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Variety, Growth, and Trade

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I. Introduction

This paper addresses an issue which has for a long time occupied a central place among policy makers: the issue of optimal trade regimes. A case for free trade is made in the international literature. In models of international trade, such as in Ricardian models or Heckscher-Ohlin models, the competitive equilibrium is Pareto optimal. Krueger [1978] argues that free trade leads to larger growth than protectionism. Her claim is supported by data on the relationship between various measures of the freeness of trade and economic growth. Another potential determinant of growth and development, a determinant, which has received scant attention, is the variety of goods produced or the variety of skills present in an economy. Narrow specialization, especially narrow specialization in the production of raw materials or agricultural products, is often considered a symptom of under-development. Narrow specialization may also be an early sign of problems ahead. Jacobs [1985], for example, argues that the drop in the standard of living in Uruguay after World War II, when Uruguay's former markets in Europe became more self-sufficient in wheat and beef, was the result of narrow specialization in agriculture. She claims that had Uruguay not been narrowly specialized in agriculture, the presence of expertise in a variety of activities would have prevented or at least ameliorated the drop in Uruguay's standard of living.

These issues are not well addressed by existing theoretical models. In Ricardian models of international trade, in Heckscher-Ohlin models, and in models which base trade on increasing returns to scale, changes in the trade regime lead to level effects. If a country changes to a free trade regime and specializes in an industry where it has, say a comparative advantage in

production, the level of per capita GNP may increase in the short-run, but there is no increase in the average rate of growth of per capita GNP.

Welfare analysis of policy measures in static models can only bring to light level effects of policy. Such static welfare losses are typically small.[1] Lucas [1985, 1986] has suggested that growth effects on welfare are potentially very large. Lucas [1985] introduces a dynamic model of international trade with a learning-by-doing technology. In such an environment policy, if desirable, can bring about large welfare gains.

In this paper, I present a dynamic model of international trade where governmental choice of a trade regime may have potentially large welfare effects. In order to enhance their skills, workers have to communicate with other workers who have related, but different kinds of expertise. The variety of accessible skills represented in a country increases the returns to learning. If a country is narrowly specialized, say because of a comparative advantage, workers in that country do not have access to a large variety of skills. An implication of the theory developed in this paper is that economies lacking a variety of skills exhibit low rates of growth and perform inefficiently in a dynamic sense. Free trade leads to narrow specialization and is not an optimal trade regime.

There is a variety of evidence supporting the importance of externalities across activities. McNeil [1982], for example, reports that in fifteenth century Europe producers of cannons ("bombards") employed "metal casting techniques which European bellmakers had already developed to a high degree of perfection" (p. 86). Some more evidence from more recent periods for these externalities is due to Jaffe [1986], Rosenberg [1963], and Sabel [1982]. Jaffe takes the number of patents per dollar expenditure on research and development as a measure of the return to R&D. Using patent data, he

constructs a metric in technology space. One of his findings is that firms which are, in the technology metric, located near other firms with high research expenditure have a higher return to R&D than firms with neighboring firms with lower levels of R&D spending. Jaffe admits that this association between the returns on R&D of a firm and the expenditure on R&D of other neighboring firms allows alternative interpretations; for example competition among firms may provide incentives for firms in close proximity to firms with high R&D spending to patent more of these inventions. In that case, Jaffe's measure of returns to R&D is biased. Jaffe, however, finds the coexistence of these competitive effects and of technological spill-overs a "compelling interpretation".

Rosenberg observed that "most machinery production poses a broadly similar set of problems and involves a broadly similar set of skills and technological knowledge in their solution (p. 418)...The use of machinery in the cutting of metal into precise shapes involves, to begin with, a relatively small number of operations (and therefore machine types): turning, boring, drilling, milling, planning, grinding, polishing, etc. Moreover, all machines performing such operations confront a similar collection of technical problems, dealing with such matters as power transmission (gearing, belting, shafting), control devices, feed mechanisms, friction reduction, and a broad array of problems connected with the properties of metals (such as the ability to withstand stresses and heat resistance).

It is because these processes and problems became common to the production of a wide range of disparate commodities that industries which were apparently unrelated from the point of view of the nature and uses of the final product became very closely related (technologically convergent) on a

technological basis - for example firearms, sewing machines, and bicycles" (p. 423).

Rosenberg then goes on to claim that economies which produce a variety of goods which are technologically convergent exhibit higher degrees of specialization and are characterized by more efficient learning (p. 425). For the purposes of this paper the most important conclusion of Rosenberg's paper is that there seems to be a connection between variety of skills in an economy and the rate of growth of the economy.

Sabel [1982] found evidence for spill-overs across activities in small regions in Northern Italy. In the 1970's small firms, most of which employed between 5 and 50 workers, clustered in and around urban areas such as Bologna. Goods produced by these small firms are, among others, automatic machines, machine tools, parts for automobiles and buses, and agricultural equipment. Workers in these small firms can access the skill of workers in other firms which produce goods utilizing related skills. For example, a worker who wants to modify the design of a tractor transmission to suit the needs of a small manufacturer of high quality seeders, may require precision parts which are not available in the market. He may lack the skills to produce these parts, but may draw on the experience of another worker in another shop, who has the requisite skills. Sabel argues that the growth of small firms and economic growth, as measured by per capita income, in the region in general is due to the close relation of firms across industries and on the collaboration of workers with different, but related kinds of expertise.

If skills in different activities matter the issue of the transfer of skills becomes important. In Jaffe the transfer of skills is possible because of the public goods aspect of information. In this paper skills are embodied

in the workers and are only transferable when face-to-face communication between workers of different skills occurs. This paper makes no effort at explaining the industrial organization of communication between workers. Communication between workers may occur within large organizations or between individuals who own small shops. The crucial element of this paper is that skills in different but related activities are important in the sense that private returns to learning are enhanced if other workers performing similar tasks have a high level of skill.

II. Description of the Environment

In this section the environment is specified. The environment consists of two countries.

Each of the two countries is inhabited by a large number of infinitely lived agents; more precisely the population of each country is given by an interval of length one and agents are uniformly distributed over that interval. To distinguish between the two countries, all variables pertaining to country 2 will be superscripted by * whenever that is necessary to avoid confusion.

There are two different kinds of goods which are non-storable. All consumers value both goods. The preferences of all consumers are identical and are given by

$$(1) \qquad \sum_{t=0}^{\infty} \beta^t (\log x_{1t} + \log x_{2t}) ,$$

where x_{jt}, $j = 1,2$, denotes the amount of good j consumed at time t and the discount factor β is contained in the open interval (0,1).

Each one of the two goods can be produced with one distinct production technology. Each one of these production functions is given by

$$(2) \qquad y_j = z_j v_j \ , \quad j = 1,2 \ .$$

Here, y_j denotes per capita output of a worker who produces good j; v_j is the time that worker allocates to the production of the good; and z_j is the skill or human capital of that worker. Skill is embodied in the worker and is industry specific; that is if a worker has a stock of skill z_1, say, but his stock of the second type of skill is zero, his output of the second good is also zero.

