PN-ABI-193
71963

SCHEDULING IMPORT SUBSTITUTION IN A TWO-GAP DEVELOPMENT MODEL

by

Lance Taylor

Economic Development Report No. 58
May 1967.

PROJECT FOR QUANTITATIVE RESEARCH
IN ECONOMIC DEVELOPMENT
Center for International Affairs,
Harvard University,
Cambridge, Massachusetts.

A.I.D. Contract # 1543

PN-ABI 193

Scheduling Import Substitution in a Two-Gap Development...
330 Harvard Univ. Center for International Affairs.
T243a Scheduling Import Substitution in a Two-Gap Development Model. Lance Taylor. May 1967.
9 p. appendix.
Bibliography: 1 p.
Economic Development Report no. 58.
Proj. 931-17-995-476.
AID/csd-1543.

1.Economic development.2.Import substitution.3.Models, Economic.I.Taylor, Lance.II.Contract.III.Title.

SCHEDULING IMPORT SUBSTITUTION IN A TWO-GAP DEVELOPMENT MODEL[1/]

Introduction

Since the publication in 1962 of the basic paper by Chenery and Bruno [3], two-gap models of economic development have received considerable attention in the literature.[2/] With the exception of a linear programming analysis by Chenery and MacEwan [4], most of the published studies have been descriptive, and have not considered explicitly the optimizing problem inherent in two-gap models, viz., the coordination of foreign aid received with the investment possibilities provided by increased saving out of increased income. The purpose of this note is to provide an analytic discussion of optimal timing of "import-substituting" investment in the important "trade-limited" case of two-gap analysis, and to illustrate the nature of optimal growth paths under a range of realistic assumptions.

Section I discusses the model used, and presents a useful diagram for analysis of optimal paths. Section II summarizes the characteristics of optimal import substitution paths for a number of less-developed countries and compares results with a linear (in time) schedule for import substitution. The

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1/ The author is a graduate student in economics at Harvard University, working with the Project for Quantitative Research in Economic Development. He is grateful to Hollis Chenery for discussion of this note, and to the Project for subsidizing computer time.

2/ A representative sample of papers would include case studies of Greece [2], Pakistan [4], and Colombia [7], and a general comparative study [5]. A rather more elaborate theoretical model has been worked out by McKinnon [6].

Appendix uses the Pontryagin Minimum Principle to give a semi-rigorous derivation of the analytic results stated in Section I.

I. The Problem of Optimal Import Substitution

The heart of the two-gap argument is that developing countries will have a tendency toward balance-of-payments deficits so long as they do not carry out sufficient investment to enable them to produce currently imported goods domestically.[1/] In a planning context, a reasonable policy goal is that the trade gap between import demands and export revenues should be closed at some date in the future. Another reasonable goal is that the integral of some weighted average of domestic savings and foreign deficit finance -- the two scarce resources in the model -- should be minimized between now and that date. This minimization is easily shown to be equivalent to the minimization of the integral of investment plus "foreign aid."

The integral of investment is determined by the planned path of income growth. The integral of foreign aid is obviously minimized by early import substitution, since foreign aid at any time is higher as import substitution is not completed. Therefore, the planners of a developing country ought to concentrate on import substitution at the beginning of the planning period. In fact (as shown in the Appendix) they should push investment in import substitution to the greatest extent that potential domestic savings allow until

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1/ Assume that demand for goods which are "import-substitutable" rises linearly with income, and that export revenues are given exogenously. Also assume for now a constant geometric growth rate of national income. (This is relaxed in the Appendix.)

the end-of-period trade gap is closed, and then settle back and invest only enough to keep up the planned income growth rate.

This last point can more easily be seen in terms of Figure 1, which represents the constraints in a simple two-gap economy at any time. The line C_1 corresponds to a domestic savings constraint

$$C_1 \quad I - F - S_o - \alpha V \leq 0$$

which states that investment (I) must not be greater than the total of foreign aid (F) and potential domestic savings $(S_o + \alpha V)$.

Constraint C_2 represents the minimum trade gap,

$$C_2 = M_o + \mu V - M_m - E_o e^{\varepsilon t} - F \leq 0$$

where import demand is $M_o + \mu V$, M_m is import substitution completed at any time, and the $E_o e^{\varepsilon t}$ term gives export revenues (assumed to rise exponentially over time). As import substitution proceeds, C_2 will shift down to a position like C_2', and must finally coincide with the I-axis at the end of the planning period.

