PN-ABI-336
ISN 72156

PROGRAM OF DEVELOPMENT STUDIES
121 Sewall Hall
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Houston, Texas 77001

On the General Equilibrium Analysis of Tax Incidence.

336.294 Rice Univ. Program of Development Studies.NTIS
B191 On the General Equilibrium Analysis of
Tax Incidence. J. Gregory Ballentine and
INU db Ibrahim Eris. 1973.
ISN=6591 34 p.
P343/08261 Contract AID/csd-3302.
Proj. 931-17-995-534.

1. Taxation. 2. Economic research. 3. Models - Taxation.
I. Ballentine, J. Gregory. II. Eris, Ibrahim. III.
Contract. IV. Title.

Paper No. 38

ON THE GENERAL EQUILIBRIUM ANALYSIS OF TAX INCIDENCE

By

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Spring, 1973

The authors are graduate student and Instructor of Economics, respectively, at Rice University. This paper reports research related to AID contract No. AID/csd-3302, on Distribution of Gains, Wealth and Income from Development.

Program Discussion Papers are preliminary materials circulated to stimulate discussion and critical comment. References in publications to Discussion Papers should be cleared with the authors to protect the tentative character of these papers.

On the General Equilibrium Analysis of Tax Incidence

In this paper we give a simple general equilibrium analysis of tax incidence. Harberger, in his well-known 1962 paper on the corporation income tax, purports to do the same.[1] His model has been used extensively to analyze various aspects of taxation.[2] We show that his approach is fundamentally inadequate for analysis of taxation, even within the confines of a static, two-factor, two-good model, which we, like Harberger, employ.[3]

At the risk of being pedantic, let us consider the use of differential equations for analysis of finite parameter (in this case, tax) changes, which will enable us to indicate the general source of the inadequacy of the Harberger model. A differential equation which determines the slope of the relationship between some tax and some variable and which is valid for all tax values may be integrated to obtain the exact change in the variable given some finite tax change. Unfortunately integration of such equations is rarely feasible. However, since it is only the dif-

*We wish to thank Professors Charles McLure and Wayne Thirsk, both of Rice University, for their helpful suggestions.

[1] Arnold C. Harberger, "The Incidence of the Corporation Income Tax," Journal of Political Economy (June, 1962), pp. 215-40.

[2] See, for example, Arnold Harberger, "Efficiency Effects of Taxes on Income from Capital," in Marian Krzyzaniak, ed., Effects of Corporation Income Tax (Detroit: Wayne State University Press, 1966), pp. 107-17; Melvyn Krauss, "Differential Tax Incidence: Large Versus Small Tax Changes," Journal of Political Economy (Jan./Feb., 1972), pp. 193-97; Charles E. McLure, Jr., "Tax Incidence, Macroeconomic Policy, and Absolute Prices," Quarterly Journal of Economics (May, 1970), pp. 254-67; Charles E. McLure, Jr., "Taxation, Substitution, and Industrial Location," Journal of Political Economy (Jan./Feb., 1970), pp. 112-32; and Peter M. Mieszkowski, "On the Theory of Tax Incidence," Journal of Political Economy (June, 1967), pp. 250-62.

[3] Our model makes the same behavioral and technical assumptions as Harberger's, except that we allow the government's marginal propensity to consume to differ from the private one. However, when we show the inadequacy of Harberger's approach we assume these propensities to be the same. We include the possibility of government and private propensities to consume differing to indicate the difference between those conditions which rule out the effects of income redistribution on demand and those which rule out the effect of overall income changes on demand. As will be made clear below, the latter conditions are of crucial importance.

ferential equation which is integrated, that equation can at least be used to (1) list all the variables which influence the incidence of any finite tax change, (2) indicate, where possible, how each variable influences incidence, and (3) determine conditions under which the sign of the differential equation is unambiguous over some range of tax values making the sign of the integral unambiguous over that range.

The methodology outlined above is essentially that followed in the Harberger literature.[1] However, it is precisely such analysis which cannot be performed using Harberger's model. The basic reason for this is that that model is stated and solved under the assumption that all taxes (including the tax whose incidence is being analyzed) are zero. As shown below (section 2) it is a special property of the zero tax point that a term, which we call the net income term, does not appear in the differential equations. Since the net income term includes variables which do not enter Harberger's solutions, his analysis cannot list all of the variables which influence the incidence of any partial tax, much less make statements about their influence.[2]

The source and consequences of these additional variables which influence tax incidence are discussed in sections (2)-(5). In section 2 it is shown that the absence of the net income term in Harberger's solution stems from a simplification he makes in the demand function. This simplification is in general not valid, though it is so at the point he considers.

The net income term provides a useful and interesting link between tax incidence and income changes due to the inefficiency effects of taxes (section 3). This link

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[1] See, for example, Harberger, "The Corporation Income Tax," Section VI, pp. 227-30 and Mieszkowski, "Theory of Tax Incidence," pp. 253-4.

[2] By a partial tax we mean a tax on one good or on one factor in one of its uses.

has been neglected in the Harberger literature and the possibility of it appearing in a model employing differential equations has even been denied.

Omission of the net income term has important consequences for the usefulness of Harberger's model, beyond that discussed above. For example, a topic which has caused considerable controversy in the field of public finance is the issue of the shifting of the U.S. corporation income tax. In section 5 we consider the empirical relevance of the net income terms in the context of this issue, showing that omission of them results in a consistent underestimation of shifting. Another example deals with the incidence of a tax when there are existing non-tax price or factor return distortions, such as monopoly power, union wage differentials, or tariffs. In analyzing such cases it is invalid to use Harberger's model <u>even if one is solely interested in the rate of change of some variable with respect to a tax at only the zero tax point</u>. Harberger attempted to do this in his discussion of corporate tax incidence under monopoly mark-up pricing.[1] However, as is also shown in section 5, this and any similar use of his model is incorrect, for under such conditions the net income term influences incidence even when all taxes are zero.

Finally, in the last section we examine differential incidence. We show that there are important factors limiting the usefulness of Mieszkowski's analysis of this subject in addition to those applicable to the Harberger model itself.[2]

1

In this section we start with a set of equations determining the equilibrium values of the economic variables under consideration with fixed initial tax values.[3]

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[1] Harberger, "Corporation Income Tax," pp. 238-39.

[2] Mieszkowski, "Theory of Tax Incidence," pp. 250-62.

[3] J. E. Meade and Murray C. Kemp both present models which assume initial

Later by differentiating these equations implicitly we solve for elasticities of some important variables with respect to the tax parameters at the equilibrium point.

We consider two partial factor taxes which are levied on capital used in the two sectors at the rates t_{KX} and t_{KY}, and an excise tax in one of the sectors at the rate t_X.

We assume that the production functions for two goods, X and Y,

$$X = f(K_X, L_X) \tag{1}$$

$$Y = g(K_Y, L_Y) \tag{2}$$

are homogeneous of degree one with the usual concavity properties. The two factors, capital and labor, are assumed to be fixed in supply. Thus

$$L_X + L_Y = L \tag{3}$$

$$K_X + K_Y = K \tag{4}$$

Assuming perfectly competitive factor and product markets and labor to be the numeraire good, the profit maximization conditions are

$$r(1 + t_{KX}) = f_K p_X \tag{5}$$

$$r(1 + t_{KY}) = g_K p_Y \tag{6}$$

$$1 = f_L p_X \tag{7}$$

$$1 = g_L p_Y \tag{8}$$

where f_K, f_L, g_K, g_L are the partial derivatives of the production functions with respect to the two factors; p_X, p_Y are the prices of the products the producers face; r is the price of capital net of taxes.

