PN-ABF-124
66154

STOCHASTIC LINEAR RESPONSE AND PLATEAU FUNCTIONS IN MODELING CROP RESPONSE TO FERTILIZATION IN NIGER, WEST AFRICA

BY:

Patrick A. Jomini; Robert R. Deuson;
J. Lowenberg-DeBoer; André Bationo[1]

90-8 December 1989

[1] Respectively: Research Assistant, Purdue University; Assistant Professor, Purdue University; Economist, National Agronomic Institute of Niger; Soil Scientist, International Fertilizer Development Center.

This research is supported by the U.S. Agency for International Development, Science and Technology Bureau, Technology of Soil Moisture Management Project under USDA PASA No. BST-4021-P-AG-108D-00.

FOREWORD

This paper was submitted as a poster presentation to the CSIRO International Symposium on Climatic Risk in Crop Production: Models and Management for the Semi-Arid Tropics and Sub-Tropics, held at the University of Queensland, St Lucia, Brisbane, Queensland, Australia, July 2-9, 1990. The paper itself is followed by a reproduction of the submitted poster.

I. Introduction

The low phosphorus (P) and nitrogen (N) content of many sandy soils of the West African Semi-Arid Tropics (WASAT) constitutes a major constraint to increasing food production in the region (Jones & Wild, 1975). Many soils in Niger are sandy (> 90% sand), low in organic matter (< 1%) and plant-available P (Bray P1 test of 3 ppm), and have a low moisture holding capacity with a 3% moisture content measured at field capacity (Mahaman, 1988). This low soil fertility is complemented by low and erratic rainfall which occurs in a single cropping season from June to October (Sivakumar, 1988). Chemical fertilization is believed to be a viable way of raising soil fertility, and thus food production and farm income. However, recent estimates (Ministère de l'Agriculture, 1985; FAO, 1988), put average fertilizer use at less than 1 kilogram per hectare of cultivated land. Though millet, the main food crop in Niger, response to P and N fertilization is significant and well documented (Bationo et al., 1986; Jones and Wild, 1975; Mughogho et al., 1986; Roesch and Pichot, 1985) little work has been done to determine optimal application rates. There has also been little effort to link fertilizer response and application rates to specific environmental conditions such as natural soil fertility and rainfall (Bationo et al., 1986). Carryover effects especially of P, though widely recognized by researchers (e.g. Jones and Wild, 1975) and farmers have not been taken into consideration when making fertilization recommendations. The main objective of this paper is to determine economically viable levels of P and N fertilization for millet grown on sandy soils in Niger while taking environmental conditions and carryover effects into account.

Economists have recently recognized the value of using Linear Response and Plateau (LRP) functions in crop response analysis (e.g. Perrin, 1976; Lanzer and Paris, 1985; Grimm et al., 1988). This functional form is based on agronomic principles first advocated by the German chemist Justus von Liebig (1863). The LRP function is based on the Law of the Minimum according to which crop growth is proportional to the availability of the most limiting nutrient until another factor becomes limiting. Any increase in an input other than the limiting input does not result in any response and the function displays a horizontal plateau over a large range of the inputs considered (Redman and Allen, 1954). The two-input LRP function shown in Figure 1 illustrates these principles. Recent advances in computing capacity and the development of derivative-free solution algorithms for compartment analysis problems (Ralston and Jennrich, 1978) have made the estimation of LRP surfaces less costly. In this paper, an LRP surface for millet response to N, P and a measure of

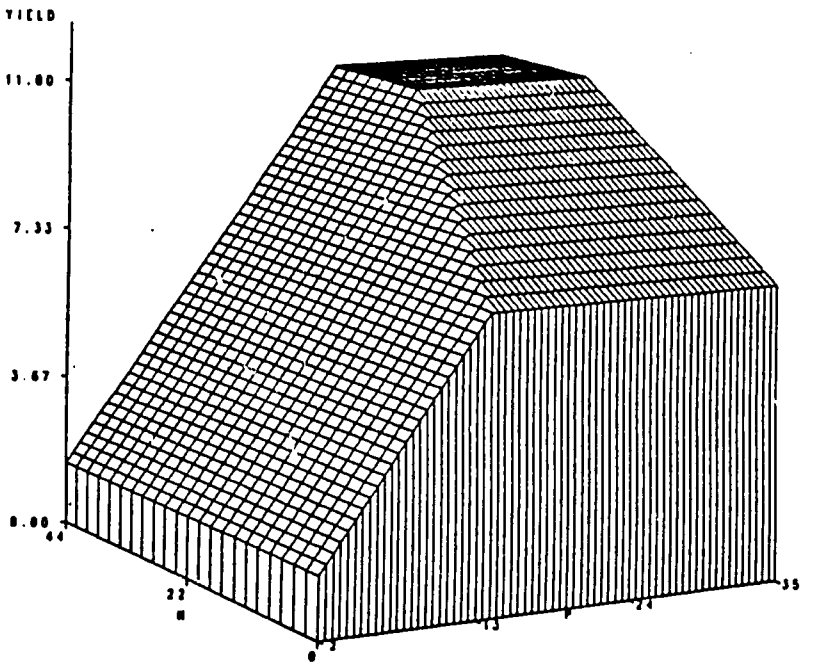

Figure 1. Two-input linear response and plateau function for modeling millet response to nitrogen and phosphorus

available moisture is estimated and used to illustrate an important property of these functions in modeling uncertain crop response. A stochastic formulation of the LRP is specified and used in a dynamic optimization model to provide viable fertilization strategies.

In the following section, previous uses of the LRP concept and of relevant dynamic models for the fertilization problem are reviewed. A model of fertilizer decision-making under uncertainty, taking carryover into account is presented in section three. Conditions for optimality are derived in section four, while data from experiments conducted in Western Niger are described in section five. Results and recommendations are discussed in section six before conclusions are offered in section seven.

II. Previous LRP function applications

The LRP specification arises from von Liebig's "Law of the Minimum" according to which plant growth is proportional to an increase in the supply of the most limiting factor called the "minimum factor" (Redman and Allen, 1954). Anderson and Nelson (1975) have shown that the concept of the LRP function provides a useful framework from the design of experiments to the analysis of data and the formulation of recommendations.

While the concept of proportional response was criticized by various authors (e.g. Mitscherlich, 1909), the plateau concept is generally well accepted among biological researchers. Waggoner and Norvell (1979) found the LRP formulation to be an adequate approximation for corn (*Zea mays* L.) and red clover (*Trifolium pratense* L.) grown in Iowa. Cate and Nelson (1971); Anderson and Nelson (1975); Ackello Ogutu et al. (1987) and Grimm et al. (1987) found the LRP specification to be a good yield-predicting model by various statistical and goodness-of-fit criteria for a wide variety of crops grown in temperate areas. Lanzer and Paris (1981) showed that maintenance fertilization recommendations for a wheat-soybean cropping system in Southern Brazil could be significantly improved from a cost reduction standpoint by using a LRP framework. While most of these authors' discussion centers on the superiority of the LRP specification over continuously differentiable functions, this will not be done here. The reader is referred to Jomini, et al. (1988) and Jomini (1990) for detailed procedures and criteria for discriminating between response models.

