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A SIMULATION MODEL OF MILLET AND COWPEA INTERCROP

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ABSTRACT

A simulation model for millet-cowpea intercrop is presented. The model is developed within the generic GROWIT framework and uses a spreadsheet. In addition, a spreadsheet template is presented for making stochastic dominance comparisons. The millet-cowpea model and the stochastic dominance spreadsheet are applied to the evaluation of millet and cowpea production strategies in Niger, West Africa.

Experience with use of the GROWIT-based simulation model to evaluate production strategies supports the potential of using generic models for analyzing crop innovations. The model can be adapted to new situations quickly. Over a period of several months, the GROWIT model was modified for intercropped annuals by two researchers working part-time. Preliminary calibration and validation took about 4 person-weeks. The validations suggest that the model captures the main tendencies of the yield distribution, but that accuracy, in terms of simulated yields as a percentage of observed yields, leaves something to be desired, especially in low rainfall years. In the economic analysis, simulation-derived distributions were used to rank crop production strategies according to risk characteristics using the stochastic dominance template. The conclusions obtained with the help of simulation complemented information from other sources.

The model is expected to be of use to biological and economic researchers working on the development of new intercrop technologies. The millet-cowpea crop association is grown widely in West Africa; thus, the model can be used directly in areas other than Niger. In addition, the generic nature of the model makes it possible to adapt it to other grain/legume intercrop systems.

UN MODELE DE SIMULATION DE LA CULTURE INTERCALAIRE MIL-NIEBE

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RESUME

Un modèle de simulation de la culture intercalaire mil-niébé est décrit. Le modèle est développé sur la base du progiciel générique GROWIT et utilise un tableur. Un gabarit de tableur est présenté pour permettre de faire des comparaisons de dominance stochastique. Le modèle mil-niébé et le tableur de dominance stochastique sont utilisés pour évaluer des stratégies de production de mil et de niébé au Niger, en Afrique de l'Ouest.

L'expérience acquise à l'utilisation du modèle de simulation basé sur le progiciel GROWIT pour évaluer les stratégies de production confirme le potentiel de l'utilisation des modèles génériques aux fins d'évaluer des innovations agricoles. Le modèle peut être adapté rapidement à de nouvelles situations. Sur une période de plusieurs mois, le modèle GROWIT fut adapté aux cultures intercalaires annuelles par deux chercheurs travaillant à temps partiel. Le calibrage et la validation préliminaires nécessitèrent quatre personne-semaines. Les validations successives suggèrent que le modèle saisit bien les tendances principales de la distribution des rendements. Cependant la précision du modèle, mesurée par le pourcentage des rendements simulés par rapport aux rendements observé, laisse quelque peu à désirer, particulièrement dans les années à faible pluviométrie. Au cours de l'analyse économique, on utilisa les distributions dérivées au moyen de la simulation pour classer les stratégies de production de cultures suivant des caractéristiques de risque, et ce en utilisant le gabarit de dominance stochastique. Les résultats obtenus au moyen de la simulation sont complémentaires aux informations provenant d'autres sources.

Le modèle sera utile aux chercheurs en agronomie et en économie qui travaillent au développement de nouvelles technologies de cultures intercalaires. La culture intercalaire mil-niébé est cultivée partout en Afrique de l'Ouest, par conséquent le modèle peut être utilisé ailleurs qu'au Niger. De plus la qualité générique du modèle permet son adaptation à d'autres cultures intercalaires céréales-légumineuses.

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1. INTRODUCTION

A key problem in assessing the agronomic and economic potential of a new cropping technology is to determine the distribution of outcomes. The answer to the question, "Does it work?", is usually not a simple yes or no, but an explanation of how often the technology performs and to what degree. The traditional way to determine the performance of a new cropping technology is to do field trials on several sites over a period of years. This method is reliable, but time-consuming and expensive. Simulation has been suggested as an additional tool for assessing the distribution of yields and other factors (Barrett and Peart, Lowenberg-DeBoer et al.). With the advent of generic plant growth models in a spreadsheet format (Lowenberg-DeBoer and Cherney), the development and use of such models has been facilitated. This document describes the use of a millet and cowpea intercrop model with examples drawn from work in Niger, West Africa. The document is expected to be of use to biological scientists and economists working on the development of new crop technologies. The millet and cowpea crop association is widely grown in West Africa; thus, the model may be used directly in areas other than Niger. In addition, the generic nature of the model makes it possible to adapt it to other grain/legume intercrop systems.

The document is organized first to provide background information, second, to describe the model structure, and third, to provide an example of its use. The rest of this first section is a brief overview of intercropping generally and millet and cowpea production specifically. The second section provides background information on biological modeling and the GROWIT model, including a general overview of the model structure. The third section explains the spreadsheet adaptation of the version of GROWIT used in this research. The fourth section outlines validation and calibration procedures. The fifth section applies the model to an analysis of the risk characteristics of certain millet and cowpea production innovations.

1.1 Intercropped Background

Intercropping is the practice of planting two or more crops in a field interspersed with each other. Numerous patterns of planting geometry may be used. For instance, the traditional American corn and pumpkin intercrop involved planting pumpkins or squash at intervals in the same row as the corn. The West African millet and cowpea intercrop usually involves planting one or more rows of cowpeas between the millet rows. All crops in an intercrop may be planted at the same time or there may be some degree of relay cropping which involves planting additional crops in an already-growing crop. The later planted crop can use any residual resources (water, sunlight, soil fertility etc.) that are not used by the initial crop and may continue to grow after the initial crop matures, thereby making fuller use of the growing season.

Many different crops are intercropped depending on the climate, soil, price relationships, food preferences and other factors. In the Americas, corn is traditionally intercropped with pumpkins and various types of beans. In the Sahel, millet and cowpea intercrop is the most common cropping pattern. In the Sudano-Sahelian zone, the rainfall permits a sorghum/peanut intercrop.

Sometimes millet and sorghum are intercropped. In the Sudanian zone, intercrops involving corn are used. In many areas of Africa, cassava is intercropped with grains and vegetables.

Crop associations have traditionally been used by low income farmers to make the most of their resources and to reduce risk. There is some evidence that intercropping can increase overall average yields. Vine crops can spread to make use of any space and sunlight left by taller grain crops. Intercrops can provide ground cover and reduce evaporation losses. In certain cases, leguminous crops can provide nitrogen for their associated crops. But even if overall production is not increased, the risk reduction alone may be enough to make intercropping worthwhile. With two or more crops growing in the field, crop total failure is less likely than one crop. With multiple crops and multiple planting, flowering, and grain fill periods, the chances of at least one crop having favorable conditions are increased.

Intercropping is most commonly found on unmechanized subsistence farms. With hand planting, weeding and harvesting, the presence of two or more crop species does not create major problems. With mechanization, intercropping becomes more difficult. For example, planting a second crop into a growing first crop is delicate and difficult to accomplish mechanically without damaging the first crop. Very little of the commercial crop production in the U.S. or Canada is currently intercropped, but some farmers are trying intercrops involving small grains and soybeans. In this system, soybeans are seeded into growing winter wheat or barley; the grain is harvested in early summer and the soybeans continue growing until fall. As is the case with the subsistence farmer, commercial farmers look to intercropping as a way of increasing production and managing risk.

1.2 Millet and Cowpea Intercrop

The millet and cowpea intercrop is well-adapted to poor soils and low rainfall of the Sahelian zone of West Africa (Swinton). Over 70 percent of the cropland in Niger is intercropped, and millet and cowpeas are the primary crop association. Millet is planted as soon as the rainy season starts in June or July. Millet is planted in hills in wide rows. The rows may be 1 to 2 meters apart and the hills may be spaced to 1.5 meters apart in the row. It is hand-weeded once or twice in the season. The long, compact spike heads are hand-harvested at the end of the rainy season in September and October. The stalks are used as a construction material and for fodder. Most millet is consumed on the farms where it is grown, but some enters commercial channels to be sold in local cities. In Niger, millet is not typically rotated with other crops, but is in a long-term crop/bush fallow rotation. With increasing population density, the fallow period is being shortened and, in some areas, is disappearing.

Cowpeas are a moderately drought-tolerant legume grown in many parts of the world (Duke). They are typically planted four to six weeks after millet, though the exact planting date depends on rainfall. They are typically planted in between the millet rows and are weeded along with the millet. Cowpeas are indeterminate. If soil moisture is adequate, cowpeas will continue to grow and flower after the millet is harvested. Some cowpeas are harvested at an

immature stage for direct consumption. Mature cowpeas may be harvested two or three times as the pods mature and dry. In some areas, cowpeas are grown primarily as fodder. In that case, the vines are cut while still green, dried and bound into bundles. Both cowpeas and cowpea hay are important items in commerce. The cowpea hay is sold in Nigerien cities to feed goats and other domestic livestock. The cowpeas may be sold locally, but are also exported to Nigeria.

In Niger and other areas of West Africa, the impetus to improve millet and cowpea production comes from the interrelated goals of food security and economic development. Millet is the primary food staple in the Sahel. If the millet crop fails, many people in the region risk starvation. In the 1970s and 1980s, food aid from foreign donors was used to cover the gap between production and consumption needs, but it is a stated goal of the government of Niger to achieve food self-sufficiency. In an area with over 90 percent of its population engaged in agriculture, economic development can be aided by making agriculture more productive. Better-fed farmers with more grain and legumes to sell could be the foundation for economic development.

Scientists at the Institut National de Recherches Agronomiques du Niger (INRAN), the International Crops Research Institute for the Semi-Arid Tropics (ICRISAT), and other organizations have been continuously involved in developing and testing new millet and cowpea technologies. Innovations have centered on new varieties of both crops, increased density, changes in planting geometry and improving soil fertility. A major element in this research has been on-farm trials (see for instance Ly et al.)

Relatively little of this newly-developed millet and cowpea technology has moved beyond the trial stage. One commonly-mentioned reason for the reluctance to extend the new technology is a general lack of confidence about the robustness of these innovations. In the highly variable climate of the Sahel with poverty-stricken subsistence farmers, very few researchers, extension agents, or government officials have been willing to whole-heartedly endorse any technological innovation. The next year might reveal the weakness of the proposed technology and create a disaster if it were widely adopted.

The millet and cowpea simulation model described here was developed as a tool to help researchers screen innovations and identify problems. Simulation does not replace field trials, but supplements them. The model can help extract the full information from the field trials. By combining field trial information with information about plant physiology and growth patterns, and with the 50 or more years of weather data available in some parts of the Sahel, researchers can begin to understand more fully how the technology behaves under the broad range of circumstances that might be encountered.

2. MODELING BACKGROUND

Simulation of plant growth has been widely used in agricultural research (Barrett and Peart), primarily for the major food crops. Because of the investment time and money required to develop a detailed model, simulation has been less commonly used in the analysis of "niche crops." These are crops or

cropping systems that are very important in some relatively limited geographic area, but do not have broad areas with common production patterns that characterize rice, wheat, maize and cotton. For a crop like maize, it is relatively easy to justify the cost of model development by spreading the cost over millions of hectares and millions of farmers. African agriculture is a maze of environmental niches, each with its own cropping pattern. It is harder to justify a major modeling effort for the relatively smaller area. The problem is compounded in Africa by the general lack of trained research personnel, main frame computers for large-scale models and the relative scarcity of research funding. The crop associations commonly grown in tropical Africa pose special modeling problems because of interactions between crops.

The development of generic plant growth models may change this situation. Generic plant growth models provide a framework of general biological principles which are applicable to a wide range of plants. The special characteristics of a particular crop can then be incorporated in the general framework. With a generic model, there is no need to construct each plant model from scratch. The primary disadvantage of a generic model is that it usually does not capture detailed differences in plant growth processes as well as a single-purpose model. Use of a generic model involves evaluating the tradeoff between accuracy and timeliness of results.

Model development can also be facilitated by the use of models in a spreadsheet format (Lowenberg-DeBoer and Cherney). Spreadsheets are easy to modify and do not require special programming expertise. Spreadsheet software is widely available and relatively standardized. For example, LOTUS1-2-3® is used almost everywhere microcomputers are used. This facilitates model portability. Spreadsheets have the disadvantage of being relatively slow and of being cumbersome for the manipulation of large quantities of data (for example, a long series of historical weather data).

2.1 Other Millet and Cowpea Models

At least two other millet models are in the process of being developed. A millet submodel was developed by Mishoe and Jones as part of a larger effort to model to examine Raghuva population dynamics. Raghuva is an insect pest which infests millet heads. A detailed millet phenology model is currently being developed through a United States Agency for International Development (USAID) project with International Benchmark Sites Network for Agro-technology Transfer (IBSNAT), in cooperation with Dr. J. Ritchie of Michigan State University and the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT). Neither of these models includes intercrop effects.

Under the IBSNAT program, the SOYGRO model (Jones et al.) is supposed to be the base for modeling the various members of the bean and pea families. To the best of the authors' knowledge a cowpea adaptation has not been made. The SOYGRO model does not incorporate intercrop effects.

The purpose of the model described here is not to compete with or replace the other millet or cowpea models. When they are available and incorporate the technology under study, the detailed biological models are superior; they

synthesize more biological information than the relatively simple model described here. But detailed biological models are not always available and do not always incorporate the plant characteristic under study. For example, available models do not include intercrop effects. Because of their complex structures, intercropping is not simply a matter of linking two of the existing detailed biological models. Parts of the models are usually highly interdependent, and changing one part of the model (for example, by inserting the effect of competition for water) may have repercussions in many other parts of the model. Modifying the detailed models entails time, expertise and funds that are sometimes not available.

In addition, the detailed models sometimes can be demanding in terms of the data required. For example, the SOYGRO model requires data on solar radiation which is not always readily available. For many sites, weather information is limited to maximum and minimum temperatures and rainfall.

The GROWIT type models in spreadsheet format can fill part of the research gap in modeling "niche crops" or "niche cropping systems." Because of the relatively simple structure and spreadsheet format, this type of model can be modified with less time, expertise and funds than would be required to modify a detailed model. In addition, the GROWIT type simulations can be used where data are not adequate to support the detailed models.

2.2 GROWIT Overview

The GROWIT model calculates daily plant growth based on temperature, rainfall and soil characteristics. The structure of the GROWIT model was originally developed by Smith and Loewer. The model used here was adapted from the spreadsheet template for switchgrass simulation developed by Lowenberg-DeBoer and Cherney, which in turn followed closely the FORTRAN GROWIT version worked on by Nebraska researchers (Brown et al).

GROWIT has been used to simulate a wide variety of crop species. It has been used most extensively for perennial crops, but some annuals have been modeled. Crops modeled include: Tall Fescue and Coastal Bermuda (Smith and Loewer), Red Clover and Tall Fescue in a mixed sward (Ewen), Alfalfa (Neels), Birdsfoot Trefoil and Tall Fescue separately and in a mixed sward (Musil), Switchgrass, Reed Canarygrass, Sweet Sorghum, and Sorghum Sudangrass (Cherney et al.), Tobacco (Binger et al.).

Potential growth is estimated by integrating over a continuous rate growth function which depends on air temperature. Air temperature in turn is modeled as a function of time, with the daily low at sunrise, the high at noon and the harmonic mean at sunset (Figure 2.1). The Brown et al. and Lowenberg-DeBoer and Cherney versions of the model use two quadratic functions to model this nonsymmetric relationship:

$$\text{(Eq. 2.1.1)} \qquad T = \begin{cases} a + b*DH + c*DH^2, & \text{for Sunrise<=DH<=Midday} \\ d + e*DH + f*DH^2, & \text{for Midday<DH<=Sunset} \end{cases}$$

where: T is temperature, DH is time of day in hours with DH=0 at sunrise and a, b, c, d, e, f are coefficients.

Midday here refers to solar noon. The coefficients of the temperature equation must satisfy:

$$\text{(Eq. 2.2.1)} \qquad Tmin = a + b*0 + c*0^2 = a = g$$

$$\text{(Eq. 2.2.2)} \qquad Tmax = a + b*DL/2 + c*(DL/2)^2$$

$$\text{(Eq. 2.2.3)} \qquad 0 = b + 2*c*DL/2 = b + c*DL$$

$$\text{(Eq. 2.2.4)} \qquad (Tmin + Tmax)/2 = d + e*DL + f*DL^2$$

$$\text{(Eq. 2.2.5)} \qquad Tmax = d + e*DL/2 + f*(DL/2)^2$$

$$\text{(Eq. 2.2.6)} \qquad 0 = e + 2*f*DL/2 = e + f*DL$$

where: Tmax = the daily maximum temperature,
Tmin = the daily minimum temperature, and
DL = total length of daylight in hours.

Equation (2.2.1) requires the morning quadratic segment to equal the daily low temperature at sunrise (DH = 0). Equation (2.2.2) forces the morning segment to equal the daily high temperature at solar noon (DH = DL/2); Equation (2.2.5) makes an equivalent requirement for the afternoon quadratic segment. Equations (2.2.3) and (2.2.6) are the first derivatives of their respective functions; they are set to zero at solar noon to insure that both functions have their maximum at that time. Equation (2.2.4) requires the afternoon quadratic to equal the harmonic mean temperature at sunset (DH= DL). Solving equations 2.2.1 through 2.2.3 and 2.2.4 through 2.2.6 simultaneously yields:

(Eq. 2.3.1) $$a = Tmin$$

(Eq. 2.3.2) $$b = 4*(Tmax-Tmin)/DL$$

(Eq. 2.3.3) $$c = -4*(Tmax-Tmin)/DL^2$$

(Eq. 2.3.4) $$d = Tmean$$

(Eq. 2.3.5) $$e = 4*(Tmax-Tmean)/DL$$

(Eq. 2.3.6) $$f = -4*(Tmax-Tmean/DL^2$$

where: Tmean = (Tmax + Tmin)/2.

The growth rate function also uses two quadratic equations; one for the low-temperature growth and the other for high-temperature growth:

(Eq. 2.4.1) $$GR = \begin{cases} g + h*T + i*T^2, & \text{for } TG1<=T<=TG3 \\ m + n*T + q*T^2, & \text{for } TG3<T<=TG2 \end{cases}$$

where: GR = growth rate in kg/ha/hr,
T = temperature in degrees Centigrade,
TG1 = the minimum temperature for growth,
TG2 = the maximum temperature for growth,
TG3 = the optimal temperature for growth, and
g,h,i,m,n,q are coefficients.

The equations are joined at the optimal growth temperature. Two equations are required to allow for nonsymmetry in the growth response. For example, growth may start at some low temperature with the rate increasing very slowly up to an optimal temperature, but thereafter the growth rate may drop off sharply with higher temperatures (Figure 2.2). Both the high- and low-temperature growth rate functions have coefficients calculated to give both functions a maximum at the maximum growth rate and optimal temperature. The low-temperature growth rate function coefficients are calculated to pass the function through the zero at the minimum temperature for growth. The higher-temperature growth rate function coefficients are calculated to pass the function through the temperature at which growth stops because of heat. Thus the low-temperature coefficients must satisfy:

(Eq. 2.5.1) $$0 = g + h*TG1 + i*TG1^2$$

(Eq. 2.5.2) $$GRMAX = g + h*TG3 + i*TG3^2$$

(Eq. 2.5.3) $$0 = h + 2*i*TG3$$

Equation (2.5.1) requires the low-temperature function to equal zero at the temperature at which growth ceases because of cold (TG1). Equation (2.5.2) insures that the low-temperature function will equal the maximum growth rate (GRMAX) at the optimal temperature. Equation (2.5.3) is the first derivative of the low-temperature function, and setting it to zero at the optimal temperature enforces the requirement that the function have a maximum at the optimal temperature.

And the high-temperature coefficients must satisfy:

(Eq. 2.5.4) $$0 = m + n*TG2 + q*TG2^2$$

(Eq. 2.5.5) $$GRMAX = m + n*TG3 + q*TG3^2$$

(Eq. 2.5.6) $$0 = n + 2*q*TG3$$

Equation (2.5.4) requires the high-temperature function to equal zero at temperature at which growth ceases because of heat (TG2). Equation (2.5.5) insures that the high-temperature function will equal the maximum growth rate at the optimal temperature. Equation (2.5.6) is the first derivative of the high-temperature function, and setting it to zero at the optimal temperature enforces the requirement that the function have a maximum at the optimal temperature.

The simultaneous solution of Equations 2.5.1 through 2.5.3 and 2.5.4 through 2.5.6 yields:

(Eq. 2.6.1) $$g = -GRMAX*TG1*(2*TG3-TG1)/(TG3-TG1)^2$$

(Eq. 2.6.2) $$h = 2*GRMAX*TG3/(TG3-TG1)^2$$

(Eq. 2.6.3) $$i = -GRMAX/(TG3-TG1)^2$$

(Eq. 2.6.4) $$m = -GRMAX*TG2*2*TG3-TG2)/(TG3-TG2)^2$$

(Eq. 2.6.5) $$n = 2*GRMAX*TG3/(TG3-TG2)^2$$

(Eq. 2.6.6) $$q = -GRMAX/(TG3-TG2)^2$$

It should be noted that the coefficient expressions 2.6.1 through 2.6.6 are linear in the maximum growth rate (GRMAX).

(Eq. 2.4.2) $$GR = \begin{cases} GRMAX*(\hat{g} + \hat{h}*T + \hat{i}*T^2), & \text{for } TG1<=T<=TG3.5 \\ GRMAX*(\hat{m} + \hat{n}*T + \hat{q}*T^2), & \text{for } TG3<T<=TG2 \end{cases}$$

where: $\hat{g}=g/GRMAX$, $\hat{h}=h/GRMAX$, $\hat{i}=i/GRMAX$, $\hat{m}=m/GRMAX$, $\hat{n}=n/GRMAX$, $\hat{q}=q/GRMAX$.

