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PUBLIC POLICY AND ECONOMIC GROWTH:
Developing Neoclassical Implications

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This paper is a substantially revised version of one prepared for the conference on "The Problem of Development" held at the State University of New York at Buffalo in May 1988. We have benefited from comments by Stanley Fischer and Arnold Harberger—who discussed the paper at the Buffalo conference—as well as by Kenneth Judd, who discussed the paper at the Summer Econometric Society meetings.

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Economists have long suspected that there is a link between national economic policies and the long term rates of growth of countries. For example, Schultz (1981) suggests that many public policies contain disincentives for growth, in that they act to reduce the rewards to accumulation of a comprehensive concept of capital that contains human as well as tangible resources. In this paper, we show that a basic Schultzian model—constructed following leads provided by Uzawa (1965), Lucas (1988b), and Rebelo (1987)—can yield a wide disparity of growth rates in response to modest variations in taxation. Thus, in our analysis, changes in public policy can potentially explain the emergence of periods of secular stagnation or high economic growth.

The specific model that we construct is part of a larger program of research into models of endogenous economic growth, i.e., models in which sustained positive average rates of growth in percapita income arise in the absence of exogenous sources of productivity increase. Generally, these new models depart from the basic neoclassical model of Solow (1956), Cass (1965) and Koopmans (1965) by altering intertemporal technology in ways that make sustained growth feasible in the absence of time dependent elements in technology. The specific model that we study is an important class that originates in Uzawa (1965) and retains many of the key properties of the basic neoclassical model: (i) asymptotic growth occurs at a constant rate; and (ii) competitive and optimal allocations coincide in the absence of public interventions. The crucial attributes of the class—as developed in Rebelo (1987)—is that there are (i) constant returns to scale; and (ii) that all factors necessary for economic growth can be reproduced and accumulated. Recent work indicates that this class of models is very large, including

structures with many capital stocks in the growth "core" and with nonreproducible factors outside the growth "core" (Rebelo (1987)); or with steady states that are only asymptotically obtained (Jones and Manuelli (1988)). An essential attribute of these models is that labor be viewed as reproducible, i.e., as human capital in the sense of Schultz (1961) and Becker (1964).[1]

In addition to its main conclusion, our investigation is notable in two other ways. First, in examining the potential relation between public policy and long term growth, we require that the parameters of the model economy—governing aspects of preferences and production technology—are restricted to accord with existing microeconomic and macroeconomic evidence, a methodology that has proven to provide a powerful organizing tool in other areas of research in aggregate economics.[2] Second, we apply our methodology to versions of the basic neoclassical model, which has served as the workhorse of public finance theory for the last several decades. We find

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[1]An alternative class of endogenous growth models stemming from Romer (1986) stresses social increasing returns to capital formation in the core of capital stocks necessary for economic growth. Since models constructed along these lines generally also feature productive externalities—to reconcile existence of competitive equilibrium with increasing social returns—one need not require that nonreproducible factors are unimportant. Further, these alternative models generally differ from the class studied here in that: (i) growth rates accelerate over time (due to increasing returns); and (ii) policies that intervene in competitive markets to promote growth are socially desirable.

[2]Lucas (1980) provides cogent arguments for combining aggregate and microeconomic evidence to restrict dynamic macroeconomic models of business fluctuations. Other applications of this strategy include Mehra and Prescott's (1985)) work on asset pricing and recent work on real business cycles as surveyed by King, Plosser and Rebelo (1988a,b).

that neoclassical models have positive implications that are grossly at variance with major features of long term economic growth. Notably, with a central role for nonreproducible factors, the basic neoclassical model cannot readily explain why there are relatively small secular movements in market indicators of the return to capital. Thus, our paper simultaneously provides (i) a demonstration of the feasibility and desirability of exploring a new class of economic models; and (ii) an indication of the limitations of further pursuing conventional directions.

The details of our investigation of public policy and economic growth are as follows. In section I, we provide some stylized facts about the process of economic development, drawing on prior work by Kaldor (1961). In section II, we provide a discussion of the consistency of these facts with the basic neoclassical model of Solow (1956), Cass (1965), and Koopmans (1965). For the past thirty years this framework has served as a basic analytical tool for studying aspects of long term growth and as a laboratory in which to undertake policy experiments. This lengthy tradition makes many of us unwilling to discard the neoclassical model without a demonstration of central deficiencies, so we begin our paper with a thorough critical review. We begin by discussing the conventional set of cross-sectional empirical tests that have been used to explore a major implication of the basic neoclassical model, which is that in the absence of different rates of technological progress countries should converge to the same long run growth rate. We express a critique of the power of these tests in the presence of cross-country differences in public policies. However, we then provide simulations of parametric economies that demonstrate what we believe to be the major empirical failing of the basic neoclassical model: it cannot account for the pattern of cross-country diversity in rates of growth without

introducing substantial heterogeneity in the rate of technological progress across countries or generating counterfactual predictions for the behavior of the real rate of return.

Section III provides our analysis of growth through human capital accumulation and the incentive effects of public policy on this process. Our analysis proceeds in three stages. Following Rosen (1976) and Heckman (1976), we discuss optimal individual accumulation of human capital and the influence of various taxes on it. To highlight the role of taxes and to conform to prior microeconomic studies, our analysis takes key prices as exogenous—the wage rate per unit of human capital; the price of investing in human capital; and the real interest rate on consumption loans. Our next two stages make these relative prices endogenous. First, we specify technologies for producing consumption and investment goods, while retaining an exogenous borrowing and lending rate. This provides a framework to discuss a small open economy's accumulation of nontraded human capital. In this section, policies can alter prices through changes in factor mix, as well as through the direct tax effects analyzed in the individual's problem. Second, we study a full general equilibrium in which the rate of return adjusts to equate borrowing and lending or, equivalently, savings and investment. This provides a final set of policy influences on the growth rate.

Not surprisingly, the magnitude of policy effects on the rate of growth depend on the specification of preferences and technology. But a virtue of our approach is that it indicates microeconomic measurements that would be valuable in isolating the effects of policies on growth.

I. Stylized Facts of Economic Growth

A natural reference point in terms of summarizing the empirical evidence on the growth process is the set of stylized facts identified by Kaldor (1961). These empirical regularities constitute the motivation for the basic neoclassical model, and hence, can also be used to provide an informal assessment of of the adequacy of the new theories we discuss.

Kaldor observed that three variables seem to be relatively constant over time: the rate of return on capital, the capital and labor shares in national income, and the capital-output ratio. In contrast, he identified a tendency for the productivity of labor and for the capital-labor ratio to rise at a steady pace. Finally, Kaldor observed that the growth rate of labor productivity differed significantly across countries.

Romer (1988a) recently reviewed the evidence on Kaldor's descriptions of the behavior of labor productivity, labor and capital shares, and of the capital-output ratio. He concluded that, except for some evidence of a downward trend in the share of capital income, the stylized facts described above continue to be reasonable descriptions of the data.[3]

Our discussion focuses on the behavior of the real interest rate and of the growth rate of percapita real gross domestic product, relying on data from Summers and Heston (1984) and Maddison (1982).

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[3]Maddison (1987), p. 660, also reports evidence that points to a secular decline in the share of capital in GDP. However, he discusses several forms of measurement error that may have caused this decline and proceeds to use a constant capital share in his growth accounting calculations.

I.1. Real Gross Domestic Product Percapita

We provide three descriptions of the behavior of the growth rates of percapita real GDP in the Summers and Heston data set. The first consists of Table 1 which provides a list of average growth rates, and their standard deviation by country for the period 1950 to 1981. The number of observations available for each country is also reported. The second is Figure 1 which associates the level of percapita real GDP in 1960 with its average rate of change over the period from 1960 to 1981. The third is Figure 2 which shows the relationship between growth rates for two consecutive subperiods: 1950-1969 and 1970-1981.

To complement this evidence we study the long run behavior of percapita real GDP for sixteen advanced market economies using data form Maddison (1982).

Table 1 documents the large diversity in average rates of growth to which we alluded in the introduction. There seems to be a small negative relationship between growth and its volatility—the correlation between the average rate of change of GDP and its standard deviation is -.07.

Figure 1 shows that the level of income percapita in the beginning of the period seems to be unrelated to whether a country subsequently grows slowly or fast. The estimate of the correlation between the rate of growth and the level of income percapita in the beginning of the period is .13. This estimate indicates that if any relationship exists between the level of economic activity and the subsequent growth rate it is likely to be positive. This fact is often invoked to dismiss the convergence predictions of the neoclassical model.

Figure 2 documents the extent to which there is persistence in the growth process.[4] It considers two subperiods, 1950–1969 and 1970–1981. The correlation between growth rates in the first subperiod and in the second is .39, indicating a tendency for an initial period of high (low) growth to be followed by another period of high (low) growth.

Table 2 uses data from Maddison (1982) to examine the presence of trends in the rate of change of percapita GDP. The sample includes the US, Canada Japan, Australia, and twelve European countries. These countries were chosen for having data on real GDP and population available for most of the 1870 to 1970 period (in some cases this time interval had to be divided in two parts due to missing observations). The evidence is hard to interpret—the data for the 19th century and beginning of the 20th century is not very reliable and the sample is influenced by three major events, the two World Wars and the Great Depression. Nevertheless, the vast majority of the trend estimates is positive although not statistically different from zero at conventional significance levels. Among the countries with data available for the full sample period Italy and Norway are the only cases in which the (positive) trend estimates are significantly different from zero. The four countries with data for only part of the sample (Austria, Belgium, Japan, and Switzerland) show higher average growth rates in the second part of the sample than in the first part and (except for Japan) exhibit trends in the rate of growth not significantly different from zero in each subperiod. We interpret these results as supporting the view that there seems to be no

[4]In Figures 2, 3 and 4 the symbol * denotes the mean across countries of the two variables represented on the axes.

general long run tendency for the rate of growth to decrease in advanced economies.

I.2 Public Policy and Growth

One of our main focuses in the theoretical sections will be on policy implications of public policy for economic growth. The Summers and Heston data set contains only one policy measure, the share of government in GDP. The relationship between this share of the rate of growth GDP percapita is illustrated in Figure 3. For the full Summers and Heston sample the correlation is -.13. However, over subsamples this correlation is not stable: it is basically nil in the period 1950-1969 and -.33 in the period 1970-1981. This is associated with the relatively small persistence in the share of government expenditure in GDP. Figure 3 suggests that public policy does not offset growth through the expenditure channel.

Finally, Figure 4 plots the rate of growth in real per capita GDP against the share of national product devoted to investment. The correlation between these two variables for the entire sample is .46. Further, there is considerable persistence in the share of GDP that countries devote to investment. Thus, if policy affects growth, its influence may well derive from an influence on the share of investment.

I.3. Real Interest Rates

As will be clear from the theoretical discussion, evidence on the behavior of the real interest rate plays a crucial role in allowing us to distinguish between different models of economic development.

The modern evidence largely supports Kaldor's contention that the real interest rate has been relatively constant over time. Even though there are

well-known difficulties in obtaining meaningful estimates of real rates of return, namely controlling for risk and correcting for anticipated inflation, we believe that if pronounced trends were present in the data these would eventually surface despite measurement error problems.

Our first source of evidence is Homer's (1963) historical study of interest rates, which examines the behavior of interest rates since Ancient times until the 20th century. From the spectrum of interest rates existing in any country at any given time, Homer considers the lowest regularly reported rate to be the one suitable for studies of long run trends. This is preferable to the average rate which bundles together the returns to assets with different risk characteristics.

Although the rates of return reported are nominal and the nature of the underlying assets extremely diverse, we believe that the evidence presented by Homer and summarized in Table 3 is useful in terms of judging whether or not Kaldor's description of the behavior of the real interest rate is adequate. Table 3 makes it hard to view the early stages of development as being accompanied by very high or very low real interest rates. Further, it indicates that for most countries the real interest rate seems to have decreased over time. However, its rate of decrease is extremely small so that for one hundred year periods—the time frame we will consider—Kaldor's contention that the real interest rate appears to be constant is not inconsistent with Homer's evidence.

Our second source of information on the behavior of the real rate of return is Ibbotson and Sinquefield's (1982) compilation of returns on various U.S. securities in the period 1926 to 1981. Table 4 tests for the presence of trends in the real returns to common stocks, small stocks, long-term corporate bonds, long-term government bonds and U.S. Treasury Bills. At

standard significance levels, we cannot reject the hypothesis of no trend in the returns to stocks and treasury bills. The trend coefficients on the real rate of return on long-run government and corporate bonds are negative, of the order of −.02 percent and significantly different from zero at levels of significance of 2 and 10 percent, respectively.

We interpret these two sets of evidence as being broadly consistent with the view that growth in percapita income is not accompanied by systematic increases or decreases in the real rate of return on capital.

II. Counterfactual Implications of the Neoclassical Model

This section is devoted to our critique of the neoclassical model. The essence of our argument is that the transitional dynamics associated with capital accumulation are inconsistent with Kaldor's facts, so long as there is an empirically reasonable amount of diminishing returns to capital.

II.1 Outline of the Model

We start by reviewing a discrete time version of the standard neoclassical model with exogenous technical progress. Our discussion focuses on general properties of this model economy as a framework for explaining long term growth in percapita quantities, so our exposition abstracts from population growth.

Output, denoted by Y_t, is produced according to a constant returns to scale production function with the usual neoclassical properties.[5] The factors of production are capital (K_t) and labor (N_t), augmented by the productivity index X_t. We often refer to X_t as representing "technological progress" but, as discussed in section III, this is really a catch-all phrase to summarize all the sources of productivity growth exogenous to the model. For simplicity, labor supply is viewed as exogenous.

$$Y_t = F(K_t, N_t X_t) \tag{II.1}$$

Output can be used for consumption (C_t) or devoted to investment (I_t).

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[5]The production function is assumed to be constant returns to scale, concave, twice continuously differentiable, to satisfy the Inada conditions, and be such that both factors are essential in production.

$$Y_t = C_t + I_t \tag{II.2}$$

Capital depreciates at rate δ.

$$K_{t+1} = I_t + (1-\delta)K_t \tag{II.3}$$

Technical progress grows at rate $\gamma_x - 1$.

$$X_{t+1} = \gamma_x X_t \tag{II.4}$$

Finally, the initial conditions K_o and X_o are given.

It is well known that the only sustainable growth path is one in which Y_t, C_t, I_t, and K_t grow at the rate of expansion of technical progress, γ_x-1. This trajectory is an example of a steady state path, which we define as a trajectory along which the rate of growth of labor supply and of the production of capital and consumption goods are constant.[6]

II.1. The Convergence Implication

When a description of savings behavior is added to the basic neoclassical model, there is generally a strong implication that all economies should converge to the steady state growth path. For example, if we follow Solow

[6]This concept of steady state allows for changes in the number of hours worked. Although this is beyond the objectives of this paper, the motivation for this is to accommodate the secular decrease in percapita hours worked documented in Maddison (1982). In endogenous labor models this phenomenon can be incorporated by choosing preferences that imply a (extremely small) constant decrease in the fraction of time devoted to work in the market.

(1956) by assuming a fixed savings rate and constant labor supply, then the dynamics of accumulation are given by

$$K_{t+1} - K_t = sF(K_t, NX_t) - \delta K_t, \tag{II.5}$$

where s is the savings rate. Relative to the steady state growth path this process is just

$$\gamma_x k_{t+1} - k_t = sF(k_t, N) - \delta k_t, \tag{II.6}$$

where $k_t = K_t/X_t$. From any initial capital stock k_o, this difference equation converges monotonically to a unique stationary value k^* satisfying $(\gamma_x - 1)k^* = sF(k^*, N) - \delta k^*$. Thus, if countries begin with a relatively low capital and, hence, low income, they will initially grow faster. However, in Figure 2, we saw little tendency for a low initial level of income (in 1960) to be followed by high rates of expansion over the subsequent two decades. This fact is often taken to be a strong refutation of the neoclassical model but we are skeptical about relying on it in a world with potential heterogeneity in production possibilities, preferences and public policies. The basis for our skepticism can be illustrated by using a version of the Solow (1956) model that incorporates heterogeneity by adding a country superscript j and specializing the production function to the Cobb-Douglas form. Solow's difference equation then takes the form:

$$K_{jt+1} - K_{jt} = s_j A_j K_{jt}^{\alpha_k} (N_{jt} X_t)^{\alpha_n} - \delta_j K_{jt} \tag{II.5'}$$

with $\alpha_k + \alpha_n = 1$. The implied dynamics of transformed capital are:

$$\gamma_x k_{jt+1} - k_{jt} = s_j A_j k_{jt}^{\alpha_k} N_{jt}^{\alpha_n} - \delta_j k_{jt} \qquad \text{(II.6')}$$

with a stationary value $k^* = [(\gamma_x+\delta_j-1)/(s_j A_j N_j^{\alpha_n})]^{1/(\alpha_k-1)}$.

In this simple extension of the Solow model there is potential heterogeneity in initial conditions (k_o) and terminal conditions (k^*) so that one might expect levels and growth rates to be roughly uncorrelated as in Figure 2.

Thus, we believe that one cannot understand the deficiencies of the basic neoclassical model without undertaking a detailed quantitative evaluation of its properties when its parameters are restricted by empirical evidence.

II.3 The Real Interest Rate and Technology

One can extract some information about the behavior of the real interest rate in the neoclassical model without specifying preferences. Using the fact that percapita GDP increased in the U.S. by a factor of 7 in the last century (see Maddison (1987), Table A-4) and the knowledge of today's real interest rate, we can compute what the real interest rate should have been one century ago according to the model. Since in 1960 the percapita GDP of most underdeveloped countries was about 1/7 of that of the U.S. (see Figure 1) this calculation also provides information about the interest rate differential that would have to exist between the U.S. and those countries in the absence of international capital markets.