We assume there exists n individuals in the economy each of whom is endowed with some capital (K_i) and labor (L_i) so that the ith individual's demand for the

taxes are arbitrary. They only consider excise taxes, however, and only Kemp analyzes the effect of these taxes on the wage-rental ratio, the main subject of Harberger's and our analysis. See James E. Meade, Mathematical Supplement to Trade and Welfare (Oxford: Oxford University Press, 1955), pp. 34-46 and Murray C. Kemp, The Pure Theory of International Trade and Investment (Englewood Cliffs, New Jersey: Prentice-Hall, Inc., 1967), pp. 21-26.

two products can be characterized by the demand function

$$X_i = D_i[p_X(1+t_X),\ p_Y,\ L_i + rK_i] \tag{9}$$

and the budget constraint

$$p_X(1 + t_X)\ X_i + p_Y Y_i = L_i + rK_i\ . \tag{10}$$

We also have:

$$\sum_{i=1}^{n} K_i = K \tag{11}$$

$$\sum_{i=1}^{n} L_i = L. \tag{12}$$

The government is assumed to have a demand function which can be characterized by

$$X_G = D_G[p_X(1 + t_X),\ p_Y,\ I_G] \text{ and} \tag{13}$$

$$p_X(1+t_X)X_G + p_Y Y_G = I_G \tag{14}$$

where I_G is government revenue and thus

$$I_G = rt_{KX}K_X + rt_{KY}K_Y + p_X t_X X.$$

Finally we have for the product markets:

$$X = X_G + \sum_{i=1}^{n} X_i \tag{15}$$

$$Y = Y_G + \sum_{i=1}^{n} Y_i \tag{16}$$

The above equations are assumed to determine unique equilibrium values of the variables as functions of the parameters, t_{KX}, t_{KY} and t_X. Now we can proceed to implicitly differentiate (1)-(16) with respect to any of these paramters, to explicitly solve for the desired slopes or elasticities. To avoid repetition, in our differential equations we shall not specify with respect to which parameter the derivative is taken. Later we shall do the proper substitutions.

If we use the ($\hat{}$) notation to denote the percentage change in a variable, we obtain equations (1') - (8') from (1) - (8):

$$\hat{X} = \theta_{LX}\hat{L}_X + \theta_{KX}\hat{K}_X \tag{1'}$$

$$\hat{Y} = \theta_{LY}\hat{L}_Y + \theta_{KY}\hat{K}_Y \tag{2'}$$

$$dL_X + dL_Y = 0 \tag{3'}$$

$$dK_X + dK_Y = 0 \tag{4'}$$

$$\hat{f}_K + \hat{p}_X = \hat{r} + \widehat{(1 + t_{KX})} \tag{5'}$$

$$\hat{g}_K + \hat{p}_Y = \hat{r} + \widehat{(1 + t_{KY})} \tag{6'}$$

$$\hat{f}_L + \hat{p}_Y = 0 \tag{7'}$$

$$\hat{g}_L + \hat{f}_Y = 0 \tag{8'}$$

where $\theta_{KX} = [r(1 + t_{KX})/p_X X] = f_K K_X/X$, etc.

Denoting the elasticity of commodity X with respect to its own price along the income compensated demand curve for individual i by ϵ_i, his marginal propensity to consume commodity X by m_i and similar quantities for the government by ϵ_G and M_G, the differentials of equations (9), (10), (13) and (14) may be written as

$$\hat{X}_i = \epsilon_i[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] + (m_i/X_i)\{K_i dr - X_i d[p_X(1+t_X)] - Y_i dp_Y\} \tag{9'}$$

$$p_X(1+t_X)dY_i + Y_i d[p_X(1+t_X)] + p_Y dY_i + Y_i dp_Y = K_i dr \tag{10'}$$

$$\hat{X}_G = \epsilon_G[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] + (m_G/X_G)\{dI_G - X_G d[p_X(1+t_X)] - Y_G dp_Y\} \tag{13'}$$

$$dI_G = X_G d[p_X(1+t_X)] + p_X(1+t_X)dX_G + p_Y dY_G + Y_G dp_Y \tag{14'}$$

where

$$dI_G = (K_X t_{KX} + K_Y t_{KY})dr + (t_{KX} - t_{KY})rdK_X + rK_X dt_{KX} + rK_Y dt_{KY} + p_X X dt_X$$

$$+ t_X X dp_X + p_X t_X dX.$$

In this paper we assume that all private consumers have a common marginal propensity to consume, say, m_p. This and equations (15) and (16) enable us to write the total demand for commodity X as

$$\hat{X} = \epsilon[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] + (m_p/X)\{Kdr - X_p d[p_X(1+t_X)] - Y_p dp_Y\}$$

$$+ (m_G/X)\{dI_G - X_G d[p_X(1+t_X)] - Y_G dp_Y\} \tag{17}$$

where

$$X_p = \sum_{i=1}^{n} X_i$$

and

$$\epsilon = (X_G/X)\epsilon_G + \sum_{i=1}^{n} (X_i/X)\epsilon_i.$$

Solving these equations, one can obtain the following three elasticities:[1]

$$\eta_{r,t_{KX}} = [\hat{r}/\widehat{(1+t_{KX})}]$$

$$= \frac{\epsilon\theta_{KX}A + \sigma_X B - m_G p_X C_1 + (m_G - m_p)p_X E_1 A}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y + m_G p_X(C_1 + C_2) + (m_G - m_p)p_X(E_3 - E_2 - E_1)A} \tag{18}$$

$$\eta_{r,t_{KY}} = [\hat{r}/\widehat{(1+t_{KY})}] = \frac{-\epsilon\theta_{KY}A + \sigma_Y - m_G p_X C_2 + (m_G - m_p)p_X E_2 A}{D} \tag{19}$$

$$\eta_{r,t_X} = [\hat{r}/\widehat{(1+t_X)}] = \frac{\epsilon A + (m_G - m_p)(E_1/\theta_{KX})}{D} \tag{20}$$

where D is the same denominator as in $\eta_{r,t_{KX}}$ and

$$A = (K_X/K_Y) - (L_X/L_Y)$$

$$B = \theta_{LX}(K_X/K_Y) + \theta_{KX}(L_X/L_Y)$$

$$C_1 = \{1 - [(1 + t_{KY})/(1 + t_{KX})]\}\theta_{KX}\sigma_X(L_X/L_Y) + t_X\sigma_X B$$

$$C_2 = \{1 - [(1 + t_{KY})/(1 + t_{KX})]\}\theta_{KX}\sigma_y + t_X\sigma_Y$$

$$E_1 = (1 + t_X)(X_p/X)\theta_{KX}$$

$$E_2 = (p_Y Y_p/p_X X)\theta_{KY}$$

$$E_3 = [\theta_{KX}/(1+t_{KX})] + [\theta_{KX}/(1+t_{KY})](p_Y Y/p_X X)$$

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[1]Several other elasticities are presented in the appendix.

For ease of reference, we define $\sigma_X B$ and σ_Y as <u>substitution terms</u>, $\epsilon\theta_{KX}A$, $\epsilon\theta_{KY}A$ and ϵA as <u>output terms</u>, $m_G p_X C_1$ and $m_G p_X C_2$ as <u>net income terms</u>, and $(m_G - m_p)p_X E_1 A$, $(m_G - m_p)p_X E_2 A$ and $(m_G - m_p)p_X E_3 A$ as <u>demand terms</u>, whether they appear in the numerator or the denominator.[1]

The reader should immediately note that if, as is assumed in the Harberger model, $t_{KX} = t_{KY} = t_X = 0$ and $m_G = m_p$ then the net income terms as well as the demand terms disappear and our equations reduce exactly to those of the Harberger model. However, if one is interested in, say, the incidence of a finite partial factor tax in industry X alone (which is the main subject of the corporation income tax literature) then one must consider the net income terms, since imposition of such a tax requires that $t_{KX} \neq t_{KY}$.

It is important to note further that the marginal propensity to consume in no way affects Harberger's solution. Our equation indicates that, even when $m_p = m_G$, the incidence of a tax is influenced by the marginal propensity to consume.[2]

2

In the appendix we prove that the equations (18)-(20) are independent of the net income terms (i.e., can be written as is done in the Harberger model) <u>if and only if</u>

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[1]Our definitions of output and substitution terms correspond roughly to Mieszkowski's. (See Mieszkowski, "Theory of Tax Incidence," p. 254.) Inexplicably, however, he seems to consider these definitions to hold only when these terms appear in the numerator of his equations. This causes him to argue that,"There is no factor-substitution effect for commodity taxes." (<u>Ibid</u>., p. 259.) This is quite misleading since both $\sigma_X B$ and σ_Y appear in the denominator of his (and our) equation determining the incidence of that tax.

[2]Our net income term includes only m_G, and thus m_p and m_G enter the equations asymmetrically. This is purely a matter of the way the terms were collected, in effect one could rewrite the equation so that the net income term would include only m_p rather than m_G. The important point is that if $m_p = m_G = m$, the demand term drops but the net income term remains with the common marginal propensity to consume, m.

the income effect in the total demand equation (17) is zero. We now show that Harberger, by restricting himself to the zero tax case ,implicitly assumes the income effect in (17) to be zero, thus eliminating the net income term from his incidence analysis. While we note some other special cases in which the income effect is zero, we show that in general it is non-zero.

