Y'}{\partial L'} / \frac{\partial Y}{\partial L}\right) \left(\frac{\mu'}{\mu}\right)^{\theta-1} \frac{\lambda}{\lambda'} \qquad (i=1,\ldots,8)$$

i.e. $\quad y_i = a + b\, x_i$

where

$$y_i = \log(p_i'/p_i)$$

$$x_i = \log(q_i'/q_i)$$

$$a = \log\left[(L'/L)^{1-\theta} \left(\frac{\partial Y'}{\partial L'} / \frac{\partial Y}{\partial L}\right) \left(\frac{\mu'}{\mu}\right)^{\theta-1} \frac{\lambda}{\lambda'}\right]$$

$$b = \theta - 1 = \frac{\sigma-1}{\sigma} - 1 = -\frac{1}{\sigma}$$

where σ is the elasticity of substitution.

Thus if one regresses the logarithm of the ratio of p_i' to p_i (the figures in column 4 of the table above) on the logarithms of the ratio of q_i' to q_i (the figures in column 7) one obtains an estimate of $-\frac{1}{\sigma}$.

The value of this estimate was -0.276 with a standard error of 0.045. This implies a value of 3.63 for σ. The estimate of the parameter a was also highly significant. The value of R^2 was 0.862.

Appendix 3

The Projections of Primary and Higher Education

Primary Education

The constitution of Colombia states that primary education is to be free and compulsory. At present this goal is far from being attained since only approximately 22% of the children who enter primary school graduate from it, but it is reasonable to assume that the situation will improve fairly rapidly. One of the major causes of desertion is the failure of a large number of schools to offer all five grades of primary education. Many, especially in rural areas, offer just the first two grades and the students in these schools have little opportunity for further education. Accordingly an increase in the supply of places in primary school should produce immediate results. There are of course additional, and probably less tractable, problems on the demand side, especially in rural areas where the need for sending children to primary school may not be sufficiently appreciated and the pressure to withdraw them for agricultural work may be strong.

It has therefore been necessary to make some assumption about the way in which the drop-out rate for each grade of primary school will fall over time. It has been assumed that the graduation rate will increase from 22% to 90% over a period of fifteen years, in such a way that the non-graduation rate will decline by a constant percentage each year. The drop-out rates for the five individual grades were kept in constant proportions to one another but their absolute values for each year were adjusted in order to be consistent with the non-graduation rate for primary education as a whole for that year.

In view of the fact that the drop-out rates are thus functions of time the "impact table" for primary education had to be computed for each calendar year. This has the same form as Table 3 in Section II. It shows for each year total enrolment , enrolment in each grade, the number of teachers required (all primary school teachers are assumed to have secondary school training and to be drawn from labour category C 5), the

number of drop-outs from the first three grades (potential recruits to C 2), the number of drop-outs from the last two grades (potential recruits to C 3), the number of graduates, and the capital requirements.

This series of impact tables is then used to calculate the consequences of the projection of the number of students entering the first grade in each year. The projection has two components: (a) the number of seven-year-olds in the country for each year, it being assumed that every child at least enters primary school even if his chances of completing it are poor, and (b) an adjustment for the "backlog effect". At present the enrolment of students in the first grade is considerably greater than the number of seven-year-olds. This is partly due to the large repeating rate for this grade, partly due to the late entry of part of the backlog of older persons who did not enter primary school when aged seven. As can be seen from Table 3, repetition is automatically taken into account in the calculations, but a separate adjustment has to be made for the backlog.

In the reference analysis the projection of seven-year-olds was made by taking a figure of 700,000 for 1969 and assuming that this grows at a rate of 3.2% per year, the present population growth rate. The additional enrolment in first grade due to the backlog was estimated to be 300,000 in 1969 and it was assumed that it would decline at a rate of 30,000 per year as a result of the increasing coverage of primary education. Section VIII describes the effects of taking an alternative hypothesis in which the projection of seven-year-olds starts with a figure of 750,000 for 1969 and has the same growth rate as before, and the projection of the additional enrolment due to the backlog starts at 400,000 in 1969, declining at a rate of 40,000 per year. The rate of return to primary education is not much affected by the variation.

Higher Education

Higher education in Colombia is divided between the "carreras largas", university courses leading to a degree, and the "carreras cortas", shorter vocational courses for which a diploma is granted. Both classifications encompass a large variety of studies and it is therefore difficult to speak of the student-teacher ratio, the student-capital ratio, even of the length of the course.

The problem of the differing course lengths has been partially overcome by assuming that a certain proportion of the students terminate their studies at the end of each grade.[1] 8.15% terminate at the end of the first grade, and 9.54%, 9.27%, 12.95%, 28.14%. and 31.94% at the end of grades two through six. These figures are based on the present structure of the labour force with higher education. Those students terminating their studies during the first three grades are assigned to labour category C 6, and those terminating in later years are assigned to C 7.

The estimates of the other basic parameters, i.e. the student-teacher ratio and the student-capital ratio, are the observed averages over higher education as a whole. All teachers are assumed to have university degrees and are thus withdrawn from C 7. Since relatively few university teachers teach full time, the student - equivalent-full-time-teacher ratio has been computed and the requirements of teachers are measured in equivalent-full-time-teacher years.

With these data an "impact table" similar to that for secondary education was constructed and used to evaluate the consequences of projections of the future enrolment in first grade.

The rate of return to higher education should therefore be interpreted as a weighted average of the rates of return to the different types of course of which it is composed. The rate of return for years in the future is calculated using the implicit assumption that the mix of courses will not change enough to affect it significantly.

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1 No attempt was made to distinguish between those whose studies were completed at that grade and those dropping out from courses of greater duration.

Appendix 4

The Derived Projections for Labour Categories C 1 - C 4 , C 6 , C 7

In Section III it was shown how the projection of the future size of the labour category C 5 was derived from underlying projections of the enrolments in the first grades of the different branches of the educational system. The projections of the remaining categories are made in similar fashion. As in the case of C 5 , the estimate of the potential supply of each category is multiplied by an adjustment factor which takes into account the fact that not all people qualified for that category actually enter the labour force.

C 1 comprises illiterates. It is assumed that in future all Colombian children will receive at least some primary education and that therefore there will be no additions to this category. The projection is thus that of the decline, due to retirement or death, of the stock existing in 1969.

C 2 has two components - the residue from the 1969 initial stock and the cumulative total of drop-outs from the first three grades of primary school. The latter is dependent on the projection of the enrolment in primary school and the projections of the declines in the drop-out rates for the first three grades.

C 3 is composed of (i) the projection of the residue from the 1969 initial stock; (ii) the projection of the cumulative total of drop-outs from the last two grades of primary school; and (iii) the projection of the cumulative number of primary school graduates who do not enter secondary school. (ii) and (iii) are calculated using the impact tables for primary education and the enrolment projections for primary and secondary school.

C 4 comprises the residue from the 1969 initial stock and the cumulative total of those students who drop out during the "ciclo básico" of secondary school, i.e. its first four grades.

C 6 is composed of the residue from the 1969 initial stock and the cumulative total of those students who end their studies during the first three years of higher education.

C 7 has four components, of which two are positive: (i) the projection of the residue from the 1969 initial stock, and (ii) the projection of those students who leave grades four through six of higher education; and two negative: (iii) the projection of university trained teachers required by secondary school, and (iv) the projection of university trained teachers required by higher education itself. Components (ii) and (iv) are calculated using the enrolment projection and impact table for higher education, and component (iii) is calculated using the enrolment projection and impact table for secondary education.

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