Suppose that technology is Cobb-Douglas, labor supply exogenous and that there is no technological progress. The capital stock that rationalizes a

given value r_T of the real interest rate today is $K_T = [\alpha_k A N^{\alpha_n}/(\delta+r_T)]^{1/(\alpha_k-1)}$. In this expression (as above), α_n is labor's share, α_k is capital's share, N is the supply of labor, A is the level parameter of the production function, and δ is the depreciation rate. Given K_T and the fact that output increased 7 times over the course of a century we can compute the capital stock that should have been in place one century ago, $K_o = K_T/(7^{1/\alpha_k})$, and the associated real interest rate, $r_o = \alpha_k A K_o^{(\alpha_k-1)} N^{\alpha_n} - \delta$. To construct a baseline scenario, we choose parameter values that reflect the U.S. experience in the post-war period. We choose the capital share to be 1/3, which accords with the estimates reported in Maddison (1987), Table 8. The labor supply is set to .2 which is the average fraction of time devoted to work in the period 1948-1986.[7] The real interest rate in the end of the period is set equal to the average return to equity in the 1948-1986 period—6.5% per annum. The rate of capital depreciation is set to 10%, which is within the range reported by Maddison (1987), Table 7. This parameterization of the neoclassical model implies that the real interest rate one century ago, r_o, should have been a staggering 789.5% a year!

Table 5A summarizes the predicted values for r_o under different hypotheses for the growth rate of exogenous technical progress. The value of the real interest rate in the beginning of the period is lower when a smaller fraction of growth is associated with transitional dynamics. Naturally, if the economy is at the steady state at time zero, so that there are no transitional dynamics, the rate of interest is the same in the beginning and

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[7]King, Plosser and Rebelo (1988) discuss derivation of this number from the Household Survey published by the Bureau of Labor Statistics.

in the end of the period. The first column of Table 5A is devoted to the baseline model which has the technology we just described. Columns 2, 4 and 5 consider perturbations of this baseline scenario which involve different rates of depreciation, capital shares and terminal real interest rates. In Column 3 labor supply is taken to be .36 in the beginning of the period and .2 at the end of the period, so as to reflect the decrease in hours devoted to market work occurred in the last century (see Maddison (1987), table A-9).

The message implicit in Table 5A is clear: transitional dynamics cannot account for a large fraction of the expansion in output without generating implausible values for the real interest rate in the beginning of the period. In order for all the output expansion to be associated with transitional dynamics the real interest rate one century ago should have been higher than 100%, unless we postulate an implausibly high share of capital in production.

Table 5B explores a variation of the baseline model in which the elasticities of substitution in production are different from the unitary elasticity associated with the Cobb-Douglas production function. All economies have a CES production function with elasticity of substitution ρ and the same terminal capital stock, $K_T = 100$. The remaining two parameters of the production function are chosen so that r_T is 6.5% and the share of capital in output at time T is 1/3. This ensures that at time T all the economies have the same capital stock, real interest rate, production and capital share but different elasticities of substitution.

One might expect that with elasticities of substitution lower than one the value of r_o would be higher than that associated with Cobb-Douglas production. This is not necessarily true as the last line of Table 5B shows—without technological progress, a decrease in the elasticity of substitution from .9 to .5 actually decreases r_o. This is partly due to the

fact that the marginal products schedules for these two CES production functions intersect twice, at K_T and at a value of the capital stock between zero and K_T.

With non-unitary elasticity of substitution in production the capital share is no longer constant over time when the economy is not following a steady state path. It decreases over time when $\rho < 1$ and it increases for $\rho > 1$. Table 5B indicates that varying the elasticity of factor substitution away from one moderates in some cases the predicted values for r_o—with no exogenous productivity growth the value of r_o associated with $\rho = .5$ is 111.6, roughly seven times smaller than that associated with Cobb-Douglas production. However, the values of r_o continue to be extremely high in light of the historical evidence (see Table 3) when the role of transition dynamics is significant. Furthermore, varying the elasticity of substitution generates implausible implications for the evaluation of the share of capital in production (for instance, with $\rho = .5$ the share of capital decreases by roughly 3 fold over the course of a century).

II. 3. Preferences and Dynamic Competitive Equilibrium

To discuss the form of preferences consistent with steady state growth in the presence of labor augmenting technical progress, it is easiest to examine the economy's competitive equilibrium. For expositional purposes, we consider a market structure with spot labor markets and one-period loan markets under perfect foresight.

Profit maximization by firms implies that in any period the real interest rate (r_t) must equal the net marginal product of capital: $r_t = D_1F(K_t, N_tX_t) - \delta$, where $D_iF(.)$ denotes the ith partial derivative of the function F(.). In the steady state the real interest rate is constant since K and X grow at the

same rate. This implies that the steady state path will only be an equilibrium if households choose to expand their consumption at a constant rate when faced with a constant interest rate. If preferences are time separable this implies that momentary utility must be isoelastic, i.e. life-time utility must take the form:

$$U = \sum_{t=o}^{\infty} \beta^t u(C_t) \qquad \text{(II.7)}$$

with $u(C_t) = \frac{C_t^{1-\sigma}-1}{1-\sigma}$.

Given these preferences, households choose the growth rate of consumption to be $\gamma_C = [\beta(1+r)]^{1/\sigma}$. Since feasibility requires that $\gamma_C = \gamma_x$, the steady state real interest rate must be $r = (\gamma_x^{\sigma}/\beta) - 1$. This pins down the steady state capital-labor ratio (K/NX). The real interest rate, the output-capital ratio, the shares of capital and labor in income and the consumption and investment shares in output are constant along the steady state trajectory.

Transitional Dynamics

Specifying preferences allows us to go beyond the simple calculations of the real interest rate at time zero. We analyze the transitional dynamics of the model by studying the optimal (and competitive) path of an economy whose capital stock has associated a level of percapita output seven times smaller than that associated with the steady state. This analysis adopts the baseline parameters for technology that underlies Table 5A; specifies preferences to be (II.7); and chooses β to be consistent with a steady state real interest rate of 6.5% (the steady state real interest rate is

$r_s = (1/\beta) - 1)$. We experiment with various forms for the momentary utility function u(C).

Figure 5 depicts the trajectories of capital, output, consumption and investment for the case of logarithmic monentary utility. The dynamic equilibrium displays high real interest rates and fast expansion in early stages of development.

Table 6A lists the average annual rates of growth of output and capital and the average annual real interest rate for successive 5 year periods within the first 50 years.[8] Line one of the table reports numerical results that correspond to Figure 5, i.e., for the case in which the intertemporal elasticity of substitution $(1/\sigma)$ is equal to one. The transitional dynamics are remarkably rapid in this model—after 8 years half of the adjustment toward the steady state is completed. Average real interest rates are very high in early stages of development, that is approximately 100% per annum.

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[8]The numerical results reported in Tables 6 and 7 were obtained by shooting methods. The easiest way to study the model's transitional dynamics is to consider a planning problem in which a benevolent planner maximizes welfare of the representative agent subject to the technological constraints spelled out in the main text. The first order conditions for this planning problem can be expressed as a system of two autonomous first order non-linear difference equations in K and λ, the Lagrange multiplier associated with the resource constraint. Given knowledge of λ_o and K_o we can solve this system of equations forward. The value of λ_o has to be such that the "transversality condition" $\lim_{t\to\infty} \beta^t \lambda_t K_{t+1} = 0$ is satisfied. This condition is the analogue of the complementary slackness condition that arises in T period problems associated with the restriction $K_{T+1} \geq 0$. Shooting methods are iterative procedures that start with a guess for λ_o, solve the system of difference equations forward, and check whether the transversality condition is satisfied. In case it is not, the initial guess is revised, using methods for finding zeros of equations (e.g., Newton-Raphson). A detailed discussion of shooting methods can be found in Lipton et al (1982) and Roberts and Shipman (1972).

With lower intertemporal substitution (σ = 20) the growth process is much more protracted, as the second line of Table 6A demonstrates. The half life of the system dynamics is now 61 years. The low elasticity of intertemporal substitution leads agents to prefer a smoother path for consumption than in the baseline model (when σ = 1). On the one hand, less intertemporal substitution leads to lower rates of growth in early development periods than in the baseline scenario. However, on the other, it implies even higher values of the real interest rate than in the case of σ = 1. This example demonstrates how reparameterizing the neoclassical model simply shifts the difficulty to another area. We will repeatedly encounter this theme as we proceed through this section.

The third parameterization modifies momentary utility to the form: $u(c) = \log(c-\bar{c})$, where $\bar{c}$ denotes the subsistence consumption level. In this model, there is an unstable steady state at the level of sustainable capital stock compatible with $\bar{c}$. This low level steady state resembles somewhat the "poverty trap" familiar from the development literature. That is, despite the good investment opportunities the country does not invest because production is barely enough to attend to subsistence consumption and to the replacement of the depreciated capital stock. In the parameterization examined, we chose $\bar{c}$ to be .04 which represents 95% of the resources available to the economy in the end of the first period (production plus non-depreciated capital). This economy displays a "hump-shaped" growth path (although this cannot be concluded from the averages reported in the table)—growth rates are low in the initial periods, subsequently rise and then decline. But again, altering preferences to produce more protracted transitional dynamics also generates higher real interest rates in initial stages of development than those associated with the baseline scenario.

In line 4, we perturb the basic model by altering capital's share to be a value of $\alpha_k = .5$, which we think is a plausible upper bound. Transitional dynamics are more persistent relative to the baseline model but real interest rates, though lower, are still high in the early stages.

The experiment reported in the last line of Table 6A is suggestive. If the share of capital is moved to $\alpha_k = .9$, so that the production function comes close to being constant returns in the factor that can be accumulated, the process of transitional dynamics is protracted and the real interest rate assumes more moderate values in early stages of development. But $\alpha_k = .9$ is counterfactual, if we maintain that capital earns a competitive factor share.

II.5. The two-sector neoclassical model

One might think that the results of the experiments above are peculiar to the one-sector nature of the model. Table 6B sheds light on this conjecture. It summarizes the adjustment path for a two-sector model in which both production functions are Cobb-Douglas with level parameters normalized to one. The labor share in the capital sector is taken to be .5. We experiment with two alternative values of the labor share for the consumption industry, $\omega_k = .25$ and $\omega_k = .75$. (When $\omega_k = .50$ the two sector model generates results identical to those reported on line 4 of Table 6A since it reduces to a one-sector economy.) The remaining parameters and initial capital stock of the economies that underlie Table 6B are identical to those of Table 6A.[9] As

[9]Output measured in terms of the consumption good is given by $Y_t = p_t I_t + C_t$. Since p_t, the relative price of investment in terms of consumption is endogenous, we cannot compute directly the value of K_o that has associated a value of output seven times larger than that of the steady state. We used instead the value of K_o implied by the one-sector model so that Table 6B can be readily compared with Table 6A. This choice did not affect the results

one might expect, separating out the consumption sector and making its production function more linear in capital makes the interest rate implications less dramatic but still generates implausible values for r_0. In order to generate empirically plausible values for r_0 one has to postulate that the production function of the capital sector is close to linear in capital.

II.6. Adjustment Costs

Costs of changing the capital stock are another potential avenue for eliminating the counterfactual implications for the behavior of the real interest rate. We consider below a version of the neoclassical model with adjustment costs similar to the one developed by Abel and Blanchard (1983). To preview the results of this investigation, it is true that if one freely chooses the adjustment cost function, then one can overturn the implication for the beginning of period real interest rates. But there are then other undesirable implications. Moreover, we would like to employ adjustment cost functions that are empirically reasonable on other grounds. For this purpose, we draw on work by Hayashi (1982) that develops the connection between adjustment costs and Tobin's q—the ratio of stock market valuation of existing capital to its replacement cost. We conclude that one can only overturn the implication of implausibly high interest rates at the cost of generating counterfactual values for Tobin's q. That is, initial period q falls well outside the range of values that have been estimated in the literature on empirical investment equations.

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discussed in the main text.

The introduction of adjustment costs requires that we alter the resource constraints of the neoclassical model as follows

$$Y_t = C_t + Z_t[1 + h(Z_t/K_t)] \tag{II.2'}$$

$$K_{t+1} = Z_t + (1-\delta)K_t. \tag{II.3'}$$

In the standard model, one unit of investment increases the capital stock by one additional unit. Now it is necessary to invest $1 + h(Z_t/K_t) + (Z_t/K_t)Dh(Z_t/K_t)$, where $Dh(.)$ denotes the derivative of $h(.)$.

The adjustment cost function $h(.)$ is assumed to be homogeneous of degree zero in Z and K. As Hayashi (1982) has shown, this makes the theory operational since it allows us to determine Tobin's marginal q by measuring average q. We assume that $h(\delta) = 0$ and $Dh(\delta) = 0$, so that the steady state capital stock is not affected by the introduction of adjustment costs. Without this assumption the adjustment costs economy would have a lower steady state capital stock than the comparable standard model. This would contribute to an *increase* in r_o. To make clear that our conclusions do not hinge on this effect, we chose to eliminate it.

Finally, we postulate that both the adjustment costs and the total cost of investing are increasing: $Dh(.) \geq 0$, and $2Dh(Z_t/K_t)) + (Z_t/K_t)D^2h(Z_t/K_t) > 0$, where $D^2h(.)$ denotes the second derivative of $h(.)$.

The value of Tobin's (marginal) q implied by this model is

$$q_t = 1 + h(Z_t/K_t) + (Z_t/K_t)Dh(Z_t/K_t) \tag{II.8}$$

and the real interest rate is given by:

$$r_t = [DF(K_t) + (Z_t/K_t)Dh(Z_t/K_t)]/q_{t-1} + (1-\delta)q_t/q_{t-1} - 1 \qquad \text{(II.9)}$$

The consideration of adjustment costs introduces two conflicting effects on the real interest rate. First, the fact that the cost of increasing capital by an extra unit is now higher than one ($q_{t-1} = 1 + h(Z_{t-t}/K_{t-t}) + Dh(Z_{t-t}/K_{t-t})(Z_{t-t}/K_{t-t}) \geq 1$) lowers the real interest rate relative to the non-adjustment cost case. Second, the fact that an additional unit of capital lowers adjustment costs ($(Z_t/K_t)Dh(Z_t/K_t) \geq 0$) contributes to a higher value of the real interest rate. Equation (II.9) makes clear that low values of the real interest rate can only be obtained by introducing adjustment costs that imply large values of q.

Summers (1981) showed that when h(.) takes the functional form (II.10), the model described above predicts a linear relationship between Z_t/K_t and q_t.

$$h(Z_t/K_t) = \frac{(b/2)(Z_t/K_t - a)^2}{Z_t/K_t} \qquad \text{when } Z_t/K_t \geq a$$

$$h(Z_t/K_t) = 0 \qquad \text{when } Z_t/K_t < a \qquad \text{(II.10)}$$

Estimating this linear relationship correcting q_t for the effects of taxation, Summers (1981) obtained the following estimates: b = 32.2 and a = .088. The requirement of no adjustment costs at the steady state implies that the rate of depreciation must be equal to a, so we set δ equal to .088.

With these parameter values in hand we can study the model's implications for the behavior of real interest rate. Table 6C summarizes the transitional dynamics of a version of the baseline model in which we introduced the form

of adjustment costs described above. This Table confirms what expression (II.9) led us to expect. While the introduction of adjustment costs moderates the implications of the model for r_o, it does so by simultaneously generating implausibly high values for Tobin's q. The average value of q in the first five years of the simulation is 13.4. This value is well outside the range of values for q estimated in the investment literature (the highest value of q reported by Summers for the period 1933-1978 is barely above 2).

The conclusion that low values of r_o can only be obtained by postulating empirically unacceptable adjustment costs is independent of the connection between adjustment costs and Tobin's q which we used to organize our discussion. To demonstrate the implausibility of the adjustment costs that underlie the first line of Table 6C it is sufficient to cite the fact that they imply that—at time zero—the marginal adjustment costs associated with increasing installed capital by one unit are equal to 20.3 units of output.

II. 7. Implications for open economies

The numerical results reported so far have been interpreted using the neoclassical model as a model of a closed economy or alternatively of the world as a whole. Taken together, the versions of the model considered involved implausibly high real interest rates for the beginning of this century. Alternatively, one might view the neoclassical model as predicting how the real interest rates should be related to the level of development in the absence of international capital markets. Under this interpretation $r_o - r_T$ becomes the differential between the rate of return to capital in developed and underdeveloped countries predicted by the model. This differential is so large that it is hard to believe that investment flows

from rich to poor countries would not take place, even taking into account such factors as political risk, transaction costs, etc.

In fact, in the standard open economy neoclassical view, capital flows would instantaneously equalize the rate of return in all countries so the process of adjustment would be instantaneous. Again, one might think that introducing adjustment costs would eliminate this unrealistic implication by creating a wedge between the marginal product of capital and the real rate of return to capital. In other words, making the cost of investment increasing in the rate of expansion of the capital stock might potentially smooth out the flow of investment from rich to poor countries so that the transition period might be very long. Table 7 summarizes the transition path of an economy with adjustment costs identical to the one that underlies Table 6C but that can borrow and lend in the international capital market at the rate of 6.5% per year. This Table shows that, even with adjustment costs that imply values for Tobin's q greater than 20, the model still predicts a fast process of convergence—the average growth rate of output in the first five years is 18% per annum. This leads us to conclude that it is not possible to attribute an important role to transitional dynamics in accounting for the expansion of percapita income observed in the last century. On the basis of the neoclassical model, we cannot reconcile the presence of (possibly imperfect) international capital markets, with the absence of a very rapid process of cross-country convergence.

II.8. The Bottom Line

While we studied the most commonly used versions of the neoclassical growth model, there are (of course) many other variations we could have considered.[10] However, we believe that the inconsistency of the neoclassical transitional dynamics with Kaldor's stylized facts is a basic feature of this class of environments.

Thus, the neoclassical models discussed to this point are deficient as an explanation of economic growth. Steady state growth is compatible with the stylized facts presented by Kaldor but the long run growth rate is determined exogenously. In order to explain a given pattern of diversity in rates of growth as the outcome of steady state dynamics, one would have to assume that countries that grow faster have higher rates of exogenous "technical progress," i.e. we have in effect assume a priori the pattern of diversity we are trying to explain.

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[10]One commonly used extension of the standard neoclassical model which we have not considered in the main text makes the supply of labor endogenous. This model leads, in general, to higher values of r_0 than the baseline model since labor supply tends to move inversely with the capital stock (see King, Plosser and Rebelo (1988)).

III. Public Policy and Endogenous Economic Growth

In the neoclassical model reviewed in the previous section, there were two sources of economic growth, transitional dynamics and exogenous increases in productivity. We argued that the empirical importance of transitional dynamics is limited. The presence of sharply diminishing returns to capital leads to counterfactual implications about real interest rates during transition periods. Hence, variations in the growth rate have to be mainly a result of increases in productivity that are exogenous to the neoclassical model. For the neoclassical model to possess relative constancy of growth and interest rates—Kaldor's stylized facts—it must be the case that exogenous productivity growth are expressible in labor augmenting form.