III. An economic model of the fertilization problem

The model developed in this section presents 3 important characteristics:

1. it is developed in a stochastic framework to account for the risk surrounding response to fertilization in Niger;
2. it is developed in a dynamic framework to account for the carryover effects of some fertilizers;
3. it is based on the LRP specification for modeling crop response.

As a first approximation, the farmer is assumed to maximize in each period t the expected income from cultivating millet on one hectare, with input and output prices assumed known:

$$(1) \qquad \max \ \pi_t^e = p_y Y_t^e - \sum_{i \in I} c_i X_{it}$$

where

c_i and p_y are input and output prices respectively,

Y_t^e is expected yield,

X_{it} is the amount of input i purchased and

π_t^e is expected profit.

The farmer's decisions are constrained by the technology he is using and the natural conditions (in this case soil fertility and moisture) under which he operates. The following constraints define the feasible set:

$$(2) \qquad Y_t^e = \sum_{s \in S} p^s Y_t^s$$

$$(3) \qquad Y_t^s = \min_{i \in I} [\alpha_{i0}^s + \alpha_{i1}^s W_{it}, M^s]$$

$$(4) \qquad b_{it} = a_{i0} + a_{i1} W_{it} + a_{i2} Y_t^e$$

$$(5) \qquad W_{it} = b_{it} + \lambda_i X_{it}$$

$$(6) \qquad b_{i0} = \underline{b}_i$$

where new variables include:

Y_t^s the expected yield in state of nature s,

W_{it} the amount of input i available for plant uptake during cropping season t,

b_{it} the plant-available stock of input i uptake at the beginning of the cropping season.

New parameters are:

α^s_{i0} and α^s_{i1} the linear response parameters to input i which may depend on state of nature s,

M^s the maximum yield plateau associated with state of nature s,

a_{i0}, a_{i1}, and a_{i2}, the carryover parameters for input i, and

λ_i the rate at which applied input i is transformed into plant available form during the cropping season.

The relations between soil and inputs are described in the carryover equations (3). In the problem at hand, the inputs considered in set I are N, P and useful rainfall (R). Of these three factors, carryover is most important for P. In Niger, sandy soils dry up for all practical purposes between two cropping seasons. This means $b_{Rt} = b_{Rt+1} = 0$. Different measures of available moisture are discussed later. In the case of N, losses through volatilization are very high due to high temperatures (Jones and Wild, 1975). This means a large part of any applied N is lost before the following cropping season. In addition, the accurate measurement of plant available N is difficult. For these reasons, the carryover function for N is not estimated, but is assumed to be $W_{Nt} = X_{Nt}$.

The only carryover function left in the specific model therefore describes the behavior of plant-available P. This function is based on a very simplified nutrient balance approach (Frissel, 1978), accounting for additions and decreases to the nutrient stock. The interpretation of the carryover coefficients is:

a_{P0} the net rate by which the stock of plant-available P is increased from one year to the next. This coefficient results from the sum of the rate at which plant-available P is released by the natural weathering of soil particles, and the amount of P brought with wind-borne dust, less the losses due to erosion and limited leaching.

$a_{P1} = \partial b_{Pt+1} / \partial b_{Pt}$ the rate at which plant-available P carries over into the following year,

$a_{P2} = \partial b_{Pt+1} / \partial Y^e_{Pt}$ the average rate at which P is taken up by harvested plant matter.

The LRP concept is applied in defining millet grain yield response (Y^s_t) to the inputs considered (equation 2). This function is defined for sandy soils similar to those on which the experiments were conducted and for meteorological conditions prevailing in state of nature s. States of nature are defined in terms of moisture availability.

While it is possible that the slope parameters α^s_{i0} and α^s_{i1} may change with the state of nature, the largest effect would probably be the changes in plateau level because the limiting factor would often depend on the state of nature. Thus it is assumed here that only the plateau (M^s) varies with the state of nature. The response to a given nutrient is therefore assumed not to be affected by the moisture supply if this supply is sufficient to promote adequate plant growth. This assumption is consistent with the implications of the Law of the Minimum.

The expected yield response (Y^e_t) is defined in equation 3 as a linear combination of the response functions characterizing each state of nature (Y^s_t). This linear combination is itself a minimum function with at most $(i + 1)^S$ terms. In order to illustrate this algebraically, a single-input, two states of nature example is shown below. The expected yield function is:

$$(7) \qquad Y^e = \sum_{s=1}^{2} p^s \min [\alpha^s_0 + \alpha^s_1 X, M^s]$$

where

Y^e is expected yield,

p^s is the probability associated with state of nature s,

α^s_0 and α^s_1 are the linear response parameters,

M^s is the plateau level associated with state of nature s, and

X is the plant-available level of input.

The linear combination of minimum functions can be rewritten as a minimum function which is composed of 2^2 terms:

$$(8) \qquad Y^e = \min [p^1 (\alpha^1_0 + \alpha^1_1 X) + p^2 (\alpha^2_0 + \alpha^2_1 X), p^1 (\alpha^1_0 + \alpha^1_1 X) + p^2 M^2, \\ p^1 M^1 + p^2 (\alpha^2_0 + a^2_1 X), p^1 M^1 + p^2 M^2]$$

Each term of the minimum expected yield function (Y^e) is a linear combination of terms of the original yield functions specific to each state of nature (Y^s). Since some terms may be redundant, the graphical representation of the expected yield function may not be composed of as many planes as would be expected from the number of terms in the algebraic expression.

