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FISCAL POLICIES AND OPTIMAL ECONOMIC GROWTH

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THE MODEL

The utility function is

$$(1) \quad U = \int_0^\infty e^{-\rho t}[\ (1-a)\mathrm{Log}(c_t) + a\mathrm{Log}(1-\ell_t)\ ]dt$$

The production technology is represented by

$$(2) \quad y = f(k_t^*, k_t, A_t\ell_t, g_t)\ ,$$

where k* represents an externality factor. It is equal to the level capital in each firm (k=k*), but it is taken as exogenous by investors. The term A increases with technological progress (which is labor augmenting). More specifically, technological progress occurs by doing, and its rate of growth n depends on the amount of time spent in production activities:

$$(3) \quad \dot{A} = An(\ell)$$

Capital accumulation takes place according to the equation:

$$(4) \quad \dot{k} = f(k^*, k, A\ell, g) - c$$

It will be convenient to redefine all variables by their ratio with the

scale factor of technology A. The capital accumulation equation is of the form:

(5) $\dot{k} = f(k^*,k,\ell,g) - c - n(\ell)$

The first order conditions of the private sector a

(6) $c = ((1-a)A\bar{w}/a)(1-\ell)$, for the intra-temporal condition and

(7) $\dot{c} = c(\bar{r} - \rho - n(\ell))$ for the inter-temporal condition,

where $A\bar{w}$ and $\bar{r}$ are the net of tax wage rate per efficiency unit of labor and the net rate of return, respectively.

From the first of these two equations one can represent the level of labor ℓ as a function of consumption and of the net wage rate $\bar{w}$:

(8) $\ell = 1 - (a/(1-a))c/\bar{w}$.

EFFICIENT POLICY

The instruments of fiscal policy are linear taxes on the wage rate, the private rate of return to capital, the level of public borrowing. Further more is assumed that pure profits, if they exist, can be taxed at the rate τ. In order to simplify the exposition, the production function is Cobb-Douglas, and that the shares of income to capital and labor, which accrue to the private sector, are equal to α and β respectively. The share of pure profit is equal to $1-\alpha-\beta$.

The second-best policy optimizes the fiscal instruments to maximize the welfare function U subject to the private sector's optimizing behavior and the governrment budget constraint. One can assume that the government controls the net factor prices $\bar{w}$ and $\bar{r}$, respectively.

The main policy questions are the taxation of capital income, the efficiency cost of taxation and the determination of the public debt.

The analysis of second-best policies relies on the

current value Hamiltonian, which is defined by:

$$(9)\quad H = \mathrm{Log}(c) - a\mathrm{Log}(\bar{w}) + \mathrm{Log}(A) + \lambda(f(k^*,k,\ell) - c - g - n(\ell))$$
$$+ \mu((\bar{r}-n)b + \bar{r}k + \bar{w}\ell - (\alpha+\beta+\tau\gamma)f(k^*,k,\ell)$$
$$+ \nu c(\bar{r}-\rho-n(\ell))$$
$$+ \xi An(\ell),$$

where ℓ is a function of consumption c, and of the net wage rate $\bar{w}$.

THE PRIVATE RATE OF RETURN IN THE SECOND BEST

It is determined by the first order condition with respect to k:

$$(10)\quad \lambda(f_0' + r - n) + \mu(\bar{r} - (\alpha + \beta + \theta(1-\alpha-\beta)(f'_0 + r) = \rho\lambda - \dot{\lambda} \quad ,$$

where r is the gross private rate of return and $f_0' + r$ is the social marginal

return. The term f_0' represents the externality factor of capital.

In the steady state, the variable $\dot{\lambda}$ is constant, and this equation is equivalent (after some manipulations) to:

$$(11)\quad (\lambda-\mu)(f_0' + r - \bar{r}) = -\mu(1-\tau)(1-\alpha-\beta)(f_0' + r)$$

(Note that the private intertemporal optimization requires that $\bar{r}$ is equal to $\rho + n$ on the balanced path.

The term $\lambda-\mu$ is positive because λ is the social marginal value of capital and μ is the social marginal value of the public debt, and is negative.

This formula implies therefore the following result:

<u>Proposition 1</u>

The private rate of return to capital, net of tax, is equal to the social return in the long-run when there are no pure profits, or pure profits can be completely taxed. When this condition is not satisfied, the net private rate of return is smaller than the social rate of return.