Harberger writes his demand equation for X in differential form as[1]

$$\hat{X} = \epsilon[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] \tag{21}$$

Although he never explicitly states it, it is clear from the context of Harberger's work and Mieszkowski has sketched a proof that ϵ, in this equation, must be the income compensated elasticity of demand for commodity X.[2]

An alternative proof may be given as follows. Assuming, as Harberger does, $m_p = m_G = m$, the total demand function (17) can be written as

$$\hat{X} = \epsilon[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] + m/X\{[(1+t_{KX})K_X + (1+t_{KY})K_Y]dr + rK_X dt_{KX} + rK_Y dt_{KY} - Xdp_X - Ydp_Y + p_X t_X dX + (t_{KX} - t_{KY})rdK_X\}$$

But since from (1')-(8')

$$\hat{p}_X = \theta_{KX}[\hat{r} + \widehat{(1+t_{KX})}] \tag{22}$$

and

$$\hat{p}_Y = \theta_{KY}[\hat{r} + \widehat{(1+t_{KY})}] \tag{23}$$

then, remembering that $\theta_{KX} = r(1+t_{KX})K_X/p_X X$ and similarly for θ_{KY}, we obtain

$$Xdp_X + Ydp_Y = [(1+t_{KX})K_X + (1+t_{KY})K_Y]\, dr + rK_X dt_{KX} + rK_Y dt_{KY}. \tag{24}$$

Inserting this into the above equation for $\hat{X}$, we can write the total demand function as

$$\hat{X} = \epsilon[\hat{p}_X + \widehat{(1+t_X)} - \hat{p}_Y] + m/X[(t_{KX} - t_{KY})rdK_X + p_X t_X dX]. \tag{25}$$

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[1]Harberger's price for X, p_X, contains no taxes because he assumes them to be zero. Since we are considering the validity of Harberger's model at points where partial taxes are not zero, we use this formulation (i.e. $\hat{p}_X + (1+t_X)$ instead of $\hat{p}_X$). As will be clear, our objection holds even if $t_X = 0$ as long as $t_{KX} \neq t_{KY}$.

[2]See Mieszkowski, "Theory of Tax Incidence," p. 260, fn. 10.

If $t_{KX} = t_{KY} = t_X = 0$, as Harberger assumes, or more generally if $t_{KX} = t_{KY}$ and $t_X = 0$, then the income effect in this equation is zero independent of the values of dK_X and dX. Thus one can use the demand equation (21) with the interpretation that ϵ is the income compensated price elasticity.[1]

However, clearly when $t_{KX} \neq t_{KY}$ and/or $t_X \neq 0$, the income effect is in general not equal to zero and its value depends on the values of dK_X and dX and thus on the tax with respect to which the derivatives are taken. In the appendix to this paper we explicitly solve for the value of the income effect yielded by each tax and show that in general they will be different. However, our equations there do not rule out the possibility of one or all the income effects becoming zero at particular values of t_X, t_{KX}, and t_{KY}.

To examine such a possibility we first note that from equations (1')-(6')

$$p_X dX + p_Y dY = (t_{KX} - t_{KY}) r dK_X \tag{26}$$

Thus the income effect yielded by some given tax is zero, i.e.

$$(t_{KX} - t_{KY}) r dK_X + p_X t_X dX = 0$$

if and only if

$$-(dY/dX) = p_X(1+t_X)/p_Y. \tag{27}$$

The right hand side of this equation is simply the consumer's price ratio. The term on the left denotes the rate at which X and Y can be exchanged along the equilibrium path generated by varying one of the taxes. This path is not the

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[1] Harberger, and those using his model, consistently give the impression that all that is necessary to be able to formulate the demand equation as in (21) is that $m_p = m_G = m$. See, for example, Harberger, "Corporation Income Tax," p. 228, fn. 9, Mieszkowski, "Theory of Tax Incidence," p. 253, and McLure, "The Theory of Tax Incidence with Imperfect Factor Mobility," Finanzarchiv (Band 30, Heft 1, 1971), p. 33. We initially wrote m_p and m_G separately to explicitly indicate the role the assumption that they are equal plays in the analysis. Examination of (25) and (18)-(20) indicates that the assumption of equal propensities to consume does not, in general, allow one to write the demand equation as (21) or to ignore the net income terms in (18)-(20).

same for each of the taxes. Varying t_X, while t_{KX} and t_{KY} are held constant, induces movement along a transformation curve and in that case dY/dX is the slope of that transformation curve. However, varying t_{KX} or t_{KY} causes the transformation curve to shift in and the equilibrium path they generate runs across such curves.[1] In this case dY/dX is the slope of that path, not the slope of a transformation curve.[2] Thus what equation (27) states is that the income effect yielded by a given tax is zero at a point if and only if at that point the rate at which X and Y can be exchanged along the equilibrium path generated by varying the tax under consideration is equal to the consumers' price ratio.

It should be added that if $t_{KX} \neq t_{KY}$ then such a situation implies $t_X \neq 0$. To see this, first note that, independent of the value of the income effect, from (26),

$$-(dY/dX) = (p_X/p_Y) + [(t_{KX} - t_{KY})rdK_X/dX].$$

But, when the income effect is zero (27) also holds. Thus t_X must be non-zero if $t_{KX} \neq t_{KY}$ and the income effect is zero.

3

It has long been recognized that imposition of partial taxes reduces real income. In addition, it has been pointed out that this change in real income will influence the incidence of such taxes.[3]

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[1] See, for a discussion of this point, H. G. Johnson, "Factor Market Distortions and the Shape of the Transformation Curve," Econometrica (July, 1966), pp. 686-98, and R. W. Jones, "Distortions in Factor Markets and the General Equilibrium Model of Production," Journal of Political Economy (May/June, 1971), pp. 437-59.

[2] However, when $t_{KX} = t_{KY}$, as is shown in the appendix, the path generated by varying t_{KX} or t_{KY} is tangent to the outermost transformation curve.

[3] See, for example, Richard A. Musgrave, The Theory of Public Finance (New York: McGraw-Hill, 1959), pp. 207-8.

However, changes in real income are related to the income effect in the total demand equation. When the income effect is zero real income is constant at that point, otherwise real income is changing.[1] Since the net income terms appear in the incidence equations if and only if the income effect is non-zero, they provide the link between the inefficiency of taxes and their incidence.

The absence of this link in the Harberger model has been subjected to various interpretations. In his early writings Mieszkowski has, in effect, argued that this link does not appear because incidence is independent of the inefficiency effects of taxes on demand. For example, in 1967 he stated, without qualification,[2]

> "Incidence, then depends on three separable effects: the source of income (demand) effect, the output (factor intensity) effect and the factor substitution effect. . . . The demand effect has no bearing on incidence when all groups have the same spending propensities."

The demand effect which Mieszkowski mentions is essentially similar to our demand term and thus is due to differences in marginal propensities to consume and occurs as a given level of real income is redistributed among groups with differing propensities. Thus this statement seems to argue that the incidence of a tax does not depend on the change in aggregate real income induced by imposing that tax.[3]

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[1] What we have called the income effect is often referred to as excess burden. Since, in the public finance literature excess burden and incidence are often treated as two different subjects of tax analysis and since we wish to emphasize that we are here examining only the incidence of taxes we shall retain the standard demand theory terminology. The reader will note that our results indicate that one cannot usefully analyze the incidence of a tax without noting its excess burden.

[2] Mieszkowski, "Theory of Tax Incidence," p. 253.

[3] This listing seems to be regarded as complete in all of the Harberger literature. A most revealing example of this and the entire misconception of Harberger's model is Melvyn Krauss' recent article, "Differential Tax Incidence." Krauss attempts to obtain explicit equations for the change in r given a finite change in t_{KX} or t_X by integrating the differential equations supplied by Harberger. As should be clear from the discussion above, such a procedure is <u>impossible</u>. Integration of an equation over some interval, say, $(0, t^*_{KX})$ requires that that equation be valid for all values of t_{KX} in that interval, thus only our equations can be used for such integration.

This is not the only example in Krauss' paper of his acceptance of the

On another occasion Mieszkowski, in reviewing his own work, states that[1]

> "Mieszkowski disregards excess burdens and analyzes the differential incidence of a wide range of taxes from the point of view of capitalists."

