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by

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ECONOMIES OF SCALE AND INVESTMENT OVER TIME

Hollis Chenery and Larry Westphal

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ECONOMIES OF SCALE AND INVESTMENT OVER TIME[1/]

The allocation and coordination of investment is a central feature of development policy. Half the total investment activity in underdeveloped countries is typically carried out by the government itself; much of the remainder is influenced through licensing, price policies and import controls. When development policies are successful, large changes can be made in the economic structure and the stock of capital doubled in ten or fifteen years.

Problems of investment policy have usually been discussed in terms appropriate to a mature economy in which the growth of the capital stock is slower and economies of scale are relatively unimportant. The framework of partial analysis applied to individual projects that is justifiable under these circumstances breaks down when there are significant economies of scale in related sectors of the economy. A more comprehensive economic model is then needed to take account of the multiple limitations on both the composition of output and the supply of factors. Since new investment may produce relatively large changes in the economic structure over a period of a few years, partial analysis based on existing prices can be quite misleading. Not only do marginal rules of allocation break down, but even the possibility of calculating numerical solutions to problems of any complexity is only now being developed.

1/ We are indebted to David Kendrick, Stephen Marglin and Paul Roberts for helpful suggestions on the formulation of the problem and computational possibilities. Andrew Szasz assisted in the computations. Our research has been supported by the Project for Quantitative Research in Economic Development, Center for International Affairs, Harvard University.

In the absence of a general theory of investment allocation for developing countries, a variety of special theories and intuitive suggestions have been put forward. Among the best known are the Rosenstein-Rodan theory of the Big Push /18, 19/, the Nurkse-Lewis theories of Balanced Growth /12, 15, 17/, Leibenstein's concept of Critical Minimum Effort /11/, Hirschman's theory of Unbalanced Growth /9/, and the attempts of authors such as Fleming /7/, Lipton /13/, Scitovsky /20/, Streeten /22/, and Sutcliffe /23/ to reconcile some of the conflicting conclusions. Among the central issues in this debate are:

(i) the effects of limited export possibilities and economies of scale on the optimal pattern of investment;

(ii) the relations between investments in overhead facilities and in commodity production;

(iii) the importance of utilizing external resources to secure an initial spurt in investment and growth.

The discussion has served to clarify the assumptions underlying the several approaches, but it has not yet provided an adequate basis for empirical analysis.

These questions cannot be answered without a more precise description of the economic structure and specification of policy objectives. We therefore propose to reformulate several earlier approaches in such a way that formal optimizing procedures can be applied to the resulting model. A series of experiments will then be undertaken to determine the

optimal investment patterns that are characteristic of situations involving economies of scale in related sectors.

I. FORMULATION OF THE PROBLEM

In partial analysis, the effects of economies of scale in one sector on investment in related sectors are traditionally discussed under the heading of "external economies."[1/] However, determination of the optimal choice of investments in interrelated sectors involves a simultaneous allocation among them. This requires a more comprehensive formulation in which welfare is maximized (or cost minimized) subject to constraints on demand and factor use. In general equilibrium analysis, it is not necessary (and often not possible) to allocate welfare gains or cost savings to individual investment decisions. For this reason, the concept of "external economies" has never acquired much operational value.

With allowance for economies of scale, our formulation of the problem of investment choice will follow the modified input-output approach to development programming that is exemplified in the models of Chenery and Kretschmer /3/, Manne /15/, Bruno /1/, and Eckaus and Parikh /6/.[2/] These programming models determine the optimal pattern of investment when the composition of domestic final demand is given and the main choices are between domestic

---

[1/] The clearest formulation of dynamic externalities is that of Scitovsky /20/.

[2/] The principal differences among these analyses are in the degree of disaggregation, the treatment of investment and other resource limitations, and the number of time periods considered.

production and imports. The problem of vertical and horizontal interdependence is thus posed in an empirical context and simple form -- i.e., with both input coefficients and the composition of demand fixed.

The effects of economies of scale in a single sector on the pattern of investment over time have been analysed in a partial equilibrium framework by Chenery /2/ and Manne /14/. The growth of demand and the supply cost of inputs are taken as given, the optimum plant size and the intervals between investments are determined to be a function of the scale economies, the rate of growth of demand, the cost of imports, and the interest rate.

Our approach incorporates elements of this single - sector analysis into a simplified dynamic interindustry programming model. The model has the following structural characteristics:

(i) linear production activities with economies of scale in the use of capital and constant coefficients for other inputs;

(ii) import and export activities for traded commodities;

(iii) two scarce factors -- domestic investment resources and imported goods;

(iv) domestic consumption of each commodity as a function of income;

(v) limited access to foreign loans;

(vi) a welfare function which depends on consumption over time, terminal capacity, and the amount of terminal debt.

These structural elemebts have been chosen to include most of the determinants of the investment patterns described in the literature. The principal patterns previously discussed may be identified as:

(i) Rodan -- Nurkse (Balanced) Growth, which is characterized by simultaneous investment in many sectors and a large capital inflow (big push) in early periods;

(ii) Scitovsky -- Streeten (Unbalanced) Growth, in which there is an alternation of investment among sectors with imports filling the gaps between supply and demand.[1/]

(iii) Specialization according to existing comparative advantage, normally in the export of primary products and import of manufactured goods.

Since the arguments for balanced growth are based on the limited possibilities of expanding exports, we have found it useful to define the problem as that of jointly allocating investible resources and foreign exchange over time. The lumpiness of efficient investments makes it necessary to borrow and invest irregularly, so that the balance of payments becomes as important in determining the optimal investment pattern as the balance of savings and investment.

There are several tradeoffs to be considered in a framework of significant scale economies and two scarce factors:

[1/] We refer to Streeten's analysis of the effects of economies of scale /22, pp. 176-177/, in which he cites an example taken from Scitovsky. Hirschman's concept of unbalanced growth /9/ is based on psychological reactions that cannot readily be included in this type of analysis.

(i) the cost of borrowing vs. the gains from exploiting economies of scale;

(ii) the cost of deferring investment vs. the gains from larger plants;

(iii) the loss of present consumption vs. the creation of greater future capacity;

(iv) the loss of current commodity output vs. the gain from exploiting scale economies in overhead (non-importable) services.

Optimal patterns of investment involve balancing all of these factors. They rarely produce the extreme solutions implied by more partial analyses.

II. THE MODEL

Previous studies of Chenery /4/ and Haldi /8/ have used simple interindustry models to explore some of the effects of economies of scale on investment patterns. These attempts were severely limited by the lack of an efficient method of solving programming problems including economies of scale short of enumerating all the feasible solutions. The recent development of integer programming algorithms makes it feasible to work with models containing a greater number of activities characterized by economies of scale. To exploit these possibilities, we have designed a four-sector model containing scale economies in two industries which is solved for up to ten time periods. After an extensive process of trial and error, we have developed a computable model which includes most of the significant features assumed in previous studies

A. Activities in the Model

The first cycle of investment and production activity and the parameter values for the basic model are represented in activity analysis form in Table I. The magnitudes of the most important parameters are intended to be realistic although the degree of aggregation makes the results of illustrative value only. Table A-I of the Appendix gives a list of the symbols used in the paper.

Production and trade activity and consumption levels in period zero (the initial period) are derived from specified initial capital endowments under the assumption of no excess capacity. From these activity levels the initial endowments of savings and foreign exchange available for investment are determined.[1/] Consequently, the investment variables of period zero are the first set of activities in the dynamic model.

The four production sectors in the model are intended to represent the manufacturing complex and the export sector of a dual economy. The four sectors include finished, intermediate, and primary goods-producing industries (sectors one through three respectively) as well as a sector providing overhead facilities (sector four). With the exception of indirect investment activity, the rest of the economy -- i.e., handicraft industry and traditional primary production not included in sector three -- is excluded from the model.[2/]

1/ Table II below illustrates these calculations.

2/ As an alternative interpretation, the four sectors in the model might be understood to represent a highly integrated set of sectors within an economy. One such set is the complex described by Chenery /4/, which includes electric power, coal and iron ore mining, steel and finished metal products.

TABLE 1

Tableau of Basic Activity Set

Activities / Constraints	Equation Number	T = 0 Investment I_1^o	F_2^o	I_2^o	I_3^o	F_4^o	I_4^o	AI_1^o	AI_2^o	Transfer of Primary Inputs D^o	UFE^o	MS^o	T = 1 Production X_1'	X_2'	X_3'	X_4'	Trade M_1'	M_2'	E_3'	CN CN'	RHS
T = 0																					
Primary Resources																					
FE	1	-.20	-19.0	-.15	-.60	-57.6	-1.28	-.10	-.05	1.0	-1.0	-1.0									≥ -40.0
DS	2	-.53	-42.0	-.45	-7.00	-108.0	-2.40	-.27	-.15	1.0		.8									≥ -82.3
T = 1																					
Capacity																					
1. Finished	4-1	1.0											-1.0								≥ -50.0
2. Intermed.	4-2			1.0										-1.0							≥ -75.0
3. Primary	4-3				1.0										-1.0						≥ -160.0
4. Overhead	4-4						1.0									-1.0					≥ -126.7
Auxiliary (1)	5-1							1.0					-1.0								≥ -50.0
Auxiliary (2)	5-2								1.0					-1.0							≥ -75.0
Production																					
1. Finished	6												1.0				1.0			-2.4	= 90.0
2. Intermed.	7												-.40	1.0				1.0		-1.7	= 75.0
3. Primary	8												-.1	-.48	1.0				-1.0	-.2	= 11.5
4. Overhead	9												-.10	-.21	-.35	1.0				-1.0	= 50.0
Primary Resources																					
FE	12									-1.07	1.0		-.04	-.06			-1.0	-1.0	1.0		≥ *
DS	13									-1.07			.2	.2	.2	.2					≥ **
Fixed Charge Constraints			+1.0																		= 1 or 0
			200.0	-1.0																	≥ 0
						+ 1.0															= 1 or 0
						200.0	-1.0														≥ 0
Ø																					

Notes: Equation numbers refer to formal statement of model in appendix. For convenience the following constraints are not shown:

3: Debt limit

10: Production cannot decrease in Sector 3

11: Consumption Growth Constraints

$$* = \sum_j f_{1j} F_j' + \sum_{i=1}^{4} f_{2i} I_i' + \sum_{i=1}^{2} f_{3i} AI_i' - B' + UFE' + MS'$$

$$** = \sum_j u_{1j} F_j' + \sum_{i=1}^{4} u_{2i} I_i' + \sum_{i=1}^{2} u_{3i} AI_i' - B' - .8MS'$$

So that a positive entry in the matrix would denote provision and a negative entry use, all equations have been multiplied by minus one

However, the production of finished and intermediate goods requires inputs from sectors outside the model. The "auxiliary investment" (AI) activities show the cost of the increased capacity necessary to provide the other inputs required for production in sectors one and two.[1/]

The intersection of the production and trade activities with the production and primary resource constraints of period one in Table I gives the standard input-output portrayal of economic activity. Due to our hypothetical economy's stage of development, it is assumed that finished and intermediate goods cannot be exported profitably and primary products are not imported. The output of the overhead sector is not traded. Production in sectors one and two also requires non-competitive imports as intermediate inputs, as shown in the "FE" row for period one.

Domestic savings (DS) are generated by production in each sector, a formulation that avoids the necessity of measuring total income separately. We assume that the traditional sector of the economy does not supply net savings and that public and private saving are rigidly linked to production, as specified by the entries in the "DS" row in the production columns (X_1 through X_4).[2/]

[1/] The importance of "auxiliary investment" in this model is the role it plays in matching the pattern of investment to the supply of investible resources over time. Over-investment in a sector to realize economies of scale need not be accompanied by over-investment to provide the external inputs.

[2/] This formulation allows for differences in taxes and savings rates among sectors, but we have not utilized this possibility in our numerical estimates.

The consumption activity (CN), which enters the objective function, assumes rigid complementarity of demand between the four goods.[1] The consumption activity is stated in terms of the growth of consumption above its initial level.

We turn now to the investment variables of period zero. A one-period gestation lag from investment to usable capacity is assumed for each sector, so that investments in period zero come on line in period one (see the capacity constraints). Economies of scale enter the model in the investment cost functions of sectors two and four (activities F_2, I_2, and F_4, I_4). These cost functions are characterized by a "fixed charge" (F_i) incurred if investment takes place and a "variable charge" (I_i) which depends on the level of investment. The resulting cost function exhibits constant marginal cost and declining average cost.[2] The fixed charge constraints assure that the fixed charge will be incurred if capacity is built through the operation of the variable charge activity.

Investment in sectors one and three and auxiliary investment take place at constant average cost. Investment in capacity in any sector requires two factors, foreign exchange (FE) and domestic savings (DS). The domestic savings charge associated with an investment activity gives the total resource cost of investment, while the foreign exchange charge expresses its import component separately. The productive mechanism whereby savings are

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[1] Lack of substitution in final consumption is the source of "horizontal interdependence" assumed by Nurkse and other balanced growth theorists.

[2] The two cost functions are shown in figure 4 below.

translated into capacity through investment is not detailed in the model.

The block of activities titled "Transfer of Primary Inputs" provides for the transfer of "FE" and "DS" both between periods and within the period. In this model constraints on the transfer of investible resources over time assume vital importance because they limit the extent to which scale economies can be realized through the concentration of investment in one period. Without activities providing for transfer over time of investible resources, investment in a given period would be limited to the investible resources generated in that period. In the real world, such activities include changes in foreign exchange reserves and stocks of commodities, external borrowing, and changes in the proportions of income that is saved. We include two such activities, borrowing and changes in reserves. The debt activity (D), in accord with the conceptual framework of national income accounting, yields a unit of both foreign exchange and domestic savings at the cost of repaying both resources in a later period. The UFE activity (representing reserve changes) transfers unused foreign exchange from one period to the next. We neglect stockpiling of commodities, so unused domestic savings cannot be similarly transferred without being combined with imports as completed investment projects.

A third transfer activity (MS) converts foreign exchange into domestic resources for investment. The increase in savings from this activity results from reducing accumulated foreign exchange reserves. The additional resources imported may be thought of as investment goods --

such as cement -- that are normally produced domestically. The latter aspect has led us to assume that 1 unit of foreign exchange can replace only 0.8 units of domestic savings (DS).

The full model consists of ten cycles of activity like that shown in Table I; it begins with investment in period zero and ends with production, trade and consumption in period ten. In some experiments the model has also been computed for shorter periods, which only necessitates a change in the valuation of terminal capacity.

B. Constraints on the System.

This section gives a brief verbal description of the principal constraints on the system. The Appendix contains a formal algebraic statement of the optimization model.

