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Working Paper No. 2
"Geometric Programming Applied to Some
Optimal Harvesting Problems"
by

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NOTE

This manuscript has been submitted to the journal "Fishery Bulletin" for publication. It is being disseminated at this time as a "Working Paper" in order to provide quick dissemination to all interested collaborators in the fisheries stock assessment CRSP and to obtain your comments and advice on its content.

Although this manuscript deals explicitly with a single species stock assessment problem, it is considered relevant to our overall objective for the following reasons:

1) An improved method for the selection of the optimal harvest problem is described.

2) Our applications of the method will relate to maximizing reproductive value of certain tropical fisheries.

Introduction

In harvesting a population, the age structure is usually ignored but there may be occasions when specific age classes can be cropped. In the latter case, a number of investigators have shown that the greatest harvest would require cropping of no more than two ages (Beddington and Taylor 1973; Rorres and Fair 1975; Doubleday 1975; Rorres 1976). Algorithms for finding the optimal harvesting strategy with regard to age structured populations have been published (Beddington and Taylor 1973; Rorres 1976; Law 1979). Methods, such as linear programming (Doubleday 1975; Cullen 1986) and dynamic programming (Mendelssohn 1976) have been used to solve the optimization problem .

Linear programming is the most commonly used method for solving optimal harvesting problems in Leslie-type models. For example, Doubleday (1975) maximized yield subject to the constraints that population size and age structure are restored after each time period. In matrix notation, $c'(I-\Theta)Av$ is maximized over Θ and v . A is the Leslie matrix, v is the age distribution vector normalized so that the sum of all individuals is 1, I is the identity matrix, c' is a vector of weights, and Θ is a harvesting matrix, the diagonal elements of which represent the proportions of each age class remaining after harvesting. Weights are assigned depending on what particular aspect of the harvest is

to be maximized. For example, if $c'=1$ the object is to maximize yield in numbers. Since one of the constraints is that age structure is restored ($\Theta Av = v$), maximizing $c'(I-\Theta)Av$ and $c'(Av-v)$ are equivalent. If the harvesting matrix is of the form:

$$\begin{vmatrix} \Theta_1 & & & & \\ & \Theta_2 & & & \\ & & \cdot & & \\ & & & \cdot & \\ & & & & \Theta_n \end{vmatrix}$$

and $\Theta_i \neq 1$, this is a standard linear programming problem and the objective function $c'(Av-v)$ is maximized over v subject to:

$$v >= 0$$

$$Av >= v$$

$$1'Av = 1$$

$$\Sigma v_i = 1$$

After solving for v, the relationship $\Theta Av = v$ is used to obtain Θ.

In operational fisheries, it is seldom possible to fish one age class only . Of greater interest and relevance is the case where a constant proportion of all individuals over a certain size are captured while none of the smaller individuals are harvested. In this case the harvesting matrix takes the form:

$$\Theta_k = \begin{vmatrix} 1 & & & & & \\ & \Theta & & & & \\ & & \Theta & & & \\ & & & \cdot & & \\ & & & & \cdot & \\ & & & & & \Theta \end{vmatrix}$$

where $\Theta = 1$ for age classes not fished and is some constant between 0 and 1 for all harvestable ages. This more realistic fisheries problem cannot be solved with linear programming. Doubleday (1975) showed that if $AV >= V >= 0$, $\Theta_k AV = V$ has at most n-k+1 solutions since Θ_k must satisfy a polynomial of degree n-k+1. When n is large, generating the corresponding polynomial is a tedious operation involving many substitutions and a computer program must be used to solve the polynomial.

The purpose of this paper is to present a new method which solves for V and Θ at the same time and to illustrate the method with some examples from the literature. The method, geometric programming, can solve certain problems involving multivariable, nonlinear objective functions and constraints of geometric form. See, for example Rao (1984) for details. Using the notation of Kuester and Mize (1973), the constrained maximization problem is:

$$\text{maximize:} \quad y_o(X) = \sum_{t=1}^{T_o} \sigma_{ot} \; C_{ot} \prod_{n=1}^{N} X_n^{a_{otn}}$$

$$\text{subject to:} \quad \sum_{t=1}^{T_m} \sigma_{mt} \; C_{mt} \prod_{n=1}^{N} X_n^{a_{mtn}} <= \sigma_m \quad \text{for } m=1,\ldots,M$$

where:

σ_{ot} and $\sigma_{mt} = \pm 1$ (the sign of each term in the objective function and m^{th} constraint; respectively),

C_{ot} and $C_{mt} > 0$ (the coefficients of each term in the objective function and the mth constraint; respectively),

$X_n > 0$ (the independent variables),

$\sigma_m = \pm 1$ (the limit bound of the mth constraint),

a_{otn} and a_{mtn} are the exponents of the nth independent variable of the Tth term of the objective fuction and the mth constraint respectively.

M is the number of constraints.

T_o is the number of terms in the objective function.

$T_1, T_2, \ldots, T_M$ are the number of terms in each constraint, 1 to M respectively.