2.3 Integrating Over the Growth Function

By substituting the temperature equation (2.1.1) into the growth rate equation (2.4.1), the growth rate can be expressed as a fourth degree polynomial function of the time of day. By integrating over the growth rate function during the period when temperatures are between TG1 and TG2 for each day, the daily potential growth can be calculated. The variable of integration is the time of day (DH). Because the growth rate function is a polynomial, the integration has a closed form solution, a fifth degree polynomial. Because two separate functions are used to model growth and two separate functions are used to model morning and afternoon temperatures, the integration must be calculated in four segments: morning low-temperature growth, morning high-temperature growth, afternoon low-temperature growth and afternoon high-temperature growth. On most days, some segments will not be used; it is relatively unusual for temperatures to be below the growth range in the morning, rise beyond the growth range by solar noon and fall below the growth range again before sunset. The time in the morning at which integration starts is:

$$0, \text{ if } TG1<Tmin<TG2,$$

(Eq. 2.7.1) $$B1 = DL/2, \text{ if } Tmin>=TG2 \text{ or } Tmax<=TG1,$$

$$(DL/2)*(1 - [1 + (Tmin - TG1)/(Tmax - Tmin)]^{0.5}), \text{ otherwise.}$$

If the daily minimum temperature is greater than the maximum temperature at which growth occurs (Tmin>TG2) or the daily maximum temperature is less than

the temperature at which growth occurs (Tmax<TG1), then no growth occurs. The third alternative in Equation 2.7.1 occurs if temperatures rise into the growth range some time during the morning. It should be noted that as Tmin approaches TG1, the term in square brackets goes to 1 and B1 goes to zero. As Tmax approaches TG1, the term in square brackets goes to zero and B1 approaches DL/2. The equation for the third alternative is found by solving for DH in the temperature equation:

(Eq. 2.8.1) $$TG1 = g + h*DH + i*DH^2$$

The time in the morning at which low-temperature integration ends is either noon if the temperature never rises above TG3 or the time at which temperature reaches TG3 and integration switches from the low-temperature equation to the high-temperature equation in expression 2.1.1. The low-temperature upper bound (B2*) is:

(Eq. 2.7.2) $$B2^* = \begin{cases} 0, \text{ for } TG3<Tmin<TG2, \\ DL/2, \text{ for } Tmax<=TG3 \text{ or } Tmin>=TG2, \\ (DL/2)*\{1 - [1 + (Tmin - TG3)/(Tmax - Tmin)]^{0.5}\}, \text{ otherwise.} \end{cases}$$

B2* is also the morning high-temperature lower bound. The morning high-temperature upper bound is either solar noon or the time at which temperatures reach TG2. The morning high-temperature upper bound (B2) is:

(Eq. 2.7.3) $$B2 = \begin{cases} DL/2, \text{ for } Tmax<=TG2 \text{ or } Tmin>=TG2, \\ (DL/2)*\{1 - [1 + (Tmin - TG2)/(Tmax - Tmin)]^{0.5}\}. \end{cases}$$

The second alternative in (2.7.3) occurs if the temperature rises above the growth range some time in the morning. Thus, morning low-temperature integration is from B1 to B2* and morning high-temperature integration is from B2* to B2. In many cases, either B1=B2* or B2*=B2. For example, if temperature never rises above the optimal temperature (Tmax<TG3), then B2*=B2=DL/2.

In the afternoon, a similar set of bounds of integration must be calculated. The lower bound for the afternoon high-temperature integration (B3) is:

$$\text{(Eq. 2.7.4)} \quad B3 = \begin{cases} DL, \text{ for } Tmax <= TG1 \text{ or } Tmean >= TG2, \\ DL/2, \text{ for } TG1 < Tmax < TG2 \\ (DL/2)*\{1 + [1 + (Tmean - TG2)/(Tmax - Tmean)]^{0.5}\}, \text{ otherwise.} \end{cases}$$

Note that for all the morning bounds, the relevant sign of the square root term is negative; for all afternoon bounds the relevant sign is positive. The upper bound for the afternoon high-temperature integration and lower bound for the afternoon low-temperature integration (B4*) is:

$$\text{(Eq. 2.7.5)} \quad B4^* = \begin{cases} DL, \text{ for } Tmax <= TG1 \text{ or } Tmean >= TG3, \\ DL/2, \text{ for } TG1 < Tmax < TG3 \\ (DL/2)*\{1 + [1 + (Tmean - TG3)/(Tmax - Tmean)]^{0.5}\}, \text{ otherwise.} \end{cases}$$

The upper bound for the afternoon low-temperature integration (B4) is:

$$\text{(Eq. 2.7.6)} \quad B4^* = \begin{cases} DL, \text{ for } Tmax <= TG1 \text{ or } Tmean >= TG1, \\ (DL/2)*\{1 + [1 + (Tmean = TG1)/(Tmax = Tmean)]^{0.5}\}, \text{ otherwise.} \end{cases}$$

For instance, the expression for morning low-temperature growth (GML) is:

$$\text{(Eq. 2.9.1)} \quad GML = \int_{B1}^{B2^*} [g+h*(a+b*DH+c*DH^2)+i*(a+b*DH+c*DH^2)^2]\,dDH$$

(Eq. 2.9.2) $$GML = \int_{B1}^{B2^*} [(g+h*a)+b*(h+2*i*a)DH+(h*c+i*b^2+2*i*a*c)DH^2 +2*i*b*DH +i*c *DH]dDH$$

(Eq. 2.9.3) $$GML = [(g+h*a)*(B2^*-B1)+b*(h+2*i*a)*(B2^*-B1)^2/2 +(h*c+i*b^2+2*i*a*c)*(B2^*-B1) /3+2*i*b*(B2^*-B1)^4/4 +i*c^2*(B2^*-B1)^5/5]$$

Similarly, the morning high-temperature growth (GMH), the afternoon high-temperature growth (GAH) and the afternoon low-temperature growth (GAL) are:

(Eq. 2.9.4) $$GMH = [(m+n*a)*(B2-B2^*)+b*(n+2*q*a)*(B2-B2^*)^2/2 +(n*c+q*b^2+2*q*a*c)*(B2-B2^*)^3/3+2*q*b*(B2-B2^*)^4/4 +q*c^2*(B2-B2^*)^5/5]$$

(Eq. 2.9.5) $$GAH = [(m+n*d)*(B2-B2^*)+e*(n+2*q*d)*(B2-B2^*)^2/2 +(n*f+q*e^2+2*q*d*f)*(B2-B2^*)^2/3+2*q*e*(B2-B2^*)^4/4 +q*f^2*(B2-B2^*)^5/5]$$

(Eq. 2.9.6) $$GAL = [(g+h*d)*(B2^*-B1)+e*(h+2*i*d)*(B2^*-B1)^2/2 +(h*f+i*e^2+2*i*d*f)*(B2^*-B1)^3/3+2*i*e*(B2^*-B1)^4/4 +i*f^2*(B2*B1)^5/5]$$

The sum of potential growth in all four segments equals potential daily growth (GDP):

(Eq. 2.10.1) $$GDP = GML + GMH + GAH + GAL$$

Because the potential growth expression is linear in the growth rate coefficients (see 2.4.2) and the growth rate coefficients are linear in the maximum growth rate (GRMAX), the potential growth can be written as the maximum growth rate multiplied by a temperature factor:

(Eq. 2.10.2) $$GDP = GRMAX * TEMPFAC$$

where: $TEMPFAC = GML/GRMAX + GMH/GRMAX + GAH/GRMAX + GAL/GRMAX$.

2.4 Limiting Factors

Factors other than temperature affect growth through the maximum growth rate, which is limited to the growth that can be achieved with the most limiting factor. This approach ignores interactions among factors, but it is widely recognized as a good starting point in plant growth modeling (Salisbury and Ross, 1978). The limiting factors are leaf area, soil water and fertility:

(Eq. 2.11.1) $$GRMAX = R_0 * \min[LAF, SWF, FERTFAC]$$

where: R_0 = maximum possible growth rate under ideal conditions,
LAF = leaf area factor,
SWF = soil water factor and
FERTFAC = soil fertility factor.

The leaf area factor, soil water factor and soil fertility factor are expressed as a proportion of the maximum potential growth (R_0). Thus, they are between zero and one.

Leaf area is important because the amount of photosynthesis that can take place is limited by the surface available for collecting solar energy. The leaf area factor in GROWIT has been modeled as a linear (Brown et al.) or quadratic (Smith and Loewer) response function with a plateau that is achieved when leaf area is sufficient to make full use of available sunlight (Figure 2.3). As part of the effort to keep the model simple, leaf area is not calculated directly, but the leaf area effect is expressed as a function of accumulated dry matter. This is based on the assumption that there is a close relationship between dry matter and leaf area for any given plant variety, determined by leaf structure and geometry of leaf placement on the plant. In some versions of the GROWIT model, senescence is accounted for by including a downward sloping segment in the leaf area response at high dry matter accumulations. In the Lowenberg-DeBoer and Cherney version, senescence is modeled separately; thus to avoid double counting senescence effects, no downward sloping segment is included. The leaf area can be expressed as the proportion of maximum potential growth attainable with current leaf area. For instance:

(Eq. 2.12.1) $$LAF = \min[LAINT + L1*ADM + L2*ADM^2, 1]$$

where: LAINT = leaf area intercept,
L1,L2 = coefficients and
ADM = accumulated dry matter.

The quadratic response curve is chosen here because it is reasonable that at low leaf area coverage increases in efficiency of sunlight use occur more

rapidly than they do at high dry matter levels when more overlap and shading occur. Biological scientists often have information concerning two points on this function: the leaf area intercept and the accumulated dry matter at which leaf area is sufficient to fully use available sunlight (QQ1). The leaf area intercept indicates the proportion of the maximum potential growth rate that is achievable with no dry matter accumulation; for example, the growth achievable by a temperate climate perennial in the spring before appreciable leaf area has developed. The perennial growth would be based on energy in root reserves. Similarly for an annual plant, the leaf area intercept might be the proportion of the maximum growth rate possible with energy drawn from the seed. Density of the stand also affects the leaf area intercept. If the stand is very sparse, a smaller proportion of the potential growth is achievable from seed or root sources, even if later on the full grown plants are able to fully use the sunlight. To drive the sloping segment in expression 2.11.1 through the 1 at ADM=QQ1, the following equations must hold:

(Eq. 2.12.2) $$1 = LAINT + L1*QQ1 + L2*QQ1^2$$

(Eq. 2.12.3) $$0 = L1 + 2*L2*QQ1$$

Simultaneous solution of Equations 2.12.2 through 2.12.3 yields:

(Eq. 2.12.4) $$L1 = 2*(1-LAINT)/QQ1$$

(Eq. 2.12.5) $$L2 = (1-LAINT)/QQ1^2$$

The resulting function allows the leaf area factor to increase at a decreasing rate with increases in accumulated dry matter up to a maximum at QQ1 Figure 2.3).

The effect of moisture availability on growth rate is represented by an sigmoid curve with an inflection point at 50 percent of soil moisture holding capacity (Figure 2.4). This reflects the observation that at soil moisture levels below 50 percent of capacity, plant growth slows markedly, while at levels over 50 percent, growth rapidly approaches the potential. The sigmoid specification contains more information about the plant response mechanism than the entirely empirical function suggested by Smith and Loewer, but it is no more than a rough approximation of a very complex phenomenon. Because the model has been used on well-drained sites, no water logging effect is included. Because the curve is only a rough approximation, the form of the curves is based on programming convenience and calibration. Brown et al. specified this function as a series of linear segments. Lowenberg-DeBoer and Cherney specify the function as two exponential curves because this permits a more concise formulation that fits into the limited programming capacity of a spreadsheet cell. The sigmoid curve can be written as:

$$\text{(Eq. 2.13.1)} \qquad SWF = \begin{cases} u_1 + \exp(u_2*W + u_3*W), & \text{for } W<=0.5 \\ u_4 + \exp(u_5*(W - 0.5) + u_6*(W - 0.5)^2), & \text{for } W>0.5 \end{cases}$$

where: SWF is the soil-water function,
W = soil moisture as a proportion of water holding capacity and
u_i = coefficients, i=1...6.

The lowest soil moisture at which plant growth is possible and the soil moisture level at which plant growth reaches the full potential are not well-studied. The moisture levels probably differ by soil type and plant species. For lack of more specific information, the Brown et al. and Lowenberg-DeBoer and Cherney models specify the end points of the sigmoid curve at zero and 100 percent of soil moisture holding capacity. Both models assume 32, 88, and 98 percent of potential growth can be achieved at 30, 50 and 80 percent of soil water holding capacity respectively. Thus, the coefficients must satisfy:

$$\text{(Eq. 2.13.2)} \qquad 0 = u_1 + \exp(u_2*0 + u_3*0^2),$$

$$\text{(Eq. 2.13.3)} \qquad 0.32 = u_1 + \exp(u_2*0.3 + u_3*0.3^2),$$

$$\text{(Eq. 2.13.4)} \qquad 0.88 = u_1 + \exp(u_2*0.5 + u_3*0.5^2),$$

$$\text{(Eq. 2.13.5)} \qquad 0.88 = u_4 + \exp(u_5*(0.5 - 0.5) + u_6*(0.5 - 0.5)^2),$$

$$\text{(Eq. 2.13.6)} \qquad 0.98 = u_4 + \exp(u_5*(0.8 - 0.5) + u_6*(0.8 - 0.5)^2),$$

$$\text{(Eq. 2.13.7)} \qquad 1.00 = u_4 + \exp(u_5*(1.0 - 0.5) + u_6*(1.0 - 0.5)^2).$$

Simultaneous solution of Equations 2.13.2 through 2.13.4 and 2.13.5 through 2.13.7 yields:

$$\text{(Eq. 2.13.8)} \qquad u_1 = -1$$

$$\text{(Eq. 2.13.9)} \qquad u_2 = 0.4197825$$

$$\text{(Eq. 2.13.10)} \qquad u_3 = 0.0168552$$

(Eq. 2.13.11) $u_1 = -0.12$

(Eq. 2.13.12) $u_2 = 0.4542654$

(Eq. 2.13.13) $u_3 = -0.4552161$

Soil moisture is estimated with a water budget: the new soil moisture level equals the previous day's level plus rainfall minus evapotranspiration. If this budget results in a negative value, the soil moisture is set to zero. If this budget results in a value greater than the soil water holding capacity, the capacity level is used. Capacity is defined in terms of the water holding capacity in the usual rooting zone of the plant. Excess rainfall is assumed to be lost in runoff. Thus the soil moisture might be expressed as:

(Eq. 2.14.1)
$$W_t = \begin{cases} 0, \text{ for } W_{t-1} + RAIN - EVAP <= 0 \\ W_{t-1} + RAIN - EVAP, \text{ for } 0 <= W_{t-1} + RAIN - EVAP <= \bar{W} \\ \bar{W} \end{cases}$$

where: RAIN = recorded rainfall for day and
EVAP = estimated evapotranspiration.
$\bar{W}$ = soil water at capacity.

In the Brown et al. and Lowenberg-DeBoer and Cherney versions of the model, the evapotranspiration is estimated using the Blaney-Criddle formula. This estimator is based on the observation that temperature and sunlight are the most important factors in determining evapotranspiration (Blaney and Criddle). Because records of actual sunlight are often not available, the estimator uses day length as a proxy. A crop and location specific factor is used to account for plant specific transpiration patterns, wind, relative humidity and other variables. The Blaney-Criddle formula was originally developed to estimate monthly evapotranspiration. Use of the estimator for daily evapotranspiration will often be quite inaccurate for any given day, though following the general trend of evapotranspiration quite accurately over periods of several weeks. A comparison of the Blaney-Criddle estimator with other evapotranspiration formulas can be found in Jensen. The Blaney-Criddle formula is one of the most widely-used evapotranspiration estimators because it uses commonly-available data. Daily evapotranspiration might be estimated as:

(Eq. 2.15.1) $$EVAP = CU*VEGFAC*SWF$$

where: CU = Blaney-Criddle consumptive use factor and
VEGFAC = a vegetation factor.

The Blaney-Criddle consumptive use factor is the product of mean temperature multiplied by the proportion of annual daylight hours occurring in the period under study. For metric applications, the factor can be written:

(Eq. 2.15.2) $$CU = Tmean*DL/TDL$$

where: TDL = the total daylight hours in a year.

In this formulation the crop and location specific factor (often denoted as "k") is the product of a vegetation component and a soil water component. Evidence related to the general form of these two factors is reviewed by Hanks and Ashcroft. In the Brown et al. and Lowenberg-DeBoer and Cherney models, the vegetation factor is modeled as a linear segmented function of vegetative dry matter (Figure 2.5). At low levels of vegetative cover, evaporation from the soil surface is the most important part of total evapotranspiration. Evaporation from a wet soil surface is high, but this surface dries rather quickly, thus limiting water loss to a small portion of the potential. As the plant grows, transpiration comes to dominate the evapotranspiration. This relationship might be expressed as:

(Eq. 2.15.3) $$VEGFAC = \begin{cases} 1, \text{ if } RAIN > 1 \\ 0.8, \text{ for } RAIN < 1 \text{ and } DM > 1500 \text{ kg/ha} \\ 0.14 + DM*0.00044, \text{ otherwise.} \end{cases}$$

This assumes that if the plant and soil surface is wet from rain, evapotranspiration will be at potential; if there is little or no vegetative dry matter and the surfaces are dry, there will be low evapotranspiration; and for full vegetative cover and dry surfaces, the evapotranspiration is somewhat below potential because of the shading and mulching effect of the vegetation. The choice of the specific values of VEGFAC=0.14 at DM=0 and VEGFAC=0.8 at DM>=1500 kg/ha are based on a review of the literature and discussions with agronomic researchers at Purdue University.

The soil water factor (SWF) used for the evapotranspiration estimation is the same as is used for modeling the effect of soil moisture on growth (12.13.1). The sigmoid form of this function fits the general principles of

soil moisture effects on evapotranspiration presented by Hanks and Ashcroft. Because general principles did not suggest a functional form significantly different from the soil moisture growth effect, model simplicity was maintained by using the same function for both factors.

The fertilizer response factor assumes a positive, but diminishing, response as fertilizer levels are increased (Figure 2.6). Brown et al. use a quadratic response function. Lowenberg-DeBoer and Cherney use a negative exponential and plateau function as follows:

(Eq. 2.16.1) $$FERTFAC = \min[1,\ Y_{N=0} + (1 - \exp(\ln(Y_{N=0})*N/N_{Y=max})]$$

where:

$Y_{N=0}$ = proportion of potential yield achieved with no fertilizer,
N_{Ymax} = fertilizer level for maximum yield and
N = fertilizer level.

More than one fertilizer factor can be used if data is available on separate nutrients.

2.5 Partitioning Growth

Growth may be partitioned in various ways between plant parts, leaves, stems, roots, etc. The earliest GROWIT versions (Smith and Loewer, Brown et al.) only partitioned growth between root and top growth as a function of photoperiod. This was used to model the phenomenon observed in perennial crops which store carbohydrates in the roots late in the growing season. The stored carbohydrates are used to initiate growth after the dormancy period. Following Holt et al., this photoperiod effect can be modeled as a linear function which is triggered by decreasing day length and reaches its full effect at another specified day length. This can be written as:

(Eq. 2.17.1) $$PF = \begin{cases} 1 + (C - 1)*(DL - A)/(B - A), & \text{for } JD>172 \text{ and } DL<A, \\ 1, & \text{otherwise.} \end{cases}$$

where: PF = photoperiod factor,
A = the day length at which plant begins root storage,
B = the day length at which the proportion of carbohydrates going into root storage reaches a maximum, and
C = the proportion of total carbohydrates going into root storage at day length B.

Another form of partitioning is senescence. The senescence modeling used by Smith et al. and by Lowenberg-DeBoer and Cherney assumes that plant growth lives for a certain period of time, dies and the dead material decays at a fixed rate. The decay may be expressed as a proportion of the total senescent material. The daily change in senescent material may be written:

(Eq. 12.17.1) $$SM_t = AG_{t-s} + SM_{t-s} * DF$$

where: SM_t = senescent material at time t,
AG = estimated actual growth at time t-s
s = the number of days plant growth lives, and
DF = decay factor.

The rate of decay is most easily calculated from the half-life estimate, that is from the time it takes for half of the dead material to decay:

(Eq. 2.17.2) $$0.5*SM = DF^{HL}*SM$$

or

(Eq. 2.17.3) $$DF = (1/HL)^{0.5}$$

where: HL = half-life.

2.6 Daily Biomass Estimates

From the model outlined above, daily estimates of the changes in vegetative dry matter can be made:

(Eq. 2.18.1) $$AG_t = GRMAX*TEMPFAC*PF - AG_{t-s}$$

The effects of temperature and sunlight are incorporated in the TEMPFAC term. The limiting factors, leaf area, soil moisture and soil fertility, as well as the genetic potential of the plant, are included in the GRMAX. The photoperiod effect (PF) incorporates the root storage observed in perennial plants. The final term subtracts any senescent vegetative material. Total biomass on any day is the sum of accumulated growth estimates plus undecayed senescent material.

3. THE MILLET-COWPEA MODEL

The basic GROWIT model outlined in Section II was modified to account for the interaction between the two crops grown in association and for the annual nature of both crops. Added to the model were planting rules triggered by the onset of the rainy season and explicit modeling of the stages of growth, germination, vegetative growth, flowering and grain fill. The calculation of the temperature factor (2.1.1 through 2.9.6) remain the same as in the basic model. The modifications are concentrated on the leaf area factor, the soil water budget and the partitioning of carbohydrate production.

The competition between the two crops for available sunlight was modeled by assuming that millet was the dominant crop and cowpeas used residual space. This assumption was made because millet is tall and therefore could shade the cowpeas and because millet is customarily planted earlier than the cowpeas in Niger. Total leaf area factor was limited to 1. The millet plant was assumed to be able to shade up to one-half meter in any direction. Under this assumption if the millet was planted in 1.5-meter row widths with 1.5 meter between plants in the row, 0.5 meter would be available between the rows and 0.5 meter would be available between plants in the row for cowpea growth even when the millet reached full vegetative growth. Likewise under this assumption, with row widths of less than one meter and more than 10,000 plants per hectare, no space is available for cowpeas under fully grown millet. The dense planting of millet does not, however, totally eliminate cowpea production in the model because cowpeas can make use of unused millet space early in the season when the millet plants are small, late in the season when millet leaves have become senescent and if poor conditions at germination reduce the surviving plant population. Cowpeas were also assumed to be able to spread up to one-half meter on any side of the plant. Given this difference between vining and nonvining cowpea varieties, the spreading assumption may be an important characteristic for distinguishing between varieties in the model.

The soil water budget is calculated assuming that both crops draw from a common soil water resource. In particular, the vegetative cover of both crops affects the VEGFAC and, in turn, the evapotranspiration calculation. A separate rainfall factor is calculated for each crop because the crops may react differently to soil moisture levels. In calibrating the model, it was noted that excessive cowpea growth occurred at low soil moisture levels using the rainfall factor outlined in Section II. The rainfall factor function for cowpeas was modified to stop growth below 25 percent of soil moisture holding capacity. The base model allowed some growth as long as some moisture was available. The millet model used the base model rainfall factor calculation. Four plant growth stages were used in both the millet and cowpea submodels: 1) germination, 2) vegetative growth, 3) flowering and 4) grain fill. This modeling is a compromise between the desire for simplicity and for the need to include crucial phases in the plant development. Many plant growth models have a much more detailed modeling of development; for instance, the appearance of individual leaves might be specified. At each of the stages included in the modified GROWIT model, weather conditions may limit development and thus reduce the potential for development and eventual yield.

The germination period starts at planting and continues until 100 percent of seed is germinated or up to eight days. Planting is triggered by a rain of 10 mm or more and continues only if at least 10 mm of rain has fallen in the last week. The model also permits the user to specify a planting day. Evidence suggests that temperature is the primary factor determining millet germination in moist soil (Garcia-Huidobro et al.). Thus germination was modeled as a function of the temperature factor. The 10 mm rainfall requirement limits germination to relatively moist soil conditions. Germination was determined as the previous germination percentage plus the temperature factor divided by 100.

Vegetative growth begins after germination and continues until flowering is triggered. During the vegetative growth stage, all carbohydrate production is allocated to the leaves and stems. If growth is limited during this period, the leaf area factor may not reach 1 and the leaf area during flowering and grain fill may be inadequate to support high yields.