In the first period the total use of foreign exchange on investment account and the amount of savings to be invested are limited to the initial endowment of investible resources plus borrowing. Investment in each succeeding period is similarly constrained by the production, trade and the transfer activities of that period. The debt limit is formulated in such a way that the borrowing of the previous period must be repaid along with the interest charge, either through renewed borrowing or from savings and foreign exchange generated by current production.

The availability of foreign borrowing to a less developed country is one of the central limitations on the system. Criticisms of the balanced growth theories have stressed its importance in the real world. We show this limit in the form of a ceiling on the allowable debt of each period.

The debt limit is varied parametrically to show its effect on the optimal pattern of investment during the plan period.

Since labor is a free good in the model, production is limited only by available capacity. In sectors one and two production is further limited by the auxiliary capacity needed in sectors outside the model to provide intermediate inputs. Since resources devoted to primary production are not readily transferable to other sectors, we require that the production level in sector three not fall. The consumption of each product, both on final and intermediate account, is equal to production plus imports (less exports) in each period.[1/]

The final constraint is a means of incorporating the non-linear element of diminishing marginal utility in the objective function. Consumption growth (i.e., the level of the CN activity) is constrained to be at least two units in each period. Without this constraint, optimization often leads to a concentration of consumption growth in a short period.

C. The Welfare Function and Terminal Conditions.[2/]

Since the operation of the model terminates with production and consumption in the final period, it is necessary to insure sufficient foreign exchange and savings for post-plan growth. The economy is required to provide these resources for post-plan investment in amounts at least equal to their initial endowments.

1/ Equality is imposed on two accounts: first, to preclude the simultaneous production and disposal of a product to obtain the savings thereby created (excess capacity in sector four would give rise to this kind of behavior if product disposal were allowed); and second, to avoid problems associated with the valuation of utilized and excess capacity.

2/ The Appendix gives a more detailed discussion of the terminal conditions and their derivation.

Value is given in the objective function (in terms of the contribution to post-terminal consumption) to the "excess" foreign exchange and domestic savings passed into the post-terminal future.

External debt is allowed in the terminal year, but its level cannot be increased above that of the previous year. The cost of terminal debt in the objective function is the opportunity cost in terms of consumption of the foreign exchange and domestic savings required for repayment. (The optimal solutions to the model as initially specified always involve repayment of debt.)

Since over-investment and excess capacity are expected as a result of scale economies, it is necessary to value terminal excess capacity in the objective function. Excess capacity in the terminal period reduces the necessity for investment in that sector in the immediate future. Seen in this light, the value of excess capacity in the terminal period is the opportunity cost of investible resources in periods beyond the plan horizon needed to supply the expected demands in these sectors. Transferring excess capacity to post-plan periods is thus similar to providing foreign exchange and domestic savings for post-plan investment. Since capacity is specific to a given sector, some type of optimization must be utilized to determine the opportunity cost of the future foreign exchange and savings whose use is released by virtue of terminal excess capacity. The procedure adopted is described in the appendix. It should be noted that terminal excess capacity can not be restricted *a priori* to those sectors in which scale economies are present.

Having assessed the value of excess capacity in the terminal period, it is also necessary to value capacity utilized in production in that period. The composite of terminal capacity in each sector necessary to sustain a unit of consumption is therefore given the value of one unit of consumption sustained through an infinite time horizon (ignoring depreciation). Consumption growth in each period is similarly given a value in the objective function equal to the value of a stream of consumption beginning in that period and continued through the terminal period.

In summary, the function to be maximized is the sum of four elements: (1) the discounted value of consumption during the plan period; (2) the value of capacity utilized in the terminal period which was not present at the beginning of the plan period; (3) the value of unused capacity in each sector; (4) the cost of terminal debt. The activities in the welfare function are all valued in terms of composite units of consumption over an infinite time horizon.

D. Solutions to the Model

In solving the model we have used the Land and Doig algorithm /10/. This algorithm is a sophisticated, if time consuming, means of searching among the feasible solutions for the highest value of the objective functions. In this context a feasible solution results from a process of sub-optimization. Each integer variable (the F_i^T) is set at either zero or one and the optimal values all other activities are

determined by linear programming. In principle there are 2^n such feasible solutions (where "n" is the number of integer variables), each of which is optimal for the pattern of fixed charges incurred. The optimal solutions for several model specifications are given in Appendix tables A-III, A-IV and A-V.

The prices associated with the feasible solutions are similar to shadow prices of linear programming in that they represent the marginal value products of the resources, given the "basis" and the pattern of fixed charges.[1/] However, the prices cannot be used to judge the profitability of a scale economy activity not already in the model. For such activities the model must be solved again whenever a new integer variable is introduced. The question of whether or not to invest in a sector with scale economies can thus be determined only by comparing the welfare value of the solutions in which that investment does and does not take place. It is only valid to use prices to test the profitability of other activities for a given pattern of investment in scale economy sectors.

In the process of obtaining the model in its present form we worked with similarly specified models which were run for three, four and seven periods. To solve directly the ten period model with its twenty integer variables for the optimal solution would have required days of computer time. Therefore we relied on knowledge gained from these earlier runs to select solutions for the ten period model.

[1/] Two differences should be noted: (i) unlike linear programming, prices associated with one set of activity levels may also be associated with a different set of activity levels (hence the problem with using prices as decision guides in scale-economy models); (ii) the minimum value of the dual associated with any integer solution will exceed the maximum value of the primal

The following procedure was used. A four period model having the same specifications as the ten period model (except that terminal valuations differ) was solved for the optimal pattern of fixed charges.[1/] Next a seven period model with the integer variables of the first four periods set at the values of the optimal four period solution was solved for the optimal pattern of fixed charges in the last three periods. This process was then repeated in going on to the ten period solution.

The best solution to the ten period model obtained in this iterative fashion is not known to be the optimal solution (with respect to the pattern of fixed charges incurred). However, a number of alternative solutions to the ten period model were computed using knowledge gained from the shorter planning period models to set the integer variables at different values.[2/] Since little improvement was achieved, we are confident that our "best" solutions are at least close to optimal.

[1/] This process required from three to four hours of time on the IBM 7094 computer for each solution.

[2/] The investment patterns and welfare values of 41 experimental solutions are summarized in table IV below.

III. INVESTMENT PATTERNS

In order to derive some broad conclusions from these experiments in optimization it is first necessary to identify some general properties of the solutions. Since a complete description of a ten-period solution requires several hundred variables, we have sought key elements that determine the dominant features of the pattern of resource allocation. For most purposes a solution is adequately described by the yearly amounts of capacity, production and imports in each sector. The pattern of investment is dominated by the size and timing of capacity increases in the scale-economy sectors.[1/] The simplest characterization of an investment pattern, therefore, is by the number and size of plants built in sectors 2 and 4 during the ten year period.

Our experiments with the ten-period model lead to the classification of investment patterns shown in Table II. The original set of parameter specifications is identified as Model I. Its optimal solution involves the construction of one plant in sector 2 (steel) and two in sector 4 (overhead facilities) during the ten year period. This investment pattern is identified as B2 for Model I, or IB2. A number of alternatives must be examined for each such pattern to determine the optimal time for construction of each plant.

[1/] Our cost function implies that capacity is increased only by building new plants.

The characteristics of the optimal solution for each model can be brought out by comparison to a number of the more promising alternatives. Two alternative specifications of the model have been made in which the amount of external resources available was increased to make possible larger investments. A complete listing of the solutions is given in Table IV. They cover eight of the nine possible patterns indicated in Table II. [1/]

A. The Basic Pattern

The features typical of an optimal investment pattern will be shown by considering the best solutions to Models I and II. Their assumptions differ only in the increase in external debt that is allowed during the plan period, which is 75 units for Model I and 150 units for Model II. Model II is taken as representative of developing countries in the past decade, in which the net deficit on current account has typically amounted to about a quarter of gross investment.[2/] The increase in borrowing of 150 units in Model II finances about a quarter of investment in the first five years of the optimal solution. Model III (discussed below) is designed to show the effects of an equal increase in borrowing in the second five years.

[1/] These were selected after extensive experimentation with the three-period version of the model.

[2/] This is the median value found by Chenery and Strout /5 , p. 684/ for a sample of 31 underdeveloped countries.

TABLE II.

Investment Patterns in the Ten Year Model[1/]

Number of Steel Plants (Sector 2)	Number of Overhead Investments (Sector 4) 1	2	3	Total Solutions
0 (A)	A1 (0)	A2 (2)	A3 (1)	(3)
1 (B)	B1 (2)	B2 (15)	B3 (10)	(27)
2 (C)	C1 (2)	C2 (6)	C3 (3)	(11)
Total solutions	(4)	(23)	(14)	(41)
Model I	0	4	11	15
Model II	3	7	2	12
Model III	1	12	1	14

[1/] Based on Table IV. Figures in parentheses show the number of solutions corresponding to each pattern in the first series of experiments. The three Models are distinguished as follows:

Model I: Debt limit 75

Model II: Debt limit 150

Model III: Debt limit 150 for years 1-5, 300 thereafter.

The principal features of the optimal solutions to Models I and II are shown in figures 1 and 2.[1/] Part "a" of each figure brings out the relations between increases in capacity, production, imports and exports in each of the four sectors. Because capacity is fully utilized in period 0, the cycle starts with investment in overhead facilities (symbolized by electric power), which are necessary for increased production in all other sectors. A steel plant is built in period 2 and a second power plant in period 6. These three plants account for about 40% of total investment in this segment of the economy.

To adjust to these lumpy investments, borrowing is increased in periods 0, 2 and 6, and investment in other sectors is curtailed. In fact the low debt ceiling assumed in Model I causes other investment to be virtually eliminated in these three periods, as shown in figure 1C. The investment and borrowing patterns are dominated by the timing of the overhead investments, which take up 30% of the investment resources and are concentrated in two periods. Scale economies make it optimal to build plants large enough to take care of the growth of demand for the following five years even when it is necessary to defer almost all other investment in order to do so.

1/ The values of the main variables in the solutions are given in Appendix B.

Investment in intermediate products (which we symbolize by steel) takes place only once; the increase in capacity in both models is equal to about twelve years growth of total demand. As Manne has shown /14/, the availability of imports makes it efficient to postpone construction in order to build a larger plant with lower unit costs.

The inclusion of vertical and horizontal interdependence in our model brings out a further feature not previously noted: it is efficient in Model I (and to a lesser extent in Model II) to reduce steel production in periods 5 and 6 in order to postpone the construction of added power capacity so that a more efficient plant can be built in period 6. In this way, the importation of steel indirectly postpones the requirement to expand power production and makes possible a more economical plant. The expansion of primary production and exports during these years provides a more economical alternative to increasing power and steel production, even though long run comparative advantage favors steel when a larger plant can be built.

As a further reflection of the importance of realizing greater economies of scale, consumption increases are held to a minimum until after the second power plant has been built. In period 7 production in all sectors expands, final consumption rises more rapidly, and primary production is diverted from exports to the production of intermediate products and finished goods. Once the bottleneck has been removed, there is a rapid increase in power use.

Because of the evident effects of the power shortage in both models, it might be supposed that advancing the construction of the second power plant by a year or two

might be advantageous. This turns out not to be true, as shown in Table IV below, because the smaller plant that would have to be built would hamper growth and raise costs in later years.

The general pattern of investment revealed by these solutions is much closer to the Scitovsky-Streeten concept of alternating investments in different sectors than it is to any of the versions of balanced growth. Exports are essential to this pattern. In the short run it is efficient to increase investment in primary production even though long run comparative advantage favors import substitution in the two manufacturing sectors. The latter factor is outweighed by the increased flexibility made possible by imports during the transitional period when capital is relatively scarce.

The effects of alternating scale economy investments on the financing of total investment and imports are shown in the last three figures for each model. Irregular investment requires the repayment of debt (and building up of exchange reserves) in order to finance the bulge in investment and imports of investment goods in period 6 by again increasing debt and reducing reserves.

The need to balance investment over time also leads to investment in finished goods (in Model I) in advance of the increase in demand, even though there are no economies of scale in this sector. Without this further adaptation to the lumpiness of investment in other sectors, the increase in total consumption would have to be deferred even longer.

B. Effects of Varying External Resources

It is apparent from the preceding discussion that external resources are exceedingly important to the realization of economies of scale because they make possible periodic bulges in total investment considerably in excess of domestic savings. In the first two models, the cost assigned to external debt in the terminal year caused it to be reduced to zero at the end of the period. The marked increase in capacity in all sectors between Models I and II is therefore due entirely to temporary borrowing, which makes it possible to construct larger and more efficient plants.

To explore this phenomenon in greater detail, we have assumed in Model III a further doubling in the availability of external resources. To avoid an unrealistic bulge in investment in the first period, however, the debt limit is raised from 150 to 300 only in period 6. The cost of terminal debt has also been lowered so that it is no longer advantageous to repay debt within the ten-year period.

The optimal investment and production pattern for Model III is shown in figure 3a. It has the same timing of investment as Models I and II, but there is now a more substantial "big push" in period 7 after the debt limit is raised.[1/] The power plant completed in this period is 50% larger than that in Model II and at the same time there is a big jump in finished goods capacity. These increases permit final consumption to increase much more

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[1/] A more gradual rise in the debt limit would not greatly affect the result, since this investment pattern is optimal even without the further increase in external capital.

rapidly than before and the excess steel and power capacity are fully used by the end of the period.

The feasibility of this investment pattern requires that the future spurt in demand for steel be anticipated at the beginning of the period when the steel plant is built. Although all conditions in Models II and III are identical for the first five years, the steel plant constructed in the first five year plan under Model III must be 30 percent larger to satisfy the future demand for steel five or six years later from plants that will be built only when final demand increases.

The direct and indirect effects of increasing the availability of foreign resources on total welfare are summed up in figure 4 and table III. The direct effect is the cost saving resulting from constructing larger plants, which is shown in figure 4. The total reduction in investment cost for Models II and III (compared with the unit costs of Model I) is shown in table III, line 7. The indirect effects of external borrowing are even greater, however. The more rapid growth that is produced by more efficient allocation of resources as well as lower-cost capacity results in a substantial increase in total savings. Table III shows that for Model II, where there is no increase in total debt compared to Model I, these indirect effects account for over 60% of the substantial welfare gain (21%) that results from increasing the debt limit.

The increase in external borrowing in Model III produces a gain in welfare of 60% above Model I. Cost savings from larger plants and increased savings from more rapid growth again account for the bulk of the total. Comparing Model II and Model III,

TABLE III

Effects of Mineral Resources[1]

	Model I	Model II	Change from I	Model III	Change from I
1. Terminal Consumption (CN_9)	28	40	+12	54	+26
2. Total Value (W)	3436	4165	+729	5538	+2102
3. Total Investment (ΣI)	1237	1398	+161	1758	+521
4. Capital Inflow (ΣF)	-27	-66	-39	200	+227
5. Total Savings (3-4)	1264	1464	+200	1558	+294
6. Average Productivity of Investment (2 ÷ 3)	2.78	2.98		3.15	
Sources of Improvement over Model I					
7. Reduction in Investment Cost[2]		102		185	
8. Increased Savings		200		294	
9. Increased Capital Inflow		-39		227	
0. Total (7+8+9)		263		706	
1. Productivity[(ΔW ÷ (7+8+9)]		2.77		2.98	

[1] Derived from optimal solutions to each model as given in Tables A-III, A-IV, and A-V below.