This is further illustrated in Figure 2 in which a single-input, three-states of nature combination generated an expected yield function composed of 4 splines (assuming parameters α_0 and α_1 do not depend on the

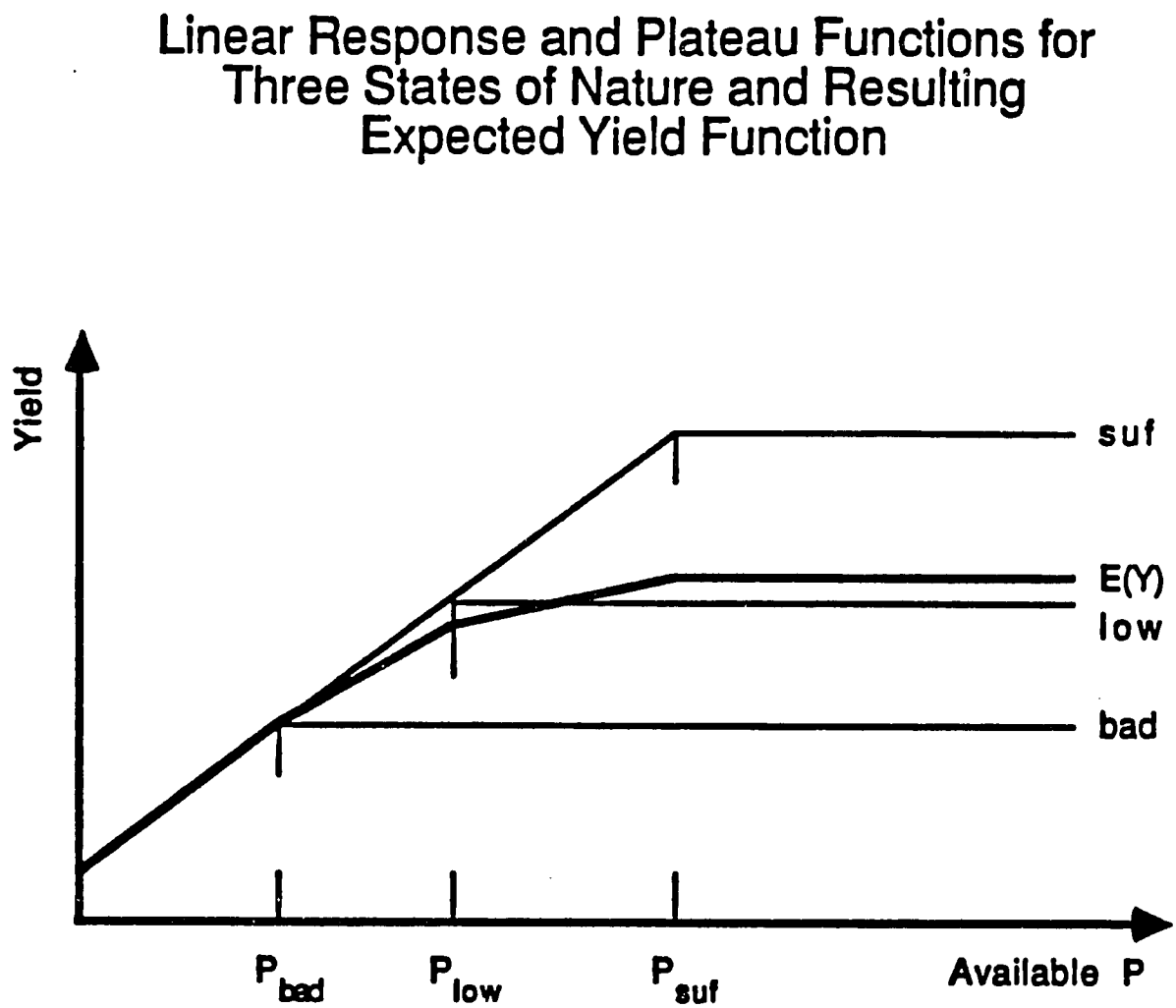

Figure 2. Single-input linear response and plateau functions for three states of nature and corresponding expected yield function

states of nature). It is worth noting that multiplying the number of states of nature results in an expected yield function that approaches a smooth form on a certain interval (between the lowest and highest plateaus). The origin of this smoothness is however specifically linked to the existing states of nature, not to the input response. This smoothness will be shown in the following section to have important consequences on the optimization process.

IV. Optimality conditions.

Optimality conditions for the fertilizer problem have been reviewed by Dillon (1977) and Kennedy et al. (1973). When nutrients carry over into following production periods, the problem is to choose application rates and timing that maximize the net present value of the stream of income generated by the cropping activity. In general, the cost of fertilization must be covered by the value of the marginal yield increases over the entire planning period that are due to the application.

A schematic relating the different parameters and variables is shown in Figure 3. The same notation as was used in the model specification is used here. The problem of determining optimal fertilization rates can be divided into two sub-problems once parameters are known. In a first step, the optimization process (equation 1), in combination with the biological response function (equation 3), determines jointly the optimal yield (Y_t^e) and fertility level (W_{Pt} and W_{Nt}) to achieve. These results serve in turn as input to finding the optimal fertilizer application levels (X_{Pt} and X_{Nt}) given an initial soil fertility (b_{Pt}) level.

Assuming the farmer maximizes current and future benefits due to the current fertilizer application, the problem can be restated in a recurrance relation:

$$(9) \qquad \pi_t(b_{Pt}) = \max_{W_{Pt},W_{Nt}} \left[p_y Y_t^e - \sum_{i \in I} c_i X_{it} + (1+r)^{-1} \pi_{t+1}(b_{Pt+1}) \right]$$

where:

π_t is the maximum expected profit that results from the optimal application choice given the pre-season soil test level of b_t and assuming all future decisions are taken optimally, and

r is the private rate of discount reflecting the decision-maker's rate of time preference.

Since time enters the problem only through the discount term, this is an autonomous problem, which tends toward a steady state equilibrium in the long run (Kamien and Schwartz, 1981). The steady state