Typically, we refer to this exogenous component as "technical progress", but in fact the residual factor summarizes a broad range of activities responsible for secular growth in productivity. These activities include formal education, on-the-job training, basic scientific research, product development, innovation in systems and management, etc. The natural starting point for quantitative model building is to assume that the result of these activities can be summarized by a composite capital good. Thus, one of the societal decisions is to determine the rate at which it increases its stock of labor in efficiency units, by varying the composite capital good that augments its workers' time.[11]

Analysis of human capital investment following Schultz (1961) and Becker (1964) similarly investigates the accumulation of capital that augments the

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[11]An important assumption in this approach is that changes in productivity summarized by the evolution of the composite capital good are embodied in the representative worker. See Romer (1986, 1988b) for analyses that do not rely on this embodiment assumption.

productivity of time, so we make use of this approach in developing our analysis of the effects of public policy on economic growth below. However, our sense is that our analysis will carry over to the more comprehensive concept of technical progress discussed above.

In our analysis of the neoclassical model, we studied transitional dynamics for two alternative economies. First, we considered a closed economy in which there was general equilibrium determination of output growth and the interest rate. Second, we studied output growth for a small open economy, taking the real interest rate as given from the rest of the world. We follow that procedure in the present investigation of endogenous economic growth.

Our interest is in models of endogenous growth that are compatible with Kaldor's stylized facts. For this reason, we focus on economies in which the real interest rate is constant along steady state paths. This requires that the production of both physical and human capital goods take place according to constant returns to scale technology. We also carry along the assumptions on preferences that lead individuals to choose a constant growth of consumption when faced with a constant interest rate.

Discipline on aggregate model building requires that we be explicit about microeconomic underpinnings so that key parameters can potentially be estimated from other than aggregate relations. For this reason, we begin in section III.1 by developing a variant of the dynamic microeconomic model of human capital accumulation, drawing on prior work by Becker (1964), Ghez and Becker (1975), Heckman (1976) and Rosen (1976). Motivated by our empirical section, we look for specifications of preferences and technologies that make steady state growth an outcome when there is a constant wage rate (in

efficiency units), a constant price of investment in human capital, and a constant real interest rate. When human capital occurs in an environment with borrowing and lending (or physical capital accumulation), there is the standard potential for knife-edge investment decisions, i.e., either specialization or indeterminacy of portfolio fractions. To break this result, we require that human capital accumulation be subject to diminishing returns at each point-in-time, while maintaining the feasibility of sustained growth. As in Rosen (1976), this is accomplished by specification of a suitable technology for transforming investment in human capital into changes in the stock of human capital. Then, in section III.2, we consider a "small open economy" reinterpretation of this consumer choice problem. As in Baxter's (1988) analysis of the Heckscher-Ohlin model with international borrowing and lending, it turns out that the world interest rate, national taxes and production structures dictate the wage rate and price of investment in human capital independent of the rate of human capital investment. This analysis moves us part way to a full general equilibrium but retains an exogenous interest rate. Section III.3 then indicates that a full general equilibrium can be obtained by imposing the Fisherian requirement that the interest rate equal the marginal rate of substitution in consumption, with consumption growing at the rate of human capital accumulation.

The model economy that we construct in these three sections is of the class studied previously by Uzawa (1965), Lucas (1988b) and Rebelo (1987) in that it highlights the societal allocation of resources between current consumption and a comprehensive accumulation (physical and human capital) under constant returns to scale. However, because we want to understand (i) the decentralization of accumulation decisions and (ii) growth in an open economy with traded physical capital and nontraded human capital, we require

that the rate of human capital investment is subject to diminishing point-in-time returns as in Rosen (1976).

III. 1. Individual Human Capital Accumulation

Our analysis of individual human capital accumulation follows the now standard microeconomic treatment, presenting a discrete time version of the analyses of Heckman (1976) and Rosen (1976).

Costs and Benefits of Human Capital Accumulation

The basic treatment of human capital as developed by Schultz (1961) and Becker (1964) starts by analyzing the rate of return to its accumulation. If human capital augments time then the value of an additional unit of human capital just for next period will be

$$w_{t+1}N_{t+1} + (1-\delta_H)\, p_{IH,t+1},$$

where w_{t+1} is the wage rate per efficiency unit of labor; N_{t+1} is the number of units of time, which represents the utilization base for human capital; and $(1-\delta_H)\, p_{IH,t+1}$ is the value of reducing next period's investments in human capital to keep the subsequent path unchanged, involving the standard components of depreciation $(1-\delta_H)$ and the price of the investment good $p_{IH,t+1}$. The gross rate of return on a date t investment in human capital—which costs $p_{IH,t}$—is thus

$$[w_{t+1}N_{t+1} + (1-\delta_H)\, p_{IH,t+1}]/[p_{IH,t}].$$

If the wage and investment good price is independent of the scale of human capital investment, then—in a perfect capital market—there are three possibilities: (a) no investment in human capital (specialization in other investments); (b) indifference between quantities invested in human capital and other assets; and (c) specialization in human capital investment. Letting the interest rate between t and t+1 be r_t, these situations occur depending on the relative magnitudes of rates of return,

$$1+ r_t \gtreqless [w_{t+1}N_{t+1} + (1-\delta_H)\, p_{IH,t+1}]/[p_{IH,t}].$$

To avoid these extreme implications in the analysis below, we will study a formulation of the human capital process that blends the investment "adjustment cost" technology of Rosen (1976) with the investment production function of Heckman (1976). At each point-in-time human capital accumulation will occur according to

$$H_{t+1} - H_t = \Theta(I_{Ht}/H_t)H_t - \delta_H H_t, \qquad \text{(III.1)}$$

where I_{Ht} is human capital investment and where the Θ function reflects diminishing point-in-time capacity to grow, as in Rosen (1976), i.e., $D\Theta > 0$ and $D^2\Theta < 0$. This structure permits steady state growth if I_{Ht} and H_t grow at the same rate. Further, it is consistent with the view that investment in human capital combines time and other inputs according to a production function as in Heckman (1976), in ways that we detail in section III.2 below.

We start by studying the individual's decision problem when there is a given wage rate and price of investing in human capital. These are assumed constant over time, as they will be in a steady state.

Optimal Growth of Consumption and Human Capital

The individual under study seeks to maximize lifetime utility subject to an intertemporal budget constraint with the interest factor $R=(1+r)$. That is, the formal problem is:

$$\max \sum_{t=0}^{\infty} (\beta)^t u(C_t).$$

subject to
$$\sum_{t=0}^{\infty} (\tfrac{1}{R})^t C_t \leq B_0 + \sum_{t=0}^{\infty} (\tfrac{1}{R})^t [wNH_t - p_{IH} I_{Ht}]$$

and
$$[H_{t+1} - H_t]/H_t = \Theta(I_{Ht}/H_t) - \delta_H,$$

where B_0 represents initial financial assets. As in section II above, we restrict momentary utility to constant elasticity form so that consumption will grow at a constant rate when the interest rate is constant ($u(C_t) = [C_t^{1-\sigma} - 1]/[1-\sigma]$).

To examine the determination of the optimal growth rate, consider increasing investment in human capital by a small amount (dI_{Ht}). This increases human capital at t+1 by $dH_{t+1} = D\Theta_t \, dI_{Ht}$, where to simplify the notation in this expression and those that follow we define $D\Theta_t = D\Theta(I_{Ht}/H_t)$ and $\Theta_t = \Theta(I_{Ht}/H_t)$. In order for human capital to remain unchanged in periods t+2 and later, there must be a change in investment at date t+1. The amount of this change is given by:

$$dI_{H,t+1} = \{[(I_{H,t+1}/H_{t+1})D\Theta_{t+1} - \Theta_{t+1} - (1-\delta_H)]/D\Theta_{t+1}\} \, dH_{t+1}.$$

he absence of diminishing returns to investment ($\Theta = I_H/H$), so that tion C_t^*

$_{t+1} = -(1-\delta_H)\, dH_{t+1}$. But with diminishing returns to investment, an ɜn,

red saving on future investment occurs, which incorporates changes in ɛe of

ginal adjustment costs." As previously, the marginal return associated at

the change in investment is expressible as ɜl

$$[w\, N\, dH_{t+1} - p_{IH}\, dI_{H,t+1}]/p_{IH}.$$

$\Theta = I_H/H$, this collapses to $[w\, N + p_{IH}\,(1-\delta_H)]/p_{IH}$, the rate of return

he previous section. this

The efficient rate of investment (I_H/H) is obtained by equating this

s rate of return to $R = 1+r$. Equivalently, using ɜd part

$H_t = \Theta^{-1}[(H_{t+1}/H_t) - (1-\delta_H)]$, we can solve for the efficient rate of n that

n capital growth (H_{t+1}/H_t) from this requirement. The optimal rate of (1988)

n capital growth thus depends on (i) the relative price (w/p_{IH}); (ii) the n

discount factor R; (iii) the depreciation rate of human capital δ_H; and uman

the parameters of the Θ function. dd a

There is a separation of consumption and production decisions in this

l, so that preferences do not influence the rate of human capital ɜ

ation. Consumption growth occurs at the familiar rate of

$_1/C_t) = [R\beta]^{1/\sigma}$, reflecting the relative magnitude of the force of

rest and time preference along with the intertemporal elasticity of

titution ($1/\sigma$). I.3)

Finally, suppose that we follow Heckman (1976) in modifying the

umer's preferences to the form $u(C_t^*)$ with C_t^* being a composite good that labor

only

income

at wage rate A_1). The second produces an intermediate product that represents gross investment in human capital accumulation,

$$I_{Ht} = F_2(K_{2t}, N_{2t}H_t), \tag{III.4}$$

where K_{2t} is the amount of capital allocated to investment in human capital and $N_{2t}H_t$ is the amount of labor in efficiency units allocated to this activity.[12]

The constraint on allocation of time is $N_{1t} + N_{2t} \leq 1$, where the endowment of time is normalized to one.

Capital in each of these activities evolves according to standard accumulation technologies, possibly with differential rates of depreciation. That is, the capital stocks accumulate according to $K_{1,t+1} - K_{1,t} = I_{K1t} - \delta_{K1} K_{1t}$ and $K_{2,t+1} - K_{2,t} = I_{K2t} - \delta_{K2} K_{2t}$, where $0 \leq \delta_{K1} \leq 1$ and $0 \leq \delta_{K2} \leq 1$. Similarly, as in the preceding discussion, human capital evolves according to $H_{t+1} - H_t = \Theta(I_{Ht}/H_t)H_t - \delta_H H_t$.

With borrowing and lending, financial assets evolve according to $B_{t+1} = [1 + r(1-\tau_r)]B_t + b_t$, where τ_r is the tax rate on income on financial assets and b_t is the flow of acquisitions. These acquisitions obey

$$b_t = [(1-\tau_Y)Y_t - (1+\tau_C)C_t - (1+\tau_{I1})I_{K1,t} - (1+\tau_{I2})I_{K2,t} + T_t],$$

where τ_Y is the tax on income; τ_C, τ_{I1}, and τ_{I2} are taxes on alternative uses of incomes; and T_t is transfer payments.

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[12]Uzawa (1965) and Lucas (1988) study a special case with only human capital used to produce human capital: some implications of this assumption are discussed in Rebelo (1987) and section III.3 below.

Maximizing Wealth with Given Growth

It is useful to begin by considering the small country's static competitive equilibrium or, equivalently, an individual decision maker's efficient production decisions with a given path of human capital accumulation (i.e., a given sequence of investments $\{I_{Ht}\}_{t=0}^{\infty}$). Maximizing wealth requires that one solve the following income maximization problem for each date $t > 0$.

$$\max\ [(1-\tau_Y)\ F_1(K_{1t},N_{1t}H_t) + T_t - q_1K_{1t} - q_2K_{2t}]$$

$$\text{subject to} \qquad F_2(K_{2T},\ N_{2t}H_t) = I_{Ht}$$

$$\text{and} \qquad N_{1t} + N_{2t} \leq N,$$

where the maximization is undertaken with respect to N_{1T}, N_{2t}, K_{1T}, K_{2t}. The rental prices of capital are $q_1 = [r(1-\tau_r) + \delta_{K1}]\ (1 + \tau_{I1})$ and $q_2 = [r(1-\tau_r) + \delta_{K2})](1 + \tau_{I2})$.

The structure of this "two sector" problem can be best understood as by following the line of argument developed in Baxter's (1988) analysis of the two-sector neoclassical (exogenous growth) model. First, for any positive amount of labor input allocated to production of commodities, maximizing income with respect to K_{1t} requires that its value marginal product equal its rental price, i.e., $(1 - \tau_Y)D_1F_1(K_{1t},\ N_{1t}H_t) = q_1$. This efficiency condition implies an optimal capital labor ratio, call it ρ_1, which is a function of $q_1/(1-\tau_Y)$ with $D\rho_1 < 0$. That is,

$$K_{1t}/(N_{1t}H_t) = \rho_1[q_1/(1-\tau_Y)].$$

In turn, this equilibrium input ratio implies a commodity wage rate

$$w = (1-\tau_Y)D_2F_1(K_{1t}, N_{1t}H_t) = (1-\tau_Y)D_2F_1(\rho_1(\frac{q_1}{(1-\tau_Y)}), 1),$$

using the zero degree homogeneity of marginal products.

Thus, the marginal activity of producing for the market sets a key relative price that partly determines the optimal mix of factors in the growth activity. From standard arguments about production in constant returns-to-scale industries, we know that we can express the factor demands as

$$N_{2t}H_t = \nu_N(w/q_2)I_{Ht}$$

$$K_{2t} = \nu_K(w/q_2)I_{Ht}$$

and derive an expression for the implicit unit price of investment in human capital production

$$p_{IH} = q_2\ \nu_K + w\ \nu_N.$$

Clearly, from a public policy perspective, the tax rates on capital and labor inputs will influence the price of investing in human capital. This channel is one that was omitted in the earlier analysis of section III.1.

III. 3 General Equilibrium Determination of Interest and Growth Rates

In a closed economy general equilibrium, the interest rate will be tied to the growth rate according to the Fisherian requirement that the gross rate of return equal the marginal rate of substitution in consumption. In particular, given the specification of preferences consistent with steady state growth, it must be the case that

$$1 = R\beta[Du(C_{t+1})/Du(C_t)] = R\beta(\gamma_H)^{-\sigma},$$

since consumption and human capital will be growing at the same rate. Thus, increasing the growth rate will raise the interest rate, inducing additional affects on the rental prices not present in the previous section.

In this general equilibrium, the specification of preferences will influence the response to tax policies, in contrast to the results in section III.1 and III.2, where the choice of the efficient rate of accumulation of human capital was dictated solely by productive efficiency.

As in our section II discussion of the neoclassical model, we interpret this general equilibrium in two alternative ways: as an autarkik equilibrium and as a world general equilibrium of countries that are identical in terms of preferences, production possibilities and policies.

III. 4 Positive Implications of Taxation for Economic Growth

In the Summers and Heston (1984) panel of country experiences summarized in Table 1 and Figure 1, countries grew at widely different rates. In section II of the paper, we argued that the transitional dynamics of the neoclassical model could not account for this disparity of growth experiences without producing counterfactual implications for interest rates, asset

prices or capital flows. By extension, our argument implied that public tax and expenditure policies could not not account for the cross country disparity in observed growth rates, since such policies influence growth in the neoclassical model by setting in motion a transition from one steady state path to another.

In this section, we focus on cross country differences in tax policies as a potential explanation of the disparity of observed growth rates. Many other policies that affect the incentives for economic growth—including government provision of education, and property rights—operate via similar channels to those that we describe. The focus is entirely on the influence of public policies on the steady state growth rate, rather than on transitional dynamics. As we have seen, transitional dynamics in the neoclassical model are short lived and, in endogenous growth models, they are generally more limited in duration because agents have additional margins of choice. Further, we interpret the stylized facts of economic growth as indicating that one *must* explain sustained cross country differences in growth rates. (Since we are using technology and preferences consistent with constant interest rates along such paths, we will focus on the model's implications for the steady state growth rate rather than its implications for interest rates in the early stages of development.)

Method

Our objective in this section is to explore the effects of policies on the steady state growth rates in models that account for these general equilibrium effects on the real rate of return. For this purpose, as in the previous sections, we need to specify aspects of the investment technologies (parameters of the Θ function, δ_H, δ_{K1}, δ_{K2}); production technologies

(parameters of the functions F_1 and F_2); and tax structure. Further, we need to specify aspects of preferences—β and σ—since these influence the equilibrium interest and growth rates. Our procedure is as follows: we choose a value of the intertemporal elasticity of substitution in consumption ($1/\sigma$) and a baseline value for the interest rate and growth rate. Then, from above, we can compute the discount factor β. Fixing this discount factor, we then explore how steady state interest rates and growth rates vary as the tax structure is altered. (The details of this procedure are reported in Appendix B).

Throughout our analysis, we concentrate on the case where the production functions F_1 and F_2 are Cobb-Douglas, with $F_i = A_i K^{(1-\alpha_i)} (NH)^{\alpha_i}$. We uniformly assume that the share of labor (α_1) in sector 1 is 2/3, so that our results are compatible with those for the neoclassical models explored in section II above. Further, as in section II, we normalize the constant term in the sector 1 production function to unity ($A_1 = 1$). Our benchmark preference case is to assume that the intertemporal elasticity of substitution is unity ($1/\sigma = 1$). Throughout, we use the steady state real interest rate of .065 ($R = 1.065$) and the depreciation rate of $\delta_K = .10$. We employ a parameterization of the human capital accumulation technology which assumes that there are locally no adjustment costs at zero gross investment so that $D\Theta(I_H/H) = 1$ at $I_{Ht} = 0$. Further, we use a fairly linear form of this technology, setting the parameter θ equal to .8 in the functional form $\Theta(I_H/H) = [I/H + \theta^{\frac{1}{1-\theta}}]^{\theta} - \theta^{\frac{1}{1-\theta}}$. See Appendix A for a graph of this function and discussion of its derivation. Our intent is to keep the individual choice problem well defined, without letting specification of the Θ function dominate the results.