[2] Difference between actual investment cost and cost of building the same size plants at the average unit costs of Model I.

it can be seen that a net increase in capital inflow amounting to 15% of total investment is highly productive because of these indirect effects. Further increases would be less productive because the size of investment in Model III exploits most of the available economies of scale.

C. Deviations from the Optimal Pattern

Since perfect coordination of investment cannot be achieved in either a planned or a market economy, it is important to determine the effects on total welfare of departures from optimality in the size and timing of investments. As these costs have been investigated by Manne and others /14/ in a partial equilibrium context, we will concentrate on the adjustments that are necessary in the pattern as a whole. A detailed examination of one alternative solution illustrates the adjustments that can be made and their net cost.

The optimal pattern for all three models requires sufficient foresight to accept periodic power shortages in order to build larger plants. A plausible "real world" alternative would be to keep up with demand at the cost of building smaller and more expensive plants. Such an investment policy is illustrated in figure 5., in which the investment cycle in sector 4 is cut from 6 to 3-4 years and 3 plants are built instead of 2. Optimization for Model I under these restrictions gives the investment and production patterns of figure 5. Keeping up with the demand for power leads to higher steel and power production over the period and greater power capacity at the end. The cost of this policy is reflected in smaller power plants that cost 15% more per unit

of capacity and hence reduce the investment available for sectors 1 and 3. The welfare cost to the economy of this single change in investment policy is 3% of the total value of Model I, as shown in table IV.[1/] This pattern requires less advance planning of investment, however, and is less dependent on steel imports.

Table IV gives a number of other examples of the cost of changes in the investment pattern or of departures from optimal timing. Timing of investment is more critical in Model I because limited external resources reduce the possibilities for adjustments in other sectors. The investment patterns that are most closely competitive to the optimal (B2) are B3 for Model I and C2 for Model III. Continued specialization on primary exports instead of import substitution in steel is quite costly in all models under the given assumptions, although the construction of the steel plant can be postponed 2 or 3 years without great loss (as shown by solution 30).

Apart from the ten-period solutions given in table IV, we have calculated the effects of a number of different specifications of the parameters in the model for shorter periods. Among the changes tested were higher discount rates, greater returns from exporting, and possible export of manufactured goods. Since the results were essentially those predictable by partial analysis, they will not be reproduced.

1/ The optimal Model I solution for pattern B3 is No. 24 (shown in figure 5), which has a value of 3342. Solution No. 11 is optimal for pattern B2 and has a value or 3436.

TABLE IV.

Values of Experimental Solutions.[1]

Investment Pattern	Timing of Investment in Sectors: 2	4	Model I	Model II	Model III
A 2	-	0,5			(1) 3950
	-	0,7		(2) 3266	
A 3	-	0,4,7	(3) 2572		
B 1	2	0		(4) 3553	(5) 4291
B 2	1	0,5			(6) 5068
	1	0,7		(7) 4130	
	2	0,4	(8) 3265		
	2	0,5	(9) 3405		(10) 5512
	2	0,6	(11) 3436 *	(12) 4165 *	(13) 5538 *
	2	0,7	(14) 3382	(15) 4157	(16) 5511
	2	0,8		(17) 4080	(18) 5398
	3	0,5			(19) 5008
	3	0.7		(20) 4117	
B 3	1	0,4,7	(21) 3254		
	2	0,3,7	(22) 3148		
	2	0,4,6	(23) 3234		
	2	0,4,7	(24) 3342[2]	(25) 3711	
	2	0,5,6			(26) 5034
	2	0,5,7	(27) 3295		
	2	0,7,8		(28) 3746	
	3	0,4,7	(29) 3255		
	5	0,4,7	(30) 3221		
C 1	2,7	0		(31) 3601	
	2,8	0		(32) 3644	
C 2	1,3	0,5			(33) 4912
	2,6	0,7			(34) 5471
	2,7	0,7		(35) 4068	(36) 5474
	2,7	0,8			(37) 5372
	2,8	0,7			(38) 5438
C 3	1,3	1,4,7	(39) 3114		
	2,6	0,4,7	(40) 3179		
	2,7	0,4,7	(41) 3165		

[1] Each value is calculated from the optimal solution for the specified timing of investments in sectors 2 and 4. Solution numbers in parentheses. Best solutions for each Model are starred.

[2] Optimal solution for pattern B3, Model I.

D. Implication for Developing Planning

These experiments bring out an aspect of development plans that is only beginning to be adequately appreciated: their function in balancing investment over time. Scitovsky /21, 213-214/ reached a very similar conclusion in trying to reconcile the arguments for balanced and unbalanced growth:

"... several-year investment plans, of which one hears so much nowadays, may be regarded as plans for unbalanced growth, extending to several years so as to restore balance by the end of the period for which the plans are made. In the interim, imbalance manifests itself by the completion of productive capacity before the demand for its full utilization has arisen, or by the creation of consumers' or producers' demand before the capacity to fill this demand is completed. The temporary excess capacity may have to be accepted in most cases as an inevitable cost of (temporarily) unbalanced growth for the sake of securing economies of scale; the temporary excess demand may be filled by imports, which is one reason why the availability of foreign loans or foreign exchange is so strategic a factor in investment planning. Such dependence on foreign trade, however, is very different from that which accompanies unbalanced growth concentrated on industries with a comparative advantage. For one thing, this is a temporary dependence, while that is permanent; for another, the dependence here is primarily on foreign import supplies, there on foreign export markets."

The costs of errors in judging plant size that may result from poor forecasts of future demand and of improper timing of interrelated investments are suggested by the solutions in table IV. The welfare costs shown understate the actual losses likely to be incurred because they assume that the rest of the economy adjusts optimally to the investment pattern in the scale-economy sectors, which is most unlikely.

IV. CONCLUSIONS

This study has attempted to give greater precision to the discussion of alternative investment patterns by restating several of the principal hypotheses in programming terms. We have shown that under realistic assumptions as to the nature of horizontal and vertical interdependence, the timing of investment in scale-economy sectors has a substantial effect on the timing in other sectors and hence on the whole investment pattern. The optimal pattern balances gains from larger plants against costs of deferring investment in other sectors and the resulting loss of growth in income and savings. The general characteristics of this pattern include both the type of alternation envisaged by Scitovsky and Streeten and the exploitation of an integrated spurt in investment foreseen by Rodan.

The use of a comprehensive optimizing model has also provided some new perspectives on the characteristics of optimal allocation patterns. In a rapidly growing economy the timing of investment is often more important than the choice of sectors along static comparative-advantage lines. In order to exploit economies of scale in some sectors, it is necessary to limit investment in others, if necessary by using small-scale techniques that will prove to be inefficient at a later date. In our model, primary production performs this function; it is expanded whenever necessary to secure imports, even though these imports are periodically replaced with lower cost domestic production.

Although the concept of "optimal overcapacity" was derived from a partial equilibrium analysis, an explicit consideration of the effects of interdependence leads to an analogous concept of "optimal shortages". The gains from building a bigger power plant next year must be weighed against the cost of lost production this year in power-using sectors. These opportunity costs can only be determined in a more comprehensive analysis. Our optimal power cycle consists of 3-4 years of excess capacity and 1-2 years of power shortages.

Even though the choice between domestic production and imports is central to the present model, comparative advantage is hard to define or measure in a system containing economies of scale and limited investment resources. In the scale-economy sectors, the critical question is the plant size and timing at which import substitution becomes profitable. There is no possibility of a time-less ranking of projects along partial equilibrium lines because the need to accommodate the lumpy investments causes relative profitability to vary year by year. While this phenomenon is exaggerated by considering only part of the whole economy as we have done, it is of considerable empirical importance in underdeveloped countries.

The policy significance of these theoretical and illustrative findings can only be evaluated when more realistic models are tested. Our results emphasize the importance of jointly planning the allocation of investment and foreign exchange over time in order to allow for alternating production and imports. The advantages of this broader approach to development planning appear to be substantial as compared to more traditional methods of allocating each scarce factor separately.

APPENDIX A: Statement of the Model

TABLE A-I

Explanation of Symbols Used

Symbol	Usage
Activities	
F_j^T	Fixed charge investment activity; i = 2, 4
I_i^T	Variable charge investment activity in sector i
A_i^T	"Auxiliary" investment in capital in rest of economy necessary for the operation of sector i; i = 1, 2
D^T	Level of debt in time T
UFE^T	Transfer of foreign exchange to next period
MS^T	Diversion of foreign exchange earnings to savings
X_i^T	Level of production in the i-th sector
M_i^T	Import level of product i; i = 1, 2
E_i^T	Exports of product i; i = 3
CN^T	Consumption activity, stated as increment over last period's consumption
EC_i^T	Excess productive capacity in sector i
EAC_i^T	Excess capacity in capital required in rest of the economy for production of good i; i = 1, 2
VR_k^T	Activity giving value to provision of foreign exchange and domestic savings for post-plan investment in excess of initial endowment
VEC_k^T	Activity giving value to terminal excess capacity

TABLE A-I (continued)

Symbol	Usage
VUC_k^T	Gives value to capacity utilized in the terminal period
C_i^T	Consumption of the i-th product
The following variables have shadow prices associated with them: the equations from which the prices derive are noted.	
K_i^T	Capital available in the i-th sector: Eq. (4-i)
AK_i^T	Capital required in the rest of the economy for production of good i; i = 1, 2: eq. (5-i)
P_i^T	Supply of product i: Eqs. (6) through (9)
BC^T	Borrowing constraint: Eqs. (3) and (16)
UC_i^T	Capacity utilization constraint: Eqs. (10)
CC^T	Consumption growth constraint: Eq. (11)
FE^T	Foreign exchange: Eqs. (1), (12) and (14)
DS^T	Domestic savings: Eqs. (2), (13), and (15)
Objective Values	
Ø	Total value of solution
ØC	Value of consumption increase during plan period
ØTR	Value of terminal foreign exchange and domestic savings less cost of terminal borrowing
ØEC	Consumption value of terminal excess capacity
ØC'	Value of post plan consumption obtained from utilized capacity [illegible] during the plan period

TABLE A-I (continued)

Symbol	Usage
Parameters	
r	Rate of interest on external borrowing
$u_{nj} f_{nj}$	Investment cost parameters (u_{nj} gives foreign exchange component, f_{nj} total resource cost)

Notes: Subscripts are used to denote sectors (i) and investment patterns (k). i = 1, 2, 3, 4 and k = 1, 2, 3, 4 unless otherwise stated.
Superscripts denote time periods; T = 0, ... 10
Throughout J will be used to designate the set of integer variables
Other small Roman and Greek letters are associated with terminal valuations, see the section on derivation of terminal conditions in the Appendix for their meaning.

APPENDIX A.

A formal statement of the model and a description of the terminal conditions is given in this section. Table A-I contains a list of the symbols and their explanation.

Initial Conditions

(1) $\sum_J f_{1j} F_j^0 + \sum_{i=1}^{4} f_{2i} I_i^0 + \sum_{i=1}^{2} f_{3i} AI_i^0 - D^0 + UFE^0 + MS^0 \quad FE^0$

(2) $\sum_J u_{1j} F_j^0 + \sum_{i=1}^{4} u_{2i} I_i^0 + \sum_{i=1}^{2} u_{3i} AI_i^0 - D^0 - .8MS^0 - DS^0$

Debt Limit Constraint

(3) $D^T \leq$ debt limit $\quad T = 0, \ldots 9$

The debt limit is as follows: Model I, 75 units; Model II, 150 units; Model III, 150 units for periods 0-5 and 300 units thereafter.

Capacity Constraints

(4-i) $X_i^T - \sum_{t=0}^{T-1} I_i^T + EC_i^T = K_i^0: \quad T = 1, \ldots 10; \; i = 1, 2, 3, 4$

(5-i) $X_i^T - \sum_{t=0}^{T-1} AI_i^T + EAC_i^T = AK_i^0: \quad T = 1, \ldots 10; \; i = 1, 2$

Production Constraints

(6) $-X_1^T - M_1^T + 2.4CN^T = -C_1^{T-1} : \quad T = 1, \ldots 10$

(7) $.40X_1^T - X_2^T - M_2^T + 1.7\,CN^T = -C_2^{T-1} : \quad T = 1, \ldots 10$

(8) $.12X_1^T + .48X_2^T - X_3^T + F_3^T + .2CN^T = -C_3^{T-1} : \quad T = 1, \ldots 10$

(9) $.10X_1^T + .21X_2^T + .28X_3^T - X_4^T + 1.0CN^T = -C_4^{T-1} : \quad T=1, \ldots 10$

(10) $X_i^{T-1} - X_i^T \leq 0$: $T = 1, \ldots 10$ for $i = 3$

Consumption Constraint

(11) $- CN^T \leq - 2.0$: $T = 1, \ldots 10$

Investible Resource Constraints

(12) $$(1.07)\, D^{T-1} - UFE^{T-1} + (.04X_1^T + .06X_2^T) + M_1^T + M_2^T - E_3^T + \sum_J f_{1j}F_j^T + \sum_{i=1}^{u} f_{2i}I_i^T + \sum_{i=1}^{2} f_{3i}AI_i^T - D^T + UFE^T + MS^T \leq 0: T = 1, \ldots 9$$

(13) $$(1.07)\, D^{T-1} - \sum_{i=1}^{4} .2X_i^T + \sum_J u_{1j}F_j^T + \sum_{i=1}^{4} u_{2i}I_i^T + \sum_{i=1}^{2} u_{3i}AI_i^T - D^T - .8\, MS^T \leq 0: T = 1, \ldots 9$$

Derivation of Objective Function and Terminal Conditions

The specification of terminal conditions is a major difficulty in programming models; our model is certainly no exception to this unfortunate circumstance. However, experience with three, four, seven and ten period runs of our model indicates that the terminal conditions specified in the model nearly correctly summarize the requisite post-terminal information needed for the "correct" solution of the finite-plan-period model.

The welfare function values a plan of activity on the basis of its contribution to consumption levels during the plan and in the post-terminal future. The valuation of consumption growth is straightforward and needs no comment.

$$\phi_c = \sum_{T=1}^{10} \sum_{t=T}^{10} \frac{5.3}{(1.05)^t} \quad CN^T$$

The terminal valuations α_k, β_k, β'_k, ρ and γ are best discussed with reference to Table A-II. One of the central features of our model is the existence of four distinct patterns of production which are the only possible means of satisfying consumption demands: (I) production of all four goods domestically; (II) import of intermediate goods with domestic production of the remaining products: (III) production at home of goods two, three and four with importation of good one; and (IV) domestic provision of primary goods and overhead facilities with imports of finished and intermediate goods. Within any period the model will have a comparative advantage in one of these patterns (or, less likely, a combination). Given the consumption vector, the input-output matrix yields, for each pattern, the production levels necessary to sustain one unit of consumption. These production levels are given below.