Constraint C_3 represents the limitation imposed by external lenders that foreign aid should make up at most a certain fraction of total investment,

$$C_3 = F - \beta I \leq 0 \qquad \beta < 1.$$

Constraint C_4 represents the investment necessary to sustain the growth of income,

$$C_4 = -I + rkV \leq 0$$

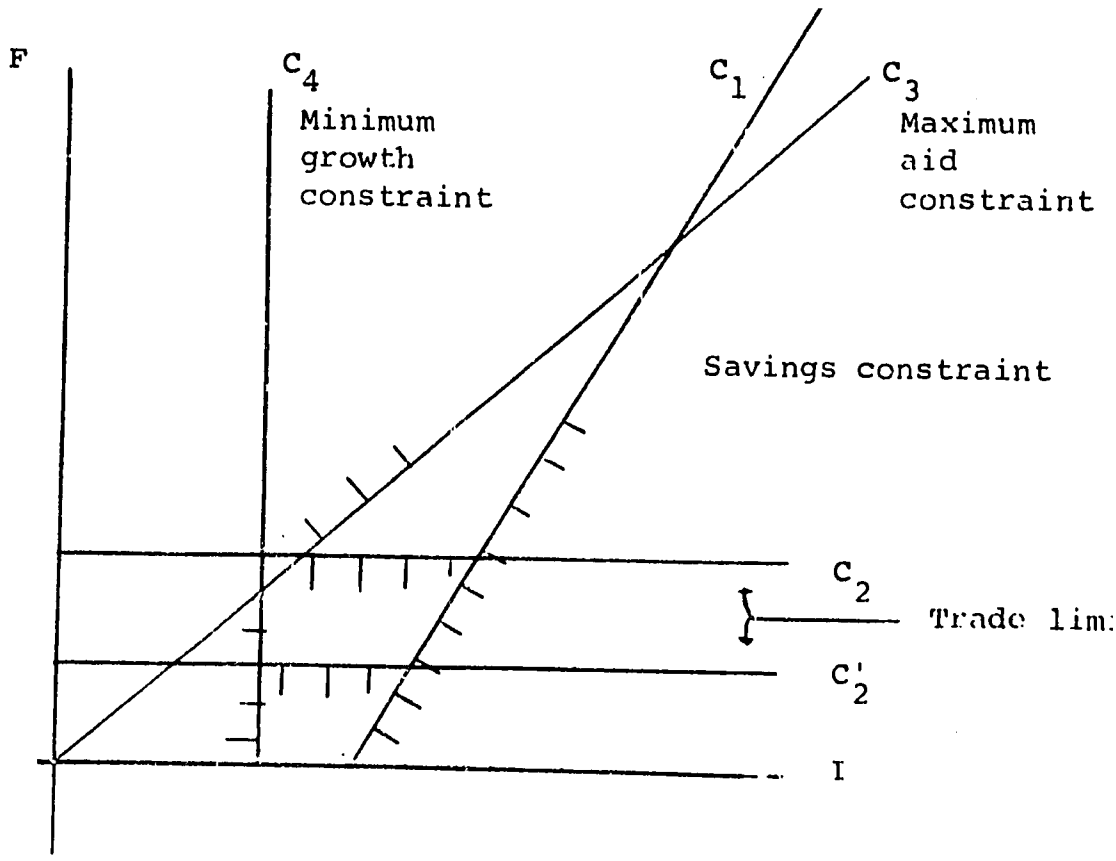
F
C_4
Minimum growth constraint
C_1
C_3
Maximum aid constraint
Savings constraint
C_2
Trade lim
C'_2
I

Figure 1

where r is the growth rate and k is the capital-output ratio.[1/]

Since the planners minimize F at all times, constraint C_2 -- the floor under F -- is binding throughout the planning period. The range of feasible investment levels is bounded by C_1 and C_3 when C_2 is "high", and by C_1 and C_4 when C_2 is "low." Concentration of import substitution at the beginning of the planning period means that investment should be pushed up against C_1 during the early years (with the excess investment over C_4 being devoted to import substitution per se), and then allowed to fall back to C_3 or C_4 when the initial push is completed. In the later years of the plan, the economy coasts down on the minimum investment contraint C_4, having completed the investment necessary to close the trade gap by the end of the planning period.[2/]