Influences of Production Structure

The first set of calibration experiments reported in Table 8 indicates the influence of labor's share (α_2) in sector 2, the sector producing the human capital investment good. Our motivation for exploring this parameter's influence is based on practical and theoretical considerations: (i) it is a parameter about which there is considerable uncertainty, since it is unclear exactly what sectoral output is being measured; and (ii) Uzawa (1965) and Lucas (1988b) assume that only labor is necessary to produce human capital ($\alpha_2 = 1$), which Rebelo (1987) shows theoretically restricts certain policies to have no effect on the steady state growth rate.

The calibration activity undertaken in Table 8 indicates that the productivity parameter A_2 adjusts in response to changes in α_2. Roughly, the chain of influence is as follows, using the results of our analyses in sections III.1 and III.2. First, the specified values of sector 1 production pin down the required value of the capital-labor ratio in sector 1, since $D_1F_i = A_i[K/(NH)]^{-\alpha_i} = (R - (1-\delta_K)) = .165$. In turn, this implies a steady state real wage rate (in units of sector 1 output), w. Second, specification of the growth rate of the economy (γ_H) and the adjustment technology (Θ) implies a level of marginal adjustment costs. Third, from our analysis of the individual's problem, we know that the (constant) growth of human capital depends on marginal adjustment costs, on the real interest rate, on the real wage rate (per unit of human capital) and on the price of investing in human capital (p_{IH}). But the first three of these have already been determined, so we know the relative price p_{IH} without reference to the specification of the sector 2 technology parameters (and, more generally, without reference to its parametric form). Fourth, from our analysis of the small open economy problem, we know that the factor intensity in sector 2 is

pinned down from the sector 1 wage rate (w) and the rental price of capital ($R - (1-\delta_H)$), without reference to the "neutral" technical parameter A_2. Thus, as we vary α_2—which influences the efficient factor intensity—A_2 must change to keep the unit cost of production equal to the price p_{IH}.

These considerations are reflected in the columns of Table 8, panels A and B. In addition to the influence of α_2 on A_2 stressed above, we can see that there is an invariance of the wage rate w and the price of human capital investment good across the values of α_2. Further, as α_2 increases, the quantity of capital employed in sector 2 declines, which lowers the two investment share measures. These measures differ in their denominators: s_i scales investment relative to sector 1 output (y_1) and s_i^* scales by the more comprehensive measure ($y_1+p_{IH}y_2$). Notably, the former investment share measure is high relative to the U.S. experience, for which Summers and Heston (1984) report an average of 24% over 1950-1981. Throughout, taxes on sectoral outputs are held constant. Finally, the fact that Table 8 reflects a calibration exercise to match growth (γ_H) and interest rates (R) means that the last two columns are constant as well.

Comparing the two panels, isolates the impact of changing the rate of depreciation on human capital. Panel A treats the case of equal depreciation rates for physical and human capital ($\delta_k = \delta_H = .10$); Panel B uses Mincer's (1974) estimate of 1.28% per annum. Varying the depreciation rate in this way lowers the marginal cost of investing in human capital $D\Theta(\gamma_H - (1-\delta_H))$ used in the calibration, so that the steady state relative price of human capital must rise, following the arguments above. Lowering the depreciation rate also lowers the scale of production in sector 2 (less replacement investment) and this scaling back of capital in sector 2 is reflected in

lower physical capital investment shares (s_i and s_i^*) in panel B relative to panel A.

Implications of Taxation

The three panels of Table 9 indicates the potential magnitude of policy influence on economic growth in this general equilibrium setting. Although the panels reflect different assumptions about preferences and technology, there is a common method employed in the construction of each. First, the system is calibrated under the assumption that there are no taxes to match growth and interest rate values ($R = 1.065$ and $\gamma_H = 1.02$) for specified parameter values. Then, the tax parameters are altered, resulting in new values of growth and interest rates; of investment shares; and the relative prices (the price of investment in human capital and the wage rate).

The tax parameters are output taxes on the two sectoral outputs with the assumption being that the proceeds are rebated as lump sum transfers. We chose this method of taxation for its simplicity, although taxation of income from factor supply would also be of interest and would be perhaps more realistic. Variations are from a subsidy of −10 % through a tax of 50%. Some of the experiments considered in Table 9 involve only taxation of the commodity output sector (sector 1). A literal interpretation of the output of sector 2 as new human capital would lead us to view this sector as untaxed. However, we study as well the effects of changes in τ_2 which can be interpreted more broadly as summarizing aspects of the education system, patent policy, etc. Throughout the panels, increases in taxation lower the growth rate and this decline is accompanied by a reduction in the real interest rate.

Panel A provides results for a benchmark case, in which there is essentially identical technology in sector 1 and 2 (i.e., we assume that $\alpha_1 = \alpha_2 = 2/3$ so that the technologies are the same up to the constant A_2). The influence of taxation on the growth rate can be dramatic When there are uniform tax rates of 50%, the economy exhibits growth of -6% per year. When the tax is restricted to sector 1, the growth rate is -2%. The decline in growth is reflected in a decline in the rate of investment according to either investment share measure (s_i and s_i^*), as seen in Figure 4. Since the results of this benchmark experiment suggest a fairly strong role for the incentive effects of public policy, the next two panels indicate perturbations in preferences and technology that mitigate the effects of policy.

When there is a relatively high value of labor's share (human capital's share) in the production of human capital as in Panel B, then these results change in the direction suggested by the prior analyses of Lucas (1988b) and Rebelo (1987). Using a value of $\alpha_2 = .9$, Panel B shows that taxation of only sector 1 output at a 50 % rate leads to a growth rate of 1% when there is a 2% growth rate in the untaxed economy. This indicates that isolation of aspects of the production structure will be central to increasing the precision of the incentive effects of policy on growth. However, there remain substantial welfare implications of variations in growth rates of the magnitude obtained under this extreme value for α_2, as indicated by the analysis of section IV below.

Panel C indicates that preferences also play a role in the determination of the response of the growth rate to the incentive effects of policy. When there is smaller intertemporal substitution in consumption (σ larger), then there is a smaller magnitude influence of taxation on growth, although this

influence mains negative. For an individual consumer, the value of σ controls the response of the growth rate of consumption to a change in the interest rate, according to the approximate formula, $\Delta \gamma_c = (1/\sigma) \Delta R$. Thus, the value of σ used in panel C implies that a change in the rate of return of 100 basis points results in a change in the growth rate of consumption of only .2% per annum. This reduction in substitution implies that the introduction of a 50% tax in both sectors leads to a decline in the growth rate of 1%, in contrast to the 8% decline shown in panels A and B.

Summary of Closed Economy Results

The results of Tables 8 and 9 indicate the potential explanatory power of the class of endogenous growth models developed—following the path of Uzawa (1965)—in recent work by Jones and Manuelli (1988), Lucas (1988b), and Rebelo (1987). In these models, the influence of policy on growth is governed by preferences—notably the intertemporal rate of substitution—and production possibilities. Hence, further research must focus on refining these specifications, particularly the latter.

Open Economy Experiments

With borrowing and lending possible at constant interest rates, as for a small open economy, policy in a single country may occasion changes in the desirable patterns of borrowing to support profiles of physical and capital accumulation.

In open economy versions of the environment discussed in the previous section, there are dramatic implications of borrowing and lending for growth and capital accumulation, as illustrated by Table 10. To maximize compatibility with the preceding analysis, we start the open economy in a

setting in which it would not trade in the absence of policy, calibrating it so that its domestic interest rate and growth rate are the same as the world's, i.e., R = 1.065 and $\gamma_H = 1.02$. In all other ways, the economy matches panel A of Table 9 ($\sigma = 1$ and $\alpha_1 = \alpha_2 = 2/3$). Then, we tax the economy's output of sector 1 or of both sectors 1 and 2, permitting it unrestricted borrowing and lending at the given world interest rate. A few percentage points of taxation are typically sufficient to shut down the accumulation of domestic human and physical capital reducing the growth of the economy to its minimum feasible level ($\gamma_H = 1-\delta_H = .988$). Although few countries with substantial government intervention actually permit unrestricted access to international capital markets, these examples do indicate the potential power of these environments for explaining the "development trap," i.e., economies that appear to be stuck with low levels of national income and slow growth.

IV. Welfare Implications of Taxation

In this section we evaluate the welfare consequences of alternate policies within representative models drawn from the prior two sections. Our objective is to illustrate the general principle that there are larger welfare effects in endogenous growth models.

Method

The method that we employ is based on Lucas (1988a) and is given as follows. Suppose that there is a baseline consumption growth path given by $C_t = C_0\gamma^t$, where γ is the gross growth rate and C_0 is the period zero level of consumption. Now, consider an arbitrary alternative path $\{C_t'\}$. A utility equivalent path with growth rate γ, involves the constant $\tilde{C}_0$ that satisfies

$$U(\{\tilde{C}_0\gamma^t\}) = U(\{C_t'\}). \tag{IV.1}$$

Then, a comparison of the baseline path and the alternative path $\{C_t'\}$ can be reduced to a single number, the ratio $\tilde{C}_0/C_0$. This ratio provides the answer to the question, "at what fraction of initial consumption—growing at rate γ—would the individual just be indifferent between a constant growth path and the alternative path."

IV.1 A Linear Technology Example

The following simple economy provides an opportunity to illustrate our methodology for evaluating welfare implications of taxation, as well as providing a convenient benchmark for the more complex models considered below. Let production at date t depend only on the stock of capital,

$Y_t = A\,K_t$, and let capital evolve according to $K_{t+1} = (1-\delta_K)\,K_t + I_t$. The constraint on an individual's sources and uses of funds is just

$$C_t + I_t \leq (1-\tau)Y_t + T_t,$$

where τ is the tax rate and T is a lump sum transfer payment. In this simple environment, the after tax gross rate of return—which we call $R(\tau)$—is determined solely by technology and tax rates, $R(\tau)=(1-\tau)A + 1-\delta_K$. With a constant elasticity of intertemporal substitution utility function, this specification then implies that the growth rate of consumption is

$$\gamma(\tau) = C_{t+1}/C_t = [\beta R(\tau)]^{(1/\sigma)},$$

where β is the subjective discount factor. Under the assumption that the government rebates the proceeds of commodity taxation, it follows that that $T_t = \tau Y_t$, so that equilibrium accumulation is just $K_{t+1} = (1-\delta_K)\,K_t + AK_t - C_t = R(0)\,K_t - C_t$. This accumulation equation then implies that the initial consumption level satisfies $C_0 = [R(0)-\gamma(\tau)]K_0$, using basic present value conditions. Thus, the equations for $\gamma(\tau)$ and C_0 determine the time profile of consumption for any specification of the tax rate. Notably, this model implies that a permanent change in the tax rate implies an immediate shift in the level of the consumption path and a change in its rate of growth—there are no transitional dynamics.

In this context, then, the method works as follows. First, fix a baseline value of the tax rate τ and compute $\gamma(\tau)$. Second, choose a new value of the tax rate τ', and compute $\gamma(\tau')$ and $C_0(\tau')$. Third, find the constant $\bar{C}_0$ that provides utility equivalent to $U(\{C_0(\tau')\gamma(\tau')^t\})$ when

consumption is growing at the baseline rate $\gamma(\tau)$. Fourth, compute the ratio $\tilde{C}_0/C_0(\tau)$, as a measure of the cost or benefit of the tax variation.[13]

Table 11 provides some experiments with this simple model that provide background to interpreting our subsequent results. In this table, as in our subsequent analysis, we vary $1/\sigma$—the intertemporal elasticity of substitution—and tax wedges. In reading the table, one should recognize that for each value of σ, we chose β so that the economy without taxes displays the same growth rate of consumption (using $\gamma = [\beta R]^{(1/\sigma)}$).

Looking first at the effects of taxation on steady state growth rates (moving across one of the rows in panel A), we find that taxation reduces the growth rate. Looking down the columns, we find that this effect is more pronounced when individuals are more willing to substitute over time (i.e., when σ is small). Given the society's resource constraint ($C_0 = [R(0) - \gamma(\tau)]K_0$), lower growth is associated with a higher initial level of consumption. In panel C, we find that these effects are potentially large. For example, with $\sigma = 1$, there is a 35% increase in initial consumption—relative to the path with no taxes—when τ is increased from 0 to 10%. With relatively high rates of intertemporal substitution, zero gross investment is optimal so that the economy's growth rate is $-\delta = -.10$.

With lower growth due to taxation, welfare unambiguously declines in this economy. Panel B of Table 11 shows the percentage decline in the utility equivalent consumption path—relative to the path with no taxes—when a tax is imposed (this corresponds to the percentage decline in $\tilde{C}_0/C_0$ in the

[13]As Ken Judd has pointed out to us, this measure of the welfare effects of taxation would not be appropriate if we were trying to address normative questions, such as the design of an optimal taxation system. In that case, the adequate measure of the welfare effects of taxation would be the marginal deadweight loss—see Judd (1987).

terminology above). Generally, the welfare cost is higher when there is either more taxation (τ high) or greater substitution in consumption (σ low).

Under some restrictions on technology, the outcomes of our linear technology example will be identical to those of the two sector economy constructed in section III if both sectors are taxed at the same rate. Sufficient technology restrictions are (i) no point in time diminishing returns in human capital (Θ linear); and (ii) sectoral production functions that are Cobb-Douglas with identical shares (but with possibly different multiplicative constants). Thus, the linear technology example provides a benchmark case for the two sector endogenous growth economy.

IV.2 Taxation in the Neoclassical Model

The neoclassical model has been a workhorse of modern public finance, so that the influence of taxation on real activity within this environment is relatively well understood. The imposition of an output tax, for example, occasions a shift in the level of the steady state path—but not its slope—and sets in motion the transitional dynamics discussed in section II.

Table 12 provides a welfare analysis of an increase in the output/income tax rate from .20 to .30 under some alternative assumptions about intertemporal substitution in preferences. The rate of time preference (β) is calibrated so that the steady state real interest rate is 6.5% for each value of σ, so that varying σ has no influence on the steady state path. The remaining values of the model's parameters are those of the baseline economy constructed in section II ($A = 1$; $\delta_K = .10$; $\alpha = 2/3$; $\gamma_X = 1.02$; and $N = .2$).

In the steady state, capital falls by 18.2% and consumption fall by 3.6% due to the increase in the tax rate from 20% to 30%. As Judd (1987) and

Jorgenson and Yun (1988) have stressed, it is inappropriate to evaluate tax policies solely on the basis of these long run effects. This is highlighted in our analysis by the fact that the welfare costs in the neoclassical model would be independent of preferences if the steady state comparison were utilized.

Table 12 presents the results of the policy of raising the income tax rate from 20% to 30% on the welfare measure described above. For example, when $\sigma = 1$, the tax increase lowers the welfare measure by 1.6%. That is, the tax rate increase is equivalent—in utility terms—to an immediate 1.6% shift down in the steady state consumption path. When the intertemporal elasticity of substitution in consumption is increased (σ reduced), then there is an increase in the welfare cost of the tax increase—when $\sigma = 1/10$, it is 1.84%. Symmetrically, when σ is increased, the welfare costs falls—when $\sigma = 10$, it is .97%.

In all cases, the initial consumption level rises in response to the tax increase, so that the economy "works off" the capital stock through lower net investment and transitorily high levels of consumption. With low intertemporal substitution in consumption (high σ), there is a relatively small initial upward jump in consumption, since individuals seek to avoid temporally uneven consumption profiles. The initial upward jump in consumption is highest (roughly 20% of the initial steady state) when there is very high intertemporal substitution ($\sigma = 1/10$).

Relative to the example, the calibrated neoclassical model involves a much smaller welfare loss of taxation. For the example economy, with $\sigma = 1$, the effect of changing τ from .20 to .30 is a decline in the utility equivalent consumption path equal to 30.18% − 16.59% = 13.59%. For the neoclassical model, the effect is 1.60%. Fundamentally, this difference

reflects the fact that the long run growth rate is affected in the linear technology economy but not in the basic neoclassical model.

IV.3 Taxation in the Two-Sector Endogenous Growth Model

The two sector endogenous growth model that we explored in section III involves accumulation of physical capital and human capital, with the latter subject to a diminishing point in time investment technology. In this section, we provide an evaluation of the welfare cost of taxation within this model.

From section III, we know that the steady state rate of economic growth is generally affected by taxation in either sector. For simplicity, we wish to focus on evaluating public policies that induce an immediate shift in the steady state path in the two sector model, as in the example considered in section IV.1. In general, within endogenous growth models, changes in public policies occasion both a shift in the steady state path—both its level and growth rate—as well as setting off transitional dynamics as the economy adjusts toward an altered steady state ratio of physical to human capital. Transitional dynamics do not arise if the preferences, technologies and policy interventions are such that there is no required variation in this ratio. Thus, for the purpose of this section, we restrict the policy rules and technologies: (i) we require that both sectors are taxed at the same rate; and (ii) we require that the depreciation rates of the two capital goods are identical and that the production functions in the two sectors are the same up to a factor of proportionality. With this configuration of restrictions, we can then show that the introduction of diminishing point in time returns to human capital accumulation—necessary to a unique competitive decentralization of aggregate outcomes—does not overturn the comparison of

the endogenous growth and neoclassical models that we made in the previous sections.

Table 13 presents the results of these experiments. The economy is parameterized so that there is equal factor intensity in each sector ($\alpha_1 = \alpha_2 = 2/3$) and the depreciation rates are taken equal ($\delta_K = \delta_H = .10$). Further, as in section III, we assume modest diminishing point in time returns (parameterized by $\theta = .8$, as discussed in appendix A.) In comparison with table 11, the effect of the diminishing point in time investment technology is to mitigate the influence of taxation on growth rates. However, the influence of taxation continues to be important, both in the panel A implications for growth rates and the panel B implications for welfare.

Earlier, we saw that it was possible for taxation to move the example economy to a situation in which no gross investment was undertaken, i.e., the economy contracted at the rate $-\delta$. In the current analysis, this possibility will continue to arise but in more restricted circumstances due to the form of the human capital investment technology.

V. Conclusions

For a lengthy paper, the conclusions should be brief, so we simply list four basic lessons from the paper.

First, there is substantial cross country variation in average rates of growth of percapita income—which occurs without accompanying trends in interest rates, asset prices or factor shares—as noted by Kaldor more than 20 years ago. To us, this suggests that the growth process is potentially described by models in which there are endogenously determined differences in steady state growth rates.