In each time period each agent is endowed with one unit of time. This one unit of time can be allocated to the production of goods or for accessing the skill of other agents. Let u_{jk} be the number of workers (more precisely the measure of workers) in industry k = 1,2 contacted by an agent in industry j. Each agent can contact as many workers as there are in each industry, but each worker can be contacted only once. If n_k is the measure of workers employed in industry k then it is clear that the measure of workers contacted in industry k by any worker cannot exceed n_k, that is $u_{jk} \leq n_k$. Contacting other agents requires time. The time required to contact other workers is linear: If u_{j1} and u_{j2} are the measures of agents in industry 1 and 2, respectively, contacted by an agent from industry j, the time required for such contacts is $\tau(u_{j1} + u_{j2})$. Here τ is a technology parameter; its units of measurement are hours per person contacted. The time allocated each period cannot exceed the agent's endowment of time. Each period an agent employed in industry j faces the time constraint

$$(3) \qquad \tau(u_{j1} + u_{j2}) + v_j \leq 1 \ .$$

Contacting other agents and accessing their skills increases the agent's own skill. The returns to such contacts are functions of the average level of skill in each industry. Let Z_j, $j = 1,2$ denote the average level of skill in industry j as opposed to z_j which denotes the level of skill of an individual agent who is employed in industry j.

If an agent in industry j accesses the expertise of u_{jj} agents in industry j and of u_{jk} agents in industry k, $k \neq j$, then the agent's human capital applicable to industry j next period, z_j' is given by

$$(4) \qquad z_j' = z_j + \sigma Z_j u_{jj}^{\alpha} + \omega Z_k u_{jk}^{\alpha} \ .$$

In equation (4), α is a constant which is strictly between zero and one; there are diminishing returns to communicating with any one type of agent. The coefficients σ and ω are both positive and satisfy $0 < \omega \leq \sigma$. If $\omega < \sigma$, then the returns to accessing expertise in the other industry are less than the returns to accessing skills in the same industry. The relationship between σ and ω induces a "distance" between two industries. We say that two industries are close if the skills in the two industries are close. Skills in the two industries are close if ω/σ is close to unity. Skills can only be accumulated in the industry in which the agent is employed. Since skills can only be accumulated in the industry in which the agent is employed, skills in the other industry fall behind and agents will never switch industries. Equation (4) says then that externalities across industries are present only when the size of the other industry exceeds measure zero.

At the beginning of time 0 each agent is endowed with a 2-vector of skills. The time zero endowment of skill of an agent in country 1 is given by

$$(6) \qquad z_0^1 = (z_{10}^1 \; z_{20}^1) = (\zeta_1 \; \zeta_2)$$

the time zero endowment of an agent in country 2 is

$$(7) \qquad z_0^2 = (z_{10}^2 \; z_{20}^2) = (\zeta_2 \; \zeta_1)$$

where it is assumed that $\zeta_1 \geq \zeta_2$. Consequently agents born in country 1 have a comparative advantage in industry 1 while agents born in country 2 have a comparative advantage in industry 2.

Migration of labor across countries is not feasible. Furthermore, agents can enter only one industry and agents cannot change industries over time. The assumption that agents cannot switch industries is only made for convenience; it will not be binding in equilibrium. Agents can only accumulate skills in industries in which they produce physical output; the other skill remains at the time zero level. If an agent enters, say industry 1, it suffices to keep track only of that agent's skill in industry 1. Similarly, for an agent who enters industry 2, it is enough to keep track only of that agent's skill in industry 2.

III. Definition, Existence, and Uniqueness of Competitive Equilibrium under Free Trade

In this section the competitive equilibrium and stationary competitive equilibrium under a free trade regime are defined. This section also contains

a theorem guaranteeing the existence and uniqueness of a stationary competitive equilibrium under free trade. The proof of this theorem is contained in the appendix.

Agents who behave competitively choose an industry of employment, a consumption plan and a production plan to maximize life-time utility subject to a budget constraint, taking as given a pair of price sequences and a pair of average capital stock sequences. Let p_{jt} be the price of good j at time t, let $p_t = (p_{1t}, p_{2t})$, and let $p = \{p_t\}_{t=0}^{\infty}$. Prices are normalized so that $p_{10} = 1$. Agents in the two industries will choose the same consumption vector; therefore the consumption plan at time t $x_t = (x_{1t}, x_{2t})$ does not need a subscript indicating the industry in which the agent is employed. Also, $x = \{x_t\}_{t=0}^{\infty}$ denotes the consumption plan of an agent regardless of the industry in which the agent is employed. Finally, let $u_{jt} = (u_{j1t}, u_{j2t})$, let $u_j = \{u_{jt}\}_{t=0}^{\infty}$ and let $v_j = \{v_{jt}\}_{t=0}^{\infty}$; then (u_j, v_j) is the production plan of a worker employed in industry j.

As there are no restrictions on borrowing and lending the budget constraint of a worker in industry j is

$$(8) \qquad \sum_{t=0}^{\infty} p_t \cdot x_t \leq \sum_{t=0}^{\infty} p_{jt} y_{jt} \cdot$$

Equation (8) just says that life-time expenditure cannot exceed life-time income.

The maximization problem to be solved by the agents is similar across industries and countries. An agent in country 1 and industry 1, for example, solves the following problem:

$$\text{(P)} \qquad \max_{\{x,u,v\}} \sum_{t=0}^{\infty} \beta^t [\log x_{1t} + \log x_{2t}]$$

$$\text{subject to} \quad \sum_{t=0}^{\infty} p_t \cdot x_t \leq \sum_{t=0}^{\infty} p_{1t} y_{1t}$$

$$y_{1t} = z_{1t} y_{1t}$$

$$z_{1t+1} = z_{1t} + \sigma Z_{1t} u_{11t}^{\alpha} + \omega Z_{2t} u_{12t}^{\alpha}$$

$$\tau(u_{11t} + u_{12t}) + v_{1t} \leq 1$$

given prices p and average stocks of capital $Z = (Z_{1t}, Z_{2t})_{t=0}^{\infty}$.

Workers in the other country and other industries solve problems analogous to problem (P). We can now define a competitive equilibrium for this economy under free trade.

<u>Definition 1</u>: A competitive equilibrium under free trade is

a) a consumption plan x and a production plan (u_1, u_2, v) for each agent,

b) a pair of sequences of specialized human capital (z_1, z_2),

c) a pair of sequences of average human capital (Z_1, Z_2),

d) a pair of price sequences (p_1, p_2), and

e) a 4-vector of employment for each industry in each country (n_1, n_2, n_1^*, n_2^*);

satisfying

(i) $\{x, u_1, u_2, v\}$ solve the agent's maximization problem,

(ii) $z_j = Z_j$, $z_j^* = Z_j^*$, $j = 1,2$,

(iii) $x_{jt} + x_{jt}^* = n_j y_{jt} + n_j^* y_{jt}^*$,

(iv) if $V_j(x)$ denotes utility of a worker in industry j, then $V_j(x) \geq V_k(x)$, $k \neq j$ with equality if $n_j > 0$ and $n_k > 0$.

In definition 1 condition (i) is the requirement of maximizing behavior, as is condition (iv); condition (ii) requires that expectations are self-fulfilling, and condition (iii) is the usual market-clearing condition.