Pattern	Production in Sector 1	2	3	4
I	2.40	2.66	2.02	2.51
II	2.40	.0	3.24	2.37
III	.0	1.70	3.52	2.59
IV	.0	.0	4.30	2.51

Using the production data, it is possible to determine the average investment cost per unit of consumption associated with a given level of the consumption vector for each of the production patterns. The second and third columns of

TABLE A - II

Derivation of Terminal Conditions

Pattern of Investment	Average Use at CN = 50		Parameters of Model			
			r_k	α_k	β_k	β'_k
	FE	DS	$\frac{\text{aver DS}}{\text{aver FE}}$	$\frac{V^{11}}{\text{aver FE}}$	$\frac{V^{13}}{\text{aver FE}}$	$\frac{V^{16}}{\text{aver FE}}$
I	7.21	26.68	3.70	9.48	8.62	6.75
II	6.85	32.45	4.74	9.97	9.07	7.11
III	7.30	34.88	4.78	9.94	8.51	6.67
IV	6.94	38.28	5.51	9.85	8.95	7.01

Variable	Production Level	x_i, y_i	x'_i, y'_i
		average FE	average DS
X_1	120.0	.200	.530
X_2	133.0	.293	.766
X_3	101.0	.600	1.000
X_4	125.5	1.739	3.261
AI_1	120.0	.100	.270
AI_2	133.0	.050	.150

$V^{11} = 68.33$, $V^{13} = 61.99$, $V^{18} = 48.53$;

where $V^T = \sum_{t=T}^{\infty} \frac{5.3}{(1.05)^T}$

Table A-II give the average investment of foreign exchange and domestic savings associated with each pattern of production for a consumption level of fifty units. These costs yield the starting point of the valuation procedure.

In the context of the model there are two principal means of providing within the plan period for post-terminal growth. Either investible resources may be passed into the post-terminal future or over-investment within the plan period may provide excess capacity to be used for future growth. The welfare value associated with each mode of guaranteeing post-terminal growth is the value of the post-plan consumption which it makes possible.

The value of one unit of consumption sustained from period eleven through an infinite horizon is given by V^{11}. Division of V^{11} by the average investment use of foreign exchange by production pattern "k" gives the average value of foreign exchange invested in that pattern (α_k). "r_k" gives the total investment of resources necessary to yield the value α_k. Thus, the investment of 3.70 units of resources (foreign exchange component of one) in pattern I in period 10 gives rise to 9.48 units of welfare value. Value is given to the provision of terminal foreign exchange and investment resources only if the provision exceeds the initial endowments of these resources. Equations (14) and (15) are used to value terminal investible resources.

$$(14) \quad (1.07)D^9 - UFE^9 + .04X^{10} + .06X^{10} + M_1^{10} + M_2^{10} - E_1^{10} - D^{10} + \sum_k VR_k - FE^0$$

$$(15) \quad (1.07)\, D^9 - \sum_{i=1}^{4} .2X_i^{10} - D^{10} + \sum_{k=1}^{4} r_k\, VR_k\, s - DS^0$$

Terminal debt is restricted to repayment of borrowing in the ninth period and its cost is given by the value of the investible resources in the post-terminal period necessary for repayment (and interest charges). The value of foreign exchange and savings used to determine the cost of borrowing represents an educated guess on the basis of computations in Table A-II. The cost of terminal borrowing in the basic model is 9.19.

$$(16) \quad D^{10} - (1.07)D^9 \leq 0$$

$\emptyset_{TR}$ is the value of terminal foreign exchange and domestic savings less the cost of terminal borrowing:

$$\emptyset_{TR} = \sum_{k=1}^{4} r_k\, VR_k - 0.1\, D^{10}$$

The valuation of terminal excess capacity is similar to the valuation of terminal investible resources. The existence of excess capacity available for use post-plan represents a reduction in the investment needed to obtain a given level of post-terminal growth. Excess capacity should be valued in terms of the investible resources which it "releases" from post-plan investment. One assumption is necessary to make the valuation: the consumption value of excess capacity depends upon how quickly it is put to use. We have assumed a rate of post-terminal growth [illegible] two consumption units per period (this assumption is crucial only for the first five post-terminal periods). Any excess capacity that

would come on line in the first five post-terminal periods under this assumption is valued as if it came on line in the third post-terminal period. Any remaining excess capacity in the terminal period which would come on line after the fifth post-terminal period is valued as if it came on line in the eighth post-terminal period. A step function like that used here is necessary to approximate the algebraically correct, but non-linear, function which would value terminal excess capacity in terms of the period in which it came on line. Two steps were sufficient for the linear approximation. Some runs were made in which excess capacity was valued as though it came on line in the first post-terminal period; this specification appeared highly unreasonable because of the tendency for plants of the maximum allowable size to be constructed in sector two in the terminal period.

The value of a unit of consumption available from period thirteen to infinity is V^{13}. The value of a unit of foreign exchange and r_k units of investment funds released from investment in pattern "k" by the presence of excess capacity is β_k. To ascertain the amount of investible resources released by excess capacity within a sector we assumed that pattern I is the relevant production mode. The second column in the lower half of Table A-II gives the production level in each sector associated with the operation of the consumption activity at level fifty. The third and fourth columns give the [illegible] foreign exchange investment component and total investment needed for the level of production in column two in each sector.

On the first step activities EC_1 through EC_4, EAC_1 and EAC_2 are constrained to be less than or equal to 24.0, 26.6, 20.2, 25.1, 24.0 and 26.6 respectively. Valuations along the second step are derived in a similar manner and they are given by β_1'.

Equations (17) and (18) transform excess capacity in the various sectors into the investment cost associated with this capacity in the manner explained above. Only the first step of the function valuing terminal excess capacity is shown. $\emptyset_{EC}$ is the value of terminal excess capacity.

$$(17) \quad -\sum_{i=1}^{4} x_i \, EC_i - \sum_{i=1}^{2} y_i EAC_i + \sum_{k=1}^{4} VEC_k \leq 0$$

$$(18) \quad -\sum_{i=1}^{4} x_i' \, EC_i - \sum_{i=1}^{2} y_i' EAC_i + \sum_{k=1}^{4} r_k VEC_k \leq 0$$

The value of terminal utilized capacity is determined as follows: In the immediate post-terminal periods production in the four sectors will take place at the ratios given by the production levels of one (or, a combination) of the patterns necessary to provide a unit of consumption. This is so by virtue of the assumed unchanging incremental pattern of consumption demand with growth in income. The activity VUC_k gives the value V^{1j} (=γ) to the utilized capacity over and above that which existed at the beginning of the plan period necessary to yield a unit of consumption under production pattern "j". Using optimization to choose the pattern or patterns in which terminal utilized capacity (beyond that available in the initial period) is valued

allows the "intermediate" run to have some influence upon investment in the plan period. Equations (19) through (22) are used to determine the value of terminal utilized capacity ($\phi_{C'}$):

$$(19)\quad -X_1^{10} + 2.40\ VUC_1 + 2.40\ VUC_2 \leq 0$$

$$(20)\quad -X_2^{10} + 2.66\ VUC_1 + 1.70\ VUC_3 \leq 0$$

$$(21)\quad -X_3^{10} + 2.02\ VUC_1 + 3.24\ VUC_2 + 3.52\ VUC_3 + 4.30\ VUC_4 \leq 0$$

$$(22)\quad -X_4^{10} + 2.51\ VUC_1 + 2.37\ VUC_2 + 2.59\ VUC_3 + 2.51\ VUC_4 \leq 0$$

$$\phi_{C'} = \sum_{k=1}^{4} \gamma\ VUC_k$$

In conclusion, it is <u>a priori</u> impossible to determine the post-plan pattern of production; it is necessary to consider each pattern as a possible means of achieving post-plan growth (at least with reference to the immediate post-plan future). Inclusion of all possible patterns allows the model to determine terminal values within reasonable limits and without their being specified in a more arbitrary fashion than that used here. And, explicit statement of the post-plan investment alternatives allows the model to optimize within the plan period with partial knowledge of post-plan alternatives.

The most arbitrary assumptions in the valuation of terminal resources concern the appropriate level of consumption from which average capacity costs are determined, the rate of post-plan growth, and the date on which terminal borrowing must be repaid. The first two assumptions are necessary because average investment cost is the relevant cost from which to determine the consumption value of the investible resources: maximum welfare requires that the average cost of providing the consumption bundle over the long run be minimized. The assumption of a consumption level of fifty units yields production levels requiring investment in plants of approximately the same size as those built within the plan period in the various solutions. It is also in keeping with the derivation of terminal values by a means which stresses intermediate run alternatives.

A notable feature of the model resulting from these assumptions is the consistent failure to carry excess capacity into the post-terminal future in sector four. This failure, interestingly enough, persists even if the size of plant in sector four used to determine the terminal valuations is cut in half with all else remaining as specified in the basic model (a seven period run was used). This feature of the model is doubtlessly linked to the assumption that debt must be repaid in the first period after the plan. However

in one run debt was assumed to be paid off in the twentieth post-terminal period (decreasing by more than half the cost of terminal debt) and the assumed post-terminal plant size in sector four was again half as large as that given in Table A-II. The solution to this (seven period) run did not differ from the solution to the model with terminal specifications as outlined here. Experience with the models of various plan period lengths suggests that the aversion to terminal debt and excess capacity in sector four is unreasonable.

The Objective Function

Maximize $\emptyset = \emptyset_C + \emptyset_{TR} + \emptyset_{EC} + \emptyset_{C'}$

Variable Restrictions

As in standard linear programming, all variables are restricted to non-negative levels. However, optimization within the constraints is transformed into a mixed integer programming problem by the following requirement:

$$(23)\quad F_j^T = 1 \text{ or } 0;\ I_j^T > 0 \Longleftrightarrow F_j^T = 1;\quad j \in J;\ T = 0, \ldots 9$$

Equations (24) yield a "fixed charge" investment cost function for sectors two and four.

APPENDIX B. TABLE A-III

Optimal Solution to Model I

		PERIOD										
		0	1	2	3	4	5	6	7	8	9	10
A. CAPACITY BY SECTOR												
Finished Goods	K_1	50	50	100	104	109	114	148	148	152	157	317
Intermediate Goods	K_2	75	75	75	180	180	180	180	180	180	180	180
Primary Products	K_3	160	160	164	164	168	183	194	194	217	228	238
Overhead Facilities	K_4	127	155	155	155	155	155	155	211	211	211	211
B. PRODUCTION												
Finished Goods	X_1	50	50	100	104	109	114	119	148	152	157	131
Intermediate Goods	X_2	75	75	75	127	128	92	62	175	180	180	164
Primary Products	X_3	160	160	164	164	168	183	194	194	217	228	228
Overhead Facilities	X_4	127	129	137	150	155	155	155	193	205	211	211
C. IMPORTS												
Finished Goods	M_1	40	45	0	0	0	0	0	0	0	0	40
Int. Goods	M_2	20	23	47	0	4	46	81	0	0	5	20
D. CONSUMPTION	CN	0	2	4	6	8	10	12	24	26	28	34
E. SOURCES OF INVESTMENT FINANCING												
Savings	S	82	83	95	109	112	109	106	142	151	155	
Excess Imports	.8 MS	18	0	0	0	0	0	62	30	0	13	
Capital Inflow 2/	F	75	-13	3	-76	-5	0	75	-5	-70	-11	
Total Sources 2/		175	70	98	33	107	109	243	167	81	157	
F. USES OF INVESTMENT												
Sector 2	$(42 - 45\ I_2)$	0	0	88	0	0	0	0	0	0	0	
Sector 4	$(108 - 2.4I_4)$	175	0	0	0	0	0	243	0	0	0	
Other Sectors 2/	(1)	0	69	9	33	107	109	0	167	80	157	
Total Uses 2/		175	69	97	33	107	109	243	167	80	157	
G. SOURCES OF FOREIGN EXCHANGE												
Change in Reserves $(UFE^T - UFE^{T-1})$		0	0	-14	14	-49	-37	86	0	0	-18	
Exports	E_3	107	106	104	78	80	111	135	76	95	105	
Foreign Capital 2/	F	75	-13	3	-76	-5	0	75	-5	-70	-11	
Total Sources 2/		182	93	93	16	26	74	296	71	25	76	
H. USES OF FOREIGN EXCHANGE												
Investment Imports	(3)	93	18	38	4	10	18	129	17	8	38	
Other Imports	(3)	67	75	55	12	16	56	89	16	17	22	
Transfer to Savings	MS	22	0	0	0	0	0	78	38	0	16	
Total Uses 2/		181	93	93	16	26	74	296	71	25	76	
WELFARE VALUES												
Consumption increase	Øc	598										
Post plan Consumption	Øc	2298										
Terminal excess	ØEC	375										
Transfer of DS and FE	ØTR	165										
Total value	Ø	3437										

1/ $(.53I_1 + 1.0I_3 + .27AI_1 + .15\ AI_2)$

2/ Total sources equal total uses except for deviations due to rounding.

3/ See equation Appendix.

APPENDIX B. TABLE A-IV Optimal Solution to Model II.