$\sigma = \pm 1$ (the assumed sign of the objective function)

Applications

We first illustrate the method by maximizing $c'(Av-v)$ subject to $\Theta Av = v$ and $\Sigma v_i = 1$ using Williamson's matrix A_1 with

$c' = 1$. . The objective function, the constraints and the harvesting matrix for the three possible harvesting strategies are shown. The initial matrix is:

$$A_1 = \begin{vmatrix} 0 & 9 & 12 \\ .33 & 0 & 0 \\ 0 & .5 & 0 \end{vmatrix}$$

a) For all ages subject to harvesting:

$$\Theta_k = \begin{vmatrix} \Theta & 0 & 0 \\ 0 & \Theta & 0 \\ 0 & 0 & \Theta \end{vmatrix}$$

maximize: $-.667v_1 + 8.5v_2 + 11v_3$

subject to :

$$v_1 + v_2 + v_3 = 1$$
$$9v_1^{-1}v_2\Theta + 12v_1^{-1}v_3\Theta = 1$$
$$.33v_1v_2^{-1}\Theta = 1$$
$$.5v_2v_3^{-1}\Theta = 1$$

b) For the first age not harvested:

$$\Theta_k = \begin{vmatrix} 1 & 0 & 0 \\ 0 & \Theta & 0 \\ 0 & 0 & \Theta \end{vmatrix}$$

maximize: $-.33v_1 - .5v_2 - v_3$

subject to :

$$v_1 + v_2 + v_3 = 1$$
$$9v_1^{-1}v_2 + 12v_1^{-1}v_3 = 1$$
$$.33v_1v_2^{-1}\Theta = 1$$
$$.5v_2v_3^{-1}\Theta = 1$$

c) For the first two ages not harvested::

$$\theta_k = \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & \theta \end{vmatrix}$$

maximize: $.5v_2 - v_3$

subject to :

$$v_1 + v_2 + v_3 = 1$$
$$9v_1^{-1}v_2 + 12v_1^{-1}v_3 = 1$$
$$.33v_1v_2^{-1} = 1$$
$$.5v_2v_3^{-1}\theta = 1$$

To illustrate the utility of this method in a more realistic case, the optimal yield in numbers, age structure and θ's for six different harvesting policies were calculated for an Atlantic menhaden (Brevoortia tyrranus) population. The Leslie matrix was taken from Horst (1977) and it's elements are given in Table 1. The BLAU (GOMTRY Algorithm) program from Kuester and Mize (1973) was implemented in BASICA to perform the calculations.

Results and Discussion

Results of the different harvesting strategies are given in Table 2. The constraints were not satisfied for the strategies which involved fishing only age classes 6-7 or 7. No solution was possible for these harvesting strategies. Initially, decreases in θ_k were small with increasing age-at-first-capture. However, after the first three age classes were not harvested, the proportion of the remaining age classes which had to be harvested

in order to satisfy the constraints increased significantly with age-at-first-capture. Yield in numbers also decreased with age-at-first-capture. The drop in yield from all ages fished to young-of-the-year not fished was particularly significant and reflects the numerical importance of age class 0 fish.

These examples have shown that geometric programming is a useful method for solving fisheries-type optimal harvesting problems. Optimal Θ_k, yield, and age structure are solved for simultaneously. The method is particularly useful when the population of intrerest has a large number of age or size classes. In comparison, the method of generating and solving the polynomial in Θ_k is not suitable when there are more than a few age or size classes.

This method has some other potential applications in fisheries. For species which are difficult to age, length data can be substituted for numbers of individuals. Solving the fisheries-type optimal harvesting problem can provide some meaningful insights into fisheries management problems. For example, different characteristics of the yield such as weight or reproductive value can be maximized and different harvesting strategies compared.

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Table 1. Elements of the Leslie matrix for Atlantic menhaden (*Brevoortia tyrranus*) after Horst(1977).

Age (i)	Eggs per adult (F_i)	Survivorship (S_i)
0	0	0.000077
1	7497	0.2015
2	19597	0.2015
3	29567	0.2015
4	37647	0.2015
5	43537	0.2015
6	47617	0.2015
7	51355	0

Table 2. Optimal yield, age structure and Θ for various Atlantic menhaden harvesting strategies.

	Age Structure					
Age Class	0-7	1-7	2-7	3-7	4-7	5-7
			(Age classes fished)			
v_0	0.999904	0.999904	0.999904	0.999904	0.999904	0.999904
v_1	7.69E-05	7.68E-05	7.70E-05	7.70E-05	7.70E-05	7.70E-05
v_2	1.55E-05	1.54E-05	1.54E-05	1.55E-05	1.55E-05	1.55E-05
v_3	3.11E-06	3.10E-06	3.08E-06	3.04E-06	3.13E-06	3.13E-06
v_4	6.26E-07	6.23E-07	6.16E-07	5.96E-07	5.56E-07	6.30E-07
v_5	1.26E-07	1.25E-07	1.23E-07	1.17E-07	9.89E-08	5.26E-08
v_6	2.53E-08	2.51E-08	2.46E-08	2.28E-08	1.76E-08	4.40E-09
v_7	5.09E-09	5.05E-09	4.92E-09	4.47E-09	3.13E-09	3.67E-10
Θ	0.9983	0.9972	0.9924	0.9722	0.8826	0.4147
Yield	1.73E-03	2.71E-07	1.47E-07	1.05E-07	7.93E-08	3.36E-08