In the private market equilibrium with no tax, the rate of return to capital is smaller than the social return because individuals do not take into account the factor of externality. It is clear that this discrepancy calls for a subsidization of capital when the subsidy can be financed with a lump-sum taxation. The previous result shows that this may also be the case in the second-best.

THE EFFICIENT COST OF TAXATION

The marginal efficiency cost of taxation in welfare terms is equal to the shadow price of the public debt μ. It can be expressed in terms of produced goods by taking its ratio with the marginal utility of consumption q. This ratio μ/q is constant over time as can be seen by the combination of the intertemporal optimization constraints of the private sector and of the public sector. The latter is the first order condition with respect to the level of the public debt in the optimal program.

The efficient cost in terms of produced goods can therefore be found by computing its value in the steady state. (All endogenous variables in the steady state depend on the full history of the economy on the transition path). There is no simple equation for the computation of this efficiency cost, and we have to consider all the first order conditions that have not been used yet.

(i) <u>With respect to A</u>, the first order condition is

$$(12)\quad \frac{1}{A} + \xi n = \rho\xi - \dot{\xi}$$

This expression can be written as

$$(13)\quad (\dot{\xi A}) = \rho(A\xi) - 1$$

Its solution is that ξA is constant and equal to $1/\rho$. This value can be substituted in the last term of the Hamiltonian.

The other first order conditions are

(ii) <u>with respect to $\bar{r}$</u>

(14) $\mu(b+k) + \nu c = 0$

(iii) <u>with respect to c</u>

(15) $\frac{1}{c} - \lambda + G\ell_c = \rho\nu - \dot{\nu}$, with $\dot{\nu} = 0$ in the steady state,

and where G is the expression

(16) $G = \lambda w + \mu(\bar{w}-w) - \sigma[\lambda k + \mu b + \nu c - \frac{1}{\rho}]$,

where σ is the elasticity of the growth rate with respect to the level of labor measure in units of time, ℓ:

(17) $\sigma = \frac{\ell}{n}\frac{dn}{d\ell}$

Substituting for the value of νc in the previous equation

(18) $1 - \lambda c + Gc\ell_c + \mu\rho(b+k) = 0$

(iv) <u>with respect to $\bar{w}$</u>

multiplying both sides by $\bar{w}$,

(19) $-a + G\bar{w}l_{\bar{w}} + \mu\bar{w}\ell = 0$

The separability of the utility function beween consumption and leisure implies that

(20) $c\ell_c = -\bar{w}\ell_{\bar{w}}$.

Using this identity and summing up the first order conditions with respect to c and $\bar{w}$, and using the equation $\bar{r} = \rho+n$ in the steady state, one finds that

$$(21)\quad \lambda = q + \mu$$

The social marginal value of capital is equal to the sum of the private value and of the marginal eficiency cost of taxation. Following some manipulations of these equations,

Proposition 2

The efficiency cost of taxation is given by the following equation, where the values of all endogenous variables are values on the balanced growth path:

$$(22)\quad -\frac{\mu}{q} = \frac{1}{1+\epsilon}\left[\frac{\theta}{1-\theta} + \sigma\left(\frac{n}{\rho}\left(\epsilon+\frac{c}{\bar{w}\ell}\right) - \frac{nk}{\bar{w}\ell}\right)\right]$$

where θ is the wage tax rate.

The right hand side can be decomposed in two parts; the first is the efficiency cost with exogenous growth. The second is due to the impact of the tax on the level of labor and on the rate of growth.

Numerical Example

The numerical value of the efficiency cost can be determined as a function of the stylized parameters of the economy. Representative values of these parameters are:

$n = .02$, $\bar{r} = .04$. These two values imply that $\rho = .02$. The shares of labor and capital are .75 and .25. The share of government revenues is equal to .25. Since all government income is derived from the wage tax the ratio between $\bar{w}$ and w is equal to .75. With these values, the expression () takes the special form:

$$(23) \quad -\frac{\mu}{q} = \frac{\epsilon}{1+\epsilon}\left[\ .5 + \sigma\left(\ \epsilon + \frac{3}{8}\ \right)\ \right]$$