Definition 2: A stationary competitive equilibrium is a competitive equilibrium which in addition to conditions (i) - (iv) of definition 1 also satisfies

(v) $x_{1t}, x_{2t}, z_{1t}, z_{2t}, Z_{1t}, Z_{2t}$ grow at the same constant rate,

(vi) u_{1t}, u_{2t}, v_t are constant over time,

(vii) p_{1t}, p_{2t} decline at constant rates.

The following theorem guarantees the existence of a stationary competitive equilibrium and that in the class of stationary competitive equilibria it is unique.

Theorem 1: There exists exactly one stationary competitive equilibrium under free trade if $1 + \alpha\sigma - \frac{\alpha\sigma}{\tau} - \frac{\beta}{1+\sigma} > 0$.

The proof of theorem 1 is contained in the appendix. To find a stationary competitive equilibrium I only need to find an algorithm which allows me to calculate the growth rates and the initial values of the variables in definition 2. Since the stationary competitive equilibrium is completely characterized by the rates of growth and the initial values of a finite number of variables I can use finite dimensional arguments to establish existence of a stationary competitive equilibrium. The algorithm ensures that the equilibrium is unique in the class of stationary competitive equilibria.

To prove existence of competitive equilibrium in growth models in which the capital stock grows without bounds one typically needs a condition which ensures that the growth rate is not too high so that agents' utility remains finite. Since in this model the growth rate is bounded above by some geometric rate, since the discount factor is strictly less than one, and since per period utility is logarithmic, such a condition is not needed here. The condition in theorem 1 ensures that agents' choices of how many other agents to contact remain within the interval [0,1].

IV. Characterization of the Stationary Competitive Equilibrium under Free Trade

This section contains the characterization of the stationary competitive equilibrium under free trade. First it will be established that countries specialize according to comparative advantage: all agents in country 1 will enter industry 1 and all workers in country 2 will be employed in industry 2. Then the growth rate in the stationary equilibrium will be characterized, using optimal decision rules derived from the agents' maximization problems. The analysis will concentrate on agents in country 1; for agents in country 2 the analysis is analogous.

<u>Proposition 1</u>: If $\zeta_1 > \zeta_2$ then in the stationary competitive equilibrium under free trade $n_1 = 1$, $n_2 = 0$, $n_1^* = 0$, $n_2^* = 1$.

The proof of proposition 1 follows from the proof of theorem 1. Proposition 1 just says if country 1 has a comparative advantage in the production of good 1 it will produce only good 1 and will import good 2 from country 2 which in turn produces only good 2. The specialization result for economies with Ricardian technologies, such as, for example in Dornbusch, Fischer, Samuelson [1977] carries over to this environment.

If country 1 specializes in the production of good 1 only skills of type 1 will be present in equilibrium. The agent's maximization problem can then be reformulated as:

$$(P') \qquad \max_{\{x,u_1,v\}} \sum_{t=0}^{\infty} \beta^t [\log x_{1t} + \log x_{2t}]$$

$$\text{subject to } \sum_{t=0}^{\infty} p_t \cdot x_t \leq \sum_{t=0}^{\infty} p_{1t} y_{1t}$$

$$y_{1t} = z_{1t} v_{1t}$$

$$z_{1t+1} = z_{1t} + \sigma Z_{1t} u_{11t}^{\alpha}$$

$$\tau u_{11t} + v_{1t} \leq 1$$

given prices $p \in P$ and the average stock of capital $Z_1 = (Z_{1t})_{t=0}^{\infty}$.

Since problem (P') is a concave program any solution to the problem can be calculated using first-order conditions derived from the Lagrangean function. The Lagrangean for problem (P') is:[1,2]

$$L = \sum_{t=0}^{\infty} \beta^t [\log x_{1t} + \log x_{2t}]$$

$$+ \mu\{\ldots p_{1t} z_{1t}(1 - \tau u_{1t}) + p_{1t+1}(z_{1t} + \sigma Z_{1t} u_{1t}^{\alpha}) v_{t+1}$$

$$+ p_{1t+2}(z_{1t} + \sigma Z_{1t} u_{1t}^{\alpha} + \sigma Z_{1t+1} u_{1t+1}^{\alpha}) v_{t+2}$$

$$+ \ldots - \sum_{t=0}^{\infty} p_t \cdot x_t\} .$$

Since preferences are strictly monotone and since the utility function is logarithmic the solution to problem (P') is not a corner solution. Any interior solution to problem (P') must satisfy the following first-order conditions:

$$\text{(9)} \qquad \frac{\partial L}{\partial x_{1t}} = \beta^t \frac{1}{x_{1t}} - \mu p_{1t} = 0 , \quad t = 0,1,2,\ldots,$$

$$\text{(10)} \qquad \frac{\partial L}{\partial x_{2t}} = \beta^t \frac{1}{x_{2t}} - \mu p_{2t} = 0 , \quad t = 0,1,2,\ldots,$$

$$\text{(11)} \qquad \frac{\partial L}{\partial u_{1t}} = p_{1t} z_{1t}(-\tau) + p_{1t+1} \alpha\sigma Z_{1t} u_{1t}^{\alpha-1} v_{t+1}$$

$$+ p_{1t+2} \alpha\sigma Z_{1t} u_{1t}^{\alpha-1} v_{t+2} + \ldots = 0 , \; t = 0,1,2,\ldots.$$

In equilibrium, expectations are self-fulfilling, that is $z_{1t} = Z_{1t}$; and in a stationary equilibrium $u_{1t} = u_1$ and $v_t = v$ for all t. Then equation (11) can

be written as

$$(12) \qquad p_{1t}\tau = \alpha\sigma u_1^{\alpha-1} v \sum_{s=1}^{\infty} p_{1t+s} \; .$$

Also, in a stationary equilibrium prices decline at a constant rate. Let Γ_p be that rate; that is $p_{jt+1} = \Gamma_p p_{jt}$ for all $j = 1,2$ and for all $t = 0,1,2,\ldots$ Then equation (12) can be rewritten as

$$(13) \qquad \tau = \alpha\sigma u_1^{\alpha-1} v(1 - \Gamma_p)^{-1} \; ,$$

where the fact that prices are normalized so that $p_{10} = 1$ is used. Using equation (9) for two consecutive time periods yields

$$\frac{x_{1t+1}}{x_{1t}} \cdot \frac{p_{1t+1}}{p_{1t}} = \beta$$

or, letting Γ_x denote the growth rate of x_j, $j = 1,2$ plus one, that is $x_{jt+1} = \Gamma_x x_{jt}$, equation (9) implies

$$(14) \qquad \Gamma_x \Gamma_p = \beta \; .$$

Since consumption of good j, x_j, and skill in industry j, z_j, grow at the same rate equation (14) implies

$$(15) \qquad \Gamma_z \Gamma_p = \beta \; .$$

The law of motion for skill in a stationary equilibrium is

(16) $$\Gamma_z = 1 + \sigma u_1^\alpha \ .$$

Now equations (13), (15), and (16) yield the following equation in u_1:

(17) $$1 + \alpha\sigma u_1^\alpha - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1} - \beta(1 + \sigma u_1^\alpha)^{-1} = 0 \ .$$

To see that equation (17) has a solution, first let $u_1 \to 0$. As u_1 converges to zero the left side of equation (17) converges to negative infinity. Now, let $u_1 \to 1$. As u_1 converges to one, the left side of equation (17) converges to $1 + \alpha\sigma - \frac{\alpha\sigma}{\tau} - \frac{\beta}{1+\sigma}$. If $1 + \alpha\sigma - \frac{\alpha\sigma}{\tau} - \frac{\beta}{1+\sigma} > 0$ equation (17) has a solution since the left side of equation (17) considered as a function of u_1 is continuous in the interval (0,1). To see that this solution is unique, define the function $F(u_1) = 1 + \alpha\sigma u_1^\alpha - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1} - \beta(1 + \sigma u_1^\alpha)^{-1}$. The derivative of F is given by $F'(u_1) = \alpha^2\sigma u_1^{\alpha-1} - (\alpha-1)\frac{\alpha\sigma}{\tau} u_1^{\alpha-2} + \beta(1 + \sigma u_1^\alpha)\alpha\sigma u_1^{\alpha-1}$. Since $F'(u_1) > 0$ whenever $0 < u_1 < 1$, the function F crosses the axis at most once. The solution to equation (17) is therefore unique.