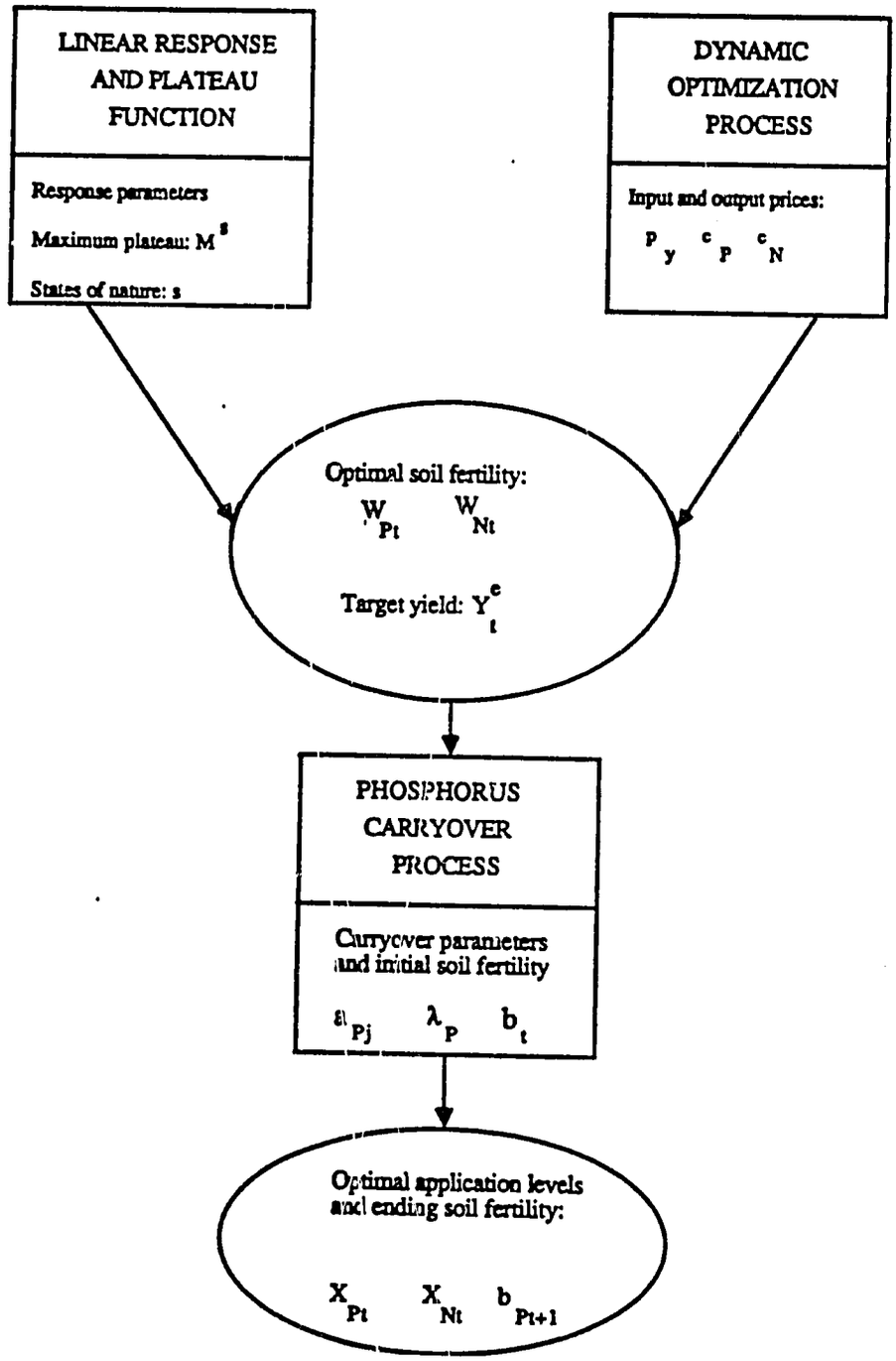

Figure 3. Schematic relating parameters and variables in the dynamic fertilization problem

equilibrium is attained when the state variable b_{pt} is maintained constant at b_p^*. The corresponding fertilization rate X_p^* and X_N^* are held constant in order to replenish the nutrient stock and maintain the long term equilibrium expected yield (Y^e).

The steady-state P fertilizer application necessary to maintaining soil fertility constant at b_p^* is found by solving carryover equation (4) for X_p^*, replacing W_p^* by its expression in terms of native and applied P:

$$(10) \qquad X_p^* = (a_{p1}\ \lambda_p)^{-1}\ [\ b_p^*\ (1 - a_{p1})\ -\ a_{p0}\ -\ a_{p3}Y^e\ ]$$

The steady state fertilization X_p^* corresponds to the net losses incurred by the soil P stock, adjusted for the fertilizer's efficiency in raising soil P(λ_p). For simplification, an average yield is included instead of the actual yield.

All variables in (10) are dependent on the optimal long-term soil fertility b_p^*. This optimal level is obtained by solving the maximization problem stated in (9). Noting that in the steady state $\pi_t(b_t) = \pi_{t+1}(b_{t+1}) = \pi^*(b^*)$, the recusive equation in (9) can be rewritten as:

$$(11) \qquad \pi^*(b_p^*) = \max_{W_N^*, W_p^*} (r^{-1} + 1)\ [\ p_y\ Y^e - \sum_{i \in I} c_i X_i^*\ ]$$

i.e. the sum of all current and future contributions to the sum of steady state profits due to current decisions.

For N, the response function is expressed in terms of nutrients applied, while for P, the soil level of P is the argument to the response function. Marginal conditions for the problem are:

$$(12) \qquad \partial\pi^* / \partial W_p^* = \partial\pi^* / \partial b_p^* = (r^{-1} + 1)\ [\ p_y(\ \partial Y^e / \partial b_p^*) - c_p\ ] \geq 0$$

$$(13) \qquad \partial\pi^* / \partial W_N^* = \partial\pi^* / \partial X_N^* = (r^{-1} + 1)\ [\ p_y(\ \partial Y^e / \partial X_N^*) - c_N\ ] \geq 0$$

or

$$(14) \qquad \partial Y^e / \partial W_i^* = \sum_{s \in S} p^s\ (\ \partial Y^s / \partial W_i^*) \geq c_i / p_y \qquad \forall\ i \text{ simultaneously}$$

which is clearly analogous to the first order conditions for the single-period profit maximization problem.

This is further illustrated graphically with the two-input expected minimum yield function shown in Figure 4. The optimal plant-available nutrient combination to maintain in the steady state is found at the tangency of a plane with slopes c_N/p_y and c_p/p_y and the planes of the response function. Given our earlier assumptions about nutrients, the optimal application of N is $X_N^* = W_N^*$. The optimal P fertilization level is obtained by solving equations (10) and (5) simultaneously for X_p^* and b_p^*.