II. Applications of the Model

Table 1 summarizes the results of applying our import substitution strategy to a selected group of real and dummy countries.[3/] The sample is fairly representative of the total

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1/ The statement of the problem here is quite similar to that of Chenery and Bruno [3], except that constraint C_3 has been added to their model, and no explicit consideration is given to labor force growth. Figure 1 here corresponds to a translation of Chenery and Bruno's Figures 1-3 to the I-F plane after omitting constraints due to differing assumptions about the exchange rate and labor force growth, and adding C_3.

2/ Chenery and MacEwan get this same time pattern in their linear programming model, although the results are somewhat complicated by an early "skill-limited" phase of economic growth.

3/ The source of data for the countries used is the comparative analysis article by Chenery and Strout [5]. In most cases, I used "plan" values as stated in [5] for parameters. The two dummy countries use the upper quartile and median parameters of the sample of 31 less developed countries in [5].

group of less developed countries which satisfy three conditions implicitly assumed by the model: (i) the value of the marginal savings rate divided by the capital-output ratio exceeds the planned growth rate, so that there is not an ever-widening gap between savings and growth-sustaining investment; (ii) the rate of growth of exports exceeds the rate of growth of income, so the trade gap can ultimately close; (iii) growth is trade-limited, in the sense that the trade gap plus potential domestic savings exceed the investment needed to sustain income growth.[1/]

The planning periods of Table 1 are chosen so that the actual increase of exports at the end of the period exceeds the increase in imports. Otherwise, closing the trade gap makes no sense -- it only opens wider the next year.

Analysis of Table 1 leads to the following conclusions:

(i) The time needed to close the trade gap is highly dependent on initial levels of exports and imports, and on relative growth rates of exports and income. Pakistan and Mexico are favored by high initial export/import ratios, and can close the gap in five years, while Korea -- handicapped by slow export growth -- takes 55 years. The median country of the Chenery-Strout sample, which is perhaps "representative" in some sense, takes 40 years to close the gap -- a distressingly long period of being on the international dole.

1/ The third criterion explains why the model is applied to Pakistan beginning in 1972. The implicit assumption is that import substitution investment is not done until that date. The model could easily be generalized to handle import-substitution during savings-limited growth as well, although there is no natural upper bound on import substitution, such as is provided by the savings constraint in trade-limited growth.

TABLE 1

Country Examples

Country	Planning period (years)	Time using total savings	Capital-output ratio	Initial levels ($ millions) Exports	Imports	GNP	Marginal rates % Imports	Savings	Growth rates % GNP	Exports	Total Foreign Debt ($ millions) Linear import sub.	Optimal import sub.	Efficiency 1/
Pakistan 2/	5	4	3.0	1246	1551	15513	.10	.24	6.0	7.9	1523	683	2.2
Mexico	5	1	2.5	1547	1701	14175	.11	.17	4.0	5.0	524	256	2.0
Nigeria	15	2	3.8	522	688	3434	.28	.125	3.0	5.4	1814	1171	1.5
Israel	25	17	3.0	441	864	2107	.40	.30	9.0	11.2	11921	6747	1.8
Korea	55	51	3.3	146	448	2178	.26	.15	4.0	5.8	13443	8698	1.5
Upper Quartile 3/	10	3	2.8	2.5	16	100	.01	.26	6.2	8.0	100	24	4.2
Median 3/	40	12	3.5	15	20	100	.20	.19	4.6	5.0	163	61	2.7

1/ Ratio of total debt on linear import substitution path to total debt on optimal path

2/ Initial values are projections for 1972 based on Table 3 of [5].

3/ These "countries" use parameter values from a sample of 31 countries in [5].

Source: Tables 1, 3, 6, A-1, A-3 of [5].