Here the interpretation seems to be that taxes have two effects, incidence and excess burden, and that either of these can be usefully analyzed independent of the other.

A more recent interpretation of the omission of the influence of real income changes on incidence by the Harberger model is provided by Johnson and Mieszkowski. While recognizing the importance of such factors, they argue that they cannot appear in a model based on differential equations. Referring to their use of a general equilibrium model similar to Harberger's and ours to analyze a case involving existing distortions, they state,[2]

> "One of the shortcomings of this approach is that it cannot account for the effect on demand of a removal or introduction of a distortion in factor markets. All outputs are evaluated on the basis of original marginal products, and the increased or decreased real income arising out of a more or less efficient allocation of resources is not accounted for. The basic difficulty is that differentiation is a first order approximation."

From the results of the previous section we know that, since they are examining the case of an existing distortion (equivalent in our terms to the case $t_{KX} \neq t_{KY}$), the income effects will not in general cancel out. Apparently because they did not split their demand function into substitution and income effects they did not realize this fact.[3]

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conclusions derived in the Harberger literature for the zero tax point as being valid for finite tax changes. For his analysis of large tax changes he uses "equal yield" conditions derived by Mieszkowski on the assumption of zero taxes. See section 6 below where we show that these conditions are valid only at the zero tax point, and thus cannot be used for the type of analysis Krauss uses.

As a result of both of these errors, none of the assertions made in Krauss' paper can be proven using his analysis.

[1] Peter M. Mieszkowski, "Tax Incidence Theory: The Effects of Taxes on the Distribution of Income," Journal of Economic Literature (December, 1969), p. 1109.

[2] Harry G. Johnson and Peter M. Mieszkowski, "The Effects of Unionization on the Distribution of Income: A General Equilibrium Approach," Quarterly Journal of Economics (November, 1970), p. 547.

[3] The model for which Johnson and Mieszkowski explicitly show their solution

Our results clearly show that all of these interpretations are false. The link between the inefficiency of a tax and its incidence is lost in the Harberger model only because of the assumption that all taxes are zero. The issue has nothing to do with the properties of differential equations. One can list all of the factors which determine the incidence of a finite tax from a set of differential equations, as we have done, and such a list will include the inefficiency effects of taxes on real income (as embodied in the net income terms).[1]

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differs from ours only in that they use a specific demand function (Graham's demand function). In this case they do not split that demand function into income and substitution effects, however later in their paper they indicate that they solved the model for a general demand function similar to ours with at least the substitution effect explicitly mentioned. (They do not show their solution for this case. See ibid., p. 555, fn. 5.) If they stated and solved the model properly for such a case the income effects of the general demand equation would not drop and the net income term would appear in their solution, which makes the statement we quote above even more difficult to understand. But they state, referring to their use of Graham's demand function in their initial model, that "except for the specialized demand function, this is the same model used by Harberger in work on the incidence of the corporation income tax." (Ibid., p. 548, fn. 5.) This gives the impression that, when they used a general demand function, they duplicated Harberger's model, implying the use of (21) instead of (25), the correct equation for analysis of cases involving existing distortions. This error would explain their lack of reference to a net income term and would be consistent with their contention that income effects are not accounted for.

[1]Krauss and Johnson allege a similar limitation of the use of differential calculus. They state:

> "Because he [Mieszkowski] does not consider the factor-substitution effect in terms of output changes, and further, because the techniques of the differential calculus is strictly valid only for infinitesimal small parametric changes, there is a strong implication in Mieszkowski's analysis that as a result of a partial factor tax the output of the commodity which uses the differentially burdened factor intensively in its production falls while the output of the other rises. A geometric analysis of the problem reveals that this need not be the case; . . . the output of both goods can decrease as a result of the inefficient factor allocation induced by a partial factor tax." Melvyn Krauss and H. G. Johnson, "The Theory of Tax Incidence: A Diagrammatic Analysis," Economica (forthcoming), pp. 374-5.

Krauss and Johnson are correct in stating that Mieszkowski's equations (i.e.; those of the Harberger model) do not admit of the possibility of dY/dX being positive (that is both X and Y decreasing or increasing as a partial factor tax is changed). This again has nothing to do with any limitations of differential calculus, but results from the fact that Mieszkowski has assumed that all taxes are zero. In that case dY/dX, for a change in t_{KX} or t_{KY}, corresponds to the slope of the outermost transformation curve which we know to be negative. However, as examination of the equations for dY/dX presented in the appendix reveals, whenever $t_{KX} \neq t_{KY}$, this ratio can be positive.

4

If all consumers are assumed to have the same average propensity to consume and differing endowments of labor and capital, then the distribution of income (the subject of incidence analysis) changes only as the wage-rental ratio changes. In our model, since the wage rate is chosen as numeraire, this means changes in the distribution of income are determined by the size and sign of η_r. This issue has been explored thoroughly in the context of Harberger's model.

Before considering similar issues in the context of the model presented here, one more important difference between Harberger's solutions and ours should be noted. In both sets of equations the term

$$(\theta_{KY} - \theta_{KX})\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right)$$

appears. As was pointed out by Jones,[1] Magee,[2] and others, the sign of this term is negative as long as no factor price distortions exist, but is ambiguous if there are such distortions. This can be seen easily since

$$(\theta_{KY} - \theta_{KX})\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right) = \frac{L_X K_X r(1+t_{KY})}{p_X X p_Y}\left[\frac{K_Y}{K_X} - \frac{(1+t_{KX})}{(1+t_{KY})}\frac{L_Y}{L_X}\right]\left(\frac{K_X}{K_Y} - \frac{L_X}{L_Y}\right).$$

Thus if the industry producing commodity X is the capital intensive one, then the term is negative, zero or positive depending on whether

$$\frac{(K_Y/L_Y)}{(K_X/L_X)} \lesseqgtr \frac{1+t_{KX}}{1+t_{KY}}$$

An analogous statement holds if Y is produced by the capital intensive industry.[3]

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[1] R. W. Jones, "Distortions in Factor Markets," p. 440.

[2] Stephen P. Magee, "Factor Market Distortions, Production, Distribution and the Pure Theory of International Trade," Quarterly Journal of Economics (Nov., 1971), p. 641.

[3] Johnson and Mieszkowski, in dealing with a case with factor market distortions, seem to rule out this sign reversal in their equations. Since they do not provide the complete equations for their model we are unable to determine the source of this discrepancy. See Johnson and Mieszkowski, "The Effects of Unionization," p. 549.

As Harberger points out, and as can easily be seen from our equations, the three elasticities can be of any sign, even if one considers the special case where all taxes are zero. However, Harberger states and proves the following theorem:[1]

If $t_{KX} = t_{KY} = t_X = 0$, $m_p = m_G$ and $(K_X/L_X) \geq (K_Y/L_Y)$, then $\eta_{r,t_{KX}} \leq 0$.

Actually from equation (18) above one can prove the following more general theorem:

If $t_X = 0$, $m_p = m_G$ and $(K_X/L_X) \geq (K_Y/L_Y)$, then $\eta_{r,t_{KX}} \leq 0$.

Proof of this statement is quite straightforward. Noting the fact that $m_G p_X < 1$ always, one can show that the hypotheses are sufficient to guarantee that the numerator in (18) is negative and the denominator is positive.[2] This proposition states that as long as there are no partial excise taxes, the net income term, even though non-zero under the hypothesis, does not reverse the sign rule given by Harberger. Interestingly enough, if the excise tax is not zero, then the net income term can lead to a positive value for $\eta_{r,t_{KX}}$ even if $t_{KX} > t_{KY}$ and X is the capital intensive industry.

An important corollary to this proposition is that if $t_{KY} = t_X = 0$ and if X remains the capital intensive industry throughout the application of a <u>finite</u> tax on capital in that industry, then r must fall, since the sign of the derivative will be unambiguous over that interval. Obviously, analogous statements can be made about a tax on capital in industry Y.

Further, confirming a result which can be easily demonstrated geometrically,

If $t_{KX} = t_{KY}$, $m_G = m_p$ then $\eta_{r,t_X} \lesseqgtr 0$, as $(K_X/L_X) \gtreqless (K_Y/L_Y)$.

Again, as the sign of η_{r,t_X} is unambiguous for any value of t_X under these assumptions, r will rise (fall) as any <u>finite</u> t_X is applied, if industry X is labor (capital) intensive.