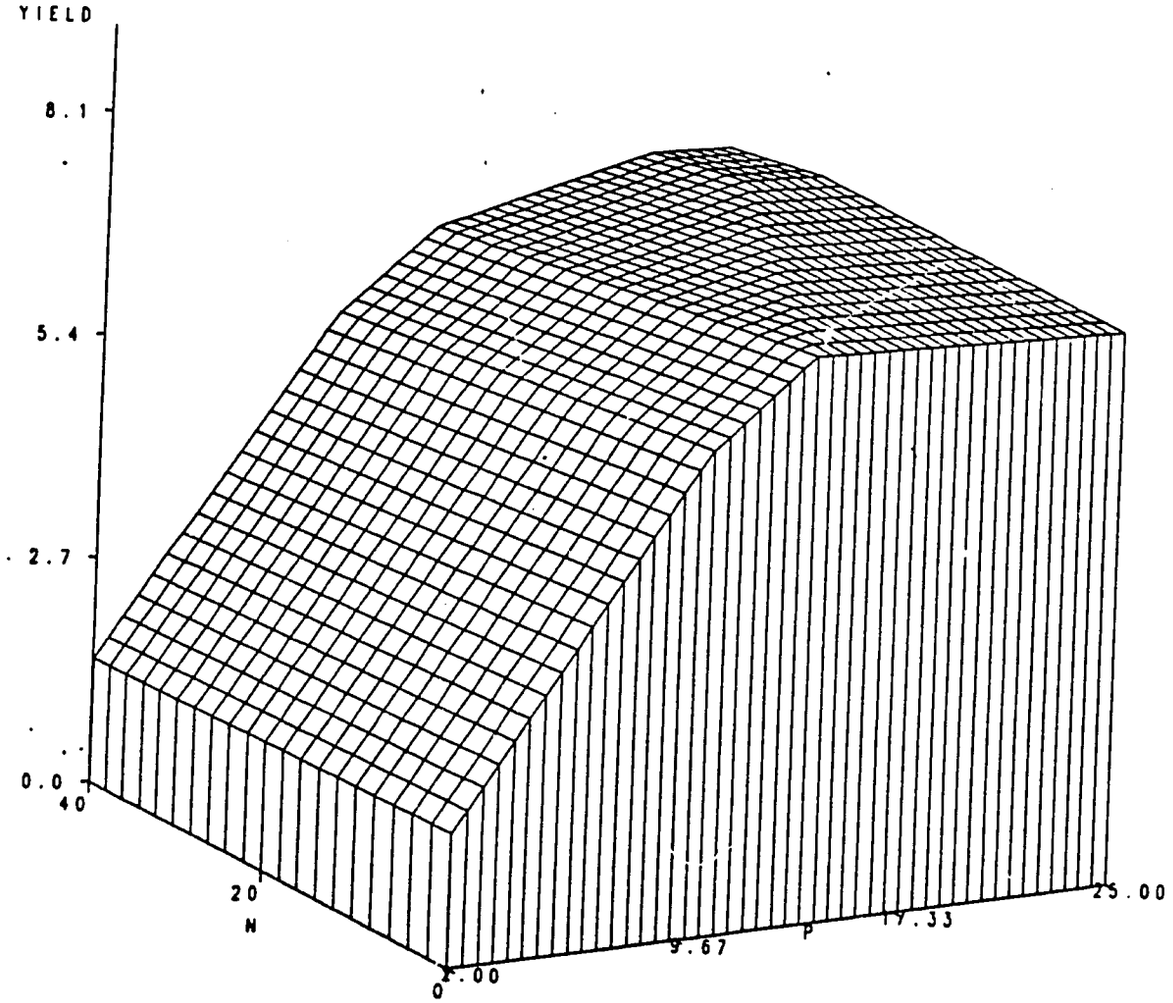

Figure 4. Expected linear response and plateau function for millet response to nitrogen and phosphorus under three states of nature

V. Fertilizer response data from Niger.

Since 1982, the International Fertilizer Development Center (IFDC) has collaborated with the International Crop Research Institute for the Semi-Arid Tropics (ICRISAT) on an extensive fertilizer research program. The data used in this study were collected by these institutes at ICRISAT's Sahelian Center located in Sadoré, near Say, 40 km southeast of Niamey, Niger. Soils at the experiment station are well characterized by the data presented in the introduction. Data collected at this site during 1982, 1984, 1985 and 1986 were used. Nineteen different combinations of N and P were used in the ranges 0-45 kg P_2O_5 ha^{-1} in the form of simple superphosphate (SSP) and 0-120 kg N ha^{-1} in the form of urea. A detailed presentation of the data is found in Jomini (1990).

VI. Results and discussion

The expression for W_{Pt} (equation 5) was substituted into (4) to estimate the annual P carryover equation. This equation was corrected for heteroskedasticity after a Breush-Pagan test (Judge et al., 1988) showed variance increasing with higher soil test levels. The estimated carryover equation for P is:

$$(15) \quad b_{Pt+1} = 1.95 + 0.45\, b_{Pt} + 0.14\, X_{Pt} + 0.14\, Y_t$$

(standard errors) (0.25) (0.07) (0.02) (0.03)

where

b_{Pt} is the Bray P1 soil test level, expressed in ppm,

X_{Pt} is the level of P fertilizer application, expressed in kg P_2O_5 ha^{-1}, and

Y_t^e is millet grain yield, expressed in 100 kg millet grain ha^{-1}.

All parameters are significantly different from zero as seen from standard errors supplied in parentheses. A high R^2 = 85% indicates this model was able to explain a large part of the observed data. Parameters are reasonable and of expected signs. As a matter of quick validation, assuming no fertilization, the low-level equilibrium yield is about 250 kg/ha and carryover soil P is 3 ppm, which is typical of unfertilized sandy soils under cereal cultivation in Niger (Mahaman, 1988).

The estimated intercept indicates that 2 ppm are added each year to the stock of plant-available P through natural causes (i.e. wind-blown dust, weathering of soil particles, breakdown of organic matter). According to the coefficients on Y_t^e, each 100 kg in grain yield removes 0.14 ppm P from the soil. This

removal rate is consistent with rates found through plant analysis (Jomini, 1990; Mahaman, 1988). Removal is also reflected in the parameter on b_{Pt} which shows that nearly 45% of the P available at the beginning of any season is available in the following season.

Finally, each kg P_2O_5 ha^{-1} applied increases the soil P in the following year by 0.14 ppm. Assuming a relevant soil depth of 15 cm. and a soil bulk density of 1.6 g cm^{-3}, 1 kg P_2O_5 ha^{-1} is converted to $10^6(2.4\ 10^6)^{-1}$, or 0.42 ppm. Multiplying X_{Pt} by this factor results in an application rate expressed in ppm. The corresponding level of a_{P2} is therefore 0.29. An immediate rate of P availability is obtained by assuming P from the soil and from fertilizer carryover in a similar fashion. Replacing W_{Pt} in equation (4) by its expression in (5), parameter λ_P is then obtained by dividing the parameter on X_{Pt} by the parameter on b_{Pt} (i.e. a_{P1}). This immediate rate of P solubility is found to be 70%, which again is reasonable since SSP is a soluble fertilizer.

Results for a multiple input LRP function are presented in Table 1. The inputs included are useful rainfall, P and N. Useful rainfall is defined as the rainfall occurring during a period of 80 days following the first occurrence of a 3-day rainfall exceeding 20 mm. The 80 day period is assumed to cover the critical growth periods of the millet variety used (CIVT), including plant establishment and flowering. Although other factors affect crop growth, especially early sand storms which impede proper stand establishments and dry spells during the cropping season, this measure of available moisture was found to be useful in the estimation of crop response. The estimation shows that in 1984, a catastrophic year in this respect with only 196 mm of useful rain, yield was significantly limited by rainfall. Among the 140 observations made during that year, predicted yield is limited by rainfall in 109 cases; otherwise yield is limited by very low available P of the order of 2.5-5.0 ppm. According to these results, no response to N is expected before the soil P level reaches 14 ppm. This result is consistent with observations by Bationo et al. (1986) and especially Jones and Wild who refer to the West African savana (1975, p. 150):

> Most cereals show some response to nitrogen, but this element is the first limiting nutrient only in some of the more humid parts of the region (...), where long grass fallows have just been cleared (...). Elsewhere, phosphate deficiencies must be remedied before applied nitrogen can give substantial yield increases.