If u_1 is the unique solution to equation (17) the stationary equilibrium rate of growth for human capital in industry 1 is given by

(18) $$\Gamma_z = \frac{z_{1t+1}}{z_{1t}} = 1 + \sigma u_1^\alpha \ .$$

Now the rate of growth of per capita GNP in country 1 can be calculated. The value of output in country 1 in period t evaluated in period t prices is

(19) $$GNP_t = p_{1t} y_{1t} \ .$$

The value of output in country 1 in period (t + 1) evaluated in period t prices is

$$GNP_{t+1} = p_{1t} y_{1t+1} \cdot$$

Then the real rate of growth of GNP is

$$(20) \qquad \frac{\Delta\ GNP_{t+1}}{GNP} \equiv \frac{GNP_{t+1} - GNP_t}{GNP_t}$$

$$= \frac{y_{1t+1} - y_{1t}}{y_{1t}}$$

$$= \frac{z_1'}{z_1} - 1$$

$$= \sigma(u_1)^{\alpha}$$

where the second to last equality holds only in the stationary competitive equilibrium.

How does the rate of growth of GNP depend upon the parameters α, β, σ, and τ? To find the dependence of the rate of growth of real GNP on the parameters it suffices to show how u_1 depends upon these parameters. Equation (17) defines a function G: $R^4 \times R^1 \to R^1$. Since $\frac{\partial G}{\partial u_1} = F' > 0$ the implicit function theorem can be used to solve (locally) for u_1 in terms of α, β, σ, and τ. Furthermore, the implicit function theorem implies that

$$(21) \qquad (u_{1\alpha}\ u_{1\beta}\ u_{1\sigma}\ u_{1\tau}) = -(G_u)^{-1}(G_\sigma\ G_\beta\ G_\sigma\ G_\tau)$$

where the variables subscripted by α, β, σ, τ, and u denote partial derivatives. The partial derivatives are:

$$G_\alpha = \sigma u_1^\alpha + \alpha\sigma u_1^\alpha \log \alpha - \frac{\sigma}{\tau} u_1^{\alpha-1} - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1} \log \alpha$$

$$+ \beta(1 + \sigma u_1^\alpha)^{-2} \sigma u_1^\alpha \log \alpha$$

$$= \sigma u_1^\alpha (1 + \alpha \log \alpha) - \frac{\sigma}{\tau} u_1^{\alpha-1} (1 + \alpha \log \alpha)$$

$$+ \beta(1 + \sigma u_1^\alpha)^{-2} \sigma u_1^\alpha \log \alpha$$

$$= (1 + \alpha \log \alpha)\sigma u_1^{\alpha-1}(u_1 - \frac{1}{\tau}) + \beta(1 + \sigma u_1^\alpha)^{-2} \sigma u_1^\alpha \log \alpha \ ,$$

$$G_\beta = -(1 + \sigma u_1^\alpha)^{-1} \ ,$$

$$G_\sigma = \alpha u_1^\alpha - \frac{\alpha}{\tau} u_1^{\alpha-1} + \beta(1 + \sigma u_1^\alpha)^{-2} u_1^\alpha$$

$$= \alpha u_1^{\alpha-1}(u_1 - \frac{1}{\tau}) + \beta(1 + \sigma u_1^\alpha)^{-2} u_1^\alpha \ ,$$

$$G_\tau = \frac{\alpha\sigma}{\tau^2} u_1^{\alpha-1} \ .$$

It is important to know how the rate of growth of the economy depends upon the preference and technology parameters. The effects of the parameters β and τ on the growth rate are easy to see. Since $G_u > 0$ and since $G_\beta < 0$, equation (21) implies that $u_{1\beta} > 0$, which together with equation (20) implies that the rate of growth of per capita GNP is an increasing function of

the discount factor β. Also, $G_\tau > 0$ so that equations (20) and (21) imply that the rate of growth of per capita GNP is a decreasing function of time required to access the skill of other agents. These two results are not surprising; economies in which consumers are more patient are expected to grow faster than economies in which consumers are impatient; also economies in which the cost to learning, in this case the cost to accessing expertise, are high are expected to grow more slowly than economies with low learning costs. The effects of changes in α and σ on the rate of growth are not unambiguous without further restrictions on the environment.

Since $\beta(1 + \sigma u_1^\alpha)^{-2}\sigma u_1^\alpha \log \alpha$ is negative, a sufficient condition for G_α to be negative is that $(1 + \alpha \log \alpha)\sigma u_1^{\alpha-1}(u_1 - \frac{1}{\tau})$ is also negative. Since for all $\alpha \in (0,1)$, $\log \alpha > -\frac{1}{\alpha}$, if $\tau < 1$, $(1 + \alpha \log \alpha)\sigma u_1^{\alpha-1}(u_1 - \frac{1}{\tau}) < 0$. Thus, if $\tau < 1$, $G_\alpha < 0$ which implies together with equations (20) and (21) that the rate of growth is an increasing function of α. Economies in which the diminishing returns to learning within one industry are less severe have higher rates of growth than economies with severe diminishing returns to learning within one industry.

Since

$$G_\sigma < \alpha u_1^{\alpha-1}(u_1 - \frac{1}{\tau}) + \beta u_1^\alpha$$

$$= (\alpha + \beta)u_1^\alpha - \frac{\alpha}{\tau} u_1^{\alpha-1} ,$$

if $\tau < \frac{\alpha}{\alpha+\beta}$, for all $u \in (0,1)$, $G_\sigma < 0$. But then equation (21) implies that $u_{1\sigma} > 0$. This says that in economies in which learning, in a sense, is more productive than physical production the rate of growth is higher than in

economies in which physical production is more productive than learning.