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[1] Harberger, "The Corporation Income Tax," p. 227.

[2] From the discussion given at the beginning of this section it should be clear that $(\theta_{KY} - \theta_{KX})[(K_X/K_Y) - (L_X/L_Y)] \leq 0$ under the hypothesis of this theorem.

A case which was extensively analyzed by Harberger is based on the assumptions that the elasticities of substitution in production are constant and equal. Further the elasticity of substitution in consumption is assumed to be constant and equal to the production elasticities. This latter assumption implies that the market demand function is derived from a C.E.S. utility function, and thus $m_p = m_G = m$. Harberger has shown that at the zero tax point under these assumptions $\eta_{r,t_{KX}} = (K_X/K)$.[1] Using our equations it can be shown that this equality holds for any value of t_{KX} and t_{KY} if $t_X = 0$. This result is of particular interest since it implies that capital will bear the full burden of any <u>finite</u> tax.

The proof of this theorem is fairly straightforward. The reader should first note that if σ_D is the elasticity of substitution in consumption

$$\epsilon = \frac{p_Y Y}{p_X X + p_Y Y} \sigma_D.$$

Further, by the homotheticity property of C.E.S. utility functions

$$m_G p_X = \frac{p_X X}{p_X X + p_Y Y} .$$

Substituting these into (18) one can easily show the difference between it and (K_X/K) to be zero.

Another important aspect of our elasticity equations is that, independent of the values of t_{KX}, t_{KY} or t_X, if $m_p = m_G$ then

$$\eta_{r,t_{KX}} + \eta_{r,t_{KY}} = -1$$

i.e., if $(1 + t_{KX})$ and $(1 + t_{KY})$ are simultaneously increased by, say, one percent, then r decreases by exactly one percent. With the help of this result one can also

[1] Harberger, "Incidence of the Corporation Income Tax," p. 230.

show that for any other variable in the system the sum of the elasticities of that variable with respect to t_{KX} and t_{KY} is equal to zero, which implies that, if t_{KX} and t_{KY} are increased as specified above, nothing else in the economy changes as long as $m_p = m_c$.[1]

Conversely, if $(1 + t_{KX})$ and $(1 + t_{KY})$ are increased in unequal percentages, then at least one variable in the economy must change. To see this, note from (22) and (23) that $\hat{p}_X = \hat{p}_Y = 0$ if and only if $\widehat{(1 + t_{KX})} = \widehat{(1 + t_{KY})}$. But $\widehat{(1 + t_{KX})} = \widehat{(1 + t_{KY})}$ implies the ratio $(1 + t_{KX})/(1 + t_{KY})$ is kept constant. Thus, except for determining the variable r, $(1 + t_{KX})$ and $(1 + t_{KY})$ need not enter the model separately, but rather in a ratio. This ratio, $(1 + t_{KX})/(1 + t_{KY})$, which is also the ratio of the price of capital in the two industries, is the factor market distortion.[2] Therefore the above propositions can now be restated: the equilibrium values of the variables, excepting r, will remain constant if and only if the factor market distortion is not changed (assuming, of course, $\hat{t}_X = 0$).

This proposition has some interesting implications for the properties of a general factor tax (i.e., a tax on a factor in all of its uses). A commonly stated proposition for simple models such as ours is that a general factor tax will be borne entirely by the taxed factor and imposition of that tax will not change the equilibrium point. However, the above discussion indicates that this is only true if there is no partial factor tax on the factor which is to bear a general tax. This can be seen by noting how a general factor tax on capital at the rate t_K enters our

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[1] This summation property of elasticities is shown for some variables in the appendix. The reader should not have difficulty in obtaining the same result for others.

[2] In accordance with this our model, except for the variable r, could have been written solely in terms of $(1+t_{KX})/(1+t_{KY})$ and t_X. If one is only interested in the prices of capital $r(1+t_{KX})$ and $r(1+t_{KY})$, this can be done, but if one wishes to solve explicitly for r, then $(1+t_{KX})$ and $(1+t_{KY})$ must enter the model separately. This was also pointed out by Magee, "Factor Market Distortions," Johnson, "Factor Market Distortions and the Shape of the Transformation Curve," and Jones, "Distortions in Factor Markets."

model. Before the general tax, the price of capital in X relative to that in Y is

$$\frac{r(1 + t_{KX})}{r(1 + t_{KY})} = \frac{1 + t_{KX}}{1 + t_{KY}} .$$

After imposition of t_K this becomes

$$\frac{1 + t_{KX} + t_K}{1 + t_{KY} + t_K} .$$

As the ratio has changed the distortion level has changed and thus imposition of the general tax will change the equilibrium value of at least one variable other than r.

In this paper we have not considered an excise tax on good Y. Had we done so we could have made statements similar to those above for a general consumption tax, showing that such a tax does not alter the equilibrium point only if there is no pre-existing partial excise tax.

Our proposition also has consequences for the equivalence of certain taxes. It is often stated that, in models such as ours, an income tax, a general factor tax on <u>both</u> factors, and a consumption tax are all equivalent. This is true in the following sense. If we consider three identical economies with no taxes and introduce an income tax into one of these economies, a consumption tax into another and a general factor tax on <u>both</u> factors into the other, then the three economies will remain identical.

On the other hand, again consider the three economies before taxes and introduce into all three the <u>same</u> partial taxes at the rates $t_{KX} \neq t_{KY}$ and $t_X \neq 0$. They will, of course, remain identical to each other. Now apply an income tax, a consumption tax, and a general factor tax on both factors in the three economies respectively. They will no longer be identical. The economy receiving the income tax will not change. In the one receiving the general factor tax, the factor market distortion

will change, inducing a change in other variables. The economy receiving the general consumption tax will also change its equilibrium point. For that economy the factor market distortion will remain unchanged, but the distortion between relative producer prices and relative consumer prices will change. Before the tax that distortion is $(1+t_X)/1$ and after the tax it is $(1 + t_X + t_C)/(1 + t_C)$ where t_C is the rate of the consumption tax. Thus the equivalence of these taxes is not a general statement, but is applicable in a specific case only.

5

Probably the most important empirical application of Harberger's model has been his own use of it to estimate the shifting of the corporation income tax.[1] As a result of his analysis, he concluded that that tax is fully borne by capital. This result is in sharp contrast to the conclusions of the Krzyzaniak-Musgrave study which found substantial shifting.[2] However, Harberger's analysis is based on a misuse of his model.

Both Harberger's model and the one presented here provide only equations for the slope of the relationship between, say, r and t_{KX}. If t_{KX} is "small" then an empirical estimate of its incidence may use only one such equation, e.g., pre-tax or post-tax values (though it is incumbent on the analyst to justify his criteria for determining the smallness of a tax). On the other hand, if the tax is "large," then the estimate of the slope at one point may not provide a good approximation for the incidence of that tax. In such a case it is useful to provide estimates based on,

[1] Harberger, "Corporation Income Tax," pp. 230-35.

[2] Marian Krzyzaniak and Richard Musgrave, The Shifting of the Corporation Income Tax (Baltimore: Johns Hopkins Press, 1963). See also Marian Krzyzaniak, "The Burden of a Differential Tax on Profits in a Neoclassical World," Public Finance (#4, 1968), pp. 462-66.

at least, pre-tax and post-tax values. Harberger raised this same issue, stating

> "If the corporation income tax were of small magnitude the pre-tax values of [the variables] . . . would all be very close to their post-tax values. . . However, the tax is in fact substantial in the United States. I have accordingly decided to use two alternative sets of values [pre-tax and post-tax]."[1]

He then proceeds to insert the post-tax values into the equation derived from his model, which, as we have seen, is invalid when there are taxes.

To indicate the empirical relevance of the net income terms it would be useful to compare Harberger's post-tax estimates which ignore those terms with estimates based on equation(18). However, such an approach would not isolate the influence of those terms since (18) differs from the equation Harberger uses in one other respect. For his post-tax estimates Harberger calculates θ_{KX} as net of taxes. He does not justify doing this and as he never writes the model with existing taxes it is not clear why he concluded that this is the relevant formulation. The reader may remember that in our model θ_{KX} was defined as equal to $(f_K K_X/X)$ which is in turn equal to $[r(1+t_{KX})K_X/X]$. Thus a simple comparison of Harberger's estimates with those based on (18) would combine the effects of the net income terms and conversion of θ_{KX} from net to gross of taxes. Therefore we shall redo Harberger's estimates only converting θ_{KX} to gross of taxes and compare those values with values obtained using (18).