Twenty-two observations for the 4 other years were in the area where none of the inputs considered (N, P, R) was limiting. For these observations the yield is expected to reach 1087 kg/ha and, it is presumed

Table 1. Linear Response and Plateau functions relating millet yield [q/ha] to rainfall, available P from SSP, and N

Parameters	Estimate	Asymptotic Std. Error
α_{R0}	-2.4674	1.0387
α_{R1} [cm]	0.4014	0.0462
α_{P0}	0.5448	0.0361
α_{P1} [ppm]	0.4697	0.0347
α_{N0}	7.0932	0.3443
α_{N1} [kg N/ha]	0.1261	0.2808
M	10.8754	1.1726
observations	440	
MSE	6.87	

that either some other nutrient, management or the genetic potential of CIVT limit the yield.

The volatility of the individual rainfall coefficients should be noted however (high standard error), as they vary considerably with the starting point used in the estimation process. Their interpretation may not be reliable since we do not have enough points to correctly estimate the spline corresponding to this input to the crop response function. In spite of these shortcomings, the function presented in this section was used to determine the states of nature used in the analysis. The states of nature were determined by using the structure of the LRP function.

According to the function's parameters, the minimum level of rainfall needed to obtain the maximum plateau of 1089 kg/ha is 332 mm in the 80 days assumed to be relevant to plant growth. Similarly, if the rainfall is 238 mm in the first 80 days of the season, the maximum obtainable yield is 709 kg/ha. Rainfall of 196 mm (as in 1984) further limits yield to 540 kg/ha. To each of these yield levels corresponds a minimum level of P and N. These minimum levels are found in Table 2 along with the corresponding useful rainfall. These results show that relatively low useful rainfall of less than 350 mm is sufficient to produce 1000 kg grain millet yields. Whenever this level of moisture is attained (through reasonably distributed useful rainfall), soil fertility is expected to limit yield. This confirms recent suspicions formulated by agronomists who have identified soil fertility as the major impediment to increasing food crop production in the WASAT in most years. The rainfall limits presented in Table 2 are used next to define intervals of R for the purpose of identifying probability distributions of various moisture conditions.

Two different distributions of three moisture intervals are shown in Table 3. The distribution of useful rainfall for Sadoré (in the second column) is based on the data used in estimating the response function. The other distribution (third column) is based on daily rainfall data from Niamey Airport. For the two locations presented, the probability of sufficient moisture (R>332) is seen in Table 3 to be much lower for Niamey. This may well reflect a difference in the years sampled, but also illustrates the worsening in moisture conditions that can be expected as latitude increased in the region. Since the response estimates are supposed to reflect physiological reactions of a given millet variety on sandy soils to varying levels of N, P, and R, they can be used along with a variety of rainfall patterns stemming from different locations. The expected yield function (Y_t^e) is then fully specified. This function is used next in the optimization model to determine optimal fertilizer application rates.

Table 2. Minimum input requirements for various yield levels using SSP.

Target yield [kg/ha]	Minimum		
	Useful Rainfall [mm]	P_2O_5 [ppm]	Nitrogen [kg/ha]
200	111	3.1	0
540	196	10.3	0
709	238	13.9	0
1087	332	22.0	30

Table 3. Frequency distributions of useful rainfall for 2 different sites.

Rainfall intervals [mm]	Sadoré 1982-1986 [%]	Niamey Airport 1968-1987 [%]
less than 238	40	35
238 - 332	20	45
greater than 332	40	20

It is assumed that a substantial amount of the organic matter produced under improved soil fertility conditions is left on the field. This is assumed to maintain the level of N if none is applied, and to avoid some of the acidification problems that has been observed by Pichot et al. (1981) and others with continued N application on soils with low organic matter.

Fertilizer is assumed to be available at 50 FCFA kg^{-1} SSP (18% P_2O_5) and 65 FCFA kg^{-1} urea (40% N), or US$ 0.17 and 0.22, respectively. These are the prices for each of these inputs during the period 1986-1989. Millet grain prices vary widely from place to place and season to season. Three prices are used in this analysis to represent this wide range: 100 FCFA kg^{-1} grain, a high price that may be found after a poor crop, 10 FCFA kg^{-1}, a low price that may be found in an isolated area after a large crop; and 50 FCFA kg^{-1}, a common price in rural areas at harvest time. These three output prices are used to illustrate the interaction between the biological expected yield function and expected prices.

Results using these three output prices are shown in Table 4. In the first column, output price is assumed to be 100 FCFA kg^{-1} grain. This high price results in a high optimal steady-state fertilization rate. Soil fertility is maintained at a relatively high level (b^* = 7.5 ppm) This level of soil fertility is obtained by applying at least 21 kg P_2O_5 ha^{-1} (or 117 kg SSP ha^{-1}) and 30 kg N ha^{-1} (or 63 kg urea ha^{-1}). The expected yield is 726 kg ha^{-1} with a high variability (coefficient of variation, CV - 27%). While a relatively high per hectare income is expected (64,812 FCFA or US $216), cash outlays of 7,788 FCFA (US $26) are relatively high for farmers with cash constraints. At the lower output price level, the farmer's expected yield and soil fertility goals are lower at 650 kg ha^{-1} grain and 5.3 ppm, respectively. This is obtained by applying yearly 12 kg P_2O_5 ha^{-1} (67 kg SSP ha^{-1}) and no N. By giving up higher yield, the variability of yield drops (CV = 12%). Expected income is also much lower at 29,164 FCFA (US $97) and so are fertilizer costs (3,336 FCFA, or US $11). In spite of this fall in income, it is still more than double the income expected from unfertilized yields (12500 FCFA or US$ 42).

When the price of millet drops to 10 FCFA kg^{-1} grain, the steady state of fertilization level is 8 kg P_2O_5 ha^{-1} and no N. This results in a drastic drop in expected income to 3,176 FCFA (US $11). These expected benefits are less than twice the additional costs linked to fertilization (2,224 FCFA, or US $8). According to the FAO rule of thumb requiring that benefits be at least twice the extra costs entailed by a new technology, this would lead to rejecting the use of fertilizer at this low price of output. In addition, no

Table 4. Summary of optimal steady-state fertilization under various input-output price assumptions.

Millet price [FCFA/kg grain]	100	50	10
Relative cost of P (c_P/p_y)	2.78	5.56	27.80
Soil phosphorus			
carryover (b_P^*) [ppm]	7.5	5.3	4.5
available (W_P^*) [ppm]	22.0	13.9	10.3
Fertilization			
phosphorus (X_P^*) [kg P_2O_5/ha]	21	12	8
nitrogen (X_N^*) [kg N/ha]	30	0	0
Yield			
expected (Y^e) [kg/ha]	726	650	540
coeff. var. [%][1]	27	12	0[2]
Net income			
expected (π^*) [FCFA]	64,812	29,164	3,176
coeff. var. [%][1]	31	14	0[2]
Total fertilizer cost ($\Sigma_i\ c_i X_i^*$) [FCFA]	7,788	3,336	2,224
Rate of return [%][3]	8.3	8.7	1.42

notes: [1] coefficients of variation (CV) are calculated as s_x / E(x) where x is the variable in question, s_x is the standard deviation of x over the states of nature, and E(x) is the mean of variable x over the states of nature.