Second, analysis of calibrated versions of the neoclassical model reveals that its transitional dynamics are inconsistent with the stylized facts of growth. Because of the presence of sharply diminishing returns to capital accumulation in these models, comparisons of early and later stages of development imply counterfactual trends in interest rates, asset prices or factor shares. The only way to rescue the neoclassical model is to introduce cross sectional differences in rates of exogenous "technical progress," which augments the productivity of labor, but this does not represent an *explanation* of differences in growth rates.

Third, following Uzawa (1965), Lucas (1988b) and Rebelo (1987), there are a variety of ways to make a comprehensive measure of "technical progress" endogenous in neoclassical analysis. Interpreting this comprehensive measure as social investment in "human capital" provides an important parallel to the work of Schultz (1961, 1981) on economic development. Our analysis uses this link to build some explicit microfoundations for a two sector model of endogenous economic growth. When we calibrate this growth model, we find that public policies can have quantitatively large influences on average rates of economic growth in economies that operate in isolation. Policies

display these effects—as in Schultz (1981)—because they influence private incentives for accumulation of physical and human capital. These incentive effects of taxation are reinforced in open economies that have access to international capital markets.

Fourth, since policies have the potential to influence the growth rate in models with endogenous long run growth, there is potentially a much larger quantitative influence of policies on welfare than in the neoclassical model where the growth rate is governed by the exogenous rate of technical progress. Some experiments comparing neoclassical and endogenous growth models suggest that this difference can be quantitatively important. It is relatively easy to generate situations in which countries stagnate or even regress for lengthy periods, if policies eliminate the incentives for growth that are provided by technology. This explanation of "no growth steady states" contrasts with that offered by Becker, Murphy and Tamura (1988) and Azariadis and Drazen (1988), where aspects of the technology give rise to multiple steady states so that economies with different initial conditions may converge to steady states with different rates of growth even in the absence of cross-country heterogeneity in public policy.

This paper demonstrates that the most basic class of models of endogenous economic growth—stemming from Uzawa (1965)—contains quantitatively important influences of public policy on steady state economic growth. The precise magnitudes of these incentive effects of taxation and analogous policies depends—as in any equilibrium framework—on the specification of preferences, technology and policy rules. As in other macroeconomic contexts, then, our analysis of growth models points to sets of microeconomic measurements that will prove valuable in determining the magnitudes of policy effects. Further, we find that the extent of access to international capital

markets substantially influences the impact of tax policy on economic growth. In summary, with the results of the present paper, we find new promise for the hypotheses of Schultz (1981) that incentive effects of policy can influence economic activity—taxation can readily lead to development traps and growth miracles. Models of endogenous economic growth thus provide new analytical paths for old problems in the economics of development.

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TABLE 1

GROWTH RATE OF PERCAPITA REAL GDP : 1950-1981

Country	*Mean*	*Standard Deviation*	*Number of Observations*
AFRICA			
ALGERIA	0.0243	0.0876	21
ANGOLA	-0.0129	0.1055	21
BENIN	-0.0006	0.0382	22
BOTSWANA	0.0755	0.0668	21
BURUNDI	-0.0109	0.0704	21
CAMEROON	0.0275	0.0310	21
CENTRAL AFRICAN REP.	-0.0078	0.0313	21
CHAD	-0.0249	0.0817	21
CONGO, PEOP. REP. OF	0.0239	0.0487	21
EGYPT, ARAB REP. OF	0.0311	0.0352	31
ETHIOPIA	0.0138	0.0250	31
GABON	0.0776	0.1174	21
GAMBIA, THE	0.0180	0.1602	21
GHANA	-0.0084	0.0625	26
GUINEA	-0.0008	0.0382	22
IVORY COAST	0.0271	0.0451	21
KENYA	0.0066	0.0533	31
LESOTHO	0.0500	0.0761	21
LIBERIA	0.0061	0.0509	21
MADAGASCAR	-0.0098	0.0328	21
MALAWI	0.0247	0.0439	28
MALI	-0.0079	0.0419	21
MAURITANIA	0.0112	0.0716	21
MAURITIUS	0.0112	0.0553	31
MOROCCO	0.0234	0.0520	31
MOZAMBIQUE	-0.0044	0.0617	21
NIGER	0.0199	0.0843	21
NIGERIA	0.0298	0.0736	31
RWANDA	0.0259	0.1075	21
SENEGAL	-0.0170	0.0504	21
SIERRA LEONE	0.0126	0.0458	21
SOMALIA	0.0009	0.0665	21
S. AFRICA	0.0192	0.0338	31
SUDAN	0.0130	0.0905	26
SWAZILAND	0.0541	0.0617	21
TANZANIA, UNITED REP	0.0225	0.0456	21
TOGO	0.0265	0.0545	21
TUNISIA	0.0439	0.0349	21
UGANDA	0.0034	0.0502	31

UPPER VOLTA	0.0047	0.0790	21
ZAIRE	0.0104	0.0773	31
ZAMBIA	0.0055	0.1002	26
ZIMBABWE	0.0085	0.0490	28
ASIA			
AFGHANISTAN	-0.0073	0.0519	21
BANGLADESH	0.0103	0.0590	22
BURMA	0.0238	0.0571	31
HONG KONG	0.0741	0.0431	21
INDIA	0.0141	0.0427	31
IRAN	0.0263	0.1025	26
IRAQ	0.0160	0.1186	28
ISRAEL	0.0355	0.0466	31
JAPAN	0.0653	0.0381	31
JORDAN	0.0286	0.1000	28
KOREA, REP. OF	0.0467	0.0464	28
MALAYSIA	0.0411	0.0362	26
NEPAL	0.0123	0.0407	21
PAKISTAN	0.0189	0.0365	31
PHILIPPINES	0.0275	0.0268	31
SINGAPORE	0.0666	0.0387	21
SRI LANKA	0.0089	0.0704	31
SYRIAN ARAB REP.	0.0535	0.1034	21
TAIWAN	0.0532	0.0283	31
THAILAND	0.0303	0.0433	31
EUROPE			
AUSTRIA	0.0414	0.0284	31
BELGIUM	0.0297	0.0240	31
CYPRUS	0.0411	0.1072	31
DENMARK	0.0274	0.0328	31
FINLAND	0.0361	0.0313	31
FRANCE	0.0355	0.0187	31
GERMANY, FED. REP.	0.0422	0.0341	31
GREECE	0.0464	0.0364	31
ICELAND	0.0288	0.0526	31
IRELAND	0.0266	0.0230	31
ITALY	0.0389	0.0313	31
LUXEMBOURG	0.0195	0.0389	31
MALTA	0.0566	0.0483	28
NETHERLANDS	0.0294	0.0344	31
NORWAY	0.0337	0.0199	31
PORTUGAL	0.0469	0.0507	31
SPAIN	0.0415	0.0369	31
SWEDEN	0.0257	0.0211	31
SWITZERLAND	0.0246	0.0330	31
TURKEY	0.0354	0.0451	31
UNITED KINGDOM	0.0189	0.0218	31

NORTH & CENTRAL AMERICA			
BARBADOS	0.0350	0.0520	31
CANADA	0.0247	0.0286	31
COSTA RICA	0.0271	0.0440	31
DOMINICAN REP.	0.0254	0.0564	31
EL SALVADOR	0.0086	0.0489	31
GUATEMALA	0.0168	0.0216	31
HAITI	0.0085	0.0353	21
HONDURAS	0.0121	0.0265	31
JAMAICA	0.0211	0.0514	28
MEXICO	0.0301	0.0240	31
NICARAGUA	0.0210	0.0772	31
PANAMA	0.0276	0.0293	31
TRINIDAD AND TOBAGO	0.0383	0.0339	31
UNITED STATES	0.0190	0.0270	31
SOUTH AMERICA			
ARGENTINA	0.0141	0.0529	31
BOLIVIA	0.0109	0.0415	31
BRAZIL	0.0376	0.0341	31
CHILE	0.0187	0.0451	31
COLOMBIA	0.0224	0.0337	31
ECUADOR	0.0292	0.0352	31
GUYANA	0.0035	0.0617	31
PARAGUAY	0.0241	0.0369	31
PERU	0.0201	0.0308	31
SURINAM	0.0276	0.0787	21
URUGUAY	0.0122	0.0493	31
VENEZUELA	0.0130	0.0383	31
OCEANIA			
AUSTRALIA	0.0212	0.0304	31
FIJI	0.0267	0.0594	21
INDONESIA	0.0364	0.0426	21
NEW ZEALAND	0.0137	0.0316	31
PAPUA NEW GUINEA	0.0161	0.0522	21

Source: Summers and Heston (1984).

TABLE 2

THE RATE OF GROWTH OF GNP PERCAPITA
1870 - 1979: SUMMARY STATISTICS

Countries	Time Period	Average Growth Rate	Trend	R^2
Australia	1870–1979	1.1	2.1 E-2 (1.4 E-2)	.02
Austria	1870–1912	1.5	1.2 E-2 (2.9 E-2)	.00
	1920–1979	2.9	6.0 E-2 (10.3 E-2)	.01
Belgium	1870–1913	1.0	-.2 E-2 (1.7 E-2)	.00
	1949–1979	3.3	.8 E-2 (4.3 E-2)	.00
Canada	1870–1979	2.0	1.5 E-2 (2.7 E-2)	.00
Denmark	1870–1979	1.8	1.9 E-2 (1.3 E-2)	.02
Finland	1900–1979	2.5	5.0 E-2 (3.2 E-2)	.03
France	1870–1979	1.9	(a) 2.7 E-2 (1.9 E-2)	.02
Germany	1870–1979	2.5	3.7 E-2 (3.0 E-2)	.01
Italy	1870–1979	1.6	3.9 E-2 (1.8 E-2)	.04
Japan	1886-1944	1.7	-2.3 E-2 (4.6 E-2)	.00
	1948-1979	7.2	-14.9 E-2 (6.5 E-2)	.15
Netherlands	1900–1979	1.8	4.5 E-2 (4.4 E-2)	.01
Norway	1870–1979	2.1	2.8 E-2 (1.1 E-2)	.05
Sweden	1870–1979	2.3	(b) 1.1 E-2 (1.2 E-2)	.00
Switzerland	1925-1944	.9	-.2 E-2 (1.2 E-2)	.19
	1948-1979	2.2	-6.7 E-2 (6.2 E-2)	.04
United Kingdom	1870–1979	1.3	1.4 E-2 (1.0 E-2)	.02
USA	1870–1979	1.9	(a) .4 E-2 (1.6 E-2)	.00

Source: Maddison (1982). E-2 denotes 10^{-2}. Units are percentage points. Standard errors in parenthesis. Time periods dictated by data availability. (a) corrected for MA(1) residuals; (b) corrected for AR(1) residuals.

TABLE 3

HISTORICAL EVIDENCE ON LONG-TERM INTEREST RATES

Date	England	France	Dutch Republic · Holland—	Spanish Netherlands— Belgium	Germany	Sweden	Spain	Switzerland	Italy	United States	Canada
	Minimum Rates on Best Credits by Half Centuries*										
13th century											
1st half				8.00					6 5/8		
2nd half		14.00		8.00							
14th century											
1st half				8.00					4 7/8		
2nd half				8.00					5 1/4		
15th century											
1st half		10.00		8.00			4.00		6.00		
2nd half		10.00		8.00	5.00		4.00		5.00		
16th century											
1st half	10.00	8 1/3	8 1/3	4.00	4.00				4.00		
2nd half	10.00	8 1/3	6 1/6	4.00	4.00				4.00		
17th century											
1st half	8.00	8 1/3	5.00								
2nd half	4.00	5.00	3.00								
18th century											
1st half	3.05	5.00	3.00		4.00			5.00	4.00		
2nd half	3.13	5.00	2.50		4.00			4.00	4.00		
19th century											
1st half	3.26	4.06	4.53	4.53	3.84					4.55	
2nd half	2.47	3.03	2.93	2.71	3.53	3.68				3.23	
20th century											
1st half	2.79	3.06	3.20	2.94	3.64	3.04		3.31		2.31	2.93

* Lowest decennial average where available; otherwise lowest reported rate. Lowest yield or yields are underlined for each time period. All yields in annual terms and expressed in percentage points.
Source: Homer: (1963)

TABLE 3 (Continued)

HISTORICAL EVIDENCE ON SHORT-TERM INTEREST RATES

Date	England	France	Dutch Republic Holland—	Spanish Netherlands—Belgium	Germany	Sweden	Switzerland	Italy	United States	Canada
	Minimum Rates on Best Credits by Half Centuries*									
13th century										
1st half				10.00				20.00		
2nd half		15.00		10.00				8.00		
14th century										
1st half		15.00		10.00				7.00		
2nd half		15.00		10.00				5.00		
15th century										
1st half	10.00			5.00	5.00			5.00		
2nd half	10.00			5.00	5.00			5.00		
16th century										
1st half		5.00		4.00	4.50			4.00		
2nd half		8.00		6.25	5.00			5.00		
17th century										
1st half	6.00		6.00							
2nd half	3.00[a]		1.75			6.00				
18th century										
1st half	3.00[b]	4.00	1.75			4.00				
2nd half	5.00[a]	4.00	2.00			3.00				
19th century										
1st half	3.57[b]	4.00[a]	2.87	3.71	3.90	4.00	4.11		7.99	
2nd half	2.09[b]	2.09[b]	2.04	2.29	2.97	4.17	3.35		4.51	
20th century										
1st half	0.83[b]	1.69[b]	1.26	1.83	2.18	2.80	1.33		0.87	1.90

* Lowest decennial average where available; otherwise lowest reported rate. Lowest yield or yields are underlined for each time period. All yields in annual terms and expressed in percentage points. [a] official discount rate, [b] free open market rate.
Source: Homer: (1963)

TABLE 4

ANNUAL REAL RATES OF RETURN: SUMMARY STATISTICS

US Securities, 1926 - 1981

Series	*Average Real Rate of Return*	*Trend*	*D-W*	R^2
Common Stocks	8.3	-.19 (.18)	2.00	.02
Small Stocks	14.9	-.06 (.31)	1.84	.00
Long Term Corporate Bonds*	.8	-.23 (.09)	1.94	.27
US Treasury Bills*	.01	-.00 (.008)	1.76	.40
Long Term Government Bonds	.08	-.22 (.07)	1.82	.17

Source: Ibbotson and Sinquefield (1982). Standard errors in parenthesis. D-W denotes the Durbin-Watson test. Units are percentage points.

* Corrected for first order autocorrelation.

Table 5A
Behavior of Real Interest Rates in the Neoclassical Model
Real Interest Rate 100 years ago—Cobb-Douglas Production Function

Fraction of Growth Due to Transitional Dynamics	τ_x	Baseline Model (%)	Depreciation Rate $\delta = 0$ (%)	Depreciation Rate $\delta = .25$ (%)	Labor Supply $N_0 = .36$ (%)	Capital Share $\alpha_k = .5$ (%)	Capital Share $\alpha_k = .9$ (%)	End of Period Real Interest Rate $r_T = 4\%$ (%)	End of Period Real Interest Rate $r_T = 9\%$ (%)
0%	2%	6.5	6.5	6.5	43.5	6.5	6.5	4.0	9.0
25%	1.5%	16.7	10.5	26.0	76.6	11.0	6.9	12.7	20.8
50%	1%	40.5	19.9	71.5	153.7	18.9	7.6	32.9	48.2
75%	.5%	119.4	51.0	222.0	409.1	36.2	8.5	99.8	139.0
100%	0%	798.5	318.5	1518.5	2609.2	105.5	10.5	676.0	921.0

Table 5B
Real Interest Rate (r_o) and Capital Share (α_{ko}) 100 years ago
CES Production Function with Elasticity of Substitution ρ

Fraction of Growth Due o Transitional Dynamics	τ_x	$\rho = .9$ r_0	$\rho = .9$ α_{k0}	$\rho = .5$ r_0	$\rho = .5$ α_{k0}	$\rho = 1.1$ r_0	$\rho = 1.1$ α_{k0}	$\rho = 1.25$ r_0	$\rho = 1.25$ α_{k0}
0%	2%	6.5	33.3	6.5	33.3	6.5	33.3	6.5	33.3
25%	1.5%	17.6	35.1	23.7	47.6	16.0	31.9	15.0	30.0
50%	1%	41.5	37.4	46.9	61.9	39.7	29.9	38.6	25.4
75%	.5%	106.8	40.5	76.2	76.2	136.6	26.8	180.2	18.1
100%	0%	430.7	46.3	111.6	90.5	2193.2	20.4	2993474.3	1.6

Table 6A
Transitional Dynamics, One-Sector Neoclassical Model

Parameterizations		Time Period (each period has 5 years) 1	2	3	4	5	6	7	8	9	10
1) BASELINE MODEL	Half life = 8 years										
	Average Growth of Capital	165.1	13.3	4.2	1.6	0.7	0.3	0.1	0.1	0.0	0.0
	Average Growth Rate of Output	38.4	4.2	1.4	0.5	0.2	0.1	0.0	0.0	0.0	0.0
	Average Real Interest Rate	129.2	15.7	9.4	7.6	7.0	6.7	6.6	6.6	6.5	6.5
2) LOW INTERTEMPORAL SUBSTITUTION (σ = 20)	Half life = 61 years										
	Average Growth of Capital	29.4	19.3	13.6	10.1	7.8	6.2	5.1	4.3	3.7	3.4
	Average Growth Rate of Output	9.0	6.1	4.3	3.3	2.5	2.0	1.7	1.4	1.2	1.1
	Averaage Real Interest Rate	550.5	257.3	149.2	99.0	71.5	54.8	43.8	36.1	30.4	26.0
3) STONE-GEARY UTILITY ($\bar{c}$ = .04)	Half life = 12 years										
	Average Growth Rate of Capital	104.1	35.4	9.3	3.6	1.5	0.7	0.3	0.1	0.1	0.0
	Averaage Growth Rate of Output	26.8	10.6	3.0	1.2	0.5	0.2	0.1	0.0	0.0	0.0
	Average Real Interest Rate	394.6	36.3	13.8	9.3	7.7	7.0	6.8	6.6	6.6	6.5
4) HIGH CAPITAL SHARE (α_k = .5)	Half life = 13 years										
	Average Growth Rate of Capital	60.4	16.3	7.2	3.7	2.1	1.2	0.7	0.4	0.2	0.1
	Average Growth Rate of Output	26.6	7.8	3.6	1.8	1.0	0.6	0.3	0.2	0.1	0.1
	Average Real Interest Rate	55.7	19.9	12.5	9.6	8.2	7.5	7.1	6.8	6.7	6.6
5) VERY HIGH CAPITAL SHARE (α_k = .9)	Half life = 80 years										
	Average Growth Rate of Capital	3.7	3.3	3.0	2.8	2.5	2.3	2.1	1.9	1.8	1.6
	Average Growth Rate of Output	3.3	3.0	2.7	2.5	2.3	2.1	1.9	1.7	1.6	1.5
	Average Real Interest Rate	10.3	10.0	9.7	9.4	9.1	8.9	8.7	8.5	8.3	8.2

Table 6B
Transitional Dynamics, Two-Sector Neoclassical Model

Parameterizations		Time Period (each period has 5 years) 1	2	3	4	5	6	7	8	9	10
1) LABOR SHARE Capital sector = .5 Consumption sector = .75	Half life = 11 years										
	Average Growth of Capital	66.3	15.7	6.4	3.0	1.5	0.8	0.4	0.2	0.1	0.1
	Average Growth Rate of Output	13.3	3.6	1.5	0.7	0.4	0.2	0.1	0.1	0.0	0.0
	Average Real Interest Rate	59.9	19.1	11.7	9.0	7.8	7.2	6.9	6.7	6.6	6.6
2) LABOR SHARE Capital sector = .5 Consumption sector = .25	Half life = 15 years										
	Average Growth Rate of Capital	54.3	16.6	8.0	4.4	2.6	1.6	1.0	0.7	0.4	0.3
	Average Growth Rate of Output	38.1	12.0	5.8	3.2	1.9	1.2	0.8	0.5	0.3	0.2
	Average Real Interest Rate	51.3	20.4	13.2	10.3	8.7	7.9	7.4	7.1	6.9	6.7

Each period has 5 years. All rates reported are the average over the five-year period in annual terms and expressed in percentage points.