		PERIOD 0	1	2	3	4	5	6	7	8	9	10
A. CAPACITY BY SECTOR												
Finished Goods	K_1	50	95	100	105	110	115	120	175	180	185	185
Intermediate Goods	K_2	75	75	75	216	216	216	216	216	216	216	216
Primary Products	K_3	160	164	176	176	196	207	207	207	228	250	255
Overhead Facilities	K_4	127	173	173	173	173	173	173	245	245	245	245
B. PRODUCTION												
Finished Goods	X_1	50	95	100	105	110	115	120	175	180	185	185
Intermediate Goods	X_2	75	75	75	127	132	138	126	206	211	216	216
Primary Products	X_3	160	164	176	176	196	206	206	206	228	250	255
Overhead Facilities	X_4	127	134	141	155	165	173	173	218	230	241	245
C. IMPORTS												
Finished Goods	M_1	40	0	0	0	0	0	0	1	0	0	5
Int. Goods	M_2	20	41	47	0	0	0	17	0	0	0	3
D. CONSUMPTION	CN	0	2	4	6	8	10	12	36	38	40	42
E. SOURCES OF INVESTMENT FINANCING												
Savings	S	82	94	98	113	121	126	125	161	170	178	
Excess Imports	.8 MS	46	18	15	42	11	0	56	2	0	0	
Capital Inflow	F	150	-24	3	-11	-52	-117	150	-11	-11	-143	
Total Sources 2/		278	88	116	144	80	9	331	152	159	35	
F. USES OF INVESTMENT												
Sector 2	$(42 + .45\ I_2)$	0	0	105	0	0	0	0	0	0	0	
Sector 4	$(108 + 2.4\ I_4)$	218	0	0	0	0	0	281	0	0	0	
Other Sectors	(1)	61	88	12	145	80	10	50	153	159	36	
Total Uses 2/		279	88	117	145	80	10	331	153	159	36	
G. SOURCES OF FOREIGN EXCHANGE												
Change in Reserves	$(UFE^T - UFE^{T-1})$	0	0	0	0	-20	20	0	-21	-41	61	
Exports	E_3	107	104	115	89	106	113	118	68	86	104	
Foreign Capital	F	150	-24	3	-11	-52	-117	150	-11	-11	-143	
Total Sources 2/		257	80	118	78	34	16	268	36	34	22	
H. USES OF FOREIGN EXCHANGE												
Investment Imports	(3)	131	9	44	14	10	3	168	14	15	3	
Other Imports	(3)	67	50	55	12	12	13	29	19	20	20	
Transfer to Savings	MS	58	23	19	53	14	0	70	3	0	0	
Total Uses 2/		256	82	118	78	36	16	267	36	35	23	

WELFARE VALUES		
Consumption increase	Øc	755
Post Plan consumption	Øc	3209
Terminal excess	ØEC	0
Transfer of DS and FE	ØTR	201
Total value	Ø	4166

1/ $(.53\ I_1 + 1.0I_3 + .27AI_1 + .15AI_2)$

2/ Total sources equal total uses except for deviations due to rounding.

3/ See equation (12), Appendix.

APPENDIX B. TABLE IV. Optimal Solution to Model III

		PERIOD 0	1	2	3	4	5	6	7	8	9	10
A. CAPACITY BY SECTOR												
Finished Goods	K_1	50	95	100	104	109	114	119	211	216	221	221
Intermediate Goods	K_2	75	75	75	256	256	256	256	256	256	256	256
Primary Products	K_3	160	163	171	171	191	211	211	213	235	260	286
Overhead Facilities	K_4	127	174	174	174	174	174	174	283	283	283	283
B. PRODUCTION												
Finished Goods	X_1	50	95	100	104	109	114	119	211	216	221	201
Intermediate Goods	X_2	75	75	75	127	132	138	126	245	251	256	256
Primary Products	X_3	160	163	171	171	191	211	211	213	235	260	286
Overhead Facilities	X_4	127	134	140	153	163	174	174	248	259	271	283
C. IMPORTS												
Finished Goods	M_1	40	0	0	0	0	0	0	0	0	0	31
Int. Goods	M_2	20	41	47	0	0	0	29	0	0	0	0
D. CONSUMPTION	CN	0	2	4	6	8	10	12	50	52	54	59
E. SOURCES OF INVESTMENT FINANCING												
Savings	S	82	93	97	111	119	127	126	183	192	2	
Excess Imports	.8 MS	46	8	19	40	36	0	89	0	5		
Capital Inflow	F	150	-38	18	-11	-11	-123	260	-21	-21	-	
Total Sources 2/		278	63	134	140	144	4	475	162	176	1	
F. USES OF INVESTMENT												
Sector 2	$(42 + .45\ I_2)$	0	0	123	0	0	0	0	0	0		
Sector 4	$(108 + 2.4\ I_4)$	222	0	0	0	0	0	370	0	0		
Other Sectors	(1)	56	64	12	140	144	4	105	162	176	1	
G. SOURCES OF FOREIGN EXCHANGE												
Change in Reserves	$(UFE^T - UFE^{T-1})$	0	0	0	0	-20	20	-11	11	0	-28	
Exports	E_3	107	104	111	85	101	118	122	48	67	88	
Foreign Capital	F	150	-38	18	-11	-11	-123	260	-21	-21	-21	
Total Sources 2/		257	66	129	74	70	15	371	38	46	39	
H. USES OF FOREIGN EXCHANGE												
Investment Imports	(3)	133	7	49	12	13	2	219	15	16	15	
Other Imports	(3)	67	49	56	12	12	13	41	23	24	24	
Transfer to Savings	MS	57	10	24	50	45	0	111	0	6	0	
Total Uses 2/		257	66	129	74	70	15	371	38	46	39	

WELFARE VALUES

Consumption increase	Øc	985
Post Plan consumption	Øc	4255
Terminal excess	ØEC	37
Transfer of DS and FE	ØTR	-39
Total value	Ø	4166

1/ $(.53I_1 + 1.0I_3 - .27AI_1 + .15AI_2)$

2/ Total sources equal total uses except for deviations due to rounding.

3/ See Equation (12), Appendix.

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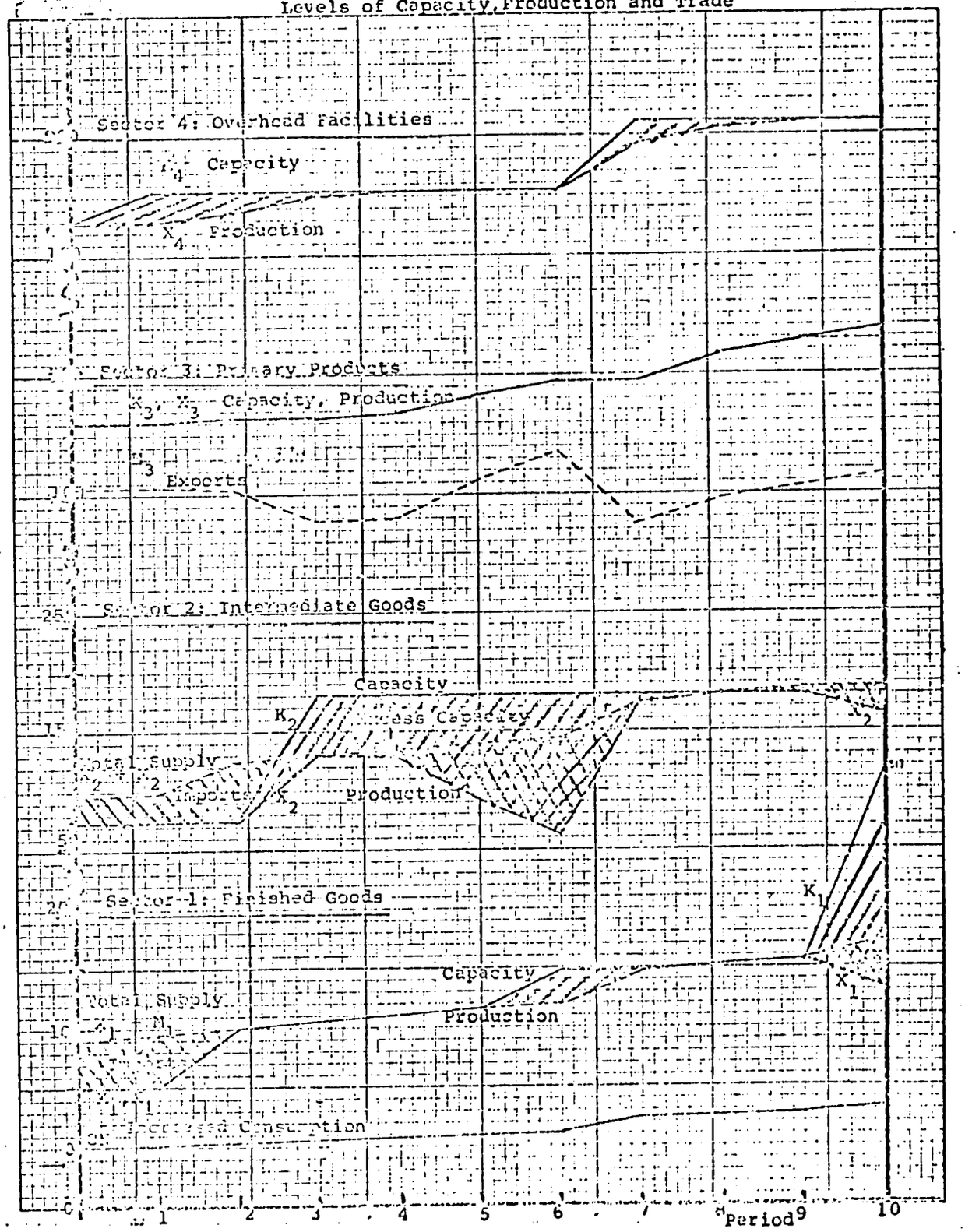
FIGURE 1a.

Optimal Solution for Model I:

Levels of Capacity, Production and Trade



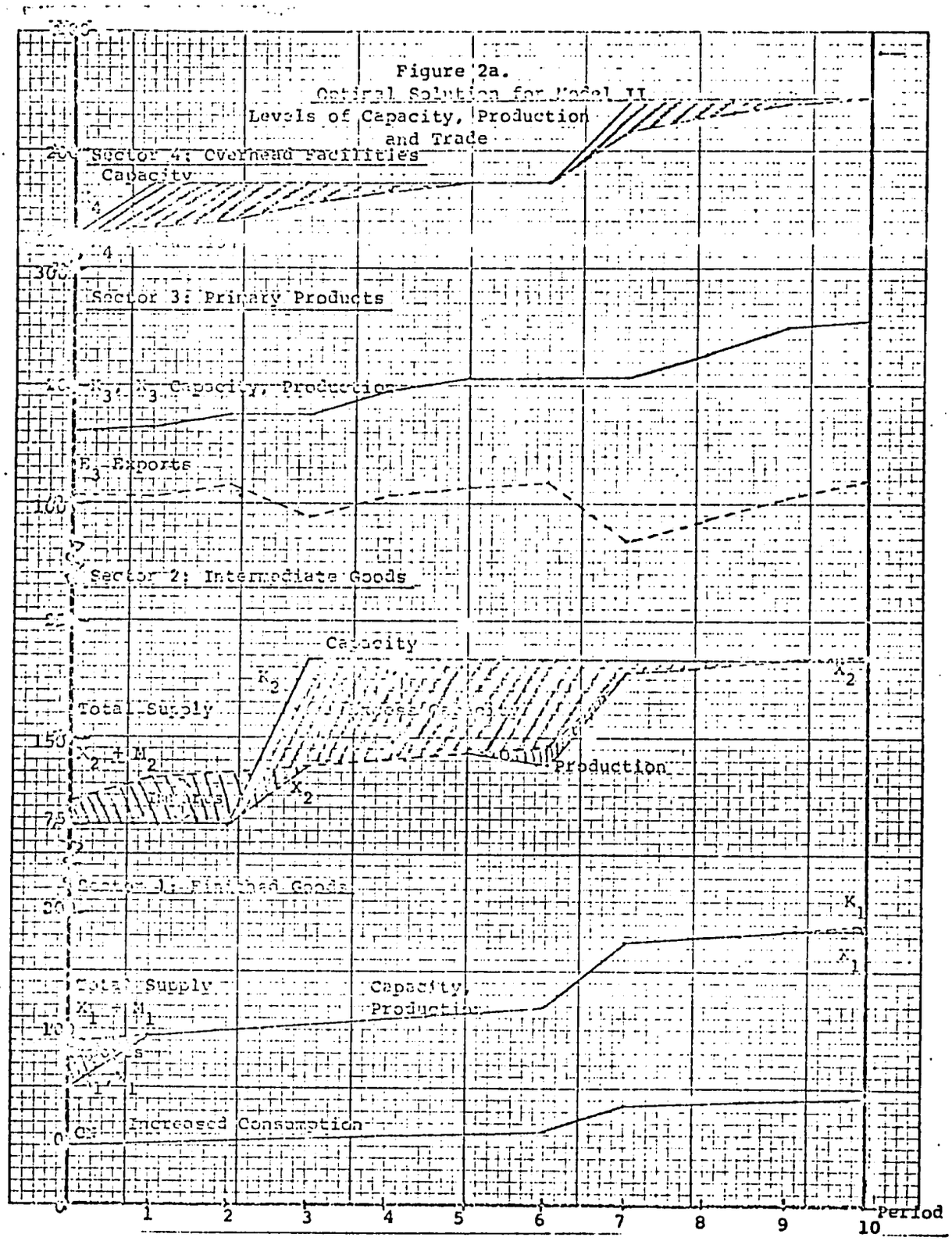


Figure 2a.
Optimal Solution for Model II
Levels of Capacity, Production and Trade