[2] the CV = 0 because the supply of P is just sufficient to sustain the maximum yield obtainable in the worst state of nature. While this may seem unreasonable, it is reasonable to expect a lower distribution of yields if a nutrient is limiting than when it is not limiting and the yield distribution is governed by the distribution of states of nature.

[3] Rate of return is Net benefit / Cost

allowance was made for procurement and application costs. It is worth noting however, that the expected yield is double the level expected from a low fertility field ($b^* = 3$ ppm) and 140 kg above the national average during 1980-1985 (Ministère de l'Agriculture, 1986).

This latter result indicates that even when rainfall is limiting, yield increases can be expected from the improvement of fertility in the poorest soils of Niger. Although a distribution of soils by classes of P content is not available, it is suspected that P-starved soils (soil P < 5 ppm) prevail over large areas of the country. Moderate fertilization in these areas would lead to increased yields, food production and income.

While the optimal approach path to maintenance fertility and application levels is not investigated here, Dillon (1977) and Kennedy et al. (1973) show that when no constraints impede it and the rate of time preference is positive, the best path to the steady-state is one by which the target soil fertility is reached the quickest. In this case, the optimal short run strategy starting with a low fertility soil would be to apply the amounts of P and N necessary to obtain the optimal soil fertility in the current period.

When cast in terms of rates of return, the best return to cash investment is shown when the price of 50 FCFA kg^{-1} millet grain is assumed. However, returns to this resource are comparable at the higher output price. Presumably, at this higher price, returns to other factors of production such as labor and land would also be higher. Both higher price situations are superior to the situation in which farmers can only expect the low output price.

VII. Conclusions

A dynamic model of millet response to P, N and available moisture was developed within a linear response and plateau framework. Results from the agronomic response model indicate first that little response to N applications are to be expected unless a soil P of at least 14 ppm is obtained. Secondly, relatively little moisture (350 mm during the first 80 days of a cropping season) is expected to be sufficient to support a yield of 1 metric ton of millet, well above unfertilized yield levels.

From an economic stand point, moderate P fertilization is found to be optimal to maintain soil fertility at a level that permits yield and income increases over the unfertilized situation. Only at relatively high output prices are the returns to N sufficient to warrant its use.

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POSTER

STOCHASTIC LINEAR RESPONSE AND PLATEAU FUNCTIONS IN MODELING CROP RESPONSE TO FERTILIZATION IN NIGER, WEST-AFRICA

Patrick A. Jomini (Purdue University, USA), Robert Deuson (WINROCK International)
J. Lowenberg-DeBoer (INRAN, Niger) and Andre Bationo (IFDC, Niger)

Introduction

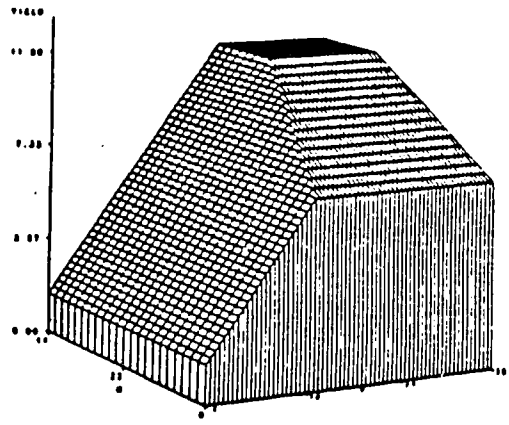
The low phosphorus (P) and nitrogen (N) content of many sandy soils of the West African Semi-Arid Tropics (WASAT) constitutes a major constraint to increasing food production in the region. This low soil fertility is complemented by low and erratic rainfall which occurs in a single cropping season from June to October (Sivakumar, 1988). Chemical fertilization is believed to be a viable way of raising soil fertility, and thus food production and farm income. Though millet, the main food crop in Niger, response to P and N fertilization is significant and well documented (...) little work has been done to determine optimal application rates. There has also been little effort to link fertilizer response and application rates to specific environmental conditions such as natural soil fertility and rainfall (...) Carryover effects especially of P, though widely recognized by researchers (e.g. Jones and Wild, 1975) and farmers have not been taken into consideration when making fertilization recommendations. The main objective of this paper is to determine economically viable levels of P and N fertilization for millet grown on sandy soils in Niger while taking environmental conditions and carryover effects into account.

The linear response and plateau yield function

Economists have recently recognized the value of using Linear Response and Plateau (LRP) functions in crop response analysis (e.g. Perrin, 1976; Lanzer and Paris, 1985; Grimm et al., 1988). This functional form is based on agronomic principles first advocated by the German chemist Justus von Liebig (1863). The LRP function is based on the Law of the Minimum according to which crop growth is proportional to the availability of the most limiting nutrient until another factor becomes limiting. Any increase in an input other than the limiting input does not result in any response and the function displays a horizontal plateau over a large range of the inputs considered (Redman and Allen, 1954). The two-input LRP function shown in Figure 1 illustrates these principles.

Figure 1. Two-input linear response and plateau function for modeling millet response to nitrogen and phosphorus under state of nature s:

$$Y^s = \min [\alpha_{i0} + \alpha_{i1} X_i , M^s]$$

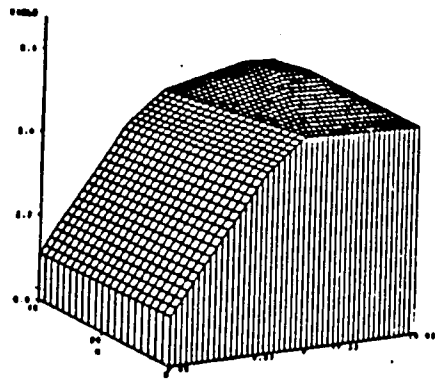


An expected linear response and plateau yield function

Each term of the minimum expected yield function (Y^e) is a linear combination of terms of the original yield functions specific to each state of nature (Y^s). It is worth noting that multiplying the number of states of nature results in an expected yield function that approaches a smooth form on a certain interval (between the lowest and highest plateaus). The origin of this smoothness is however specifically linked to the existing states of nature, not to the input response (Figure 2).