(ii) In most of the computer runs, investment started on the savings constraint C_1, and then jumped to the minimum growth constraint C_4. The maximum aid constraint C_3 was not effective, except when β was set at 0.1 or lower, in which case investment followed the C_3-C_1 sequence of constraints. (That is, C_3 put a high lower bound on 1 for a given β and savings up to the limit of C_1 were not required.) In the runs summarized in Table 1, investment followed the C_1-C_4 sequence. The time spent on the savings constraint C_1 is another measure of the difficulty in closing the trade gap. For many of the countries, maximum savings are required for much of the planning period, even with continuous foreign aid inflows. For the countries such as Mexico and Nigeria[1/] which do not utilize their maximum savings potential for very long, the planning period might be shortened (although in no case can it be reduced to the next-lowest multiple of five years).

(iii) The savings in foreign aid achieved by early import substitution as opposed to following a path where M_m (completed import substitution) increases linearly with time are striking, especially in countries whose export potential allows short "gap-closing" periods. Even in Korea, the use of aid on a linear import substitution path is one and one-half times the use on an optimizing path, and aid use is better than cut in half in the more favored countries.

The general picture from the empirical results is that the trade gap is likely to be a nagging problem for a long

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1/ Note that Nigeria has a fairly easy time closing its trade gap because the growth rate of income allowed by its low savings rate and high capital-output ratio is so low.

time for many underdeveloped countries. It is clear that any policies which can increase the growth rate of exports would be highly desirable in terms of reducing foreign dependence, and that immediate increases in domestic savings and immediate investment in import substitution will pay off very well indeed in terms of future savings of foreign aid. This last conclusion is amplified by the consideration that the model presented here -- like all aggregate models -- undoubtedly underestimates the difficulties of adjusting production facilities to carry out import substitution investment. It is doubtless impossible to put all excess savings into import substitution, but the analysis here indicates that planners should come as close to realizing this goal as they can.

APPENDIX

The model used in this note can be stated more formally as follows.

Assume the country has a planned national income growth rate

$$r = \dot{V}/V$$

which is strictly followed through the planning period,[1/] subject to constraints C_1-C_3. The amount of import substitution completed at any time is determined by the differential equation

$$(1) \qquad \dot{M}_m = \frac{1-a}{bk} I u \qquad 0 \leq u \leq 1$$

where k is the marginal capital-output ratio, b is a factor indicating the excess capital-intensiveness of import substitution investment, and a is the import content of import substitution above the economy average. The "control variable u indicates what fraction of investment goes to import substitution.

Since the economy is assumed to stick to its planned growth at all times, the following relation must hold:

$$(2) \qquad \dot{V} = rV_o e^{rt} = Iu/bk + I(1-u)/k.$$

Eliminating u between (1) and (2) gives another differential equation for M_m:

$$(3) \qquad \dot{M}_m = \xi(I-rkV)$$

where $\xi = (1-a)/k(b-1)$, corresponding to $u = 0$, one has the constraint C_4, putting a lower bound on I. For simplicity,

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1/ There is a certain amount of low cunning in setting r = constant, since the problem is then easy to analyze using Figure 1. See the end of this Appendix for a brief discussion of the case where r can vary.

assume the constraint corresponding to $u \leq 1$, which puts an upper bound on I, is superseded by the savings constraint C_1.

To construct a welfare functional, assume the economy's planners are simultaneously interested in maximizing consumption and minimizing total foreign assistance. For a given planned path of V, maximizing consumption and minimizing savings (=I-F) are equivalent, so assume the planners minimize the following integral:

$$\int_0^T [(I-F) + \gamma F]dt$$

where T is the planning period, and is the relative worth of foreign assistance in terms of local currency. To simplify notation, let $\gamma = 2$, and rewrite the welfare functional as

$$J = \int_0^T (I + F)dt.$$

The problem is to minimize J subject to the constraints C_1-C_4 and the differential equation (3). To complete the statement of the problem the following boundary conditions should be imposed:

$$M_m(0) = 0$$

$$M_m(T) = M(T) - E(T) = M_0 + \mu V_0 e^{rT} - E_o e^{\varepsilon t}.$$

The condition that import substitution at time zero is equal to zero is definitional. The condition at time T simply says that foreign assistance must equal zero at the end of the planning period.