Letting X be the corporate sector and Y the non-corporate, and using Harberger's 1953-55 data, in billions of dollars we have,

$rK_X = 20$

$rt_{KX}K_X = 20$

$wL_X = 200$

$rK_Y = 20$

$wL_Y = 20$

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[1] Harberger, "Corporation Income Tax," p. 231.

From these one can calculate every variable in (18) except σ_X, σ_Y, ϵ and mp_X (assuming as Harberger does that $m_G = m_p = m$). Harberger considers different values for σ_X, σ_Y, and σ_D (the elasticity of substitution in consumption). From σ_D he estimates ϵ by the formula presented in the previous section. Since mp_X does not appear in his equations we must provide a separate estimate for it. However, Harberger's treatment of σ_D as a constant for before and after tax values is consistent with a C.E.S. utility function which enables us to use the formula

$$mp_X = \frac{p_X X}{p_Y Y + p_X X} .$$

The table below gives values for $\eta_{r,t_{KX}}$ and capital's share of the tax burden based on various values of the elasticities considered by Harberger. These shares of the burden are calculated using Harberger's formula, $(-\eta_{r,t_{KX}} K/K_X)$:

Table 1

	$\eta_{r,t_{KX}}$ omitting the net income terms	Capital's share of burden omitting net inc. terms	$\tilde{\eta}_{r,t_{KX}}$	Capital's share of the burden
I. $\sigma_X = \sigma_Y = \sigma_D = -1$	-.58	116%	-.50	100%
II. $\sigma_X = \sigma_D = -1$; $\sigma_Y = -.5$	-.67	134%	-.58	116%
III. $\sigma_X = \sigma_Y = -1$; $\sigma_D = -.5$	-.64	128%	-.56	112%
IV. $\sigma_X = \sigma_Y = -1$; $\sigma_D = -1.5$	-.53	106%	-.44	88%
V. $\sigma_Y = \sigma_D = -1$; $\sigma_X = -1.2$	-.63	126%	-.55	110%
VI. $\sigma_Y = \sigma_D = -1$; $\sigma_X = -.8$	-.52	109%	-.44	88%
VII. $\sigma_Y = \sigma_X = -.66$; $\sigma_D = -1$	-.53	106%	-.44	88%

The most striking result of this comparison is that, for this set of data, omission of the net income term results in 16-20% over estimation of $\eta_{r,t_{KX}}$ (in absolute value) and capital's share of the burden. In fact, using different combinations of the elasticities appearing in the table, the overestimation can be much higher (e.g. when

$\sigma_X = -.66$ $\sigma_Y = -1$ and $\sigma_D = -1.5$ $\eta_{r,t_{KX}}$ is overestimated by 33%).

Another important consequence of the omission of the net income term has to do with the use of Harberger's model to discuss tax incidence when there are non-tax price distortions. In the previous sections we have shown that Harberger's equations, though valid at the point $t_{KY} = t_{KX} = t_X = 0$, do not capture all the variables which influence the incidence of partial taxes. However, if there are <u>any</u> price or factor return distortions Harberger's equations are not correct even at that point, for in such cases the net income term must enter those equations. Thus for economies with distortions such as monopolistic pricing, union wage differentials, or tariffs, the Harberger model cannot determine, say, $\eta_{r,t_{KX}}$ even when all taxes are zero.

To see this, and how our model might be applied to such cases, consider Harberger's analysis of tax incidence under monopoly mark-up pricing.[1] He wishes to determine the incidence of a partial factor tax on capital in X when K_X is paid a monopoly mark-up (M). He retains his demand equation (21) stating that only equation (22) above need be altered making it[2]

$$\hat{p}_X = \theta_{KX}[\hat{r} + \widehat{(1+t_{KX})}](1 + M).$$

However (21) is not the proper demand equation even though $t_{KX} = t_{KY} = t_X = 0$ and $m_p = m_G = m$. To see this we derive the total demand equation analogous to (17). Remembering that $dt_{KY} = dt_X = 0$ and that payments to capital in industry X are $r(1 + M)K_X$, one obtains from (9')-(14')

$$\hat{X} = \epsilon(\hat{p}_X - \hat{p}_Y) + m/X\{[(1 + M)K_X + K_Y]dr + MrdK_X + (1 + M)rK_X dt_{KX} - Xdp_X - Ydp_Y\}$$

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[1] Harberger, "The Corporation Income Tax," pp. 239-40.

[2] Except for this equation, throughout this paper $\theta_{KX} = \frac{f_K K_X}{X}$. Had we retained this definition in this instance also (1 + M) would be subsumed in θ_{KX}. As Harberger introduces (1 + M) in the form shown above we have followed that example.

Equation (24) now becomes

$$Ydp_Y + Xdp_X = [(1 + M)K_X + K_Y]dr + (1 + M)rK_X dt_{KX}.$$

Inserting this into the previous equation we obtain

$$\hat{X} = \varepsilon(\hat{p}_X - \hat{p}_Y) + m/X(MrdK_X)$$

Since $dK_X \neq 0$, the income effect is non-zero. Thus one cannot write the demand equation as in (21), and the net income term must enter the equation determining $\eta_{r,t_{KX}}$.[1] Therefore one cannot insert even pre-tax data into his solution for this case.

6

So far in this model we have discussed only balanced budget incidence. We now consider differential incidence showing its relation to balanced budget incidence and compare our analysis with that provided by Mieszkowski.[1]

There are, essentially, two ways of examining differential incidence. One is to determine the effects of substituting one tax for another while holding the government budget constant at some given level (in "real" or "money" terms). The other is to compare the effects of two taxes which provide the same yield to the government. To simplify the algebra we shall employ the former. This is not a serious limitation, however, since the same factors influence both analyses.[2] Also for

[1] Mieszkowski, "Theory of Tax Incidence," pp. 252-55.

[2] A word here may be necessary on the algebra of the second type of analysis. For such analysis one stipulates that the two taxes provide equal yield. To see how such a condition must be formulated algebraically, first consider the derivatives, (dI_G/dt_{KY}) and (dI_G/dt_X) evaluated at the existing equilibrium point. Obviously, these two derivatives cannot be arbitrarily set equal to each other. Thus the only way to determine two taxes which provide <u>exactly</u> equal yields would be as follows: if t^*_{KX} and t^*_X are initial taxed rates and ΔI_G is the desired increase in yield, then one could solve for the tax rates t^{**}_{KX} and t^{**}_X such that

$$\Delta I_G = \int_{t^*_{KX}}^{t^{**}_{KX}} (dI_G/dt_{KX})dt_{KX} = \int_{t^*_X}^{t^{**}_X} (dI_G/dt_X)dt_X.$$

The analyst can then proceed to compare the incidence of these tax changes. (See below, for a discussion of Mieszkowski's use of approximations to these integrals.)

Though the algebra for this type of analysis differs from what we use in the text, the reader will note that the same terms (i.e. dI_G/dt_{KX} and dI_G/dt_X) appear in both

simplicity we shall only consider the differential incidence of t_{KX} and t_X.

The first step is to determine the relationship between t_{KX} and t_X which leaves the government budget unchanged at a certain level. To specify that level we shall require that the government always be able to just purchase the bundle (X_G^*, Y_G^*).[1] This relationship then may be formulated as

$$r(t_{KX}K_X + t_{KY}K_Y) + p_X X t_X - X_G^* p_X(1 + t_X) - Y_G^* p_Y = 0.$$

Implicitly, differentiating

$$r(t_{KX}K_X + t_{KY}K_Y)\hat{r} + r(1 + t_{KX})K_X\widehat{(1 + t_{KX})} + r(1 + t_{KY})K_Y\widehat{(1 + t_{KY})}$$

$$+ rK_X(t_{KX} - t_{KY})\hat{K}_X + p_X(1 + t_X)(X - X_G^*)\widehat{(1 + t_X)} + p_X X t_X \hat{p}_X + p_X X t_X \hat{X}$$

$$- p_X X_G^* (1 + t_X)\hat{p}_X - p_Y Y_G^* \hat{p}_Y = 0$$

and substituting from (1')-(16') (or the elasticity equations supplied in the appendix) we obtain

$$\frac{(1 + t_X)}{(1 + t_{KX})} \frac{dt_{KX}}{dt_X} = \frac{\widehat{(1 + t_{KX})}}{\widehat{(1 + t_X)}}$$

$$= - \frac{[F + (p_X X/A)(C_1 + C_2)]\eta_{r,t_X} - [p_X(X - X_G^*)(1 + t_X)]}{[F + (p_X X/A)(C_1 + C_2)]\eta_{r,t_{KX}} + [F + rK_X - rt_{KY}K_Y + (p_X X/A)C_1]} \tag{30}$$

where $F = \theta_{LX} r(t_{KX}K_X + t_{KY}K_Y) + Y_G^* p_Y(\theta_{KX} - \theta_{KY})$. This equation shows the rate at which t_{KX} must be substituted for t_X to keep the government budget constant.