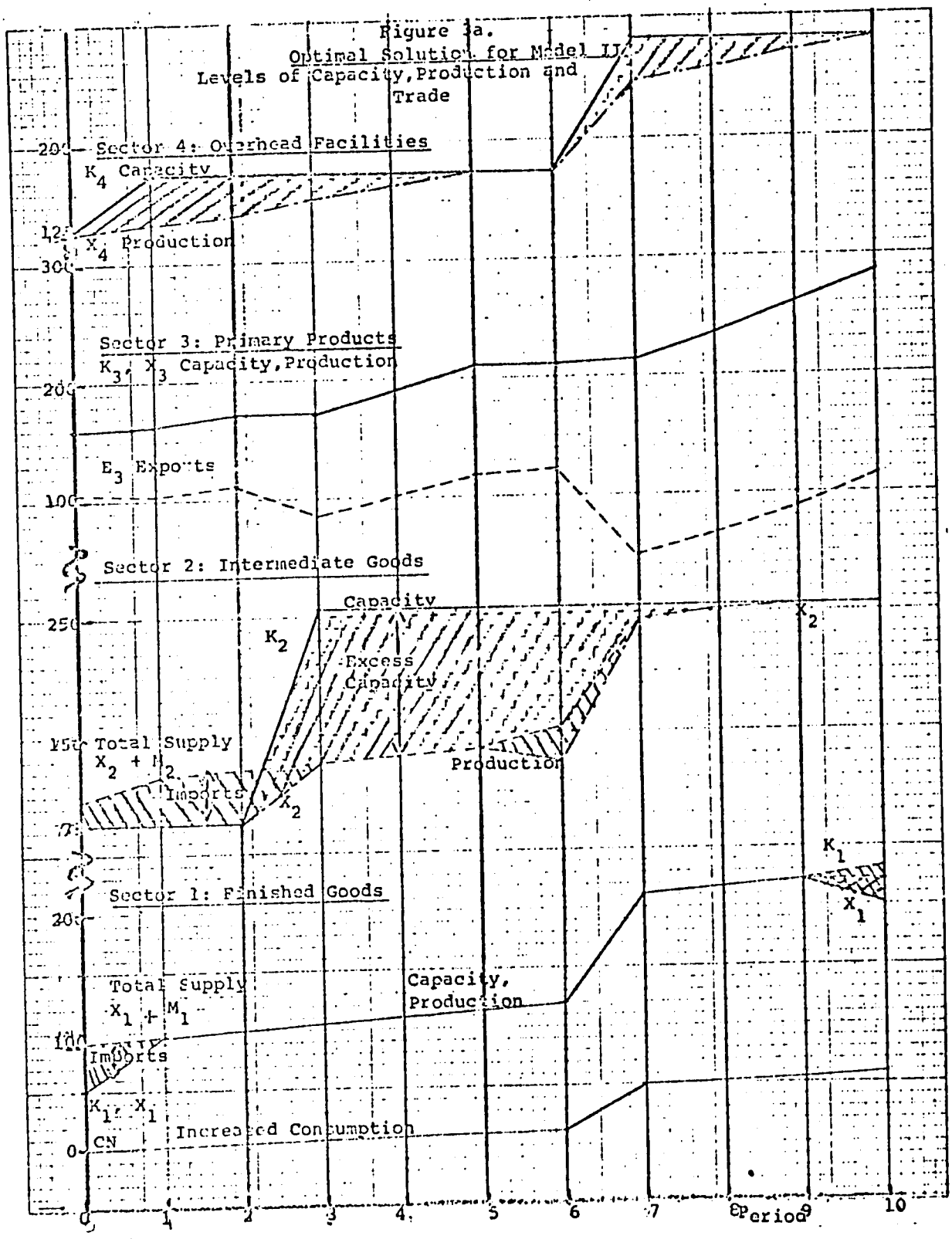
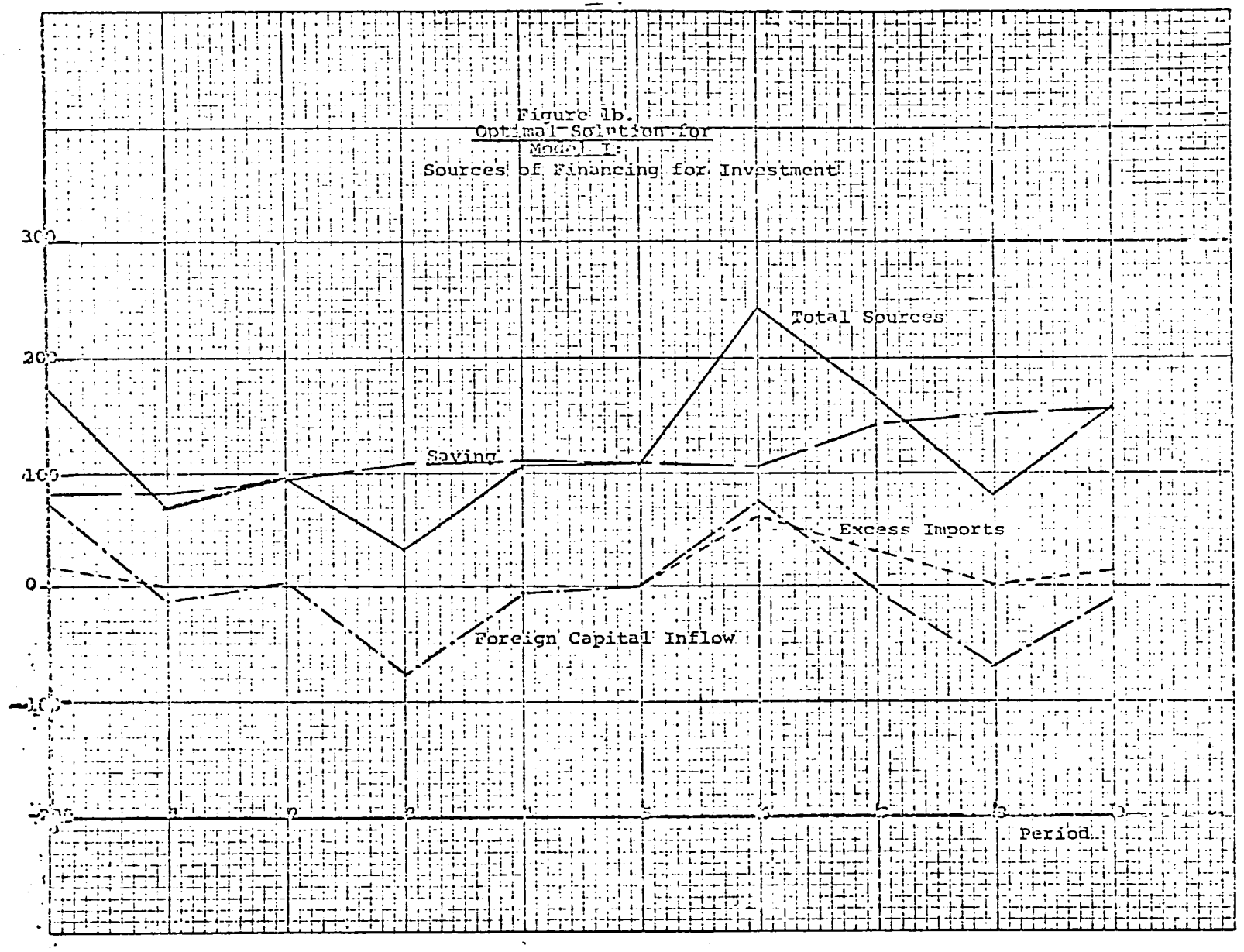


Figure 3a.
Optimal Solution for Model II
Levels of Capacity, Production and Trade

Figure 1b.
Optimal Solution for
Model I:
Sources of Financing for Investment



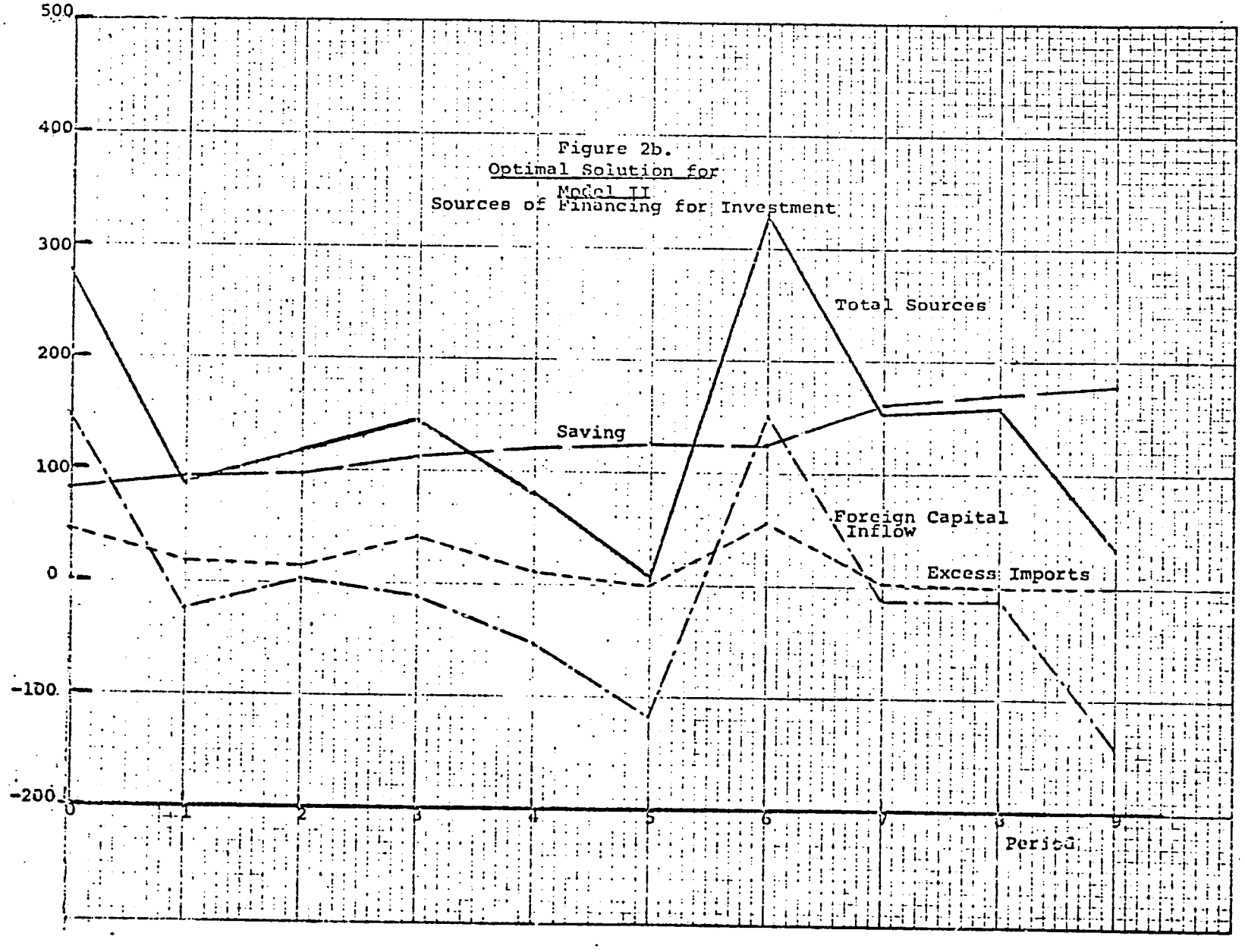


Figure 2b.
Optimal Solution for
Model II
Sources of Financing for Investment

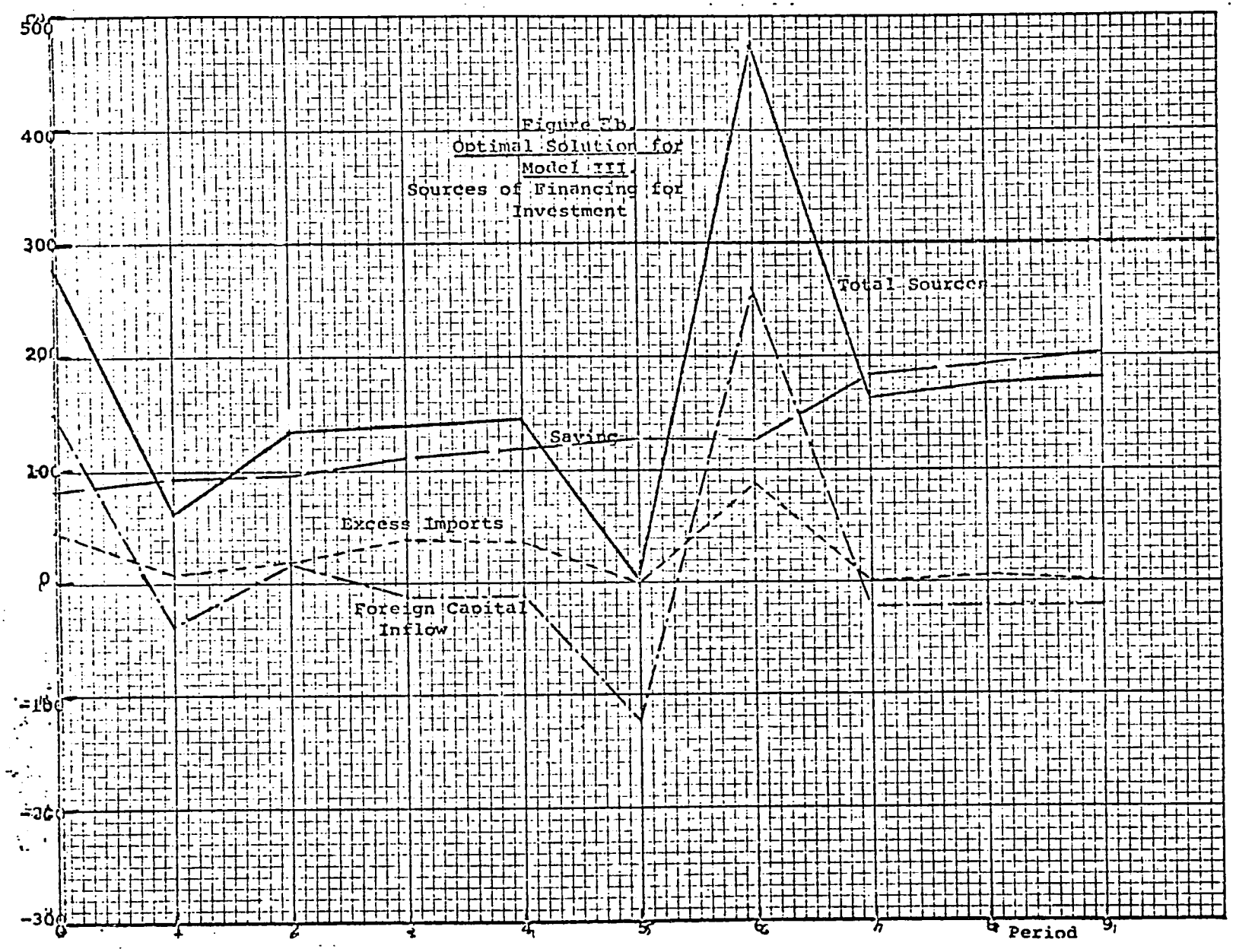


Figure 3b
Optimal Solution for
Model III
Sources of Financing for
Investment
Total Sources
Saving
Excess Imports
Foreign Capital
Inflow
500
400
300
200
100
0
-100
-200
-300
0
1
2
3
4
5
6
7
8
9
Period