V. Definition, Existence, and Uniqueness of the Stationary Competitive Equilibrium when Trade is Prohibited

In the previous section the competitive equilibrium was characterized. Due to the presence of externalities the competitive equilibrium is not Pareto optimal. In the model described in section 2 there are two reasons why private returns differ from social returns: First, when workers make their time allocation decisions they will not take into consideration how their skill influences the average level of skill. Secondly, since countries specialize according to comparative advantage, the other related skill will not be accessible. Under specialization the law of motion of skill is

$$z_j' = z_j + \sigma Z_j u_j^{\alpha}$$

rather than

$$z_j' = z_j + \sigma Z_j u_j^{\alpha} + \omega Z_k u_k^{\alpha} .$$

In the presence of externalities which make the competitive equilibrium suboptimal, it is natural to ask how economic agents can internalize these externalities. In this model "entrepreneurs" could gather agents, instruct some agents to produce good 1, and other agents to produce good 2, and instruct agents to allocate their time between physical production and contacting other agents so as to achieve the optimal rate of growth. Unless the coalition thus formed is the grand coalition within a country agents belonging to the coalition may contact other agents both in and outside of the

coalition. In this paper I abstract from these contacts within such a coalition which can be optimally calculated by the entrepreneur. In this model, contacts outside the coalition are still suboptimal. When the competitive equilibrium is suboptimal due to externalities, the relevant standard for comparison is the solution to the social planner's problem. In this environment the social planner's problem is not a concave program and its analysis is therefore difficult. In the following sections I will consider an intermediate step: I will consider the effects of a policy which insures that both skills are present in each country. If the government of say, country 1, closes down trade among the two countries and imposes autarky the nature of preferences in this model insures that both countries will produce both goods.

This section contains definitions analogous to definitions 1 and 2 for the autarkical trade regime and concludes with a theorem which guarantees the existence of a stationary competitive equilibrium when trade among countries is prohibited.

Definition 3: A competitive equilibrium when trade among countries is prohibited is

a) a consumption plan x and a production plan (u_1,u_2,v) for each agent,

b) a pair of sequences of individual human capital (z_1,z_2),

c) a pair of sequences of average human capital (Z_1,Z_2),

d) a pair of price sequences (p_1,p_2),

e) a 2-vector of employment for each industry (n_1,n_2)

satisfying

(i) $\{x, u_1, u_2, v\}$ solve the agent's maximization problem

(ii) $z_j = Z_j$, $j = 1,2$

(iii) $x^1_{jt} + x^2_{jt} = n_{jt} y_{jt}$, $j = 1,2$; $t = 0,1,2,\ldots$, where x^i_{jt} denotes the amount of good j consumed at time t by a person in industry i

(iv) $V_1(x) = V_2(x)$, where as in definition 1, $V_j(x)$ denotes utility of a worker in industry j.

Definition 3 is analogous to definition 1 with the exception that the market-clearing condition in definition 3 incorporates the restriction that there is no trade between the countries.

Definition 4: A stationary competitive equilibrium when trade is prohibited is a competitive equilibrium which in addition to conditions (i) - (iv) of definition 3 also satisfies

(v) x_{jt}, z_{jt}, Z_{jt}, $j = 1,2$ grow at a common constant rate,

(vi) n_{jt}, v_t $j = 1,2$ are constant over time,

(vii) p_{1t}, p_{2t} decline at a common constant rate.

Definition 4 is analogous to definition 2.

Theorem 2: There exists exactly one stationary competitive equilibrium when trade is prohibited if $\zeta_1 = \zeta_2$ and if

$$1 + \alpha\sigma\left(1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{1-\alpha}} - \frac{1}{\tau}\right) - \beta\left(1 + \sigma + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}\right)^{-1} > 0.$$

The proof of theorem 2 is similar to the proof of theorem 1 and is therefore omitted.

VI. Characterization of the Stationary Competitive Equilibrium when Trade is Prohibited

This section contains the characterization of the stationary competitive equilibrium when trade is prohibited. As in section 4 growth rates of human capital stocks and of GNP will be characterized using the maximizing decision rules derived from the agents' maximization problem. Since preferences are logarithmic each country will produce both goods when trade among countries is prohibited. I will only consider the closed economy of country 1.

For a worker in industry 1 the Lagrangean function is

$$
\begin{aligned}
L = \sum_{t=0}^{\infty} \beta^t [\log x_{1t} + \log x_{2t}] \\
+ \mu\{\ldots p_{1t} z_{1t}(1 - \tau u_{1t} - \tau u_2) \\
+ p_{1t+1}(z_{1t} + \sigma Z_{1t} u_{1t}^{\alpha} + \omega Z_{2t} u_{2t}^{\alpha}) v_{t+1} \\
+ p_{1t+2}(z_{1t} + \sigma Z_{1t} u_{1t}^{\alpha} + \omega Z_{2t} u_{2t}^{\alpha} + \sigma Z_{1t+1} u_{1t+1}^{\alpha} \\
+ \omega Z_{2t+1} u_{2t+1}^{\alpha}) v_{t+2} + \ldots - \sum_{t=0}^{\infty} p_t \cdot x_t\} .
\end{aligned}
$$

As in section 4 since the utility function is logarithmic the solution to the agent's problem is not a corner solution. Any interior solution to the agent's problem must satisfy the following first-order conditions.

$$(22) \qquad \frac{\partial L}{\partial x_{1t}} = \beta^t \frac{1}{x_{1t}} - \mu p_{1t} = 0 \ , \quad t = 0,1,2,\ldots,$$

$$(23) \qquad \frac{\partial L}{\partial x_{2t}} = \beta^t \frac{1}{x_{2t}} - \mu p_{2t} = 0 \ , \quad t = 0,1,2,\ldots,$$

$$(24) \qquad \frac{\partial L}{\partial u_{1t}} = p_{1t} z_{1t}(-\tau) + p_{1t+1}\alpha\sigma Z_{1t} u_{1t}^{\alpha-1} v_{t+1}$$

$$+ p_{1t+2}\alpha\sigma Z_{1t} u_{1t}^{\alpha-1} v_{t+2} + \ldots = 0 \ , \quad t = 0,1,2,\ldots,$$

$$(25) \qquad \frac{\partial L}{\partial u_{2t}} = p_{1t} z_{1t}(-\tau) + p_{1t+1}\alpha\omega Z_{2t} u_{2t}^{\alpha-1} v_{t+1}$$

$$+ p_{1t+2}\alpha\omega Z_{2t} u_{2t}^{\alpha-1} v_{t+2} + \ldots = 0 \ , \quad t = 0,1,2,\ldots.$$

In equilibrium, expectations are self-fulfilling, that is $z_{jt} = Z_{jt}$ and in a stationary equilibrium all capital stocks grow at a constant rate so that we have $z_{1t} = Z_{1t} = Z_{2t} = z_{2t}$. Furthermore, in a stationary equilibrium $u_{jt} = u_j$ and $v_t = v$ for all t and prices decline at the common rate Γ_p so that $p_{jt+1} = \Gamma_p p_{jt}$. Equations (24) and (25) can now be written as

$$(26) \qquad p_{1t} z_{1t} \tau = \alpha\sigma Z_{1t} u_1^{\alpha-1} v \frac{\Gamma_p^t}{1 - \Gamma_p}$$

$$(27) \qquad p_{1t} z_{1t} \tau = \alpha\omega Z_{2t} u_2^{\alpha-1} v \frac{\Gamma_p^t}{1 - \Gamma_p}$$

or as

(28) $$\tau = \alpha\sigma u_1^{\alpha-1} v \frac{1}{1 - \Gamma_p}$$

(29) $$\tau = \alpha\omega u_2^{\alpha-1} v \frac{1}{1 - \Gamma_p} \quad .$$

Division of equation (28) by equation (29) yields

$$1 = \frac{\sigma}{\omega} \cdot \frac{u_1^{\alpha-1}}{u_2^{\alpha-1}}$$

or

(30) $$u_2 = \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}} u_1 \quad .$$