Flowering is probably the most sensitive period because it is a relatively short period and because the number of seeds set in this period effectively limits grain production. The plant has a limited capacity to compensate for reduced seed set by increasing the size of individual seeds. Millet flowering is triggered by photoperiod or by a certain number of days after planting, whichever comes first (Ong and Monteith, Ong and Everard). The number of days after planting and the day length for flowering differ between millet varieties. In the model, the flowering period is limited to three weeks. Because continued vegetative growth is observed in millet during flowering and grain fill periods, only a portion of carbohydrate production is allocated to produce the flowering parts. By calibration, this proportion was set at 20 percent. The flowering parts are recognizable in the final yield as the chaff and other materials that are separated from the grain at threshing.

In the model, the grain fill period starts three weeks after the beginning of flowering. During the grain fill period, a portion of the carbohydrate production is allocated to the seed. The proportion used is the same as for the partition of growth during the flowering period (20 percent). Total dry weight of grain is limited by the seed set in the flower period. The dry weight of flowering parts is taken as a proxy for seed set. The weight of the flowering parts divided by 1 minus the threshing percentage is used as the upper limit on grain production. Because the threshing percentage is used to define an upper limit, the upper limit of the interval for threshing percentage was used. In addition to direct allocation of growth, the model permits mobilization of carbohydrates stored in the stem and leaves to complete grain fill under adverse conditions. In the model, mobilization of stem reserves occurs when the rainfall factor drops below 0.5. The amount of carbohydrate mobilized is determined by the growth that would occur if rainfall were adequate. Vegetative dry matter is reduced by the amount of the mobilized carbohydrate. In the context of the plant's ability to mobilize stem and leaf reserves, the continued vegetative growth during flowering and grain fill can be understood as a strategy to insure that at least some seed reaches maturity. If adverse conditions strike during the grain fill period, the plant can draw on the vegetative reserves.

3.1 Guide to the Model Templates

The intercrop model is based on a two-template, two-step GROWIT version. The step 1 template (Screens 1-4) contains the day length and temperature-related calculations. It produces daily day length, mean temperature and temperature factor (see Equation 2.10.2) which are used as input in the second step template. This division of calculation is useful because the maximum, minimum and optimum temperatures for growth are often quite well-studied by biological scientists, but are not often manipulated in the development of new varieties. Other aspects of the plant growth, fertilizer response, senescence, etc., are often less well-studied, but more often manipulated to develop improved production techniques. Thus, in many cases, the temperature factor template can be calculated once for a given season's temperatures and the calibration or simulation activities can be concentrated on the second template. As a result of the two-template system, it is possible to work with smaller templates and achieve faster recalculation times.

The basic data used by the temperature factor template are the maximum and minimum temperatures for the period under study. The parameters needed are latitude of the study site, minimum temperature for growth (TG1), maximum temperature for growth (TG2), and the optimum temperature for growth (TG3). The initial parameters used in the millet calibration are listed in the top left-hand corner of the spreadsheet (Screen 1). For both millet and cowpeas, the initial temperature values were drawn from a review of the literature. For millet, Ong and Monteith provided the basic information. For cowpeas, Duke, Singh and Wien and Summerfield were the primary sources of temperature information. To the right of the parameter list, the values of constants are calculated (Screen 2). The SINLAT (I5) and COSLAT (L5) terms are used in the calculation of day length. The i/R, h/R, g/R, q/R, n/R, and m/R terms correspond to the terms with "hats" in expression 2.4.2. They are the growth equation coefficients with the maximum growth rate factored out. Below the parameter list and calculation of constants, the calculations of the day length, mean temperature and temperature factor are made on a daily basis, with one line for each day.

The daily calculation is broken down into 30 columns (screens 1-4). The cells formulas are given in Table 1. Column A contains the 1 Julian day. The Julian day series can be changed by changing the starting value in A15. Columns B, C and D are devoted to the day length calculation. The day length formulas are drawn from the *Almanac for Computers* developed by the U.S. Naval Observatory. Because the model needs only the day length, not the time of sunrise and sunset, the longitudinal correction drops out of the expression. Columns E and F are temperature data. Column G is the calculation of the harmonic mean temperature. The remaining columns are devoted to the integration over the temperature functions as described in Section II. Columns H-J are the morning integration boundaries (B1<=B2*<=B2). Columns K and L are temperature function coefficients. Column M (Step I) contains the linear and squared terms in the morning low-temperature integration expression. Column N contains the cubic and fourth degree terms.

Column O contains the fifth degree term and the sum of all terms for the morning low-temperature integration. The morning high-temperature integration

is organized similarly in Columns P-R. It should be noted that the temperature functions coefficients (a,b) in columns K and L are used for both the high and low-temperature integration. The afternoon integration is organized in the same pattern in columns S-AC. The temperature factor is calculated in column AD as the sum of columns O, R, Z and AC.

The basic GROWIT spreadsheet is organized with parameters listed in the upper left-hand corner of the template, constants calculated to the right of the parameter list, and the daily growth estimate below the parameter list, with one row per day (Screens 5-11). The millet and cowpea model is organized as two of these individual models side by side. The millet model occupies columns A-U and the cowpea model uses columns V-AK. The cowpea model use fewer columns than the millet model because some columns do not have to be repeated. For instance, day length, mean temperature, rainfall and soil moisture are the same for both cowpeas and millet.

The millet parameter list (Screen 5) includes latitude (D5), planting and leaf area information (D7..D11), the soil moisture parameters (D13..D14), flowering and grain set parameters (D16..D19), temperature and ideal growth rate information (D21..D24), soil fertility information (D26..D31), and senescence half-life (D33). The latitude is used in the calculation of the total annual daylight hours (H5), which is in turn used in the evapotranspiration calculation (column I). The total daylight hours are estimated as the product of the day length at the vernal equinox multiplied by the number of days in a year using the day length formula from the *Almanac for Computers* (Table 3). The constant 0.516813 is the solar angle at the vernal equinox, approximately julian day 81.25 (see Table 2 for solar angle formula). This estimation is effective because day length varies symmetrically around the day length at the equinox, with each day shorter than the day length at the equinox matched by a day longer than the day length at the equinox.

The millet planting date and leaf area parameters include: planting date, leaf area intercept, QQ1, planted hills per hectare and row width. Planting in the model (column J) is triggered by a rainfall of 10 mm or more, or by the planting date in cell D7, whichever comes first. The entry (365) in D7 in screen 5 is made to insure that planting was triggered only by rainfall for initial model runs. As all dates in the model, the planting date is specified in julian days. The leaf area intercept is the percentage of growth possible from energy stored in the seed (see Equation 2.12.1). QQ1 is the level of vegetative dry matter per hectare at which full utilization of sunlight is possible. The leaf area intercept and QQ1 are used in the leaf area factor calculation (column E). The row width and planted hills per hectare are used in calculating the leaf area factor and the vegetation factor (column H). The length of row per hectare is calculated in cell H10 (formula in Table 3). The density of plants in the row is calculated in cell H11 (formula in Table 3). Using the assumption that the millet plant can use up to a one square meter space, the density is expressed as the ratio of planted hills to row length. If there is more than one hill per meter, the model assumes that full use of the space in the row is possible. If there is less than one hill per meter of row, the model assumes that there are gaps which can be bare soil or be filled by cowpea vines.

The soil moisture conditions are described with two parameters: the moisture holding capacity in the top meter of soil (D13) and the initial moisture in the top meter of soil (D14). Millet roots extend up to 3 meters into the soil, but according to Dancette, 75 percent of the water uptake occurs in the top 30 cm. The 28 mm holding capacity is based on measurements reported by Mahamane. Because the model starts at the end of the dry season, it was assumed that there was no initial soil moisture.

The millet flowering and grain set parameters include: the day length at which flowering is triggered (D16), the number of days after sowing that flowering is begun if not triggered earlier by the photoperiod (D17), the maximum threshing ratio (D18) and the proportion of carbohydrate production diverted to flowering and grain fill (D19). The "Flower Hour" and "Flower Onset" parameters are used in the determination of the growth stage (column J). The primary reference used in setting the photoperiod was Ong and Everard. The maximum threshing ratio is used in determining the upper limit on grain production in column U. The maximum threshing ratio of 70 percent was based on discussions with INRAN agronomists (C. Reddy and A. Berrada, personal communication). The grain fill factor is used in determining partitioning of dry matter production in vegetative growth (column N), flower dry matter (column R) and grain dry matter (column U). The grain fill factor was determined by calibration.

The millet growth rate under ideal conditions was calibrated with initial values based on previous work with forage sorghums (Cherney et al.). The temperatures are given here for information. All temperature-related calculations are done in the temperature factor template and summarized in the temperature factor which is copied into column M from the temperature factor template.

The millet soil fertility parameters (D26..D31) include both nitrogen and phosphate response parameters. Nitrogen and phosphate are the two major nutrients lacking in Nigerien soils; there is little evidence of a potassium response in Niger. The general fertilizer response function is given in Equation (2.16.1). The parameter labeled 'Y at 0 N" is the percentage of potential growth achievable without nitrogen application and if all other factors are nonlimiting. It is the intercept of the nitrogen fertilizer factor calculated in H28. "N for Max" is the nitrogen required for maximum yield; this is the lowest nitrogen level at which the plateau level yield is achieved. The nitrogen fertilizer application for this simulation is entered in cell D28. The phosphate level in the soil before fertilization expressed in parts per million is entered in D29. The soil phosphate level of 3 ppm is based on soil tests reported by Mahaman and by Bationo et al. The soil phosphate level required for maximum yield is entered in D30. The phosphate level required for maximum yield was calibrated with initial values drawn from work by Jomini et al. (1988a and 1988b). The phosphate fertilizer application in kg/ha is entered in D31. To calculate the fertilizer factor, fertilizer application and soil tests must be expressed in common units; thus the soil test levels are converted to kg/ha in cells H29 and H30. The conversion to kg/ha assumes a depth of 15 cm and a bulk density of 1.6 g/cm^3. The phosphate response function calculation differs from the nitrogen response calculation in H28, because it is based on soil phosphate, not on applied fertilizer. It

is assumed that if the soil phosphate level was really zero, there would be no growth; thus the function passes through the origin. The effectiveness of phosphate fertilizer in raising soil phosphate levels (0.8 in the first year) is based on work by Jomini and colleagues. The soil phosphate level after fertilization in the phosphate fertility factor (H31) is composed of the original soil level, plus a portion of the applied phosphate. The 0.8 factor assumes use of commercial phosphate fertilizer. Use of rock phosphate would require a lower effectiveness factor.

The half-life of senescent millet vegetation is entered in cell D33. The proportion of the dead material which is lost to decay each day is denoted as "Delta." The proportion of the dead material which remains each day is therefore one minus Delta. This remaining proportion is calculated in cell H33 and used in column T.

The cowpea parameters are organized in a similar fashion in cells Z5 to Z33 (screen 9). The cowpea constants are calculated in cells AD5, AD8, AD10, AD11, AD17 and AD28-AD33 (screen 10, formulas in Table 3). The cowpea planting is defined with respect to millet planting. The minimum number of days to wait for cowpea planting after planting millet is entered in cell Z5. In cell AD 5, minimum number of days millet planting occurs after millet germination is calculated, assuming complete germination of millet requires seven days. The cell AD5 constant is used in the cowpea growth stage calculation (column X).

The density and leaf area parameters are entered in cells Z8 to Z11. Both cowpea leaf area intercept (Z8) and the QQ1 (Z9) value are determined by calibration. The 1 minus the leaf area intercept (AD8) and QQ1 value are used in the leaf area factor calculation in column W. The planted hills per hectare (Z10) and the row width (Z11) are used in determining the density factor (AD11), which is used in the leaf area factor calculation (column W) and in the cowpea vegetation factor for evapotranspiration purposes (column AJ).

The cowpea flowering and grain set parameters are entered in cells Z16 to Z19. The day length which triggers flowering is entered in cell Z16. The 11.5-hour day length used is drawn from Singh (see also Wien and Summerfield). The day length for flowering onset is used in the growth stage calculation in column X. The number of days after sowing that flowering starts (if not triggered by the photoperiod) is entered in cell Z17. The maximum threshing ratio is entered in Z18. The Z17 value is used in the growth stage calculation in column X. The maximum threshing ratio is used in the grain fill column (AK) and the mobilization for grain fill calculation (column AD). Both the number of days after sowing for flowering and the maximum threshing ratio is based on discussions with INRAN agronomists (C. Reddy and A. Berrada, personal communication). The grain fill factor is entered in Z19. It is used in the vegetative growth calculation in column AC, the mobilization for grain fill calculation in column AD, the pod and flower calculation in column AF and the grain fill calculation in column AK. The grain fill factor is defined by calibration.

The cowpea growth rate and temperature parameters are entered in cells Z21 to Z24. The temperature calculations for cowpea are done in the

temperature factor template and copied to column AA. The temperatures are repeated in Z22-Z24 for information. The ideal growth rate (R_0) is entered in Z21. The rate used in the model was defined with calibration, with the initial value defined somewhat lower than the millet growth rate as suggested by Singh.

The cowpea soil fertility parameters are entered in cells Z26 to Z31. The nitrogen response section is included here for completeness, but no nitrogen response was used in the cowpea model. The nitrogen application required for maximum yield was set at 1 kg/ha (Z27) to permit evaluation of the logarithm in the nitrogen fertilizer factor calculation (AD28); a maximum yield at zero nitrogen application would generate errors throughout the model. The phosphate response was defined as with millet. The phosphate level required for maximum yield was refined with calibration.

The half-life of senescent cowpea vegetation is entered in Z33. The proportion of senescent vegetation which remains after each day's decay is calculated in AD33.

3.2 Millet Daily Calculations

The daily calculations for millet are in columns A through U starting at line 39 (Screens 5-11). Cell formulas are given in Table 4. Five of these columns are input data: A, the Julian Day; B, the day length; C, the rainfall; D, the harmonic mean temperature; and M, the temperature factor. Daily rainfall in millimeters should be imported or copied into the growth model. The other data columns are copied in from the temperature factor spreadsheet.

Columns E through I and K are used to calculate the factors affecting the maximum possible growth rate (Column L). Column E is the millet leaf area factor calculation. The millet leaf area factor in the basic model assumes that 100 percent of the field surface can be covered with vegetation (2.12.1). With certain planting patterns and germination conditions, millet may not cover the entire surface area. Thus the expression (2.12.1) is modified, replacing 1 with the production of the proportion of the field covered by the row at full vegetation (1/D11), the proportion of planting hills with surviving plants (Column K) and the density in the row (H11) (Table 4).

The water aspects of the millet model are treated in columns F-I. The soil moisture as a proportion of capacity is calculated in column F using the unmodified expression 2.14.1. The rainfall factor is calculated in column G using the expression from 2.13.1. The vegetation factor for the evapotranspiration estimate is calculated in column H. This vegetation factor applied only to the proportion of the field which can potentially be covered by millet vegetation; thus the entire factor is multiplied by the proportion of the field covered by 1-meter-width rows (1/D11). The cowpea vegetation factor is in column AJ. The model assumes that the narrowest row width considered is 1 meter. The factor is calculated as the sum of the factor for bare soil, that is 0.14 for dry soil surface or 1 after a rain, and the factor for the millet vegetation (2.15.3). The proportion of the row actually covered with vegetation is estimated as the product of the planted density in the row (H11) and the proportion of the planted hills surviving (column K). Evapotranspiration

(column I) is estimated as in expression (2.15.1) with the single modification that the vegetation factor is the sum of the millet and cowpea factors (Column H + Column AJ).

The proportion of hills germinating and surviving is calculated in column K. If the growth stage is "pre" (preplant), the value returned is zero. If the growth stage is "GERM" (germination) and the rainfall in the last seven days is 10 mm or more, then germination is accumulated at the rate defined by the temperature factor divided by 100. This calculation allows germination to be cut short by lack of rain and to be reduced by high temperature because the temperature factor would be reduced by temperatures over the optimal level. The germination differs from the vegetative growth calculation because the effect of water availability is treated as an all or nothing effect, instead of a continuous rainfall factor, and the seed is assumed to supply nutrients for growth, so no fertility factors are included.

The growth stage calculation in column J affects the germination and survival factor (column K), the maximum possible growth (column L) and the partitioning of carbohydrate production in columns O, P, R, and U. Planting is triggered by a rain of 10 mm or more or by the planting day specified in D7, whichever comes first. The germination period continues as long as the germinating proportion is less than 1 and the rainfall in the last week is 10 mm or more, but not more than eight days. The vegetative growth stage (1) continues until flowering is triggered by the numbers of days after emergence (H17) or by the photoperiod (D16). Flowering continues for 21 days. The grain fill stage occupies the rest of the growing period.

The maximum possible growth (Column M) is defined to be zero before planting and expression (2.11.1) after planting. The effects of planting density and survival during germination are through the leaf area factor (E40). Because two soil nutrients are considered, two fertility factors are included (F28, F31).

The estimate of actual millet growth (column N) is the product of the maximum possible growth (column L) and the temperature factor (column M).

Millet carbohydrate production is partitioned in columns O through U. During germination ("GERM") and the vegetative growth stage (1), all carbohydrate production is allocated to vegetative growth (Column O). During flowering (2) and the grain fill period (3), the proportion one minus the grain fill factor (D19) is allocated to vegetation. Stages 2 and 3 are treated separately in the if statement because during calibration, partition proportions were varied between periods. Vegetative dry matter is accumulated in column Q as the sum of last period dry (Column Q), plus vegetative growth (column O), minus carbohydrates mobilized for grain fill (column P) and senescence (Column S). The accumulated vegetative dry matter is an input into the leaf area factor calculation (Column E) and the millet vegetation factor (Column H). Flower dry matter is accumulated in column R during the flowering period (2). The proportion D19 (grain fill factor) is allocated to flowering. The flower dry matter is an input into the grain dry matter calculation (Column U) and the mobilization for grain fill calculation (Column P). Daily senescence is shown in column S. In the line 40 formulas given in Table 4, no

senescence formula is included because there is not growth (or even cell) 60 days earlier. The senescence formula starts in cell S98 with the entry +O39. The dead dry matter in column T is the sum of the newly senescent material (column S) and the remaining proportion of dead material from the day before (Column T multiplied by H33). The grain dry matter in column U is accumulated only if the growth stage is 3 and only if the grain dry matter already accumulated is less than the upper limit defined by dividing the flower dry matter by one minus the threshing ratio (Column R/(1-D18)). Grain dry matter is accumulated as the sum of the previous day's grain dry matter, plus the proportion of carbohydrate production allocated to grain (Column N multiplied by D19) and plus the mobilization for grain fill.

3.3 Cowpea Daily Calculations

The daily calculations for cowpeas are shown in columns V through AK (Screens 8-11). The cell formulas are shown in Table 5. The only column of data among the cowpea columns is the temperature factor in column AA. This is copied in from the temperature factor spreadsheet. The cowpea model uses the julian day, day length and rainfall data from the millet columns.

Factors affecting the maximum possible cowpea growth (Column Z) are found in columns V, W, and Y. The cowpea rainfall factor in column V uses the principles discussed for the basic model (2.13.1), but with the coefficients recalculated to bring the factor to zero at 25 percent of soil moisture holding capacity. The basic model assumed the factor reached zero only as the soil moisture reached zero. The cowpea leaf area factor calculation (Column W) uses one minus the millet leaf area factor (column E) as the upper limit, under the assumption that the leaf area factor can at most take a value of one. The calculation of the leaf area expression is multiplied by the density factor (AD11) and the surviving proportion (column Y). The cowpea survival proportion (Column Y) uses the same logic as the millet survival proportion (Column K).

As in the millet submodel, the cowpea growth stage calculation (Column X) is used as input in determining survival (Column Y), maximum possible growth (Column Z) and partitioning of carbohydrates (Columns AC-AI, AK). The cowpea growth stage calculation differs from the millet calculation in that it is triggered by the number of days after millet emergence. The number of days is a constant specified in AD5. In general, the cowpea maximum possible growth (Column Z), the actual growth estimate (Column AB), and the carbohydrate partitioning (Columns AC-AI, AK) follow the same logic as the millet submodel. It differs from the millet submodel in that flower dry matter continues to be accumulated in the grain fill stage at a rate that is 10 percent of the rate during the flower period proper (Column AF). This continued flowering is based on the observation that cowpea will continue to flower after the initial flowering if moisture is adequate. In addition, the mobilization for grain fill is more limited in the cowpea submodel (Column AD). The accumulated mobilization (@SUM(AD$39...AD39)) must be less than or equal to the current vegetative dry matter. Because mobilization is subtracted from the vegetative dry matter amount, the mobilization is effectively limited to about half of vegetative dry matter. Mobilization is also limited to, at most, one percent of vegetative dry matter per day. These mobilization limits were added in

calibration. Cowpea hay is calculated as the material that could be used as animal feed, including here the vegetative dry matter (Column AE), the senescent material (Column AH) and the flowers and pods (Column AH). In some cases, the cowpea grain would be harvested along with the other material and the grain column (AK) should be added to the hay calculation. The grain accumulation is limited to 90 percent of the carbohydrates allocated to reproduction (the grain fill factor multiplied by the actual growth estimate, or Column AB*Z19) because 10 percent of the growth continues to go into flowering during growth stage 3.

The cowpea vegetation factor (Column AJ) follows the same logic as the millet vegetation factor (Column H), with the additional limitation that the total vegetation factor has an upper limit of one.

4. CALIBRATION AND VALIDATION

The calibration and validation of simulation models is the subject of a large literature (see, for instance, Anderson, Gass, McCarl, Naylor and Finger). Calibration is the process of determining which coefficient values permit the model to mimic observed patterns most closely. It is used because some model coefficients are defined by previous research only up to a range of values; the task of calibration is to find which value in the range is most appropriate. Validation asks if the model results are "good enough." Does the model mimic observed patterns in a way that permits the desired use with some confidence?

Both calibration and validation may be divided into general categories: informal and formal. Informal calibration or validation involves confronting someone who is well-acquainted with the real process with the model and its results. Does the model and its results appear plausible to the "expert?" For example, a plant growth model may be shown to an agronomist who has studied that species in the field. The criteria in the informal validation are subjective. Informal validation model results are compared to actual observations, and statistical criteria are applied. For example, statistical tests might be applied to determine if the simulation results can be considered to be from the same distribution as the field observation.

Calibration and validation (especially validation) is an ongoing process. No model is ever totally validated without a doubt. A model is validated for a specific use and is validated as being better than the next best alternative analysis. This document is part of the validation of the millet and cowpea model. The major purpose of the document is to give the reader enough information to judge the model and its output.

A generic plant growth model designed for rapid modification and timely delivery of analyses should have a straightforward and standardized validation procedure. Lowenberg-DeBoer and Cherney suggest three criteria: 1) most of the simulated yields should be within the range of observed yields under the same conditions, 2) simulated yields should be close to the mean observed yields, and 3) a regression of the observed yields on the simulated yields should have an intercept of zero and a slope coefficient of one. Criteria 1)

asks, if observed yield values and simulated yield values were mixed, could a trained observer pick out the simulated values. Obviously, if the simulated results are within the range of observed values, there would be nothing to distinguish the simulated yields. This criteria can be easily applied if the observed data are in the form of an agronomic experiment with replication. The criteria becomes: is the simulated yield within the range of the replication yields? In some cases this criteria can be misleading. For instance, if the range of replications is very narrow for a given treatment, the simulated yield may be very close to the mean yield in percentage terms, but still outside the range of replication. In this case, the simulated value as a percentage of the mean or the interval of two standard deviations provides alternative criteria. Experience suggests that if all simulated yields are within 10 percent of the observed means yields, the simulation is quite good. If the overall within-treatment standard deviation is used (as might be calculated with ANOVA) to calculate the two-standard deviation interval, the problem of a few abnormally narrow replication ranges is avoided because information from all the replications is used to determine the likely distribution of observations. The two-standard deviation interval is suggested instead of the more precise normal or t statistic confidence interval because of the unknown nature of the underlying distribution. Chebychev's theorem indicates that regardless of the distribution, at least 75 percent of the observations should be within two standard deviations. In the case of data with a very wide range among replication results, the criteria based on the range of observation or the two-standard deviation interval is very weak. For instance, on-farm trial data are characterized by a much higher variability within replications than would be the case for on-station experiments. In the case of high variability among replications, the percentage criteria or the regression test is a better indicator of adequacy.