Table 6C

Transitional Dynamics, Quadratic Adjustment Costs Model

Parameterizations		Time Period (each period has 5 years) 1	2	3	4	5	6	7	8	9	10
1) b = 32.2	Half life = 47										
	Average Growth Rate of Capital	45.0	22.8	14.2	9.8	7.2	5.5	4.3	3.4	2.7	2.2
	Average Growth Rate of Output	13.2	7.1	4.5	3.2	2.3	1.8	1.4	1.1	0.9	0.7
	Average Real Interest Rate	22.7	15.2	12.0	10.2	9.2	8.5	8.0	7.7	7.4	7.2
	Average Tobin's Q	13.4	7.6	5.2	4.0	3.2	2.7	2.3	2.0	1.8	1.7
2) b = 2.0	Half life = 14										
	Average Growth Rate of Capital	108.8	25.7	10.3	4.9	2.6	1.4	0.8	0.4	0.2	0.1
	Average Growth Rate of Output	27.8	7.9	3.3	1.6	0.8	0.5	0.3	0.1	0.1	0.0
	Average Real Interest Rate	37.6	16.5	11.0	8.8	7.7	7.2	6.9	6.7	6.6	6.6
	Average Tobin's Q	2.6	1.4	1.2	1.1	1.0	1.0	1.0	1.0	1.0	1.0

Each period has 5 years. All rates reported are the average over the five-year period in annual terms and expressed in percentage points.

Table 7
Transitional Dynamics, Open Economy with Adjustment Costs

Parameterizations		Time Period (each period has 5 years) 1	2	3	4	5	6	7	8	9	10
1) b = 32.2	Half life = 37										
	Average Growth Rate of Capital	64.1	22.6	12.5	8.1	5.7	4.3	3.3	2.6	2.0	1.6
	Average Growth Rate of Output	18.0	7.0	4.0	2.6	1.9	1.4	1.1	0.8	0.7	0.5
	Average Tobin's Q	22.4	8.3	5.0	3.6	2.8	2.4	2.0	1.8	1.7	1.5
2) b = 2.0	Half life = 9										
	Average Growth Rate of Capital	146.3	19.5	5.8	2.0	0.7	0.3	0.1	0.0	0.0	0.0
	Average Growth Rate of Output	35.0	6.1	1.9	0.7	0.2	0.1	0.0	0.0	0.0	0.0
	Average Tobin's Q	4.6	1.4	1.1	1.0	1.0	1.0	1.0	1.0	1.0	1.0

Each period has 5 years. All rates reported are the average over the five-year period in annual terms and expressed in percentage points.

Table 8
Effects of Technology Parameters on Calibrated Steady State

A. Alternate Values of Labor's Share (α_2) with $\delta_H = \delta_k = .10$

Technology Parameters				Tax Rates		Investment Shares		Prices		Interest Rate	Growth Rate
A1	A2	α_1	α_2	τ_1	τ_2	s_i	s_i^*	p	w	R	γ_H
1.0000	0.0323	0.6667	0.0100	0	0	0.5911	0.3982	5.5061	0.9476	1.0650	1.0200
1.0000	0.0494	0.6667	0.1000	0	0	0.5816	0.3831	5.5061	0.9476	1.0650	1.0200
1.0000	0.0701	0.6667	0.2000	0	0	0.5693	0.3645	5.5061	0.9476	1.0650	1.0200
1.0000	0.0933	0.6667	0.3000	0	0	0.5548	0.3438	5.5061	0.9476	1.0650	1.0200
1.0000	0.1182	0.6667	0.4000	0	0	0.5373	0.3206	5.5061	0.9476	1.0650	1.0200
1.0000	0.1436	0.6667	0.5000	0	0	0.5158	0.2944	5.5061	0.9476	1.0650	1.0200
1.0000	0.1676	0.6667	0.6000	0	0	0.4890	0.2647	5.5061	0.9476	1.0650	1.0200
1.0000	0.1823	0.6667	0.6700	0	0	0.4657	0.2413	5.5061	0.9476	1.0650	1.0200
1.0000	0.1876	0.6667	0.7000	0	0	0.4543	0.2305	5.5061	0.9476	1.0650	1.0200
1.0000	0.2001	0.6667	0.8000	0	0	0.4077	0.1908	5.5061	0.9476	1.0650	1.0200
1.0000	0.2000	0.6667	0.9000	0	0	0.3421	0.1443	5.5061	0.9476	1.0650	1.0200
1.0000	0.1789	0.6667	0.9900	0	0	0.2546	0.0950	5.5061	0.9476	1.0650	1.0200

B. Alternate Values of Labor's Share (α_2) with $\delta_H = .0128$ and $\delta_k = .10$

Technology Parameters				Tax Rates		Investment Shares		Prices		Interest Rate	Growth Rate
A1	A2	α_1	α_2	τ_1	τ_2	s_i	s_i^*	p	w	R	γ_H
1.0000	0.0148	0.6667	0.0100	0	0	0.4445	0.3471	11.9949	0.9476	1.0650	1.0200
1.0000	0.0227	0.6667	0.1000	0	0	0.4334	0.3355	11.9949	0.9476	1.0650	1.0200
1.0000	0.0322	0.6667	0.2000	0	0	0.4199	0.3218	11.9949	0.9476	1.0650	1.0200
1.0000	0.0428	0.6667	0.3000	0	0	0.4052	0.3070	11.9949	0.9476	1.0650	1.0200
1.0000	0.0543	0.6667	0.4000	0	0	0.3890	0.2912	11.9949	0.9476	1.0650	1.0200
1.0000	0.0659	0.6667	0.5000	0	0	0.3710	0.2741	11.9949	0.9476	1.0650	1.0200
1.0000	0.0770	0.6667	0.6000	0	0	0.3511	0.2556	11.9949	0.9476	1.0650	1.0200
1.0000	0.0837	0.6667	0.6700	0	0	0.3357	0.2417	11.9949	0.9476	1.0650	1.0200
1.0000	0.0861	0.6667	0.7000	0	0	0.3287	0.2356	11.9949	0.9476	1.0650	1.0200
1.0000	0.0919	0.6667	0.8000	0	0	0.3036	0.2137	11.9949	0.9476	1.0650	1.0200
1.0000	0.0918	0.6667	0.9000	0	0	0.2751	0.1898	11.9949	0.9476	1.0650	1.0200
1.0000	0.0821	0.6667	0.9900	0	0	0.2459	0.1664	11.9949	0.9476	1.0650	1.0200

Effects of Taxation on Steady State Growth

A. Alternate Tax Rates with α_2 = .67 and σ = 1

Technology Parameters				Tax Rates		Investment Shares		Prices		Interest Rate	Growth Rate
A1	A2	α_1	α_2	τ_1	τ_2	s_1	s_1^*	p	w	R	γ_H
1.0000	0.0834	0.6667	0.6667	0.5000	0.5000	0.0969	0.1069	11.9949	0.3926	1.0202	0.9771
1.0000	0.0834	0.6667	0.6667	0.5000	0	0.1394	0.1268	5.9974	0.3560	1.0461	1.0019
1.0000	0.0834	0.6667	0.6667	0.4000	0.4000	0.1302	0.1325	11.9949	0.4984	1.0288	0.9853
1.0000	0.0834	0.6667	0.6667	0.4000	0	0.1762	0.1533	7.1969	0.4609	1.0506	1.0062
1.0000	0.0834	0.6667	0.6667	0.3000	0.3000	0.1694	0.1590	11.9949	0.6076	1.0377	0.9938
1.0000	0.0834	0.6667	0.6667	0.3000	0	0.2145	0.1782	8.3964	0.5731	1.0547	1.0101
1.0000	0.0834	0.6667	0.6667	0.2000	0.2000	0.2157	0.1863	11.9949	0.7191	1.0467	1.0025
1.0000	0.0834	0.6667	0.6667	0.2000	0	0.2541	0.2013	9.5959	0.6920	1.0584	1.0137
1.0000	0.0834	0.6667	0.6667	0.1000	0.1000	0.2706	0.2141	11.9949	0.8326	1.0558	1.0112
1.0000	0.0834	0.6667	0.6667	0.1000	0	0.2948	0.2227	10.7954	0.8170	1.0618	1.0170
1.0000	0.0834	0.6667	0.6667	0	0	0.3365	0.2424	11.9949	0.9476	1.0650	1.0200
1.0000	0.0834	0.6667	0.6667	-0.1000	-0.1000	0.4164	0.2712	11.9949	1.0638	1.0743	1.0289

B. Alternate Tax Rates with α_2 = .9 and σ = 1

Technology Parameters				Tax Rates		Investment Shares		Prices		Interest Rate	Growth Rate
A1	A2	α_1	α_2	τ_1	τ_2	s_1	s_1^*	p	w	R	γ_H
1.0000	0.0918	0.6667	0.9000	0.5000	0.5000	0.1071	0.1127	10.3864	0.3857	1.0245	0.9812
1.0000	0.0918	0.6667	0.9000	0.5000	0	0.1326	0.1121	4.7778	0.3424	1.0580	1.0133
1.0000	0.0918	0.6667	0.9000	0.4000	0.4000	0.1348	0.1327	10.8309	0.4914	1.0325	0.9889
1.0000	0.0918	0.6667	0.9000	0.4000	0	0.1606	0.1304	6.0871	0.4476	1.0596	1.0150
1.0000	0.0918	0.6667	0.9000	0.3000	0.3000	0.1648	0.1509	11.1967	0.6012	1.0406	0.9966
1.0000	0.0918	0.6667	0.9000	0.3000	0	0.1889	0.1472	7.4701	0.5613	1.0613	1.0165
1.0000	0.0918	0.6667	0.9000	0.2000	0.2000	0.1977	0.1667	11.5043	0.7141	1.0487	1.0044
1.0000	0.0918	0.6667	0.9000	0.2000	0	0.2175	0.1626	8.9194	0.6829	1.0627	1.0178
1.0000	0.0918	0.6667	0.9000	0.1000	0.1000	0.2340	0.1798	11.7671	0.8298	1.0569	1.0122
1.0000	0.0918	0.6667	0.9000	0.1000	0	0.2462	0.1768	10.4292	0.8118	1.0639	1.0189
1.0000	0.0918	0.6667	0.9000	0	0	0.2751	0.1898	11.9949	0.9476	1.0650	1.0200
1.0000	0.0918	0.6667	0.9000	-0.1000	-0.1000	0.3229	0.1965	12.1945	1.0672	1.0731	1.0278

C. Alternate Tax Rates with α_2 = .67 and σ = 5

Technology Parameters				Tax Rates		Investment Shares		Prices		Interest Rate	Growth Rate
A1	A2	α_1	α_2	τ_1	τ_2	s_1	s_1^*	p	w	R	γ_H
1.0000	0.0834	0.6667	0.6667	0.5000	0.5000	0.2051	0.1548	11.9949	0.3933	1.0197	1.0112
1.0000	0.0834	0.6667	0.6667	0.5000	0	0.1982	0.1591	5.9974	0.3563	1.0458	1.0163
1.0000	0.0834	0.6667	0.6667	0.4000	0.4000	0.2341	0.1759	11.9949	0.4992	1.0284	1.0129
1.0000	0.0834	0.6667	0.6667	0.4000	0	0.2274	0.1783	7.1969	0.4612	1.0504	1.0172
1.0000	0.0834	0.6667	0.6667	0.3000	0.3000	0.2611	0.1943	11.9949	0.6073	1.0378	1.0147
1.0000	0.0834	0.6667	0.6667	0.3000	0	0.2557	0.1960	8.3964	0.5734	1.0545	1.0180
1.0000	0.0834	0.6667	0.6667	0.2000	0.2000	0.2873	0.2120	11.9949	0.7197	1.0464	1.0164
1.0000	0.0834	0.6667	0.6667	0.2000	0	0.2833	0.2125	9.5959	0.6922	1.0583	1.0187
1.0000	0.0834	0.6667	0.6667	0.1000	0.1000	0.3122	0.2278	11.9949	0.8329	1.0557	1.0182
1.0000	0.0834	0.6667	0.6667	0.1000	0	0.3101	0.2279	10.7954	0.8171	1.0618	1.0194
1.0000	0.0834	0.6667	0.6667	0	0	0.3365	0.2424	11.9949	0.9476	1.0650	1.0200
1.0000	0.0834	0.6667	0.6667	-0.1000	-0.1000	0.3602	0.2561	11.9949	1.0633	1.0744	1.0218

Table 10
Effects of Taxation on Small Open Economy

Tax Rate τ_1	Tax Rate τ_2	Growth Rate γ_H
.050	.050	.9872*
.050	0	.9872*
.025	.025	.9872*
.025	0	.9923
.010	.010	.9881
.010	0	1.0080
0	0	1.0200

Economy is calibrated so that closed economy version is equivalent to panel A, Table 9, i.e., $A_1 = 1$, $A_2 = .0834$, $\alpha_1 = \alpha_2 = 2/3$, $\delta_K = .10$, $\delta_H = .0128$, and $\sigma = 1$. An asterisk indicates human capital investment is zero.

Table 11
Tax Effects on Growth in Example Economy
A. Growth of Consumption (γ)

σ \ τ	0	0.10	0.20	0.30
10.00	2.00	1.84	1.68	1.52
5.00	2.00	1.68	1.36	1.03
2.00	2.00	1.21	0.41	-0.40
1.00	2.00	0.42	-1.16	-2.74
0.50	2.00	-1.14	-4.22	-7.26
0.20	2.00	-5.66	-10.00	-10.00
0.10	2.00	-10.00	-10.00	-10.00

B. Welfare Effect

$$100\ [1 - (\tilde{C}_0/C_0)]$$

σ \ τ	0	0.10	0.20	0.30
10.00	0	0.82	4.71	35.23
5.00	0	1.54	7.94	28.46
2.00	0	3.27	13.29	30.38
1.00	0	5.16	16.59	30.18
0.50	0	7.05	17.77	27.48
0.20	0	8.22	-----	-----
0.10	0	----	-----	-----

C. Initial Consumption Effect

$$100\ [(C_0'/C_0) - 1]$$

σ \ τ	0	0.10	0.20	0.30
10.00	0	3.54	7.12	10.76
5.00	0	7.07	14.22	21.47
2.00	0	17.63	35.39	53.30
1.00	0	35.12	70.23	105.35
0.50	0	69.69	138.29	205.81
0.20	0	170.23	------	------
0.10	0	------	------	------

----- indicates economy contracts at rate $-\delta$. Hence, welfare and initial consumption effects are not calculated.

Table 12
Taxation in the Neoclassical Model
Increase in Tax Rate from $\tau = .20$ to $\tau = .30$

A. Steady State Effects

	c	% Decline	k	% Decline
$\tau = .20$	.2049		.4109	
$\tau = .30$	.1975	3.6	.3363	18.2

B. Full Transition Path Effects

Preference Parameter	Initial Consumption Level	% Decline in Utility Equivalent Consumption Path
$\sigma = 10$	.2080	.97
$\sigma = 5$	.2160	1.21
$\sigma = 2$	.2140	1.46
$\sigma = 1$	.2184	1.60
$\sigma = 1/2$	.2243	1.71
$\sigma = 1/5$	.2346	1.80
$\sigma = 1/10$	.2439	1.84

Table 13
Tax Effects on Growth in Two-Sector Economy
A. Growth of Consumption (γ)

σ \ τ	0	0.10	0.20	0.30
10.00	2.00	1.84	1.68	1.52
5.00	2.00	1.69	1.37	1.05
2.00	2.00	1.22	0.44	-0.36
1.00	2.00	0.46	-1.09	-2.64
0.50	2.00	-1.01	-3.97	-6.89
0.20	2.00	-4.87	-10.00	-10.00
0.10	2.00	-9.45	-10.00	-10.00

B. Welfare Effect
$100\ [1 - (\tilde{C}_0/C_0)]$

σ \ τ	0	0.10	0.20	0.30
10.00	0	0.88	4.68	27.42
5.00	0	1.58	7.81	26.96
2.00	0	3.26	13.06	29.72
1.00	0	5.12	16.40	29.90
0.50	0	7.02	17.79	27.72
0.20	0	8.44	-----	-----
0.10	0	8.53	-----	-----

C. Initial Consumption Effect
$100\ [(C_0'/C_0) - 1]$

σ \ τ	0	0.10	0.20	0.30
10.00	0	3.39	6.81	10.25
5.00	0	6.85	13.76	20.74
2.00	0	17.13	34.37	51.73
1.00	0	33.94	67.77	101.48
0.50	0	66.16	130.25	192.08
0.20	0	149.57	------	------
0.10	0	245.07	------	------

----- indicates economy contracts at rate $-\delta$. Hence, welfare and initial consumption effects are not calculated.