Using equation (22) for consecutive time periods yields

$$\frac{x_{1t+1}}{x_{1t}} \cdot \frac{p_{1t+1}}{p_{1t}} = \beta$$

which can be written in terms of growth rates as

(31) $$\Gamma_x \Gamma_p = \beta \quad .$$

Substitution of equation (30) and the time constraint (3) into equation (28) yields

(32) $$\Gamma_p = 1 - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1}\left(1 - \tau u_1 - \tau\left(\frac{\alpha}{\omega}\right)^{\frac{1}{\alpha-1}} u_1\right)$$

Equations (31) and (32) together imply that

$$(33) \qquad \frac{\beta}{\Gamma_x} = 1 - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1}(1 - \tau u_1 - \tau(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1) \ .$$

Since consumption of each good j and skill in production of good j grow at the same rate, that is $\Gamma_x = \Gamma_z$, equation (33) can be written as

$$(34) \qquad \frac{\beta}{\Gamma_z} = 1 - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1}(1 - \tau u_1 - \tau(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1) \ .$$

In a stationary competitive equilibrium the rate of growth of z_1 is given by

$$(35) \qquad \Gamma_z = \frac{z_{1t+1}}{z_{1t}} = 1 + \sigma u_1^\alpha + \omega u_2^\alpha \ .$$

Substitution of (35) into (34) yields

$$(36) \qquad 1 + \alpha\sigma u_1^\alpha + \alpha\sigma(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^\alpha - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1}$$

$$- \beta(1 + \alpha u_1^\alpha + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^\alpha)^{-1} = 0 \ .$$

Again it is easy to check that equation (36) has a solution: If u_1 converges to zero, the left side of equation (36) converges to negative infinity and if u_1 converges to one, then the left side of equation (36) converges to

$1 + \alpha\sigma(1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} - \frac{1}{\tau}) - \beta(1 + \sigma + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}})^{-1}$ which is positive by assumption.

As in section 3.4, let $F(u_1) = 1 + \alpha\sigma u_1^\alpha + \alpha\sigma(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^\alpha - \frac{\alpha\sigma}{\tau} u_1^{\alpha-1} -$

$\beta(1 + \sigma u_1^\alpha + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^\alpha)^{-1}$. Since F is continuous at every $u_1 \varepsilon (0,1)$, equation (36) has a solution. Since $F'(u_1) = \alpha^2\sigma_1^{\alpha-1} + \alpha^2\sigma(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^{\alpha-1} + \alpha(1-\alpha)\frac{\sigma}{\tau} u_1^{\alpha-2} + \beta(1 + \sigma u_1^\alpha + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_1^\alpha)^{-1}\omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} \alpha u_1^{\alpha-1} > 0$, the solution to equation (36) is unique.

An equation analogous to equation (36) can be derived for a worker in industry 2. The derivative of this equation is omitted. The equation is

$$(37) \qquad 1 + \alpha\omega\bar{u}_1^{\alpha} + \alpha\omega(\frac{\omega}{\sigma})^{\frac{1}{\alpha-1}} \bar{u}_1^{\alpha} - \frac{\alpha\omega}{\tau} \bar{u}^{\alpha-1} - \beta(1 + \omega u_1^\alpha + \sigma(\frac{\omega}{\sigma})^{\frac{1}{\alpha-1}} \bar{u}_1^{\alpha})^{-1} = 0 ,$$

where $\bar{u}_1$ denotes the measure of agents in industry 1 contacted by an agent in industry 2.

Equation (37) has a solution by the same arguments used to establish a solution for equation (36).

In the remainder of this section I show that the growth rate of the stationary competitive equilibrium when international trade is restricted exceeds the growth rate of the stationary equilibrium under free trade.

<u>Proposition 2</u>: The growth rate of per capita GNP in the stationary equilibrium when trade is prohibited exceeds the growth rate of per capita GNP in the stationary equilibrium under free trade.

Proof: When international trade is prohibited the workers in country one are divided evenly over the two industries. The growth rate per capita GNP in the stationary competitive equilibrium is equal to the rate of growth of human capital. The rate of growth of human capital in the stationary equilibrium when trade is restricted is

$$\Gamma_a = 1 + \sigma u_a^\alpha + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}} u_a^\alpha ,$$

where u_a is the solution to equation (36); under free trade the rate of growth of human capital is

$$\Gamma_f = 1 + \sigma u_f^\alpha ,$$

where u_f is the solution to equation (17). I need to show that

$$(38) \qquad \left(\sigma + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}}\right)u_a^\alpha > \sigma u_f^\alpha .$$

The following two equations are just reproductions of equations (17) and (36):

$$(39) \qquad 1 - \frac{\alpha\sigma}{\tau} u_f^{\alpha-1} + \alpha\sigma u_f^\alpha - \beta(1 + \sigma u_f^\alpha) = 0$$

$$(40) \qquad 1 - \frac{\alpha\sigma}{\tau} u_a^{\alpha-1} + \alpha\sigma u_a^\alpha + \alpha\sigma\left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}} u_a^\alpha - \beta\left(1 + \sigma u_a^\alpha + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}} u_a^\alpha\right)^{-1} = 0$$

Subtracting equation (39) from equation (40) yields:

$$(41) \qquad \frac{\alpha\sigma}{\tau}(u_f^{\alpha-1} - u_a^{\alpha-1}) + \alpha\sigma(1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}})u_a^{\alpha}$$

$$- \alpha\sigma u_f^{\alpha} + \frac{\beta}{1 + \sigma u_f^{\alpha}} - \frac{\beta}{1 + \sigma u_a^{\alpha} + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_a^{\alpha}} = 0 .$$

Let $u_a = u_f$; then the left side of equation (41) can be written as

$$(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_a^{\alpha} + \beta(1 + \sigma u_a^{\alpha})^{-1} - \beta(1 + \sigma u_a^{\alpha} + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_a^{\alpha})^{-1}$$

which is strictly positive. Let $A = \frac{\alpha\sigma}{\tau}(u_f^{\alpha-1} - u_a^{\alpha-1})$,

$B = \alpha\sigma(1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}})u_a^{\alpha} - \alpha\sigma u_f^{\alpha}$, and let $C = \beta(1 + \sigma u_f^{\alpha})^{-1} -$

$\beta(1 + \sigma u_a^{\alpha} + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} u_a^{\alpha})^{-1}$. Now I will show that as

$$u_a \downarrow \frac{\sigma}{\sigma + \omega(\frac{\sigma}{\omega})^{\frac{\alpha}{\alpha-1}}}^{\frac{1}{\alpha}} u_f$$

the sum A + B + C converges to a negative number, so that for equation (41) to be satisfied it must be the case that inequality (38) holds. First, consider A:

$$A = \frac{\alpha\sigma}{\tau} u_f^{\alpha-1} - \frac{\alpha\sigma}{\tau} u_a^{\alpha-1} \downarrow \frac{\alpha\sigma}{\tau} u_f^{\alpha-1} - \frac{\alpha\sigma}{\tau} u_f^{\alpha-1}(\frac{\sigma}{\sigma + \omega(\frac{\sigma}{\omega})^{\frac{\alpha}{\alpha-1}}})^{\frac{\alpha-1}{\alpha}}$$

$$= \frac{\alpha\sigma}{\tau} u_f^{\alpha-1}(1 - (\frac{\sigma}{\sigma + \omega(\frac{\sigma}{\omega})^{\frac{\alpha-1}{\alpha}}})^{\frac{\alpha-1}{\alpha}}) < 0 .$$