The regression test can be implemented by using the usual t test supplied by most regression software packages. The test should be:

$$Ho: intercept=0 \text{ and } Ha: intercept<>0$$

$$Ho: slope=1, Ha: slope<>1$$

A positive (negative) intercept with a slope of 1 indicates that the model results follow the observed trend, but are generally too low (high) so that the observation is the simulated result plus (minus) a constant. A zero intercept with a slope of greater (less) than one suggests that the model generally under-(over) estimates the observed value. Results derived by Aigner suggest that the regression test is appropriate for deterministic models like the GROWIT version outlined here. Aigner indicates that the regression test is not appropriate for stochastic models.

Calibration and validation often use the same criteria, but different data. Data used for calibration should not be used for validation. Given a large enough number of parameters, it is possible to adjust any model to any set of calibration data with any desired level of accuracy. The real test of

a model is if it will mimic results outside of the sample of observations used in calibration. Ideally, a set of validation data is only used once. If validation fails, the next attempt at validation should use another data set, because the modeler invariably learns something about the models' problems from the failed validation, and this information is incorporated in improved versions of the model. Thus, the data set from the failed validation becomes part of the sample. In practice, the shortage of appropriate data may force relaxation of the rule requiring a single use of validation data.

4.1 Calibrating the Model

Calibration of a simulation model can be treated as a nonlinear programming problem (Talpaz et al.), but given the highly irregular and noncontinuous (because of the logical expressions) nature of many models, finding globally optimal coefficients is difficult. There are often many local optima. In the context of the Lotus 123 spreadsheet package, optimization can be handled by a macro (for instance, based on a grid search procedure). Such macros have been tried on earlier GROWIT versions with disappointing results. The optimization process is relatively slow; simulation models written in high level programming languages recalculate faster and therefore can be optimized more rapidly. More importantly, adapting GROWIT to a new environment or plant often requires a combination of calibration and model development. For instance, adapting the general intercrop model to the millet and cowpea context required changing the rainfall factors for both crops. Optimizing model structure is much more difficult to handle than the optimization of a set of parameters.

The millet and cowpea model was calibrated with a grid search technique guided by knowledge of model structure and experience with model responses. Given the initial search over a broad grid, further search could be focused around parameters that gave the best results and in directions that were consistent with the model structure. Ordinarily it is best to focus the grid search on one parameter at a time; when searching more parameters, it is hard to distinguish the effects between parameters. But which parameter is the focus of the search should depend on the difference between the model and observed patterns. For example, if the model results generally follow the observed trend but are too low, it may be well to concentrate on the ideal growth rate (R_0), because it affects every day's growth at all input levels. If, for example, the low fertility simulations are on target, but the higher fertility results underestimate observed patterns, the fertility levels for maximum yields may need to be adjusted (D27 and D30 for millet, Z30 for cowpea). The calibration should be done on data from several sites and several years, if possible. Parameters which are perfect for the weather in one year may be wrong for the next year's conditions. Parameters are needed which give good results under a wide range of conditions. The wider the range of calibration data, the more likely validation will be successful. Calibration (and validation) can be aided by developing a macro which recalculates the model at the input levels for which there are data, saves the appropriate yield values and calculates the test criteria. Thus the calibration procedure is to:

1) Recalculate the model for one site and one growing season at the input levels for which there are data and over relatively broad intervals of some key parameters,

2) Based on these results, the model parameters are adjusted to improve the match between simulation and observed results,

3) When "good" results are obtained, the same parameters should be tried on another site or another year,

4) If "good" results are again obtained, step 3 should be repeated as time and data allow. If the results in step 3 are not "good", the model should be adjusted to fit the data and the resulting parameters tried on the original calibration data set.

5) The process stops when a set of parameters is found that gives good results on several sites and in several growing seasons.

In practice, the temperature parameters (TG1 ,TG2, TG3) are often quite well-known, so that the temperature factor template need only be calculated once for any site and year. Thus the calibration can concentrate on the growth model template. It is a good idea to keep a record of the results with parameters that give less than the best results for a given year and site. If best parameters from step 2 do not give "good" results in step 3, the record of the results can guide further calibration. For example, there may be a set of parameters which is less than the best for the first year or site, but is nonetheless relatively "good", which is also relatively good for the second or subsequent data set. As mentioned above, "good" can be defined in formal and informal ways. Calibration is as much an art as a science, and the formal criteria should not be treated as rigid rules.

4.2 Millet and Cowpea Model Calibration

The millet and cowpea model was calibrated with data from the ICRISAT Sahelian Center at Sadoré, Niger, and with on-farm trial data collected by INRAN/DECOR researchers in the area of Maradi, Niger. It should be emphasized that this calibration is preliminary, and because of time constraints, it used the data that were at hand. The millet and cowpea submodels were initially individually calibrated with pure crop data collected on phosphate and nitrogen trials conducted by IFDC researcher, A. Bationo, at the ICRISAT Sahelian Center at Sadoré, Niger (some details on this data are given by Bationo et al. and by Jomini et al., 1988a). Rain and temperature data for the Sahelian Center were obtained from ICRISAT agroclimatologist, M.V.K. Sivakumar.

The intercrop model was calibrated using on-farm trial data from Maiguéro, Niger, during the 1986 and 1987 crop seasons. These trials were a continuation of the work described by Ly et al. The calibration focused on four treatments: T1, the farmer's traditional crop varieties and methods; T2, improved crop varieties with traditional agronomic practices; T3, improved crop varieties with increased planting density; and T4, improved crop varieties, with increased planting density and nitrogen and phosphate

fertilization. In terms of model parameters, the differences between treatments are primarily reflected in the millet and cowpea days to flowering, in the planting density and in the fertilization level (Table 6). During calibration, the phosphate level for maximum yield and the cowpea grainfill factor were also varied between treatments. Because the model does not contain a soil nutrient budget, the additional phosphate requirement at the higher density was modeled as a higher required level for maximum yield.

Rainfall data were collected at the on-farm trial village, but temperature data were not recorded. Temperature data for many areas in Niger are available through the national meteorological service, but these data were not available when the calibration was made at Purdue University. Therefore, the decision was made to make an initial calibration using the Sadore temperature data. This decision was based on the observation that rainfall is the crucial weather variable in Nigerien agriculture and that temperature varies relatively little from place to place (Sivakumar, personal communication).

The parameter levels identified after many iterations of the calibration procedure outlined above are given in Table 7. The calibration criteria levels are given in Tables 8 and 9. The calibration occupied the time of two researchers working full time for about two weeks. All of the simulated yields are within the range of replication values. In this case, farmers were taken as replications because the objective of the research was to estimate the distribution of yields for the various technologies for the representative farmer. Because of individual differences in soil quality, management capacity, etc., each farmer will have a somewhat different distribution of yields under each technology. In this case, being within the replication range is a rather weak criteria because of the high within-treatment variance.

The millet model appears to follow the central tendency of the observed yields better than the cowpea model. All of the millet model yields are within 25 percent of the mean and in both years the regression shows that the hypothesis of a zero intercept and slope of 1 cannot be rejected. In part, the problem with the cowpea yields expressed as a percentage of observed mean yields is because of the small base. For example, for Maiguéro in 1987, the model treatment 1 yield is 4 kg/ha higher than the observed yield. At the yield levels of 1986, the 4 kg difference is less than 5 percent, but because of the low 1987 yield, the difference is a high proportion of the observed yield. The calibration results show a pattern common when models are calibrated over more than one season; the model underestimates 1986 yields and overestimates 1987 yields. The simple model does not capture all of the difference between years and treatments, but rather the basic pattern of the differences. The 1986 yields are higher than the 1987 yields. Treatment 2 shows some advantage in millet yield over treatment 1. For millet, treatment 3 is not as advantageous as treatment 2, but for cowpea, treatment 3 results is a yield increase. A relatively large yield increase for millet and cowpea occurs with the fertilization in treatment 4.

Is the calibrated model "good enough?" Some of the simulated yields as a percentage of observed yields are far from 100 percent. One of the regression slope coefficients is statistically different from 1. However, further calibration did not appear to improve the results. Increased accuracy would

probably be gained only with increased model complexity. Because the model appeared to capture the main pattern of difference between years and treatments, it was decided to attempt validation.

4.3 Validating the Model

The model was validated with on-farm trial data from Maiguéro and Rigial Oubandawaki (Tables 10 and 11). Again all the model yields are within the range of the replications. The model yields as a percentage of observed yields leave something to be desired; the millet yields are underestimated and the cowpea yields in 1987 tend to be overestimated. As in the calibration case, the model appears to capture the pattern of differences between treatments and years. The cowpea crop failure in Maiguéro in 1985 is captured by the model. The hypothesis of a zero intercept and slope of 1 is not rejected in any of the regressions at a 5% significance level. The results appear to show a "qualified validation." Future work may lead to improved modeling and parameter values, but the model appears to capture general patterns well enough to provide some information about yield distributions under the various technologies studied.

5. USING THE MILLET AND COWPEA MODEL IN ECONOMIC ANALYSIS

The primary direct uses of the model are exploration of the production surface and development of yield distributions for stochastic dominance analysis. The model may also be used to develop yield distribution parameters to be used in programming models (see, for instance, Lowenberg-DeBoer and Lopez-Pereira, or Krause et al.). Lowenberg-Deboer and Cherney, and Cherney et al. report production surface exploration with GROWIT models with respect to fertilization levels and harvest strategies. This was possible because calibration and validation data included several fertilization levels and alternative harvest dates. Because the calibration and validation data for the millet and cowpea model focused on all or nothing changes in agronomic practices, it is not reliable for looking at small changes in those parameters. The surface would be determined more by the underlying assumption on functional forms in the model than by calibration data. The millet and cowpea model is more useful in looking at the long-run distribution of yields under each treatment and the corresponding distribution of farm returns. This is true because the model is validated for those agronomic conditions and appears to capture the general patterns for the technologies involved.

The first step in this analysis requires simulation of millet and cowpea yields over many seasons. Either simulated or historical weather can be used. Simulated weather can be useful if a long historical series is not available or if the weather patterns are changing so that historical weather does not represent current conditions. Simulated weather is more convenient because it eliminates the data handling chores associated with many years of weather data, but historical weather data may provide more credibility among certain groups. To some observers, the use of simulated weather removes the model one more step from reality, while use of historical data helps maintain that connection to the real world. In particular, biological scientists appear to find use of historical data more credible. Because the model was developed

for use in collaboration with agronomists and other biological scientists, historical rainfall data (Sivakumar, personal communication) was used in the simulations. As discussed in the calibration section, temperature data for the Maradi area were not available at Purdue University at the time when the model was developed. To permit preliminary yield distribution estimations, the average temperatures from Sadoré in 1986 and 1987 were used in all the years simulated. Thus, the same daily temperature factors were used in all simulations. Yields were simulated from 1961-1974 and 1979-1981 (Tables 12 and 13). No Maradi rainfall information was available for 1975-1978. Rainfall information was available for the period before 1961, but because of concern about changing weather patterns in the Sahel, it was thought that the earlier period might be uncharacteristic of current conditions. The model was recalculated four times for each year, one recalculation for each treatment.

The simulated yield distributions for millet indicate that treatment 2 may increase millet yields with only a small increase in variability, but that treatment 3 may sacrifice millet yield for improved cowpea production (Tables 12 and 13, Figure 5.1). The fertilizer in treatment 4 more than doubles average yields and almost triples the standard deviation of yields. The simulations indicate that the variability of yields under treatment 4 increases largely because of increased yields in the years of high rainfall. In the simulations, the treatment 4 yields are never below the yields in the other treatments (Figure 5.1). It should be noted that this is consistent with the observed yields at Maiguéro (Table 14). In both the observed and simulated millet yields, treatment 4 is always superior to all other treatments, and treatment 2 is always superior to treatments 1 and 3. The rank of treatments 1 and 3 depends on the year; sometimes treatment 1 has the higher millet yield, and at other times treatment 3 has the higher millet yield.

The simulated yield distributions for cowpea indicate a substantial increase in yield and variability with the increased density in treatment 3 (Table 13, Figure 5.2). As in the millet distributions, the fertilizer in treatment 4 generates a higher mean yield and greater standard deviation than the other treatments. In the model, the introduction of an improved millet variety at the traditional density (treatment 2) has little effect on the cowpea yield; thus, cowpea yield distributions for treatments 1 and 2 are almost identical. In each year simulated, the yield under treatment 4 is greater than the yield under treatment 3 and, in turn, the yields of both treatments 3 and 4 are greater than the yield for treatments 1 and 2. This ranking of the treatments does not always hold in the observed yields. The yields for Maiguéro in 1987 show a treatment 3 cowpea yield higher than the treatment 4 cowpea yield (Table 14).

In general, both the millet and cowpea simulations show somewhat higher mean yields and wider variability than the observed yields. The higher standard deviations are to be expected because the simulations occur under a wider range of weather conditions than was experienced during 1985-1987. The higher mean yields could indicate a bias in the model, or it could be due to generally favorable weather conditions that occurred in Niger in the 1960s. Also, it should be noted that there may be some small weather difference between Maradi and Maiguéro.

Further analysis of these yield distributions requires some knowledge of the criteria for choosing between crop production technologies. The key variables must be identified and the rules for choosing between them. On a subsistence farm the technique which produces the most food most regularly may be preferred. However, yield distributions do not take into account the cost of inputs or the possibility of a marketed surplus. Thus, a monetary criteria, such as income per hectare, might be appropriate. The farmer may have multiple objectives; therefore, it is useful to look at several criteria. A widely-adopted technology is likely to have strong rankings under several criteria. For this example analysis, the focus will be on millet yield per hectare and net cash income per hectare. Millet yield is important because millet is the staple food for many Nigerien farmers. Cowpeas play a much smaller role in the Nigerien diet than millet and are often treated as a cash crop.

The choice between risky activities has been the subject of much research and discussion, but no one set of rules for this choice has yet been universally accepted. Among the many options, stochastic dominance analysis is one of the most widely-accepted because it enables the researcher to determine the acceptability of activities with minimal assumptions about risk preferences (Anderson et al.). Usually, the researcher will try to distinguish three groups: 1) dominated activities, 2) the first degree stochastic dominance (FDSD) set and 3) the second degree stochastic dominance (SDSD) set. The criteria uses pairwise comparisons between the cumulative distributions functions of the activities under study.

An activity is said to be dominated if the cumulative distribution function (CDF) of that activity is always to the left of the cumulative distribution function of the other activity. The intuition behind this rule is that if the cumulative distribution function of activity 1 is always to the left of the cumulative distribution function of activity 2, then the level of benefit achievable with a certain probability is always less for activity 1. For example, in Figure 5.3, an income of at least 15,000 FCFA is achievable 50 percent of the time with activity 1, but an income of 20,000 is achievable 50 percent of the time with activity 2. Given the shape and position of the cumulative distributions functions in Figure 5.3, activity 2 has a higher income at every level of probability. The only preference assumption required for this rule is that "more is better." All activities that are not dominated under this "more is better" rule are part of the FDSD set. For activities in the FDSD set, neither activity has the higher benefit for all levels of probability. The cumulative distribution functions cross and, at some probability levels, one activity has the higher benefit and at other probability levels, the other activity has the higher benefit. For example, in Figure 5.4, activity 3 has a lower probability of low income levels, but it also has a lower probability of high income levels.

To distinguish among the activities in the FDSD set, another rule is used. The SDSD rule specifies that an activity is dominated if it has the larger area under the CDF at every benefit level. The intuition behind this rule is that for the risk-averse individual, a higher probability of outcomes in the middle part of the benefit range may be acceptable in the case where that activity has a lower probability of low outcomes. For example, in Figure

5.5, the probability of outcomes below 10,000 FCFA is zero for activity 6, but about 30 percent for activity 5. However, the FDSD rule cannot distinguish between these two activities, because the activity 6 CDF lies to the left of the activity 5 CDF between 15,000 and 20,000 FCFA. The SDSD rule indicates that activity 6 is to be preferred as long as the cross-hatched area C is smaller than the area B. That is to say, activity 6 is preferred as long as the higher probability levels between 15,000 and 20,000 FCFA is outweighed by the lower probability of low income outcomes. If area C is larger than area B, both activities are part of the set that cannot be distinguished by SDSD. The SDSD rule requires the additional behavioral assumption of risk aversion.

When many equally likely observations on the process are available, the CDF can be estimated relatively simply by a piecewise linear function. The procedure is to:

1) rank the benefit levels from lowest to highest,

2) assign each observation the probability of the inverse of the number of observations (1/n),

3) accumulate the probability according to the rank of the observation (so that observation ranking 5 has a cumulative probability of 5/n), and

4) make linear interpolations between the observation points for activities with continuous outcomes.

Activities with discrete outcome levels would have stepwise CDFs. For the nondiscrete case, as the number of observations becomes large, the piecewise linear function will approach a smooth CDF.

FDSD comparisons can be made graphically, but the SDSD rule often requires a numerical estimate because visual assessment of areas is sometimes difficult. To do this, another Lotus template was developed. The template is organized in five sections: 1) original data, Columns A-H, 2) ranked data, Columns I-P, 3) precalculations, Columns Q-AF, 4) area calculations, Columns AG-AW, and 5) FDSD and SDSD indicators, Columns AW-BD. The template is arranged horizontally so that it can easily accommodate data sets of various sizes. Cell formulas in sections 3, 4, and 5 can simply be copied down to fit the number of observations. The data can be ranked in section 2 by using the /DataSort command. In section 3, the preliminary calculations and the rearrangement of data so that it can be used by the Lotus table look-up function are accomplished. Only two activities can be compared at a time in any stochastic dominance analysis. In the template, the column of ranked data relating to alternative 1 is copied to Column Q, and the column of data relating to alternative 2 is copied to Column Y. The precalculation sections for each alternative has the sorted data (Columns Q and Y), the next higher observations (Columns R and Z), the cumulative probability associated with each observation (Columns S and AA), the area under the CDF between the observation and the last observation (Columns T and AB) and the cumulative area up to each observation point (Columns U and AC). The layout of the precalculation section can be seen in Screen 12, and the cell formulae are in

Table 15. The number of observations should be entered in cell R3 for use in the probability estimation. The column of next higher observations is required so that the table look-up function can associate each observation with the next higher observation in the calculation of the CDF and cumulative area.

The precalculation of cumulative probabilities and areas at each observation is required by the way the template is organized. In section 4, the CDF and the cumulative area are calculated at given intervals based on the slope or accumulated area from the last observation lower than the current value. Alternative estimation techniques would calculate the CDF and cumulative area just at the observation points or would calculate them at the given interval by accumulating the probability or area from the previous intervals. The CDF and cumulative area are calculated at given intervals rather than just at observation points because the FDSD and SDSD rules are specified to hold at every point. The intervals can be made as small as desired within the limits of LOTUS1-2-3® capacity. The CDF and area are calculated from the last observation point rather than the last interval level, because regardless of the size of the interval, it is possible for two or more observation points to occur within one interval and the formulas required to do CDF and area calculations for several observations within an interval would be very complicated.

In section 3, the area between each observation and the next is calculated as the sum of a rectangle and a triangle (Figure 5.6). For example, if the first three observations are 1000, 1500, and 1600, the cumulative area at 1000 is zero. The cumulative area at 1500 is:

(Eq. 5.1) $$(1500-1000)*1/n + 0.5*(1500-1000)*1/n.$$

The first term in expression (5.1) is the area of the rectangle A and the second term is the area of the triangle B. The cumulative area at 1600 is the area at 1500 plus the area of rectangle D and triangle C. The objective of the calculations in section 4 is the probability level at any level of benefit (Columns AL and AT) and the cumulative area at any point (Columns AN and AV). The probability level at any scale value is used in graphing the CDF. Because the LOTUS1-2-3® line graph commands lay out the X axis values at even spatial intervals, it is necessary to place the variable with even intervals on the X axis. Thus, the probability level at each observed benefit level as calculated in section 3 (Column Q vs. S or Column Y vs. AA) can be meaningfully graphed only in an inverted form, with the probability on the X axis. To put the graph into a more usual perspective, it is necessary to calculate the probability level at even intervals of benefit.

The first four columns of the section 4 calculations for each alternative (Columns AG-AJ or AO-AR) are used to estimate the slope of the CDF at the interval values (Screen 13). The first column in each alternative (AG or AO) contains the scale values. The starting point and the interval must be entered in cells AJ3 and AJ4, respectively. The starting point for the scale should be less than the minimum value for either alternative. The calculation

of the probability level requires the slope of the CDF. Calculation of slope using the form:

slope = rise/run

demands the difference between first observation lower than the current scale value and the next observation higher than the scale value (the run). Therefore, the next two columns after the scale (Columns AH-AI or AP-AQ) are used to determine the next lower and next higher observations using the table look-up function on the precalculation values. The if statement in each column sets the value to the scale value (Column AG or AO) if the scale value is below the range of observations. If the value is above the range of observations, the LOTUS1-2-3® table look-up functions choose the highest range value. Because of the assumption that each observation is equally likely, the rise is always 1/n.

The fifth and sixth columns in section 4 for each alternative (Columns AK-AL or AS-AT) use the previously-calculated slope to find the CDF value at the given interval point. The CDF value is calculated (Column AL or AT) as the probability at the last observation, plus the slope multiplied by the difference between the scale value and the last observations (Table 15). Columns AK and AS are used to find the probability of the last observation for their respective alternatives.

The area calculation in Columns AM-AN or AU-AV follows a similar pattern. The area is the area at the last lower observation, plus the area under the CDF between the last observation and the current scale point. The table look-up function is used to find the area at the last observation (Column AM or AU). The area under the CDF between the last observation and the current scale value is the sum of the area of the rectangle:

(Eq. 5.2) Probability at last observation x (Scale Value - Last Observation)

plus the area of the triangle:

(Eq. 5.3) (Probability at current value - Probability at last observation) x (Scale Value - Last Observation) x 0.5

Section 5 of the stochastic dominance template (Columns AW-BC, Screen 14) tallies the results and indicates which, if any, activity is dominant. In Columns AW-AY, the FDSD is determined. In Column AW, the difference in benefit at each probability increment is returned. If this difference is always one sign, there is FDSD. The next two columns check the signs. If alternative 1 is always greater or equal to alternative 2, Column AX will contain all ones. A zero in column AX is returned if alternative 1 is smaller

than alternative 2. A similar calculation is made in Column AY; a 1 is returned if alternative 2 is larger than or equal to alternative 1 and zero if it is smaller. The results are tallied in the cells AZ2-AZ4. AZ2 is the sum of column AX. AZ3 is the sum of column AY. AZ4 is the number of rows in columns AX and AY. If one of the columns AX and AY is all "1's, there is FDSD. Cell AZ6 returns the label "Alt 1 FDSD" if cell AZ2 equals AZ4 and the label "Alt 2FDSD" if cell AZ3 equals AZ4. If neither column AX nor AY is all "1's," the label "undominated" is returned. If there are probability levels at which benefits from the two alternatives are equal, one of the columns may be all "1's", while the other column has some "1's." To reduce problems with circular references, it is useful to set the recalculation order for the stochastic dominance template to "columnwise" because all of the calculations required for the conclusion in AZ6 are to the left or above that cell.