We now seek the elasticity of r with respect to changes in both t_{KX} and t_X as specified by (30). This will then indicate the differential incidence of these two taxes. Denoting this elasticity by $\eta^D_{r,t_X,t_{KX}}$ it may be obtained as follows: since

$$\hat{r} = \eta_{r,t_X}\widehat{(1 + t_X)} + \eta_{r,t_{KX}}\widehat{(1 + t_{KX})},$$

[1] By this we are not assuming that the government actually consumes (X_G^*, Y_G^*) but, only that it is just able to do so.

letting the numerator and denominator of the right hand side of (30) be α_1 and α_2 respectively, we can substitute and obtain

$$\eta^D_{r,t_X,t_{KX}} = \eta_{r,t_X} - \eta_{r,t_{KX}} (\alpha_1/\alpha_2). \tag{31}$$

Thus the differential incidence of two taxes depends on their balanced budget incidence and the rate at which they can be substituted so as to leave the government budget unchanged. We have already shown that η_{r,t_X} and $\eta_{r,t_{KX}}$ can be of any sign. One might expect that $(-\alpha_1/\alpha_2)$ would be negative, since in general to leave the budget unchanged one must increase one tax and decrease another. However, examination of (30) reveals that this ratio may be positive. Such a situation implies that increasing one tax actually decreases government revenues and thus to hold the budget constant another tax must also be increased.

For discussion of the differential incidence of finite tax changes, (30) and (31) are useful only insofar as they indicate the factors which influence such incidence. In this respect it is interesting to contrast the factors which enter our analysis with those which enter Mieszkowski's. Mieszkowski uses a variant of the second approach to differential incidence. Rather than performing integration of the form mentioned in footnote 1 above, he approximates the change in revenue resulting from the introduction of two arbitrarily small taxes and sets these two equal. In effect, he writes

$$\Delta I_G^{t_{KX}} \approx (dI_G)/(dt_{KX})\ \Delta t_{KX} = (dI_G/dt_X)\ \Delta t_X \approx \Delta I_G^{t_X}$$

where Δt_{KX} and Δt_X are arbitrarily small changes in the two taxes.[1] Evaluating (dI_G/dt_{KX}) and (dI_G/dt_X) at the zero tax point, as Mieszkowski does, one obtains

[1] We use here the Δ notation to distinguish arbitrarily small changes in a variable from the derivative used in our analysis which employs the symbol "d". The literature using the Harberger model uses "d" where we use Δ.

$$rK_X \Delta t_{KX} = p_X X \Delta t_X.$$

Thus he solves for the size of Δt_{KX} in terms of Δt_X

$$\Delta t_{KX} = \frac{p_X X}{rK_X} \Delta t_X.$$

He then compares the two equations

$$\frac{\Delta r}{r} \approx \eta_{r,t_X} \Delta t_X$$

and

$$\frac{\Delta r}{r} \approx \eta_{r,t_{KX}} \frac{p_X X}{rK_X} \Delta t_X$$

where η_{r,t_X} and $\eta_{r,t_{KX}}$ are evaluated at the zero tax point. Returning to our formulation and evaluating (α_1/α_2) at the zero tax point (31) becomes

$$\eta^D_{r,t_X,t_{KX}} = \eta_{r,t_X} - \eta_{r,t_{KX}} \frac{p_X X}{rK_X}$$

The reader will note the similarity between this and the algebraic difference of the two equations Mieszkowski compares.

More important than this similarity at the zero tax point is the difference between the general form of α_1/α_2 and what is essentially a simplified form of that provided by Mieszkowski (i.e. $p_X X/rK_X$). Thus, beyond ignoring the net income terms, Mieszkowski's equal yield conditions capture few of the variables which influence the yield of any tax.[1] Any results derived from so simplified an analysis, such as his ranking of taxes, can only be considered tenuous.

[1]Mieszkowski's equations are first order approximations. He suggests that those terms which do not appear in his equations are of second order (ibid., p. 254, fn. 4). This is quite misleading, for one should carefully distinguish between second order terms, which one might ignore, and first order terms, which drop only when all taxes are zero. For example, the government budget for any tax rates t_{KX} and t_X can be written as

$$r(t_{KX}K_X) + p_X X t_X$$

Using Taylor's expansion and retaining only the first order terms one gets

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In this paper we have argued that the standard general equilibrium analysis of tax incidence initiated by Arnold Harberger in 1962 is of limited usefulness. The pervasive impression given by the literature based on that model is either (1) that it captures all the variables which influence the incidence of finite partial taxes or (2) that although there are other important variables, no comparative statics analysis based on differential equations can capture them. Using a model identical to Harberger's, except that initial tax values are left arbitrary, we have shown both these impressions to be false.

Specifically, it was shown that, by assuming all taxes to be zero, Harberger's incidence analysis cannot take account of the effects of taxes on real income. The model presented here, on the other hand, captures such effects and thus explicitly indicates the link between the inefficiency (burden) of taxes and their incidence. Because of the omission of this link (the net income term) the analysis given in the literature based on the Harberger model has been incomplete and at times invalid. In this paper we have tried to indicate some of the modifications which must be made in the existing analysis for it to be relevant to the issues of taxation.

$$\Delta I_G = t_{KX}K_X\Delta r + t_{KX}r\Delta K_X + rK_X\Delta t_{KX} + p_X X\Delta t_X + p_X t_X\Delta X + t_X X\Delta p_X.$$

When taxes are zero, the only first order terms which appear are $rK_X\,\Delta t_X + p_X X\Delta t_X$. A similar confusion may be the source of the erroneous statements made by Johnson and Mieszkowski, and Johnson and Krauss quoted above, p. 13 and p. 14, fn. 1.

APPENDIX

In this appendix we prove some of the more technical assertions made in the text.[1] In particular we (i) show that the rate at which X and Y will be exchanged at a point, (dY/dX), differs depending on whether a partial factor tax or excise tax is being changed except when $t_{KX} = t_{KY}$, (ii) solve for the income effect induced by each tax, showing that they, in general, differ, and (iii) prove that Harberger's solution for each tax is valid if and only if the income effect of that tax is zero.

We will need the following elasticities, which may be obtained from (1')-(16'), remembering that $\eta_{r,t_{KX}} + \eta_{r,t_{KY}} = -1$.

$$\eta_{K_X,t_{KX}} = -\eta_{K_X,t_{KY}} = (\beta_1 + \beta_2)\eta_{r,t_{KX}} + \beta_2$$

$$\eta_{K_X,t_X} = (\beta_1 + \beta_2)\eta_{r,t_X}$$

$$\eta_{L_X,t_{KX}} = -\eta_{L_X,t_{KY}} = (\beta_1 + \beta_3)\eta_{r,t_{KX}} + \beta_3$$

$$\eta_{L_X,t_X} = (\beta_1 + \beta_3)\eta_{r,t_X}$$

$$\eta_{X,t_{KX}} = -\eta_{X,t_{KY}} = (\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)\eta_{r,t_{KX}} + (\theta_{LX}\beta_2 + \theta_{KX}\beta_3)$$

$$\eta_{X,t_X} = (\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)\eta_{r,t_X}$$

$$\eta_{Y,t_{KX}} = -\eta_{Y,t_{KY}} = -\{[\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y)]\beta_1 + \theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3\}\eta_{r,t_{KX}}$$
$$- [\theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3]$$

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[1] To simplify our proofs, throughout this appendix we assume $[(K_X/L_X)-(K_Y/L_Y)] \neq 0$, though one can extend the proofs to include this case.