Secondly, consider B:

$$B = \alpha\sigma\left(1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}\right)u_a^\alpha - \alpha\sigma u_f^\alpha + \alpha\sigma\left[\left(1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}\right)\frac{\sigma}{\sigma + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}}} - 1\right]u_f^\alpha$$

$$= \alpha\sigma\left(\frac{1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}}{1 + \left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}} \cdot \frac{\omega}{\sigma}} - 1\right)u_f^\alpha$$

$$= \alpha\sigma\left(\frac{1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}}{1 + \left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}}} - 1\right)u_f^\alpha$$

$$= 0 .$$

Finally, for C we have

$$C = \beta(1 + \sigma u_f^\alpha)^{-1} - \beta\left(1 + \sigma u_a^\alpha + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}} u_a^\alpha\right)^{-1} \tag{42}$$

$$+ \beta(1 + \sigma u_f^\alpha)^{-1} - \beta\left(1 + \sigma \frac{\sigma}{\sigma + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}}} u_f^\alpha\right.$$

$$\left. + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{1}{\alpha-1}} \frac{\sigma}{\sigma + \omega\left(\frac{\sigma}{\omega}\right)^{\frac{\alpha}{\alpha-1}}} \cdot u_f^\alpha\right)^{-1} .$$

For the limit in (42) to be negative it must be the case that

$$\sigma u_f^\alpha > \frac{\sigma}{1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}} u_f^\alpha + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} \frac{1}{1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}} u_f^\alpha$$

$$= u_f^\alpha \frac{\sigma + \omega(\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}}{1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}}$$

$$= u_f^\alpha \sigma \frac{1 + \frac{\omega}{\sigma} (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}}{1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}}$$

But since $\frac{\omega}{\sigma} \leq 1$, $1 + \frac{\omega}{\sigma} (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}} \leq 1 + (\frac{\sigma}{\omega})^{\frac{1}{\alpha-1}}$ so that $C \leq 0$. Finally, since in the limit $A < 0$, $B = 0$, and $C \leq 0$, in the limit $A + B + C < 0$, which establishes the desired result.

VII. Conclusion

The model presented in this paper is an attempt to formalize Jane Jacobs' [1985] idea that only some cities perform the function of engines of growth, namely those cities which provide a variety of goods. It is difficult to see how variety of goods produced in a city is an essential element of an engine of growth without externalities like the ones described in this paper.

The model presented in this paper has profound implications for trade policy. In this model there are circumstances under which the conventional wisdom that free trade is optimal is no longer true. The measurement issue is crucial: Before variety can be measured and before these externalities are found to be significant this model cannot be taken as a basis for policy recommendations. If these externalities, however, are found to be significant

policy recommendations against appeal exclusively to the benefits of free trade in a Ricardian or Heckscher-Ohlin world.

Economic theories which provide justification for protectionistic measures are open to abuse: Special interest groups may use these theories to push for inefficient protection of obsolete specialized human capital. The theory presented here provides no justification for such protection.

Over-specialization as a symptom of under-development has long been recognized in the literature. The discussion of over-specialization typically, however, centers on the exposure of specialized economies to high risk. In the presence of shocks to productivity, income in highly specialized economies would exhibit large fluctuations. The theory developed in this paper points out another possible undesirable effect of over-specialization: Narrowly specialized economies in this model are not characterized by large fluctuations but by low rates of growth. The welfare losses calculated by Lucas [1987] indicate that growth retardation may be a more important effect of over-specialization than increased fluctuations.

Appendix

In this appendix the existence of a stationary competitive equilibrium under free trade will be established. The proof of existence of a stationary competitive equilibrium utilizes an algorithm. This algorithm will also show that the stationary equilibrium is unique in the class of stationary equilibria. Furthermore this algorithm shows that country 1 specializes in the production of good 1 and that country 2 specializes in the production of good 2 and that this specialization is complete: Each country only produces one good.

The first step in proving existence of a stationary competitive equilibrium is to show that each agent's maximization problem has a solution. This is accomplished by showing that the utility function is continuous and that the agent's constraint set is compact.

If the agent's maximization problem has a solution, first-order conditions to the constraint maximization problem can be used to characterize the solution.

Strict concavity of the utility function implies that the solution to the agent's maximization problem is unique.

The second step in proving existence of a stationary competitive equilibrium is the algorithm which takes as an input the first-order conditions to the agents' maximization problems.

Let $Y_t = \{(u_{1t}, u_{2t}, v_t) \in R^3_+ : u_{1t} + u_{2t} + v_t \leq 1\}$ and let $Y = \prod_{t=0}^{\infty} Y_t$. Let $X_t = \{x_t = (x_{1t}, x_{2t}) \in R^2_+ : x_{jt} \leq y_{jt},\ j = 1,2\}$ and let $X = \prod_{t=0}^{\infty} X_t$. The rate of growth of capital and hence output

is bounded above by $1 + \sigma + \omega$. With capital stocks which grow without bounds, Arrow-Debreu prices need to fall sufficiently fast so that values in the budget constraint remain finite. Therefore, let $P = \{p \in (R_+^\infty)^2 : p_{10} = 1$ and $\sum_{t=0}^{\infty} p_{jt}(1 + \sigma + \omega)^t < \infty,\ j = 1,2\}$.

<u>Lemma 1</u>: For all $p \in P$ and for all y satisfying $y_{jt} \leq (1 + \sigma + \omega)^t \zeta_j$, $j = 1,2$, $t = 0,1,2,\ldots$ the budget set $\phi(p,y_j) = \{x \in X : \sum_{t=0}^{\infty} p_t x_t \leq \sum_{t=0}^{\infty} p_{jt} y_{jt}\}$ is a closed subset of a compact set and is therefore compact (in the product topology).

<u>Proof</u>: For each j,t, y_{jt} is finite. Therefore X_t is a compact subset of R^2. By Tychonoff's theorem, X is compact in the product topology.

Let $x^n \in \phi(p,y_j)$, $n = 1,2,\ldots$ such that $x^n \to x$ pointwise. I need to show that $x \in \phi(p,y_j)$.

Suppose $x \in \phi(p,y_j)$. Then $\sum_{t=0}^{\infty} p_t x_t > \sum_{t=0}^{\infty} p_{jt} y_{jt}$. Since, for all $p \in P$, $\sum_{t=0}^{T} p_t x_t \to \sum_{t=0}^{\infty} p_t x_t$ as $T \to \infty$, for any $\varepsilon > 0$, there exists $\bar{T}$ such that

$$\sum_{t=0}^{T} p_t x_t > \sum_{t=0}^{\infty} p_{jt} y_{jt} + 3\varepsilon .$$

Since $x_t^n \to x_t$ for all $t = 0,1,2,\ldots$, for any $\varepsilon > 0$, there exists N such that

$$n \geq N \to \sum_{t=0}^{\bar{T}} p_t x_t^n > \sum_{t=0}^{\bar{T}} p_t x_t - 2\varepsilon .$$

But then for $n \geq N$ we have

$$\sum_{t=0}^{\infty} p_t x_t^n \geq \sum_{t=0}^{\bar{T}} p_t x_t^n > \sum_{t=0}^{\bar{T}} p_t x_t - 2\varepsilon > \sum_{t=0}^{\infty} p_{jt} y_{jt} + \varepsilon .$$

This last statement contradicts the fact that $x^n \in \phi(p,y_j)$ for all $n \geq 1$.