A similar procedure is used to check for SDSD in columns BA-BC. The difference in areas is calculated in column BA. Columns BB and BC are indicators of the signs of the differences. The results are tallied in cells BD2-BD4. The conclusion is given in cell BD6.

The stochastic dominance template was used to compare the risk characteristics of the millet yield for each possible pair of treatments: T1 vs. T2, T1 vs. T3, T1 vs. T4, T2 vs. T3, T2 vs. T4, and T3 vs. T4. The CDFs of each treatment are graphed in Figure 5.7. In this analysis T4 clearly dominates the other treatments by FDSD. This can be seen in Figure 5.7 by noting that the T4 CDF is always to the right of the other CDFs. Treatment 2 dominates treatments 1 and 3 by FDSD, but is dominated by treatment 4. Treatments 1 and 3 are dominated by treatments 2 and 4, but the analysis cannot distinguish between T1 and T3. T1 cannot be dominant because it has the very lowest yield outcome, but T3 does not dominate by SDSD because it has less favorable outcomes in the upper end of the yield range. Above about 210 kg/ha, the T3 CDF is to the left of the T1 CDF. It should be noted that the formal stochastic dominance analysis confirms what was suggested by the time series plot of the simulated millet yields (Figure 5.1). The dominance of treatment 4 over the others and treatment 2 over treatments 1 and 3 was suggested by the time series plot, because in that plot treatment 4 yields are always above the other yields and treatment 2 yields are always above the treatment 1 and 3 yields. The analysis suggests that the risk characteristics of treatments 2 and 4 by themselves should not prevent adoption of these technologies, though other constraints such as lack of capital to purchase inputs or lack of access to improved seed might.

The risk characteristics of the net cash income per hectare (Table 16) were also compared. The potential net cash income for each year was calculated as the market value of the millet and cowpea grain minus the cost of any inputs that must be purchased. The details of the income estimations are found in Krause et al. The market value was calculated at the annual average real price for the year of the simulation. This annual average price is relevant if the marketings of grain are spread evenly throughout the year. Based on the average relationship between market prices and official prices of improved seed, improved millet seed was valued at 1.33 times the market value of millet, and improved cowpea seed was valued at 1.14 times the market value

of cowpeas. The budgets assume that only family labor was used and that there is no cash cost for using family labor.

The analysis of the net cash income distribution indicates that treatment 4 dominates by FDSD when the target variable is income. This can be seen in the CDF graph (Figure 5.8) by noting that the treatment 4 CDF is always to the right of the others. Treatment 1 is dominated by all other treatments; its CDF is to the left of the other three. Treatments 2 and 3 cannot be distinguished, even by SDSD in this case. Treatment 3 shows up better relative to treatment 2 in this case, because the improved cowpea production under treatment 3 increases income substantially in some years. Treatment 2 cannot dominate treatment 3 because the treatment 2 lowest value is less than the treatment 3 lowest value. The analysis of cash income suggests that treatment 4 has relatively desirable risk characteristics, and that either treatments 2 or 3 may dominate treatment 1, the traditional practices.

It should be noted that this conclusion in the stochastic dominance analysis of cash income can also be seen in the time series plot of income (Figure 5.9). The treatment 4 estimated income is above the incomes for the other treatments in every year. Though it is difficult to see in the figure because the lines are close together, the treatment 1 estimated income is always below the income estimates for the other treatments.

5.1 Comparison to a Simpler Analysis

If the goal is risk analysis, there are other sometimes simpler approaches than the use of simulation. In particular, the sparse data techniques suggested by Anderson et al. could use the observed data (Table 16) directly to do stochastic dominance analysis. It is useful to compare the sparse data and simulation results to better understand the place of simulation.

Sparse data techniques can be used to sketch a CDF based on a few observations. The observations are ranked and the kth observation is taken as the best estimate of the k/n+1 fractile. A smooth CDF is then sketched through the points, taking into account any other information such as the non-negativity of some variables or the unimodal nature of many empirical distributions (Anderson et al.). The CDF of a unimodal, two-tailed distribution has an S shape. Thus, to sketch a CDF for millet yields under treatment 1 at Maiguéro, the lowest yield (154 kg/ha) is plotted at the 25 percent probability because k=1 for the lowest observation, and with three observations, n+1=4 (Figure 5.10). The middle observation is plotted at 50 percent and the highest observation at 75 percent. Assuming that the yield is non-negative, but very low yields are possible, the CDF is started at zero. Assuming that the millet yield is unimodal, the CDF is sketched in an "S" shape. A similar procedure is followed for the other treatments.

The sparse data CDFs for millet yield suggest the same conclusion as the simulation-derived CDFs. The T4 CDF is always to the right of the others. The T2 CDF is to the right of the T1 and T3 CDFs, but rank of T1 relative to T3 is unclear. The simulation analysis strengthens an argument which could also be made with CDFs calculated with sparse data techniques. Most resear-

chers would not make a very strong conclusion based on a CDF sketched from three observations. The simulation analysis permits the conclusions to be drawn with greater certainty.

The sparse data CDF was also drawn for net cash income per hectare (Figure 5.11). Again, the conclusions are generally the same as in the simulations analysis, though the problem in ranking treatments 2 and 3 is not as clear here as in the simulation CDFs. Treatment 4 clearly dominates the others; the treatment 4 CDF is always to the right. Treatment 1 appears to be dominated by the others.

6. CONCLUSIONS

A simulation model for millet-cowpea intercrop has been presented. The model was developed within the generic GROWIT framework and uses a spreadsheet format. In addition, a spreadsheet template was presented for making stochastic dominance comparisons. The millet-cowpea model and the stochastic dominance spreadsheet are applied in the evaluation of millet and cowpea production strategies in Niger, West Africa.

Experience with use of the GROWIT-based simulation model to evaluate production strategies supports the potential of using generic models for analyzing crop innovations. The model can be adapted to new situations quickly. Over a period of several months, the GROWIT model was modified for intercropped annuals by two researchers working part-time. Preliminary calibration and validation took about 4 person-weeks. The validation suggests that the model captures the main tendencies of the yield distribution, but that accuracy, in terms of simulated yields as a percentage of observed yields, leaves something to be desired, especially in low rainfall years. In the economic analysis, simulation-derived distributions were used to rank crop production strategies according to risk characteristics using the stochastic dominance template. The conclusions obtained with the help of simulation complemented information from other sources.

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Table 1. Temperature Factor Template Cell Formulas, Constants.

```
J5: SINLAT = @SIN(D5*@PI/180)
M5: COSLAT = @COS(D5*@PI/180)
H7: i/R = -1/(D9-D7)^2
J7: q/R = -1/(D9-D8)^2
H8: h/R = -2*D9*H7
J8: n/R = -2*D9*J7
H9: g/R = +D7*(2*D9-D7)*H7
J9: m/R = +D8*(2*D9-D8)*J7
```

Table 2. Temperature Factor Template Cell Formulas, Daily Values.

Day Length and Temperature, Columns A-G:

A16: Julian Day = +A15+1

B16: Julian Factor = 0.9856*A16-3.289

C16: Solar Angle = 180/@PI*@ASIN(@SIN((@PI/180)*(B16+1.916*@SIN(B16*@PI/180)+0.02*@SIN(2*(@PI/180)*B16)+282.634))*0.39782) D16: Daylength = 24/@PI*@ACOS((-0.01454-@SIN(C16*@PI/180)*$J$5)/(@COS(C16*@PI/180)*$M$5))

G16: Harmonic Mean Temperature C = (E16+F16)/2

Morning Integration, Columns H-R:

H16: B1 = @IF($E16<=$D$7#OR#$F16>=$D$8,D16/2,@IF($D$7<F16,0,D16/2*¡(1-@SQRT(@ABS(1+((F16-$D$7)/(E16-F16)))))))

I16: B2 = @IF($E16<=$D$9#OR#$F16>=$D$8,$D16/2,@IF($D$9<$F16,0,$D16/2*(-1-@SQRT(@ABS(1+(($F16-$D$9)/($E16-$F16)))))))

J16: B2* = @IF($E16<=$D$8#OR#$F16>=$D$8,$D16/2,$D16/2*(1-@SQRT(@ABS(1+(($F16-$D$8)/($E16-$F16))))),,

K16: b = 4*($E16-$F16)/$D16

L16: c = -K16/$D16

M16: Step I = ($H$9+$H$8*$F16+$H$7*$F16^2)*(I16-H16)+0.5*$K16*($H$8+2*$H$7*$F16)*($I16^2-$H16^2)

N16: Step II = 1/3*($H$8*$L16+$H$7*$K16^2+2*$H$7*$F16*$L16)*($I16^3-$H16^3)+0.5*$H$7*$K16*$L16*($I16^4-$H16^4)

O16: Low Temp Growth = (M16+N16+1/5*$H$7*$L16^2*($I16^5-$H16^5))

P16: Step I = ($J$9+$J$8*$F16+$J$7*$F16^2)*(J16-I16)+0.5*$K16*($J$8+2*$J$7*$F16)*($J16^2-$I16^2)

Q16: Step II = 1/3*($J$8*$L16+$J$7*$K16^2+2*$J$7*$F16*$L16)*($J16^3-$I16^3)+0.5*$J$7*$K16*$L16*($J16^4-$I16^4)

R16: High Temp Growth = (P16+Q16+1/5*$J$7*$L16^2*($J16^5-$I16^5))

Afternoon Integration, Columns S-AD:

S16: B3 = @IF($E16<=$D$7#OR#$G16>=$D$8,$D16,@IF($E16<=$D$8,$D16/2,$D16/2*(1+@SQRT(@ABS(1+($G16-$D$8)/($E16-$G16))))))

T16: B4* = @IF(E16<=$D$7#OR#$G16>=$D$9,$D16,@IF($D$9>=E16,D16/2,D16/2*(1+@SQRT(@ABS(1+((G16-$D$9)/(E16-G16)))))))

U16: B4 = @IF(E16<=$D$7#OR#$G16>=$D$7,$D16,D16/2*(1+@SQRT(@ABS(1+((G16-$D$7)/(E16-G16))))))

V16: d = 4*($E16-$G16)/$D16

W16: e = -V16/$D16

X16: Step I = ($H$9+$H$8*$G16+$H$7*$G16^2)*(U16-T16)+0.25*$V16*($H$8+2*$H$7*$G16)*($U16^2-$T16^2)

Y16: Step II = 1/6*($H$8*$W16+0.5*$H$7*$V16^2+2*$H$7*$G16*$W16)*($U16^3-$T16^3)+0.125*$H$7*$W16*$V16*($U16^4-$T16^4)

Z16: Low Temp Growth = (X16+Y16+1/20*$H$7*$W16^2*($U16^5-$T16^5))

AA16: Step I = ($J$9+$J$8*$G16+$J$7*$G16^2)*(T16-S16)+0.25*$V16*($J$8+2*$J$7*$G16)*($T16^2-$S16^2)

AB16: Step II = 1/6*($J$8*$W16+0.5*$J$7*$V16^2+2*$J$7*$G16*$W16)*($T16^3-$S16^3)+0.125*$J$7*$W16*$V16*($T16^4-$S16^4)

AC16: High Temp Growth = (AA16+AB16+1/20*$J$7*$W16^2*($T16^5-$S16^5)) AD16: Temp Factor = +$O16+$R16+$Z16+$AC16

Table 3. Millet and Cowpea Growth Model Template, Cell Formulas for Calculated Constants.

Millet Constants:

```
H5: Total DL = 365.25*(24/@PI)*@ACOS((-0.01454-
@SIN(0.516813*@PI/180)*@SIN($D$5*@PI/180))/(@COS(0.516813*@PI/180)*@COS(
$D$5*@PI/180)))
H8: 1 - LAI =  1-D8
H10: Row Length = 10000/D11
H11: Density = @IF(D10>=H10,1,D10/H10)
H17: Flower DAE = +D17-7
H28: N Fertfac = @MIN(1,D26+(1-@EXP(@LN(D26)*D28/D27)))
H29: Soil P = +D29*2.4
H30: P for max = +D30*2.4
H31: P Fertfac = @MIN(1,((H29+D31*0.8)/H30))
H33: 1 - Delta = 0.5^(1/$D$33)
```

Cowpea Constants:

```
AD5: DASM Germ = +Z5-7
AD8: 1 - LAI = 1-Z8
AD10: Row Length = 10000/Z$11
AD11: Density = @IF(10000/Z11<Z10,1/Z11,1)
AD17: Flower DAE = +Z17-7
AD28: N Fertfac = @MIN(1,Z26+(1-@EXP(@LN(Z26)*Z28/Z27)))
AD29: Soil P = +Z29*2.4
AD30: P for max = +Z30*2.4
AD31: P Fertfac = @MIN(1,((AD29+Z31*0.8)/AD30))
AD33: 1 - Delta = 0.5^(1/$D$33)
```

Table 4. Millet and Cowpea Growth Model Template, Millet Daily Cell Formulas.

```
E40: Leaf Area Factor = @MIN((1/$D$11)*K39*$H$11,@IF(Q39<D$9,
$D$8+2*$H$8*Q39/$D$9-$H$8*(Q39/$D$9)^2,1))
F40: Soil Moisture =  [W12] @IF(F39*$D$13+C39-
I39>$D$13,1,@IF(F39*$D$13+C39-C39<0,0,(F39*$D$13+C39-I39)/$D$13))
G40: Rainfall Factor =@IF($F40<0.1,0,@IF($F40<=0.5,@EXP((0.4197825*$F40)
+(1.68552*($F40^2)))-1,@EXP(0.4542654*($F40-0.5)-0.45522*($¡F40-0.5)^2)-
0.12))
H40: Millet Veg Factor = @IF($C39>=1,1,0.14)*(1-
$H$11*K39)+@IF(($C39>=1)#AND#(Q39<=1500),1,@IF(($C39<1)#AND#(Q39<=1500),
0.25+((Q39-250)*0.55/1250),0.8))*$H$11*K39)*1/$D$11
I40: Evapotranspiration = @MAX(0,((45.72*D40+812.8)*B40/($H$5))*G40*
(@MIN(1,H40+AJ40)))
J40: Growth Stage = @IF(J39="PRE",@IF($C40>=10#OR#A40=$D$7,"GERM"¡"PRE")
,@IF(J39="GERM",@IF(K39<1#AND#@SUM($C34..$C40)>=10#AND#J33="PRE","GERM",
1),@IF(J39=1,@IF($A40>172#AND#$B
K40: Survival Factor = @IF($J40="PRE",0,@IF($J40="GERM",
@IF(@SUM(C34..C40)>=10, @MIN(1,K39+M39/100),0),K39))
L40: Max Possible Growth =
@IF($J40="PRE",0,$D$21*$H$11*@MIN(E40,G40,1,$H$28,$H$31))
N40: Actual Growth Estimate = @MAX(0,M40*L40)
O40: Veg Growth =  @MAX(0,@IF(J40="GERM"#OR#J40=1,N40,@IF(J40=2,N40*(1-
$D$19),N40*(1-$D$19))))
P40: Mobilize for Grain Fill = @IF(G40<0.5#AND#J40=3#AND#U39<R39/$D$18,
$D$19*M40*$D$21*@MIN(1,E40,$H$28,$H$31),0)
Q40: Veg Dry Matter = @MAX(0,+Q39+O40-P40-S39)
R40: Flower Dry Matter = @IF(J40=2,R39+N40*$D$19,R39)
T40: Dead Dry Matter = +T39*$H$33+O40
U40: Grain Dry Matter =  @IF(J40=3,@MIN(R40/(1-
$D$18),U39+$D$19*N40+P40),0)
```

Table 5. Millet and Cowpea Growth Model Template, Cowpea Daily Cell Formulas.

```
V40: Cowpea Rainfall Factor = @IF($F40<0.25,0,@IF($F40<=0.5,-
@EXP((4.160124*$F40)+(-16.801303*($F40^2)))+1,@EXP(0.4542654*($F40-0.5)-
0.45522*($F40-0.5)^2)-0.12))
W40: Cowpea Leaf Area Factor =  @MIN(1-
E40,$AD$11*Y39*@IF(AE39<Z$9,Z$8+2*AD$8*AE39/$Z$9-AD$8*(AE39/$Z$9)^2,1))
X40: Cowpea Growth Stage =  @IF(X39="PRE",@IF($C40>=10#AND#
@SUM(J26..J39)>AD$5,"GERM","PRE"),@IF(X39="GERM",@IF(Y39<1#AND#@SUM(C34.
.C40)>=10#AND#X33="PRE","GERM",1),@IF(X39=1,@IF($A40>
Y40: Cowpea Survival Factor = @IF($X40="PRE",0,@IF($X40="GERM",
@IF(@SUM(C34..C40)>=10,@MIN(1,Y39+AA39/100),0),Y39))
Z40: Max Possible Growth =
@IF($X40="PRE",0,$Z$21*@MIN(V40,W40,1,$AD$28,$AD$31))
AB40: Actual Growth Estimate =  @MAX(0,AA40*Z40)
AC40: Cowpea Veg Growth = @MAX(0,@IF(X40="GERM"#OR#X40=1,AB40,
@IF(X40=2,AB40*(1-$Z$19),@IF(X40=3,AB40*(1-$Z$19),0))))
AD40: Mobilize for Grain Fill = @IF(AK39<AF39/(1-$Z$18),
@IF(F40<0.25#AND#X40=3#AND#@SUM(AD$39..AD39)<AE39,@MIN(AE39/100,$Z$19*AA
40*$Z$21*@MIN(1,W40,$AD$28,$AD$31)),0),0)
AE40: Veg Dry Matter = @MAX(0,+AE39+AC40-AD40-AG39)
AF40: Flower and Pod Dry Matter =
@IF(X40=2,AF39+AB40*$Z$19,@IF(X40=3,AF39+AB40*$Z$19*0.1,$AF39))
AH40: Dead Dry Matter = +AH39*$AD$33+AG40
AI40: Cowpea Hay = +AE40+AF40+AH40
AJ40: Cowpea Veg Factor = @MIN(1-$H40,(@IF($C39>=1,1,0.14)*(1-
$AD$11*Y39)+@IF(($C39>=1)#AND#(AE39<=1500),1,@IF(($C39<1)#AND#(AE39<=150
0),0.25+((AE39-250)*0.55/1250),0.8))*$AD$11
AK40: Cowpea Grain = @IF(X40=3,@MIN(AF40/(1-
$Z$18),$AK39+0.9*$AB40*$Z$19+$AD40),AK39)
```

Table 6. Millet and Cowpea Model Parameters that Vary by Treatment.

Variable	Unit	Treatment T1	T2	T3	T4
Millet Days to Flowering	days	70	62	62	62
Cowpea Days to Flowering	days	0	56	56	56
Millet Density	hills/ha	6670	6670	8888	8888
Cowpea Density	hills/ha	2290	2290	17777	17777
Fertilizer Response:					
Optimal P for millet	ppm.	14.6	14.6	18	18
Optimal P for cowpea	ppm.	11.2	11.2	14	14
Applied N	kg/ha	-	-	-	21
Applied P	kg/ha	-	-	-	19
Cowpea Grain Fill Factor	percent	15	15	14	14

Table 7. Millet and Cowpea Model Parameters.

Variable	Unit	Value	Cell[1]
Temperatures:			
Minimum, millet and cowpea	degrees C	10	Tempfac D7
Maximum, millet and cowpea	degrees C	45	Tempfac D8
Optimum for millet	degrees C	33	Tempfac D9
Optimum for cowpea	degrees C	27	Tempfac D9
Soil Characteristics:			
Moisture holding capacity	mm/m	28	Growth D13
Initial moisture	mm/m	0	Growth D14
Soil phosphate	ppm	3	Growth D29
Millet Vegetative Growth:			
Maximum Growth Rate	kg/ha/hr	16.1	Growth D21
Senescence Half Life	days	15	Growth D33
Cowpea Vegetative Growth:			
Maximum Growth Rate	kg/ha/hr	10.75	Growth Z21
Semescences Half Life	days	30	Growth Z33
Millet Grain Fill:			
Day length for flowering	hours	12.26	Growth D16
Maximum threshing ratio	percent	70	Growth D18
Grain fill factor	percent	20	Growth D19
Cowpea Seed Formation:			
Day length for flowering	hours	10.75	Growth Z16
Maximum threshing ratio	percent	70	Growth D18
Nitrogen Fertilizer Response:			
Millet yield potential at N=0	percent	40	Growth D26
N for maximum millet yield	kg/ha	21	Growth D27

[1]"Tempfac" refers to the temperature factor template. "Growth" refers to the plant growth template. The same parameter may also appear elsewhere in the templates. For example, the temperature levels should be copied to the growth model parameter list (D22-D24, Z22-Z24). The soil parameters also appear in the cowpea parameter list.

Table 8. Calibration: Comparison of Millet and Cowpea Observed and Simulated Yields.

Year, Treatment and Species	Observed Minimum	Observed Maximum	Observed Mean	Model	Model as percent of Mean
	----------kg/ha----------				
Millet, 1986:					
T1	19	669	279	274	98
T2	36	735	310	308	99
T3	26	537	238	255	107
T4	268	965	627	624	100
Millet, 1987:					
T1	0	686	154	190	123
T2	0	567	180	205	114
T3	6	447	146	171	117
T4	36	819	323	332	103
Cowpea, 1986:					
T1	9	240	67	61	91
T2	10	310	84	61	73
T3	20	306	150	125	83
T4	30	435	245	228	93
Cowpea, 1987:					
T1	0	40	4	8	200
T2	0	45	12	15	125
T3	0	190	40	60	150
T4[1]	0	110	65	100	154

[1]In 1987, severe insect problems were experienced in the cowpea plots. Because the model does not include an insect submodel, this complicating factor was reduced by using the T5 yields for comparison. T5 is like T4 except that an insecticide is applied to cowpeas.

Table 9. Calibration: Regression of Observed Yields on Simulated Yields.

Coefficient, Year and Species	Estimate	Std. Error	t Statistic
Millet, 1986:			
intercept	-9.82	33.68	-0.29
slope	1.02	0.09	0.26
Millet, 1987:			
intercept	-55.62	43.96	-1.24
slope	1.14	0.19	0.73
Cowpea, 1986:			
intercept	16.10	12.01	1.34
slope	1.01	0.09	0.15
Cowpea, 1987:			
intercept	0.53	4.44	0.12
slope	0.67	0.08	-4.17

Table 10. Validation: Comparison of Millet and Cowpea Observed and Simulated Yields.