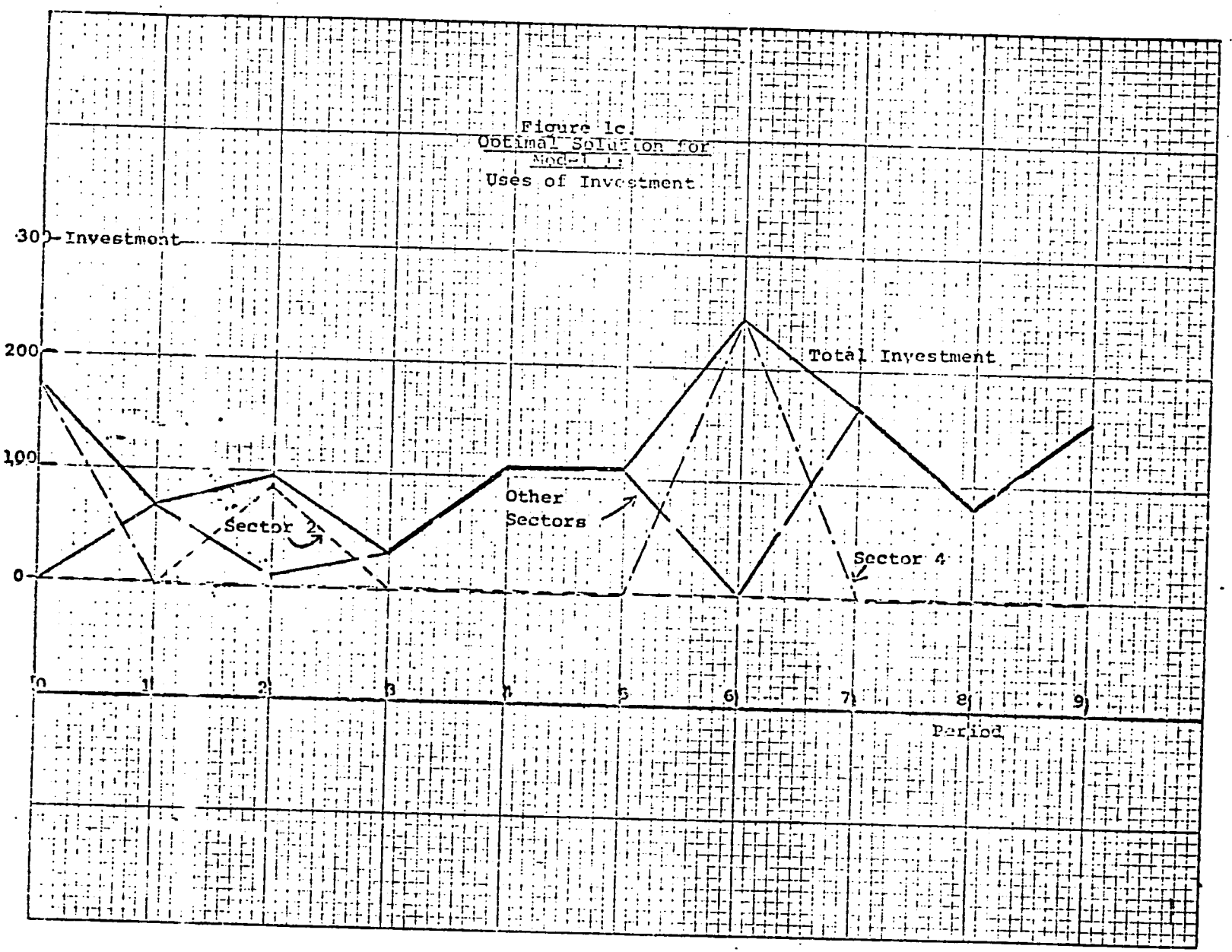


Figure 1c
Optimal Solution for
Model 1
Uses of Investment

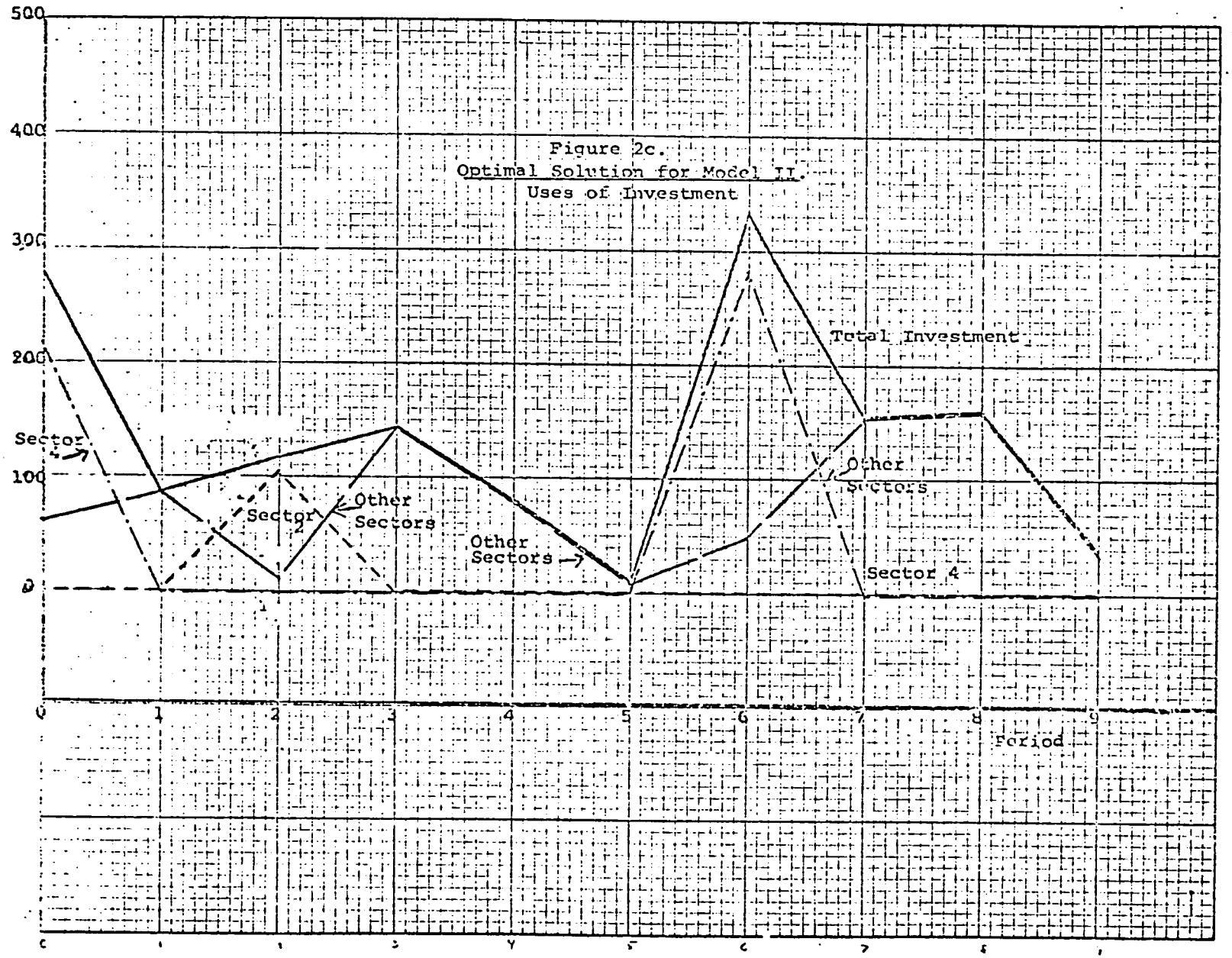


Figure 2c.
Optimal Solution for Model II.
Uses of Investment

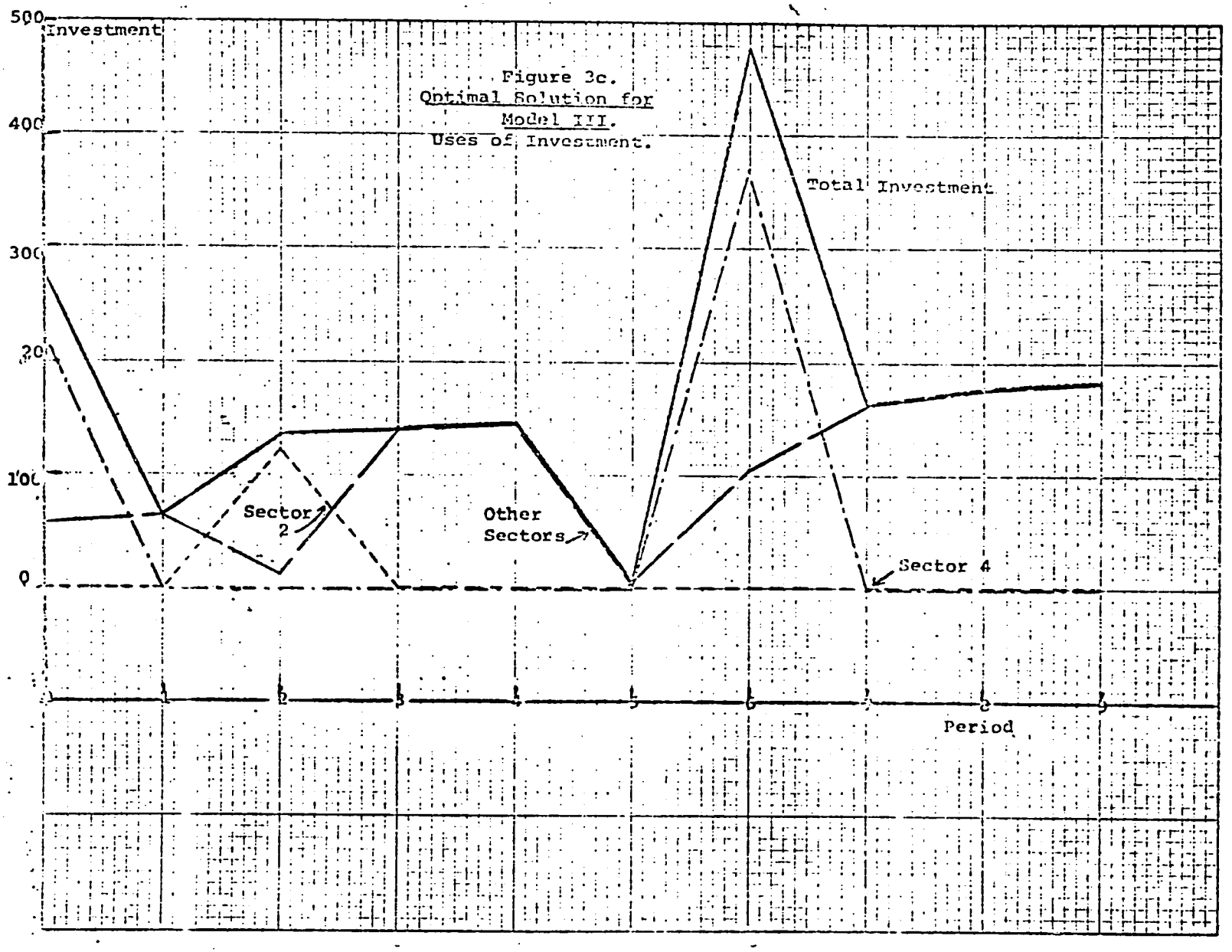


Figure 3c.
Optimal Solution for
Model III.
Uses of Investment.

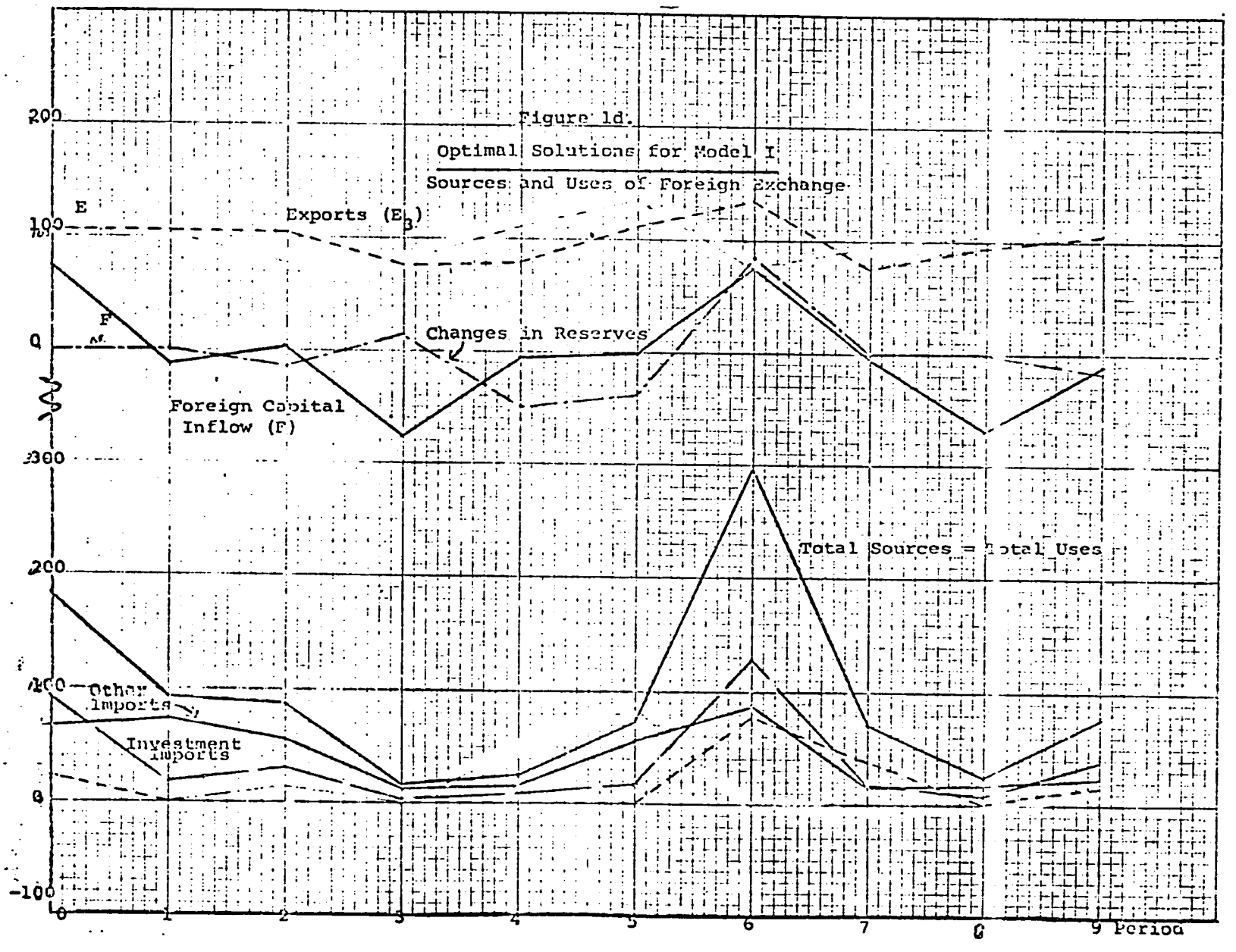


Figure 1d.
Optimal Solutions for Model I
Sources and Uses of Foreign Exchange
E
Exports (E_3)
F
Changes in Reserves
Foreign Capital Inflow (F)
Total Sources = Total Uses
Other Imports
Investment Imports
200
100
0
300
200
100
0
-100
0
1
2
3
4
5
6
7
8
9 Period