□

Extend the utility functional $\sum_{t=0}^{\infty} \beta^t[\log x_{1t} + \log x_{2t}]$ to a functional $V: X \times Y \to R$ by setting $V(s) = \sum_{t=0}^{\infty} \beta^t[\log x_{1t} + \log x_{2t}]$ where $s = (x,u_1,u_2,v)$.

<u>Lemma 2</u>: The functional $V: X \times Y \to R$ is continuous in the product topology.

<u>Proof</u>: Let $x^n \to x$ pointwise and let $s_t^n = (x_t^n,u_{1t},u_{2t},v_t)$ for some $(u_{1t},u_{2t},v_t) \in Y_t$. I need to show that $V(s^n) \to V(s)$. Let $V_T(s) = \sum_{t=0}^{T} \beta^t[\log x_{1t} + \log x_{2t}]$ for any $T \geq 0$. Then

$$|V(s^n) - V(s)| \leq |V(s^n) - V_T(s^n)| + |V_T(s^n) - V_T(s)|$$

$$+ |V_T(s) - V(s)| .$$

Since $V(s) < \infty$ for any feasible s, for all $\eta > 0$ there exists T_η such that $|V(s^n) - V_{T_\eta}(s^n)| < \eta$ and $|V_{T_\eta}(s) - V(s)| < \eta$. It remains to show that $|V_T(s^n) - V_T(s)|$ can be made arbitrarily small. Since $\log x_{1t} + \log x_{2t}$ is continuous, for any $\varepsilon_t > 0$ there exists N_t such that

$$n_t \geq N_t \rightarrow |\log x_{1t}^n + \log x_{2t}^n - \log x_{1t} - \log x_{2t}| < \varepsilon_t \ .$$

But then, for sufficiently large n

$$\begin{aligned} &\left|\sum_{t=0}^{T_n} \beta^t[\log x_{1t}^n + \log x_{2t}^n] - \sum_{t=0}^{T_n} \beta^t[\log x_{1t} + \log x_{2t}]\right. \\ &\quad \leq \sum_{t=0}^{T_n} \beta^t \varepsilon_t \\ &\quad \leq \max_{0 \leq t \leq T_n} \{\varepsilon_t\} \sum_{t=0}^{T_n} \beta^t \\ &\quad = \varepsilon \cdot \frac{1 - \beta^{T_n}}{1 - \beta} \end{aligned}$$

where $\varepsilon = \max_{0 \leq t \leq T_n} \{\varepsilon_t\}$. □

Lemmas 1 and 2 imply that the agent's maximization problem has a solution. Strict concavity of the utility function implies that the solution is unique. Since the utility function is logarithmic the solution will be interior. Any interior solution to the agent's maximization problem must satisfy the following first-order conditions:

(A1) $$\frac{\partial L}{\partial x_{1t}} = \beta^t \frac{1}{x_{1t}} - \mu p_{1t} = 0$$

(A2) $$\frac{\partial L}{\partial x_{2t}} = \beta^t \frac{1}{x_{2t}} - \mu p_{2t} = 0$$

$$\text{(A3)} \qquad \frac{\partial L}{\partial u_{1t}} = p_{1t}z_{1t}(-\tau) + p_{1t+1}\alpha\sigma Z_{1t}u_{1t}^{\alpha-1}v_{t+1}$$

$$+ p_{1t+2}\alpha\sigma Z_{1t}u_{1t}^{\alpha-1}v_{t+2} + \ldots = 0$$

$$\text{(A4)} \qquad \frac{\partial L}{\partial u_{2t}} = p_{1t}z_{1t}(-\tau) + p_{1t+1}\alpha\omega Z_{2t}u_{2t}^{\alpha-1}v_{t+1}$$

$$+ p_{1t+2}\alpha\omega Z_{2t}u_{2t}^{\alpha-1}v_{t+2} + \ldots = 0 \ .$$

Of course, in equation (A4) $u_{jt} \leq n_j$. This implies that if it turns out that $n_2 = 0$, that $u_{2t} = 0$ for all t and the solution to the agent's maximization problem only has to satisfy equations (A1) - (A3).

The following lemma establishes that, if skills in the two industries grow at the same rate, the two countries must be completely specialized; that is country 1 produces only good 1 and country 2 produces only good 2.

<u>Lemma 3</u> If $r_{z_1} = r_{z_2}$, then $n_1 = 1$, $n_2 = 0$, $n_1^* = 0$, $n_2^* = 1$.

<u>Proof</u>: If in country 1 both n_1 and n_2 are positive, per capita income in the two industries must be equal; that is

$$\text{(A5)} \qquad \sum_{t=0}^{\infty} p_{1t}z_{1t}v_1 = \sum_{t=0}^{\infty} p_{2t}z_{2t}v_2 \ .$$

In a stationary competitive equilibrium, human capital grows at a constant rate, prices decline at a constant rate, and the time allocated to physical production remains constant over time. Therefore equation (A5) reduces to

(A6) $$p_{10}z_{10}v_1 \sum_{t=0}^{\infty} \Gamma_p \Gamma_z = p_{20}z_{20}v_2 \sum_{t=0}^{\infty} \Gamma_p \Gamma_z \ .$$

Since the sums in (A6) are finite equation (A6) becomes

(A7) $$p_{10}z_{10}v_1 = p_{20}z_{20}v_2 \ .$$

Since $z_{10} > z_{20}$, it must be true that $p_{10}v_1 < p_{20}v_2$. Suppose that $p_{10} < p_{20}$. Then a similar argument for country 2 establishes that $p_{10} > p_{20}$ which yields a contradiction. Now suppose that $v_1 < v_2$. Since the growth rate of skill is inversely proportional to the time allocated to physical production $v_1 < v_2$ implies that $\Gamma_{z_1} < \Gamma_{z_2}$ which again yields a contradiction. This establishes that both countries are completely specialized. Since country 1 has a comparative advantage in good 1 it produces good 1 and country 2 produces good 2.

□

The marginal condition for u_1 determines u_1. The agent's time constraint determines v. Once u_1 is known the law of motion for human capital determines the rate of growth of human capital which is equal to the rate of growth of per capita output and of consumption. The marginal conditions for good j imply that $\Gamma_{p_j} \cdot \Gamma_{x_j} = \beta$ so that once Γ_{x_j} is determined the rate of decay of prices is also determined. Finally, the initial levels of consumption are determined by the market-clearing conditions for t = 0.

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Footnotes

1. Harberger's estimate of welfare losses due to monopolistic resource misallocation is about 0.1 percent of GNP.
2. In the remainder of this paper the subscript denoting the industry where the agent is employed is dropped whenever this is possible without creating confusion.
3. The first-order conditions are necessary but not sufficient conditions for a maximum. Since there exists a solution to the agent's maximization problem it suffices to use the necessary conditions to characterize the competitive equilibrium.