Location, Year, Species and Treatment	Observed Minimum	Observed Maximum	Observed Mean	Model	Model as percent of Mean
	----------kg/ha----------				
Maiguéro					
Millet, 1985:					
T1	40	1167	284	216	76
T2	80	1315	358	264	74
T3	18	1547	309	214	69
T4	140	2330	598	527	88
Rigial O.					
Millet, 1987:[1]					
T1	8	411	202	159	79
T2	79	571	323	253	78
T3	28	348	198	202	102
T4	72	546	335	318	95
Maiguéro					
Cowpea, 1985:					
T1	0	0	0	0	100
T2	0	0	0	0	100
T3	0	0	0	0	100
T4	0	0	0	0	100
Rigial O.					
Cowpea, 1987:[1]					
T1	0	66	3	10	333
T2	0	199	16	10	63
T3	0	146	28	81	289
T4[1]	0	244	65	102	157

[1]In 1987, severe insect problems were experienced in the cowpea plots. Because the model does not include an insect submodel, this complicating factor was reduced by using the T5 yields for comparison. T5 is like T4 except that an insecticide is applied to cowpeas.

Table 11. Validation: Regression of Observed Yields on Simulated Yields.

Location, Year, Species and Coefficient	Estimate	Std. Error	t Statistic
Maiguéro			
Millet, 1985:			
intercept	95.23	98.21	0.97
slope	0.96	0.30	-0.15
Rigial O.			
Millet, 1987:			
intercept	35.30	55.66	0.63
slope	0.98	0.23	-0.07
Rigial O.			
Cowpea, 1987:			
intercept	-14.44	10.74	-1.34
slope	1.31	0.16	1.87

Table 12. Simulated Millet Yields, 1961-1974, 1979-1985.

	Treatment Yield (kg/ha)			
Year	T1	T2	T3	T4
1961	248.20	273.40	227.00	621.10
1962	299.10	331.80	277.50	713.40
1963	445.80	518.80	432.80	1131.10
1964	252.20	285.40	225.00	633.20
1965	347.70	380.40	315.60	792.30
1966	257.30	321.50	266.70	786.00
1967	131.40	194.40	167.60	455.60
1968	290.30	307.40	251.40	474.00
1969	286.30	334.40	270.10	750.40
1970	247.40	273.50	214.60	626.80
1971	379.50	427.90	343.00	829.60
1972	175.40	185.80	146.70	208.50
1973	233.80	255.80	206.90	471.90
1974	144.50	232.40	193.00	467.50
1979	361.60	393.90	326.80	753.70
1980	255.20	264.40	216.20	582.60
1981	357.80	414.50	344.00	822.00
1982	250.50	282.40	239.90	618.80
1983	324.80	397.70	338.00	765.00
1984	314.70	355.40	285.80	627.60
1985	209.40	279.70	233.20	405.60
mean	276.8	319.6	262.9	644.6
std	77.4	80.0	67.1	190.9
max	445.8	518.8	432.8	1131.1
min	131.4	185.8	146.7	208.5

Table 13. Simulated Cowpea Yields, 1961-1974, 1979-1985.

	Treatment Yield (kg/ha)			
Year	T1	T2	T3	T4
1961	59.70	59.70	160.10	304.90
1962	57.80	57.80	178.40	318.10
1963	50.80	50.80	164.80	279.40
1964	57.20	57.20	156.70	306.60
1965	61.60	61.60	165.40	338.50
1966	39.20	39.20	77.10	150.40
1967	38.40	38.40	83.10	162.50
1968	22.90	23.00	66.00	98.30
1969	60.60	60.60	126.10	233.90
1970	69.50	69.50	128.40	249.70
1971	43.90	43.90	63.40	109.00
1972	10.80	10.80	21.00	31.20
1973	35.00	35.00	87.80	161.50
1974	23.40	23.40	66.90	129.00
1979	58.70	58.70	156.00	308.50
1980	50.50	50.50	130.00	23[illegible] 80
1981	51.10	51.10	147.90	299.90
1982	22.50	22.50	85.60	123.90
1983	48.90	48.90	124.40	245.60
1984	20.50	20.50	66.60	113.90
1985	17.60	17.60	66.60	98.10
mean	42.9	42.9	110.6	204.6
std	16.9	16.9	44.0	90.2
max	69.5	69.5	178.4	338.5
min	10.8	10.8	21.0	31.2

Table 14. Observed Yields and Net Cash Income per Hectare Estimated with Observed Yields, Maiguéro, 1985-1987.

	Treatment			
	T1	T2	T3	T4
	----------kg/ha----------			
Yield:				
Millet:				
1985	283.5	357.8	309.1	597.7
1986	279.0	310.1	238.1	626.6
1987	154.1	180.1	145.6	323.0
mean	238.9	282.7	230.9	515.8
std	60.0	75.1	66.9	136.8
max	283.5	357.8	309.1	626.6
min	154.1	180.1	145.6	323.0
Cowpeas:				
1985	0.0	0.0	0.0	0.0
1986	66.9	83.8	149.7	244.7
1987	3.8	11.9	39.7	30.8
mean	23.6	31.9	63.1	91.8
std	30.7	37.0	63.3	108.8
max	66.9	83.8	149.7	244.7
min	0.0	0.0	0.0	0.0
	----------FCFA/ha----------			
Estimated Net Cash Income:				
1985	30904	33796	35892	58536
1986	12986	15007	17039	28044
1987	8199	8711	9850	10977
mean	17363	19171	20927	32519
std	9772	10656	10981	19672
max	30904	33796	35892	58536
min	8199	8711	9850	10977

Table 15. Stochastic Dominance Template Formulas.

```
Precalculation Formulas:
  R14: @IF(Q15=0,Q14,Q15)
  S14: +$R$4+S13
  T14: (Q14-Q13)*$S13+(Q14-Q13)*$R$4*0.5
  U14: +U13+T14
  Y14: 21196
  Z14: @IF(Y15=0,Y14,Y15)
  AA14: +$Z$4+AA13
  AB14: (Y14-Y13)*AA13+(Y14-Y13)*$Z$4*0.5
  AC14: +AC13+AB14

Calculation Formulas:
  AG9: +AG8+$AJ$3
  AH9: @IF(AG9<$Q$13,AG9,@VLOOKUP(AG9,$Q$13..$Q$33,0))
  AI9: @IF(AG9<$Q$13,AG9,@VLOOKUP(AG9,$Q$13..$R$33,1))
  AJ9: @IF(AI9>AH9,$R$4/(AI9-AH9),0)
  AK9: @IF(AG9<$Q$13,0,@VLOOKUP(AG9,$Q$13..$S$33,2))
  AL9: +AK9+AJ9*(AG9-AH9)
  AM9: @IF(AG9<$Q$13,0,@VLOOKUP(AG9,$Q$13..$U$33,4))
  AN9: +AM9+(AG9-AH9)*AK9+(AG9-AH9)*(AL9-AK9)*0.5
  AO9: +AO8+$AR$3
  AP9: @IF(AO9<$Y$13,AO9,@VLOOKUP(AO9,$Y$13..$AC$33,0))
  AQ9: @IF(AO9<$Y$13,AO9,@VLOOKUP(AO9,$Y$13..$AC$33,1))
  AR9: @IF(AQ9>AP9,$Z$4/(AQ9-AP9),0)
  AS9: @IF(AO9<$Y$13,0,@VLOOKUP(AO9,$Y$13..$AC$33,2))
  AT9: +AS9+AR9*(AO9-AP9)
  AU9: @IF(AO9<$Y$13,0,@VLOOKUP(AO9,$Y$13..$AC$33,4))
  AV9: +AU9+(AO9-AP9)*AS9+(AO9-AP9)*(AT9-AS9)*0.5

Stochastic Dominance Indicator Formulas:
  AW13: +Q13-Y13
  AX13: @IF(AW13>=0,1,0)
  AY13: @IF(AW13<=0,1,0)
  BA13: +AN8-AV8
  BB13: @IF(BA13>=0,1,0)
  BC13: @IF(BA13<=0,1,0)
```

Table 16. Net Cash Income Per Hectare, Estimated with Simulated Yields, 1961-1974, 1979-1985.

	Treatmemt Income (FCFA/ha)			
Year	T1	T2	T3	T4
Estimated with Simulated Yields:				
1961	28543	30490	37782	82993
1962	29246	31466	41260	84287
1963	34296	38908	43994	94946
1964	19357	21196	23368	52514
1965	48424	52329	50987	117766
1966	27923	33483	32502	82028
1967	10429	13927	15160	31736
1968	21991	22977	20300	30739
1969	29714	33606	33076	77156
1970	28367	30474	29462	70868
1971	41683	45998	39478	84468
1972	26877	28124	22989	25871
1973	39558	42351	46149	91735
1974	21233	30594	36976	76088
1979	77708	82267	111722	231456
1980	52937	53925	75060	155879
1981	97735	110843	119952	267261
1982	55084	61189	68089	145953
1983	58257	68219	79530	165188
1984	72551	80857	81600	162489
1985	21805	27821	34118	48847
mean	40177	44812	49693	103922
std	21562	23564	28357	62341
max	97735	110843	119952	267261
min	10429	13927	15160	25871

Screen 1. Temperature Factor Template, A1...F17

Temperature Factor Calculation

Parameters
Latitude
LAT - 13.48 degrees
Ideal Growth
TG1(min) - 10.00 degrees C
TG2(max) - 45.00 degrees C
TG3(opt) - 35 degrees C
-Tempfac-Tempfac-Tempfac-Tempfac-Tempfac-----Tempfac-----

Julian Day	Julian Factor	Solar Angle	Day Length	Max Temp C	Min Temp C
120	114.98	14.61	12.60	39.90	27.40
121	115.97	14.91	12.61	41.40	28.50

Screen 2. Temperature Factor Template, G1...N17

Calculated Constants

SINLAT - 0.23 COSLAT - 0.97

1/R - -0.0016 q/R - -0.0100
h/R - 0.11 n/R - 0.70
g/R - -0.96 m/R - -11.25
-Tempfac----Tempfac-Tempfac----Tempfac-Tempfac-Tempfac-Tempfac-Tempfac

Harmonic Mean Temp C	Morning Integration: B1	B2*	B2	b	c	Step I	Step II
33.6500	0.0000	2.3548	6.2980	3.97	-0.32	2.40	-0.11
34.9500	0.0000	1.8635	6.3033	4.09	-0.32	1.89	-0.06

Screen 3. Temperature Factor Template, Columns O10-W15

-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac
Low Temp Growth	Step I	Step II	High Temp Growth	Afternoon Integration: B3	B4*	B4	e	f
2.29	11.96	-6.57	3.44	6.2980	11.8745	12.5960	1.98	-0.16
1.83	12.21	-6.65	3.47	6.3033	12.5822	12.6067	2.05	-0.16

Screen 4. Temperature Factor Template, Columns X10-AD15

-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac	-Tempfac
Step I	Step II	Low Temp Growth	Step I	StepII	High Temp Growth	Temp Factor
0.76	0.12	0.72	6.83	1.47	5.50	11.94
0.02	0.01	0.02	6.34	3.63	5.94	11.26

Screen 5. Millet and Cowpea Growth Model Template, A1...F40

Millet and Cowpea, Maiguéro, Niger - Step 2 With Growth Stages and Yield Partitioning

Millet Parameters

Latitude

LAT -	13.48	degrees

Density and Leaf Area

Planting Date-	365.00	
Leaf Area Intercept-	0.0500	
QQ1 -	6000.00	kg/ha
Planted Hills/ha-	8,838.00	
Row Width-	1.50	meter

Soil Moisture

CAP -	28.00	mm
SMinit -	0.00	mm

Flowering and Grain Set

Flower Hour	12.26	hours
Flower Onset	65	days
Max Grain/Head Ratio	70.00	%percent
Grain Fill Factor	0.20	

Ideal Growth

Ro -	16.10	kg/ha/hr
TG1(min) -	10.00	degrees C
TG2(max) -	45.00	degrees C
TG3(opt) -	35.00	degrees C

Fertility

Y at 0 N -	40.00	%percent
N for Max -	30.00	kg/ha
N -	21.00	kg/ha
P Soil Test-	3.00	ppm
P for Max-	15.00	ppm
P applied-	19.00	kg./ha.

Senescence

Half Life -	15.00	

-Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86-

Julian Day	Day length	Actual Rain	Harmonic Mean Temp C	Leaf Area Factor	Soil Moisture
120	12.60	0.00	33.6000	0.0000	0.0000
121	12.61	0.00	34.9500	0.0000	0.0000

Screen 6. Millet and Cowpea Growth Model Template, F1...L40

Calculated Constants

Total DL = 4430.76

1-LAI= 0.9500

Row Length= 6666.67
Density = 1.00

Flower DAE= 58.00

N Fertfac= 0.87
Soil P 7.20 kg/ha
P for max 36.00 kg/ha
P Fertfac= 0.62

1-Delta = 0.9548

-Maiguéro86-- Rainfall Factor	-Maiguéro86-- Veg. Factor	-Maiguéro86-- Evapo-transp-iration	-Maiguéro86-- Growth Stage	-Maiguéro86-- Survival Factor	-Maiguéro86- Max. Possible Growth
0.0000	0.0933	0.0000 PRE		0.00	0.0000
0.0000	0.0933	0.0000 PRE		0.00	0.0000

Screen 7. Millet and Cowpea Growth Model Template, M34...R40

-Maiguéro86-- Temp Factor	-Maiguéro86-- Actual Growth Estimate	-Maiguéro86-- Veg Growth	-Maiguéro86-- Mobilize For Grain Fill	-Maiguéro86-- Veg Dry Matter	-Maiguéro86- Flower Dry Matter
11.94	0.00	0.00	0.00	0.00	0.00
11.26	0.00	0.00	0.00	0.00	0.00

Screen 8. Millet and Cowpea Growth Model Template, S34...W40

-Maiguéro86--	-Maiguéro86--	-Maiguéro86--	-Maiguéro86-
Daily Senescence	Dead Dry Matter	Grain Dry Matter	Cowpea Rainfall Factor
0.00	0.00	0.00	0.0000
0.00	0.00	0.00	0.0000

Screen 9. Millet and Cowpea Growth Model Template, Wl...AB40

```
Cowpea Parameters
            Sowing DASM-                    7.00
Density and Leaf Area

            Leaf Area Intercept-          0.0500
            QQl -                       2000.00 kg./ha.
            Planted Hills/ha-          17,777.00
            Row Width-                      1.50 meter
Soil Moisture
            CAP -                          28.00 mm
            SMinit -                        0.00 mm
Flowering and Grain Set
            Flower Hour                    10.75 hours
            Flower Onset                      56 days
            Max Grain/Head Ratio           70.00%percent
            Grain Fill Factor               0.15
Ideal Growth
            Ro -                           11.00 kg/hr/hr
            TGl(min) -                     10.00 degrees C
            TG2(max) -                     45.00 degrees C
            TG3(opt) -                     35.00 degrees C
Fertility
            Y at 0 N -                    100.00%percent
            N for Max -                     1.00 kg/ha
            N -                            21.00 kg/ha
            P Soil Test-                    3.00 ppm
            P for Max-                     12.00 ppm
            P applied-                     19.00 kg/ha
Senescence
            Half Life -                    30.00
-Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86-
   Cowpea      Cowpea      Cowpea       Max.       Cowpea       Actual
  Leaf Area    Growth     Survival    Possible     Temp        Growth
   Factor      Stage       Factor      Growth      Factor       Estimate
-----------------------------------------------------------------------
 0.0000 PRE                0.00        0.0000      9.44          0.00
 0.0000 PRE                0.00        0.0000      9.76          0.00
```

Screen 10. Millet and Cowpea Growth Model Template, C5...AH40

```
DASM Germ=          0.00

1-LAI=            0.9500

Row Length=      6666.67
Density =           0.67


Flower DAE=        49.00


N Fertfac=          1.00
Soil P              7.20 kg/ha
P for max          28.80 kg/ha
P Fertfac=          0.78

1-Delta =         0.9548
-Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86--Maiguéro86-
Cowpea      Mobilize       Veg       Flower, Pod   Cowpea      Dead Veg
                For          Dry       and Grain     Daily        Dry
   Growth    Grain Fill    Matter    Dry Matter  Senescence    Matter
-----------------------------------------------------------------------
   0.00        0.00         0.00        0.00        0.00        0.00
   0.00        0.00         0.00        0.00        0.00        0.00
```

Screen 11. Millet and Cowpea Growth Model Template, AI34...AK40

```
-Maiguéro86--Maiguéro86--Maiguéro86-
               Cowpea
   Cowpea       Veg        Cowpea
    Hay        Factor      Grain
-----------------------------------
    0.00      0.0933        0.00
    0.00      0.0933        0.00
```

Screen 12. Stochastic Dominance Template, Q1...X14

Precalculations, Alt 1:
#obs alt1 21
Interval1 0.0476

Alt 1 T4	Next Obs.	Prob. Level	Area This Obs.	Cum. Area
25871	30739	0.0476	0.00	0.00
30739	31736	0.0952	347.71	347.71

Screen 13. Stochastic Dominance Template, AG1...AN9

Alt 1 Calculations
Scale 1 Initial Value- 10000.00
Scale 1 Interval- 1000.00

Next 1 Lower	Next Higher	Current slope	Last Prob.	Current Point	Last Area	Current Area	Scale
10000	10000.00	10000.00	0.000000	0.0000	0.0000	0.0	0.0
11000	11000.00	11000.00	0.000000	0.0000	0.0000	0.0	0.0

Screen 14. Stochastic Dominance Template, AW1...BC13

Stochastic Dominance Indicator

Sum AX-	21.00	Sum BB-	4.00
Sum AY-	0.00	Sum BC-	195.00
Rows-	21.00	Rows -	195.00

Alternative FDSD- Alt 1 Alternative SDSD- Alt1

First Degree SD: Level Diff	Alt 1 Indicator	Alt 2 Indicator	Second Degree SD: Area Diff	Alt 1 Indicator	Alt 2 Indicator
11944	1.00	0.00	0.00	1.00	1.00
9543	1.00	0.00	0.00	1.00	1.00