$$\eta_{Y,t_X} = -\{[\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y)]\beta_1 + \theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3\}\eta_{r,t_X}$$

where

$$\beta_1 = \frac{K_Y\sigma_Y}{L_X[(K_X/L_X) - (K_Y/L_Y)]}$$

$$\beta_2 = \frac{K_Y\sigma_X}{L_Y[(K_X/L_X) - (K_Y/L_Y)]}$$

$$\beta_3 = \frac{K_X\sigma_X}{L_X\ [(K_X/L_X) - (K_Y/L_Y)]}\ .$$

From these we obtain

$$-(\frac{X}{Y})\frac{(\partial Y/\partial t_{KX})}{(\partial X/\partial t_{KX})} = -\frac{\eta_{Y,t_{KX}}}{\eta_{X,t_{KX}}} = -\frac{\eta_{Y,t_{KY}}}{\eta_{X,t_{KY}}} = -(\frac{X}{Y})\ \frac{(\partial Y/\partial t_{KY})}{(\partial X/\partial t_{KY})}$$

$$= \frac{\{[\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y)]\beta_1 + \theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3\}\eta_{r,t_{KX}}}{(\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)\eta_{r,t_{KX}} + (\theta_{KX}\beta_2 + \theta_{LX}\beta_3)}$$

$$+ \frac{[\theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3]}{(\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)\eta_{r,t_{KX}} + (\theta_{KX}\beta_2 + \theta_{LX}\beta_3)} \tag{A1}$$

and

$$-(\frac{X}{Y})\ \frac{(\partial Y/\partial t_X)}{(\partial X/\partial t_X)} = -\ \frac{\eta_{Y,t_X}}{\eta_{X,t_X}}$$

$$= \frac{[\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y)]\beta_1 + \theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3}{(\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)}. \tag{A2}$$

Clearly, in general, (A1) is not equal to (A2). However, if $t_{KX} = t_{KY}$, then they are equal. To see this, note that the numerator and the denominator of (A2) are the coefficients of $\eta_{r,t_{KX}}$ in the numerator and the denominator of (A1) respectively. Thus to show (A1) = (A2) we need to show that the ratio of the second term in the numerator of (A1) to the second term in the denominator of (A1) is equal to (A2). Similarly to prove this it is sufficient to show

$$\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y) = \frac{\theta_{KY}(K_X/K_Y)\beta_2 + \theta_{LY}(L_X/L_Y)\beta_3}{\theta_{KX}\beta_2 + \theta_{LX}\beta_3} .$$

Substituting for β_2 and β_3 this can be written as

$$\theta_{KY}(K_X/K_Y) + \theta_{LY}(L_X/L_Y) = \frac{1}{\theta_{KX}(K_Y/K_X) + \theta_{LX}(L_Y/L_X)} .$$

Opening the θ's we get

$$\frac{r(1 + t_{KY})K_X + wL_X}{p_Y Y} = \frac{p_X X}{r(1 + t_{KX})K_Y + wL_Y} .$$

But, clearly if $t_{KX} = t_{KY}$ this equality holds since both ratios then equal $(p_X X/p_Y Y)$. This proves that if $t_{KX} = t_{KY}$ then

$$\frac{\partial Y/\partial t_X}{\partial X/\partial t_X} = \frac{\partial Y/\partial t_{KX}}{\partial X/\partial t_{KX}} .$$

Furthermore, since (A2) is always negative (A1) will also be negative at this point; however, in general, (A1) may be of any sign.[1]

We now proceed to solve for the income effects induced by each tax. Denoting the income effect induced by t_{KX} as $(IE)_{t_{KX}}$ from (25) we have

$$(IE)_{t_{KX}} = (t_{KX} - t_{KY})r(\partial K_X/\partial t_{KX}) + p_X t_X(\partial X/\partial t_{KX})$$

$$= \frac{(t_{KX} - t_{KY})}{(1 + t_{KX})} rK_X \eta_{K_X, t_{KX}} + \frac{p_X t_X X}{(1+t_{KX})} \eta_{X, t_{KX}}$$

$$= (t_{KX} - t_{KY})rK_X(\beta_1 + \theta_{KX}\beta_2 + \theta_{LX}\beta_3)\ \eta_{r, t_{KX}} + \frac{(t_{KX} - t_{KY})rK_X\beta_2}{(1 + t_{KX})}$$

$$+ \frac{t_X p_X X}{(1+t_{KX})} (\theta_{KX}\beta_2 + \theta_{LX}\beta_3) .$$

It is interesting to note that this can be related to the net income terms mentioned

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[1] Cf. fn. 1, p. 14 above.

in the text. From the equation above we can write

$$(IE)_{t_{KX}} = \frac{p_X X}{A(1+t_{KX})} [(C_1 + C_2)\eta_{r,t_{KX}} + C_1]$$

where A, C_1 and C_2 are defined as in the text. Similarly,

$$(IE)_{t_{KY}} = \frac{p_X X}{A(1+t_{KY})} [(C_1 + C_2)\eta_{r,t_{KX}} + C_2]$$

and

$$(IE)_{t_X} = \frac{p_X X}{A(1+t_X)} [(C_1 + C_2)\eta_{r,t_X}] .$$

The reader may have noted that the elasticities with respect to $(1 + t_{KX})$ and $(1 + t_{KY})$, of all the variables we have considered, except r, have summed to zero. This is indicative of the proposition mentioned in the text, that these variables are invariant with respect to equal percentage increases of both $(1 + t_{KX})$ and $(1 + t_{KY})$. One would expect a similar result to hold for the income effects. Indeed it does, although it is difficult to show because the income effect is not the derivative of any one variable in the model. To get around this let us call the income effect the derivative of, say, W. Then

$$\eta_{W,t_{KX}} = (IE)_{t_{KX}} \frac{1 + t_{KX}}{W} = \frac{p_X X}{AW} [(C_1 + C_2)\eta_{r,t_{KX}} + C_1]$$

and

$$\eta_{W,t_{KY}} = (IE)_{t_{KY}} \frac{1 + t_{KY}}{W} = [(C_1 + C_2)\eta_{r,t_{KX}} + C_1].$$

Thus $\eta_{W,t_{KX}} = -\eta_{W,t_{KY}}$.

Examination of the three income effects reveals that the income effect of an excise tax is in general different from that of a partial factor tax.

The final proposition to be considered is that the form of the incidence equation for a given tax, derived from the Harberger model, is valid if and only if the income effect induced by that tax is zero. We shall offer a proof only for the incidence of t_{KX} and t_X, since the proof for t_{KY} is similar to that for t_{KX}.

The correct equation for η_{r,t_X}, assuming $m_p = m_G \neq 0$ is[1]

$$\eta_{r,t_X} = \frac{\epsilon A}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y + m_G p_X (C_1 + C_2)} .$$

The form of Harberger's solution on the other hand is

$$\frac{\epsilon A}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y} .$$

These two will be equal at a point if and only if at that point either $\epsilon A = 0$ or $(C_1 + C_2) = 0$. But examination of $(IE)_{t_X}$ reveals that it also is zero if and only if either $\epsilon A = 0$ or $(C_1 + C_2) = 0$. Hence η_{r,t_X} is independent of the net income terms if and only if $(IE)_{t_X} = 0$.

Now considering $\eta_{r,t_{KX}}$, the correct equation is

$$\eta_{r,t_{KX}} = \frac{\epsilon\theta_{KX}A + \sigma_X B - m_G p_X C_1}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y + m_G p_X (C_1 + C_2)}$$

while Harberger supplies the equation

$$\frac{\epsilon\theta_{KX}A + \sigma_X B}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y} .$$

These two will be equal if and only if either $C_1 = C_2 = 0$ or

$$-\frac{C_1}{(C_1 + C_2)} = \frac{\epsilon\theta_{KX}A + \sigma_X B}{\epsilon(\theta_{KY} - \theta_{KX})A - \sigma_X B - \sigma_Y} . \tag{A3}$$

But $(IE)_{t_{KX}} = 0$ if and only if either $C_1 = C_2 = 0$ or

$$\eta_{r,t_{KX}} = -\frac{C_1}{C_1 + C_2} . \tag{A4}$$

However (A4) is true if and only if (A3) is true. Thus $\eta_{r,t_{KX}}$ is independent of the net income terms if and only if $(IE)_{t_{KX}} = 0$.

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[1] We ignore the case $m_p = m_G = 0$, in which case the following two equations are identical, independent of the income effect.

Note that the income effect being zero does not imply that the net income terms are zero but only that they do not influence tax incidence.

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