FIGURE 1

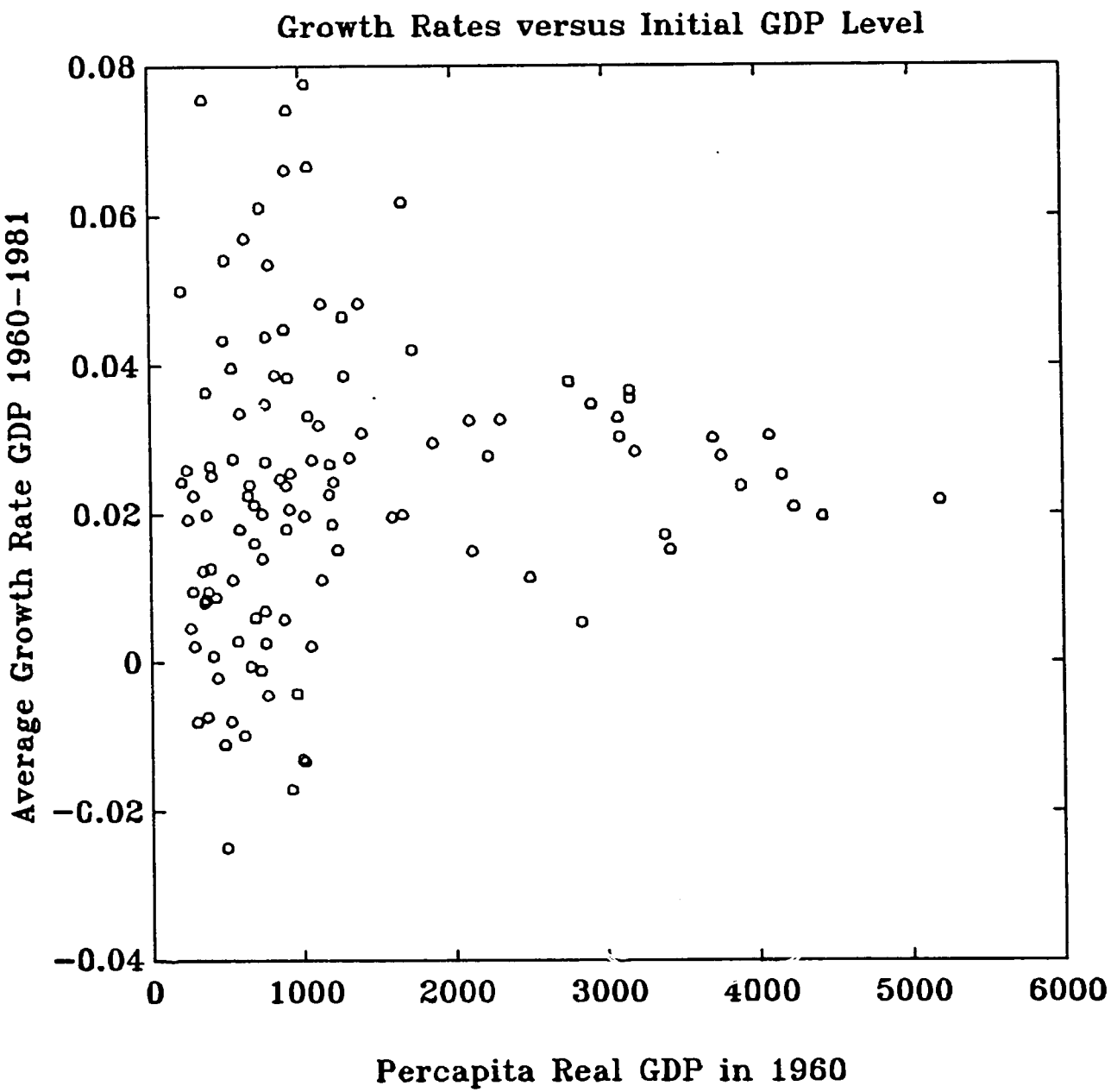

Growth Rates versus Initial GDP Level
0.08
0.06
0.04
0.02
0
−0.02
−0.04
Average Growth Rate GDP 1960−1981
0
1000
2000
3000
4000
5000
6000
Percapita Real GDP in 1960

FIGURE 2

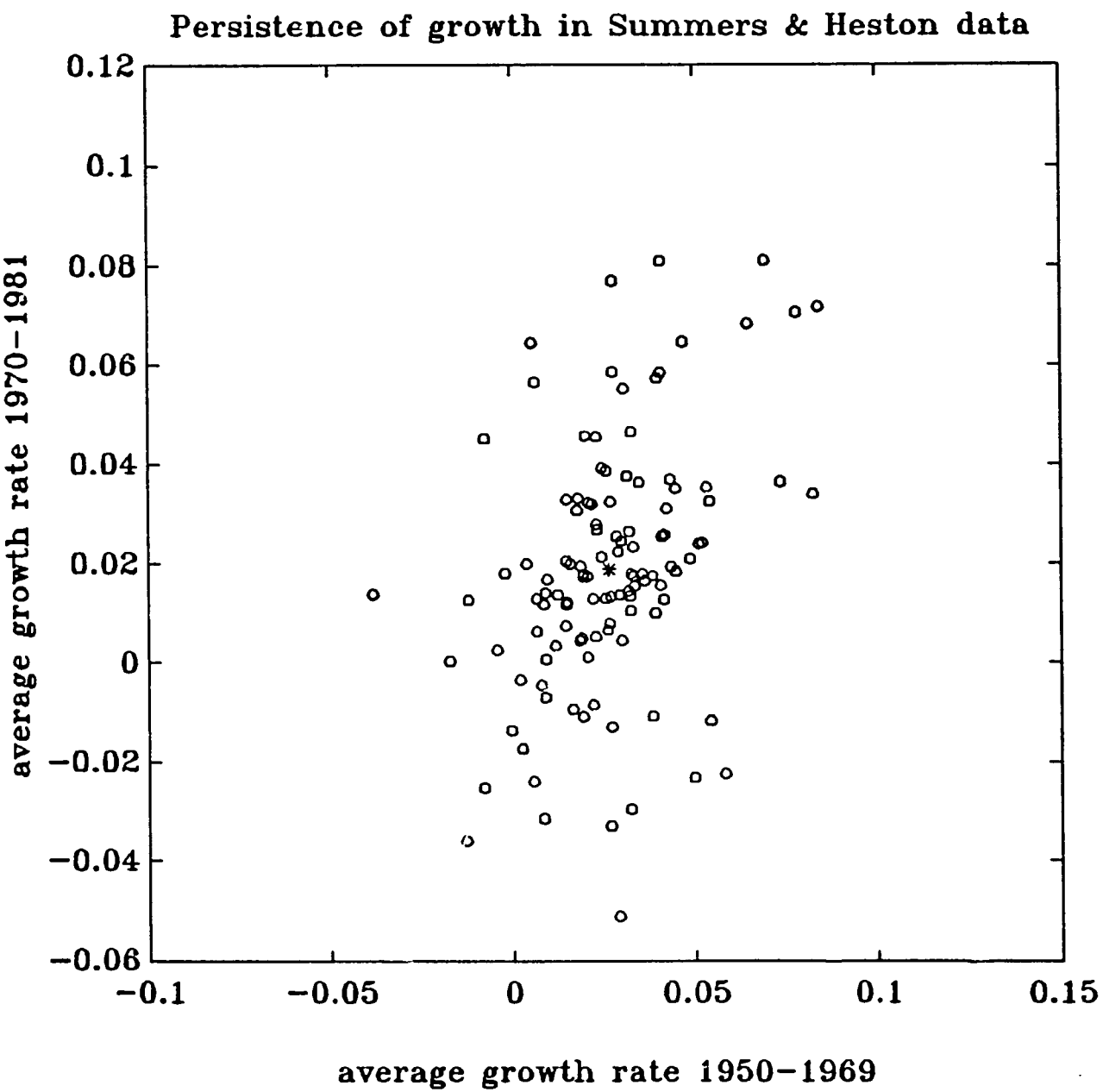

Persistence of growth in Summers & Heston data
0.12
0.1
0.08
0.06
0.04
0.02
0
−0.02
−0.04
−0.06
average growth rate 1970−1981
−0.1
−0.05
0
0.05
0.1
0.15
average growth rate 1950−1969

FIGURE 3

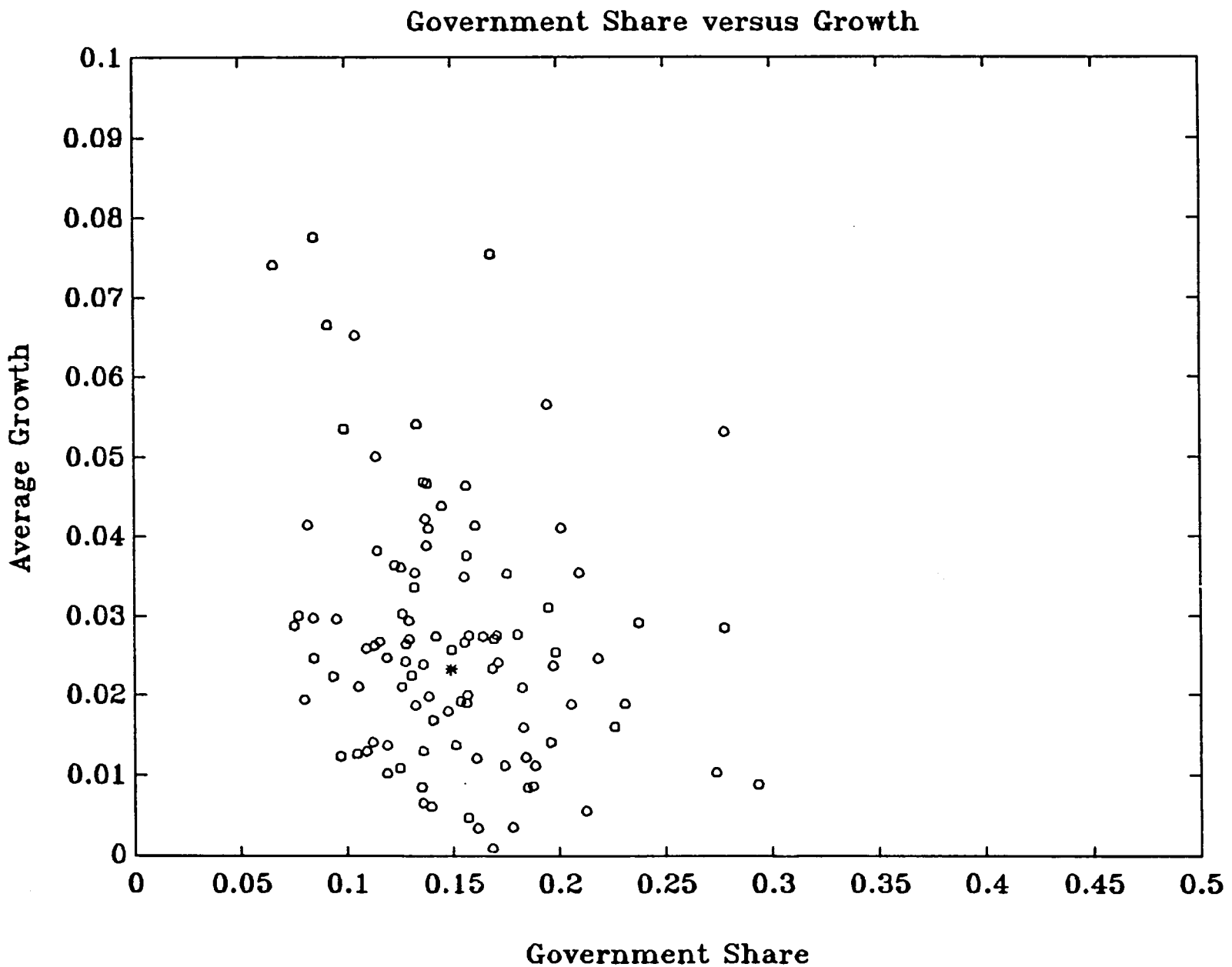

Government Share versus Growth
Average Growth
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Government Share

FIGURE 4

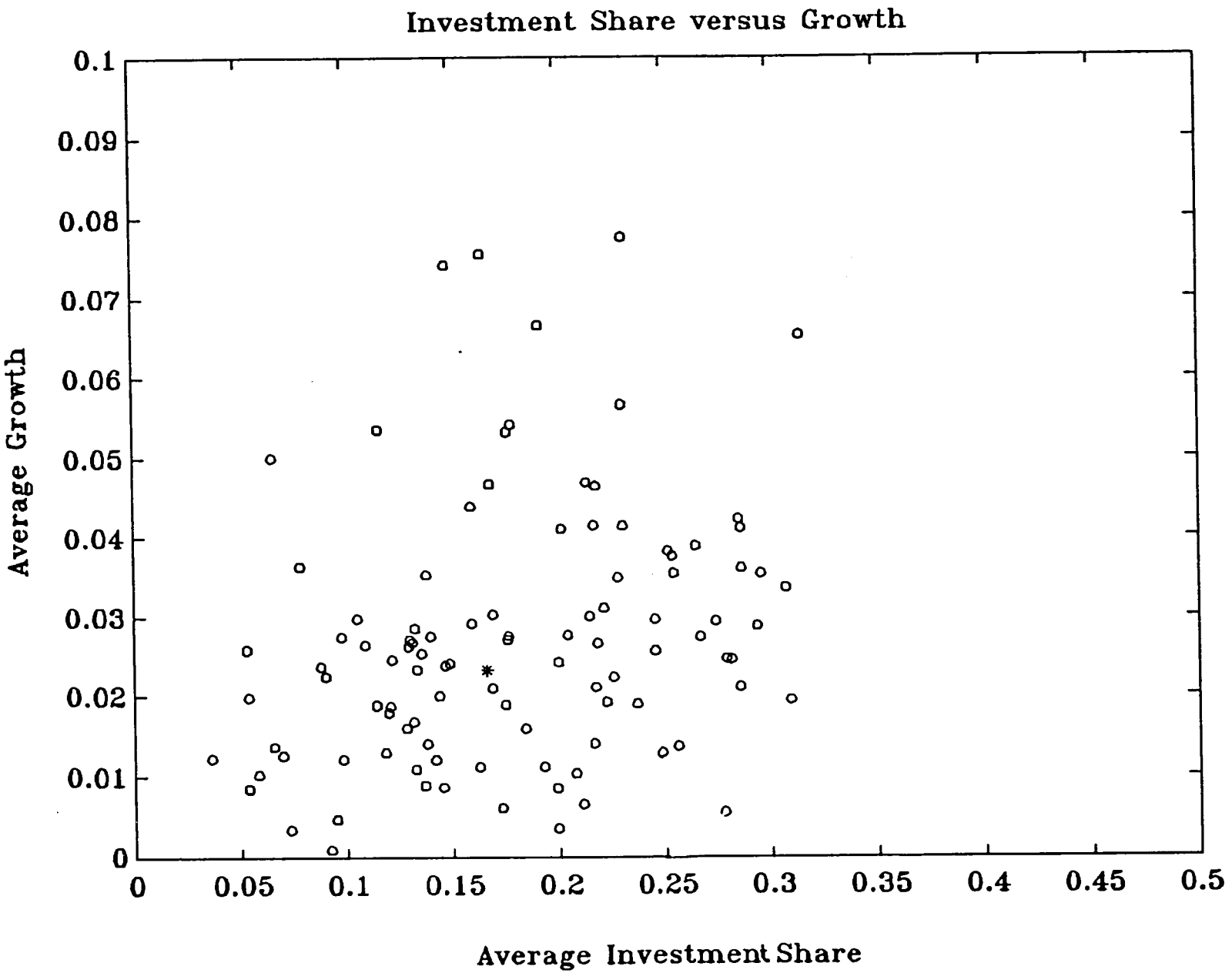
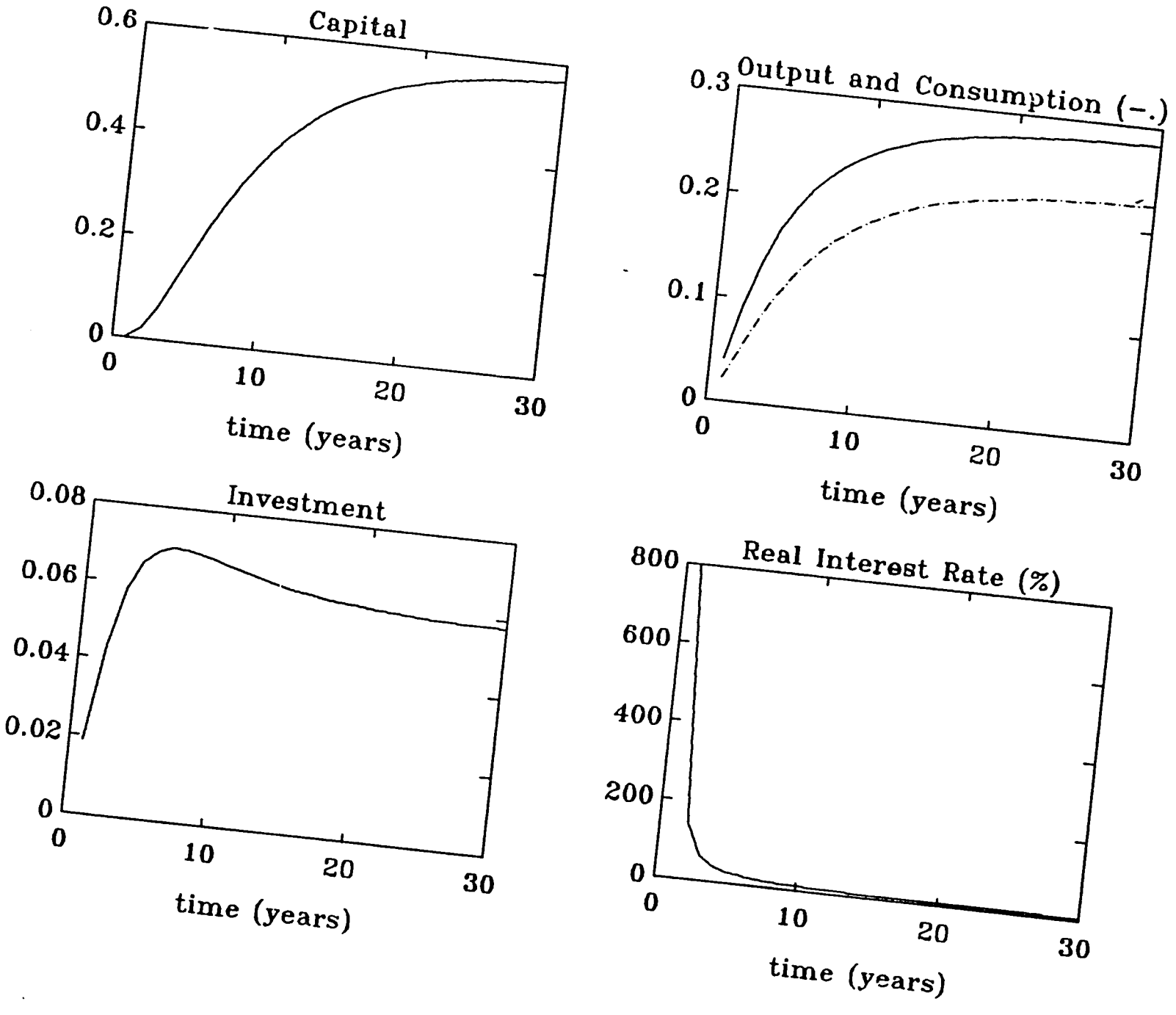

Investment Share versus Growth
Average Growth
0.1
0.09
0.08
0.07
0.06
0.05
0.04
0.03
0.02
0.01
0
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
Average Investment Share

FIGURE 5


Capital
0.6
0.4
0.2
0
0
10
20
30
time (years)
Output and Consumption (-.)
0.3
0.2
0.1
0
0
10
20
30
time (years)
Investment
0.08
0.06
0.04
0.02
0
0
10
20
30
time (years)
Real Interest Rate (%)
800
600
400
200
0
0
10
20
30
time (years)

Appendix A

Properties of Human Capital Accumulation

The growth of human capital is given by

$$\frac{H_{t+1} - H_t}{H_t} = \Theta\left[\frac{I_t}{H_t}\right] - \delta_H,$$

where for the purpose of this appendix we consider only human capital investment and, thus, do not subscript the variable I. Here, we explore parameterizations of the Θ function, which is assumed to be increasing and concave. One natural property is that $\Theta(0) = 0$. That is, without investment in human capital, human capital grows at gross rate $\gamma_H = 1 - \delta_H$. A constant elasticity form for Θ involves

$$\Theta(\tfrac{I}{H}) = A[(\tfrac{I}{H}) + B]^{\theta} - C$$

where θ, A, B, and C are parameters to be determined.