Figure 2. Two-input expected linear response and plateau function for modeling millet response to nitrogen and phosphorus:

$$Y^e = \Sigma_s p^s \min [\alpha_{i0} + \alpha_{i1} X_i , M^s]$$



Optimality conditions

In general, the cost of fertilization must be covered by the value of the marginal yield increases over the entire planning period that are due to the application. Assuming the farmer maximizes current and future benefits due to the current fertilizer application, the steady-state solution (...) is analogous to the solution for the single-period profit maximization problem. This is illustrated graphically with the two-input expected minimum yield function shown in Figure 4. The optimal plant-available nutrient combination to maintain in the steady state is found at the tangency of a plane with slopes corresponding to the input-output price ratios and the planes of the response function.

Results and discussion

Results for a multiple input LRP function are presented in Table 1. Inputs included are useful rainfall(R), P and N. Useful rainfall is defined as the rainfall occurring during a preiod of 80 days following the first occurrence of a 3-day rainfall exceeding 20mm. The 80-day period is assumed to cover the critical growth stages of the millet variety used (CIVT).

Table 1. Linear Response and Plateau functions relating millet yield [q/ha] to rainfall (R), available P from SSP, and N

Parameters	Estimates	Asymptotic St.Error
$\alpha_r 0$	-2.4674	1.0387
$\alpha_r 1$	0.4014	0.0462
$\alpha_r 0$	0.5448	0.0361
$\alpha_r 1$	0.4697	0.0347
$\alpha_r 0$	7.0932	0.3443
$\alpha_r 1$	0.1261	0.2808
M	10.8754	1.1726

Table 2. Minimum levels of useful rainfall, P and N necessary to obtain various target yields

Target Yield [kg/ha]	Useful Rainfall [mm]	P2O5 [ppm]	Nitrogen [kg/ha]
200	111	3.1	0
540	196	10.3	0
709	238	13.9	0
1087	332	22.0	30

The estimation shows that in 1984, a catastrophic year in this respect with only 196 mm of useful rain, yield was significantly limited by rainfall. Among the 140 observations made during that year, predicted yield is limited by rainfall in 109 cases; otherwise yield is limited by very low available P of the order of 2.5-5.0 ppm. According to these results, no response to N is expected before the soil P level reaches 14 ppm. This result is consistent with observations by Bationo et al. (1986) and especially Jones and Wild who refer to the West African savana (1975, p. 150):

> Most cereals show some response to nitrogen, but this element is the first limiting nutrient only in some of the more humid parts of the region (...), where long grass fallows have just been cleared (...). Elsewhere, phosphate deficiencies must be remedied before applied nitrogen can give substantial yield increases.

Twenty-two observations for the 4 other years were in the area where none of the inputs considered (N, P, R) was limiting. For these observations the yield is expected to reach 1087 kg/ha and, it is presumed that either some other nutrient, management or the genetic potential of CIVT limit the yield. According to the function's parameters, the minimum level of rainfall needed to obtain the maximum plateau of 1089 kg/ha is 332 mm in the 80 days assumed to be relevant to plant growth. Similarly, if the rainfall is 238 mm in the first 80 days of the season, the maximum obtainable yield is 709 kg/ha. Rainfall of 196 mm (as in 1984) further limits yield to 540 kg/ha. To each of these yield levels corresponds a minimum level of P and N. These minimum levels are found in Table 2 along with the corresponding useful rainfall. These results show that relatively low useful rainfall of less than 350 mm is sufficient to produce 1000 kg grain millet yields.

Fertilizer is assumed to be available at 50 FCFA/kg SSP (18% P205) and 65 FCFA/kg urea (46% N), or US$ 0.17 and 0.22, respectively. These are the prices for each of these inputs during the period 1986-1989. Millet grain prices vary widely from place to place and season to season. Three prices are used in this analysis to represent this wide range: 100 FCFA/kg grain, a high price that may be found after a poor crop, 10 FCFA/kg, a low price that may be found in an isolated area after a large crop; and 50 FCFA/kg, a common price in rural areas at harvest time. These three output prices are used to illustrate the interaction between the biological expected yield function and expected prices.

Results using these three output prices are shown in Table 4. In the first column, output price is assumed to be 100 FCFA kg grain. This high price results in a high optimal steady-state fertilization rate. Soil fertility is maintained at a relatively high level (7.5 ppm compared to native soil fertility of 3 ppm). This level of soil fertility is obtained by applying at least 21 kg P205/ha (or 117 kg SSP/ha) and 30 kg N/ha (or 63 kg urea/ha). The expected yield is 726 kg ha with a high variability (coefficient of variation, CV = 27%). (...) At the lower output price level, the farmer's expected yield and soil fertility goals are lower at 650 kg ha grain and 5.3 ppm, respectively. This is obtained by applying yearly 12 kg P205/ha (67 kg SSP/ha) and no N. By giving up higher yield, the variability of yield drops (CV = 12%). (...) When the price of millet drops to 10 FCFA kg grain, the steady state level of fertilization is 8 kg P205/ha and no N. (...) expected yield is double the level expected from a low fertility field (b* = 3 ppm) and 140 kg above the national average during 1980-1985 (Ministere de l'Agriculture, 1986). This latter result indicates that even when rainfall is limiting, yield increases can be expected from the improvement of fertility in the poorest soils of Niger.

Conclusions

This research has shown that both rainfall and soil fertility are major constraints to food crop production in Niger. Although a distribution of soils by classes of P content is not available, it is suspected that P-starved soils (soil P 5 ppm) prevail over large areas of the country. Moderate fertilization in these areas would lead to increased yields, food production and income.

Table 4. Summary of optimal steady-state fertilization under various hypothetical grain assumptions.

Millet price (FCFA/kg grain)	100	50	10
Relative cost of P (c_p/p_y)	2.38	5.36	27.80
Soil phosphorus			
carryover (b_t) (ppm)	7.5	5.3	4.5
available (W_p) (ppm)	22.0	13.9	10.3
Fertilization			
phosphorus (X_p^*) (kg P_2O_5/ha)	21	12	8
nitrogen (X_n^*) (kg N/ha)	30	0	0
Yield			
expected (Y^*) (kg/ha)	726	650	540
coeff. var. (%)[1]	27	12	[2]
Net income			
expected (π^*) (FCFA)	64,522	28,344	3,176
coeff. var. (%)[1]	31	14	[2]
Total fertilizer			
cost ($X_p^* c_p + X_n^* c_n$) (FCFA)	2,280	2,206	2,204
Rate of return (%)[3]	0.3	0.7	1.45

[1] coefficients of variation (CV) are calculated as s_x / E(x) where x is the variable in question, s_x is the standard deviation of x over the states of nature, and E(x) is the mean of variable x over the states of nature.

[2] the CV = 0 because the supply of P is just sufficient to sustain the maximum yield obtainable in the worst state of nature. While this may seem unreasonable, it is reasonable to expect a lower distribution of yields if a nutrient is limiting than when it is not limiting and the yield distribution is governed by the distribution of states of nature.

[3] Rate of return is Net Income / Cost