Figure 2.1 Temperature Function

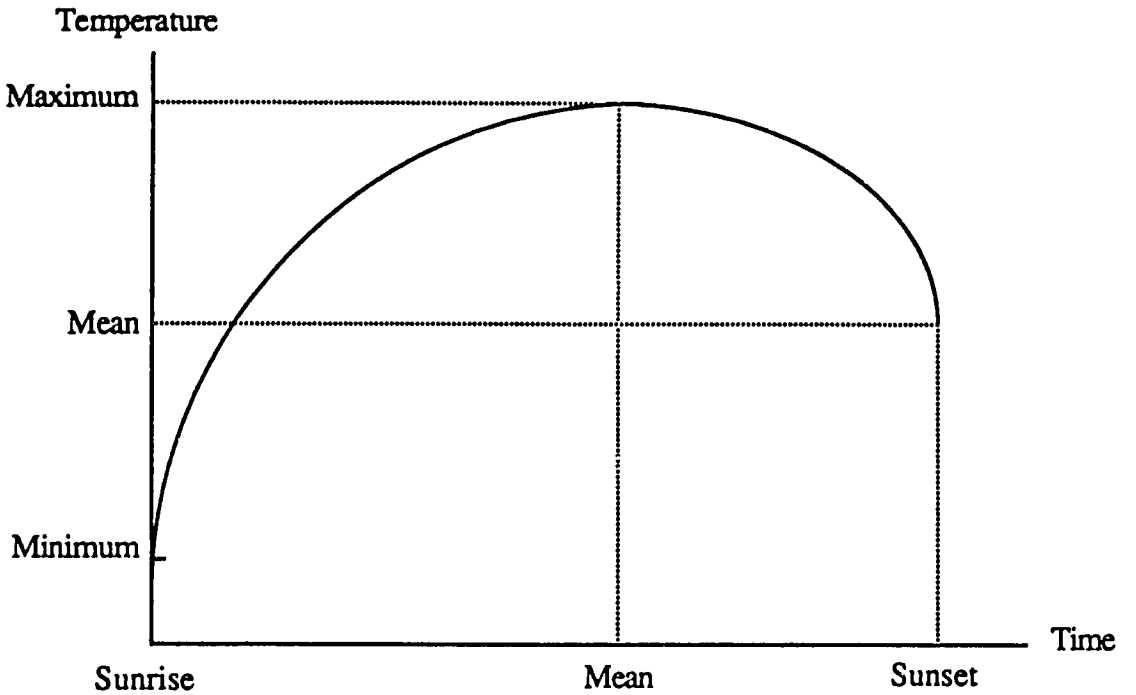

Figure 2.2 Growth Function

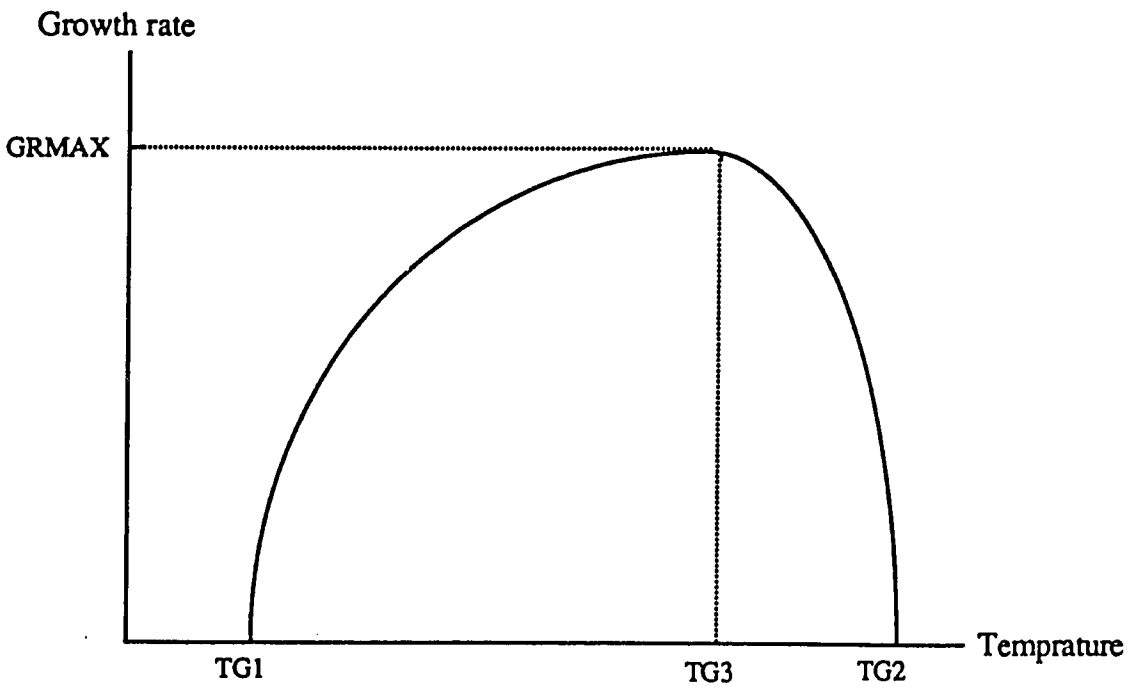

Figure 2.3 Leaf Area Factor

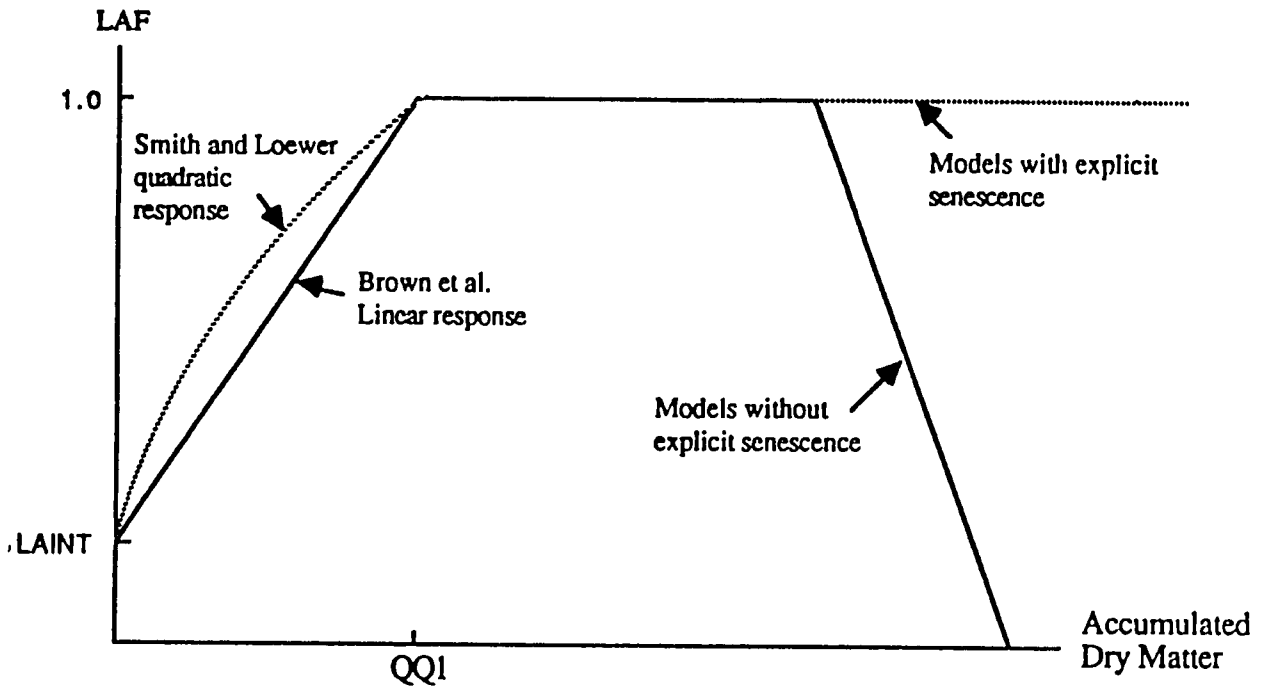

Figure 2.4 Soil Water Factor

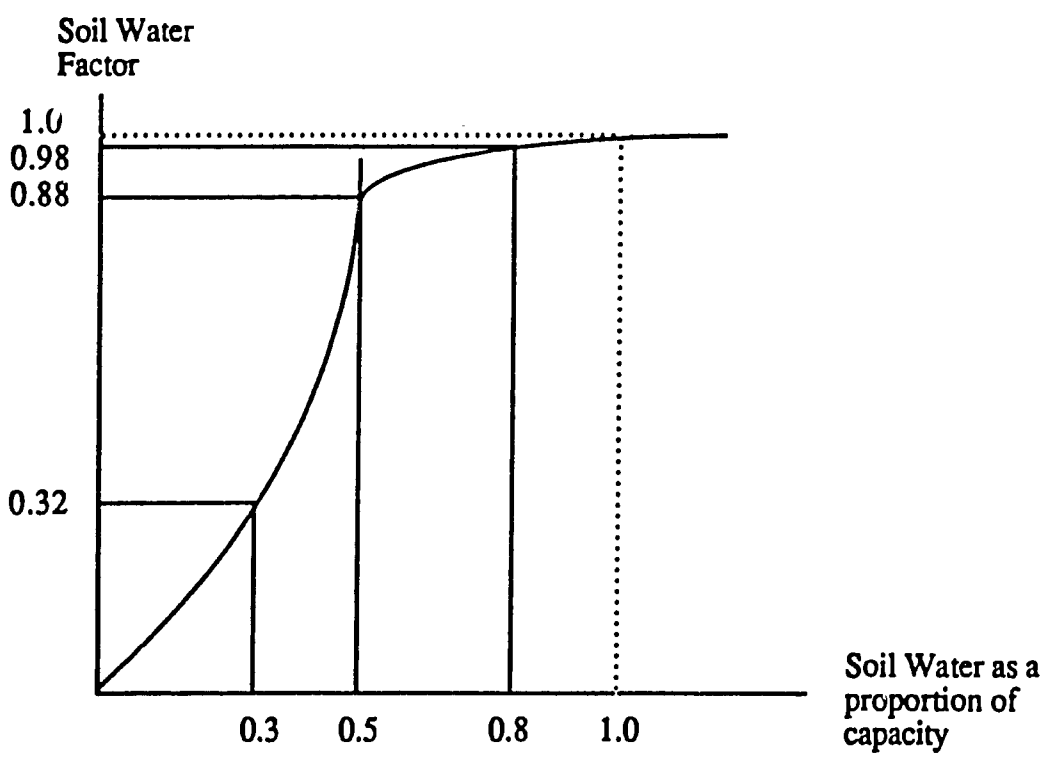

Figure 2.5 Vegetation Factor

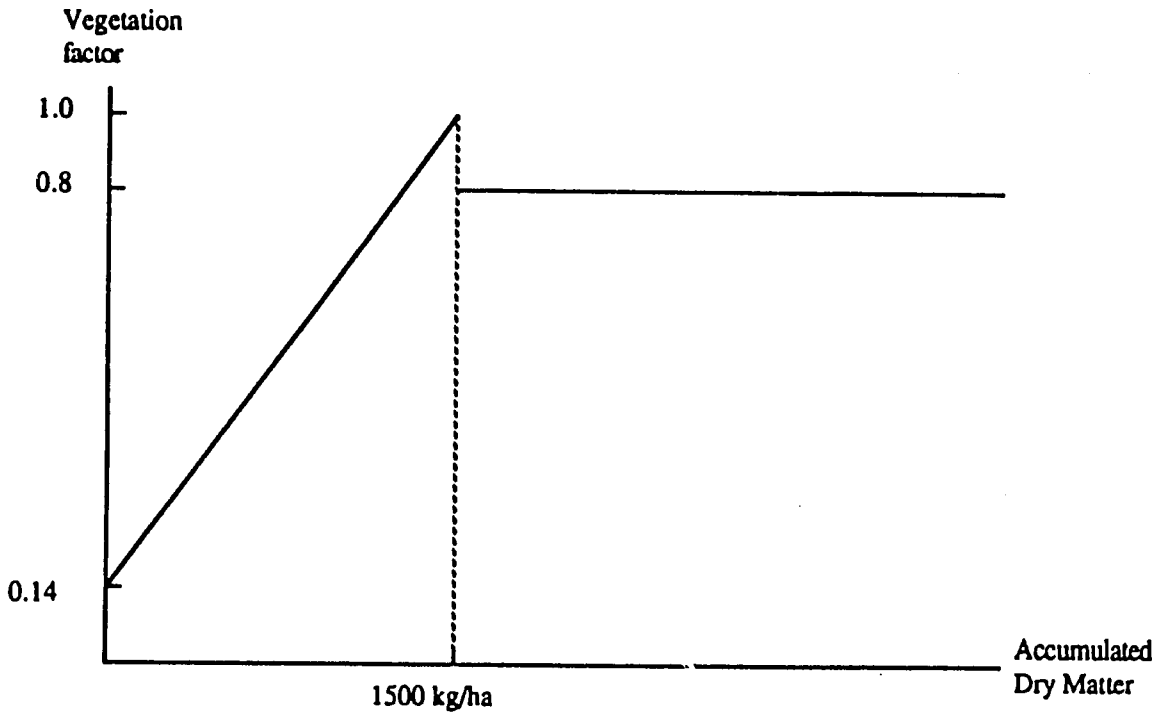

Figure 2.6 Fertilizer Response Function

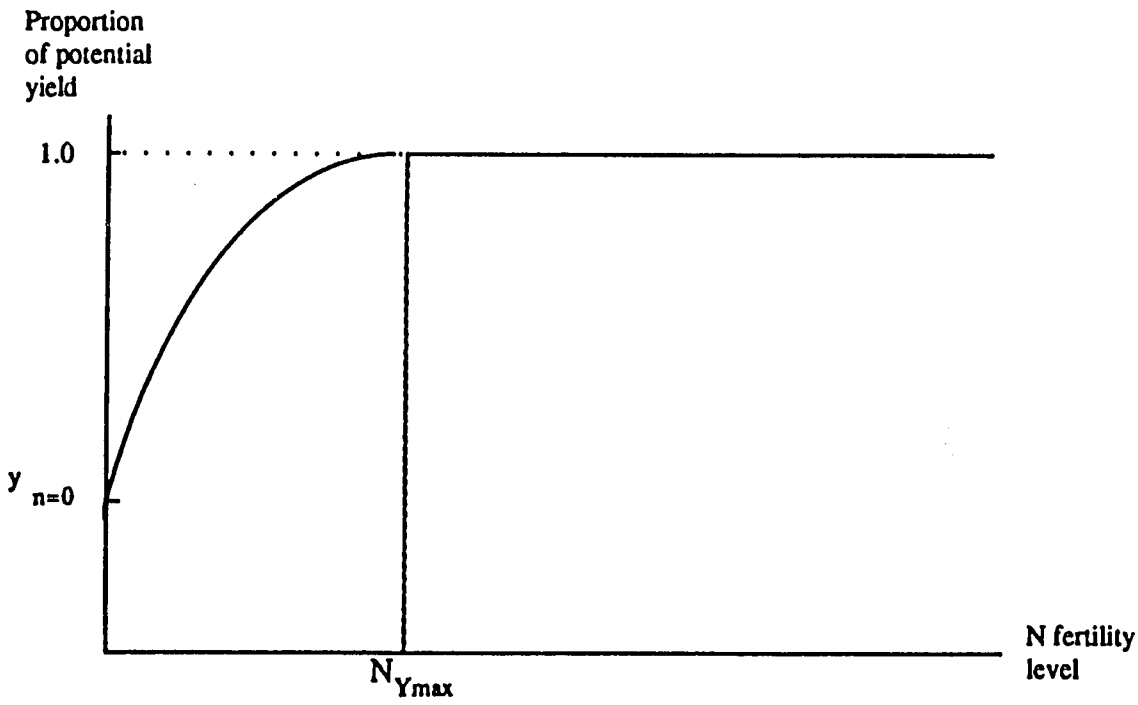

Figure 5.1 Simulated Millet Yield, 1961 - 1974 and 1979 - 1985, by Treatment.

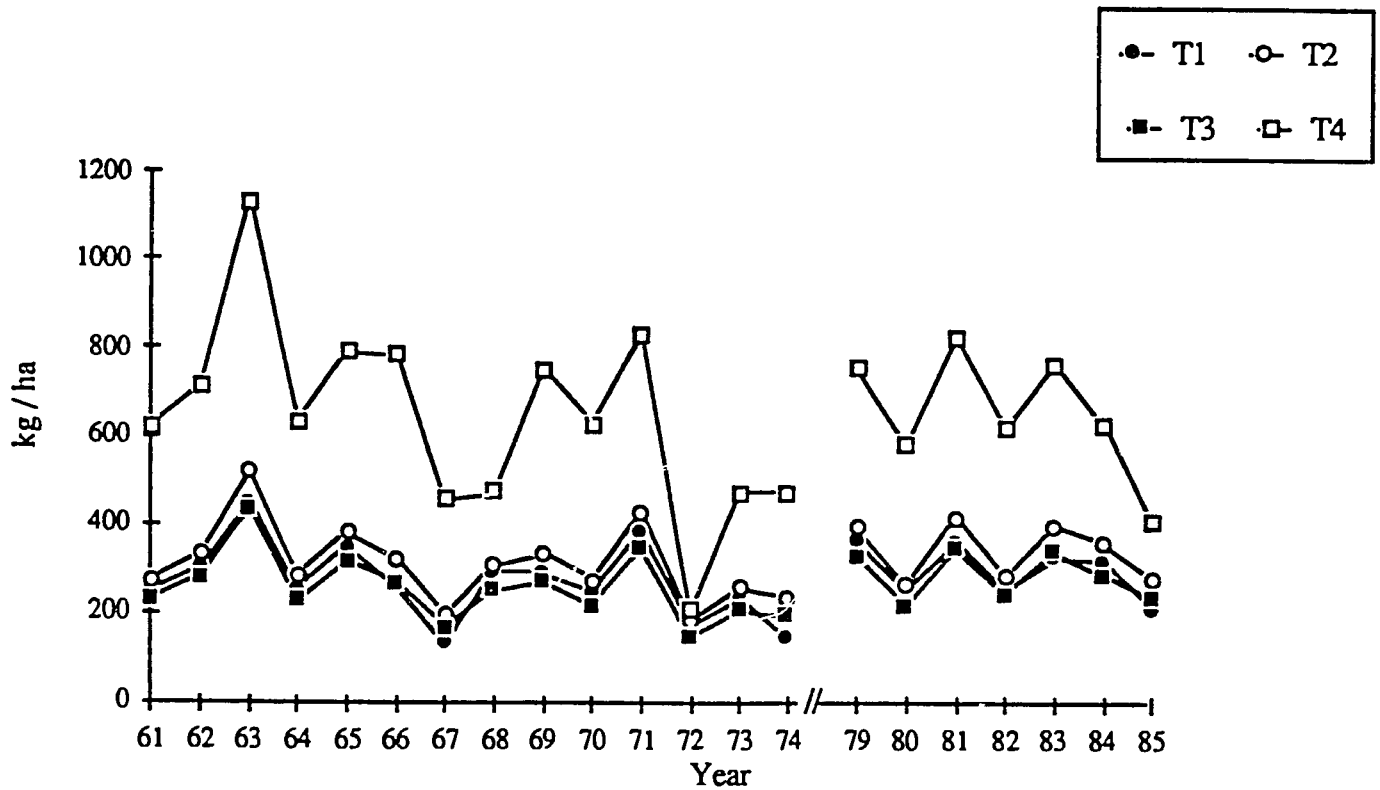

Figure 5.2 Simulated Cowpea Yield, 1961 - 1974 and 1979 - 1985, by Treatment.

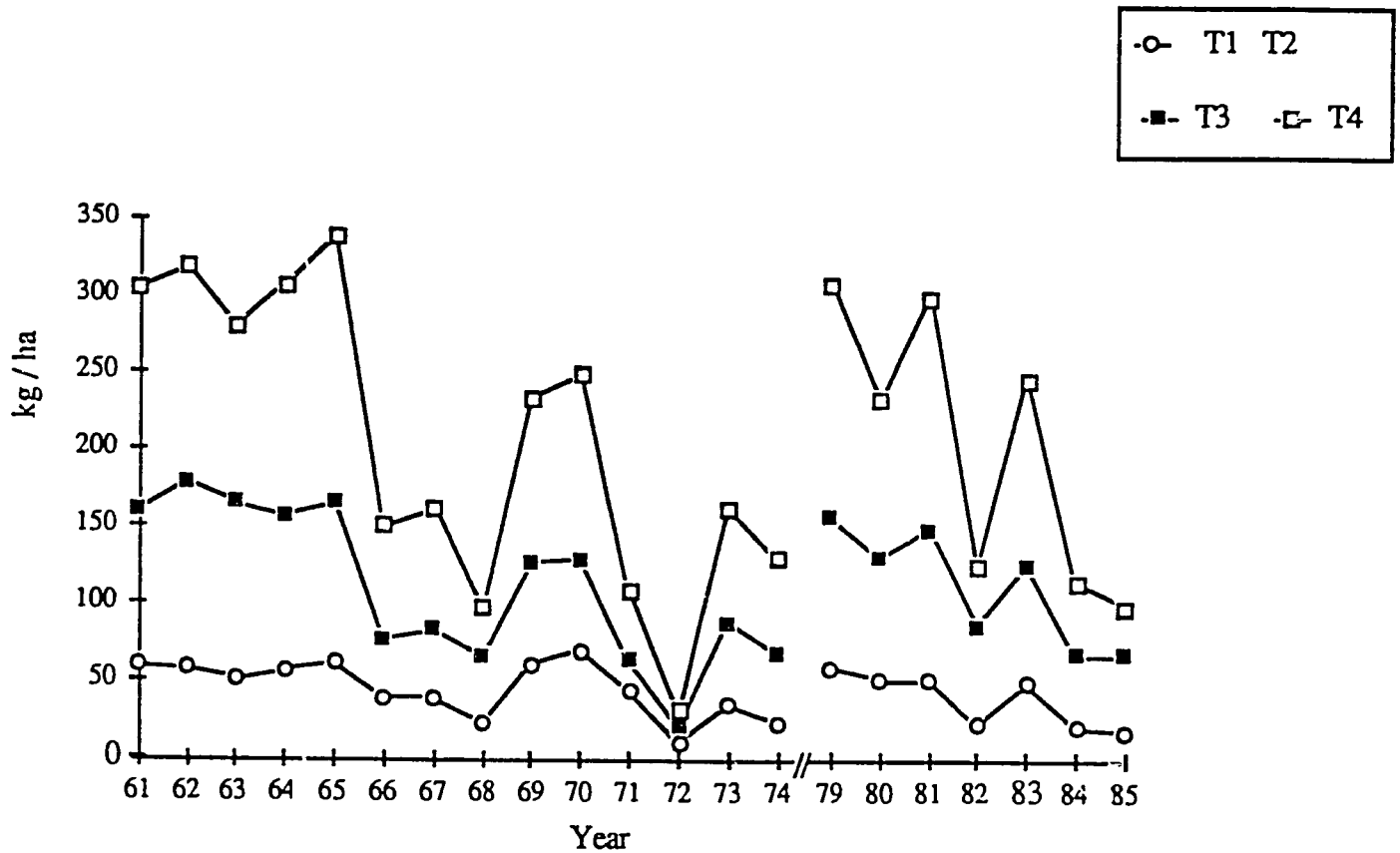

Figure 5.3 First Degree Stochastic Dominance: Case 1

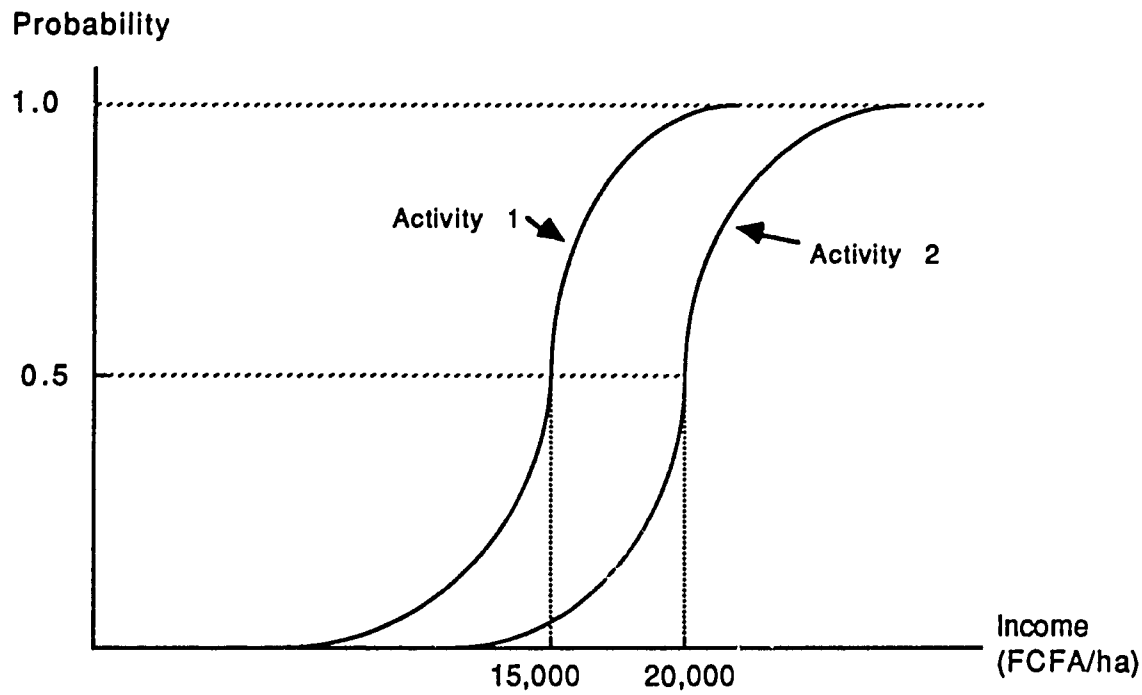

Figure 5.4 First Degree Stochastic Dominance: Case 2

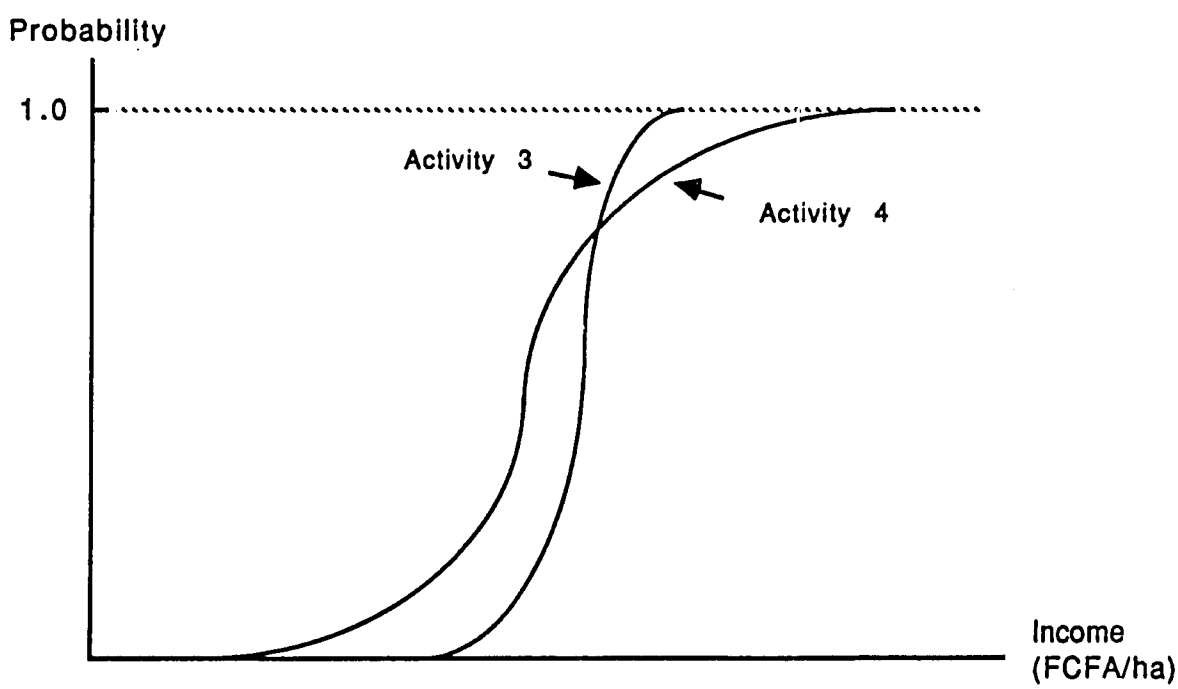

Figure 5.5 Second Degree Stochastic Dominance

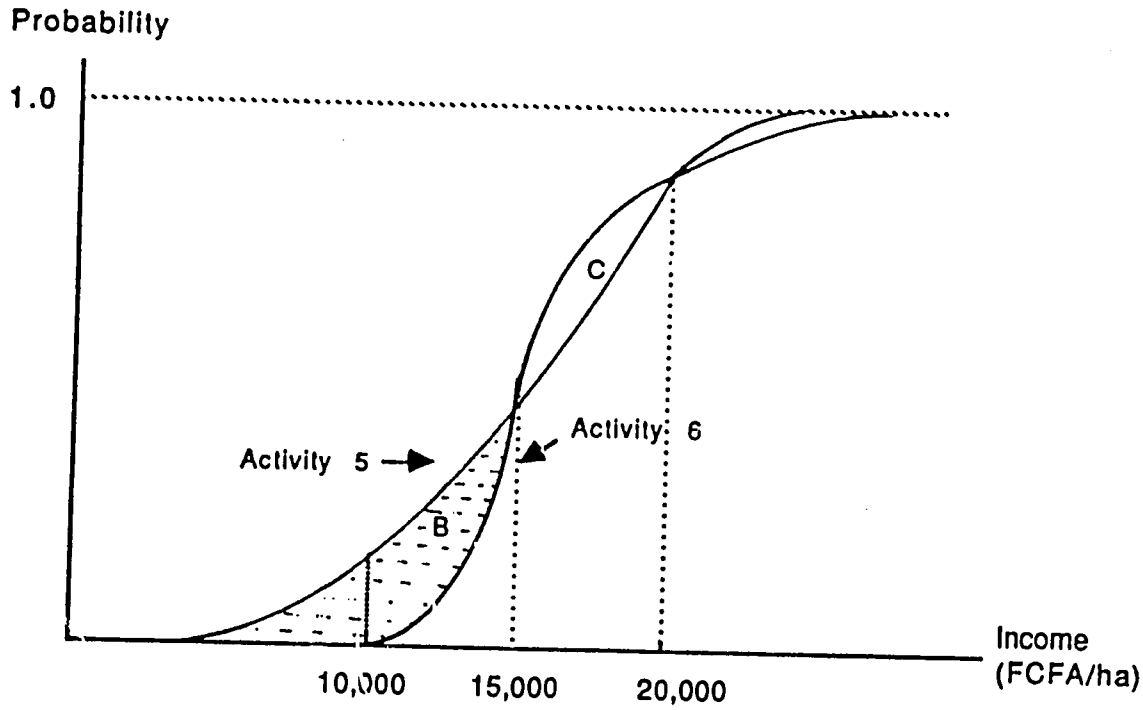

Figure 5.6 Calculation of Area Under a Piecewise Linear Cumulative Distribution Function