The restriction that $\Theta()$ is increasing and concave implies $D\Theta > 0$ and $D^2\Theta < 0$, for $(I/H) > 0$.

$$D\Theta = A\theta[(I/H) + B]^{\theta-1}$$

$$D^2\Theta = A\theta(\theta-1)[(I/H) + B]^{\theta-2}$$

so that $A\theta > 0$ and $(\theta-1) < 0$. The restriction that $\Theta(0) = 0$ implies that

$$AB^{\theta} - C = 0.$$

Thus, to this point, we have required that $0 < \theta < 1$, $A > 0$ and that

$$\Theta(\tfrac{I}{H}) = A[\tfrac{I}{H} + B]^{\theta} - AB^{\theta}.$$

Restrictions from Slopes

Another restriction that can be imposed is that there are locally no "costs of adjustment" at some point. One possibility is that this is zero at $I/H = 0$ and another is that this is at $\Theta(I/H) = \delta_H$. In either case the requirement translates into $D\Theta = 1$, at the appropriate point. We consider these two cases in turn.

(a) requirement at $I/H = 0$.

$$D\Theta(0) = A\theta B^{\theta-1} = 1 \Rightarrow B = (A\theta)^{\frac{1}{1-\theta}}$$

$$\Rightarrow A(A\theta)^{\frac{\theta}{1-\theta}} = C \qquad \text{using the } \Theta(0) = 0 \text{ requirement}$$

$$\Rightarrow A^{\frac{1}{1-\theta}} C^{\frac{1}{1-\theta}} = C$$

so that the function $\Theta(.)$ takes the form

$$\Theta(\tfrac{I}{H}) = A\left[(\tfrac{I}{H}) + (A\theta)^{\frac{1}{1-\theta}}\right]^{\theta} - \left[A^{\frac{1}{1-\theta}}\ \theta^{\frac{1}{1-\theta}}\right]$$

(b) requirement at $\Theta(I/H) = \delta_H$.

$$\Theta(\tfrac{I}{H}) = \delta_H \Rightarrow A((\tfrac{I}{H})_r + B)^{\theta} = C + \delta_H.$$

$$D\Theta = A\theta((\tfrac{I}{H})_r + B)^{\theta-1} = 1$$

$$\Rightarrow A\theta(\frac{C + \delta_H}{A})^{\frac{\theta-1}{\theta}} = 1$$

$$\Rightarrow A^{1/\theta}\ \theta\ (C + \delta_H)^{\frac{\theta-1}{\theta}} = 1$$

$$(C + \delta_H) = [A^{1/\theta}\ \theta]^{\frac{\theta}{1-\theta}} = A^{\frac{1}{1-\theta}}\ \theta^{\frac{\theta}{1-\theta}}$$

so that the function takes the form

$$\Theta(\tfrac{I}{H}) = A\left[(\tfrac{I}{H}) + B\right]^{\theta} - A^{\frac{1}{1-\theta}}\ \theta^{\frac{\theta}{1-\theta}} + \delta_H$$

$$= A\left[(\tfrac{I}{H}) + \left[\frac{A^{\frac{1}{1-\theta}}\theta^{\frac{\theta}{1-\theta}} - \delta_H}{A}\right]^{1/\theta}\right]^{\theta} - A^{(\frac{1}{1-\theta})}\ \theta^{\frac{\theta}{1-\theta}} + \delta_H$$

$$= A\left[(\tfrac{I}{H}) + (A^{\frac{\theta}{1-\theta}}\ \theta^{\frac{\theta}{1-\theta}} - \delta_H)^{1/\theta}\right]^{\theta} - A^{\frac{1}{1-\theta}}\ \theta^{\frac{\theta}{1-\theta}} + \delta_H$$

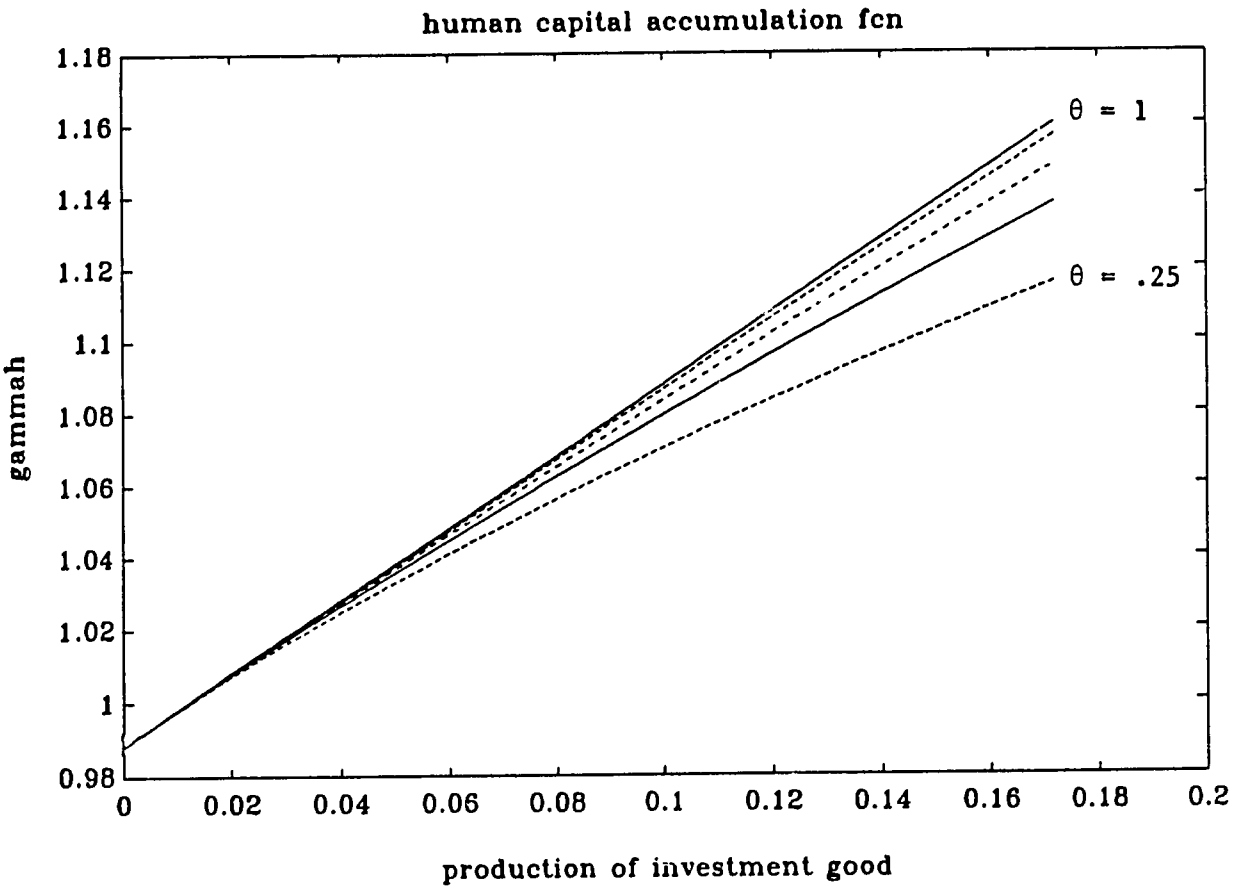
Note that the preceding functions coincide if $\delta_H = 0$. Experiments in this paper use a version of the former specification with A = 1

$$\Theta_1\left(\frac{I}{H}\right) = \left[\left(\frac{I}{H}\right) + \theta^{\frac{1}{1-\theta}}\right]^{\theta} - \theta^{\frac{\theta}{1-\theta}}$$

which are single parameter functions.

Figure A-1 provides a graph of this function for some alternative values of θ.

Figure A-1



Function Graphed for

$\theta = [1\ .9\ .8\ .7\ .5\ .25]$

Appendix B

Calibration of Two Sector Endogenous Growth Model

This appendix discusses the formal structure of the two-sector endogenous growth model outlined in the main text, as well as our procedures for calibrating its steady state and exploring policy implications.

The representative agent in this economy solves the dynamic optimization problem

$$\text{Max} \sum_{t=0}^{\infty} \beta^t \, u(C_t)$$

subject to the accumulation constraints

$$K_{t+1} = (1-\delta_K)K_t + I_{Kt}$$

$$H_{t+1} = \Theta(I_{Ht}/H_t)H_t + (1-\delta_H)H_t$$

the resource constraints

$$C_t + I_{Kt} \leq \Omega_1 F_1(K_{1t}, M_{1t}) + T_{1t}$$

$$I_{Ht} \leq \Omega_2 F_2(K_{2t}, M_{2t}) + T_{2t}$$

and the factor allocation constraints

$$M_{1t} + M_{2t} \leq NH_t$$

$$K_{1t} + K_{2t} \leq K_t$$

To analyze equilibrium behavior, we form the Lagrangian

$$\mathcal{L} = \sum_{t=0}^{\infty} \beta^t \, u(C_t)$$

$$+ \sum_{t=0}^{\infty} \tilde{\Lambda}_{1t}[\Omega_1 F_{1t} + T_{1t} + (1-\delta_K)K_t - K_{t+1} - C_t]$$

$$+ \sum_{t=0}^{\infty} \tilde{\Lambda}_{2t}[\Theta(\frac{\Omega_2 F_{2t} + T_{2t}}{H_t})H_t + (1-\delta_H)H_t - H_{t+1}]$$

$$+ \sum_{t=0}^{\infty} \tilde{Q}_{Kt}[K_t - K_{1t} - K_{2t}]$$

$$+ \sum_{t=0}^{\infty} \tilde{Q}_{Mt}[NH_t - M_{1t} - M_{2t}]$$

The efficiency conditions take the following forms. For consumption, we have the familiar requirement that

$$\beta^t Du(C_t) = \tilde{\Lambda}_{1t}.$$

For the cross-sectoral allocations of factor stocks, we have the four conditions

$$\tilde{\Lambda}_{1t}\Omega_1 D_1 F_1(K_{1t}, M_{1t}) = \tilde{Q}_{Kt}$$

$$\tilde{\Lambda}_{1t}\Omega_1 D_2 F_1(K_{1t}, M_{1t}) = \tilde{Q}_{MT}$$

$$\tilde{\Lambda}_{2t} D\Theta_t \Omega_2 D_1 F_2(K_{2t}, M_{2t}) = \tilde{Q}_{Kt}$$

$$\tilde{\Lambda}_{2t} D\Theta_t \Omega_2 D_2 F_2(K_{2t}, M_{2t}) = \tilde{Q}_{Mt}$$

For the efficient evolution of capital stocks, we have the two shadow price requirements

$$\tilde{\Lambda}_{1t} = \tilde{\Lambda}_{1,t+1}(1-\delta_K) + Q_{K,t+1}$$

$$\tilde{\Lambda}_{2t} = \tilde{\Lambda}_{2,T+1}(1-\delta_H) + Q_{1t,T+1}N + \Lambda_{2,T+1}[\Theta_{t+1} - D\Theta_{t+1}(\frac{\Omega_2 F_{2,t+1} + T_{2,t+1}}{H_{t+1}})]$$

and the transversality conditions

$$\lim_{t\to\infty} \tilde{\Lambda}_{1t} K_{t+1}$$

$$\lim_{t\to\infty} \tilde{\Lambda}_{2t} H_{t+1}$$

Finally, we have the four resource constraints

$$K_{t+1} = \Omega_1 F_{1t} + T_{1t} + (1-\delta_K)K_t - C_t$$

$$H_{t+1} = \Theta(\frac{\Omega_2 F_{2t} + T_{2t}}{H_t})H_t + (1-\delta_H)H_t$$

$$K_t = K_{1t} + K_{2t}$$

$$NH_t = M_{1t} + M_{2t}$$

Steady State Requirements

Consolidating the preceding conditions, we find that the steady state is described by

(SS1) $$\beta(\gamma_H)^{-\sigma} = 1/(1+r).$$

(SS2) $$(1+r) = (1-\delta_K) + \Omega_1 D_1 F_1(K_1/M_1, 1)$$

(SS3) $$(1+r) = [(1-\delta_H) + N\ D\Theta\ \Omega_2 D_2 F_2 + \Theta - (F_2/H)\ D\Theta]$$

(SS4) $$\frac{D_1 F_1(K_1/M_1, 1)}{D_2 F_1(K_1/M_1, 1)} = \frac{D_1 F_2(K_2/M_2, 1)}{D_2 F_2(K_2/M_2, 1)}$$

(SS5) $$1 = \frac{M_1}{NH} + \frac{M_2}{NH}$$

(SS6) $$\frac{K}{NH} = (\frac{K_1}{M_1})(\frac{M_1}{NH}) + (\frac{K_2}{M_2})(\frac{M_2}{NH})$$

(SS7) $$\gamma_H = \Theta(I_E/H) + 1-\delta_H$$

(SS8) $$I_E/H = F_2(\frac{K_2}{M_2}, 1)\ \frac{M_2}{H}$$

(SS9) $$\left(\frac{Y}{NH}\right) = F_1(K_1/M_1,\ 1)\left(\frac{M_1}{NH}\right)$$

(SS10) $$\frac{C}{NH} = \frac{Y}{NH} - (\gamma_H + 1-\delta_K)\frac{K}{NH}$$

This system is 10 equations in the 10 unknowns γ_H, $(1+r)$, (K_1/M_1), (K_2/M_2), (M_1/NH), (K/NH), (I_E/H), (Y/NH) and (C/NH).

Calibration

For the purpose of determining the parameters of the steady state to match observed average growth and real interest rates, we proceed as follows. First, we postulate CES forms for the F_1 and F_2 functions, so that F_i

$$F_i(K_i,\ M_i;\ A_i,\ \alpha_i,\ \rho_i) = A_i[(1-\alpha_i)K_i^{-\rho_i} + \alpha_i M_i^{-\rho_i}]^{-1/\rho_i}$$

Then, we compute the steady state according to the following algorithm:

Step 1: Given $A_1 = 1$, α_1, and ρ_1, invert $(1+r) = D_1F_1(K_1/M_1,1) + 1-\delta_K$ to obtain steady state K_1/M_1 ratio using (SS2).

Step 2: Given α_1, α_2, ρ_1 and ρ_2 calculate K_2/M_2 from requirement that marginal rates of transformation are equated in the two sectors, using (SS4).

Step 3: Given the parameters of Θ function—the coefficients developed in Appendix A (i.e., θ and A_3)—compute (I_E/H) consistent with specified γ_H using (SS7).

Step 4: Given the results of the preceding steps, (SS3) permits—with specification of Ω_2—solution for the parameter A_2

$$A_2 = \frac{r+\delta_H - [\Theta(I_E/H) - D\Theta(I_E/H)(I_E/H)]}{ND\Theta(I_E/H)\Omega_2[D_2F_2(K_2/M_2,\ 1)/A_2]}$$

Step 5: Use (SS8) to compute the fraction of time in efficiency units allocated to investment in human capital, given previously determined (I_E/H) and (K_2/M_2) with specified N.

$$(M_2/NH) = I_E/[F_2(\frac{K_2}{M_2},1)]$$

Step 6: Compute $M_1/NH = 1-(M_2/NH)$, using (SS5).

Step 7: Compute $\frac{Y}{NH} = (\frac{Y}{M1})(\frac{M_1}{NH}) = F_2[(\frac{K_1}{M_1}),\ 1;\ A_1,\ \alpha_1,\ \rho_1]\bullet(\frac{M_1}{NH})$, using (SS8) and the results above.

Step 8: Compute $\frac{K}{NH} = (\frac{K_1}{M_1}\frac{M_1}{NH} + \frac{K_2}{M_2}\frac{M_2}{NH})$, using (SS6) and the results above.

Step 9: Compute $\frac{C}{NH} = \frac{Y}{NH} - \gamma_H\frac{K}{NH} + (1-\delta_K)\frac{K}{NH}$, using (SS9) and the results above.

Step 10: Compute $\beta = \gamma_H^{\sigma}/(1+r)$ using (SS1) and the results above.