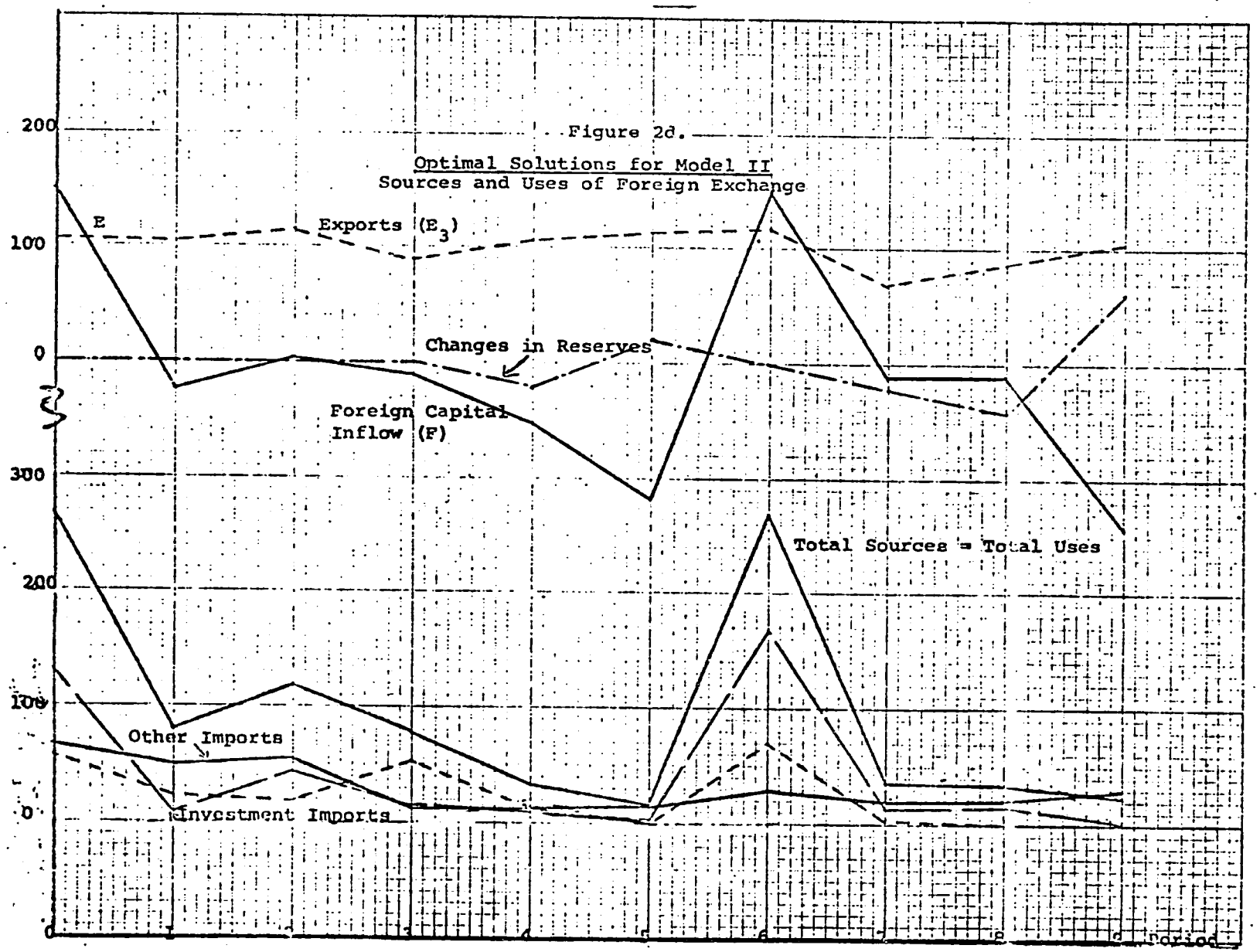


Figure 2d.
Optimal Solutions for Model II
Sources and Uses of Foreign Exchange
E
Exports (E_3)
Changes in Reserves
Foreign Capital Inflow (F)
Total Sources = Total Uses
Other Imports
Investment Imports
200
100
0
300
200
100
0
0
Period

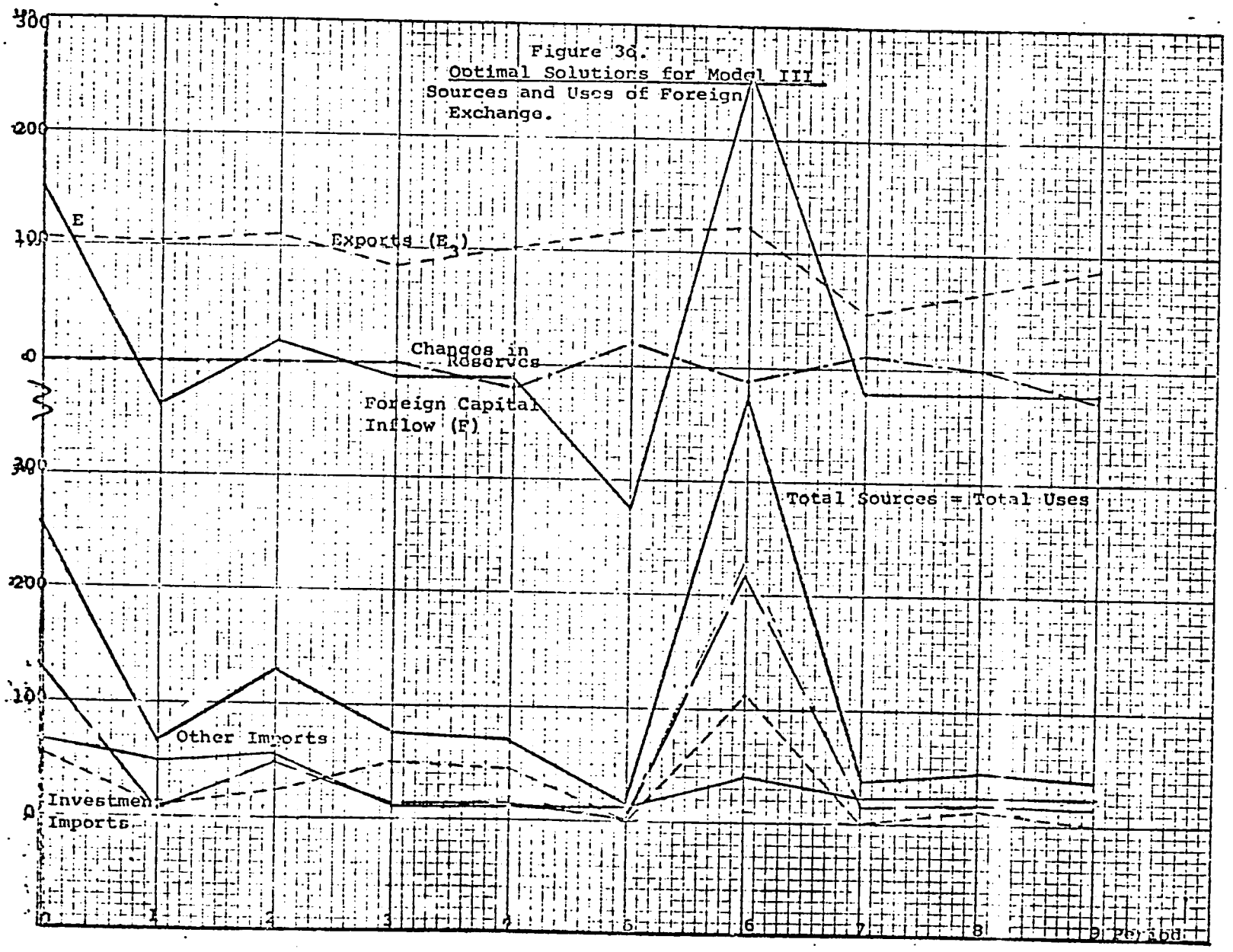


Figure 3c.
Optimal Solutions for Model III
Sources and Uses of Foreign Exchange.
E
Exports (E_3)
Changes in Reserves
Foreign Capital Inflow (F)
Total Sources = Total Uses
Other Imports
Investment Imports
300
200
100
0
300
200
100
0
0
1
2
3
4
5
6
7
8
9 Period

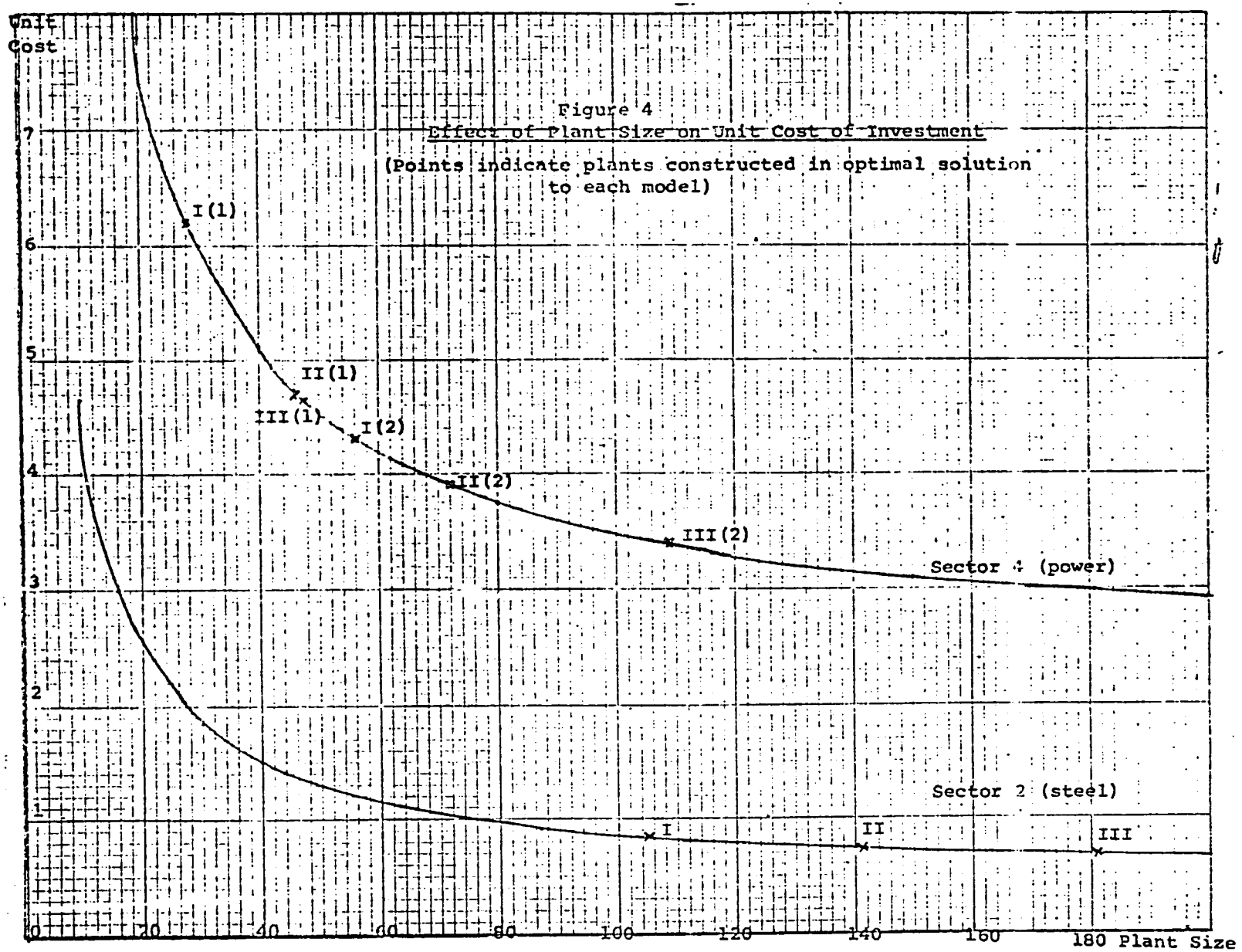


Figure 4
Effect of Plant Size on Unit Cost of Investment
(Points indicate plants constructed in optimal solution to each model)

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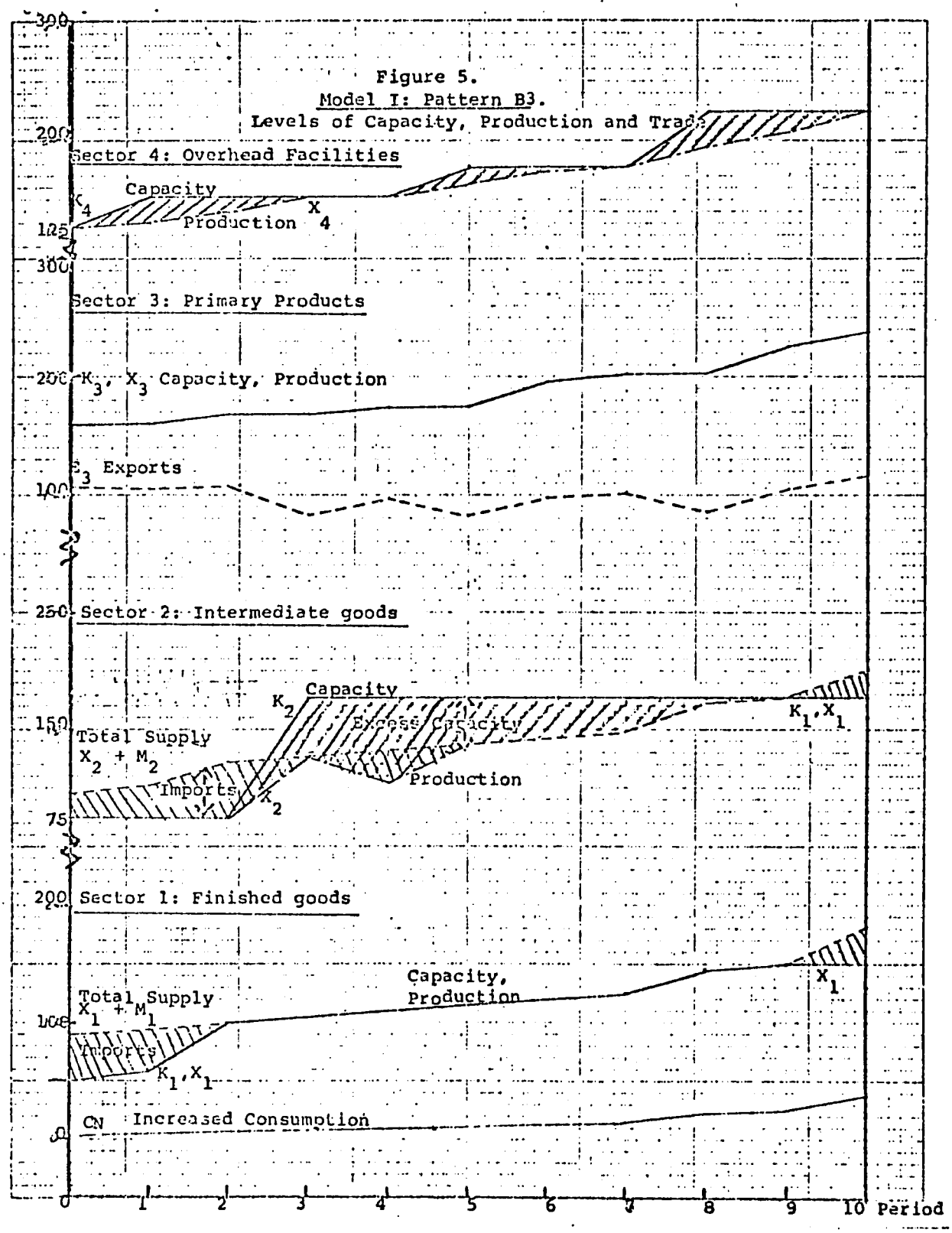

Figure 5.
Model I: Pattern B3.
Levels of Capacity, Production and Trade