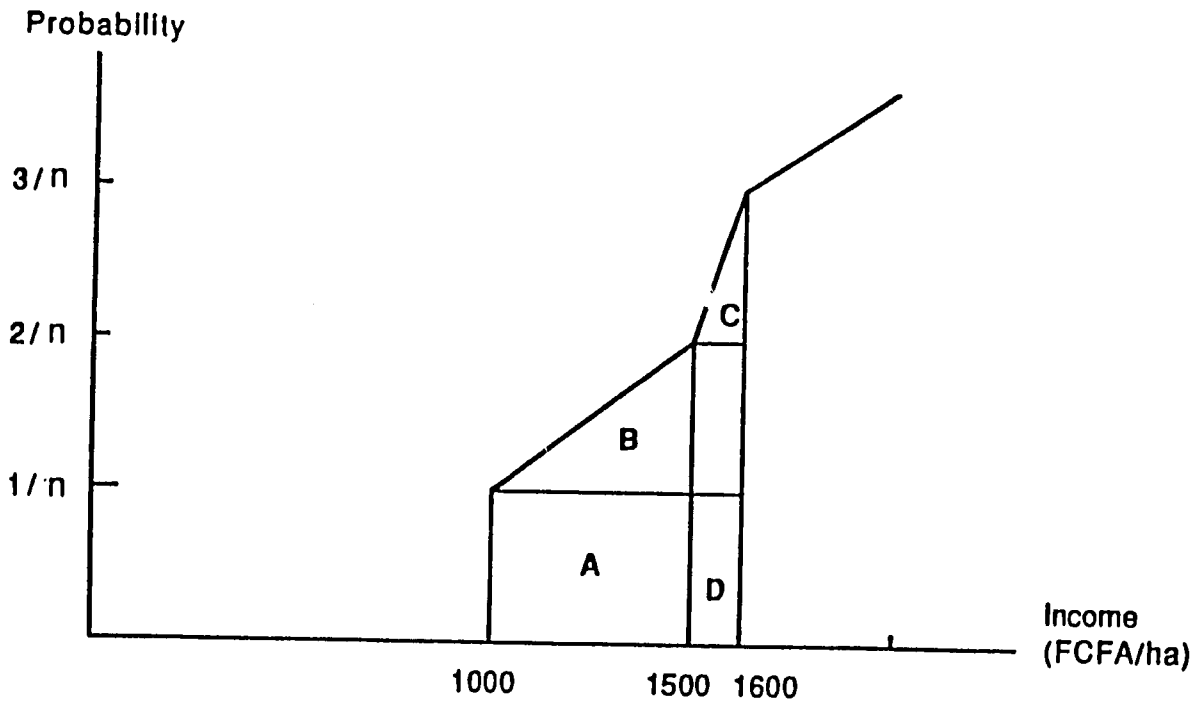

Figure 5.7 Cumulative Distribution of Millet Yields Per Hectare, Per Treatment.

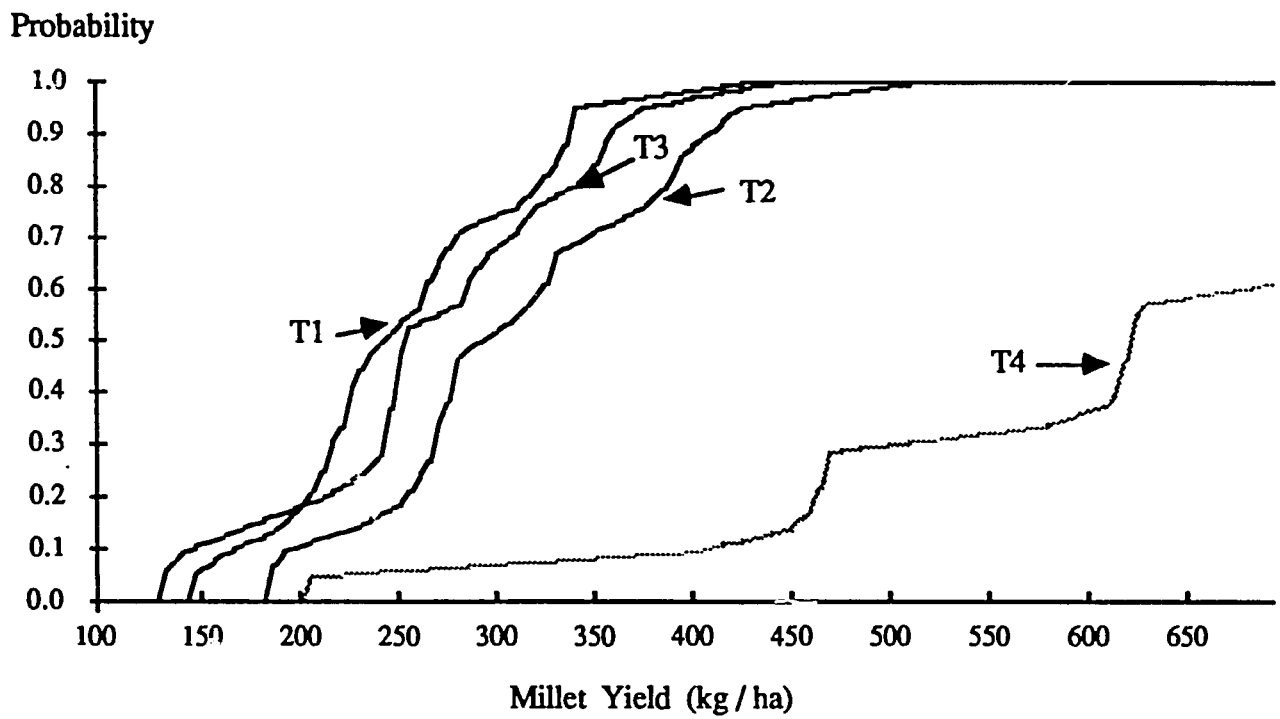

Figure 5.8 Cumulative Distribution of Net Returns Per Hectare, Per Treatment.

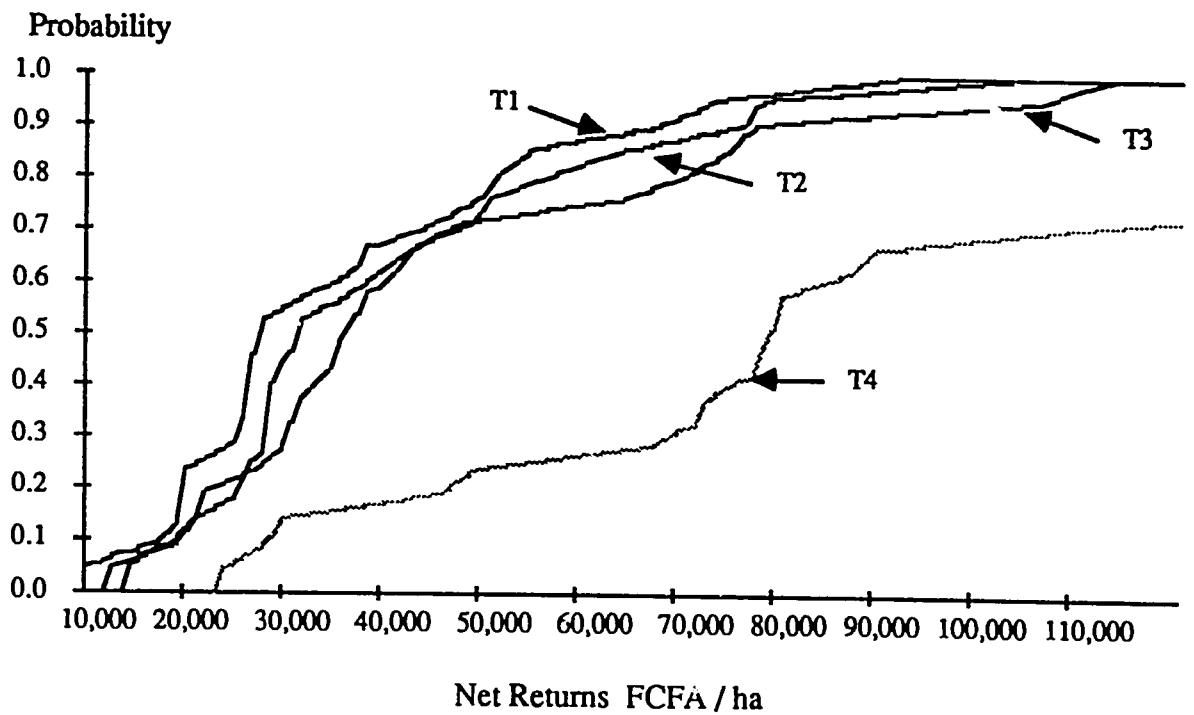

Figure 5.9 Net Cash Income Per Hectare, 1961 - 1985, Per Treatment.

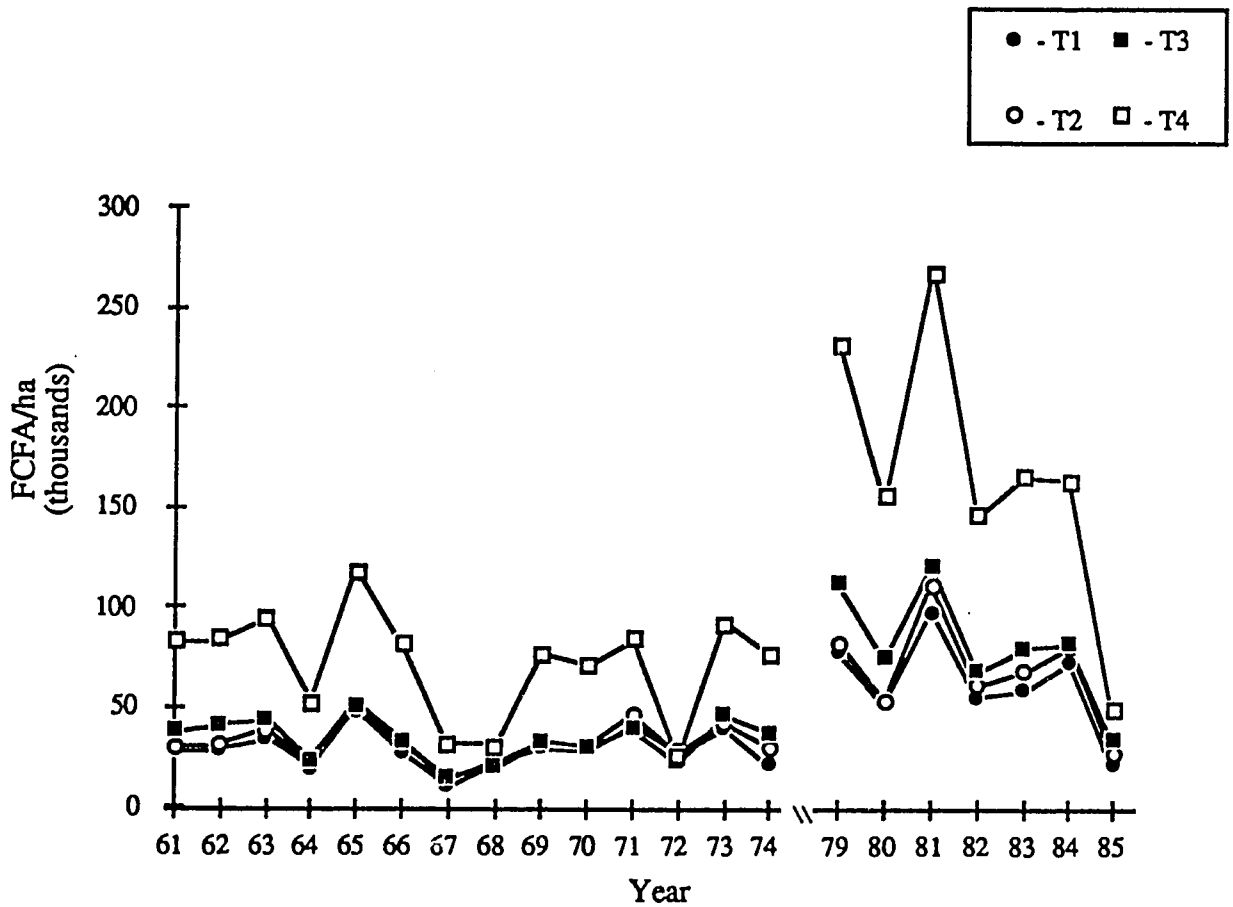

Figure 5.10 Cumulative Distributions of Millet Yield at Maiguero for Treatments 1-4

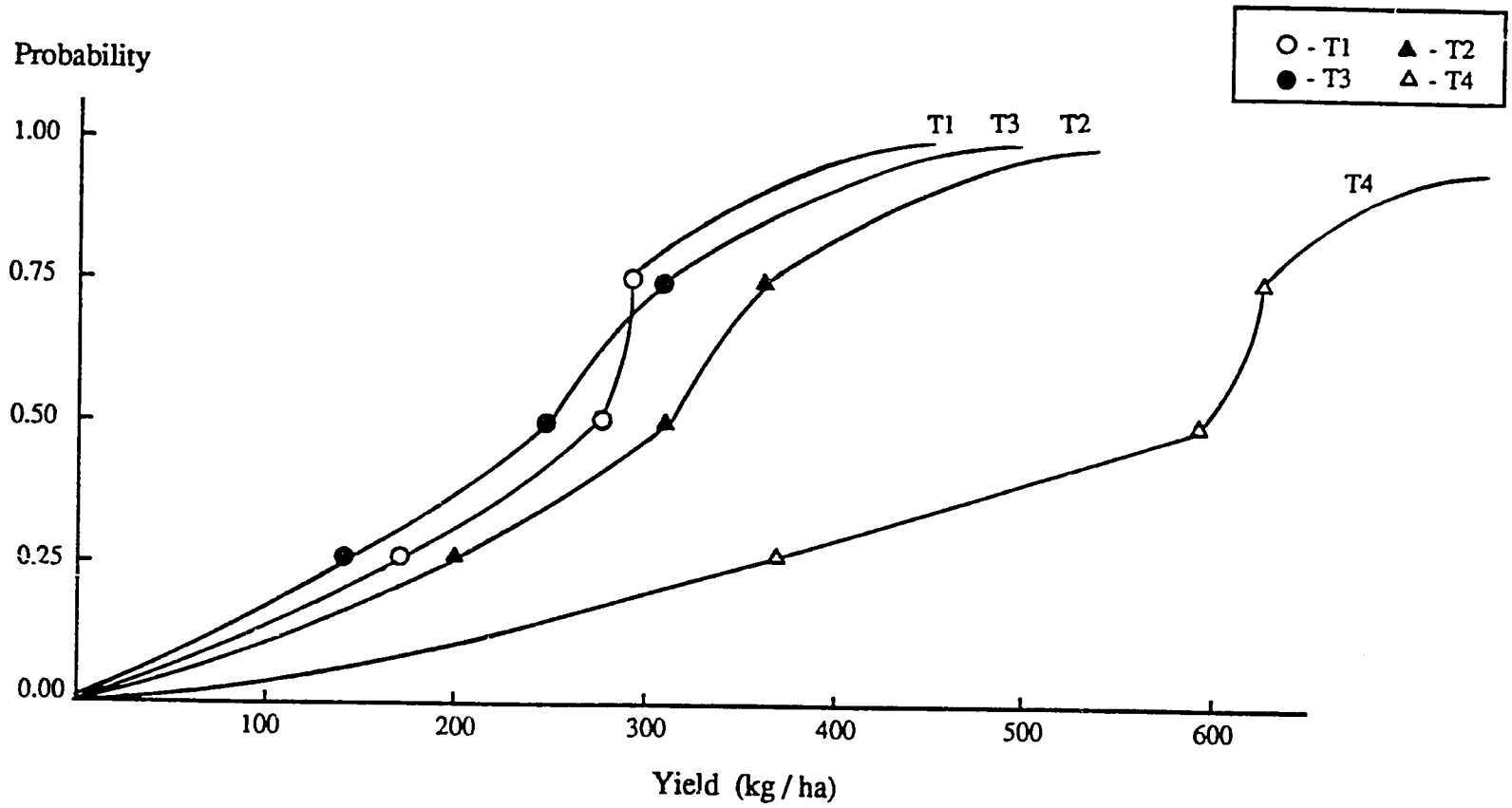
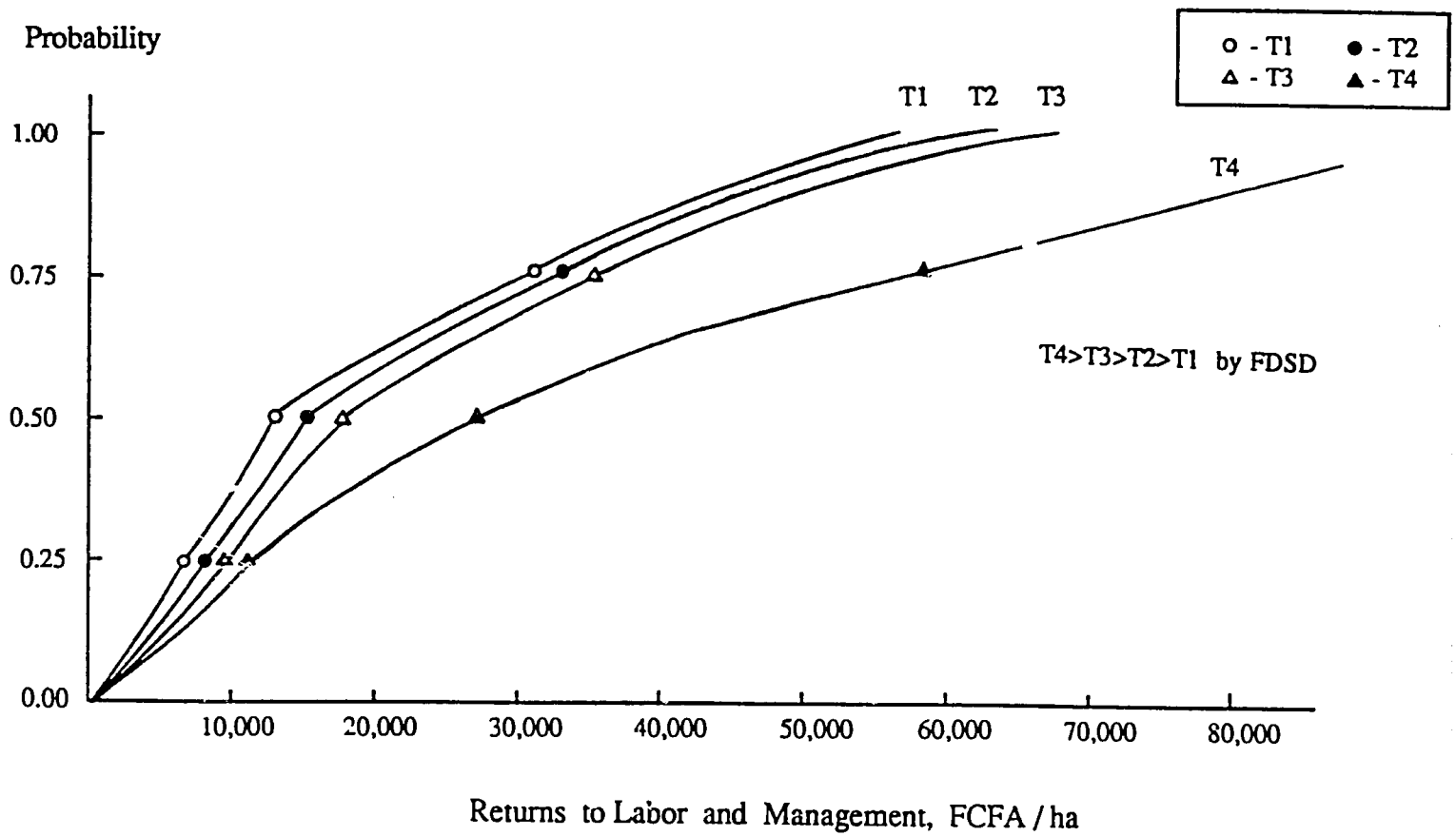
Figure 5.11 Cumulative Distributions of Returns to Labor and Management for Treatments 1-4