Using the Minimum Principle as stated by Bryson and Ho [1], one can determine optimal levels of I and F at any time by minimizing the Hamiltonian function

$$H = (I+F) + \lambda \dot{M}_m$$

or

$$H = (1+\lambda\xi)I + F - \lambda\xi rkV$$

subject to C_1-C_4. The "influence function" $\lambda(t)$ is equal to $\frac{\partial J}{\partial M_m(t)}$ on the optimal path, and is determined from the minimization of H, in a way to be shown shortly.

At most points in time, H will be minimized subject to two effective constraints in standard linear-programming fashion. This minimization will result in two non-negative shadow prices, p_i, p_j determined by the equations

$$\frac{\partial H}{\partial I} + p_i \frac{\partial C_i}{\partial I} + p_j \frac{\partial C_j}{\partial I} = 0$$

$$\frac{\partial H}{\partial F} + p_i \frac{\partial C_i}{\partial F} + p_j \frac{\partial C_j}{\partial F} = 0$$

when constraints C_i and C_j are effective. The influence function λ is then given by

$$\dot{\lambda} = - p_i \frac{\partial C_i}{\partial M_m} - p_j \frac{\partial C_j}{\partial M_m} = \begin{cases} p_2 & \text{when } C_2 \text{ is effective,} \\ 0 & \text{otherwise,} \end{cases}$$

with $\lambda(T) = \kappa$, a constant determined by the required level of $M_m(T)$.

Since $\partial H/\partial F > 0$, minimization of H with respect to F imples that C_2 is always effective, so $p_2 > 0$. Hence, $\dot{\lambda} > 0$, so λ takes on any specific value only once during the planning period.

Minimization of H with respect to I depends on the sign of the quantity $1 + \lambda\xi$. When $1 + \lambda\xi$ is positive, H is minimized by making I as small as possible, so constraints (C_2, C_3) or (C_2, C_4) will be effective. When $1 + \lambda\xi$ is negative, I should be as large as possible, implying that C_1 and C_2 are binding. When $1 + \lambda\xi = 0$, only C_2 binds and I is indeterminate, but since $\dot{\lambda}$ is positive, λ does not equal $-1/\xi$ for a finite time, and this "singular" condition causes no problems.

The terminal boundary condition requires that the economy be at the intersection of C_4 with the I-axis at time T, so $1 + \lambda\xi$ at time T must be positive. Working backwards in time, λ must decrease. If $1 + \lambda\xi$ stays positive, basis sequence (C_2, C_3) to (C_2, C_4) is followed. Otherwise, more import substitution is required, and basis (C_1, C_2) is used early in the planning period. Monotonicity of λ prohibits more than one jump across the feasible area from C_1 to C_3 or C_4.[1/]

The case where r can vary is analyzed in a similar, though more tedious, fashion.[2/] Suppress the $rV_o e^{rt}$ term in

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1/ For the record, note that the strictly linear model stated here necessarily gives rise to a "bang-bang" solution -- the control variables I and F bang around from constraint to constraint but never stay in the "middle" of the feasible region defined by the constraints for any finite time. A non-linear functional J, the integral to be minimized, would allow the possibility of being away from the constraint boundaries for a finite time.

2/ The tedium arises from the possibility of many more switches of control in this more complicated model. A computer analysis of empirical data for the more general model was not undertaken for the same reasons -- a large amount of computer logic would be required to keep track of all the possible control variable cases.

equation (2) above, and add a terminal level for V. The system now becomes a variational problem with two state variables -- M_m and V -- governed by differential equations (1) and (2). The control variables are now I, F, and u, and the feasible values of the controls are limited by C_1-C_3 and the additional constraints 0 u 1. The feasible region is still given by Figure 1, with a third dimension added to make a triangular box with its floor on the u = 0 plane and top coinciding with the u = 1 plane. Analysis of the differential equations for the (now) two influence functions usually gives an optimal path qualitatively like the one discussed above -- u starts on the ceiling and at some point jumps down to the floor, while I starts on the constraint C_1 in most cases and eventually jumps across to C_3 or to some minimum value. (This value will be zero unless some lower bound constraint is put on I -- the model has no desire to maintain capital to support consumption beyond its planning horizon.) Again, it pays off in terms of the cost integral to invest early and heavily in import substitution. This should result in a fairly high early rate of growth, which slackens off toward the end of the planning period -- a result again broadly consistent with the linear programming analysis of [4].

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